Earthquake Analysis of MDOF Systems

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Presentation outline

- Earthquake analysis of linear systems
- Basic Structural dynamics for MDOF systems
- Modal Analysis
- Modal response history analysis (RHA)
- Modal response spectrum analysis (RSA)
- Numerical examples
  1. RSA and ESA for 3 story two shear frames
  2. RHA and RSA for 5 story two shear frames

Earthquake Analysis of Linear Systems

Type of structure	Method of Analysis
Regular (simple) structures	Equivalent static analysis
↓	Response spectrum analysis
	Response history analysis
Irreguar (complex) structures	Nonlinear time history analysis

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Earthquake Analysis of Linear Systems (contd.)

- Equivalent static analysis (ESA) or (ELF)
    - acceptable results for regular structures
- Dynamic analysis
    - Response spectrum analysis (RSA)
        - satisfactory for majority of the cases
    - Response history analysis (RHA or THA)
        - can be used to model linear & nonlinear behavior depending on the nature of the site, size and sensitivity of the structures
    - Nonlinear THA (may include soil-structure interaction)
        - only for special structures

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Basic structural dynamics - MDOF

For a MDOF system

$$\begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1n} \\ m_{21} & m_{22} & \cdots & m_{2n} \\ \cdots & \cdots & \ddots & \cdots \\ m_{n1} & m_{n2} & \cdots & m_{nn} \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \cdots \\ \ddot{u}_3 \end{Bmatrix} + \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \cdots & \cdots & \ddots & \cdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \cdots \\ \dot{u}_3 \end{Bmatrix} + \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \cdots & \cdots & \ddots & \cdots \\ k_{n1} & k_{n2} & \cdots & k_{nn} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ \cdots \\ u_3 \end{Bmatrix} = \begin{Bmatrix} p_1 \\ p_2 \\ \cdots \\ p_3 \end{Bmatrix}$$

Writing the matrices compactly

$$[m]\{\ddot{u}(t)\} + [c]\{\dot{u}(t)\} + [k]\{u(t)\} = \{p(t)\}$$

Let $\{u\} = [\Phi]\{q\}$

$$\{u_n(t)\} = \{\emptyset_n\}q_n(t)$$

$$\{u_n(t)\} = \{\emptyset_n\}(A_n \cos \omega_n t + B_n \sin \omega_n t)$$

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Basic structural dynamics-MDOF (contd.)

- solving the differential equation

$$\left[[k] - \omega_n^2[m]\right] \{\emptyset_n\} = \{0\}$$

➡ Matrix eigenvalue problem

- the non-trivial solution is obtained from:

$$\det|[k] - \omega_n^2[m]| = 0$$

➡ characteristic equation (polynomial of n[th] degree)

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Basic structural dynamics-MDOF (contd.)

- solving the real roots for the characteristic equation
  - natural frequencies of vibration (eigenvlalues)

    ω_n (n=1, 2, …, N) e.g. $\omega_1, \omega_2, \omega_3, \ldots$

    natural periods: $T_n = 2\pi/\omega_n$ e.g. $T_1, T_2, T_3, \ldots$
  - natural mode shapes of vibration (eigenvectors)

    Φ_n (n=1, 2, …, N)

$$\phi_1 = \begin{Bmatrix} \phi_{11} \\ \phi_{21} \\ \phi_{31} \end{Bmatrix}, \quad \phi_2 = \begin{Bmatrix} \phi_{12} \\ \phi_{22} \\ \phi_{32} \end{Bmatrix}, \quad \phi_3 = \begin{Bmatrix} \phi_{13} \\ \phi_{23} \\ \phi_{33} \end{Bmatrix}, \quad \ldots$$

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Modal Analysis
Modal transformation

$$[m]\{\ddot{u}(t)\} + [c]\{\dot{u}(t)\} + [k]\{u(t)\} = \{p(t)\}$$

Let $\{u(t)\} = [\phi]\{q(t)\}$

$$[m][\Phi]\{\ddot{q}(t)\} + [c][\Phi]\{\dot{q}(t)\} + [k][\Phi]\{q(t)\} = \{p(t)\}$$

Pre-multiplying all by $[\Phi]^T$

$$[\Phi]^T[m][\Phi]\{\ddot{q}(t)\} + [\Phi]^T[c][\Phi]\{\dot{q}(t)\} + [\Phi]^T[k][\Phi]\{q(t)\} = [\Phi]^T\{p(t)\}$$

Let $\{p(t)\} = -[m]\{i\}\ddot{u}_g$

$$[M]\{\ddot{q}(t)\} + [C]\{\dot{q}(t)\} + [K]\{q(t)\} = -[\Phi]^T[m]\{i\}\ddot{u}_g$$

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Modal Analysis (Cont'd)
Modal transformation

$$\begin{bmatrix} M_1 & & \\ & M_2 & \\ & & M_N \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_N \end{Bmatrix} + \begin{bmatrix} C_1 & & \\ & C_2 & \\ & & C_N \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_N \end{Bmatrix} + \begin{bmatrix} K_1 & & \\ & K_2 & \\ & & K_N \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_N \end{Bmatrix} = -[\varphi]^T \begin{Bmatrix} m_1 \\ m_2 \\ m_N \end{Bmatrix} \ddot{u}_g(t)$$

For each Mode

$$M_n \ddot{q}_n + C_n \dot{q}_n + K_n q_n = -L_n^h \ddot{u}_g(t) \qquad n = 1,2,..,N$$

Where $L^h_n = \{\phi_n\}^T [m] \{i\}$

Dividing by M_n yields $\ddot{q}_n + 2\xi_n \omega_n \dot{q}_n + \omega_n^2 q_n = -\Gamma_n \ddot{u}_g(t)$

where $\Gamma_n = \dfrac{L_n^h}{M_n}$ is the modal participation factor

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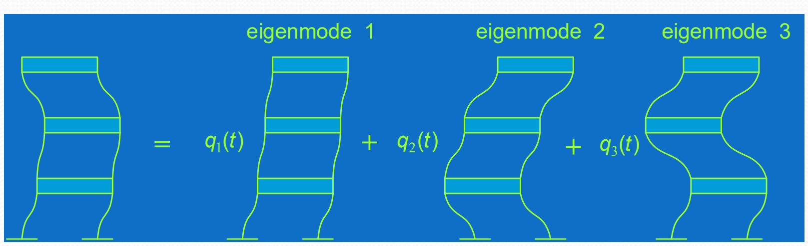
Modal Analysis (Contd.)
Interpretation of modal superposition

Modal expansion of displacement $u(t) = \sum \emptyset_n q_n(t)$

$$\begin{Bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{Bmatrix} = [\emptyset] \begin{Bmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \end{Bmatrix} = \begin{Bmatrix} \emptyset_{11} \\ \emptyset_{21} \\ \emptyset_{31} \end{Bmatrix} q_1(t) + \begin{Bmatrix} \emptyset_{12} \\ \emptyset_{22} \\ \emptyset_{32} \end{Bmatrix} q_2(t) + \begin{Bmatrix} \emptyset_{13} \\ \emptyset_{23} \\ \emptyset_{33} \end{Bmatrix} q_3(t)$$



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Modal Analysis (contd.)
Modal Expansion of Inertia Force

- Modal expansion of $\{p_{eff}(t)\} = -[m]\{i\}\ddot{u}_g(t)$

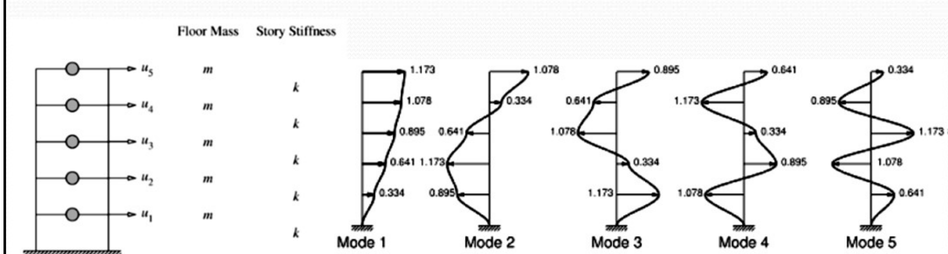
$$[m]\{i\} = \sum_{n=1}^{N} \{S_n\} = \sum_{n=1}^{N} \Gamma_n [m]\{\emptyset_n\}$$

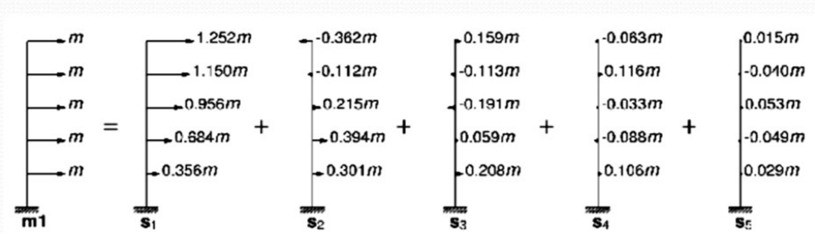
where
$$\Gamma_n = \frac{L_n^h}{M_n} = \frac{\{\emptyset_n\}^T [m]\{i\}}{\{\emptyset_n\}^T [m]\{\emptyset_n\}} = \frac{\sum m_j \emptyset_{jn}}{\sum m_j \emptyset_{jn}^2}$$

n^{th} mode components

$$\{p_{eff,n}(t)\} = -\{S_n\}\ddot{u}_g(t)$$

Modal Analysis (contd.)
Modal Expansion of Inertia Force





Effective modal mass and modal height

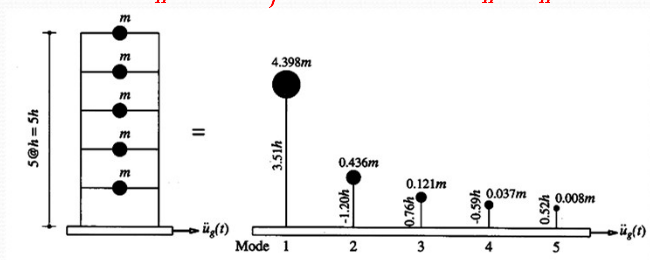
Base shear effective modal mass, Mn*

$$M_n^* = \Gamma_n L_n^h = \frac{\left(L_n^h\right)^2}{M_n} = \frac{\left(\sum m_j \phi_{jn}\right)^2}{\sum m_j \phi_{jn}^2}$$

Base moment effective modal height, hn*

$$h_n^* = \frac{L_n^\theta}{L_n^h} = \frac{\sum h_j m_j \phi_{jn}}{\sum m_j \phi_{jn}}$$

Note: $\Sigma M_n^* = \Sigma m_j$ **and** $\Sigma h_n^* M_n^* = \Sigma m_j h_j$



Effective modal masses and effective modal heights.

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Modal Response History Analysis (RHA)

- Response quantity: $r_n(t) = r_n^{st} A_n(t)$

  where r_n^{st} is modal static response due to S_n

- Displacement response: $u_n(t) = \frac{\Gamma_n}{\omega_n^2} \phi_n A_n(t)$

All quantities are computed for each mode (n = 1, 2, …, N)

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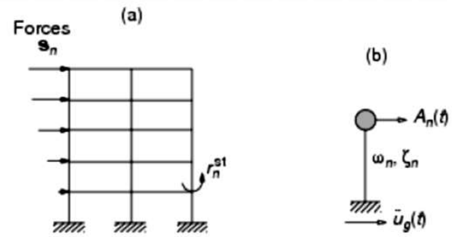
Modal Response History Analysis (RHA)

- Modal Response

$$r_n(t) = r_n^{st} A_n(t)$$



- Total Response

$$r(t) = \sum_{n=1}^{N} r_n(t) = \sum_{n=1}^{N} r_n^{st} A_n(t)$$

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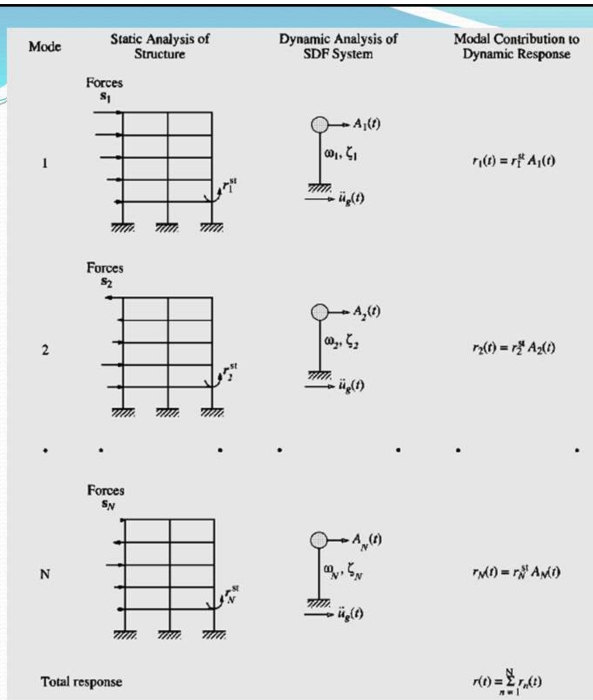
Modal Response history analysis (contd.)



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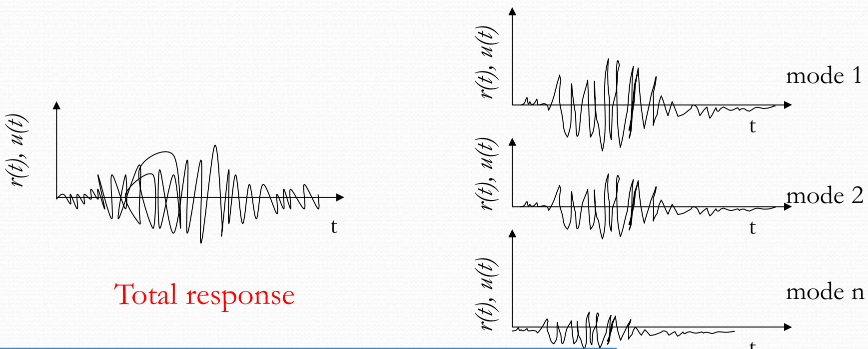
Modal Response history analysis (contd.)

- Total response
  - Combine the response of all modes
  - $u(t) = \Sigma u_n(t)$ and $r(t) = \Sigma r_n(t)$



Total response

Note that peak values can occur at different times

Modal Analysis
Response Spectrum Analysis (RSA)

- Instead of calculating the response r(t) as a function of time in RHA, only peak values are calculated in RSA.
  - Modal peak values $r_{no} = r_n^{st} A_{no}$

    where $A_{no} = \max_t |A(T_n, \zeta_n)|$ is the peak ordinate of the design spectrum corresponding to natural period T_n & damping ratio ζ_n.

Modal Response spectrum analysis (contd.)

- Modal combination rules (ABSSUM, SRSS & CQC)
  - Absolute sum (ABSSUM) $r_{no} \leq \sum_{n=1}^{N} |r_{no}|$

Upper bound result

  - Square-root-of-sum-of-squares (SRSS)

$$r_{no} \cong \left(\sum_{n=1}^{N} r_{no}^2 \right)^{1/2}$$

Good results for most structures with well separated natural frequencies

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Modal Response spectrum analysis (contd.)

- complete quadratic combination (CQC)

$$r_{no} \cong \left(\sum_{i=1}^{N} \sum_{n=1}^{N} \rho_{in} r_{io} r_{no} \right)^{1/2} \cong \left(\sum_{n=1}^{N} r_{no}^2 + \underbrace{\sum_{i=1}^{N} \sum_{n=1}^{N}}_{i \neq n} \rho_{in} r_{io} r_{no} \right)^{1/2}$$

where correlation coefficient ρ_{in} is

$$\rho_{in} = \frac{8\zeta^2 (1+\beta_{in}) \beta_{in}^{3/2}}{\left(1-\beta_{in}^2\right)^2 + 4\zeta^2 \beta_{in} (1+\beta_{in})^2} \quad and \quad \beta_{in} = \frac{\omega_i}{\omega_n}$$

Good results even for structures with closely spaced natural frequencies

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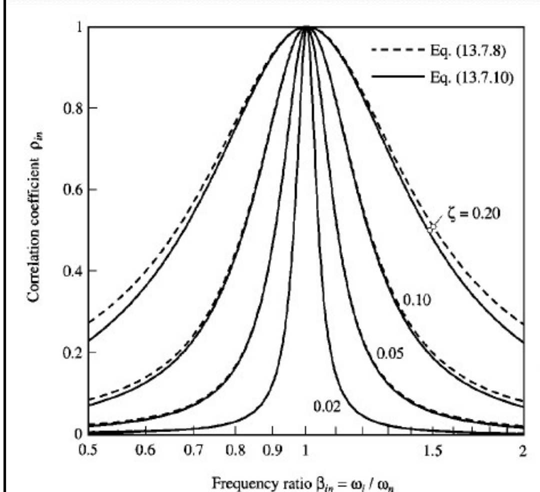
Correlation Coefficient Equations, ρ_{in}



Rosenblueth-Elorduy

$$\rho_{in} = \frac{\zeta^2(1+\beta_{in})^2}{(1-\beta_{in})^2 + 4\zeta^2\beta_{in}} \qquad (13.7.8)$$

Der Kiureghian

$$\rho_{in} = \frac{8\zeta^2(1+\beta_{in})\beta_{in}^{3/2}}{(1-\beta_{in}^2)^2 + 4\zeta^2\beta_{in}(1+\beta_{in})^2} \qquad (13.7.10)$$

Numerical example 1

Equivalent static analysis as per EBCS 8: 1995

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Dynamic response spectrum analysis

Examples of equivalent static analysis:

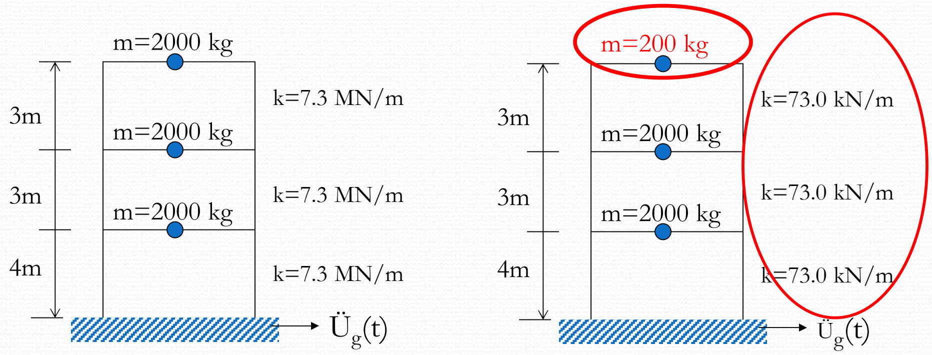
Frame 1

Regular elevation

Frame 2

Irregular elevation

Flexible columns



The structures are subjected to Üg(t) = 0.3g bedrock acceleration, soil class A

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Equivalent static analysis as per EBCS 8:1995

- Design spectrum coefficients
  - Fundamental period: $T_1 = C_1 H^{3/4}$

    $T_1 = 0.075*(10)^{3/4} = 0.422$ sec
  - Response factor $\beta = \frac{1.2S}{T_1^{2/3}} \leq 2.5$

    $\beta = 2.84 > 2.5 \Longrightarrow \beta = 2.50$
  - $\alpha = 0.3$
  - $\gamma = 1$ assume the behavior factor to be 1 (elastic RS)
- $S_d(T_1) = \alpha\beta\gamma = 0.75$

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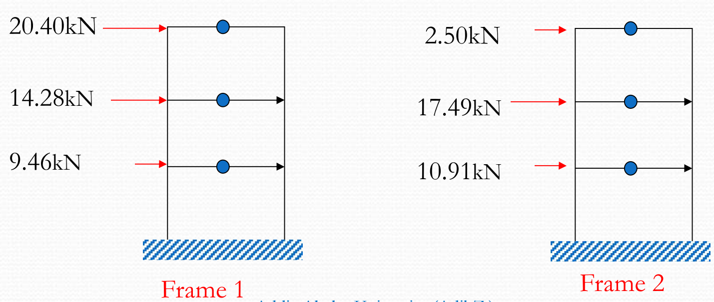
Equivalent static analysis as per EBCS 8:1995

- Total Base shear $F_b = S_d(T1).W$
  - $F_b = 44.15$ kN for frame 1 since m = 6000 kg, i.e. W = 58.56 kN
  - $F_b = 30.90$ kN for frame 2 since m = 4200 kg, i.e. W = 41.20 kN
- Distribution of lateral force $F_i = \frac{(F_b - F_t)W_i h_i}{\sum W_j h_j}$ *and* $F_t = 0.07T_1F_b$



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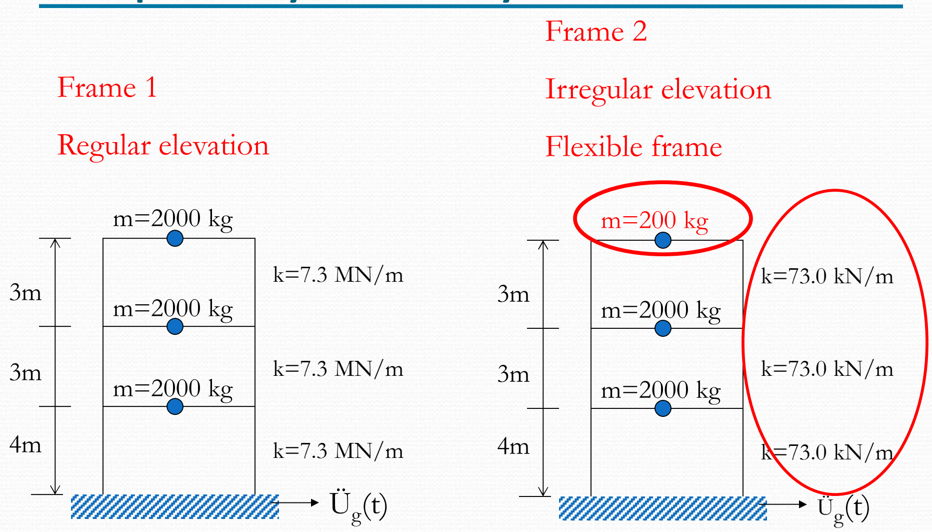
Examples of dynamic analysis:



The structures are subjected to Üg(t) = 0.3g bedrock acceleration, soil class A

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Solution of the eigenvalue problem for frame 1

$$[m] = m\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \qquad [k] = k\begin{bmatrix}2 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}$$

where m = 2000kg

k = 7.3MN/m

$$\det|[k] - \omega_n^2[m]| = \det\begin{bmatrix}2k-\omega_n^2 m & -k & 0\\ -k & 2k-\omega_n^2 m & -k\\ 0 & -k & k-\omega_n^2 m\end{bmatrix} = 0$$

The characteristic equation after simplification

$$k^3 - 6mk^2\omega_n^2 + 5m^2k\omega_n^4 - m^3\omega_n^6 = 0 \Rightarrow \begin{aligned}\omega_1 &= 0.445\sqrt{k/m}\\ \omega_2 &= 1.247\sqrt{k/m}\\ \omega_3 &= 1.802\sqrt{k/m}\end{aligned}$$

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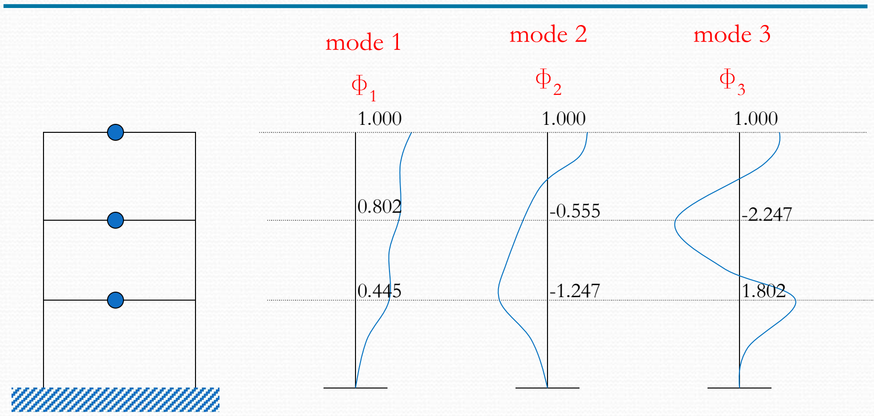
Natural frequency and mode shapes: Frame 1



	mode 1	mode 2	mode 3
	ω_1=26.88rad/s	ω_2=75.34rad/s	ω_3=108.86rad/s
	T_1=0.234sec	T_2=0.083sec	T_3=0.058sec
From EBCS design spectrum	A_1=7.358m/s^2	A_2=6.607m/s^2	A_3=1.834m/s^2

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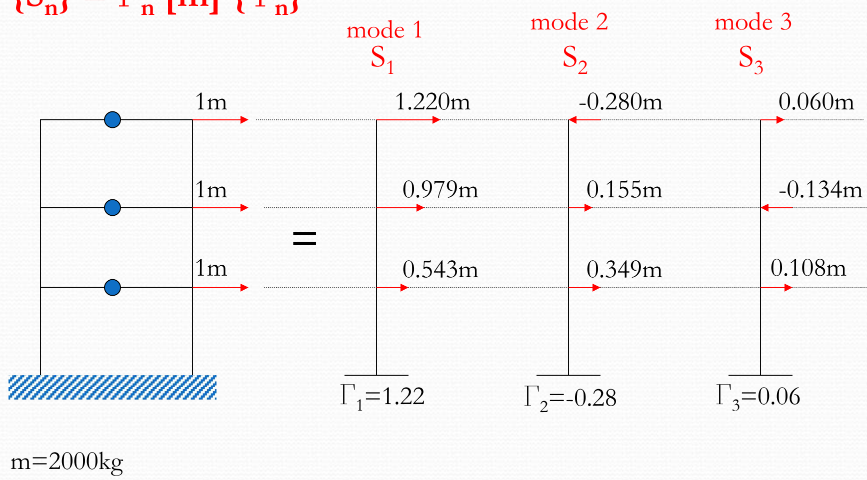
Modal mass distribution vectors: Frame 1

$\{S_n\} = \Gamma_n [m] \{\Phi_n\}$



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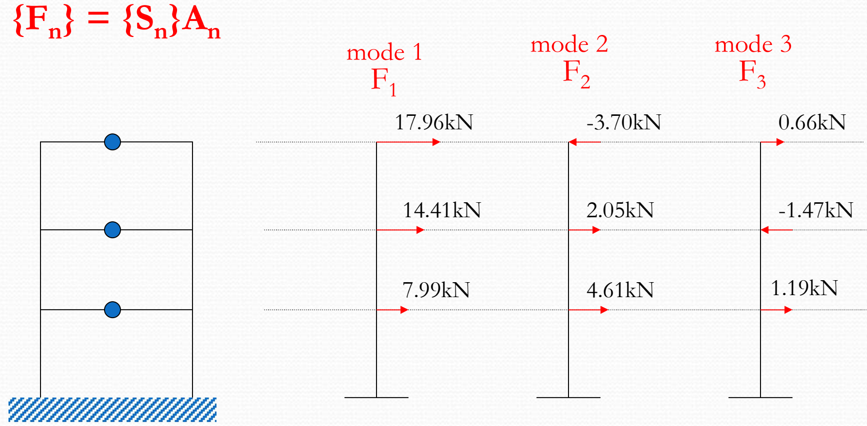
Equivalent modal forces: Frame 1 Regular Elevation

$\{F_n\} = \{S_n\}A_n$



modal base shear= 40.36kN 2.96kN 0.38kN

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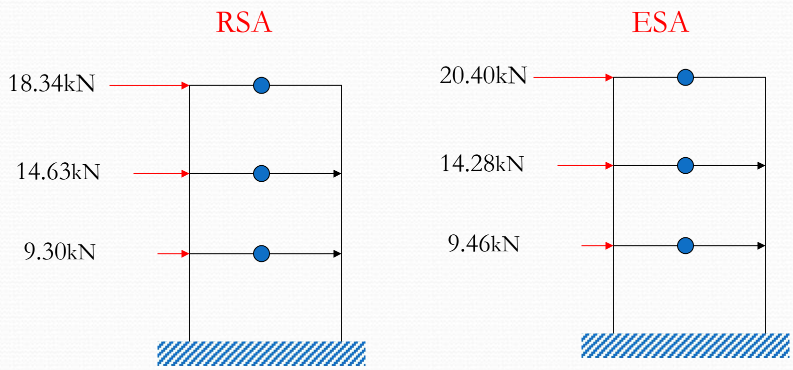
Equivalent static lateral forces from all modes: Frame 1



F_b = 42.27 kN F_b = 44.15 kN

M_b = 323.01 kNm M_b = 341.83 kNm

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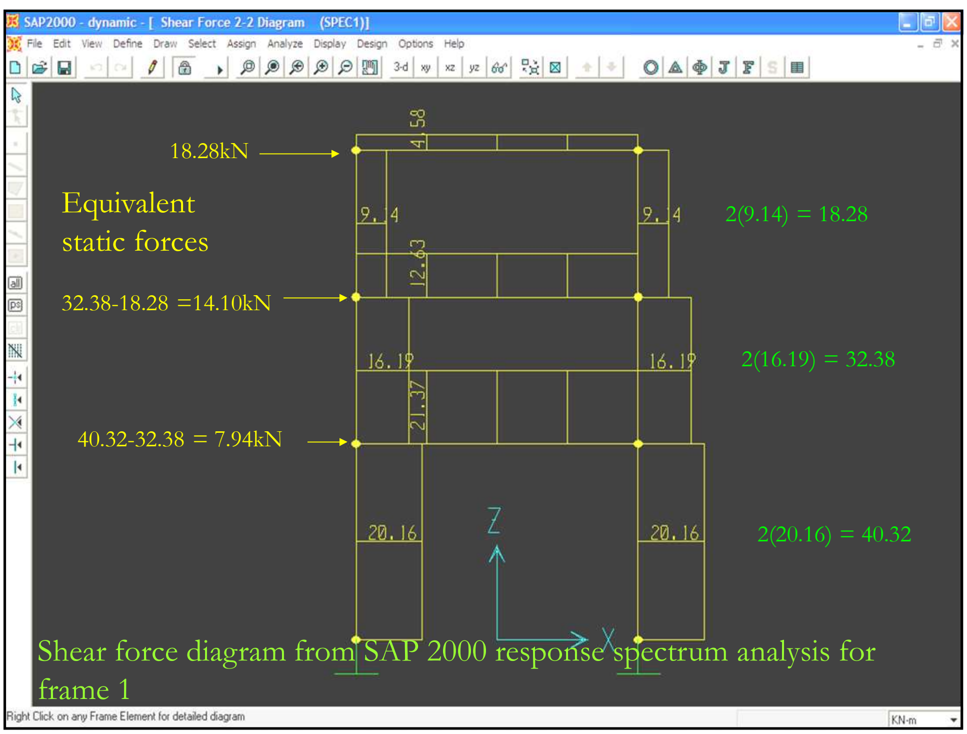

Solution of the eigenvalue problem for frame 2

$$[m] = m\begin{bmatrix}10 & 0 & 0\\ 0 & 10 & 0\\ 0 & 0 & 1\end{bmatrix} \qquad [k] = k\begin{bmatrix}2 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}$$

where m = 200kg

k = 73kN/m

$$\det|[k]-\omega_n^2[m]| = \det\begin{bmatrix}2k-\omega_n^2 10m & -k & 0\\ -k & 2k-\omega_n^2 10m & -k\\ 0 & -k & k-\omega_n^2 m\end{bmatrix} = 0$$

The characteristic equation after simplification

$$k^3 - 33mk^2\omega_n^2 + 140m^2k\omega_n^4 - 100m^3\omega_n^6 = 0$$

$\omega_1 = 0.1884\sqrt{k/m}$

$\omega_2 = 0.5034\sqrt{k/m}$

$\omega_3 = 1.0541\sqrt{k/m}$

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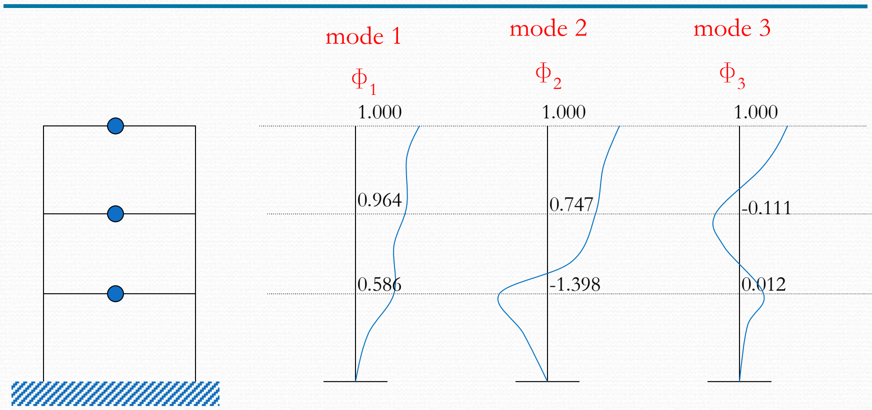
Natural frequency and mode shapes Irregular elevation



	mode 1	mode 2	mode 3
	ω_1=3.60rad/s	ω_2=9.62rad/s	ω_3=20.14rad/s
	T_1=1.75sec	T_2=0.65sec	T_3=0.31sec
From EBCS design spectrum	A_1=1.682m/s^2	A_2=4.528m/s^2	A_3=7.358m/s^2

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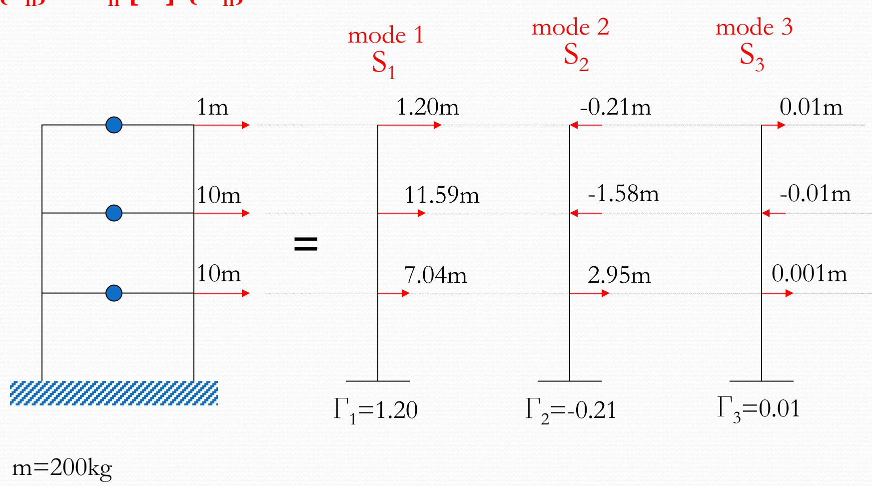
Modal mass distribution vectors: Irregular elevation

$\{S_n\} = \Gamma_n [m] \{\Phi_n\}$



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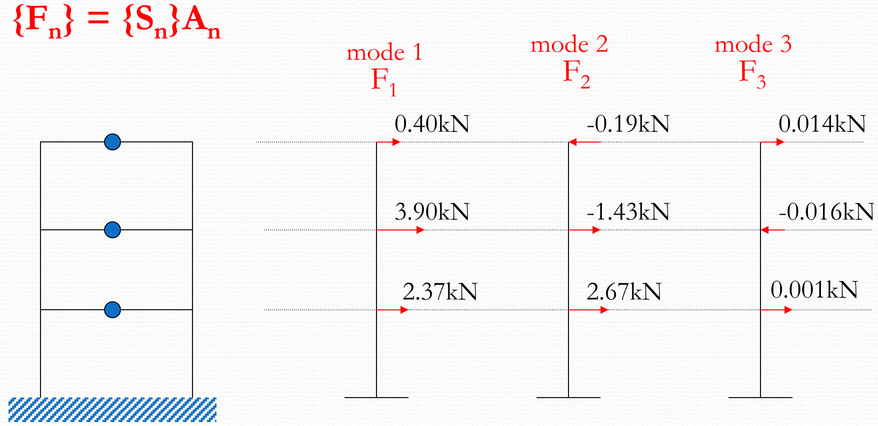
Equivalent modal forces: Frame 2 Irregular elevation

$\{F_n\} = \{S_n\}A_n$



modal base shear= 6.67kN -1.05kN -0.001kN

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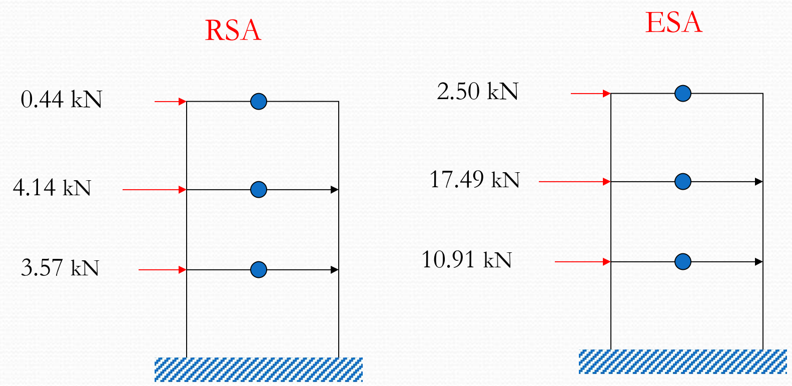
Equivalent static lateral forces from all modes: Irregular elevation



$F_b = 8.15$ kN

$M_b = 47.66$ kNm

$F_b = 30.90$ kN

$M_b = 191.08$ kNm

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Numerical example 2

Response History Analysis

&

Response spectrum analysis

NB: Refer Chopra's Dynamics of Structures book, chapter 13 for the details

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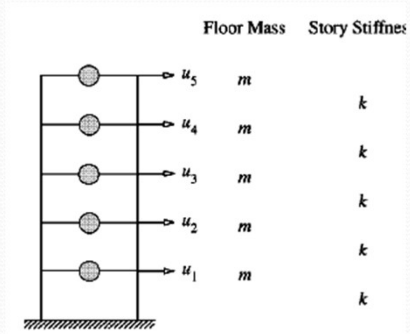
RHA for 5 story shear frame



$$m_j = m = 100 \text{ kips/g}$$

$$k_j = k = 31.54 \text{ kips/in.}$$

$$\mathbf{m} = m\begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{bmatrix} \qquad \mathbf{k} = k\begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & -1 & 2 & -1 \\ & & & -1 & 1 \end{bmatrix}$$

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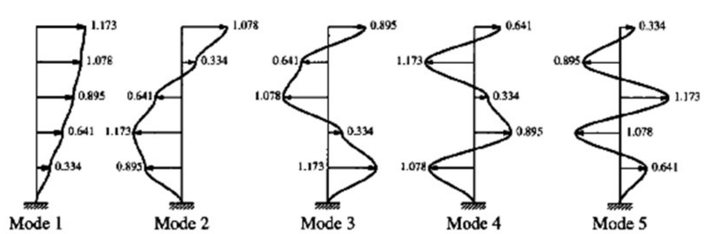
RHA for 5 story shear frame

$$\phi_1 = \begin{Bmatrix} 0.334 \\ 0.641 \\ 0.895 \\ 1.078 \\ 1.173 \end{Bmatrix} \quad \phi_2 = \begin{Bmatrix} -0.895 \\ -1.173 \\ -0.641 \\ 0.334 \\ 1.078 \end{Bmatrix} \quad \phi_3 = \begin{Bmatrix} 1.173 \\ 0.334 \\ -1.078 \\ -0.641 \\ 0.895 \end{Bmatrix} \quad \phi_4 = \begin{Bmatrix} -1.078 \\ 0.895 \\ 0.334 \\ -1.173 \\ 0.641 \end{Bmatrix} \quad \phi_5 = \begin{Bmatrix} 0.641 \\ -1.078 \\ 1.173 \\ -0.895 \\ 0.334 \end{Bmatrix}$$



$$\omega_n = \alpha_n \left(\frac{k}{m}\right)^{1/2}$$

$\alpha_1 = 0.285$, $\alpha_2 = 0.831$, $\alpha_3 = 1.310$, $\alpha_4 = 1.682$, and $\alpha_5 = 1.919$.

$T_n = 2.0$, 0.6852, 0.4346, 0.3383, and 0.2966 sec.

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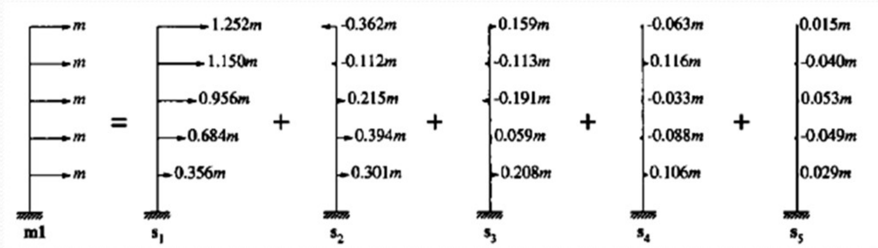
RHA for 5 story shear frame



$\{S_n\} = \Gamma_n \, [m] \, \{\Phi_n\}$

Mode	V_{bn}^{st}/m	V_{5n}^{st}/m	M_{bn}^{st}/mh	u_{5n}^{st}
1	4.398	1.252	15.45	0.127
2	0.436	−0.362	−0.525	−0.004
3	0.121	0.159	0.092	0.0008
4	0.037	−0.063	−0.022	−0.0002
5	0.008	0.015	0.004	0.00003

RHA for 5 story shear frame

Effective modal masses and effective modal heights

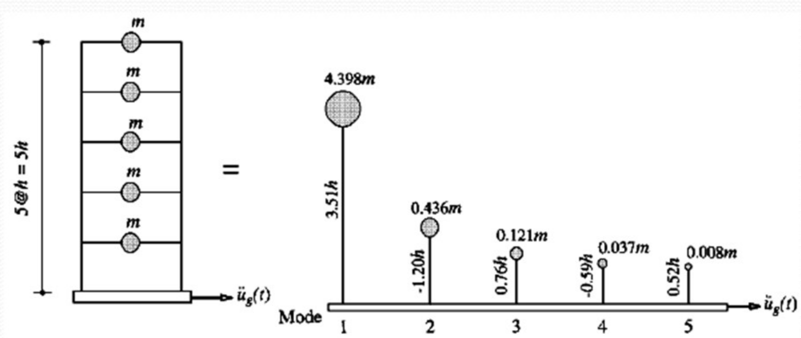


Figure 13.2.5 Effective modal masses and effective modal heights.

RHA for 5 story shear frame

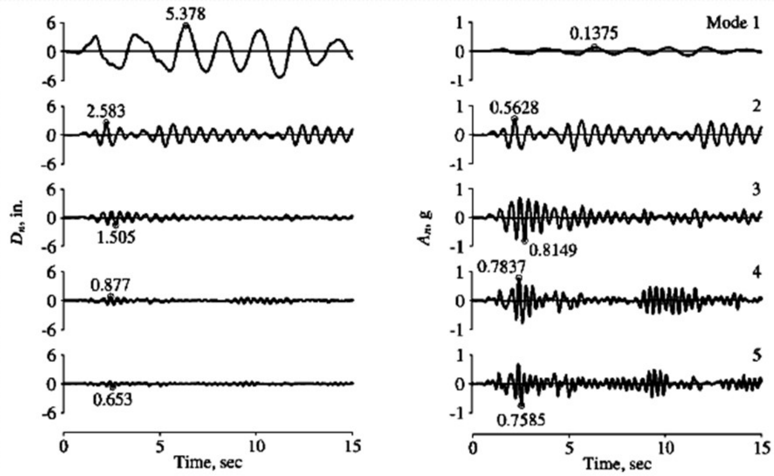


Figure 13.2.6 Displacement $D_n(t)$ and pseudo-acceleration $A_n(t)$ responses of modal SDF systems.

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RHA for 5 story shear frame

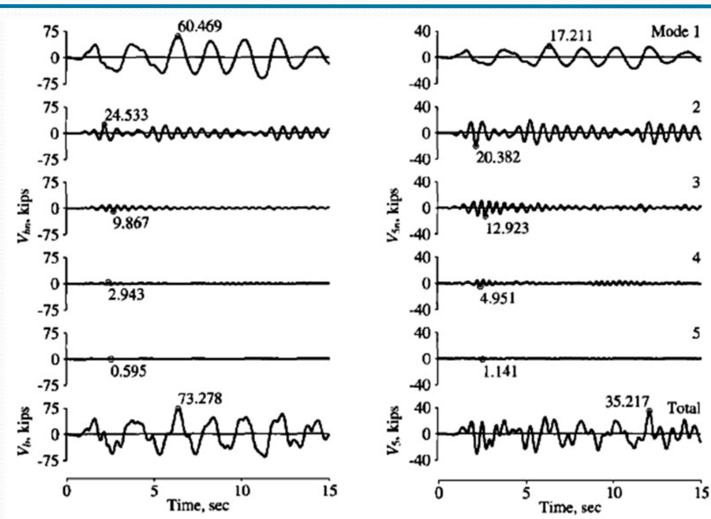

Figure 13.2.7 Base shear and fifth-story shear: modal contributions, $V_{bn}(t)$ and $V_{5n}(t)$, and total responses, $V_b(t)$ and $V_5(t)$.

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RHA for 5 story shear frame

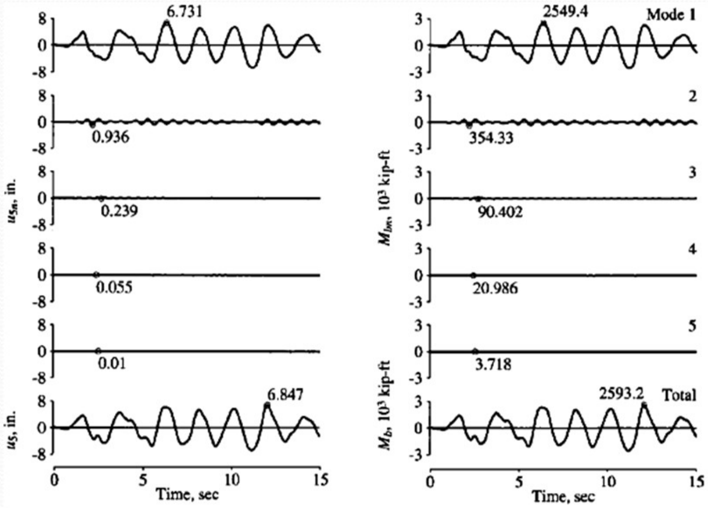


Figure 13.2.8 Roof displacement and base overturning moment: modal contributions, $u_{5n}(t)$ and $M_{bn}(t)$, and total responses, $u_5(t)$ and $M_b(t)$.

RSA for 5 story shear frame

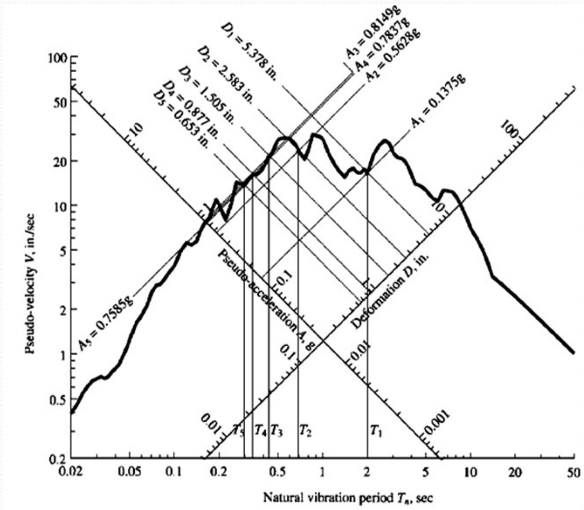


Figure 13.8.2 Earthquake response spectrum with natural vibration periods T_n of example structure shown together with spectral values D_n and A_n.

RSA for 5 story shear frame

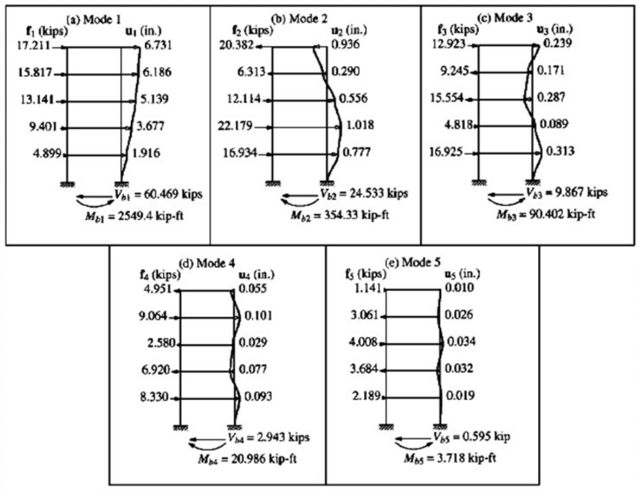


Figure 13.8.3 Peak values of displacements and equivalent static lateral forces due to the five natural vibration modes.

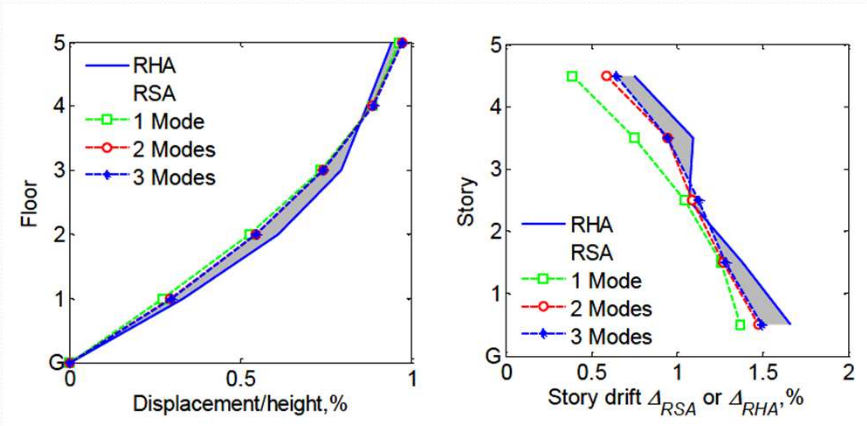
5 Story frame RSA & RHA

Mode	V_b (kips)	V_5 (kips)	M_b (kip-ft)	u_5 (in.)
1	60.469	17.211	2549.4	6.731
2	24.533	−20.382	−354.33	−0.936
3	9.867	12.923	90.402	0.239
4	2.943	−4.951	−20.986	−0.055
5	0.595	1.141	3.718	0.010

TABLE 13.8.5 RSA AND RHA VALUES OF PEAK RESPONSE

	V_b (kips)	V_5 (kips)	M_b (kip-ft)	u_5 (in.)
ABSSUM	98.407	56.608	3018.8	7.971
SRSS	66.066	30.074	2575.6	6.800
CQC	66.507	29.338	2572.7	6.793
RHA	73.278	35.217	2593.2	6.847

RHA vs RSA: one ground motion



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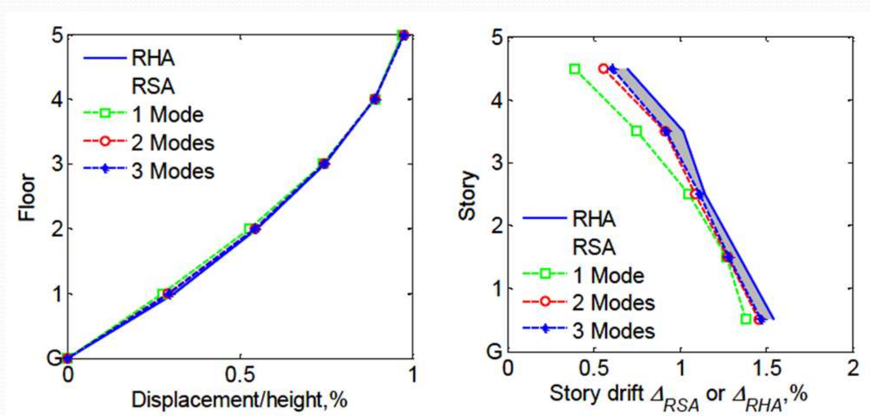
RHA vs RSA: 20 ground motions



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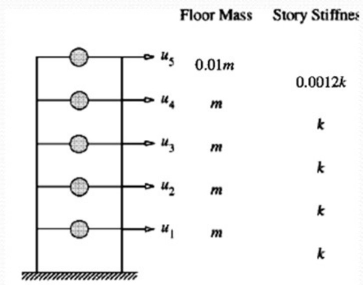
RHA for 4 story frame with appendage



$m = 100$ kips/g.

$k = 22.599$ kips/in.

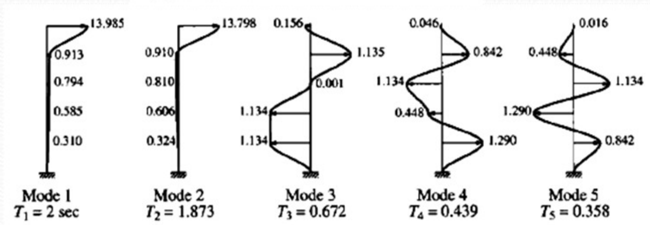


Figure 13.2.9 Natural periods and modes of vibration of building with appendage.

RHA for 4 story frame with appendage

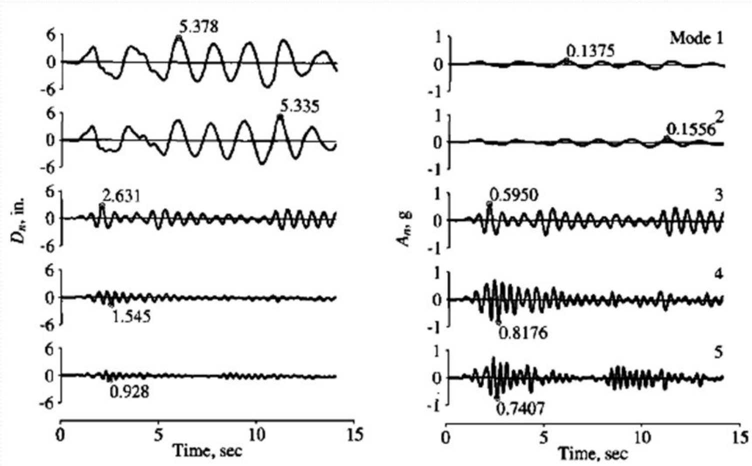


Figure 13.2.10 Displacement $D_n(t)$ and pseudo-acceleration $A_n(t)$ responses of modal SDF systems.

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RHA for 4 story frame with appendage

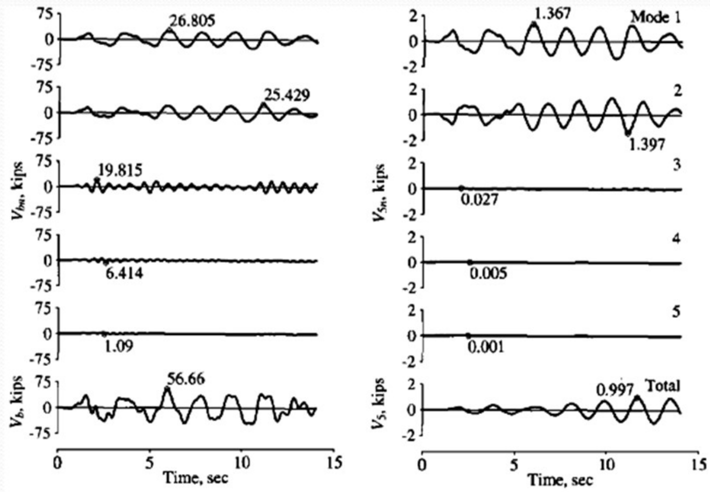

Figure 13.2.11 Base shear and appendage shear: modal contributions, $V_{bn}(t)$ and $V_{5n}(t)$, and total responses, $V_b(t)$ and $V_5(t)$.

4 Story frame with an appendage RSA & RHA

TABLE 13.8.6 SPECTRAL VALUES AND PEAK MODAL RESPONSES

Mode	T_n (sec)	D_n (in.)	A_n/g	V_b (kips)	V_5 (kips)
1	2.000	5.378	0.1375	26.805	1.367
2	1.873	5.335	0.1556	25.429	−1.397
3	0.672	2.631	0.5950	19.816	0.027
4	0.439	1.545	0.8176	6.414	−0.005
5	0.358	0.928	0.7407	1.090	0.001

TABLE 13.8.11 RSA AND RHA VALUES OF PEAK RESPONSE

	V_b (kips)	V_5 (kips)
ABSSUM	79.554	2.797
SRSS	42.428	1.954
CQC	52.774	1.074
RHA	56.660	0.997