CHAPTER 1 DISCONTINUITY REGIONS AND STRUT-AND-TIE MODELS

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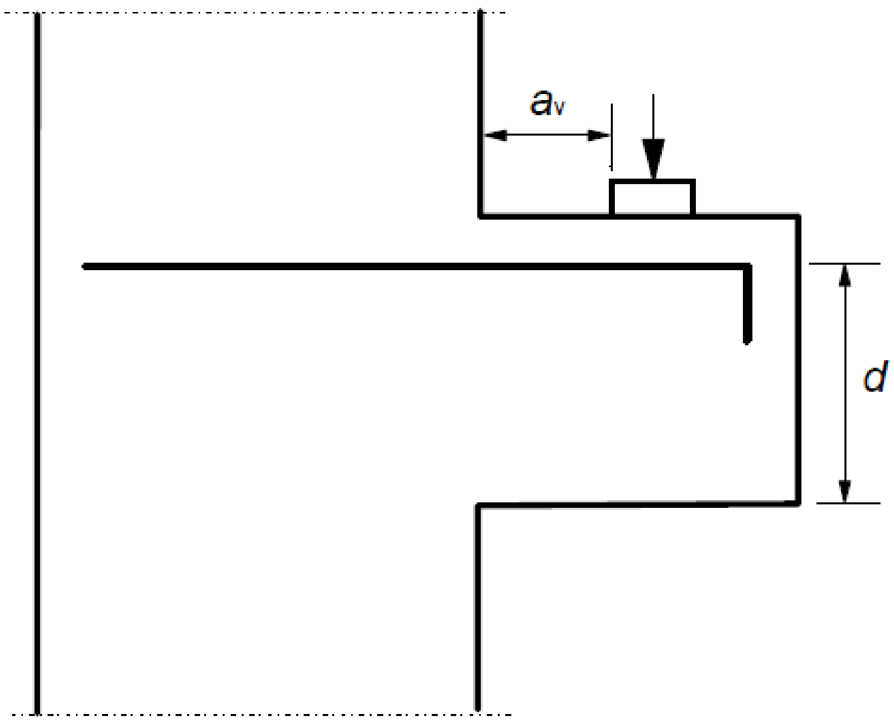
WHEN DO WE DESIGN WITH STRUT AND TIE MODEL?

- Are the following structures different?

CORBEL



(b) Corbel

SINGLE-SPAN DEEP BEAM

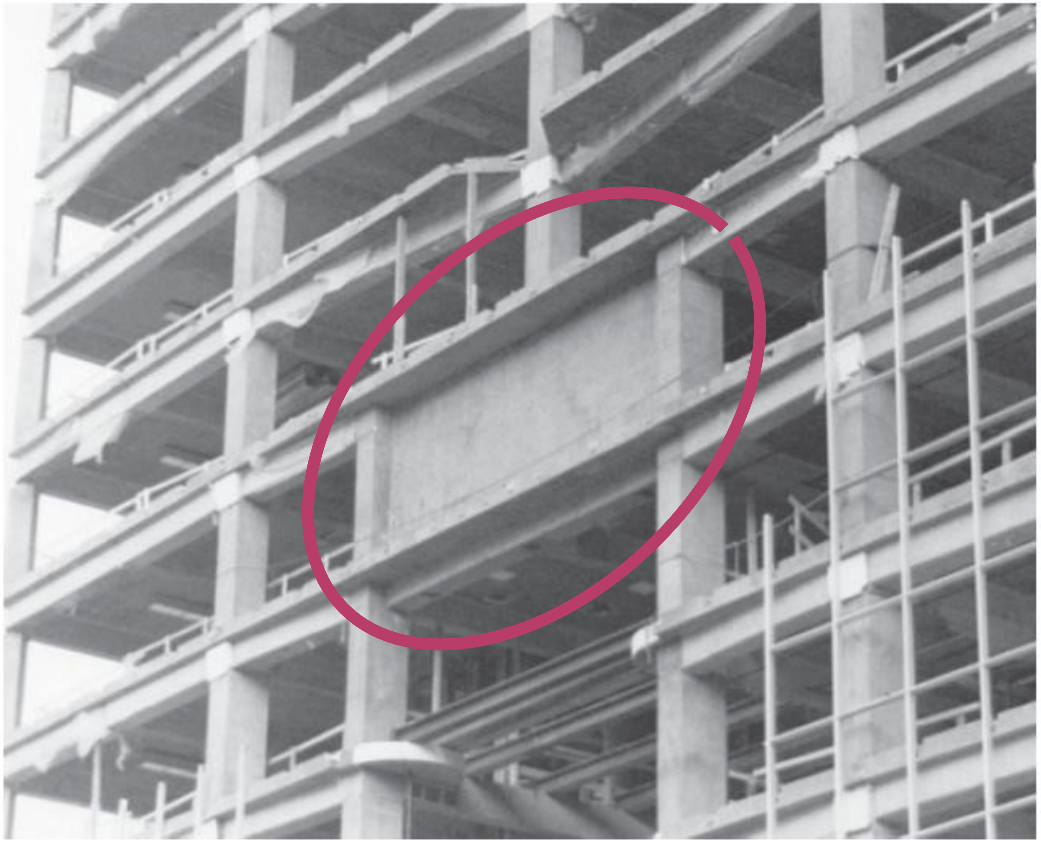
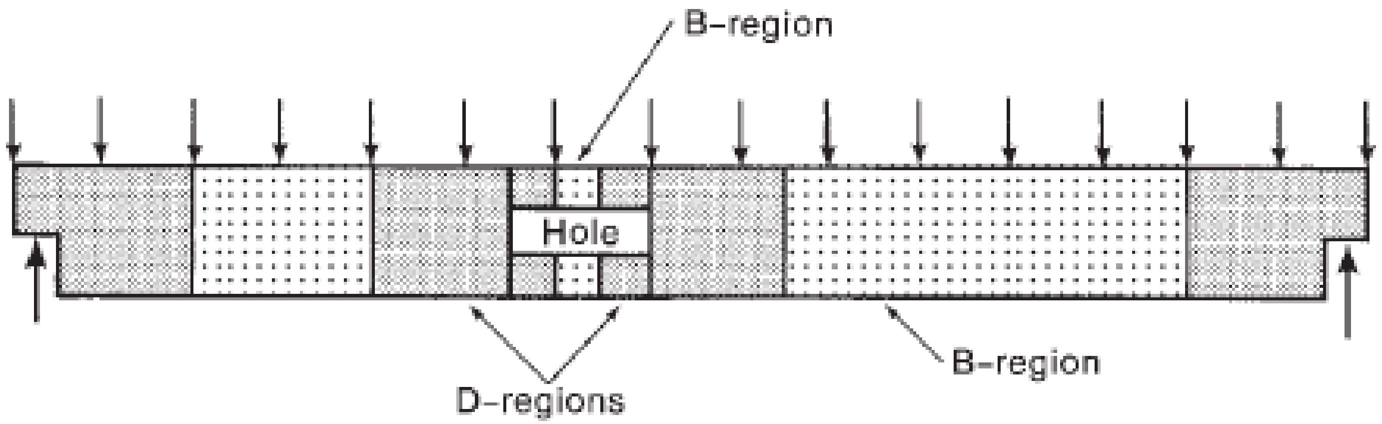


Fig. 17-25
Single-span deep beam.
(Photograph courtesy of J. G. MacGregor.)

DAPPED END BEAM WITH HOLE



(c)

1.1 INTRODUCTION

- Definition of Discontinuity Regions
  - Structural members may be divided into portions called B-regions (B = Beams, or BERNOULLI), in which beam theory applies (linear strain distribution) and other portions called D-regions (D = Discontinuity), where beam theory does not apply and whose load flow should be examined differently.
  - D-regions can be geometric discontinuities, adjacent to holes, abrupt changes in cross section, etc. or statical discontinuities, which are regions near concentrated loads and reactions

1.1 INTRODUCTION

- Corbels, dapped ends, and joints (beam-column) are affected by both statical and geometric discontinuities.
- For many years, D-region design has been by "good practice," by rule of thumb, and empirical.
- Three landmark papers by Prof. Schlaich (1982, 1987, 1991) changed this (according to MacGregor)
- Will present rules and guidance for the design of D-regions based largely on these and other recent papers.

1.1 INTRODUCTION

- Saint Venant's Principle and Extent of D-regions
  - Saint Venant's Principle suggests that the localized effect of a disturbance dies out about one member-depth from the point of the disturbance. It may be the case, however, that the whole structure represents a D-region such as in deep beams or short walls.
  - Figures (next slide) show D-regions in a number of structures (discuss extent of D-Region above hole→ depth of D-region is small → causing small B-region)
  - New ES EN 1992-1-1:2015 allows design with strut and tie models (Section 6.5: 6.5.1 General, 6.5.2 Struts, 6.5.3 Ties, 6.5.4 Nodes S. 107)

1.1 INTRODUCTION

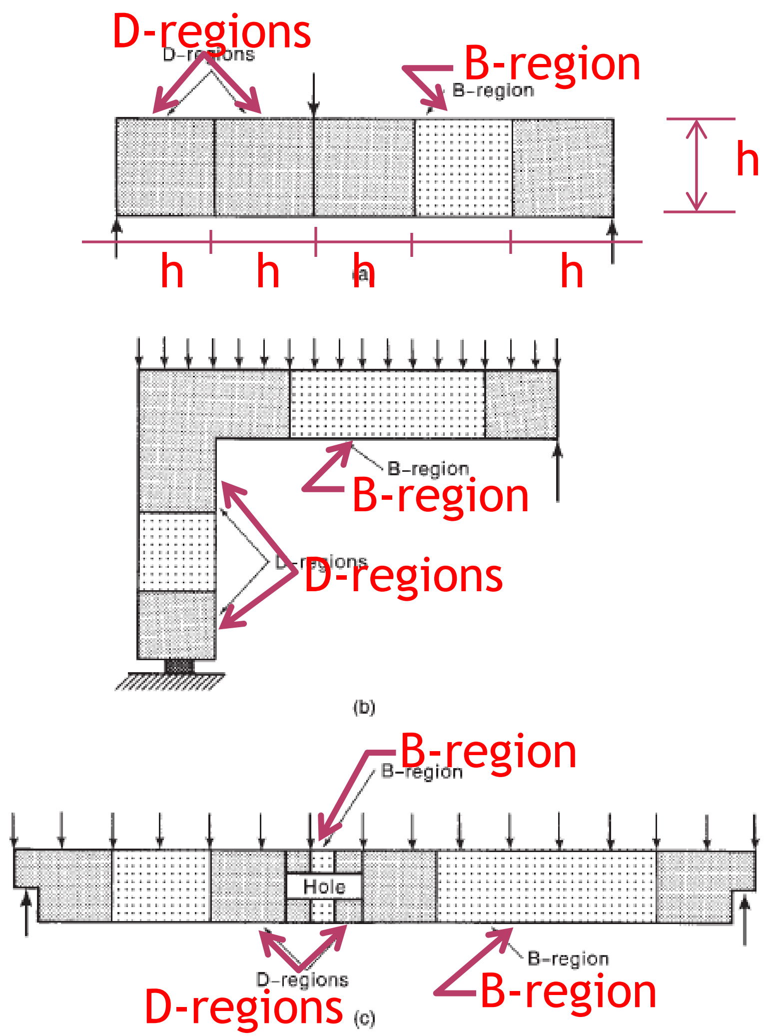


Fig. B-regions and D-regions

Fig. 17-1
B-regions and D-regions.

1.2 STRUT-AND-TIE MODELS

- Strut-and-Tie Models
  - Prior to any cracking, an elastic stress field exists, which can be quantified with an elastic analysis, such as a finite-element analysis.
  - Cracking disrupts this stress field, causing a major reorientation of the internal forces.
  - After cracking, the internal forces can be modeled via a strut-and-tie model consisting of concrete compression **struts**, steel tension **ties**, and joints referred to as **nodal zones**.
  - If the compression struts are narrower at their ends than they are at midsection, the struts may, in turn, crack longitudinally (cracked compression zones → may lead to failure if unreinforced

1.2 STRUT-AND-TIE MODELS

- On the other hand, struts with transverse reinforcement to restrain the cracking can carry further load and will fail by crushing (see figure next slide and ES EN-2 provisions for struts in cracked compression zones)
- With reinforcement for crack control, strut is designed with reduced f_{cd}. (See Figure 6.24: Design strength of concrete struts with transverse tension in ES EN-2: 2015).
- Failure may also occur by yielding of the tension ties, failure of the bar anchorage, or failure of the nodal zones. As always, failure initiated by yield of the steel tension ties tends to be more ductile and is desirable

1.2 STRUT-AND-TIE MODELS

Compression struts failing by crushing

Compression fan in D-region
(Example of struts in cracked compression zone)

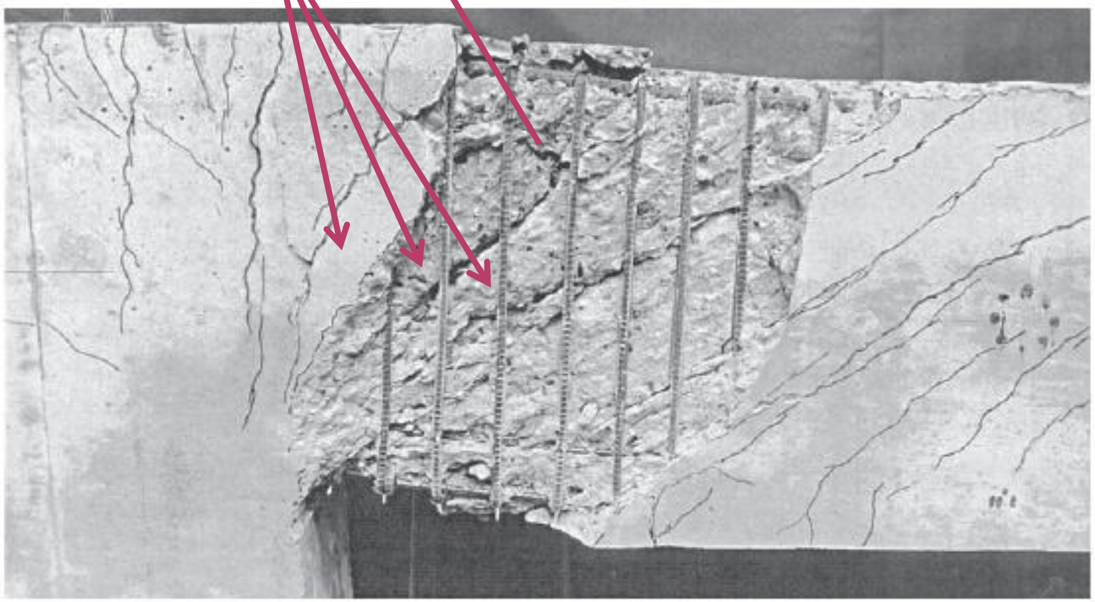
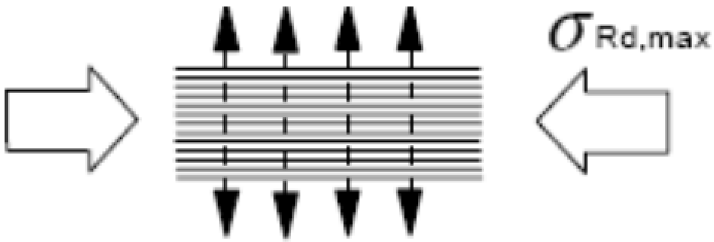


Fig. 6-22
Compression fan at interior support of the beam shown in Fig. 6-21b. (Photograph courtesy of J. G. MacGregor.)

1.2 STRUT-AND-TIE MODELS (ES EN 1992-1-1)

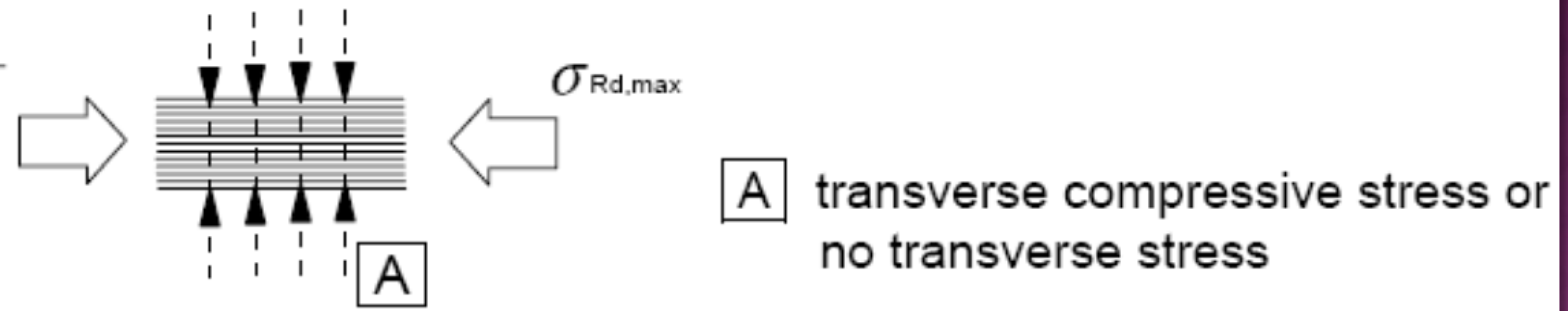


Struts in cracked compression zones, with transverse tension

$$\sigma_{Rd,max} = \upsilon f_{cd}$$

Recommended value $\upsilon = 0{,}60\ (1 - f_{ck}/250)$

1.2 STRUT-AND-TIE MODELS (ES EN 1992-1-1)



Struts with transverse compression stress or zero stress:

$$\sigma_{Rd,max} = f_{cd}$$

1.2 STRUT-AND-TIE MODELS

- A strut-and-tie model for a deep beam is shown in figure (next slide). It consists of two concrete compressive struts, longitudinal reinforcement serving as a tension tie, and joints referred to as nodes.
- The concrete around a node is called a nodal zone.
- The nodal zones transfer the forces from the inclined struts to other struts, to ties and to the reactions.
- ACI Section 11.1.1 (also ES EN-2: 2015) allows D-regions to be designed using strut-and-tie model according to the requirements in ACI Appendix A (2002)(according to the requirements in ES EN-2: 2015) .

1.2 STRUT-AND-TIE MODELS

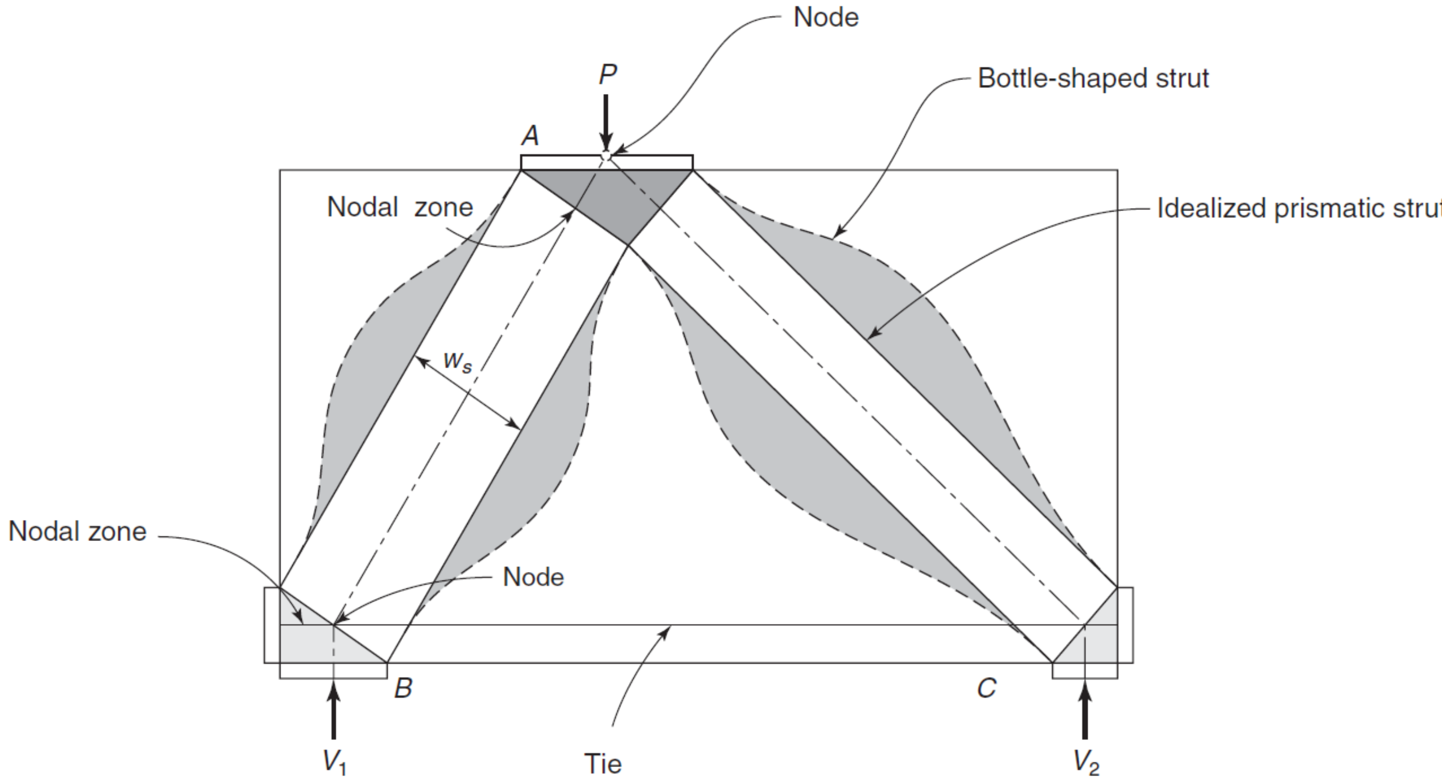


Fig. 17-3
Strut-and-tie model of a deep beam.

1.2 STRUT-AND-TIE MODELS

- A strut-and-tie model is a model of a portion of the structure that satisfies the following:
  - (a) it embodies a system of internal forces that is in equilibrium with a given set of external loads, and
  - (b) the factored-internal member forces at every section in the struts, ties, and nodal zones do not exceed the corresponding factored-member strengths (resistances) for the same sections
- The lower-bound theorem of plasticity states that the capacity of a system of members, supports, and applied forces that satisfies both (a) and (b) is a lower bound on the strength of the structure. (see alternative next slide)

1.2 STRUT-AND-TIE MODELS

- If for a given external loading we can find an internal force system (in the struts, ties, and nodal zones) that satisfies the condition of equilibrium and boundary condition at all points, and the factored-internal member forces at every section in the struts, ties, and nodal zones do not exceed the corresponding factored-member strengths for the same sections, then the applied loads represent a lower bound. (recall parallel for internal force distribution was moment field in slabs)

1.2 STRUT-AND-TIE MODELS

- For the lower-bound theorem to apply, the structure must have:
  - (c) sufficient ductility to make the transition from elastic behavior to enough plastic behavior to redistribute the factored internal forces into a set of forces that satisfy items (a) and (b)
  - The combination of factored loads acting on the structure and the distribution of factored internal forces is a lower bound on the strength of the structure, provided that no element is loaded beyond its capacity.

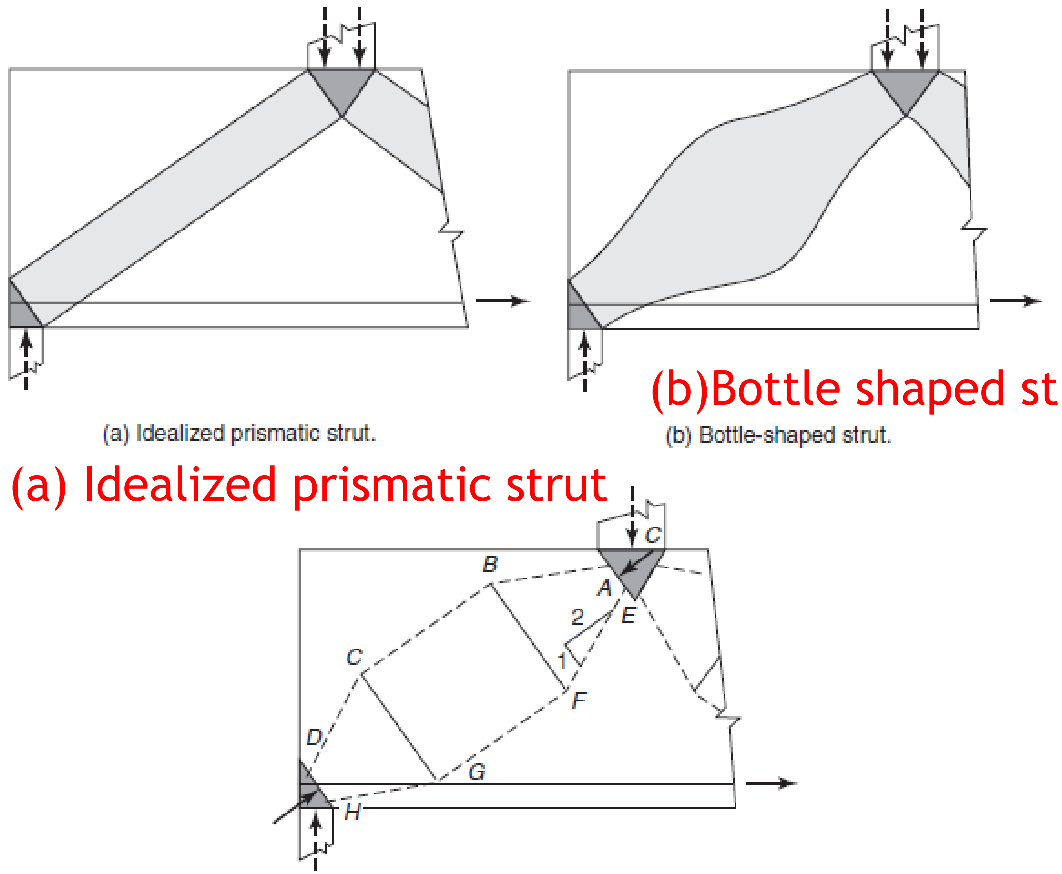
1.2 STRUT-AND-TIE MODELS

- In most applications of strut-and-tie models the internal forces, F_{Ed}, due to the factored loads are determined, and the struts, ties, and nodal zones are proportioned using
  - $F_{Ed} < F_{Rd}$ (design format according to ES EN 1992-1-1) (Design values of the nominal strengths of the struts, ties, and nodal zones are greater than the design value of the internal action effects)

1.3 STRUTS

- In a strut-and-tie model, the struts represent concrete compressive stress fields with the compression stresses acting parallel to the strut. Although they are frequently idealized as prismatic or uniformly tapering members (see fig next slide), struts generally vary in cross section along their length (see figure).
- The spreading of the compression forces gives rise to transverse tensions in the strut which may cause it to crack longitudinally. A strut w/o transverse reinforcement may fail after this cracking occurs. If adequate transverse reinforcement is provided, the strength of the strut will be governed by crushing

1.3 STRUTS



(a) Idealized prismatic strut.

(b) Bottle-shaped strut.

(b)Bottle shaped strut

(a) Idealized prismatic strut

(c) Strut-and-tie model of a bottle-shaped strut.

Fig. 17-4
Bottle-shaped strut.

(c) Strut-and-tie model of a bottle-shaped strut

1.3 STRUT FAILURE BY LONGITUDINAL CRACKING

- 1.3.1 Strut Failure by Longitudinal Cracking
  - Figure (next slide) shows one end of a bottle-shaped strut. The width of the bearing area is a, and the thickness of the strut (into the page) is t.
  - At midlength the strut has an effective width b_{ef}. According to Prof. Schlaich, $b_{ef}=\ell/3$ but not less than a, and ℓ is the length of the strut from face to face of the nodes.
  - $b_{ef} = 0.5H + 0.65a$; $a \leq h$ *(according to ES EN; Fig 6.25; H= length of strut)*

1.3 STRUT FAILURE BY LONGITUDINAL CRACKING

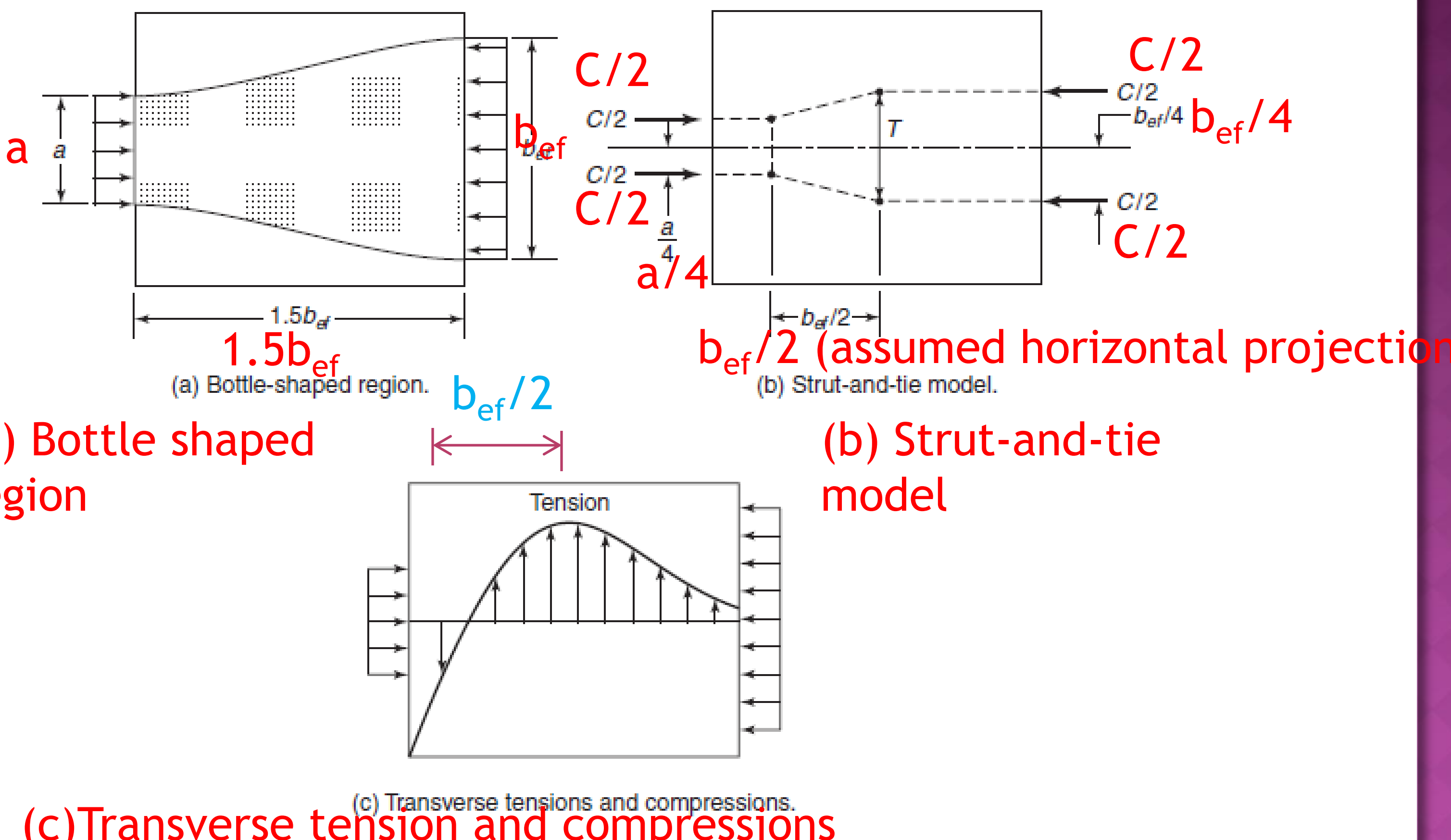


Fig. 17-5
Spread of stresses and transverse tensions in a strut.

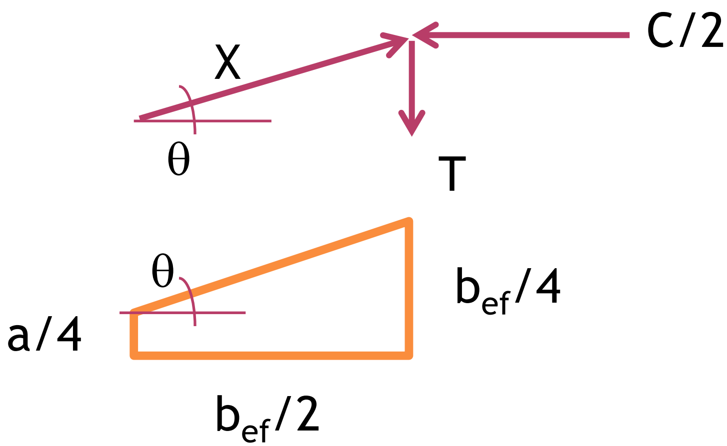
Fig. Spread of stresses and transverse tensions in a strut

1.3 STRUT FAILURE BY LONGITUDINAL CRACKING

- According to the ACI, $\rightarrow$ $b_{ef} = a + \ell/6$ but not more than the available width

- Figs 17-5b show strut-and-tie models for the bottle-shaped region. It is based on the assumption that the longitudinal projection of the inclined struts is equal to $b_{ef}/2$.
- The transverse tension force T at one end of the strut is: [from $\tan\theta = V/H = T/(C/2) = ((b_{ef}/4) - (a/4))/(b_{ef}/2) \rightarrow$ see FBD of the upper node]

1.3 STRUT FAILURE BY LONGITUDINAL CRACKING



- $T = (C/2)((b_{ef}/4)-(a/4))/(b_{ef}/2)$ or simplified
- $T = (C/4)(1-(a/b_{ef}))$ (Eq. 17.3)
- The force T causes transverse stresses in the concrete, which may cause cracking (provide skin or crack control reinforcement as discussed later).
- The distribution of the transverse tensile stress field is shown by the curved line in Fig 17.5c.

1.3 STRUT FAILURE BY LONGITUDINAL CRACKING (ES EN-1992-1-1)

Distinction is made b/n partial and full discontinuity

(3) Reinforcement required to resist the forces at the concentrated nodes may be smeared over a length (see Figure 6.25 a) and b)). When the reinforcement in the node area extends over a considerable length of an element, the reinforcement should be distributed over the length where the compression trajectories are curved (ties and struts). The tensile force T may be obtained by:

a) for partial discontinuity regions $\left(b \le \frac{H}{2}\right)$, see Figure 6.25 a:

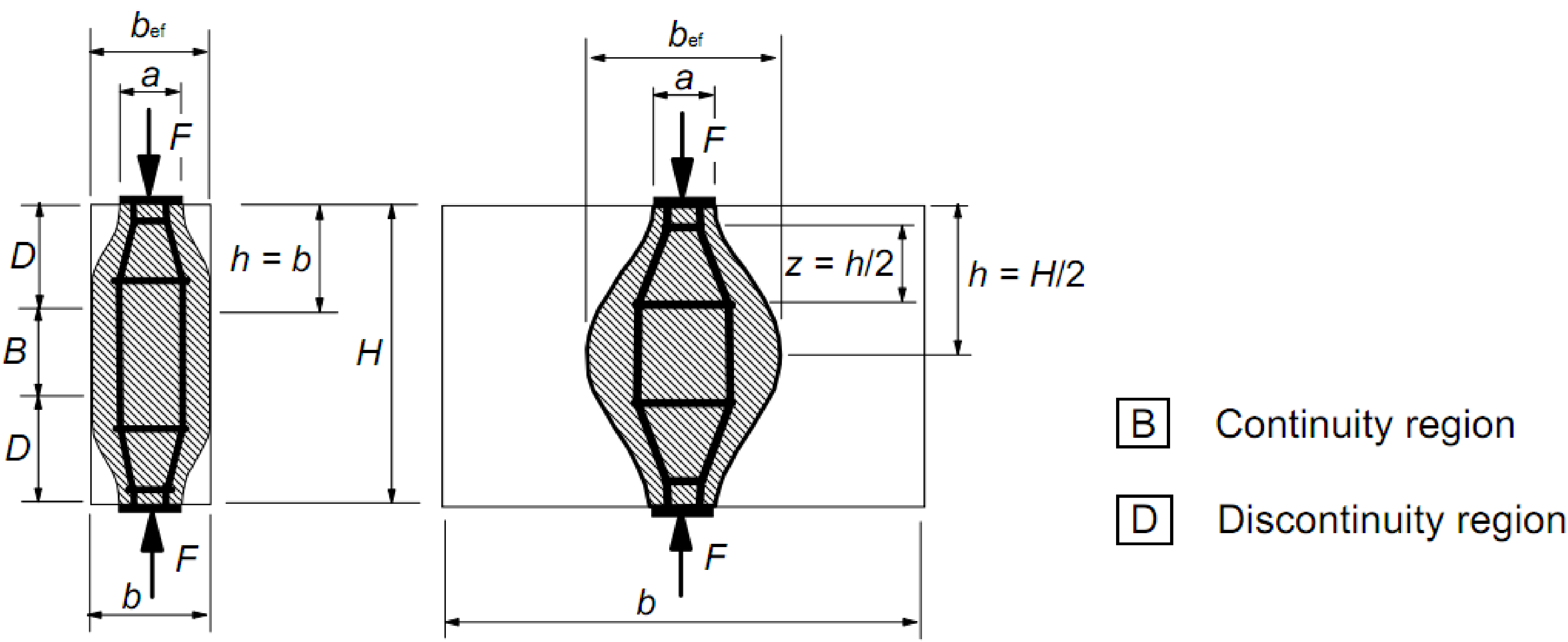
$$T = \frac{1}{4}\frac{b-a}{b}F \qquad (6.58)$$

Identical to Eq. 17.3

b) for full discontinuity regions $\left(b > \frac{H}{2}\right)$, see Figure 6.25 b:

$$T = \frac{1}{4}\left(1 - 0{,}7\frac{a}{h}\right)F \qquad (6.59)$$

1.3 STRUT FAILURE BY LONGITUDINAL CRACKING (ES EN 1992-1-1)



$b_{ef} = b$

$b_{ef} = 0{,}5H + 0{,}65a;\ a \leq h$

a) Partial discontinuity b) Full discontinuity

Figure 6.25: Parameters for the determination of transverse tensile forces in a compression field with smeared reinforcement

1.3 STRUT FAILURE BY LONGITUDINAL CRACKING

- The maximum load on an unreinforced strut in a wall-like member such as the deep beam in Fig 17-3, if governed by cracking of the concrete in the strut, is given by Eq. 17-3.
- This assumes that the compression force spreads in only one direction. If the bearing area does not extend over the full thickness of the member, there will also be transverse tensile stresses through the thickness of the strut that will require reinforcement through the thickness as shown in figure (next slide)

1.3 STRUT FAILURE BY LONGITUDINAL CRACKING

≡ Cracking

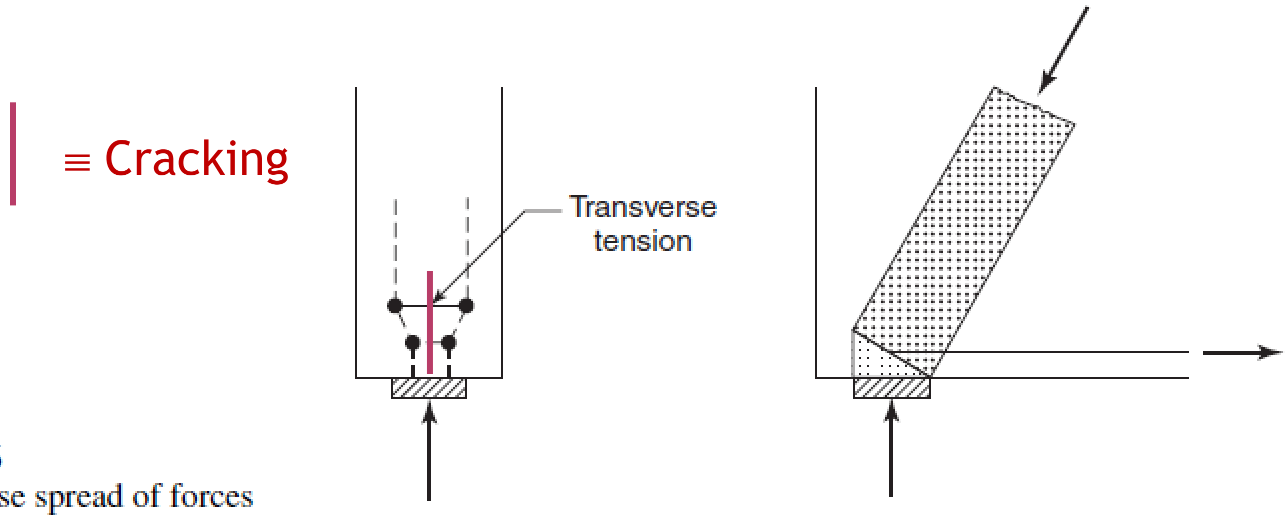


Fig. 17-6
Transverse spread of forces through the thickness of a strut.

(a) End view.

(b) Side view.

1.3 STRUT COMPRESSION FAILURE

- **1.3.2 compression failure of Struts**
- The crushing strength of the concrete in a strut is referred to as the effective strength (design compressive strength $\sigma_{Rd,max}$ in ES EN-2), $f_{ce} = \beta_{(s/n)} f_c'$, where β = the efficiency factor b/n 0 and 1. (Note: $f_c' \equiv f_{ck}$).
- The major factors affecting the effective compressive strength are:
  - (1) **The concrete strength** $\rightarrow$ concrete becomes more brittle and β tends to decrease as the concrete strength increases (same as in ES EN-2 $\rightarrow$ $\sigma_{Rd,max} = k_i \cdot \nu' \cdot f_{cd}$, where $\nu' = 1 - f_{ck}/250$)

1.3 STRUT COMPRESSION FAILURE

- (2) **Load duration effects** $\rightarrow$ The strength of concrete beams and columns tends to be less than the cylinder compressive strength, f_c'. Various reasons are given for this including the observed reduction in compressive strength under sustained load. For struts, load duration effects are accounted for by rewriting Eqn above as: $f_{ce} = 0.85\beta_s f_c'$. For nodal zones β_n is used (**same as in ES EN-2$\rightarrow$0,85 in f_{cd}**)
- (3) **Tensile strains transverse to the strut** resulting from tensile forces in the reinforcement crossing the cracks (**same as in ES EN-2$\rightarrow$** $\sigma_{Rd,max} = 0{,}6 \cdot \nu' \cdot f_{cd}$, where $\nu' = 1 - f_{ck}/250$)

1.3 STRUTS COMPRESSION FAILURE

- (4) **cracked struts:** Struts crossed by cracks inclined to the axis of the strut are weakened by the cracks.

- ACI presents the nominal compressive strength of a strut as: $F_{ns} = f_{ce}A_c$ (n=nominal, s=strut, A_c=cross sectional area at the end of the strut, and $f_{ce}= 0,85\beta_s f_c'$). For nodal zones:$\rightarrow$ $F_{nn} = f_{ce}A_n$, where $f_{ce}= 0,85\beta_n f_c'$). Values of β_s and β_n are given in Table 17-1 (next slide)

1.3 STRUTS AND NODES EFFECTIVE COMPRESSIVE STRENGTH

TABLE 17-1 ACI Code Values of β_s and β_n for Struts and Nodal Zones

Struts, $f_{ce} = 0.85\beta_s f'_c$	
ACI Section A.3.2.1 For struts in which the area of the midsection cross section is the same as the area at the nodes, such as the compression zone of a beam.	$\beta_s = 1.0$
ACI Section A.3.2.2 For struts located such that the width of the midsection of the strut is larger than the width at the nodes (bottle-shaped struts):	
(a) with reinforcement satisfying A.3.3.	$\beta_s = 0.75$
(b) without reinforcement satisfying A.3.3.	$\beta_s = 0.60\lambda$
ACI Section A.3.2.3 For struts in tension members or the tension flanges of members.	$\beta_s = 0.40$
ACI Section A.3.2.4 For all other cases	$\beta_s = 0.60\lambda$
Nodal zones, $f_{ce} = 0.85\beta_n f'_c$	
ACI Section A.5.2.1 In nodal zones bounded on all sides by struts or bearing areas, or both . .	$\beta_n = 1.0$
ACI Section A.5.2.2 In nodal zones anchoring a tie in one direction.	$\beta_n = 0.80$
ACI Section A.5.2.3 In nodal zones anchoring two or more ties.	$\beta_n = 0.60$

Cracking compressive force C_{cr}

Not bottle shaped or not in tension flanges (parallel struts in beam webs)

λ = 1.0 for normal weight concrete. Self reading for table explanation

GOTO S50 after design compressive strengths in nodes according to ES EN 1992-1-1

1.3 STRUTS AND NODES EFFECTIVE COMPRESSIVE STRENGTH (ES EN 1992-1-1)

(4) The design values for the compressive stresses within nodes may be determined by:

a) in compression nodes where no ties are anchored at the node (see Figure 6.26)

$$\sigma_{Rd,max} = k_1 \; \nu' f_{cd} \qquad (6.60)$$

Note: The value of k_1 for use in a Country may be found in its National Annex. The recommended value is 1,0.

where $\sigma_{Rd,max}$ is the maximum stress which can be applied at the edges of the node. See 6.5.2 (2) for definition of ν'.

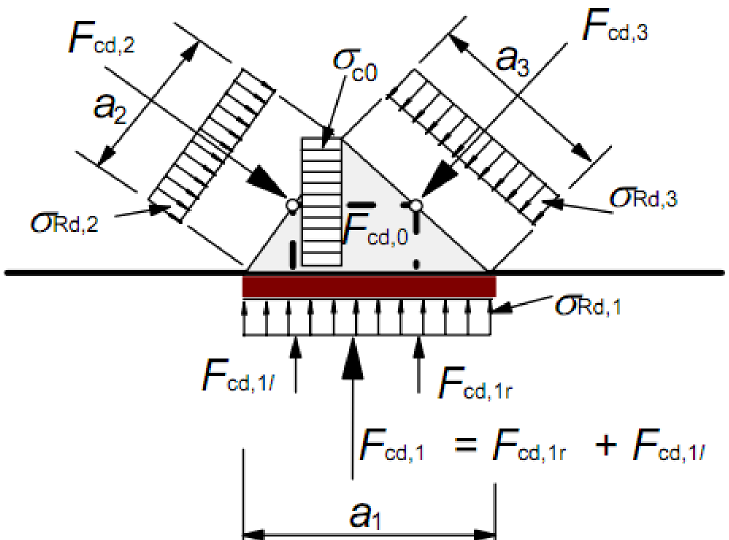


Figure 6.26: Compression node without ties

1.3 STRUTS AND NODES EFFECTIVE COMPRESSIVE STRENGTH (ES EN 1992-1-1)

b) in compression - tension nodes with anchored ties provided in one direction (see Figure 6.27),

$$\sigma_{Rd,max} = k_2 \nu' f_{cd} \qquad (6.61)$$

Recommended k_2 = 0,85

where $\sigma_{Rd,max}$ is the maximum of $\sigma_{Rd,1}$ and $\sigma_{Rd,2}$, See 6.5.2 (2) for definition of ν'.

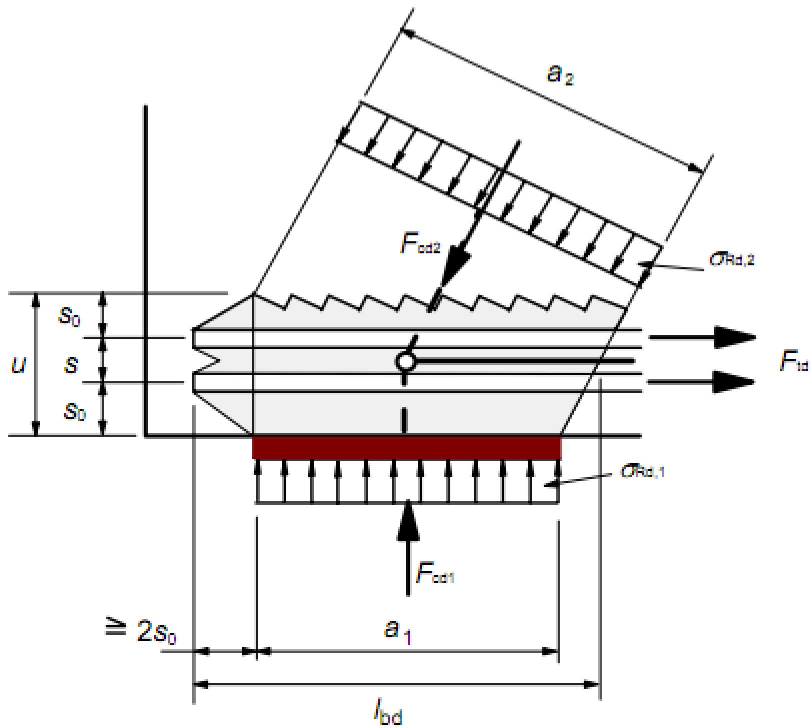


Figure 6.27: Compression tension node with reinforcement provided in one direction

1.3 STRUTS AND NODES EFFECTIVE COMPRESSIVE STRENGTH (ES EN 1992-1-1)

c) in compression - tension nodes with anchored ties provided in more than one direction (see Figure 6.28),

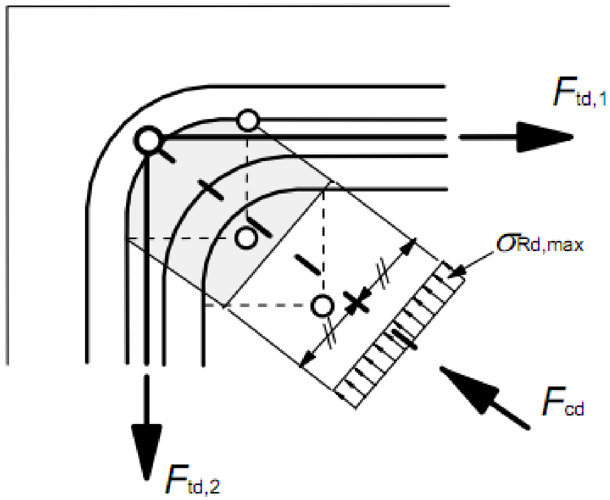


Figure 6.28: Compression tension node with reinforcement provided in two directions

$$\sigma_{Rd,max} = k_3 \ \nu' f_{cd} \tag{6.62}$$

Note: The value of k_3 for use in a Country may be found in its National Annex. The recommended value is 0,75.

1.3 STRUTS COMPRESSION FAILURE

- Explanation of Types of Struts Described in Table 17-1
  - **Case A.3.2.1** applies to a strut equivalent to a rectangular stress block of depth, a, and thickness, b, as occurs in the compression zones of beams or eccentrically loaded columns. In this case $\beta_s = 1.0$. The strut is assumed to have a depth of a and the resultant compressive force in the rectangular stress block, $C=f_{ce}ab$, acts at a/2 as shown in figure (next slide)

1.3 STRUTS COMPRESSION FAILURE

$f_{ce}=0.85\beta_s f_c' = 0.85f_c'$

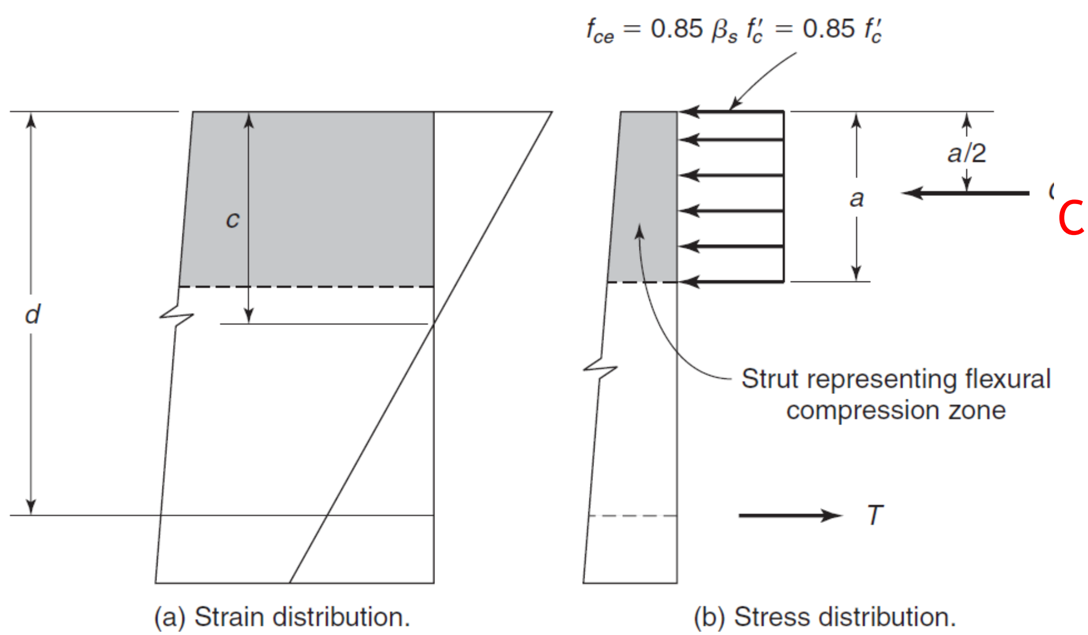


Fig. 17-7
Strut representing the compression stress block in a beam

1.3 STRUTS COMPRESSION FAILURE

- **Case A.3.2.2(a)** applies to bottle-shaped struts similar to those in Fig. 17-4b which contain reinforcement crossing the potential splitting cracks. Although such struts tend to split longitudinally, the opening of a splitting crack is restrained by the reinforcement allowing the strut to carry additional load after the splitting cracks develop. For this case $\beta_s = 0.75$. If there is no reinforcement to restrain the opening of the crack, the strut is assumed to fail upon cracking, or shortly after, and a lower value of β_s is used.

1.3 STRUTS COMPRESSION FAILURE

- **Case A.3.2.2(a)** cont'd →
- The yield strength of the reinforcement required to restrain the crack is computed using a localized strut-and-tie model of the cracking in the strut as shown in Fig. 17-4c. The slope of the load-spreading struts is taken as slightly less than 2 to 1 (parallel to axis of strut, to ⊥ to axis): $T_n = (C_n/4)\{1-(a/b_{ef})\}$.
- Setting T_n equal to $A_s f_y$ gives the transverse tension force T_n at the **ends** of the bottle-shaped strut at cracking as: $A_s f_y \geq \Sigma\ (C_n/4)\{1-(a/b_{ef})\}$.(*)

1.3 STRUTS COMPRESSION FAILURE

- **Case A.3.2.2(a)** cont'd $\rightarrow$
- Where C_n is the nominal compressive force in the strut and a is the width of the bearing area at the end of the strut as shown in Fig 17-5a.If the reinforcement is at an angle θ to the axis of the strut, A_s should be multiplied by sin θ. This reinforcement will be referred to as crack control reinforcement.
- In lieu of using a strut-and-tie model to compute the necessary amount of crack control reinforcement, ACI allows A_s determined from: $\Sigma\ (A_{si}/b_{si})\ \sin \gamma_i \geq 0.003$ (**) [e.g. $\Sigma\ (A_{s1}/s_1)\ \sin \gamma_1 \geq 0.003$ where Σ means total at the two faces] etc.
- Compare with ES EN 1992-1-1 (minimum skin reinforcement in deep beams, etc).

1.3 STRUTS COMPRESSION FAILURE

- **Case A.3.2.2(a)** cont'd $\rightarrow$
- Where A_{si} refers to the crack control reinforcement adjacent to the two faces of the member at an angle γ_i to the crack, as shown in Figure (next slide)
- The ACI Eq for the crack control reinforcement is written in terms of a reinforcement ratio rather than the force to simplify the presentation. This is acceptable for concrete strengths not exceeding 40 MPa. For higher concrete strengths the committee felt the load-spreading should be computed. NB: A tensile strain in bar 1, ε_{s1}, in Figure results in a tensile strain of $\varepsilon_{s1} \sin\gamma_1 \perp$ to the axis of the strut. Similarly for bar 2 strain $\perp$ to the axis of the strut is $\varepsilon_{s2} \sin\gamma_2$, where $\gamma_1 + \gamma_2 = 90°$

1.3 STRUTS COMPRESSION FAILURE

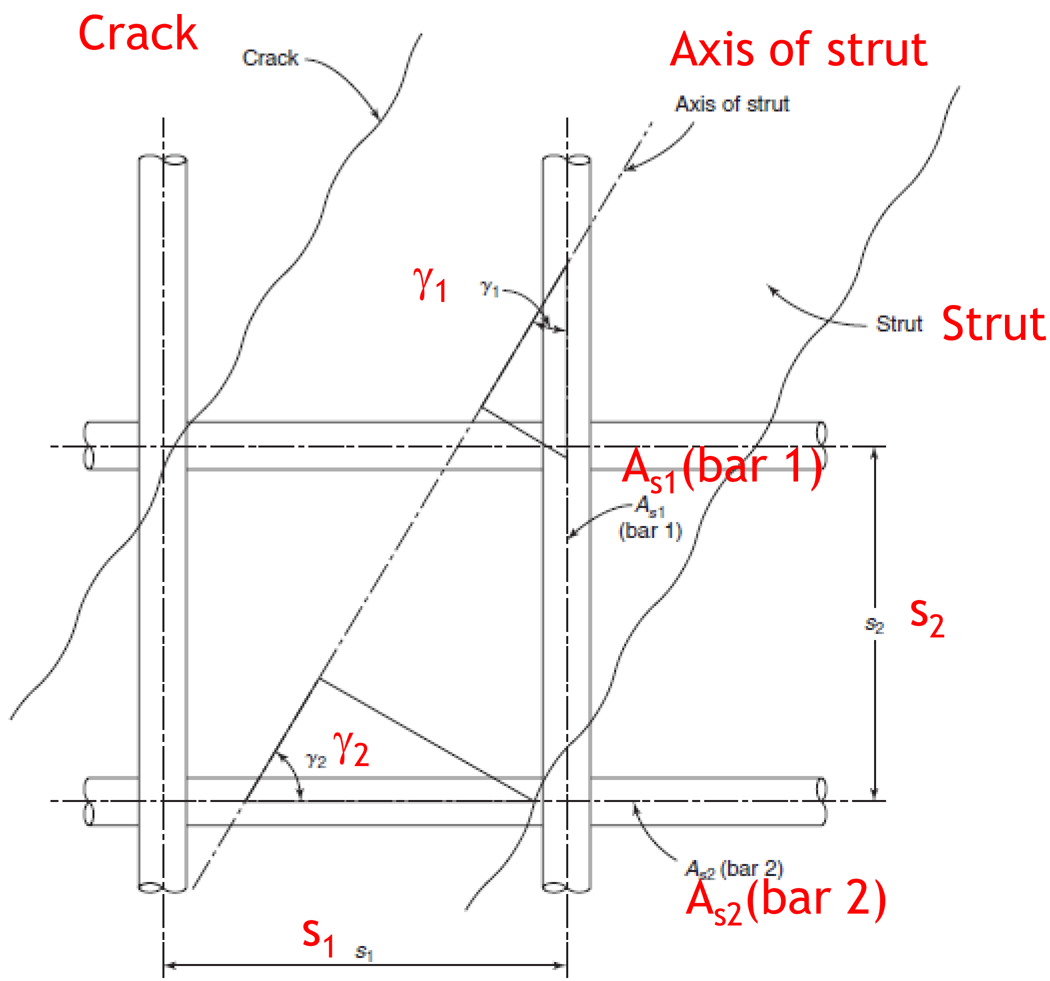


Fig. 17-8
Crack control reinforcement crossing a strut in a cracked web.

1.3 STRUTS COMPRESSION FAILURE

- **Case A.3.2.2(b)** In mass concrete members such as pile caps for more than two piles it may be difficult to place the crack control reinforcement. ACI Section A.3.2.2(b) specifies a lower value of f_{cu} in such cases.
- Because the struts are assumed to fail shortly after longitudinal cracking occurs, β_s is multiplied by the correction factors, λ, for light weight concrete when such concrete is used. Values of λ are given in ACI Section 11.7.4.3. It is 1.0 for normal- weight concrete.

1.3 STRUTS COMPRESSION FAILURE

- **Case A.3.2.3** The value of β_s in Section A.3.2(3) is used in proportioning struts in strut-and-tie models used to design the reinforcement for the tension flanges of beams, box girders and the like. It accounts for the fact that such cracks will tend to be wider than the cracks in beam webs.
- **Case A.3.2.4** The value of β_s in Section A.3.2(4) applies to all other types of struts not covered above. This includes struts in the web of a beam where more or less parallel cracks divide the web into parallel struts. It also includes struts likely to be crossed by cracks at an angle to the struts (recent publication in ACI)

1.3 STRUTS DESIGN

- **1.3.3 Design of struts**
- Once the strut-and-tie model has been laid out, the strength of a strut is computed as follows:
  - (1) If there is no transverse reinforcement in the strut, the strength should be taken as the compression causing cracking, computed as: $C_{cr} = 0.85(0.60\lambda)f'_c at$ (see ACI Section A.3.2.2(b)), where a and t, respectively are the width and thickness of the nodal zone.
  - (2) If the strut is crossed by reinforcement satisfying minimum requirements ($A_{s,min}$) the nominal strength of the strut is determined based on the smallest cross-sectional area of the strut and on the applicable concrete strength from Table 18-1

1.4 TIES

- The 2nd major component of a strut-and-tie model is the tie. A tie represents one or several layers of reinforcement in the same direction.
- Design is based on: $\phi F_{nt} \geq F_{ut}$; where t refers to "tie," and F_{nt} is the nominal (strength) resistance of the tie, taken as: $F_{nt} = A_{st}f_y + A_{ps}(f_{se} + \Delta f_p)$. The 2nd term on the RHS is for prestressed ties.
- ACI Section A.4.2 requires that the axis of the reinforcement in a tie coincide with the axis of the tie (Also EN ES EN 1992-1-1:2015)

1.4 TIES

- In the layout of the strut-and-tie model, ties consist of the reinforcement plus a prism of concrete concentric with the longitudinal reinforcement making up the tie.
- The width of the concrete prism surrounding the tie is referred to as the effective width of the tie, w_t.
- ACI sections A.4.2 and R.A.4.2 give limits on w_t (to meet geometric and stress conditions). Geometric: a width equal to twice the distance from the surface of the concrete to the centroid of the tie reinforcement

1.4 TIES

- In a hydrostatic C-C-T nodal zone, the stresses on all faces of the nodal zone should be equal (i.e., the effective strength of the node f_{ce}). As a result the stress condition on the width of a tie is taken equal to $w_t = F_{nt}/(f_{ce}b)$ (where $F_{nt} = A_s f_{yd}$ is the nominal strength of tie)
- The concrete is included in the tie to establish the widths of the faces of the nodal zones acted on by ties. The concrete in a tie does not resist any load. It aids in the transfer of loads from struts to ties or to bearing areas through bond with reinforcement

1.4 TIES

- The concrete surrounding the tie steel increases the axial stiffness of the tie by tension stiffening. Tension stiffening may be used in modeling the axial stiffness of the ties in a serviceability analysis.
- Ties may fail due to lack of end anchorage. The anchorage of the ties in the nodal zones is a major part of the design of a D-region using the strut-and-tie model. Ties are shown as solid lines in strut-and tie models.

1.5 NODES AND NODAL ZONES

- The points at which the forces in the struts-and-ties meet in a strut-and-tie model are referred to as nodes. Conceptually, they are idealized as pinned joints.
- The concrete in and surrounding a node is referred to as a nodal zone.
- In a planar structure, 3 or more forces must meet at a node for the node to be in equilibrium. This requires that: $\Sigma F_x = 0$; $\Sigma F_y = 0$; and $\Sigma M = 0$. The condition $\Sigma M = 0$ is identically satisfied (SNS).

1.5 NODES AND NODAL ZONES

Forces acting on nodes

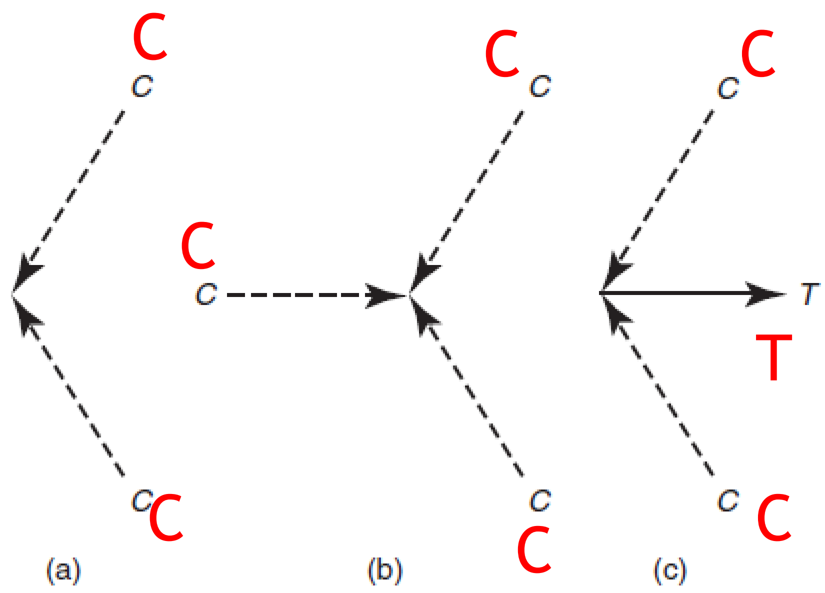
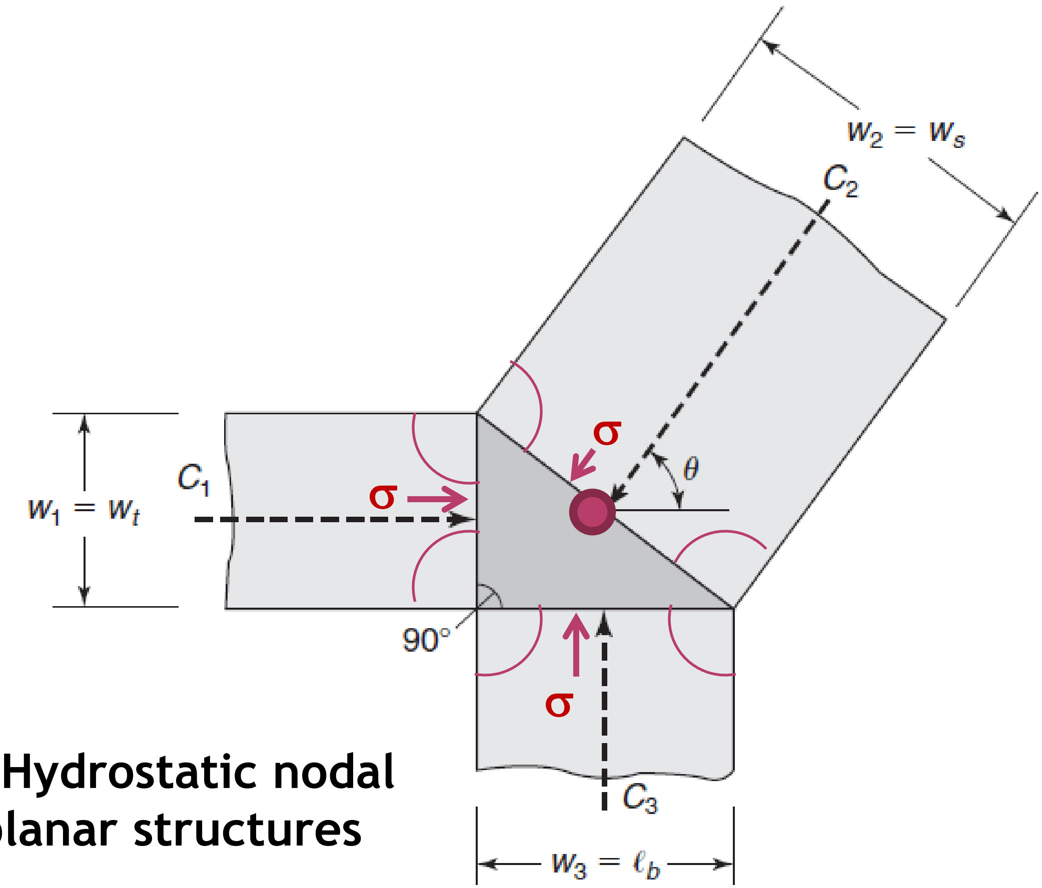


Fig. 17-9
Forces acting on nodes.

1.5 NODES AND NODAL ZONES

- Nodal zones are classified as C-C-C if 3 compressive forces meet (Fig b), and as C-C-T if one of the forces is tensile (Fig c)
- C-T-T joints may also occur.
- **1.5.1 Hydrostatic Nodal Zones**
- Two common ways of laying out nodal zones are shown in Figure 17-10 and 17-11 (SNS). The prismatic compression struts are assumed to be stressed in uniaxial compression. A section ⊥ to the axis of a strut is acted on only by compression stresses, while sections at any other angle have combined comp and shear stresses.

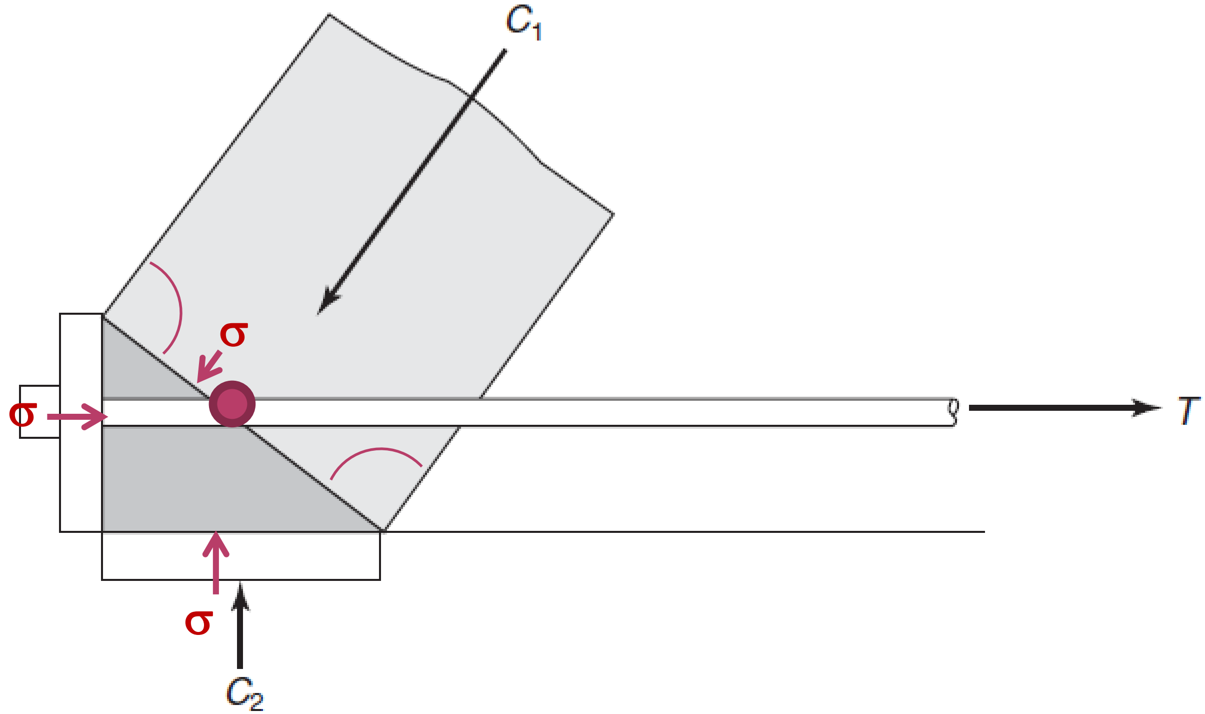
1.5 NODES AND NODAL ZONES



(a) C–C–C node.

Fig 17-10 Hydrostatic nodal zones in planar structures

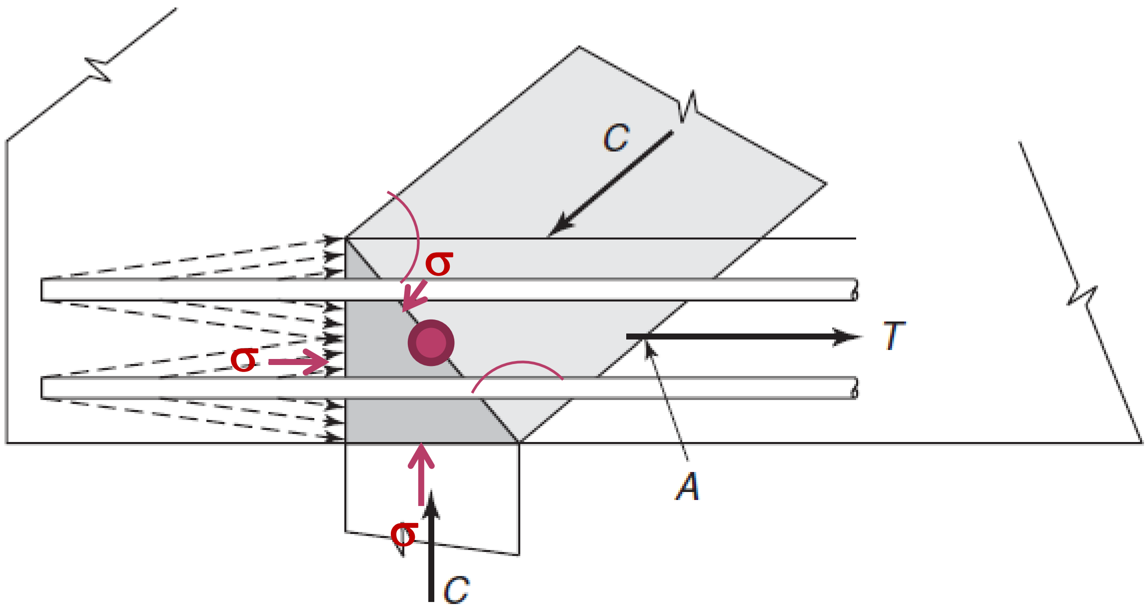
1.5 NODES AND NODAL ZONES



(b) C–C–T node.

Fig 17-10 Hydrostatic nodal zones in planar structures

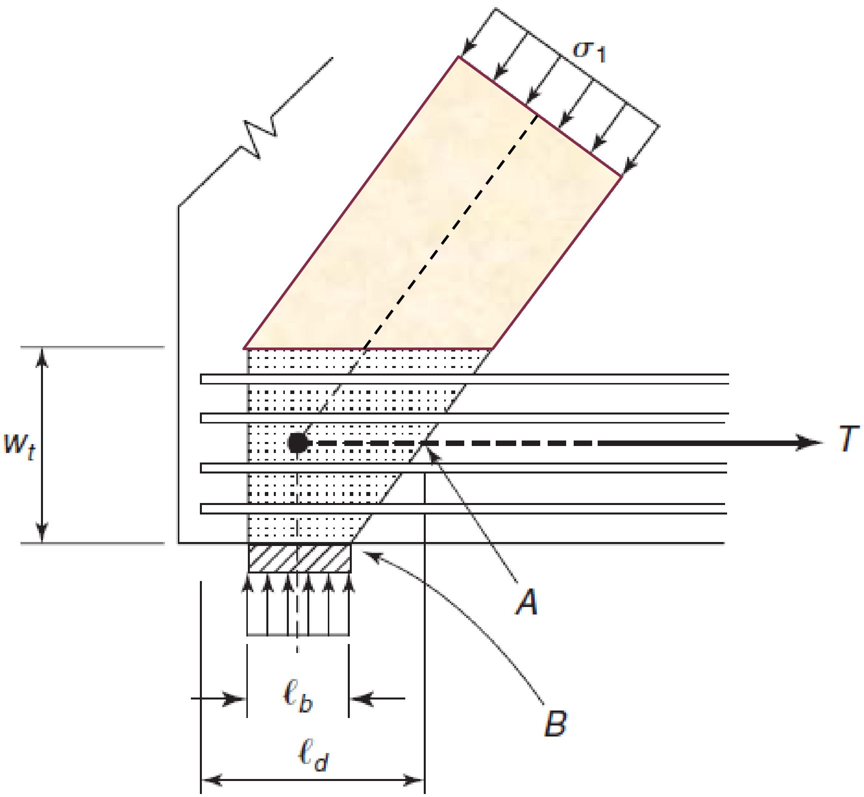
1.5 NODES AND NODAL ZONES



(c) C–C–T node, T anchored by bond.

Fig 17-10 Hydrostatic nodal zones in planar structures

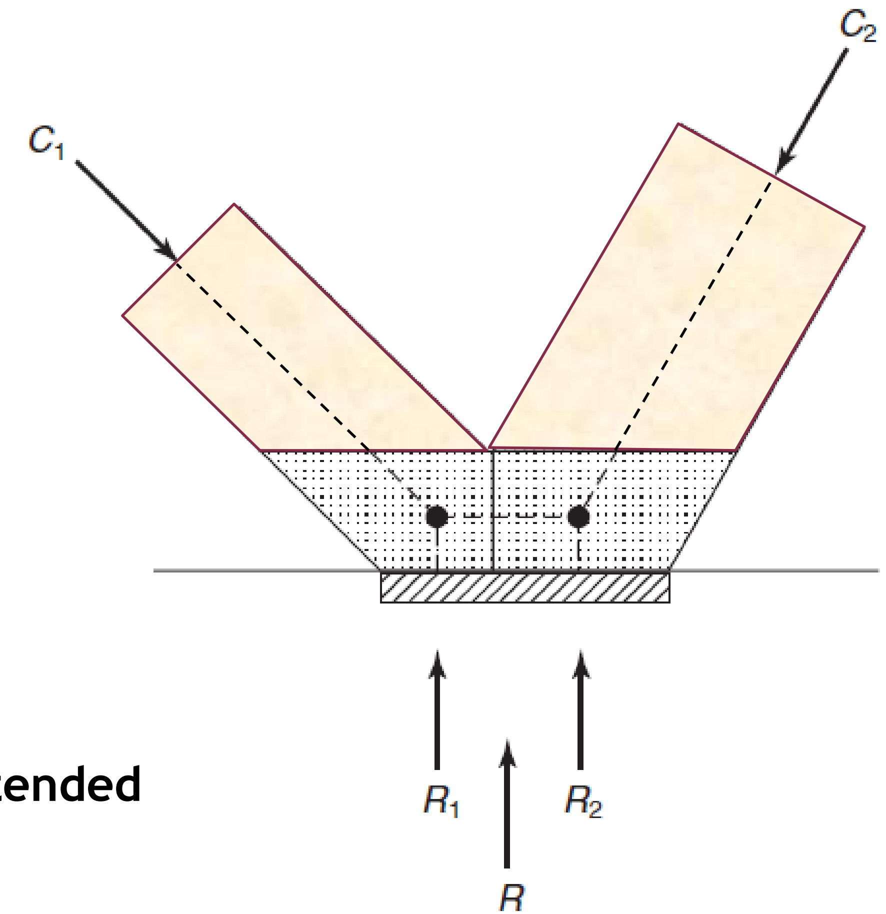
1.5 NODES AND NODAL ZONES



(a) $C-C-T$ node.

Fig. 17-11
Extended nodal zones.

1.5 NODES AND NODAL ZONES



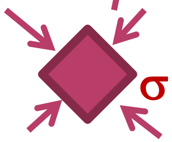
(b) Subdivision of a nodal zone.

Fig 17-11 Extended nodal zones

1.5 NODES AND NODAL ZONES

- One way of laying out nodal zones is to orient the sides of the nodal zones at right angles to the axes of the struts or ties meeting at that node (see Fig. 17-10) and to have the same bearing pressure σ on each side of the nodal zone.
- When this is done for a C-C-C nodal zone, the ratio of the lengths of the sides of the nodal zone, $w_1:w_2:w_3$, is the same as the ratio of the forces in the 3 members meeting at the node, $C_1:C_2:C_3$ as shown in Fig 17-10(a).

Biaxial plane stress state with the same value σ as the boundary also inside the nodal zone . That was the reason to look for hydrostatic nodal zones because checking the boundary covers also the inside of the nodal zone



1.5 NODES AND NODAL ZONES

- Nodal zones laid out in this fashion are sometimes referred to as hydrostatic nodal zones since the in-plane stresses in the nodal zones are the same in all directions. In such a case, the Mohr's circle for the in-plane stresses reduces to a point.
- If one of the forces is tensile, the width of that side of the zone is calculated from a hypothetical bearing plate on the end of the tie, which is assumed to exert a bearing pressure on the node equal to the compressive stress at that node as shown in Fig 10-7(b). GOTO S66.

P. MARTI, ACI JOURNAL, 1985

- More discussion (P. Marti, ACI Journal, 1985)
  - Figure (SNS) shows an equilibrium system of 3 strut forces. When strut widths are chosen, 3 different uniaxial compressive stress fields are created. The intersection pts A, B, and C of the strut edges define a biaxially stressed triangular zone. The stress state in this zone is found by drawing parallel lines to the sides BC, CA, and AB of the triangle through the poles QA, QB, an QC of the individual struts' Mohr's circles in Fig(b). The intersection pts of A, B, and C of these lines with the corresponding Mohr's circles define the Mohr's circle for the biaxial compressive stress state in the triangle ABC of Fig(a). Read Paper

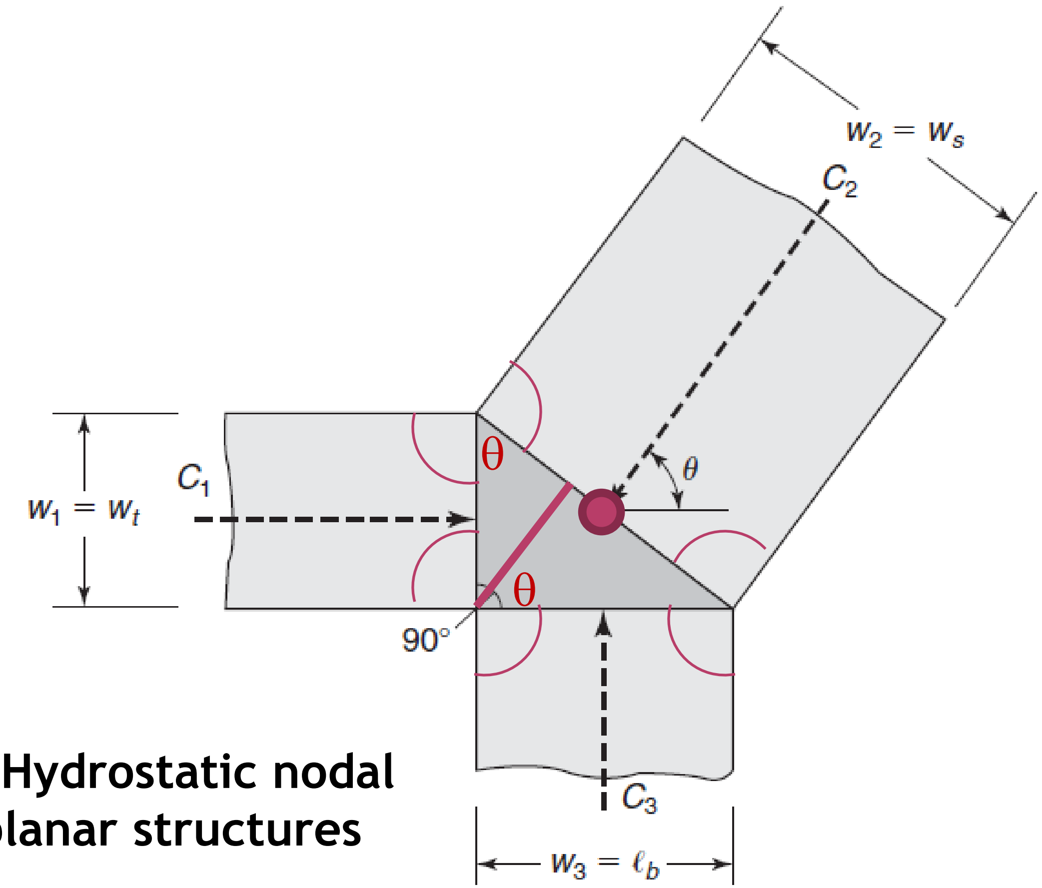
1.5 NODES AND NODAL ZONES

- Alternatively, the reinforcement may extend through the nodal zone to be anchored by bond, hooks, or mechanical anchorage before the reinforcement reaches point A on the RHS of the extended nodal zone as shown in Fig 17-10(c)(SNo 60).
- Such a nodal zone approaches being a hydrostatic C-C-C nodal zone. However, the strain incompatibility resulting from the tensile steel strain adjacent to the compressive concrete strain reduces the strength of the nodal zone. Thus, this type of joint should be designed as a C-C-T joint with $\beta_n = 0.80$

1.5 NODES AND NODAL ZONES

- **1.5.2 Geometry of Hydrostatic Nodal Zones**
- As described earlier, the stresses are equal or close to equal on all faces of a hydrostatic nodal zone. $\rightarrow$ **Geometry of Hydrostatic Nodal Zones** is easily determined as follows.
- Given tie force and reaction determine the tie width and bearing width such that the stresses on both faces are equal (f_{ce}). From geometry determine the strut side and check the stress. If it is greater than f_{ce} increase w_t, or ℓ_b or both. But then the stresses are slightly different from f_{ce})
- Figure 17-10a (SNS) shows a hydrostatic C-C-C node. For a nodal zone with a 90° corner, as shown:

1.5 NODES AND NODAL ZONES



(a) C–C–C node.

Fig 17-10 Hydrostatic nodal zones in planar structures

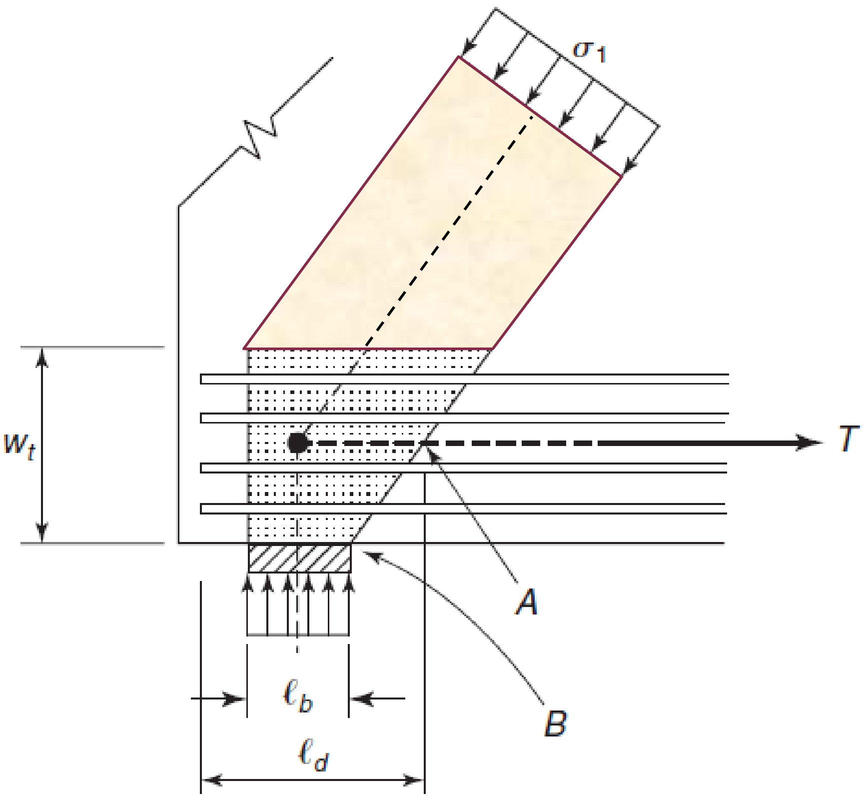
1.5 NODES AND NODAL ZONES

- Given the angle b/n the axis of the inclined strut and the horizontal is θ, and determining the tie width $w_1 = w_t$ and the base ℓ_b from conditions of equal stress, f_{ce}, the width of the 3rd side, the strut, $w_2 = w_s$, can be computed as:
- $w_s = w_t \cos\theta + l_b \sin\theta$. (Eq. 17-15)
- This equation can also be applied to a C-C-T node, as shown in Figure 17-10b.
- If the width of the strut, w_s, computed from the strut force is larger than the width calculated using the above eqn, it is necessary to increase either w_t or ℓ_b or both, until the width equals or exceeds the width calculated from the strut forces.

1.5 NODES AND NODAL ZONES

- **1.5.3 Extended Nodal Zones**
- The use of hydrostatic nodes can be tedious in design. More recently, the design of nodal zones has been simplified by considering the nodal zone to comprise that concrete lying within extensions of the members meeting at the joint as shown in Fig 17-11(SNS) (J. Schlaich/FIP).
- This allows different stresses to be assumed in the struts and over bearing plates and the stress state inside the nodal zone is assumed safe if the surface stresses are checked to be safe and the conditions on SNo 74 are fulfilled.

1.5 NODES AND NODAL ZONES



(a) C–C–T node.

Fig. 17-11
Extended nodal zones.

1.5 NODES AND NODAL ZONES

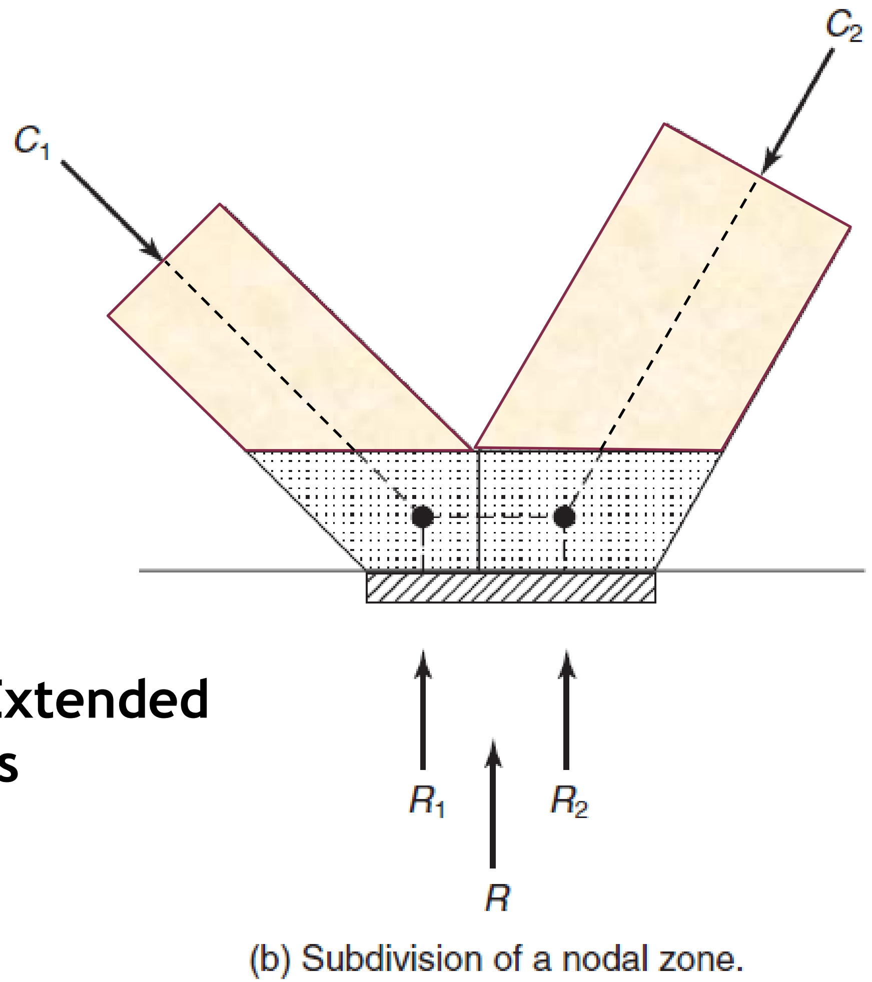


Fig 17-11 Extended nodal zones

1.5 NODES AND NODAL ZONES

- Two examples are given in Fig 17-11. Fig 17-11a shows a C-C-T node. The bars must be anchored within the nodal zone or to the left of point A, which ACI Section A.4.3.2 describes as "the point where the centroid of the reinforcement in the tie leaves the extended nodal zone."
- The length, ℓ_d, in which the bars of the tie must be developed is shown.
- The vertical face of the node is acted on by stress equal to the tie force T divided by the area of the vertical face.

1.5 NODES AND NODAL ZONES

- The stresses on the three faces of the node can all be different, provided that
  - 1. the resultants of the 3 forces coincide
  - 2. the stresses are within the limits given in Table 17-1
  - 3. the stress is constant on any one face
- Eq 17-15 is used to compute the widths $\perp$ to the axis of inclined struts in extended nodal zones, as shown in Fig (SNS), even though these equations are derived for hydrostatic nodal zones.

1.5 NODES AND NODAL ZONES

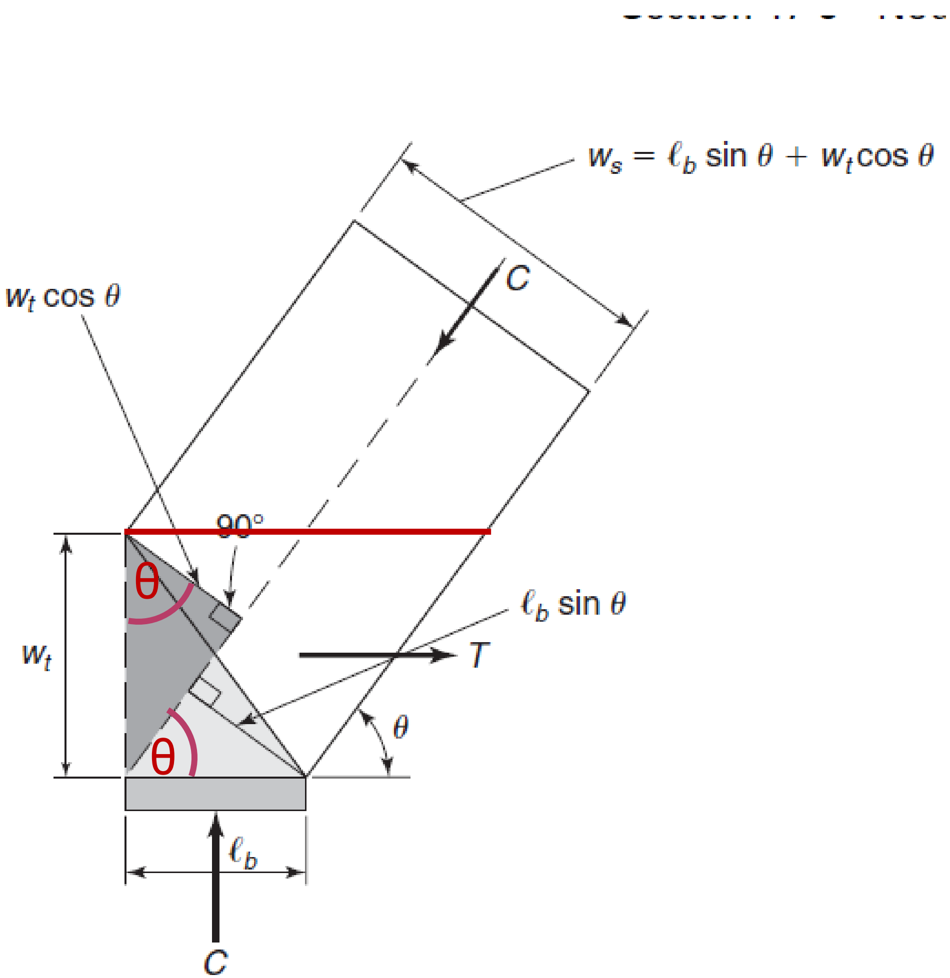


Fig. 17-12
Width of inclined strut at a *C–C–T* nodal zone.

1.5 NODES AND NODAL ZONES

- The advantage of the extended nodal zones in Fig 17-11a and b comes from the fact that ACI Section A.4.3.2 allows the length available for bar development to anchor the tie bars to be taken out to point A rather than point B at the edge of the bearing plate.
- This extended anchorage length recognizes the beneficial effect of the compression from the reaction and the struts improving bond b/n the concrete and the tie reinforcement

1.5 NODES AND NODAL ZONES

- **1.5.4.Strength of Nodal Zones**
- Nodal zones are assumed to fail by crushing.
- A tension tie is anchored in a nodal zone and tends to weaken the nodal zone.
- ACI Section A.5.1 limits the effective concrete strengths, f_{ce}, for nodal zones as: $F_{nn} = f_{ce}A_n$; where A_n is the area of the face of the node that the strut or tie acts on, taken $\perp$ to the axis of the strut or tie, or the area of a section through the nodal zone and $f_{ce} = 0.85\beta_n f_c'$

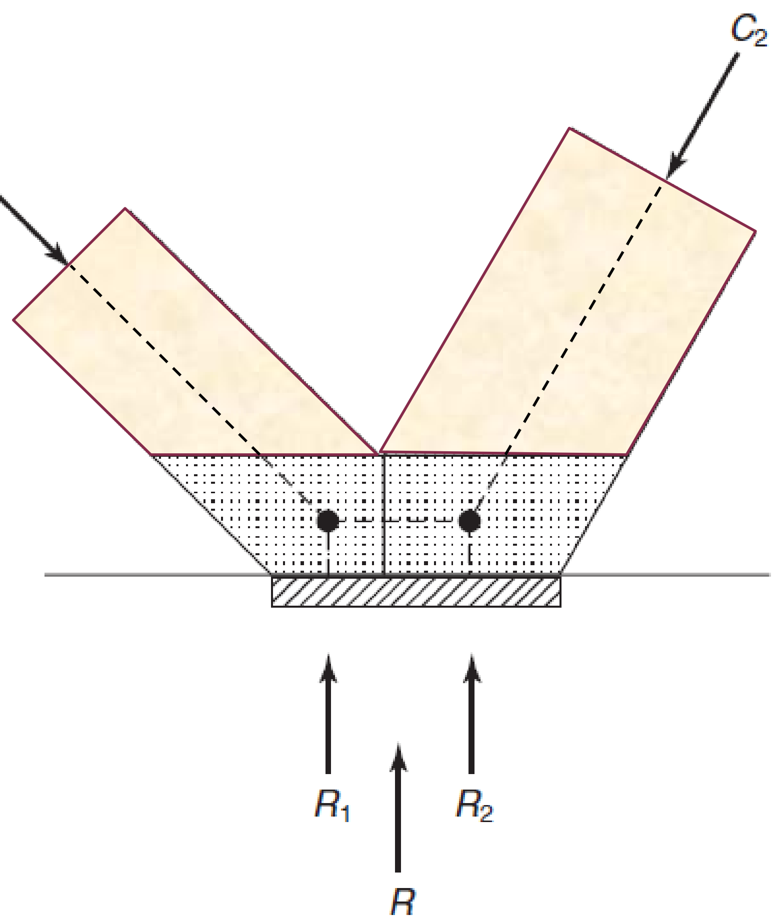
1.5 NODES AND NODAL ZONES

- ACI Section A.5.1 gives the following 3 values for nodal zones (See also Table 17-1)
  - 1. β_n = 1.0 in C-C-C nodal zones bounded by compressive struts and bearing areas
  - 2. β_n = 0.8 in C-C-T nodal zones anchoring a tension tie in only one direction
  - 3. β_n = 0.60 in C-T-T nodal zones anchoring tension ties in more than one direction

1.5 NODES AND NODAL ZONES

- Tests of C-C-T and C-T-T nodes reported in lit developed β_n = 0.95 in properly detailed nodal zones
- **1.5.5 Subdivision of Nodal Zones**
- Frequently it is easier to lay out the size and location of nodal zones if they are subdivided into several parts, each of which is assumed to transfer a particular component of the load through the nodal zone. (SNS-Fig 17-11b)

1.5 NODES AND NODAL ZONES



(b) Subdivision of a nodal zone.

1.5 NODES AND NODAL ZONES

- The reaction R has been divided into 2 components R_1, which equilibrates the vertical component of C_1, and R_2, which equilibrates C_2.

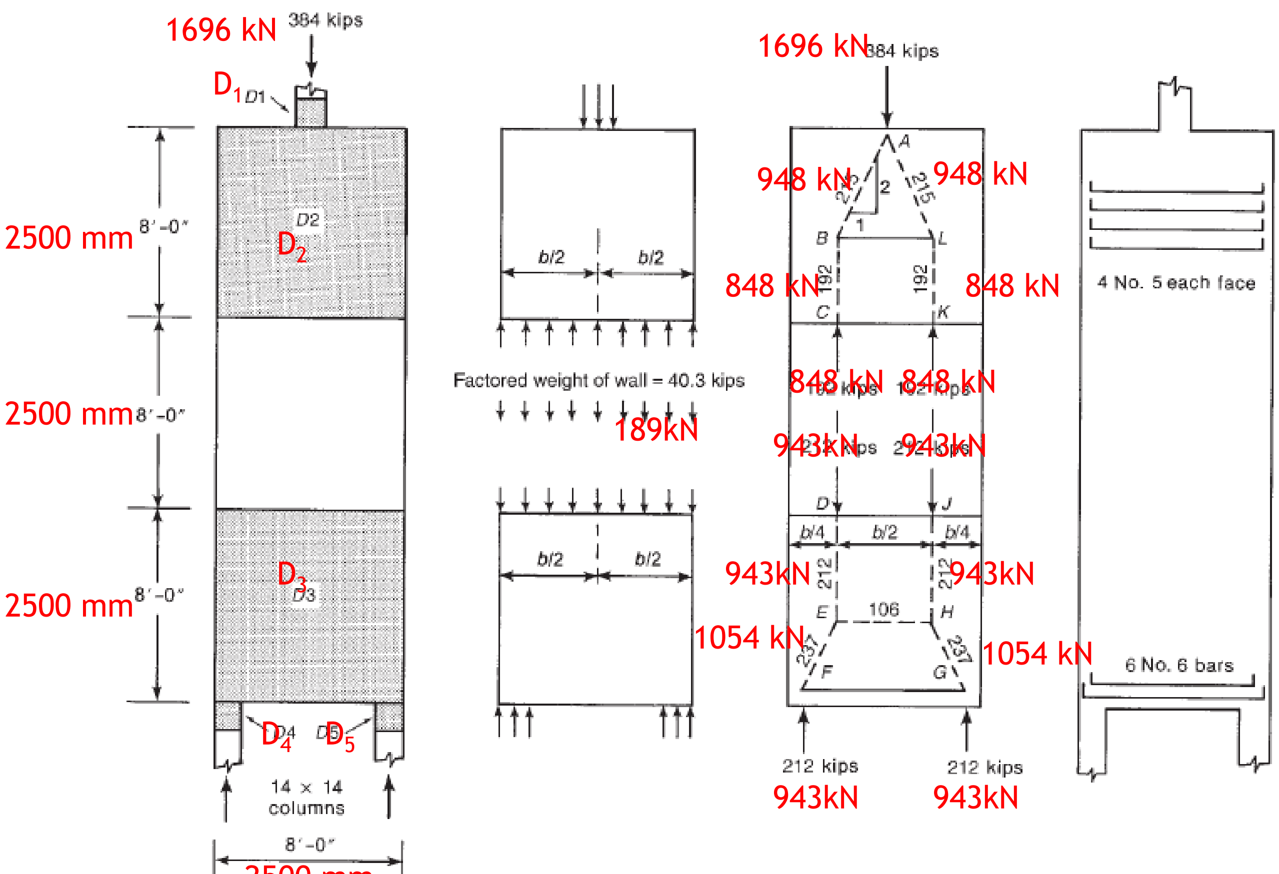
1.5 NODES AND NODAL ZONES

- **1.5.6 Anchorage of Ties in a Nodal Zone**
- A challenge in design using strut and tie models is the anchorage of the tie forces in the nodal zones at the edges or ends of a strut-and-tie model. This pm is independent of the type of analysis used in design. It occurs equally in structures designed by elastic analysis or strut-and-tie models. In fact, one of the advantages of strut-and-tie models comes from the attention that the strut-and-tie model places on the anchorage of ties as described in ACI, Section A.4.3.

1.6 EXAMPLE

- **Design of a wall loaded and supported by columns**
- The 350 mm thick wall shown in Fig (SNS) supports a 350 mm by 550 mm column carrying unfactored loads of 440 kN DL 730 kN LL, plus the wt of the wall. The wall supports this column and is supported on 2 other columns which are 350 mm by 350 mm. The floor slabs (not shown) provide stiffness against out-of-plane buckling. Design the reinforcement. Use f_c'=20 MPa and f_y= 420MPa

1.6 EXAMPLE



(a) Wall and *D*-regions. (b) Stresses on *D*-regions. (c) Truss model. (d) Reinforcement.

Fig. 17-17
D-regions in a wall—Example 17-1.

1.6 Example

- Solution
  - 1. **Isolate the D-Regions**. The loading discontinuities (wall has 2 statical discontinuities at the top and bottom) dissipate in a distance approximately one member dimension from the location of the discontinuity. $\rightarrow$ wall is divided into 2 D-regions separated by a B-region. There are 3 more D-regions at the ends of the columns which have little effect on the wall $\rightarrow$ not considered.
  - 2. Compute factored loads. $U=1.2\times440 + 1.6\times730 = 1696$ kN

1.6 EXAMPLE

- **3. Subdivide the boundaries of the D-regions and compute the force resultants on the boundaries of the D-region (Load Path Method)**
  - For D2, we can represent the load on the top boundary by a single force of 1696 kN at the center of the column, or as two forces of 848 kN acting at the quarter pts of the width of the column at the interface with the wall. Strut-and-tie is drawn using one force. The bottom boundary of Region D2 is divided into 2 segments of equal lengths, b/2 each with its resultant force of 848 kN acting along the struts at the quarter pts. This gives uniform stress on the bottom of D2.

1.6 EXAMPLE

- **4. Lay out the strut-and-tie models**
  - Two strut-and-tie models are needed, one in each of D2 and D3. The function of the upper strut-and-tie model of D2 is to transfer the column load from the center of the top of D2 to the bottom of D2. The compression stresses fan out from the column (2:1), approaching a uds at the ht where the struts pass through the quarter pts of the section. In D2 this occurs at level B-L. Below this level struts B-C and L-K are vertical and pass through the quarter pts of the width of the section. This gives uniform compression stresses over the width

1.6 EXAMPLE

- **For D3**, similarly, the load on top of D-region D3 will be represented by struts at the quarter pts of the top of the D-region. The strut-and-tie model in D3 transfers the uniformly distributed loads, including the dead load of the wall, from the top of D3 down to the 2 concentrated reactions where the wall is supported by the columns.

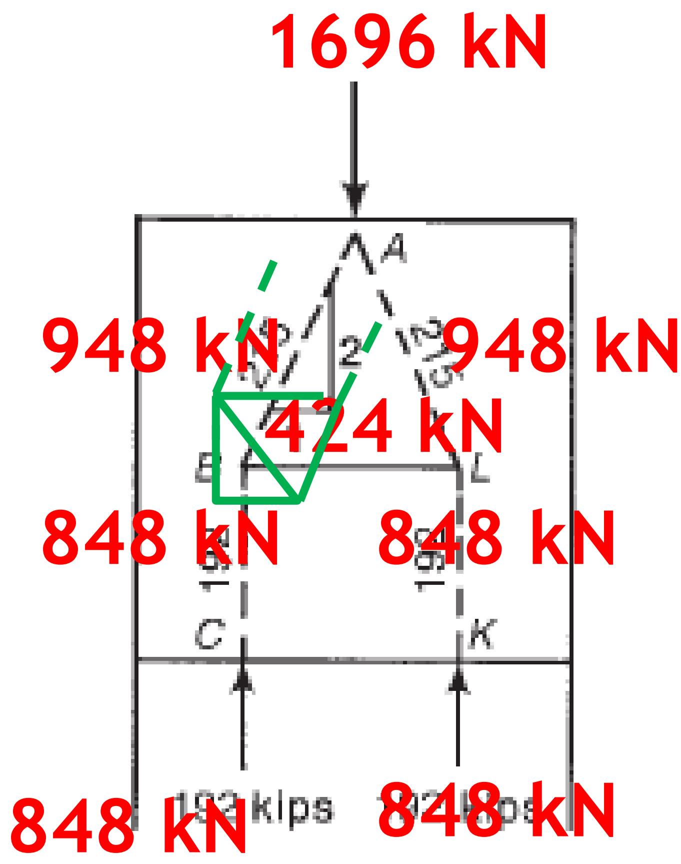
5. Draw the strut-and-tie models

- Recall recommendation that load-spreading struts at a (2 to 1) slope relative to the axis of the applied load $\rightarrow \theta = \arctan(1/2) = 26.6°$

1.6 EXAMPLE

- **D-region D2**
  - (i) **Node A and struts A-B and A-L**: Treating Node A as a hydrostatic node (constant uniform stress on all faces), either the node or one of the struts A-B and A-L will control.
    - Node A: β_n = 1.0 (node is compressed on all faces) $\rightarrow f_{ce}$ = $0.85 \times 1.0 \times 20$ = 17 MPa
    - Struts A-B and A-L: Bottle shaped struts $\rightarrow \beta_s = 0.75 \rightarrow f_{ce}$ = $0.85 \times 0.75 \times 20$ = 12.8 MPa $\rightarrow$ governs $\rightarrow$ an area of $(1696 \times 1000)/(0.75 \times 12.8)$ = 176667 mm^2 is required. Column area = 350×550 = 192500 mm^2 $\rightarrow$ ok (capacity reduction factor ϕ = 0.75)
  - (ii) **Minimum dimensions for Nodes B and L**: These are C-C-T nodes $\rightarrow f_{ce} = 0.85 \times 0.80 \times 20$ = 13.6 MPa (they are not treated as hydrostatic nodes)

1.6 EXAMPLE


1696 kN
A
948 kN
2
948 kN
424 kN
B
L
848 kN
848 kN
C
K
848 kN
kips
848 kN

1.6 EXAMPLE

- **6. Compute the forces and strut widths**
  - The calculations are given in tables below

Calculation of forces and strut widths in regions D2

D-Region	Member	V-compo	H-compo	Axial force	f_{ce}	Width of strut or nodal zone
D2	Node A	1696	0	1696	12.8	504.8
	A-B	848	424	948	12.8	282.1
	B-C	848	0	848	13.6	237.5
	A-L	848	424	948	12.8	282.1
	L-K	848	0	848	13.6	237.5
	B-L	0	424	424	13.6	118.8

$1696\times10^3/(0.75\times12.8\times350)$

1.6 EXAMPLE

- This will control the base dimension of Nodes B and L because struts B-C and L-K are prismatic struts that could be designed by using $\beta_s = 1.0$. → The base dimension of node B and the width of strut B-C is:
  - $w_s = 848000/(0.75 \times 13.6 \times 350) = 237.5$ mm.
    - This is much less than b/2 (1250 mm), so the node easily fits within the dimensions of the wall
- The height of node B is of interest for tie B-L. So,
  - $w_t = 424000/(0.75 \times 13.6 \times 350) = 118.8$ mm.
    - This is a very small dimension and the reinforcement for tie B-L will be spread over a larger distance and the dimensions of nodes B and L will be much larger than the minimum values calculated here

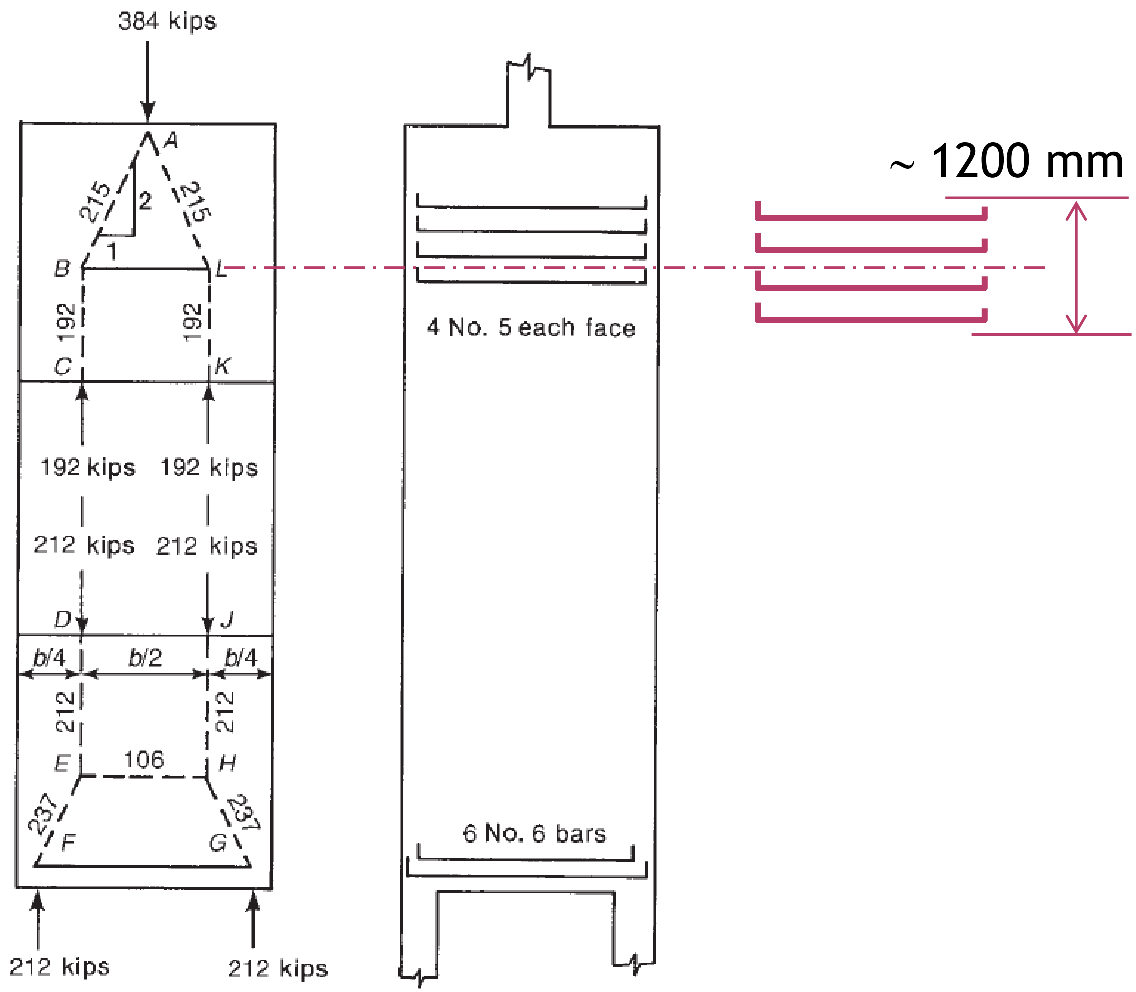
1.6 EXAMPLE

- Check stress on the side of the nodal zone common with the bottle shaped strut. It should not exceed f_{ce} = 12.8 MPa. Since we have much bigger nodes this will be satisfied easily. (Note the use of different strengths f_{ce}, here on strut adjacent face because of the extended nodal zone). If we had to design it as a hydrostatic nodal zone then we had to use the governing stress f_{ce} = 12.8 MPa on all faces and in the nodal zone itself.

1.6 EXAMPLE

- (iii) **Required area of reinforcement**
  - Tie force $T_u = 848 \times \tan\theta = 424$ kN
  - Required $A_s = T_u/\phi f_y = 424000/(0.75 \times 420) = 1346\ mm^2$
    - Essentially a band of transverse steel having this area should be provided across the full width of the wall extending about 25% of the width of the wall above and below the position of tie B-L so that the centroid of the areas of the bars is close to tie B-L. See fig. Both ends of each bar should be hooked.
    - **Use 8 No. 16 M bars (A_s = 1592 mm^2) at a vertical spacing of 300 mm, half in each face and hooked at both ends** (See figure SNS). This spacing provides a tie width of approx 1200 mm, as recommended above.

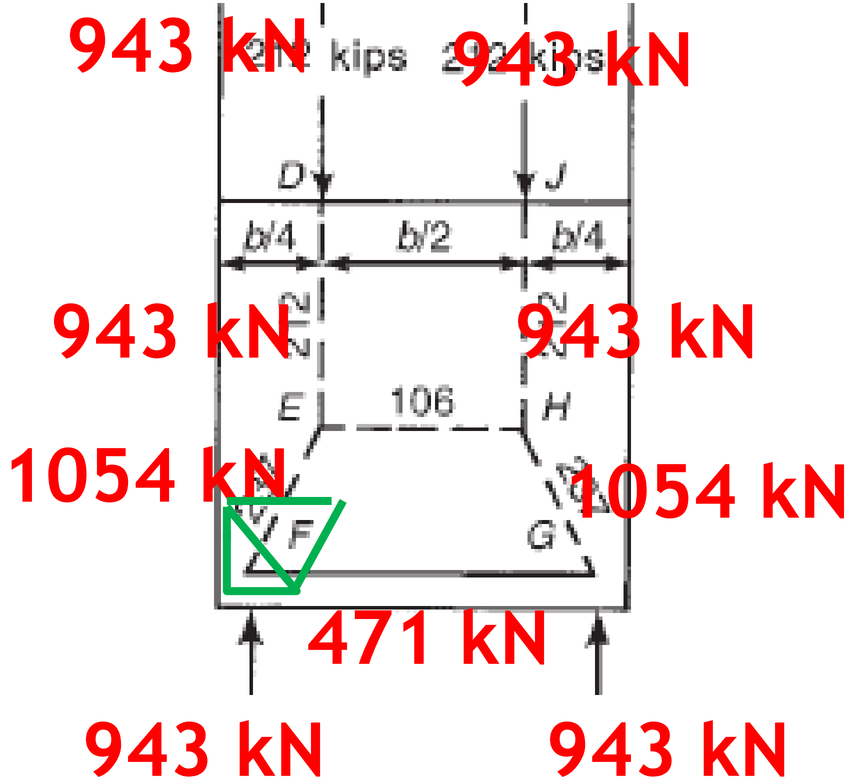
1.6 EXAMPLE


384 kips
A
215
2
215
1
B
L
192
192
C
K
192 kips
192 kips
212 kips
212 kips
D
J
b/4
b/2
b/4
212
212
E
106
H
237
237
F
G
212 kips
212 kips
4 No. 5 each face
6 No. 6 bars
~ 1200 mm

1.6 Example

- D-region D3
  - Nodes F and G are C-C-T nodes, similar to nodes B and L. → Use f_{ce} = 13.6 MPa, to determine the min dimensions for all struts, ties, and nodes in D-region D3. → See result in column (7) of Table (CHECK as EXERCISE). All element dimensions fit within the wall and supporting column dimensions
  - **(i) Required area of reinforcement for tie F-G**
    - Tie force T_u = 471 kN
    - Required $A_s = 471000/(0.75\times420) = 1495\ mm^2$
      - **Thus, use 6 No. 19M bars, $A_s = 1704\ mm^2$, placed in two layers of three bars per layer**. This would put the centroid of these bars approximately at mid-height of tie F-G whose height (width)(131.9mm) is given in table below. All of these bars must be hooked at the edges of the wall as shown in figure.

1.6 EXAMPLE


943 kN
212 kips
212 kips
943 kN
D
J
b/4
b/2
b/4
943 kN
943 kN
E
106
H
1054 kN
1054 kN
F
G
471 kN
943 kN
943 kN

1.6 EXAMPLE

Calculation of forces and strut widths in region D3						
D-Region	Member	V-compo	H-compo	Axial force	f_{ce}	Width of strut or nodal zone
D3	D-E	943	0	943	13.6	264.1
	E-F	943	471	1054	12.8	295.2
	F-G	0	471	471	13.6	131.9
	G-H	943	471	1054	12.8	295.2
	H-J	943	0	943	13.6	264.1
	E-H	0	471	471	13.6	131.9

1.7 COMMON STRUT-AND-TIE MODELS

- Compression Fans
  - A compression fan is a series of compression struts that radiate out from a concentrated applied force to distribute that force to a series of localized tension ties, such as the stirrups. Fans are shown over the reaction and under the load in Fig (SNS). The failure of a compression fan is shown in Fig 6-22.
- Compression Fields
  - A compression field is a series of parallel compression struts combined with appropriate tension ties and compression chords as shown in Fig (SNS). Compression fields are shown b/n the compression fans

1.7 COMMON STRUT-AND-TIE MODELS

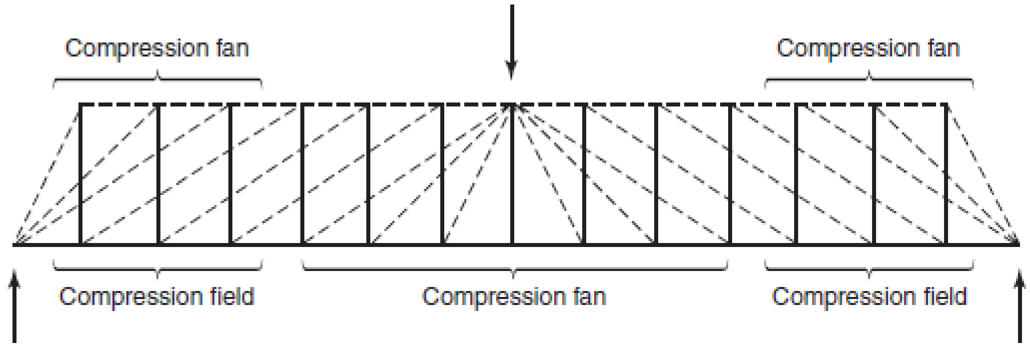


Fig. 17-18
Compression fans and compression fields.

Compression fans and compression fields

1.7 COMMON STRUT-AND-TIE MODELS

Compression struts failing by crushing

Compression fan in D-region

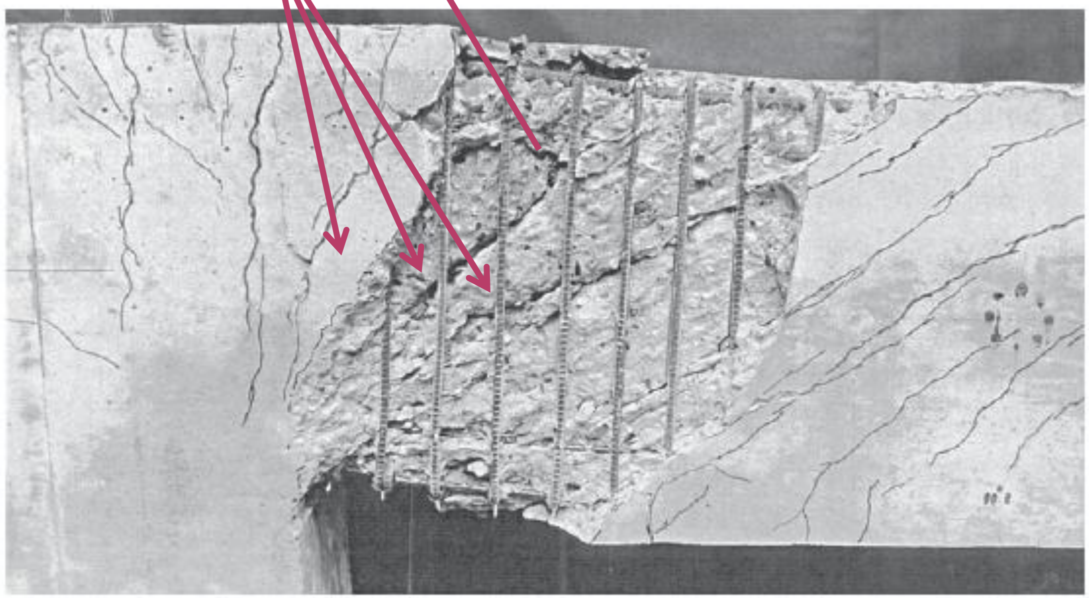


Fig. 6-22
Compression fan at interior support of the beam shown in Fig. 6-21b. (Photograph courtesy of J. G. MacGregor.)

1.7 COMMON STRUT-AND-TIE MODELS

- Force Whirls, U-Turns
  - In Fig 17-19 (SNS), the column load causes the stresses shown by the shaded area at the bottom of the D-region. The stresses on the bottom edge, A-I are computed using flexure formula (P/A-(My/I)). NA is at G, 667 mm from the edge. The widths of EG and GI have been chosen as 667 mm so that the upward force at F equals the downward force at H. The widths of the other two parts (AC and CE) were chosen so that the forces in them are equal
  - The right hand two reactions are 186 kN each. They cause the force whirl made up of compression members **FO** and **OP** and tension tie **HP**. The reinforcement computed from the strut-and-tie model is shown in Fig 17-20

1.7 COMMON STRUT-AND-TIE MODELS

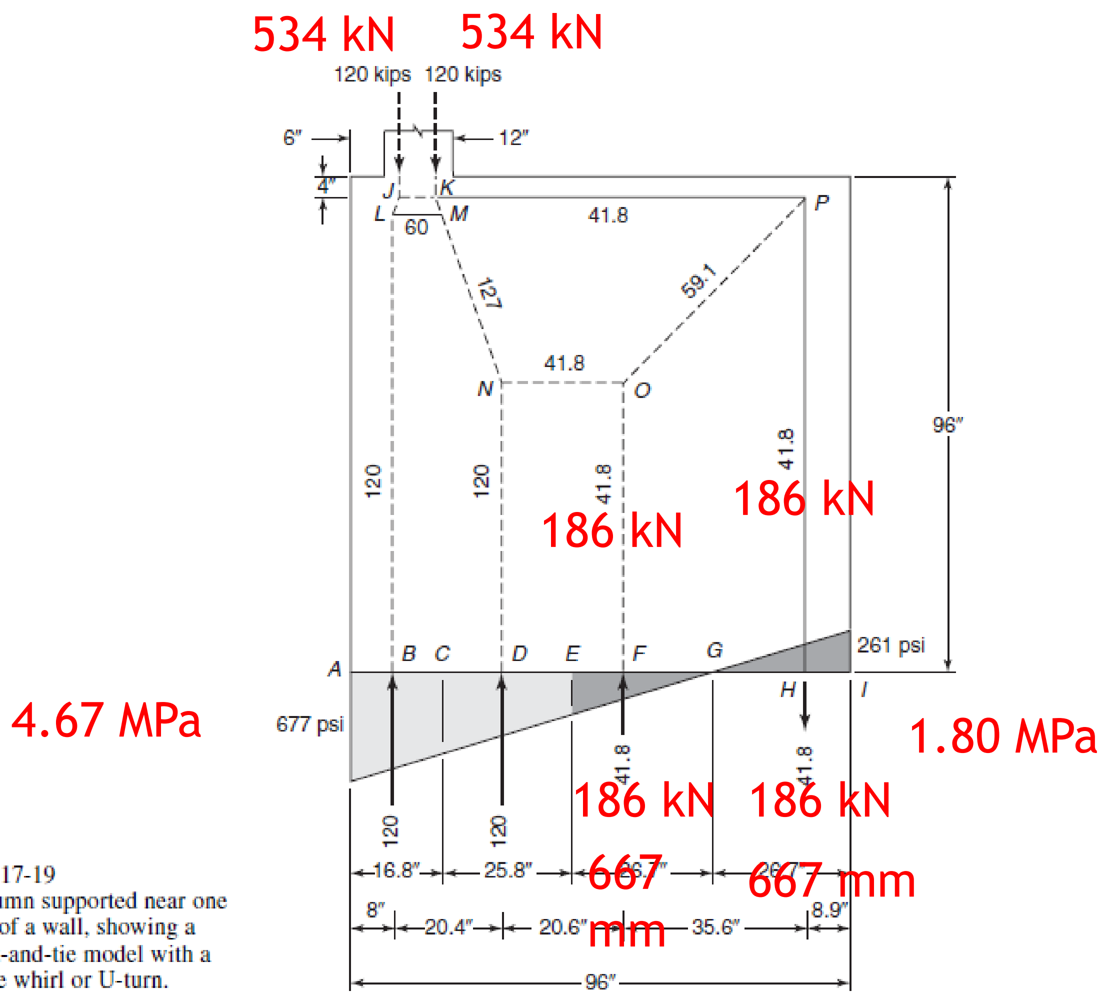


Fig. 17-19
Column supported near one end of a wall, showing a strut-and-tie model with a force whirl or U-turn.

(FIP)

1.7 COMMON STRUT-AND-TIE MODELS

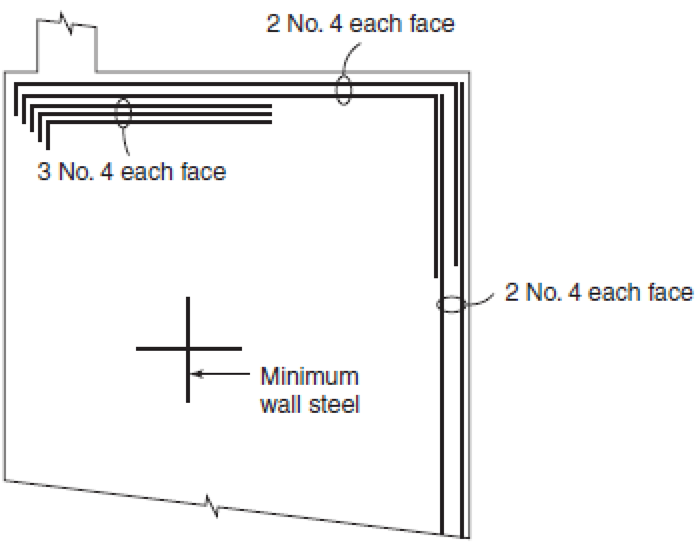


Fig. 17-20
Reinforcement in the wall in
Fig. 17-19.

(FIP)

1.9 DEEP BEAMS

- Definition of deep beam
  - The term deep beam is defined in ACI section 10.7.1 as a member
    - (a) loaded on one face and supported on the opposite face so that compression struts can develop b/n the loads and the supports and
    - (b) having either
      - (i) clear spans, ℓ_n, equal to or less than four times the overall member height h,(ES EN-2:2015 span less than 3 times the overall section depth) or
      - (ii) regions loaded with concentrated loads within 2d from the face of the support

Most typically deep beams occur as transfer girders, which may be single span or continuous (SNS)

1.9 DEEP BEAMS

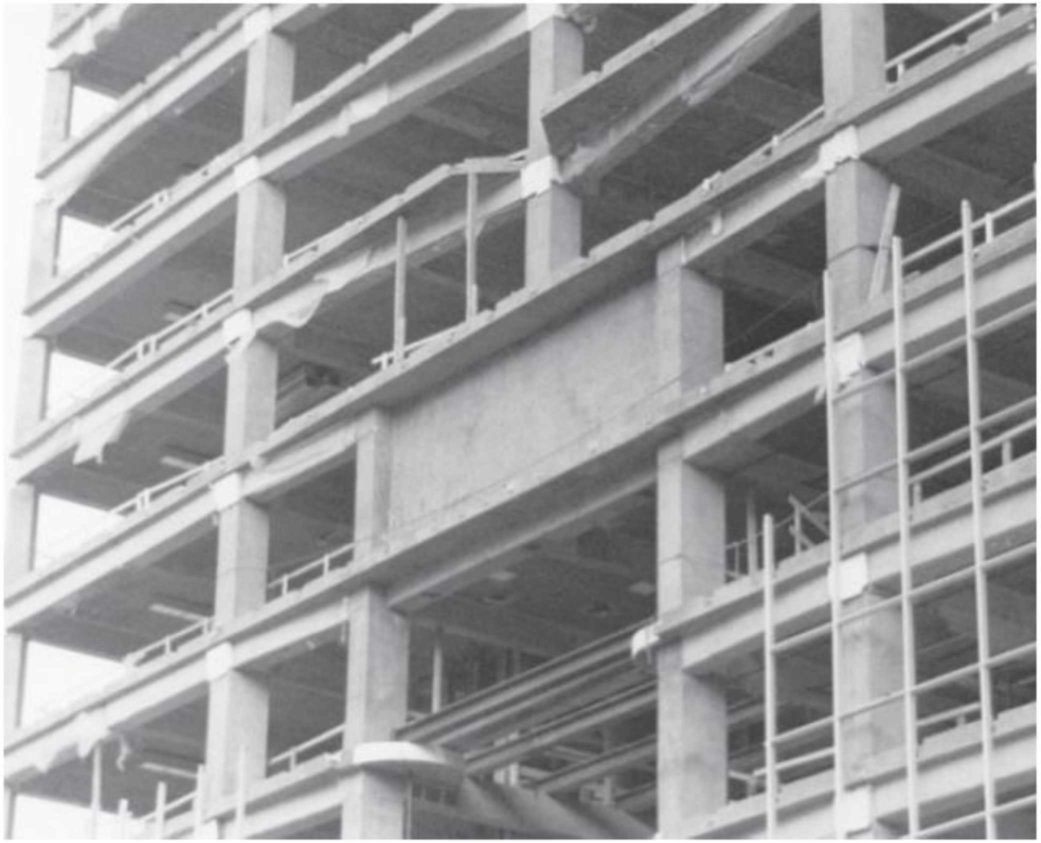


Fig. 17-25
Single-span deep beam.
(Photograph courtesy of J. G. MacGregor.)

1.9 DEEP BEAMS

Fig. 17-26
Three-span deep beam,
Brunswick building, Chicago.
(Photograph courtesy of
J. G. MacGregor.)

1.9 DEEP BEAMS

- 1.9.1 Analysis and Behavior of Deep Beams
  - Elastic analysis of deep beams in the uncracked state are meaningful only prior to cracking. In a deep beam, cracking will occur at one-third to one-half of the ultimate load.
  - After cracks develop, a major redistribution of stresses is necessary since there can be no tension across the cracks. The results of elastic analysis are of interest primarily because they show the distribution of stresses which cause cracking and hence give guidance as to the direction of cracking and the flow of forces after cracking.

1.9 DEEP BEAMS

- An uncracked, elastic, single span beam supporting a uniform load at the top has the stress trajectories shown in Figure (SNS). The distribution of horizontal stresses on vertical sections at midspan and quarter pt are shown in Fig b.
- The stress trajectories can be represented by the simple truss in Fig c (SNS). In the 1st case, the uniform load is divided into two parts, each represented by its resultant. In the 2nd case, 4 parts are used. The angle θ varies from about 68° for $l/d = 1.0$ or smaller to about 55° for $l/d = 2.0$
- The crack pattern is shown in Fig e.

1.9 DEEP BEAMS

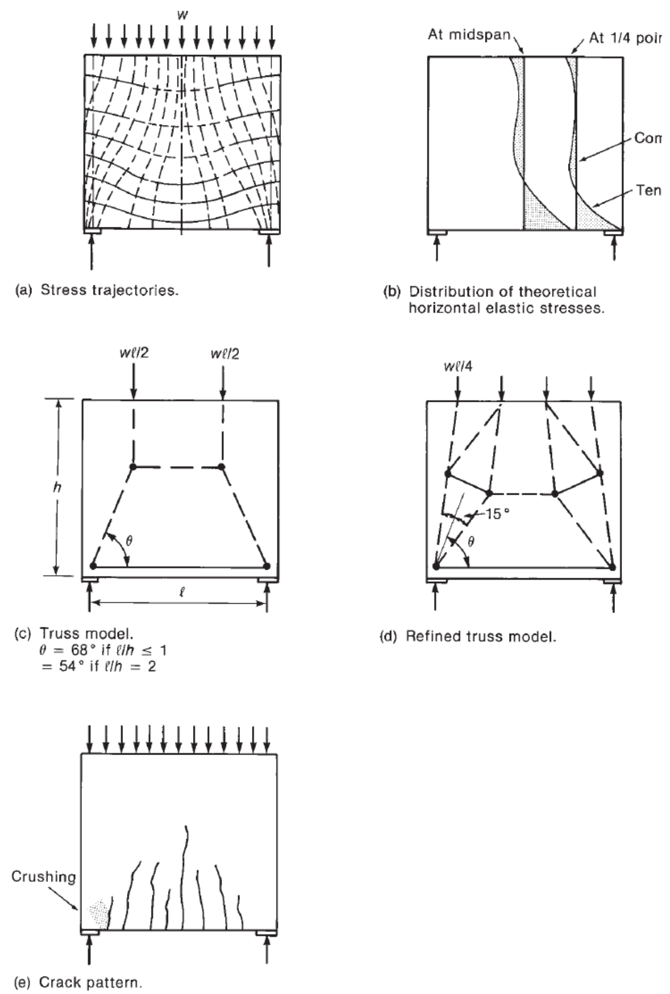
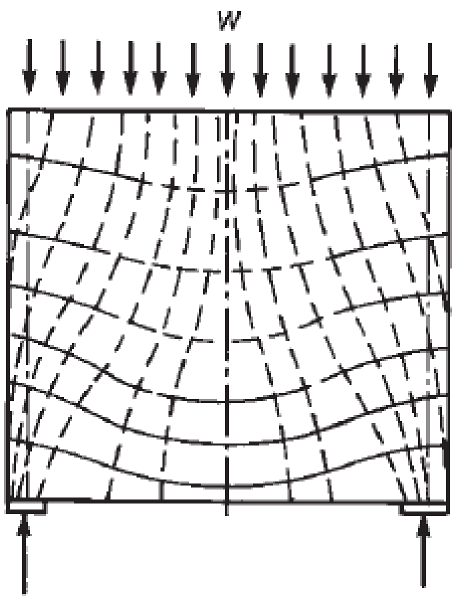
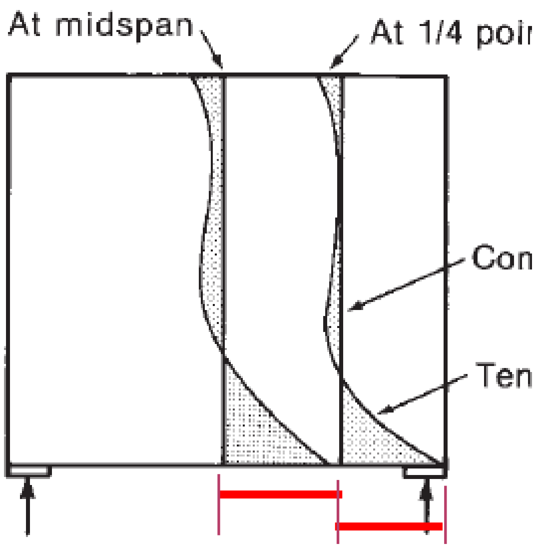


Fig. 17-27
Uniformly loaded deep beam.
(Adapted from [17-1].)

1.9 DEEP BEAMS

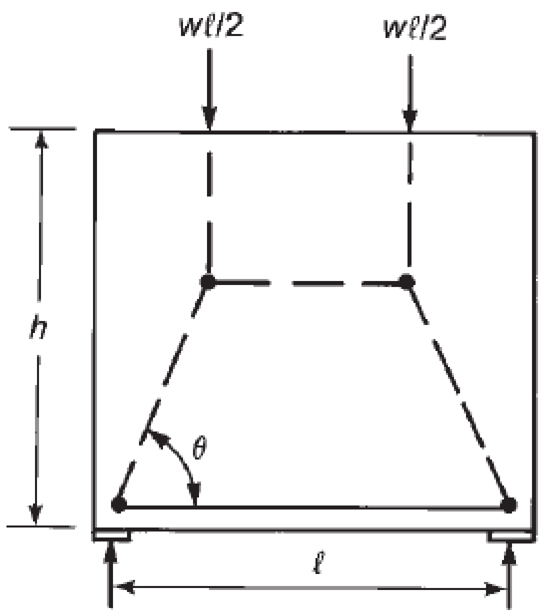


(a) Stress trajectories.

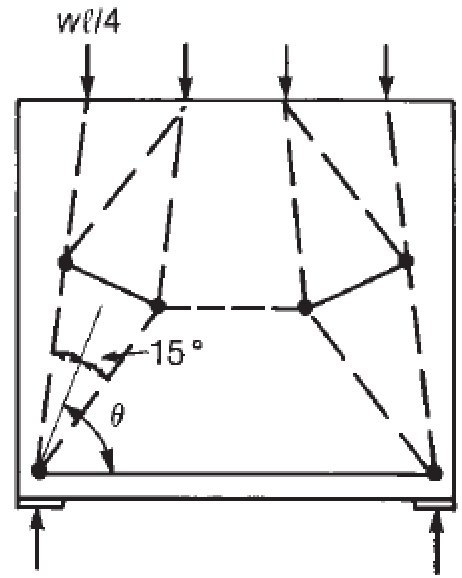


(b) Distribution of theoretical horizontal elastic stresses.

1.9 DEEP BEAMS

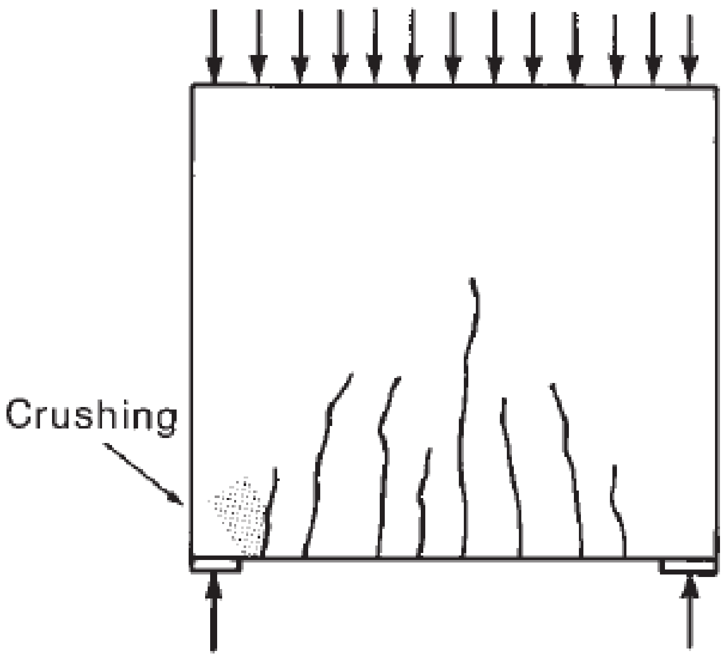


(c) Truss model.
$\theta = 68°$ if $\ell/h \leq 1$
$= 54°$ if $\ell/h = 2$



(d) Refined truss model.

1.9 DEEP BEAMS



(e) Crack pattern.

1.9 DEEP BEAMS

- Fig a shows the stress trajectories for a deep beam supporting a uniform load acting on edges at the lower face of the beam (SNS).
- The compression trajectories form an arch, with loads hanging from it, as shown in Figs b and c.
- The crack pattern in Fig d clearly shows that the load is transferred upward by the reinforcement until it acts on the compression arch, which then transfers the load down to the supports.
- The distribution of the flexural stresses and the truss models all suggest that the force in the longitudinal tension ties will be constant along the length of the deep beam. This implies that this force must be anchored at the joints over the reactions. Failure to do so is a major cause of distress in deep.

1.9 DEEP BEAMS

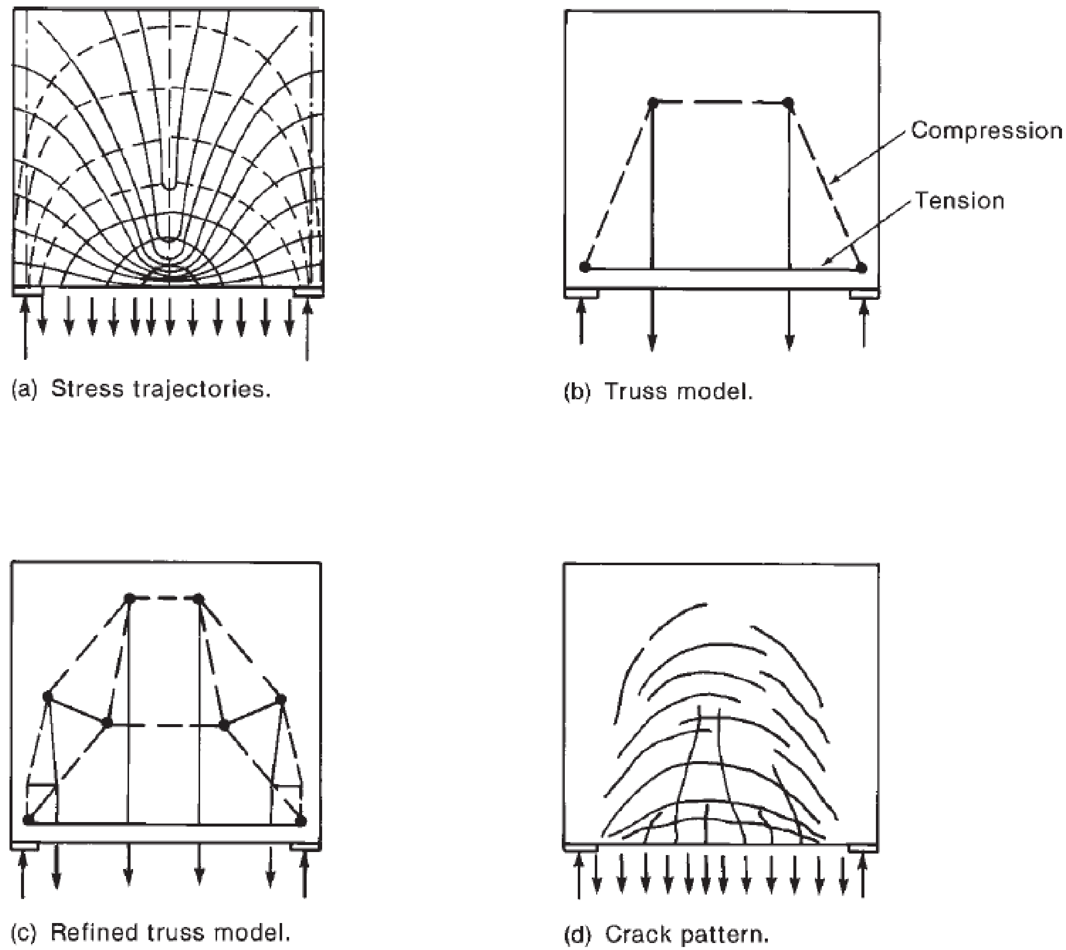


Fig. 17-28
Deep beam uniformly loaded on the bottom edge.
(Adapted from [17-1].)

1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM

- Design a deep beam to support an unfactored column load of 1330 kN dead load and 1500 kN live load from a 500 by 500 mm column. The axes of the supporting columns are 2m and 3m from the axis of the loading columns as shown in Figure (SNS). The supporting columns are also 500 mm square. Use 30-Mpa concrete and Grade-420 steel.

1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM

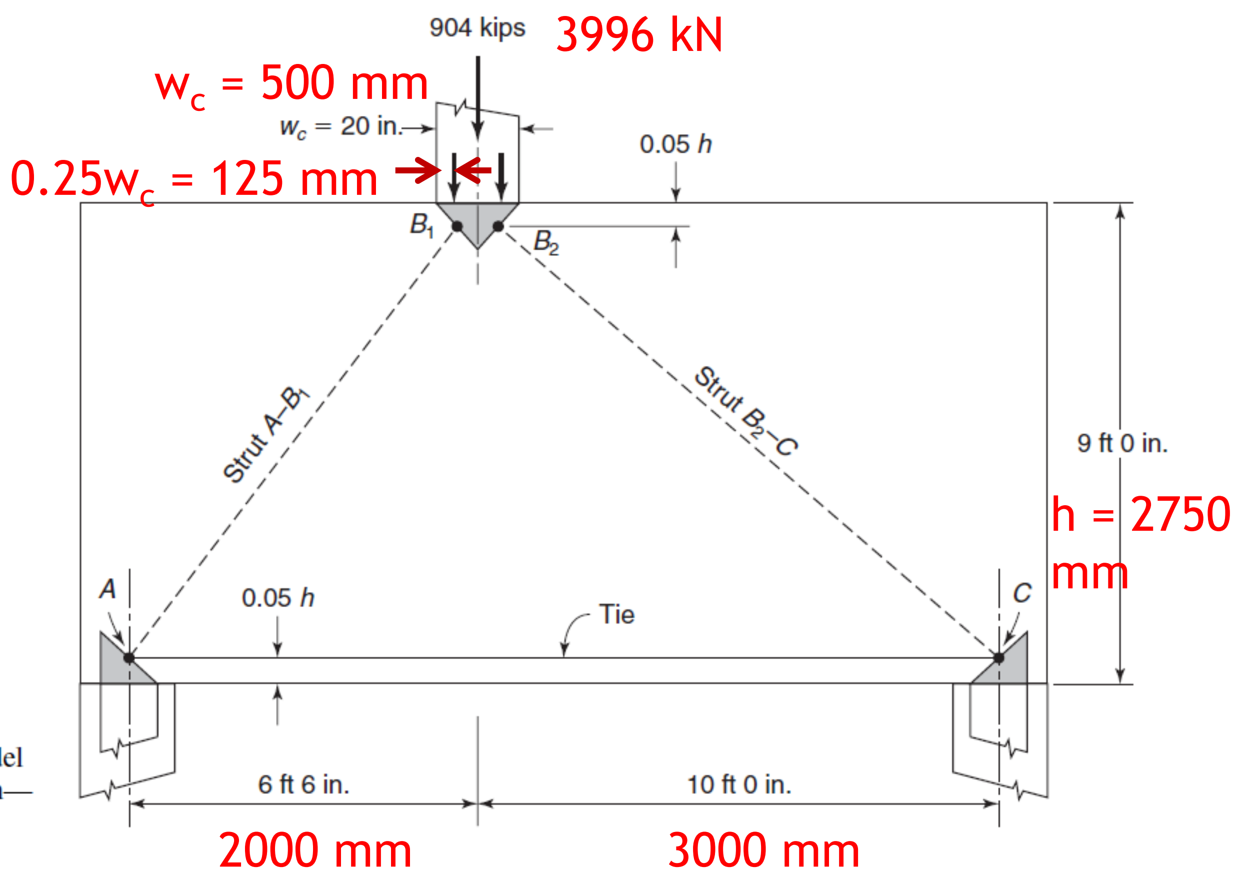


Fig. 17-29
First trial strut-and-tie model of a single-span deep beam—Example 17-2.

First trial …..

1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM

- 1. Select a strut-and-tie model
  - **1(a) Select shape and flow of forces**. The strut-and-tie model is shown in Figure. We shall assume that a portion of the column load equal to the left reaction flows down through strut A-B to the reaction at A. The rest of the column load is assumed to flow down strut B-C to the reaction at the right end of the beam.
  - 1(b) Compute factored load and reactions
    - $P_u = 1.2D + 1.6L = 1.2\times1330 + 1.6\times1500 = 3996$ kN
    - $R_A = 2398$ kN; $R_C = 1598$kN

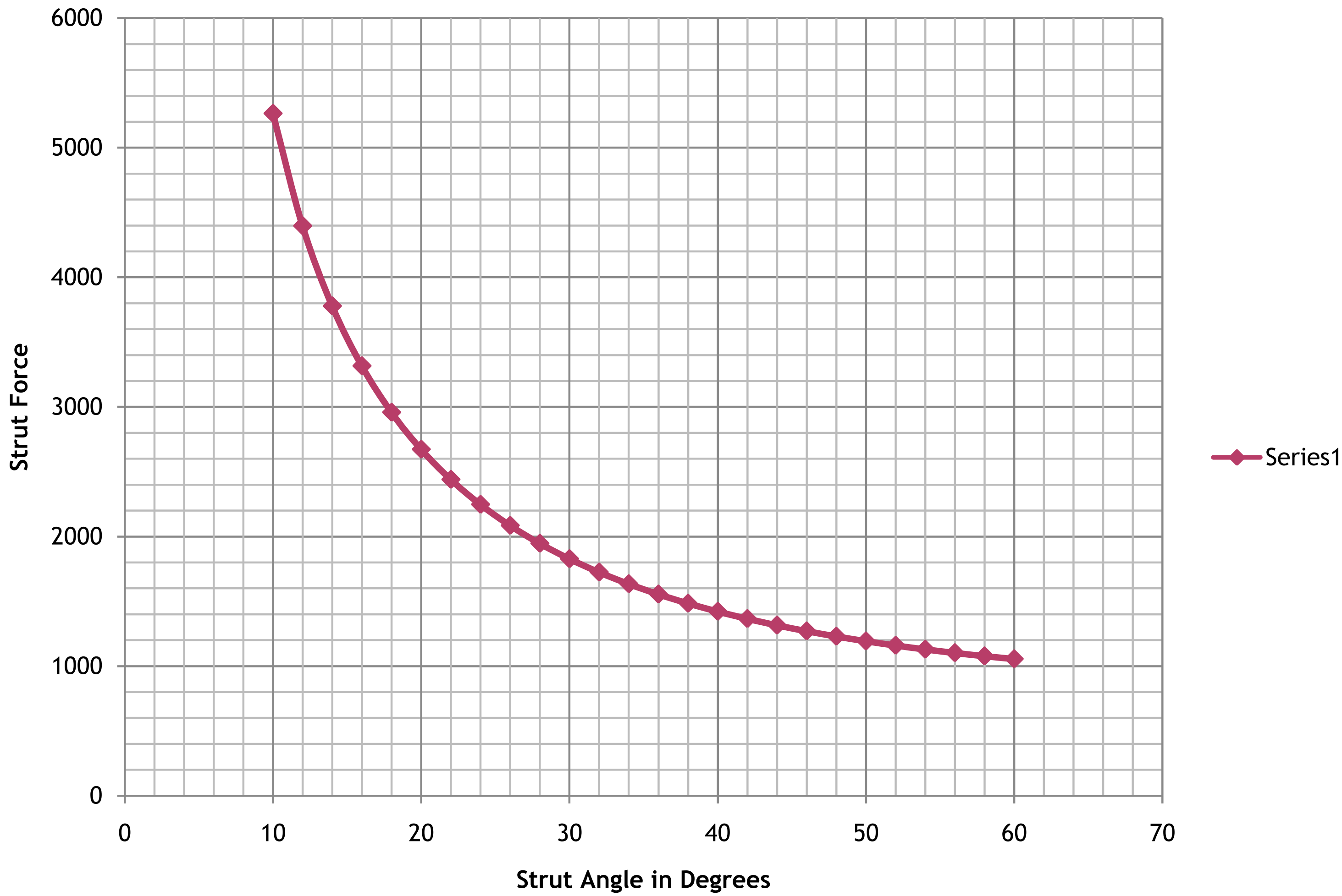
1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM

- 2. Estimate the size of the beam. This will be done in 2 different ways
  - 2(a) ACI limits the shear in a deep beam to $(10/12)\sqrt{f'_c}b_wd$ →Limit V_n to $\phi\times(10/12)\sqrt{f'_c}b_wd$. The maximum shear ignoring the dead load of the beam is 2398 kN, so $b_wd=V_n/((\phi\times10/12)\sqrt{f'_c})$ = $2398000/((0.75\times10/12)\sqrt{30})$ = 1000715 mm². Using b_w equal to the width of the columns, b_w = 500 mm. → this gives d = 2001 mm. Assuming that $h\approx d/0.9$ gives h = 2223 mm. Try a 500 by 2450 mm beam.

1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM

- 2(b) Select height to keep the flattest strut at an angle of ≈ 40° from the tie. Note that a limit of 40° is more restrictive than the limit of 25° in ACI Section A.2.5. The strut-and-tie force begin to grow rapidly for angles less than 40° (SNS). For the 3000-mm shear span, the minimum height center to center of nodes is 3000 tan 40° = 2517 mm. Assuming that the nodes at the bottom of this strut are located at midheight of a tie with an effective tie height of 0.1h, this puts the lower nodes at 0.05h from the bottom of the beam. We shall locate the node at B the same distance below the top, giving the required height of the beam as h ≈ 2517/0.9 = 2797 mm

1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM



1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM

- → **For a 1st trial, we shall assume b_w = 500 mm and h = 2750 mm with the lower chord located at 0.05h = 138 mm above the bottom of the beam and the center of the top node at 0.05h = 138 mm below the top of the beam**. The factored wt of the beam is: 217.8 kN. To simplify the calculations we shall add this to the column load giving a total factored load of 4214 kN. The corresponding reactions are 2528 kN at A and 1686 kN at C.

1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM

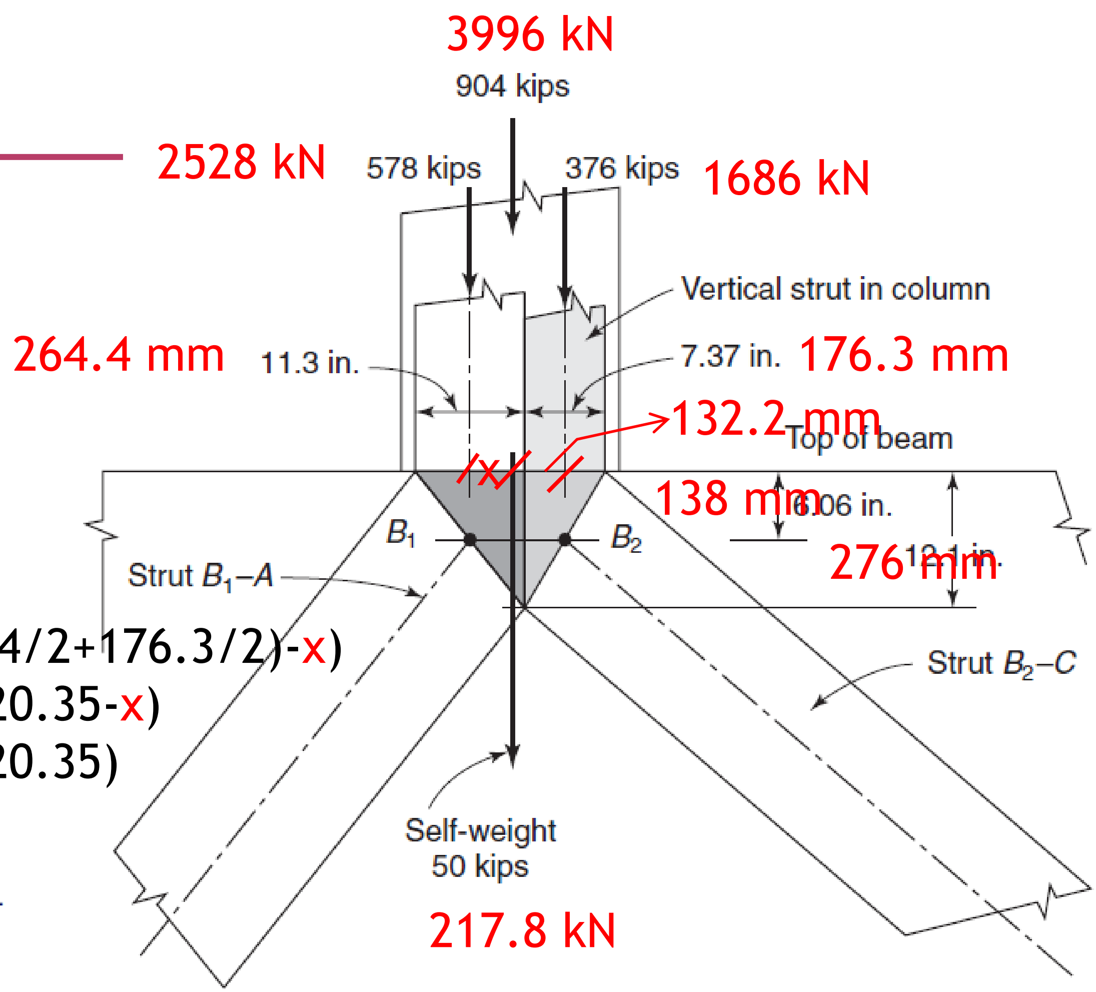
- 3. Compute effective compression strengths, f_{ce}, for the nodal zones and struts
  - **3(a) Nodal zones**
    - Nodal zone at A. This is a CCT node. $\rightarrow f_{ce} = 0.85\beta_n f'_c = 0.85 \times 0.80 \times 30 = 20.4$ Mpa
    - Nodal zone at B. This is a CCC node. $\rightarrow f_{ce} = 0.85\beta_n f'_c = 0.85 \times 1.0 \times 30 = 25.5$ Mpa
    - Nodal zone at C. This is a CCT node. $\rightarrow f_{ce} = 20.4$ Mpa
  - **3(b) Struts**
    - Strut A-B: This strut has a room for the width of the strut to be bottle-shaped. We will provide skin reinforcement satisfying ACI Section A.3.3. $\rightarrow f_{ce} = 0.85\beta_s f'_c = 0.85 \times 0.75 \times 30 = 19.1$ Mpa

1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM

- 4. Locate nodes-1st Trial
  - Nodal zones at A and C: The nodes at A and C are located at 0.05h = 138 mm above the bottom of the beam. We use this in the 1st trial strut-and tie model.
  - Nodal zone at B: node B is located at 0.05h = 138 mm below the top of the beam. The node at B is subdivided into 2 subnodes, as shown in Figure (SNS)

1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM

Including self wt



2528(x)=1686((264.4/2+176.3/2)-x)

→ 2528(x)=1686(220.35-x)

→ 4214(x)=1686(220.35)

→ x=88.16

Fig. 17-30
Struts and nodal zones adjacent to the column at B—Example 17-2.

1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM

- Subnode at B_1 is assumed to transmit 2528 kN to the left reaction and the other 1686 kN to the right reaction. Each of the subnodes is assumed to receive load from a vertical strut in the column over B. Both of the subnodes are CCC nodes. The effective strength of the nodal zone at B is f_{ce} = 25.5 Mpa. For struts A-B_1 and B_2-C, f_{ce} = 19.1 Mpa. We shall use f_{ce} = 19.1 Mpa for the faces of the nodal zone loaded by bottle-shaped struts, and f_{ce} = 25.5 Mpa for the face of the nodal zone that is loaded by compression stresses in the column and for the vertical struts in the column over point B.

1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM

- $\rightarrow w_{B1} = 2528000/(0.75 \times 25.5 \times 500) = 264.4$ mm; $w_{B2} = 176.3$ mm. (NB! Are also widths of vertical struts in column)
- The total width of the vertical struts in the column at B is $\Sigma w_{Bs} = 440.7$ mm. This will fit in the 500 mm square column.
- Because the vertical load components acting at B_1 and B_2 are different, the vertical division of node B is not located at the center of the column. B_1 is at 88.16 mm to the left of the center of the column, and B_2 at 132.2 mm to rt. With this the 1st truss geometry is established. See First Trial S. No. 130)

1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM

- **5. Compute the lengths and widths of the struts and ties and draw the 1st trial strut-and tie model**
  - Table: Geometry and Forces in Struts and Tie

Strut or Tie	Horiz Proj. (mm)	Vert Proj. (mm)	Angle	Vert. Comp. (kN)	Horiz Comp. (kN)	Axial Force (kN)	Width of Strut(mm)
Vert at A	0.0	-	Vertical	2528	0	2528	353
A-B1	2000-0-88.2=1911.8	2750-138-138=2474 mm	52.3	2528	1954	3195	446
B2-C	3000-132.2-0=2867.8	2474 mm	40.8	1686	1954	2581	360
A-C	-	-	-	-	1954 at A 1954 at C	1954 at A 1954 at C	w_t = 255
Vert at C	0.0	-	Vertical	1686	0	1686	235

$3195000/500/19.1/\phi = 446$

1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM

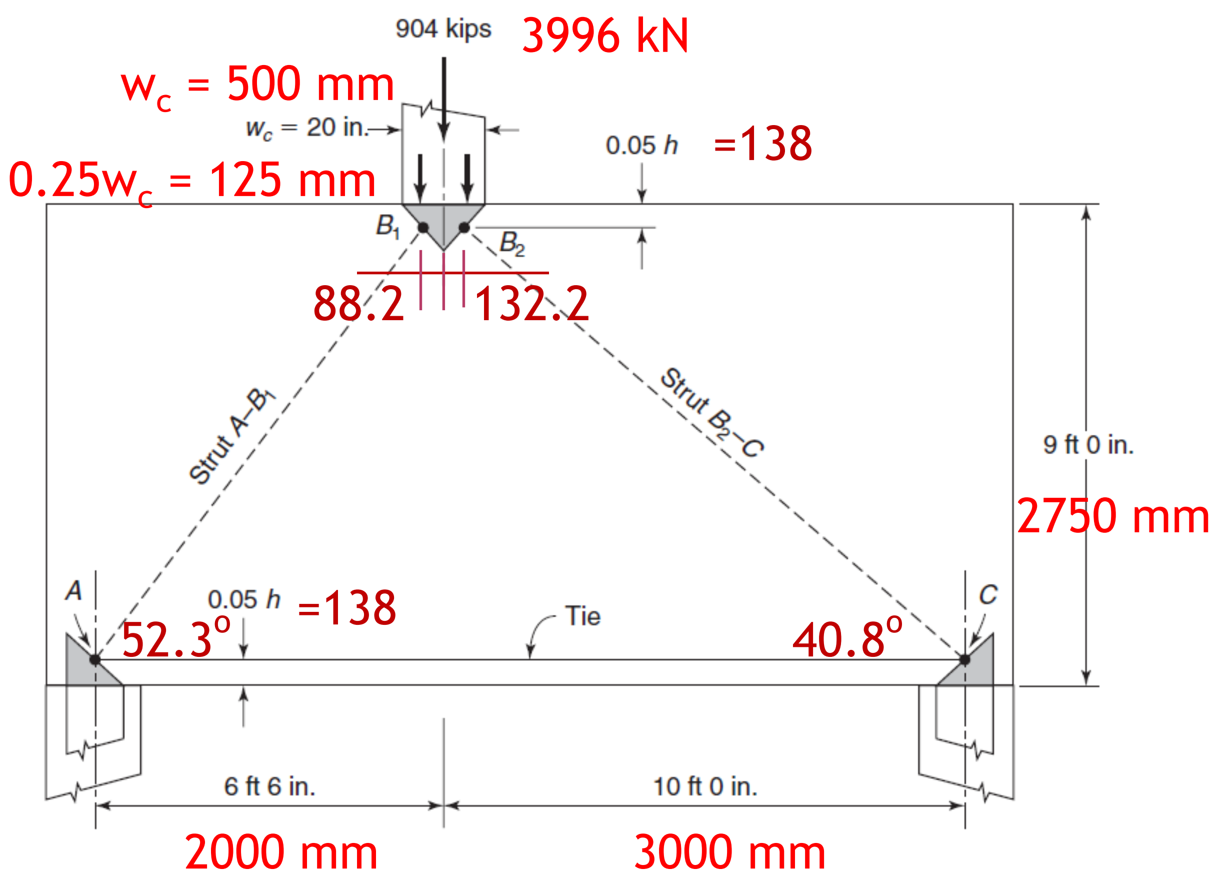


Fig. 17-29
First trial strut-and-tie model of a single-span deep beam—Example 17-2.

First trial …..

1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM

- **5(a) Geometry and forces in struts and tie** (See Table)
- Check some entries: width of strut $AB_1 \rightarrow w_s$ = 3195000/(0.75×19.1×500) = 446 mm

Vertical strut at A

- The width of the strut under the node at A is: $\rightarrow w_{sA}$ = 2528000/(0.75×19.1×500) = 353 mm
- Check the width of strut AB_1 at A, using θ = 52.3°, ℓ_b = 353 mm and w_t = 255 → Geometry of node dictates $w_s = w_t \cos\theta + \ell_b \sin\theta$ = 255 cos 52.3° + 353 sin 52.3° = 435 mm
- Because w_s = 435 mm is less than the 446 mm required from column 8 of Table, we should increase h_t or l_b to increase w_s to at least 446 mm. By increasing the width, l_b, of the vertical strut under node A from 353 mm to 367, the 446 mm width of strut A-B1 can be accommodated

1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM

- **Vertical strut at C**
  - The width of the strut under the node at C is : w_s = 1686000/(0.75×19.1×500) = 235 mm
- **Strut B_2-C**
  - See calculation results in Table

- **6. Compute tie forces, reinforcement, and the effective width of the tie**
  - 6(a) Trial choice of reinforcement in tie A-C.→
    - As = 1954000/(0.75×420) = 6203 mm^2

1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM

- Try 13 No. 25M bars. We will use one layer of 5 bars and two layers of 4 bars No. 25M bars with No. 32M bars as spacers b/n layers.

- **6(b) Minimum flexural reinforcement** does not govern(refer)
- **6(c) Effective height of tie A-C, w_t**. ACI Section R.A.4.2 suggests that the tie be spread over a ht ranging from a max equal to the tie force divided by the effective compression strength for the nodes at A and C to a minimum of half of this. → The max distribution ht of the tension tie is → $w_t = 1954000 / (0.75 \times 20.4 \times 500) = 255$ mm → The max ht of the centroid of the reinf is 255/2 = 127.5 mm above the bottom of the beam where as the centroid of the 3 layers is 118.7 above the bottom (ok)

1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM

- **6(d) Anchor the tie reinforcement at A.** The tie reinforcement must be anchored for the full yield strength at the point where the axis of the tie enters the struts at A and C. Assuming an extended nodal zone with node A directly over the center of the column, the length available to anchor the tie is (SNS)
- Distance from the center of the 500 mm column to the outside face of the cutoff or hooked bars = 250-(40+12.7) = 197.3 mm. → The anchorage length available is:
- = 197.3 + (353/2) + (127.5/tan52.4°) = 472 mm (SNS). Since ℓ_d = 1169 mm, the length available to develop anchorage of straight bars is insufficient. → Use hooks or similar method (hair pin) to anchor the bars.

1.10 Example- Design of a Single-Span Deep Beam

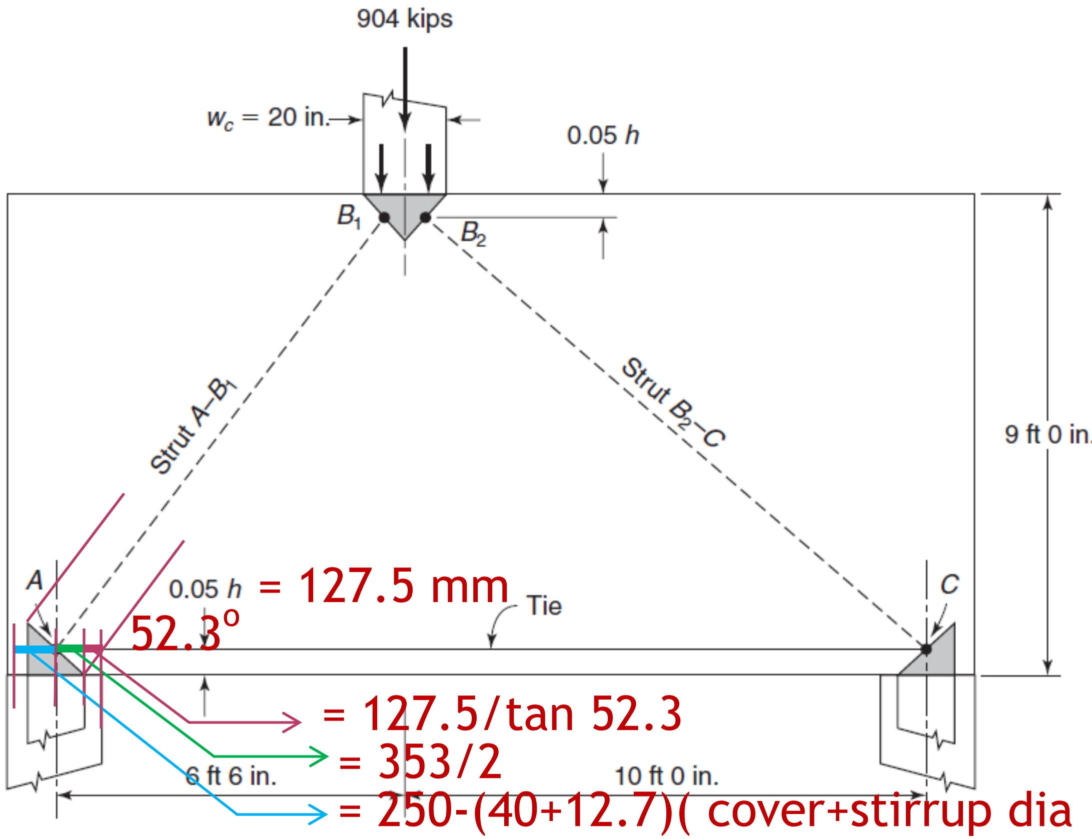


Fig. 17-29
First trial strut-and-tie model of a single-span deep beam—Example 17-2.

1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM

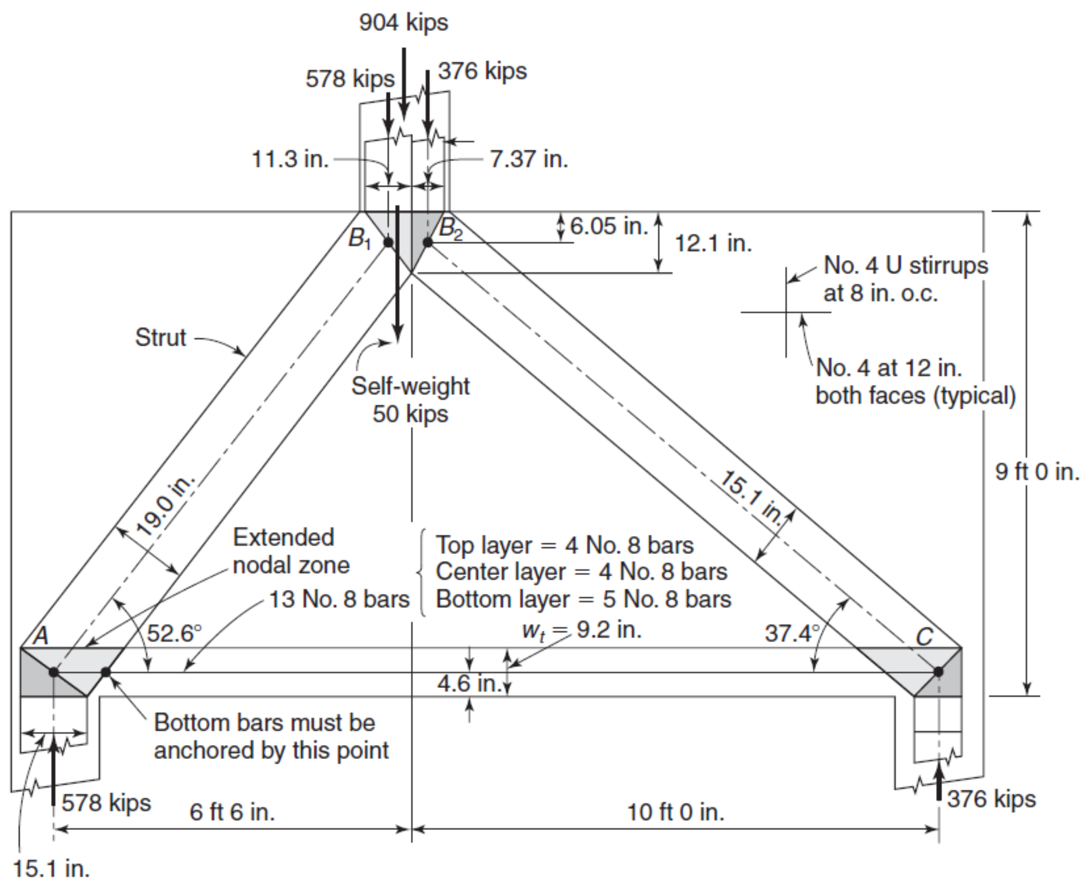


Fig. 17-31
Final design single-span deep beam—Example 17-2.

1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM

- **6(e) Check height of Node B**. Summing horizontal forces on a vertical section through the entire beam and the nodal zone at B shows that the horizontal force in the struts meeting at B equals the horizontal force in tie A-C which will be taken equal to $\phi A_s f_y$, thus, ϕF_{nn} = $0.75 \times 6630 \times 420 \times 10^{-3}$= 2088 kN. Total depth of the nodal zone at B is $2088000/(0.75 \times 19.1 \times 500)$ = 292 mm. The top nodes, B1 and B2, are located at 292/2 = 146 mm below the top of the beam

1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM

- **7. Draw second trial strut-and-tie model**. This step is required if there is a significant change in geometry of the assumed truss. In this example there is little change (e.g. Locations of top nodes B1 and B2, are 146 mm below the top edge instead of the earlier 138 mm)
- 8. Recompute strut-and-tie model using computed locations of the nodes from the second trial (See Table)
  - Nodes A and C will be assumed to be at the centers of the columns and 118.7 mm above the bottom of the beam
- 9. **Compute the required crack control reinforcement** (refer)

1.10 EXAMPLE- DESIGN OF A SINGLE-SPAN DEEP BEAM

Strut or Tie	Horiz Proj. (mm)	Vert Proj. (mm)	Angle	Vert. Comp. (kN)	Horiz Comp. (kN)	Axial Force (kN)	Width of Strut(mm)
Vert at A	0.0	-	Vertical	2528	0	2528	353
A-B1	2000-0-88.2=1911.8	2750-146-118.7=2474 mm	52.4	2528	1947	3191	446
B2-C	3000-132.2-0=2867.8	2474 mm	40.9	1686	1947	2576	360
A-C	-	-	-	-	1947 at A 1947 at C	1947 at A 1947 at C	w_t = 255
Vert at C	0.0	-	Vertical	1686	0	1686	235

CORBELS/ DAPPED ENDS

- **Assignment (involves reading and submission)**
- Other structure designed using the strut and tie method are corbels and beams with dapped ends. Read and rework examples with SI units.
- **Submit corbel example. Apply EN ES 1992-1-1 recommendations**