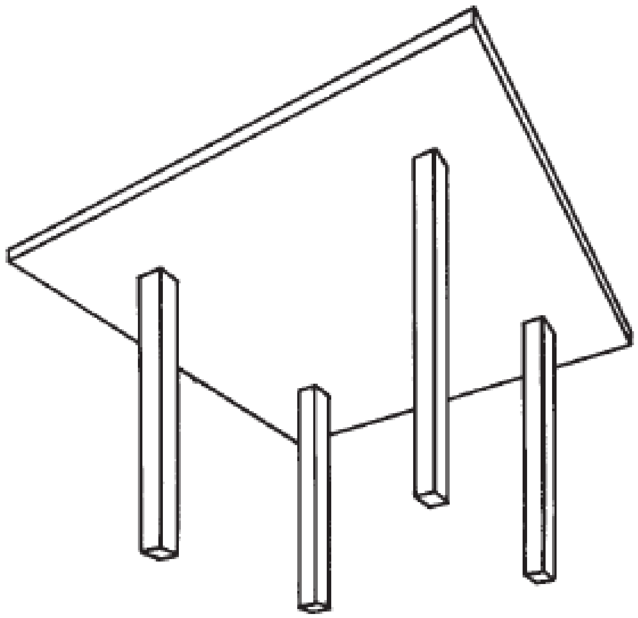
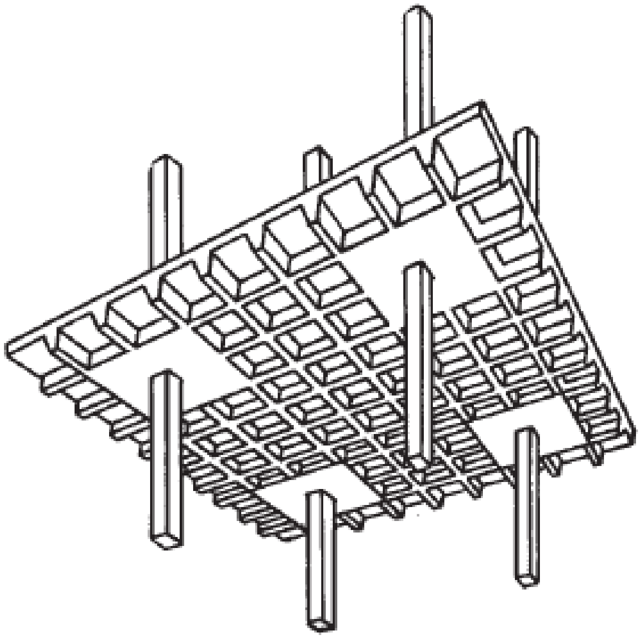
CHAPTER 4
ANALYSIS AND DESIGN OF TWO-WAY SLABS WITH EMPHASIS ON EQUIVALENT FRAME METHOD

4.0 INTRODUCTION

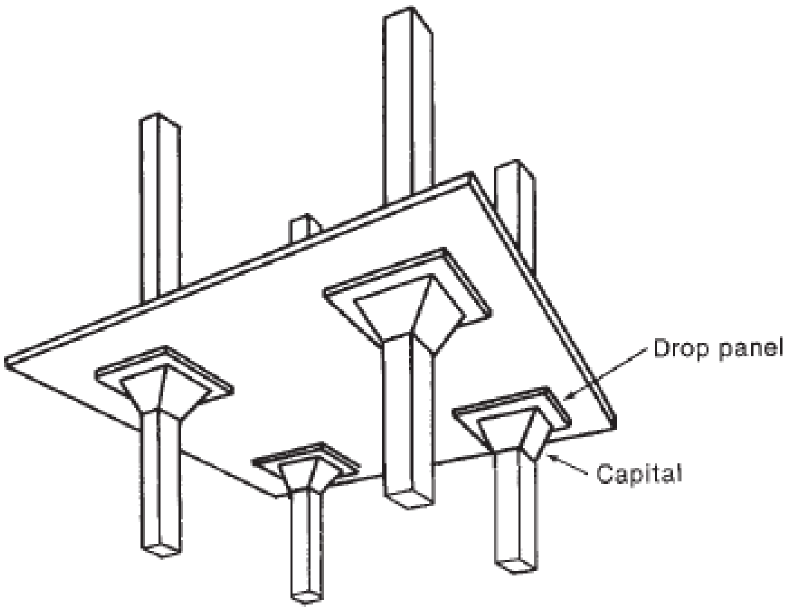


(a) Flat plate.

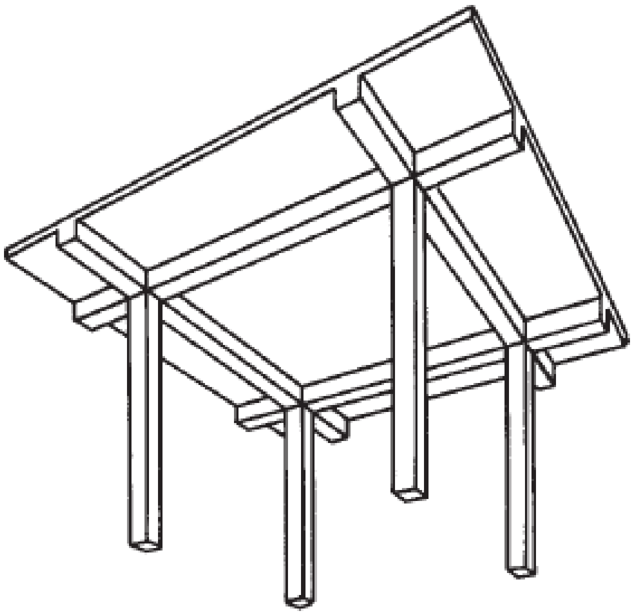


(b) Waffle slab.

4.0 INTRODUCTION



(c) Flat slab.



(d) Two-way slab with beams.

4.0 INTRODUCTION

- Two-way slabs transmit loads in two directions (compare with one-way slabs)
- They are efficient, economical, and widely used structural system
- In practice two-way slabs take various forms
- For relatively light loads (appt bldgs), flat plates are used.
- For longer spans, waffle slabs (or two way joist system) are used (formed with fiberglass or metal "dome" forms)

4.0 INTRODUCTION

- For heavy industrial loads, the flat slab system shown in Figure (c) may be used
- Shear transfer to the column is accomplished by thickening the slab near the column with drop panels or flaring the top of the column top to form a column capital
- Slab systems may incorporate beams between some or all of the columns. The resulting structure is referred to as two-way slabs with beams.

4.1 ELASTIC ANALYSIS OF TWO-WAY SLABS

- **Elastic Analysis of Slabs**
- Slabs are 2D structures
- The concepts involved in the elastic analysis is discussed in chapter 2 $\rightarrow$ Action is proportional to action effect (F=k z)
- The same principle holds for linear elastic analysis of slabs, bearing in mind that the analysis is much more complicated than for linear elements

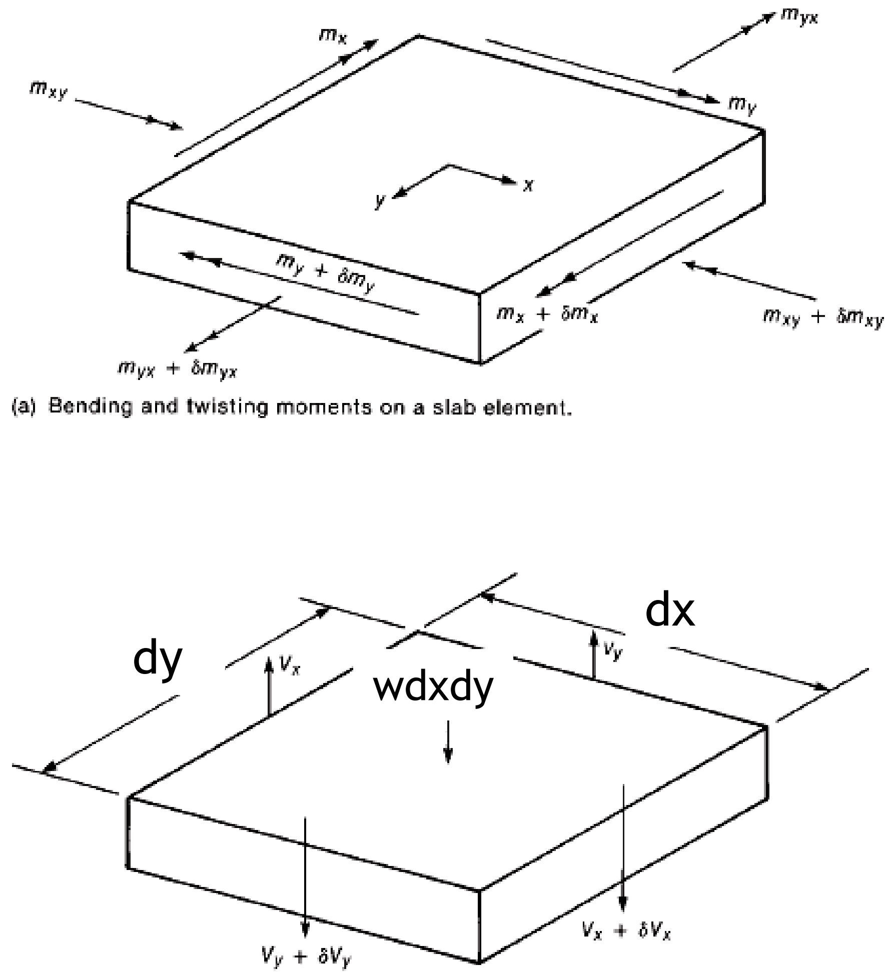
4.1 ELASTIC ANALYSIS OF TWO-WAY SLABS

- Slabs may be subdivided into:
  - Thick slabs $\rightarrow$ thickness greater than about 1/10th of the span (500 mm for a 5000 mm span)
  - Thick slab transmit a portion of the loads as a flat arch and have significant in-plane-compressive forces, with the result that the internal resisting compressive force C is larger than the internal tensile force T.
  - Thin slabs transmit a portion of the loads by acting as a tension membrane; hence T is larger than C
  - A medium thick slab does not exhibit either arch action or membrane action and thus T=C

4.1 ELASTIC ANALYSIS OF TWO-WAY SLABS

- Figure (next slide) shows an element cut from a medium thick, two-way slab.
- This element is acted on by the moments shown in Figure (a) and by shears and loads shown in Figure (b) (Figures are separated for clarity)
- Two types of moments $\boldsymbol{m_x}$ and $\boldsymbol{m_y}$ about axes parallel to the edges, and twisting moments $\boldsymbol{m_{xy}}$ and $\boldsymbol{m_{yx}}$ about axes $\perp$ to the edges.

4.1 ELASTIC ANALYSIS OF TWO-WAY SLABS



(a) Bending and twisting moments on a slab element.

(b) Shears and loads on a slab element.

Fig. 14-1
Moments and forces in a medium-thick plate.

4.1 ELASTIC ANALYSIS OF TWO-WAY SLABS

- NB: m_x, m_y, m_{xy}, and m_{yx} are moments and twisting moments per meter width
- V_y, and V_x are forces per meter width
- δm_x is change in m_x over a distance of $dx \rightarrow$

$$\delta m_x = \left(\frac{\partial m_x}{\partial x}\right) dx$$

- Similarly δV_y is change in V_y over a distance of $\mathrm{dy} \rightarrow$

$$\delta V_y = \left(\frac{\partial V_y}{\partial y}\right) dy$$

- and so on

4.1 ELASTIC ANALYSIS OF TWO-WAY SLABS

- Summing vertical forces→

$$-wdxdy + V_y dx - \left[V_y + \left(\frac{\partial V_y}{\partial y}\right)dy\right]dx + V_x dy - \left[V_x + \left(\frac{\partial V_x}{\partial x}\right)dx\right]dy = 0$$

$$\rightarrow \quad -wdxdy - \left(\frac{\partial V_y}{\partial y}\right)dydx - \left(\frac{\partial V_x}{\partial x}\right)dxdy = 0$$

$$\rightarrow \quad \left(\frac{\partial V_y}{\partial y}\right) + \left(\frac{\partial V_x}{\partial x}\right) = -w \qquad \text{........(4.1)}$$

4.1 ELASTIC ANALYSIS OF TWO-WAY SLABS

- Summing moments about lines parallel to the x and y axes and neglecting higher order terms gives:

$$\left(\frac{\partial m_y}{\partial y}\right)+\left(\frac{\partial m_{xy}}{\partial x}\right)=V_y \qquad and \qquad \text{....(4.2)}$$

$$\left(\frac{\partial m_x}{\partial x}\right)+\left(\frac{\partial m_{yx}}{\partial y}\right)=V_x$$

- It can be shown that $m_{xy} = m_{yx}$ (theory of elasticity)

4.1 ELASTIC ANALYSIS OF TWO-WAY SLABS

- Differentiating (4.2) and substituting in (4.1) gives the basic equilibrium equation for medium thick slabs:

$$\left(\frac{\partial^2 m_x}{\partial x^2}\right)+2\left(\frac{\partial^2 m_{xy}}{\partial x \partial y}\right)+\left(\frac{\partial^2 m_y}{\partial y^2}\right)=-w \quad \ldots(4.3)$$

- This is purely an equation of statics and applies regardless of the behavior of the plate material. (discuss interpretation)

4.1 ELASTIC ANALYSIS OF TWO-WAY SLABS

- For an elastic plate, the deflection, z, can be related to the applied load by means of:

$$\left(\frac{\partial^4 z}{\partial x^4}\right) + 2\left(\frac{\partial^4 z}{\partial x^2 \partial y^2}\right) + \left(\frac{\partial^4 z}{\partial y^4}\right) = -\frac{w}{D} \quad \text{.........(4.4)}$$

$$or \qquad \nabla^4 z = -\frac{w}{D}$$

- where the plate rigidity is : $D = \dfrac{Et^3}{12\left(1-\upsilon^2\right)}$ ..(4.5)

- and ν is Poisson's ratio

4.1 ELASTIC ANALYSIS OF TWO-WAY SLABS

- D is comparable to the EI value of a unit width of the slab
- Recall that for linear elements $\rightarrow$ $\dfrac{d^4z}{dx^4} = -\dfrac{w}{EI}$
- Solution of the 4th order PDE for a UDL as solved by Navier's method is:

$$z = \frac{1}{\pi^4 D} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{a_{mn}}{\left(\dfrac{m^2}{L_x^2} + \dfrac{n^2}{L_y^2} \right)} \sin \frac{m\pi x}{L_x} \sin \frac{n\pi y}{L_y}$$

$$where \qquad a_{mn} = \frac{4}{L_x L_y} \int_0^{L_x} \int_0^{L_y} w \sin \frac{m\pi x}{L_x} \sin \frac{n\pi y}{L_y} \, dy \, dx$$

4.1 ELASTIC ANALYSIS OF TWO-WAY SLABS

- So in an elastic plate analysis, Eqn. (4.4) is solved to determine the deflection, z, and the moments are calculated from:

$$m_x = -D\left[\frac{\partial^2 z}{\partial x^2} + \upsilon\left(\frac{\partial^2 z}{\partial y^2}\right)\right]$$

$$m_y = -D\left[\frac{\partial^2 z}{\partial y^2} + \upsilon\left(\frac{\partial^2 z}{\partial x^2}\right)\right] \qquad \text{.....(4.6)}$$

$$m_{xy} = -D(1-\upsilon)\left(\frac{\partial^2 z}{\partial x \partial y}\right)$$

4.1 ELASTIC ANALYSIS OF TWO-WAY SLABS

- Discussion about closed form solution of the governing PDE of elastic plates
- Discussion about non-linear material
- Distribution of moments in slabs (qualitative discussion)

4.2 DISTRIBUTION OF MOMENTS IN SLABS SUPPORTED ON STIFF BEAMS AND WALLS

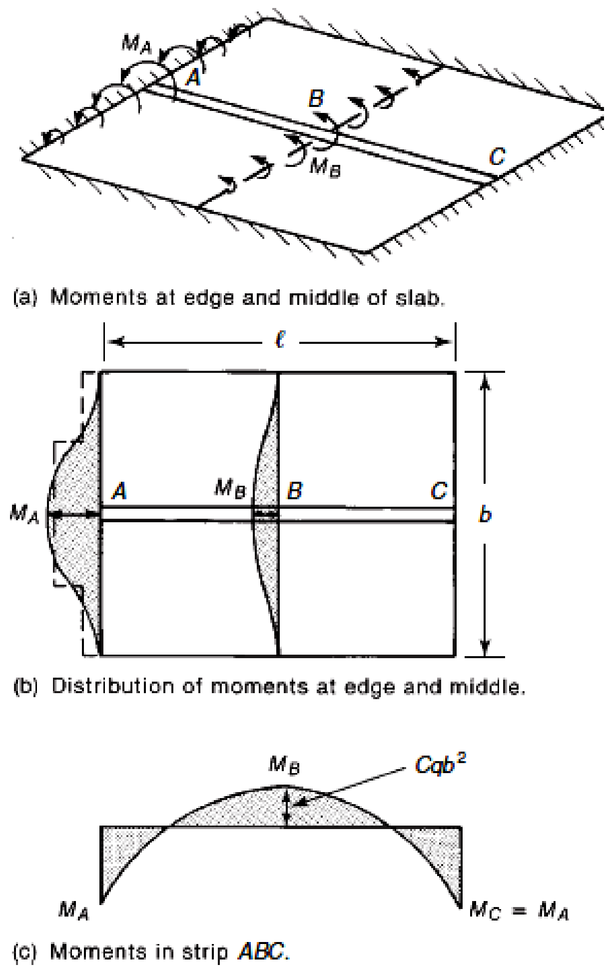


Fig. 13-11
Types of moment diagrams:
four-edged fixed slab.

4.2 DISTRIBUTION OF MOMENTS IN SLABS SUPPORTED ON STIFF BEAMS AND WALLS

- The distributions of moments will be presented in one of two graphical treatments
- The distribution of the negative moments, m_A, or of the positive moments, m_B, along lines across the slab will be depicted as shown in Figure (b)
- These distributions may be shown as continuous curves, as shown by the solid lines and shaded areas, or as a series of steps, as shown by the dashed line.

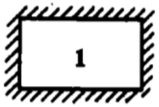
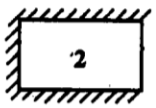
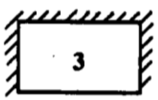
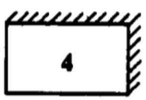
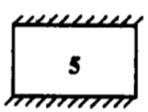
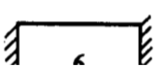
4.2 DISTRIBUTION OF MOMENTS IN SLABS SUPPORTED ON STIFF BEAMS AND WALLS

- The height of the curve at any point indicates the magnitude of the moment at that point
- Discussion why the moments m_A and m_B decrease towards the support
- Occasionally, the distribution of BMs in a strip A-B-C across the slab will be plotted as shown in Figure (c)
- The moments will be expressed in terms of CwL_x^2, where L_x is the short dimension of the panel. The unit is kNm/m

4.3 ANALYSIS OF BEAM/WALL SUPPORTED TWO WAY SLABS ACCORDING TO EBCS-2

$$m_{ij} = \alpha_{ij} w_d L_x^2$$

Table A-1 Bending Moment Coefficients for Rectangular Panels Supported on Four Sides with Provision for Torsion at Corners

Support Condition	Coeff.	Values of L_y/L_x								Long span coefficients, α_{ys} and α_{yf}, for all values of L_y/L_x
		1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	
 1	α_{xs} α_{xf}	0.032 0.024	0.037 0.028	0.042 0.032	0.046 0.035	0.050 0.037	0.053 0.040	0.059 0.044	0.063 0.048	0.032 0.024
 2	α_{xs} α_{xf}	0.039 0.029	0.044 0.033	0.048 0.036	0.052 0.039	0.055 0.041	0.058 0.043	0.063 0.047	0.067 0.050	0.039 0.029
 3	α_{xs} α_{xf}	0.039 0.030	0.049 0.036	0.056 0.042	0.062 0.047	0.068 0.051	0.073 0.055	0.082 0.062	0.089 0.067	0.039 0.030
 4	α_{xs} α_{xf}	0.047 0.036	0.056 0.042	0.063 0.047	0.069 0.051	0.074 0.055	0.078 0.059	0.087 0.065	0.093 0.070	0.047 0.036
 5	α_{xs} α_{xf}	0.046 0.034	0.050 0.038	0.054 0.040	0.057 0.043	0.060 0.045	0.062 0.047	0.067 0.050	0.070 0.053	- 0.034
 6	α_{xs}	-	-	-	-	-	-	-	-	0.045

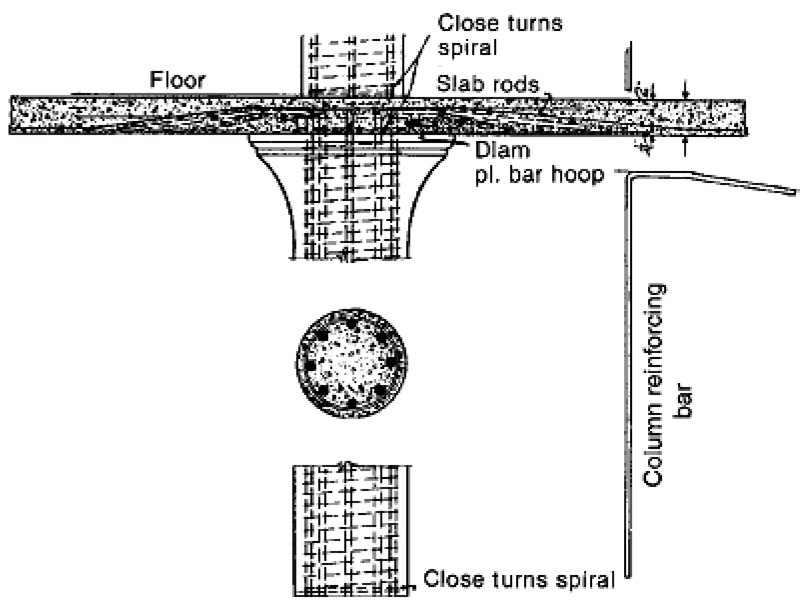
4.4 HISTORY OF TWO-WAY SLABS

One of the most interesting chapters in the development of reinforced concrete structures concerns the two-way slab. Because the mechanics of slab action were not understood when the first slabs were built, a number of patented systems developed alongside a number of semi-empirical design methods. The early American papers on slabs attracted copious and very colorful discussion, each patent holder attempting to prove that his theories were right and that all others were wrong.

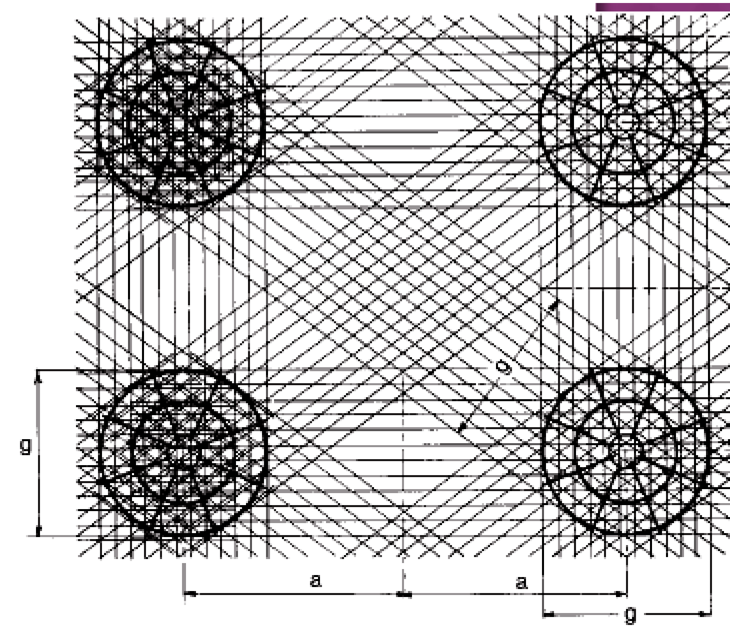
It is not clear who built the first flat slabs. In their excellent review of the history of slabs, Sozen and Siess claim that the first American true flat slab was built by C. A. P. Turner in 1906 in Minneapolis [13-1]. In the same year, Maillart built a flat slab in Switzerland. Turner's slabs were known as mushroom slabs because the columns flared out to join the slab, which had steel running in bands in four directions (i.e., the two orthogonal directions and the diagonals). These bands draped down from the top of the slab over the columns to the bottom of the slab at midspan. Some of the slab bars were bent down into the columns, and other bars were bent into a circle and placed around the columns (Fig. 13-3).

4.4 HISTORY OF TWO-WAY SLABS

The early slab buildings were built at the risk of the designer, who frequently had to put up a bond for several years and often had to load-test the slabs before the owners would accept them. Turner based his designs on analyses carried out by H. T. Eddy, which were based on an incomplete plate-analysis theory. During this period, the use of the crossing-beam analogy in design led to a mistaken feeling that only part of the load had to be carried in each direction, so that statics somehow did not apply to slab construction.



(a) Section through slab and mushroom head.



(b) Plan of reinforcement.

4.4 HISTORY OF TWO-WAY SLABS

In 1914, J. R. Nichols [13-2] used statics to compute the total moment in a slab panel. This analysis forms the basis of slab design in the current ACI Code and is presented later in this chapter. The first sentence of his paper stated "Although statics will not suffice to determine the stresses in a flat slab floor of reinforced concrete, it does impose certain lower limits on these stresses." Eddy [13-3] attacked this concept, saying "The fundamental erroneous assumption of this paper appears in the first sentence . . . " Turner [13-3] thought the paper "to involve the most unique combination of multifarious absurdities imaginable from either a logical, practical or theoretical standpoint." A. W. Buel [13-3] stated that he was "unable to find a single fact in the paper nor even an explanation of facts." Rather, he felt that it was "contradicted by facts." Nichols' analysis suggested that the then current slab designs underestimated the moments by 30 to 50 percent. The emotions expressed by the reviewers appear to be proportional to the amount of under-design in their favorite slab design system.

Although Nichols' analysis is correct and generally was accepted as being correct by the mid-1920s, it was not until 1971 that the ACI Code fully recognized it and required flat slabs to be designed for 100 percent of the moments predicted from statics.

4.5 BEHAVIOR OF SLABS LOADED TO FAILURE IN FLEXURE

- Four or more stages:

i. Before cracking the slab acts as an elastic plate, and for short time loads, the deformations, stresses and strains can be predicted from an elastic analysis.

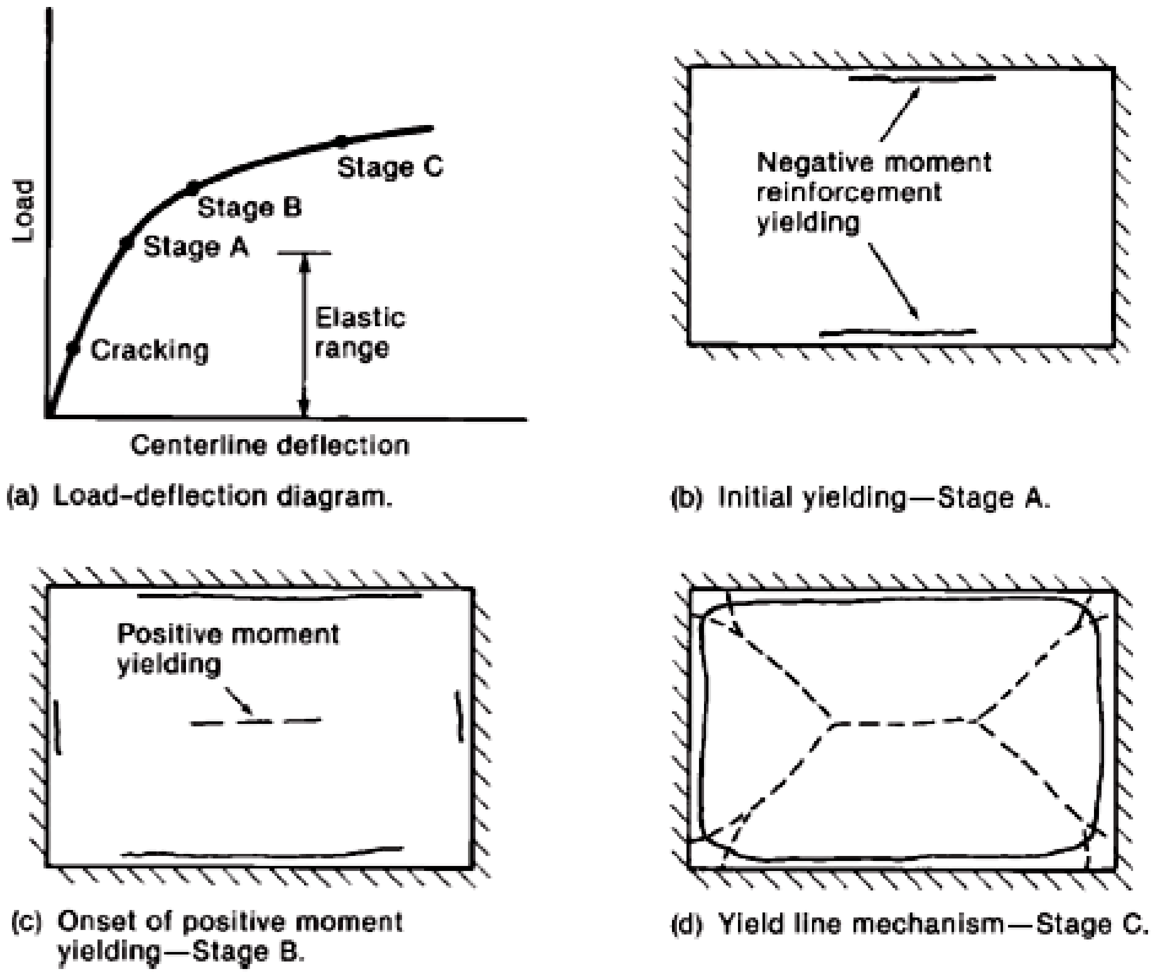
ii. After cracking and before yielding of the reinforcement, the slab no longer has a constant stiffness, because the cracked regions have a lower flexural stiffness EI than the uncracked regions and the slab is no longer isotropic because the crack pattern may differ in the two directions.

4.5 BEHAVIOR OF SLABS LOADED TO FAILURE IN FLEXURE

- Although these conditions violate the assumptions in elastic theory, tests indicate that the elastic theory still predicts the moments adequately. Generally normal building slabs are partially cracked under service loads.

iii. yielding of reinforcement eventually starts in one or more region of high moment and spreads through the slab as the moments are redistributed from yielded regions to areas that are still elastic. The progression of yielding through a slab fixed on four edges is illustrated in Figure (next slide)

4.5 BEHAVIOR OF SLABS LOADED TO FAILURE IN FLEXURE

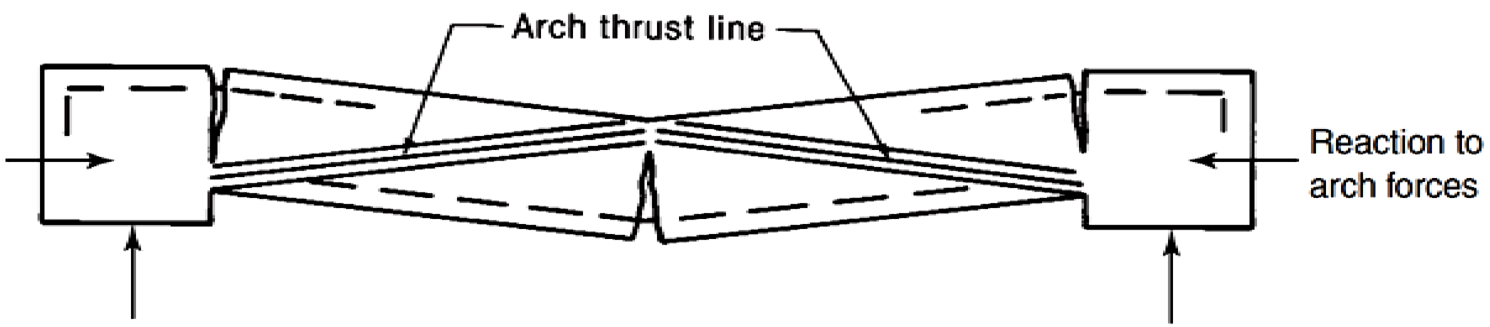


4.5 BEHAVIOR OF SLABS LOADED TO FAILURE IN FLEXURE

- With further load, the regions of yielding known as yield lines, divide the slab into a series of trapezoidal and triangular elastic plates as shown in Figure (d) above. The loads corresponding to this stage of loading can be estimated by using yield-line analysis (plastic method analysis)

iv. Although the yield lines divide the slab to form a mechanism, the hinges jam with increased deformation, and the slab forms a very flat compression arch as shown in Figure (next slide)(avail stiff support). This stage of loading usually is not considered in design

4.5 BEHAVIOR OF SLABS LOADED TO FAILURE IN FLEXURE



4.5 ANALYSIS OF MOMENTS IN TWO-WAY SLABS

- Figure (next slide) shows a floor made of simply supported planks supported by simply supported beams. The floor carries a load of q kN/m^2.
- The moment per meter width in the planks at section A-A is: $m = ql_1^2/8$ kNm/m
- The total moment in the entire width of the floor is: $M_{A\text{-}A} = (ql_2)l_1^2/8$ kNm
- This is the familiar equation for the maximum moment in a simply supported floor of width l_2 and span l_1.

4.5 ANALYSIS OF MOMENTS IN TWO-WAY SLABS

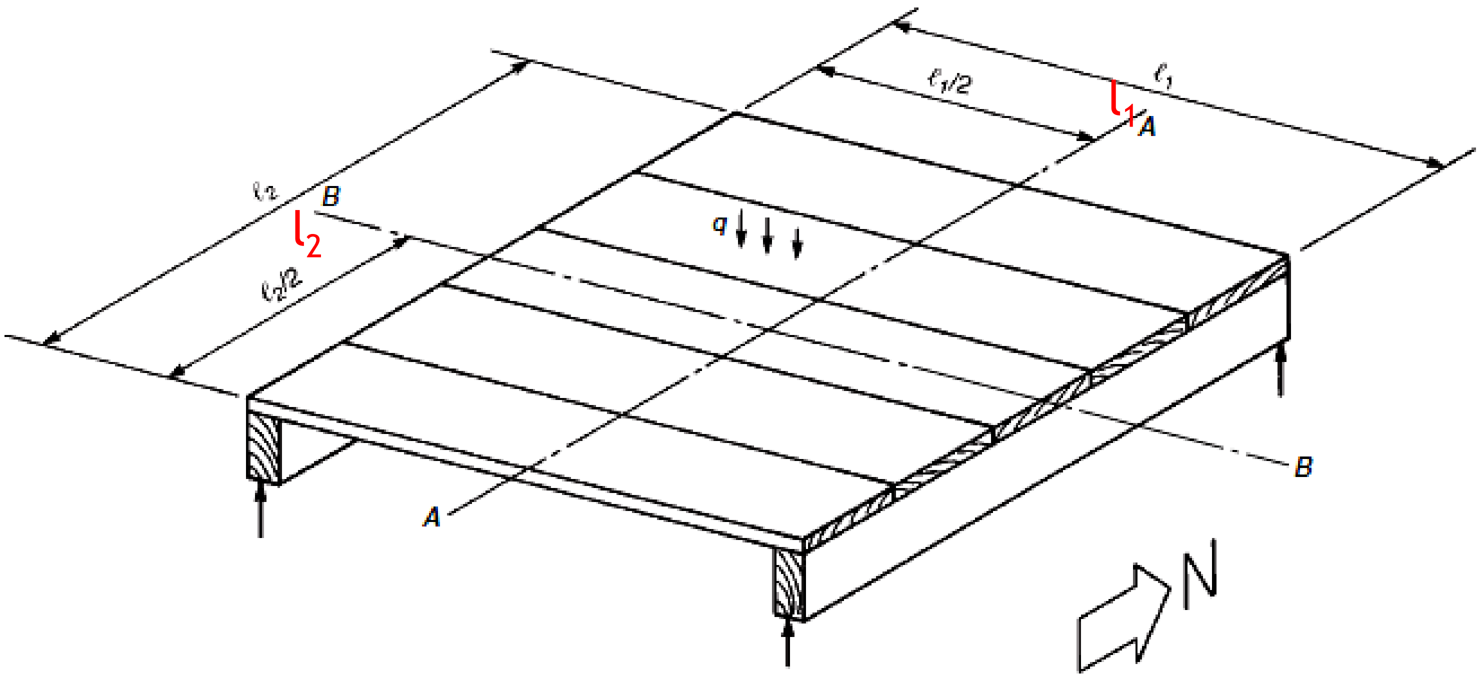


Fig. 13-6
Moments in a plank-and-beam floor.

4.5 ANALYSIS OF MOMENTS IN TWO-WAY SLABS

- The planks apply a uniform load of $ql_1/2$ kN/m on each beam.
- The moment at section B-B in one beam is thus: $M_{1b} = (ql_1/2)l_2^2/8$ kNm/m
- The total moment in both beams is: $M_{B\text{-}B} = (ql_1)l_2^2/8$
- It is important to note that the full load was transferred east and west by the planks, causing a moment equivalent to $wl_1^2/8$ in the planks where $w = ql_2$. Then the full load was transferred north and south by the beams, causing a similar moment in the beams.

4.5 ANALYSIS OF MOMENTS IN TWO-WAY SLABS

- Exactly the same thing happens in the two way slab shown in Figure (next slide).
- The total moments required along sections A-A and B-B are: $M_{A\text{-}A} = (ql_2)l_1^2/8$ and $M_{B\text{-}B} = (ql_1)l_2^2/8$
- Again, the full load was transferred east and west and then the full load was transferred north and south- this time by the slab in both cases.
- This, of course always must be true regardless of whether the structure has one-way slabs and beams, two-way slabs or some other system

4.5 ANALYSIS OF MOMENTS IN TWO-WAY SLABS

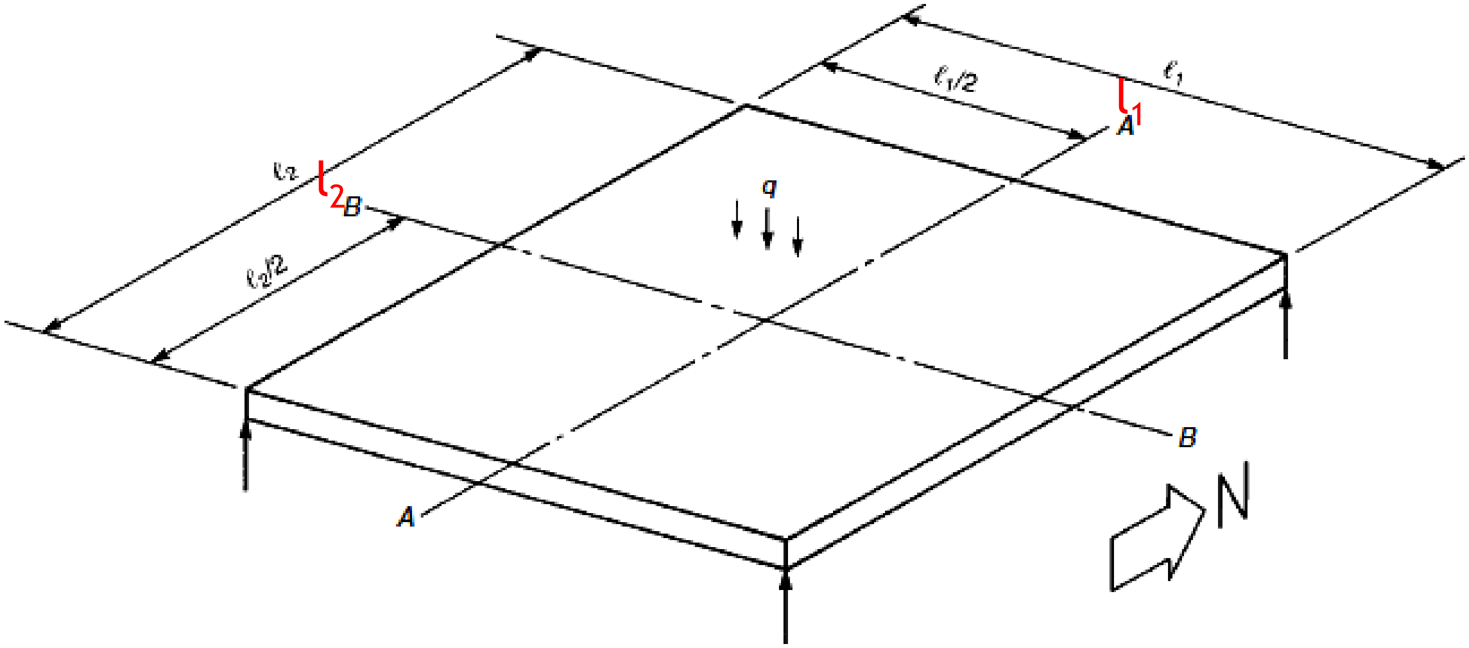
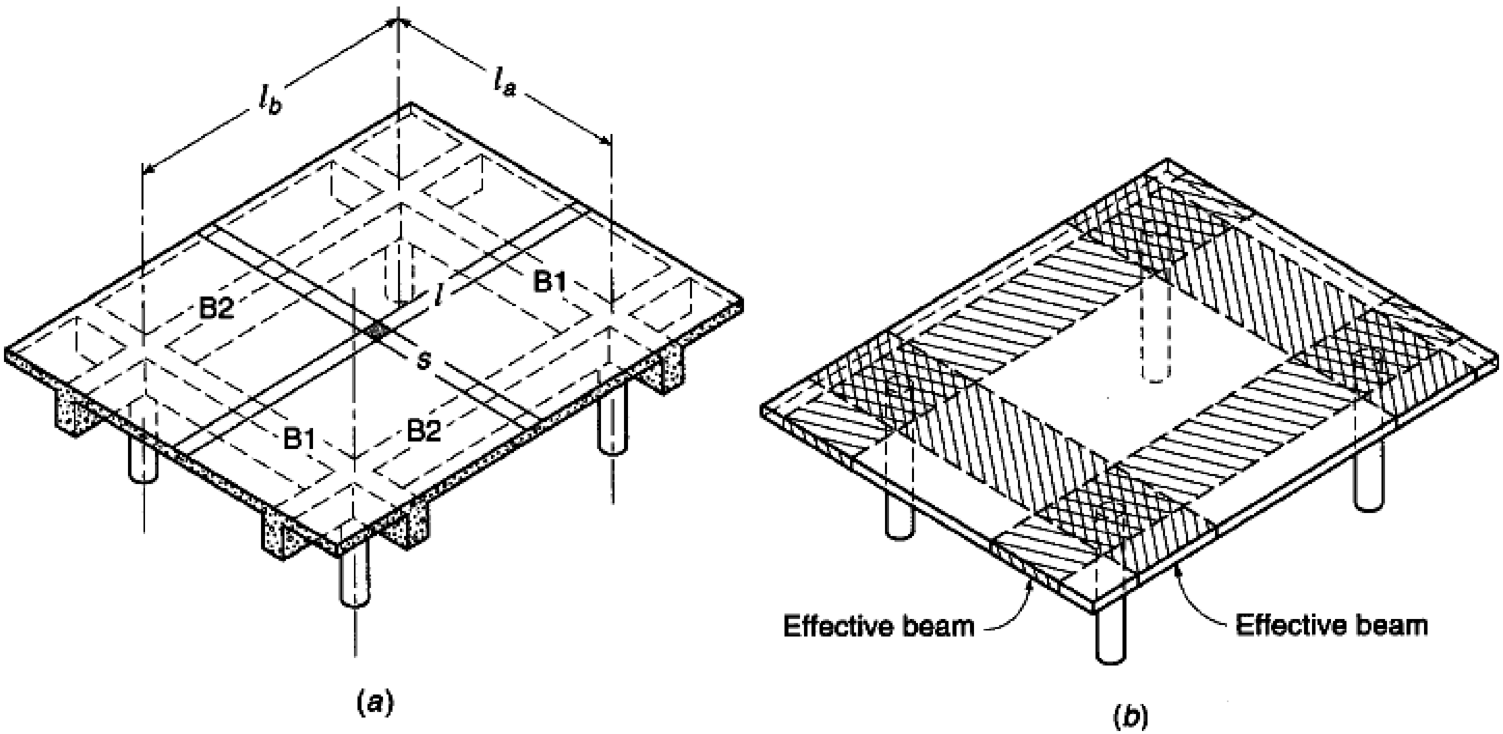


Fig. 13-7
Moments in a two-way slab.

4.5 ANALYSIS OF MOMENTS IN TWO-WAY SLABS

- To emphasize load transfer mechanism in two way slabs using the column supported two-way slabs in Figure (next slide)
- If a surface load is applied, it is shared between imaginary slab strips l_a in the short direction and l_b in the longer direction.
- Note that the portion of the load that is carried by the long strips l_b is delivered to the beams B_1, which in turn carries it in the short direction. That portion of the load plus that directly carried in the short direction by the slab strips l_a, sum up to 100% of the load applied to the panel. The same is true in the other direction

4.5 ANALYSIS OF MOMENTS IN TWO-WAY SLABS



4.5 ANALYSIS OF MOMENTS IN TWO-WAY SLABS

- A similar situation is obtained in the flat plate floor where broad strips of the slab centered on the column lines in each direction serve the same function as the beams
- Therefore, for column supported construction (one-way or two-way), 100% of the applied load must be carried in each direction, in the case of two-way beam supported slabs, jointly by the slab and its supporting beams

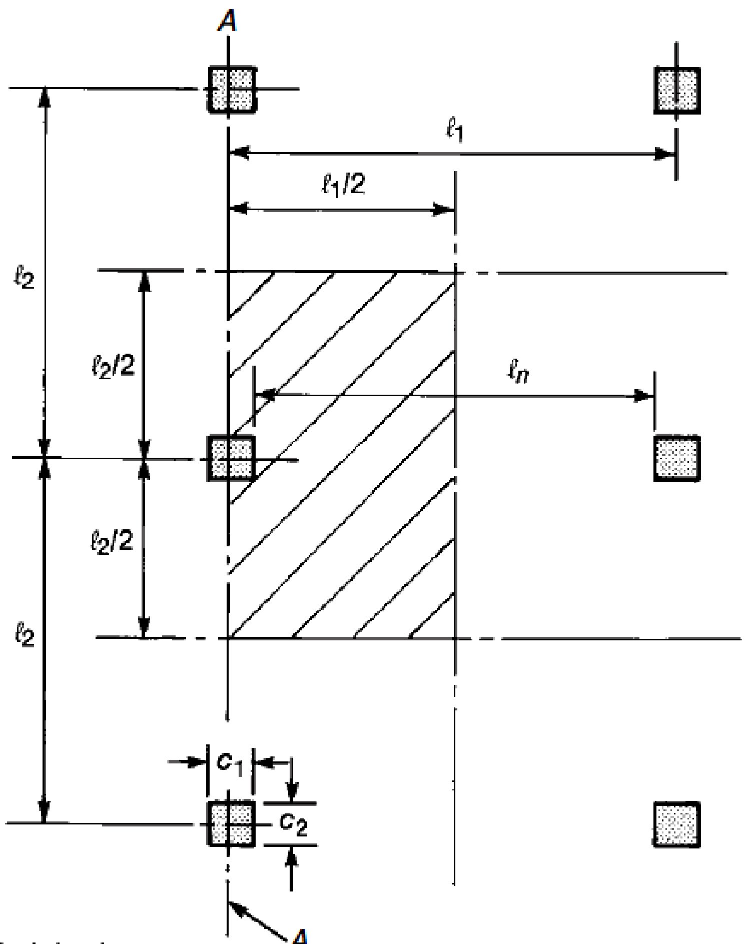
4.5.1 NICHOL'S ANALYSIS OF MOMENTS IN SLABS

- The analysis used to derive the moments in two way slabs was 1st published by Nichol in 1914.
- The derivation using rectangular columns (instead of the original circular columns by Nichol) will be shown.
- Assume : (1) A typical rectangular, interior panel in a large structure and (2) that all the panels in the structure are uniformly loaded with the same load.

4.5.1 NICHOL'S ANALYSIS OF MOMENTS IN SLABS

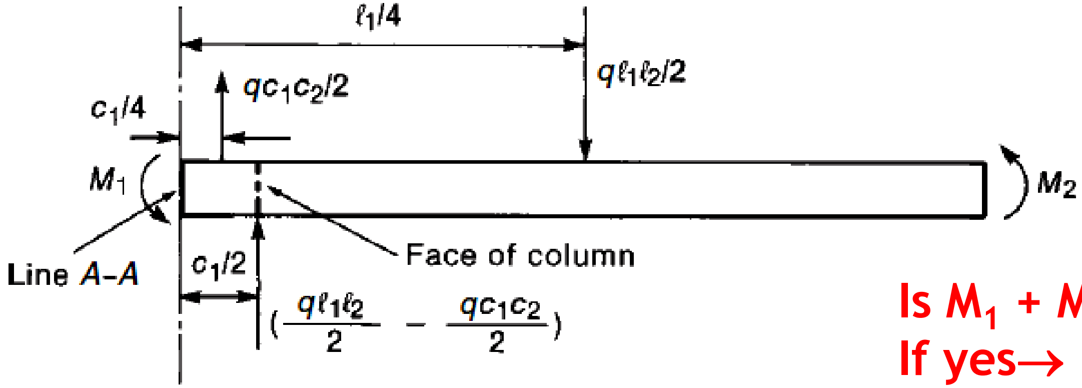
- The two assumptions ensure that the lines of maximum moment, and hence the lines on which the shears and twisting moments are equal to zero, will be lines of symmetry in the structure.
- This allows one to isolate the portion of the slab shown shaded in Figure (next slide). This portion is bounded by lines of symmetry located at the center of panels on three sides and along column axis on the fourth side. Shears and twisting moments are zero on these sections

4.5.1 NICHOL'S ANALYSIS OF MOMENTS IN SLABS

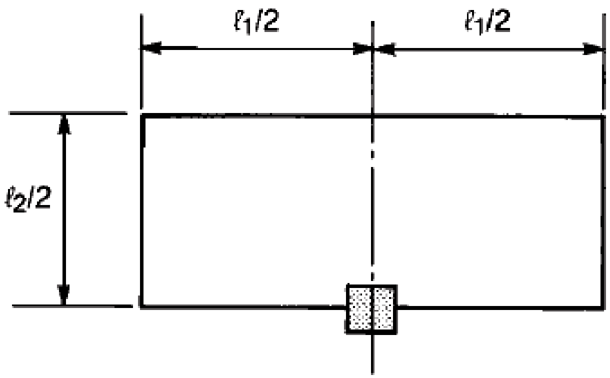


(a) Plan of slab element.

4.5.1 NICHOL'S ANALYSIS OF MOMENTS IN SLABS



(b) Side view of slab element.



(c) Plan of second slab element.

Is $M_1 + M_2 = wl_2l_1^2/8$?
If yes→ 100% of the loading is carried in the l_1 direction.
Similarly in the l_2 direction

4.5.1 NICHOL'S ANALYSIS OF MOMENTS IN SLABS

- The reactions to the vertical loads are transmitted to the slab by shear around the face of the columns. It is necessary to know, or assume, the distribution of this shear to compute the moments in this slab panel
- The maximum shear transfer occurs at the corners of the column, with lesser amounts transferred in the middle of the sides of the column. For this reason we shall assume that (3) the column reactions are concentrated at the four corners of each column

4.5.1 NICHOL'S ANALYSIS OF MOMENTS IN SLABS

- Figure (b) shows a FBD, a side view of the slab element with the forces and moments acting on it
- The applied load is $(wl_1l_2/2)$ at the center of the shaded panel, minus the load on the area occupied by the column $(wc_1c_2/2)$ ($\therefore$) shown upward in the FBD is equilibrated by the upward reaction at the corners of the columns $(wl_1l_2/2 - wc_1c_2/2)$.
- The total statical moment, M_o, is the sum of the negative moment, M_1, and the positive moment, M_2.

4.5.1 NICHOL'S ANALYSIS OF MOMENTS IN SLABS

- The magnitude of M_o may be obtained by summing moments about axis A-A. $\rightarrow$

$$M_o = M_1 + M_2 = \left(\frac{wl_1l_2}{2}\right)\frac{l_1}{4} - \left(\frac{wc_1c_2}{2}\right)\frac{c_1}{4}$$

$$-\left(\frac{wl_1l_2}{2} - \frac{wc_1c_2}{2}\right)\frac{c_1}{2}$$

  - NB: 1st term from slab load, 2nd term from -ve load on column, 3rd term from reaction at edges of column. After simplifications $\rightarrow$

4.5.1 NICHOL'S ANALYSIS OF MOMENTS IN SLABS

$$M_o = \left(\frac{wl_2}{8}\right)\left[l_1^2\left(1-2\frac{c_1}{l_1}+\frac{c_2}{l_2}\left(\frac{c_1^2}{l_1^2}\right)\right)\right]$$

- Note that this is almost equal to $\frac{wl_2 l_1^2}{8} \rightarrow$ full load is carried in the design direction l_1 by a strip width equal to the width of the panel, i.e. l_2

4.5.1 NICHOL'S ANALYSIS OF MOMENTS IN SLABS

- The ACI Code has simplified this expression slightly by replacing the term in the square bracket with l_n^2, where l_n is the clear span between the faces of the columns, given by

  $$l_n = l_1 - c_1 \quad \text{because} \quad l_n^2 = l_1^2\left(1 - 2\frac{c_1}{l_1} + \frac{c_1^2}{l_1^2}\right)$$

  differs only slightly from the terms in the square bracket
- The statical moment $M_o = wl_2l_n^2/8$ (ACI) (A)

4.5.1 NICHOL'S ANALYSIS OF MOMENTS IN SLABS

- If the equilibrium of the element shown in Figure (c) were studied, a similar equation for M_o would result, but one having l_1 and l_2 interchanged and c_1 and c_2 interchanged
- This indicates once again that the slab in flat plates and the slabs and supporting beams in beam supported two-way slabs must be good for 100% of the loading in both directions.
- Analysis of moments according to the ACI is a unified approach that is applicable to both flat slabs and beam-supported two-way slabs

4.6 DISTRIBUTION OF MOMENTS IN SLABS SUPPORTED BY COLUMNS

- In a plate the slab is supported directly on the columns w/o any beams. Here the stiffest portions of the slab are those running from column to column along the four sides of a panel. As a result, the moments are largest in these parts of the slab.
- (Go to s.55)
- Figure (next slide) illustrates the moments in a typical interior panel of a very large slab in which all panels are uniformly loaded with equal loads. The slab is supported on circular columns with a diameter $c = 0.1l$

4.6 DISTRIBUTION OF MOMENTS IN SLABS SUPPORTED BY COLUMNS

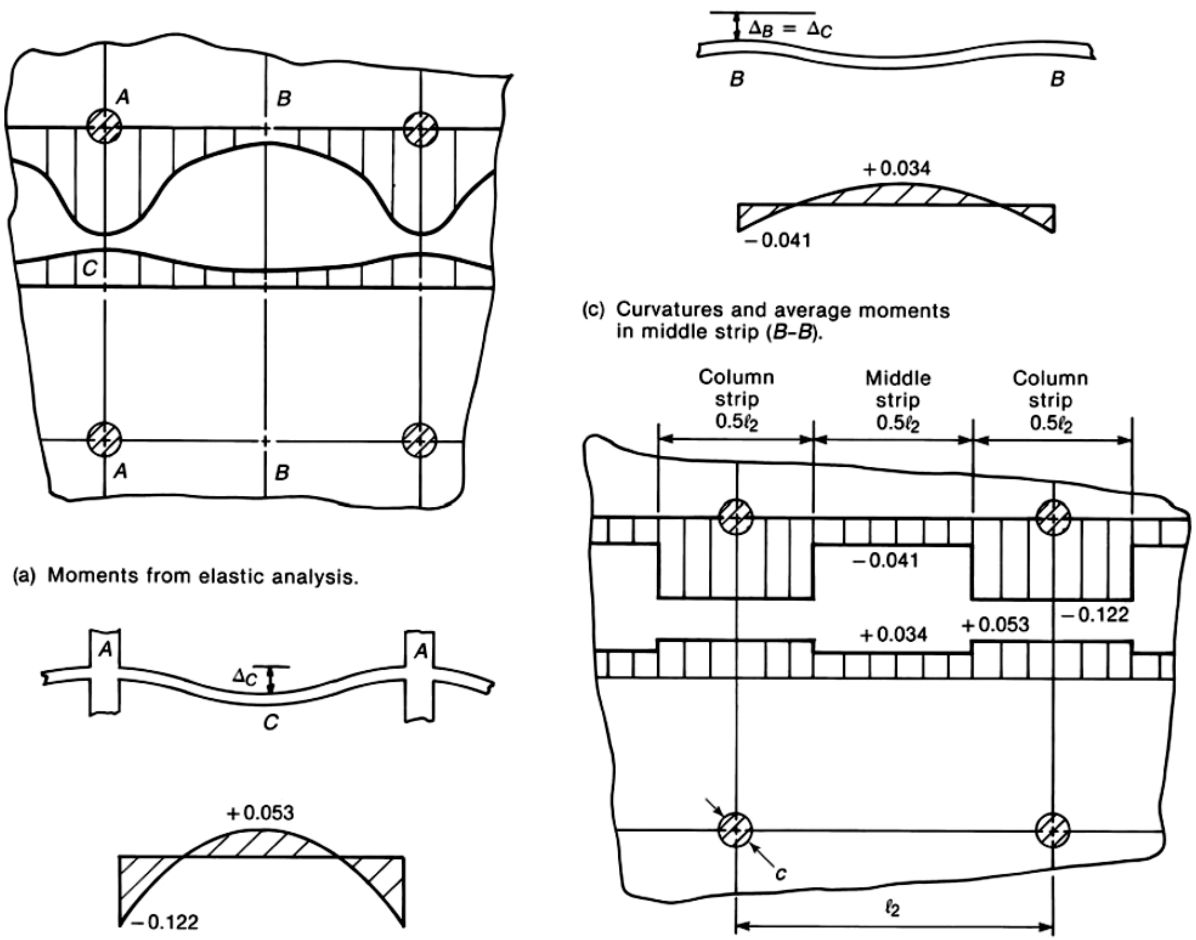


Fig 4.5.1

(a) Moments from elastic analysis.

(b) Curvatures and average moments in column strip (A–A).

(c) Curvatures and average moments in middle strip (B–B).

(d) Elastic moments averaged over strips.

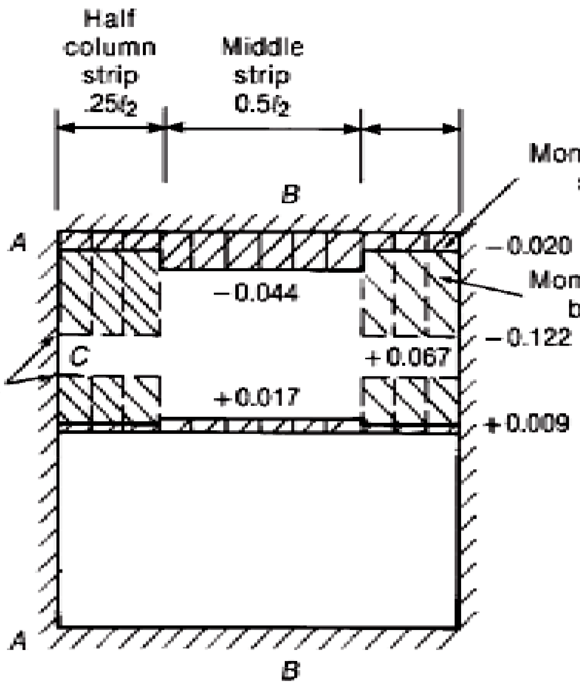
4.6 DISTRIBUTION OF MOMENTS IN SLABS SUPPORTED BY COLUMNS

- The largest negative and positive moments occur in the strips spanning from column to column in Figures 4.5.1(b) and 4.5.1(c).
- The curvatures and moment diagrams are shown for strips along lines A-A and B-B.
- Both strips have –ve moments adjacent to the columns and +ve moments at mid-span.
- In Figure 4.5.1(d) the moment diagram from 4.5.1(a) is re-plotted to show the average moments over the width of the middle and column strips

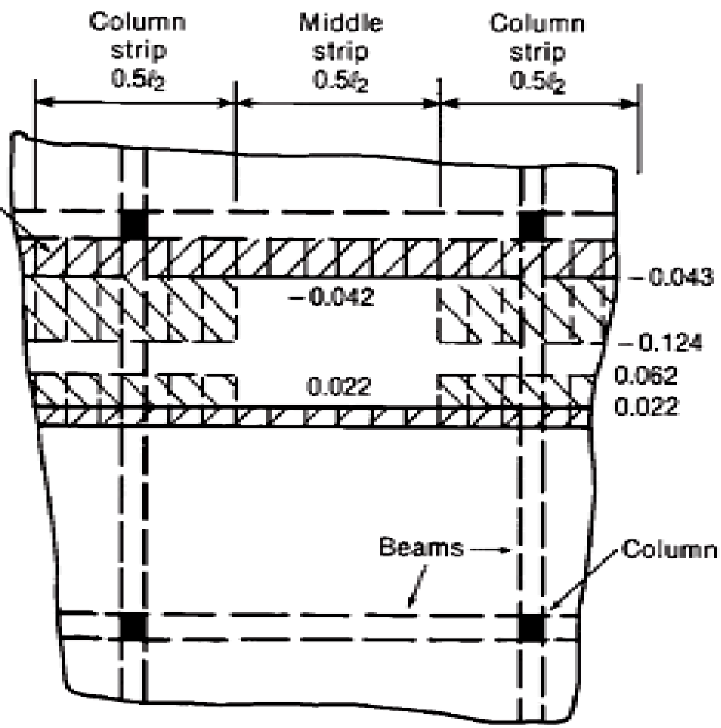
4.6 DISTRIBUTION OF MOMENTS IN SLABS SUPPORTED BY COLUMNS

- The total static moment, M_o, accounted for here is (NB: *Factor*$\times ql_n^2$ *gives moment per meter width)*
- $M_o = ql_n^2[(0.122\times0.5l_2) + (0.041\times0.5l_2) + (0.053\times0.5l_2) + (0.034\times0.5l_2)] = 0.125ql_2l_n^2$
- The distribution of moments given in Figure (next slide) for a square slab supported on rigid beams is shown in (a) with the moments averaged over column-strip and middle-strip bands in the same way as the flat-plate moments shown earlier

4.6 DISTRIBUTION OF MOMENTS IN SLABS SUPPORTED BY COLUMNS



(a) Four edges fixed and supported on nondeflecting edges.



(b) Four edges supported on flexible beams.

Fig 4.5.2

4.6 DISTRIBUTION OF MOMENTS IN SLABS SUPPORTED BY COLUMNS

- In addition, the sum of the beam moments and the column-strip slab moments has been divided by the width of the column strip and plotted as the total column-strip moment.
- The distribution of moments in Figure 4.5.1(d) of the flat plates closely resembles the distribution of middle-strip and total column-strip moments in Figure 4.5.2 (a).
- An intermediate case in which the beam stiffness, I_b, equal the stiffness, I_s, of a slab of width, l_2, is shown in Figure 4.5.2 (b).

4.6 DISTRIBUTION OF MOMENTS IN SLABS SUPPORTED BY COLUMNS

- Although the division of moments b/n slab and beams differs, the distribution of the total moments is again similar to that shown in Figures (d) and (a)
- The slab design procedures in the ACI Code take advantage of this similarity in the distributions of the total moments by presenting a unified design procedure for the whole spectrum of slab and edge-beam stiffness from slabs supported on isolated columns to slabs supported on stiff beams in two directions

4.7 DESIGN OF SLABS

- Two slab design procedures are allowed by the ACI (EBCS EN 1992-1-1). These are the direct design method and the equivalent frame design method. The two methods differ primarily in the way in which the slab moments are computed.
- The calculation of the moments in the direct design method is based on the statical moment M_o. **($M_o = wl_2l_n^2/8$** (ACI) (A))
- In this method, the slab is considered panel by panel, and Eq. (A) is used to compute the total moment in each panel

4.7 DESIGN OF SLABS

- The statical moment is then divided up b/n positive and negative moments, and these are divided b/n middle strip and column strips.
- In the equivalent frame method, the slab is divided into a series of two-dimensional frames, and the positive and negative moments are computed via an elastic frame analysis. Once the +ve and –ve moments are known, they are divided up b/n middle strips and column strips in exactly the same way as in the direct design methods.

4.7.1 BEAM-TO-SLAB STIFFNESS RATIO, α_f

- Slabs are frequently built with beams from column to column around the perimeter of the building. These beams act to stiffen the edge of the slab and help to reduce the deflections of the exterior panels of the slabs. (Very heavily loaded slabs and long-span waffle slabs sometimes have beams joining all columns in the structure)

4.7.1 BEAM-TO-SLAB STIFFNESS RATIO, α_f

- The effects of beam stiffness on deflections and the distribution of moments are expressed as a function of α_f, defined as the flexural stiffness, 4EI/l, of the beam divided by the flexural stiffness of a width of slab bounded by the centerlines of the adjacent panels on each side of the beam.
- $\alpha_f = (4E_{cb}I_b/l)/(4E_{cs}I_s/l)$
- Since the length, l, of the beam and the slab are equal, this quantity is simplified and expressed in the Code (ACI) as:
- $\alpha_f = (E_{cb}I_b)/(E_{cs}I_s)$

4.7.1 BEAM-TO-SLAB STIFFNESS RATIO, α_f

- If there is no beam, α_f = 0. (mostly the case except at the edges where beams are provided for stiffening edge panels)
- The sections considered in computing I_b and I_s are shown in Figure (next slide). (NB. Span direction is l_1)
- ACI, Section 14.2.4 defines a beam in monolithic or fully composite construction as the beam stem plus a portion of the slab on each side of the beam extending a distance equal to the projection of the beam above or below the slab whichever is greater , but not greater than four times the slab thickness (next next slide).

4.7.1 BEAM-TO-SLAB STIFFNESS RATIO, α_f

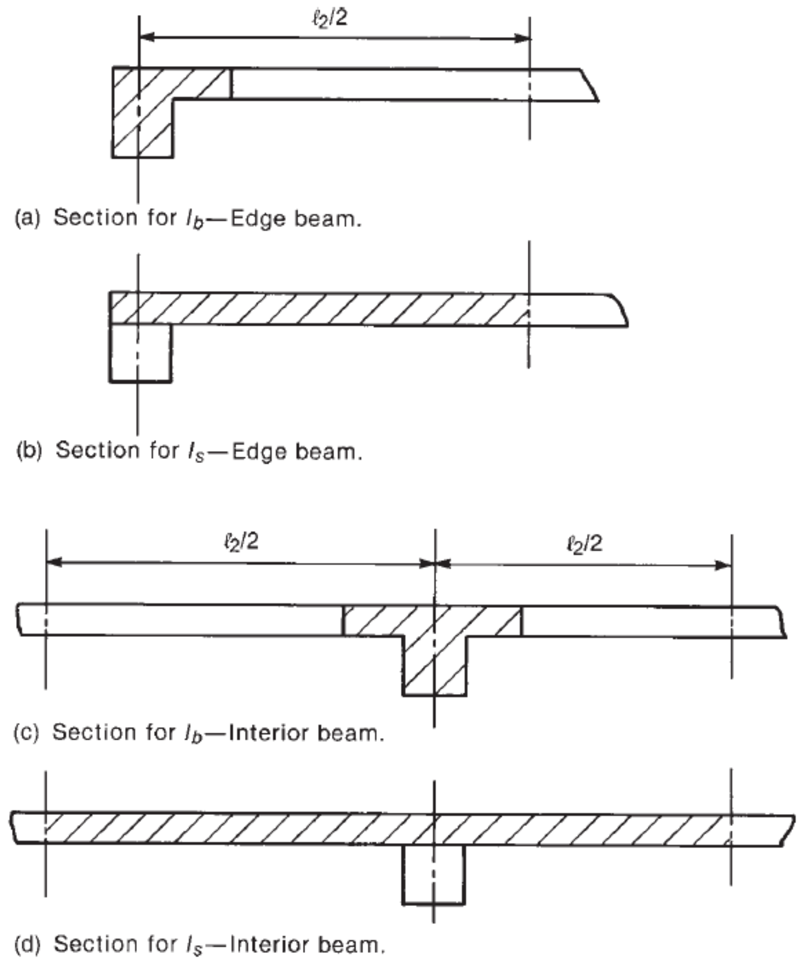


Fig. Beam and slab sections for calculations of α_f

Fig. 13-17
Beam and slab sections for calculations of α_f.

4.7.1 BEAM-TO-SLAB STIFFNESS RATIO, α_f

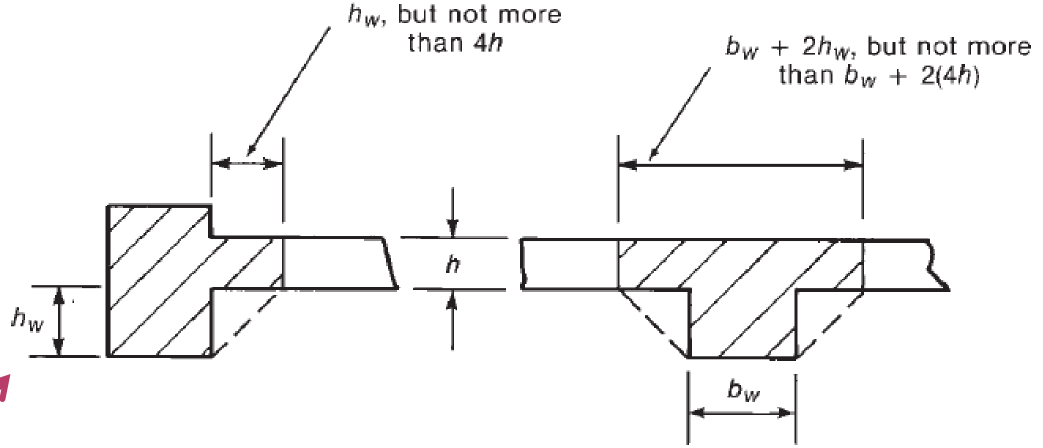


Fig. 13-18
Cross section of beams as defined in ACI Code Section 13.2.4.

Governing projection

4.7.1 BEAM-TO-SLAB STIFFNESS RATIO, α_f

- Example: Calculation of α for an edge beam
- Go to S.66
- A 200 mm-thick slab is provided with an edge beam that has a total depth of 400 mm and a width of 300 mm as shown in Figure (next slide). The slab and beam were cast monolithically and have the same concrete strength and the same E_c. Compute α_f.

4.7.1 BEAM-TO-SLAB STIFFNESS RATIO, α_f

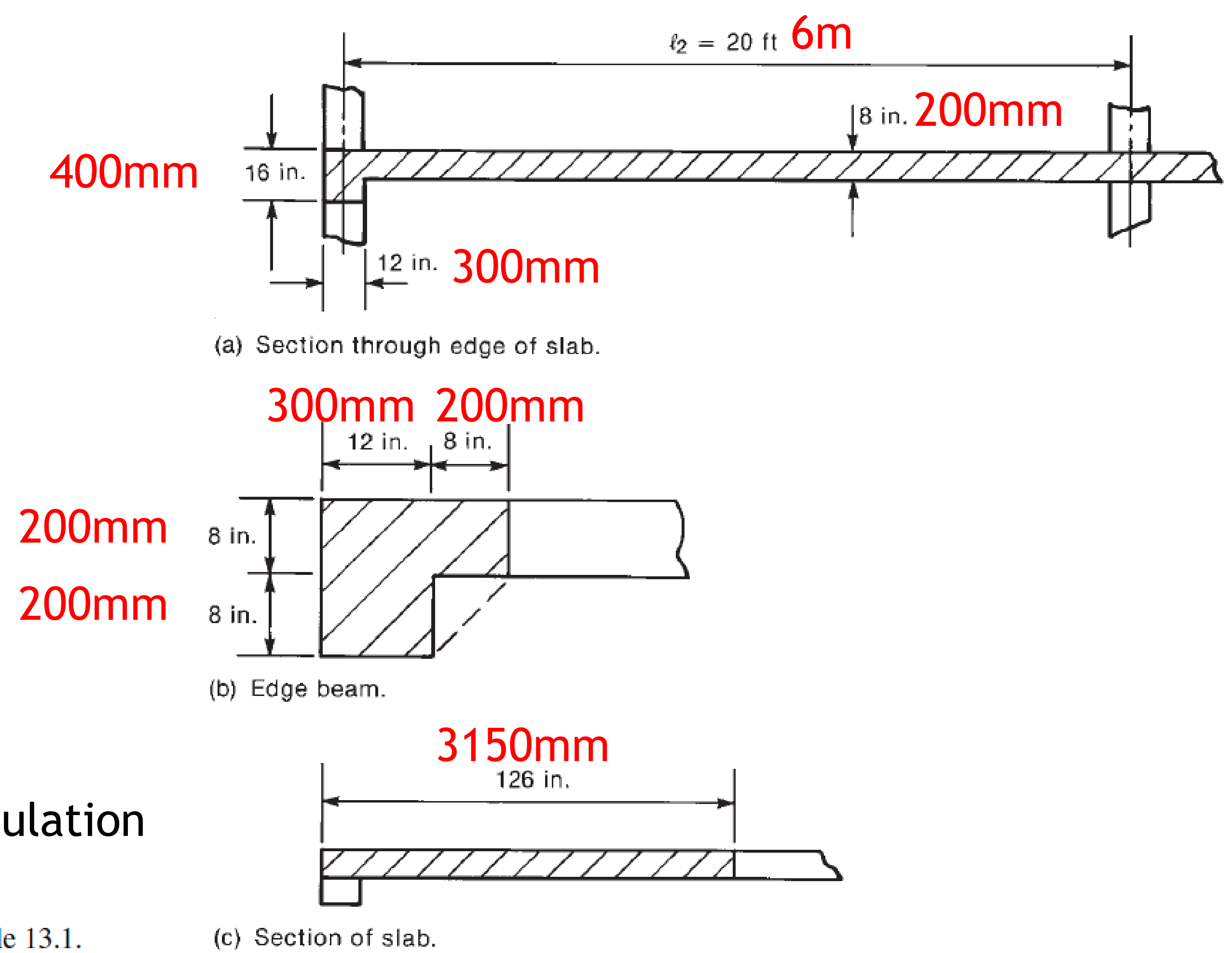


Fig: α_f calculation

Fig. 13-19
Slab for Example 13.1.

4.7.1 BEAM-TO-SLAB STIFFNESS RATIO, α_f

- Solution:
- $\alpha_f = I_b/I_s$
- (1) Compute I_b: The cross section of the beam is as shown in Figure (slide above). The centroid of the beam is located 175 mm from the top of the slab. $\rightarrow$ moment of inertia of the beam is: $I_b = (300\times400^3/12) + (300\times400)\times25^2 + (200\times200^3)/12) + (200\times200)\times75^2 = 2.0333\times10^9$ mm^4
- (2) Compute I_s: $I_s = 3150\times200^3/12 = 2.1\times10^9$ mm^4
- (3) compute $\alpha_f = 2.0333\times10^9/2.1\times10^9 = 0.968$

4.7.2 MINIMUM THICKNESSES OF TWO-WAY SLABS

- ACI code defines minimum thicknesses that are generally sufficient to limit slab deflections to acceptable values (same as in EBCS-2). Thinner slabs can be used if it can be shown that the computed slab deflections will not be excessive.
- Slabs without beams between interior columns……. (SI Version)
- Slabs with beams between the interior supports…….. (SI Version)

4.8 DIRECT DESIGN METHOD

The direct-design method also could have been called “the direct-analysis method,” because this method essentially prescribes values for moments in various parts of the slab panel without the need for a structural analysis. The reader should be aware that this design method was introduced in an era when most engineering calculations were made with a slide rule and computer software was not available to do the repetitive calculations required to analyze a continuous-floor slab system. Thus, for continuous slab panels with relatively uniform lengths and subjected to distributed loading, a series of moment coefficients were developed that would lead to safe flexural designs of two-way floor systems.

4.8.1 LIMITATIONS ON THE USE OF THE DIRECT DESIGN METHOD

- **Limitations on the use of the DDM**

1) there must be a minimum of 3 continuous spans in each direction. Thus a nine-panel structure (3 by 3) is the smallest that can be divided.
2) rectangular panels must have a long-span/short-span ratio not greater than 2. one-way action predominates as the span ratio reaches and exceeds 2
3) successive span lengths in each direction shall not differ by more than one-third of the longer span

4.8.1 LIMITATIONS ON THE USE OF THE DIRECT DESIGN METHOD

4) columns may be offset from the basic rectangular grid of the building by up to 0.1 times the span parallel to the offset
5) all loads must be due to gravity only. The direct design method can not be used for unbraced laterally loaded frames, foundation mats, or prestressed slabs.
6) the service live load shall not exceed two times the service dead load.
7) for a panel with beams b/n supports on all sides, the relative stiffness of the beams in the two $\perp$ directions given by $(\alpha_{f1} l_2^2)/(\alpha_{f2} l_1^2)$ shall not be less than 0.2 or greater than 5. (α is the beam-to-slab stiffness ratio defined earlier

4.8.2 DISTRIBUTION OF MOMENTS WITHIN PANELS-SLABS W/O BEAMS B/N ALL SUPPORTS

- For design, the slab is considered to be a series of frames in the two directions, as shown in Figure (next slide). These frames extend to the middle of the panels on each side of the column
- In each span of each of the frames, it is necessary to compute the total statical moment M_o: $M_o = q_u l_2 l_n^2/8$; where q_u = factored load; l_2 = transverse width of the strip; l_n = clear span between columns

4.8.2 DISTRIBUTION OF MOMENTS WITHIN PANELS-SLABS W/O BEAMS B/N ALL SUPPORTS

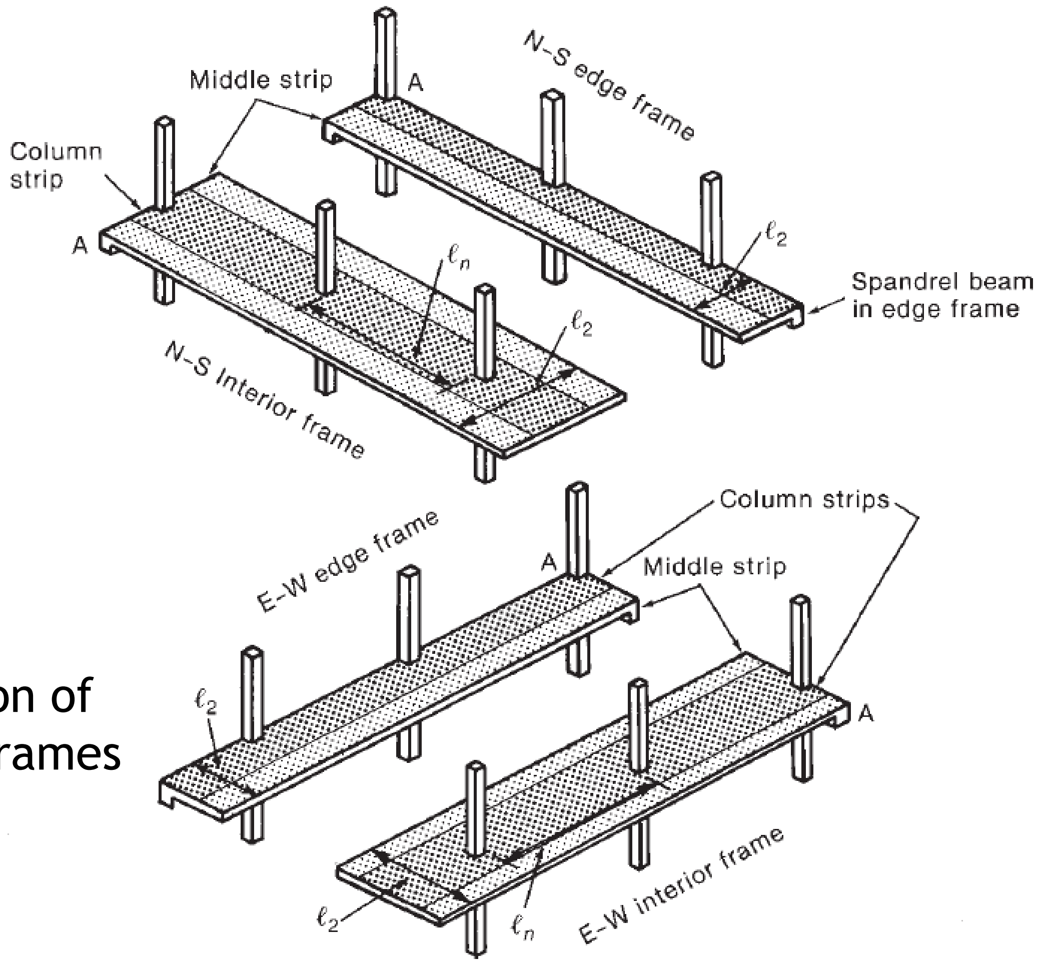
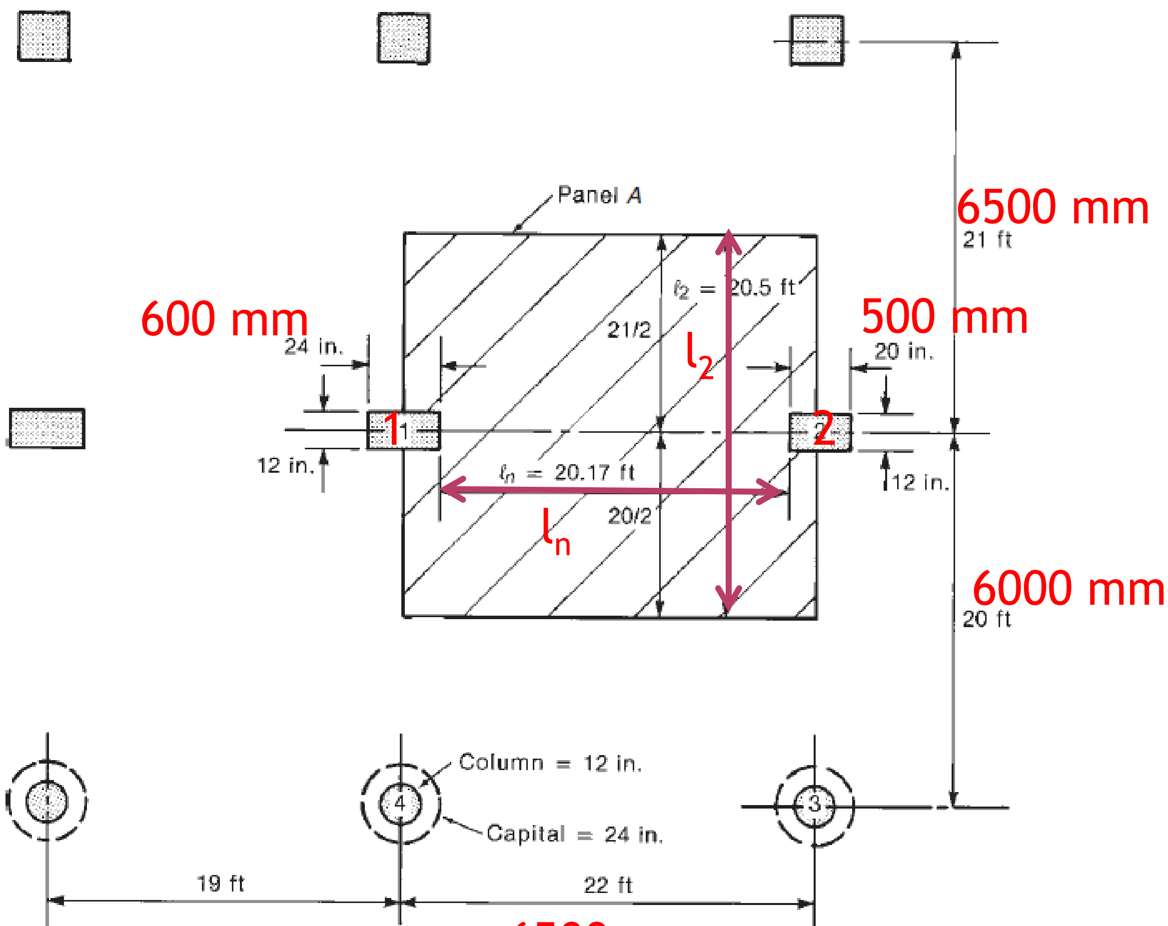


Fig: Division of slab into frames for design

4.8.2 DISTRIBUTION OF MOMENTS WITHIN PANELS-SLABS W/O BEAMS B/N ALL SUPPORTS

- Example: Compute the statical moment, M_o, in the slab panels in Figure (next 2 slides). The slab is 200 mm thick and supports a live load of 4.53 kN/m^2
- **Sol:** (1) Compute the design load: q_d = 1.3×0.2×25 + 1.6×4.54 = 14.76 kN/m^2
- (2) Consider panel A spanning from column 1 to column 2. Slab panel A is shown shaded in Figure (next slide). The moments computed here would be used to design the reinforcement parallel to lines 1-2 in this panel

4.8.2 DISTRIBUTION OF MOMENTS WITHIN PANELS-SLABS W/O BEAMS B/N ALL SUPPORTS

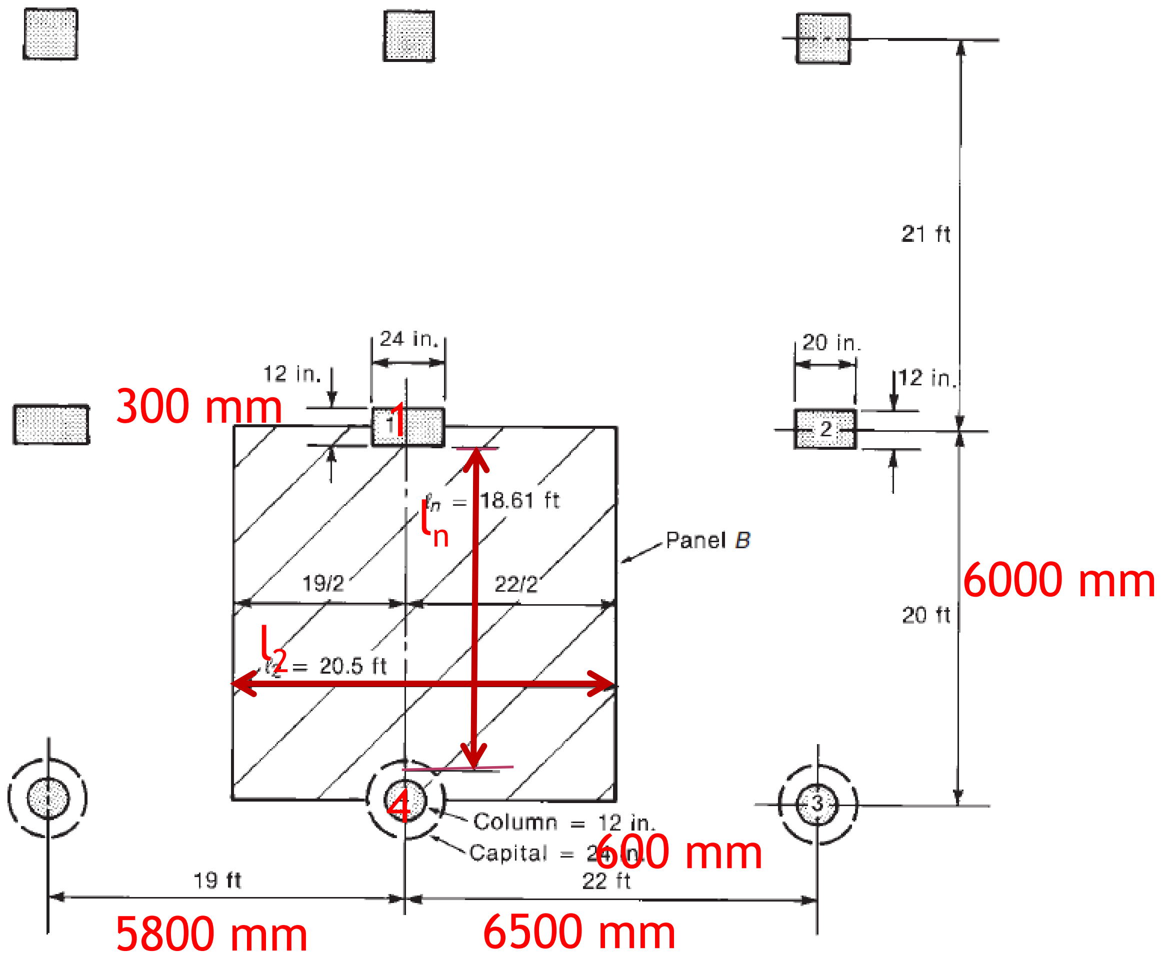


(a) Slab panel A.

4.8.2 DISTRIBUTION OF MOMENTS WITHIN PANELS-SLABS W/O BEAMS B/N ALL SUPPORTS

- Now $M_o = (q_d l_2) l_n^2/8$; where l_n = clear span of slab panel = 6.5-1/2(0.5)-1/2(0.6) = 5.95m ; l_2 = width of panel = 6.5/2 + 6.0/2 = 6.25m $\rightarrow M_o = (14.76 \times 6.25 \times 5.95^2)/8 =$ **381 kNm**
- Consider panel B, spanning from column 1 to column 4 (next slide). The moments computed here would be used to design the reinforcement parallel to lines 1-4 in this panel. For the purpose of computing l_n, the circular supports are replaced by equivalent square columns having a side length $c_1 = 0.886 d_c$.

4.8.2 DISTRIBUTION OF MOMENTS WITHIN PANELS-SLABS W/O BEAMS B/N ALL SUPPORTS



(b) Slab panel *B*.

4.8.2 DISTRIBUTION OF MOMENTS WITHIN PANELS-SLABS W/O BEAMS B/N ALL SUPPORTS

- $\rightarrow l_n = 6.0\text{-}1/2(0.3)\text{-}1/2(0.886 \times 0.6) = 5.59\text{m}$; $l_2 = 5.8/2 + 6.5/2 = 6.15\text{m}$; $M_o = (14.76 \times 6.15 \times 5.59^2)/8 =$ **331 kNm**
- Now the total statical moment will be divided between the negative and positive sections of the panel

4.8.3 POSITIVE AND NEGATIVE MOMENTS IN PANELS

- In the DDM, the total factored statical moment M_o is divided into +ve and –ve factored moments according to the rules given in ACI Code, Section 14.6.30.
- These are illustrated in the Figure (next slide)
- In interior spans, 65% of M_o is assigned to the negative moment region and 35% to the +ve moment region
- The exterior end of an exterior span has considerably less fixity than the end at the interior support.

4.8.3 POSITIVE AND NEGATIVE MOMENTS IN PANELS

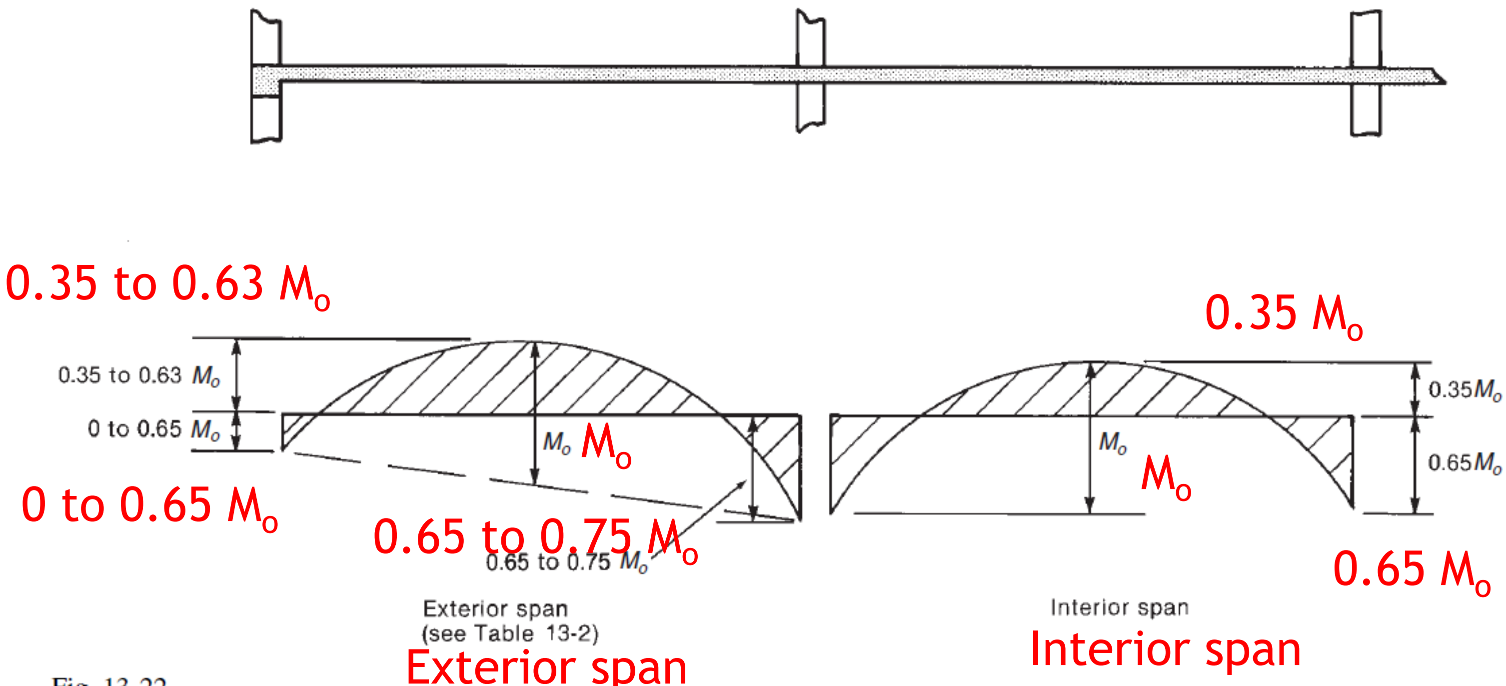


Fig. 13-22
Assignment of positive- and negative-moment regions.

Assignment of positive- and negative-moment regions

4.8.3 POSITIVE AND NEGATIVE MOMENTS IN PANELS

- The division of M_o in an end span into +ve and –ve moment regions is given in Table 14.2 (next slide). In this table, “exterior edge unrestrained” refers to a slab whose edge rests on, but is not attached to, for example, a masonry wall. “Exterior edge fully restrained” refers to a slab whose exterior edge is supported by, and is continuous with, a concrete wall with a flexural stiffness as large or larger than that of the slab. If the computed –ve moments on two sides of an interior support are different, the –ve moment section of the slab is designed for the larger of the two.

4.8.3 POSITIVE AND NEGATIVE MOMENTS IN PANELS

TABLE 13-2 Distribution of Total Factored Static Moment, M_o, in an Exterior Span

	(1) Exterior Edge Unrestrained	(2) Slab *with* Beams between *All* Supports	(3) Slab *without* Beams between *Interior* Supports: Without Edge Beam	(4) Slab *without* Beams between *Interior* Supports: With Edge Beam	(5) Exterior Edge Fully Restrained
Interior Negative Factored Moment	0.75	0.70	0.70	0.70	0.65
Midspan Positive Factored Moment	0.63	0.57	0.52	0.50	0.35
Exterior Negative Factored Moment	0	0.16	0.26	0.30	0.65

Source: ACI Code Section 13.6.3.3.

4.8.4 DEFINITION OF COLUMN STRIPS AND MIDDLE STRIPS

- The moments vary continuously across the width of the slab panels. To aid in steel placement, the design moments are averaged over the width of column strips over the columns and middle strips between the column strips $\rightarrow$ define column and middle strips
- Column strips in both directions extend one-fourth of the smaller span, l_{min}, each way from the column line.
- Middle strips are the strips between the column strips.

4.8.4 DEFINITION OF COLUMN STRIPS AND MIDDLE STRIPS

l_{max}

l_{min}

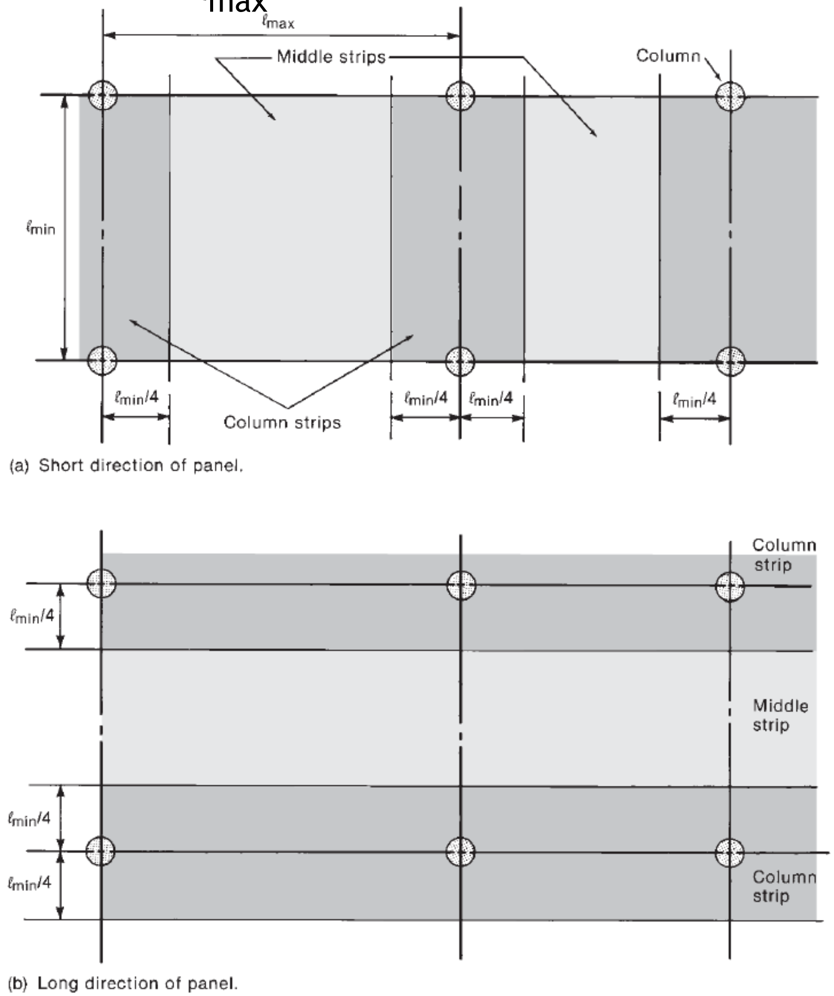


Fig. 13-23
Definitions of columns and middle strips.

4.8.5 DISTRIBUTION OF MOMENTS B/N MIDDLE STRIPS AND COLUMN STRIPS

- ACI Section 14.6.4 defines the fraction of the negative and positive moments assigned to the columns strips. The remaining amount of negative and positive moment is assigned to the adjacent half-middle strips. Table (next slide) gives the percentage distribution of –ve factored moment to the column strip at all interior supports
- The division is a function of $(\alpha_{f1} l_2 / l_1)$,which depends on the aspect ratio of the panel, l_2/l_1, and the relative stiffness, α_{f1}, of the beams (if any) spanning parallel to and within the column strip

4.8.5 DISTRIBUTION OF MOMENTS B/N MIDDLE STRIPS AND COLUMN STRIPS

TABLE 13-3 Percentage Distribution of Interior Negative Factored Moment to Column Strip

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

4.8.5 DISTRIBUTION OF MOMENTS B/N MIDDLE STRIPS AND COLUMN STRIPS

- For floor systems w/o interior beams, $(\alpha_{f1} l_2/l_1)$ is taken equal to zero, since $\alpha_{f1} = 0$. In this case 75% of the negative moment is distributed in the column strip, and the remaining 25% is divided equally b/n the two adjacent half middle strips
- For cases where a beam is present in a column strip (spanning in the direction of l_1) and $(\alpha_{f1} l_2/l_1) \geq 1.0$, the second row in table 14.3 applies.
- For $0 \leq (\alpha_{f1} l_2/l_1) \leq 1.0 \rightarrow$ use linear interpolation

4.8.5 DISTRIBUTION OF MOMENTS B/N MIDDLE STRIPS AND COLUMN STRIPS

- Table 13-4 gives the percentage distribution of +ve factored moment to the column strip at mid span for both interior and exterior spans.
- For floor systems w/o interior beams, 60% of the +ve moment is assigned to the column strip and the remaining 40% is divided equally b/n the adjacent half middle strips.
- If a beam is present in the column strip (spanning in the direction of l_1), either the percentages in the 2nd row or a linear interpolation b/n the percentages given in the 1st or 2nd row in Table 13-4 will apply

4.8.5 DISTRIBUTION OF MOMENTS B/N MIDDLE STRIPS AND COLUMN STRIPS

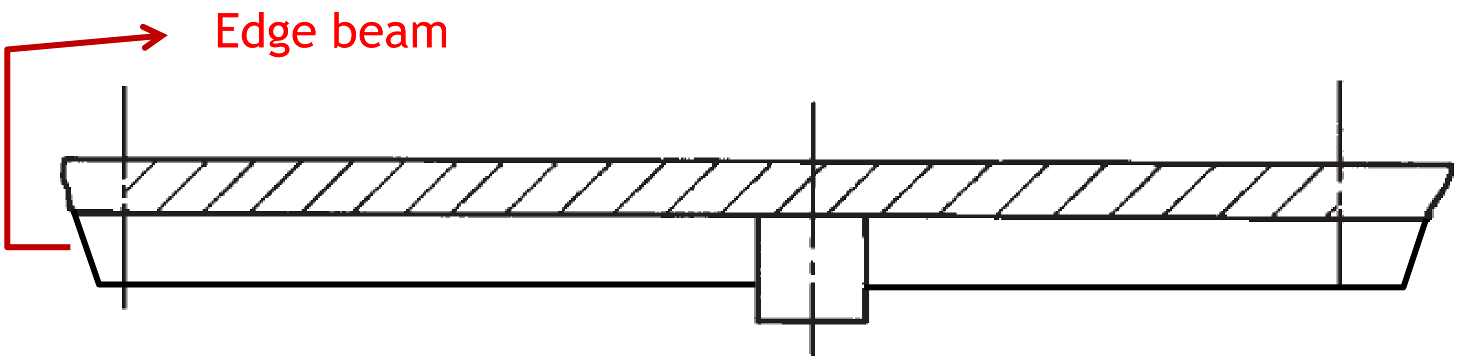
TABLE 13-4 Percentage Distribution of Midspan Positive Factored Moment to Column Strip

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	60	60	60
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

4.8.5 DISTRIBUTION OF MOMENTS B/N MIDDLE STRIPS AND COLUMN STRIPS

- At an exterior edge, the division of the exterior-end factored negative moment distributed to the column and middle strips spanning ⊥ to the edge also depends on the torsional stiffness of the edge beam, calculated as the shear modulus, G, times the torsional constant of the edge beam, C, divided by the flexural stiffness of the slab spanning ⊥ to the edge beam (i.e., EI for a slab having a width equal to the length of the edge beam from the center of one span to the center of the other span) designated by β_t (see next slide)

4.8.5 DISTRIBUTION OF MOMENTS B/N MIDDLE STRIPS AND COLUMN STRIPS



(d) Section for I_s—Interior beam.

Width of slab for the calculation of relative torsional stiffness β_t of edge beam

4.8.5 DISTRIBUTION OF MOMENTS B/N MIDDLE STRIPS AND COLUMN STRIPS

- Assuming that $\nu = 0 \rightarrow G = E/2$ so that $\beta_t = (E_{cb}C/2E_{cs}I_s)$
- The term C is the torsional constant of the edge beam which is calculated by subdividing the cross section into rectangles and carrying out the summation: $C=\Sigma[(1-0.63x/y)x^3y/3]$; where x = shorter side of a rectangle and y = longer side (NB: Several possible combination of rectangles have to be tried to get the maximum value of C. To do so wide rectangles should be made as large as possible. See Slide 90)

4.8.5 DISTRIBUTION OF MOMENTS B/N MIDDLE STRIPS AND COLUMN STRIPS

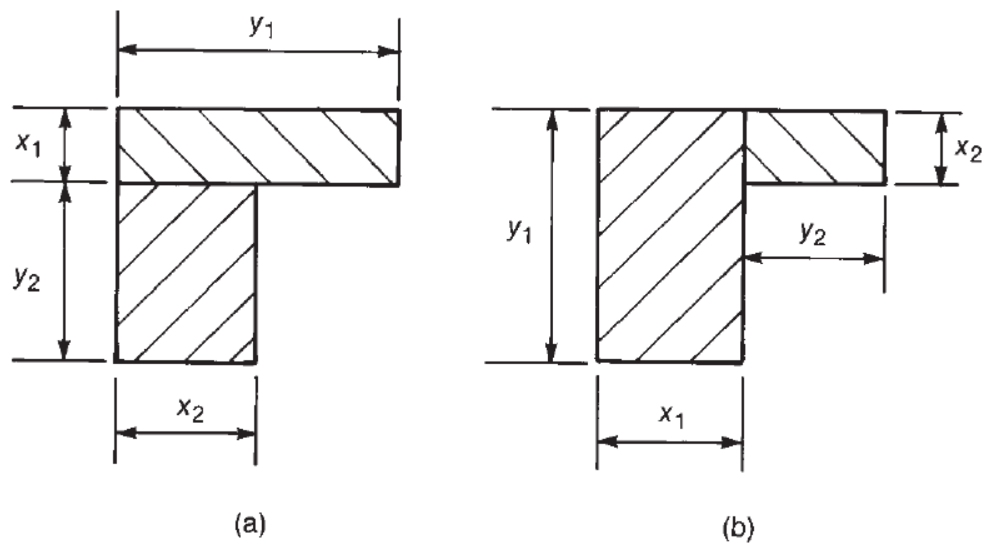


Fig. 13-24
Division of edge members for calculation of torsional constant, *C*.

4.8.5 DISTRIBUTION OF MOMENTS B/N MIDDLE STRIPS AND COLUMN STRIPS

- Table 13-5 gives percentage distribution of negative factored moment to column strip at exterior supports. The set up of this table is similar to the previous ones (tables 14.3 and 14.4) with the addition of two rows to account for presence or absence of an edge beam working in torsion to transfer some of the slab negative moment into the column.
- When there is no edge beam (β_t = 0), all of the negative moment is assigned to the column strips. This is reasonable because there is no torsional edge member to transfer moment from the middle strips all the way back to the columns.

4.8.5 DISTRIBUTION OF MOMENTS B/N MIDDLE STRIPS AND COLUMN STRIPS

TABLE 13-5 Percentage Distribution of Exterior Negative Factored Moment to Column Strip

ℓ_2/ℓ_1		0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	90	75	45

4.8.5 DISTRIBUTION OF MOMENTS B/N MIDDLE STRIPS AND COLUMN STRIPS

- If a stiff beam is present ($\beta_t \geq 2.5$), table gives specific percentages to be assigned to the column strip, depending on the value of α_{f1} and the l_2/l_1 ratio, as was done in the previous tables.
- For values of β_t between 2.5 and 0.0 and values of $(\alpha_{f1} l_2/l_1)$ b/n 1.0 and 0.0, two or three levels of linear interpolation may be required to determine the percentage distribution of negative moment assigned to the column strip.

4.8.5 DISTRIBUTION OF MOMENTS B/N MIDDLE STRIPS AND COLUMN STRIPS

- If a beam is present in the column strip (spanning in the direction of l_1), a portion of the column-strip moment is assigned to the beam (ACI Code, Section 14.6.5).
- If the beam has $(\alpha_{f1} l_2/l_1) > 1$, 85% of the column-strip moment is assigned to the beam and 15% to the slab.

4.8.6 EXAMPLE

- **Calculation of moments in an exterior panel of a flat plate**
- The slab is 200 mm thick and supports a superimposed service dead load of 1.2 kN/m^2 and a service live load of 3 kN/m^2. the beam is 300 mm wide by 400 mm in overall depth and is cast monolithically with the slab.
- Go to S.159
- (1) Compute the factored loads: Let q_d = 12 kN/m^2

4.8.6 EXAMPLE

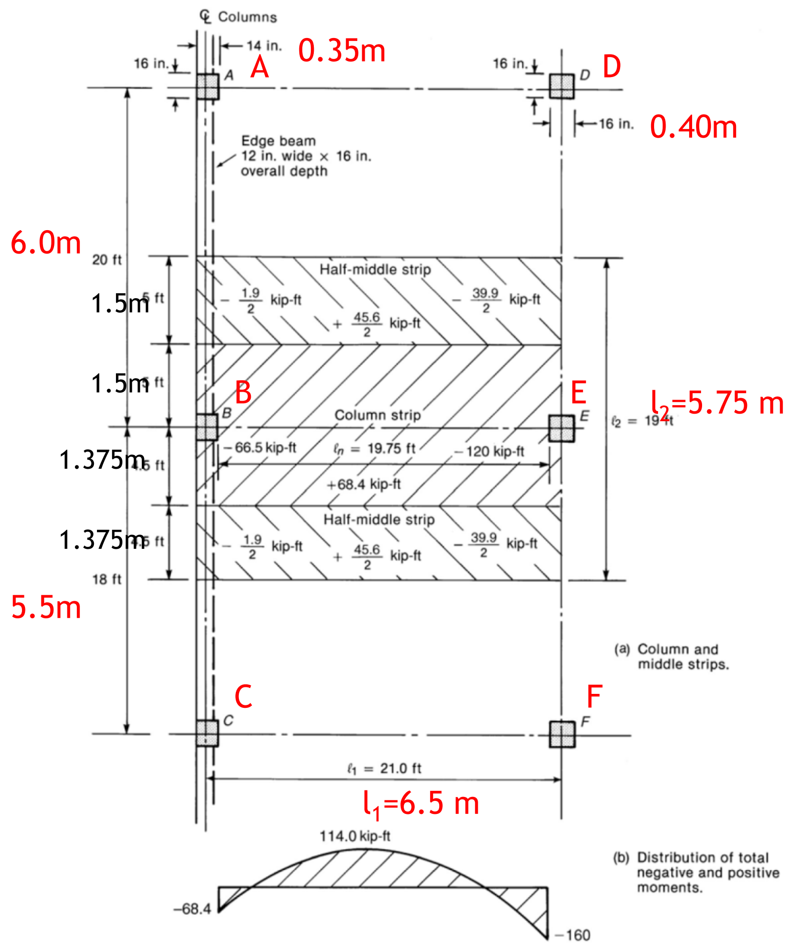


Fig. 13-28
Calculation of moments in an end span—Example 13-4.

4.8.6 EXAMPLE

- (2) Compute the moments in span BE.
- $\rightarrow$ (a) Compute l_n and l_2 and divide the slab into middle and column strips. $\rightarrow$ l_n = 6.5-1/2(0.35)-1/2(0.4) = 6.125m; l_2 = 5.75m. The column strip extends the smaller of $l_2/4$ or $l_1/4$ on each side of the column centerline. $\rightarrow$ The column strip extends 6/4 = 1.5 m toward AD and 5.5/4 = 1.275 m toward CF from line BE as shown in Slide 96. $\rightarrow$ The total width of the column strip is 2.875 m. The half middle strip b/n BE and CF has a width of 1.375 m, and the other one is 1.5 m

4.8.6 EXAMPLE

- $\rightarrow$ (b) Compute M_o: $M_o = q_d l_2 l_n^2/8 = 12 \times 5.85 \times 6.125^2/8 = 324.6$ kNm
- $\rightarrow$ (c) Divide M_o into positive and negative moments. The distribution of the total factored moment to the negative and the positive moment regions is as given in Table 13-2 under the column "slabs w/o beams b/n interior supports with edge beam"

4.8.6 EXAMPLE

- From Table 13-2, the total moment is divided as follows:
  - $\rightarrow$Interior negative: $M_u = 0.70M_o$ = -226.5 KNm
  - $\rightarrow$Positive: $M_u = 0.50M_o$ = +161.8 KNm
  - $\rightarrow$Exterior negative: $M_u = 0.30M_o$ = -97.1 KNm
- (d) Divide the moments b/n the column and middle strips
  - Interior negative moments (Table 13-3): This division is a function of $\alpha_{f1} l_2/l_1$, which is equal to zero, since there are no beams || to BE
    - $\rightarrow$ Interior column-strip negative moment: $0.75\times$ -226.5 = -169.9 kNm = -59.1 kNm/m width of column strip

4.8.6 EXAMPLE

  - $\rightarrow$Interior middle-strip negative moment = -56.6 kNm. Half of this goes to each of the half middle strips
- Positive moments: (Table 13-4) :
  - $\rightarrow$Column-strip positive moment: 0.60×161.8 = 97.1 kNm$\rightarrow$ 34.8 kNm/m
  - $\rightarrow$Middle-strip positive moment = 64.7 kNm. Half of this goes to each half-middle strip.
- Exterior negative moment: From ACI Section 14.6.4.2, the exterior negative moment is divided as a function of $\alpha_{f1} l_2/l_1$ (again equal to zero, since there is no beam || to l_1) and β_t. See next slide for attached torsional member for which β_t will be calculated

4.8.6 Example

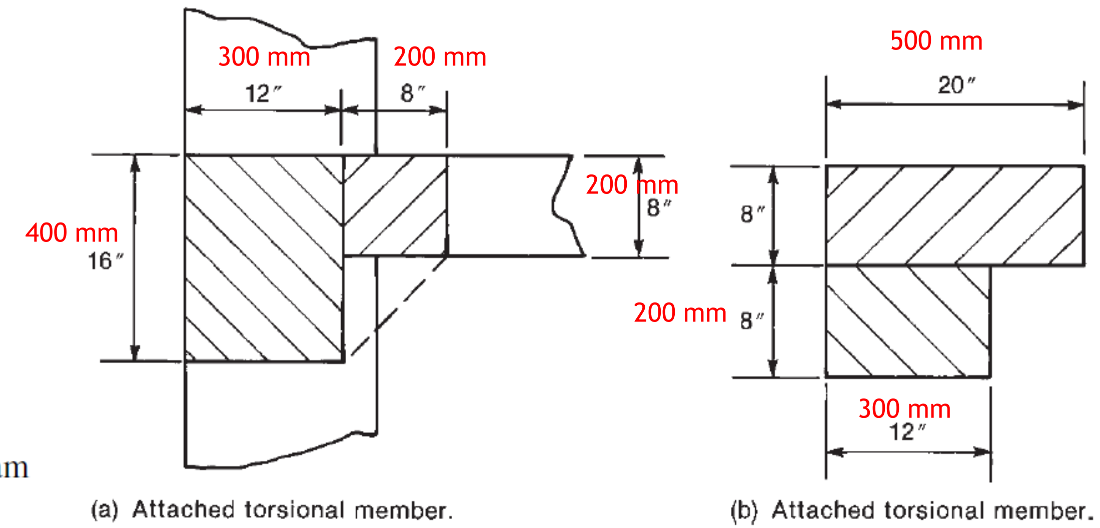


Fig. 13-30
Slab, column, and edge beam for Example 13-4.

4.8.6 EXAMPLE

- For Fig (a): $C=[(1-0.63\times300/400)300^3\times400/3+(1-0.63\times200/200)200^3\times200/3] = 2096.3\times10^6$ mm^4
- For Fig (b): $C = 1461.3\times10^6$ mm^4. The larger of the values is used; $\rightarrow C = 2096.3\times10^6$ mm^4
- I_s the moment of inertia of the strip of slab being designed, which has b=5.75m and h=200mm.
  - $\rightarrow I_s=5750\times200^3/12 = 3834.3\times10^6$ mm^4
  - Since f_{ck} is the same in the slab and beam, $E_{cb}=E_{cs}$ and $\beta_t= 2096.3\times10^6 /(2\times 3834.3\times10^6) = 0.273$
- Interpolating in Table 13-5, we have:
  - For $\beta_t=0 \rightarrow$ 100% to column strip
  - For $\beta_t=2.5 \rightarrow$ 75% to column strip
  - $\rightarrow$ for $\beta_t=0.273 \rightarrow$ 97.3% to column strip and we have:

4.8.6 EXAMPLE

- Exterior column-strip negative moment: 0.973(-97.1)= -94.5 kNm = -32.9 kNm/m
- Exterior middle-strip negative moment: -2.6 kNm

4.8.7 MOMENTS IN COLUMNS AND TRANSFER OF MOMENTS TO COLUMNS

- Exterior Columns: When design is carried out by the DDM, ACI specifies that the moment that is transferred from a slab w/o interior beams to an edge column is 0.26 to 0.30 M_o, as given in Table 13-2.
- This moment is used to compute the shear stresses due to moment transfer to the edge column (discussed later)
- The exterior negative moment from the DDM calculation is divided b/n the columns above and below the slab in proportion to the column stiffness, 4EI/l. the resulting column moments are used in the design of the columns

4.8.7 MOMENTS IN COLUMNS AND TRANSFER OF MOMENTS TO COLUMNS

- Interior Columns: At interior supports the column moments are determined from unbalanced moment resulting from an uneven distribution of live load.
- The unbalanced moment is computed by assuming that the longer span adjacent to the column is loaded with the factored dead load and half the factored live load, while the shorter span carries only the factored dead load

4.8.7 MOMENTS IN COLUMNS AND TRANSFER OF MOMENTS TO COLUMNS

- The total unbalanced negative moment at the joint is thus: $M = 0.65(1/8)\{(w_d+0.5w_\ell)\ell_2\ell_n^2 - w'_d\ell'_2(\ell'_n)^2\}$; where w_d and w_ℓ refer to the factored dead and live loads on the longer span and w'_d, ℓ'_2, and ℓ'_n refer to the shorter span adjacent to the column
- A portion of the unbalanced moment is distributed to the slabs and the rest goes to the columns. →ACI gives $M_{col} = 0.07\{(w_d+0.5w_\ell)\ell_2\ell_n^2 - w'_d\ell'_2(\ell'_n)^2\}$

4.8.8 DESIGN OF EDGE BEAMS FOR SHEAR AND MOMENT

- When a slab panel contains a beam, either an edge beam or an interior beam b/n the columns, the moments in the panel are divided b/n the slab and the beam the same way the moments are divided b/n the slab and the beam for interior beams (refer to literature (Macgregor) as alternative to design of two-way beam supported slabs using the coefficients in EBCS-2).

4.9 SHEAR STRENGTH IN TWO-WAY SLABS

- A shear failure in a beam results from an inclined crack caused by flexural and shearing stresses. This crack starts at the tensile face of a beam and extends diagonally to the compression zone.
- In the case of a two-way slab or footing, the two shear-failure mechanisms shown in Figure (next slide) are possible.
- One-way shear or beam-action shear (Fig a) involves an inclined crack extending across the entire width of the structure.

4.9 SHEAR STRENGTH IN TWO-WAY SLABS

- Two-way shear or punching shear involves a truncated cone or pyramid-shaped surface around the column as shown in Fig b. Generally, the punching-shear capacity of a slab or footing will be considerably less than the one-way shear capacity.
- This section is limited to footings and slabs w/o beams. Refer to literature (Macgregor) for shear strength of slabs with beams.

4.9 SHEAR STRENGTH IN TWO-WAY SLABS

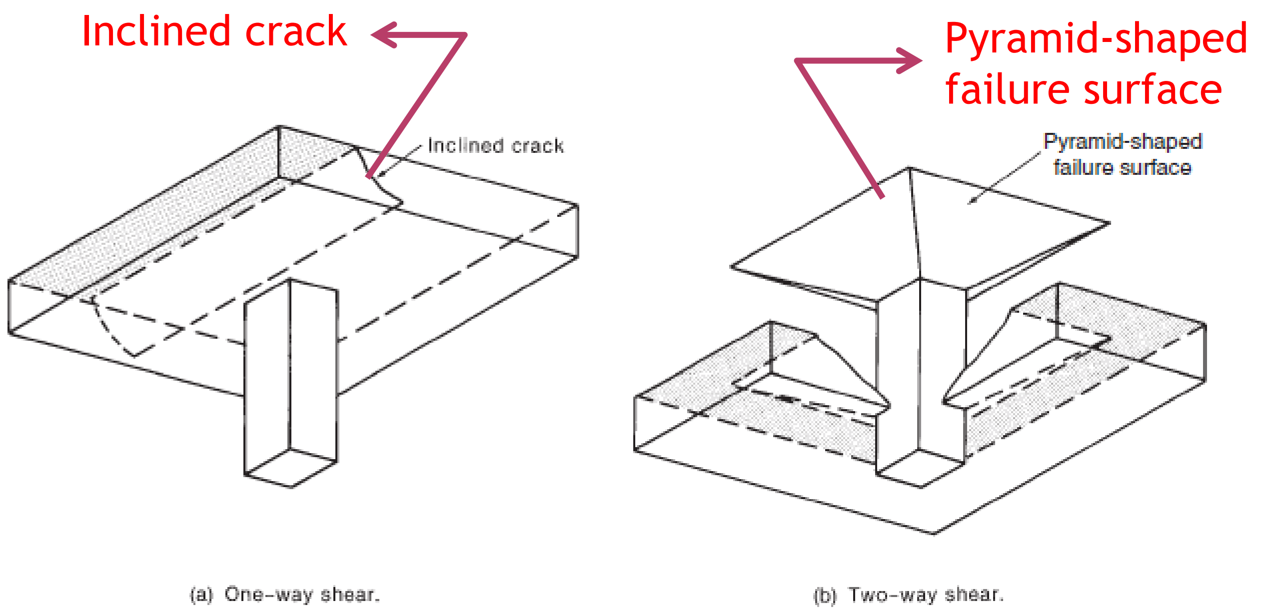


Fig. 13-52
Shear failure in a slab.

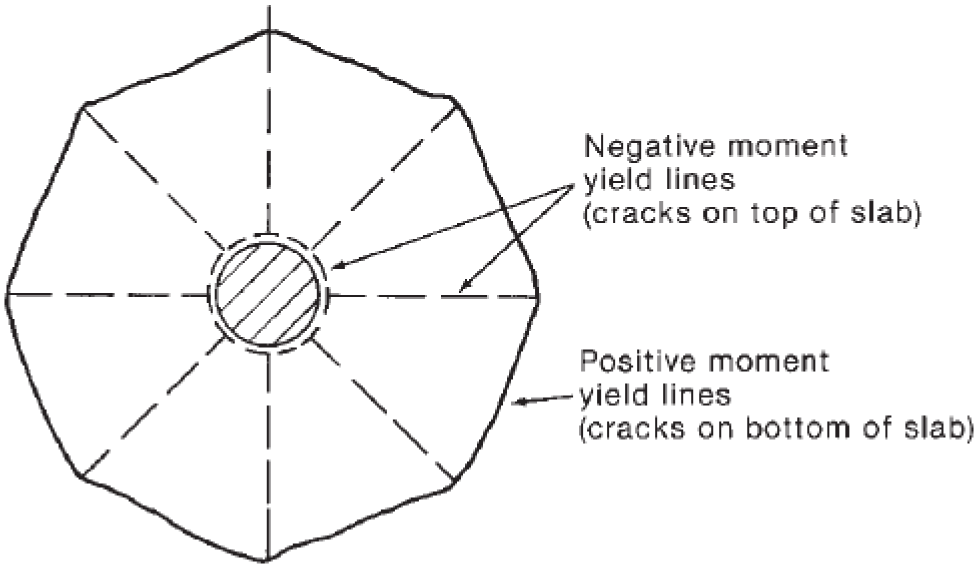
(a) One-way shear

(b)Two -way shear

4.9 SHEAR STRENGTH IN TWO-WAY SLABS

- **Behavior of slabs failing in two way shear**
- As discussed in DDM, the maximum moments in a uniformly loaded plate occur around the columns and lead to a circular crack around each column. After additional loading, the cracks necessary to form a fan yield-line mechanism develop (see next slide), and at about the same time, inclined or shear cracks form on the truncated conical surface shown in Fig b. These cracks can be shown in Fig (slide after next), which shows a slab that has been sawn through along two sides of the column after the slab had failed in **two-way shear**

4.9 SHEAR STRENGTH IN TWO-WAY SLABS



(a) Fan yield line at column in a flat plate.

Fan yield line at a column in a flat plate

4.9 SHEAR STRENGTH IN TWO-WAY SLABS

Inclined cracks in a slab after shear failure

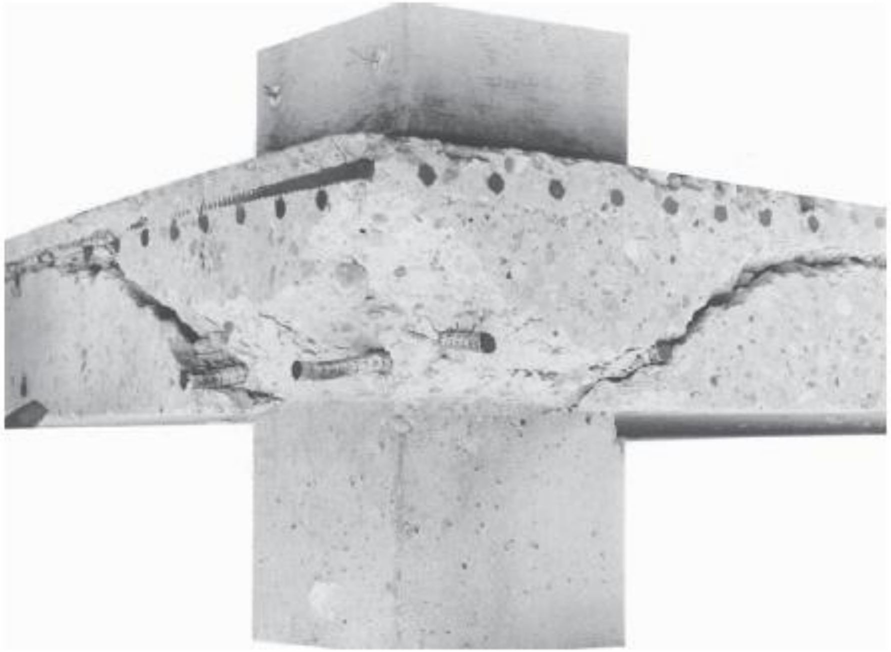


Fig. 13-53
Inclined cracks in a slab after a shear failure. (Photograph courtesy of J. G. MacGregor.)

4.9 SHEAR STRENGTH IN TWO-WAY SLABS

- Alexander and Simonds used the truss model in Fig (see next slide) to analyze punching-shear failures. Prior to the formation of the inclined cracks shown in Fig 13-53, the shear is transferred by shear stresses in the concrete. Once the cracks have formed, only relatively small shear stresses can be transferred across them. Now the majority of the vertical shear is transferred by inclined struts A-B and C-D extending from the compression zone at the bottom of the slab to the reinforcement at the top of the slab.

4.9 SHEAR STRENGTH IN TWO-WAY SLABS

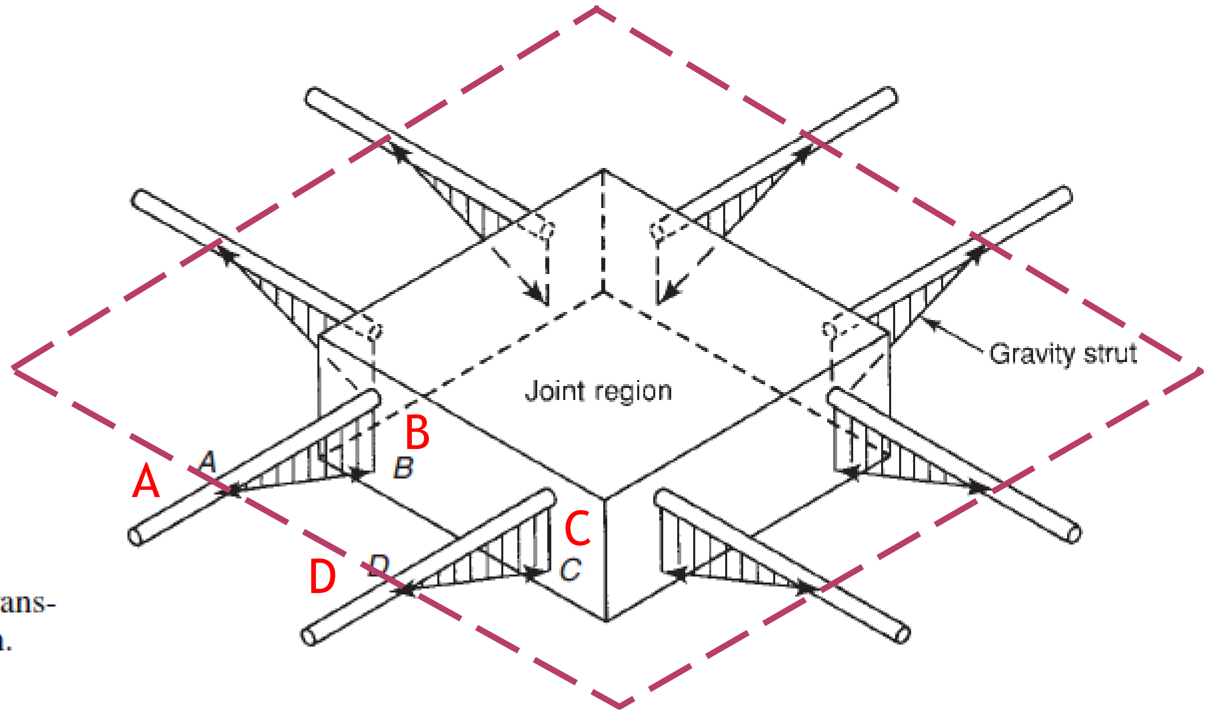


Fig. 13-54
Truss model for shear transfer at an interior column.

4.9 SHEAR STRENGTH IN TWO-WAY SLABS

- Similar struts exist on all four sides of the column. The horizontal component of the force in the struts causes a change in the force in the reinforcement at A and D, and the vertical pushes up on the bar and is resisted by the tensile stresses in the concrete b/n the bars. Eventually, this concrete cracks in the plane of the bars, and a punching failure results.
- Such a failure occurs suddenly, with little, if any, warning.

4.9 DESIGN OF TWO-WAY SHEAR

- We will consider the case of shear transfer w/o appreciable moment transfer. The case when both shear and moment are transferred from the slab to the column is discussed in subsequent sections.
- Location of critical perimeters
  - Two-way shear is assumed critical on a vertical section through the slab or footing extending around the column. According to the ACI at d/2 1.5d according to EBCS-2. See Figs (next slides)

4.9 DESIGN OF TWO-WAY SHEAR

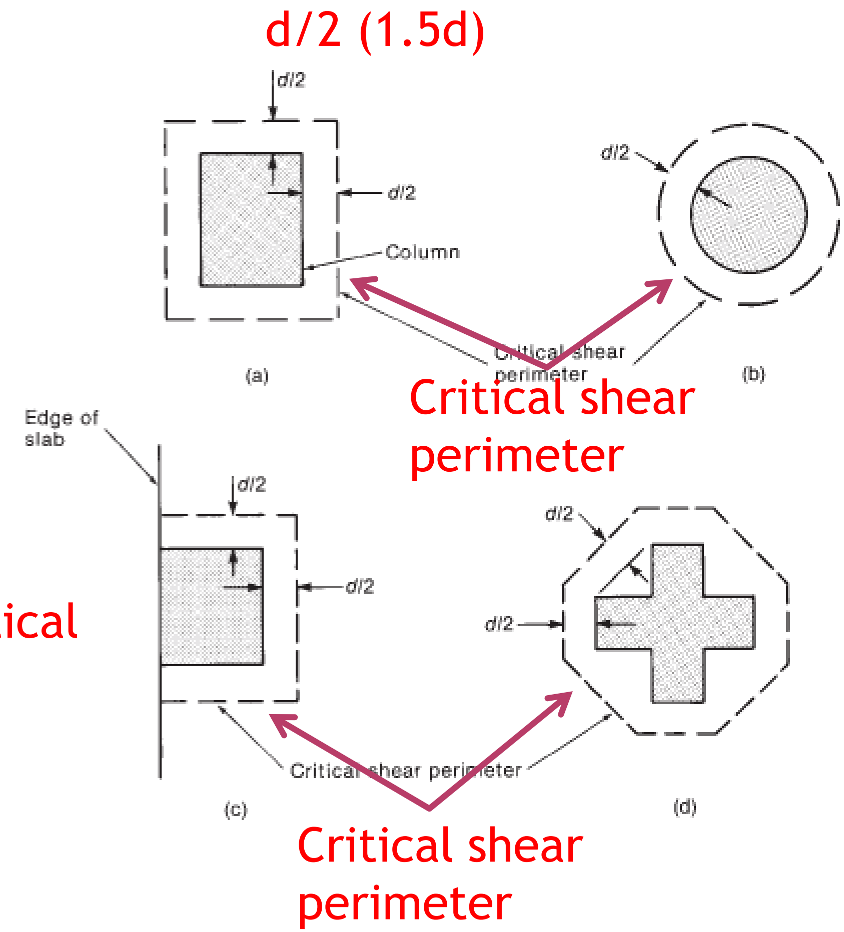


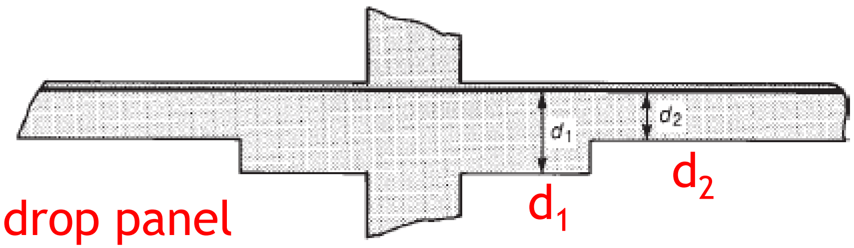
Fig. Location of critical shear perimeter

Fig. 13-55
Location of critical shear perimeters.

4.9 DESIGN OF TWO-WAY SHEAR

- **Critical sections for slabs with drop panels**
- When high shear forces are being transferred at a slab-column connection, the slab shear strength can be increased locally by using a drop panel to locally increase the thickness of the slab. ACI requires that the total thickness of the slab and drop panel to be at least 1.25 times the thickness of the slab adjacent to the drop panel.
- In slab with drop panels, two critical sections should be considered, as shown in Figure (next slide)

4.9 DESIGN OF TWO-WAY SHEAR



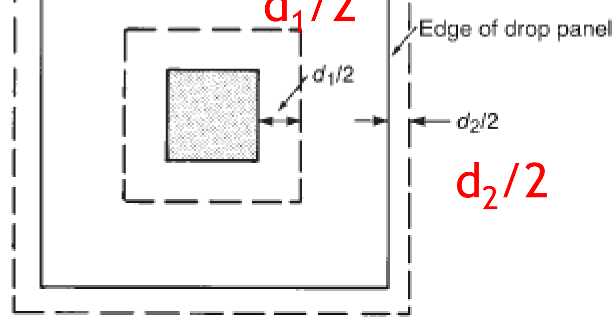
d_2

d_1

(a) Section through a drop panel

(a) Section through drop panel.

Edge of drop panel



$d_2/2$

Fig. Critical sections in a slab with drop panels

Fig. 13-56
Critical sections in a slab
with drop panels.

(b) Critical sections.

(b) Critical sections

4.9 DESIGN OF TWO-WAY SHEAR

- If a drop panel is also used to control deflections or reduce the amount of flexural reinforcement required in the slab, the drop panel must satisfy the length requirements given in ACI Code, Section 14.2.5.
- **Critical sections near holes and at edges**
- When openings are located at less than 10 times the slab thickness from a column, ACI Code Section 11.11.6 requires that the critical perimeter be reduced as shown in Figure (next slide)

4.9 DESIGN OF TWO-WAY SHEAR

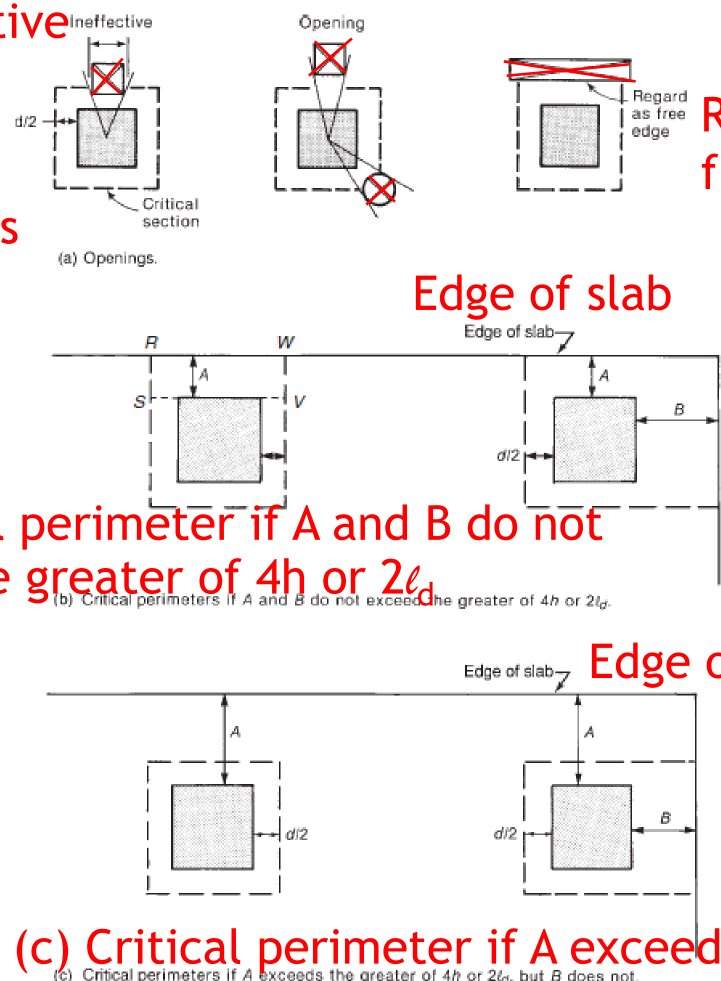


Fig. Effects of openings and edges on the critical shear perimeter

Fig. 13-57
Effects of openings and edges on the critical shear perimeter.

4.9 DESIGN OF TWO-WAY SHEAR

- **Tributary areas for shear in two-way slabs**
- For uniformly loaded two-way slabs, the tributary areas used to calculate V_u are bounded by lines of zero shear. For interior panels, these lines can be assumed to pass through the center of the panel. For edge panels, lines of zero shear are approximately at 0.42l to 0.45l from the center of the exterior column, where l is the span measured from center-to-center of the cols.

4.9 DESIGN OF TWO-WAY SHEAR

- However to be conservative in design, ACI Code Section 8.4.3 requires that the exterior supports must resist a shear force due to loads acting on half of the span (0.5l). Also to account for the larger tributary area for the 1st interior support, ACI Code, Section 8.4.3 requires that the shear force from loads acting on half the span must be increased by 15%. See Figure (next slide)

4.9 DESIGN OF TWO-WAY SHEAR

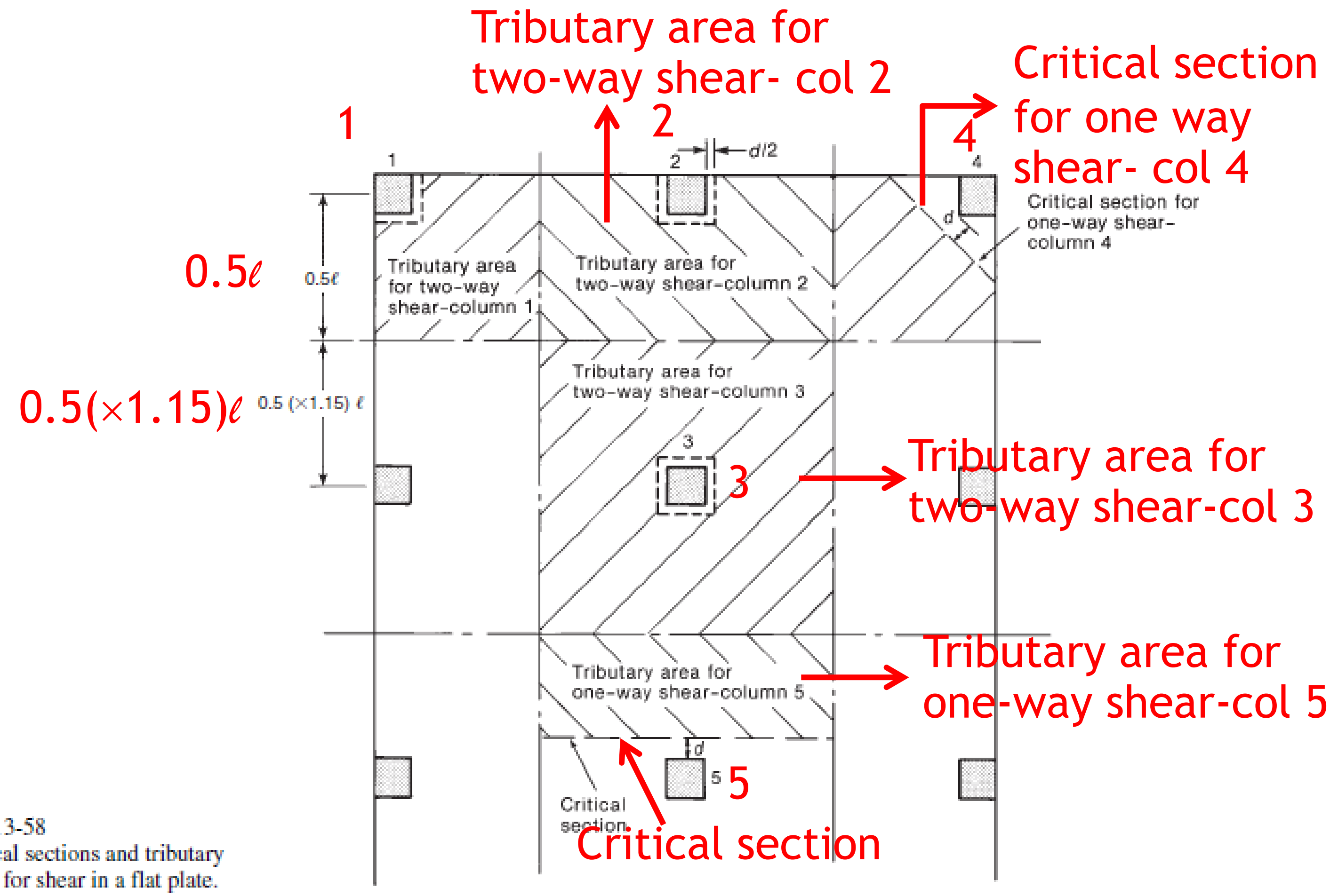


Fig. 13-58
Critical sections and tributary areas for shear in a flat plate.

Fig. Critical sections and tributary areas for shear in a flat plate

4.9 DESIGN OF TWO-WAY SHEAR

- **Design Equations: Two-way shear with negligible moment transfer**
- Lateral loads and unbalanced floor loads, on a flat-plate building require that both moments and shears be transferred from the slab to the columns. In the case of interior columns in a braced flat-plate building, the worst loading case for shear generally corresponds to a negligible moment transfer from the slab to the column. Similarly, columns generally transfer little or no moment to footings
- Design for two-way shear w/o moment transfer is carried out by using EBCS-2 Eq (…)

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- **6.4 Punching**
- **6.4.1 General**
- (1)P The rules in this section complement those given in 6.2 (Shear) and cover punching shear in solid slabs, waffle slabs with solid areas over columns, and foundations.
- (2)P Punching shear can result from a concentrated load or reaction acting on a relatively small area, called the loaded area A_{load} of a slab or foundation.
- (3) An appropriate verification model for checking punching failure at the ULS is shown in Figure (next slide)

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

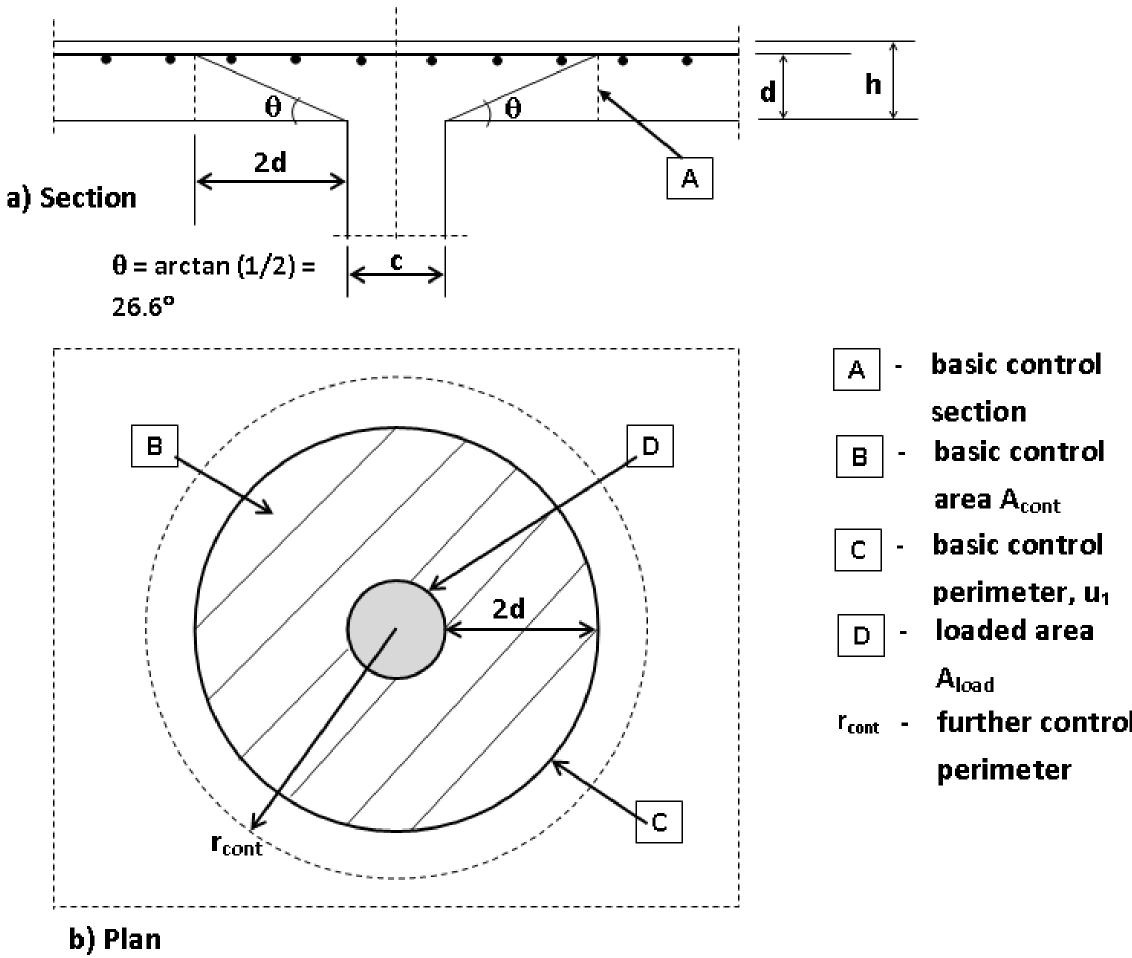


Fig: Verification model for punching shear at the ultimate limit state

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- (4) The shear resistance should be checked along defined control perimeters
- (5) The rules in 6.4 are principally formulated for the case of uniformly distributed loading. In special case, such as footings, the load within the control perimeter adds to the resistance of the structural system, and may be subtracted when determining the design punching shear stress

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- **6.4.2 Load distribution and basic control perimeter**
- (1) The basic control perimeter u_1 may normally be taken to be at a distance 2.0d from the loaded area and should be constructed so as to minimize its length (see Figure next slide). The effective depth of the slab is assumed constant and may normally be taken as: $d_{eff} = (d_y + d_z)/2$; where d_y and d_z are the effective depths of the reinforcement in two orthogonal directions

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

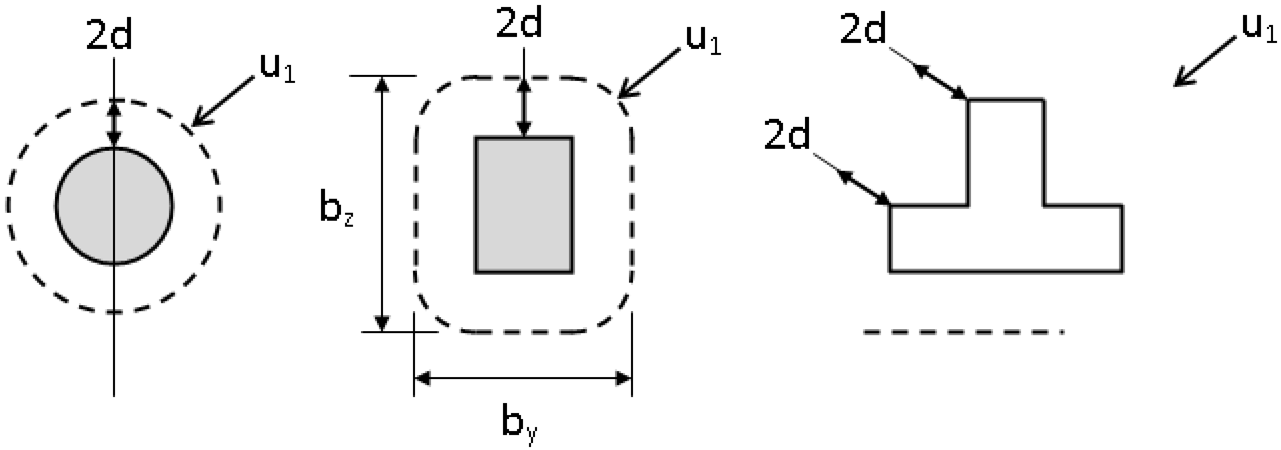


Fig: Typical basic control perimeters around loaded areas

Figure 6.13

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- (2) Control perimeters at a distance less than 2d should be considered where the concentrated force is opposed by a high distributed pressure (e.g. soil pressure in a base), or by the effects of a load or reaction within a distance 2d of the periphery of area of application of the force
- (3) For loaded areas situated near openings, if the shortest distance b/n the perimeter of the loaded area and the edge of the opening does not exceed 6d, that part of the control perimeter contained b/n two tangents drawn to the outline of the opening from the center of the loaded area is considered to be ineffective (SNS)

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

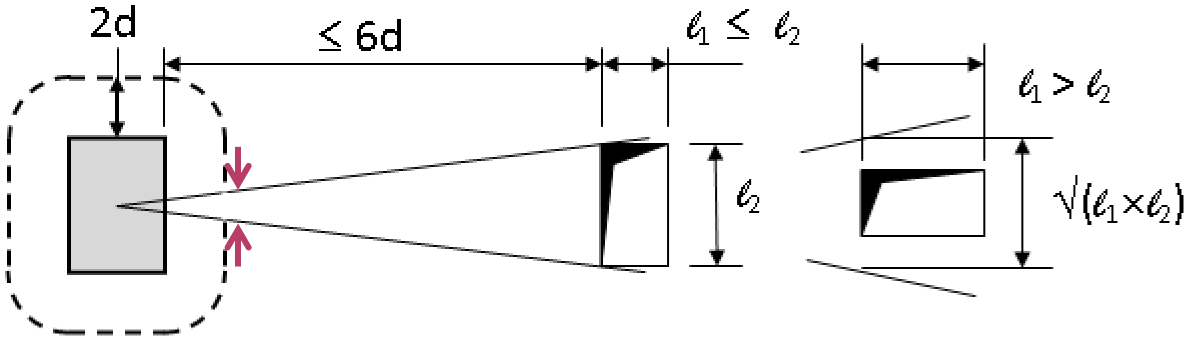


Fig: Control perimeter near an opening

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- (4) For a loaded area situated near an edge or a corner, the control perimeter should be taken as shown in Figure (SNS), if this gives a perimeter (excluding the unsupported edges) smaller than that obtained from (1) and (2) above.

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

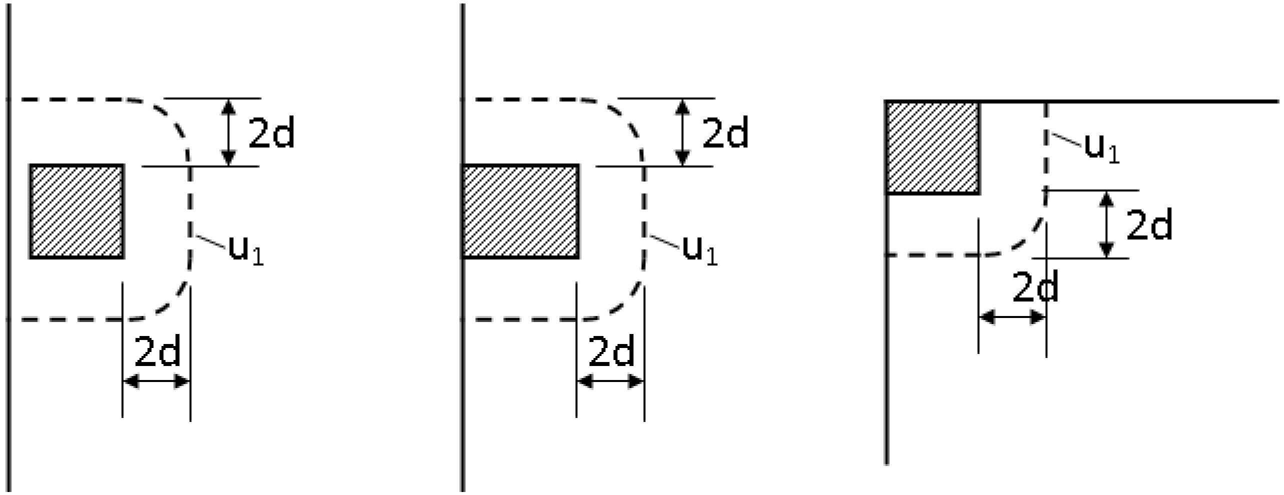


Fig: Control perimeters for loaded areas close to or at edge or corner

Figure 6.15

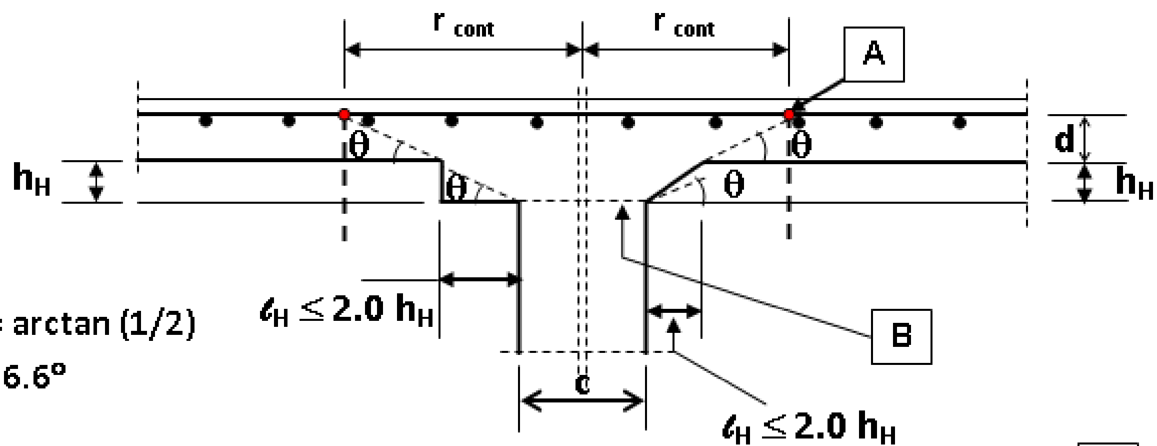
4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- (5) For loaded areas situated near or on an edge or corner, i.e. at a distance smaller than d, special edge reinforcement should always be provided, see 9.4.1.4
- (6) The control section is that which follows the control perimeter and extends over the effective depth d. For slabs of constant depth, the control section is ⊥ to the middle plane of the slab. For slabs or footings of variable depth, the effective depth may be assumed to be the depth at the perimeter of the loaded area

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- (7) Further perimeters, u_i, inside and outside the control area should have the same shape as the basic control perimeter.
- (8) For slabs with circular column heads for which $\ell_h \leq 2h_H$ (see Fig next slide) a check of the punching shear stresses according to 6.4.3 is only required on the control section outside the column head. The distance of this section from the centroid of the column r_{cont} may be taken as: $r_{cont} = 2d + \ell_H + 0.5c$; where ℓ_H is the distance from the column face to the edge of the column head and c is the diameter of a circular column

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2



$\theta = \arctan(1/2)$
$= 26.6^{\circ}$

A - basic control section

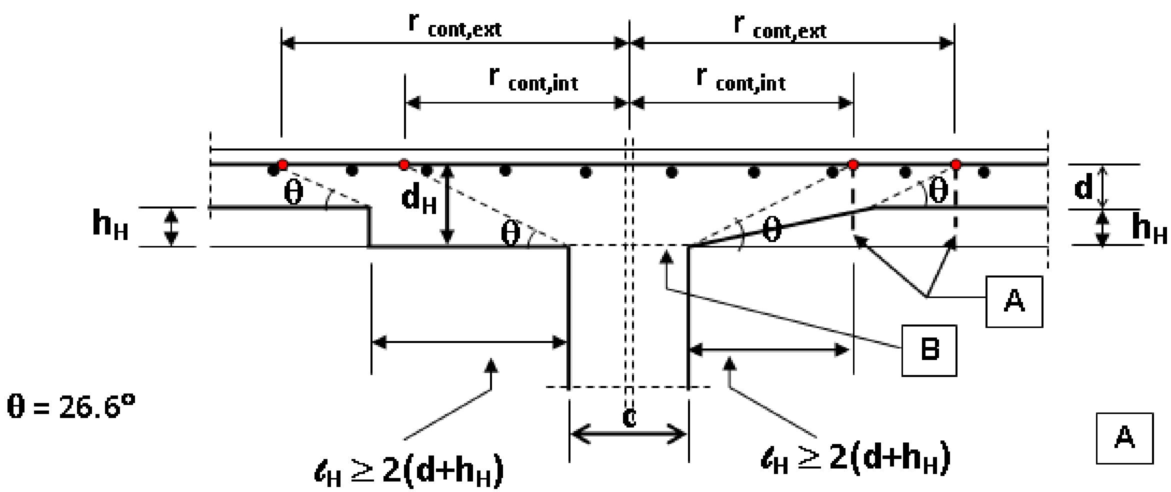
B - Loaded area A_{load}

Fig: Slab with enlarged column head where $\ell_H < 2.0\ h_H$

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- For a rectangular column with a rectangular head with $\ell_h \leq 2d$ and overall dimensions ℓ_1 and ℓ_2 ($\ell_1 = c_1 + 2\ell_{H1}$, $\ell_2 = c_2 + 2\ell_{H2}$, $\ell_1 \leq \ell_2$), the value r_{cont} may be taken as the lesser of: $r_{cont} = 2d + 0.56\sqrt{(\ell_1 \ell_2)}$ and $r_{cont} = 2d + 0.69\,\ell_1$
- (9) For slabs with enlarged column heads where $\ell_h \geq 2h_H$ (see Figure NS) the critical sections both within the head and in the slab should be checked.
- (10) The provisions of 6.4.2 and 6.4.3 also apply for checks within the column head with d taken as d_H according to Figure in NS.

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2



A	-	basic control sections for circular columns
B	-	Loaded area A_{load}

Fig: Slab with enlarged column head where $\ell_H > 2(d+h_H)$

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- (11) For circular columns the distances from the centroid of the column to the control sections in Figure (SPS) may be taken as: $r_{cont,ext} = \ell_H + 2d + 0.5c$; $r_{cont,int} = 2(d + h_H) + 0.5c$
- **6.4.3 Punching shear calculation**
- (1)P The design procedure for punching shear is based on checks at a series of control sections, which have a similar shape as the basic control section. The following design shear stresses, per unit area along the control sections, are defined:

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- $v_{Rd,c}$ is the design value of the punching shear resistance of a slab w/o punching shear reinforcement along the control section considered
- $v_{Rd,cs}$ is the design value of the punching shear resistance of a slab with punching shear reinforcement along the control section considered
- $v_{Rd,max}$ is the design value of the maximum punching shear resistance along the control section considered

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- (2) The following checks should be carried out;
  - (a) At the column perimeter, or the perimeter of the loaded area, the maximum punching shear stress should not be exceeded: → $v_{Ed} < v_{Rd,max}$
  - (b) Punching shear reinforcement is not necessary if: $v_{Ed} < v_{Rd,c}$
  - (c) Where v_{Ed} exceeds the value $v_{Rd,c}$ for the control section considered, punching shear reinforcement should be provided according to 6.4.5

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- (3) where the support reaction is eccentric with regard to the control perimeter, the maximum shear stress should be taken as: $v_{Ed} = \beta(V_{Ed}/u_i d)$; where d is mean effective depth of slab, taken as $(d_y + d_z)/2$; u_i is length of control perimeter being considered; and β is given by: $\beta = 1 + k(M_{Ed}/V_{Ed})(u_1/W_1)$ **Eq.(6.39)**; where u_1 is the length of the basic control perimeter; k is a coefficient dependent on the ratios b/n the column dimensions c_1 and c_2: its value is a function of the proportions of the unbalanced moment transmitted by uneven shear and by bending and torsion (see Table 6.2)

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- W_1 corresponds to a distribution of shear as shown in Figure (SNS) and is a function of the basic control perimeter u_1: $W_1 = \int_0^u |e| \, d\ell$; where $d\ell$ is the a length increment of the perimeter; and e is the distance of $d\ell$ from the axis about which the moment M_{Ed} acts.
- **Table 6.2: Values of k for rectangular loaded areas**

c_1/c_2	≤ 0.5	1.0	2.0	≥ 4.0
k	0.45	0.60	0.70	0.80

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

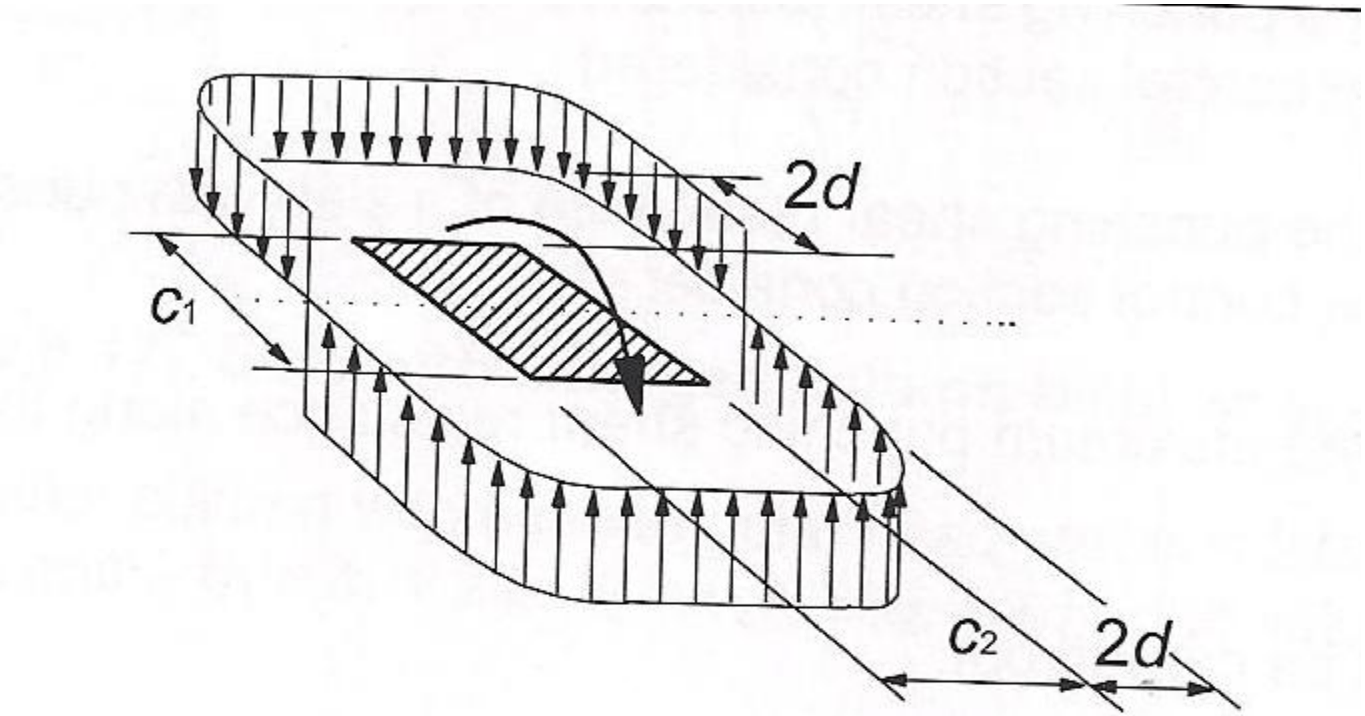


Fig: Shear distribution due to an unbalanced moment at a slab-internal column connection

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- For a rectangular column: $W_1 = (c_1^2/2) + c_1c_2 + 4c_2d + 16d^2 + 2\pi dc_1$; where c_1 is the column dimension parallel to the eccentricity of the load; and c_2 is the column dimension $\perp$ to the eccentricity of the load.
- For internal columns β follows from: $\beta = 1 + 0.6\pi(e/(D+4d))$
- For an internal rectangular column where the loading is eccentric to both axes, the following approximate expression for β may be used: $\beta = 1 + 1.8(\surd(e_y/b_z)^2 + (e_z/b_y)^2$; where

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- e_y and e_z are the eccentricities M_{Ed}/V_{Ed} along y and z axes respectively; b_y and b_z is the dimensions of the control perimeter (see figure 6.13); D is the diameter of the circular column. (Note: e_y results from a moment about the z axis and e_z from a moment about the y axis)
- (4) For edge column connections, where the eccentricity ⊥ to the slab edge (resulting from a moment about an axis || to the slab edge) is toward the interior and there is no ecc || to the edge, the punching force may be considered to be uniformly distributed along the control perimeter u_1^* (See NS).

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

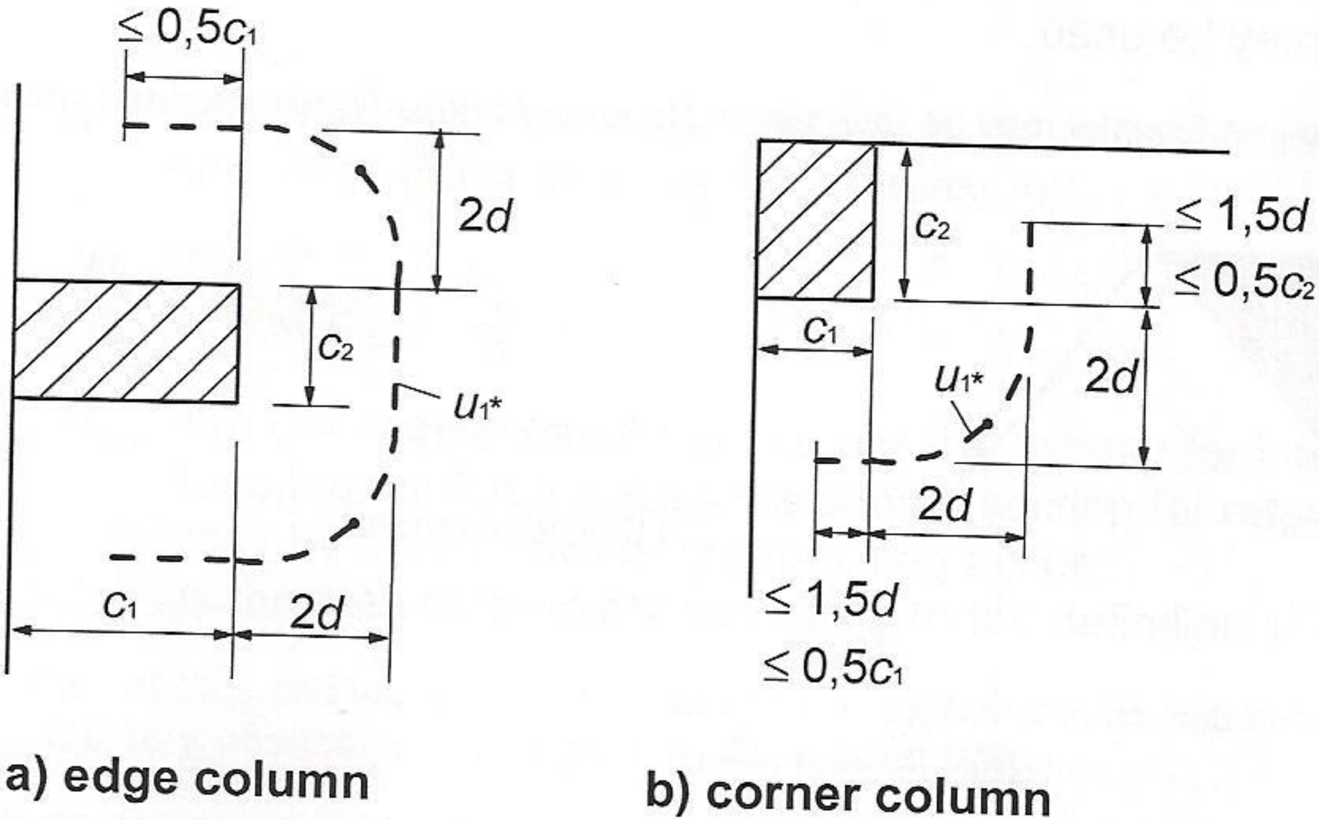


Figure 6.20: Equivalent control perimeter u_{1*}

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- Where there are ecc in both orthogonal directions, β may be determined using the following expression: $\beta = (u_1/u_1^*) + k(u_1/W_1)e_{par}$; where u_1 is the full control perimeter (see Fig 6.15); u_1^* is the reduced control perimeter (See Fig 6.20 (a)); e_{par} is the ecc $||$ to the slab edge resulting from a moment about an axis $\perp$ to the slab edge; k may be determined from Table 6.2 with the ratio c_1/c_2 replaced by $c_1/2c_2$; and W_1 is calculated for the full perimeter (see fig 6.13

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- For a rectangular column as shown in Figure 6.20(a): $W_1 = (c_2^2/4) + c_1c_2 + 4c_1d + 8d^2 + \pi dc_2$
- If the ecc $\perp$ to the slab edge is not toward the interior, Expression (6.39) applies. When calculating W_1 the ecc e should be measured from the centroid of the control perimeter.
- (5) For corner column connections, where the ecc is toward the interior of the slab, it is assumed that the punching force is uniformly distributed along the reduced control perimeter u1*, as defined in Fig 6.20b

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- The β-value may then be considered as: $\beta = u_1/u_1^*$. If the ecc is toward the exterior, Expression (6.39) applies
- (6) For structures where the lateral stability does not depend on frame action b/n the slabs and the columns, and where the adjacent spans do not differ in length by more than 25%, approximate values for β may be used (β = 1.15 for internal columns; β = 1.4 for edge columns).

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- (7) Where a concentrated load is applied close to a flat slab column support, the resistance enhancement according to 6.2.2(5) is not valid and should not be included.
- (8)The punching shear force V_{Ed} in a foundation slab may be reduced due to the favorable action of the soil pressure.
- (9) The vertical component V_{pd} resulting from inclined prestressing tendons crossing the control section may be taken into account as a favorable action where relevant.

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- **6.4.4 Punching shear resistance for slabs or column bases w/o shear reinforcement**
- (1) The punching shear resistance of a slab should be assessed for the basic control section according to 6.4.2. The design punching stress (resistance) is given by: $v_{Rd,c} = C_{Rd,c}k(100\rho_\ell f_{ck})^{1/3} + 0.10\sigma_{cp} \geq (v_{min} + 0.10\sigma_{cp})$; where f_{ck} is n MPa; $k = 1 + \sqrt{(200/d)} \leq 2.0$ d in mm; $\rho_\ell = \sqrt{(\rho_{\ell y} \times \rho_{\ell z})} \leq 0.02$; $\rho_{\ell y}$, $\rho_{\ell z}$ relate to the bonded tension steel in the y- and z- directions respectively. The values $\rho_{\ell y}$, $\rho_{\ell z}$) should be calculated as mean values taking into account a slab width equal to the column width plus 3d each side.

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- σ_{cp}= $(\sigma_{cy} + \sigma_{cz})/2$; where σ_{cy}, σ_{cz} are the normal concrete stresses in the critical section in y- and z-directions (MPa, positive if compression): σ_{cy}= $N_{Ed,y}/A_{cy}$ and σ_{cy}= $N_{Ed,z}/A_{cz}$ where $N_{Ed,y}$, $N_{Ed,z}$/ are the longitudinal forces across the full bay for internal columns and the longitudinal forces across the control section for edge columns. The force may be from a load or prestressing action; A_c is the area of concrete according to the definition of N_{Ed}(Note: the values of $C_{Rd,c}$ and v_{min} for use in a Country may…

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- The recommended value for $C_{Rd,c}$ is $0.18/\gamma_c$ and that for v_{min} is given by Expression (6.3N)
- (2) The punching resistance of column bases should be verified at control perimeters within 2d from the periphery of the column. The lowest value of resistance found in this way should control the design. For concertric loading the net applied force is: $V_{Ed,red} = V_{Ed} - \Delta V_{Ed}$; where V_{Ed} is the column load; and ΔV_{ed} is the net upward force within the control perimeter considered, i.e., upward pressure from soil minus self weight of base

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- $v_{Ed} = V_{Ed,red}/ud$
- $v_{Rd} = C_{Rd,c}k(100\rho f_{ck})^{1/3} + 2d/a \geq v_{min} \times (2d/a)$; where a is the distance from the periphery of the column to the control perimeter considered; $C_{Rd,c}$ defined in 6.4.4(1); v_{min} defined in 6.4.4(1)
- For eccentric loading: $v_{Ed} = (V_{Ed,red}/ud)[1 + k(M_{Ed}u/V_{Ed,red}W)]$; where k is defined in 6.4.3(4)

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- **6.4.5 Punching shear resistance of slabs or column bases with shear reinforcement**
- Read!

4.10 EQUIVALENT-FRAME METHODS

- **4.10.0 Equivalent-Frame Methods**
- The ACI Code presents two general methods for calculating the longitudinal distribution of moments in two-way slab systems. These are the direct-design method and the equivalent-frame methods.
- Equivalent-frame methods are intended for use in analyzing moments in any practical slab-column frame. Their scope is thus wider than the direct-design method, which is subject to the limitations presented in Section 13-7.

4.10 EQUIVALENT-FRAME METHODS

- In the direct-design method, the statical moment M_0, is calculated for each slab span. This moment is then distributed b/n positive- and negative- moment regions using arbitrary moment coefficients, which are adjusted to reflect pattern loadings.
- For equivalent-frame methods, a stiffness analyses of a slab-column frame is used to determine the longitudinal distribution of bending moments, including possible pattern loadings. The transverse distribution of moments to column and middle strips, is the same for both methods

4.10 EQUIVALENT-FRAME METHODS

- **4.10.1 Classic Equivalent-Frame Analysis of Slab Systems for Vertical Loads**
- The slab is divided into a series of equivalent frames running in two directions of the building as shown in Figure (SNS).
- These frames consist of the slab, any beams that are present, and columns above and below the slab.
- For gravity load analysis, the code allows analysis of entire equivalent frame extending over the height of the building, or each floor can be considered separately with the far ends of the columns being fixed.

4.10 EQUIVALENT-FRAME METHODS

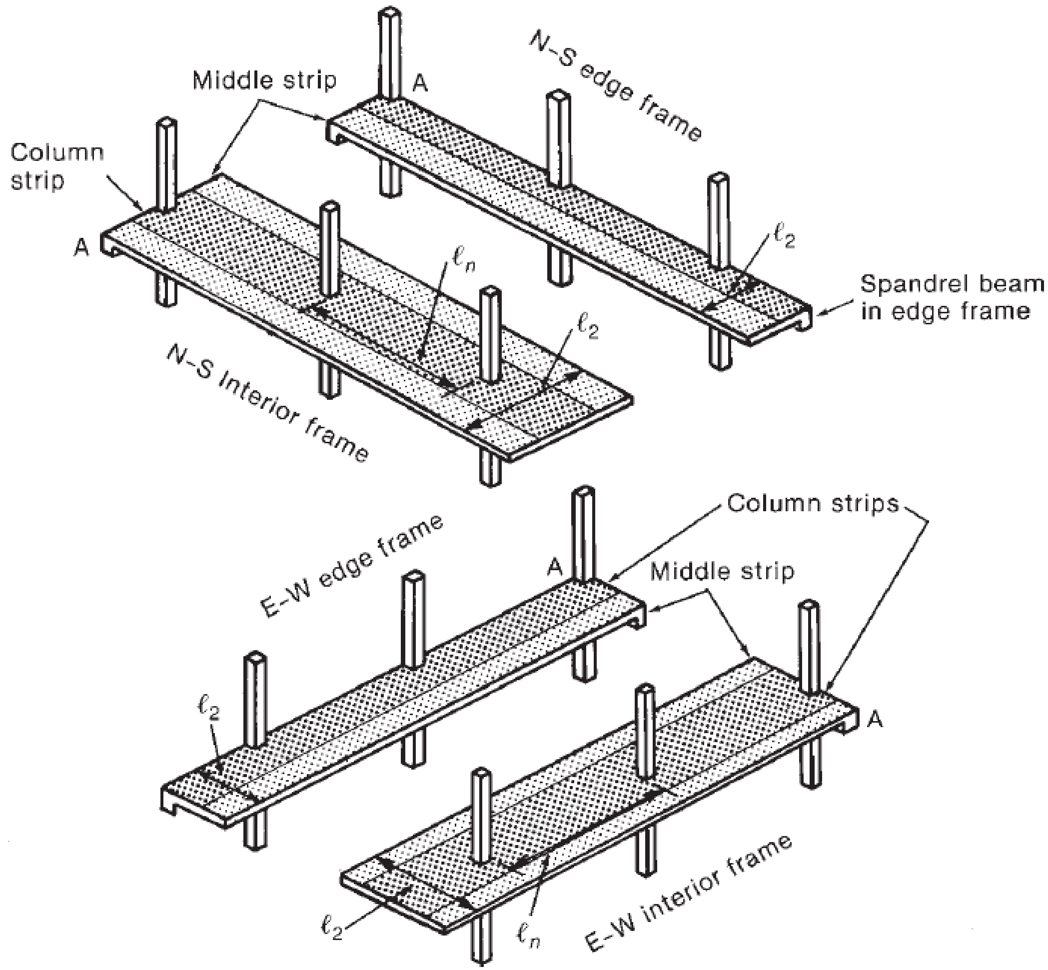


Fig. 13-20
Division of slab into frames for design.

4.10 EQUIVALENT-FRAME METHODS

- The original derivation of the classic equivalent-frame method assumed that the moment distribution would be the calculation procedure used to analyze the continuous-slab system, so some of the concepts in the method are awkward to adapt to other methods of analysis.
- **(i) Calculation of Stiffness, Carryover, and Fixed-End Moments**
- In the moment distribution method, it is necessary to compute flexural stiffnesses, K; carry-over factors, COF; distribution factors, DF; and fixed-end moments, for each members in the structure (read..)

4.10 EQUIVALENT-FRAME METHODS

- In the equivalent-frame method, the increased stiffness of members within the column-slab joint region is accounted for, as is the variation in cross section at drop panels. As a result, all members have a stiffer section at each end, as shown in Figure (SNS)
- **(ii) Properties of Slab-Beams**
- The horizontal members in the equivalent frame are referred to as slab-beams. These consist of either only a slab, or a slab and a drop panel, or a slab with a beam running parallel to the equivalent frame

4.10 EQUIVALENT-FRAME METHODS

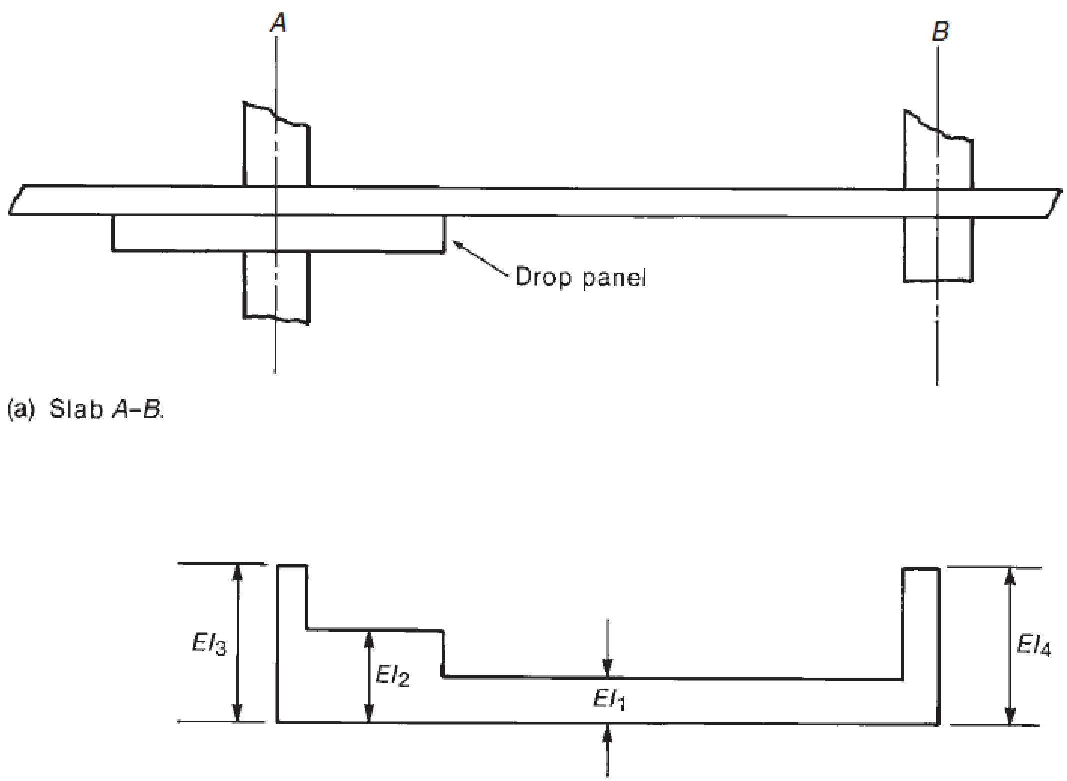


Fig. 13-31
Variation in stiffness along a span.

(b) Distribution of EI along slab.

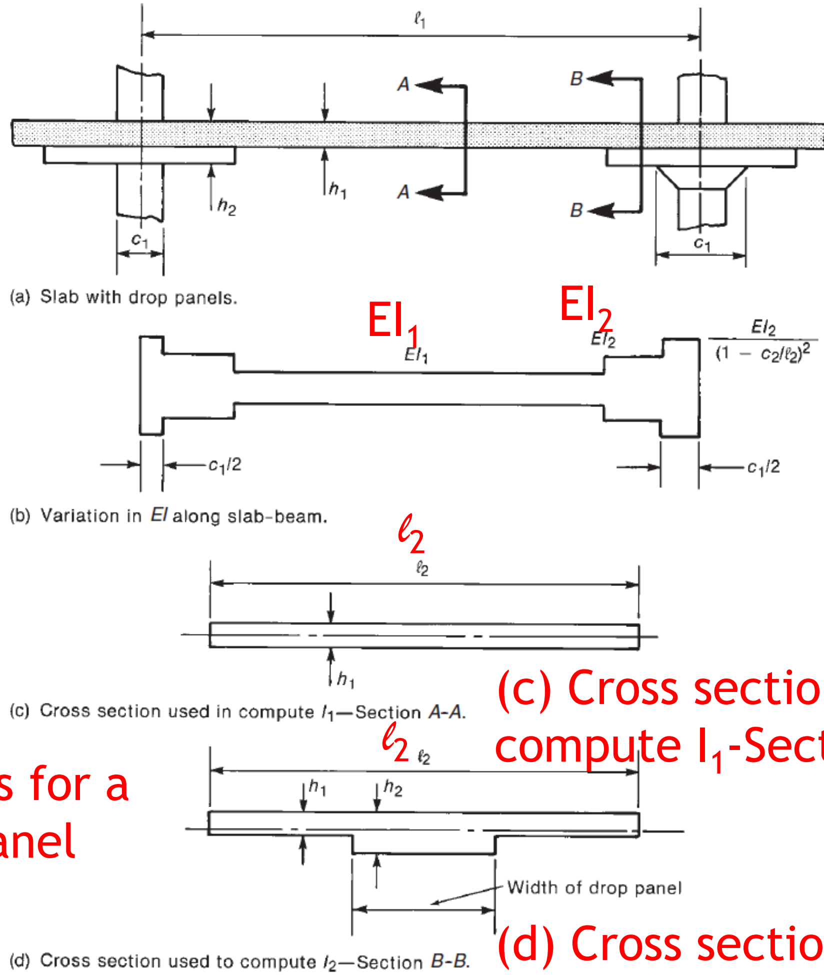
(b) Distribution of EI along slab

Fig. 13-31 Variation in stiffness along a span

4.10 EQUIVALENT-FRAME METHODS

- ACI Code Section 14.7.3 explains how these nonprismatic beams are to be modeled for analysis: (Read)
- The application of the approach is illustrated in Figures (SNS). Tables A-14 through A-16 etc present moment-distribution constants for flat plates and for slabs with drop panels.
- Example- Calculation of the Moment-Distribution Constants for Flat-Plate Floor (Read pp 670, and 672)

4.10 EQUIVALENT-FRAME METHODS



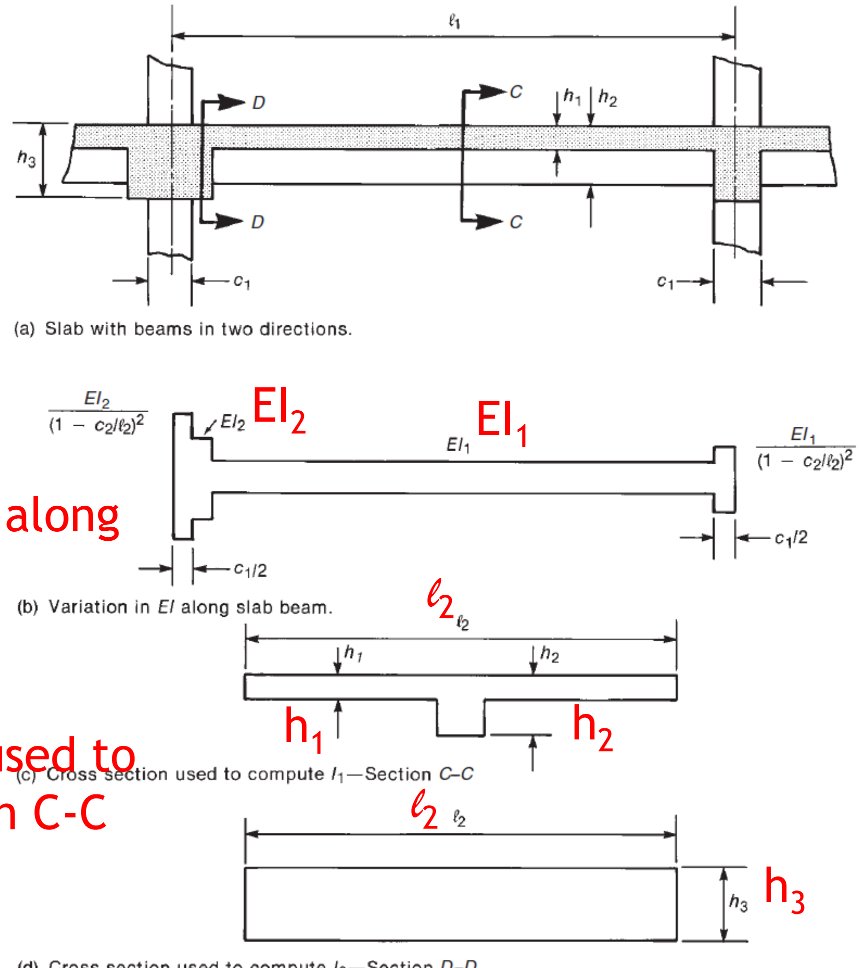
(c) Cross section used to compute I_1-Section A-A

Fig. 13-32 EI values for a slab with a drop panel

Fig. 13-32
EI values for a slab with a drop panel.

(d) Cross section used to compute I_2-Section B-B

4.10 EQUIVALENT-FRAME METHODS



(b) Variation of EI along slab beam

(c) Cross section used to compute I_1-Section C-C

Fig. 13-33
EI values for a slab and beam.

(d) Cross section used to compute I_1-Section D-D

4.9 DESIGN OF TWO-WAY SHEAR ACCORDING TO EC2

- **(iii) Properties of Columns**
- In computing the stiffnesses and carryover factors for columns, ACI Code Section 14.7.4 states the following:
- 1. The moment of inertia of columns at any section outside of the joints or column capitals may be based on the gross area of the concrete, allowing for variations in the actual moment of inertia due to changes in the column cross section along the length of the column (SNS) (read)

4.10 EQUIVALENT-FRAME METHODS

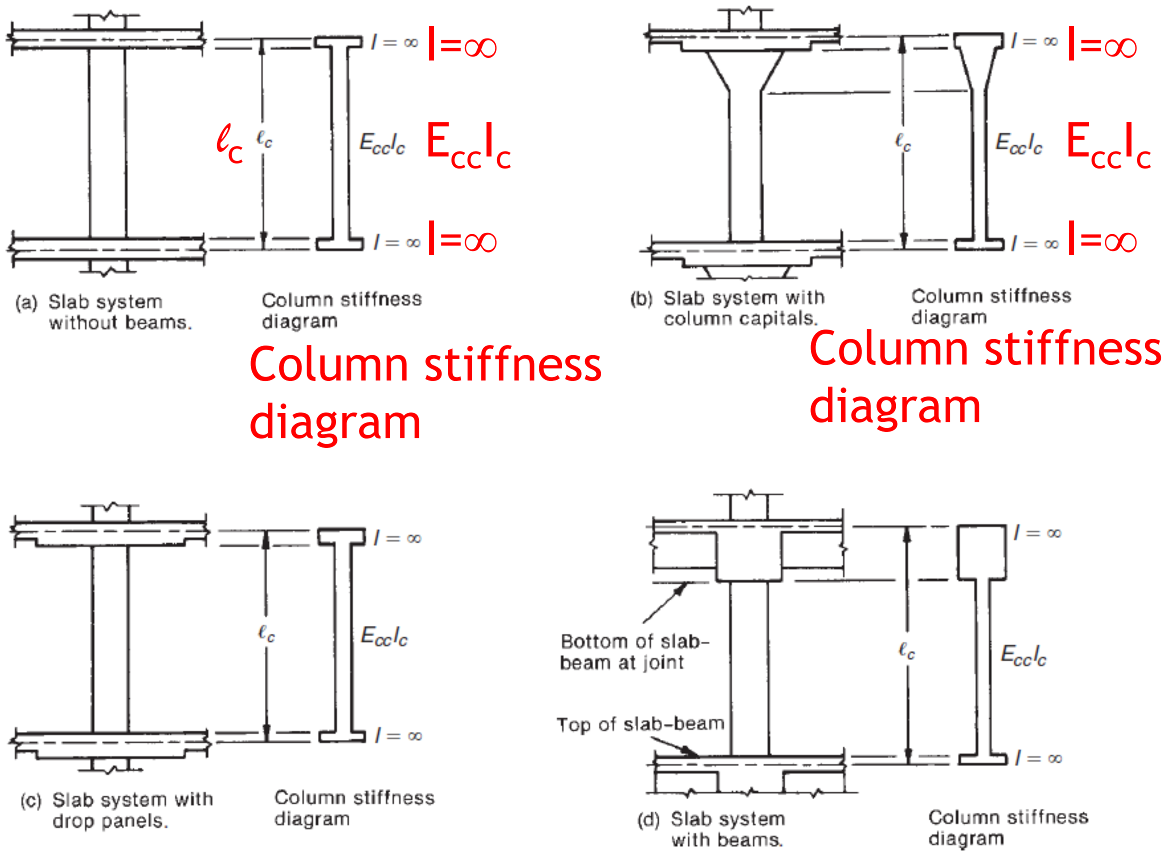


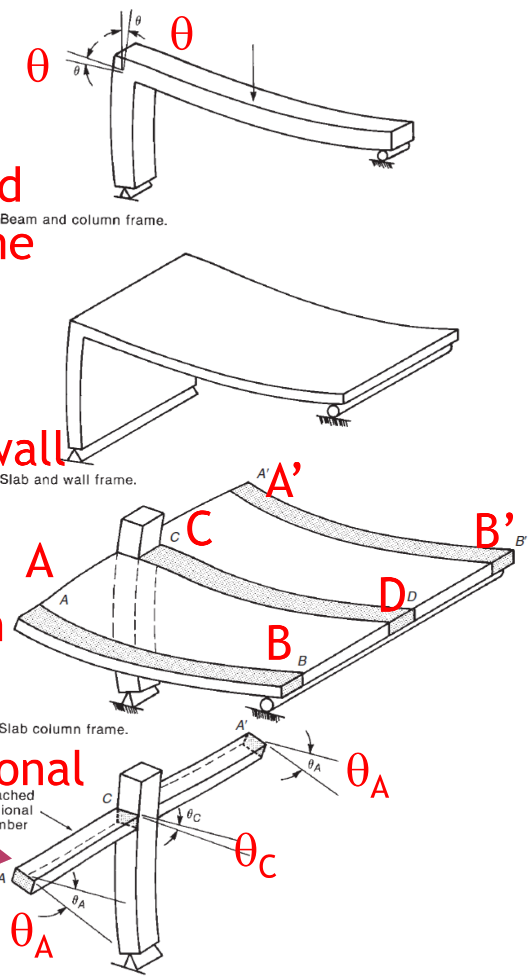
Fig. 13-37 Sections for the calculations of column stiffness

Fig. 13-37
Sections for the calculations of column stiffness, K_c.

4.10 EQUIVALENT-FRAME METHODS

- **(iv) Torsional Members and Equivalent Columns**
- When the beam and column frame shown in Figure (SNS) is loaded, the ends of the column and beam undergo equal rotations where they meet at the joint. If the flexural stiffness, $K=M/\theta$, is known for the two members, it is possible to calculate the joint rotations and the end moments in the members. Similarly, in the case shown in Figure (b), the ends of the slab and the wall both undergo equal end rotations when the slab is loaded

4.10 EQUIVALENT-FRAME METHODS



(a) Beam and column frame

(b) Slab and wall frame

(c) Slab column frame

Attached torsional member

Fig. 13-38 Frame action and twisting of edge member

Fig. 13-38
Frame action and twisting of edge member.

4.10 EQUIVALENT-FRAME METHODS

- When a flat plate is connected to a column, as shown in Fig(c), the end rotation of the column is equal to the end rotation of the strip of slab C-D, which is attached to the column.
- The rotation at A of the strip A-B is greater than the rotation at point C, however, because there is less restraint to the rotation of the slab at this point
- In effect the edge of the slab is twisted, as shown in Fig (d)

4.10 EQUIVALENT-FRAME METHODS

- As a result, the average rotation of the edge of the slab is greater than the rotation of the end of the column
- To account for this effect in slab analysis, the column is assumed to be attached to the slab-beam by the transverse torsional members A-C and C-A’. One way of including these members in the analysis is by use of the concept of an equivalent column, which is a single element consisting of the columns above and below the floor and attached torsional members, as shown in Figure (d).
- Go to S.187

4.10 EQUIVALENT-FRAME METHODS

- The stiffness of the equivalent column, K_{ec}, represents the combined stiffness of the columns and attached torsional members:
  - $K_{ec} = M/$(average rotation of the edge beam)
- The flexibility of the equivalent column, $1/K_{ec}$, is equal to the average rotation of the joint b/n the "edge beam" and the rest of the slab when a unit moment is transferred from the slab to the equivalent column.
- This average rotation is the rotation of the end of the columns, θ_c, plus the average twist of the beam, $\theta_{t,avg}$, with both computed for a unit moment
  - $\theta_{ec} = \theta_c + \theta_{t,avg}$

4.10 EQUIVALENT-FRAME METHODS

- The value of θ_c for a unit moment is $1/\Sigma K_c$, where ΣK_c refers to the sum of the flexural stiffnesses of the columns above and below the slab.
- Similarly, the value $\theta_{t,avg}$ for a unit moment is $1/K_t$, where K_t is the torsional stiffness of the attached torsional members. Substituting:

$$\frac{1}{K_{ec}} = \frac{1}{\Sigma K_c} + \frac{1}{K_t}$$

4.10 EQUIVALENT-FRAME METHODS

- If the torsional stiffness of the attached torsional members is small, K_{ec} will be much smaller than ΣK_c
- The derivation of the torsional stiffness of the torsional members (or edge beams) is illustrated in Figure (SNS).
- Figure (a) shows an equivalent column with attached torsional members that extend halfway to the next column in each direction.
- A unit torque, T=1, is applied to the equivalent column with half going to each arm. Linear torque distribution t, per unit length is assumed as shown in Figure (b)

4.10 EQUIVALENT-FRAME METHODS

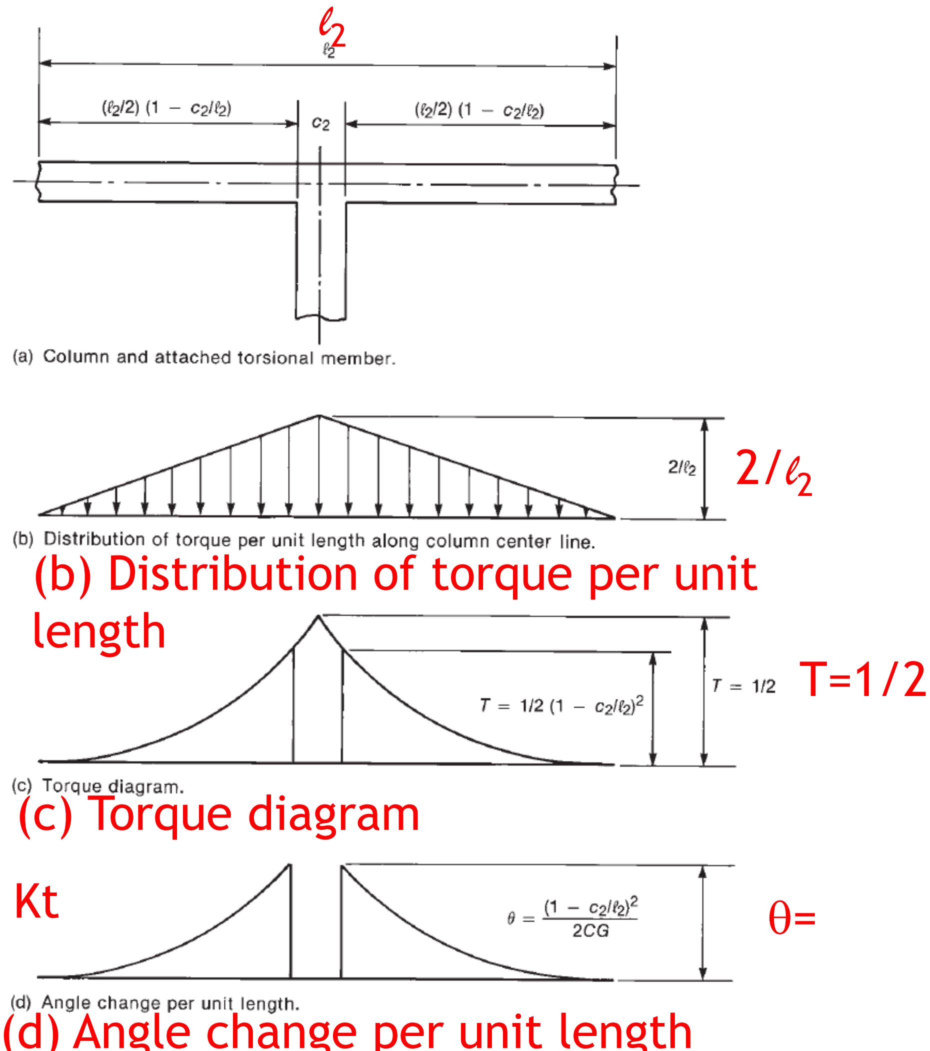


Fig. 13-39 Calculation of Kt

Fig. 13-39
Calculation of K_t.
(From [13-17].)

4.10 EQUIVALENT-FRAME METHODS

- The applied torques give rise to the twisting-moment diagram shown in Figure (c). Because half of the torque is applied to each arm, the maximum twisting moment is ½.
- The twist angle per unit length is shown in Figure (d). This is calculated by dividing the twisting moment at any point by CG, the product of the torsional constant, C (similar to a polar moment of inertia), and the modulus of rigidity, G.
- The total twist of the end of an arm relative to the column is the summation of the twists per unit length and is equal to the area of the diagram in Figure (d) (diagram is parabolic) →

4.10 EQUIVALENT-FRAME METHODS

- Area equals 1/3 of the height times the length of the diagram $\rightarrow$

$$\theta_{t,end} = \frac{1}{3}\frac{(1-c_2/l_2)^2}{2CG}\left(\frac{l_2}{2}\left(1-\frac{c_2}{l_2}\right)\right)$$

  - Replacing G with E/2

$$\theta_{t,end} = \frac{l_2(1-c_2/l_2)^3}{6CE}$$

- This is the rotation of the end of the arm. The rotation required for use in Eqn above is the average rotation of the arm, which is assumed to be a third of the end rotations.

4.10 EQUIVALENT-FRAME METHODS

- $\rightarrow$

$$\theta_{t,avg} = \frac{l_2(1-c_2/l_2)^3}{18CE}$$

- Finally, the torsional stiffness of one arm is calculated as $K_t = M/\theta_{t,avg}$, where the moment resisted by one arm is taken as ½, giving:

$$K_t(one-arm) = \frac{9EC}{l_2(1-c_2/l_2)^3}$$

- ACI expresses the torsional stiffness of the two arms as $\rightarrow$

4.10 EQUIVALENT-FRAME METHODS

- $\rightarrow \quad K_t = \Sigma \dfrac{9E_{cs}C}{l_2(1-c_2/l_2)^3}$
- For a corner column there is only one term in the summation.
- The cross section of the torsional members is defined in ACI Code Section 13.7.5 and is illustrated in Figure (SNS)

4.10 EQUIVALENT-FRAME METHODS

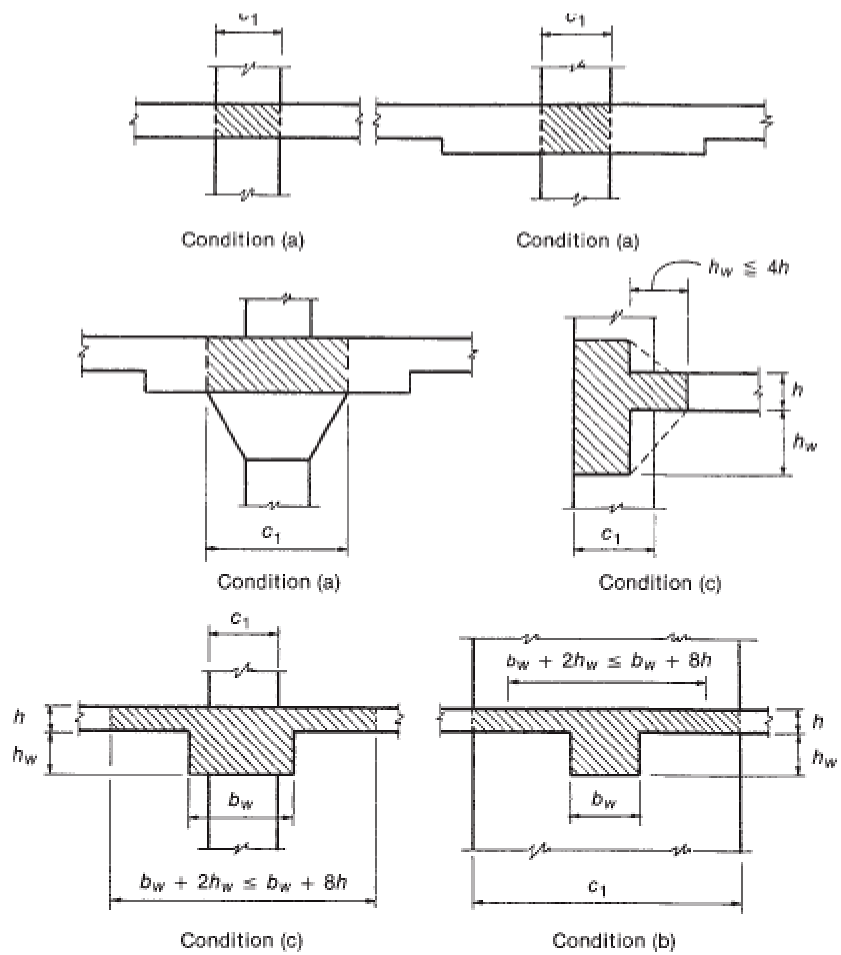


Fig. 13-40
Torsional members. (From [13-15], courtesy of the Portland Cement Association.)

4.10 EQUIVALENT-FRAME METHODS

- The constant C in Eqns above is calculated by subdividing the cross section into rectangles and carrying the out the summation

$$C = \sum \left((1 - 0.63\frac{x}{y})\frac{x^3 y}{3} \right)$$

  - Where x is the shorter side of a rectangle and y is the longer side.

- Read example 13-7 and 13-8 –Calculation of K_t, ΣK_c, and K_{ec}

4.10 EQUIVALENT-FRAME METHODS

- If a beam parallel to the ℓ_1 direction (a beam along C-D in Figure 13-38) frames into the column, a major fraction of the exterior negative moment is transferred directly to the column w/o involving the attached torsional member. In such a case, K_{ec} underestimates the stiffness of the column.
- This is allowed for empirically by multiplying K_t by the ratio I_{sb}/I_s, where I_{sb} is the moment of inertia of the slab and beam together and I_s is the moment of inertia of the slab neglecting the beam stem (ACI Code Section13.7.5.2)

4.10 EQUIVALENT-FRAME METHODS

- Arrangement of live loads for structural analysis and moments at face of supports → See example 13-9 for Analysis of a Flat-Plate using the Classic Equivalent-Frame Method
- **Distribution of Moments to Column Strips, Middle Strips, and Beams**
  - Once the negative and positive moments have been determined for each equivalent frame, these are distributed to column and middle strips in the same way as in the DDM.
  - For panels with beams b/n the columns on all sides, the distribution of moments to the column and middle strips according to ACI Code Sections 13.6.4 and 13.6.6 is valid only if $\alpha_{f1} l_2^2 / \alpha_{f2} l_1^2$ falls b/n 0.2 and 5.0. Cases falling outside of this range tend to approach one-way action, and other methods of slab analysis are required

4.10 EQUIVALENT-FRAME METHODS

- **4.10.2 Use of Computers for an Equivalent-Frame Analysis**
- The classic EFM was derived by assuming that the structural analysis would be carried out by hand using the moment-distribution method.
- Thus tables were developed to evaluate fixed-end moments, stiffnesses, and equivalent-column stiffnesses for use in such analysis
- If standard frame analysis software based on the stiffness method is to be used, the torsional member (and the resulting equivalent-column stiffness) defined in the classic EFM will need to be incorporated into the stiffness of either the slab-beam or column elements.

4.10 EQUIVALENT-FRAME METHODS

- The general research direction has been to modify the stiffness of the slab-beam element by defining an effective slab width to reduce the element stiffness, particularly at connections.
- The frame analysis results for gravity loading, obtained using the modified slab-beam elements, should be in reasonable agreement with those obtained from the classic EFM.
- Several researchers have worked on the development of effective slab width models that could be used to define the stiffness of an equivalent beam in a standard frame analysis program for the analysis of slab-column frame subjected to combined vertical and lateral loading. (based on plate theory and exp results)

4.10 EQUIVALENT-FRAME METHODS

- Hueste and Wight$\rightarrow$ the 1st step in building a slab-column frame analysis model is to select an effective slab width that is a fraction, α, of the total slab width, l_2 (avg) as shown in Figs. 13-34 and 13-35 (SNS).
- A wide range of α values have been suggested by various researchers, but Wight prefers to simply use $\alpha = 0.5$ for all positive-bending regions and for negative-bending regions at interior supports.

4.10 EQUIVALENT-FRAME METHODS

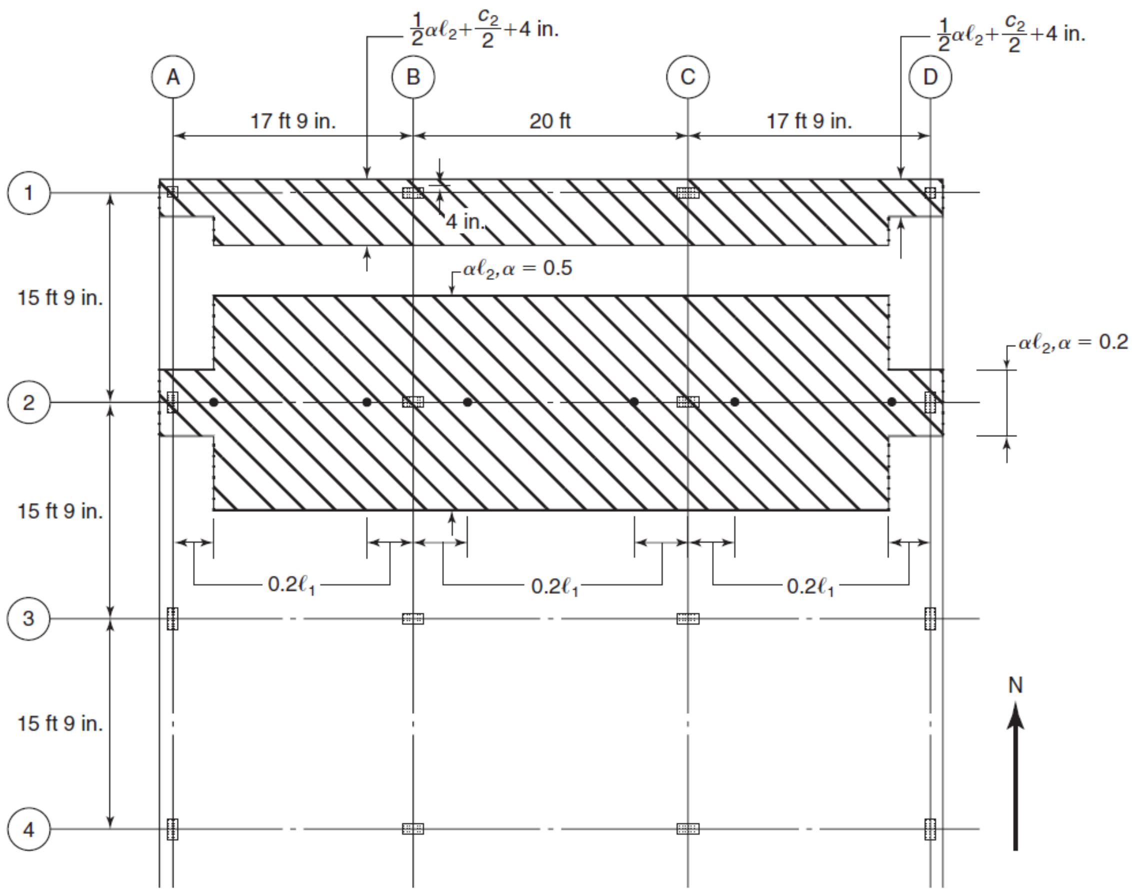


Fig. 13-49
Effective slab width, $\alpha\ell_2$, and locations for intermediate nodes along the span.

4.10 EQUIVALENT-FRAME METHODS

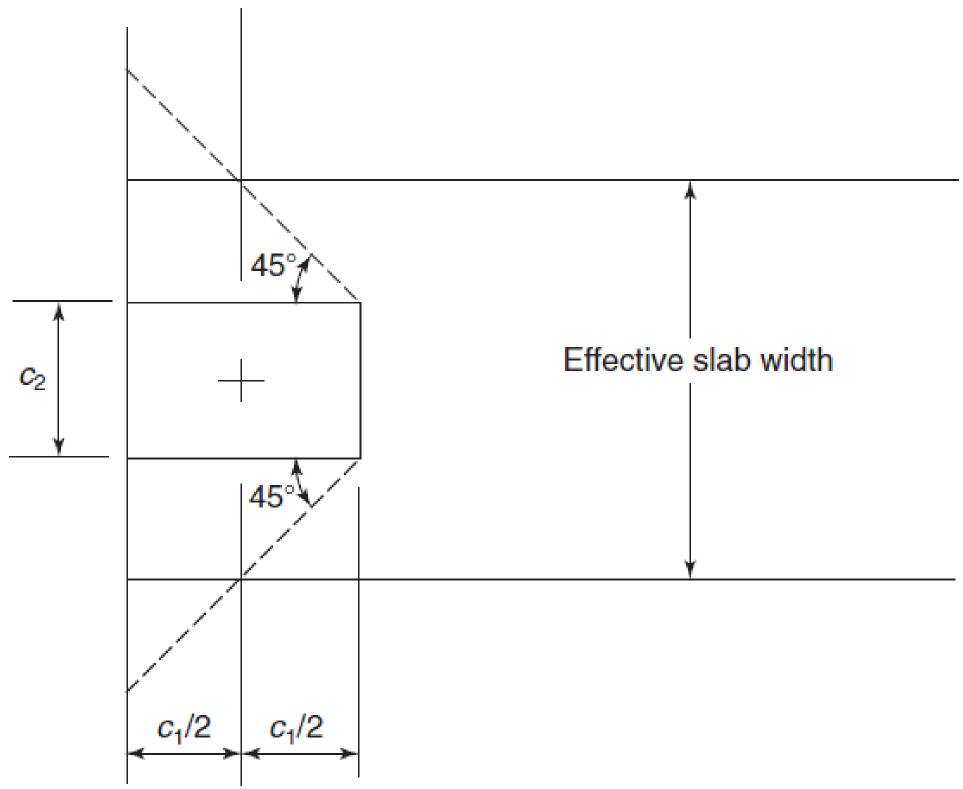


Fig. 13-48
Minimum value for effective slab width at exterior slab-to-column connections.

4.10 EQUIVALENT-FRAME METHODS

- For negative-bending regions at exterior supports, the effective slab width depends on the torsional stiffness at the edge of the slab.
- If no edge beam is present, then an α value of 0.2 is recommended.
- If an edge beam is present and has a torsional stiffness such that β_t, as defined in Eqn (13-12), is greater than or equal to 2.5, then the recommended α value is 0.5

4.10 EQUIVALENT-FRAME METHODS

- If the value of β_t is b/n 0.0 and 2.5, a linear interpolation can be used to find an α value b/n 0.2 and 0.5.
- For low values of α, the effective slab width should not be taken to be less than the column width, c_2, plus one-half of the column total depth, c_1, on each side of the column (Fig. 13-48).
- For slab-column frame along a column line at the edge of a floor plan, the effective slab widths are reduced accordingly.

4.10 EQUIVALENT-FRAME METHODS

- The resulting models for one exterior and one interior column line is shown in Fig. 13-49
- As indicated in Fig. 13-49, the negative-bending region at the exterior connection is assumed to extend over 20 percent ($0.2\ell_1$) of the span. The authors recommend that the same assumption be used for negative-bending regions at all interior and exterior connections.
- This assumption essentially creates extra node points within the span and becomes important when assigning cracked-stiffness values to the positive and negative moment regions

4.10 EQUIVALENT-FRAME METHODS

- After the effective slab width, $\alpha\ell_2$, has been established, the gross moment of inertia for the slab-beam can be calculated using either a section similar to Fig. 13-32 c (if no beam is present) or a section similar to that in Fig 13-33c (if a beam is present). For both cases, the effective slab width, $\alpha\ell_2$, is to be used in place of the ℓ_2 value shown in those figures.
- If a drop panel is present in the negative-bending region, then a section similar to that used in Fig. 13-32d (with $\alpha\ell_2$, in place of ℓ_2) is to be used. Read more.

4.10 EQUIVALENT-FRAME METHODS

- A final modification is to be made to the slab-beam stiffness to account for flexural cracking. In general, the cracked moment of inertia for a slab-beam section, I_{cr}, is some fraction of the gross moment of inertia for that section. Because slabs normally have lower reinforcement ratios than beams, their cracked moment of inertia is usually a smaller fraction of the gross moment of inertia than for a typical beam section. However, because large portions along the slab-beam will remain uncracked and the flexural cracks that do occur usually will not propagate over the entire width of the slab, an effective moment of inertia, I_e, needs to be defined for different portions of the slab-beam span.

4.10 EQUIVALENT-FRAME METHODS

- Commonly, a factor β is used to define the effective moment of inertia as some fraction of the gross moment of inertia ($I_e = \beta I_g$). For all positive-bending regions of the slab, the author recommends $\beta=0.5$.
- Because larger moments typically occur near interior connections, and in order to not overestimate the slab-to-edge beam-to-column stiffness at an exterior connection, whether or not an edge beam is present, the author recommends a β factor of 0.33 for all negative bending regions. (see summary SNS)

4.10 EQUIVALENT-FRAME METHODS

TABLE 13-7 Recommended α and β Values for the Flexural Stiffness of Slab-Beam Elements

Region of the Slab	α-Value (For Effective Width $\alpha\,\ell_2$)	β-Value (For $I_e = \beta\, I_g$)
Positive-bending regions	0.5	0.5
Negative-bending regions (interior columns)	0.5	0.33
Negative-bending regions (exterior columns)	0.2 to 0.5 (function of edge beam stiffness)	0.33

4.10 EQUIVALENT-FRAME METHODS

- For analysis of post-tensioned slabs, wight etal have recommended the use of a β value equal to 0.67 because of the reduced flexural cracking expected in a post tensioned slab.
- For a gravity load analysis, the slab-beam elements can be assembled with column elements that extend one story above and one story below the floor system (F9g 13-50), as permitted by ACI Code Section 13.7.2.5
- The column lengths should be set equal to the center-to-center dimensions from one floor level to the next, and the gross moment of inertia of the column sections can be used as input to the structural analysis software. (moment at face of support etc. Read)

4.10 EQUIVALENT-FRAME METHODS

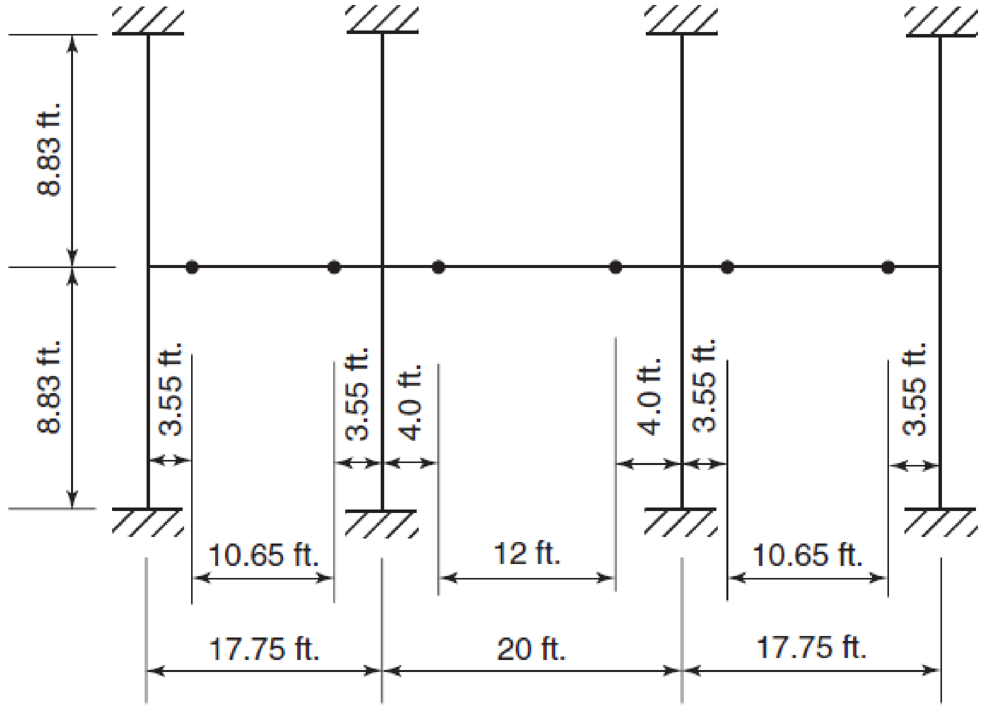


Fig. 13-50
Dimensions and node point locations for slab–column frame analyzed in Example 13-10.

4.10 EQUIVALENT-FRAME METHODS

- **Analysis of Slab-Column Frames for Combined Gravity and Lateral Loads**
- A frame consisting of columns and either flat plates or flat slabs but lacking shear walls or other bracing elements is inefficient in resisting lateral loads and may be subject to significant lateral drift deflections.
- As a result, slab-column frame structures of more than two or three stories are generally braced by shear walls.

4.10 EQUIVALENT-FRAME METHODS

- When unbraced slab-column frames are used, it is necessary to analyze equivalent frame structures for both gravity and lateral loads.
- The general equivalent-frame analysis method discussed previously can be used by simply extending the slab-column frame over the full height of the structure, as shown in Fig. 13-51.
- In order not to overestimate the lateral stiffness of the slab-column frame (and thus underestimate the lateral deflections), the author recommends that the effective moment of inertia of the column sections should be taken as 70 percent of the gross moment of inertia, as required in ACI Code Section 10.10.4.1 for lateral stability analysis.

4.10 EQUIVALENT-FRAME METHODS

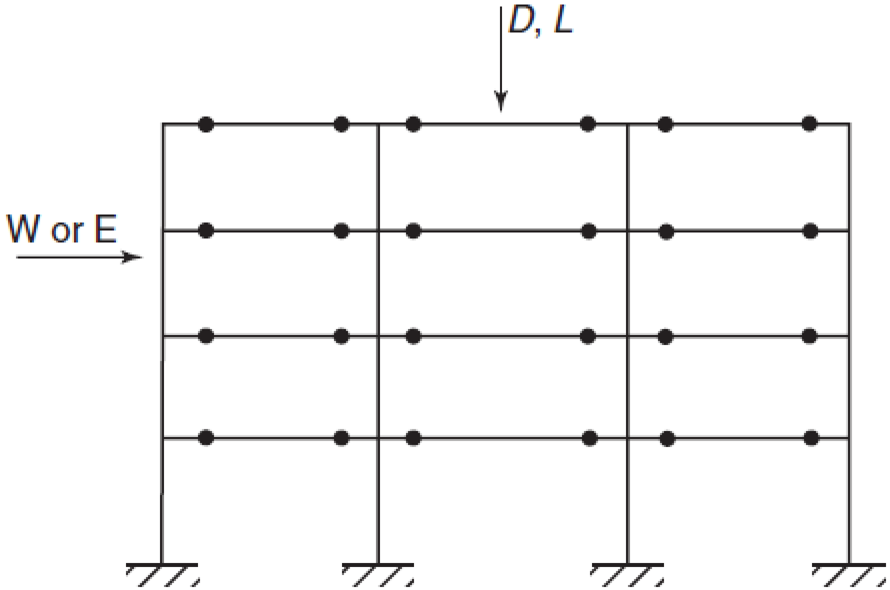


Fig. 13-51
Equivalent-frame model for analysis of slab–column frame subjected to gravity and lateral loads.