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Advanced modelling and simulation of water distribution systems with discontinuous control elements

by

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I dedicate this thesis to my beloved Marta, whom is my inspiration and the dearest friend. And to my children Yan, Hania and Alex without whom this work would have been completed at least two years earlier.

Abstract

Water distribution systems are large and complex structures. Hence, their construction, management and improvements are time consuming and expensive. But nearly all the optimisation methods, whether aimed at design or operation, suffer from the need for simulation models necessary to evaluate the performance of solutions to the problem. These simulation models, however, are increasing in size and complexity, and especially for operational control purposes, where there is a need to regularly update the control strategy to account for the fluctuations in demands, the combination of a hydraulic simulation model and optimisation is likely to be computationally excessive for all but the simplest of networks.

The work presented in this thesis has been motivated by the need for reduced, whilst at the same time appropriately accurate, models to replicate the complex and nonlinear nature of water distribution systems in order to optimise their operation. This thesis attempts to establish the ground rules to form an underpinning basis for the formulation and subsequent evaluation of such models.

Part I of this thesis introduces some of the modelling, simulation and optimisation problems currently faced by water industry. A case study is given to emphasise one particular subject, namely reduction of water distribution system models. A systematic research resulted in development of a new methodology which encapsulate not only the system mass balance but also the system energy distribution within the model reduction process. The methodology incorporates the energy audits concepts into the model reduction algorithm allowing the preservation of the original model energy distribution by imposing new pressure constraints in the reduced model. The appropriateness of the new methodology is illustrated on the theoretical and industrial case studies. Outcomes from these studies demonstrate that the new extension to the model reduction technique can simplify the inherent complexity of water networks while preserving the completeness of original information.

An underlying premise which forms a common thread running through the thesis, linking Parts I and II, is in recognition of the need for the more efficient paradigm to model and simulate water networks; effectively accounting for the discontinuous behaviour exhibited by water network components.

Motivated largely by the potential of contemplating a new paradigm to water distribution system modelling and simulation, a further major research area, which forms the basis of Part II, leads to a study of the discrete event specification formalism and quantised state systems to formulate a framework within which water distribution systems can be modelled and simulated. In contrast to the classic time-slicing simulators, depending on the numerical integration algorithms, the quantisation of system states would allow accounting for the discontinuities exhibited by control elements in a more efficient manner, and thereby, offer a significant increase in speed of the simulation of water network models. The proposed approach is evaluated on a number of case studies and compared with results obtained from the Epanet2 simulator and OpenModelica. Although the current state-of-art of the simulation tools utilising the quantised state systems do not allow to fully exploit their potential, the results from comparison demonstrate that, if the second or third order quantised-based integrations are used, the quantised state systems approach can outperform the conventional water network simulation methods in terms of simulation accuracy and run-time.

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Lastly, but not least, my very special thanks go to my love, Marta, as this is as much a result of my work as her, and without her support and sacrifice I would not completed this thesis.

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Listings

List of acronyms

ACV	altitude control valve
ANN	artificial neural network
BFS	breadth-first search
CLR	Common Language Runtime
CMK	Cuthill-McKee
CV	check valve
DAE	differential algebraic equation
DASSL	differential algebraic system solver
DEVS	discrete-event specification
DFS	depth-first search
DMA	district metering area
EGGA	enhanced global gradient algorithm
EPS	extended-period simulation
FCV	flow control valve
FSP	fixed-speed pump
GA	genetic algorithm
GGA	global gradient algorithm
GIS	geographic information system
GUI	graphical user interface
ICT	information and communications technology
MAE	mean absolute error
MBE	mass balance error
MD	minimum degree
MPC	model predictive control
MRE	mean relative error
MWRE	mean weighted relative error
ND	nested dissection
NRV	non-return valve
NSE	Nash-Sutcliffe efficiency
ODE	ordinary differential equation

OMC	OpenModelica compiler
PRV	pressure reducing valve
PSV	pressure sustaining valve
QSS	quantised state system
RCMK	reversed Cuthill-McKee
RMSE	root mean squared error
SCADA	supervisory control and data acquisition
TCV	throttle control valve
TRE	tank relative error
VSP	variable-speed pump
WDN	water distribution network
WDS	water distribution system
WSS	Water Software Systems

Introduction

Chapter 1

General introduction

1.1 Introduction

The work documented in this thesis represents a first step towards developing and re-engineering a new approach to reduce water distribution system (WDS) models for the purpose of their subsequent use in the optimisation studies. In addition, the author has been driven by a recognition of the acute need to redress some of the concepts and contemplations of water networks modelling and analysis to align better with the new and perceived emerging demands of industry. The development of such a new modelling and simulation framework is a much larger task than that of an individual research programme, but nevertheless the thesis does attempt to lay down some of the ground rules for such a new development; indeed further work is continuing and is currently progressing, being undertaken by colleagues within the Water Software Systems (WSS) group at Leicester, UK.

This chapter is focused towards introducing the formulation of the above ideas and is structured as follows: Firstly, an overview of the above new concepts and re-alignment for water network analysis is developed. Then, a short review of the work to be presented is given. The research aims and objectives are summarised in Section 1.2. To simplify the reading of the thesis, an outline of the flow of the research carried out and the structure of the thesis is described in Section 1.3. Section 1.4 provides a list of the author's claims in terms of main achievements and novel contributions to the field of hydroinformatics, together with links to the appropriate chapters where the material can be found. Section 1.5 and Section 1.6 enumerate the publications and the research projects associated with the work presented in this thesis.

1.1.1 Enhanced model reduction methodology

Water distribution systems are designed to provide water for domestic, commercial, industrial and fire fighting purposes while simultaneously satisfying demand, pressure and water quality requirements. The typical components of WDS are reservoirs, nodes, pipes, valves and pumps. Each of these interconnected elements is interrelated with its neighbours thus the entire WDS behaviour depends on each of its elements. Nowadays, it is common that WDS models consist of thousands of such elements to accurately replicate hydraulic behaviour and topographical layout of real WDSs. Such models are appropriate for simulation purposes; however, online optimisation tasks are much more computationally demanding, hence, simplified models are required. There are different techniques of model reduction; the outcome of most of these methods is a hydraulic model with a smaller number of components than the prototype. The accuracy of the simplification depends on the model complexity and the selected method such as skeletonization (Walski *et al.*, 2003; Saldarriaga *et al.*, 2008; Iglesias-Rey *et al.*, 2012), parameter-fitting (Anderson and Al-Jamal, 1995), graph decomposition (Deuerlein, 2008), enhanced global gradient algorithm (EGGA) (Giustolisi and Todini, 2009), metamodelling (Rao and Alvarruiz, 2007; Broad *et al.*, 2010; Behandish and Wu, 2014) and variables elimination (Ulanicki *et al.*, 1996; Alzamora *et al.*, 2014).

While some of the techniques demonstrated a potential for implementation in real time optimisation strategies, a more insightful analysis has exposed a potential for further improvements. Whereas in the aforementioned simplification methods, the obtained reduced models replicate, to a specific degree, the original hydraulic water network characteristics, the energy distribution of the original system is usually not considered in the simplification process. This could cause a situation, where the pump speed required to satisfy the minimum pressure constraints is different for the reduced model and the original model. To alleviate this mismatch it is proposed in this thesis to incorporate the concept of the energy audits of water networks introduced by Cabrera *et al.* (2010) into the model reduction algorithm.

Use of geographic information system (GIS) and supervisory control and data acquisition (SCADA) in water industry resulted in an increasing amount of information about actual network topology and service that can be incorporated into a model. Hence, to ensure that the model reduction algorithm is able to cope with complex topologies of large size networks a research is necessary in order to improve the numerical efficiency of the algorithm. Having in mind the properties of water networks and model reduction techniques, the investigation to be conducted can be narrowed to the following topics: (i) an efficient way to manage large sparse matrices representing water distribution network (WDN) topologies, (ii) exploitation of the multi-thread computing aimed at distributing the computational load on multi-core processors, and (iii) analysis of water networks aimed at improving the understanding of network functioning, and eventually, reducing the computational effort while managing or even improving the accuracy of the model reduction algorithm. It is apparent that none of these research directions prevails on the others, but rather their combined development would provide means to enhance, and ultimately, create a practical, reliable and efficient tool.

It is considered important by the author to highlight the impact of research work on the practical systems as well as to reflect the constant motivation during this work to appeal and collaborate with various companies and industrial bodies in order to ensure the viability and applicability of the resulting concepts that are developed. Hence, within this thesis the author investigates the applicability of his research findings in a real case study of determining optimal pump schedules. The case study is based on the project carried out by WSS aimed to optimise operation of a large-scale WDS. The data used in the project concern an actual WDS being part of a major water company in area of southern United Kingdom. The objective is to reduce the cost of energy used for water pumping whilst satisfying all operational constraints, including the pressure constraints in different parts of the water network.

1.1.2 New paradigm for water network modelling and simulation

Applying research in practice has bidirectional benefits; not only enables to evaluate the developed concept in a real world but also it is likely that new research directions are to be generated to solve the subsequent problems encountered in the real world experience. Indeed, the application of research findings in the project that concerned optimisation of operation in the real WDS has allowed the assessment of the solutions proposed by the author in the first part of this thesis. However, the author has came across a number of further problems still to be addressed. For example, if the calculated optimal schedules to be applied to a real network are continuous they have to be converted to their discrete equivalents; continuous schedules cannot be directly implemented as one cannot have e.g. “0.7 of pump ON”, thus a further processing called discretisation is needed. But the process of discretisation of continuous schedules presents a challenge. In (Bounds *et al.*, 2006) it was addressed that a discrete schedule for a pump, which is a part of a big pump station, may differ significantly from the corresponding continuous schedule, because the aggregated flow is achieved by a combination of many pumps. This renders research questions whether is it possible to convert the continuous results to the integer solutions more effectively? Or maybe, the optimal discrete solutions can be found directly within a time interval that allow their real-time implementation?

In addition to above questions, a number of challenges in water network analysis are still present; e.g. majority of water pipe networks analysis methods and simulators are based on the time-slicing approach i.e. numerical methods used in computer simulation of a system characterised by differential equations require the system to be approximated by discrete quantities. The solution of difference equation is calculated at fixed points in time. However, some elements in WDN models (e.g. valves) may cause numerical difficulties (convergence problems) in simulation due to their inherent non-smooth and discontinuous characteristics (Filion and Karney, 2003; Afshar and Rohani, 2009; Rivera *et al.*, 2010; Kovalenko *et al.*, 2010). This is mainly due to the fact that the switching events may not happen at the pre-selected time steps and then additional intermediate time steps need to be introduced. Such an approach is used in the water network simulator Epanet2 which introduces the intermediate steps when simulating water network models containing control elements.

To address the above challenges it is proposed in the second part of this thesis to model and simulate WDSs within the discrete-event specification (DEVS) formalism framework with use of the quantised state system (QSS) methods. Instead of the time-slicing approach as in the majority of water network simulators, the quantisation of states approach is to be investigated in order to obtain an asynchronous discrete-event simulation model of WDS. Such an approach in which water distribution systems modelled within the DEVS framework are simulated using the quantisation-based integration methods has not been applied to WDSs.

1.2 Aims and objectives

The major aim of thesis is to consider models of WDS for control and optimal operation. To do so a simplified model of WDS and a fast method of optimal scheduling are needed. There are two fundamental approaches to optimal control of WDS; time based optimal scheduling and feedback based through rules which operate on states of the system. The latter requires efficient simulation methods considering discontinues control elements and this inspired the author to investigate the event-based methods dedicated for hybrid systems. It is expected that the following objectives would lead to fulfilment of the major aim. These objectives are:

1. To provide a broad introduction to the hydraulics theory necessary for WDS analysis, emphasised on the terminology and mathematical formulations necessary to formulate a WDS simulation model. This objective is addressed in Chapter 2.

2. To conduct an extensive and systematic evaluation of WDS model reduction techniques with the focus placed on the preservation of energy distribution of the original model. This shall be addressed in three phases:

   (a) Investigation of methods and techniques aimed to obtain a reduced WDS model that can accurately replicate the hydraulic behaviour of the original model.

   (b) Study to improve the model reduction algorithm addressing shortcomings uncovered in the existing algorithm.

   (c) Development of a software prototype to allow the utilisation of the model reduction algorithm in the associated research projects.

   The Objective 2 is addressed mainly in Chapter 3 and partially in Chapter 4.

3. To investigate different numeric metrics for the assessment of accuracy of the model reduction technique. This will include the hydraulic-oriented measures used to compare the simplified and original models. This objective is addressed in Chapter 3.

4. To implement the extended model reduction algorithm in a widely used programming language, yielding a software which may be incorporated into other WDS management-related software. This objective is addressed in Chapter 4.

5. To test and examine the developed approach on the case study based on a real urban water network as a proof of concept. This objective is addressed in Chapter 5.

6. To investigate alternative modelling and simulation paradigms able to handle in a more efficient manner the inherent nonlinear and discontinuous characteristics exhibited by some components of water distribution networks. This objective is addressed in Chapter 6.

7. To evaluate appropriateness of the proposed modelling and simulation paradigm on a set of representative case studies. This objective is addressed in Chapter 7.

1.3 Outline of this thesis

The description of the research and developments presented in this thesis is organised as illustrated in Figure 1.1. The thesis comprises the following chapters and appendices:

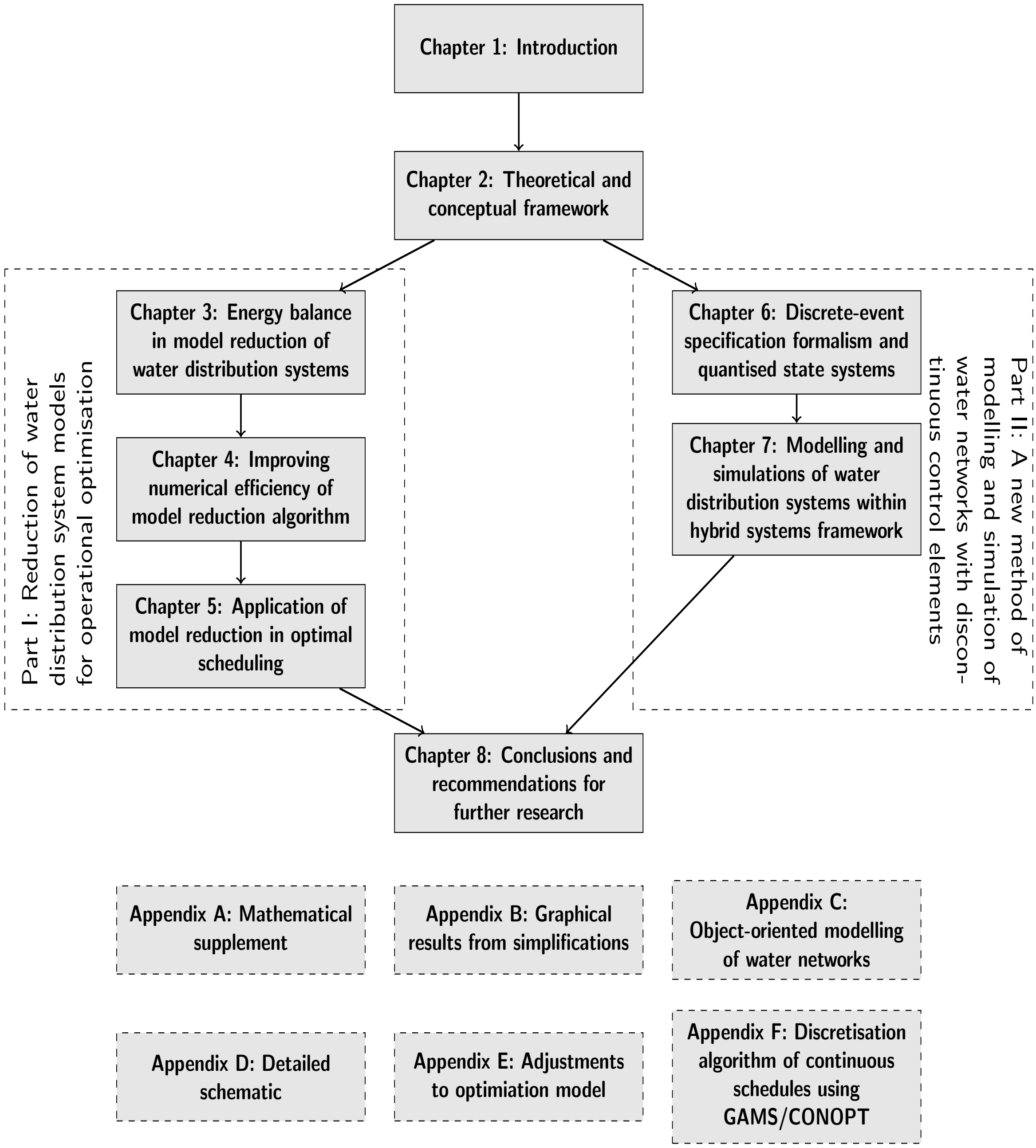


FIGURE 1.1: Schematic representation of the structure of this thesis.

Chapter 1 corresponds to this introduction.

Chapter 2 provides the foundation required to understand the hydraulic network modelling and simulation theories. This is accompanied by a literature survey highlighting issues in the existing modelling, simulation and optimisation of WDSs.

Chapter 3 reviews the existing WDS model reduction methods through analysis of their strengths and weaknesses. The attention is placed on the suitability of the particular method for inclusion into the online optimisation strategy. Within this chapter a problem of an inconsistent energy distribution in the reduced model is highlighted and solution to this problem is proposed.

Chapter 4 describes the development of the extended model reduction application emphasised on the improvements to numerical efficiency of the simplification algorithm.

Chapter 5 applies the extended model reduction algorithm to a complex real case study in the project aimed to optimise the WDS operation.

Chapter 6 gives a brief introduction to the discrete event specification formalism and quantised systems methods. These concepts are subsequently used to propose a novel modelling and simulation paradigm, which can, in an efficient manner, account for nonlinearites and discontinuous behaviour exhibited by components of WDS.

Chapter 7 evaluates the new approach for modelling and simulation of WDSs on a number of representative case studies.

Chapter 8 summarises the main conclusions of author's work in preceding chapters and describes possible avenues for further research.

In addition to the main body of this thesis, there are several appendixes that are of interest for the future reference:

- Appendix A contains a description of some miscellaneous mathematical theories.
- Appendix B presents graphical results from the application of the model reduction algorithm to several case studies described in Section 3.4.2.
- Appendix C illustrates object-oriented approach to model water networks.
- Appendix D depicts a detailed schematic of the water network used as a case study in Chapter 5.

- Appendix E lists adjustments and modifications of the reduced model, which is subsequently used as the optimisation model in Chapter 5.
- Appendix F offers an overview of the discretisation algorithm of continuous schedules used in the case study described in Chapter 5.

1.4 Contributions

Throughout duration of the research programme, a number of contributions have been made to the field of hydroinformatics. Portions of the work that were due to other individuals participating in the projects and had to be included in this thesis for the sake of completeness, are clearly marked throughout this document including the names of the contributors. The main contributions claimed in this thesis are however solely the work of the author.

It is considered that there are two major contributions (one for each of Part I and Part II of the thesis) and a number of less significant contributions which are grouped here for Part I and Part II. These are:

Part I

Major contribution

The major contribution of Part I is the development of a new extension to the WDS models reduction algorithm originally proposed by Ulanicki *et al.* (1996). The extended methodology is based on the concept of energy audits which have been incorporated into the model reduction algorithm allowing the preservation of the original model energy distribution. The idea is established on the distribution of the minimum useful energy which is depended on the minimum service pressure (Cabrera *et al.*, 2010). The standard model reduction algorithm has been extended to reallocate not only demand of the removed nodes but also their minimum useful energy (pressure constraints). The simplified model retains the original model energy distribution due to new pressure constraints. Such an approach preserves accurately the hydraulic characteristic of the original water network. Part of this research was published in (Skworcow *et al.*, 2010; Paluszczyszyn *et al.*, 2011, 2013). Details are given in Chapter 3.

Subsidiary contributions

- An extensive evaluation of strengths and weaknesses of the existing methodologies aimed at reduction of WDS models has been carried out. The methods have been thoroughly evaluated, followed by an assessment of their practical application to online optimisation of WDS operation. Details are given in Chapter 3.
- The numerical efficiency of the extended model reduction algorithm has been improved. The utilisation of the parallel programming techniques and the sparse matrices ordering algorithms have drastically increased the speed of the WDS model simplification. Part of this research was published in (Paluszczyszyn *et al.*, 2014). Details are given in Chapter 4.
- The extended model reduction algorithm has been implemented in the C# programming language, yielding a functional and reliable software that has been already used in a number of research and commercial projects. Details are given in Chapter 4.
- The complete optimisation procedure aimed at obtaining optimal schedules for pumps in the real urban water network has been described in details to demonstrate how such task can been approached and solved. It has been shown that the optimal scheduling of a complex WDN is a dynamic mixed-integer problem and its solution has faced a number of difficulties such as a large number of discrete and continuous variables, nonlinearities in the components equations, modelling uncertainties and discretisation of continuous schedules. Part of this research was published in (Skworcow *et al.*, 2014b,a). Details are given in Chapter 5.

Part II

Major contribution

A new paradigm to modelling and simulation of water distribution networks is claimed as the major contribution of Part II. The new framework has been based upon combination of the DEVS formalism and the QSS methods. Such an approach brings many benefits especially to modelling and simulation of hybrids systems, such WDSs, as instead of the classical time-slicing approach in simulation, the QSS theory considers only changes in states of the system. The results obtained from the simulators based on the QSS engine have showed a number of advantages compared with the standard simulation packages such as Epanet2 (Rossman, 2000b) or OpenModelica (Fritzson, 2010). The accuracy with the DEVS and QSS approach is nearly identical to Epanet2 and OpenModelica but it took much less time to simulate a WDS model and the resulted data consist of a significantly smaller number of points. Part of this research was published in (Paluszczyszyn *et al.*, 2012). Details are given in Chapter 7.

Subsidiary contributions

- A comparison of techniques, methods and tools as well as performance and robustness between the quantised state systems and the classical time discrete simulation for a number of representative water network models. Details are given in Chapter 7.

1.5 Publications

The work in this thesis has contributed in part or full to the following publications in journals and conference proceedings:

Journal articles:

- Skworcow, P.,**Paluszczyszyn, D**., Ulanicki B., (2014). Pump schedules optimisation with pressure aspects in complex large-scale water distribution systems. *Drinking Water Engineering and Science Discussions*, 7(1):121-149.
- **Paluszczyszyn, D.**, Skworcow, P., Ulanicki, B. (2013). Online simplification of water distribution network models for optimal scheduling. *Journal of Hydroinformatics*, 15(3):652–665.

International conference proceedings:

- **Paluszczyszyn, D.**, Skworcow, P., Ulanicki, B. (2014). Improving numerical efficiency of water network models reduction algorithm. In *Computer system engineering: Theory & Applications: 10th, 11th, 12th and 13th Polish-British Workshops*, pages 46-65, Jugów, Poland.
- Skworcow, P., **Paluszczyszyn, D**., Ulanicki B., Rudek, R., Belrain, T. (2014). Optimisation of pump and valve schedules in complex large-scale water distribution systems using GAMS modelling language. *Procedia Engineering*, 70(0):1566-1574.

- **Paluszczyszyn, D.**, Skworcow, P., Ulanicki, B. (2012). A new method of modelling and simulation of water networks with discontinuous control elements. In *Proceedings of the 14th Water Distribution Systems Analysis*, pages 158-167, Adelaide, Australia.
- **Paluszczyszyn, D.**, Skworcow, P., Ulanicki, B. (2011). Online simplification of water distribution network models. In *Proceedings of the 11th International Conference on Computing and Control for the Water Industry*, volume 3, pages 749-754, Exeter, UK.
- Skworcow, P., Ulanicki, B., AbdelMeguid, H., **Paluszczyszyn, D.** (2010). Model predictive control for energy and leakage management in water distribution systems. In *Proceedings of the UKACC International Conference on Control*, pages 990-995, Coventry, UK.

1.6 Research projects

The works presented in this thesis have been partially performed in the frameworks of the following research projects:

- *Pump scheduling planning for Affinity Water (former Veolia Water)*(2012). The project carried out by WSS aimed at possible improvements and optimisation of the water pumping strategy for a real WDS located in the area of southern United Kingdom. The improvements and optimisation were focused at reducing the cost of energy used for water pumping, whilst satisfying all operational constraints, including pressure constraints in different parts of the network. The outcomes from the projects and author's contributions are described in detail in Chapter 5.
- *NEPTUNE*(2009). Neptune was a collaborative project involving two leading UK water service providers (Yorkshire Water Services and United Utilities), a major provider of automation technologies (ABB) and seven UK universities including De Montfort University. The overall aim of NEPTUNE was to advance knowledge and understanding about water supply systems in order to develop novel, robust, practical techniques and tools to optimise, via dynamic control or otherwise, efficiency and customer service. Author's contribution towards the project involved modelling, reducing and adjusting the model of water network subsequently used in the project as a case study for the energy and leakage management within a model predictive control framework. Section 2.5 briefly describes work carried out by the WSS group within the Neptune project.

- *Burst detection and leakage management for Affinity Water (former Veolia Water)*(2012). The project was aimed at the development and implementation of techniques to enhance the burst detection methods. Author's major contribution was translation a research-based methodology for bursts detection and leakage management into an industry-viable tool. Parts of this work are included in Chapter 4.

- *Development of software for energy and leakage management in water distribution systems*(2013). The HEIF (Higher Education Innovation Funding) sponsored project aimed at design and creation of a functional software that integrates various algorithms developed by the WSS group over recent years. Author's gathered a number of different research findings and methodologies developed by the WSS group over the years into the unified and standalone toolkit. Parts of this work are described in Chapter 4 but the developed toolkit was heavily utilised throughout the work presented in this thesis.

Chapter 2

Theoretical and conceptual framework

This chapter reviews the basic concepts and terminology used in this thesis and introduces the notation. The main focus is placed on subjects concerning water distribution system (WDS) that are relevant to establishing the context of the research, which breaks up in the three modules: modelling, simulation and optimisation of WDSs. Each of these topics is broad by itself, hence here only a general background is provided. While doing this, attention is placed on identifying and highlighting problems that are still unsolved in WDS analysis. In Section 2.1, some preliminaries about WDSs in general are given. Section 2.2 introduces WDS modelling issues such as WDS model formulation, nonlinearities and discontinuities exhibited by WDS components. The aim of Section 2.3 is to provide an overview of methods that allow the simulation of hydraulic behaviour of WDS. Also some convergence problems of the existing methods are highlighted. Section 2.4 concentrates mainly on the optimisation methods oriented for the improvement of design and operation of WDSs. A survey of the optimisation methods allows the reader to identify a need for optimisation-oriented models of WDS. To assist the reader further in understanding the key issues/motivations, Section 2.5 explains how the study of optimisation methods for energy and leakage management in water networks has inspired the work forming the basis of the contents of Chapter 3 and Chapter 4. Lastly, Section 2.6 summarises this introductory chapter.

2.1 Water distribution system

The primary aim of a WDS is to deliver water from water sources to intended end points while meeting the specified requirements in terms of water quantity, quality and pressure. Typically, this is achieved by means of interconnected elements, such as pumps, pipes, control and isolation valves, storage tanks and reservoirs. Each of these elements is interrelated with its neighbours thus the entire WDS behaviour depends on each of its elements.

A simple fictitious water distribution network is depicted in Figure 2.1. It is composed of a reservoir with a pump station, a storage tank, and a number of junctions linked by pipes and valves.

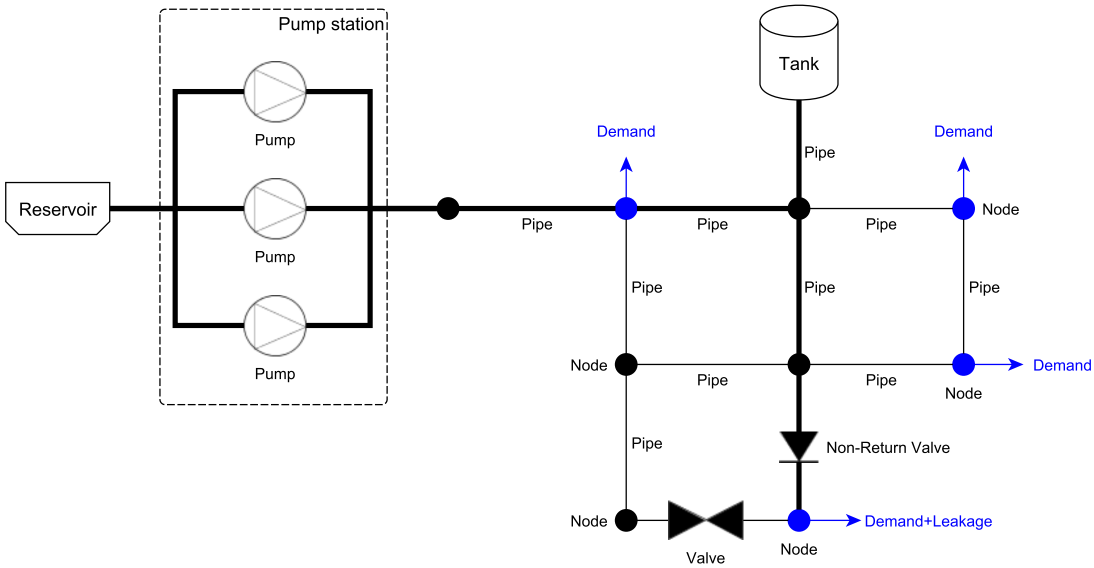


FIGURE 2.1: Illustrating a simple water distribution network. Although, in a real water network water may exit the pipe at any point along its length via service lines, in the computer models water withdrawals (demand and/or leakage) are usually aggregated at the consumption nodes (see nodes in blue).

As illustrated in Figure 2.1, a water network is predominantly represented as collection of hydraulic elements. Table 2.1 lists typical components of a WDS and their function. The layout of elements maps their topographical interdependences and is often imposed by the structure of the urban context such as roads, buildings, industrial areas, hospitals, etc. (Todini, 2000).

Table 2.1: Components of a water distribution system and their functions. Adapted from (Walski *et al.*, 2003).

Element	Type	Modelling purpose
Tank (variable head)	Node	Stores excess water within the system and releases that water at times of high usage.
Reservoir (forced head)	Node	Provides water to the system.
Junction	Node	Removes (demand) or adds (inflow) water from/to the system.
Pipe	Link	Conveys water from one node to another.
Pump	Link	Raises the hydraulic grade to overcome elevation differences and friction losses.
Control valve	Link	Regulates flow or pressure in the system based on specified criteria.

There are two main layouts of a distribution network: branched and looped (Walski *et al.*, 2003). Branched networks, or tree networks, are predominantly used to supply small areas, usually with few delivery points. For areas with many service points and high demand such as cities the looped configuration of the pipes is a more common feature of WDS. The loops provide alternative flow pathways, hence, costumers can be supplied from more than one direction. Looped networks can greatly improve the hydraulics of the distribution system in order to ensure the regularity of the water supply to the final customer. However, most of large distribution systems are essentially a combination of loops and branches with many interconnected pipes and valves. This is a result of a trade-off between loops for reliability and branches for infrastructure cost savings (Walski *et al.*, 2003). A number of examples of real-world WDS layouts are given in Section 3.4.2.

A completely satisfactory water distribution system should fulfil basic requirements such as providing the expected quality and quantity of water during its entire lifetime for the expected loading conditions while accommodating abnormal conditions such as breaks in pipes, mechanical failure of pumps, valves, and control systems, including malfunctions of storage facilities and inaccurate demand projections (Misirdali, 2003).

Hence, to effectively plan, design, maintain or optimise a water distribution system a large number of criteria must be considered. Furthermore, the complexity of this formidable problems may be augmented by a number of components that WDS may consist of; often thousands or even hundreds of thousands. As a consequence, mathematical and computerised representations, i.e. water network models, are used as a support for engineers to understand the hydraulic behaviour of the particular WDS.

In Cesario (1991) a number of different applications for WDS computer models are discussed. In general, modelling of WDS allows designers/operators to:

- understand how a WDS operates under various scenarios,
- assess the performance of a WDS in the event of various failure events e.g. pump station could be disconnected due to reallocation or maintenance service, or a pipe burst would require to isolate part of the network,
- review impacts of proposed operational modifications and developments,
- perform fire-flow studies,
- analyse the robustness and vulnerability of a WDS.
- detect changes and events in a WDS.

The above lists only some common issues that could be identified or addressed thanks to modelling. Once a representative model of WDS is built it can be used for different applications; see Table 2.2 for possible problems in WDS analysis that could be solved by modelling.

2.2 Mathematical representation of water distribution system

In the past, modelling of WDSs involved many steps and it was a tedious and laborious process. With advancement in information and communications technology (ICT), GIS and asset management systems, the process of modelling has been greatly improved as the information about the topology and components can be derived automatically from a GIS system. Time series data such as demands, pump flows, pressure levels or operational schedules can be acquired from SCADA systems. Of course, no model is perfect; any model invariably distorts in some way the very system behaviour it seeks to faithfully represent (Filion and Karney, 2003). Thereby a further calibration stage is often needed to decrease the mismatch between the real data and the model. Calibration is the process of adjusting the model parameters until the model performance reasonably agrees with measured system performance over a wide range of operating conditions (Walski *et al.*, 2003). Guidance for calibration procedures can be found in (Walski *et al.*, 2003, chap.7),

Table 2.2: Problems that could be analysed with help of modelling. Adapted from (Pilipovic, 2004).

Domain	Possible problem
Operational management	Developing and understanding of how the system operates. Training water system operators. Assessing the level of service. Assessing the carrying capacity of the existing system. Assessing the efficiency of current operational management policy. Assessing levels of pressures at critical points within the system. Identifying and resolving operational anomalies. Low pressure or high pressure fluctuation problems. Low fire flow at hydrants - if it is different from expected capacity. Daily operational use - shutting down a section of the system due to major breaks. Power outage impact on pump stations. Sizing control points subsystem metering, control valves PRV, PSV, FCV. Sizing sprinkler systems fire service and other. Assessing the available range of pressure at customer connections. Real time control of the system.
Planning	Identifying the impact of future population growth on the existing system. Identifying the impact of major new industrial or commercial developments on the existing system. Identifying key bottlenecks in current and future systems. Designing the reinforcement to the existing system to meet future demand. Designing the new distribution system. Optimizing the capital works programs. Assessing the new resource option. Assessing the effects of rehabilitation techniques. Leak control reducing losses by lowering maximum pressure. Demand management reducing the pressure related demand by lowering service pressure. Sizing elements of the system to meet fire service requirements in existing and future systems. Assessing the value and design of distribution monitoring systems. Contingency planning.
Legislative	Assessing levels of service, regulatory levels of service reporting, and options for future planning based on community consultations. Maintaining water quality within predefined regulated values. Assessing the financial contribution required for new developments. Water and pressure requirements for fire fighting purposes.
Water quality	Disinfectant residual assessments. Substance tracking, determination of age of water, water blending from various sources. Analysing water quality contamination events.

(Mays, 1999, chap.14) and (Speight, 2008; Hirrel, 2008; Takahashi *et al.*, 2010) and the references therein.

A conceptual model of a water distribution network can be presented as an input-output system as depicted in Figure 2.2 (Ulanicka *et al.*, 1998).

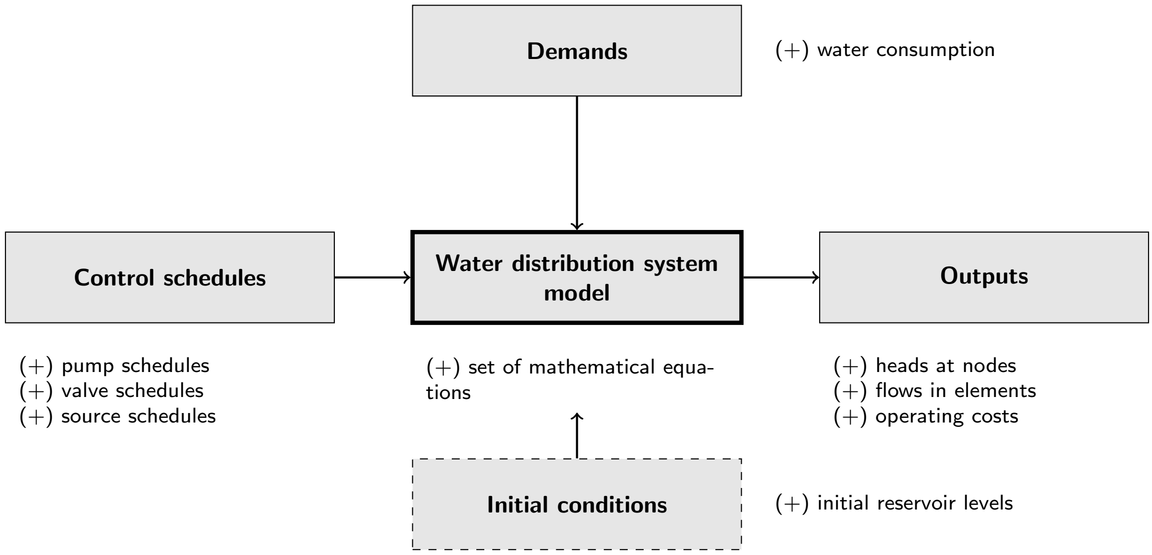


Figure 2.2: A conceptual model of water distribution system. Adapted from (Ulanicka *et al.*, 1998).

A mathematical model of a WDS can be determined by: (i) its topology, (ii) two conservation laws, namely mass balance (*flow continuity*) at nodes and energy conservation (*head loss continuity*) around hydraulic loops and paths, and (iii) equations of components (Brdys and Ulanicki, 1994). Although, here only selected equations that are fundamental for understanding the material in the next chapters are given. For a more comprehensive description the reader should refer to Brdys and Ulanicki (1994).

2.2.1 Conservation of mass and energy principles

The topology of a WDN describes the connections between its components. Such structure can be portrayed by a node-branch incidence matrix $\mathbf{\Lambda}$, where rows and columns correspond to nodes and branches (links) of the network, respectively. Two non-zero entries for each column +1 and -1 indicate the beginning and end of the link, respectively. The matrix elements (λ_{ji}) are defined as

$$\lambda_{ji} = \begin{cases} +1, & \text{if flow of branch } i \text{ enters node } j, \\ 0, & \text{if branch } i \text{ and node } j \text{ are not connected,} \\ -1, & \text{if flow of branch } i \text{ leaves node } j. \end{cases} \tag{2.1}$$

The $n \times b$ node-branch incidence matrix $\mathbf{\Lambda}$ can be reorganised into the following form:

$$\mathbf{\Lambda} = \begin{bmatrix} \mathbf{\Lambda}_r \\ \mathbf{\Lambda}_c \end{bmatrix} \tag{2.2}$$

where: $\mathbf{\Lambda}_r$ is $n_r \times b$ fixed grade node incidence matrix, $\mathbf{\Lambda}_c$ is $n_c \times b$ connection node incidence matrix, b denotes the total number of branches and n, n_r and n_c is the total number of nodes, reservoir nodes and junction nodes, respectively. Such formulation allows to distinguish reservoirs and tanks from the connection and consumption nodes.

The flow continuity law states that the sum of inflows q_{inflow} and outflows $q_{outflow}$ is equal to zero for each non-reservoir (non-storage) node.

$$\sum q_{inflow} + \sum q_{outflow} = 0 \tag{2.3}$$

For the all non-storage nodes in the water network the flow continuity law can be expressed in a matrix form as

$$\mathbf{\Lambda}_c \mathbf{q} = \mathbf{d} \tag{2.4}$$

where $\mathbf{q}$ denotes the vector of branch flows and $\mathbf{d}$ is the vector of nodal withdrawals. Note that Equation 2.4 is valid if the pressure-dependent leakage is not considered (Brdys and Ulanicki, 1994).

The head loss continuity law states that the difference in energy between two points is equal to the frictional and minor losses and, the energy added to the flow in components between these points (Mays, 1999). This condition also requires that the sum of the head losses around a loop must equal zero. This is represented as

$$\sum \pm \Delta h_i = 0 \tag{2.5}$$

where Δh is the head loss/gain across the i-th element of the loop.

The head loss continuity law for all the loops in the water network can be defined in a matrix form as

$$\Delta \mathbf{h} = \mathbf{\Lambda}^T \mathbf{h} = \mathbf{\Lambda}_r^T \mathbf{h}_r + \mathbf{\Lambda}_c^T \mathbf{h}_c \tag{2.6}$$

where $\Delta \mathbf{h}$ is the vector of branch (pipe) head losses, $\mathbf{h}$ is the vector of nodal heads, $\mathbf{h}_r$ is the vector of reservoir heads and $\mathbf{h}_c$ is the vector of junction nodes heads.

In a water network model, links are elements where the energy balance holds, while nodes are intended to represent those points where the mass balance holds (Berardi *et al.*, 2010).

The next sections describe a mathematical representation of the fundamental WDS components. Note that all lower case symbols denote time dependent variables whereas all upper case symbols represent a fixed component value.

2.2.2 Forced-head reservoir

Forced-head reservoirs are used to model a source of water where its head remains imposed and unaffected by water usage rate. It represents an infinite source, which means that it can theoretically handle any inflow or outflow rate, for any length of time, without running dry or overflowing (Walski *et al.*, 2003). Although, in reality, there is no such thing as an infinite source and modellers use reservoirs in situations where inflows and outflows have little or no effect on the head at a node. By its definition reservoir in WDS models is associated with a water surface elevation only.

2.2.3 Variable-head reservoir (tank)

Tanks are present in most real-world distribution systems. In WDS models they are associated with a volume of water in storage, a potential energy expressed in units of pressure that depends on the tank elevation, and a maximum flow rate at which they can feed the network. Additionally, tanks are described by other parameters such as physical capacity, diameter (which may be water-level dependent for non-cylindrical tanks), height and operational constraints such as minimum and maximum allowed water level. Notice that tanks vary in shapes thereby the relationship between water surface elevation and storage volume must be defined to capture tank's characteristics. The fundamental functions of tanks are to sustain pressure in adjoining parts of a water network and provide an emergency storage of water.

When a static simulation of WDS model is considered, a tank is hydraulically identical to a reservoir. In an extended-period simulation (EPS), however, the water level in the tank can vary over time what makes the tank a dynamical element. This dynamical nature can be described by the following differential equation:

$$\frac{dh_r(t)}{dt} = \frac{1}{S(h_r(t))}\left[q_r(t) + d_r(t)\right] \tag{2.7}$$

where $h_r(t)$, $q_r(t)$ and $d_r(t)$ is the reservoir head, flow into the reservoir and demand associated with the reservoir, respectively, at time instant t with $S(h_r(t))$ denoting the reservoir cross-sectional area for the head $h_r(t)$.

Equation 2.7 can be written in vector form as:

$$\frac{d\mathbf{h}_r(t)}{dt} = \mathbf{A}(\mathbf{h}_r(t))\mathbf{q}_r(t) + \mathbf{A}(\mathbf{h}_r(t))\mathbf{d}_r(t) \tag{2.8}$$

where

$$\mathbf{A}(\mathbf{h}_r) = \begin{bmatrix} S_1^- 1(h_r, 1) & \dots & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & S_{n_r}^- 1(h_r, n_r) \end{bmatrix} \tag{2.9}$$

and $\mathbf{q}_r(t)$ is the vector of reservoir inflows, $\mathbf{d}_r(t)$ is the vector of reservoir outflows, $\mathbf{h}_r$ is the vector of reservoir heads, at time instant t with $\mathbf{A}(\mathbf{h}_r)$ denoting the matrix of cross-sectional areas.

In contrast to a forced-head reservoir, in a tank the water level fluctuates according to the inflow and outflow of water, which often relates to periods of heavy pumping and high demand (Brdys and Ulanicki, 1994). This dynamical nature is a source of numerical errors in an EPS of WDSs. In an EPS it is assumed that hydraulic conditions across a time step are constant and Euler integration used to solve Equation 2.7 will track accurately the amount of water flowing in and out of a tank (Filion and Karney, 2003). However, the Euler integration calculated at the beginning of the time step does not account for continually changing (sometimes rapidly) demands. Filion and Karney (2003) demonstrated that this type of error can be sometimes significant and yet its very existence is often overlooked.

This error can be somewhat reduced either by calculating flows at the end of each time step and extrapolating water levels with a modified Euler integration or by shortening the time interval (Filion and Karney, 2003). Alternatively, in some WDS simulators (e.g. PICCOLO) the differential Equation 2.7 is solved by using the Runge-Kutta integration methods (Ułanicki, 1993). Another method, published by van Zyl *et al.* (2006), is the explicit iteration method, which attempts to decouple a network into constituent simple base systems and solve each system individually. This is done by integrating their linearised dynamic tank equations explicitly. The results are then used to estimate the dynamic behaviour of the full WDS.

Another approach proposed by Afshar and Rohani (2009) uses the modified Euler method to discretise the differential equation governing the variation of water level in tanks to obtain a nonlinear algebraic equation in terms of the tank head and inflow. These equations are subsequently embedded into the nonlinear system of equations describing the water network to obtain the final model. Next, the Newton-Raphson method of linearisation is used for the equation governing the tank water level variation along with a gradient formulation of the pipe networks.

2.2.4 Node

A node refers to either ends of a link. Nodes where the inflow or the outflow is known are referred to as junction nodes. The nodes where the outflow (demand) is present are referred to as consumption nodes. The demand at consumption nodes may vary with time or be constant (Mays, 1999). A node element in a WDS model is associated with an elevation.

2.2.5 Pipe

Pipes are the most abundantly occurring component of a WDS. Their primary objective is to convey flow as it moves from one junction node to another in a network. However, while water is transported its energy is dissipated in pipes due to friction losses. This friction head loss in a pipe, between nodes i and j, can be expressed by a nonlinear head-flow relationship as follows

$$q_{ij} = \Phi_{ij}\left(h_i - h_j\right) \tag{2.10}$$

where q_{ij} is the pipe flow and $\Phi_{ij}\left(h_i - h_j\right)$ is the nonlinear function defining the head-flow relationship.

In literature several equations are used to describe the friction head loss along a pipe. The most popular formulas amongst practitioners are: Manning formula (Manning, 1891), Colebrook-White formula (Colebrook and White, 1937), Darcy-Weisbach formula (Darcy, 1857; Weisbach, 1845) and Hazen-Williams equation (Williams and Hazen, 1933). The Manning formula has been employed extensively in open channel flow analysis. Using the Colebrook-White head loss expression one needs to cope with a difficulty that is an implicit function of the friction factor solved typically by iteration (Walski *et al.*, 2003). The Darcy-Weisbach formula was developed using dimensional analysis. This equation, combined with the Colebrook-White formula, is an accurate representation over wide ranges of flow regimes, though requires more computational effort than the Hazen-Williams equation. The Hazen-Williams formula uses a dimensionless roughness coefficient of the pipe, denoted C_{HW}. Higher values of C_{HW} represent smoother pipes and lower values of C_{HW} describe rougher pipes, see (Walski *et al.*, 2003, chap.2) for examples of C_{HW} values. Comparison of the Hazen-Williams and Darcy-Weisbach friction models can be found in (Allen, 1996; Filion and Karney, 2003). Note that throughout this work the Hazen-Williams and Darcy-Weisbach head loss expressions are employed as they are the predominant formulas used by practitioners.

It is convenient to express the head loss Δh_{ij} across a pipe located between nodes i and j in the following general form:

$$\Delta h_{ij} = h_i - h_j = R_{ij} q_{ij}^{\beta} \tag{2.11}$$

or in terms of flow q_{ij} through the pipe

$$q_{ij} = G_{ij} \Delta h_{ij}^{\alpha} \tag{2.12}$$

where h_i and h_j are the heads at nodes i and j, respectively, R_{ij} is the pipe hydraulic resistance, β is the flow power exponent (for Hazen-Williams equation, $\beta = 1.852$; for Darcy-Weisbach equation, $\beta = 2$), $G_{ij} = 1/R^{\alpha}$ is the pipe hydraulic conductance and $\alpha = 1/\beta$.

The hydraulic resistance R_{ij} has the following expressions:

Hazen-Williams equation

$$R_{ij} = \frac{K L_{ij}}{C_{HW}^{1.852} D_{ij}^{4.871}} \tag{2.13}$$

where the conversion factor K is 1.21216×10^{10} when q_{ij} is in l/s and the diameter of the pipe D_{ij} in mm. L_{ij} is the length of the pipe in metres. The factor K is 10.69 when q_{ij} is in m^3/s and the diameter D_{ij} is in metres.

Darcy-Weisbach equation

$$R_{ij} = f \frac{8 L_{ij}}{\pi^2 D_{ij}^5 g} \tag{2.14}$$

where g is the gravitational acceleration constant and f is the dimensionless Darcy-Weisbach friction factor which is a function of the Reynolds number, denoted Re, and the relative roughness, ε/D.

$$f = F\left(Re, \frac{\varepsilon}{D}\right) \tag{2.15}$$

The Darcy-Weisbach friction factor can be determined from the Moody diagram or by solving the Colebrook-White equation (Walski *et al.*, 2003). Although standard tables do exist with roughness values for commercial pipes of different materials and age categories, see e.g. (Mays, 1999).

2.2.6 Valve

A valve is a pipe element that can be opened and closed to different extents. Valves can have a profound effect on the WDS hydraulic behaviour, since they may start or stop the flow, control discharge or pressure and prevent back flow.

There are many valve types in WDN and according to Walski *et al.* (2003) they can be classified into the following five general categories: (i) isolation valves, (ii) directional valves, (iii) control valves, (iv) altitude valves and (v) air release and vacuum breaking valves.

Isolation valves are the most common valves in a typical WDS. Their primary aim is to provide means for manual or automatic closure of a link and thereby isolate part of the WDS. This might be necessary in the event of an emergency e.g. a pipe burst. In general, the isolation valves are intentionally kept in a closed position to define pressure zone boundaries.

Directional valves are used to ensure that flow is in one direction only. If conditions exist for flow reversal the valve will close and no flow will pass.

Control valves are utilised in water distribution systems to regulate flow rate or pressure. A general representation of a variable control valve can be described by a standard bidirectional pipe equation (see Equation 2.12) but with the conductivity controlled by the opening coefficient V_{ij}

$$q_{ij} = V_{ij} G_{ij} |h_i - h_j|^{\alpha} \operatorname{sign}(h_i - h_j) \tag{2.16}$$

where $0 \leqslant V_{ij} \leqslant 1$; i.e. valve is closed if $V_{ij} = 0$ and fully open if $V_{ij} = 1$.

Note that the opening coefficient does not necessarily map directly onto the operational parameter of a real valve. It must be adequately calibrated to achieve this (Rance, 1994).

In Equation 2.16 a modified pipe equation was used to model valves. But it is a practice adopted especially in the water industry to describe valves with the use of minor losses. Minor losses are due to turbulence within the bulk flow as it moves through fittings and bends (Walski *et al.*, 2003). The minor losses h_m can be calculated by multiplying the minor loss coefficient K_v by the velocity head, as follows

$$h_m = K_v \frac{v^2}{2g} \tag{2.17}$$

where v is the flow velocity and g is the gravitational acceleration constant.

Minor loss coefficients are found experimentally, and data are available for many different types of valves' fittings. Alternatively, valve manufacturers often provide a chart, (see Figure 2.3) to describe the valve's characteristic.

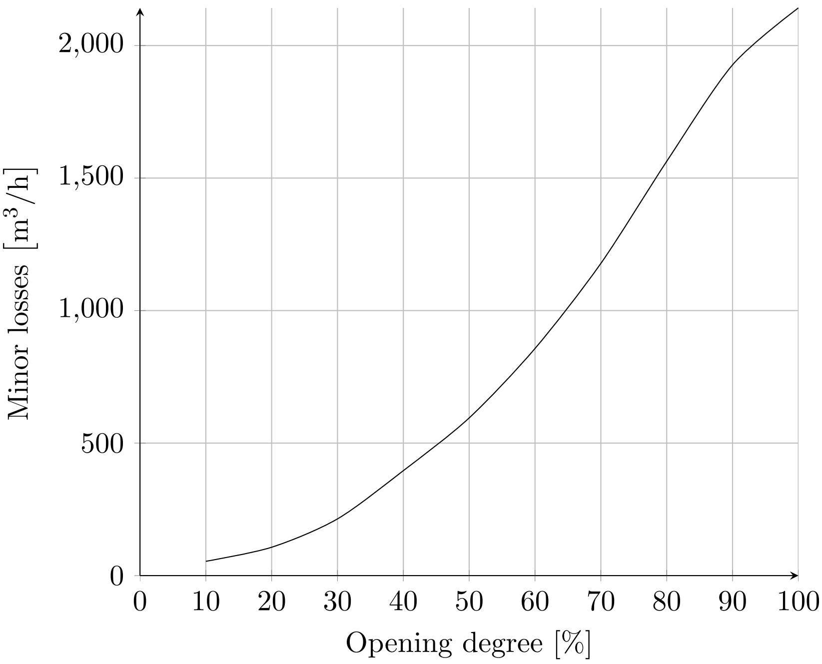


FIGURE 2.3: Example chart of valve characteristic curve.

A control valve can be in any one of the several states e.g. fully-open, fully-closed, active and inactive. Note that the states terminology may vary depending on the control valve type.

Operation of directional and control valves has a major effect on the WDS. But while these valves greatly increase flexibility of control and management of water networks, their inherent non-smooth and discontinuous characteristics may cause numerical difficulties (convergence problems) in simulation (Walski *et al.*, 2003). Some of these convergence problems are described in Rivera *et al.* (2010) and Kovalenko *et al.* (2010) and in Section 2.3.5 of this thesis.

The following sections provide an introduction to some of the most common directional and control valves types and their applications.

2.2.6.1 Non-return valve

A non-return valve (NRV) known also as a check valve (CV) allows only one direction of flow. Any water flowing backwards through the valve automatically causes it to close, and it remains closed until the flow once again begins to go through the valve in the forward direction (Walski *et al.*, 2003).

A NRV between an upstream node i and a downstream node j can be modelled as follows

$$q_{ij} = \begin{cases} G_{ij}(h_i - h_j)^{\alpha}, & \text{if } h_i > h_j \\ 0, & \text{otherwise.} \end{cases} \tag{2.18}$$

Check valves are often a built-in feature of pumps to prevent a backward flow when the pump is off.

2.2.6.2 Flow control valve

A flow control valve (FCV) is a directional valve that automatically limits the flow rate to a user-specified amount. The FCV does not guarantee that the flow will not be less than the setting value, only that the flow will not exceed the setting value. If the flow does not equal the setting, modelling packages will typically indicate so with a warning (Walski *et al.*, 2003).

The FCV is used by modellers to force the simulation to follow a set of desired (e.g. measured in a physical WDS) flows through an element. The FCV is often placed at outflow from forced-head reservoirs to limit the inflow to a WDN or before industrial costumers to maintain constant flow. To model FCV, Equation 2.16 can be used.

However, implementing a FCV and also a NRV in computer programs poses a number of issues due to the difficulty in determining states of valves a priori (Simpson, 1999) and their non-smooth head-flow relationship. Thus, these devices are modelled using heuristics incorporated in the hydraulic simulator. For example, in Epanet2 a FCV can be in *ACTIVE* or *OPEN* state (Rossman, 2000b). To determine state of a FCV, a heuristics procedure is used that includes a predefined zone, in which status switching is prohibited and hysteresis, i.e. valve's status change depends on the previous valve's status (Rivera *et al.*, 2010). Algorithm 1 illustrates how the FCV status is determined in Epanet2. Some of drawbacks of such a FCV model are discussed in (Rivera *et al.*, 2010).

Algorithm 1 Determination of the status of a flow control valve in Epanet2.

Input: S_k - current valve status, S_{k-1} - previous valve status, h_i and h_j - heads at valve terminals, q_v - flow rate through the valve, q_{set} - valve setting, q_{tol} and h_{tol} - Epanet2 solver parameters that define the prohibited zone.

$S_k \leftarrow S_{k-1}$
if $h_i - h_j < -h_{tol}$ **or** $q_v < -q_{tol}$ **then**
    $S_k \leftarrow OPEN$
else
    if $S_k = OPEN$ **or** $q_v > q_{set}$ **then**
        $S_k \leftarrow ACTIVE$
    end if
end if

Deuerlein *et al.* (2009) proposed an approach to modelling NRVs and FCVs based on the content and co-content theory. The water distribution equations are solved as a constrained nonlinear programming problem using the loop method as a simulation model algorithm. Giustolisi *et al.* (2012a) accounts for FCVs by the adjustment of the minor loss coefficient of the valve carried out outside of the hydraulic solver thus preserving the algorithm features.

2.2.6.3 Pressure reducing valve

A pressure reducing valve (PRV) is another commonly occurring control valve in a WDS, especially, in these with a varying topography. These types of valves are generally placed at pressure zone boundaries to prevent the high inlet pressure passing through trough the outlet; i.e. the PRV maintains a defined pressure at the downstream side of the valve for all flows with a pressure lower than the user-specified value. Additionally, a PRV may be used to control from which source of supply the flow comes to satisfy demand levels.

There are generally three different states a PRV can be in: (i) *active* i.e. to achieve its pressure setting on its downstream side when the upstream pressure is above the setting, (ii) *passive* i.e. fully open if the upstream pressure is below the setting and (iii) *closed* if the pressure on the downstream side exceeds that on the upstream side (Rossman, 2000b).

The PRV mathematical model is given by Equation 2.19

$$q_{ij} = \begin{cases} G_{ij}(h_{set} - h_j)^{\alpha} & \text{for} \quad h_j \leq h_{set} \leq h_i \\ G_{ij}(h_i - h_j)^{\alpha} & \text{for} \quad h_j < h_i < h_{set} \\ 0 & \text{for} \quad h_i > h_{set} \quad \text{and} \quad h_j > h_{set} \end{cases} \tag{2.19}$$

where h_{set} is the fixed PRV set point head. See Figure 2.4 for an illustration.

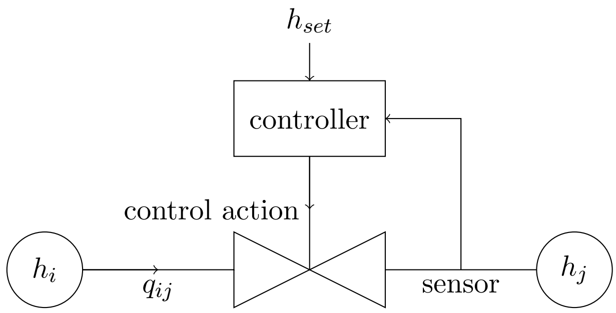


FIGURE 2.4: Model of a pressure-reducing valve.

Note that due to mathematical representation of a PRV its operation may be discontinuous, given that no flow can pass under certain conditions e.g. if the downstream pressure exceeds the head setting, h_{prv} of the valve the PRV becomes full closed and acts as CV preventing reverse flow. The operation of a pressure-reducing valve is depicted in Figure 2.5.

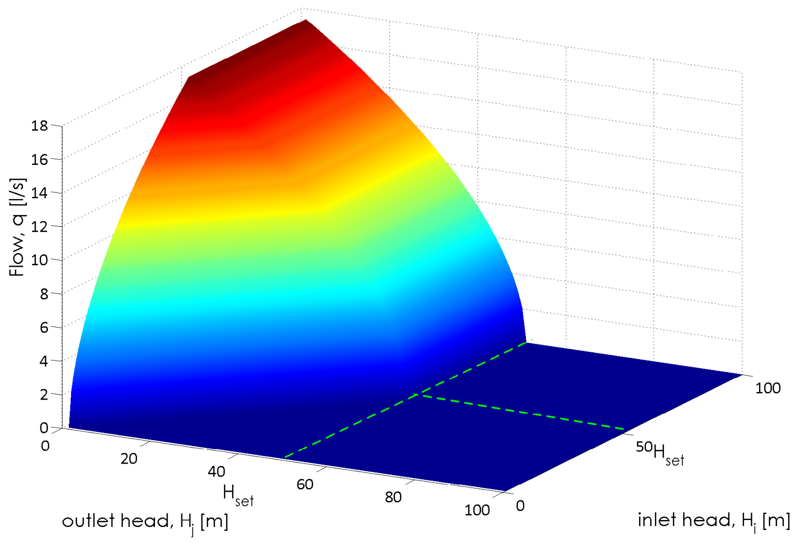


FIGURE 2.5: Illustrating the operation of a pressure-reducing valve.

2.2.7 Pump/Pump station

Pump is an active element that adds energy to a WDS in order to account for friction losses in pipes and valves and head difference in a network. Often pumps in WDS are connected in parallel to form a pump station. Pump characteristics are often presented by manufacturers in the form of data sheets with performance curves which graphically illustrate head-flow, power-flow and efficiency-flow relationships. An example of such curves is depicted in Figure 2.6.

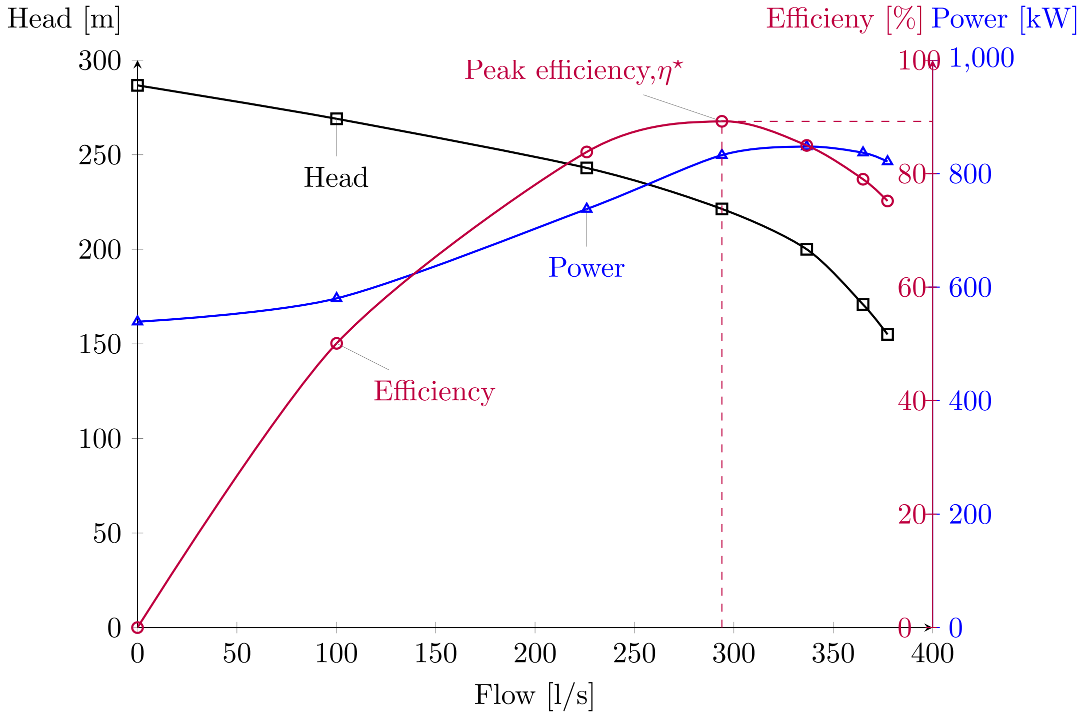


FIGURE 2.6: Example of chart of pump characteristic curves.

As can be seen in Figure 2.6 the head increase across the pump is a nonlinear function. However, for simplicity it can be approximated by a quadratic function such as

$$\frac{\Delta h}{s^2} = A\left(\frac{q}{us}\right)^2 + B\left(\frac{q}{us}\right) + C \tag{2.20}$$

where A,B and C are the quadratic equation coefficients evaluated at the nominal speed, u denotes the number of pumps that are ON, q is the pumped flow and s represents the normalised pump speed defined as a ratio

$$s = \frac{s_o}{s_n} \tag{2.21}$$

where s_o and s_n are the operating and nominal pump speed, respectively.

Alternatively, the hydraulic characteristic of a pump group can be approximated by a power law as described in (Ulanicki *et al.*, 2008):

$$\frac{\Delta h}{s^2} = A\left(\frac{q}{us}\right)^B + C \tag{2.22}$$

where A, B, and C are the pump power law head-flow coefficients with other parameters having the same interpretation as in Equation 2.20.

One of the most important parameters of a pump is the pump efficiency. While it is desirable for a pump to operate near its peak efficiency $\eta^\star$, this is not always possible. The pump efficiency is a function of the pump flow and the pump speed. As the pump efficiency is often provided by manufacturers in a form of data points or a single peak efficiency point, the pump efficiency curve needs to be approximated, e.g. using the quadratic (Coulbeck *et al.*, 1991) or cubic approximations (Ulanicki *et al.*, 2008). For the quadratic approximation the following holds

$$\eta(q,u,s) = \eta^\star\left[1 - \left(\frac{q}{usq^\star} - 1\right)^2\right] \tag{2.23}$$

where η is the pump/pump station efficiency, $\eta^\star$ is the pump/pump station peak efficiency and $q^\star$ is the corresponding peak efficiency flow.

The operation of pumps significantly affects the behaviour of a WDS. Not surprisingly the cost of pumps' operation is one of the major factors considered when determining the optimal operating strategy of WDS. Formulation of an optimal scheduling problem for a WDS requires a formulae to calculate pumping cost. The pumping cost is correlated with electrical energy consumed during pumping. The power consumed by a pump/pump station can be expressed as:

$$P(q,\Delta h,u,s) = \frac{\zeta q \Delta h}{\eta} \tag{2.24}$$

where ζ is the unit conversion factor.

However, in situations when the efficiency curve is approximated by a parabola that goes trough the origin and the peak efficiency point, the above power characteristic may have a singularity for zero efficiency flow. To prevent the singularity and to model pump efficiency closer to reality Ulanicki *et al.* (2008) recommended use of a cubic approximation of the efficiency described by Equation 2.25.

$$\eta(q,u,s) = \eta^\star q \frac{\left(\tilde{q} - 2q^\star\right)\left(\frac{q}{us}\right)^2 + \left(3\left(q^\star\right)^2 - \tilde{q}^2\right)\left(\frac{q}{us}\right) + \left(2\tilde{q}^2 q^\star - 3\left(q^\star\right)^2 \tilde{q}\right)}{(q^\star)^2 us \left(\tilde{q} - q^\star\right)^2} \tag{2.25}$$

where $\tilde{q}$ corresponds to the maximum pump flow for which the head increase across the pump is equal to zero.

Figure 2.7 shows the difference between the quadratic and cubic efficiency approximations based on a single peak efficiency point. It can be seen that the quadratic approximation has zero efficiency for any flow greater than double the peak efficiency flow limiting the operating range of the pump to $0 \leq q \leq 2q^\star$. Whereas in case of the cubic approximation the operating range is extended up to the cut-off flow $\tilde{q}$.

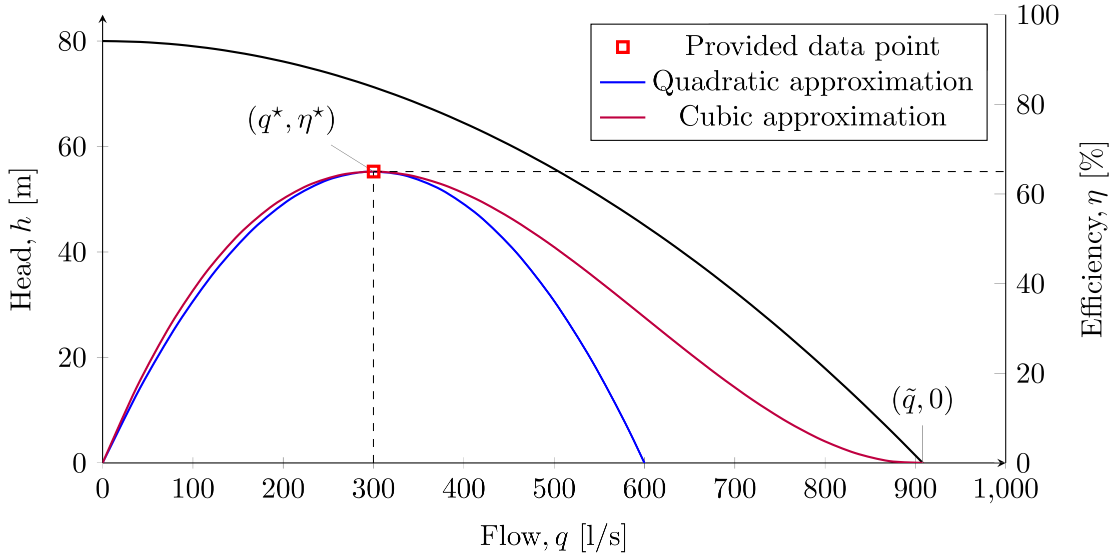


FIGURE 2.7: Illustrating the quadratic and cubic approximations of the pump efficiency when only a single peak efficiency point $(q^\star, \eta^\star)$ is provided by pump's manufacturer (adapted from (Kahler, 2006)). $q^\star$ is the pump peak efficiency flow, $\eta^\star$ is the corresponding peak efficiency and $\tilde{q}$ corresponds to the maximum pump flow for which the head increase across the pump is equal to zero.

As the hydraulic and energy behaviour of a pump is often provided by pump manufacturers in the form of data points, another recommendation by Ulanicki *et al.* (2008) is to obtain the consumed power by a direct cubic approximation of the provided power data points using Equation 2.26 instead of Equation 2.24.

$$P(q,u,s) = \begin{cases} us^3\left(E\left(\frac{q}{us}\right)^3 + F\left(\frac{q}{us}\right)^2 + G\frac{q}{us} + H\right) & \text{for} \quad u,s > 0, \\ 0 & \text{otherwise.} \end{cases} \tag{2.26}$$

where E, F, G, H are the power coefficients constant for a given pump.

Note that Equation 2.26 for a pump/pump station power consumption is used in the case study presented in Chapter 5 to determine optimal schedules for pumps and valves.

There are two basic types of pumps: (i) variable-speed pump (VSP), in which the pump speed varies and can be subjected to external control signals and (ii) fixed-speed pump (FSP), with a speed fixed. It is common that the number of pumps are combined in a parallel configuration to form a pump station in order to provide reliability and flexibility; FSP pump stations are usually used at heavy duty locations whereas VSP pump stations are useful in applications requiring operational flexibility as they allow the control of two factors, speed and number of pumps ON/OFF. Figure 2.8 illustrates changes in a pump station hydraulic behaviour due to a variation of speed and pump configuration.

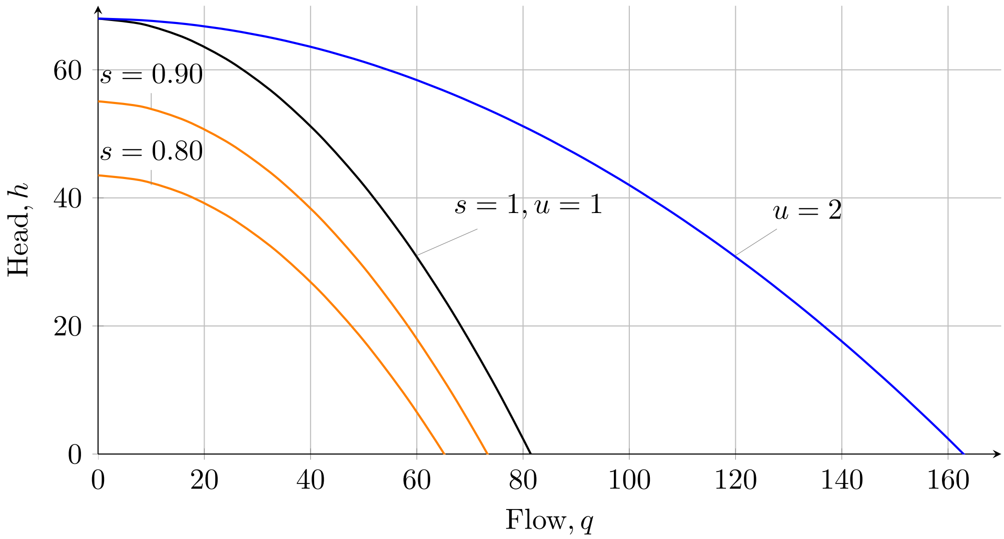


FIGURE 2.8: Illustrating changes in the pump station performance for different values of the normalised speed, s, and the number of pumps ON/OFF, u, factors.

2.2.8 Controls

Water networks often contain controls in order to achieve the desired behaviour of the network (Ulanicka *et al.*, 1998). Controls are used to automatically change the status or setting of an element based on the time of day, or in response to conditions within the network e.g a FCV with predefined time-based settings or a pump controlled by a tank level.

Controls/rules in WDS models can be represented in different ways. Some consider controls to be separate part of the model while others consider them to be an attribute of the pipe, pump, or valve being controlled (Walski *et al.*, 2003). Note that controls characteristic can be continuous or discontinuous and as such may lead to numerical difficulties in solving water networks.

A number of examples of operational controls (#1 and #3) and time-based schedules (#2) are given below. Figure 2.9 illustrates an effect of Control #1 on pump operation and correlated level of water in tank.

```
Control #1
PUMP 1 closed if TANK 1 level above 10
PUMP 1 open if TANK 1 level below 2

Time schedule #2
VALVE 2 setting 20 at time 11
VALVE 2 setting 22 at time 12
VALVE 2 setting 19 at time 13

Control #3
if TANK 2 level below 12 then PUMP 3 is open and VALVE 4 is closed
```

2.2.9 Models of water distribution networks

Combining the equations for the mass and energy conservation laws with components' equations a mathematical model of WDS can be expressed as follows:

$$\begin{aligned}
\mathbf{\Lambda}_c\mathbf{q} &= \mathbf{d} && \text{mass balance}\\
\Delta\mathbf{h} &= \Lambda^T\mathbf{h} && \text{energy conservation}\\
\mathbf{q} &= \mathbf{\Phi}(\Delta\mathbf{h}) && \text{component equation}\\
\frac{d\mathbf{h}_r(t)}{dt} &= \mathbf{A}(\mathbf{h}_r(t))\mathbf{q}_r(t) + \mathbf{A}(\mathbf{h}_r(t))\mathbf{d}_r(t) && \text{storage-reservoir dynamics}
\end{aligned} \tag{2.27}$$

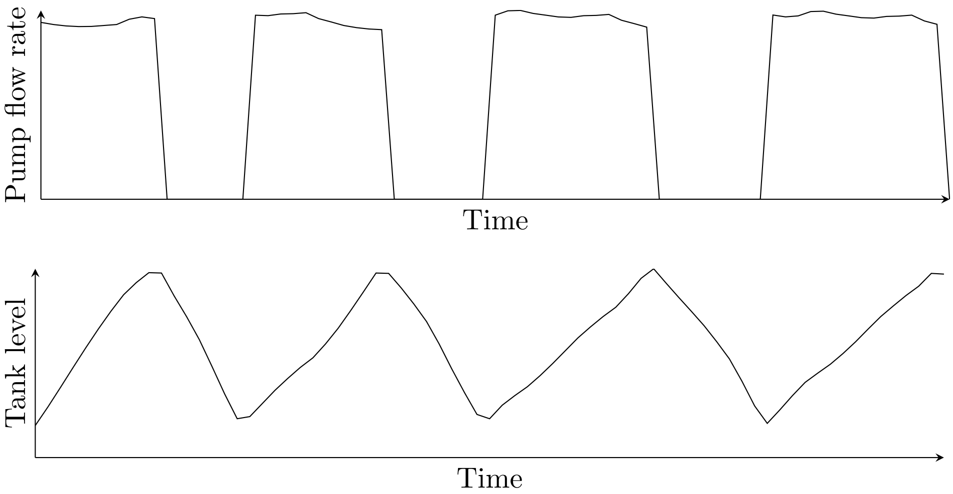


FIGURE 2.9: Illustrating effect of Control #1 on the PUMP 1 operation and the correlated TANK 1 water levels.

The above model formulation is an instance of a nonlinear differential algebraic equation (DAE) system (Brdys and Ulanicki, 1994), where the storage-reservoir dynamics equation describes the differential part of the model and the remaining equations represent the static part of the model. Whereas the mass balance equation is a linear algebraic equation, the energy conservation and component equations reflect the presence of nonlinearity due to the relationship between pipe flow rate and the pressure drop across its length. The static and dynamic parts interact through the vectors $\mathbf{h_r}$, $\mathbf{q_r}$ and $\mathbf{d_r}$.

Subsequently, several different models, in terms of either pressures (nodal formulation) or flow rates (loop and pipe formulation) can be derived by mathematical manipulations.

Nodal model

The nodal model of a water network is expressed by Equation 2.28

$$\mathbf{\Lambda}_c\mathbf{\Phi}(\mathbf{\Lambda}^T\mathbf{h}) = \mathbf{d} \tag{2.28}$$

where $\mathbf{h}$ is the vector of unknown node heads.

From the modeller perspective, it is sometimes practical to distinguish reservoirs with forced/fixed head from the other nodes. The nodal model Equation 2.28 in such form can be expressed by the following equation:

$$\mathbf{\Lambda}_c \mathbf{\Phi}(\mathbf{\Lambda}_c^T \mathbf{h}_c + \mathbf{\Lambda}_f^T \mathbf{h}_f) = \mathbf{d} \tag{2.29}$$

where $\mathbf{\Lambda}_f$ and $\mathbf{h}_f$ are the incidence matrix and vector of heads of forced-head reservoirs. For example the nodal model is used in the water network model reduction technique described in the next chapter.

Branch flow model

This model consists of two separate set of equations to be solved simultaneously: linear junction equations and nonlinear loop equations. In the branch flow model, the branch flows q are the principal unknown variables.

$$\begin{cases} \mathbf{\Lambda}_c \mathbf{q} = \mathbf{d} \\ \mathbf{\Gamma} \mathbf{R}(\mathbf{q})\mathbf{q} = \left[\begin{smallmatrix} \Delta \mathbf{h}_f \\ 0 \end{smallmatrix}\right] \end{cases} \tag{2.30}$$

where $\Delta \mathbf{h}_f$ is the vector of head losses for pseudo-loops (energy chains between reservoirs), $\mathbf{\Gamma}$ is the loop-branch incidence matrix that has a row i for every loop and a column j for every branch (link) of the water network. Two non-zero entry +1 and -1 in each row indicate orientation of the branch to go with or against the loop orientation. The element γ_{ij} of the loop-branch incidence matrix is defined as follows

$$\gamma_{ij} = \begin{cases} +1, & \text{if branch } j \text{ is in loop } i \text{ and their directions are in agreement,} \\ 0, & \text{if branch } j \text{ is not in loop } i, \\ -1, & \text{if branch } j \text{ is in loop } i \text{ and their directions are opposed.} \end{cases} \tag{2.31}$$

$\mathbf{R}(\mathbf{q})$ is the link resistance matrix defined as follows

$$\mathbf{R}(\mathbf{q}) = \begin{bmatrix} r_1(q_1) & 0 & \cdots & 0 \\ 0 & r_2(q_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & r_j(q_j) \end{bmatrix} \tag{2.32}$$

Once the branch flows are determined, the nodal heads can be subsequently calculated using the component equation from Equation 2.27.

Mixed model

In the mixed model the vector of link flows $\mathbf{q}$ and the vector of nodal heads $\mathbf{h}$ contain the unknown variables. The model is expressed by the following equation:

$$\begin{cases} \mathbf{\Lambda}_c \mathbf{q} = \mathbf{d} \\ \mathbf{R}(\mathbf{q})\mathbf{q} - \mathbf{\Lambda}^T \mathbf{h} = 0 \end{cases} \tag{2.33}$$

As it can be seen, the particular model is derived in terms of unknown nodal heads or/and unknown branch flows. On account of nonlinearity in these equations it is not possible to solve network analysis problems analytically; instead iterative numerical solution methods are used. Initial estimated values of pressure or flow are repeatedly adjusted until the difference between two successive iterates is within an acceptable tolerance. Several numerical iterative solution techniques have been suggested, from which the Newton-Raphson method is the most widely used iterative solution procedure in water network analysis. See Appendix A for the Newton-Raphson method description.

2.3 Simulation and optimisation of water distribution networks

Simulation of water distribution networks aims to provide solution of nonlinear and linear equations used to formulate the mathematical representation of the WDN. Simulation is an invaluable tool in the assessment of WDS response to different operational actions (e.g. valves opening and closing) or control strategies prior to applying the actions to a real water network.

The following simulation techniques can be distinguished: (i) steady-state simulation, (ii) extended-period simulation and (iii) transient simulation. Steady-state simulations represent a snapshot of the WDS operation i.e. demands and pressures at all nodes and flows in all pipes do not vary in time. In real systems, however, the loading conditions and states vary in time. Thus, to evaluate performance of a WDS over a period of time, an EPS is used. This type of analysis considers fluctuations of water level in tanks and demands in discrete time intervals. Notice, however, that in each time interval the system is assumed to be in steady state. The transient simulation provides the most accurate simulation of WDS as it considers naturally unsteady flow conditions incorporating transient analysis.

Due to the complexity of this approach it is not yet adopted by many practitioners; mainly limited to specialised applications such as pump design.

Simulations of WDS is not an easy task; solution cannot be obtained analytically. For tree-shaped networks one can obtain solution simply by applying the flow continuity equation at all the nodes but in practice WDSs are almost never pure tree-shaped. The analysis of a looped water network presents more challenges. Although, over the last decades a number of methods were proposed (Wood and Rayes, 1981; Ormsbee, 2006). A summary of some of the more important methods include: (i) the Hardy Cross method (Cross, 1936), (ii) the linear theory method (Wood and Charles, 1972), (iii) the Netwon-Raphson method (Shamir and Howard, 1968), (iv) the linear graph theory method (Kesavan and Chandrashekar, 1972), (v) an approach involving optimisation methods (Collins *et al.*, 1978), (vi) the global gradient method (Todini and Pilati, 1987). An overview of some of the most recognisable and utilised techniques is given in the subsequent sections.

2.3.1 Hardy Cross method

One of the very first methods for water network analysis was proposed by Cross (1936). The Hardy Cross method is an early adaptation of the Newton's method (described in Appendix A). The method was developed before the computer age to allow the solving of pipe networks by hand. The Hardy Cross method is briefly presented here to illustrate the iterative approach to solve the loop equations in water networks.

If the flow rate in each pipe is approximated as $\hat{q}$ and Δq is the error in this estimation then the actual flow rate q can be defined as follows

$$q = \hat{q} + \Delta q \tag{2.34}$$

Substituting Equation 2.34 to Equation 2.11 and after series of mathematical transformation (see Cross (1936) for details) leads to Equation 2.35 that forms the basis of the Hardy Cross method.

$$\Delta q_p = -\frac{\sum_{m=1}^{NP(p)} R_m q_m^{\beta}}{\sum_{m=1}^{NP(p)} \beta R_m |q_m|^{\beta-1}} \tag{2.35}$$

where $NP(p)$ is the number of pipes in loop p, R_m is the head-loss coefficient in pipe m (in loop p), q_m is the estimated flow in pipe m and Δq_p is the flow correction for the pipes in loop p.

Recapitulating, the Hardy Cross method initially estimates the flow conditions throughout the water network to satisfy flow continuity law at each node. As the initial approximation are not likely to satisfy the head loss continuity law, corrections to discharge Δq_p are applied for each loop to obtain zero head loss around the loop. The process is iteratively repeated until the corrections are less than a specified value.

The method is computationally more intensive than other methods and its convergence requires a "good" initial estimate of the flow (Mays, 1999). Thus nowadays it is used mainly for didactic purposes.

2.3.2 Linear theory method

Wood and Charles (1972) proposed another method in which all the equations for each loop or path are solved simultaneously to obtain the flow rate in each pipe. In this method the linearised head loss equations form a set of linear equations which, subsequently, can be iteratively solved for the unknown values of flow. At each iteration, the absolute differences between successive flow estimates are computed and compared to a convergence criterion; i.e if the differences are significant process is repeated for another iteration (Mays, 1999).

Due to combination of conservation of mass and conservation of energy equations in the linear theory method, an initial flow balance of the nodes is no longer required, but on the other hand the linear theory method can sometimes oscillate near the exact solution (Wood and Charles, 1972).

2.3.3 Gradient method

With the advent of computer hardware algorithms were developed aimed to solve water networks increasing in size and complexity. One of the most widely used algorithm, even nowadays, was proposed by Todini and Pilati (1987) and is called the gradient method. This method is another application of the Newton-Raphson technique. Although it takes into account, both, nodal heads and pipe flows to simultaneously solve the equations of conservation of mass and energy. The nonlinear energy equations are first linearised using the Taylor series expansion to produce the overall system of linear equations:

$$\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ \mathbf{h} \end{bmatrix} = \begin{bmatrix} -\mathbf{A}_{10}\,\mathbf{h}_0 \\ \mathbf{d} \end{bmatrix} \tag{2.36}$$

where $\mathbf{A}_{11}$ is the diagonal matrix containing the linearisation coefficients $R|q|^{\beta-1}$, $\mathbf{A}_{12} = \mathbf{A}_{21}^T$ is the unknown head nodes incidence matrix, $\mathbf{A}_{10}$ is the fixed head nodes incidence matrix, $\mathbf{d}$ are the nodal demands and $\mathbf{h}_0$ are the fixed nodal head.

Next, the Newton-Raphson iterative technique can be obtained by differentiating both sides

$$\begin{bmatrix} \beta\,\mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & 0 \end{bmatrix} \begin{bmatrix} d\mathbf{q} \\ d\mathbf{h} \end{bmatrix} = \begin{bmatrix} d\mathbf{E} \\ d\mathbf{d} \end{bmatrix} \tag{2.37}$$

and where

$$\begin{aligned} d\mathbf{E} &= \mathbf{A}_{11}\,\mathbf{q}^k + \mathbf{A}_{12}\,\mathbf{h}^k + \mathbf{A}_{10}\,\mathbf{h}_0 \\ d\mathbf{d} &= \mathbf{A}_{21}\,\mathbf{q}^k - \mathbf{d} \end{aligned} \tag{2.38}$$

are the residuals to be iteratively evaluated for flows $\mathbf{q}_k$ and heads $\mathbf{h}_k$ at iteration k; and $\beta\,\mathbf{A}_{11}$ is the diagonal matrix of the exponents of the pipe equations.

Subsequent updates to $\mathbf{q}_{k+1} = \mathbf{q}_k + d\mathbf{q}$ and $\mathbf{h}_{k+1} = \mathbf{h}_k + d\mathbf{h}$ continue until convergence is achieved; i.e. the residuals $d\mathbf{E}$ and $d\mathbf{d}$ are reduced to zero.

Although, the gradient algorithm requires a larger set of equations to be solved than the Hardy Cross or Linear theory methods, it has been shown to be robust and computationally efficient (Mays, 1999; Todini, 2006a).

2.3.4 Pressure-driven simulation

Nowadays, most WDN simulation models are based on the conventional methods oriented on demand-driven simulation, e.g. the gradient method. In demand-driven simulation an assumption is made that outflows from nodes (demands) are fixed and are satisfied regardless of network pressures (Tabesh *et al.*, 2002). This assumption simplifies the mathematical solution of the models but fails to take into account the relationship between pressure and demand; i.e. demand should only be satisfied provided there is sufficient pressure. Furthermore, this may lead to incongruous results while analysing the water network, especially, under subnormal pressure conditions (Tanyimboh *et al.*, 2003). Therefore, it is recommended that studies oriented on leakage minimisation should not relay on demand-driven analysis, as without accurate pressure inputs it is difficult to quantify the leakage.

The simplest form of leakage-pressure relationship is represented by Equation 2.39, which can be included in the standard hydraulic model and thereby create an extended model suitable for pressure control and leakage analysis. Inclusion of the leakage-pressure relationship into the standard hydraulic equation will be demonstrated in Section 5.4.2.

$$l = Kp^{\kappa} = K(h-z)^{\kappa} \tag{2.39}$$

where l, p, h, z are the nodal leakage, pressure, head and elevation, respectively. K and κ denote the leakage coefficient and the pressure exponent, respectively.

The leakage coefficient incorporates various flow-independent factors that depend on the shape and size of the orifice. The emitter exponent represents the sensitivity of the flow rate to pressure. The emitter exponent is often obtained experimentally and typically varies from 0.5 (pipe bursts) to 1.5 (background leakage) and depends on the type of leakage, shapes of holes, material of pipes and soil (Ulanicki *et al.*, 2000; Greyvenstein and Zyl, 2007; van Zyl and Clayton, 2007). The Equation 2.39 is often used to simulate simple pressure-dependent emitters (Rossman, 2000b).

Thus, currently, many researchers and practitioners tend to shift towards pressure driven analysis, which is physically more accurate, e.g. see (Tabesh *et al.*, 2002; Todini, 2003; Cheung *et al.*, 2005; Todini, 2006b; Giustolisi *et al.*, 2008; Wu *et al.*, 2009; Siew and Tanyimboh, 2012).

2.3.5 Simulation of water networks with controls and control devices

In real water networks the rule-based management is an indispensable element of every day network operation. The presence of rules, control loops, control elements (pumps, valves), time-based switching schedules even in a simple water network complicates the numerical calculations (Brdys and Ulanicki, 1994); at any instance a control in a network can change the state of the controlled component. Thus, simulation software need to include mechanisms for monitoring the value of the control function $c(t, x)$ and change parameters of the mathematical model accordingly. This can be done by checking the state x during the calculation time in order to execute the functions $c(t, x)$.

This procedure, however, varies with different modelling and simulation (M&S) software. Some algorithms carry on the status checks after each iteration or two iterations, others check the status only after convergence is achieved (Brdys and Ulanicki, 1994). For example, the popular hydraulic simulator Epanet2 Toolkit instead of determining the exact time step at which the control would occur, introduces additional checks around the hydraulic calculation time step. If a control condition occurs, the action is applied and an intermediate time step will be created to reflect this (Rossman, 2000a). But the quantity of status checks can have an implication on the overall simulation time, especially for network models with a large number of components such as PRVs, NRVs and FCVs. The problem associated with controls and control elements in water network simulation is well known and addressed by a number of studies (Simpson, 1999; Andersen and Powell, 1999; Arsene *et al.*, 2012b).

Simpson (1999) highlighted the issue of modelling pressure regulating devices. Simpson (1999) provided few examples of simple water networks that cannot be properly modelled and simulated in computer programs based on the early version of Epanet2 as the underlying solver. The issues reported were due to fact that the valves status is not known a priori. Simpson (1999) did not a provide remedy for these problems but rather formulated research questions yet to be addressed.

Andersen and Powell (1999) proposed an approach based on the simultaneous loop equations and graph theory. With the use of the depth-first search (DFS) algorithm (Tarjan, 1972) a spanning tree (tree with all the vertices and some or all the edges) of a network is created. The spanning tree is used to detect loops and pseudo-loops in the considered network. If a loop contains a closed NRV, its operation is analysed. The NRV is then removed from the tree and the network has to be analysed again. However, the procedure concerning PRVs is more complex, as it is crucial for the algorithm to determine a PRV location, membership to the particular loop and direction (relative to the loop direction). A logical expression is proposed to determine whether or not the PRVs will affect a link in a loop. The final solution is obtained using an algorithm based on the Netwon-Raphson method but with a linear head loss formula for the first iteration. For the subsequent iterations, the Colebrook-White head loss formula is used. The algorithm solved the model of a real network containing 26 PRVs and 4 NRVs with the convergence criterion of 10^{-8} for the maximum loop residual.

A similar method to that in (Andersen and Powell, 1999) was reported by Arsene *et al.* (2012b), where the DFS algorithm was used to build a spanning tree, topology incidence matrix and loop incidence matrix of the network. The obtained spanning tree is then partially rebuilt, if necessary, to account for NRVs and pumps (closed NRVs and pumps are handled in the same way). PRVs are treated in the same way as was proposed in (Andersen and Powell, 1999). Comparative experiments were then performed on the real network without and with PRV. Both networks were simulated in Epanet2 and in the proposed loop-based simulator. The obtained convergences were similar for both hydraulic simulators.

Gudiño Mendoza *et al.* (2012) presented an alternative approach for modelling and simulation of WDSs. As was mentioned in Section 2.2, some elements of a WDN can exhibit both discrete and continuous dynamics simultaneously e.g. a discrete pump control based on the water level in the tank. Systems where discrete and continuous dynamics are present are called hybrid systems. Gudiño Mendoza *et al.* (2012) used this property and employed timed hybrid Petri nets (THPN) as a framework within which hybrid models of WDSs are derived from the conservation of mass and energy equations. THPNs were chosen as they can cope with discrete and continuous dynamics. The approach proposed by Gudiño Mendoza *et al.* (2012) aimed to simulate both the transient and steady-state of hydraulic networks. First, the THPN model structure is obtained from the mass balance equations, and subsequently updated with other model parameters obtained from the linearised energy equations. When simulating the THPN model, first the discrete events are handled then the simulation of the continuous part is carried out. Discrete-events represent e.g. change in valve state and as such trigger necessity to modify energy equations. Despite a number of very promising studies a further development is needed as the method allows to include valves and pumps only when they are connected directly to tanks. Also, PRVs and other directional valves were not considered in the method; in fact any change in flow direction in a pipe stops the simulation. Nevertheless, the approach is unlike other aforementioned as it updates variables accordingly to the system states rather than at defined time intervals.

2.3.6 Synopsis of simulation methods

The above overview of the steady-state solution methods provides only a brief insight into simulation of water networks. It might not cover all the methods or the subsequent improvements to the already discussed but highlights that WDNs need an iterative process to be solved and that most approaches are based on the popular Newton-Raphson method.

Not surprisingly, a number of studies were conducted aimed on analysis and comparison of the particular method, e.g. (Wood and Rayes, 1981; Salgado *et al.*, 1988; Mays, 1999; Todini, 2006a) and others. For example, Wood and Charles (1972) indicated that the Newton-Raphson method solves small systems more quickly than the linear theory but may converge very slowly for large networks. Also, poor initial conditions may lead to convergence problems. According to Mays (1999) the linear theory algorithm suits the best the loop equation formulation and it does not require initialization of flows. Salgado *et al.* (1988) carried out a comparison of the Netwon-Raphson and linear theory methods with the gradient algorithm on systems with time varying demand, and concluded the gradient method to be superior to the others in each test. The computational efficiency attributed to the gradient algorithm resulted in its application in a number of simulation packages, e.g. Epanet2, AQUIS, HydrauliCAD.

One recent method for extended period simulation published by (Giustolisi *et al.*, 2012b) is the EGGA, which attempts to decrease the total number of nodes and pipes by automatically removing serial nodes i.e. nodes with only two neighbours. The reduction is done with consideration of mass and energy balance of the original model thus the “simplified” model very accurately resembles the full model. The maximum difference of nodal heads was less than 0.0005m.

Sometimes, especially for old water distribution systems, the details about their infrastructure e.g. pipes parameters may be not available or were not updated over the years. To enable simulation of such models with incomplete data Tanyimboh and Templeman (1993) proposed to use the maximum entropy to estimate the most likely values of flows in the pipes. The concept of entropy is also utilised in water distribution systems for other purposes e.g. in network designs focused on robustness and resilience e.g. (Ang and Jowitt, 2005; Saleh and Tanyimboh, 2014).

Another novel approach already discussed in Section 2.3.5 was presented in (Gudiño Mendoza *et al.*, 2012). The method treats WDS as hybrid systems and thereby uses hybrid system frameworks for modelling and simulation purposes. The approach tried to address the problem which is still present in simulation of WDS; i.e. the non-smoothness of the head-flow formula and discontinuities that present a convergence challenge for any time-slicing solver based on the Newton’s method. In fact, this problem inspired the author of this thesis to formulate a research objective aimed at modelling and simulation of WDS within discrete event specification (DEVS) framework with use of QSS methods. Results from this investigation are summarised in Chapter 6 and Chapter 7.

There are approaches to utilise the event-driven modelling in water distribution systems. However their are used to perform water quality simulation. The event-based procedure proposed by Boulos *et al.* (1994) assumes that hydraulic simulation is already conducted and all flow patterns, travel times, the network topology are always known at any point in time during the simulation. Such simulated hydraulic model works as the input to a scheduler which rearranges events associated with changes in water quality.

Other modern trends in simulation of water networks are to utilise the parallel computing techniques (Alonso *et al.*, 2000) or to include transient modelling in an EPS (Wood *et al.*, 2005; Jung *et al.*, 2007b; Ebacher *et al.*, 2011).

2.3.7 Existing modelling and simulating software

The simulation methods presented above are foundations for development of hydraulic simulator software. It is important, however, to highlight that in many cases the mathematical description of a simulation method cannot be straightforward converted into a computer program due to many restrictions in computer hardware and programming languages' specifications. Programmers often need to modify the original algorithm to account for issues or restrictions e.g. memory storage limits. To address this issue Rivera *et al.* (2010) and Kovalenko *et al.* (2010) proposed a procedure for the experimental convergence evaluation of a hydraulic-network solver.

Currently, there are many water network modelling and simulation (M&S) programs available. Table 2.3 lists some of M&S software along with their producers and websites. Some of them are free and open-sourced (e.g. Epanet2) while others are commercial products. The listed programs vary significantly in offered capabilities and features; commercial packages often offer additional capabilities beyond standard hydraulic and water quality analysis (e.g. fire-flow modelling, transient analysis, automated calibration and optimisation).

2.3.8 Epanet2 - hydraulic simulator

Many programs from Table 2.3 are in use for years with established track record. However, especially in academia, the Epanet2 software (Rossman, 2000b) sets a standard of hydraulic simulation as it has been used in hundreds of published applications. In the WSS group it has been used in many research projects; the most recent publications include e.g. (Skworcow *et al.*, 2010; Paluszczyszyn *et al.*, 2011; Skworcow and Ulanicki, 2011; Paluszczyszyn *et al.*, 2013; Skworcow *et al.*, 2014b). Also, many other researchers employed Epanet2 in different applications e.g., (Abebe and Solomatine, 1998; Alonso *et al.*, 2000; Maier *et al.*, 2003; van Zyl *et al.*, 2004; Broad *et al.*, 2005; Geem, 2006; Shen and McBean, 2010; Siew and Tanyimboh, 2012).

Epanet2 performs EPS of hydraulic and water quality behaviour of the network. It allows the application of one of three friction head loss formulas (Hazen-Williams, Darcy-Weisbach and Colebrook-White) for determining pressure loss occurring in the pipes during transport.

Epanet2 is based on the gradient method by Todini and Pilati (1987). Its simulation routine is depicted in Figure 2.10. The routine contains two main loops; *Loop A* simulates

TABLE 2.3: Water distribution system modelling software. More extensive list and reviews can be found in (Schmid, 2002) and (Balut and Urbaniak, 2011).

Software	Producer	Website
Aquadapt	Derceto, Inc.	http://www.derceto.com/Products-Services/Derceto-Aquadapt
AquaNet	INAR	http://www.inar.net/products/aquanet.htm
AQUIS	Schneider Electric	http://www.schneider-electric.com/products/ww/en/5100-software/5125-information-management/61417-aquis-software
CWSNet	University of Exeter	http://sourceforge.net/apps/trac/cwsnet/
ENCOMS	Halcrow	http://www.halcrow.com/encoms
Epanet2	US EPA	http://www.epa.gov/nrmrl/wswrd/dw/epanet.html
Eraclito	PROTEO S.p.A.	http://www.proteo.it/prodotti/eraclito.asp
Finesse	WSS	http://watersoftware.dmu.ac.uk/
H2OMAP Water	Innovyze	http://www.innovyze.com/products/h2omap_water/
H2Onet	Innovyze	http://www.innovyze.com/products/h2onet/
Helix delta-Q	Helix Technologies	http://www.helixtech.com.au/Q2Main.aspx
HYDROFLO	Tahoe Design Software	http://www.tahoesoft.com/html/hydroflo.htm
Infowater	Innovyze	http://www.innovyze.com/products/infowater/
InfoWorks WS	Innovyze	http://www.innovyze.com/products/infoworks_ws/
KYPIPE	KYPipe LLC	http://kypipe.com/kypipe
Mike Net	DHI	http://www.mikebydhi.com/Download/-MIKEByDHI2014.aspx
MISER	Tynemarch Systems Engineering Ltd.	http://www.tynemarch.co.uk/products/miser/-miser.shtml
Netbase	Crowder Consulting	http://www.crowderconsult.com/netbase-water-management-software/
optiDesigner	OptiWater	http://www.optiwater.com/optidesigner.html
Optimizer WDS	Optimatics	http://optimatics.com/software/optimizer-wds
Piccolo	Safege	http://www.safege.com/en/innovation/-modelling/piccolo/
Porteau	irstea	http://porteau.irstea.fr/
SynerGEE	GL Water	http://www.gl-group.com/en/water/-SynerGEEWater.php
Wadiso	GLS Software	http://www.gls.co.za/software/products/-wadiso.html
WatDis	Transparent Blue	http://www.watdis.com/en/about
WaterCAD V8i	Bentley	http://www.bentley.com/en-US/Products/WaterCAD/

the water network model over a desired period of time while *Loop B* is responsible for solving the system of non-linear equations.

The functions of Epanet2 software have been compiled into a library of routines, namely Epanet2 Toolkit, that can be called from other applications. Moreover, the Epanet2 Toolkit functionalities can be extended, modified or incorporated into customised applications simplifying the process of adding of hydraulic analysis capabilities.

Note that throughout the work carried out in this thesis both Epanet2 and Epanet2 Toolkit were heavily utilised for many purposes.

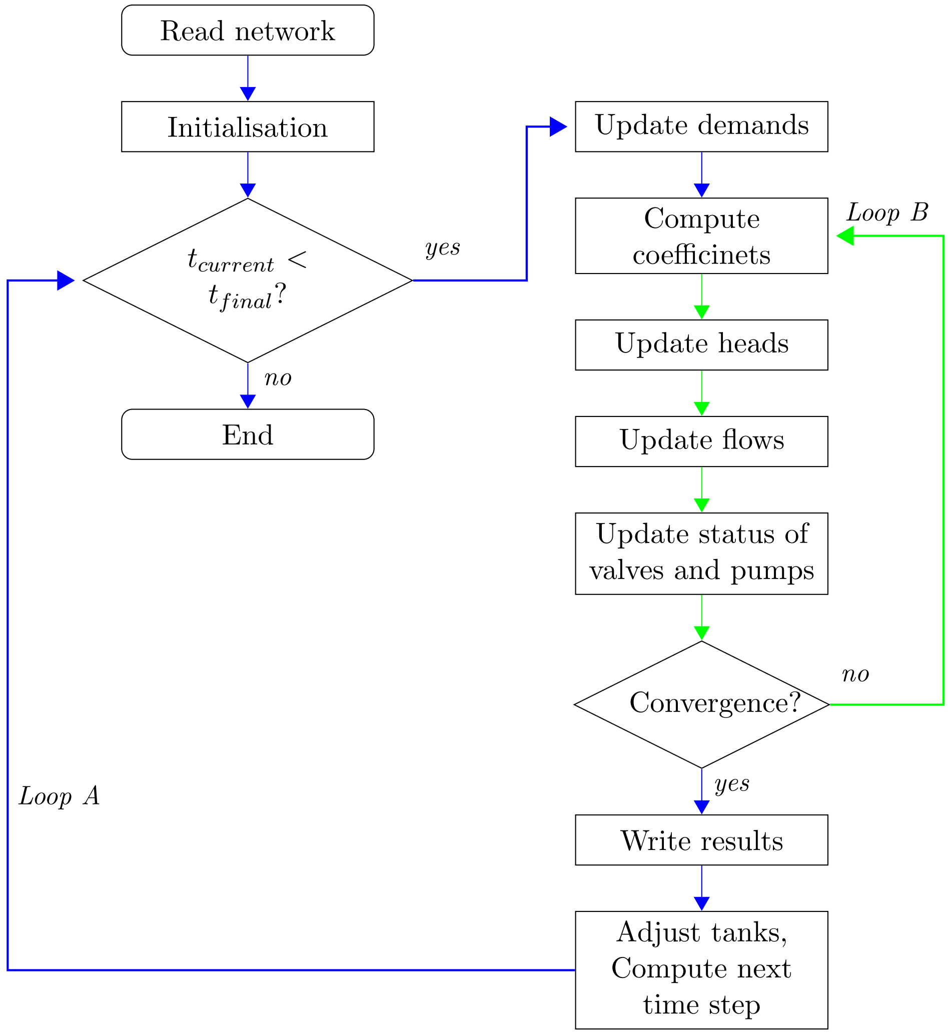


FIGURE 2.10: Hydraulic simulation routine in Epanet2. Adapted from (Alonso *et al.*, 2000).

2.4 Overview of optimisation methods for water distribution systems

In general, every optimisation problem is defined by two essential parts: an objective function and a set of constraints. The objective function describes the performance criteria of the system. Constraints describe the system or process that is being designed or analysed and can be in two forms: equality constraints and inequality constraints (Mays, 1999). These constraints are typically associated with the system hydraulic requirements such as equations of mass and energy conservation, design and/or operational parameters limits, nodal pressure bounds and other parameters dependent on both pressure and design/operational parameters (Coelho and Andrade-Campos, 2014).

A general optimisation problem can be formulated as the minimisation or maximisation of an objective function f subject to equality and/or inequality constraints and can be defined as follows

$$\min\,(\text{or}\max) \quad f(\mathbf{x}) \tag{2.40}$$

subject to constraints

$$g(\mathbf{x}) \leq 0 \tag{2.41}$$

$$h(\mathbf{x}) = 0 \tag{2.42}$$

where $\mathbf{x}$ is the vector of n decision variables $\mathbf{x} = (x_1, x_2, , x_n)$, $g(\mathbf{x})$ and $h(\mathbf{x})$ are the vectors of equations inequality and equality constraints, respectively.

A feasible solution of the above optimisation problem is a set of values of the decision variables that simultaneously satisfies the defined constraints. An optimal solution is a set of values of the decision variables that satisfies the constraints and provides an optimal value of the objective function (Mays, 1999).

Water distribution systems are large and complex structures. Hence, their construction, management and improvements are time consuming and expensive. Nevertheless, for the last few decades a significant research effort was put into improving WDS efficiency and at the same time minimising their maintenance cost. However, determining an optimal solution for WDS design and operation is not an easy task as many factors need to be considered e.g. cost of materials, topographical layout, energy cost while meeting the consumer requirements such as service pressure and water quality. To add to the overall complexity due to fluctuating demands, the WDS operation needs to be constantly adjusted which usually is carried out with the use of control devices enabling different operational modes.

In the past, design and management of water distribution networks was based on engineer/operator experience. However, constantly increasing computing power resulted in number of computerized optimisation methodologies oriented to improve design and operation of WDSs. Starting from Linear Programming (Kessler and Shamir, 1989; Samani and Zanganeh, 2010), Dynamic Programming (Yakowitz, 1982; Cervellera *et al.*, 2006; Ulanicki *et al.*, 2007), Model Predictive Control (Skworcow *et al.*, 2010), Evolutionary Algorithms (Murphy *et al.*, 1993; Savić and Walters, 1997; Gupta *et al.*, 1999), Tabu Search (Lippai *et al.*, 1999; Cunha and Ribeiro, 2004; Tospornsampan *et al.*, 2007a), Simulated Annealing (Cunha and Sousa, 1999; Tospornsampan *et al.*, 2007b), Ant Colony optimization (Maier *et al.*, 2003), Shuffled Frog-Leaping (Eusuff and Lansey, 2003), Shuffled Complex Evolution (Liong and Atiquzzaman, 2004), Harmony Search (Geem, 2006), Scatter Search (Lin *et al.*, 2007), Particle Swarm Optimization (Montalvo *et al.*, 2008), Cross Entropy (Perelman *et al.*, 2008b), Dynamically Dimensioned Search (Tolson *et al.*, 2009), Honey Bee Mating optimization (Mohan and Babu, 2010), Cuckoo Search (Wang *et al.*, 2012) and many hybrids between them. A review of some aforementioned methods can be found in (Coelho and Andrade-Campos, 2014; Vilanova *et al.*, 2014).

Nearly all the optimisation methods, whether aimed for design or operation, suffer from a need for simulation models necessary to evaluate the performance of solutions to the problem. However, the simulation models are increasing in size and complexity as more water utilities adopted the approach, with use of GIS and SCADA systems, to include each component of a large system in a WDS model. For WDS design optimisation the large models are generally acceptable, as there are seldom any limits on the computing time available. However, for operational control purposes where there is a need to regularly update the control strategy to account for the fluctuations in demands, the combination of a hydraulic simulation model and optimisation is likely to be computationally excessive for all but the simplest of networks (Jamieson *et al.*, 2007).

This provoked a necessity for substitutes for these simulation models in order to provide a speed-up in the optimisation process. A number of different techniques were proposed to generate surrogates for the large models, e.g. (Saldarriaga *et al.*, 2008; Deuerlein, 2008; Broad *et al.*, 2005; Ulanicki *et al.*, 1996). Although, even with the help of those methods in many cases preparing an optimisation-ready model is a very laborious work requiring a deep understanding of network operation. Unfortunately, such a preparation process is often omitted in the literature or only briefly described.

Hence, one of the objectives of this work is to review the model reduction techniques, focussing especially on their potential application to operational optimisation of WDSs. The next section describes an optimisation method that initiated the need for such investigation that is carried forward to Chapter 3.

2.5 Real-time optimal operation of water distribution systems

Optimisation studies of medium and large-scale water networks are typically carried out offline. This means that any changes to the water network may require significant changes in the optimisation model, which leads to high cost of system maintenance.

In turn, real-time control strategies for operation of water distribution systems are constrained by the computational time required to find an optimal solution. A number of studies were carried out to reduce the computational time and thereby enable a real-time optimal control of WDS, e.g. (Rao and Salomons, 2007; Li and Baggett, 2007; Jamieson *et al.*, 2007; Shamir and Salomons, 2008; Pasha and Lansey, 2010; Skworcow *et al.*, 2010; Kang, 2014; Odan *et al.*, 2014). While some studies proposed to use the "warm solutions" (i.e. the most recent solution) in order to reduce the computational time, others researchers recommended use of surrogate or reduced models linked with the optimisation algorithm.

The latter approach was employed by Skworcow *et al.* (2010) in the study carried out in the WSS group. Skworcow *et al.* (2010) proposed an online optimisation methodology for a real-time energy and leakage management in water networks, formulated within a model predictive control (MPC) framework. The objective was to calculate control actions, i.e. time schedules for pumps, valves and sources, which minimise the costs associated with energy used for water pumping and treatment and water losses due to leakage, whilst satisfying all operational constraints. The control scheme proposed by Skworcow *et al.* (2010) is illustrated in Figure 2.11.

The model predictive controller computes the control actions based on the telemetry readings, provided by the SCADA systems, operational constraints, boundary conditions specified by operator and future demands predicted by the demand forecaster. Inclusion of the model reduction module enables automatic adaptation to abnormal situations and structural changes in a network, e.g. isolation of part of a network due to pipe burst. In such a case an operator can change the full hydraulic model and run the model reduction

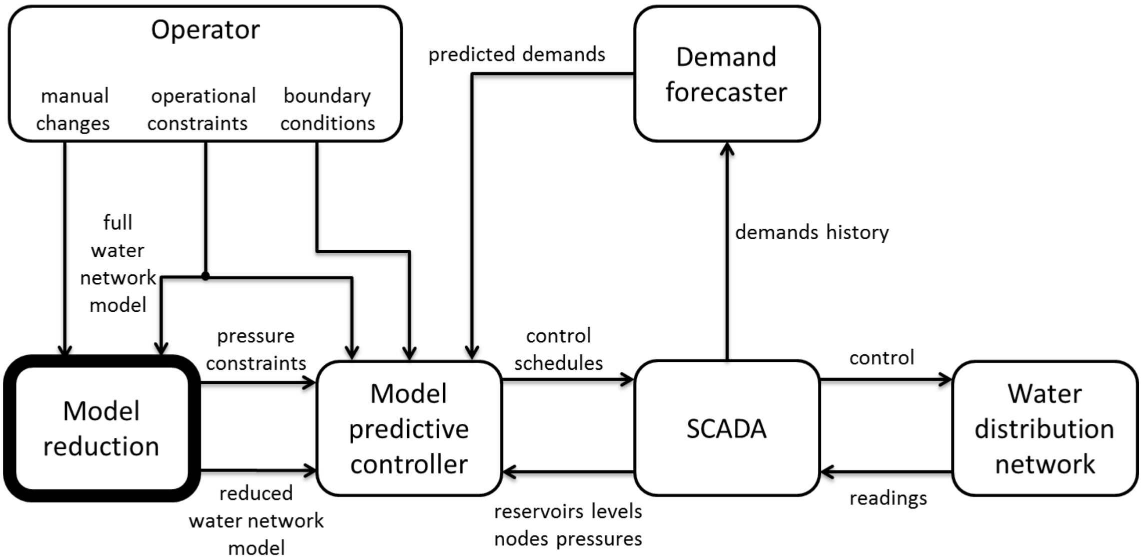


Figure 2.11: Model predictive control scheme for the online energy and leakage management (Skworcow *et al.*, 2010).

module to automatically produce the updated simplified model. In normal operation the simplified model is fixed.

The approach proposed in (Skworcow *et al.*, 2010) is model-based and water network models can consist of thousands of elements, each described by nonlinear equation; this along with the MPC algorithm computational complexity, created a need for simplified models. Also the optimisation methods, aforementioned in Section 2.4, require simplified models of WDS to perform thousands of simulations in order to evaluate solutions to the problem. Additionally, the most optimisation studies of WDS are carried out offline whereas in (Skworcow *et al.*, 2010) it was essential that the reduced model not only preserves the original water network nonlinearities but also it should be suitable for the online calculation.

Reduced water network model can be obtained in many ways. But, a technique chosen for such task will have to satisfy the major requirements gathered below:

- Reduced model should accurately replicate hydraulic behaviour of the original model.
- Algorithm should perform in sufficiently short time to allow an online adaptation to abnormal events and structural changes.
- Algorithm should keep a record how demand was aggregated and distributed in a reduced model.

- Algorithm should provide capability to retain components important from the operator perspective.

2.6 Summary

In this chapter the basic theory of WDS hydraulics has been presented. Although, in a limited and concise form it should provide the reader with a foundation for understanding the concepts and terms used in the remainder of this thesis. For a comprehensive reference on WDSs the reader may consult (Brdys and Ulanicki, 1994). Also, excellent references on WDSs modelling and simulation are Mays (1999) and Walski *et al.* (2003).

Whilst the literature review mainly concerns the principles of modelling, simulation and optimisation of WDSs some issues in those areas have been highlighted. Firstly, in Section 2.2 the different model structures, utilised to represent the nonlinear characteristics of WDSs, and specificities of their components have been presented. This included reservoirs, tanks, pumps, vales and pipes. The problems due to tank's dynamics and non-smoothness of directional and control valves have been described and typical difficulties highlighted. Few modelling and simulation techniques that account for these problems have been briefly reviewed.

In Section 2.3 aspects of WDSs simulation have also been considered. Whilst it has been established that simulation of a WDN allows to better understand behaviour of the system it has also been identified that simulation of devices with discontinuity attributes still poses a challenge in WDS analysis. If the purpose of WDSs modelling and simulation is to accurately represent hydraulic behaviour of the considered system then it is important to select an approach that accounts for both discrete and continuous characteristics of water networks. This is essentially the motivation behind this short review of the various modelling and simulation approaches. It is recognised, however, that to fully address this aspect of WDS simulation would represent a significant task in its own right. Nevertheless, the research conducted to address the aforementioned issues is summarised in Part II.

WDSs are complex and thereby require huge investments in their construction and maintenance. For these reasons, as has been shown in Section 2.4, a significant effort is put to improve their efficiency by way of minimizing their cost and maximizing the benefits accrued from them. However, optimisation of large and hydraulically complex WDSs is computationally expensive as thousands of simulations are required to evaluate the performance of candidate solutions. To minimise the optimisation search space, reduced (optimisation-oriented) models are utilised. But an initial review of different simplification approaches has risen a concern as to whether the simplified model required to simulate/replicate the behaviour of the original system allows to capture only the key nonlinear features or also the underlying dynamics characterising the system?

The major aim of Section 2.4 and Section 2.5 has been to aid the reader in understanding the formulation of the above research question. The further research and developments are continued in Part I and the corresponding appendices. It is considered important by the author of this thesis to highlight the impact of practical systems on the research work which is detailed here, as well as to reflect the constant motivation during this work to appeal and collaborate with various companies and industrial bodies in order to ensure the viability and applicability of the resulting concepts that are developed. Hence, Chapter 5 within Part I describes application of research outcomes to a real case study.

Part I

Reduction of water distribution system models for operational optimisation

Chapter 3

Energy balance in model reduction of water distribution systems

3.1 Introduction

The aim of this chapter is an investigation and development of a model reduction technique for the purpose of an online optimal strategy for WDS operation. Section 3.2 and Section 3.3 examine the model reduction methods available in the literature in the scope of utilisation for real-time optimal operation studies of WDS. Once the model reduction technique is chosen, its performance is evaluated in Section 3.4 on several models of water networks different in size and complexity. The attention is then directed to the problem of an inconsistent energy distribution in the reduced model that is exemplified in Section 3.5 which demonstrates that when node is removed from the network during the reduction process its elevation and pressure constraint are often not considered. This can cause a situation where the pump speed required to satisfy the minimum pressure constraints is different for the reduced model and the prototype. To alleviate this mismatch, a new extension to the model reduction algorithm based on the concept of energy audits is proposed in Section 3.6. The appropriateness of the new algorithm is initially demonstrated on a small hypothetical case study, and subsequently, in Section 3.7, is applied to a real water network. Section 3.7, beside the examination of the enhanced model reduction method, investigates its applicability in a study of determining optimal pump schedules. Section 3.8 summarises this chapter.

3.2 Model reduction methods

Nowadays, it is common that WDS models contain thousands of elements to accurately replicate hydraulic behaviour and topographical layout of real systems; e.g. see large-scale models in (Lippai, 2005; Ostfeld *et al.*, 2008). Such models are appropriate for the simulation purposes. But the optimisation tasks are much more computationally demanding, hence simplified models are required. Especially, in online optimisation frameworks such as in (Skworcow *et al.*, 2010) where an optimal solution has to be obtained within the defined time interval. There are different techniques of a WDS model reduction; the outcome of most of these methods is a hydraulic model with a smaller number of components than the prototype. The main aim of the reduced model is to preserve the nonlinearity of the original network and approximate its operation accurately under different conditions. The accuracy of the simplification depends on the model complexity, purpose of simplification and the selected method such as skeletonization (Walski *et al.*, 2003; Saldarriaga *et al.*, 2008; Iglesias-Rey *et al.*, 2012), parameter-fitting (Anderson and Al-Jamal, 1995), graph decomposition (Deuerlein, 2008), enhanced global gradient algorithm (EGGA) (Giustolisi and Todini, 2009), metamodelling (Rao and Alvarruiz, 2007; Broad *et al.*, 2010; Behandish and Wu, 2014) and variables elimination (Ulanicki *et al.*, 1996; Alzamora *et al.*, 2014; Paluszczyszyn *et al.*, 2013). This section examines these methods in a scope of utilisation for the online optimal operation of WDSs. Note that throughout this thesis terms reduction and simplification were used alternately to describe the process of achieving a hydraulic model with a smaller number of components than the prototype.

3.2.1 Skeletonization

Skeletonization can be defined as a reduction of data needed to represent the hydraulic performance of WDS without a significant loss of information (Hirrel, 2010). Skeletonization is not a standalone method, it is rather a process that utilises various techniques, often combined and/or applied in series to achieve a skeleton model. The widely-used techniques are: (i) inclusion in the skeleton model only the parts of the hydraulic network that have a significant impact on the behaviour of WDS (Walski *et al.*, 2003) e.g. removing all pipes that meet user-specified criteria such as diameter, roughness or other attributes, (ii) demand aggregation for neighbouring nodes; i.e. demands for nodes with similar pressure level are added and pipes between them are removed, (iii) use of equivalent pipes in place of numbers of pipes connected in parallel and/or in series. In Figure 3.1 a graphical representation of skeletonization techniques is presented.

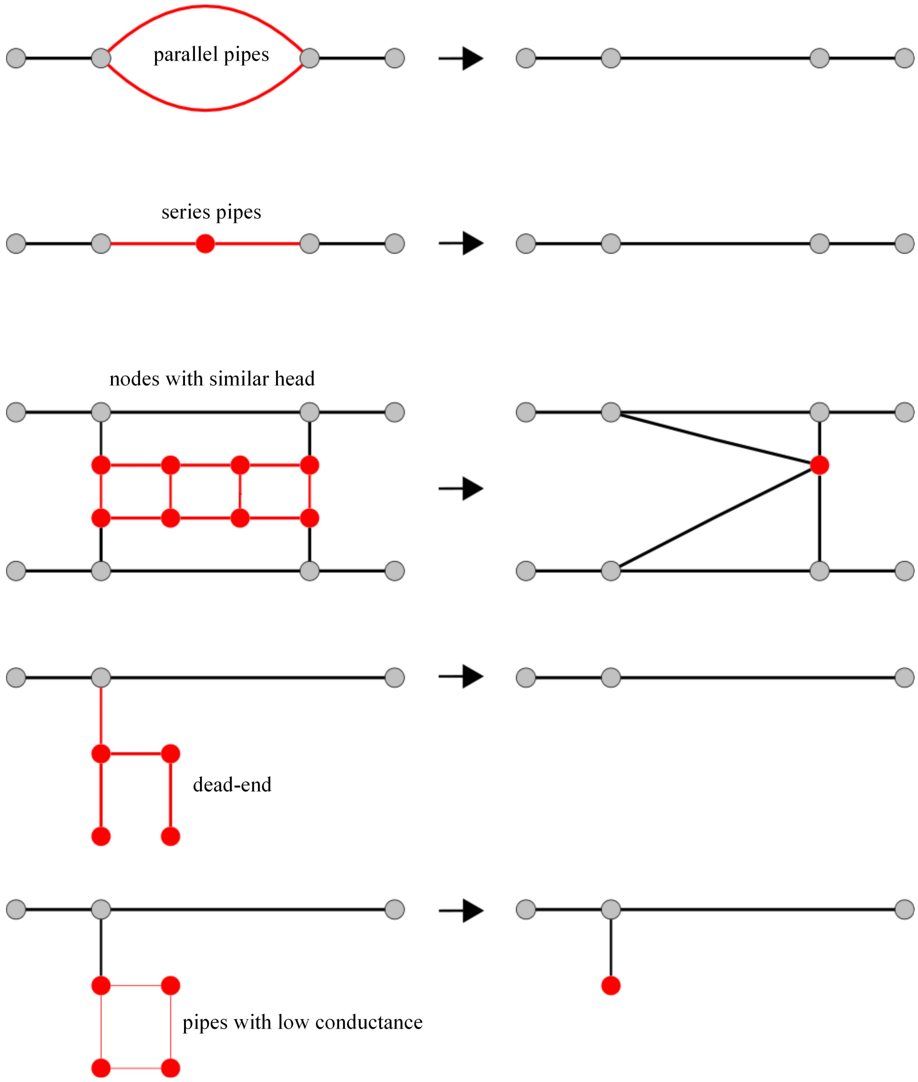


FIGURE 3.1: Illustrating different skeletonization techniques. Adapted from (Maschler and Savić, 1999).

In (Bahadur *et al.*, 2006) authors combined the following techniques: selection of pipes with certain diameter, replacement of identical pipes in series and removal of dead-end pipes. Although, the article is focused on investigation of the skeletonization impact on the water quality parameters, Bahadur *et al.* (2006) observed and highlighted issues with the demand re-allocation when performing skeletonization; i.e. consequence of re-allocation of system demands was a change in flows and velocities within the skeleton model.

Saldarriaga *et al.* (2008) presented an automated skeletonization methodology to obtain reduced models of WDS that can accurately reproduce both, the hydraulics and non-permanent water quality parameters (chlorine residual) of the original model. The proposed methodology was based on the resilience index concept introduced by Todini (2000); i.e. by using the resilience index as selection criterion to remove pipes from the prototype, reduced models that simulate the hydraulics of the original models were achieved. However, the method is focused on the pipes removal only and thereby it can be mainly applied for looped pipe networks. Moreover, the achievable degree of model reduction is not significant if pressure levels in the skeleton model are to be reflected accurately.

Another method of automatic skeletonization is described in (Hirrel, 2010). The method combines skeletonization with graph traversal algorithms. The graph traversal process is used to: (i) obtain flow balance and head at each node, (ii) identify all the paths in the network and (iii) determine a head error and a total q-prime for each path. According to Hirrel (2010) the head error is a difference between the calculated and theoretical summation of head changes along the path. The q-prime is a slope of hydraulic grade line versus flow relationship for a given pipe. Subsequently, the total q-prime for a path is the sum of q-primes for all pipes in the particular path. Having obtained head error and total q-prime, a correcting flow, to be applied to all pipes in the path, is calculated. In general, the method is aimed to minimise hydraulic computations thanks to network topology manipulation; i.e. every time that pipe is removed through the skeletonization a new set of head changes and q-primes is respectively determined and stored taking into account whether the pipe was in series, parallel or in a loop. Such approach allows to reduce the amount of data needed to simulate the performance of a hydraulic network. In the example in (Hirrel, 2010) a number of junctions was reduced from 4210 to 2658 and number of pipes from 3003 to 1625, and based on the used criteria the both solutions were almost identical. However, as Hirrel (2010) pointed out, the scope of reduction is network-specific and depends on a number of pipes that can be serialized or number of parallel pipes that can be combined.

Jiang *et al.* (2012) applied skeletonization techniques to a large-scale network model consisting of 600000 pipes and 550000 nodes. Due to size of the network, the model was initially split up into 10 sub-models, which after skeletonization, were subsequently merged back into simplified version of the original model. Jiang *et al.* (2012) employed branch collapsing, series and parallel pipe merging for three different pipe diameters thresholds of 300, 500, and 800 mm. The thresholds were used to determine which pipe in branches of the network will be retained; e.g. threshold of 300 mm retains all the pipes with diameter $\geq$300 mm in branches. However, the pipes in loop configuration with diameter $<$300 mm will be retained. Jiang *et al.* (2012) presented results from skeletonization for one of the sub-models, which contained 48660 pipes and 40178 nodes. The reduced model with pipe diameter threshold of 300 mm contained 9901 pipes and 8113 nodes. When threshold was set to 800 mm the scale of reduction was even greater; the reduced model consisted of 3699 pipes and 2538 nodes. However, Jiang *et al.* (2012) stated that such significant reduction affected ability of the reduced model to reflect the hydraulic behaviour of the original model. Only the reduced model with threshold of 300 mm was accurate enough, in terms of used criteria, to be retained. The reduced model obtained for thresholds of 500 and 800 mm were dismissed and categorised as inaccurate.

In most cases skeletonization is performed manually and accuracy of skeletonization is depended on experience and engineering judgement of modeller. One of the first commercial automatic skeletonization solution was the Skelebrator module from the Bentleys Haestad Methods WaterGEMS software (Bentley, 2013b). Skelebrator automatically skeletonizes water distribution systems using a combination of the following techniques: data scrubbing, branch trimming, series pipe removal and parallel pipes removal. According to (Bentley, 2013a) Skelebrator allowed to the city of Toronto to save over one million Canadian dollars by reducing complexity of physical network contained 307956 pipes to a smaller network with one fourth the pipes, 76989.

Number of studies were conducted in order to assess skeletonization in water networks, see (Eggener and Polkowski, 1976; Hamberg and Shamir, 1988; Cessario, 1995; Grayman and Rhee, 2000; Bahadur *et al.*, 2006; Jung *et al.*, 2007a; Cantone, 2007; Cantone and Schmidt, 2009), and, in short, the following observations can be concluded. Skeletonization is not a single process but several different low-level element removal processes that sometimes must be applied in series. Whereas for some small water networks, including mostly pipes, an accurate skeletonization is possible, for a large networks with complex topology, consisting of tanks, pumps and varied demands, a lossless skeletonization presents a challenge. Also, scope of model reduction via skeletonization is not significant if skeleton model is to preserve hydraulic behaviour of a prototype. Another major drawback is that accuracy and scope of skeletonization is network-specific what makes it very difficult to establish a set of fixed rules for skeletonization, especially in online time-constrained WDS optimisation studies.

3.2.2 Parameter fitting approach

In (Anderson and Al-Jamal, 1995), authors proposed a method for hydraulic network simplification using a parameter-fitting approach. The authors' motivation was to use simplified model in operational studies of WDS rather than for network design purposes.

The method proceeds as follows. Based on an expert knowledge of network operation the topology of the simplified network is assumed in advance. Next, the pipes conductances and the demand distribution in the simplified model are determined by minimising a measure of the difference in terms of flows between the full and simplified models performances. The objective function for the optimisation problem is minimised using a nonlinear optimization technique. The proposed objective function allows to put weights on network elements that required to be modelled more accurately than others.

Anderson and Al-Jamal (1995) claimed that they were able to reduce a model for Maiden Lane zone in London containing 66 nodes and 112 links to 3 nodes and to 4 nodes with errors in flow values within 5% of the values in the full model. But, they also admitted that there are difficulties with the use of this method when non-return valves are present in the system. Additionally, the method was tested on relatively small water network model and, as the authors observed, when the number of nodes in a simplified network increases the occurrence of multiple local minima becomes more likely what questions applicability of this technique for reduction of complex WDSs. The scope and accuracy of simplification in this method is hugely depended on the experience of the engineer, who is determining the initial layout of the simplified network. Hence, the method cannot be applicable in automatic reduction of WDS models but it can be used as additional simplification procedure to increase the accuracy of the already reduced model.

3.2.3 Graph theory based approach

Due to its nature it is common to represent a water distribution system using graph theory as was shown in (Kesavan and Chandrashekar, 1972; Gupta and Prasad, 2000). A water distribution system can be modelled as a graph $G(V, E)$ with the set of vertices/nodes, V, representing the water sources and consumption nodes, and the set of edges, E, representing pipes, pumps and valves. Number of techniques from graph theory discipline were utilised in WDSs analysis and modelling for different purposes: in prediction of contamination spread (Davidson *et al.*, 2005), sectorization (Tzatchkov *et al.*, 2006), state estimation (Kumar *et al.*, 2008), operational monitoring and control (Arsene *et al.*, 2012a), reliability assessment (Wagner *et al.*, 1988; Fragiadakis *et al.*, 2013), vulnerability and robustness analysis (Yazdani and Jeffrey, 2010, 2012), sensors placement (Deuerlein *et al.*, 2010; Perelman and Ostfeld, 2013), district metering area (DMA) identification (Ferrari and Becciu, 2012), design optimization (Zheng *et al.*, 2013). However, the following graph-theoretic applications in WDSs analysis are oriented towards methods to represent water networks in more simplistic way and yet retain their hydraulically complex behaviour.

Recently, the problem of WDSs security elicited an interest from researchers. To investigate the impact of water contamination and to prevent such events the water distribution system is subjected to a process called sectorization. The process aim is to identify or divide WDS into isolated zones (sectors) where each zone is being supplied from its water source. In (Tzatchkov *et al.*, 2006) authors employed graph traversal algorithms such DFS (Tarjan, 1972) and breadth-first search (BFS) (Pohl, 1969) to identify such isolated zones. Although, the sectorization process in (Tzatchkov *et al.*, 2006) is not aimed at reduction the number of elements in WDS, it is rather focused on achieving a better understanding of a particular WDS. It is easier for engineer to visually assess performance of WDS by sectors whereas analysis of a complex WDS with thousands of interlinked elements presents a challenge.

In (Di Nardo *et al.*, 2011), and recently updated in (Di Nardo *et al.*, 2013), enhancements to sectorization were proposed. The method combined hydraulic simulator, graph theory tools, and a specially developed genetic algorithm to take into account energy distribution in WDS. Firstly, a graph of water supply system with weighted edges (links) and vertices (nodes) is created, which subsequently is partitioned into user-defined number of sectors. Di Nardo *et al.* (2013) used performance indices such network resilience (Prasad and Park, 2004), resilience deviation index (Di Nardo *et al.*, 2013) and pressure levels at nodes to define appropriateness and accuracy of sectorization. The sectorization was tested on two case studies, which confirmed the effectiveness of the methodology in terms of employed performance indices.

Another methodology of WDS partitioning, namely clustering, that utilised graph theory principles was presented in (Perelman and Ostfeld, 2012). Using the DFS and BFS graph algorithms a water distribution system is divided into strongly and weakly connected subgraphs i.e. clusters. The algorithm resulted in a connectivity matrix that can represent the interconnections between clusters. The authors' motivation was to provide a tool that can support a response modelling plan in case of a contamination event.

In (Deuerlein, 2006) and (Deuerlein, 2008) a graph-theoretical decomposition concept of the network graph of WDS was proposed. Whereas the previous graph-based works were aimed to be applied in water network security applications the methodology in (Deuerlein, 2008) is more versatile. Deuerlein (2008) listed several potential applications such as network connectivity analysis, identification of supply areas, water network simplification, modified simulation and sensitivity, reliability, vulnerability of WDSs. In general, the approach involves several graph decomposition steps to obtain a block graph of WDS. Graph algorithms were employed to identified branched and looped sub-graphs to achieve subsequent stages of model simplification; core of the network, core network with paths only and block graph. During that process demands of the root nodes are increased by the total demand of the connected trees to ensure that the simplified network replicates the hydraulic behaviour the total network.

Although, Deuerlein (2008) provided an example of WDS simplification but in a form of graphical illustration not reinforced with numerical data which would allow to asses the hydraulic accuracy of simplification by graph decomposition. It is also difficult to assess calculation complexity of this approach without description of its implementation framework.

3.2.4 Enhanced global gradient algorithm

In (Giustolisi *et al.*, 2010) and (Berardi *et al.*, 2010) authors described how a water network simplification can be achieved by the EGGA (Giustolisi and Todini, 2009). The EGGA simplifies the network topology by automatically removing serial nodes i.e. nodes with only two neighbours. Such approach will then reduce the total number of nodes and pipes. Typically in such skeletonization technique only the mass balance is considered when distributing demand from the removed consumption nodes but in (Giustolisi *et al.*, 2010; Berardi *et al.*, 2010) an energy balance was also taken into account. It was done by introducing a hydraulic resistance correction factor into the head loss expression.

In (Giustolisi *et al.*, 2010) the method performance was demonstrated on the water network composed of 5 tanks, 1461 nodes and 1991 pipes. Giustolisi *et al.* (2010) achieved a 39% reduction, as the simplified network was composed of 5 tanks, 783 nodes and 1313 pipes. However, the main benefit of EGGA approach is that it preserves mass and energy balance of the original model thus the simplified model very accurately resembles the full model. The maximum difference of nodal heads was less than 0.0005m. Moreover as EGGA can replace the global gradient algorithm (GGA) as a WDS simulation paradigm; the simplified model will replicate the original model with a high accuracy for the whole period of a considered simulation.

In more comprehensive studies in (Berardi *et al.*, 2010) the EGGA approach was exemplified on a large-scale water network from (Ostfeld *et al.*, 2008). Albeit, a successive skeletonization step was introduced to increase the scope of reduction. Berardi *et al.* (2010) added also a feature of detection of sub-systems in the simplified network which are connected to each other by one pipe only that could be used for placement of isolation valves.

Despite that when using the EGGA the hydraulic accuracy of a model is not forfeited, the scope of the reduction might be very small as it depends on the number of serial nodes in water network. Hence, especially for looped networks, the number of elements that can be removed will be very small.

3.2.5 Metamodeling

The objective of metamodeling is to develop and utilise computationally more efficient surrogates of high-fidelity models mainly in optimisation frameworks (Razavi *et al.*, 2012b). Over the past decade metamodelling was used by researchers in water resources, see (Razavi *et al.*, 2012b) and references therein. In water resources, a metamodel serves as a surrogate or substitute for the more complex and computationally expensive simulation model (Broad *et al.*, 2005). Number of tools and techniques were developed to obtain a metamodel but it is noticeable that use of artificial neural network (ANN) was in particular interest.

In (Jamieson *et al.*, 2007) a feasibility study is provided to determine whether real-time, near-optimal control for water distribution network can be achieved. Jamieson *et al.* (2007) encountered similar difficulties as were identified in (Skworcow *et al.*, 2010); i.e. impracticability of the conventional full-scale hydraulic simulation model for online optimal control. To overcome this obstacle Jamieson *et al.* (2007) proposed the use of ANNs to capture the hydraulic behaviour of a full-scale simulation model. Subsequently, such an ANN model is the input to genetic algorithm (GA) designed specifically for a real-time use. The proposed methodology, a combination of ANN and GA, was applied to three case studies described in (Salomons *et al.*, 2007; Martínez *et al.*, 2007; Rao and Salomons, 2007).

The next paragraphs, however, are focused on the utilisation of ANNs to replicate hydraulic simulation model. This approach is described in greater detail in (Rao and Alvarruiz, 2007). An ANN is a nonlinear mathematical structure, which is capable of representing arbitrarily complex, nonlinear processes that relate the inputs and outputs of any system (Rao and Alvarruiz, 2007). The use of ANNs is well established in water resources, especially in prediction of demands, see (Maier and Dandy, 2000) and references therein. In general, an ANN is an interconnected set of neurons which are usually organised in interlinked layers. Based on the input data, the weights of neuron connections are iteratively adjusted by an algorithm in such a way that the output performance of the network is improved. ANNs are widely recognized for their accuracy and robustness. They require, however, a significant number of input/output patterns (often measured in thousands) for training and testing, what limit their use in large projects due to the computational burden.

Despite this limitation, Rao and Alvarruiz (2007) decided to capture the hydraulic characteristics of a WDS by means of ANNs. The approach requires a large number of simulations of the hydraulic model to generate different combinations of initial tank-storage levels, demands, pump and valve settings. These input/output data sets are then used to train and validate the ANN model. Root mean square error (RMSE) criteria was used to measure "goodness-of-fit" between the hydraulic simulation model and the obtained ANN model. Such an ANN model can be used to predict future tank-storage water levels, pressures and flow rates at critical points throughout the network.

To validate appropriateness, Rao and Alvarruiz (2007) applied this approach to the modified 'Any Town' water network from (Walski *et al.*, 1987). The results were presented in a form of plots of the pressure deviations on the selected node (most of results from the testing sets were in range of ± 0.1m) and the water level deviations for the chosen tank (most of results from the testing sets were in range of ± 0.03m). Unfortunately, from (Rao and Alvarruiz, 2007) one cannot determine the time complexity of this methodology, what can be significant knowing computational demands of ANN. Also, despite the relatively small network (41 pipes, 19 nodes) Rao and Alvarruiz (2007) had to address few issues such as the number of neurons in the hidden layer, the number of training sets for ANN and whether to use of a separate ANN for each of the output variables. Additionally, reader can also be concerned how to select critical elements for the input and output layers. The number of such elements or inclusion of the variable-speed pumps will increase the number of variables what in turn would have a severe impact on the computational time. Moreover, in the absence of complete information about the modelling process, the optimality of the results cannot be assessed, and it is difficult to draw meaningful conclusion about the performance of the proposed ANN metamodeling.

Indeed, similar issues were reported in (Martínez *et al.*, 2007), where the same ANN approach was used in optimisation of the operation of the real water distribution system of Valencia city in Spain. According to Martínez *et al.* (2007) the number of neurons in the hidden layer is somewhat subjective and usually based on experience, coupled with a degree of experimentation. And also determining the appropriate architecture of the ANN proved to be sometimes a frustrating procedure based on trial-and-error. The above can introduce many uncertainties into the ANN representation of the hydraulic model and, as noticed in (Broad *et al.*, 2010), even a small error in metamodel can have a significant impact on the obtained optimisation results.

Furthermore, the approach proposed by (Rao and Alvarruiz, 2007) has another disadvantage that eliminates use this technique for the control strategy described in Section 2.5. Bowden *et al.* (2012) observed that ANNs are unable to reliably extrapolate beyond the calibration range. Consequently, when deployed in real-time operation there is a need to determine if new input patterns are representative of the data used in calibrating the model. Truly, Rao and Salomons (2007) reported that in the event of disruption such as a power outage, pump failure, jammed valve or pipe burst the ANN model would require re-training what obviously increases drastically the computational time.

More comprehensive and detailed studies in using the ANN metamodels for design and operation optimisation of WDS were conducted in (Broad *et al.*, 2005) and (Broad *et al.*, 2010), respectively. The works in (Broad *et al.*, 2005) and (Broad *et al.*, 2010) are in principle very similar to the aforementioned studies in (Rao and Alvarruiz, 2007; Rao and Salomons, 2007; Martínez *et al.*, 2007). Although, both Broad *et al.* (2005) and Broad *et al.* (2010) provided much more details and numerical results that allow to assess use of ANN models to replicate behaviour of hydraulic simulation model. Additionally, Broad *et al.* (2010) in an explicit manner addressed the issue of selection of critical points for ANN model layers.

The main aim of (Broad *et al.*, 2005) was a WDS design optimisation. For this purpose authors combined ANN metamodeling with GA optimisation. The case study used to test the approach, was the well-known and popular network of New York Tunnels (NYT). While the obtained optimal solutions (similar to other works which considered the NYT case study) proved the suitability of the method, Broad *et al.* (2005) observed several issues in regard to the process of ANN metamodeling. A representative ANN needs a training data to be generated from a range of different values and for different types of variables, what became the greatest computational burden in developing the metamodel. The training time for the ANNs developed for NYT was 16 hours. This time was elapsed for only 5 critical nodes, and in case of more complicated WDS with 20 or 30 critical nodes, with a separate ANN trained for each of these nodes, the computational time might be excessive, to the point where there might be no net benefit in using a metamodeling strategy (Broad *et al.*, 2005). Additionally, values for ANN parameters such as number of hidden layers or number of hidden neurons were selected by trial and error what makes ANN metamodeling inappropriate for automatic processes. Furthermore, in (Broad *et al.*, 2005) it was observed that trained ANN are sensitive to initial weights in terms of accuracy of replicating the original model. Finally, Broad *et al.* (2005) concluded that it is unlikely that an ANN could be trained such that a perfect approximation of a Epanet2 model is obtained.

In (Broad *et al.*, 2010) the work of Broad *et al.* (2005) was extended and aimed at optimal operation of WDS. Among many enhancements to the method in (Broad *et al.*, 2005) the proposed methodology was applied to the real and fairly complex case study comprised of 1730 nodes, 2097 pipes, 10 pumps, 35 valves and 4 tanks. For such complex model, immediately a problem occurred; the time required to generate the adequate training sets ANN might be so long that additional techniques were considered such as skeletonization in order to reduce complexity of the initial model. Despite that Broad *et al.* (2010) addressed a problem of critical points selection, the previously described issues associated with the ANN metamodeling remained unsolved. However, Broad *et al.* (2010) claimed that inaccuracies in the ANN metamodel were compensated by additional correction steps at optimisation stage, but this makes the described ANN metamodeling appropriate to the proposed GA optimisation only and thereby it could not be combined with other optimisation techniques. And still it is doubtful that this improved ANN metamodeling can be applied in real-time optimisation as it required an engineering judgement at some stages. Although, authors achieved satisfactory solutions indicating a potential savings of 14% in combined pumping and chlorine costs for the considered case study, a further work needs to be conducted in order to address computational issues related to ANN metamodel development what in case of (Broad *et al.*, 2010) took 320 h.

Odan *et al.* (2014) utilised metamodels in real-time operation of water distribution systems. In contrast to aforementioned works based on ANN Odan *et al.* (2014) employed a self-adaptive ANN. The adaptive merging growing algorithm (AMGA) was used to calibrate a single hidden layer neural network. The AMGA algorithm merges and adds neurons of the hidden layer based on the progress and learning of the neurons of that same layer and, at the same time, and thereby avoids the trial and error process to determine the appropriate number of neurons of the hidden layer. The AMGA-based procedure was earlier evaluated in (Odan and Reis, 2012) on two small size water networks. While the obtained metamodels were able to accurately replicate the considered networks in terms of normalised root mean squared error (RMSE), the time spent on the metamodel calibration was significant; i.e. for the model with 19 nodes, 1 reservoir, 3 tanks, 41 pipes and 3 pumps the calibration took 100 minutes. Therefore, the optimisation strategy proposed by Odan *et al.* (2014) when initially designed for a fixed-topology network may require a prolonged metamodel recalibration when the network topology is about to change due to abnormal events such as a power outage, pump failure, etc.

Taxonomies on metamodeling frameworks, practical details, advances, challenges, and limitations are outlined by Razavi *et al.* (2012a) and Razavi *et al.* (2012b). The conclusions drawn there are similar to the observed in the references cited and discussed in this section.

While the purpose of using a metamodel is to reduce computational burden of optimisation the time demanding process of the metamodel development makes it not suitable for real-time water network optimisation where adaptation to abnormal structural changes is required. Also, numerical results in (Razavi *et al.*, 2012a) demonstrated that metamodelling is not always an efficient and reliable approach to optimizing computationally intensive problems. For simpler models, metamodelling can be very efficient and effective. However, for complex models when computational budget is not very limited, metamodelling can be misleading, and better solutions were achieved with optimizers not involving metamodels. The results in (Razavi *et al.*, 2012a) also demonstrated that ANN are not appropriate metamodelling tools for limited computational budgets. And indeed, Shamir and Salomons (2008) in similar studies to those in (Rao and Salomons, 2007; Martínez *et al.*, 2007), instead of ANN to mimic the behaviour of the simulation model used another WDS model reduction technique called variable elimination (Ulanicki *et al.*, 1996).

3.3 Variable elimination algorithm

The approach of variables elimination is based on a mathematical formalism initially presented in (Ulanicki *et al.*, 1996) and recently updated in (Alzamora *et al.*, 2014). This mathematical method allows a reduction of water network models described by a large-scale system of nonlinear differential algebraic equations. The approach is illustrated in Figure 3.2 and proceeds through the following steps: full nonlinear model formulation, model linearisation at specified operation time, linear model reduction using Gaussian elimination and nonlinear reduced model reconstruction. The approach was successfully implemented and tested on many water networks (Maschler and Savić, 1999; Rance *et al.*, 2001; Bounds *et al.*, 2006; Perelman and Ostfeld, 2006, 2008; Shamir and Salomons, 2008; Perelman *et al.*, 2008a; Preis *et al.*, 2009; Skworcow *et al.*, 2010; Preis *et al.*, 2011; Skworcow *et al.*, 2013); see Table 3.1. Especially real-time oriented studies in (Shamir and Salomons, 2008) and (Preis *et al.*, 2009) indicate that variable elimination can be effectively applied for real-time applications. The number of successful applications and automatic nature of this method clearly meets the real-time optimisation requirements outlined in Section 2.5.

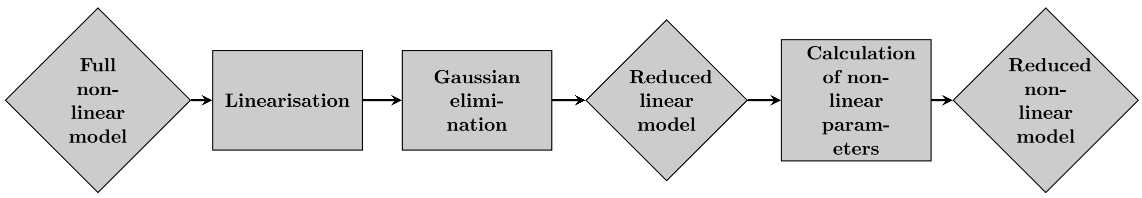


FIGURE 3.2: The variable elimination algorithm.

TABLE 3.1: Applications of variable elimination in the water resources literature.

Work by	Application	Original model statistics		Reduced model statistics		Scope of reduction	Framework
		Nodes	Pipes	Nodes	Pipes	%	
Maschler and Savić (1999)	WDS model reduction	1091	1295	134(*)	300(*)	81.81	Off-line
		500	450	15(*)	20(*)	96.32	Off-line
Bounds *et al.* (2006)	Optimal WDS operation	4248	4388	252	414	92.29	Off-line
Perelman and Ostfeld (2006)	Contamination detection	16	34	6	16	56.00	Off-line
		16	34	8	28	28.00	Off-line
Perelman and Ostfeld (2008)	Water quality analysis	36	41	11	13	68.83	Off-line
		9	12	7	9	23.81	Off-line
		92	117	36	56	55.98	Off-line
Shamir and Salomons (2008)	Optimal WDS operation	867	987	77	92	90.88	On-line
Perelman *et al.* (2008a)	Water quality simulations	14945	16056	8173	8995	44.62	Off-line
Preis *et al.* (2009)	Hydraulic state estimation	12523	14822	347	1100	94.71	Off-line
Preis *et al.* (2011)	Hydraulic state estimation	12523	14822	347(*)	1100(*)	94.71	Off-line
Skworcow *et al.* (2010)	Optimal WDS operation	2074	2212	44	42	97.99	On-line
Alzamora *et al.* (2014)	WDS model reduction	3535	3279	1023	1340	65.32	Off-line
Skworcow *et al.* (2013)	Optimal WDS operation	12363	12923	164	336	98.02	Off-line

(*) One of many obtained solutions.

As it was decided to use the variable elimination approach to reduce water distribution network models, its description is given in a greater detail. Note that some of the following definitions were already given in Chapter 2, but they are repeated here for the completeness of algorithm formulation. The algorithm proposed by Ulanicki *et al.* (1996) proceeds as follows:

Formulate a full nonlinear model

The mathematical model of water network used for model reduction procedure is in the nodal form and based on the node-branch incidence matrix Λ. The matrix Λ size is $n \times l$, where n is the number of nodes and l is the number of links. The element $\Lambda_{(n,l)} = 1$ if the link l is leaving the node n and $\Lambda_{(n,l)} = -1$ if link l is entering the node n. All other entries are 0. The mathematical model of a water network using the incidence matrix Λ is defined as follows:

$$\Lambda \mathbf{q} = \mathbf{q}^{nod} \tag{3.1}$$

$$\Delta \mathbf{h} = \Lambda^T \mathbf{h} \tag{3.2}$$

$$\mathbf{q} = \mathbf{\Phi}(\Delta \mathbf{h}) \tag{3.3}$$

where $\mathbf{q}$ denotes the vector of branch flows, $\mathbf{q}^{nod}$ is the vector of nodal demands, reservoir flows and source flows, $\mathbf{h}$ is the vector of node heads, $\Delta\mathbf{h}$ is the vector of branch head losses and $\mathbf{\Phi}(\Delta h) = (q_1(\Delta h_1), ..., q_{NL}(\Delta h_{NL}))$ is the vector of functions defining flow-head relationship, using Hazen-Williams formula, for each branch. By substituting Equations 3.1,3.2,3.3 the nodal model of a water network is obtained:

$$\Lambda \mathbf{\Phi}(\Lambda^T \mathbf{h}) = \mathbf{q}^{nod} \tag{3.4}$$

Linearise nonlinear model

The full nonlinear water network model described with Equation 3.4 is subject to linearisation around a user-specified operation point defined by the head h^0 and demand $q^{nod,0}$. The linearisation described the relationship between small changes in the head δh and flow δq^{nod} around the specified operation point. This leads to the linearised model in the following form:

$$\Lambda \frac{d\mathbf{\Phi}}{d\Delta\mathbf{h}} \Lambda^T \delta h = \delta q^{nod} \tag{3.5}$$

where

$$\Lambda \frac{d\mathbf{\Phi}}{d\Delta\mathbf{h}} \Lambda^T \tag{3.6}$$

is the symmetric $(n \times n)$ Jacobian matrix $\mathbf{J}$. The non-diagonal elements of $\mathbf{J}$ are

$$J_{n,m} = \begin{cases} 0.54\ g_{n,m}|h_n^0 - h_m^0|^{-0.46} = \tilde{g}_{n,m}, \text{for } m \in M \\ 0, \text{for } m \notin M \end{cases} \tag{3.7}$$

where M is the set of nodes connected to the node n and $\tilde{g}_{n,m}$ is the linearised link conductance. The diagonal elements of $\mathbf{J}$ are the linearised node conductances

$$\tilde{g}_n = \sum_{m \in M} 0.54\ g_{n,m}|h_n^0 - h_m^0|^{-0.46} = \sum_{m \in M} \tilde{g}_{n,m} \tag{3.8}$$

which is the sum of linearised conductances of all the links connected to the node n.

Reduce linear model using Gaussian elimination

Next, the linearised model in Equation 3.5 is reduced by means of Gaussian elimination procedure (Hammerlin and Hoffmann, 1991). Note that the Gaussian elimination is applied to the demand vector for nodes δq^{nod} as well. During this process the node n is removed from the linear model; the demand of this node is distributed to the neighbouring nodes proportionately to the conductance of the connecting pipes. Also, the pipes connecting the node n and its neighbours are removed. Finally, the new linear conductances are recalculated.

Retrieve reduced nonlinear model from the reduced linear model

The reduced linear model retains the properties of the full linear model in the form of reduced Jacobian matrix J^S. The information contained in the simplified J^S along with the mapping between the full nonlinear model and the full linear model allows calculation of nonlinear parameters for the simplified linear model. This property of preservation of nonlinearities of the input model enables to approximate the input model in a wide range of operating conditions.

Detailed illustration of the variable elimination on example water networks can be found in (Maschler and Savić, 1999) and (Alzamora *et al.*, 2014).

Over the time, as can be seen in Table 3.1, the variable elimination was applied in many applications. This resulted in adaptations and improvements introduced for needs of the particular application. Maschler and Savić (1999) tested and added pipe conductance threshold to their implementation to remove low importance pipes. Perelman and Ostfeld (2006) and Perelman and Ostfeld (2008) utilised DFS and BFS, graph-theory algorithms, to identify strongly connected components in order to select critical nodes from water quality perspective and define the order when removing nodes. Preis *et al.* (2009) and

Preis *et al.* (2011) used different pipe diameters as a parameter to define the scope of reduction.

3.4 Initial model reduction results

Prior the model reduction technique can be included into the control scheme described in Section 2.5 it needs to be demonstrated that it is capable to accurately reduce various water distribution networks within a specified time interval. Such precaution was also taken to gain experience of difficulties that might be encountered in applying the methodology to a real network. The following sections investigates the hydraulic accuracy of the selected model reduction method whereas Chapter 4 focuses on the difficulties and computational aspects encountered during the implementation.

3.4.1 Measures of quality of reduced model

It is essential that model accuracy is maintained; to validate how a reduced model replicates the hydraulic behaviour of original model a number of metrics and tools were adapted or designed. It should be noted that none of the following evaluation techniques is fully conclusive in itself and it is recommended to use at least both statistical and graphical means of models comparison. Moreover, some of the criteria are focused on comparison of the most important elements of a hydraulic model such as tanks or pumps.

- **visual evaluation**
  Graphical techniques in forms of time-series plots and plots of residuals, as can be seen in example plot in Figure 3.3, are essential in reduced model evaluation as they provide the first and often comprehensive overview of model performance.

- **mean relative error (MRE) and mean weighted relative error (MWRE)**
  Hydraulic performance is best described by flow values on links and pressure values at nodes (Anderson and Al-Jamal, 1995). Therefore the mean relative error (MRE), utilised inter alia in the calibration study in (Takahashi *et al.*, 2010), compares flow and pressure series from original and simplified model. The MRE is calculated as follows

$$\mathrm{MRE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{o_i - s_i}{o_i}\right| \times 100\% \tag{3.9}$$

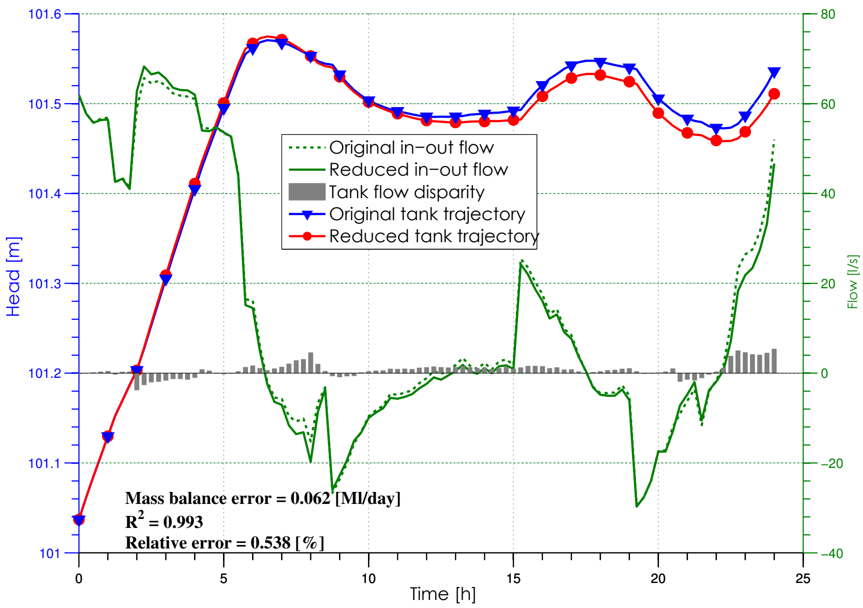


FIGURE 3.3: An example of a typical plot used for the visual inspection of the simplification algorithm accuracy.

where o_i is either the pressure or flow in the original model, s_i is the corresponding pressure or flow in the simplified model, i is the i-th sample of series and n is the number of samples. The MRE is used mostly in comparisons of flows and pressure in critical elements. Albeit, to take into account junctions or links with greater importance to model hydraulic performance the mean weighted relative error (MWRE), defined in (Takahashi *et al.*, 2010), can be employed:

$$\mathrm{MWRE} = \frac{\sum_{j=1}^{n_{\mathrm{MRE}}} p_j \mathrm{MRE}_j}{\sum_{j=1}^{n_{\mathrm{MRE}}} p_j} \tag{3.10}$$

where MRE_j is the mean relative error for series j and n_{MRE} is the number of series available. The weighting factor p_j can be used to indicate importance of series j to the overall hydraulic performance of a model.

- **mean absolute error (MAE)** and **RMSE**
  Although, MRE is a useful and practical indicator in model evaluation, sometimes, due to its 'relativity' and unitless expression it might be misinterpreted; i.e. producing large error when comparing very small values. Therefore, to aid in analysis of the

results, indices that indicate error in the units (or squared units) are included. These are the mean absolute error (MAE) and the RMSE in which values of 0 indicate a perfect fit.

$$\mathrm{MAE} = \frac{1}{n}\sum_{i=1}^{n}|o_i - s_i| \tag{3.11}$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(o_i - s_i)^2} \tag{3.12}$$

- **mass balance error (MBE)**

  One important measure of accuracy is connected with the flows from different sources within the system (Anderson and Al-Jamal, 1995). While the model reduction technique, employed in this work, ensures that total demand in reduced model is unaffected, the inflows from sources to satisfy this demand might vary. For this reason, the mass balance error (MBE), shown in Equation 3.13, is used to compare the reservoirs and tanks flow balance in the original and the reduced model. The flow is integrated over the time horizon of extended period simulation and a difference between flows in original and reduced models is calculated.

$$\mathrm{MBE} = \sum_{i=1}^{t_p}(o_i - s_i) \times t_k \tag{3.13}$$

  where o_i is the flow in the original model, s_i is the corresponding flow in the simplified model, i is the i-th sample of series, t_p is the final simulation step, and t_k is the time interval of simulation.

- **tank relative error (TRE)**

  Tanks represent system states and thereby they are important elements in a hydraulic model. Consequently, it is critical that tanks performance in original and reduced model resemble each other; often pump operation is governed by tank water level, hence, a small error in the tank level might lead to serious consequences. One way to deal with this issue is to convert such water-level based control rules for pumps into their time-based schedules equivalents.

  However, to evaluate quality of a reduced model, in the scope of tank operation, the tank relative error (TRE) is introduced in Equation 3.14. The tank flow is integrated over the time horizon of extended period simulation and denoted by T_q^O for the original model and T_q^S for the reduced model. The difference between the volumes is calculated with respect to the tank's physical capacity T_v.

$$\mathrm{TRE} = \frac{\left|T_q^O - T_q^S\right|}{T_v} \times 100\% \tag{3.14}$$

- **goodness of fit (NSE) or (R²)**
  Another mathematical measure of how well reduced model reflects hydraulic behaviour of original model is "goodness of fit" statistic often referred in the literature as Nash-Sutcliffe efficiency (NSE) (Nash and Sutcliffe, 1970) or R^2. NSE is widely used in comparison of flows, especially in hydrological models assessment; see (Moriasi *et al.*, 2007; Krause *et al.*, 2005) and the references therein. In this work, NSE is utilised mainly to compare flows of pumps and valves in original and reduced models, and sporadically, in comparison of reservoir/tanks head trajectories. The NSE formula is given in Equation 3.15. NSE ranges from $-\infty$ to 1; where 1 indicates an ideal fit. Table 3.2 displays performance ratings recommendations established to measure quality of hydraulic resemblance of evaluated element. Note that NSE ranges in Table 3.2 are established for the optimisation study described in Chapter 5 and the recommendations should be adjusted based on the purpose of employing model reduction algorithm.

$$\mathrm{NSE} = R^2 = 1 - \frac{\sum_{i=1}^{n}(o_i - s_i)^2}{\sum_{i=1}^{n}(o_i - \bar{o})^2} \tag{3.15}$$

where

$\bar{o} = \frac{1}{n}\sum_{i=1}^{n} o_i$ is the mean of trajectory o_i in original model,

s_i is the trajectory in the simplified model, i is the i-th sample of the trajectory and n is the number of samples.

TABLE 3.2: Recommended performance ratings for the R^2 (NSE) statistic.

Performance rating	NSE (R^2)
Very good	$0.85 < \mathrm{NSE} \leq 1$
Good	$0.7 < \mathrm{NSE} \leq 0.85$
Satisfactory	$0.6 < \mathrm{NSE} \leq 0.7$
Unsatisfactory	$\mathrm{NSE} \leq 0.6$

- **scope of reduction**
  The ratio of the number of elements in reduced model to the number of elements in original model, expressed in percentages, is used to quantify the scope of reduction. It is expected, that for complex water networks with many pumps, tanks and valves the scope of reduction might not be very significant. In contrast, the scope of reduction for water networks based on pipes only should be close to 99%.

- **run-time time**
  It is necessary to measure time taken by the process of simplification. The run-time may not play important role in off-line studies but the real-time studies require that the simplification is be completed within the defined time interval. This time constraint may affect the quality of the reduced model as it may imply a trade-off between the further model simplification and the model accuracy.

Following the recommendations in (Legates and McCabe, 1999) a complete assessment of reduced model performance should include at least one "goodness-of-fit" or relative error measure (e.g., mean relative error (MRE) or Nash-Sutcliffe efficiency (NSE)) and at least one absolute error measure (e.g., RMSE or mean absolute error (MAE)) with additional supporting information such as plot of time series or residuals.

3.4.2 Results from simplification of several water network models

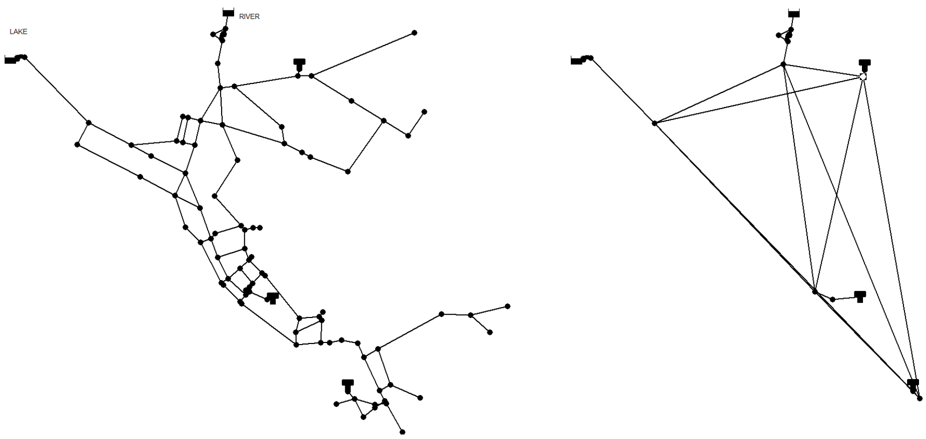
The implemented model reduction application was tested on water network models with different sizes, topologies and complexities. The details of the networks and results of simplification are summarised in Table 3.3. Note that except Epanet Net3 all other models listed in Table 3.3 are models of real water distribution systems. However, due to confidentiality issues and also for more convenient model identification the used model names are fictional.

Figures from 3.4 to 3.9 depict models' layouts before and after simplification. Additionally, in Appendix B, several simulated trajectories for each water network are included for more extensive comparison.

The results presented in Table 3.3 and Appendix B show that the variable elimination algorithm can reduce a complex water distribution network and still preserve its hydraulic behaviour. It should be highlighted that none of these models were manually prepared for the model reduction; i.e. no consideration was taken to identify and retain important elements in the particular model. The original models stored in the Epanet2 inp-format file

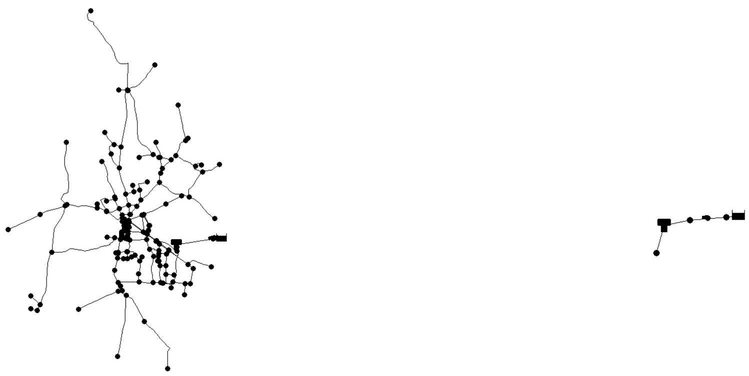
TABLE 3.3: Performance of model reduction application for several different water networks. All performance criteria were averaged for the particular water network. The relative criteria R^2 and MRE were shown only if can be calculated for over 95% of considered elements. ($\star$) Note that included run-times were obtained with use of the model reduction application described in Chapter 4.

Component	Original model statistics	Reduced model statistics	Scope of reduction %	MBE Ml/day	TRE %	R^2	MAE m or l/s	MRE %	RMSE m or l/s	Run-time* s
Epanet Net3										≤ 1
Junctions	92	12	86.96	-	-	0.987	0.072	1.820	0.095	
Pipes	117	21	82.05	-	-	-	-	-	-	
Reservoirs	2	2	0.00	0.045	-	-	-	-	-	
Tanks	3	3	0.00	-0.030	1.094	0.976	-	-	-	
Pumps	2	2	0.00	-	-	1.000	0.063	-	0.130	
Valves	0	0	0.00	-	-	-	-	-	-	
Rio										≤ 1
Junctions	164	3	98.17	-	-	1.000	0.000	0.000	0.000	
Pipes	200	3	98.50	-	-	-	-	-	-	
Reservoirs	1	1	0.00	0.000	-	-	-	-	-	
Tanks	1	1	0.00	0.000	0.000	1.000	-	-	-	
Pumps	1	1	0.00	-	-	1.000	0.000	-	0.000	
Valves	0	0	0.00	-	-	-	-	-	-	
Machu Picchu										≤ 1
Junctions	922	589	36.12	-	-	1.000	0.000	0.216	0.001	
Pipes	690	618	10.43	-	-	-	-	-	-	
Reservoirs	2	2	0.00	0.000	-	-	-	-	-	
Tanks	0	0	0.00	-	-	-	-	-	-	
Pumps	0	0	0.00	-	-	-	-	-	-	
Valves	289	289	0.00	-	-	0.967	0.001	116.573	0.002	
Rlyeh										≤ 1
Junctions	1009	39	96.13	-	-	0.763	0.223	0.574	0.295	
Pipes	1102	64	94.19	-	-	-	-	-	-	
Reservoirs	2	2	0.00	0.000	-	-	-	-	-	
Tanks	3	3	0.00	0.000	0.018	0.998	-	-	-	
Pumps	1	1	0.00	-	-	1.000	0.005	-	0.013	
Valves	11	11	0.00	-	-	0.961	0.012	1.411	0.016	
Cydonia										5
Junctions	3535	1022	71.09	-	-	0.886	0.196	191.967	0.865	
Pipes	3279	1337	59.23	-	-	-	-	-	-	
Reservoirs	5	5	0.00	-0.001	-	1.000	-	-	-	
Tanks	12	12	0.00	0.003	1.224	0.888	-	-	-	
Pumps	19	19	0.00	-	-	0.965	0.995	-	1.258	
Valves	417	417	0.00	-	-	0.288	0.272	1920.204	0.433	
Ankh-Morpork										45
Junctions	12828	7738	39.68	-	-	0.999	0.006	0.023	0.012	
Pipes	9419	6796	27.85	-	-	-	-	-	-	
Reservoirs	1	1	0.00	-0.002	-	1.000	-	-	-	
Tanks	3	3	0.00	0.001	0.011	1.000	-	-	-	
Pumps	17	17	0.00	-	-	1.000	0.003	0.007	0.003	
Valves	3858	3858	0.00	-	-	0.816	0.003	5.62e+10	0.004	



(a) Original model. (b) Reduced model.

FIGURE 3.4: Net3 network layout in the original and reduced model.

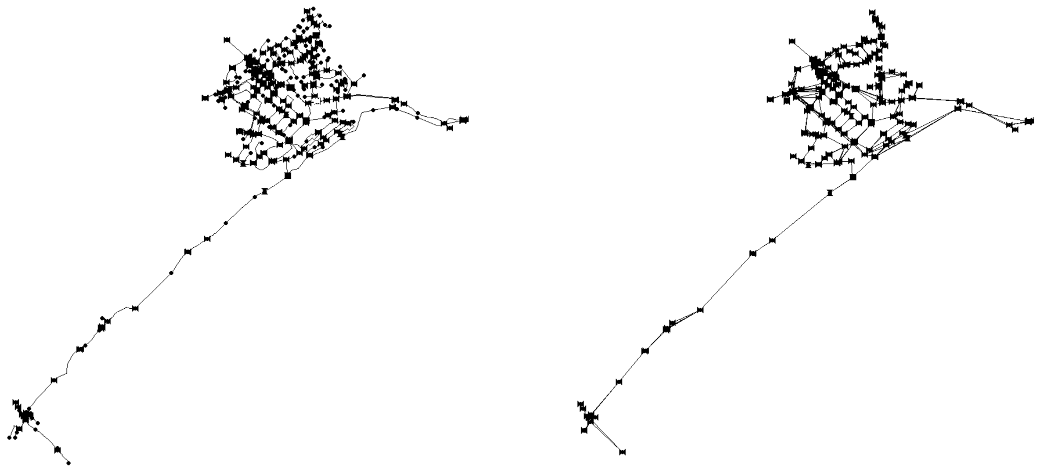


(a) Original model. (b) Reduced model.

FIGURE 3.5: Rio network layout in the original and reduced model.

were subjected to reduction directly without any modifications. Therefore it is expected that pre-processed models would resemble the original model even more accurately. Some ideas of model preparation are given in Chapter 5.

The algorithm performed very well for the tree-shaped networks such as Rio, but even for the looped water networks such as Rlyeh the reduction obtained a satisfactory hydraulic resemblance. Even for complex networks with a large number of valves such as Machu Picchu or Ankh-Morpork the results of reduction were surprisingly very accurate. But on

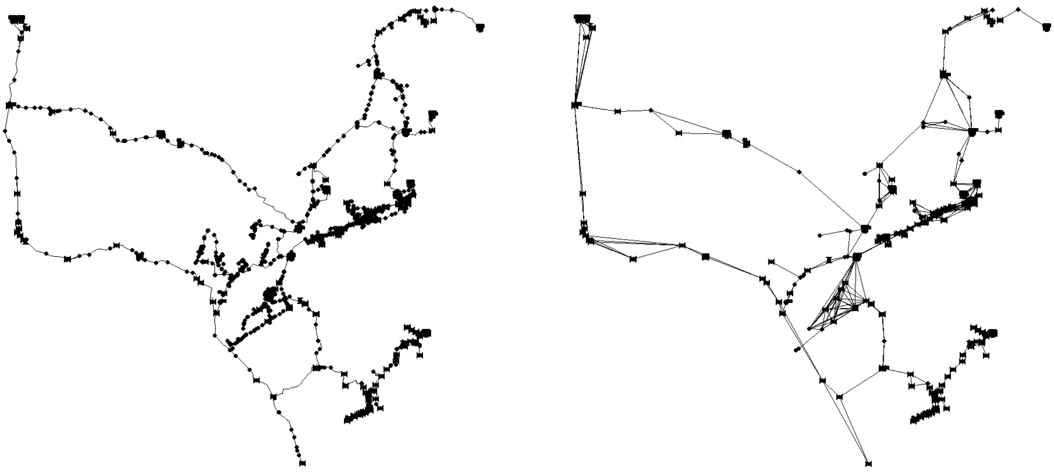


(a) Original model. (b) Reduced model.

Figure 3.6: Machu Picchu network layout in the original and reduced model.

(a) Original model. (b) Reduced model.

Figure 3.7: Rlyeh network layout in the original and reduced model.



(a) Original model. (b) Reduced model.

FIGURE 3.8: Cydonia network layout in the original and reduced model.

(a) Original model. (b) Reduced model.

FIGURE 3.9: Ankh-Morpork network layout in the original and reduced model.

the other hand due to a large number of valves that are automatically retained the scope of reduction was much smaller than in other networks.

The worst results were obtained for Cydonia network. However, the original model Cydonia simulated in Epanet2 reported dozens of warnings hence it was expected that obtained accuracy would not be in order with the other networks.

Other factors that significantly deteriorate the model reduction accuracy are control rules incorporated in the model. The pump or valve control based on the tank level might lead to significant discrepancy between the models as even small difference in tank level can switch off/on the control element in the preceding or succeeding time step. Such control rules significantly decreased the accuracy in case of reduction of the Net3 and Rlyeh models. Therefore it is recommended to replace such tank-based control rules with time schedules in order to achieve a more accurate reduced model.

It is important to point out that simplification accuracy is also affected by selection of the operating point for linearisation (Zehnpfund and Ulanicki, 1993). In the WDS simplification studies in this thesis a recommendation from (Alzamora *et al.*, 2014) was followed; i.e. operating point should be representative for normal operations of the network and should be chosen for average demand conditions while keeping at least one pumping unit working at each pumping station in order to avoid zero flow pipes. For small networks determining the operating point is a simple task but for large water networks few runs of simplification algorithm is sometimes needed to obtain the most satisfactory point.

It should be highlighted that the reduced topologies preserved the original networks layout. The retained critical elements are easy to identify, hence, the further analyses on the simplified network will be more convenient as it enhances understanding of actual network functioning.

As was aforementioned the model reduction algorithm performed as expected; i.e. all the reduced models adequately replicate hydraulic behaviour of the original model in terms of employed metrics. Although, not all the measures of quality of the reduced model proposed in Section 3.4.1 were found practical and conclusive. Especially, the MRE, used by Takahashi *et al.* (2010) in calibration study, was found impractical and misleading. Firstly, it cannot be calculated for time series with zero value, and secondly, due to its relativity a large error can be obtained as can bee seen in Figure 3.10 which shows flow time series for a valve in the original and reduced Ankh-Morpork models. Despite that the MAE and RMSE are very small the MRE is very large. Thereby, the average MRE in case of Ankh-Morpork simplification is in magnitude of 5.62e+10 while in fact the reduced model was resembling the original model very accurately as can be seen in Table 3.3 and Section B.6. The same occurred for MRE calculated for the Cydonia network.

The relative goodness-of-fit R^2(NSE) was found much more reliable and evidential, however it is recommended to used it in conjunction with other metrics such MAE and RMSE as sometimes R^2 can be also deceptive as shown in Figure 3.11.

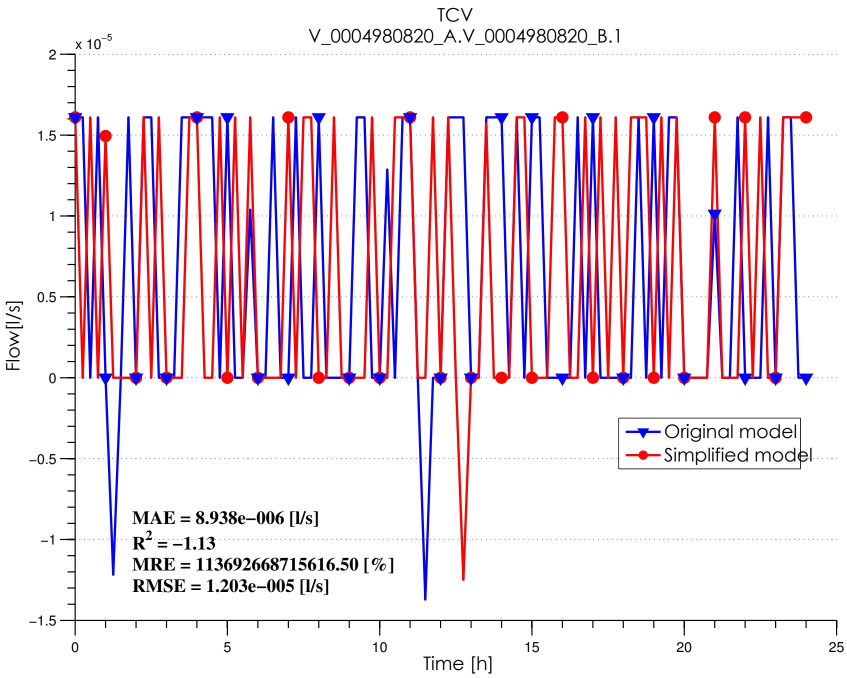


Figure 3.10: Illustrating the drawback of mean relative error. This plot depicts the valve's flows from the original and reduced Ankh-Morpork models. While the RMSE and MAE criteria indicate a very small discrepancy between depicted trajectories, the MRE suggested a huge difference between the compared flows.

3.4.3 Adaptation to abnormal situations and topographical changes

One of the main goals of the considered application was an automatic model simplification that would account for structural changes to the water network in real-time. Many abnormal situations could occur in a real water network e.g. pump station could be disconnected due to reallocation or maintenance service, tank could be under maintenance service or a pipe burst would require to isolate part of the network. Also control actions that are necessary to manage a water distribution system can result in changes of the connectivity of the network.

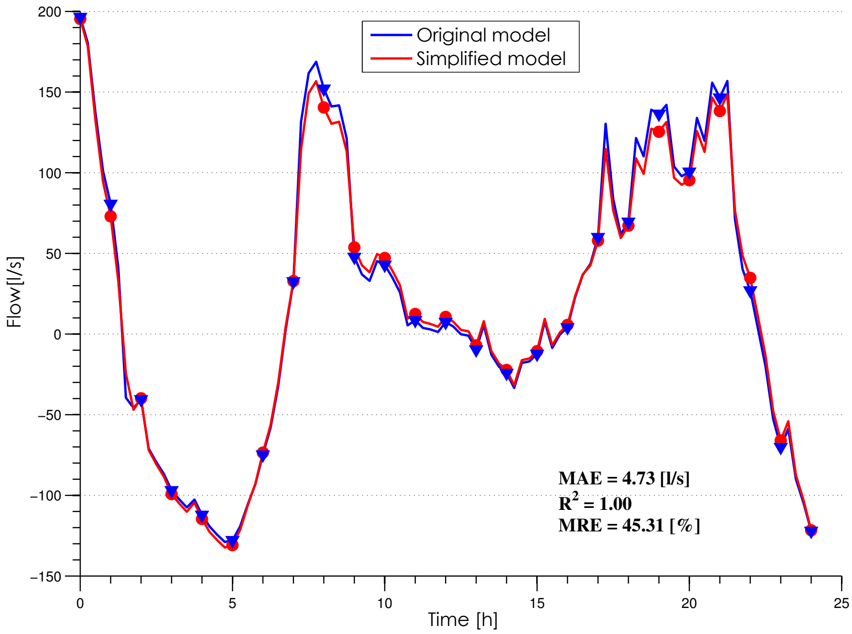


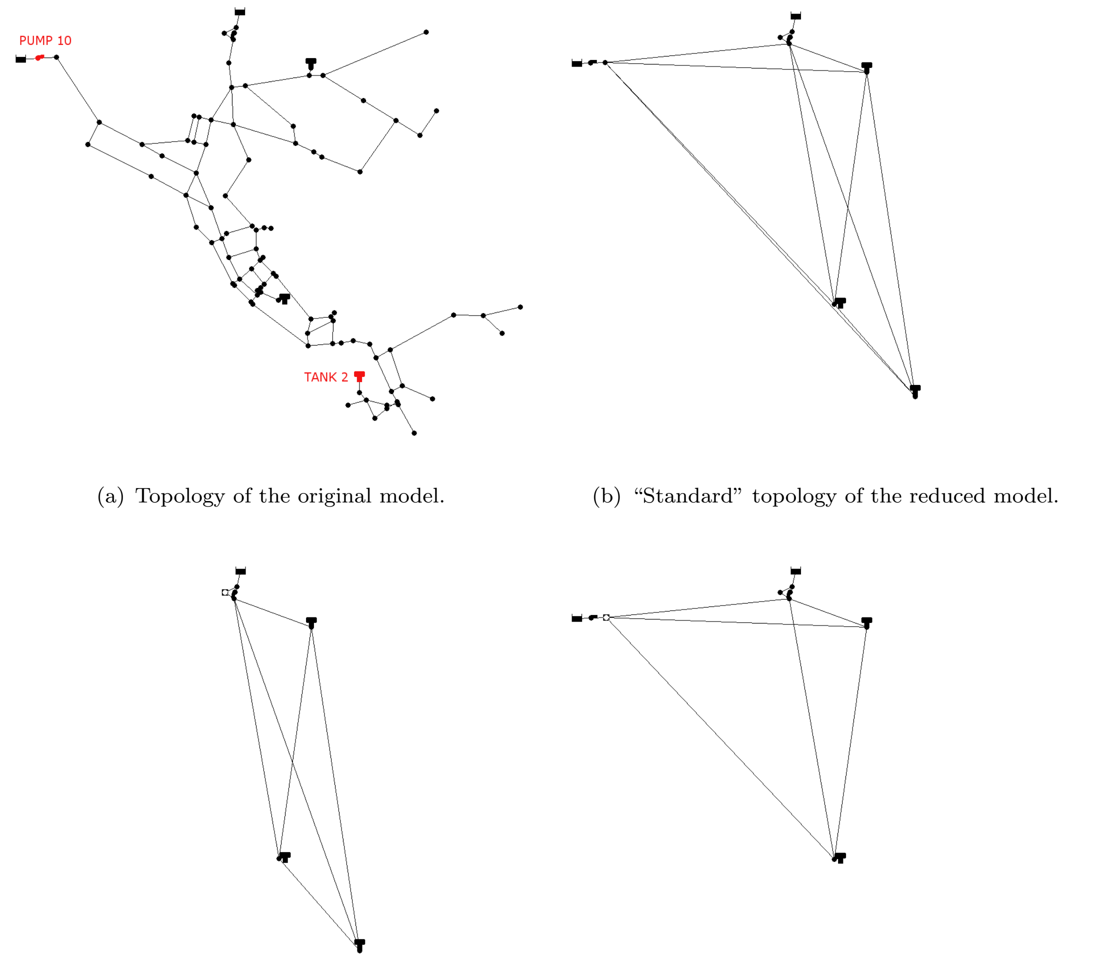
FIGURE 3.11: Illustrating drawback of R^2. The $R^2 = 1$ indicates a perfect fit but MAE and visual inspection clearly demonstrate a quite significant mismatch in terms of the flow rate magnitude.

The idea of the optimisation scheme, shown Figure 2.11, is that operator could modify the original model structure in response to the occurrence of the abnormal situation. Such modified model is subsequently simplified within time interval required to calculate new optimal schedules.

Figure 3.12 illustrates the Epanet2 Net3 benchmark model. It also depicts reduced models in response to abnormal structural changes. Figure 3.12a depicts the outcome of simplification of the original model. Figure 3.12b shows a reduced model structure when Pump 10 is out of service due to power supply failure. The reduced model in Figure 3.12c is a result of Tank 2 being removed from the original network due to e.g. maintenance service.

3.4.4 Energy distribution problem

Initially, only the hydraulic comparison was performed in order to validate accuracy of the reduced models. However, while determining the optimal pumps operation for the simplified water network in (Skworcow *et al.*, 2010) it was observed that the energy distribution was different in the full and the simplified models (Paluszczyszyn *et al.*, 2011). The reason was that the nodes elevation and the pressure constraints are not considered in the



(a) Topology of the original model.

(b) "Standard" topology of the reduced model.

(c) Topology of the reduced model when Pump 10 was removed.

(d) Topology of the reduced model when Tank 2 was removed.

FIGURE 3.12: Illustrating the adaptation to structural changes occurred in the Net3 water network.

model reduction algorithm. Subsequently, the pump speed required to satisfy minimum pressure constraints might be different for the reduced model and the prototype. This affects especially tree-shaped parts of WDS where pumps are pumping directly to satisfy demand. Such parts of water network, after simplification, are typically represented by a single node with the demand aggregated from the removed nodes. Similar observations were reported by Giustolisi and Todini (2009) and resulted in formulation of the EGGA to account for errors in energy balance equations of the standard GGA.

Therefore, to increase accuracy of the optimisation studies with the use of reduced water network models apart from hydraulic characteristics the energy balance should also be considered. The energy distribution aspect might be a significant factor when calculating optimal schedules for control elements, especially when demands at the removed nodes are being distributed in isolation from minimum service pressure constraints.

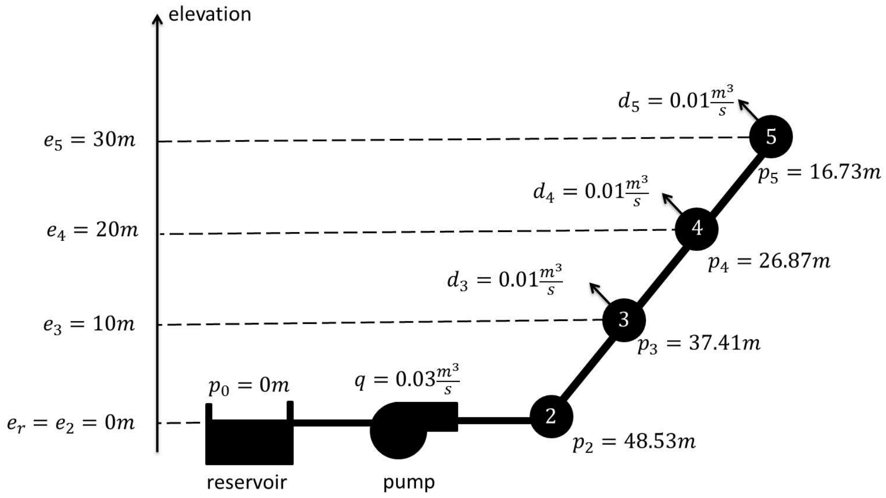
3.5 Aspect of energy distribution

To demonstrate the problem further consider a hypothetical and leak-free water network shown in Figure 3.13a. The network includes a reservoir with a water level fixed on 0 m and a variable-speed pump which pumps directly to the demands on nodes 3, 4 and 5 whilst satisfying the minimum pressure constraint of p_{min} =16 m at all nodes. The pump is described by the hydraulic curve $h_p = 53.33 - 0.005334q^2$ and all the pipes are 1000 m long, with 300 mm diameter and the HazenWilliams factor of 100. The pressure values as shown in Figure 3.13 were calculated using Epanet2. The nodal elevations and base demands are shown in Table 3.4. The EPS was conducted over 24 h with time step of 1 h.

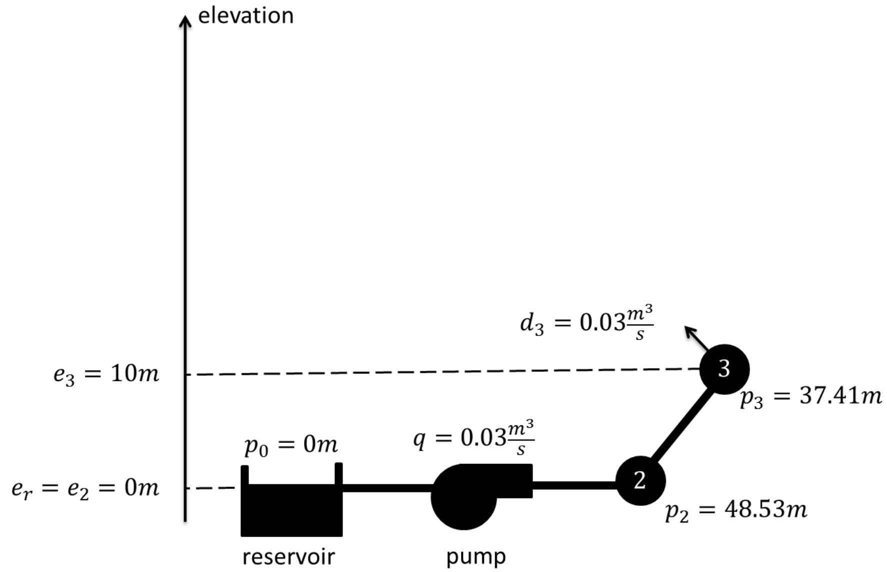
TABLE 3.4: Nodal elevations and base demands.

Node	Elevation $[m]$	Demand $[m^3/s]$
2	0	0
3	10	0.01
4	20	0.01
5	30	0.01

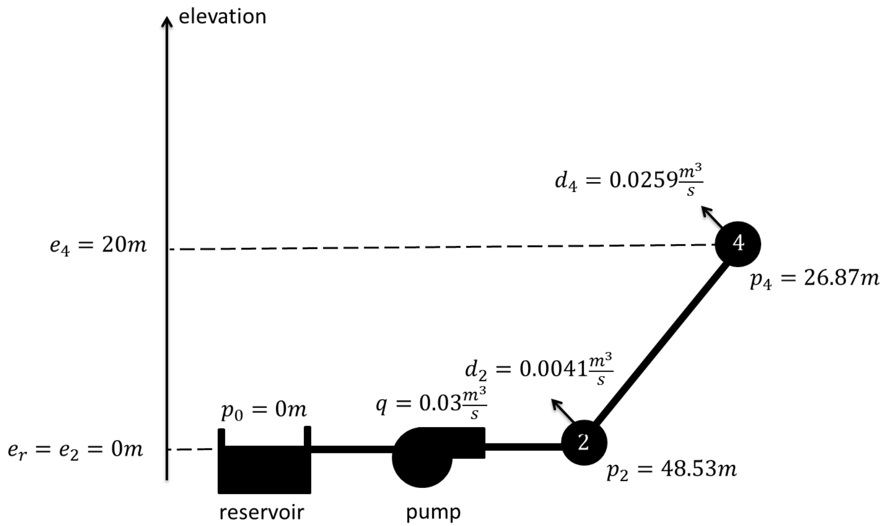
Figure 3.13b illustrates the outcome of simplification when node 3 was selected for retention. The algorithm has removed nodes 4 and 5 and transferred the demands to node 3. When both networks were compared the water volume and energy balance were similar as well as pressure and flow values in the retained components. However, the optimal pump control solutions for the two models are different if the original pressure constraints of 16 m are used. The full model still would maintain the pressure of 16 m at node 5 whereas in case (b) where the pressure at node 3 was 37.41 m; an optimisation algorithm would detect an excess of the energy supplied by the pump to the system and lower the pump speed to meet the requirements for the minimum service pressure of 16 m at node 3 (see Figure 3.14). Figures 3.13c and 3.13d show cases when nodes 4 and 5 were kept, respectively. In Figure 3.14 can be seen that the pump curve for the case (d) is the closest to the original pump head curve. Therefore only selection of the highest node 5 would give a similar optimal solution as the full model.



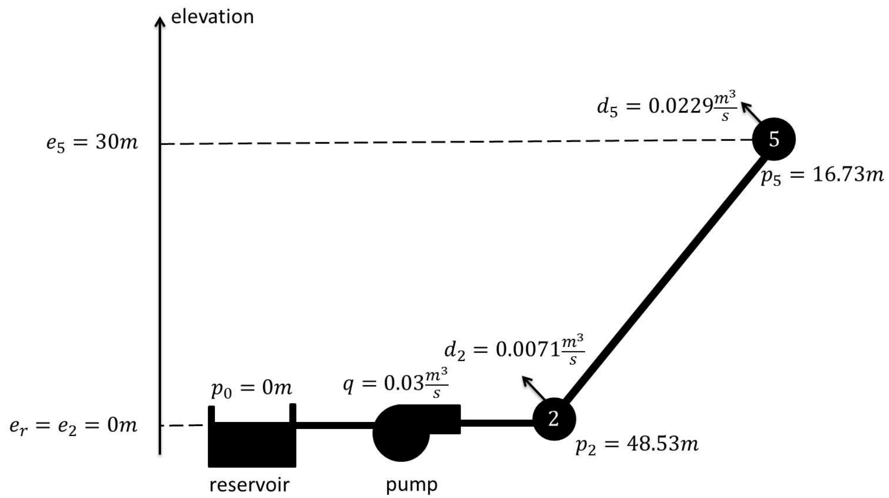
(a) The original water network to be simplified.



(b) The water network after simplification with node 3 selected to be retained.



(c) The water network after simplification with node 4 selected to be retained.



(d) The water network after simplification with node 5 selected to be retained.

FIGURE 3.13: Illustrating the energy distribution problem when reallocating demands to the nodes with a different elevation. The symbols in figure are as follows: e is the elevation, p is the pressure, d is the node demand and q is the flow.

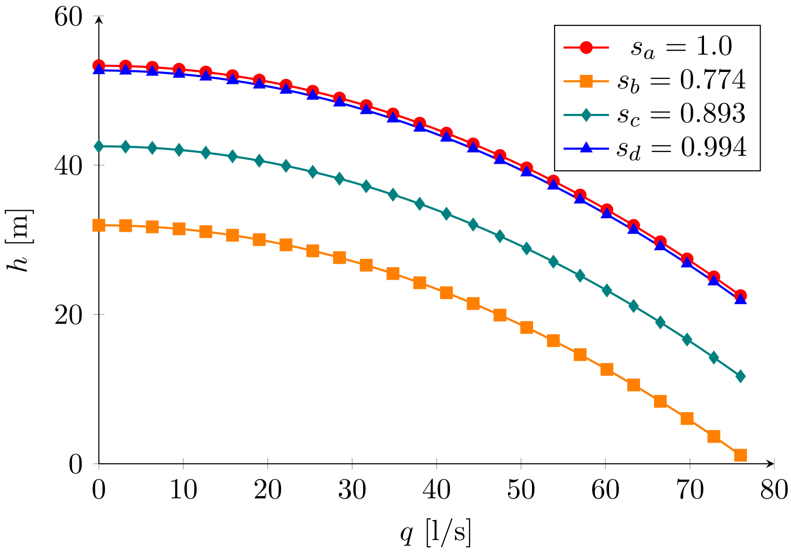


FIGURE 3.14: The original and adjusted pump head-flow curves to meet the service pressure constraint of 16m. s is the normalised pump speed.

To investigate this problem further an energy audit was carried out for the original and the reduced models. The energy audit was based on the concepts proposed in (Cabrera *et al.*, 2010) and further extended in (de Souza *et al.*, 2011; Bolognesi *et al.*, 2014; Cabrera *et al.*, 2014).

The methodology to perform an energy audit, proposed by Cabrera *et al.* (2010), includes the identification and quantification of all elements that either supply energy or consume energy to/from the water network. Elements like reservoirs or tanks supply energy in form of potential energy associated with the head of water. Also the work done by pumps introduces energy to the network. The energy output is due to energy delivered to users (this term refers to energy delivered to consumption nodes), energy dissipated due to friction and energy losses through leaks.

Here, only a concise description of the methodology and mathematical definitions of energy terms is given. For a more informative description with applications the reader should refer to (Cabrera *et al.*, 2010; Bolognesi *et al.*, 2014; Cabrera *et al.*, 2014).

In (Cabrera *et al.*, 2010) the following energies have been identified:

- Input energy supplied by the reservoirs

$$E_R(t_p) = \gamma \sum_{i=1}^{R} \left[\sum_{t_k=t_1}^{t_p} Q_{Ri}(t_k) H_{Ri}(t_k) \right] \Delta t \tag{3.16}$$

where γ is the water specific weight, R is the number of reservoirs, t_p is the total simulation time, t_k is the time interval of the simulation, $Q_{Ri}(t_k)$ is the reservoir i inflow to the network at time t_k, $H_{Ri}(t_k)$ is the reservoir i piezometric head at time t_k and Δt is the time interval of integration ($\Delta t = t_{k+1} - t_k$).

- Energy introduced by pumps

$$E_P(t_p) = \gamma \sum_{i=1}^{P} \left[\sum_{t_k=t_1}^{t_p} Q_{Pi}(t_k) H_{Pi}(t_k) \right] \Delta t \tag{3.17}$$

where P is the number of pumps, $Q_{Pi}(t_k)$ is the pump i flow rate at time t_k, $H_{Pi}(t_k)$ is the head added by pump i at time t_k.

- Energy supplied to users

$$E_U(t_p) = \gamma \sum_{i=1}^{U} \left[\sum_{t_k=t_1}^{t_p} d_{Ui}(t_k) H_{Ui}(t_k) \right] \Delta t \tag{3.18}$$

where U is the number of demand nodes, $d_{Ui}(t_k)$ is the node i demand at time t_k and $H_{Ui}(t_k)$ is the node i piezometric head at time t_k.

- Energy outgoing due to leaks

$$E_L(t_p) = \gamma \sum_{i=1}^{L} \left[\sum_{t_k=t_1}^{t_p} q_{Li}(t_k) H_{Li}(t_k) \right] \Delta t \tag{3.19}$$

where L is the number of nodes with leaks, $q_{Li}(t_k)$ is the leak's flow rate at the node i at time t_k and $H_{Li}(t_k)$ is the node i piezometric head at time t_k.

- Energy dissipated in links

$$E_D(t_p) = \gamma \sum_{i=1}^{D} \left\{ \sum_{t_k=t_1}^{t_p} \left[q_{Di}(t_k) + q_{Li}(t_k) \right] \Delta h_{Di}(t_k) \right\} \Delta t \tag{3.20}$$

where D is the number of links, $q_{Di}(t_k) + q_{Li}(t_k)$ is the link i flow rate including a leaked flow rate before leaking out and $\Delta h_{Di}(t_k)$ is the link i headloss at time t_k.

- Tanks' energy compensation

$$\Delta E_T(t_p) = \sum_{i=1}^{T} \left[E_{Ti}(t_p) - E_{Ti}(t_1) \right] = \gamma \sum_{i=1}^{T} \left\{ A_i \left[z_i^2(t_p) - z_i^2(t_1) \right] / 2 \right\} \tag{3.21}$$

where T is the number of tanks, A_i is the cross-sectional area of tank i and $z_i(t_1)$ and $z_i(t_p)$ are the water levels in tank i at initial and final time, respectively.

The energy balance for a water network, in the specified period t_p, combined all the above energies and is defined as follows:

$$E_R(t_p) + E_P(t_p) = E_U(t_p) + E_L(t_p) + E_D(t_p) + \Delta E_T(t_p) \tag{3.22}$$

Some of performance indicators developed in (Cabrera *et al.*, 2010) demanded to introduce a theoretic energy called a minimum useful energy. Its purpose is to indicate a minimum energy required to deliver water to users under specified minimum service pressure.

- Minimum useful energy

$$E_{U_{min}}(t_p) = \gamma \sum_{i=1}^{U} \left[\sum_{t_k=t_1}^{t_p} d_{Ui}(t_k) H_{U_{min}i}(t_k) \right] \Delta t \tag{3.23}$$

where $H_{U_{min}i}$ is defined by $H_{U_{min}i} = e_i + p_{min}/\gamma$ and e_i is the node i elevation with reference to the network's lowest elevation and p_{min} represents the minimum level of service pressure.

Beside the energy audit concept, Cabrera *et al.* (2010) defined a number of performance indicators in order to identify or diagnose weaknesses of the considered network. Table 3.5 lists the performance indicators that were adapted to compare both original and reduced models in terms of energy distribution. I_1 is the ratio between the real energy entering the system, and the minimum useful energy. I_5 is the direct ratio between the energy delivered to users, and the minimum useful energy. I_5 shows how average pressure levels are meeting the minimum pressure constraints. $I_5 < 1$ shows that average pressure levels are insufficient and below minimum standards. In turn, $I_5 > 1$ indicates that the pressure is kept above the minimum standards. A value closer to 1 indicates greater efficiency in satisfying the minimum pressure constraint. However, it is needed to highlight that even for $I_5 > 1$, at some nodes the pressure standard may not be satisfied (Cabrera *et al.*, 2010).

In order to preserve original energy distribution in the reduced models the calculated energy indicators should be approximately the same for both, the full model and the corresponding simplified model. The energy audits and associated performance indicators for the four cases considered in Figure 3.13 are summarised in Table 3.6. The conclusion

TABLE 3.5: Energy efficiency indicators. Where E_R is the input energy supplied by the reservoirs, E_P is the energy introduced by pumps, E_U is the energy supplied to users and $E_{U_{min}}$ is the minimum useful energy.

Indicator	Definition
Excess of supplied energy	$I_1 = (E_R(t_p) + E_P(t_p))/E_{U_{min}}(t_p)$
Excess of energy delivered to users	$I_5 = E_U(t_p)/E_{U_{min}}(t_p)$

from energy audits was much the same as from the pump head curves illustrated in Figure 3.14; i.e while energy balance was kept almost the same, the energy $E_{U_{min}}$ associated with minimum service pressure was different for each case. Thereby the indicators I_1 and I_5 were different for all three cases of the simplified models. It is evident that the model reduction algorithm does not consider the energy distribution what may lead to incorrect results in pump scheduling, hence an extension to the algorithm is needed.

TABLE 3.6: The energy audit carried out for all four cases illustrated in Figure 3.13.

Energies [kWh/day]	Models			
	a	b	c	d
E_U	345.74	348.70	346.48	346.85
E_R	0	0	0	0
E_P	356.96	356.96	356.96	356.96
E_D	11.22	8.27	10.49	10.11
$E_{U_{min}}$	264.78	191.23	244.46	286.06
$E_B = E_R + E_P - E_U - E_D$	0	0	0	0
I_1	1.35	1.87	1.46	1.25
I_5	1.31	1.82	1.42	1.21

3.6 Extension to model reduction algorithm

In order to retain the input model energy distribution an extension to the original simplification procedure, given in (Ulanicki *et al.*, 1996; Alzamora *et al.*, 2014), was proposed. The following steps were introduced by the author of this thesis into the WDS model reduction algorithm:

1. Perform an initial energy audit for the original water network as in (Cabrera *et al.*, 2010).

2. Calculate the minimum useful energy $E_{i_{U_{min}}}$ for the each node $i \in U$.

$$E_{i_{U_{min}}} = \gamma \left[\sum_{t_k=t_1}^{t_p} d_i(t_k) H_{min}{}_i(t_k) \right] \Delta t, \forall i \in U \tag{3.24}$$

   where U is the number of demand nodes.

3. The resulting vector of minimum useful energies is subjected to the Gaussian elimination in a similar way as the vector of nodal demands i.e. the nodal minimum useful energy $E_{i_{U_{min}}}$ is distributed to the neighbouring nodes proportionally to the pipes' conductance.

4. Calculate a new minimum pressure constraint $p^S_{i_{min}}$ for each node to which any demand was transferred to.

$$p^S_{i_{min}} = \frac{E^S_{i_{U_{min}}}}{\gamma D^S_i \Delta t} - e_i, \forall i \in U^S \tag{3.25}$$

   where U^S is the number of nodes in the simplified model, $E^S_{U_{min}}$ is the new i nodal minimum required energy obtained via Gaussian elimination, D^S_i is the new total demand at the node i and e_i is the node i elevation with reference to the lowest point in the water network.

5. Carry out an energy audit for the simplified network and compare it with the initial audit.

The above methodology was applied to the example water network illustrated in Figure 3.13. The results are shown in Table 3.7. It can be seen that the $E_{U_{min}}$ and indicators I_1 and I_5 for the simplified networks (b, c and d) are almost the same as for the original network (a). It can also be observed that before it would be recommended to keep the highest located node in the network to maintain initial energy distribution, whereas for the modified reduction process with inclusion of the additional steps which modify the pressure constraints the need to select such node is unnecessary. This makes the extended model reduction algorithm a straightforward process where no manual network analysis pre-processing is required to preserve the energy distribution.

Table 3.8 contains the new service pressure constraints calculated for each node. Such set of pressure constraints can be sent to the controller as the modified operational constraints.

TABLE 3.7: The energy audit with $E_{U_{min}}$ included in the reduction process.

Energies [kWh/day]	Models			
	a	b	c	d
E_U	345.74	348.70	346.48	346.85
E_R	0	0	0	0
E_P	356.96	356.96	356.96	356.96
E_D	11.22	8.27	10.49	10.11
$E_{U_{min}}$	264.78	264.78	264.76	264.79
$E_B = E_R + E_P - E_U - E_D$	0	0	0	0
I_1	1.35	1.35	1.35	1.35
I_5	1.31	1.32	1.31	1.31

TABLE 3.8: New calculated pressure constraints to be imposed on the remaining nodes for the all four cases.

Minimum service pressure [m]	Models			
	a	b	c	d
$p_{2_{min}}$	16	16	26.031	28.612
$p_{3_{min}}$	16	25.999	-	-
$p_{4_{min}}$	16	-	17.596	-
$p_{5_{min}}$	16	-	-	8.295

3.7 Case study - a small water network

The proposed methodology was applied to a model of a small DMA depicted in Figure 3.15a. The structural characteristic is similar to that in Figure 3.13a i.e. the pump is delivering water directly to the demand nodes. This leak-free network contains 165 nodes with a typical diurnal domestic demand pattern, 201 pipes with different length, diameter and Hazen-Williams coefficient parameters, 1 pump and 1 reservoir. The minimum service pressure of 20 m is assumed the same for all nodes. The EPS was conducted over a period of 24 h with the time step of 1 h.

The simplifications and energy audits were performed for the set of 10 representative nodes. The arbitrarily selected set of nodes from which a single node to be retained was selected vary in elevation with reference to the reservoir and in location in the water network model (see Figure 3.15a). The original network was simplified 10 times, resulting each time with the same topology illustrated in Figure 3.15b. The average run-time of simplification process was less than 1 second. The energy audits calculated for each simplified model are summarised in Table 3.9. Columns numbered from 1 to 10 correspond to nodes from Figure 3.15a selected to be retained in the simplified model.

TABLE 3.9: The energy audits for the original and simplified water network models. Note that elevation is in meters, energies are in kWh per day and pressures are in meters.

Model		Original network	1	2	3	4	5	6	7	8	9	10
Elevation		-	49.5	47.5	45.5	42.5	32.5	32.5	32.5	32.5	24.5	0
Energy audit	E_U	292.21	295.16	295.46	294.90	295.65	293.19	293.60	293.24	295.51	294.85	295.95
	E_R	0	0	0	0	0	0	0	0	0	0	0
	E_P	295.95	295.95	295.95	295.95	295.95	295.95	295.95	295.95	295.95	295.95	295.95
	E_D	3.74	0.79	0.50	1.06	0.30	2.76	2.35	2.71	1.44	1.10	0.0005
	$E_{U_{min}}$	215.91	221.47	95.98	202.83	89.73	172.13	157.20	146.12	110.68	94.80	77.12
	E_B	0	0	0	0	0	0	0	0	0	0	0
Performance indicators without consideration of minimum service pressure												
	I_1	1.37	1.34	3.08	1.46	3.30	1.72	1.88	2.02	2.67	3.12	3.84
	I_5	1.35	1.33	3.08	1.45	3.29	1.70	1.87	2.01	2.66	3.11	3.84
Performance indicators with consideration of minimum service pressure												
	I_1^S	1.37	1.37	1.37	1.37	1.37	1.37	1.37	1.37	1.37	1.37	1.37
	I_5^S	1.35	1.37	1.37	1.37	1.36	1.36	1.36	1.36	1.36	1.37	1.37
New pressure constraints to be imposed on nodes												
Pressure		-	6.60	9.55	10.30	14.10	22.53	22.70	22.23	22.10	30.05	55.99

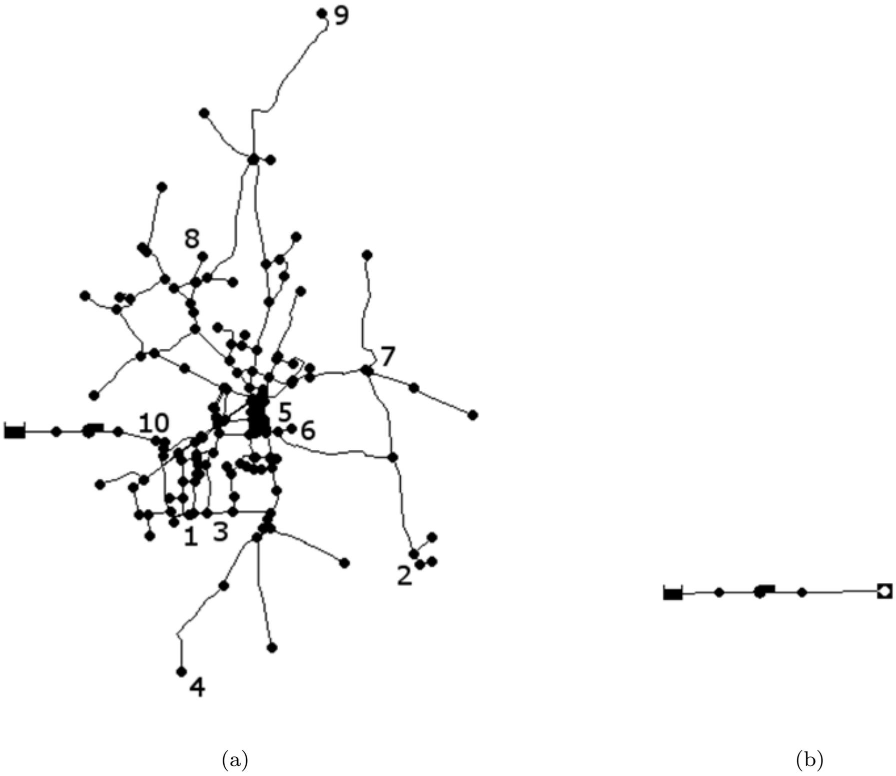


FIGURE 3.15: The water network model before (a) and after (b) simplification. Numbers indicate a single node to be retained, varied in elevation with reference to the reservoir and in location in the water network.

In Table 3.9 the performance indicators I_1, I_5 and minimum useful energy $E_{U_{min}}$, calculated for a standard model reduction procedure (i.e. not considering energy) in most cases significantly differs from the benchmarks values of $I_1 = 1.37$, $I_5 = 1.35$ and $E_{U_{min}} = 215.91$ kWh/day.

It is worth to highlight that the case with node 10 retained, which represents a case when no node has been selected to retain except nodes connected to the control elements, $E_{U_{min}} = 77.12$ kWh/day, $I_1 = I_5 = 3.84$ are almost three times higher when compared to the original water network. Such an excess of potentially recoverable energy $I_1 = 3.84$, depended on the minimum pressure constraint p_{min}, would mislead the optimiser and thereby the found optimal solution applied to the original water network would not guarantee the minimum service pressure.

Intuition suggests to keep the highest node, 1, which energy audits values are the closest to the original water network and indeed it is a standard practice to locate pressure sensors at the highest nodes in a DMA in pressure control schemes. However, when a water network contains many nodes with similar elevations the selection of the best node would be difficult. Columns labelled 5,6,7 and 8 in Table 3.9 illustrate such a case. The four nodes share the same elevation but their energy distribution and performance indicators are different. However, when the aspect of energy distribution is taken into account during the water network simplification a selection of nodes is not needed but then new pressure constraints must be imposed on those nodes. In all cases the simplified minimum useful energy was $E^S_{U_{min}} = 215.91$ kWh/day ensuring that the ratio of water energy introduced to the network to energy required to deliver water under minimum service pressure was kept the same (see bottom rows in Table 3.9).

3.8 Summary

This chapter has investigated various model reduction techniques that can be applied to models of WDSs in order to provide a tool for better understanding of water network functioning and also to decrease the computational burden associated with optimisation of WDS design and operation. The literature review conducted in Section 3.2 has revealed benefits and drawbacks of these techniques.

While some of the techniques demonstrated a potential for implementation in a real time optimisation strategies the one which has been found fast, practical and reliable was the variable elimination method proposed by Ulanicki *et al.* (1996). Its algorithmic formulation and proven record of successful applications made it a natural choice for a model reduction technique that can meet criteria outlined earlier in Section 2.5.

However, more insightful analysis conducted in Section 3.5 has exposed a potential for further improvement of this model reduction technique. Whereas in the original simplification method proposed by Ulanicki *et al.* (1996), the reduced model represented accurately the original hydraulic water network characteristics, the energy distribution was not preserved. This could cause a situation, where the pump speed required to satisfy specified minimum pressure constraints is different for the reduced model and the original model. This problem and its consequences have been illustrated in Section 3.5.

In the penultimate section a methodology based on the energy audits concepts has been incorporated into the model reduction algorithm allowing the preservation of the original model energy distribution. The idea is based on the distribution of minimum useful energy which is depended on the minimum service pressure. The standard model reduction algorithm has been extended to reallocate not only demand of the removed nodes but also their minimum useful energy (pressure constraints). The simplified model kept the original model energy distribution due to new pressure constraints. Such an approach preserves both the hydraulic and the energetic characteristic of the original water network and therefore met the requirements of the control strategy designed for a water network optimal scheduling.

Finally, this new contribution has been evaluated on the theoretical case study and the model of a small DMA. Especially, the evaluation in Section 3.7 has demonstrated the suitability of the proposed approach; the simplified model kept the original model energy distribution due to new pressure constraints. Hence, the new extension to the model reduction technique can simplify the inherent complexity of water networks while preserving the completeness of original information in order to perform correct successive optimisation.

Chapter 4

Improving numerical efficiency of model reduction algorithm

4.1 Introduction

The research outcomes of the extended model reduction algorithm developed in Chapter 3 yielded promising results. Hence, it was then decided to transform the extended algorithm into industry-viable software.

Initially, the aim of implementation was to provide means to evaluate research outcomes on the range of theoretical and real-world models. However, from the beginning it was evident that the model reduction algorithm would be needed for other R&D projects carried out in WSS e.g. it was essential for the research study described in Section 2.5 and in Chapter 5. Therefore, much effort was directed to produce a software up to the standard that resembles an off-the-shelf product.

Research described in this chapter was mainly generated from the necessity to improve the numerical efficiency of the algorithm. Use of GIS and SCADA in water industry resulted in an increasing amount of information about actual network topology and service that can be incorporated into a model. Hence, to ensure that the model reduction application would be able to cope with complex topologies of large size networks an investigation was carried out focused on: (i) an efficient way to manage large sparse matrices representing WDN topologies, (ii) exploitation of multi-thread computing aimed at distributing the computational load on multi-core processors, and (iii) analysis of water networks aimed at improving the understanding of network functioning, and eventually, reducing the computational effort while managing or even improving the accuracy of the extended model reduction algorithm. It was apparent that none of these research directions prevails on the others, but rather their combined development would provide means to enhance, and ultimately, create a practical, reliable and efficient tool.

The following sections, 4.2 and 4.3, gather all the requirements and necessary tools required to carry out the implementation. Section 4.4 outlines an initial design of the implementation process and reveals some arisen computational issues to be investigated. Section 4.5 focuses on the computational aspects arisen throughout the software development. The last section presents and briefly describes features of the final WDN model reduction application.

4.2 Application requirements

At this stage all the requirements already outlined in Section 2.5 and partially mentioned in Chapter 3 are gathered to formulate a foundation upon which the development will be carried out. Such synopsis would allow to identify not only necessary tools needed in the development process but also areas where a further research is likely to be conducted.

The following requirements were identified for the model reduction application:

Real-time or near real-time model reduction Online optimisation techniques employed in WSS research projects required that the process of WDN reduction is performed in real-time or near real-time.

Demand distribution log During the simplification process, nodes are removed and associated demands are redistributed based on the removed pipes' conductance. For the optimal scheduling purposes it was necessary to log the demands reallocation due to the need for online demand predictions and updates based on real-time telemetry; e.g. such information was necessary in the case study described in Chapter 5.

Energy distribution Operational optimisation techniques usually aim to calculate optimal control schedules for pumps and therefore it is crucial to preserve the energy distribution of a original water network.

Interaction with hydraulic simulator A hydraulic simulator is an essential tool, especially at the initial stage of the simplification process as it provides hydraulic results of the extended period simulation. It was decided to use Epanet2 Toolkit software as the hydraulic simulator. Hence, an interface between Epanet2 Toolkit and the created application was required to automatically read hydraulic data results from simulation.

User interface User interface should be plain, transparent and intuitive to ease the whole process of model reduction. Also, it should allow to define the scope of the simplification i.e. user should be able to select the WDN elements to be retained in the reduced model.

4.3 Tools and software employed

It was demonstrated in Section 3.4.2 that the simplification algorithm performed with a sufficient accuracy. But because the model reduction algorithm involved a number of matrix operations with time complexity of order $O(n^3)$ for the $n \times n$ matrix, the calculation time for large-scale networks (more than 10000 elements) could take up to several hours what is too long for utilisation in real-time applications.

Nowadays, modern computers have two or more CPU cores that allow multiple threads to be executed simultaneously. Moreover, computers in the near future are expected to have significantly more cores (Microsoft, 2012). To take advantage of this advent in IT hardware it was decided to parallelise sections of the module algorithm code with a large number of matrix operations. The workload for these compute-intensive operations was distributed among multiple processors. For this purpose a workstation powered by the six-core Intel® Core^{TM} i7 980X processor was provided as a host to perform all the necessary calculations.

The implementation was carried out with utilisation of the Microsoft Visual Studio 2010 package. Visual Studio 2010 comes with an integrated support for the .NET 4.0 framework, which enhanced the parallel programming by providing a new runtime, new class library types and new diagnostic tools (Microsoft, 2012). These features allowed for the implementation of the scalable parallel C# code without having to work directly with threads or a thread pool and and hence provide means for improving the performance of numerical calculations.

To represent a water network in a computerised form an object-oriented programming framework (Ten Dyke and Kunz, 1989) was used. The object-oriented programming paradigm is, nowadays, one of the most used and established programming language models, also utilised in creating of software for WDS analysis, e.g. see (van Zyl *et al.*, 2003; Guidolin *et al.*, 2010; van Zyl and Chang, 2010; Piller *et al.*, 2011). Figure C.1 illustrates water network data modelled with the object-oriented approach.

The input data for the model reduction algorithm are water network topology and simulated hydraulic behaviour of the considered water distribution network. For this purpose the open-source Epanet2 Toolkit (Rossman, 2000a) was used as a hydraulic simulator to perform an extended period simulation of WDN hydraulic behaviour. The library consists of set of procedures that allow to run/stop simulation, modify simulation and network parameters and read/save the simulation data. The Epanet2 Toolkit provided also a compatibility with ".inp" (INP) format as it is a commonly recognized file format used to store water network models. Unfortunately functionalities of this library are limited and a number of additional C# scripts were written to enable a dynamical hydraulic data export.

The structure of matrix representing a water distribution network is naturally sparse. Therefore, in order to exploit this sparsity feature additional open-source libraries, Math.NET Numerics (*http://mathnetnumerics.codeplex.com/*), were investigated to provide sparse matrix operations and storage implementations.

4.4 Model reduction process

The overview of the overall model reduction process is illustrated in Figure 4.1.

At first, a water network model stored in the INP file format is simulated with the aid of Epanet2 Toolkit to obtain the hydraulic results. Next, the water network model is being inspected to locate any rules or controls associated with the water network elements. Complex and large water networks modelled in Epanet2 often contain rules and controls that can decrease the accuracy of the simplification. It is highly recommended to eliminate the controls and rules and instead use time patterns resulting from the simulation of the original model (with control and rules) and associate the patterns with the water network elements. Such an approach serves as a hydraulic benchmark when original and reduced models are compared. Note that in Epanet2 user can associate rules or controls with pipes, transforming them in fact into valves. Since no time patterns can be assigned to the pipe, such rules or controls cannot be automatically eliminated. All components with controls/rules that could not be replaced with a time pattern are automatically selected for retention.

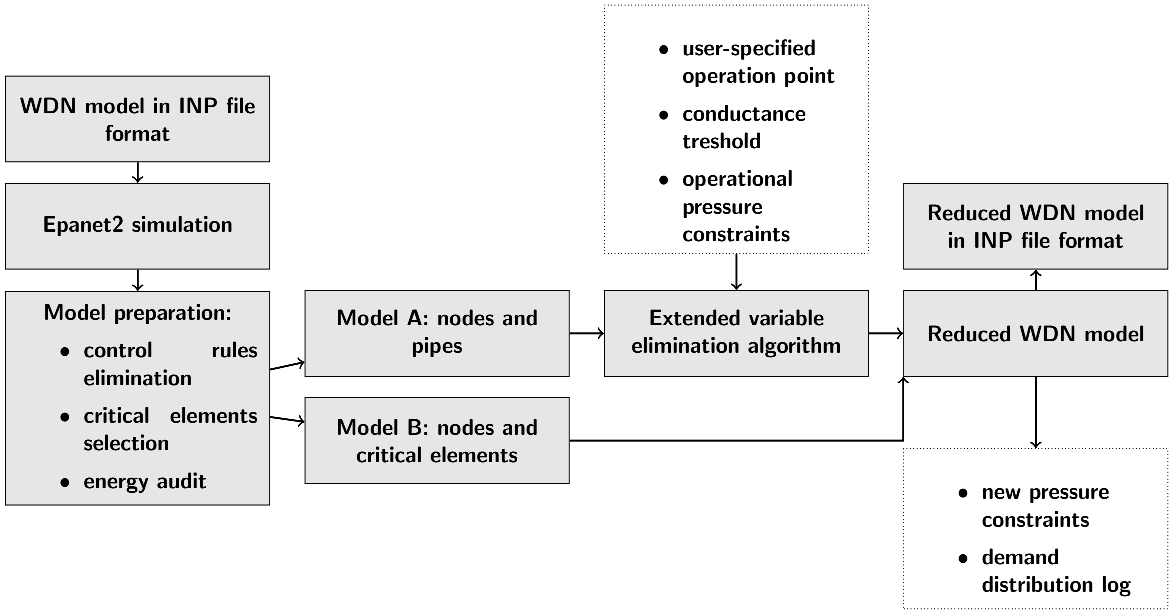


FIGURE 4.1: The overall process of the extended model reduction algorithm.

At the preparation stage the initial energy audit of the considered water network is carried out as described in (Cabrera *et al.*, 2010) and Section 3.5.

The model preparation stage involves also a selection of other important hydraulic elements to be retained. Initially, it was assumed that operator, based on his knowledge about the particular WDS would choose network elements with a significant importance in order to preserve hydraulic characteristics for wide range of operating conditions. But, even though that this operation needs to be done once for the particular model, it could be a difficult and time consuming task. Hence, the model reduction process would not be fully automatic as required.

Although a typical hydraulic simulation model contains thousands of pipes but only several tanks, pumps or control valves. Therefore, it is an adopted strategy here to reduce the number of pipes and nodes only and retain all other important elements. The identified non-pipe components of a WDS are listed in Table 4.1. The default is to retain all these elements, but alternatively, user can define a list of additional elements not to be removed.

To help out user in a decision-making process which elements should be additionally retained to replicate more accurate the hydraulic behaviour and layout of original water network few tools were introduced at the preparation stage. They allow to select nodes, based on their degree; i.e number of neighbours, or select pipes based on their diameter, an approach adapted inter alia by Preis *et al.* (2009). Nodes with many neighbours are often

TABLE 4.1: Important elements in water distribution network.

Water distribution network elements
Tanks (variable head)
Reservoirs (fixed head)
Pumps
Valves
Pipes with associated controls or rules
Nodes connected to any of the above

selected for pressure logger locations. In turn, large diameter pipes often form skeleton of the network. Hence, option to preserve critical nodes and large diameter pipes provides means to retain layout of the network, which is important in WDN design optimisations.

Taking into account the aforementioned considerations, the input model is split up into the two sub-models. Sub-model A, containing pipes and nodes, is subjected to the extended reduction algorithm described in Section 3.6 and afterwards, reunited with the other part containing non-pipe and important elements (Sub-model B) to form the complete reduced model, which is saved in the INP file format. Additional output files contain the demand distribution log and new operational pressure constraints.

4.5 Improving numerical efficiency of simplification algorithm

Once the overall simplification process was implemented it was subsequently tested on a number of water network models. While the achieved results were satisfactory from the hydraulic perspective, the computational time required to reduce large and medium size networks was in order of hours. Obviously such long computational time is not suitable for online optimisation strategies therefore the attention was directed to improve the numerical efficiency of simplification algorithm. The following computational aspects of the implementation were investigated in order to reduce the overall time of the model reduction process.

4.5.1 Parallel programming

Firstly, the focus was placed on the performance of matrix operations. The model reduction algorithm involved a number of matrix multiplications thereby the speed of these calculations is factor with a profound influence on the total algorithm calculation time.

It was decided to investigate suitability of the model reduction algorithm for a parallel programming and thereby exploit the potential of recent multi-core CPUs. The parallel programming is often employed for highly compute-intensive algorithms. It follows the basic idea of decomposition or division of data to be computed asynchronously by each processors. The process of decomposition is dependable on algorithm to be parallelised and type of parallel computing architecture. A number of concurrent programming models were developed over the years e.g. message passing interface (MPI) or multi-threading (see (Rauber and Rnger, 2010) for details). In general, all of them have a static or dynamic period for partitioning or dividing data quantity to be computed in each processor and, eventually, a subsequent utilisation period of intermediate computations to compute the final result.

There is universal agreement that writing multi-threaded code is difficult (Sutter and Larus, 2005). For example, consider snippets of C# code in Listings 4.1 and 4.2 that show sequential and parallel matrices multiplication using the multidimensional arrays (in this case two-dimensional arrays). While the numerical results of both codes are the same the way they are obtained is different as the order of iterations in the parallel variant is not necessarily sequential as can be seen in Figure 4.2. Hence, a careful consideration needs to be done while paralleling any algorithm. In fact many algorithms are not suitable for parallelisation as their overall efficiency might not be improved due to the required inter-core communication. For more information about the concurrent programming see e.g. (Pacheco, 2011) and references therein.

Fortunately, .NET 4.0 Framework enhanced the parallel programming by providing a new runtime, new class library types and new diagnostic tools (Microsoft, 2012). These features allowed the implementation of the scalable parallel C# code without having to work directly with threads or a thread pool.

Listing 4.1: Sequential multiplication of multidimensional arrays in C#.

```
for (int i = 0; i < size; i++)
{
  for (int j = 0; j < size; j++)
    {
        double tmp = 0;
        for (int k = 0; k < size; k++)
        {
          tmp += m1[k, i] * m2[j , k];
        }
        result[j,i] = tmp;
```

```
  }
}
```

Listing 4.2: Parallel multiplication of multidimensional arrays in C#.

```
Parallel.For(0, size, i =>
{
  for (int j = 0; j < size; j++)
    {
        double tmp = 0;
        for (int k = 0; k < size; k++)
        {
          tmp += m1[k, i] * m2[j , k];
        }
        result[j,i] = tmp;
  }
});
```

$$\begin{bmatrix} 1.80_{1} & 1.43_{2} & 1.35_{3} & 1.54_{4} \\ 1.52_{5} & 0.94_{6} & 1.25_{7} & 1.06_{8} \\ 2.07_{9} & 1.36_{10} & 1.54_{11} & 1.23_{12} \\ 1.14_{13} & 0.80_{14} & 0.82_{15} & 1.02_{16} \end{bmatrix}$$

(a) Sequential

$$\begin{bmatrix} 1.80_{2} & 1.43_{5} & 1.35_{9} & 1.54_{10} \\ 1.52_{1} & 0.94_{3} & 1.25_{7} & 1.06_{14} \\ 2.07_{4} & 1.36_{8} & 1.54_{11} & 1.23_{12} \\ 1.14_{6} & 0.80_{13} & 0.82_{15} & 1.02_{16} \end{bmatrix}$$

(b) Parallel

Figure 4.2: Result matrices obtained from multiplication of two 4×4 matrices with use of the code in Listings 4.1 and 4.2. Note that subscript values correspond to the order of filling in the result matrices.

Hence, only the most compute-intense and suitable parts of the model reduction algorithm were subjected to parallelisation. This includes calculation of the Jacobian matrix of a considered system and inner loops of the Gaussian elimination process. The inclusion of the parallel programming techniques drastically reduced the algorithm calculation time. Table 4.2 contains the simplification run-times for a medium size network which contained 3535 nodes, 3279 pipes, 12 tanks, 5 reservoirs, 19 pumps and 418 valves.

The obtained times of for model reduction were satisfactory for the requirements of the control strategy described in Section 2.5. The models to be considered in that study would be significantly smaller in size than model used as a benchmark in Table 4.2. The calculation time to perform all optimisation computations was under 4 minutes on average and was never longer than 5 minutes (Skworcow *et al.*, 2010). Hence, the achieved times of less than 15 minutes to reduce much more complex model is more than adequate to perform all computations (model reduction and optimisation) within the desired time interval of 1 hour.

Table 4.2: Times taken to complete the simplification process for a medium-sized water network. The benchmark network contained 3535 nodes, 3279 pipes, 12 tanks, 5 reservoirs, 19 pumps and 418 valves.

CPU threads	Simplification run-time [s]	Reduction of run-time [%]
1	5761	0
2	4417	23.33
4	2217	61.52
12	758	86.84

4.5.2 Matrix storage

Recall from Section 2.2 that topology of water network can be represented as an incidence matrix that describes the connectivity between pipes and nodes. Such a representation is also useful for the implementation purposes as the network topology can be explicitly stored in one of the available data structures in the C# language specification.

The considered C# data structures were single-dimensional arrays, multi-dimensional arrays and jagged arrays (arrays of arrays). A single-dimensional array is a list of variables where access to its elements is trough an index. A multi-dimensional array has two or more dimensions, and an individual element is accessed through the combination of two or more indices. A jagged array is an array of arrays in which the length of each array can differ. Jagged array elements are accessed also with two or more indices (Schildt, 2010).

As incidence matrices for water network topology are usually sparse (see Figure 4.3) it was decided to examine potential of the external C# library, Math.NET Numerics. This free library aims to provide methods and algorithms for numerical computations in science and supports both dense and sparse matrices (Math.NET Numerics, 2013). Sparse matrices in Math.NET Numerics are represented in the 3-array compressed-sparse-row (CSR) format (Farzaneh *et al.*, 2009).

The initial choice for a matrix representation were the the multi-dimensional arrays. Using multi-dimensional arrays a real matrix data structure can be constructed using a two-dimensional array, one dimension for rows and another one for columns. However, their overall poor performance forced a need for more effective way to store and multiply sparse matrices.

One of the techniques often used by programmers to speed up matrix operations is flattening i.e. representation of multi-dimensional arrays using single-dimensional arrays.

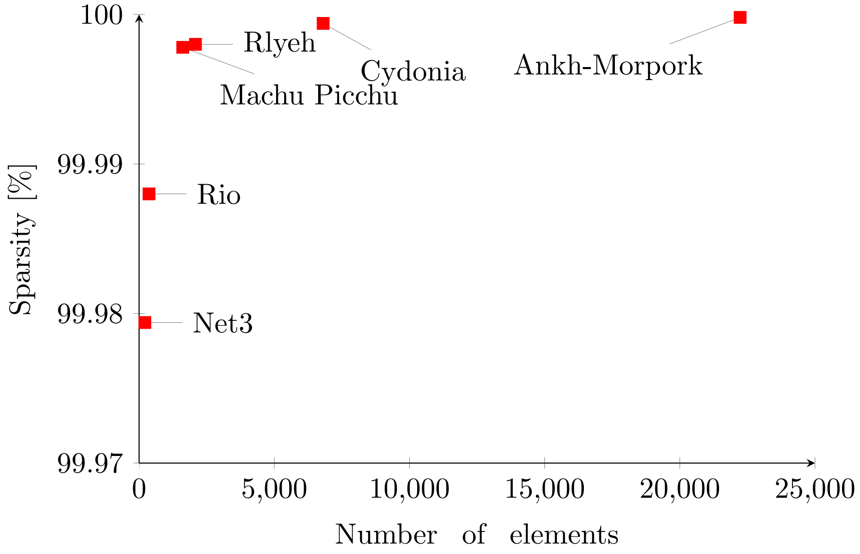


FIGURE 4.3: Sparsity of incidence matrices constructed from the topologies of water networks used in Section 3.4.2. Note that "sparsity" is defined as the ratio $\frac{n_{zeros}}{(n \times m)} \times 100\%$, where n_{zeros} is the number of zero entries for the $n \times m$ matrix.

Flattening multi-dimensional array into single-dimensional array could benefit in better performance as in the .NET Framework single-dimensional arrays have faster access to their elements, due to optimizations in Common Language Runtime (CLR). Also, usage of jagged arrays instead multi-dimensional arrays could improve matrix computations as jagged arrays are made of single-dimensional arrays.

The results of this examination are presented in Table 4.3. Table 4.3 shows average times taken to multiply sparse matrices constructed from the real water network topologies.

TABLE 4.3: Testing of performance of different C# implementations of matrices multiplication.

Matrix storage and multiplication implementation	166×166 sparse matrix with 566 non-zero elements Average time from 1000 runs [s]	3552×3552 sparse matrix with 10005 non-zero elements Average time from 10 runs [s]
1-D Arrays	0.0239	386.751
1-D Arrays (parallel programming)	0.0066	71.958
2-D Arrays	0.0493	395.336
2-D Arrays (parallel programming)	0.0132	74.770
Dense Matrix (Math.NET Numerics)	0.0095	60.879
Sparse Matrix (Math.NET Numerics)	0.2201	1467.963
Jagged Arrays	0.0289	896.050
Jagged Arrays (parallel programming)	0.0065	151.934
Jagged Arrays, (single-indexing access, ijk order, parallel programming)	0.0050	20.247

It can be seen that in all cases the introduction of parallel programming decreased the calculation time; in some cases even five times.

For the small matrix (166×166) the single-dimensional (1-D) arrays outperforms the two-dimensional (2-D) arrays, but for the large matrix the difference between these approaches is not so evident.

Also, the potential of the external C# library (Math.NET Numerics, 2013) was examined. Unfortunately, while the Math.NET dense matrices were fast especially for larger matrices the Math.NET sparse matrices due to its CSR format were slowest among the tested approaches. But, on the other hand the storage space of the Math.NET sparse matrix was the smallest.

The jagged arrays performed similar to 1-D arrays in case of $166{\times}166$ sparse matrix. But, for $3552{\times}3552$ sparse matrix the jagged arrays performed slower. Nonetheless, the jagged arrays were considered to be replacement for the multidimensional arrays rather than 1-D arrays. The main reason is behind this is the maximum storage space for C# data structures. The maximum-object size in CLR in .NET 4.0 Framework is limited to 2 GB for 32-bit application (Microsoft, 2013). Moreover, due to CLR memory overheads the actual memory limit is around 1.3 GB (Ayucar, 2013). Tests performed on the host machine reveal the memory allocation limits for data C# data structures (see Table 4.4). As can be seen in Table 4.4 the jagged arrays allowed to allocate the biggest amount of memory. It is because due to memory fragmentation it is easier to find available memory for jagged arrays, i.e. it is more likely that there will be number of blocks of smaller size available than a single, continuous block of the full size of the array which is required to allocate single and multi-dimensional arrays.

Table 4.4: Memory allocation limits for C# data structures. Tests were performed on the workstation with 4 GB RAM. Note that in C# size of *double* type is 8 bytes.

Data structure	Maximum allocated memory [Megabytes]	Maximum size of $n \times n$ matrix of *double* elements
2D array	1001	11185
Flatten array	1183	12160
Jagged arrays	1530	13829

Moreover, the performance of jagged arrays can be improved even more by employing techniques of single-indexing and ordering indices *ijk* for matrix multiplication (see (Golub and Van Loan, 2012; Momerath, 2011) for details). These modifications allowed a further reduction of the calculation time for matrices multiplication when using the jagged arrays (see the bottom row in Table 4.3).

The combination of parallel programming and jagged arrays (single-indexing access, ijk-order) reduced the overall simplification time of the benchmark water network used in Table 4.2 to 95 seconds, achieving a 99.98% decrease with the respect to the initial time.

More methods or tools such as cache optimisation (Lam *et al.*, 1991), Compute Unified Device Architecture(CUDA) (Vazquez *et al.*, 2011) or native libraries can be introduced for further improvement of jagged arrays performance. However, it was decided that further research in this direction would not provide enough gain for the effort needed. Therefore, it was decided to seek for other numerical techniques that can increase speed of calculation in the most compute-intensive algorithm of the model reduction procedure; i.e. Gaussian elimination.

4.5.3 Node removal ordering

The Gaussian elimination (Higham, 2011) is the most compute-intensive procedure of the model reduction algorithm. When dense matrices are considered one iteration of the Gaussian elimination uses $O(n^2)$ arithmetic operations and as n iterations must be performed resulting this procedure needs $O(n^3)$ arithmetic operations to complete (Lovász and Gács, 1999).

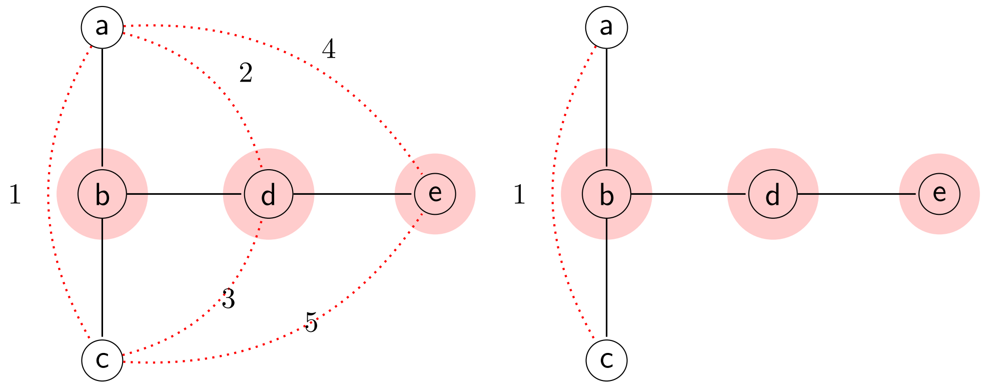
Since its introduction, Gaussian elimination and its performance is in a very strong interest for researchers from many disciplines, especially in areas where Gaussian elimination is applied to a sparse matrix. Many variations were developed over the years, often designed for a particular application (Grcar, 2011; Donfack *et al.*, 2014). The variant of Gaussian elimination used in the model reduction algorithm is given in Algorithm 2. Algorithm 2 has three nested loops with loop indices denoted k,i,j.

Saad (2003) noted that Gaussian elimination on the original matrix results in disastrous fill-ins. Fill-ins are additional non-zeros generated during the elimination. To illustrate this consider a simple network in Figure 4.4. Nodes b, d and e are to be deleted from the network. When the process of removal starts from node b and then in order d and e, additional links (indicated by dotted lines) are created between any two nodes that were adjacent to removed node. For *bde* order five links (fill-ins) were created, see Figure 4.4a. Whereas when starting removal from node e and then d and b only one fill-in (between a and c) was added, see Figure 4.4b.

Algorithm 2 Gaussian elimination used in model reduction

Require: J, n, n_r $\triangleright$ J - Jacobian matrix, n - number of nodes, n_r - number of nodes to remove

```
for k = n to n − n_r do
    if J_kk ≠ 0 then
        for i = 1 to k do
            J_ik ← m_ik = J_ik / J_kk
        end for
        for i = 1 to k do
            for j = 1 to k do
                J_ij = J_ij − m_ik × J_kj
            end for
        end for
    end if
end for
```



(a) Order of removal: *bde*. #fill-ins = 5 (b) Order of removal: *edb*. #fill-ins = 1

FIGURE 4.4: Change in the number of generated fill-ins (additional links indicated by the dotted lines) due to the order of nodes removal.

Therefore, the aim of the most researchers is to produce much less fill-ins during Gaussian elimination and thereby reduce computation time and storage space. To address the problem of fill-ins a common-used technique called reordering can be applied to sparse matrices (Pissanetzky, 1984). The idea is to permute the sparse matrix's rows or columns or both. By applying reordering algorithms, the zero and non-zero elements of a sparse matrix are rearranged such that the Gaussian elimination deals with it much more efficiently.

The amount of fill-ins depends on the chosen ordering (Pissanetzky, 1984). Because the fill-in minimisation is impossible to solve in practice heuristics are used (Davis, 2006). The most widely recognised and applied ordering algorithms are Cuthill-McKee (CMK) (Cuthill and McKee, 1969), reversed Cuthill-McKee (RCMK) (George, 1971), minimum degree (MD) (George and Liu, 1989), Gibbs-Pool-Stockmeyer (GPS) (Gibbs *et al.*, 1976a) and nested dissection (ND) (George, 1973). More details about the ordering algorithms can be found in (Gibbs *et al.*, 1976b; Pissanetzky, 1984; George *et al.*, 1994; Davis, 2006).

Nevertheless, following George (1971) observations that reversed Cuthill-McKee ordering yields a better scheme for sparse Gaussian elimination it was decided to incorporate RCMK to reorder Jacobian matrix prior Gaussian elimination. The original RCMK algorithm goal is to order nodes locally so that the adjacent nodes are ordered as close as possible.

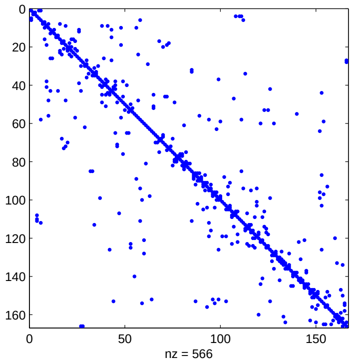
Another algorithm chosen for a closer investigation was the MD as it is perhaps the most popular strategy for reduction of amount of fill-ins during sparse Gaussian elimination (Saad, 2003) (For example the MD is utilised in Epanet2). This strategy selects the node with the smallest degree as the next pivot row which introduces the least number of non-zeros that will be introduced at the corresponding step of Gaussian elimination (Pissanetzky, 1984).

Note that Gaussian elimination, seen in Algorithm 2, is applied from the bottom in the simplification algorithm, hence the obtained RCMK and MD orderings were reversed accordingly (RCMK becomes CMK).

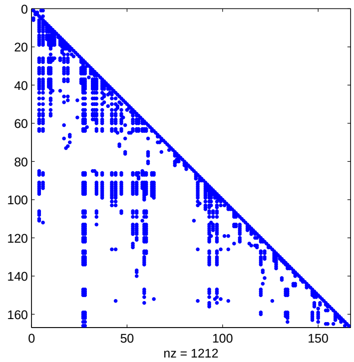
The CMK algorithm used to reorder nodes prior calculation of Jacobian matrix is given in Algorithm 3 (Saad, 2003). Its queue-based implementation was adapted for water network model reduction i.e. only nodes to be removed are ordered. Also MD, given in Algorithm 4 (Amestoy *et al.*, 1996), was modified in the same way. The effectiveness of CMK algorithm depends critically on the choice of starting node. The starting node may be one of minimum degree (Pissanetzky, 1984) or pseudo-peripheral node as proposed by George *et al.* (1994). Here, the latter heuristic was implemented to determine the best starting node for CMK ordering.

A comparison of original and ordered Jacobian matrices is shown in Figure 4.5. The original Jacobian matrix J used in this illustration was obtained from the Rio network, see Section 3.4.2. CMK ordering transformed the structure of the original sparse matrix J shown in Figure 4.5a into a band diagonal form as depicted in Figure 4.5c. While CMK is oriented on sparse matrix profile reduction the MD aims in reduction of number of fill-ins. Hence, they ordered structures are clearly distinctive; compare Figure 4.5c and Figure 4.5e.

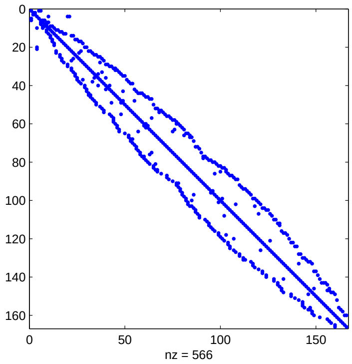
The right hand plots depict the respective Jacobian matrices after Gaussian elimination. It can been clearly seen that level of fill-ins, 1212, in the reduced unordered matrix J^S is much higher than for the ordered versions; 755 for CMK and 544 for MD, respectively.



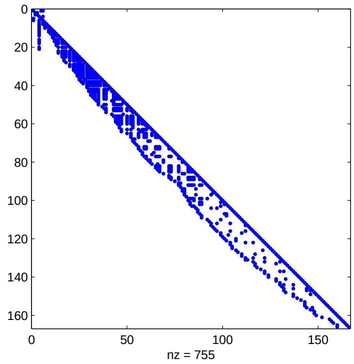
(a) Unordered Jacobian matrix J.



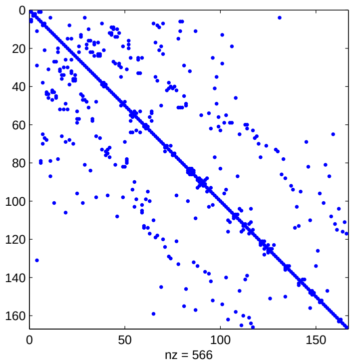
(b) Matrix J^S after Gaussian elimination.



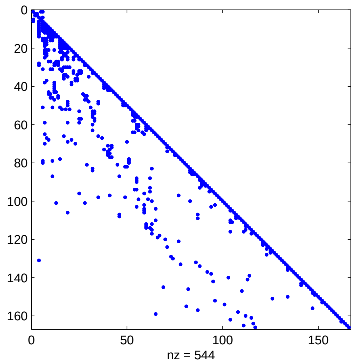
(c) Reversed Cuthill-McKee ordering of Jacobian matrix J_{RCM}.



(d) Matrix J^S_{RCM} after Gaussian elimination.



(e) Minimum degree ordering of Jacobian matrix J_{MD}.



(f) Matrix J^S_{MD} after Gaussian elimination.

FIGURE 4.5: Illustrating the original and ordered Jacobian matrices for Rio network and difference between them in terms of number of fill-ins after Gaussian elimination.

Algorithm 3 Cuthill-McKee ordering of nodes prior to Gaussian elimination

Require: V ▷ Set of all nodes
$K \subset V$ ▷ Subset of nodes to be kept
$R \subset V$ ▷ Subset nodes to be removed
$Find \quad n_0 \in R$ ▷ Find starting node n_0
$Q = \{n_0\}$ ▷ Queue of nodes
$\pi = \varnothing$ ▷ Set of ordered nodes
while $NotEmpty(Q)$ **do**
  $node = Q.Dequeue$
  if $node \notin \pi$ **then**
    $\pi.Append(node)$
    $Sort(node.neighbours)$ ▷ Sort all node neighbours based on their degree
    for all $neighbours$ **do**
      if $neighbour \notin \pi$ **then**
        $Q.Enqueue(neighbour)$
      end if
    end for
  end if
end while
return $K \cup \pi$

Algorithm 4 Minimum degree ordering of nodes prior to Gaussian elimination

Require: V ▷ Set of all nodes
$K \subset V$ ▷ Subset of nodes to be kept
$R \subset V = \{1 \dots n\}$ ▷ Subset nodes to be removed
$\pi = \varnothing$ ▷ Set of ordered nodes
for i = 1 **to** n **do**
  $Create\, elimination\, graph$
  $Find\, node\, x \in R\, with\, minimum\, degree$
  $\pi(i) = x$
  $R_{i+1} = R_i - \{x\}$
end for
return $K \cup \pi$

The reduction of fill-ins was proved correct when the number of calculations (multiplications and divisions) was tracked. The MD ordering resulted in the smallest number of calculations needed during Gaussian elimination, see Table 4.5.

Although the post-elimination matrices have a completely different structure there is no difference between the reduced networks. The obtained numerical results were the same in the parts of "reduced" Jacobian matrices that will be used in the next step to recreate the simplified nonlinear water networks.

Table 4.5: Number of calculations needed to complete Gaussian elimination for unordered and ordered matrices.

Ordering algorithm	Number of calculations
unordered	25704
reversed Cuthill-McKee	17794
minimum degree	15318

Ultimately, the choice whether to use of ordering algorithm is determined by size of the water network to be simplified. For water network models with the number of nodes $n < 500$ no ordering is applied. For larger problems the CMK ordering is chosen; despite it is not optimal it is very fast and easy to implement. Time complexity of CMK for a dense matrix is $O(q_{max}m)$ where q_{max} is the maximum degree of any node and m is the number of links (edges) (George and Liu, 1981). However, for sparse matrices the CMK time complexity is reduced to $O(n)$ (Pedroche *et al.*, 2012). Whereas in case of MD its worst-case requires a $O(n^2m)$ runtime (Heggernes *et al.*, 2001); for sparse problems MD has also much better runtime but as observed by Benzi (2002) and Duff and Meurant (1989) MD does not always succeed and can produce ordering worse than original while CMK ordering is found to be equivalent or slightly better than the natural ordering.

However, the biggest profit from the ordering was reduction of time needed for Gaussian elimination. When the model reduction algorithm was applied to the benchmark network used in Table 4.2, but preceded with CMK or MD ordering of Jacobian matrix, the computational time was reduced to less than 5 seconds. Of course, much more research can be done in this area and investigate other reordering techniques such GPS, ND, approximate minimum degree (Amestoy *et al.*, 1996), and their variations. But, such research would be beyond scope of this work. Nevertheless, it might form an interesting study to investigate effects of different orderings on performance of processing water network graphs represented by sparse matrices.

4.6 Model reduction application

The simplification process, illustrated in Figure 4.1, evolved throughout its implementation into a more sophisticated and extended tool. The final implementation took into account the outcomes from investigation of parallel programming, storage structures for sparse matrices and nodes pre-ordering.

The final application was developed to facilitate the extended simplification process of water distribution networks. The application was compiled to meet the software requirements, and specifically, to test in practice the research elaborated in Chapter 3. The present tool aims at returning a simplified WDN topology which can be still used to perform hydraulic simulation. The application can be used either as a standalone application or as an embedded module in other applications. Indeed, it forms a key module of Finesse 2 software, the successor of Finesses software (description can be found in (Rance *et al.*, 2001)), currently being developed by the WSS members.

The main user workspace of the application is pictured in Figure 4.6. The workspace includes the following elements: network map window, menu bar, status bar and water network elements selection toolbox. A concise description of main elements is provided in the next paragraphs.

The menu bar located at the top of workspace contains a collection of menus used to control the application. It provides standard commands for opening, closing, printing and setting application preferences. The menu bar includes also commands to launch tools such as water network energy audit, network system flow and scaling of total network demand.

The network map window provides means to display a schematic diagram of the objects comprising a water distribution network. The displayed topology is created upon data read from the corresponding .inp file. The crucial elements (Table 4.1) selected to retain are displayed by using different colors. Additionally, the existing objects can be clicked on for marking/unmarking. The map can be printed, zoomed and panned from one position to another. Nodes and links can be drawn at different sizes with ID labels and numerical property values displayed.

The elements selection toolbox is located on the panel right to the network map window. It provides features to mark nodes based on their degree and/or mark pipes based on diameter ranges. Note that all elements listed in Table 4.1 are marked automatically and cannot be unmarked.

The model reduction sub-window allows to choose the operating point for linearisation and provides means to log whole simplification process, log demand redistribution, save new pressure constraints and open the simplified model in Epanet2.

Since its development the model reduction application has been used to reduce many WDN models and has proven to be a practical and reliable tool. An example of utilisation the model reduction application is given in Chapter 5, where it has been used in a practical

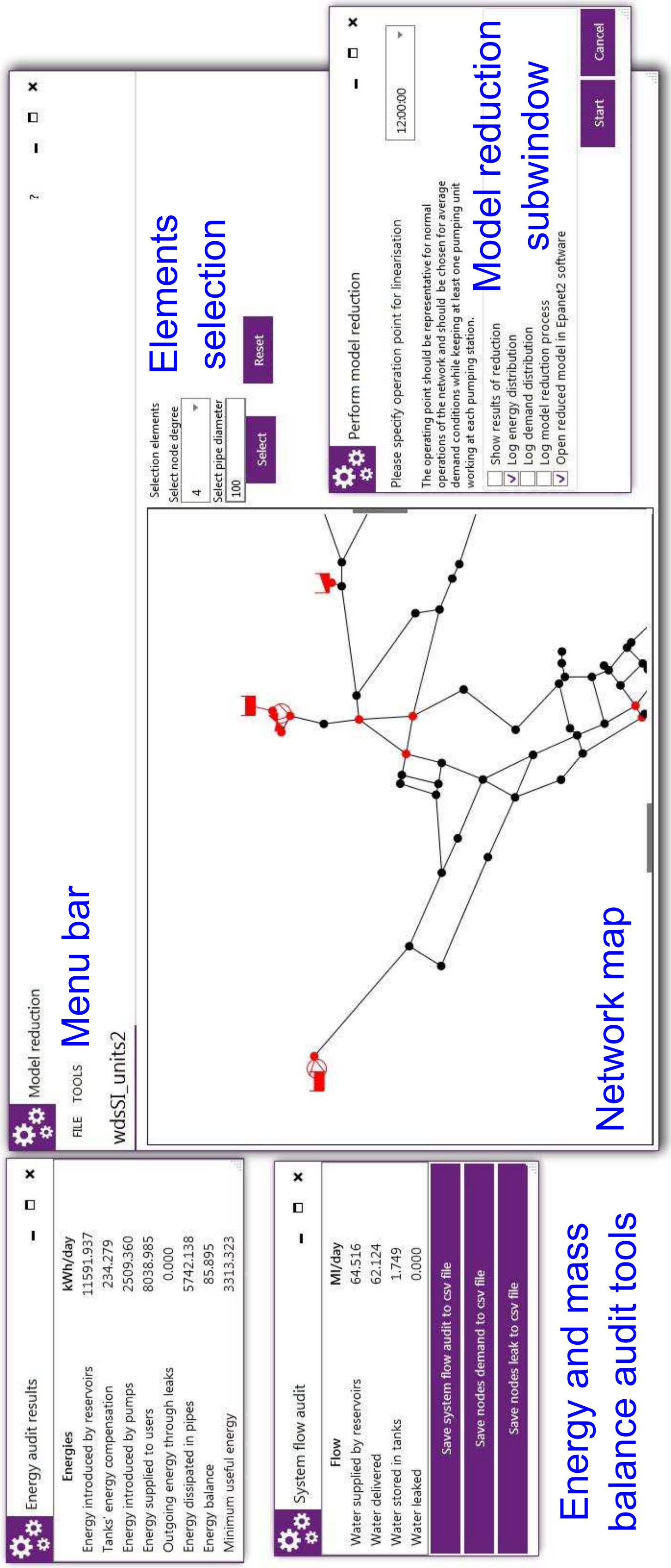


FIGURE 4.6: Illustrating the main window of the developed application.

project focused on determining optimal schedules for control elements in a real large-scale water network exhibiting highly complex topology.

Nevertheless, despite of that this is the first version of the software it has already demonstrated potential to be used in practice not only by academics but also by practitioners.

4.7 Summary

This chapter has dealt with the implementation and further improvement of the extended model reduction algorithm elaborated in Chapter 3. The process of design and development of the research software has been presented with the focus places on the emerged computational research aspects. While all the initial tasks and requirements set for the program have been accomplished, it has been decided to conduct a further research in the computer science areas in order to develop a tool that can actually be used in practical WDSs operation optimisation projects.

Hence, different implementation approaches and their limitations have been investigated. The implementation and graphical user interface (GUI) were coded in the C# programming language. The Epanet2 Toolkit has been used as a hydraulic simulator to perform an extended period simulation of WDN hydraulic behaviour. Parallel programming techniques have been employed to distribute workload of the algorithm across multiple CPU cores which nowadays are present in majority of PCs. The limitations of available data structures to store the matrix representation of water networks along with the benefits of sparse matrix reordering prior Gaussian elimination have been examined. The utilisation of parallel programming techniques and the sparse matrices ordering algorithms have drastically increased the speed of the model simplification.

There are algorithms that work well for theoretical and small systems (see Section 3.2) but they often do not consider practical constraints, hence, they are not actually suitable for real systems, whereas in this work, which started with a theoretical study, a practical tool was created. The developed software is able to simplify the water network model, consisted of several thousands elements, within seconds of calculation time. The advantage of this near real-time model reduction is that can be used to manage abnormal situations and structural changes in a water network, e.g. isolation of part of the network due to a pipe burst. In such case an operator can change the full hydraulic model and run model reduction software to automatically produce the updated simplified model.

The WDN model reduction software could be integrated with other concepts applied to the WDNs or it can be used as a standalone tool for the purpose of the model simplification only. The present tool aims at returning a simplified WDN topology which can be still used to perform hydraulic simulation. Although not necessarily relevant for the simplification process, the features of energy balance audit, system flow audit and scaling of total system demand may prove to be useful and applicable for other research purposes.

Chapter 5

Application of the extended model reduction in optimal scheduling

5.1 Introduction

The newly extended model reduction procedure introduced in Chapter 3 and implemented in Chapter 4 was so far evaluated mostly on the small-scale and hypothetical WDSs. This chapter describes application of the extended reduction method to a real large-scale water network. The study is based on the project carried out by WSS, aimed at optimisation of operation of the considered WDS. The data used in the project concern an actual WDS being part of a major water company in area of southern United Kingdom. The objective was to reduce the cost of energy used for water pumping whilst satisfying all operational constraints, including the pressure constraints in different parts of the water network. The topology of the considered WDS, namely *Water Network*, is illustrated in Figure 5.1.

The considered WDS includes complex structures and interactions between pump stations, e.g. pump stations in series without an intermediate tank, pump stations with by-passes, mixture of fixed-speed and variable-speed pump stations, valves diverting the flow from one pump station into many tanks, PRVs fed from booster pumps or a booster pump fed from a PRV. Hence, it will provide an excellent base for application of the extended model reduction technique.

The main intention in this chapter is to describe in details a procedure how the problem of optimal scheduling in a WDS can be approached and solved. Whilst doing this an emphasis is placed on the importance of appropriate pressure constraints while calculating

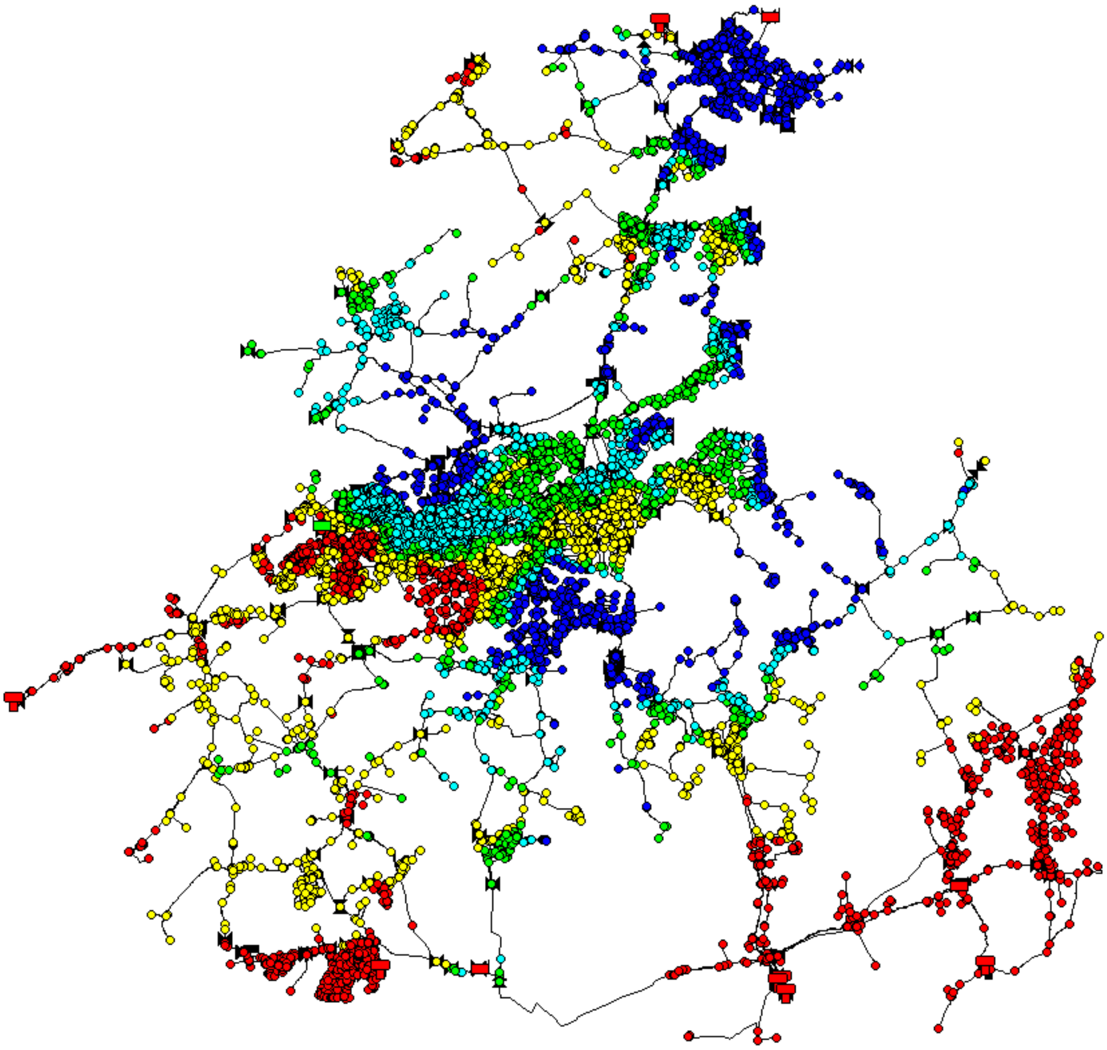


FIGURE 5.1: Model of *Water Network* in Epanet2.

optimal schedules for pumps in a real WDS. Note that in this chapter only fraction of results from the optimisation study is presented. More details can be found in (Skworcow *et al.*, 2013) and (Skworcow *et al.*, 2014a).

The research described in this chapter is a result of joint efforts of the author of this thesis and Dr Piotr Skworcow, a member of the WSS group. The specific contributions to the content of this chapter are summarised in Table 5.1.

The reminder of this chapter is organised as follows. Section 5.2 describes the overall methodology. In Section 5.3 details about obtaining the optimisation-ready model are given. Section 5.4 provides mathematical background of the methodology along with

TABLE 5.1: Contributions to the content of this chapter by the author (column 'A') and by Dr Piotr Skworcow (column 'S').

Description of contribution	A	S
Model understanding	x	
Development of model operation diagram	x	
Model simplification	x	
Model adjustments for optimisation requirements	x	
Generation of topology in the GAMS language		x
Development and implementation of optimisation method		x
Optimisation studies	x	x
Discretisation algorithm		x
Results description and presentation	x	x

outcomes from the optimisation of schedules for pumps. The emphasise is placed on the demonstration of the impact of pressure constraints on the calculated schedules for pumps. In Section 5.5 a problem associated with discretisation of continuous schedules is highlighted. Finally, Section 5.6 provides summary of this chapter.

5.2 Methodology overview

The proposed method for combined energy and pressure management, based on formulating and solving an optimisation problem, is an extension of the pump scheduling algorithms described in (Ulanicki *et al.*, 1999; Bounds *et al.*, 2006; Ulanicki *et al.*, 2007; Skworcow *et al.*, 2010). In contrast to the online optimisation method reported in (Skworcow *et al.*, 2010) here an off-line optimisation study is considered. The concept behind the approach described in (Skworcow *et al.*, 2010) is shown in Figure 5.2 which illustrates how the excessive pumping contributes to a high total cost in two ways. Firstly, it leads to exaggerated energy usage, secondly, it induces high pressure, hence increased leakage, which means that more water needs to be pumped and taken from sources. Therefore the optimiser, by minimising the total cost, attempts to reduce both the energy usage and the leakage.

The optimisation methodology described in this chapter is model-based and, as such, requires a hydraulic model of the network. Such a hydraulic model is usually developed in a modelling environment such as Epanet2, Aquis, Infoworks etc. and consists of three main components: (i) boundary conditions (sources and exports), (ii) a hydraulic nonlinear network made up of pipes, pumps, valves, and (iii) reservoir dynamics. In order to reduce the size of the optimisation problem the full hydraulic model is simplified using

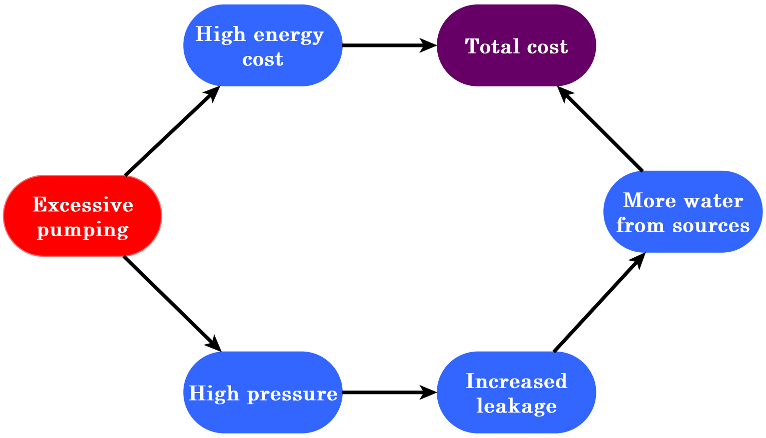


FIGURE 5.2: Illustrating how excessive pumping contributes to high total cost.

the reduction algorithm from Section 3.6. In the simplified model all reservoirs and all control elements, such as pumps and valves, remain unchanged, but the number of pipes and nodes is significantly reduced.

Subsequently, using the reduced hydraulic model and operational constraints an optimal network scheduling problem is generated in a mathematical modelling language, GAMS (Brooke *et al.*, 1998), which calls up a nonlinear programming solver, CONOPT (Drud, 1992) to calculate a continuous optimisation solution. CONOPT is a nonlinear programming solver, which uses a generalised reduced gradient algorithm. An optimal solution is then fed back from CONOPT for analysis and/or further processing. The overall optimisation process is depicted in Figure 5.3.

In the considered optimisation problem some of the decision variables are continuous (e.g. water production, pump speed, and valve position) and some are integer (e.g. number of pumps switched ON). Problems containing both continuous and integer variables are called mixed-integer problems and are hard to solve numerically. Continuous relaxation of integer variables (e.g. allowing 2.5 pumps ON) enables network scheduling to be treated initially as a continuous optimisation problem solved by a nonlinear programming algorithm. Subsequently, the continuous solution can be transformed into an integer solution by manual post-processing, or by further optimisation. For example, the result “2.5 pumps ON” can be realised by a combination of 2 and 3 pumps switched over the time step.

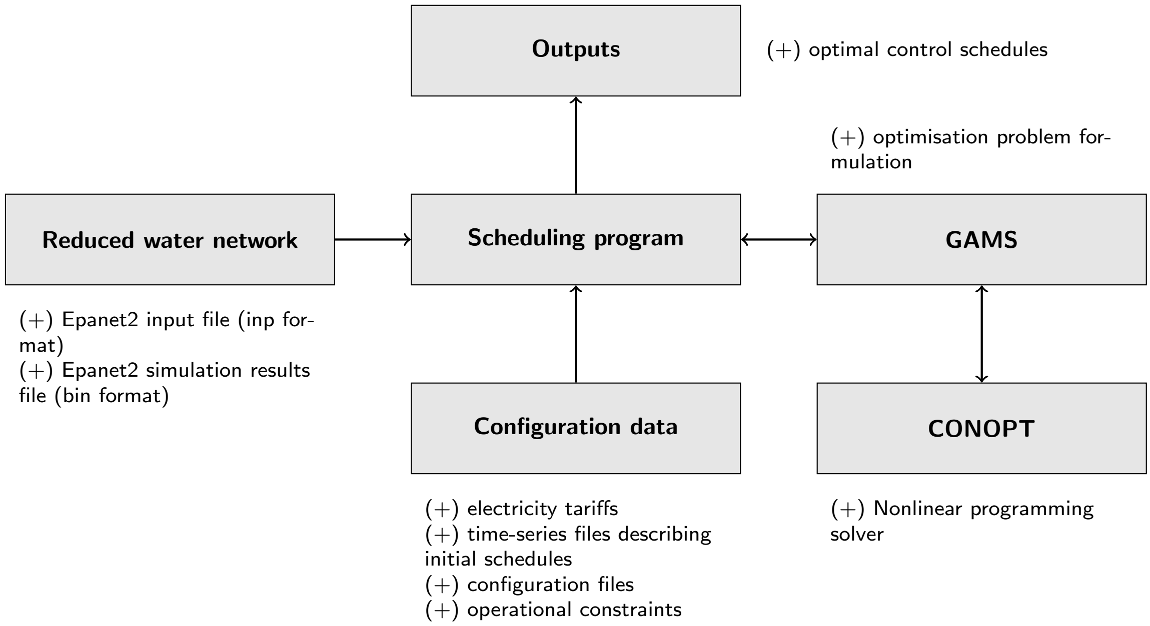


FIGURE 5.3: Overall optimisation process.

5.3 Modelling for optimal scheduling

5.3.1 Water network model

The water network model provided by the water company, henceforth referred as *Water Network*, was in the Epanet2 format (Rossman, 2000b) and contains 12363 nodes, 12923 pipes, 4 (fixed-head) reservoirs, 9 (variable-head) tanks, 10 pumps and 315 valves. The average demand in *Water Network* is 451 l/s (39 Ml/day). The considered WDN includes complex structures and interactions between pumps, e.g. pumps in series without an intermediate tank, pumps with by-passes, mixture of fixed-speed and variable-speed pumps, valves diverting the flow from one pump station into many tanks, PRVs fed from booster pumps or a booster pump fed from a PRV. Hence, to get a better understanding of *Water Network* operation a diagram, shown in Figure 5.4, was produced. More detailed diagram is provided in Appendix D.

The major source of water in the network is *Reservoir 2*. In the provided model *Reservoir 2* has the head defined as a time pattern in order to model the head from the water treatment works, namely *WTW 1*. (Note that throughout the project the initial model was several times updated with additional data from the water company e.g. in the final optimisation model, *Reservoir 2* was replaced with a pump station including 5 pumps and a buffer tank. More details about modelling of the inflow from *WTW 1* are given

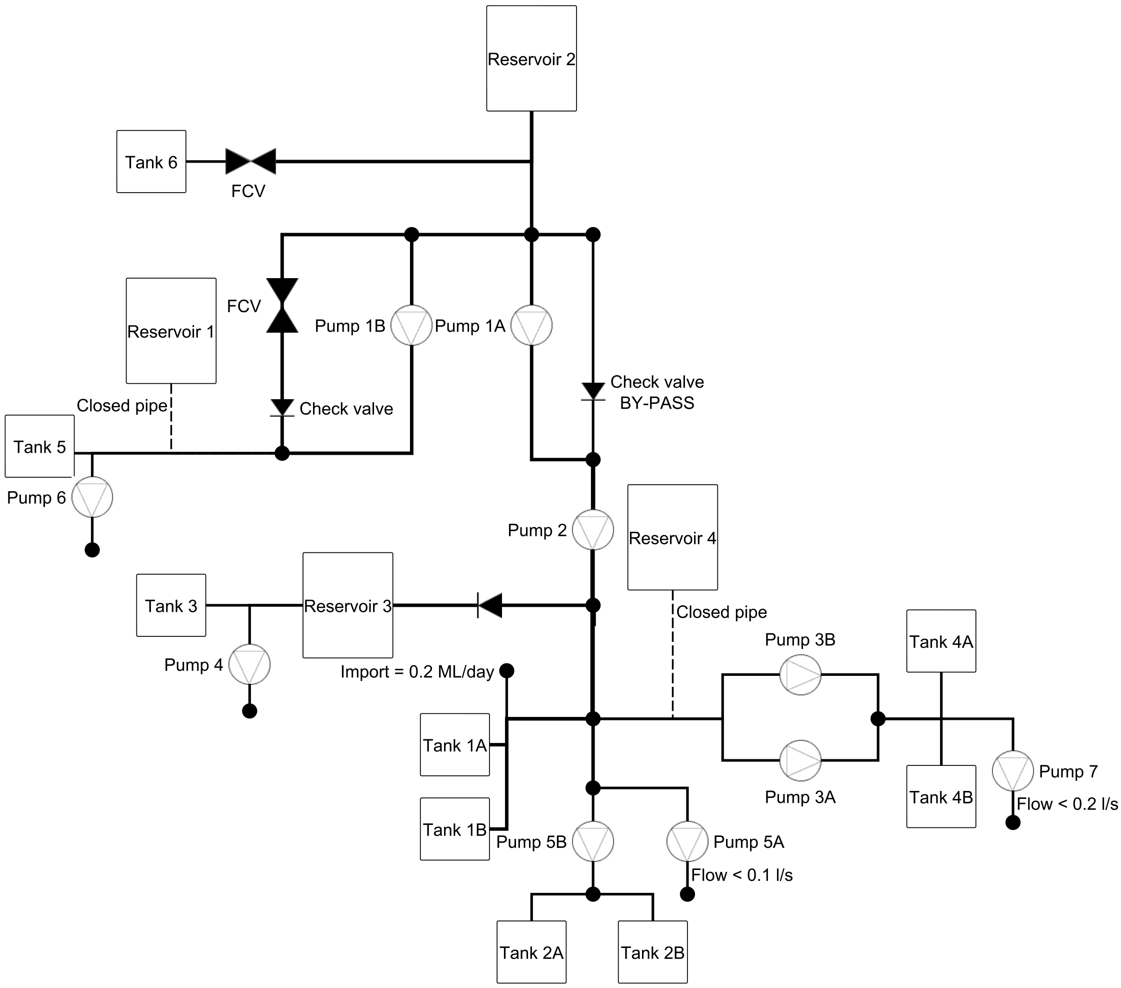


FIGURE 5.4: Initial schematic of *Water Network*.

in Section 5.3.2.3.) Another inflow to the system is from *Reservoir 3* with an imposed fixed head of 118 m. The remaining two reservoirs are disconnected from the system i.e. pipelines connecting *Reservoir 1* and *Reservoir 4* are permanently closed. It was assumed that they work as redundant reservoirs and should not be used in this optimisation study.

The major pumps in the provided model are *Pump 1A*, *Pump 1B* and *Pump 2* with the average flows of 197 l/s, 106 l/s and 103 l/s, respectively. Daily operation of all the pumps is described in Section 5.3.1.3.

The model includes 315 valves (CVs are not included in this count): 272 TCVs, 42 PRVs and 1 FCV. Details about the valves operation are given in Section 5.3.1.4.

Two time-based control rules are included in the model. The first rule controls flow of *TCV 1* towards *Tank 6*. The second control rule is applied to a pipe which along with *FCV 1* forms *Pump 1B* by-pass.

No major exports were found in the model.

All time patterns in the original model used the 15 minutes sampling interval. Note that "time pattern" is specific to Epanet2 software and it allows to vary demands at the nodes in a periodic way over the course of a day (Rossman, 2000b).

5.3.1.1 Water network operation

From the network topology, illustrated in Figure D.1, and results of simulation performed in the Epanet2 simulator the following 1-day snapshot of the system operation was concluded.

A part of the inflow from *WTW 1*, controlled by *TCV 1*, flows "gravitationally" towards *Tank 6*. However, the most of the inflow from *WTW 1* flows towards *Pump 1A* and *Pump 1B*. *Pump 1B* pumps water towards *Tank 5* and *Pump 1A* distributes the water flow via the booster pump, *Pump 2*, towards the tanks *Tank 1A*, *Tank 1B*, *Tank 2A* and *Tank 2B*. Note that to deliver water to the tanks *Tank 2A* and *Tank 2B* the booster pump, *Pump 5B*, is employed.

Reservoir 3 along with *Tank 3* and *Pump 4* form a subsystem that have very little interaction with the main water distribution system.

Two small, 0.25 Ml tanks, *Tank 4A* and *Tank 4B* along with *Pump 3A*, *Pump 3B* and *Pump 7* are used to supply water to highly elevated part of the network.

The next sections describes in greater detail the hydraulic parameters and results of a hydraulic simulation for the crucial system elements such reservoirs, tanks, pumps and valves.

5.3.1.2 Reservoirs and tanks

The original model includes 4 (forced-head) reservoirs and 9 (variable-head) tanks. The reservoirs and tanks parameters are listed in Table 5.2. Only *Reservoir 2* and *Reservoir 3* are sources of water for the systems. The pipelines connected to *Reservoir 1* and *Reservoir 4* are closed. It was assumed that these disconnected reservoirs might work as redundant reservoirs to be used in emergency cases and should not be considered in this study. Figure 5.5 illustrates the head trajectories and outflows from the reservoirs.

TABLE 5.2: Reservoirs and tanks parameters.

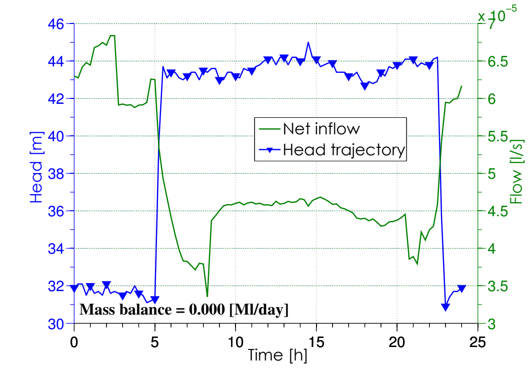
Name	Diameter [m]	Cross-sectional area [m^2]	Total volume [Ml]	Elevation [m]	Initial level [m]	Operational Min. [m]	constraints Max. [m]
Reservoir 1	-	-	-	31.9	-	-	-
Reservoir 2	-	-	-	115.48	-	-	-
Reservoir 3	-	-	-	118.00	-	-	-
Reservoir 4	-	-	-	88.43	-	-	-
Tank 1A	53.52	2249.45	10.28	97.87	3.17	0	4.57
Tank 1B	56.16	2473.12	11.50	97.79	3.34	0	4.65
Tank 2A	15.34	184.78	0.85	122.05	0	0	4.6
Tank 2B	18.87	279.57	1.30	122	3.3	0	4.65
Tank 3	41.70	1365.52	5.98	82.12	2.66	0	4.38
Tank 4A	9.73	74.41	0.25	172.47	2.50	0	3.36
Tank 4B	9.73	74.41	0.25	172.57	2.40	0	3.36
Tank 5	35.68	999.99	4.50	92.5	2.24	0	4.5
Tank 6	42.89	1444.44	6.50	63.79	3.45	0	4.5

Figure 5.6 depicts the head changes and net inflows of the tanks. Investigation of the simulation results and the model topology revealed that the following pairs of tanks: *Tank 1A* and *Tank 1B*, *Tank 2A* and *Tank 2B*, *Tank 4A* and *Tank 4B* are interlinked; i.e. the water level trajectories in the interlinked tanks are correlated, see Figure 5.6a and Figure 5.6b, and, Figure 5.6e and Figure 5.6f, respectively. Hence, it was considered that these pairs of tanks could be merged to reduce number of variables for the optimisation algorithm.

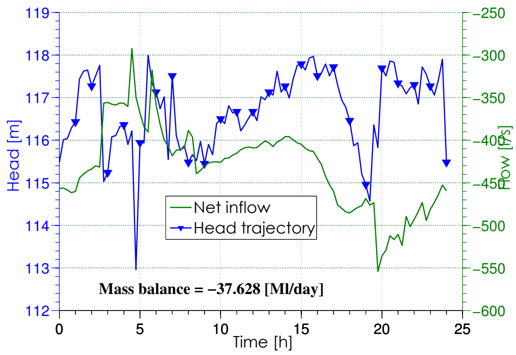
5.3.1.3 Pumps

The provided model contains 10 pumps. All the pumps were provided with the hydraulic curves (head versus flow relation). The model analysis revealed that all the pumps are modelled as separate pumps; no pump stations were found. Note that, based on the subsequent updates from the water company, in the final optimisation model the pumps' configurations were significantly modified to reflect more accurately the real *Water Network* operation. Details of these modifications are given in Section 5.3.2.3.

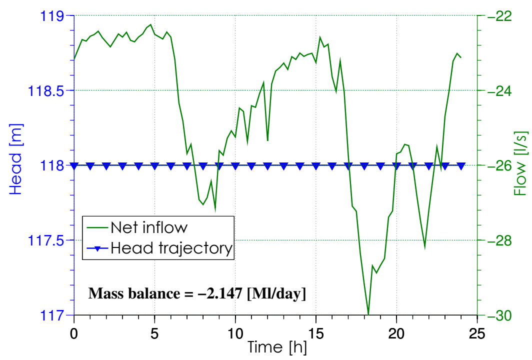
Figure 5.7 illustrates the pumps' operation for the period of 24 hours. Note that in Figure 5.7 the control schedule for a single pump corresponds to the normalised pump speed.



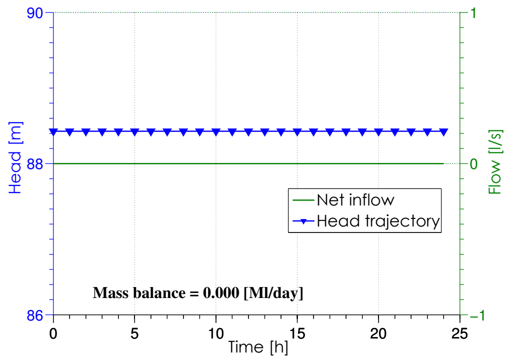
(a) Operation of *Reservoir 1*.



(b) Operation of *Reservoir 2*.



(c) Operation of *Reservoir 3*.



(d) Operation of *Reservoir 4*.

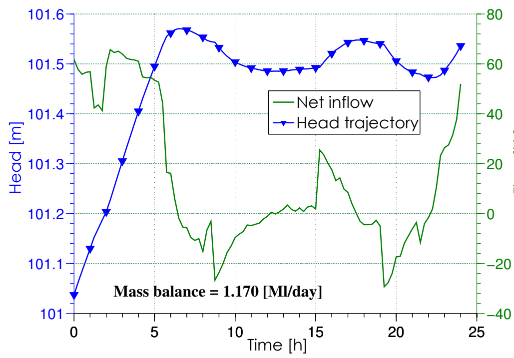
FIGURE 5.5: Operation of forced-head reservoirs in *Water Network*. The "mass balance" annotation represents the total daily inflow to the water distribution system. Note that there is no inflow to the system from *Reservoir 4* and the inflow from *Reservoir 1* is negligible.

5.3.1.4 Valves

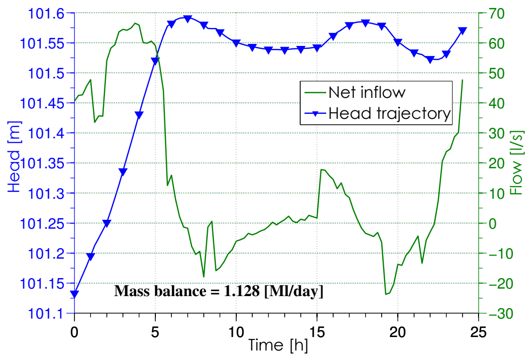
Water Network model contains of 347 valves. Table 5.3 provides an overview of the valve types used in the model.

TABLE 5.3: Valves in *Water Network*.

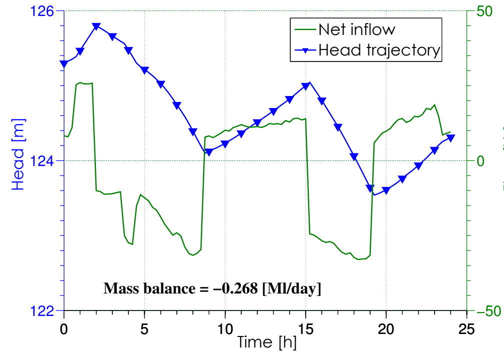
Type	Quantity
flow control valve (FCV)	1
pressure reducing valve (PRV)	42
throttle control valve (TCV)	272
check valve (CV)	32
Total	347



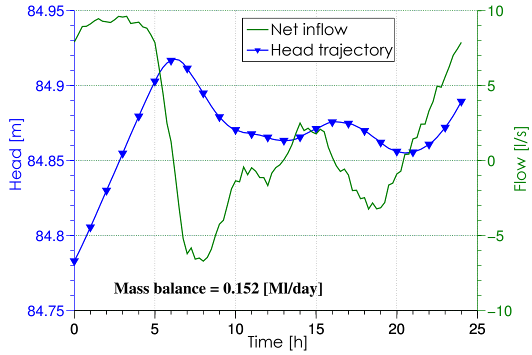
(a) Operation of *Tank 1A*.



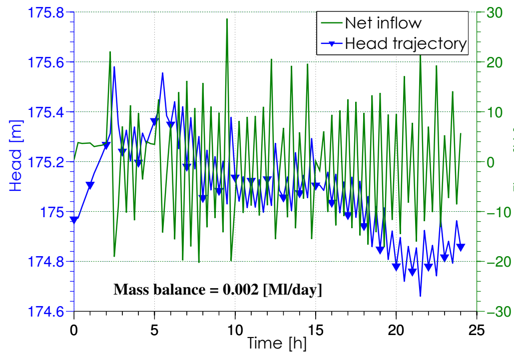
(b) Operation of *Tank 1B*.



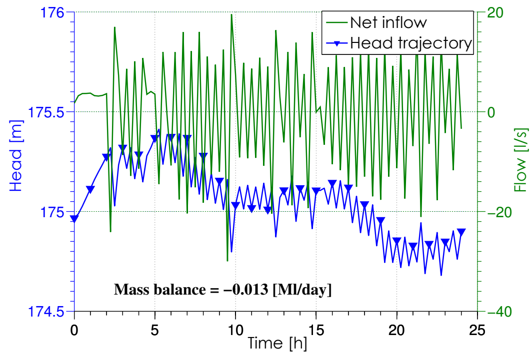
(c) Operation of *Tank 2B*.



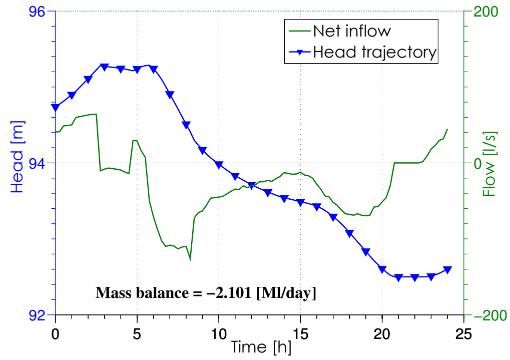
(d) Operation of *Tank 3*.



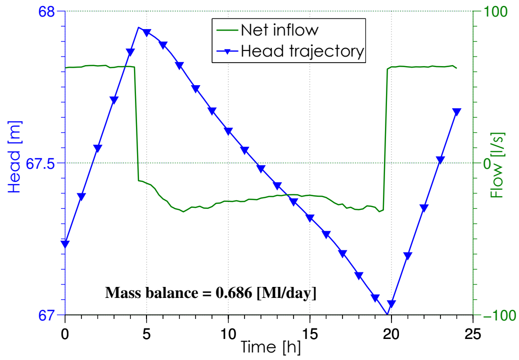
(e) Operation of *Tank 4A*.



(f) Operation of *Tank 4B*.

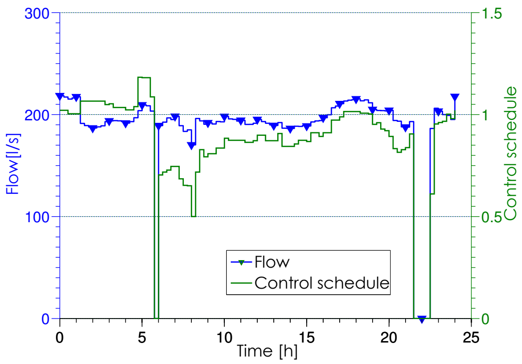


(g) Operation of *Tank 5*.

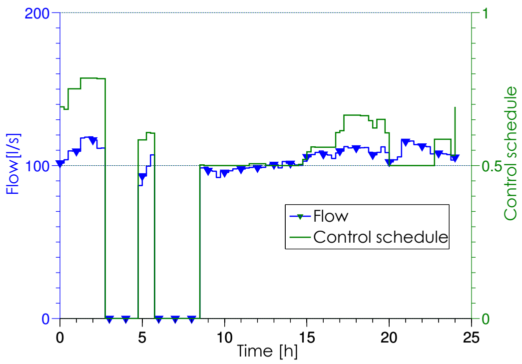


(h) Operation of *Tank 6*.

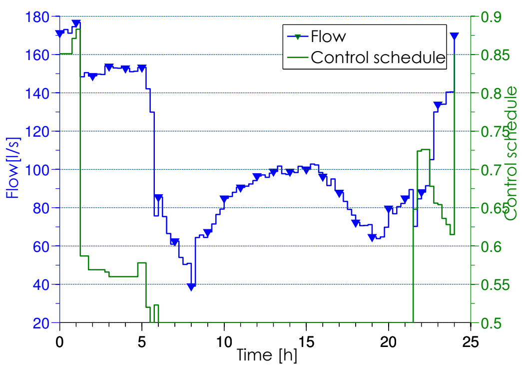
FIGURE 5.6: Operation of tanks in *Water Network*. The "mass balance" calculated for the period of 24 hours, indicates a difference between the initial and the final water level in the tank. The positive mass balance means that the tank is filling up. Consequently, the negative mass balance indicates that the tank is emptying. *Tank 2A* is connected to the system via a closed pipe therefore its operation is omitted here.



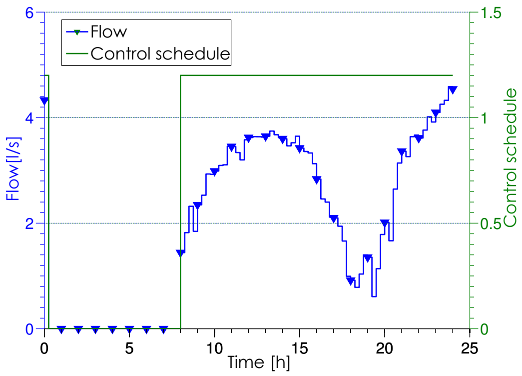
(a) Operation of *Pump 1A*.



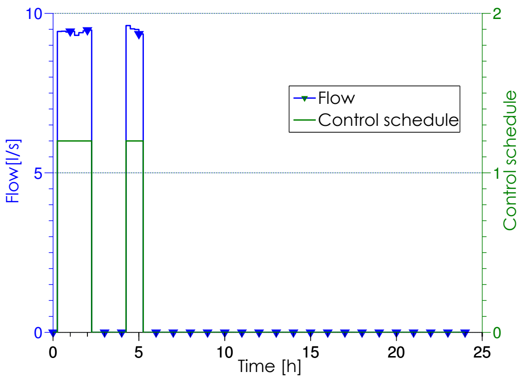
(b) Operation of *Pump 1B*.



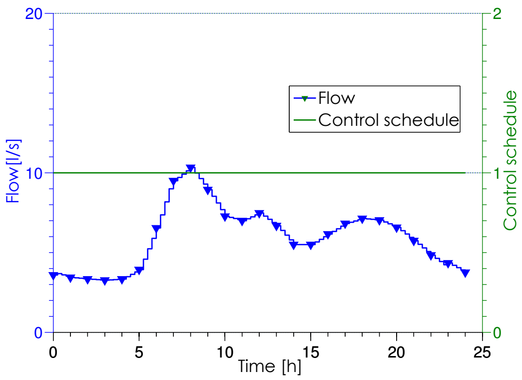
(c) Operation of *Pump 2*.



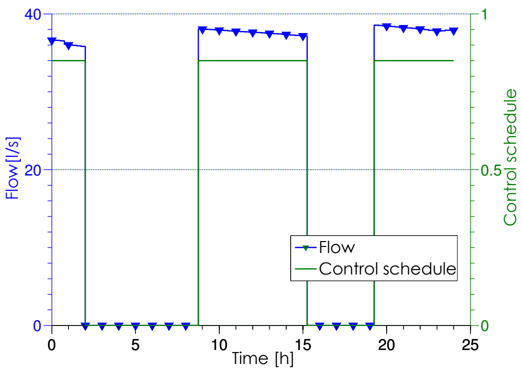
(d) Operation of *Pump 3A*.



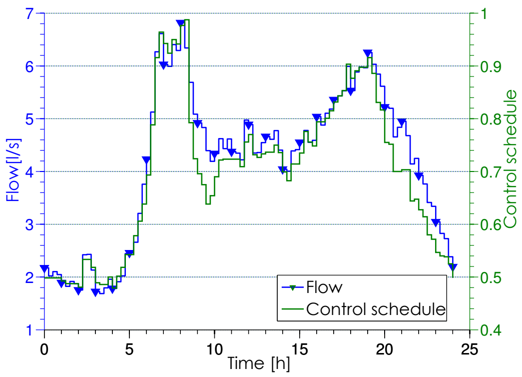
(e) Operation of *Pump 3B*.



(f) Operation of *Pump 4*.



(g) Operation of *Pump 5B*.



(h) Operation of *Pump 6*.

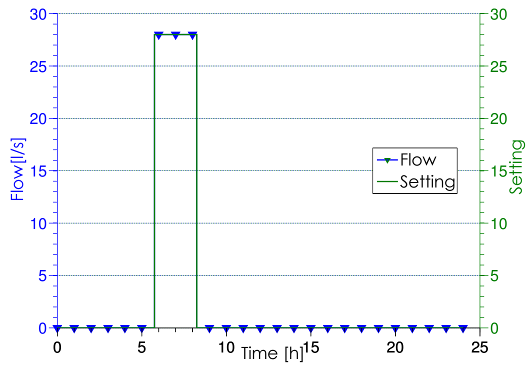
FIGURE 5.7: Operation of pumps in *Water Network*. *Pump 5A* and *Pump 7* are omitted in this Figure as their average flow is less than 0.2 l/s. Note that the term "control schedule" in this figure corresponds to the normalised pump speed.

The majority of the TCV valves, 258, were modelled with a fixed setting of 1e9. From the simulator perspective these valves are almost closed; i.e. the average flow through them is negligible ($\leq$ 0.01 l/s). Furthermore, remaining 14 TCVs have the setting parameter fixed for the simulation period. Hence, from simulation perspective such valves are not distinguishable from pipes. Therefore, it was proposed to either convert TCVs to equivalent pipes or remove "closed" valves in order to reduce the number of variables for the optimisation algorithm.

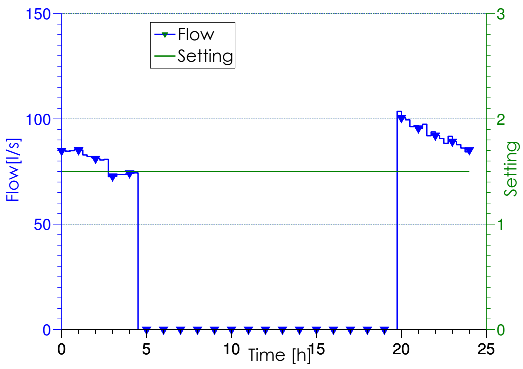
The model contains 42 PRVs. As PRVs are crucial elements of water distribution networks their performance was carefully investigated. Analysis revealed a number of issues with the pressure in the network and PRV settings, e.g. in a number of PRVs, their threshold was set to 44m but pressure at its inlet was below 44m over 3 hours.

Many valves remained fully closed for the 24h simulation period. Therefore, to reduce the number of retained elements in the simplified model, it was proposed to remove the closed valves from the model.

Two valves in the system, *FCV 1* and *TCV 1* were controlled via pre-defined time-based schedules as shown in Figure 5.8.



(a) Operation of *FCV 1*.



(b) Operation of *TCV 1*.

FIGURE 5.8: Operation of the time-controlled valves in *Water Network*. Note that for FCV term "setting" refers to allowed flow trough the valve and for throttle control valve (TCV) term "setting" refers to minor head loss coefficient of the valve.

5.3.1.5 Electricity tariff

The electricity tariff used in this case study was extracted from the provided data and is illustrated in Figure 5.9.

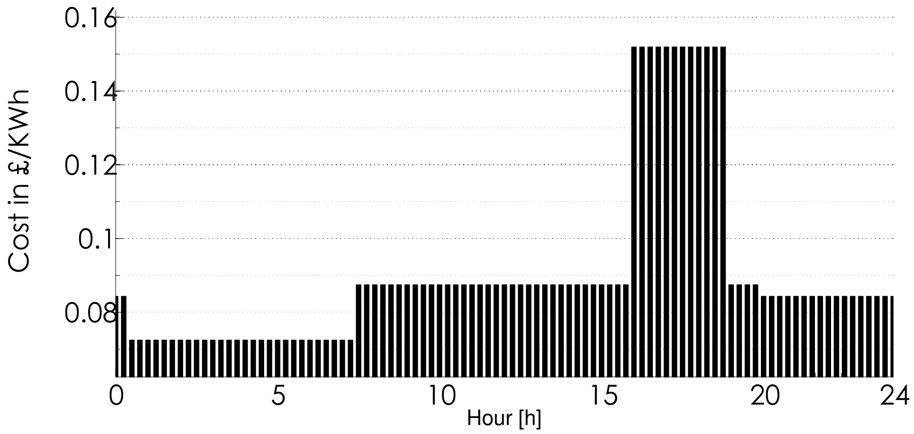


FIGURE 5.9: Electricity tariff to be used in the optimisation study.

5.3.2 Process of ***Water Network*** model reduction.

To reduce the size of the optimisation problem the model needed to be reduced. The model reduction process applied to *Water Network* proceeds trough three stages: model preparation, model reduction and final adjustments to optimisation model.

5.3.2.1 Model preparation

The model preparation stage involves an analysis of *Water Network* operation and identification of the important water network hydraulic elements to be retained. As a result, a number of issues were identified that needed to be addressed in order to meet the requirements of the simplification method. The changes and modifications to the model were as follows:

- The simplification algorithm requests a model described using the Hazen-Williams head loss formula. Therefore, the model was converted from the Darcy-Weisbach formula to the Hazen-Williams formula. However, as the conversion procedure from Darcy-Weisbach to Hazen-Williams is less accurate for low flows and it was carried out at the operating point when at least one of major pumps was ON. Hence, the obtained reduced model shall be sufficiently accurate for all operating condition present in the considered system.

- *Reservoir 1* and *Reservoir 4* are connected to the system via permanently closed pipelines. It was assumed that they should not be used in the optimisation study and they were removed from the model.

- The negative pressure level for *Reservoir 2*. The reservoir elevation was changed to account for the negative pressure. The pattern associated with *Reservoir 2* was respectively scaled.

- *Water Network* model includes a time-based control applied to a pipe. A time-controlled pipe cannot be included in an optimisation model. Pipes are not controllable elements as their head loss characteristic is fixed. The control rule was saved as a time-pattern and transferred to the adjacent FCV.

- All the permanently closed valves and pipes in the model were removed.

- Epanet2 reported *model unbalanced* or *model unstable* errors when e.g. a closed pipe was removed from the system. It was assumed that the source of problem was not in the model itself but in the Epanet2 computational engine. Epanet2 hydraulic options were iteratively adjusted in order to improve the stability of the model. The maximum number of trials used to solve the nonlinear equations that govern network hydraulics at a given point in time was increased to 500 and the convergence criterion used to signal that a solution has been found to the nonlinear equations was set to 0.005.

- TCVs set to work as isolation valves were either removed or replaced with equivalent pipes.

- Head losses at fixed-open TCVs were minimal and the valves could simply be removed, however in the study they were replaced by pipes which gave the same head loss at the representative operating point.

- A number of issues with a pressure in the network and the PRVs settings, e.g. PRV threshold was set 44m but pressure at its inlet is below 44m during 3 hours. The performance of each PRV was investigated to determine its importance to the system. The valve with low importance were either removed or converted to equivalent pipes.

- *TCV 1* was converted to equivalent FCV.

5.3.2.2 Model simplification

The prepared hydraulic model was simplified using the module reduction algorithm described in Chapter 4. In the reduced model all reservoirs and all control elements, such as pumps and valves, remained unchanged, but the number of pipes and nodes was significantly reduced. It should be noted that the connections (pipes) generated by module reduction algorithm may not represent actual physical pipes. However, as described in Section 3.3, the parameters of these connections were computed such that the simplified and full models were equivalent mathematically.

The model reduction algorithm requires an operation point around which the model will be linearised. Following the recommendation in (Alzamora *et al.*, 2014) that the operating point should be representative for normal operations of the network and should be chosen for average demand conditions while keeping at least one pumping unit working at each pumping station, the operating point was defined at 12:30 hour.

The outcome of the simplification method was a reduced model, depicted in Figure 5.10. From Table 5.4 can been seen that almost 98% elements (mostly pipes and junctions) were removed.

Table 5.4: Comparative statistics between the original and reduced models of *Water Network*.

Component	Original model statistics	Reduced model statistics	Scope of reduction %	MBE Ml/day	TRE %	R^2	MAE m or l/s	MRE %	RMSE m or l/s
Junctions	12363	164	98.67	-	-	-	0.864	2.311	1.066
Pipes	12923	335	97.41	-	-	-	-	-	-
Reservoirs	4	2	50.00	0.338	-	1	-	-	-
Tanks	9	9	0.00	-0.075	3.365	0.886	-	-	-
Pumps	10	10	0.00	-	-	0.986	1.315	-	1.657
Valves	315	43	86.35	-	-	-	1.029	-	1.226

Comparisons of the simulated pumps' operations in both models, original and simplified, are depicted in Figures from 5.11 to 5.18. The R^2 indicator calculated for the flow trajectories of each pump showed that the reduced model replicates the pumps' original performance with an appropriate accuracy that meets the requirements of the optimal scheduling method.

Subsequently, the simulated tanks' trajectories were compared (see Figures from 5.19 to 5.29). It can been seen from the illustrated trajectories and calculated indicators that the reduced model adequately replicates the hydraulic behaviour of the original model. The

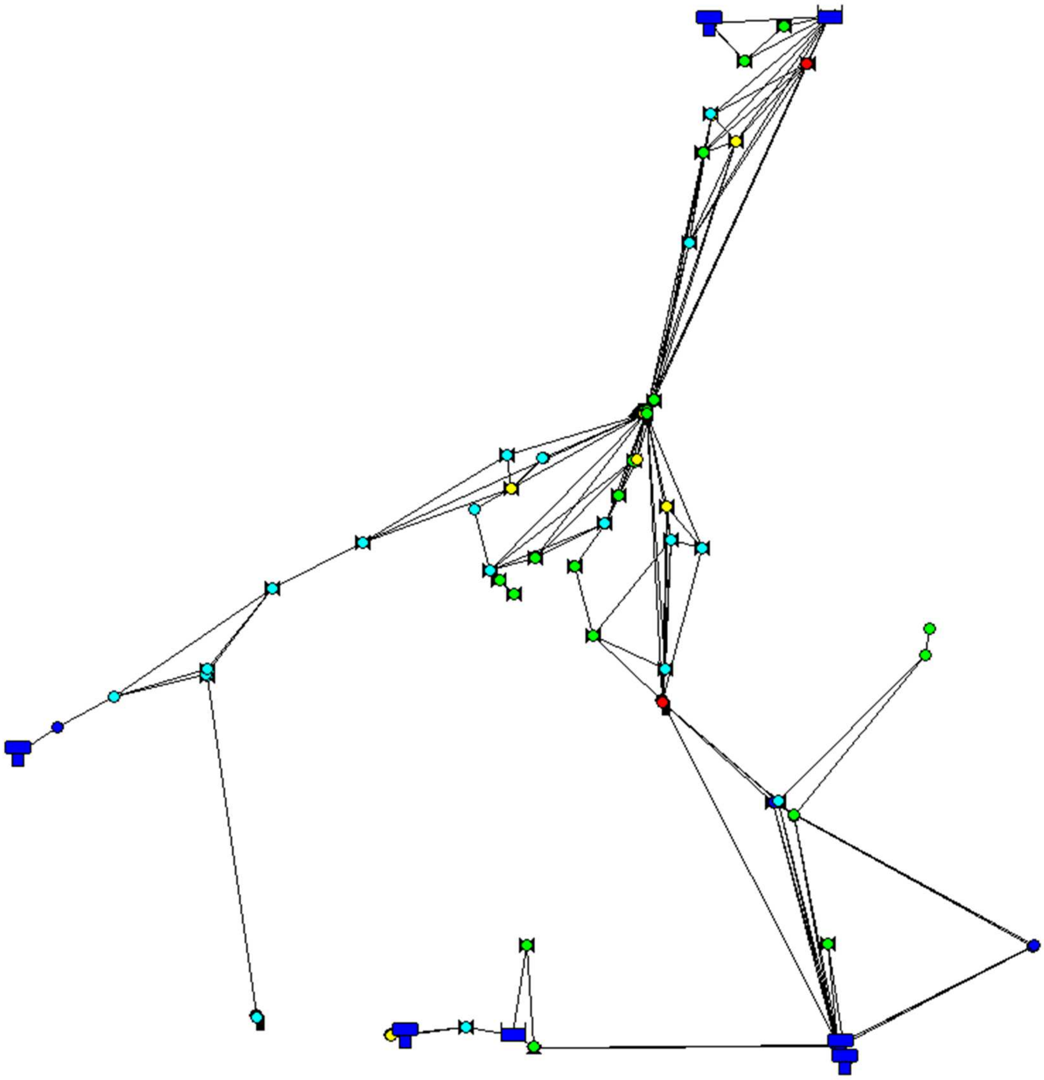


FIGURE 5.10: The reduced model of *Water Network* in Epanet2.

most significant discrepancy found was in the trajectories for *Tank 5*, see Figure 5.28. The main reason behind such discrepancy were structural changes to *Pump Station 1* in the reduced model due to nodes and pipes removal. However, it should be highlighted that the *Tank 5* trajectory in the original Epanet2 model was in fact not corresponding to the data provided subsequently by the water company in the form of screenshots from the SCADA system. Additionally, in the original model *Tank 5* was empty at 20:45, what was resulting in negative pressures at nodes nearby the tank. Therefore, as *Tank 5* trajectory in the simplified model was closer to the more recent data from the SCADA system and the tank was not emptying, it was decided to accept this discrepancy in the reduced model.

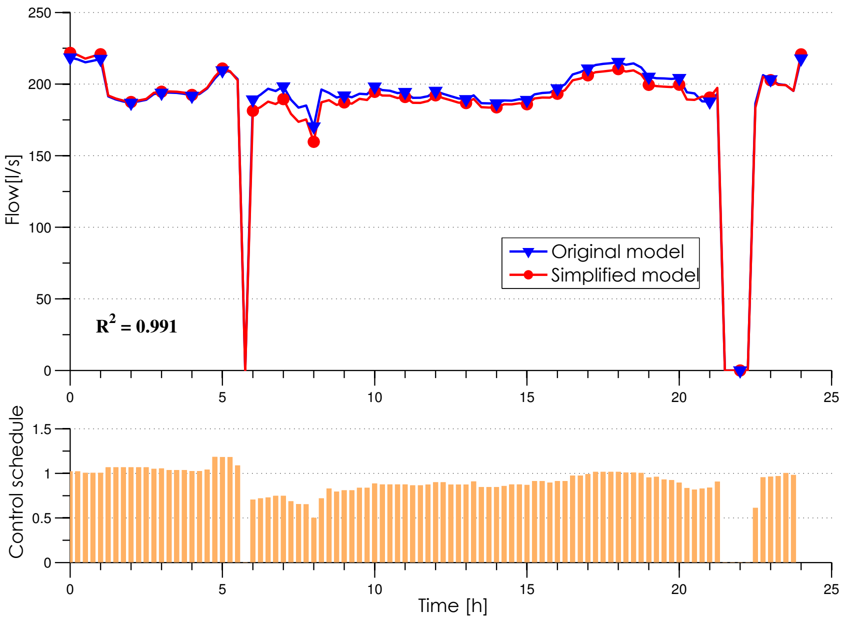


FIGURE 5.11: Performance of *Pump 1A* in the original and simplified model. The term "control schedule" corresponds to the pump normalised speed.

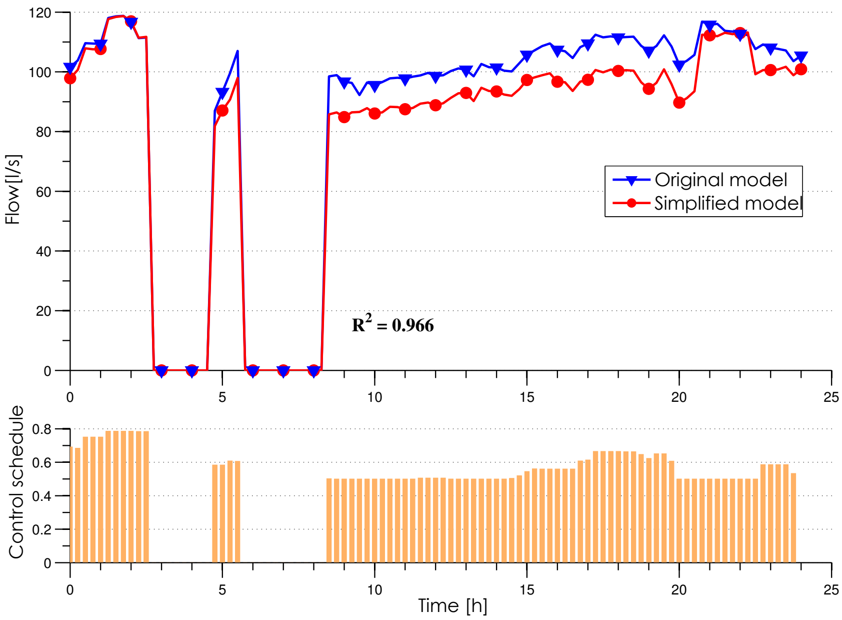


FIGURE 5.12: Performance of *Pump 1B* in the original and simplified model.

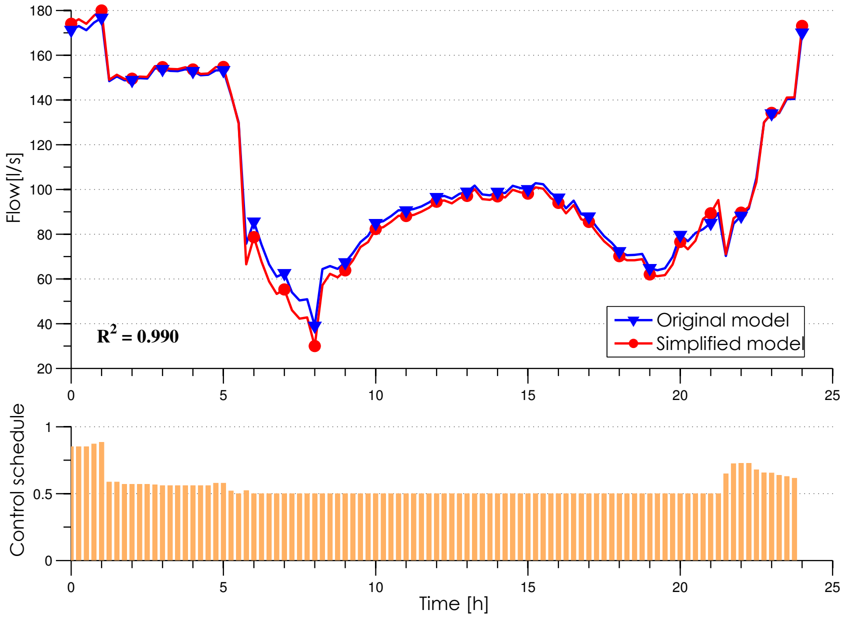


FIGURE 5.13: Performance of *Pump 2* in the original and simplified model.

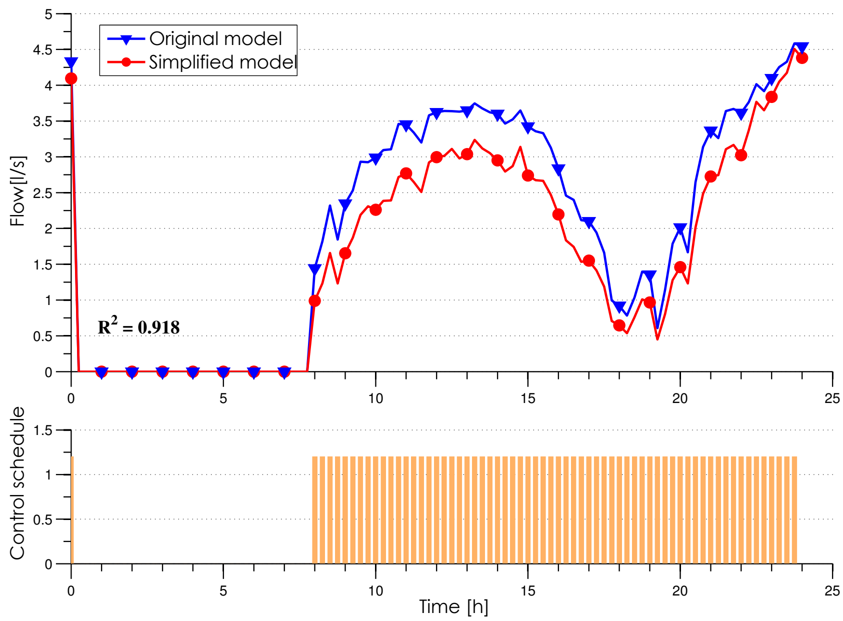


FIGURE 5.14: Performance of *Pump 3A* in the original and simplified model.

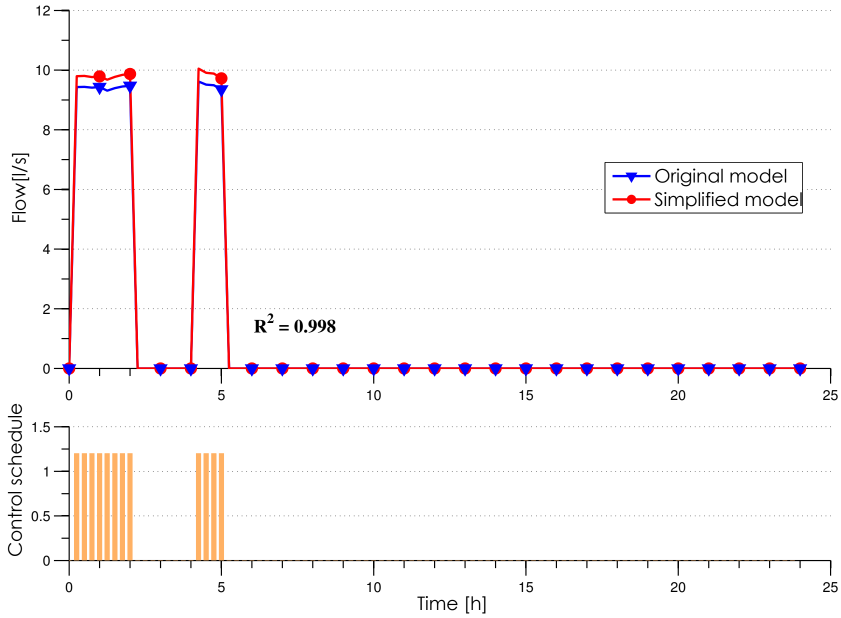


FIGURE 5.15: Performance of *Pump 3B* in the original and simplified model.

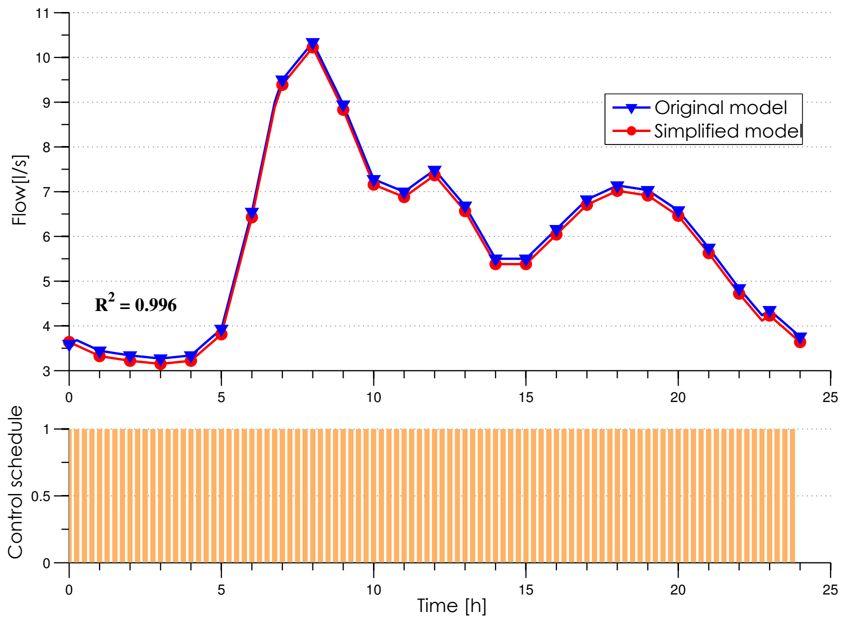


FIGURE 5.16: Performance of *Pump 4* in the original and simplified model.

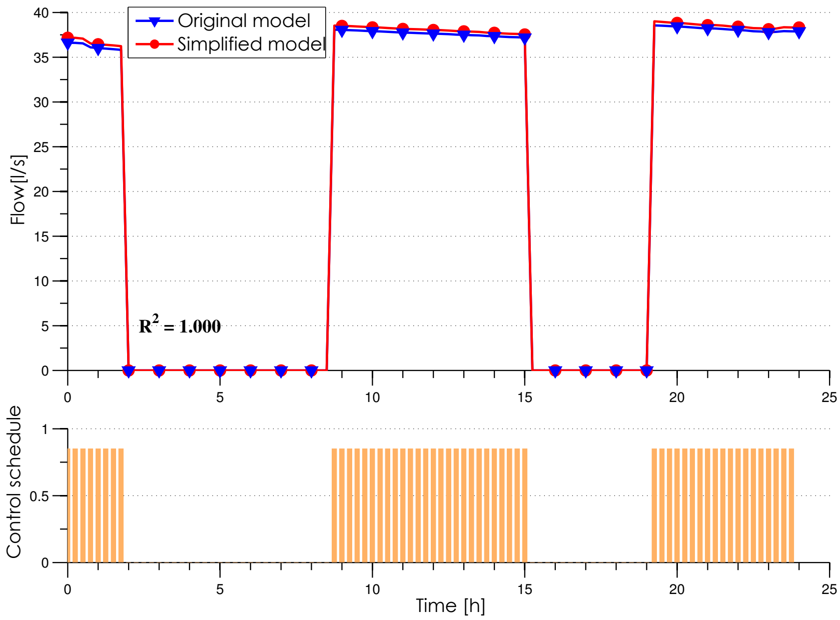


FIGURE 5.17: Performance of *Pump 5B* in the original and simplified model.

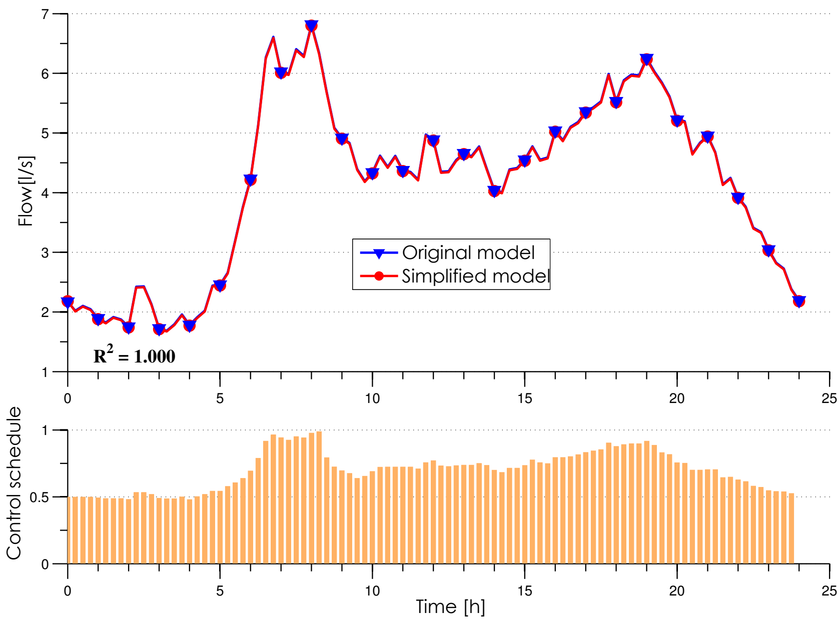


FIGURE 5.18: Performance of *Pump 6* in the original and simplified model.

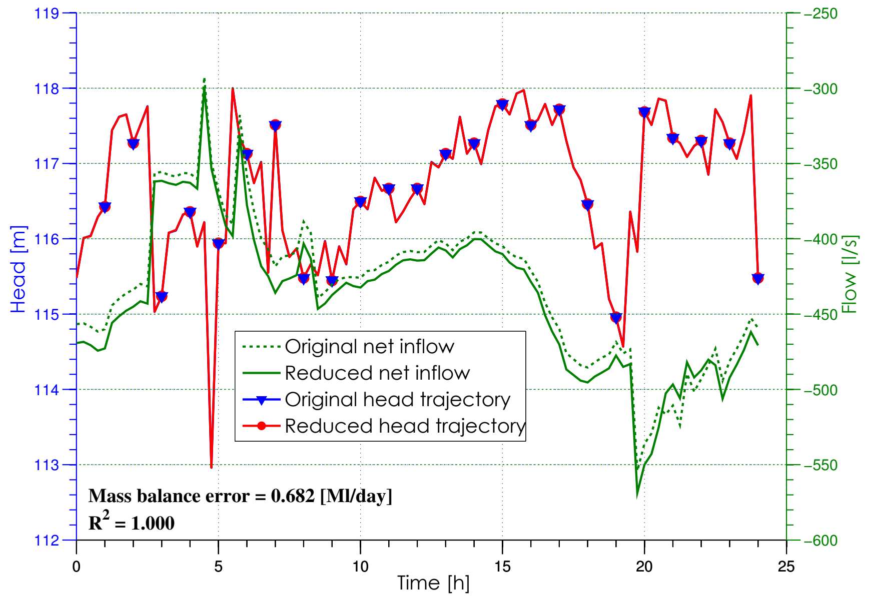


FIGURE 5.19: Comparison of simulated trajectories for *Reservoir 2*.

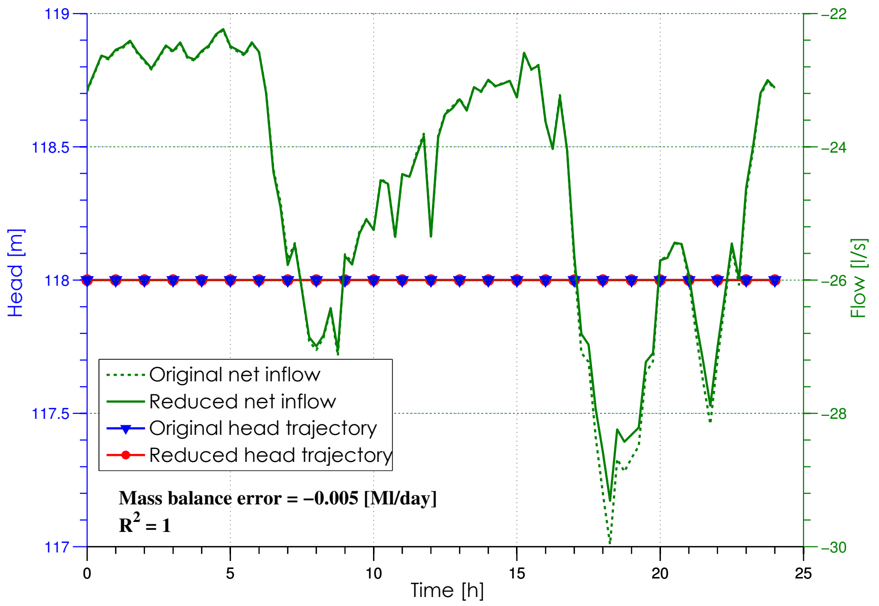


FIGURE 5.20: Comparison of simulated trajectories for *Reservoir 3*.

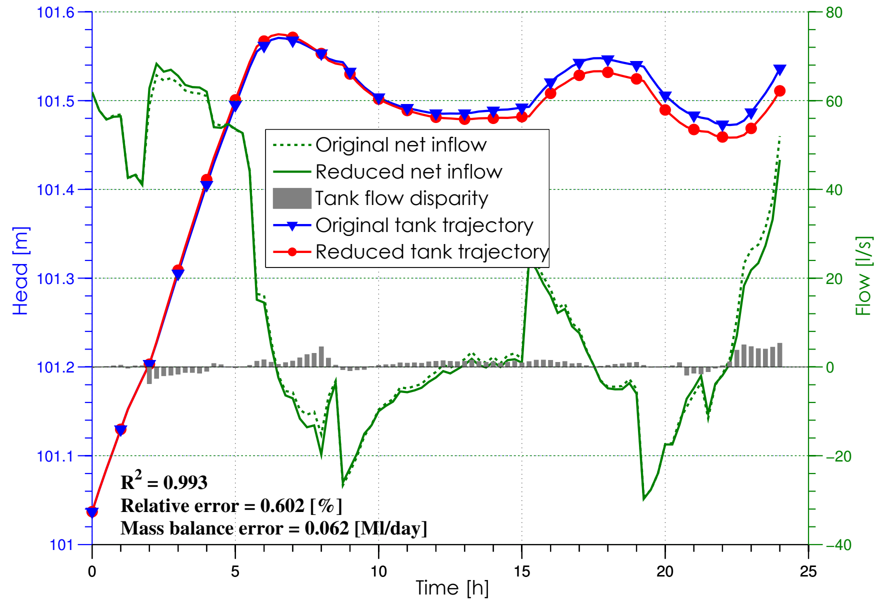


FIGURE 5.21: Comparison of simulated trajectories for *Tank 1A*.

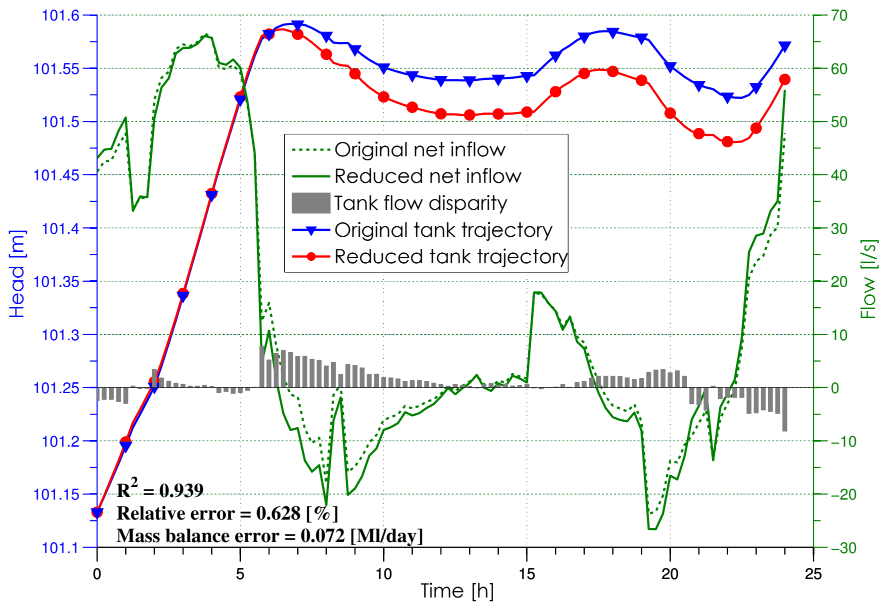


FIGURE 5.22: Comparison of simulated trajectories for *Tank 1B*.

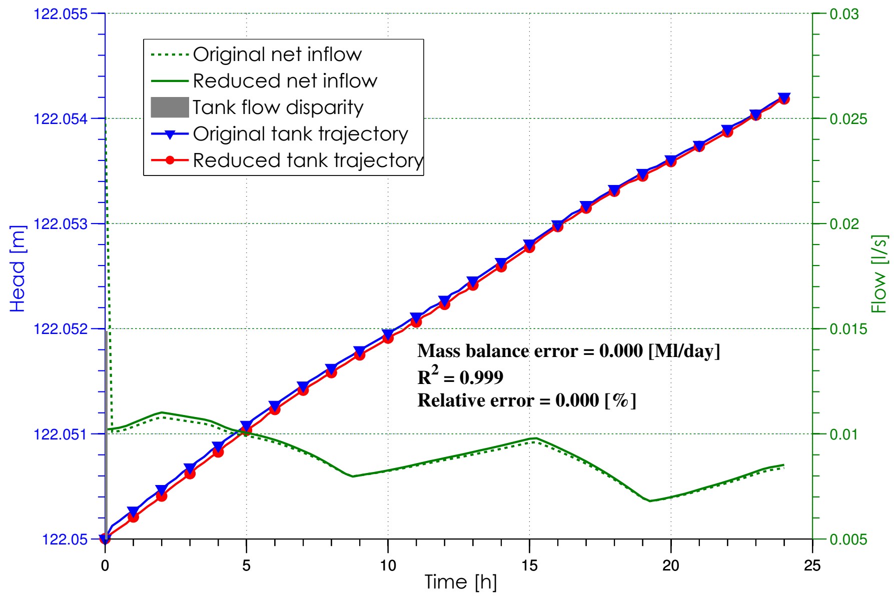


FIGURE 5.23: Comparison of simulated trajectories for *Tank 2A*.

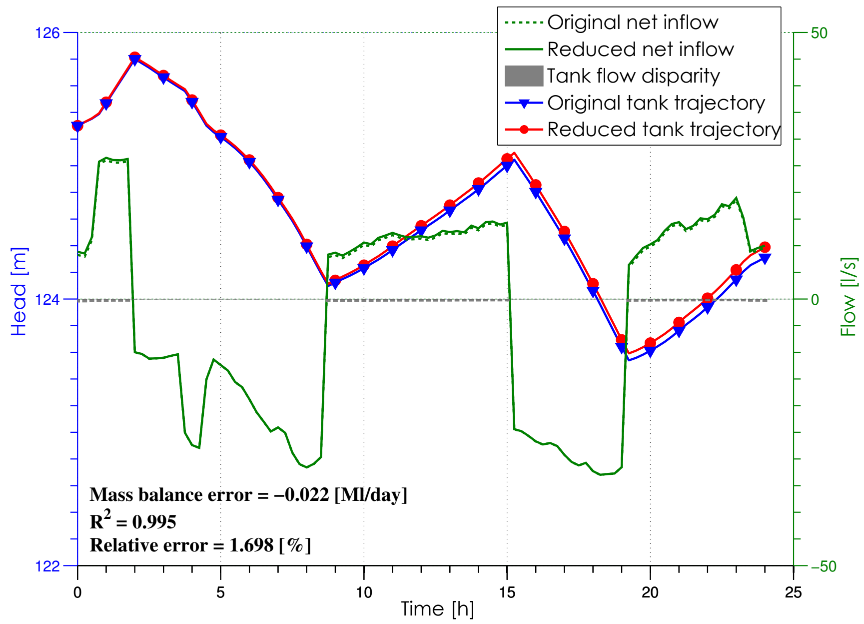


FIGURE 5.24: Comparison of simulated trajectories for *Tank 2B*.

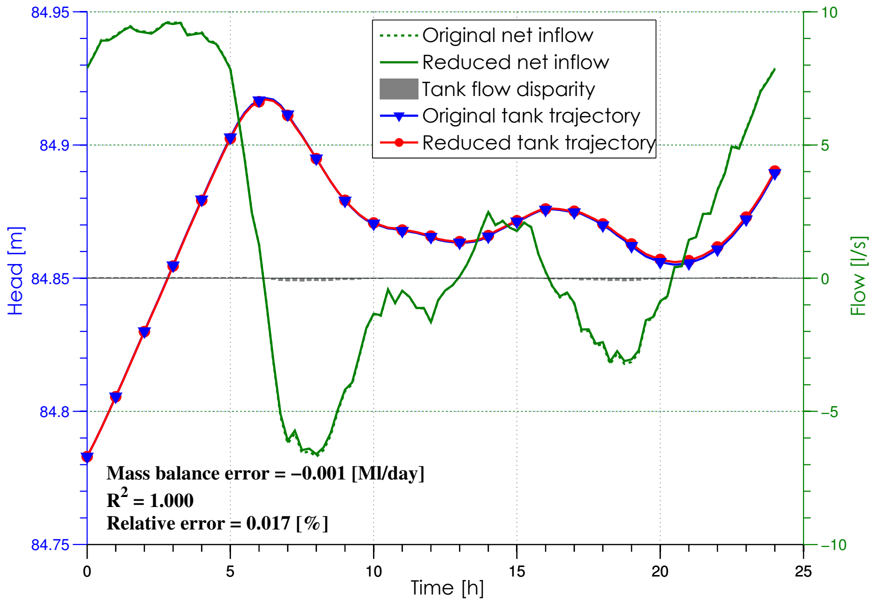


FIGURE 5.25: Comparison of simulated trajectories for *Tank 3*.

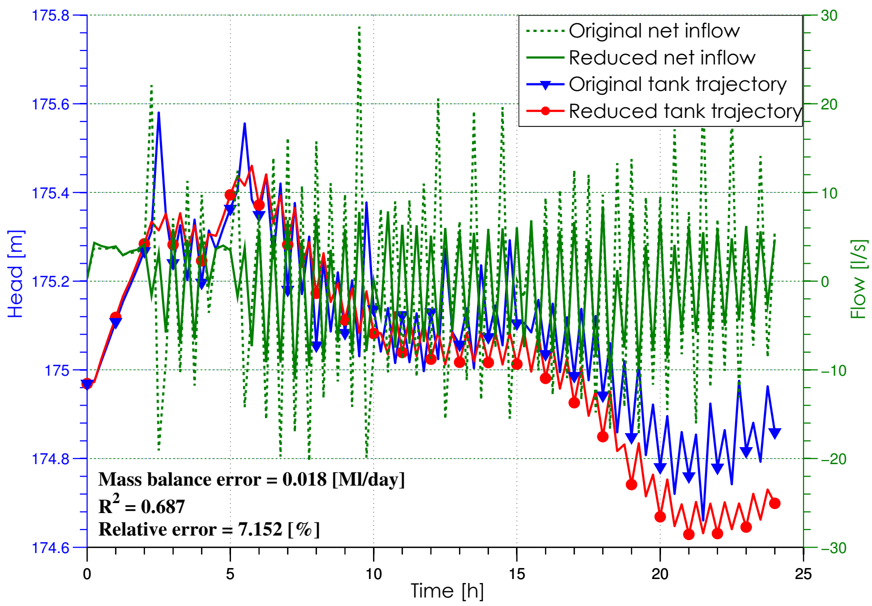


FIGURE 5.26: Comparison of simulated trajectories for *Tank 4A*.

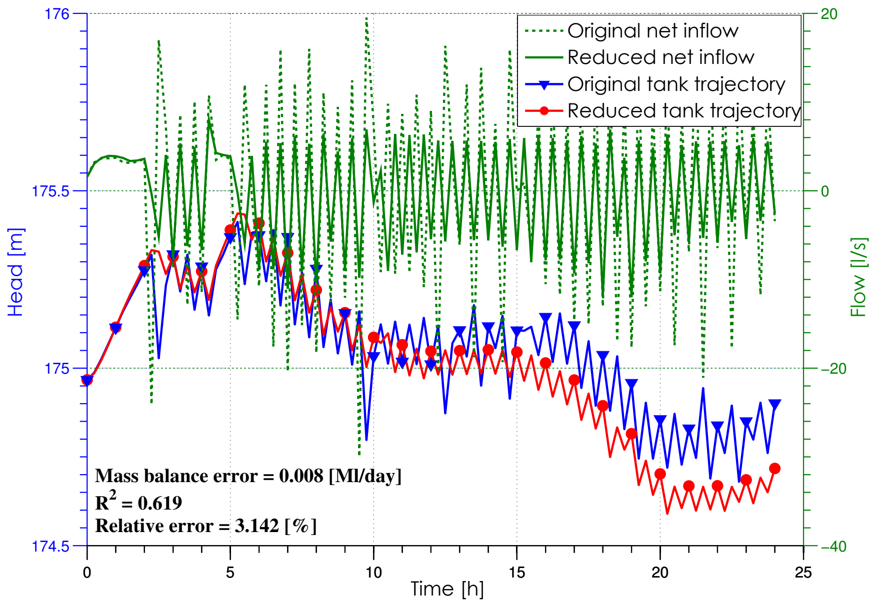


FIGURE 5.27: Comparison of simulated trajectories for *Tank 4B*.

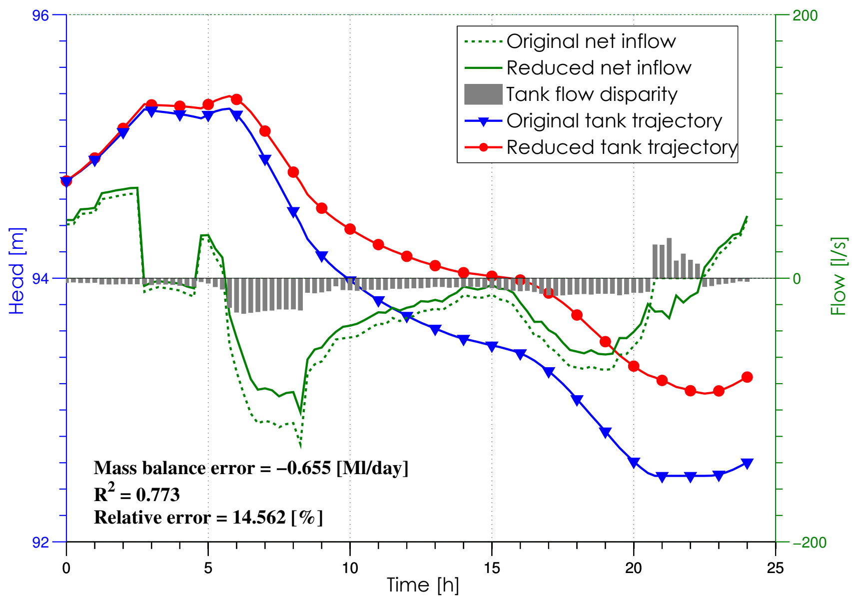


FIGURE 5.28: Comparison of simulated trajectories for *Tank 5*.

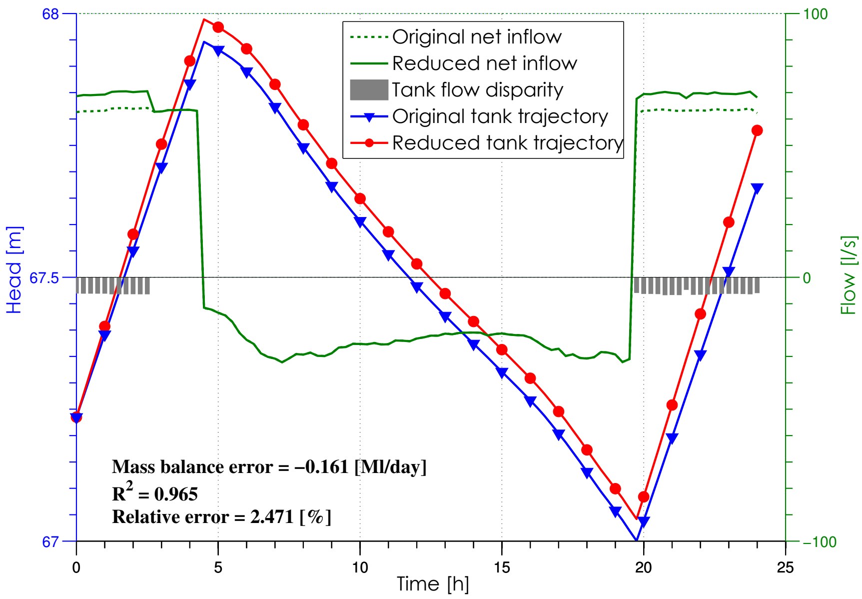


FIGURE 5.29: Comparison of simulated trajectories for *Tank 6*.

5.3.2.3 Optimisation model

Following the meeting with the water company engineers several additional discrepancies were found between the provided Epanet2 model operation and the real *Water Network* operation. In particular, the pump stations parameters, structure and schedules differed significantly from the real operation of *Water Network*. To accommodate the updates the modifications listed in Appendix E were applied to the reduced model. The updated reduced model was subsequently adjusted to meet the optimisation method requirements.

The final optimisation model contains 131 junctions, 1 reservoir, 6 tanks, 197 pipes, 13 pumps and 35 valves. The schematic of the final network configuration is depicted in Figure 5.30. The hydraulic parameters of the pumps and tanks in the optimisation model are given in Tables 5.5 and 5.6. The head-flow and power characteristics for each pump in the optimisation model are illustrated in Figure 5.31.

5.4 Optimal network scheduling: continuous optimisation

Network scheduling calculates least-cost operational schedules for pumps, valves and treatment works for a given period of time, typically 24 hours or 7 days. The decision variables are the operational schedules for control components, such as pumps, controllable valves and water works outputs. The scheduling problem was succinctly expressed as: minimise (pumping cost + treatment cost), subject to the network equations and operational constraints. The problem has the following three elements: (i) objective function, (ii) hydraulic model of the network and (iii) operational constraints. These three elements of the problem are discussed in the following sections.

5.4.1 Objective function

The objective function to be minimised is the total energy cost for water treatment and pumping. Pumping cost depends on the efficiency of the pumps used and the electricity power tariff over the pumping duration. The tariff is usually a function of time with cheaper and more expensive periods. For given time step τ_c, the objective function, ϕ, considered over a given time horizon $[k_0, k_f]$ is given by the following equation expressed in discrete-time:

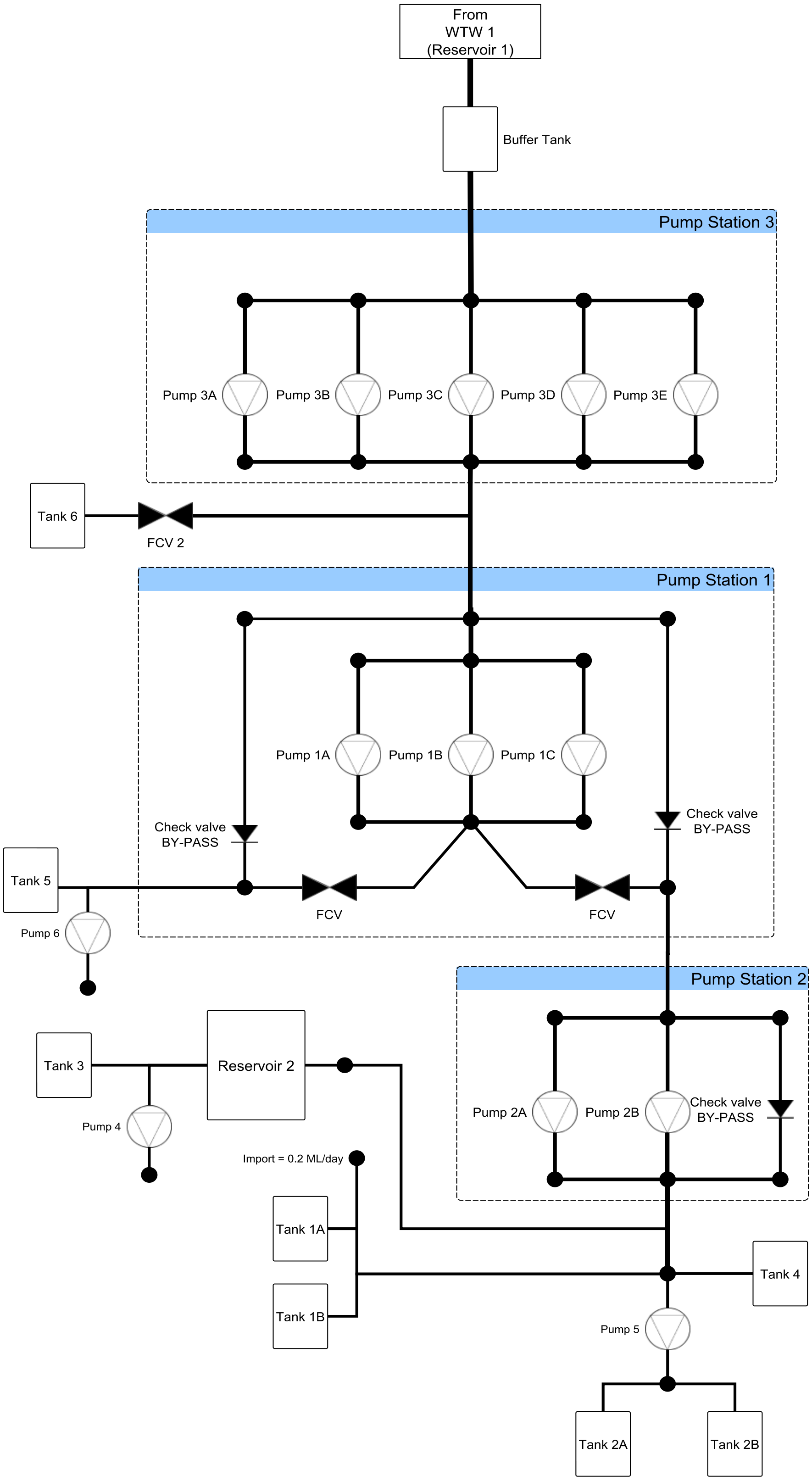
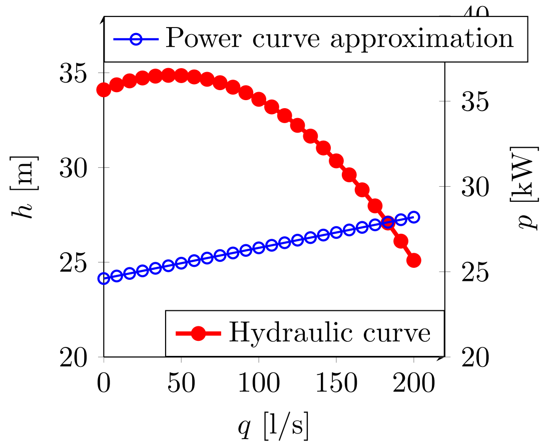


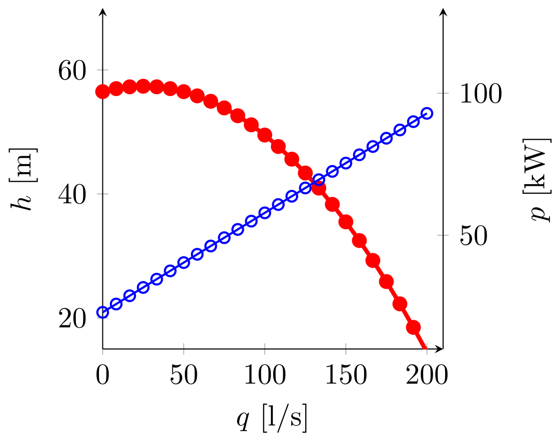
FIGURE 5.30: Schematic of the final *Water Network* configuration.

TABLE 5.5: Parameters of the retained reservoirs and tanks.

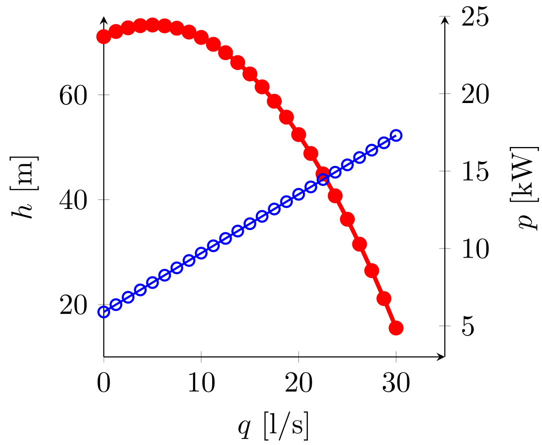
Name	Diameter $[m]$	Cross-sectional area $[m^2]$	Total volume [ML]	Elevation $[m]$	Initial level $[m]$	Operational constraints Min. $[m]$	Max. $[m]$
Reservoir 3	-	-	-	118.00	-	-	-
Tank 1A and *Tank 1B*	77.5	4722.6	21.78	97.83	3.259	0	4.61
Tank 2A and *Tank 2B*	24.3	464.4	2.15	122.03	3.300	0	4.63
Tank 3	41.7	1365.5	5.98	82.12	2.663	0	4.38
Tank 5	35.7	1000.0	4.50	92.5	2.238	0	4.5
Tank 6	42.9	1444.4	6.50	63.79	3.445	0	4.5
Buffer tank	36.0	1017.9	5.09	13.0	2.5	0	5



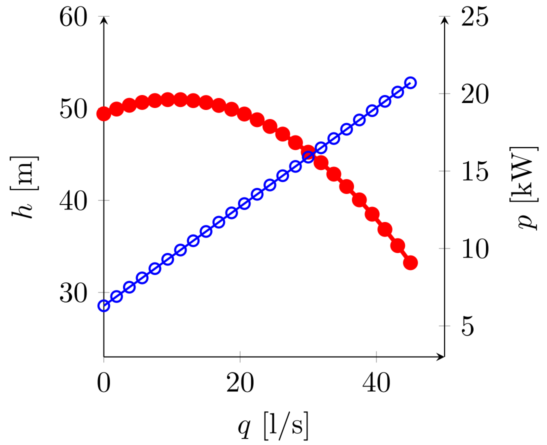
(a) Pumps in *Pump Station 1*.



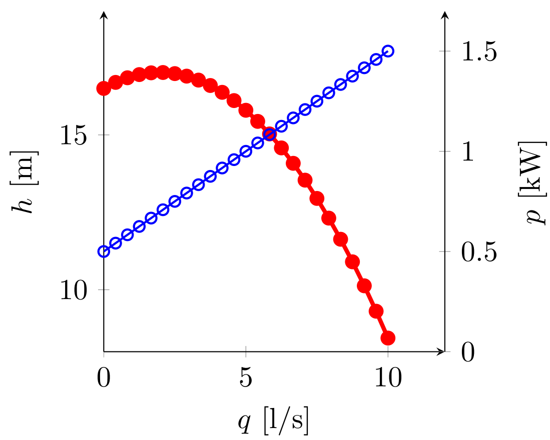
(b) Pumps in *Pump Station 2*.



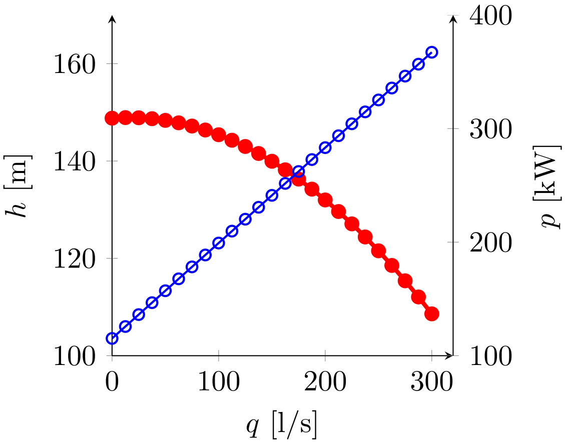
(c) Pump 4.



(d) Pump 5B.



(e) Pump 6.



(f) Pump Station 3.

FIGURE 5.31: Hydraulic and power characteristics of pumps in the optimisation model. Note that the characteristics are plotted from equations listed in Table 5.6. Pump hydraulic and power curves equations were interpolated from points provided in the Epanet2 model and pumps catalogues.

TABLE 5.6: Pumps in the optimisation model.

Name	Type	Pump station	Head-flow curve	Power consumption equation
Pump 1A	FSP	1	$h = -0.0004q^2 + 0.035q + 34.1$	$p = 0.18q + 24.6$
Pump 1B	FSP	1	$h = -0.0004q^2 + 0.035q + 34.1$	$p = 0.18q + 24.6$
Pump 1C	FSP	1	$h = -0.0004q^2 + 0.035q + 34.1$	$p = 0.18q + 24.6$
Pump 5B	VSP	-	$h = -0.0147q^2 + 0.302q + 49.4$	$p = 0.32q + 6.3$
Pump 2A	VSP	2	$h = -0.0014q^2 + 0.070q + 56.5$	$p = 0.35q + 22.9$
Pump 2B	VSP	2	$h = -0.0014q^2 + 0.070q + 56.5$	$p = 0.35q + 22.9$
Pump 4	VSP	-	$h = -0.0919q^2 + 0.904q + 71.1$	$p = 0.38q + 5.9$
Pump 6	VSP	-	$h = -0.1330q^2 + 0.524q + 16.5$	$p = 0.10q + 0.5$
Pump 8A	VSP	3	$h = -0.0005q^2 - 0.016q + 148.8$	$p = 0.84q + 115.2$
Pump 8B	VSP	3	$h = -0.0005q^2 - 0.016q + 148.8$	$p = 0.84q + 115.2$
Pump 8C	VSP	3	$h = -0.0005q^2 - 0.016q + 148.8$	$p = 0.84q + 115.2$
Pump 8D	VSP	3	$h = -0.0005q^2 - 0.016q + 148.8$	$p = 0.84q + 115.2$
Pump 8E	VSP	3	$h = -0.0005q^2 - 0.016q + 148.8$	$p = 0.84q + 115.2$

$$\phi = \left(\sum_{i \in J_p} \sum_{k=k_0}^{k_f} \gamma_{p,i}(k) P_i + \sum_{j \in J_s} \sum_{k=k_0}^{k_f} \gamma_{s,j}(k) q_{s,j}(k) \right) \tau_c \tag{5.1}$$

where J_p is the set of indices for pump stations and J_s is the set of indices for treatment works. The function $\gamma_{p,i}(k)$ represents the electrical tariff. The treatment cost for each treatment works j is proportional to the flow output $q_{s,j}(k)$ with the unit price of $\gamma_{s,j}(k)$. The term P_i represents the electrical power consumed by pump station i.

To model electricity usage, instead of using a pump efficiency equation, a direct modelling of pump station power was employed as discussed in (Ulanicki *et al.*, 2008). However, the equation was rearranged as proposed in (Skworcow *et al.*, 2013) to allow zero pumps switched on, without introducing *if-else* formulas:

$$P(k)u(k)^2 = Eq(k)^3 + Fq(k)^2u(k)s(k) + Gq(k)u(k)^2s(k)^2 + Hu(k)^3s(k)^3 \tag{5.2}$$

where E, F, G, H are the power coefficients constant for given pump station, q is the flow, P is the consumed power, s is the speed normalised to a nominal speed for which the pump hydraulic curve was obtained (see Equation 2.21). Additionally it is imposed for all pump stations that $P(k) \geq 0$, so when all pumps in a given pump station are switched off (i.e. $u(k) = 0$) the solver (due to minimising the cost) assigns $P(k) = 0$ for this pump station. Finally, Ulanicki *et al.* (2008) demonstrated that accurate approximation of the mechanical power data points can be done with use of 3^{rd} order polynomial. However, as can be seen in Ulanicki *et al.* (2008) approximated curve often resembles straight line thereby the coefficients E and F in Equation 5.2 can be neglected as they are small compared to G and H. Hence, to make a large-scale model easier to solve it was assumed that $E = 0$ and $F = 0$, i.e. consumed power depends linearly on the pump station flow.

5.4.2 Model of water distribution system

Each network component has a hydraulic equation. The fundamental requirement in an optimal scheduling problem is that all calculated variables satisfy the hydraulic model equations. The network equations are non-linear and play the role of equality constraints in the optimisation problem. The network equations used to describe reservoir dynamics, components hydraulics and mass balance at reservoirs were described in details in Chapter 2 and (Brdys and Ulanicki, 1994; Ulanicki *et al.*, 2007). Since leakage is assumed to be at connection nodes, the equation to describe mass balance at connection nodes was modified to include the leakage term:

$$\mathbf{\Lambda}_c\mathbf{q}(k) + \mathbf{d}_c(k) + \mathbf{l}_c(k) = 0 \tag{5.3}$$

where $\mathbf{\Lambda}_c$ is the node branch incidence matrix, $\mathbf{q}$ is the vector of branch flows, $\mathbf{d}_c$ denotes the vector of demands and $\mathbf{l}_c$ denotes the vector of leakages calculated as:

$$\mathbf{l}_c(k) = \mathbf{p}^{\alpha}(k)\kappa \tag{5.4}$$

with $\mathbf{p}$ denoting the vector of node pressures, $\alpha \in \langle 0.5, 1.5\rangle$ denoting the leakage exponent and κ denoting the vector of leakage coefficients, see (Ulanicki *et al.*, 2000) for details. Note that $\mathbf{p}^{\alpha}$ denotes each element of vector $\mathbf{p}$ raised to the power of α.

5.4.3 Operational constraints

In addition to equality constraints described by the hydraulic model equations, operational constraints were applied to keep the system-state within its feasible range. Practical requirements were translated from the linguistic statements into mathematical inequalities. The typical requirements of network scheduling are concerned with variable-head reservoir levels in order to prevent emptying or overflowing, and to maintain adequate storage for emergency purposes.

Similar constraints were applied to the heads at critical connection nodes in order to maintain required pressures throughout the water network. Another important constraint was on the final water level of variable-head reservoirs, such that the final level is not smaller than the initial level; without such constraint least-cost optimisation would result in emptying of reservoirs. The control variables such as the number of pumps switched ON, pump speeds or valve opening, were also constrained by lower and upper constraints determined by the features of the control components.

5.4.4 Scenarios description

The water company suggested a number of different scenarios that varied in the number of allowed pumps in pump stations, constraints on pumps' speed, tanks' maximum and minimum levels and valves' flow. However, two scenarios presented here were chosen to illustrate the significance of pressure constraints on critical nodes in an optimisation study based on a reduced model of water distribution network. Both scenarios followed the general assumptions described in the next paragraphs, but differ in terms of pressure constraints on selected nodes. In Scenario 1, the pressure constraints on selected nodes in the reduced model were set to 15 m, which was the global minimum service pressure in the original *Water Network*, whereas in Scenario 2 the pressure constraints on selected nodes were calculated with a consideration of the energy distribution as was described in Chapter 3.

In both scenarios a full tank capacity was allowed, i.e. the minimum and maximum allowed levels for each tank were as their physical limits given in the Epanet2 model, see Table 5.5. The initial tank level for each tank was assumed to be as in Table 5.5. It was assumed that the final tank level has to be at least as the initial tank level.

Additionally, it was assumed that the maximum allowed flow through each of *Pump Station 1* valves (diverting the flow towards either *Pump Station 2* or *Tank 5*) is 300 l/s, while the maximum allowed flow through valve controlling flow towards *Tank 6* was assumed to be 90 l/s. These assumptions were made based on observations of flows in the provided Epanet2 model. In all the pumping stations all pumps were allowed to be ON. Minimum and maximum normalised speed constraints for variable speed pumps were assumed based on observations of pump operation in the provided Epanet2 model and are given in Table 5.7. Note, that minimum and maximum normalised speed constraints for the pumps *4* and *6* were extended as these pumps will be used to highlight the impact of the new pressure constraints calculated for the critical nodes in order to keep the original model energy distribution.

TABLE 5.7: Normalised pump speed constraints used in scenarios.

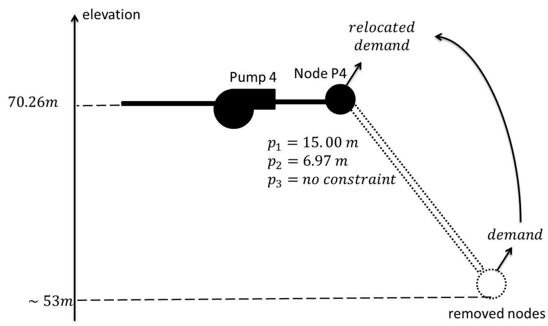
Pump station/Pump	Min	Max
Pump Station 2	0.4	1
Pump Station 3	0.7	1
Pump 4	0.1	1.2
Pump 5B	0.7	1
Pump 6	0.1	1.2

In this study, the pressure-dependent leakage was not considered since leakage data was not provided. Since the pressure dependent leakage was not included in the model, changing of the PRV setpoints does not affect the water losses and hence does not affect the cost (see Figure 5.2). Furthermore, for some PRVs the inlet pressure is lower than the required outlet pressured; this means that during the optimisation it is not possible to find a feasible solution, since the PRV equation enforces the inlet head to be higher than the outlet head if the flow through the PRV is greater than zero. For these reasons the minimum pressure constraints at non-tank nodes have been removed, thus the PRV setpoints calculated by the optimiser are not relevant and are not illustrated here.

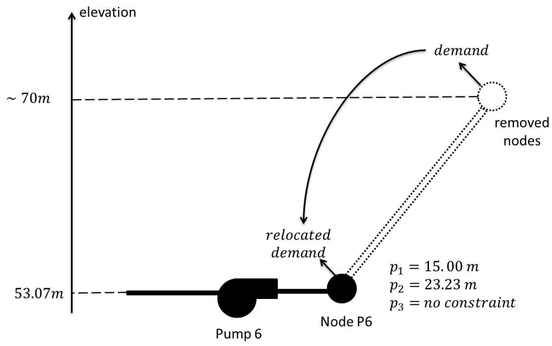
Consequently, the number of nodes where the pressure constraints could be imposed was limited. It was decided to use parts of *Water Network* where pumps were pumping directly to demand; e.g. *Pump 4* and *Pump 6*. The district metering areas behind the chosen pumps, after the model reduction process, were reduced to a single node with the demand aggregated from the removed nodes. It was noted that whereas *Pump 6* was pumping uphill, *Pump 4*, atypically, was pumping downhill. Also, *Pump 4* hydraulic characteristics significantly exceeded the actual flow requirements for the supplied area. As the original *Water Network* represents only a single day snapshot of this complex WDN, it was assumed that area supplied by *Pump 4* could contain an industrial unit, of which demands were not included in the original *Water Network*. The pumps configuration and nodes were the new pressure constraints will be imposed are illustrated in Figure 5.32.

5.4.5 Continuous optimisation outcomes

Subsequently, using the reduced hydraulic model with constraints, the optimal network scheduling problem was generated in a mathematical modelling language, GAMS (Brooke *et al.*, 1998), which called up a nonlinear programming solver, CONOPT (Drud, 1992), to calculate an optimal continuous solution.



(a) *Pump 4* configuration.



(b) *Pump 6* configuration.

FIGURE 5.32: Illustration of *Pump 4* and *Pump 6* configuration after model reduction process. The p_1, p_2 and p_3 are the pressure constraints imposed on the selected nodes P4 and P6 in the Scenario 1,2 and 3, respectively.

For both scenarios, the continuous optimisation resulted in a set of optimal heads, flows and schedules while satisfying operating constraints. Thanks to reduced optimisation model of *Water Network* the continuous optimisation process took only few minutes for the 24 h horizon and up to an hour for the 7 day period. Note that only comparison the costs between different scenarios was possible. The comparison to actual operation cost of *Water Network* could not be done, since not enough sufficient data about the current operation was received from the water company.

Figure 5.33 shows that the continuous head trajectories for all the tanks were kept within their operating constraints. It can be observed that in these 24 h horizon scenarios it was not possible to fully utilise the allowed capacity of the large tanks and their levels were far from the allowed limits. This was due to the restriction that the final tank level must be at least as the initial tank level. However, for scenarios with 7 days horizon, most tanks hit their upper or lower allowed limits. These scenarios are not included in this work but some of the results can be found in Skworcow *et al.* (2014a).

It can be noticed in Figure 5.33f that the final level in *Buffer Tank* is noticeable higher than its initial level. However, since the flow from *WTW 1* is modelled as forced inflow into *Buffer Tank*, the head produced by *WTW 1* is 'free' from the optimisation perspective. Consequently, it can be observed that in both scenarios the 24h horizon does not allow to change *Buffer Tank* water level considerably, thereby the final water level is closer to its allowed upper limit. It also can be noticed that when *Buffer Tank* is initially filling up, at the same time water needs to be delivered south to satisfy the demands, thus the level in *Tank 6* initially needs to decrease; as can be seen in the plots the flow through valve towards *Tank 6* (see Figure 5.34) is negligible during the period when *Buffer Tank* is filling up.

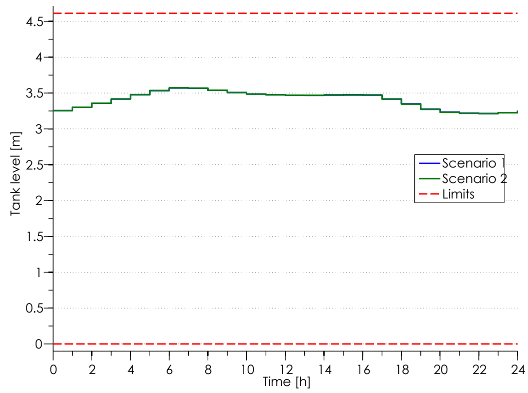
It can be observed in both scenarios that the most intensive pumping, both in terms of number of pumps ON and pump speed, occurred in the cheapest tariff period. In the most expensive tariff period the speed of variable speed pumps was reduced (e.g. *Pump Station 3*, see Figure 5.35c) and/or the number of pumps ON was decreased (e.g. *Pump Station 3* in Figure 5.35b). Furthermore, interactions between pumping stations connected in series (i.e. *Pump Station 3*, *Pump Station 1* and *Pump Station 2*) can be observed in the plots; e.g. when a pump in *Pump Station 1* was OFF, the speed of *Pump Station 2* or *Pump Station 3* was increased to produce the required flow and head increase. In both scenarios the main pump station, *Pump Station 3*, used all allowed pumps but the speed varied considerably during the considered 24h horizon.

The daily cost for the main pumps and thereby the total cost does not differ significantly in both scenarios, see Figure 5.36. However, operations of the pumps directly affected by the pressure constraints vary in each scenario.

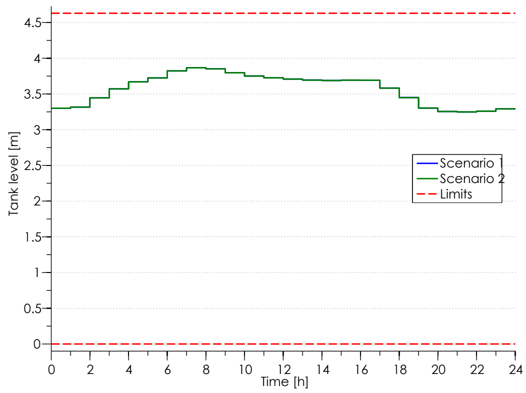
In Scenario 1 the pressure constraints on the nodes P4 and P6 were the same i.e 15 m and it resulted in the total pumping cost of £802.38. As it was mentioned, *Pump 4* hydraulic performance is more than adequate to satisfy the demand located downhill so it was excepted that the pump would operate at low speed and indeed the calculated optimal speed is below 0.4 of its normalised speed, see Figure 5.35d. In turn, *Pump 6*, which is pumping uphill, is noticeably more active, even hitting its maximum allowed speed.

In Scenario 2, new pressure constraints, calculated in order to reflect the energy distribution in the original *Water Network*, were imposed on nodes P4 and P6. New pressure constraints were: 6.97 m for the node P4 and 23.23 m for the node P6. As a consequence, the total pumping cost increased slightly to £803.59. The impact of new pressure constraints on the total cost does not seem to be significant in this particular network. This mainly due to fact that the considered pumps are the smallest in *Water Network*. However, the impact on the operation of the *Pump 4* and *Pump 6* is much more visible as shown in Figures 5.35d and 5.35f.

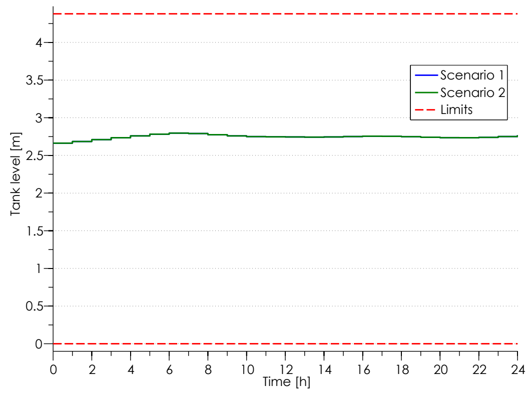
Pump 4 lowers its speed as pressure under which the water have to be delivered drops from 15 m to 6.97 m. In contrast, *Pump 6* had to increase its speed to satisfy a new pressure constraint of 23.23m. Hence, the daily cost for *Pump 4* decreased about 19% whereas the operation cost of *Pump 6* increased significantly; i.e. 116% as can bee seen in Figure 5.36.



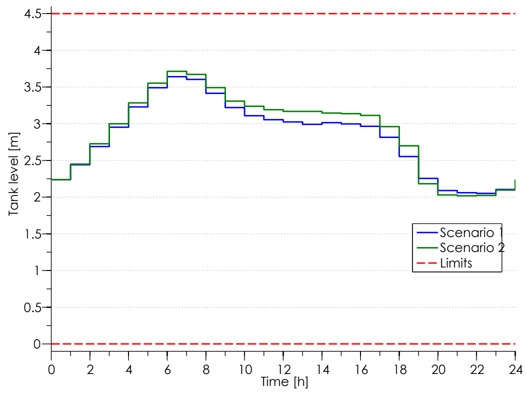
(a) Water levels in the merged *Tank 1A* and *Tank 1B*.



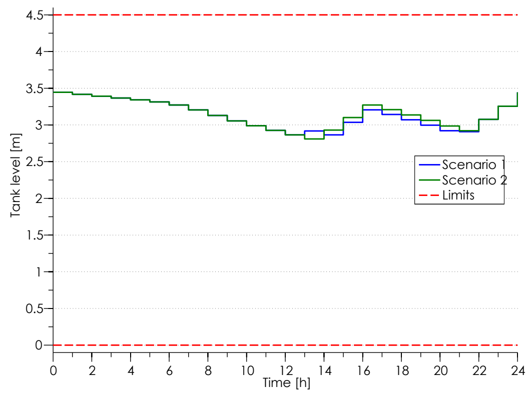
(b) Water levels in the merged *Tank 2A* and *Tank 2B*.



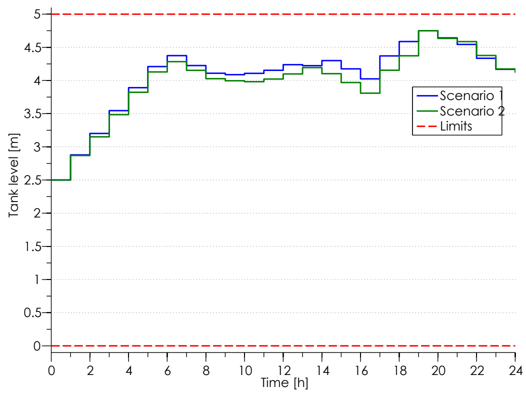
(c) Water levels in *Tank 3*.



(d) Water levels in *Tank 5*.



(e) Water levels in *Tank 6*.



(f) Water levels in *Buffer Tank*.

FIGURE 5.33: Continuous optimisation results for the tanks.

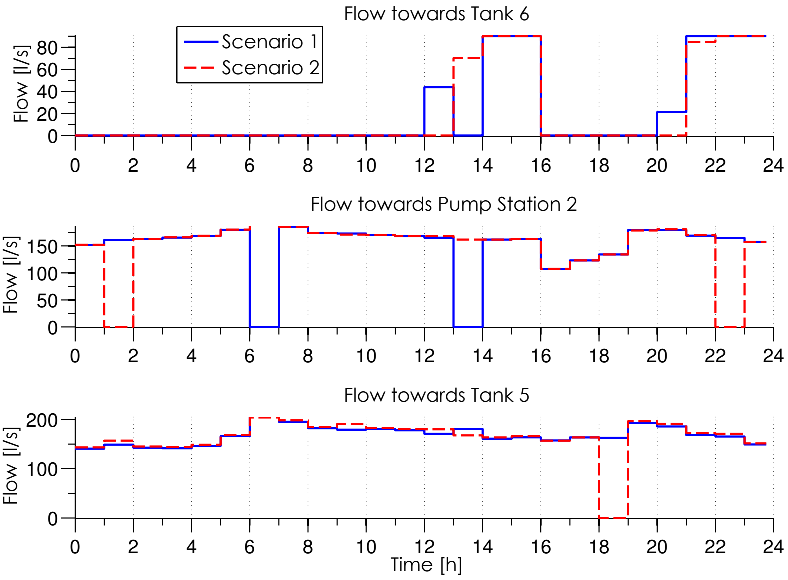
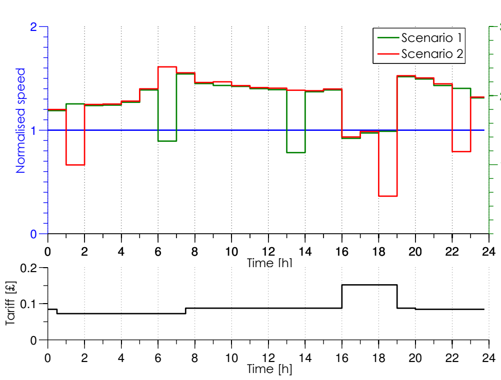


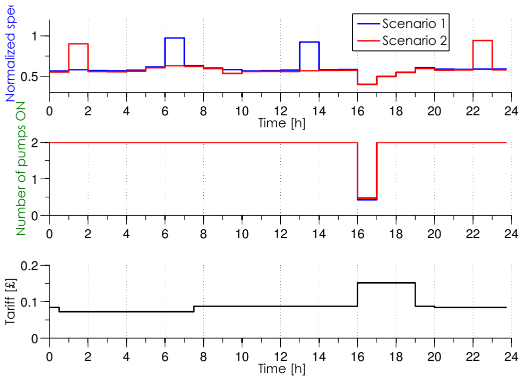
Figure 5.34: Continuous optimisation results for the valves.

As it was highlighted in (Bunn and Reynolds, 2009) pumps usually do not operate in isolation; it is typical that any change in the operating duty of one pump may affect the suction or discharge pressure of other pumps connected to the same pipe system. Indeed, in this study was observed that a change in operation, even for relatively small pumps such *Pump 4* and *Pump 6*, affected the operation of the major pump stations.

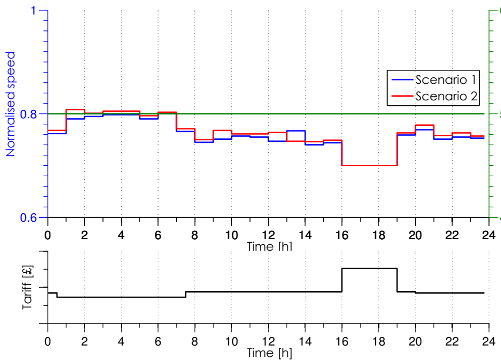
The scenarios outcomes confirmed a profound effect of the pressure constraints in the optimisation of pump schedules in water distribution network and also highlighted the importance of preserving of the energy distribution of the original network in order to not mislead the optimiser when calculating of optimal schedules. And yet in many works regarding pump operation optimisation the pressure constraints on the critical nodes do not consider the original model energy distribution. Broad *et al.* (2010) during the skeletonization process aggregated the demand from the removed nodes to the nearest nodes but it was done without consideration of the model energy balance. Instead, the pre-optimisation pressure constraints resulted from Epanet2 simulation were used. Such straightforward reduction of water distribution network model will reduce the computational cost of optimisation, but without the relevant pressure constraints it will not provide an accurate representation of the original model. And thereby, the optimal schedules applied to operation of a real WDN might not be appropriate and corrections from the operator would be necessary.



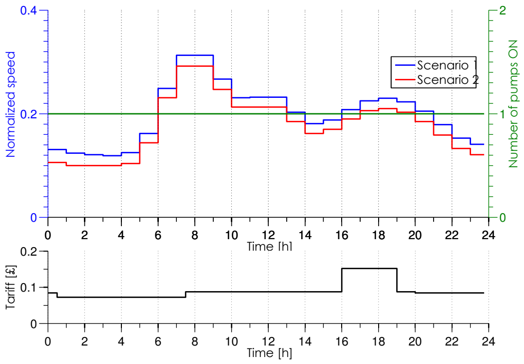
(a) Optimised operations of *Pump Station 1*.



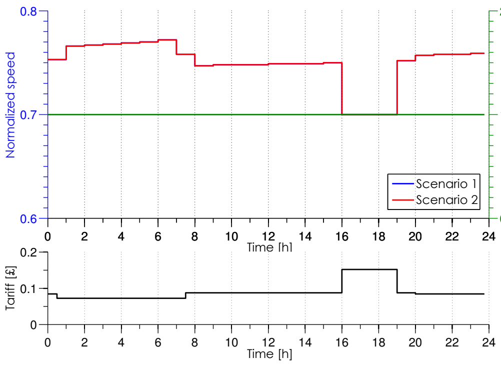
(b) Optimised operations of *Pump Station 2*.



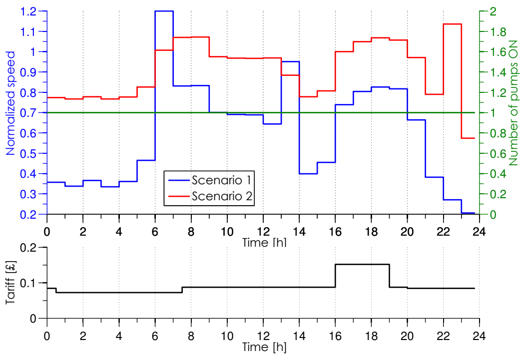
(c) Optimised operations of *Pump Station 3*.



(d) Optimised operations of *Pump 4*.



(e) Optimised operations of *Pump 5B*.



(f) Optimised operations of *Pump 6*.

FIGURE 5.35: Continuous optimisation results for the pumps.

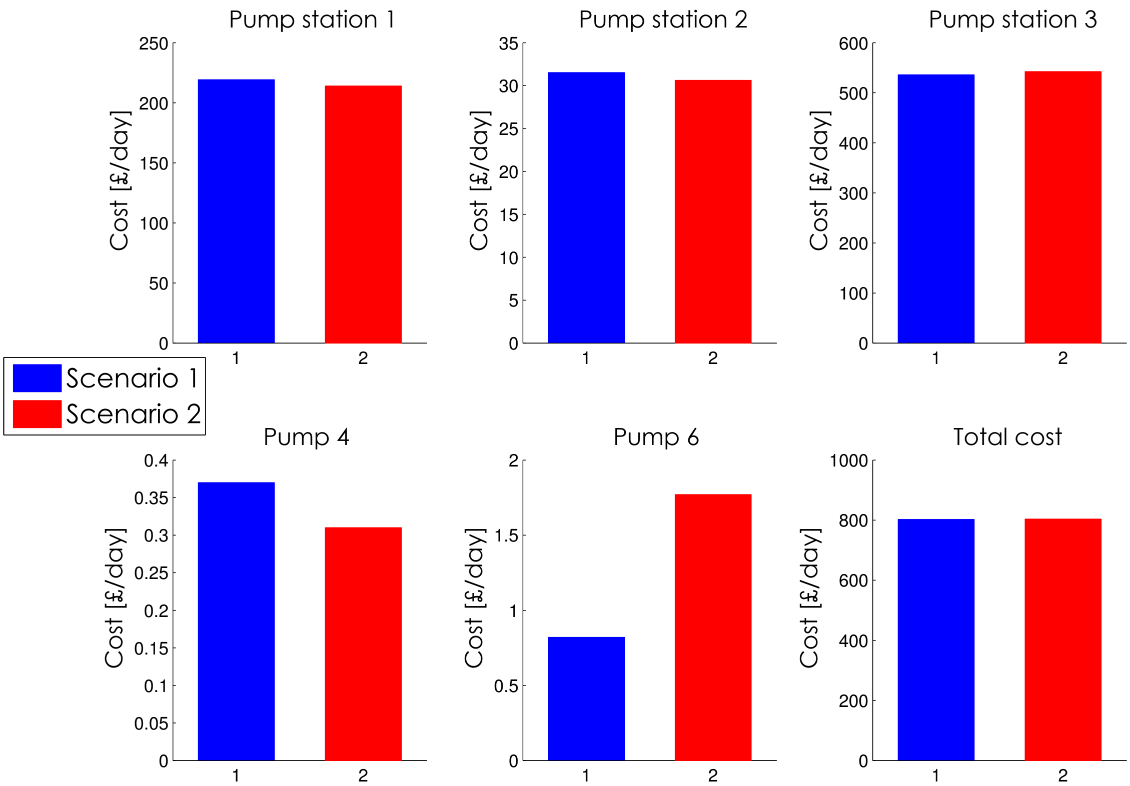


FIGURE 5.36: Illustrating the cost of optimised pumping for each pump and the total cost of pumping in each scenario. Note the daily cost of *Pump 5B* operation (not included in this plot) was £14.61 and was the same in each scenario.

5.5 Discretisation of continuous schedules

To apply the optimal schedules to a real network the continuous outcomes from the optimisation have to be converted to their discrete equivalents. Continuous schedules cannot be directly implemented in the form shown in Section 5.4.5 as one cannot have e.g. "0.7 of pump ON", thus a further processing called discretisation needs to be performed.

The process of discretisation of continuous schedules presents a challenge. Bounds *et al.* (2006) reported that a discrete schedule for a pump, which is a part of a big pump station, may differ significantly from the corresponding continuous schedule, because the aggregated flow is achieved by a combination of many pumps. In contrast, for a small isolated pump station, the discrete schedule can closely follow the continuous solution. Also, according to Bounds *et al.* (2006), for a variable speed pump, varying the speed of the pump can enable a closer approximation of the continuous solution. The discretisation issue is also present in the pipe design problem, e.g. Cunha and Sousa (2001) highlighted that conversion of the optimal continuous pipe diameters into commercial discrete equivalents may deteriorate the quality of a solution and could not always guarantee a feasible solution.

A number of approaches to schedules discretisation can be found in literature (Garcia *et al.*, 2013; Kotsiantis and Kanellopoulos, 2006). Also many meta-heuristic algorithms, such as the genetic algorithm (Murphy *et al.*, 1993), simulated annealing (Cunha and Sousa, 1999), tabu search (Lippai *et al.*, 1999), harmony search (Geem, 2006) are able to find directly discrete solutions. However, most of the listed meta-heuristics algorithms have been applied to the water distribution networks mostly operated gravitationally without utilisation of the pumps. And even if they consider pumps in the system, the case studies are rather small; see e.g (Farmani *et al.*, 2005; Geem, 2009).

Therefore a new discretisation algorithm of continuous schedules have to be developed. The method employed in this work was developed with usage of GAMS/CONOPT and is described in Appendix F.

5.6 Summary

The main aim of the project, partially presented in this chapter, was to model and optimise a large-scale water distribution network in order to reduce the total operating costs. The complete optimisation procedure has been described in details to demonstrate how the problem has been approached and solved. It has been shown that the optimal scheduling of a complex WDN is a dynamic mixed-integer problem and its solution has faced a number of difficulties: (i) a large number of discrete and continuous variables, (ii) nonlinearities in the components equations, (iii) modelling uncertainties and (iv) discretisation of continuous schedules.

The optimisation method, utilised in this chapter, has taken into account the nonlinear characteristics of the system as well as the mass balance for reservoirs. It has also employed the extended model reduction algorithm to reduce the number of elements in order to solve the optimisation problem more easily and computationally effective. The reduced model not only has provided a significant speed increase of the optimisation process but it has enabled the calculation of optimal schedules; if the full model has been considered there would be just to many of decision variables.

A number of scenarios requested by the water company have resulted in continuous optimised schedules. Subsequently, continuous optimal solution for pump control has formed the basis for a mixed-integer solution which finally has led to a full set of hydraulic results including optimal heads, flows and schedules. However, in this chapter only a fraction of obtained results have been presented. The scenarios described have been rather focused on the application of the extended model reduction method, introduced in Chapter 3, to the real case study.

The modified reduction technique has allowed the preservation of the original model hydraulic complexity and energy distribution and thereby has ensured that the computed optimised schedules for pumps will deliver water whilst satisfying the minimum service pressure limits. However, due to nature of the model, i.e. major pumps were connected and working in series, the impact of new pressure constraints on the total optimised cost has not been significant but still noticeable. Nonetheless, for the pumps pumping directly to demand the optimal schedules in the presented scenarios differed drastically with respect to each other.

The model reduction software, developed in Chapter 4, has proved to be very practical tool, enabling reduction of a complex and large water distribution system in a matter of seconds. The software is especially recommended for optimal pump scheduling in large and compound WDS with many interactions between control elements and various constraints.

Part II

A new method of modelling and simulation of water networks with discontinuous control elements

Chapter 6

Discrete-event specification formalism and quantised state systems

This part of the thesis tries to open a new paradigm for modelling and simulation of water distribution systems (WDS). It is proposed within this chapter to model and simulate WDSs within the discrete-event specification (DEVS) formalism framework with use of the quantised state systems (QSS) methods. It was highlighted in Chapter 2 that in WDS discrete and continuous dynamics can be exhibited simultaneously e.g. a discrete pump control based on the water level in a tank. Systems, such as WDS, where discrete and continuous dynamics are present are called hybrid systems. Majority of water network simulators use a time-slicing approach, typical for simulation of continuous systems, here quantisation of the states is proposed leading to an asynchronous discrete-event simulation model. Such an approach in which hybrid systems modelled within the DEVS framework are simulated using the quantisation-based integration methods has not been applied to WDSs. Section 6.1 provides an epitome of numerical difficulties may be encountered in WDSs simulation. Next, an explanatory introduction is given to the DEVS theory and the QSS methods in Section 6.2 and Section 6.3, respectively. Section 6.4 illustrates properties of the QSS on a simple nonlinear system. Section 6.5 provides arguments towards utilisation of the DEVS and QSS concepts in modelling and simulation of water distribution systems. The conclusions to this chapter are given in Section 6.6.

6.1 Time-slicing simulation of water networks

Recall from Section 2.3 that simulation of a WDS uses a mathematical representation of its nonlinear dynamics and offers a better understanding of the system behaviour. Subsequently, such an enhanced understanding enables potential improvements to WDN operation and/or design. The steady-state simulation is achieved by solving a set of hydraulic equations that include the mass and energy conservation principles. Many methods were developed for solving these equations; some of them were described in Section 2.3. Along with development of the methods for water pipe network analysis dozens hydraulic simulators were created. Table 2.3 lists some of them.

But majority of water networks analysis methods and simulators are based on a time slicing approach i.e. numerical methods, used in computer simulation of a system characterised by differential equations (e.g. tank dynamics), require the system to be approximated by discrete quantities. The solution of difference equation is calculated at fixed points in time. This feature of mapping a discrete time set to a continuous state set made the discrete time approach to simulation applicable in many fields including water networks analysis.

However, it is assumed that, in EPS of water networks, the system is in a steady state between successive time stamps. But in fact, a real WDS continually adjusts itself in response to changing requirements of the users. This rises an important issue about the model fidelity of hydraulic behaviour of a real WDS; especially a WDS with pumps operation based on the water level in tanks, as if the time interval is not appropriate the events that actually happened in the real water network might be overlooked. Figure 6.1 illustrates variation of water level in a fictitious tank for different time intervals. It is evident that length of the time interval effects significantly the hydraulic simulation results.

Furthermore, some elements included in a WDN model (see Table 6.1) may cause numerical difficulties (convergence problems) in simulation due to their inherent non-smooth and discontinuous characteristics (Filion and Karney, 2003; Afshar and Rohani, 2009; Rivera *et al.*, 2010; Kovalenko *et al.*, 2010). For example, serious convergence problems may be encountered when simulating in Epanet2 a complex and large-scale WDN consisting of hundreds of elements such as those listed in Table 6.1. This is mainly due to the fact that switching events may not happen at the pre-selected time steps and then additional intermediate time steps need to be introduced. Such an approach is used in the water network simulator Epanet2 which introduces the intermediate steps when simulating water network models containing control elements.

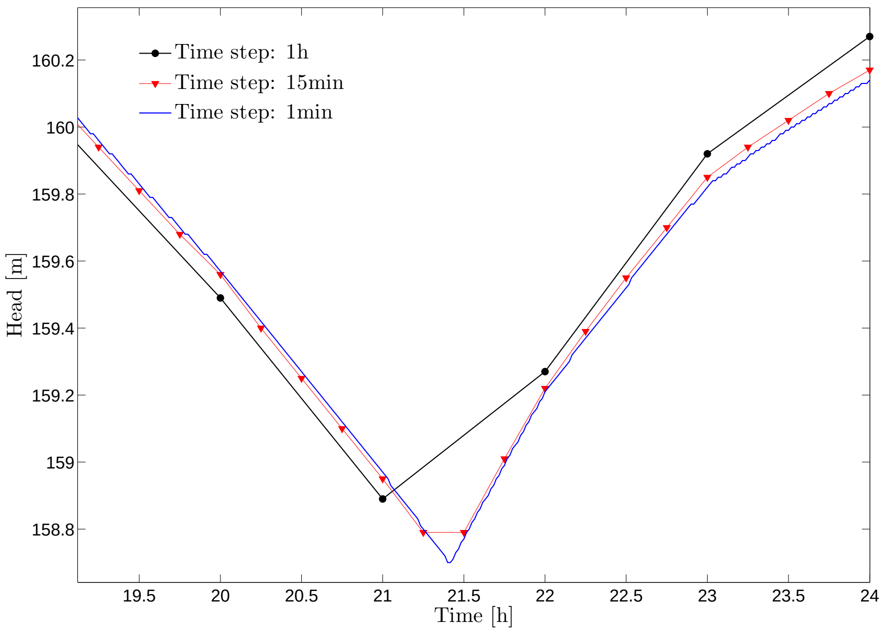
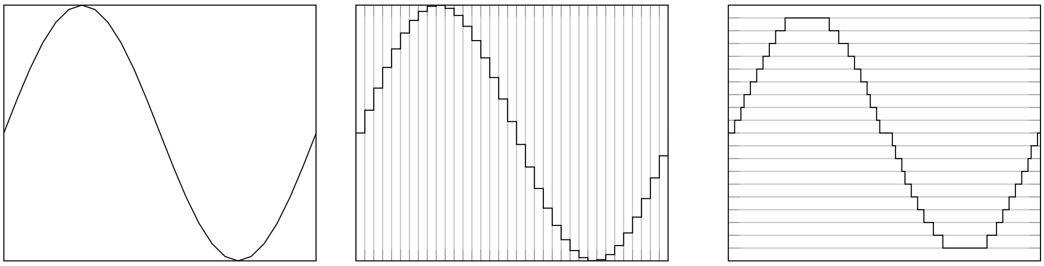


FIGURE 6.1: Illustrating the impact of different sampling time on hydraulic analysis results in Epanet2.

TABLE 6.1: Examples of non-smooth and discontinuous elements in water distribution systems.

Water distribution network components
control valves (e.g. PRV, NRV, altitude control valve (ACV), pressure sustaining valve (PSV))
pump controlled by tank level
pump controlled by many tanks' levels and time
control expressed as a computer program

In order to perform a more accurate state calculation an approach based on a discrete event solution can be used. Two aspects of the system can be made discrete, time and state (Nutaro, 2005). These two types of discretisations are illustrated in Figure 6.2. In the discrete event approach due to the event-dependent time advance, only important simulation points regarding the dynamics of the system are simulated, while idle periods of the system (intervals where no changes in states occur) are simply skipped whereas in the fixed-increment approach time advance also simulates inactive periods (Beltrame and Cellier, 2006).



(a) Continuous (b) Discrete time (c) Discrete state

FIGURE 6.2: Time and state discretisation of a continuous system.

Discrete-event systems may be modelled using Petri nets (Petri, 1962), finite-state machines (Gill, 1962), Markov chains (Norris, 1998), state charts (Harel, 1987) or the DEVS formalism (Zeigler, 1976). The latter will be presented in a greater detail in the next sections as the DEVS formalism is a general and established framework that provides means to simulate hybrid systems.

6.2 Discrete-event specification formalism

DEVS is a modular and hierarchical formalism, introduced by Zeigler (1976), for modelling discrete-event systems. DEVS can represents systems whose input/output behaviour can be described by sequence of events with the condition that the state has a finite number of changes in any finite interval of time. A DEVS model processes an input event trajectory and based on that trajectory and its own initial conditions, it produces an output event trajectory (see Figure 6.3).

FIGURE 6.3: Input/output behaviour of DEVS model.

Since its introduction the DEVS formalism was employed in many applications and fields e.g. model of human liver, sand pile model, model of forest fires spreading, snowflake formation, robot path planning, highway toll station management and many more (see (Wainer, 2009) for more details and examples). Furthermore, many extensions and modification to DEVS were proposed over the years. Extensions to the DEVS formalism include:

Parallel DEVS (Chow and Zeigler, 1994), RT-DEVS for real-time discrete-event systems (Hong *et al.*, 1997), Cell-DEVS for cellular automata (Wainer, 2004), Fuzzy-DEVS (Kwon *et al.*, 1996) and Dynamic Structuring DEVS (Barros, 1995).

DEVS uses two types of structures to describe a discrete event system: (i) atomic models describe behaviour of elementary components whereas (ii) coupled models describe collections of interacting elementary components.

A DEVS atomic model is defined by the following tuple of seven elements (Zeigler *et al.*, 2000, Chapter 4):

$$M = \langle X, Y, S, \delta_{int}, \delta_{ext}, \lambda, ta \rangle \tag{6.1}$$

where:

- X is the set of input values.
- Y is the set of output values.
- S is the set of states.
- $\delta_{int} : S \to S$ is the internal transition function. It is executed as soon as the system has elapsed the time indicated by the time-advance function.
- $\delta_{ext} : Q \times X \to S$ is the external transition function. It is executed after having received an external event $x \in X$. Q is the set of total states defined as

  $$Q = \{(s, e) : s \in S, 0 \le e \le ta\,(s)\},$$

  and e is the time elapsed since the last state transition. For example, if the system adopted state 5 at time $t = 4$ and an external event with the value $x = 7$ is received at time $t = 6.2$ then the new state is computed by $s = \delta_{ext}(5, 2.2, 7)$.
- $\lambda : S \to Y$ is the output function which specifies the output values due to the internal transitions. Note that the output function is active only before internal transitions (external transitions do no generate any output).
- $ta : S \to \mathbb{R}_0^+ \cup \{\infty\}$ is the so-called time-advance function that defines the time interval during which the model will remain in each state if no external event occurs. For example, if at time $t = 4$ the system is in state $s = 5$ and the value of $ta(s = 5) = 6$ then system will change its state at time $t = 10$. However, if in meantime an external event arrives at time $t = 7$ which will change system state to $s = 8$ then new time when the system will change is computed by $t = 7 + ta(s = 8)$.

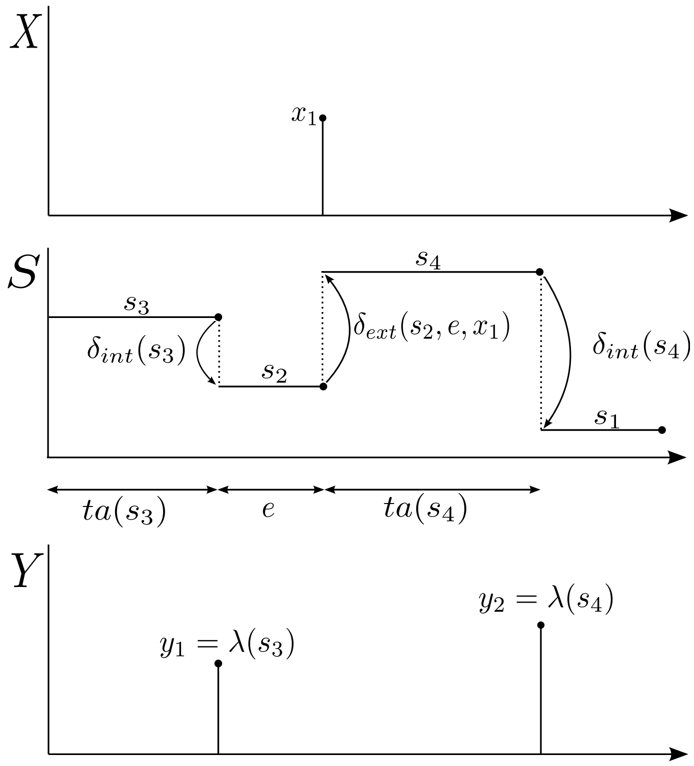


FIGURE 6.4: Example of state transitions in an atomic DEVS model. The received external inputs are in the uppermost graph. The graph in the middle shows the state trajectories of the system. The lowermost graph illustrates the produced outputs.

Figure 6.4 depicts the mechanism of state transitions of an atomic DEVS model. Apparently, the system starts in state s_3 and changes to state s_2 after the time advance $ta(s_3)$ has been elapsed. Right before executing the internal transition, it generates the output y_1. The external event x_1 occurs before e reaches $ta(s_2)$, hence the system interrupts its current behaviour and instantaneously changes its state to s_4, this time however through the external transition. Note that no output is produced. When the time advance of state $ta(s_4)$ has been elapsed, it produces an output y_2, executes the internal transition again and thereby goes to state s_1.

The main advantage of the DEVS formalism is that atomic models can be coupled in a hierarchical way i.e. the coupled model could be defined as a set of atomic or coupled models. Therefore even complex structure can be modelled as a coupled structure of simpler ones. It is only possible because DEVS models are closed under coupling (Cellier and Kofman, 2006), which means that a coupled model can can be described by the same functions as an atomic model (i.e. internal, external, time-advance and lambda functions).

Considering water network modelling the network elements such as pipes, valves, pumps could be defined as atomic models or coupled models forming a coupled model of a DMA.

There are basically two different ways of coupling DEVS models: (i) using translation functions between subsystems (see (Zeigler *et al.*, 2000, Chap.7) for details) or (ii) utilising of input and output ports. The first one is the most general but the second is simpler and more adequate to the simulation of continuous systems (Kofman, 2003).

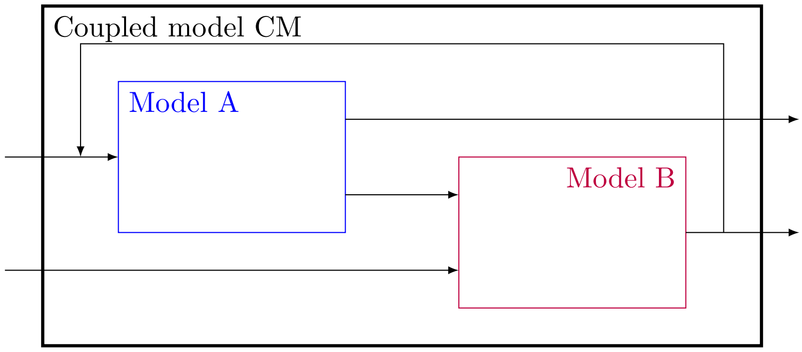


FIGURE 6.5: A coupled DEVS model.

A coupled DEVS model illustrated in Figure 6.5 is the outcome of coupling the models *Model A* and *Model B* by means of their ports, i.e. one of the output ports of *Model A* is connected to the input port of *Model B*, the output port of *Model B* is connected to the input port of *Model A*.

The utilisation of ports, however, requires an additional naming scheme to identify the port in which the event is coming. This can be done by introducing to the input and output events a new number, word or symbol representing the port associated with the event. Then, the coupling between different systems is indicated by enumerating the connections to describe it. An internal connection involves an input and an output port corresponding to different models. In the context of hierarchical coupling, there are also connections from the output ports of the subsystems to the output ports of the network, namely external output connections, and connections from the input ports of the network to the input ports of the subsystems, namely external input connections (Kofman, 2003).

Hence, when example in Figure 6.5 is considered the internal connections can be represented by [(A, 2), (B, 1)]. Other connections are [(B, 1), (A, 1)], [(CM, 1), (A, 1)], [(B, 1), (CM, 2)], etc. According to the closure property of DEVS model, the *Model CM* can be also used as an atomic DEVS and it can be coupled with other atomic or coupled models.

Beside the hierarchical construction of the models and well defined concept of coupling of components another important feature of the DEVS formalism is its ability to simulate large and complex models in a efficient way. To simulate DEVS models Zeigler *et al.* (2000) proposed a framework consisting of an abstract simulator. The concept mirrors the hierarchical structure of the DEVS model being simulated with a hierarchical framework of simulator objects. Hence, each atomic model has a corresponding DEVS simulator and a DEVS coordinator corresponds to each coupled model. At the top of the hierarchy, a root coordinator is in charge to control the progress of the simulation. See Figure 6.6 which illustrates such a mapping.

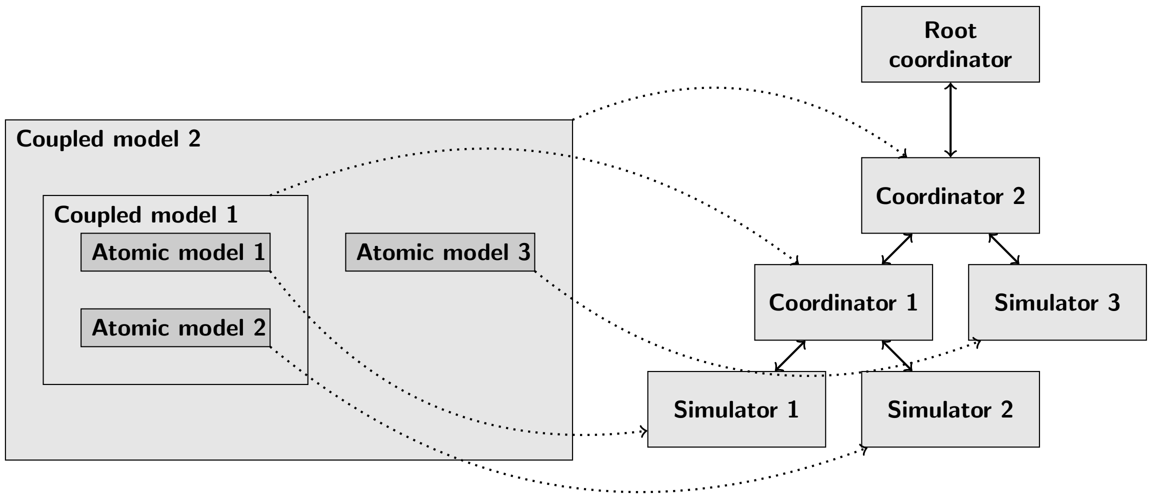


FIGURE 6.6: Mapping a hierarchical DEVS model onto a hierarchical simulator.

According to Kofman (2003) the simulation of a coupled DEVS model can be outlined as follows:

1. Search for the atomic model that is the next to execute an internal transition. Call it d^* and let tn be the time of the mentioned transition.
2. Advance the simulation time t to $t = tn$ and execute the internal transition function of d^*.
3. Propagate the output event produced by d^* to all the atomic models connected to it executing the corresponding external transition functions. Then, go back to the step 1.

The simulation of all simulator objects in the consecutive layers is coordinated by a system of messages transmitted between the associated simulators and coordinators. There are two types of messages between coordinators and simulators that are sent up and down the simulators tree. (i) Messages sent by coordinators to their children triggering the execution of different functions ($\delta_{int}, \delta_{ext}, \lambda$). (ii) When a simulator executes a transition that generates the output it sends the output value to its parent coordinator. Also, when an output event produced by one of its children has to be propagated outside the coupled model, the coordinator sends a message to its own parent coordinator carrying the output value.

Each simulator or coordinator has a local variable tn which indicates the occurrence time of its next internal transition. In the simulators, tn is calculated using the time advance function. In the coordinators, it is the minimum tn of their children. Thus, the tn of the coordinator in the top is the time in which the next event of the entire system will occur. Finally, the root coordinator only looks at this time, advances the global time t to this value and then it emits a message to its child triggering the execution of the next transition. This cycle is repeated until end of simulation is achieved. A more formal description of the simulation algorithm can be found in (Kofman *et al.*, 2003; Zeigler *et al.*, 2000).

This type of simulation has a interesting and very practical property i.e. each DEVS model, whether atomic or coupled, and the associated simulator is independent from other models. For example, if one of the atomic models in the system has its time advance set to big value or infinite the simulation will not spend any calculation with it as such atomic model will not affect other models within the system. This property of independence is especially useful in simulation of sparse systems (Kofman *et al.*, 2003).

Initially, the DEVS theory was employed mainly for discrete systems as they can be naturally and straightforward represented by sequences of events. Continuous systems can also be represented in DEVS formalism but they need to be a priori approximated by conventional numerical integration methods such as Euler, Runge Kutta etc (Bergero and Kofman, 2011). However, continuous systems can be also approximated by state quantisation and thereby systems that exhibit both continuous and discrete characteristics can be modelled and simulated within the DEVS framework.

6.3 Quantised state systems

In order to obtain a DEVS model of a continuous system a quantisation-based integration technique can be used to transform the considered continuous system into a system described by a sequence of events. The idea to approximate a continuous system by a discrete-event simulation is based on a quantisation function that enables transformation the continuous state variables into the quantised discrete valued variables. The idea initially introduced in (Zeigler and Lee, 1998) was later reformulated in (Kofman and Junco, 2001) and defined as the QSS methods.

The purpose of employing the QSS methods is to provide means to simulate the continuous part of the WDS model formulated within the DEVS framework, i.e. reservoirs dynamics. The QSS methods can be defined as follows (Cellier and Kofman, 2006)

Consider the time-invariant state equation system

$$\dot{\mathbf{x}}(t) = \mathbf{f}[\mathbf{x}(t), \mathbf{u}(t)], \tag{6.2}$$

where $\mathbf{x}(t)$ is the state vector and $\mathbf{u}(t)$ is the known piecewise constant input trajectory. The QSS method integrates an approximate system which is called the quantised state system.

$$\dot{\mathbf{x}}(t) \approx \mathbf{f}[\mathbf{q}(t), \mathbf{u}(t)], \tag{6.3}$$

where $\mathbf{q}(t)$ is the quantised version of the state vector $\mathbf{x}(t)$. The $\mathbf{q}(t)$ and $\mathbf{x}(t)$ are related components by hysteretic quantisation function (i.e each component of $q_i(t)$ is related to the corresponding state variable $x_i(t)$ by a hysteretic quantisation function). A simple quantisation function could be:

$$\mathbf{q}(t) = floor\,(\mathbf{x}(t)) \tag{6.4}$$

where $floor(x)$ is the largest integer not greater than x.

Selection of the quantisation function is often arbitrarily but it is important to highlight that wrong quantisation function can yield to illegitimate model (Cellier and Kofman, 2006) i.e. performing an infinite number of transitions in a finite time interval. To address this problem Kofman and Junco (2001) introduced a hysteresis to the quantisation function. According to (Cellier and Kofman, 2006) the definition of such a hysteretic quantisation function is as follows:

Definition 6.1. Let $Q = \{Q_0, Q_1, \ldots, Q_r\}$ be a set of real numbers, where $Q_{k-1} < Q_k$ with $1 \leq k \leq r$. Let Ω to be set of piecewise continuous trajectories, and let $x \in \Omega$ be a continuous trajectory. The mapping $b : \Omega \to \Omega$ is a hysteretic quantisation function if trajectory $q = b(x)$ satisfies:

$$q(t) = \begin{cases} Q_m & \text{if } t = t_0 \\ Q_{k+1} & \text{if } x(t) = Q_{k+1} \quad \wedge \; q(t^-) = Q_k \quad \wedge \quad k < r \\ Q_{k-1} & \text{if } x(t) = Q_k - \varepsilon \;\; \wedge \; q(t^-) = Q_k \quad \wedge \quad k > r \\ q(t^-) & \text{otherwise} \end{cases} \tag{6.5}$$

and:

$$m = \begin{cases} 0 & \text{if } x(t_0) < Q_0, \\ r & \text{if } x(t_0) \geq Q_r \\ j & \text{if } Q_j \leq x(t_0) < Q_{j+1} \end{cases}$$

The discrete values Q_k are called quantisation levels and the distance $Q_{k+1} - Q_k$ is defined as the quantum, which is usually constant. ε is the width of the hysteresis window. Q_0 and Q_r are the lower and upper saturation values, respectively.

Figure 6.7 depicts a block diagram of a QSS system. Figure 6.8 illustrates a standard quantization function $q(t)$ with uniform quantisation intervals, obtained with a hysteresis window ε. Depending on quantisation method the quantisation function can be piecewise constant (QSS1) (Kofman and Junco, 2001), linear (QSS2) (Kofman, 2002) or parabolic (QSS3) (Kofman, 2006). The family of QSS methods include also methods for stiff systems: Backward QSS and Linearly Implicit QSS (Migoni *et al.*, 2013).

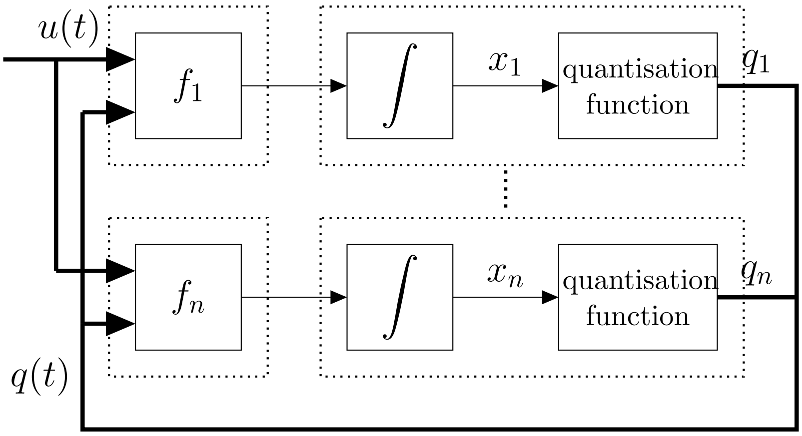


FIGURE 6.7: Block diagram of a QSS system (Kofman and Junco, 2001).

The QSS-based algorithms are of particular interest for the simulation of systems exhibiting discontinuities, as state events can be handled much more efficiently by state-quantisation

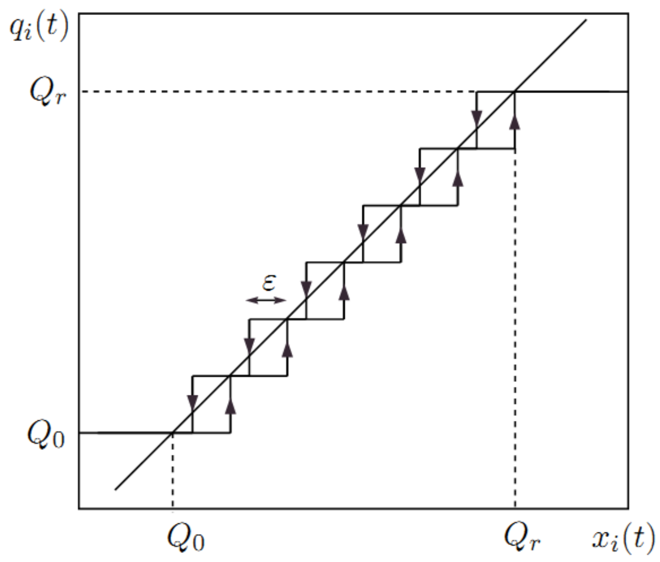


FIGURE 6.8: Typical quantisation function with uniform quantisation levels (Kofman and Junco, 2001).

algorithms compared to time-slicing algorithms. Hence, the QSS methods are well suited for the simulation of hybrid systems such as water distribution systems. Moreover, the QSS-based solvers are very promising for the simulation of large-scale models, as they exploit the sparsity inherent in these models naturally and directly. Again, this property of the QSS methods should be beneficial in simulation of water networks as they models are often large and sparse. Additionally, QSS provides features that ensure convergence and stability even for nonlinear systems (Cellier and Kofman, 2006). The QSS can be easily implemented within the DEVS formalism therefore the entire WDN can be modelled within a unified framework.

6.4 Illustrative example

In order to illustrate properties of the QSS methods consider a simple system with three states h_1, h_2 and h_3 representing heads of three cylindrical tanks with different diameters of 5, 20 and 40 m respectively. Note that for simplicity reasons these tanks are assumed to be independent. The differential equations describing dynamics of these tanks are as follows:

$$\frac{dh_1(t)}{dt} = \frac{1}{\pi \times 2.5^2}\left[-q\left(t\right)\right] \quad (6.6)$$

$$\frac{dh_2(t)}{dt} = \frac{1}{\pi \times 10^2}\left[-2q\left(t\right)\right] \quad (6.7)$$

$$\frac{dh_3(t)}{dt} = \frac{1}{\pi \times 20^2}\left[-q\left(t\right)\right] \quad (6.8)$$

where q is outflow from the tanks.

This system was simulated using the QSS2 method and the quantum of 0.001 for all states. The heads' trajectories obtained from simulation, with initial heads of $h_1(0) = 15$, $h_2(0)$=10 and $h_3(0) = 7$, are depicted in Figure 6.9.

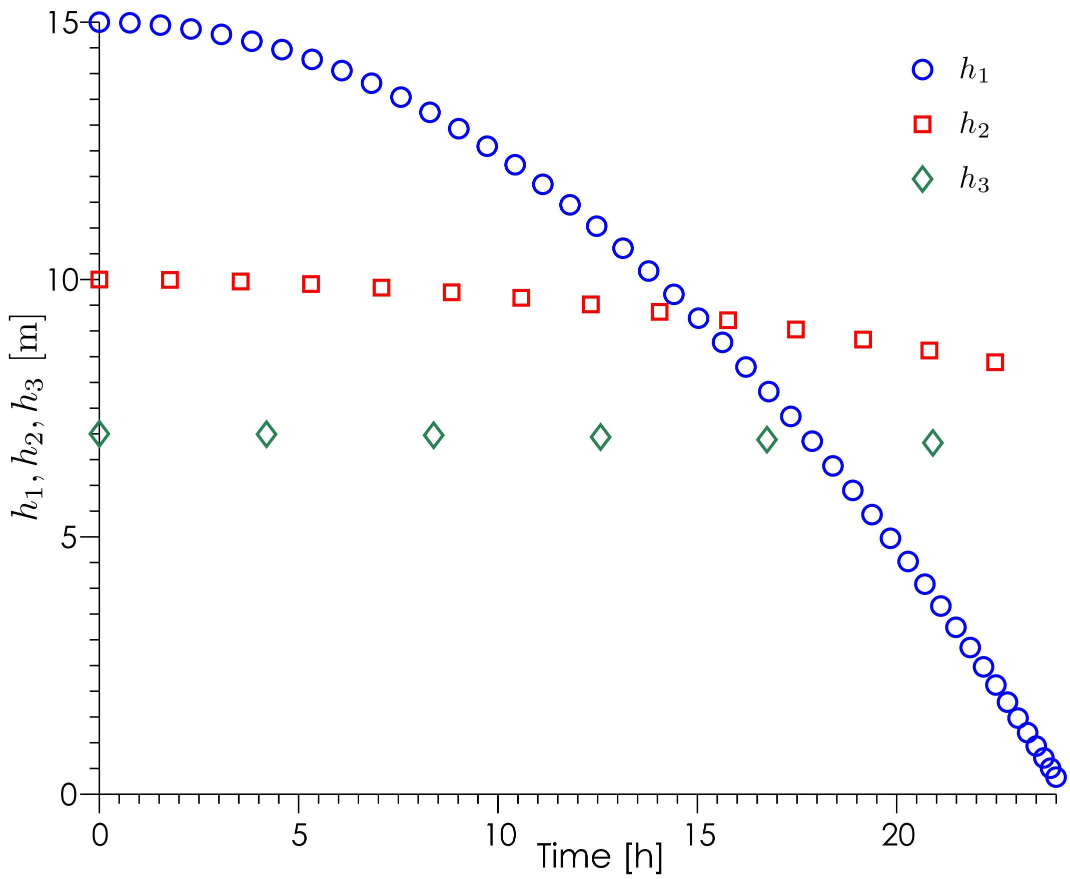


FIGURE 6.9: Illustrating properties of the QSS2 method on the example of simple non-linear system.

It can be seen that the QSS2 method adapts to the different tanks dynamics by scheduling more integrator events when the trajectory has a high curvature, while the number of integrations decrease as the trajectory resembles a straight line. This example also illustrates the asynchronous property of the QSS methods as the variable h_1 changed its state 45 times while the variables h_2 and h_3 changed 14 and 6 times, respectively. In simulation of water distribution systems with a large number of tanks with different dynamics the asynchronous property might be especially useful; tanks' states will evolve individually as there is no need to update them simultaneously.

6.5 Modelling and simulation of WDS with DEVS and QSS concepts

Section 6.1 emphasised the use of time-slicing approach in the majority of water network simulators. However, the drawbacks correlated to such an approach in simulation of hybrid systems were highlighted throughout this thesis (see Section 2.3 and Section 5.5). In summary, these problems are: non-smooth and discontinuous characteristics of components, controls based on the system states and issues with discretisation of continuous optimal schedules.

To address these issues, here quantisation of the states is proposed to create an asynchronous discrete-event simulation model of WDS. Such an approach in which hybrid system modelled within the DEVS framework is simulated using the quantisation-based integration methods has not been applied to WDSs. Having in mind the properties of the DEVS and QSS concepts, it is believed, that they shall naturally accommodate the asynchronous, concurrent and nonlinear nature of WDSs and therefore offer new contributions to the area of WDS modelling and simulation. Further investigation of the proposed approach, with the use of benchmark models available in the literature, is continued in the Chapter 7.

6.6 Summary

This chapter has introduced the reader to an alternative modelling and simulation (M&S) framework based upon combination of the DEVS formalism and the QSS methods. Such an approach brings many benefits especially to modelling and simulation of hybrids systems, such as WDSs, as in contrast to the classical time-slicing approach, the QSS methods consider only changes in states of the system.

Prior introduction to the DEVS theory, Section 6.1 has provided a synopsis of still present issues in modelling and simulation of WDSs. The highlighted problems have provoked investigation for a M&S framework that can accommodate an asynchronous, concurrent and nonlinear nature of WDSs.

The description of the DEVS and QSS concepts has been been given in Sections 6.2 and 6.3, respectively. The argument behind selection of the DEVS formalism has been that it is a formal M&S framework with established history and hundreds of applications. But what distinguish DEVS from other M&S paradigms is the separation modelling from simulation and built-in support for hierarchical, modular model development thanks to well defined coupling of components. However, the DEVS theory initially developed for discrete-time systems required additional technique in order to simulate the continuous parts of hybrid systems. Help came with introduction of the QSS methods which can be employed in a discrete event simulation of continuous systems. DEVS with QSS methods provide means to represent a full range of dynamic systems. While the focus of these sections has been put on the illustrative way to present DEVS and QSS concepts the reader may consult two excellent positions on these subjects (Zeigler *et al.*, 2000) and (Cellier and Kofman, 2006).

The introduction to DEVS and QSS concepts has been followed by an illustrative example given in Section 6.4. This example has illustrated performance of the QSS methods when simulated a simple system. The simulation outcomes have shown that in QSS each state variable is updated independently from all others, whenever it crosses through the next quantisation threshold, and also, each variable changes only at event times, i.e., when a state variable changes its quantisation level. These features of QSS shall offer increase in the computational efficiency when simulating hybrid systems with discontinuous components. Based on these preliminary considerations the next objective is the investigation of the proposed approach with the use of benchmark WDS models available in the literature.

Chapter 7

Modelling and simulations of water distribution systems within hybrid systems framework

In this chapter the results obtained from the simulations of water networks using the time-slicing and QSS methods are presented and discussed. The goal is to compare the simulation run-times and the hydraulic accuracy of the DEVS and QSS approach against the conventional time-slicing methods. More specifically, the different QSS methods in QSS Solver and PowerDEVS against the differential algebraic system solver (DASSL) solver of OpenModelica and Epanet2 simulation engine. The benchmark problems described in this chapter includes a number of representative WDSs found in the literature. However, prior evaluation of the proposed approach on the benchmark models, the investigation is carried out to establish the appropriate modelling and simulation environment for the DEVS formalism and the QSS methods. Section 7.1 describes this process. The following sections, 7.2, 7.3, 7.4 and 7.5 use the benchmark models for the systematic investigation of the accuracy and the performance of simulators. As the benchmark models are different in terms of size and complexity, the systematic investigation can reveal bottlenecks and shortcomings in the proposed approach. Section 7.6 provides the discussion of the outcomes and Section 7.7 concludes this chapter.

7.1 Modelling and simulation environments

Throughout the work described in this chapter a number of software environments and tools were used. While some of the investigated tools were fully developed and supported other were still in the development phase. Such tools offer only limited functionalities and often were released without manuals what significantly hindered work with them.

The final list of modelling and simulation environments includes: (i) Epanet2, (ii) PowerDEVS, (iii) QSS Solver and (iv) OpenModelica. Brief descriptions of these software along with reasoning behind their selection are given in the latter part of this section.

7.1.1 Epanet2

Epanet2 (Rossman, 2000b) is one of the most recognised water network solver in the water distribution research area. Within this chapter, the water network models simulated in Epanet2 are used as the benchmarks in terms of hydraulics. Note that Epanet2 was already described in Section 2.3.8. The Epanet2 version used was 2.00.12.

7.1.2 PowerDEVS

Since introduction of the DEVS formalism several implementations of this theoretical concept have been developed: DEVSim++ (Kim, 1994), DEVS-Java (Zeigler and Sarjoughian, 2000), CD++ (Wainer *et al.*, 2001), JDEVS (Filippi *et al.*, 2002), DEVS-C++ (Cho and Cho, 1997), ADEVS (Nutaro, 1999) ModelicaDEVS (Beltrame and Cellier, 2006), PowerDEVS (Bergero and Kofman, 2011) and many others. Most of them have been designed to simulate purely discrete systems but some, e.g. PowerDEVS, integrate the QSS methods, and thereby, enabling the modelling and simulation of hybrid systems.

PowerDEVS is a general-purpose modelling and simulation (M&S) software tool oriented towards the simulation of hybrid systems within the DEVS framework. PowerDEVS was one of the first, and nowadays one of the most advanced tools that allows the implementation and simulation of DEVS models. Its graphical user interface (GUI) provides user with graphical libraries of different blocks (e.g. sine, nonlinear function, ramp) that enable a quick modelling of basic systems. PowerDEVS has been successfully employed in modelling and simulation of electronic circuits (Capocchi *et al.*, 2007) and power systems (Tang and Shu, 2008). Another feature is the interconnection between PowerDEVS and the numerical package Scilab (Campbell *et al.*, 2006). PowerDEVS simulations can make use of Scilab workspace variables and functions. In turn, Scilab can be used for further processing and analysis results from the PowerDEVS simulation (Bergero and Kofman, 2011). However, modelling complex systems with thousands of elements with use of the diagram blocks can present a challenge for a PowerDEVS modeller. Also, with no manual the further modifications to the existing blocks is inconvenient and troublesome. The PowerDEVS version used was 2.3rev930.

7.1.3 OpenModelica

In addition to Epanet2, it was decided to establish the computational implementation of the benchmark networks with use of the Modelica object-oriented modelling language. Modelica is a free modelling language that supports the equation-based, object-oriented modelling methodology, and facilitates the description of large and complex systems. Models can be described in a hierarchical and modular fashion, interconnecting components similarly to the topological structure of the real system. These features along with extensive the Modelica libraries provide several modelling formalisms and ability to develop multi-domain models. In this chapter, an open-source OpenModelica (Fritzson, 2010) is used to investigate the performance of DASSL (Petzold, 1982) numerical engine in simulation of WDSs. DASSL uses the backward differentiation formulas of orders one through five and it solves the nonlinear system at each time-step by Newton's method. In the following section the simulation outcomes from OpenModelica with the use of DASSL is often used as the "analytical" benchmark against the other approaches. The OpenModelica version used was 1.9.1Beta2.

7.1.4 QSS Solver

For simplicity reasons, the most of QSS implementations were incorporated into the discrete event simulation engines such as DEVS, e.g. the QSS methods in PowerDEVS. Such an approach, however, is not fully efficient due to increased computational load within discrete event simulation mechanism. To overcome this problem Fernández and Kofman (2014) created a stand-alone QSS Solver. To model a system in QSS Solver one needs to use the μModelica language, a subset of the standard Modelica language. This tool is, however, still under development and a number functionalities such as linking to external libraries with nonlinear equations solvers were not available at the time of writing this thesis.

Table 7.1 includes a summary of the simulators features. It is important to highlight that simulators with the QSS-based integration do not support solving of simultaneous DAEs. The implications of this are discussed in the further part of this chapter.

7.1.5 Hardware

Simulations within this chapter were performed on a PC workstation powered by Intel i5-2500K processor and 8GB of RAM. The measured CPU time should not be considered as an absolute ground-truth since it will vary from one computer system to another, but the relative ordering of the methods is expected to remain the same.

7.2 Case study A: A basic network

At first, a very simple water network was considered to investigate different approaches to modelling and simulation in the described environments. The structure of the WDS model, namely Network A, is shown in Figure 7.1a. It is a simplified version of the case study utilised by Gupta and Bhave (1996); Tabesh *et al.* (2002); Cheung *et al.* (2005); Gupta *et al.* (2013). Network A contains a tank, pipe and node. In this example, the tank's water level changes over 24 hour period due to the diurnal demand at the consumption node. Data of the water network elements are listed in Table 7.2.

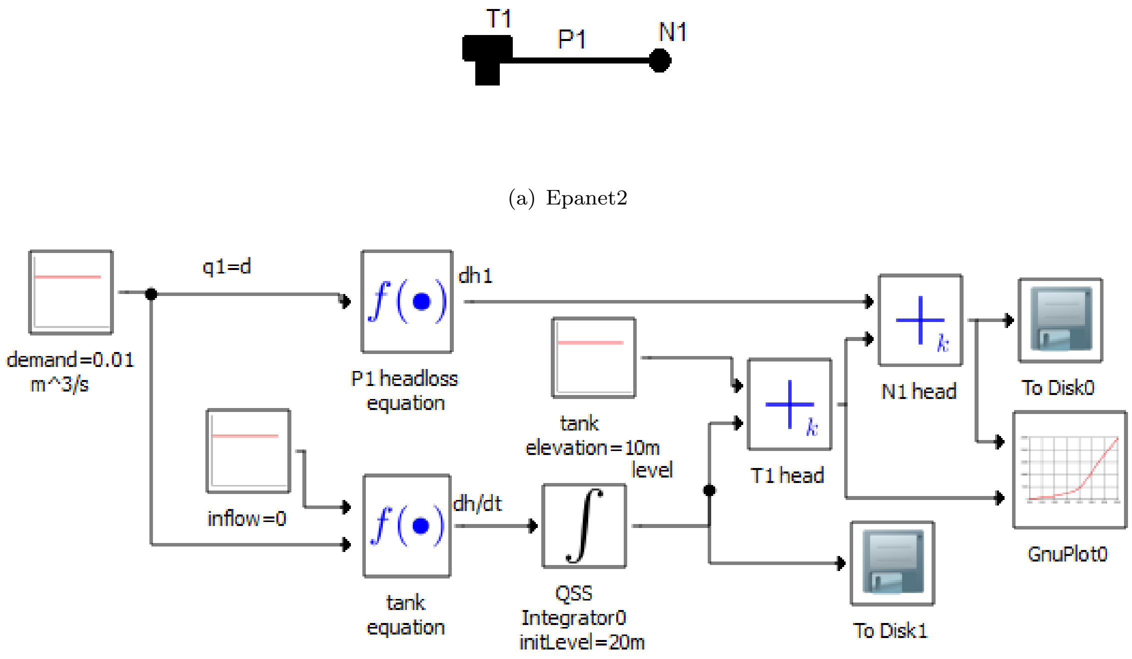
Among the employed M&S software, Epanet2, as a tool dedicated to simulation of WDSs, offers the easiest way to model Network A. PowerDEVS as a general purpose M&S tool also provides a GUI where models can be built graphically. But not surprisingly, as it was not developed for water research domain, modelling even a simple water network requires more effort than in Epanet2. Although, PowerDEVS enables creation of a customised library to speed-up the modelling process but definition of a new library might be error-prone and rather cumbersome as only a very limited manual to PowerDEVS is provided. Network A modelled in PowerDEVS is depicted in Figure 7.1b.

The OpenModelica environment is the most advanced amongst the described tools. Similarly to PowerDEVS user can build models with use of graphical blocks but its strength is in the equation-based modelling that enables a direct transformation from the mathematical representation of systems. Hence, in OpenModelica different types of WDS models; i.e. nodal, branch flow or mixed can be defined in a convenient way.

Table 7.1: Features of the employed simulators.

Simulator	User interface	Platform	Numerical integration	Simultaneous DAEs solver	Approach	Notes
Epanet2	graphical, designed specifically for WDS	open-source	Euler method	Netwon method	time-based	Epanet Toolkit provides means to add new user-defined functions
OpenModelica	Modelica language equation-based syntax and graphical with use of block diagrams	open-source	DASSL[a]	Newton method[b]	time-based	
PowerDEVS	graphical with use of block diagrams	open-source	QSS	Not supported	event-based	supports calls to external C++ functions
QSS Solver	μModelica language equation-based syntax	open-source	QSS	Not supported	event-based	the version used does not support calls to external functions

[a]OpenModelica provides a number of others methods. See (Fritzson, 2010) for details.
[b]OpenModelica provides also other methods such as KINSOL and hybrid method.



(a) Epanet2

(b) PowerDEVS

FIGURE 7.1: Network A modelled in Epanet2 and PowerDEVS.

TABLE 7.2: Nodes and links in Network A.

Tanks	Elevation [m]	Diameter [m]	Initial level [m]
T1	10	20	20
Nodes	Elevation [m]	Demand [m^3/s]	
N1	0	0.01	
Pipes	Length [m]	Diameter [m]	Roughness (Hazen-Williams)
P1	1000	0.3	100

Definition of models in QSS Solver is also the equation-based but in contrast to OpenModelica the order of equations is important; equations must be given in an explicit ordinary differential equation (ODE) form. To ease conversion from the differential algebraic equation (DAE) representation of the system to the ODE form, the developers of QSS Solver employed the built-in algorithms of the OpenModelica compiler (OMC) which simplify expressions and sort the equations. This feature of QSS Solver, however, was not fully-functional at time of writing this thesis and DAEs describing WDS models were transformed into the ODE form by author of this thesis manually.

Network A in the Modelica and μModelica standard are listed in Listings 7.1 and 7.2, respectively.

LISTING 7.1: OpenModelica model of Network A.

```
model TankSimpleModel
  //Parameters
  Real L = 1000 "Pipe length [m]";
  Real D = 0.3 "Pipe diameter [m]";
  Real C = 100 "Pipe roughness";
  Real T0 = 20 "Initial water level in tank [m]";
  Real Td = 20 "Tank diameter [m]";
  Real Te = 10 "Tank elevation [m]";
  Real d = 0.01 "Demand [m^3/s]";
  Real R,A "Resistance, Cross-sectional area";
  //Variables
  Real dh1 "Pipe 1 headloss";
  Real q1 "Pipe 1 flow";
  Real h1,h(start = 20) "Node 1 head, Tank level";
equation
  R = 10.69 * L / C ^ 1.852 / D ^ 4.871 "Resistance";
  A = 3.14159 * (Td / 2) ^ 2;
  //Mass balance
  q1 - d = 0;
  //Energy equations
  dh1 = R * q1 * abs(q1) ^ 0.852;
  h1 = h + Te - dh1;
  //Component equation
  der(h) = 1 / A * (0 - d);
end TankSimpleModel;
```

LISTING 7.2: QSS Solver model of Network A.

```
model tanksimplemodel
  annotation(
  experiment(
    description="Tank Simple model",
    solver=QSS,
    StartTime=0,
    StopTime=86400,
    Tolerance=1e-1,
```

```
    AbsTolerance=1e-9,
    QssSettings= "
      output ht;
      output h1;
    "
  ));

  //Parameters
  Real L, D, C, Td, Te, d, R, A ;
  //Variables
  Real dh1 ;
  Real q1;
  Real h1;
  Real h(start = 20);
  Real ht;
equation
  L = 1000;
  D = 0.3;
  C = 100;
  Td = 20 ;
  Te = 10 ;
  d = 0.01 ;
  R = 10.69 * L / C ^ 1.852 / D ^ 4.871 ;
  A = 3.14159 * (Td / 2) ^ 2;
  //Mass balance
  q1 = 0+d;
  //Energy equations
  dh1 = R * q1 * abs(q1) ^ 0.852;
  h1 = h + Te - dh1;
  //Component equation
  der(h) = 1 / A * (0 - d);
  ht = h + Te;
end tanksimplemodel;
```

The above representations of Network A were subsequently simulated within their respective simulation environments. Table 7.3 presents results from those simulations. The simulations and results were compared in terms of the time required for simulation, number of resulting points, and the hydraulic accuracy with respect to simulation performed in Epanet2 or Epanet2 Toolkit. The number of resulting points indicates a number of elements in the result vector. To assess the hydraulic accuracy plots of heads and flows were used.

The simulations were carried out for different tolerance (Epanet2 and OpenModelica) and quantum (PowerDEVS and QSS Solver) values to investigate their impact on the above comparison criteria.

To obtain the simulation time in Epanet2 a Matlab-based script was used to measure via Epanet2 Toolkit the average time for 10 successive simulation runs. The simulation times for PowerDEVS, OpenModelica and QSS Solver were read directly from their simulation log files. The simulation times given in Table 7.3 refer to the simulation process only; time for saving result data is not included. Note that the number of output points in OpenModelica is user-depended; in this case the number of requested points was 24. OpenModelica and QSS Solver log the simulation time with a millisecond accuracy whereas in PowerDEVS the logged times are given with the C++ float number precision.

Table 7.3: Comparison of simulations for Network A. (⋆)Note that the number of output points in OpenModelica is user-depended.

Simulator	Sampling time [s]	Tolerance/ Quantum	Number of output points	Run-time [s]
Epanet2	3600	1e-3	24	0.009
	60	1e-3	1440	0.0046
PowerDEVS	-	1e-1	2	<1e-13
	-	1e-2	15	<1e-13
	-	1e-3	148	<1e-13
	-	1e-4	1480	<1e-13
	-	1e-5	14794	0.875
QSS Solver	-	1e-1	2	<1e-1
	-	1e-2	15	<1e-1
	-	1e-3	148	<1e-1
	-	1e-4	1480	<1e-1
	-	1e-5	14794	<1e-1
OpenModelica	1	1e-3	27⋆	0.0048
	1	1e-6	27⋆	0.0088

As can be seen in Figure 7.2, the tank's trajectories obtained from the employed tools were identical. However, Table 7.3 demonstrates that the simulation run-times differs significantly between the used simulation engines. Simulations of Network A with use of the QSS methods were faster than with use of the classical time-slicing engines, except the PowerDEVS case when quantum parameter was set to 1e-5.

Although at this stage, it was difficult to notice any difference between the simulators in terms of the simulation run-times or hydraulic accuracy, this case study provided invaluable know-how knowledge which ease the implementation of the subsequent benchmark models.

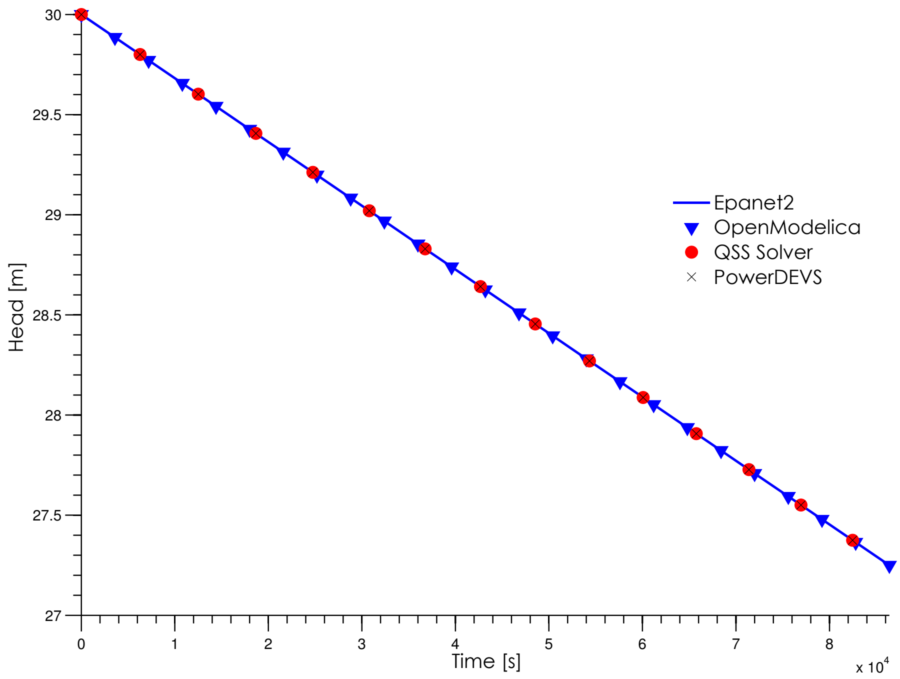


FIGURE 7.2: Head trajectories for tank T1 obtained from different simulators.

7.3 Case study B: Network with pump and tank

The next water network considered for a preliminary investigation is illustrated in Figure 7.3. The aim of this theoretical WDN, henceforth referred as Network B, is to provide with water to an industrial user. The parameters of Network B elements are given in Table 7.4 and Table 7.5. The fixed-speed pump in the network is controlled by the water level in the cylindrical tank. The pump was set to be switched off when the water level in the tank reaches 6 metres and switched on when the water level drops below 6 metres. This was done intentionally, to observe how the classical approach, represented by Epanet2, and the proposed DEVS + QSS combination, represented by PowerDEVS, can cope with a frequent switching. The corresponding models in Epanet2 and PowerDEVS are illustrated in Figure 7.4a and Figure 7.4b, respectively.

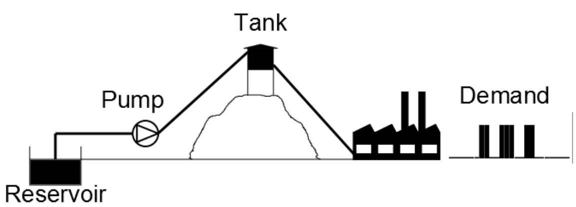


FIGURE 7.3: Illustrating a simple water distribution system.

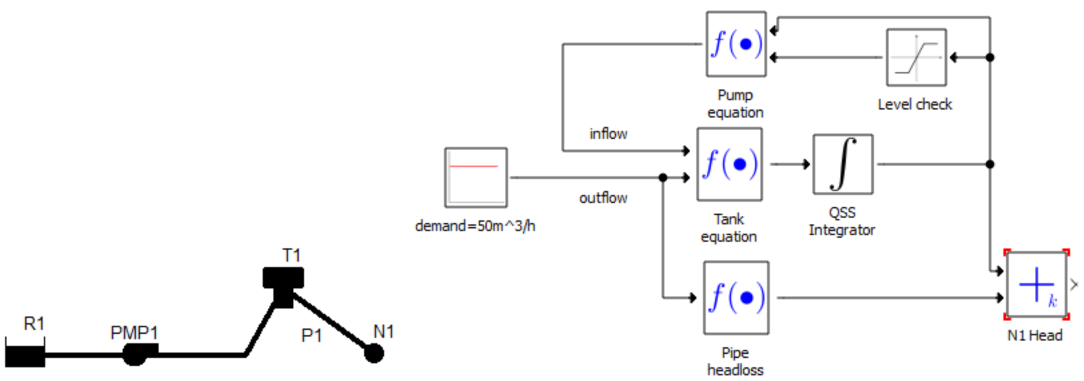
TABLE 7.4: Nodes in Network B.

Tanks & Reservoirs	Elevation [m]	Diameter [m]	Initial level [m]
T1	10	10	1
R1	0		
Nodes	Elevation [m]	Demand [m^3/h]	
N1	0	50	

TABLE 7.5: Links in Network B.

Pipes	Length [m]	Diameter [m]	Hazen-Williams factor
P1	1	1	100
Pumps	Equation	Initial status	
PMP1	$h = 80 - 0.002q^2$	closed	

While such a frequent switching may seem to be unrealistic when water distribution networks are concerned, the author of this thesis came across a real water network, in which pump controlled by water level is switched on/off frequently, e.g. in Skworcow *et al.* (2009) every 7 to 30 minutes. Furthermore, this shall evaluate the ability of simulators to detect events and assess their respective computational efficiency when simulating such a system.



(a) Epanet2 (b) PowerDEVS

FIGURE 7.4: Network B modelled in Epanet2 and PowerDEVS.

The extended period simulations of the models, illustrated in Figure 7.4, were performed for a period of 24 hours. The simulations in Epanet2 were carried out with the hydraulic steps of 1 hour and 1 min. The simulations in PowerDEVS were performed with the relative quantum step of 0.001.

The results from Epanet2 with the hydraulic step of 1 min and PowerDEVS were very similar when the pump flow and tank's water level where compared. Although, in Epanet2 detection of the event's time to switch on/off the pump was slightly inaccurate due to the sampling time. However, when the Epanet2 simulation was run again but with the hydraulic step of 1 hour, which is typical in WDS simulations, the pump missed the event where it should be switched on. This behaviour is depicted in Figure 7.5.

It was previously described in Section 2.3.5 that one way to achieve a more precise simulation of WDS is to reduce the length of time interval accordingly. Another technique, employed inter alia by Epanet2, is to introduce additional intermediate checks around the hydraulic calculation time step (Rossman, 2000a). Unfortunately, the standard Epanet2 does not provide access to such simulation data. Instead, the Epanet2 Toolkit can be used to extract the simulation data with the intermediate steps included. The trajectories obtained from Epanet2 Toolkit (with usage of intermediate steps) and PowerDEVS simulations were nearly identical as can be seen in Figures 7.5, 7.6 and 7.7. However, the consequence of introduction the intermediated steps was that the number of resulting points in the Epanet2 hydraulic results increased drastically. Table 7.6 shows the length of resulting vectors for both simulators. PowerDEVS requires significantly less points to simulate the hydraulic behaviour with the same accuracy as Epanet2. Moreover Epanet2 simulated the model in ~1400 milliseconds whereas PowerDEVS simulated the model in ~630 milliseconds.

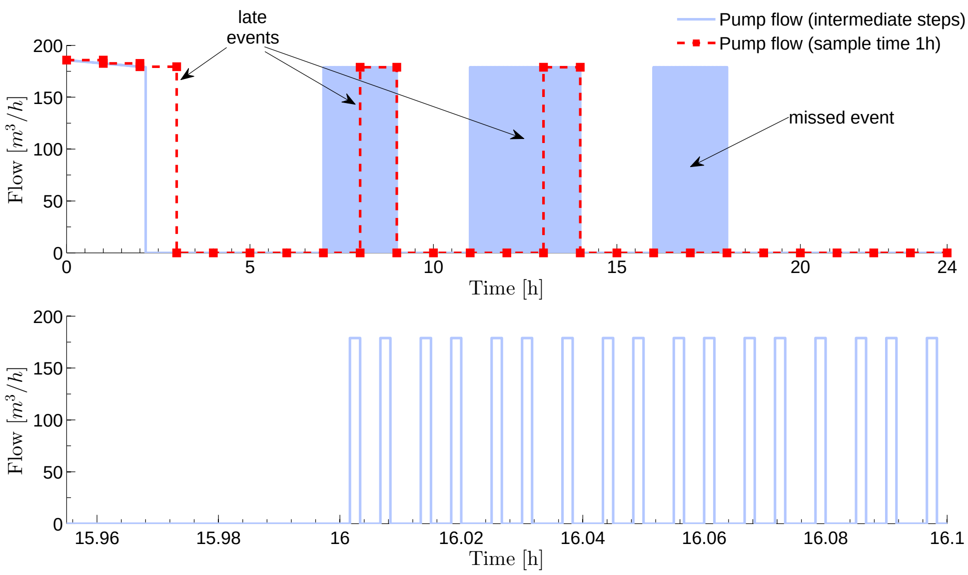


FIGURE 7.5: The upper plot shows the pump flow simulated in Epanet2 whereas the bottom plot shows a zoomed section around the time stamp of 16 hours.

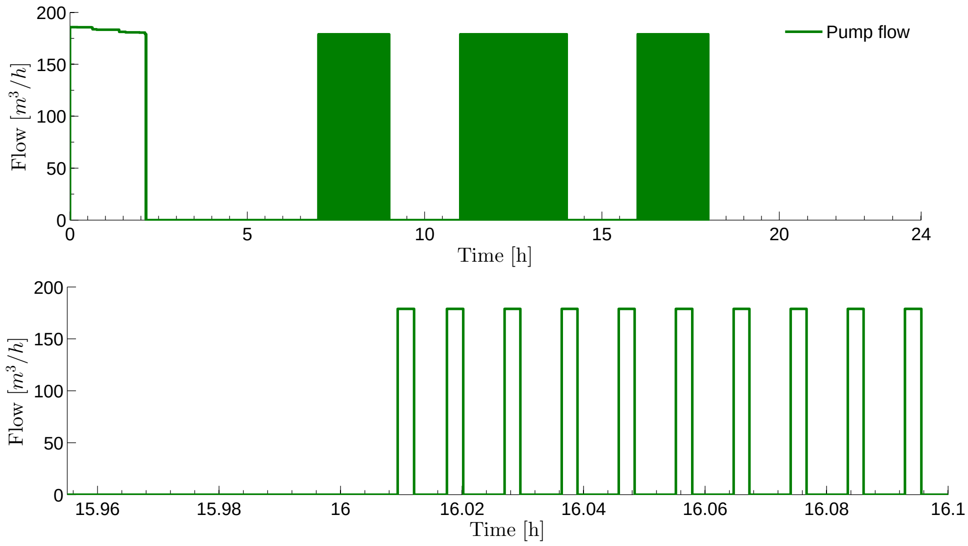


FIGURE 7.6: The upper plot shows the pump flow simulated in PowerDEVS whereas the bottom plot shows a zoomed section around the time stamp of 16 hours.

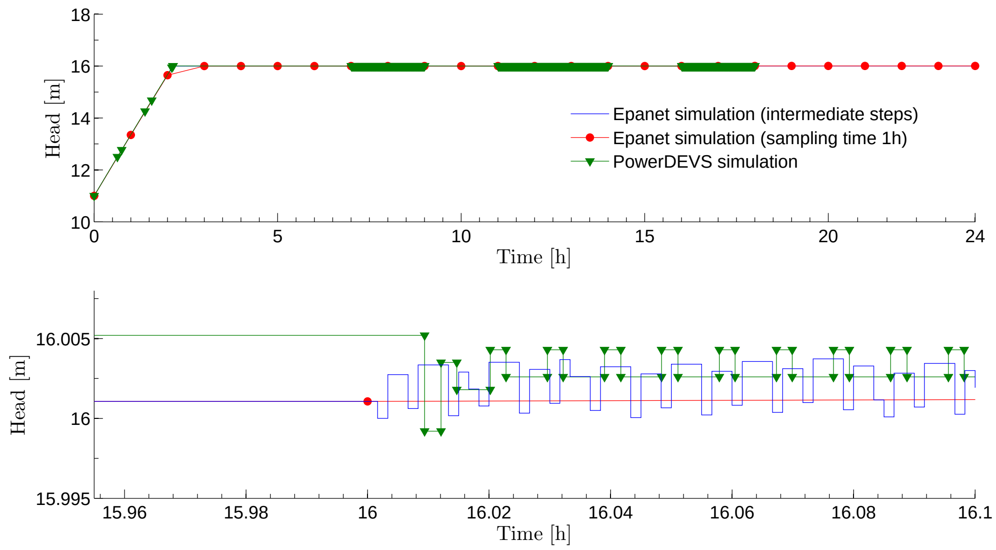


FIGURE 7.7: The upper plot depicts comparison of the water levels in the tank T1 resulted from three simulations. The bottom plot shows a zoomed section around time stamp of 16 hours.

Table 7.6: Number of resulting points for the head and flow variables.

Variable	Epanet2 Toolkit (with intermediate steps)	PowerDEVS
tank's head	3555	1408
pump's flow	3555	2999

Based on the obtained results from the PowerDEVS and Epanet2 simulators, it can be concluded that, a WDS with discontinuous elements can be simulated in PowerDEVS with a similar accuracy as in Epanet2, which for many years set the benchmark in terms of WDS analysis. But while Epanet2 needed introduction of the intermediate steps to simulate the considered WDS accurately the DEVS and QSS-based approach naturally accommodated asynchronous, concurrent and nonlinear nature of WDSs.

For the next tests, the pump behaviour, in Network B, was changed i.e. the pump was set to operate when the water level in tank reaches 15 m and when the water level drops below 10 m the pump was switched off. Additionally, OpenModelica and QSS Solver were included to model Network B. The respective models of Network B in OpenModelica and QSS Solver are presented in Listings 7.3 and 7.4, respectively.

Listing 7.3: OpenModelica model of Network B.

```
model oneTankAndPump
  Real qp,ht(start = 11),u,h1,dh1;
initial algorithm
  u:=0;
equation
  h1 = ht - dh1;
  dh1 = 50 ^ 1.852 * 1.21216 * 10 ^ 10 * 1 / 100 ^ 1.852 / 1000 ^ 4.871;
  der(ht) = 1 / 3.14159 / 25 * (qp - 50);
algorithm
  if u == 0 then
    qp:=0;
  else
    qp:=sqrt((80 - ht) / 0.002);
  end if;
  when ht > 15 then
      u:=0;
  end when;
  when ht < 10 then
      u:=1;
  end when;
end oneTankAndPump;
```

Listing 7.4: QSS Solver model of Network B.

```
model oneTankAndPump
```

```
  annotation(
  experiment(
    description="",
    solver=QSS,
    StartTime=0,
    StopTime=24,
    Tolerance=1e-3,
    AbsTolerance=1e-9,
    QssSettings= "
      output ht;
      output qp;
      output h1;
    "
  ));
Real ht(start = 11);
Real qp;
Real dh1;
Real h1;
discrete Real demand;
discrete Real u;
initial algorithm
u:=0;
demand:=1;
equation
  qp = u*sqrt((80-ht)/0.002);
  der(ht)=1/3.14159/25*(qp-50*demand);
  dh1=demand^1.852*1.21216*10^10*1 / 100^1.852 / 1000^4.871;
  h1=ht-dh1;
algorithm
  when ht>15 then
    u:=0;
  end when;
  when ht<10 then
    u:=1;
  end when;
end oneTankAndPump;
```

Table 7.7 presents the simulation results from all the simulators. The comparison includes the measured simulation run-time and the number of result points for different settings of their respective tolerance/quantum parameter.

In the time-slicing environments, Epanet2 and OpenModelica, shorter step length or increased tolerance precision improved simulation accuracy (see Figure 7.8), but, as a result of these changes, the simulation run-times were longer. Yet, all simulations in Epanet2 and OpenModelica took only several milliseconds, with OpenModelica insignificantly slower.

When the time-slicing simulators were compared against the QSS-based simulators in terms of the simulation run-time, it was noticed, that for the QSS quantum parameter

Table 7.7: Comparison of simulations for Network B.

Simulator	Interval time [s]	Tolerance/ Quantum	Number of output points	Run-time [s]
Epanet2	3600	1e-3	24	0.0011
	60	1e-3	1440	0.0069
PowerDEVS	-	1e-1	19	<1e-13
	-	1e-2	180	<1e-13
	-	1e-3	1850	0.215
	-	1e-4	18534	2.62
	-	1e-5	185388	28.2
QSS Solver	-	1e-1	19	<1e-3
	-	1e-2	180	<1e-3
	-	1e-3	1850	<1e-3
	-	1e-4	18534	0.0156
	-	1e-5	185388	0.0468
OpenModelica	3600	1e-3	46	0.0049
	3600	1e-6	46	0.0072

≥1e-3 the time-slicing approaches were significantly slower, but once the QSS quantum decreased to ≤1e-4 the simulations in PowerDEVS and QSS Solver lasted drastically longer. It is noticeable especially in the PowerDEVS simulations. Such a large overhead is expected to be due to message mechanism of the DEVS simulation engine as highlighted by Bergero *et al.* (2012), what encouraged Bergero *et al.* (2012) to develop a stand-alone QSS Solver.

From Table 7.7 in can be observed that in simulations involving the QSS methods the number of resulting points increased reverse proportionally to the quantisation step value. This was expected, as in approaches involving the QSS methods, the choice of the quantisation step is both reverse proportional to the simulation time and directly proportional to the quantisation error (Capocchi *et al.*, 2009).

It is worth to highlight that when performing simulations in OpenModelica, 24 result points were requested but the output file contained 46 points. This is due to switching events around which OpenModelica engine introduced additional points to determine the accurate time of the event.

Since the state trajectories of the model cannot be computed analytically, the accuracy of the particular simulation can only be approximated. For this purpose, results form simulations were plotted in Figure 7.8 for a graphical inspection. Figure 7.8 illustrates the head changes in tank over the period of 24 hours.

It was decided to use results from OpenModelica as a benchmark, instead the results from Epanet2, as nowadays, DASSL represents state-of-art multi-purpose DAE solver used in many commercial simulation environments e.g. in Dymola (Dassault Systems, 2014) whereas Epanet2, despite its established position, has not been improved for years.

It can be clearly seen that simulations with the QSS methods with the quantisation step of 1e-1 and 1e-2 are imprecise. This is because the QSS method used in the simulation was a first-order accurate. In this method, abbreviated QSS1, to achieve a small simulation error the tolerance needs to be decreased but then the number of output points will be larger. Also, the Epanet2 simulation with the sampling time of 1 hour cannot be treated as accurate when compared against the OpenModelica results

For a more detailed inspection the plot was zoomed around area of the first switching event occurrence (around 1h 34min). The zoomed section is pictured in Figure 7.9. Amongst the plotted trajectories only the QSS-based simulation with the quantum of 1e-5 detected the event at the same time as the reference time resulted from the OpenModelica simulation. Although, time of event occurrence in QSS 1e-5 was identical with the benchmark time this resulted in the longer simulation run-time and larger number of output points than other solutions.

The Epanet2 simulation with the interval time of 1 min along with the simulation in QSS Solver with the quantisation step of 1e-4 were ranked second when the precision of event's time detection was considered. They were followed by simulation in QSS Solver with the quantisation step of 1e-3. The differences in terms of event's detection time with respect to the OpenModelica benchmark time are shown in Table 7.8.

Table 7.8: Differences in terms of event's time detection.

Simulator	Event time [hh:mm:ss]	Difference [hh:mm:ss]
OpenModelica	01:34:25	-
QSS 1e-3	01:35:34	+00:01:09
QSS 1e-4	01:34:34	+00:00:09
QSS 1e-5	01:34:25	00:00:00
Epanet2 1min	01:34:00	-00:00:25

At this stage, the event-domain methods were not outperforming the classical time-domain approaches when simulation of Network B was considered. While the first-order QSS

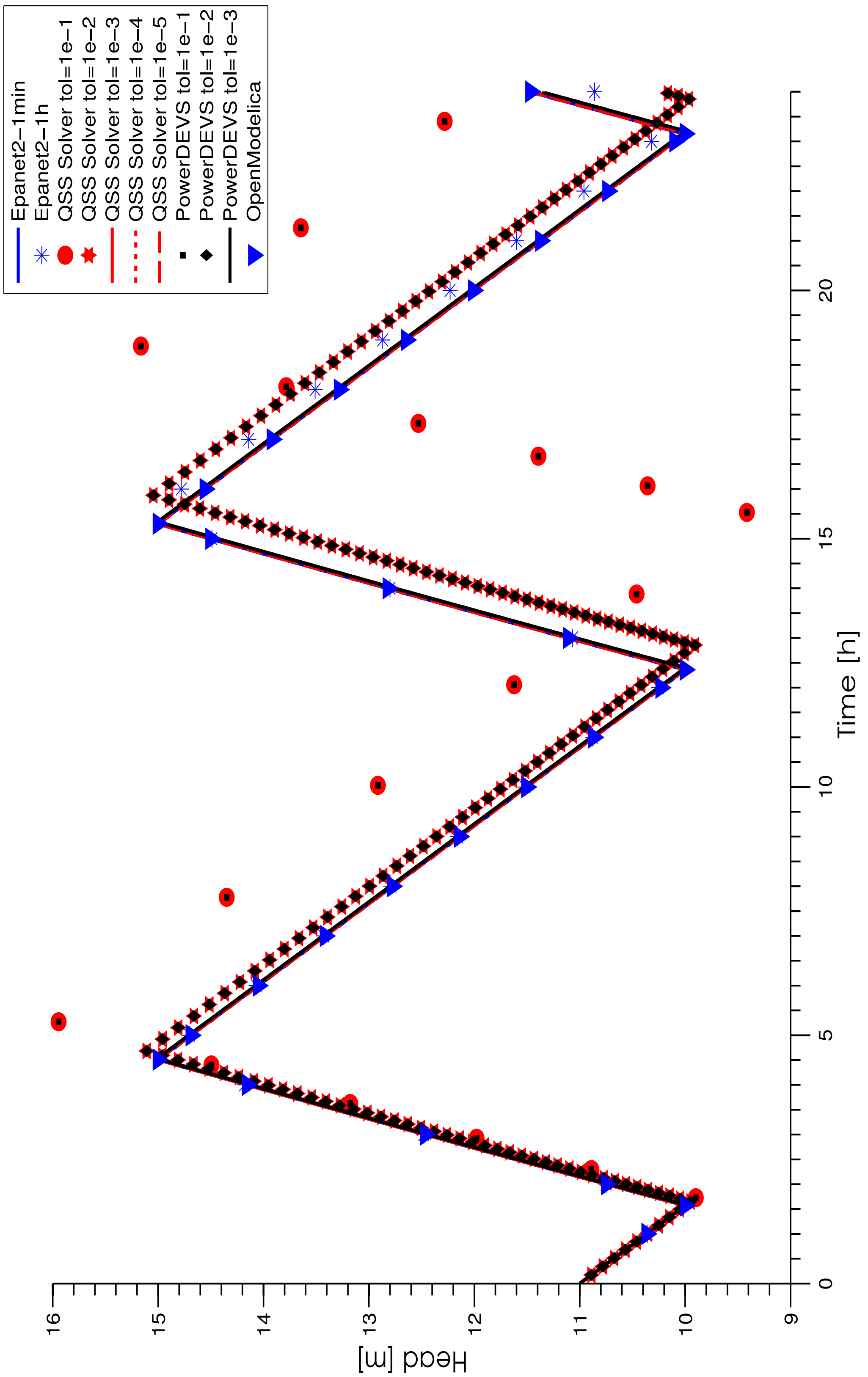


FIGURE 7.8: Simulation results for Network B depicting the tank's head trajectories.

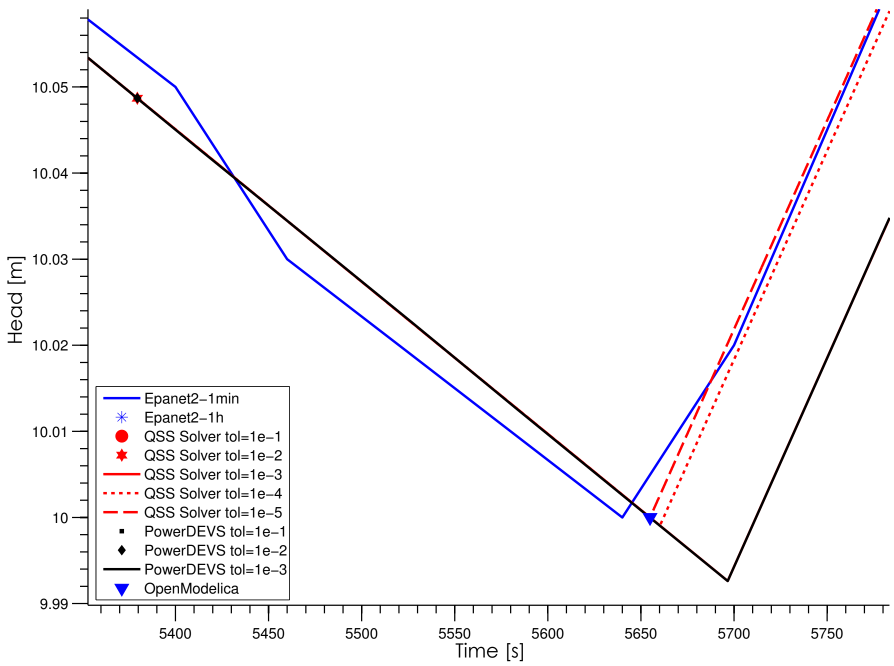


FIGURE 7.9: Simulation results for Network B depicting a zoomed section of the tank's head trajectories.

method was capable to simulate accurately the considered system it required a longer simulation run-time and produced a large number of output points. Choosing the quantisation step is not a straightforward task as its value is both reverse proportional to the simulation time and directly proportional to the quantisation error.

One way to determine the quantisation step is to know the signal magnitude and to divide it by a reasonable number to obtain acceptable results. Other solution, is to use a method defined in (Cellier and Kofman, 2006), which allows to calculate the quantisation step based on the error rate defined by the user. However, the recommenced approach is to use higher order QSS methods, especially the third order method since it provides the smaller simulation time and it gives more freedom in the choice of the quantisation step without any impact on the quantisation error (Kofman, 2006).

In QSS2 (Kofman, 2002), the quantised state variables evolve in a piecewise linear way with the state variables following piecewise parabolic trajectories. In the third-order extension, QSS3 (Kofman, 2006), the quantised states follow piecewise parabolic trajectories, while the states themselves exhibit piecewise cubic trajectories.

Results from the simulations in QSS Solver with the higher order QSS methods are presented in Table 7.9, Figure 7.10 and Figure 7.11.

TABLE 7.9: Comparison of simulations with QSS1, QSS2 and QSS3 methods.

QSS method	Quantum	Number of output points	Run-time [s]
QSS1	1e-1	19	<1e-3
	1e-2	180	<1e-3
	1e-3	1850	<1e-3
	1e-4	18534	0.0156
	1e-5	185388	0.0468
QSS2	1e-1	6	<1e-3
	1e-2	9	<1e-3
	1e-3	14	<1e-3
	1e-4	30	<1e-3
	1e-5	80	<1e-3
QSS3	1e-1	6	<1e-3
	1e-2	9	<1e-3
	1e-3	10	<1e-3
	1e-4	17	<1e-3
	1e-5	22	<1e-3

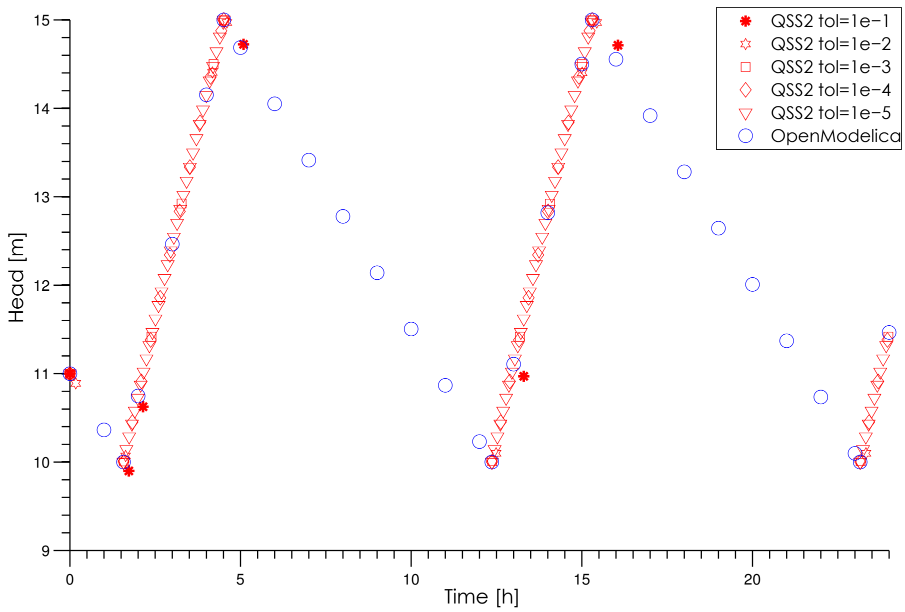


FIGURE 7.10: Comparison of QSS2 against OpenModelica.

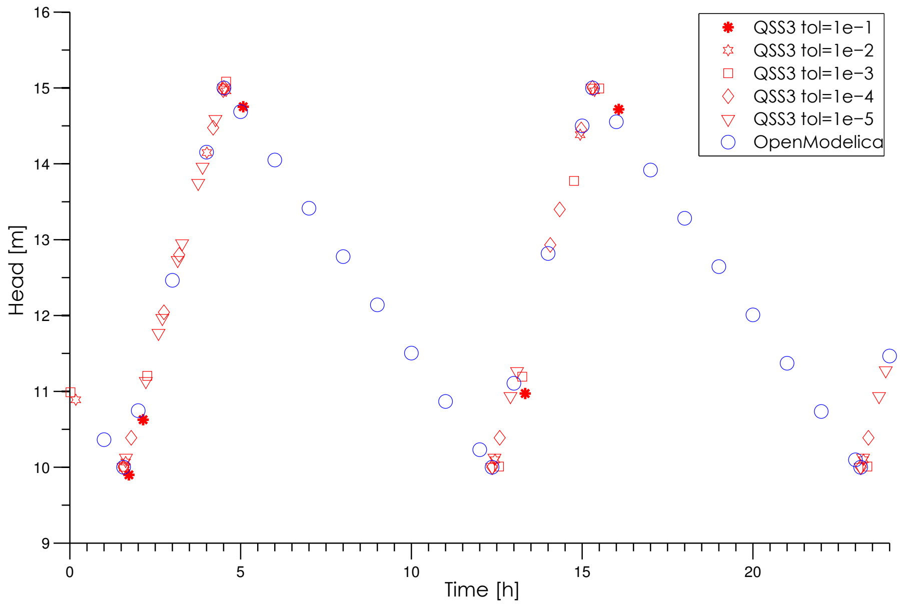


FIGURE 7.11: Comparison of QSS3 against OpenModelica.

QSS2 and QSS3 methods not only significantly decreased the simulation run-times, but also, they were much more accurate than the QSS1 method. Additionally, even for quantum values ≤1e-4, the number of the output points was significantly smaller than in the discrete-time simulators. While the first order state integration algorithm, QSS1, did not bring significant improvement over the classical time-discrete methods the higher order QSS methods outperformed the time-domain-based simulators; QSS2 and QSS3 algorithms can achieve a good accuracy without excessive increment in the number of steps.

7.4 Case study C: A looped network

So far only tree shaped water networks were considered in simulations with use of the QSS algorithms. This study, depicted in Figure 7.12, considers a one-loop WDN, which consists of three junctions, four pipes and a tank with the parameters as listed in Table 7.10 and Table 7.11. Such a model, hereafter referred as Network C, is often used in the literature to illustrate the convergence properties of different algorithms when solving water networks, see e.g. (Ulanicka *et al.*, 1998) or (Larock *et al.*, 2000).

In this network, described by Equations 7.1 - 7.6, the head loss in the pipe that connects the tank to the network can first be determined, and subsequently, this value can be subtracted from the tank's water surface elevation to determine head at node N1, see Equation 7.2.

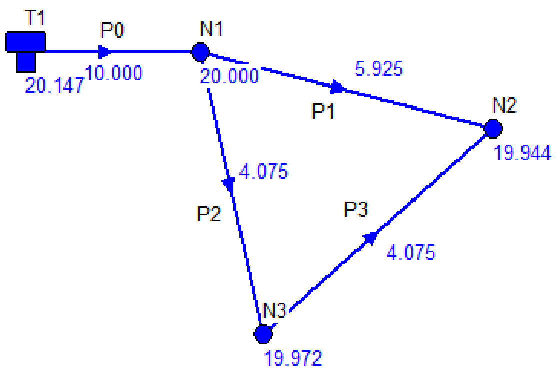


FIGURE 7.12: Model of Network C in Epanet2. Note that displayed numeric values refer to flows [l/s] in pipes and heads [m] at nodes.

TABLE 7.10: Nodes in Network C.

Tanks	Elevation [m]	Diameter [m]	Initial level [m]
T1	10.14688	20	10
Nodes	Elevation [m]	Demand [m^3/s]	
N1	0	0	
N2	0	0.01	
N3	0	0	

TABLE 7.11: Links in Network C.

Pipes	Length [m]	Diameter [m]	Hazen-Williams factor
P0	1000	0.3	100
P1	1000	0.3	100
P2	1000	0.3	100
P3	1000	0.3	100

The next step, however, requires the remaining set of nonlinear equations to be solved simultaneously as they form an algebraic loop and thereby cannot be causalised.

$$\frac{dh_t}{dt} = \frac{1}{S}(0-d) \tag{7.1}$$

$$h_1 + Rq_0|q_0|^{0.852} + h_t = 0 \tag{7.2}$$

$$q_1 + G(h_3 - h_2)|h_3 - h_2|^{-0.46} = d \tag{7.3}$$

$$q_2 - G(h_3 - h_2)|h_3 - h_2|^{-0.46} = 0 \tag{7.4}$$

$$h_1 - Rq_1|q_1|^{0.852} - h_3 = 0 \tag{7.5}$$

$$h_1 - Rq_2|q_2|^{0.852} - h_2 = 0 \tag{7.6}$$

The simulation engines in Epanet2 and OpenModelica solve such problems using the Newton iterations, see Appendix A. Epanet2 model in Figure 7.12 is simulated with use of the simulation routine shown in Figure 2.10. In OpenModelica the equations 7.1-7.6 can be directly rewritten using OpenModelica syntax as shown in Listing 7.5. Such a model is then subjected to equations sorting procedures before the Newton method is applied.

Listing 7.5: OpenModelica model of Network C.

```
model networkC
  Real q0,q1,q2,h1,h2,h3,ht(start = 20.14688);
equation
  q0 = 0.01;
  q1 + 0.02813 * (h3 - h2) * abs(h3 - h2) ^ (-0.46) = 0.01;
  q2 - 0.02813 * (h3 - h2) * abs(h3 - h2) ^ (-0.46) = 0;
  h2 + 744.59 * q1 * abs(q1) ^ 0.852 = h1;
  h3 + 744.59 * q2 * abs(q2) ^ 0.852 = h1;
  der(ht) = 1 / (3.14159 * 10 ^ 2) * (-0.01);
  h1 + 744.59 * q0 * abs(q0) ^ 0.852 = ht;
end networkC;
```

However, when Network C was converted into a block diagram representation in PowerDEVS a problem was encountered. Due to the algebraic loop, the resulting PowerDEVS model turned out to be illegitimate. To address the algebraic loop problem Cellier and Kofman (2006) proposed a solution in form of a new *Loop Breaking* atomic model. Its task is to generate an output if and only if the difference between actual and previous outputs does not exceed a threshold of tolerance specified by user. But the *Loop Breaking* model has some drawbacks i.e. introduction of ill-conditioning simulation cycles and need to define tolerance (Capocchi *et al.*, 2009). To circumvent these issues Capocchi *et al.* (2009) proposed replacement of *Loop Breaking* atomic model with coupled model of the first order low pass filter. But also in this solution the filter weighting coefficient needs to be determined in advance.

Hence, in order to simulate Network C in PowerDEVS, author of this thesis proposed to mimic idea of *Implicit Block* from the PowerDEVS continuous systems library and invoke a call to external function. But instead of solving just one nonlinear equation like *Implicit Block* does, the called external function will solve a set of nonlinnear equation simultaneously.

For this purpose, an external function was written in C++ and linked with the additional libraries from GNU Scientific Library (GSL) (Galassi *et al.*, 2013). GSL prowides with access to a number of root-finding algorithms such as Powell's hybrid method (Powell, 1964), standard Newton's method and their modifications. The external solver of nonlinear equations circumvented the limitations of PowerDEVS making it possible to simulate

Network C despite the existing restrictions. Network C represented by block diagrams in PowerDEVS is illustrated in Figure 7.13.

Unfortunately, the μModelica restrictions require models to be described by casual equations, and thereby models with algebraic loops cannot be modelled straightforward. According to Bergero *et al.* (2012) the extended OMC should be able to convert full Modelica models into their μModelica equivalents but the converted μModelica models were not replicating the original. Also, the QSS Solver version used in this work was unable to link with the GSL libraries and therefore the written function to solve nonlinear equations could not be used within QSS Solver.

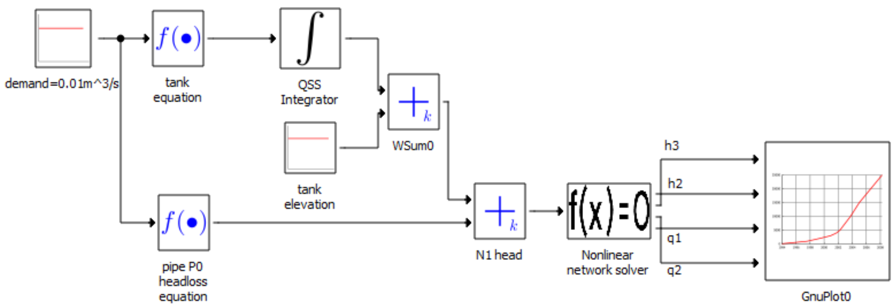


FIGURE 7.13: Model of Network C in PowerDEVS.

In this example, only a static simulation was performed to investigate whether DEVS and QSS approach is able simulate a WDS in the loop configuration. Table 7.12 shows that obtained flows q_1 and q_2 and heads h_2 and h_3 are identical in all the simulators.

TABLE 7.12: Network C static simulation results.

Simulator	q_1 [m^3/s]	q_2 [m^3/s]	h_2 [m]	h_3 [m]
Epanet2	0.005925	0.004075	19.944	19.972
OpenModelica	0.005925	0.004075	19.944	19.972
PowerDEVS	0.005925	0.004075	19.944	19.972

7.5 Case study D: Epanet2 Net 1 example

The next water network used to evaluate the proposed methodology is depicted in Figure 7.14. It is an example of water network taken from the Epanet2 manual (Rossman, 2000b). This network was used in a number of studies for different purposes, see e.g.

(Tabesh and Dolatkhahi, 2006; Cabrera *et al.*, 2010). The data of all the components are listed in Table 7.13 and Table 7.14.

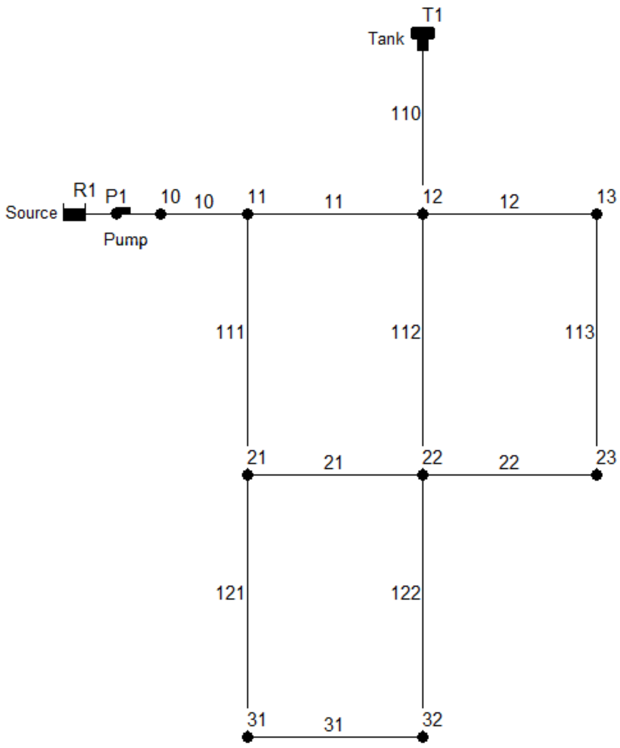


Figure 7.14: Model of Network D in Epanet2.

The network, referred henceforth as Network D, was simulated for the period of 24 hours. This example enabled to investigate the ability of the new paradigm to perform the extend period simulation of the looped WDS. Moreover, in Network D two different event types occurs: (i) time scheduled events as the demand allocated to the consumption nodes varies over the 24 hour period as is shown in Figure 7.15 and (ii) events due to changes in tank water level that govern the operation of the pump; i.e. the pump P1 is switched off when the water level in the tank T1 reaches 42.672 m and switched on when the water level drops below 33.528 m.

A similar approach to that for Network C has been employed to model and simulate Network D in PowerDEVS; i.e. a block diagram has been developed that calls the external C++ function to solve a set of nonlinear equations. Network D represented by block diagrams in PowerDEVS is illustrated in Figure 7.16. The *Check level* block detects when the water level hits the values which will switch on or off the pump. The demand over the 24 hour period has been predefined in the Scilab workspace and accessed from the model perspective via the *Demand pattern* block.

TABLE 7.13: Nodes in Network D.

Reservoirs	Elevation [m]	Initial head [m]	
R1	243.8	243.8	
Tanks	**Elevation [m]**	**Diameter [m]**	**Initial level [m]**
T1	259.08	15.3924	36.576
Nodes	**Elevation [m]**	**Demand [l/s]**	
10	216.41	0.00	
11	216.41	9.46	
12	213.36	9.46	
13	211.84	6.31	
21	213.36	9.46	
22	211.84	12.62	
23	210.31	9.46	
31	213.36	6.31	
32	216.41	6.31	

TABLE 7.14: Links in Network D.

Pipes	Length [m]	Diameter [mm]	Hazen-Williams factor
10	3209.544	457.2	100
11	1609.344	355.6	100
12	1609.344	254	100
21	1609.344	254	100
22	1609.344	304.8	100
31	1609.344	152.4	100
110	60.96	457.2	100
111	1609.344	254	100
112	1609.344	304.8	100
113	1609.344	203.2	100
121	1609.344	203.2	100
122	1609.344	152.4	100
Pumps	**Equation**	**Initial status**	
P1	$h = 101.6 - 0.002839q^2$	closed	

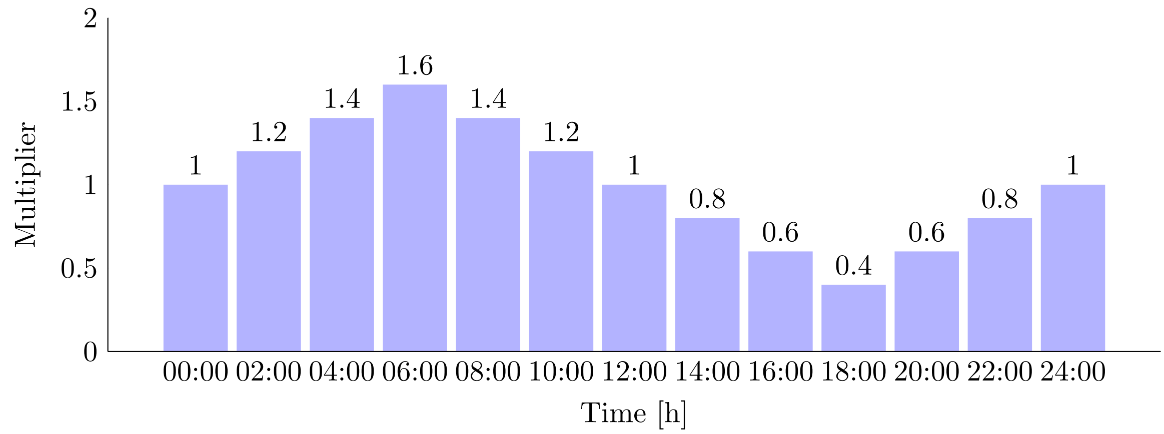


FIGURE 7.15: Demand pattern in network D.

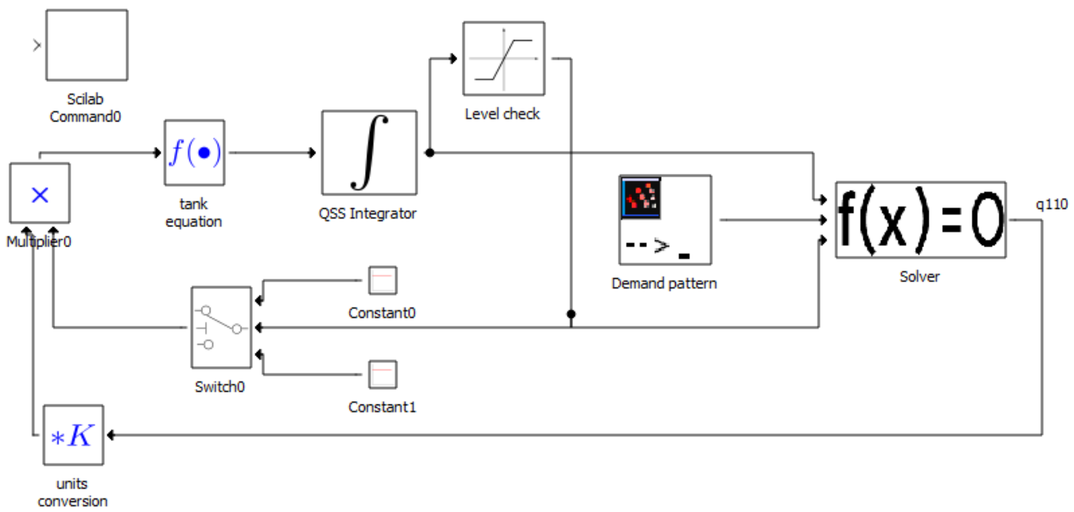


FIGURE 7.16: Model of Network D in PowerDEVS.

Network D in OpenModelica was modelled as set of equations with utilisation of the *when ... then* and *if ... then* statements to account for the time and state events. However, due to lengthy code the OpenModelica listing for Network D is omitted here.

For the purpose of comparison, Network D was simulated in Epanet2 with the 1 hour and 1 min time intervals, in PowerDEVS with use of the QSS1 method with the different quantum values, and in OpenModelica with the 1 s time interval. The results from the simulations are presented in Table 7.15 and Figure 7.17. Table 7.15 shows run-times and number of result points for each simulation and Figure 7.17 displays tank trajectories obtained from different simulation engines. It can been seen that the trajectories from nearly all the simulators are aligned and the events when the water level in tank reaches the operational constraints occurred at the similar time.

It is important to highlight, that to perform a hydraulic simulation of Network D in PowerDEVS with a satisfactory precision (i.e. to be in order with the benchmark simulation from OpenModelica), the quantum has to be set to 1e-4. The consequence of this, despite a smaller number of results points, was the much longer simulation run-time as can be seen in Table 7.15. It was assumed this was due to the drawback of the DEVS formalism, reported by Fernández and Kofman (2014), which is related to a number of messages sent between the atomic and coupled models.

Nevertheless, it is envisaged that with a new version of QSS Solver, which enables the use of external function and libraries, the simulation of Network D with use of the higher order QSS methods would result in a significant decrease of the simulation run-times.

TABLE 7.15: Network D simulation run-times.

Simulator	Interval time [s]	Tolerance/ Quantum	Number of output points	Run-time [s]
Epanet2	3600	1e-3	24	0.0035
	60	1e-3	1440	0.0398
PowerDEVS	-	1e-3	64	0.192
	-	1e-4	618	1.053
	-	1e-5	6157	9.981
OpenModelica	1	1e-6	1475	0.1348

7.6 Discussion

In the preceding sections it has been demonstrated that the functionalities included in the PowerDEVS and QSS Solver can be used to model water distribution systems using the QSS integration algorithms. The construction of models using PowerDEVS is close to the Simulink modelling paradigm; one needs to build a model using the blocks from the provided libraries. While the block diagrams are very convenient for modelling of simple physical systems they have been found not necessarily the most suitable tool for large systems such as water distribution networks. The additional drawback of PowerDEVS blocks paradigm is that it can only model systems that have been transformed into a block diagram. Unfortunately, it is not always straightforward to obtain the block diagram representation of a system. This problem is associated with the DEVS formalism, given

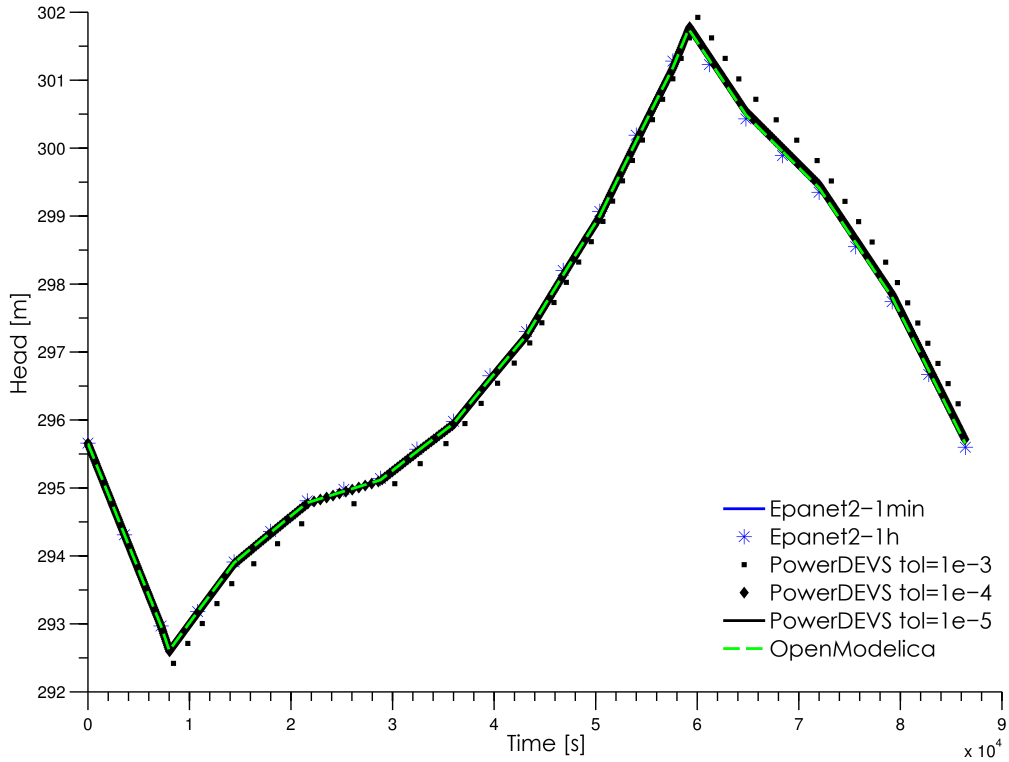


FIGURE 7.17: Simulation results for Network D depicting the tank's head trajectories.

the fact that DEVS models impose a certain input-output direction on the data flow of the simulation (Beltrame and Cellier, 2006).

Models of water networks in OpenModelica and QSS Solver can be build from their formal specification; i.e. WDS models have been implemented by describing their elements and the mass and energy conservation principles with the Modelica equation-based syntax. A significant limitation of QSS Solver is that it uses μModelica, a subset of the Modelica language, and the systems described by the DAE equations need to be transformed into the ODE form.

Another remark on PowerDEVS and QSS Solver is that they both do not facilitate solvers for sets of simultaneous nonlinear equations. This has hindered the process of simulation and additional external functions have been created by the author of this thesis to overcome this obstacle. Although, the development of additional functionalities for these simulators has been laborious and cumbersome as developers of PowerDEVS have not provided with a manual and QSS Solver is still in a development stage and thereby not fully functional.

Nevertheless, the implementation for several WDS models has permitted to test the QSS methods and appreciate their computational efficiency and accuracy.

At first, in Section 7.2, a simple WDS model has been used to illustrate modelling approaches in each of the employed software. This preliminary study has demonstrated that the QSS methods can achieve the same accuracy like in the classical time-slicing engines, represented by Epanet2 and OpenModelica, but it can be noticeable faster when the simulation run-times are considered.

In Section 7.3 the water network used for the evaluation of the proposed frameworks has comprised a discontinuous element in form of the pump operated based on the level of water in the tank. It has been assumed that this case study would ideally suit for simulations with the use of QSS methods, in which state variables (e.g. water level in tank) are updated only when the change is more than the defined quantum value. Indeed, the initial test based upon of frequent switching of the pump have illustrated asynchronism and efficiency of the QSS methods. The subsequent evaluations, with the pump operated on the water level margin, have demonstrated that the QSS methods competes well against conventional simulators in terms of accuracy. From a functional level the use of the QSS methods is analogous to ordinary time-discrete methods; if a reasonably small quantisation step is chosen then the algorithm provides a simulated trajectory that is sufficiently consistent with the analytical solution. However, the smaller quantum have resulted in increasing the simulation time. This has been especially visible in case of PowerDEVS simulations, in which the large number of integrator model activations has lead to the increased traffic within the DEVS simulation tree. Nevertheless, when the higher order QSS methods, QSS2 and QSS3, were employed the simulation run-time have decreased drastically while obtaining the same accuracy as the benchmark simulation performed in OpenModelica. It has been clearly seen that the higher order QSS methods are more efficient than conventional discrete-time solvers.

Next, in Section 7.4, the focus has been placed on the water network in a loop configuration to appreciate ability of the simulation environments to solve nonlinear equations simultaneously. While Epanet2 and OpenModelica have already built-in algorithms to solve such sets of equations (based on Newton method) the PowerDEVS and QSS Solver simulators do not provide such functionalities. To overcome this the author of this thesis has written external C/C++ functions in order to solve nonlinear equations via Newton iteration. In PowerDEVS calls to the solver have been implemented from within a block digram. Unfortunately, the QSS Solver version used has not allowed to call the external functions or link additional libraries. Nonetheless, Network C has been successfully solved in Epanet2, OpenModelica and PowerDEVS obtaining identical results in each simulator.

In Section 7.5 the water network simulated with DEVS and QSS concepts has comprised all the characteristics of the previous case studies i.e. tank dynamics, pump operated on the tank levels, looped configuration and demand pattern varied over period of time. With the Network D model created in PowerDEVS it has been demonstrated that the proposed DEVS and QSS approach can successfully simulate a fairly complex WDS model. The hydraulic results obtained in PowerDEVS have been in line with those obtained from Epanet2 and OpenModelica. However, simulations in PowerDEVS have been the slowest amongst the simulators as because of the implementations issues only the QSS1 method was used. Additionally, in PowerDEVS the QSS methods were implemented within DEVS formalism framework, which added additional overheads to the simulation run-times.

Nevertheless, it has been shown that water distribution systems can be modelled in the DEVS formalism framework with use of QSS methods. However, the current state-of-art of the associated tools do not allow to exploit fully the potential of the QSS methods. To fully appreciate the efficiency of the QSS methods a simulation tool must be developed dedicated explicitly to model and simulate water distribution networks.

7.7 Summary

Within this chapter the DEVS and QSS approach has been applied to model and simulate WDSs. Also, in the background, the implementation and issues with the used tools has been described such as algebraic loops, solving set of nonlinear equations and use of external libraries. The simulators that have been used in this study include Epanet2 and OpenModelica for the time-based domain and PowerDEVS and QSS Solver for the event-based domain.

It has been shown that DEVS-based environment allows the modelling and the simulation of WDSs. The results presented have demonstrated that the QSS methods compete well against time-slicing approach on the WDS simulation problem.

However, there still remain open problems to be addressed in the future. The proposed framework has been evaluated on few examples. A larger set of models has to be simulated and tested for correctness, as well as efficiency, of the approach. Especially the large-scale hybrid models, because their dynamics, should uncover the power and efficiency of the QSS methods. To this end, the simulators based on the QSS engine have to be extended (e.g. with DAE solvers) to handle more complex systems.

Conclusions

Chapter 8

Conclusions and recommendations for further work

This chapter summarises the work within this thesis and discusses the key outcomes of the conducted research. Section 8.1 contains a summary of the work presented in this thesis. The conclusions and research contributions for the entire thesis are listed in Section 8.2. Recommendations for further work are presented in Section 8.3.

8.1 Research summary

The opening chapter, Chapter 1, provided a brief introduction to the subject of water distribution systems, establishing the context for a new approach to reduce water distribution systems models by means of energy audits, and a new approach to model and simulate water networks within hybrid systems framework. The objectives of the thesis were outlined in Section 1.2. These objectives were fulfilled in the consequent chapters. Additionally, the first chapter outlined the organisation of material within the thesis and provided the author's contributions towards the field of hydroinformatics. The chapter concluded with a list of publications and research projects related to this thesis.

In Chapter 2, the reader has been provided with an extended theoretical and conceptual introduction to the subjects concerning water distribution systems that were relevant to the context of the research. Apart from the introduction to water distribution systems, the conducted literature review emphasised the problems that are still unsolved in WDS analysis. In particular: (i) questionable accuracy of numerical integration methods used to solve tank dynamics, (ii) non-smoothness and discontinuity exhibited by water network elements pose a challenge for time-domain solvers, (iii) appropriate substitutes for the full simulation models are still needed in optimisation studies.

Chapter 3 opened Part I of this thesis with investigation of methods aimed at simplification of WDS models. The conducted review revealed the benefits and drawbacks of the considered techniques. The methods were examined in the scope of their suitability for online operation optimisation strategies. In particular, the method proposed by Ulanicki *et al.* (1996), was examined in depth. The method was evaluated on a number of real water networks in terms of the proposed assessment criteria. An interesting discovery of the investigation conducted was the identification of a previously unrecognised problem i.e. an inconsistent energy distribution in the reduced model. It was exemplified that the node's elevation and pressure constraint was not considered when removing from a WDS model during the simplification process. This can cause a situation where the pump speed required to satisfy minimum pressure constraints is different for the reduced model and the prototype. To address this problem a new extension was proposed to the model reduction algorithm based on the energy audits. The idea was established on the distribution of minimum useful energy which is depended on the minimum service pressure. The standard model reduction algorithm was extended to reallocate not only demand of the removed nodes but also their minimum useful energy (pressure constraints). In such a way, the simplified model kept the original model energy distribution due to new pressure constraints. The appropriateness of the new algorithm was firstly demonstrated on a small hypothetical case study, and subsequently, applied to a larger network. This novel extension to the model reduction algorithm has been claimed by author of this thesis as the first major contribution to the field of hydroinformatics.

In Chapter 4, the computational aspects of the model reduction algorithm were considered. The literature review in Chapter 3 revealed that many optimisations studies for either design or operation of WDS were restricted to relatively small and often hypothetical water networks. The optimisation studies of large size water networks were hindered by the scale and complexity of the models, which nowadays may be composed of thousands or hundreds thousands components. This poses serious numerical problems in terms of computational efficiency for the WDS model reduction algorithms. The above challenges motivated author of this thesis to carry out a research oriented towards reduction of the computational time of the algorithm extended in Chapter 3. It was decided to exploit multi-thread computing and distribute the computational load of the algorithm on the multi-core processors. With the use of parallel programming techniques and appropriate data structure (single-indexed jagged arrays) for sparse matrices, the reduction in the algorithm's computational time was evident; the initial simplification time of 5761 seconds for the benchmark network was reduced to 95 seconds, achieving a 99.98% reduction. Further reduction in the algorithm's computational time was obtained by employing the reordering algorithms dedicated to sparse matrices. With use of the Cuthill-McKee algorithm the incidence matrix representing water network was rearranged for the purpose of subsequent Gaussian elimination. With the above techniques incorporated into implementation of the extended model reduction algorithm, the initial simplification time of 5761 seconds was further reduced to just under 5 seconds, achieving a 99.999% decrease. However because the parallel programming is hardware-dependent, i.e. if target computer has only one core no improvement will be provided, it is recommended that future modifications to the simplification algorithm should first consider the reordering algorithms. Additionally, it is worth to highlight that along with the research carried out in Chapter 4 a software was developed by author of this thesis that has already been used in a number of projects and has proven to be a practical and reliable tool that can be used not only by academics but also by professionals.

It was considered important by the author of this thesis to highlight the impact of practical systems on the research work which was described in this thesis, as well as to reflect the constant motivation during this work to appeal and collaborate with various companies and industrial bodies in order to ensure the viability and applicability of the resulting concepts that were developed. Hence, Chapter 5 within Part I described the application of the research outcomes from Chapter 3 and Chapter 4 to a real case study. The case study was based on the project carried out by WSS aimed at optimisation of operation of a large-scale WDS. The data used in the project concerned an actual WDS being part of a major water company in the area of southern United Kingdom. The objective was to reduce the cost of energy used for water pumping whilst satisfying all operational constraints, including the pressure constraints in different parts of the water network. The major aim of Chapter 5 was to describe the complete optimisation procedure in details to demonstrate how the optimal scheduling problem can be approached and solved. It has been shown that the optimal scheduling of a complex WDN is a dynamic mixed-integer problem and its solution faced a number of difficulties: (i) a large number of discrete and continuous variables, (ii) nonlinearities in the components equations, (iii) modelling uncertainties and (iv) discretisation of continuous schedules. This case study employed the extended model reduction algorithm to reduce the number of elements in order to solve the optimisation problem more easily and computationally effective. The reduced model not only provided a significant speed-up in the optimisation process but it enabled the calculation of optimal schedules; if the full simulation model was considered there would be just to many of decision variables. By the preservation of the original model hydraulic complexity and energy distribution, the extended reduction technique ensured that the computed optimised schedules for pumps will deliver water whilst satisfying the minimum service pressure limits.

Part II of this thesis described the research focused towards an efficient paradigm to model and simulate water networks; effectively accounting for the discontinuous behaviour exhibited by water network components. The study was based on the discrete event specification formalism and quantised state systems, as in contrast to the classic time-slicing simulators, depending on the numerical integration algorithms, the quantisation of system states can account for the discontinuities in a more efficient manner. The chapter opened with a discussion of key issues in simulations of systems with discontinuities and highlighted problems occurred by the classical discrete-time techniques when addressing these issues. Next, an explanatory description of DEVS and QSS highlighted their properties relevant to the specific needs of water network analysis.

In Chapter 7, a comparative study on hybrid systems modelling approach and simulation performance compared to traditional time-discrete methods was given for a number of representative WDS models. For the purpose of comparison, four different simulators were utilised, Epanet2 and OpenModelica for the time-based domain; PowerDEVS and QSS Solver for the event-based domain. Each simulator provided different functionalities and approaches to modelling and simulation of systems. The used benchmark models comprised all the characteristics of a typical water system; i.e. tank dynamics, pump operated on the tank levels, looped configuration and demand pattern varied over a period of time. Based on the simulation outcomes from PowerDEVS and QSS Solver it was found that the QSS methods lead to almost the same results compared to those obtained with Epanet2 and OpenModelica as long as appropriate parameters of quantiser are selected. From a functional level the use of the QSS methods was found analogous to ordinary time-discrete methods; if a reasonably small quantisation step was chosen then the algorithm provided a simulated trajectory that was sufficiently consistent with the benchmark solution from OpenModelica. However, the smaller quantum resulted in increasing the simulation time. But what differentiate the QSS methods compared to time-slicing methods is their inherent ability to detect discontinuities, and if the higher order QSS methods are used, the QSS approach can outperform the conventional methods in terms of simulation accuracy and run-time. Hence, the conclusion is that the accuracy of QSS-based integration method is enough to simulate the asynchronous, concurrent and nonlinear nature of water distribution systems. However, the current state-of-art of the associated tools do not allow to exploit fully the potential of the QSS methods. To fully appreciate the efficiency of the QSS methods a simulation tool must be developed dedicated explicitly to model and simulate water distribution networks. This new paradigm for modelling and simulation of water distribution systems with use of the QSS methods has been claimed by author of this thesis as the second major contribution to the field of hydroinformatics.

8.2 Conclusions

The conclusions reached in the development of this dissertation are the following:

- The optimisation of large and hydraulically complex WDSs is computationally expensive as thousands of simulations are required to evaluate the performance of candidate solutions. To minimise the optimisation search space, reduced models or surrogates are utilised. Especially, in online optimisation frameworks where an optimal solution has to be obtained within the defined time interval. The accuracy of the simplification depends on the model complexity, purpose of simplification and the selected method.
- A number of model reduction techniques have been reviewed in the scope of utilisation for real-time optimal WDS operation studies leading to observations that (i) skeletonization is not an automatic process and the scope of reduction is limited if the key features of the original model are to be retained; (ii) parameter-fitting requires a user-based assumption about the initial topology which is subsequently adjusted; (iii) ANN-based metamodeling is significantly burdened with the computational cost and thereby not applicable for other than small systems; (iv) variable elimination is practical and fast but it fails to account for energy distribution of the original model.
- To address these shortfalls an extension was proposed to the variable elimination algorithm based on the energy audits concepts. The energy audits concept has been incorporated into the variable reduction algorithm allowing the preservation of the original model energy distribution. The idea is based on the distribution of minimum useful energy which is depended on the minimum service pressure. The original variable elimination algorithm has been extended to reallocate not only demand of the removed nodes but also their minimum useful energy (pressure constraints). Hence, the simplified model can keep the original model energy distribution due to new pressure constraints. The extended variable elimination algorithm preserves accurately the hydraulic characteristic of the original water network and therefore enable a correct optimisation of the water network operation.

- To ensure that the extended variable elimination algorithm would be able to reduce large size networks with complex topologies within the specified real-time interval, further improvements have been introduced to the algorithm. (i) Distribution the computational load on multi-core processors, (ii) exploitation of the inherent sparsity of matrices representing WDS topologies and (iii) employing the matrix reordering algorithms have drastically reduced the model reduction run-time; from the initial 5761 seconds to just under 5 seconds for the benchmark model used.

- A new software has been designed and developed to support the conducted research. The developed software includes functionalities of the extended model reduction algorithm and is able to simplify the water network model, consisted of several thousands elements, within seconds of calculation time. The advantage of this near real-time model reduction is that can be used to manage abnormal situations and structural changes in a water network, e.g. isolation of part of the network due to a pipe burst. In such case an operator can change the full hydraulic model and run model reduction software to automatically produce the updated simplified model.

- The developed model reduction application has been used to reduce many WDS models and has proven to be a practical and reliable tool. An example of utilisation the model reduction application is given in Chapter 5, where it has been used in a practical project focused on determining optimal schedules for control elements in a real large-scale water network exhibiting highly complex topology. The optimisation method, utilised in Chapter 5, has taken into account the nonlinear characteristics of the system as well as the mass balance for reservoirs. It has also employed the extended model reduction algorithm to reduce the number of elements in order to solve the optimisation problem more easily and computationally effective. The reduced model not only has provided a significant speed increase of the optimisation process but it has enabled the calculation of optimal schedules; if the full model has been considered there would be just to many of decision variables. The modified reduction technique has allowed the preservation of the original model hydraulic complexity and energy distribution and thereby has ensured that the computed optimised schedules for pumps will deliver water whilst satisfying the minimum service pressure limits.

- Majority of water networks analysis methods and simulators are based on a time slicing approach i.e. numerical methods, used in computer simulation of a system characterised by differential equations (e.g. tank dynamics), require the system to be approximated by discrete quantities. The solution of difference equation is calculated at fixed points in time. However, it is assumed that, in the extended-period simulation of water networks, the system is in a steady state between successive time stamps. But in fact, a real WDS continually adjusts itself in response to changing requirements of the users. This rises an important issue about the model fidelity of hydraulic behaviour of a real WDS; especially a WDS with pumps operation based on the water level in tanks, as if the time interval is not appropriate the events that actually happened in the real water network might be overlooked. Furthermore, some elements included in a WDN model may cause numerical difficulties (convergence problems) in simulation due to their inherent non-smooth and discontinuous characteristics.

- To address these issues, the quantisation of the states has been proposed to create an asynchronous discrete-event simulation model of WDS. In contrast to the classic time-slicing simulators, depending on the numerical integration algorithms, the quantisation of system states allows to account for the discontinuities exhibited by control elements in a more efficient manner, and thereby, offer a significant increase in speed of the simulation of water network models. The proposed approach has been evaluated on a number of case studies and compared with results obtained from the Epanet2 simulator and OpenModelica. Although the current state-of-art of the simulation tools utilising the quantised state systems do not allow to fully exploit their potential, the results from comparison demonstrate that, if the second or third order quantised-based integrations are used, the quantised state systems approach can outperform the conventional water network simulation methods in terms of simulation accuracy and run-time.

8.3 Recommendations for further research

It is considered that the work described in this thesis represents *opening chapters* for further extensions to (i) the model reduction algorithm and (ii) modelling and simulation of water distribution systems with use of the quantised state systems. Indeed, further work based on the outcome of this research is continuing and is currently progressing, being undertaken by colleagues within the WSS group at De Montfort University. The findings made in this thesis have also supported the development of scope for further work. The key areas of further work developing contributions to the field of hydroinfotmatics are listed and discussed below.

8.3.1 Potential further work of Part I

Although, the particular model reduction algorithm, investigated in Part I, is an established research topic in field of hydroinformatics, it is believed, there are still available opportunities for research in this area. Immediate potential areas for further work are described as follows:

Proposal 1 *Identification of critical elements for purpose of water network models reduction*

At the current stage the model reduction algorithm, described in Chapter 3, retains all the reservoirs, tanks, valves, pumps and nodes at valves and pumps terminals. Additionally, the algorithm can retain all the nodes that have the number of neighbouring nodes equal or bigger than the user-specified threshold. Finally, the user can indicate directly which element of the original water network should be retained. However, there is ongoing research focused the on ranking of elements of water networks for different purposes e.g. Yazdani and Jeffrey (2010) and Yazdani and Jeffrey (2012) applied graph theory in the analysis of structural vulnerability and robustness of WDSs, Michaud and Apostolakis (2006) introduced a scenario-based methodology to rank elements of water network and Izquierdo *et al.* (2008) focused on pipes, assessing their relative importance regarding the water distribution process. Enhanced identification of critical elements will undoubtedly lead to a further refinement and inevitable improvement of the reduced models.

Proposal 2 *Further reduction of the simplification algorithm's computational time*

The proposed use of concurrent programming, exploitation of sparsity and reordering algorithms investigated in Chapter 4, should be further explored aiming at reduction of the computational time of the simplification algorithm as it is likely that future water network models will grow in size and complexity. Especially, an interesting study could be investigation of different reorderings on performance of processing water network graphs represented by sparse matrices. Also, there is a number of articles with techniques dedicated for parallerisation of Gaussian elimination that might be worth to explore with the hope to achieve further reduction in the Gaussian elimination computational time, see e.g. (Demmel *et al.*, 1999; McGinn and Shaw, 2002; Michailidis and Margaritis, 2011).

8.3.2 Potential further work of Part II

The second Part, being a new research topic in WSS, De Montfort University, and a relatively new research topic in the water community, still leaves room for a great deal of further exciting research work to be carried out. A short description of items of further work identified by the author are summarised in the following:

Proposal 3 *Fully-functional hydraulic simulator with utilisation of the QSS concepts*

The simulation study in Chapter 7, as previously stated, is considered here as being only a preliminary study. It would be interesting to extend the simulations to a wider set of benchmark models and investigate potential of the higher order QSS methods. The final aim would obviously be a fully-functional hydraulic simulator based on the QSS integration algorithms. A basic interaction scheme for the proposed simulator is depicted in Figure 8.1. Such an aim, if proved to be realisable, would be the most rewarding achievement from a scientific and practical point of view.

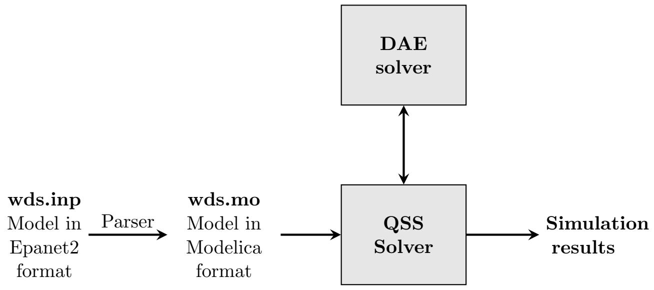


FIGURE 8.1: Hydraulic simulator with use of the QSS methods - a basic interaction scheme.

Proposal 4 *Inclusion of water quality modelling and simulation*

One of the major improvement to the paradigm based on the QSS methods would be inclusion of water quality elements to formulate a complete modelling and simulation approach for municipal water networks. This proposal is closely related to **Proposal 3** and hopefully they will lead to an industrial viable open source platform for water network analysis.

Appendix A

Mathematical supplement

This appendix provides some miscellaneous prerequisite mathematical theory.

A.1 Newton-Raphson method

The Newton-Raphson or Newton method is for solving equations of the form $f(x) = 0$. It is a very efficient iterative technique for solving equations numerically. However, it should to be noted that the Newton algorithm cannot guarantee convergence (it is very specific to the initial guess x_0 (Brenan *et al.*, 1989)).

The Newton-Raphson method is based on the simple idea of linear approximation and is defined as follows:

Let x_0 be a good initial estimate of the root r of the $f(x) = 0$ and let $r = x_0 + h$. Since the true root is r, and $h = r - x_0$, the number h measures how far the estimate x_0 is from the truth. Since h is "small", a linear (tangent line) approximation can be used to conclude that

$$0 = f(r) = f(x_0 + h) \approx f(x_0) + hf'(x_0) \tag{A.1}$$

and therefore, unless $f'(x_0)$ is close to 0,

$$h \approx -\frac{f(x_0)}{f'(x_0)} \tag{A.2}$$

it follows that

$$r = x_0 + h \approx x_0 - \frac{f(x_0)}{f'(x_0)} \tag{A.3}$$

This leads to the next estimates

$$
\begin{aligned}
x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} && \text{(A.4)}\\
x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} && \text{(A.5)}\\
&\vdots && \text{(A.6)}\\
x_{n+1} &= x_n - \frac{f(x_n)}{f'(x_n)} && \text{(A.7)}
\end{aligned}
$$

where x_n is the current estimate and x_{n+1} is the next estimate.

The Newton-Raphson method can solve also a system of nonlinear equations:

$$
\begin{aligned}
f_1(x_1, x_2, \ldots, x_n) &= 0 && \text{(A.8)}\\
f_2(x_1, x_2, \ldots, x_n) &= 0 && \text{(A.9)}\\
&\vdots && \text{(A.10)}\\
f_n(x_1, x_2, \ldots, x_n) &= 0 && \text{(A.11)}
\end{aligned}
$$

Note that the number of unknowns equals the number of equations. The above set of equationw can be rewritten using vector notation as:

$$\mathbf{f}(\mathbf{x}) = 0 \tag{A.12}$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T$ and $\mathbf{f}(\mathbf{x}) = (f_1(x), f_2(x), \ldots, f_n(x))^T$.

The Newton-Raphson iteration for this system is

$$\mathbf{x}_{n+1} = \mathbf{x} - \mathbf{J}^{-1}\mathbf{f}(\mathbf{x_n}) \tag{A.13}$$

where $\mathbf{J}$ is a Jacobian matrix i.e. a matrix of all first-order partial derivatives of the vector function $\mathbf{f}$ regarding the vector $\mathbf{x}$. An $n \times n$ Jacobian matrix $\mathbf{J}$ has the form:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \tag{A.14}$$

Appendix B

Graphical results from simplifications

B.1 Epanet Net3

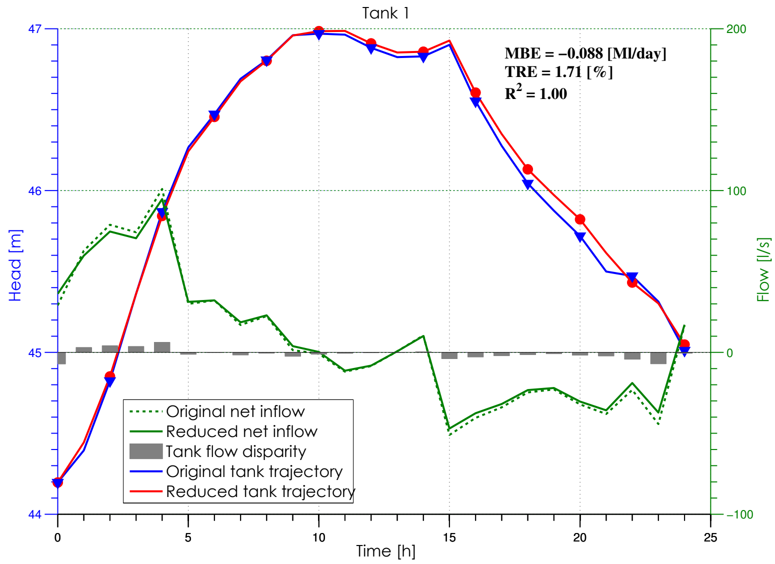


FIGURE B.1: Simulated trajectories for the selected tank in the original and reduced Net3 model.

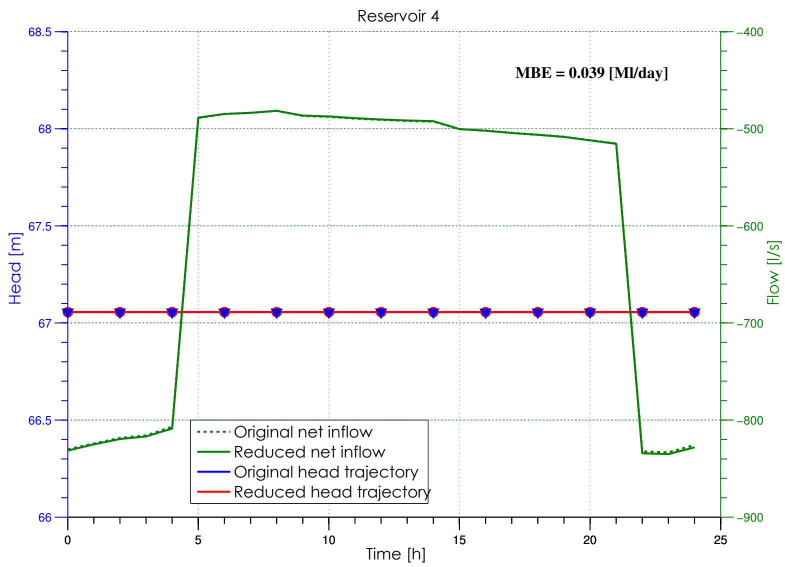


FIGURE B.2: Simulated trajectories for the selected reservoir in the original and reduced Net3 model.

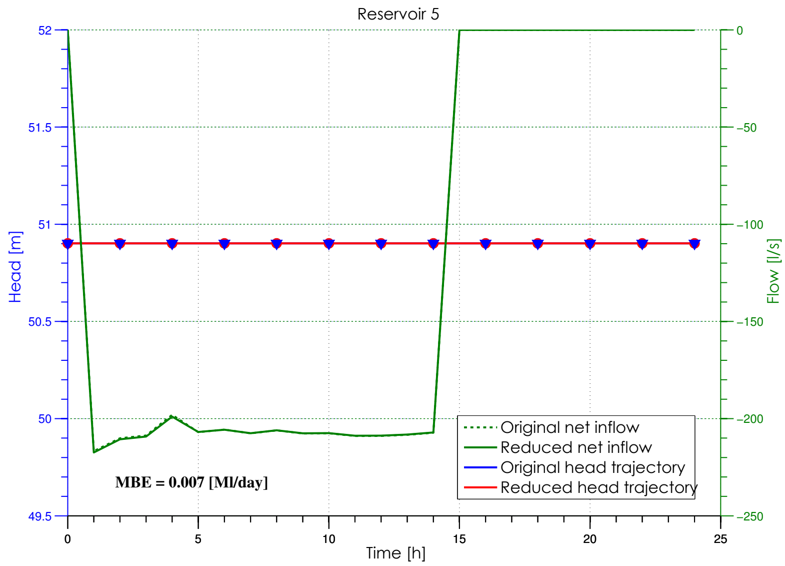


FIGURE B.3: Simulated trajectories for the selected reservoir in the original and reduced Net3 model.

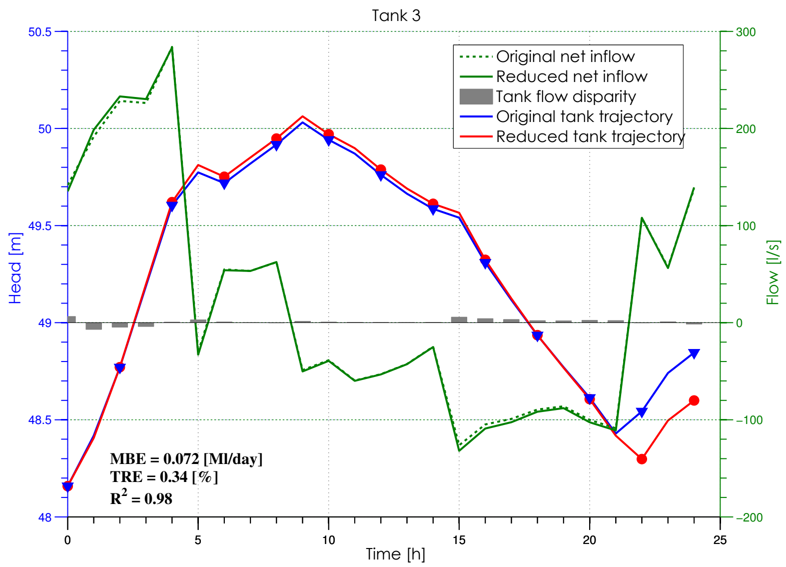


FIGURE B.4: Simulated trajectories for the selected tank in the original and reduced Net3 model.

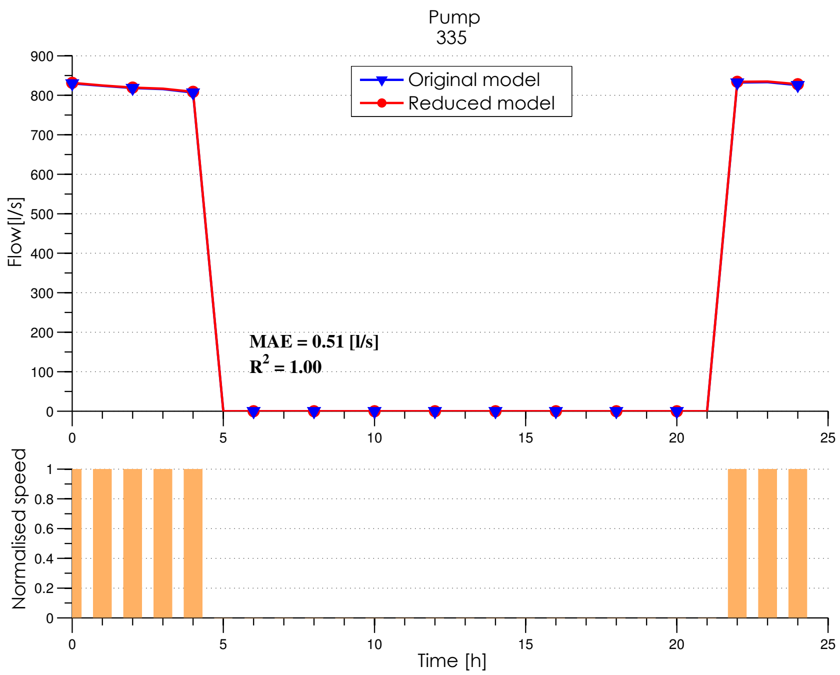


FIGURE B.5: Simulated trajectories for the selected pump in the original and reduced Net3 model.

B.2 Rio

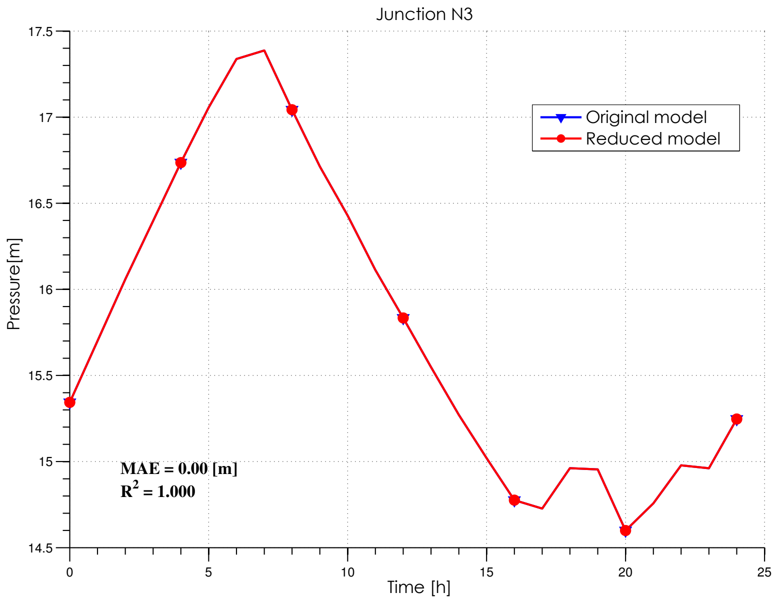


FIGURE B.6: Simulated trajectories for the selected node in the original and reduced Rio model.

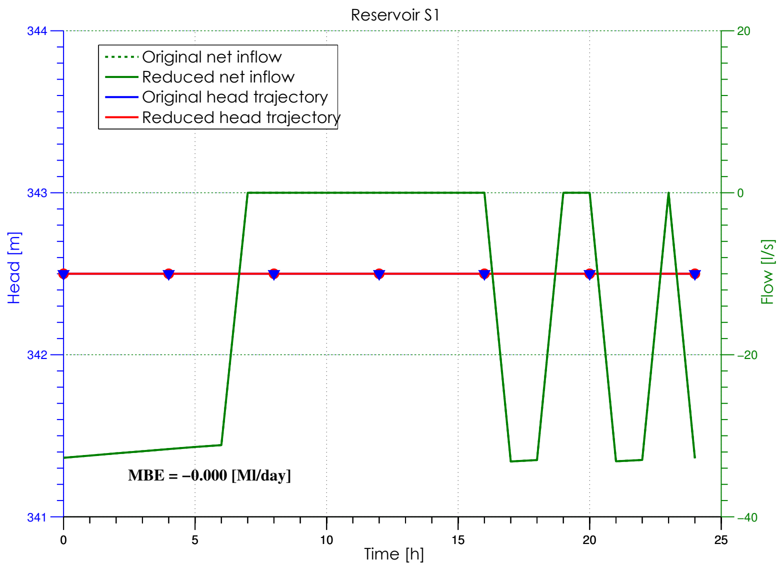


FIGURE B.7: Simulated trajectories for the selected reservoir in the original and reduced Rio model.

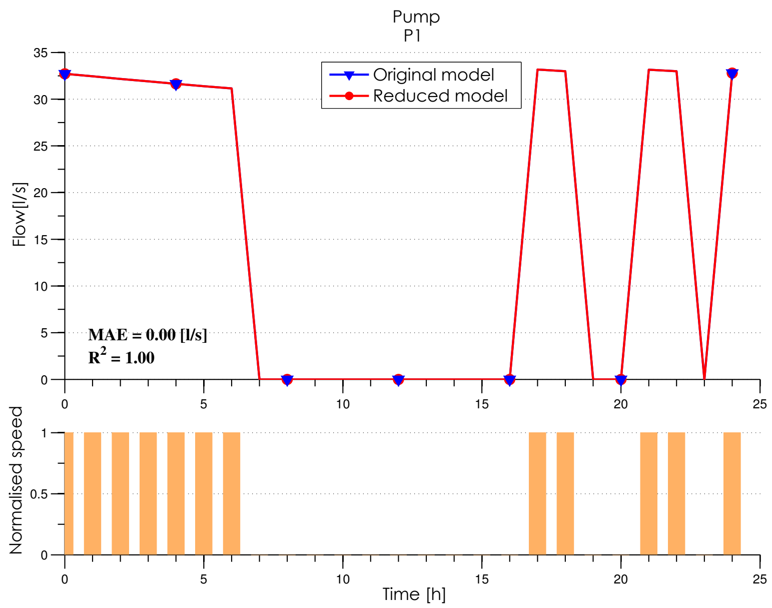


FIGURE B.8: Simulated trajectories for the selected pump in the original and reduced Rio model.

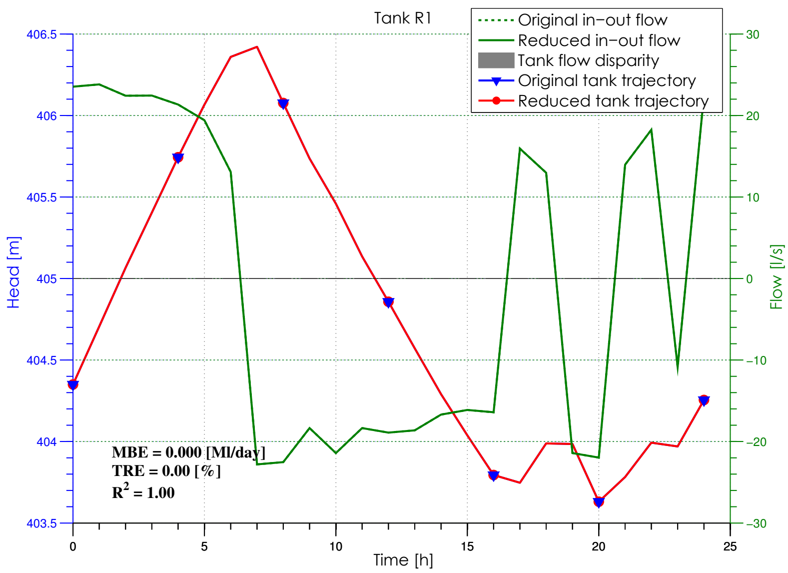


FIGURE B.9: Simulated trajectories for the selected tank in the original and reduced Rio model.

B.3 Machu Picchu

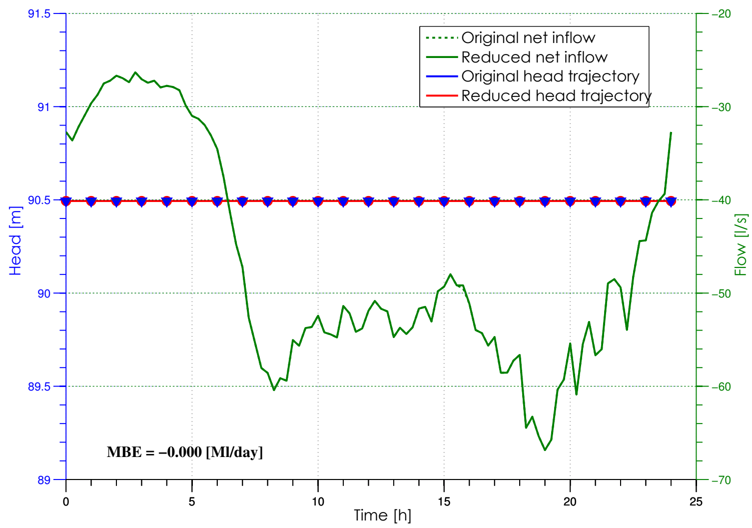


FIGURE B.10: Simulated trajectories for the selected reservoir in the original and reduced Machu Picchu model.

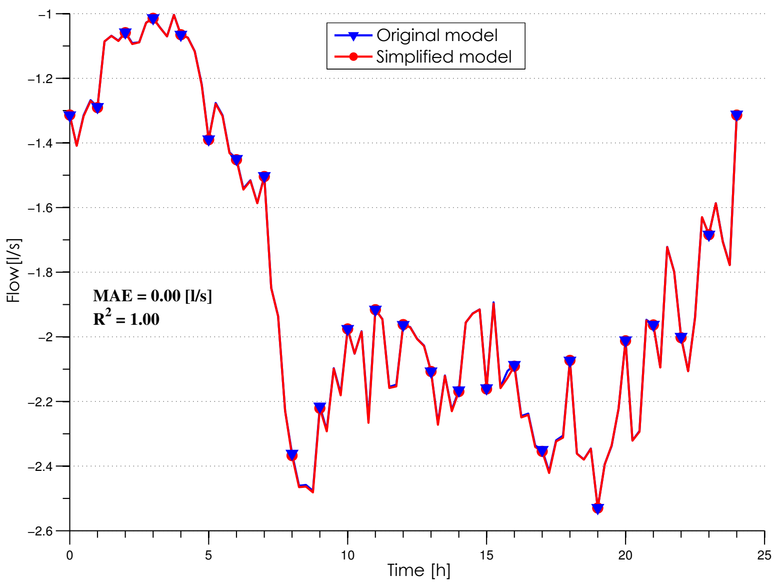


FIGURE B.11: Simulated trajectories for the selected valve in the original and reduced Machu Picchu model.

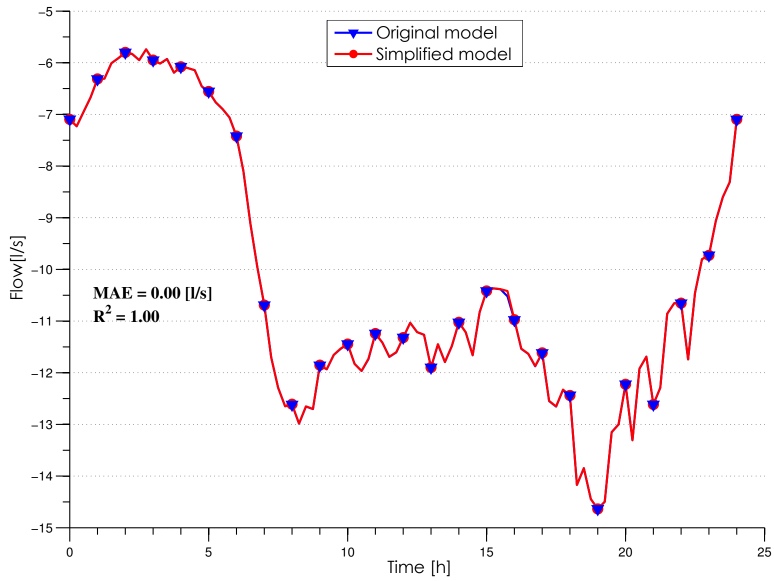


FIGURE B.12: Simulated trajectories for the selected valve in the original and reduced Machu Picchu model.

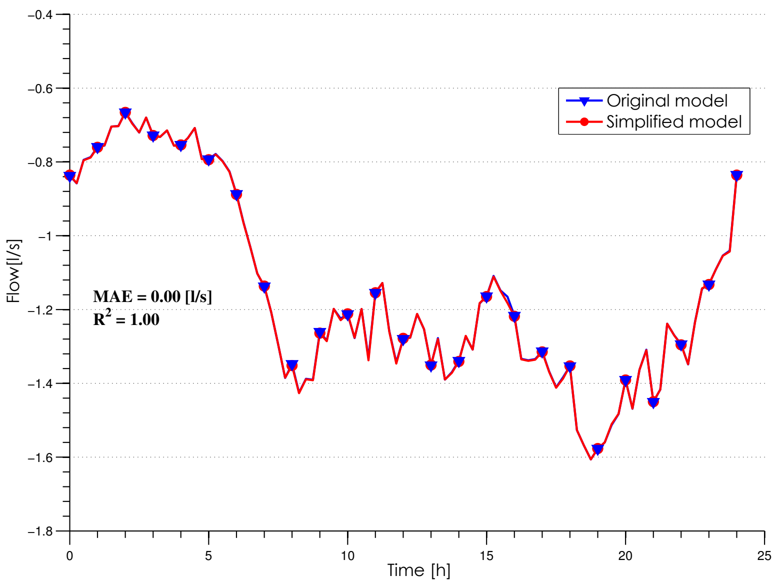


FIGURE B.13: Simulated trajectories for the selected valve in the original and reduced Machu Picchu model.

B.4 Rlyeh

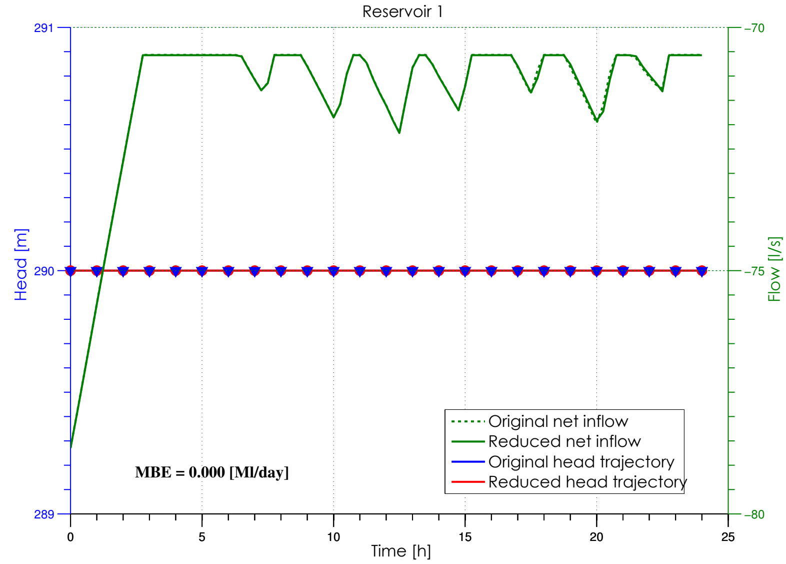


FIGURE B.14: Simulated trajectories for the selected reservoir in the original and reduced Rlyeh model.

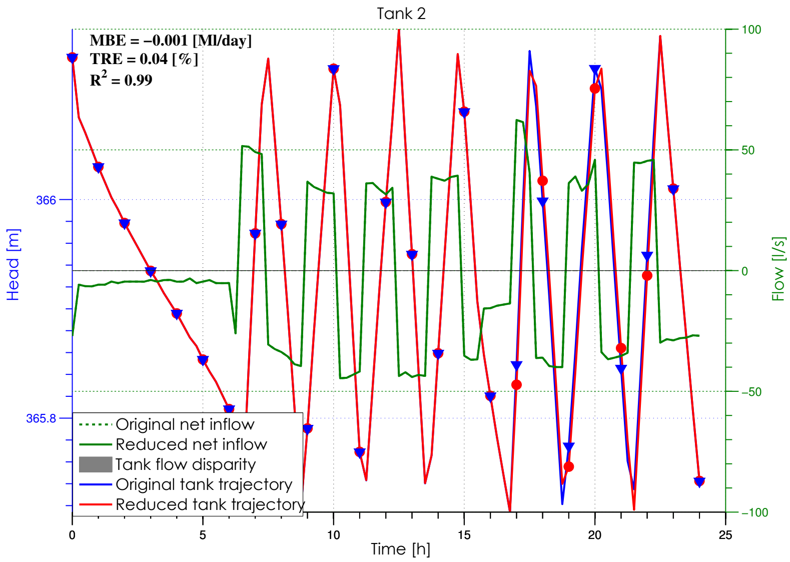


FIGURE B.15: Simulated trajectories for the selected tank in the original and reduced Rlyeh model.

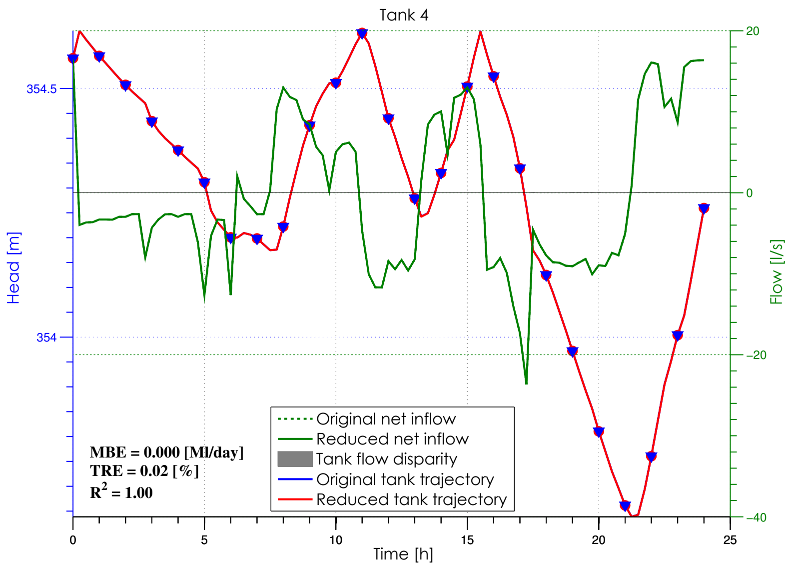


FIGURE B.16: Simulated trajectories for the selected tank in the original and reduced Rlyeh model.

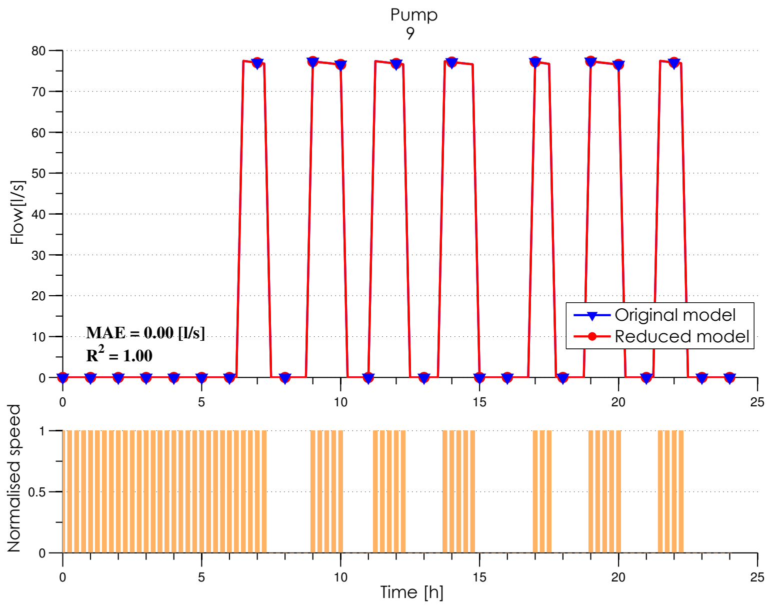


FIGURE B.17: Simulated trajectories for the selected pump in the original and reduced Rlyeh model.

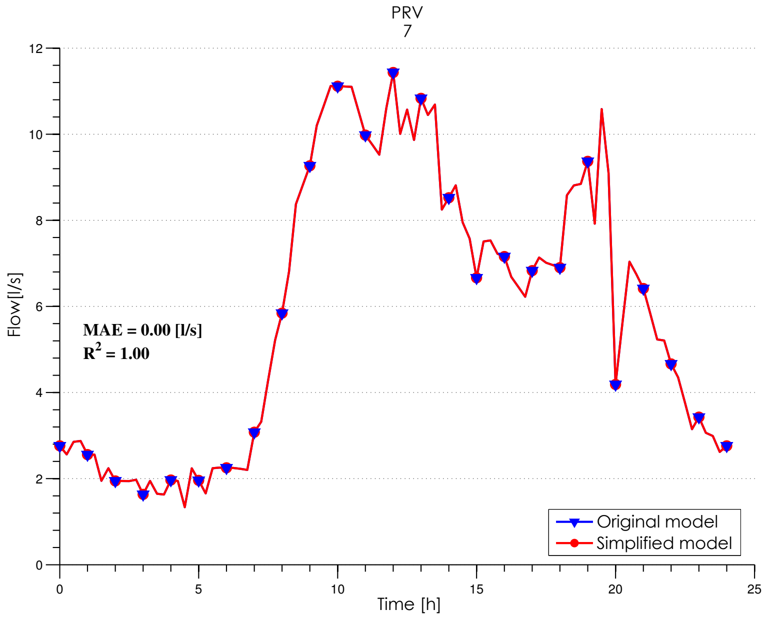


FIGURE B.18: Simulated trajectories for the selected PRV in the original and reduced Rlyeh model.

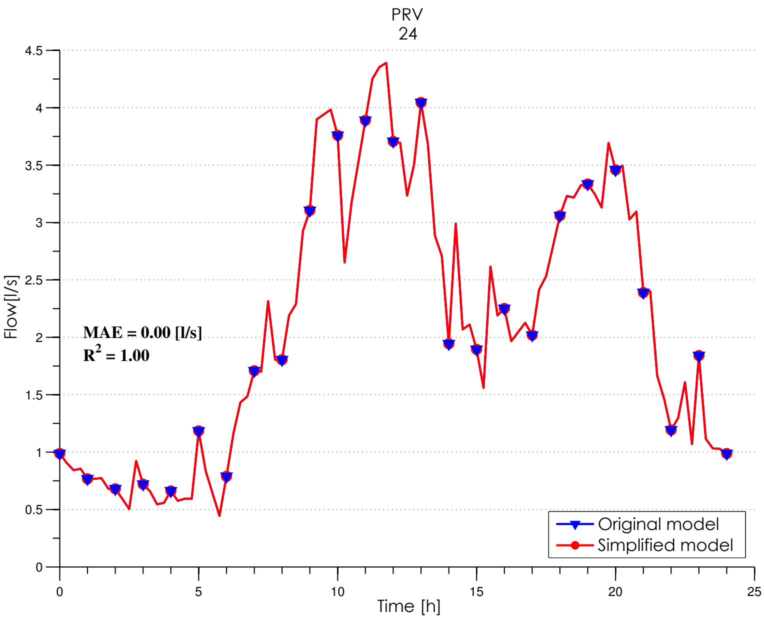


FIGURE B.19: Simulated trajectories for the selected PRV in the original and reduced Rlyeh model.

B.5 Cydonia

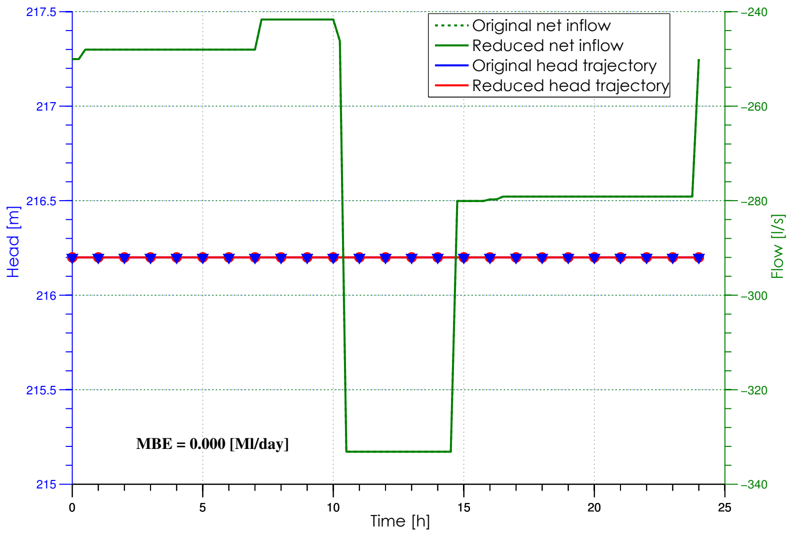


FIGURE B.20: Simulated trajectories for the selected reservoir in the original and reduced Cydonia model.

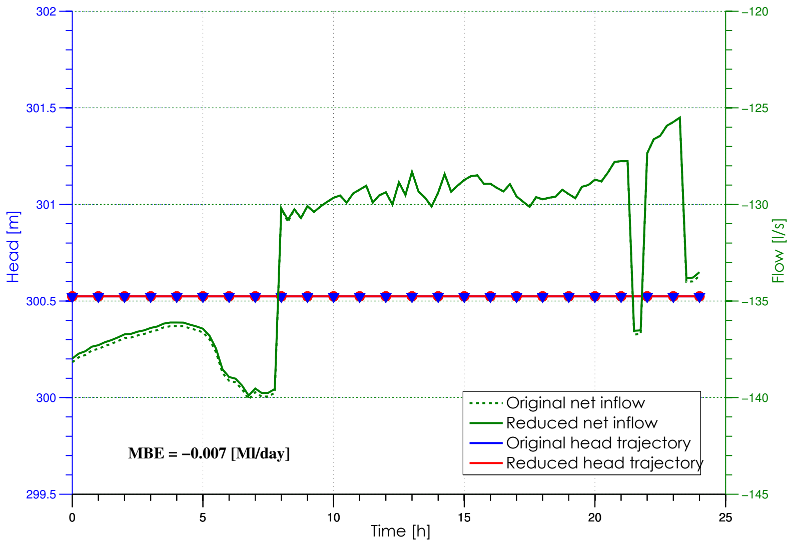


FIGURE B.21: Simulated trajectories for the selected reservoir in the original and reduced Cydonia model.

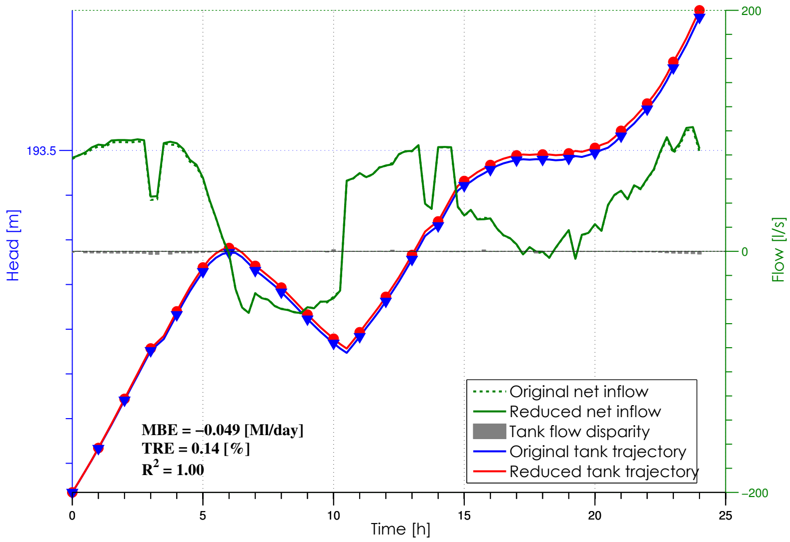


FIGURE B.22: Simulated trajectories for the selected tank in the original and reduced Cydonia model.

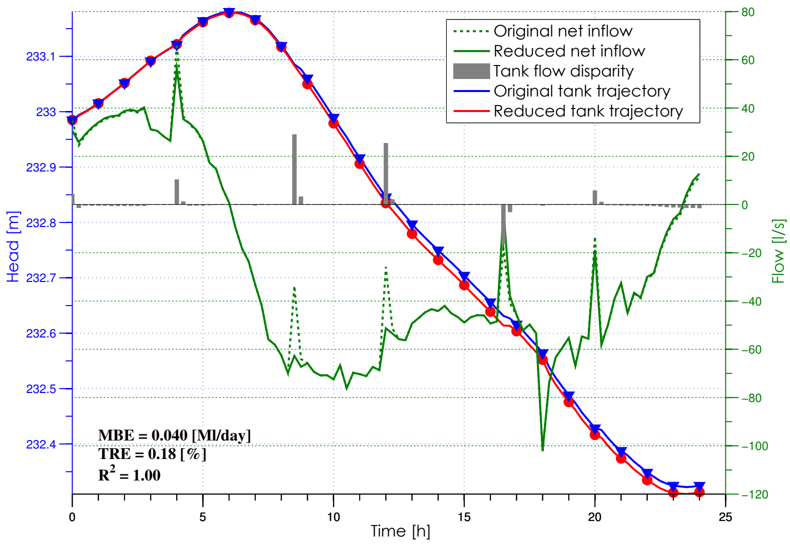


FIGURE B.23: Simulated trajectories for the selected tank in the original and reduced Cydonia model.

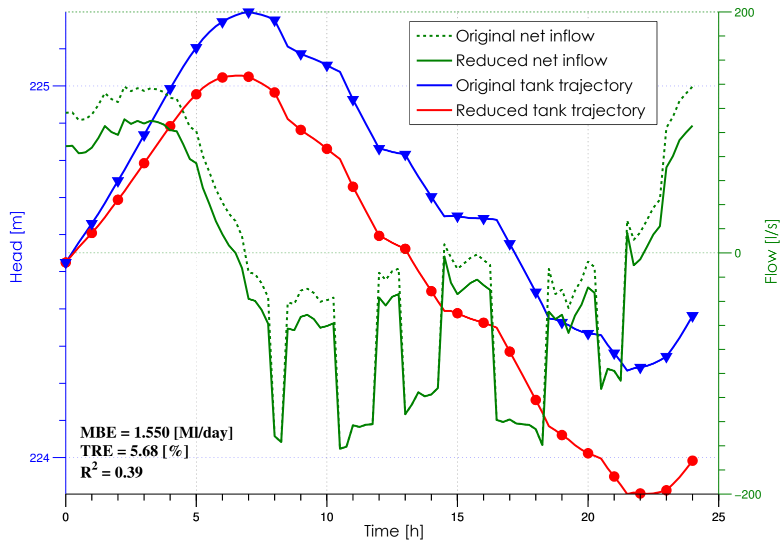


FIGURE B.24: Simulated trajectories for the selected tank in the original and reduced Cydonia model.

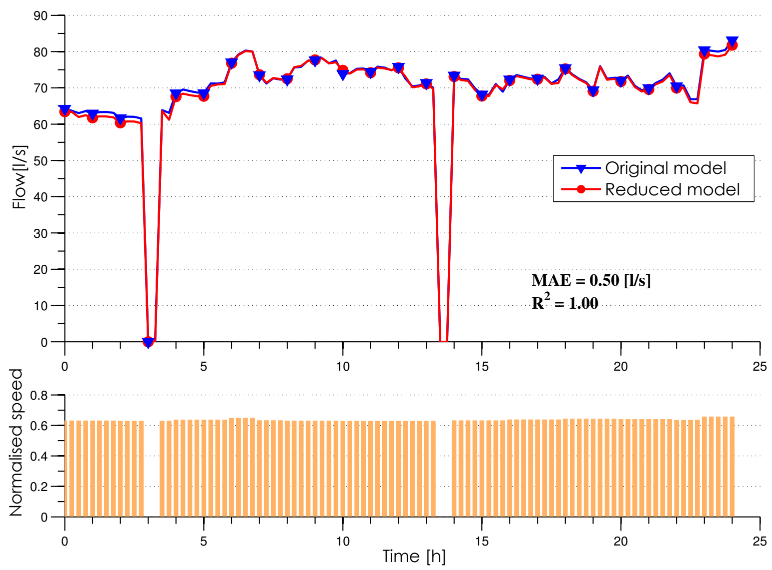


FIGURE B.25: Simulated trajectories for the selected pump in the original and reduced Cydonia model.

B.6 Ankh-Morpork

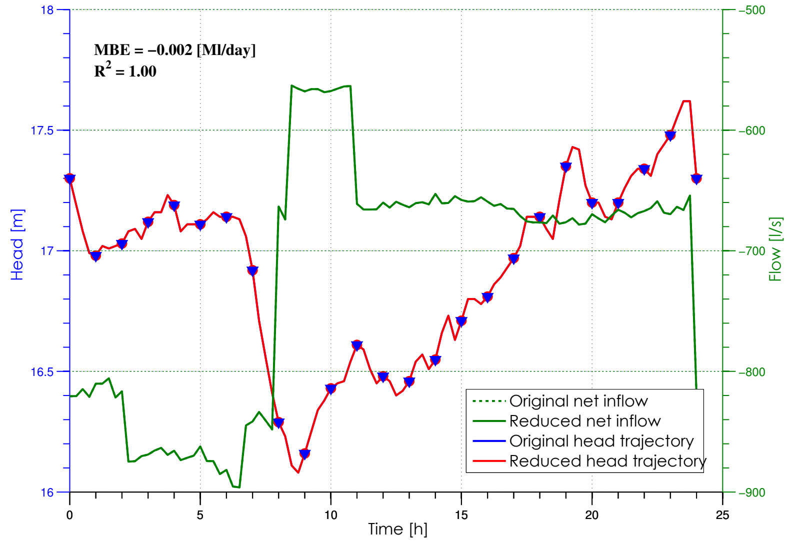


FIGURE B.26: Simulated trajectories for the selected reservoir in the original and reduced Ankh-Morpork model.

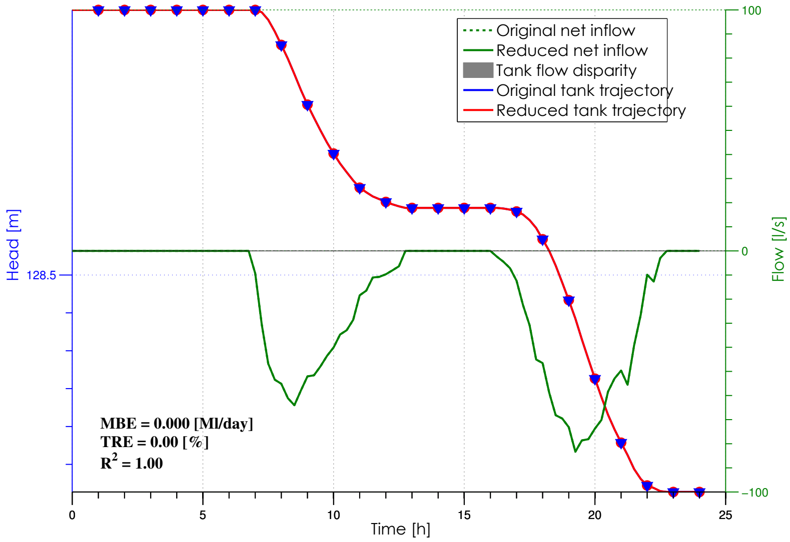


FIGURE B.27: Simulated trajectories for the selected tank in the original and reduced Ankh-Morpork model.

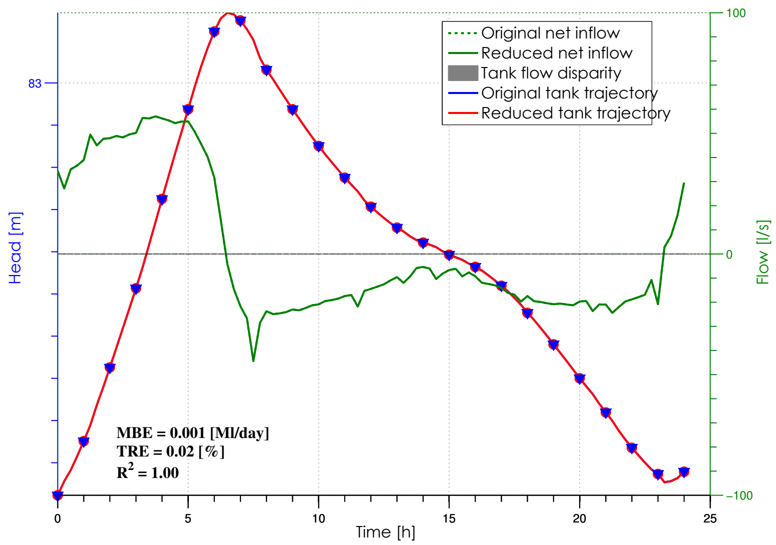


FIGURE B.28: Simulated trajectories for the selected tank in the original and reduced Ankh-Morpork model.

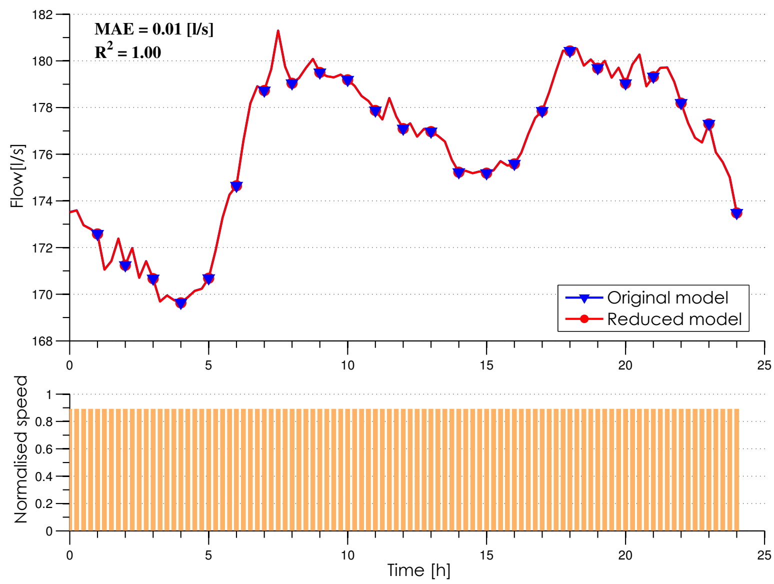


FIGURE B.29: Simulated trajectories for the selected pump in the original and reduced Ankh-Morpork model.

Appendix C

Object-oriented modelling of water networks

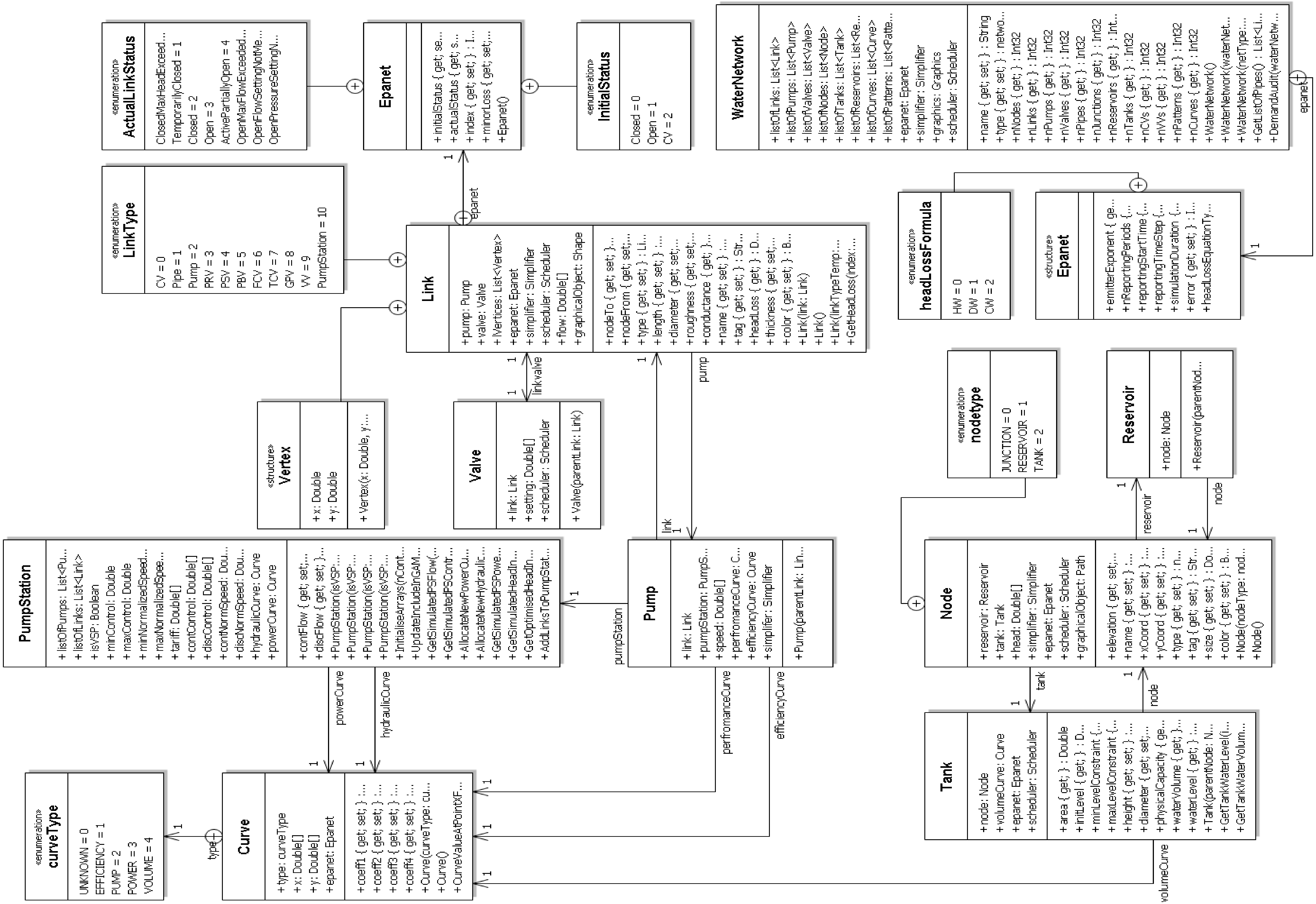


Figure C.1: Object-oriented approach to model water network data in the C# language.

Appendix D

Detailed schematic

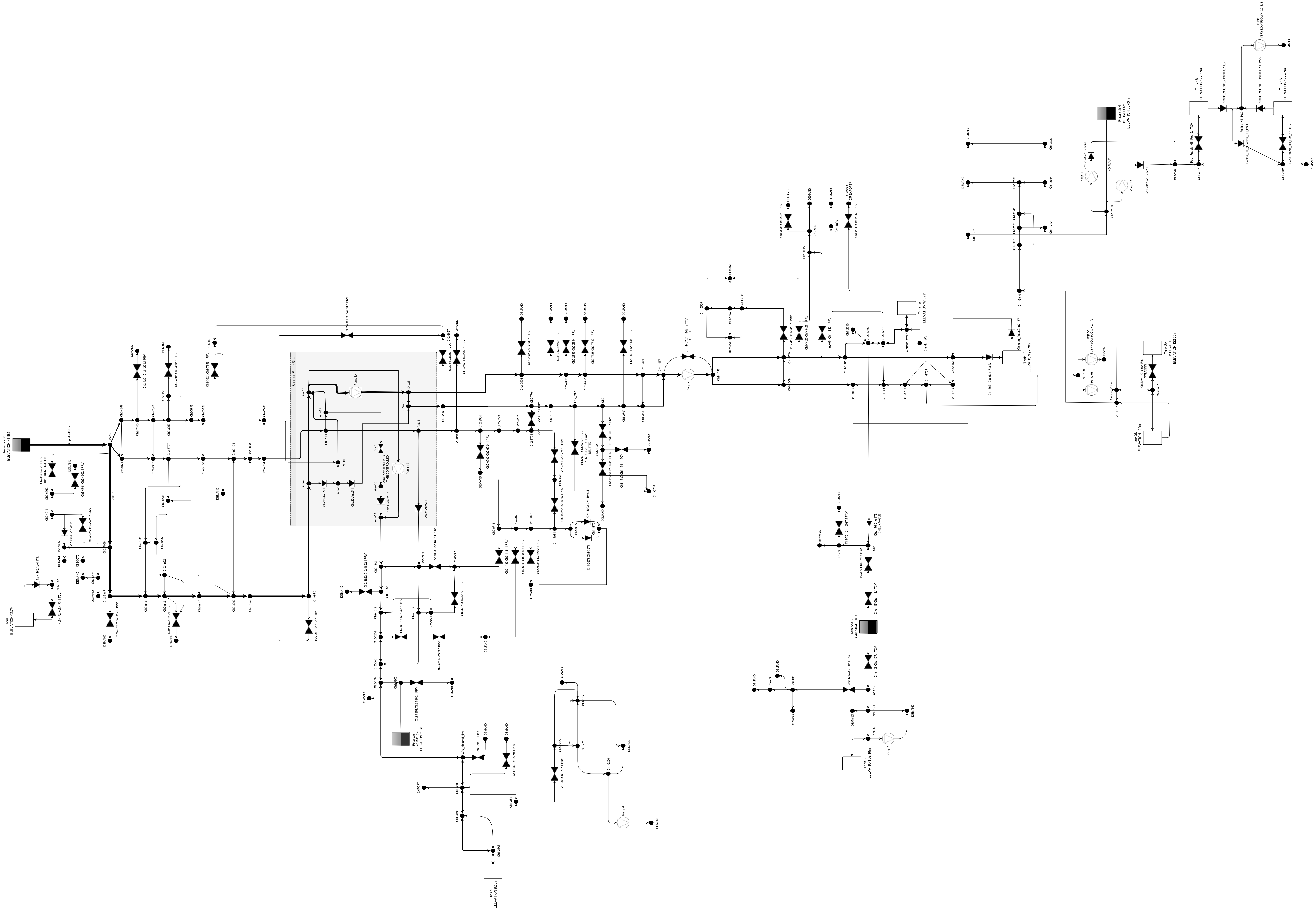


Figure 5.4: Topology of *Water Network*.

Appendix E

Adjustments to optimisation model

The modifications and adjustments to the reduced model include:

- **Pump Station 1**
  Based upon the update from the water company the way *Pump 1A* and *Pump 1B* were modelled in the provided Epanet2 model was different than in reality. In the model only 2 pumps were included and they were set to work independently of each other while in reality *Pump 1A* and *Pump 1B* are part of pump station, namely *Pump Station 1*. Moreover the pumps were set to be variable speed pumps whereas, during the meeting, it was clarified that these pumps are in fact fixed speed pumps.

  The modified structure for *Pump Station 1* is depicted in Figure E.1. Three pumps with the same hydraulic characteristics were combined into a pump station. Note that due to no data from the water company, the performance curve $h = f(q)$ for the pumps were calculated from the performance curves of *Pump 1A* and *Pump 1B* provided in the original model; i.e. the flow and head points (q_i^{new}, h_i^{new}) of the new performance curve, illustrated in Figure E.2, were calculated as follows $(\frac{q_i^{1A}+q_i^{1B}}{3}, \frac{h_i^{1A}+h_i^{1B}}{2})$. Two FCVs were included into *Pump Station 1* to control flows towards *Tank 5* and *Pump Station 2*.

- **Pump Station 3**
  In the provided original Epanet2 model *Pump Station 3* was modelled as *Reservoir 2* with a pre-defined time-based pressure pattern. However, based upon updates from the water company engineers is was concluded that the structure for this pump

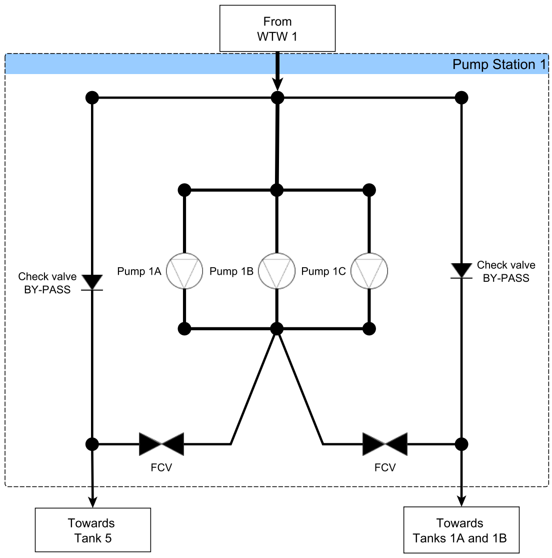


FIGURE E.1: New structure of *Pump station 1*.

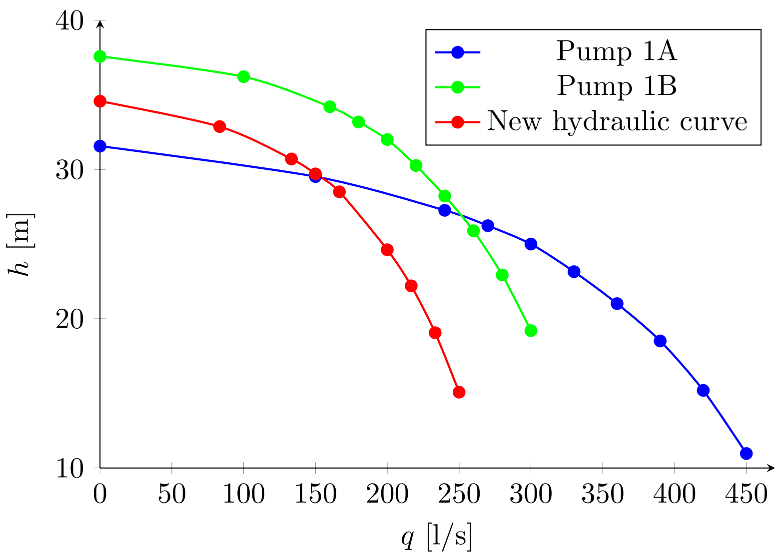


FIGURE E.2: New hydraulic curve applied to all three pumps in *Pump Station 1*. The new hydraulic curve was calculated from the hydraulic curves for pumps *1A* and *1B*.

station is as illustrated in Figure E.3. The pump station includes 5 variable-speed pumps with hydraulic characteristics extracted from the provided data sheet.

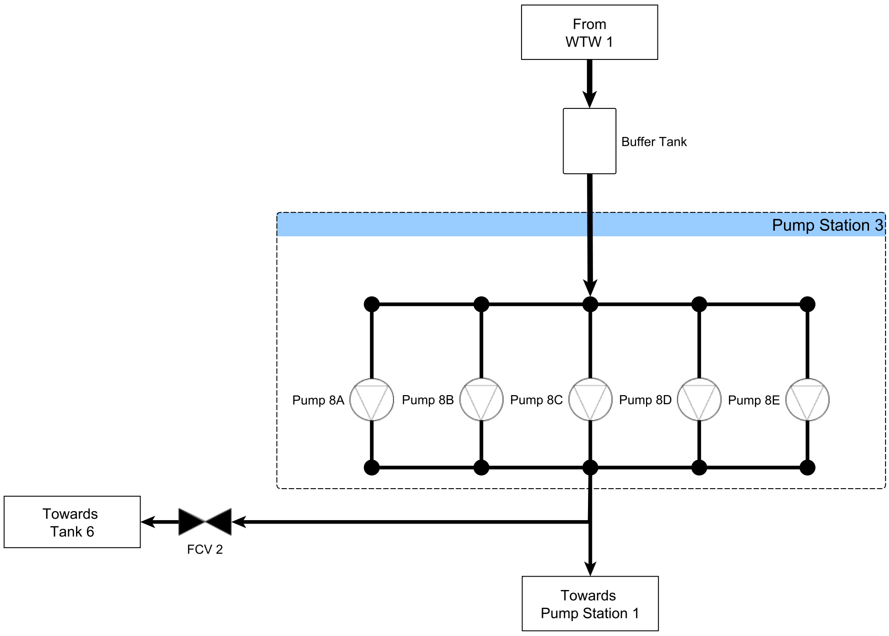


FIGURE E.3: The updated structure of the inflow from *WTW 1* including *Buffer tank* and *Pump Station 3.*

- **Water Treatment Works, WTW 1**

  As *Reservoir 2* was replaced with *Pump Station 3* a new source of water, namely *WTW 1*, was added to the model, see Figure E.3. *WTW 1* contains: (i) source node with a fixed inlet to the system of 425 l/s and (ii) a buffer tank with parameters as in Table 5.5. Note that in the original model the average inflow from *Reservoir 2* reservoir was 441 l/s, but this resulted in the WDS receiving too much water and the final level (i.e. at the end of 24 h horizon) in some tanks was significantly higher than the initial level; for 7-days simulation this resulted in overfilling of these tanks. For this reason the value of inflow of 425 l/s was chosen such that the overall system received an adequate amount of water.

- **Pump station 2**

  In the original model *Pump Station 2* included only 1 pump, *Pump 2*. The updated *Pump Station 2*, depicted in Figure E.4 contains 2 pumps and the by-pass FCV. It was assumed that *Pump 2* performance curve described combined performances of real pumps. Hence, the performance curve for the new pumps *2A* and *2B* was calculated using hydraulics principles for the parallel pumps.

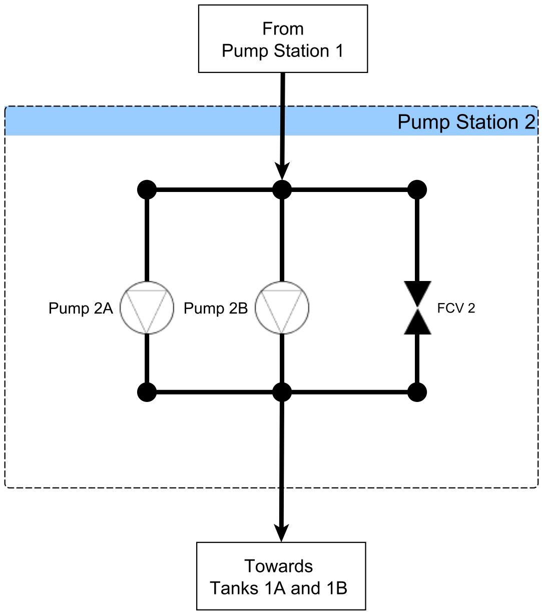


FIGURE E.4: New structure of *Pump Station 2*.

- **Interlinked tanks**

  The aim of employing the model reduction algorithm was to reduce the number of nodes and pipes. However it is also possible to replace the number of tanks situated in a close proximity by equivalent one water storage. Hence, pairs of *Tank 1A* and *Tank 1B*; and *Tank 2A* and *Tank 2B* were merged. The parameters of the new tanks were adjusted as follows:
  $t^*_A = t^1_A + t^2_A$, $t^*_d = 2\sqrt{\frac{t^*_A}{\pi}}$, $t^*_e = \frac{t^1_e + t^2_e}{2}$, $t^*_i = \frac{t^1_i + t^2_i}{2}$, $t^*_m = \frac{t^1_m + t^2_m}{2}$
  where t^* is a new merged tank, t^1 and t^2 are the tanks to be merged, and A, d, e, i, m are the tanks' parameters; cross-sectional area, diameter, elevation, initial water level, maximum water level, respectively. Table 5.5 contains parameters for the merged tanks.

- **Issue with a highly elevated area**

  Pump 3A and *Pump 3B* operate at 1.2 of normal speed, if this speed was reduced even by 1% the pumps could not deliver head and the whole model was unbalanced. More detailed analysis of this area led to conclusion that only a small fraction of the total daily demand was delivered there therefore the whole area of the following pumps and tanks: *Pump 3A*, *Pump 3B*, *Pump 7*, *Tank 4A*, *Tank 4B* was removed along with the neighbouring nodes and pipes. Any demand found on the removed nodes was transferred to the adjacent retained nodes. The retained pumps are listed in Table 5.6.

- **Pump 5A**
  The average flow trough *Pump 5A* is less than 0.2 l/s. Furthermore, the pump is supplying water to few consumption nodes only. Therefore due to a low importance to the whole system performance the pump was removed and demand from the nodes was transferred to adjacent retained nodes.

- **Pumps' power characteristics**
  The objective function, defined in Section 5.4.1, requires power characteristics for each pump. As the model provided by water company, does not contain such details the power characteristics were adopted from the data sheets for similar pumps. The approximated linear equations of power characteristics for each pump are listed in Table 5.6.

- **Pumps type undefined**
  From the provided data it could not be determined whether pump is a variable-speed or fixed-speed type. The pump types were assumed as listed in Table 5.6.

- **Control schedules**
  The defined control schedules and rules were removed.

Appendix F

Discretisation algorithm of continuous schedules using GAMS/CONOPT

The methodology can be summarised as follows:

1. Load continuous optimisation results produced by GAMS/CONOPT.
2. For each pump station round the continuous pump control (i.e. number of pumps ON) to an integer number, while calculating an accumulated rounding error at each time step. The accumulated rounding error is used at subsequent time steps to decide whether the number of pumps ON should be rounded up or down.
3. Generate GAMS code with number of pumps ON for each pump station and at each times step being fixed, i.e. as calculated in the above step. Initial conditions for all flows and pressures in the network are as calculated by GAMS/CONOPT during the continuous optimisation. Note that in this GAMS code the number of pumps ON for each pump station and at each times step are no longer decision variables but forced parameters. However, the solver (CONOPT) can still change pump speed and can adjust valve flow to match the integer number of pumps ON. The cost function to be minimised and the constraints are the same as in the continuous optimisation.
4. Call GAMS/CONOPT and subsequently load the results of integer optimised solution.

5. In the continuous optimisation pump station flow can be zero only when all pumps in this station are off. However, in integer optimisation it may rarely happen that pump station control is forced to have e.g. 1 pump ON during a particular time step, but this pump is unable to deliver the required head at that time step, hence the pump flow is zero. If such event occurs, the above steps 3 and 4 are repeated, but at the time steps when the resulting pump station flow was zero, the number of pumps ON is forced to be zero.

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