

## PF&T Practice exercises

1. The continuously stirred tank reactor (CSTR) shown in Figure below is used to treat an industrial waste, using a reaction which destroys the waste according to first-order kinetics:  $\frac{dC}{dt} = -kC$ , where  $k = 0.216 \text{ day}^{-1}$  and  $C$  is concentration inside the tank. The reactor volume is  $500 \text{ m}^3$ , the volumetric flow rate of the single inlet and exit is  $50 \text{ m}^3/\text{day}$ , and the inlet waste concentration is  $100 \text{ mg/l}$ . What is the outlet concentration?

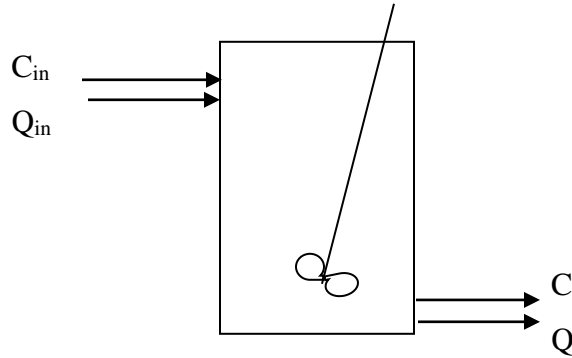


Figure Schematic of CSTR

Hint: write the mass balance equation and solve the resulting equation

2. The manufacturing process that generates the waste in example above has to be shut down, and, starting at  $t = 0$ , the concentration  $C_{in}$  entering the CSTR is set to 0. What is the outlet concentration as a function of time after the incoming concentration is set to 0? How long does it take the tank concentration to reach 10% of its initial, steady-state value?

Hint: write the mass balance equation and solve the resulting equation

3. The CSTR shown in Figure above is used with a conservative substance. The reactor is filled with clean water before it is started. After starting, waste containing a  $100 \text{ mg/l}$  of a pollutant is added at a flow rate of  $50 \text{ m}^3/\text{day}$ . The volume of the reactor is  $500 \text{ m}^3$ . What is the concentration exiting the reactor as a function of time after it is started?

Hint: write the mass balance equation and solve the resulting equation

4. A sewage pipe from a wastewater treatment plant discharges  $1.0 \text{ m}^3/\text{s}$  of effluent containing  $5.0 \text{ mg/l}$  of phosphorus compounds (reported as  $\text{mg P/l}$ ) into a river with an upstream flow rate of  $25 \text{ m}^3/\text{s}$  and a phosphorus concentration of  $0.010 \text{ mg P/l}$  (see Figure below). What is the resulting concentration of phosphorus in the river downstream of the sewage outflow, in units of  $\text{mg/l}$ ?

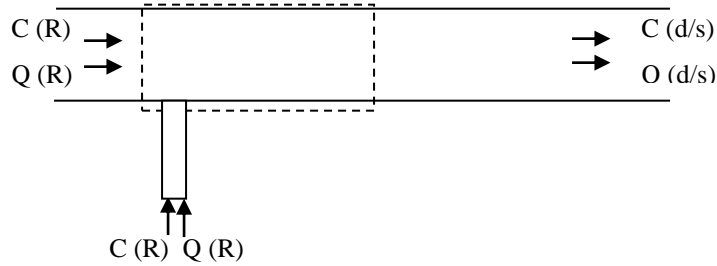
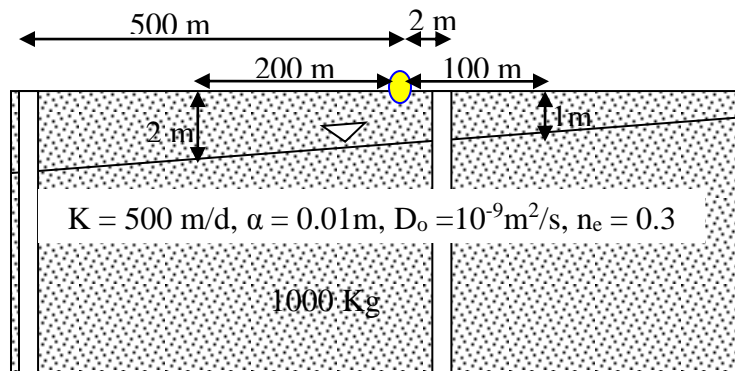


Figure Phosphorous mixing problem

Hint: write two mass balance equations (one for pollutant the other for discharge) and solve the resulting equation

5. A river with cross section  $A = 20 \text{ m}^2$  has a flow rate of  $Q = 1 \text{ m}^3/\text{s}$ . The effective mixing coefficient is  $D_d = 1 \text{ m}^2/\text{s}$ . For what distance downstream is diffusion dominant? Where does advection become dominant? What is the length of stream where diffusion and advection have about equal influence?
6. To estimate the mixing characteristics of a small stream, a scientist injects 5 g of dye instantaneously and uniformly over the river cross section ( $A = 5 \text{ m}^2$ ) at the point  $x = 0$ . A measurement station is located 1 km downstream and records a river flow rate of  $Q = 0.5 \text{ m}^3/\text{s}$ . In order to design the experiment, the scientist assumed that  $E = 0.1 \text{ m}^2/\text{s}$ . Use this value to answer the following questions.
  - a. The flurometer used to measure the dye downstream at the measuring station has a detection limit of  $0.1 \text{ }\mu\text{g/l}$ . When does the measuring station first detect the dye cloud?
  - b. When does the maximum dye concentration pass the measuring station, and what is this maximum concentration?
  - c. After the maximum concentration passes the measuring station, the measured concentration decreases again. When is the measuring station no longer able to detect the dye?
  - d. Why is the elapsed time between first detection and the maximum concentration different from the elapsed time between the last detection and the maximum concentration?
7. You are in a river where the average downstream velocity is  $1.2 \text{ m/s}$  and the longitudinal dispersion coefficient is  $0.1 \text{ m}^2/\text{s}$ . You run an experiment with a conservative tracer and a reactive tracer and measure breakthrough curves at a distance 180m downstream of the injection point. You put in equal masses of each. The peak concentration for the conservative tracer happens at 150 seconds. The concentration of the reactive tracer at the same time is 0.2 times that of the conservative tracer. Estimate the effective reaction rate.

8. A factory is dumping Chloromezalone into a stream at a concentration of 3 g/l. The stream has an average water velocity of 0.4 m/s and the total longitudinal dispersion coefficient is 0.05 m<sup>2</sup>/s. Chloromezalone is known to react in the benthos of the stream with effective reaction rate of  $k = 0.001 \text{ s}^{-1}$ . For centuries this stream has provided water to a small farm 500m downstream. Concentrations of Chloromezalone higher than 0.5 g/l are known to inhibit certain reactions essential to human environment. Should you be concerned about contamination? Consider steady state
9. A lake of 1000 m<sup>3</sup> can be broken into a surface layer of 500 m<sup>3</sup> and two deeper layers of equal size. The top layer is fed by a contaminated stream with concentration 5g/liter, which flows in at a rate of 20 m<sup>3</sup>/hr. It is drained by a similar stream. Groundwater flow in the surface layer is negligible compared to the surface water input. Both deeper layers are fed by groundwater at rates of 2 m<sup>3</sup>/hr. The groundwater in this area is pristine. The surface and middle layer have a turbulent exchange rate of 5 m<sup>3</sup>/hr, while the middle and lower layer have an exchange rate of 1 m<sup>3</sup>/hr.
- Write down evolution equations for the concentration in each layer.
  - Given enough time, what will the concentration profile in the lake look like?
10. A spill of a contaminant into an aquifer has occurred. The spill was short and over a small area. The total mass of the spill is 1000kg. After performing a pumping test you infer that the hydraulic conductivity of the aquifer is 500m/d and you know the effective porosity is 0.3. You know flow is from east to west. You have a depth to water measurement at 100m east of the spill 1m and at 200 m west of the spill it is 2m. The surface elevation is flat. There are two drinking wells, one 2 m east of the spill and another is 500 m west of the spill. Calculate the concentrations that will arrive at these wells. The molecular diffusion is  $10^{-9} \text{ m}^2/\text{s}$ . The dispersivity ( $\alpha$ ) is 0.01 m.



11. A spill of a contaminant into an aquifer has occurred. The spill was short and over a small area. The total mass of the spill is 10kg. You know from a recent study that the Darcy velocity of this aquifer is more or less uniform in one direction and of magnitude 2m/day. The effective porosity of the aquifer 0.3. 200m

downstream of the spill location is a well field. There are five wells along a line perpendicular to the direction of flow located at  $y=-20,-10,0,10,20$  (where  $y=0$  aligns with the center of mass of the plume). The molecular diffusion is  $10^{-9} \text{ m}^2/\text{s}$ . The longitudinal dispersivity is  $0.01 \text{ m}$ . Calculate concentration arriving at these well and plot breakthrough curves.

- 12.** A fully penetrating well pumps  $500 \text{ m}^3/\text{day}$  water from a confined aquifer with a thickness of  $50 \text{ m}$ . Before pumping there existed a uniform groundwater flow parallel to the  $x$ -axis in a negative direction (see figure below); the hydraulic gradient of this uniform flow field was  $0.001$ . The aquifer has a hydraulic conductivity of  $10 \text{ m/day}$ . If A land fill site is to be constructed in the vicinity (along the positive  $x$  axis) of the well, where should the site be?

