Groundwater Hydraulics

Chapter 3 – Well Hydraulics

CENG 6606

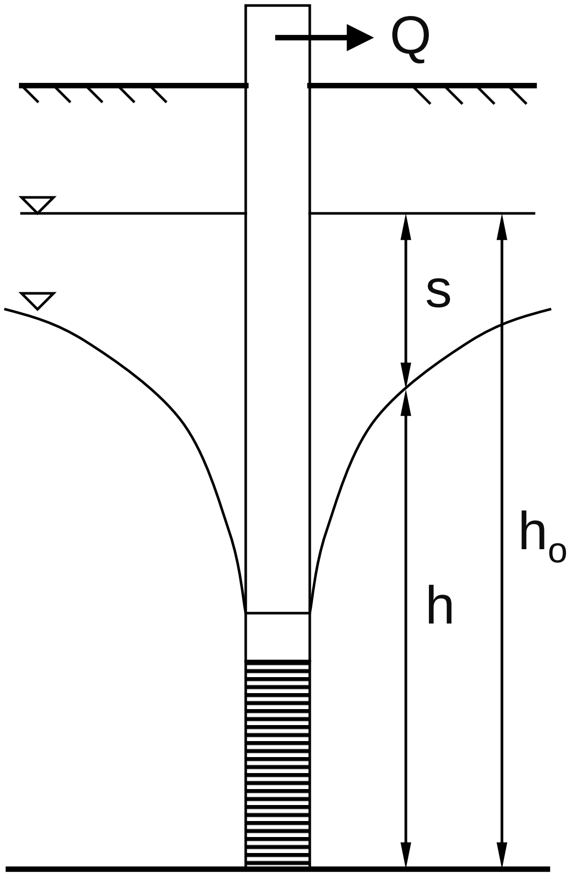
AAU

Contents

Well Hydraulics

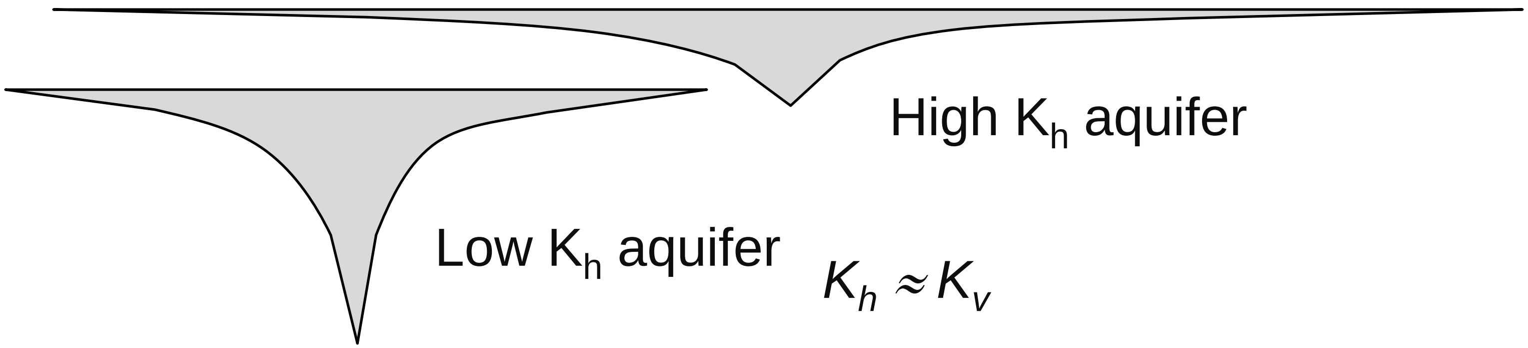
- A water well is a hydraulic structure that is designed and constructed to permit economic withdrawal of water from an aquifer
- Water well construction includes:
  - Selection of appropriate drilling methods
  - Selection of appropriate completion materials
  - Analysis and interpretation of well and aquifer performance

1. Pumping Well Terminology



- **Static Water Level** [SWL] (h_o) is the equilibrium water level before pumping commences
- **Pumping Water Level** [PWL] (h) is the water level during pumping
- **Drawdown** ($s = h_o - h$) is the difference between SWL and PWL
- **Well Yield** (Q) is the volume of water pumped per unit time
- **Specific Capacity** (Q/s) is the yield per unit drawdown

Cone of Depression

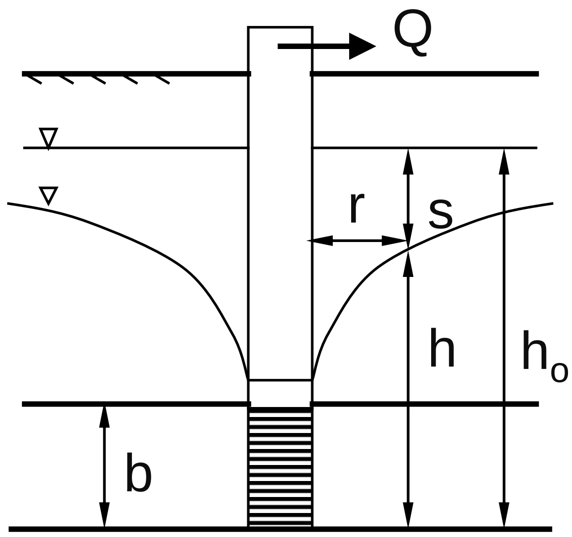


- A zone of low pressure is created centered on the pumping well
- Drawdown is maximum at the well and reduces radially
- Head gradient decreases away from the well and the pattern resembles an inverted cone called the **cone of depression**
- The cone expands over time until the inflows (from various boundaries) match the well extraction
- The shape of the equilibrium cone is controlled by hydraulic conductivity

Aquifer Characteristics

- Pump tests allow estimation of transmission and storage characteristics of aquifers
- **Transmissivity** ($T = Kb$) is the rate of flow through a vertical strip of aquifer (thickness b) of unit width under a unit hydraulic gradient
- **Storage Coefficient** ($S = S_y + S_s b$) is storage change per unit volume of aquifer per unit change in head
- **Radius of Influence** (R) for a well is the maximum horizontal extent of the cone of depression when the well is in equilibrium with inflows

2. Unsteady Radial Confined Flow



- **Assumptions**

  Isotropic, homogeneous, infinite aquifer, 2-D radial flow

- **Initial Conditions**

  $h(r,0) = h_o$ for all r

- **Boundary Conditions**

  $h(\infty,t) = h_o$ for all t

- PDE $$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right) = \frac{S}{T}\frac{\partial h}{\partial t}$$
- Solution is more complex than steady-state
- Change the dependent variable by letting

$$u = \frac{r^2 S}{4Tt}$$

- The ultimate solution is:

$$h_o - h = \frac{Q}{4\pi T}\int_u^\infty \left(\frac{e^{-u}}{u}\right)du$$

- where the integral is called the exponential integral written as the well function W(u)

This is the Theis Equation

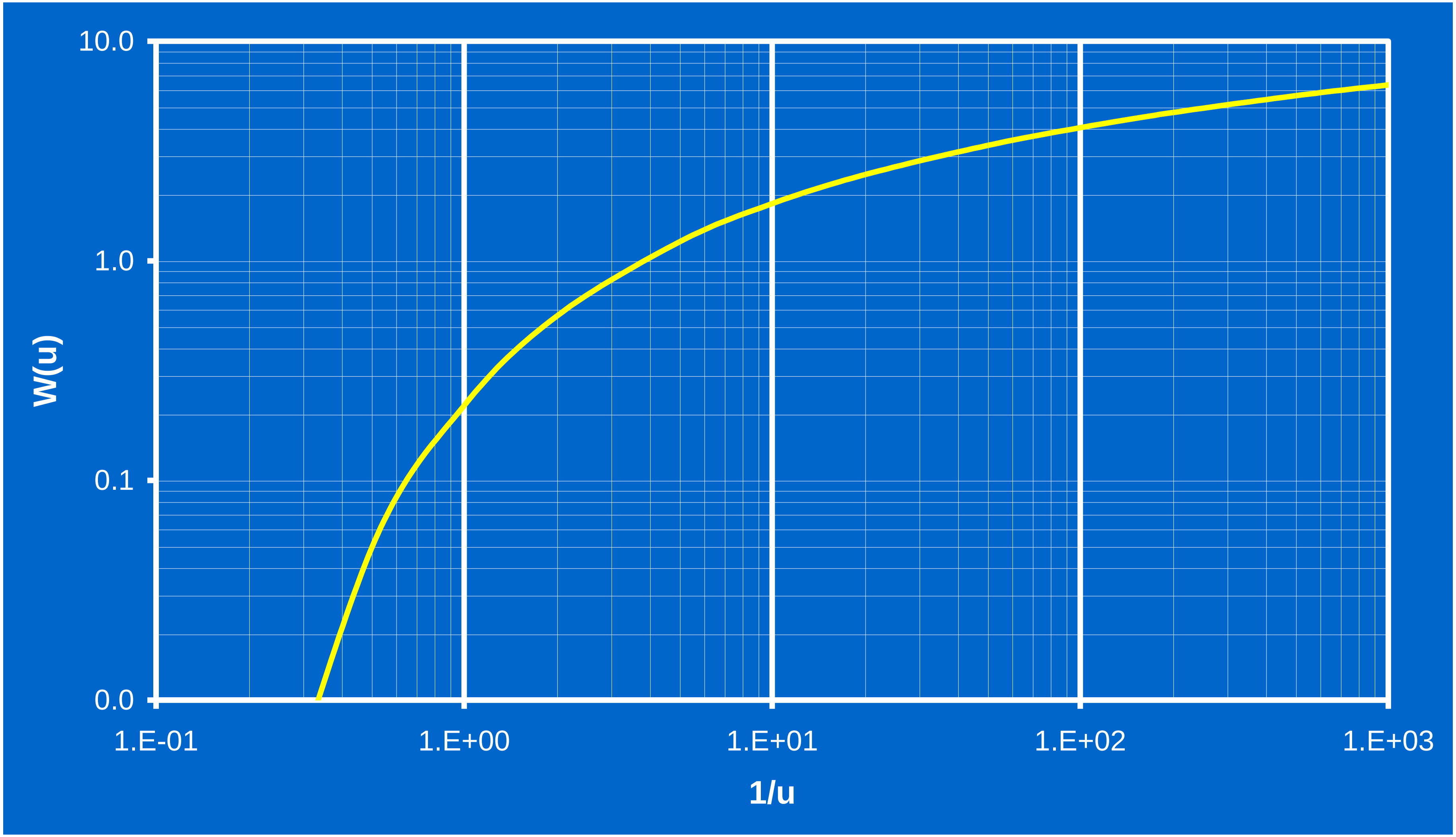
Theis PDE to ODE

- Let α = S/T (to simplify notation where α is called the inverse hydraulic diffusivity)
  - $PDE: \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right) = \alpha\frac{\partial h}{\partial t}$
  - $u = \frac{r^2 S}{4Tt} = \frac{\alpha r^2}{4t} \Rightarrow \frac{\partial u}{\partial r} = \frac{\alpha r}{2t} = \frac{2u}{r}; \frac{\partial u}{\partial t} = -\frac{\alpha r^2}{4t^2} = -\frac{u}{t}$
- Thus the PDE in terms of u
  - $\frac{1}{r}\frac{d}{du}\left(r\frac{\partial u}{\partial r}\frac{dh}{du}\right)\frac{\partial u}{\partial r} = \alpha\frac{\partial u}{\partial t}\frac{dh}{du}$
- Rewriting partial derivatives in terms of u
  - $\frac{1}{r}\frac{d}{du}\left(r\frac{2u}{r}\frac{dh}{du}\right)\frac{2u}{r} = -\alpha\frac{u}{t}\frac{dh}{du} \Rightarrow \frac{d}{du}\left(u\frac{dh}{du}\right) = -\alpha\frac{r^2}{4t}\frac{dh}{du} = -u\frac{dh}{du}$
  - $\Rightarrow u\frac{d}{du}\left(\frac{dh}{du}\right) + \frac{dh}{du} = -u\frac{dh}{du} \Rightarrow u\frac{d}{du}\left(\frac{dh}{du}\right) = -(u+1)\frac{dh}{du}$
  - $\Rightarrow u\frac{dh'}{du} + h' = -uh' where\ h' = \frac{dh}{du}$
  - $\Rightarrow \frac{dh'}{du} = -\left(1+\frac{1}{u}\right)h' \Rightarrow \frac{dh'}{h'} = -\left(1+\frac{1}{u}\right)du$

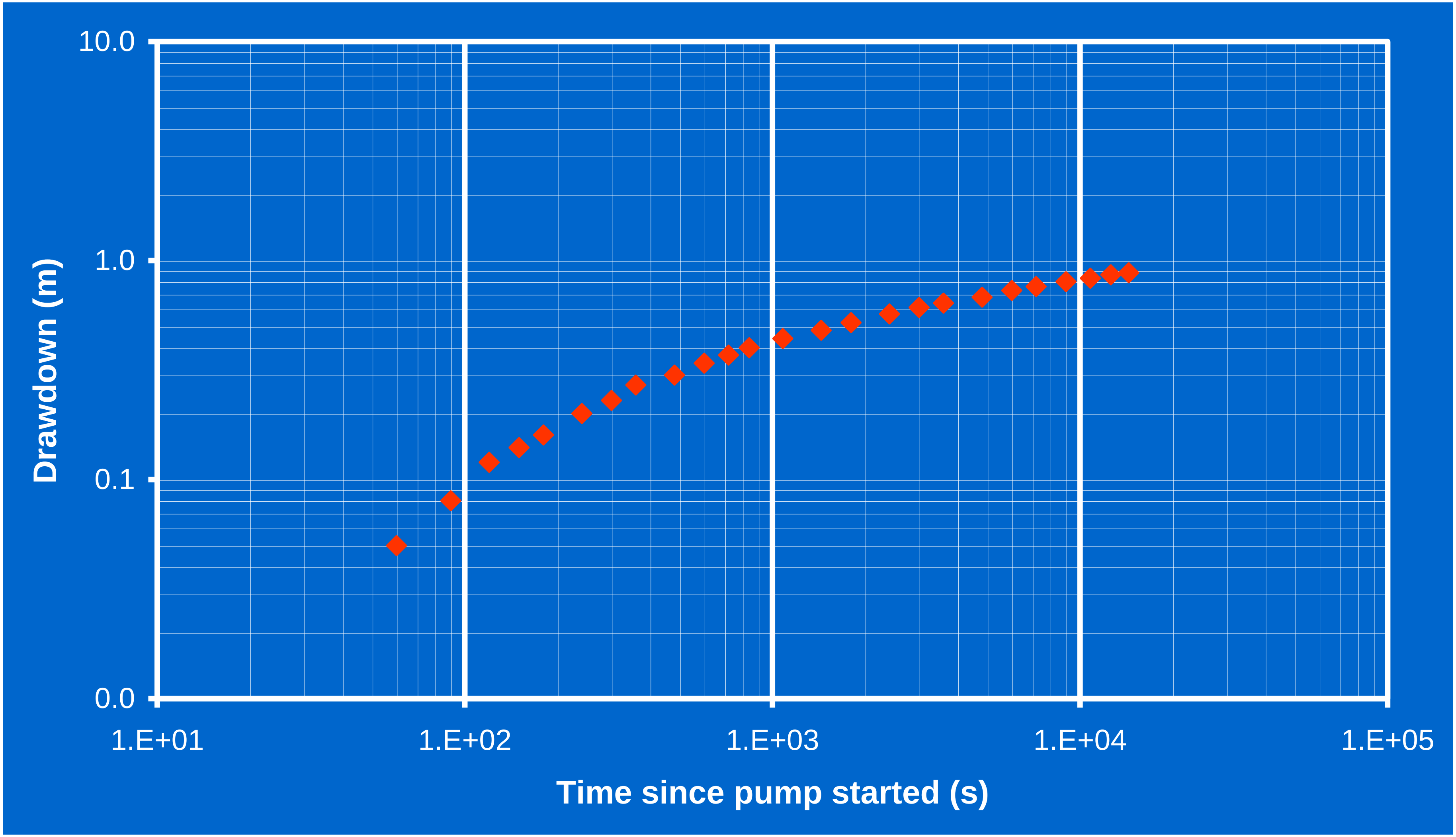
Theis Integration

- The resulting ODE is:
  - $\frac{dh'}{h'} = -\left(1 + \frac{1}{u}\right) du$
  - $\Rightarrow \ln(h'u) = c - u$
  - $\Rightarrow h'u = e^c e^{-u}$
  - $\Rightarrow \lim_{u \to 0}(h'u) = \lim_{u \to 0}(e^c e^{-u}) = e^c$
- To eliminate e^c, use Darcy's Law:
  - Remember $\lim_{r \to 0}\left(r\frac{\partial h}{\partial r}\right) = -\frac{Q}{2\pi K b} = \lim_{u \to 0}(2h'u)$
  - $r\frac{\partial h}{\partial r} = r\frac{\partial h}{\partial u}\frac{\partial u}{\partial r} = r\frac{dh}{du}\frac{2u}{r} = 2h'u$
- Simplifying: $h' = \frac{-Q}{4\pi T} \times \frac{e^{-u}}{u}$
  - $h' = \frac{dh}{du} = \frac{-Q}{4\pi T} \times \frac{e^{-u}}{u}$
  - $\Rightarrow h = \frac{-Q}{4\pi T}\int_u^\infty \left(\frac{e^{-u}}{u}\right) du + C$
- Finally, using $h(\infty,t) = h_o$ to eliminate C:
  - $h_o - h = \frac{Q}{4\pi T}\int_u^\infty \left(\frac{e^{-u}}{u}\right) du$
- The integral is called the exponential integral but is often written as the Theis well function
  - $s = h_o - h = \frac{Q}{4\pi T} W(u)$
- Well function is dimensionless

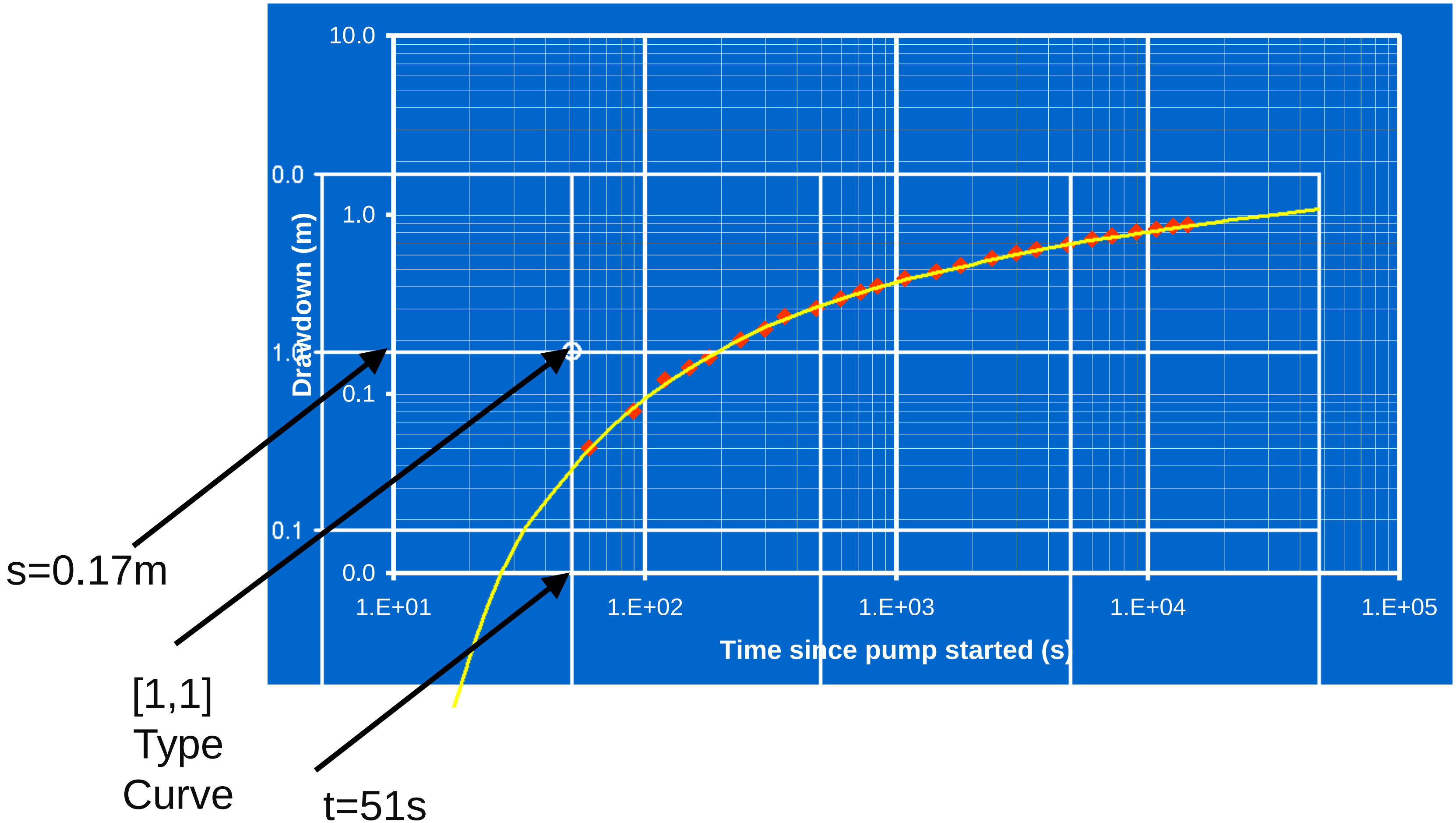
Theis Plot : 1/u vs W(u)



Theis Plot : Log(time) vs Log(drawdown)



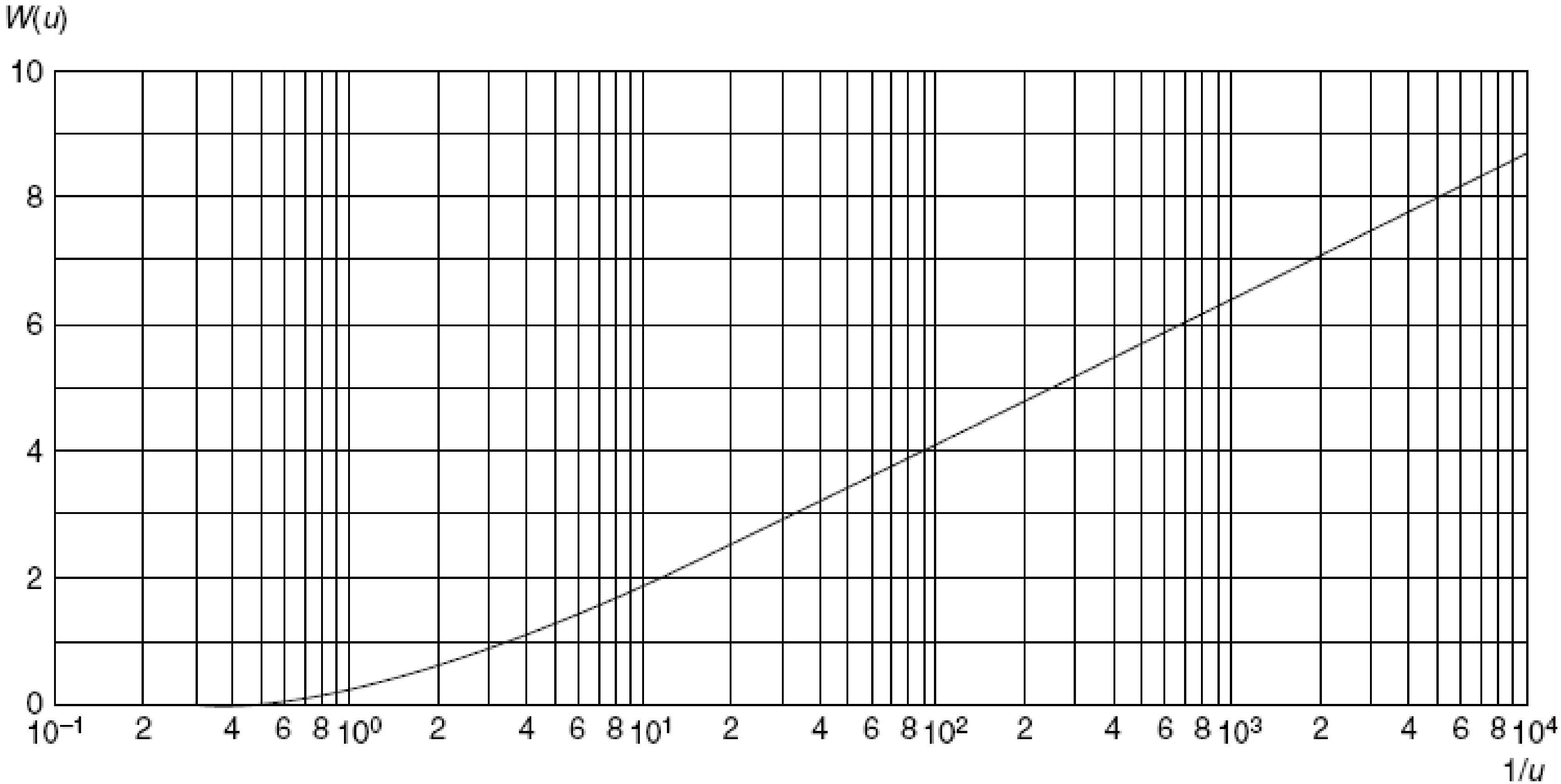
Theis Plot : Log(time) vs Log(drawdown)



Theis Analysis

1. Overlay type-curve on data-curve keeping axes parallel
2. Select a point on the type-curve (any will do but [1,1] is simplest)
3. Read off the corresponding co-ordinates on the data-curve [t_d, s_d]
4. For [1,1] on the type curve corresponding to [t_d, s_d], $T = Q/4\pi s_d$ and $S = 4Tt_d/r^2 = Qt_d/\pi r^2 s_d$
5. For the example, Q = 32 L/s or 0.032 m^3/s; r = 120 m; t_d = 51 s and s_d = 0.17 m
6. T = (0.032)/(12.56 x 0.17) = 0.015 m^2/s = 1300 m^2/d
7. S = (0.032 x 51)/(3.14 x 120 x 120 x 0.17) = 2.1 x 10^{-4}

Copper Jacob



Cooper-Jacob

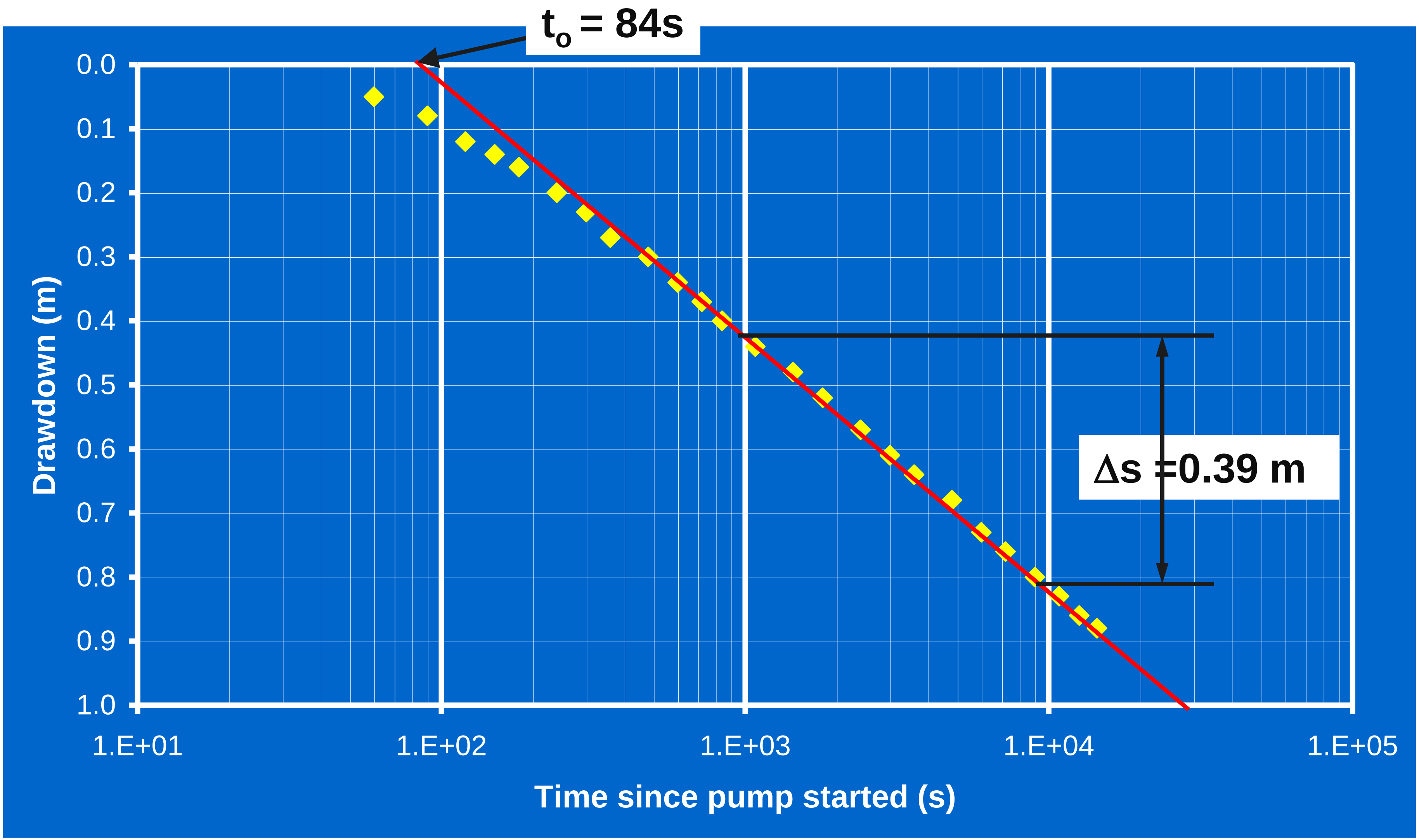
- In the above figure, the Theis well function *W(u) is plotted vs. 1/u on semi-log paper.*
- This figure shows that, for large values of 1/*u*, *the Theis well function exhibits a straight-line segment.*
- The Jacob method is based on this phenomenon. Cooper and Jacob (1946) showed that, for the straight-line segment, s can be approximated by

$$s = h_o - h = \frac{Q}{4\pi T} W(u) = \frac{Q}{4\pi T} \ln\left(\frac{2.25Tt}{r^2 S}\right) = \frac{2.3Q}{4\pi T} \log\left(\frac{2.25Tt}{r^2 S}\right)$$

with an error less than 1%, 2%, 5%, and 10% for 1/*u larger than 30*, *20*, *10*, *and 7*, *respectively*.

- The Cooper-Jacob simplification expresses drawdown (*s*) as a linear function of *ln(t)* or *log(t)*.

Cooper-Jacob Plot : Log(t) vs s



Cooper-Jacob Analysis

- Fit straight-line to data (excluding early and late times if necessary):
- Note: at early times the Cooper-Jacob approximation may not be valid and at late times boundaries may significantly influence drawdown
- Determine intercept on the time axis for s=0
- Determine drawdown increment (Δs) for one log-cycle
- For straight-line fit,

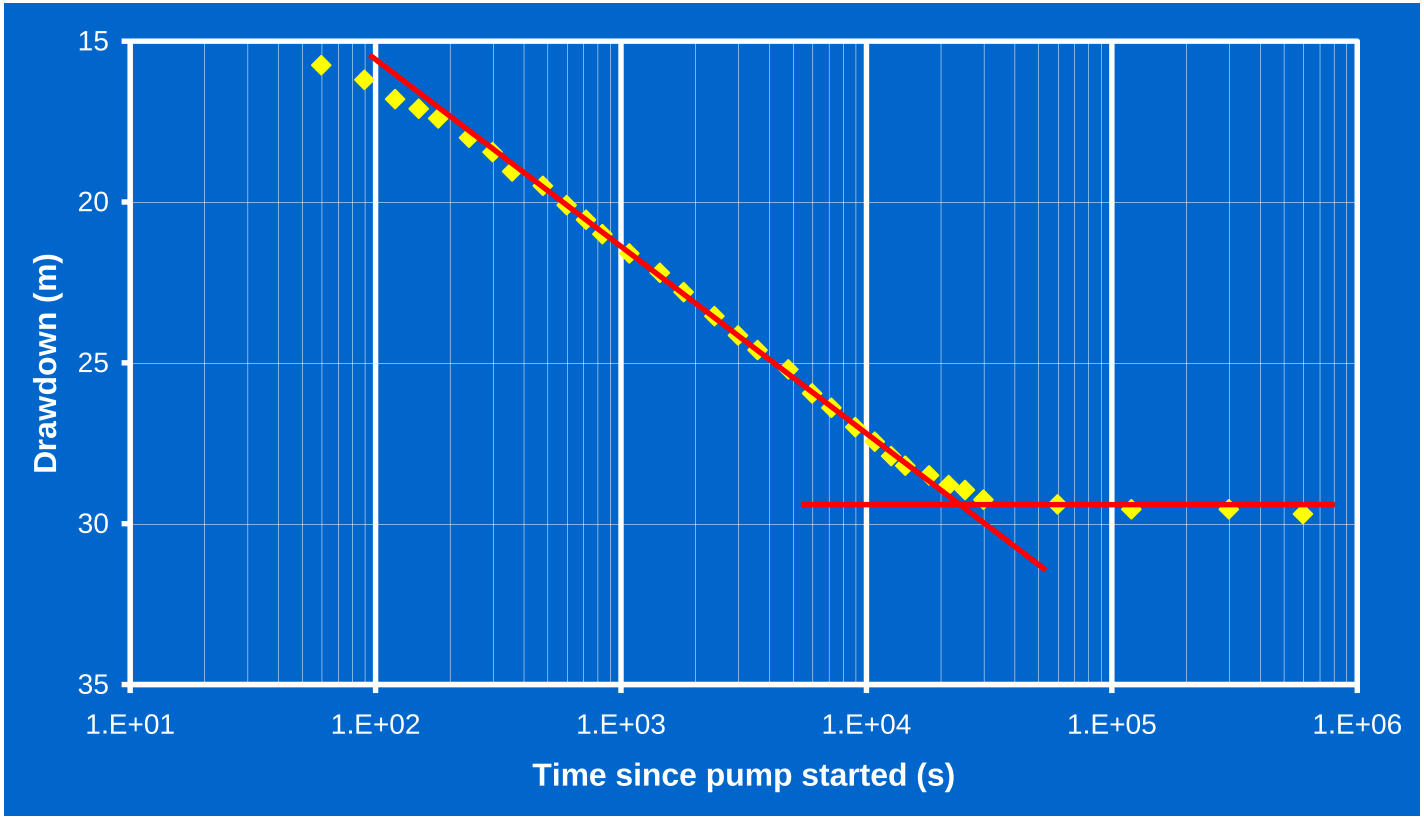
$$T = \frac{2.3Q}{4\pi\Delta s} \qquad S = \frac{2.25Tt_0}{r^2} = \frac{2.3Qt_0}{1.778\Delta s \pi r^2}$$

- For the example, Q = 32 l/s or 0.032 m^3/s; r = 120 m; t_o = 84 s and Δs = 0.39 m
- T = (2.3 x 0.032)/(12.56 x 0.39) = 0.015 m^2/s = 1300 m^2/d
- S = (2.3 x 0.032 x 84)/(1.78 x 3.14 x 120 x 120 x 0.39) = 1.9 x 10^{-4}

Theis-Cooper-Jacob Assumptions

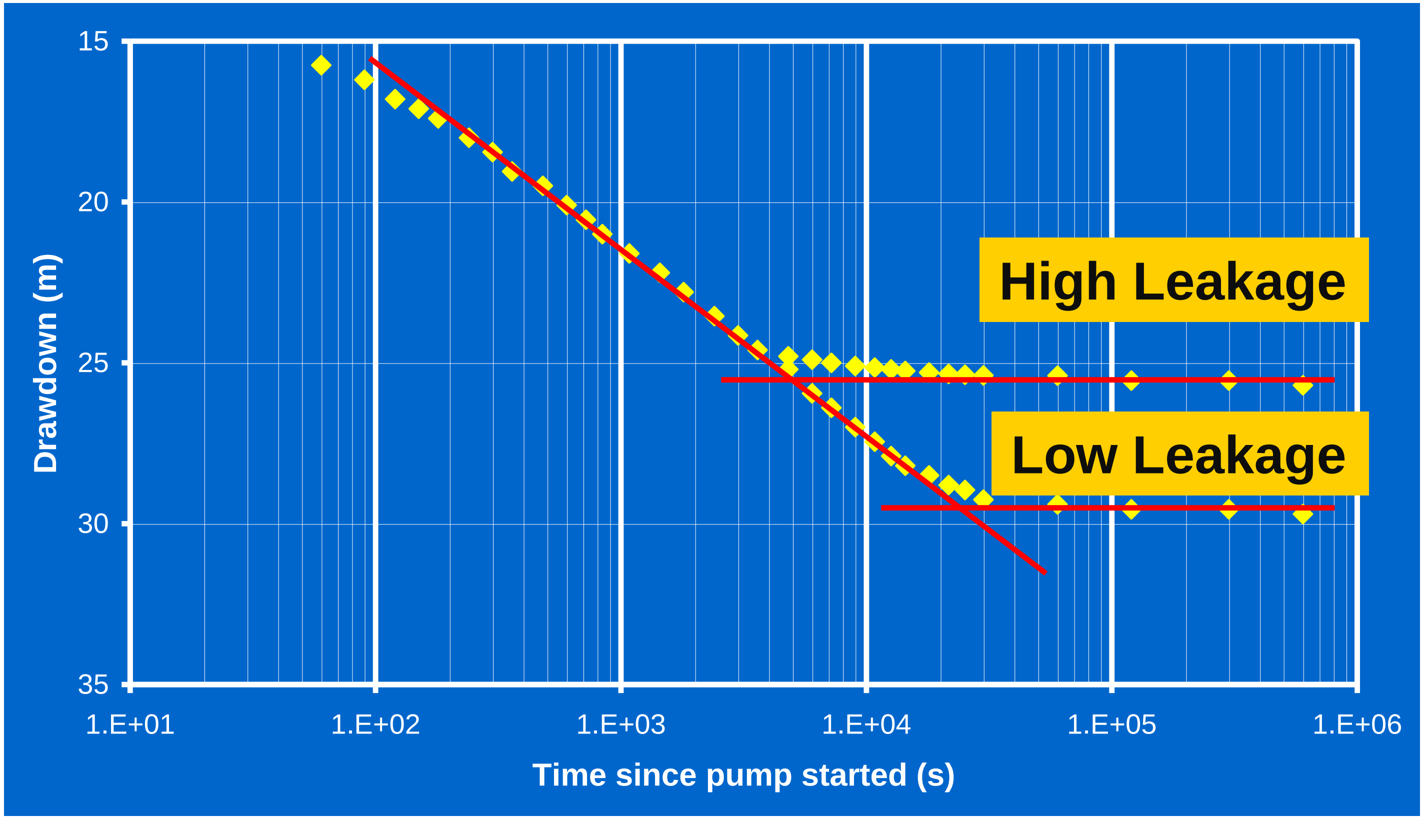
- Real aquifers rarely conform to the assumptions made for Theis-Cooper-Jacob non-equilibrium analysis
  - Isotropic, homogeneous, uniform thickness
  - Fully penetrating well
  - Laminar flow
  - Flat potentiometric surface
  - Infinite areal extent
  - No recharge
- Failure of some or all of these assumptions leads to "non-ideal" behavior and deviations from the Theis and Cooper-Jacob analytical solutions for radial unsteady flow

Recharge Effect : Recharge > Well Yield



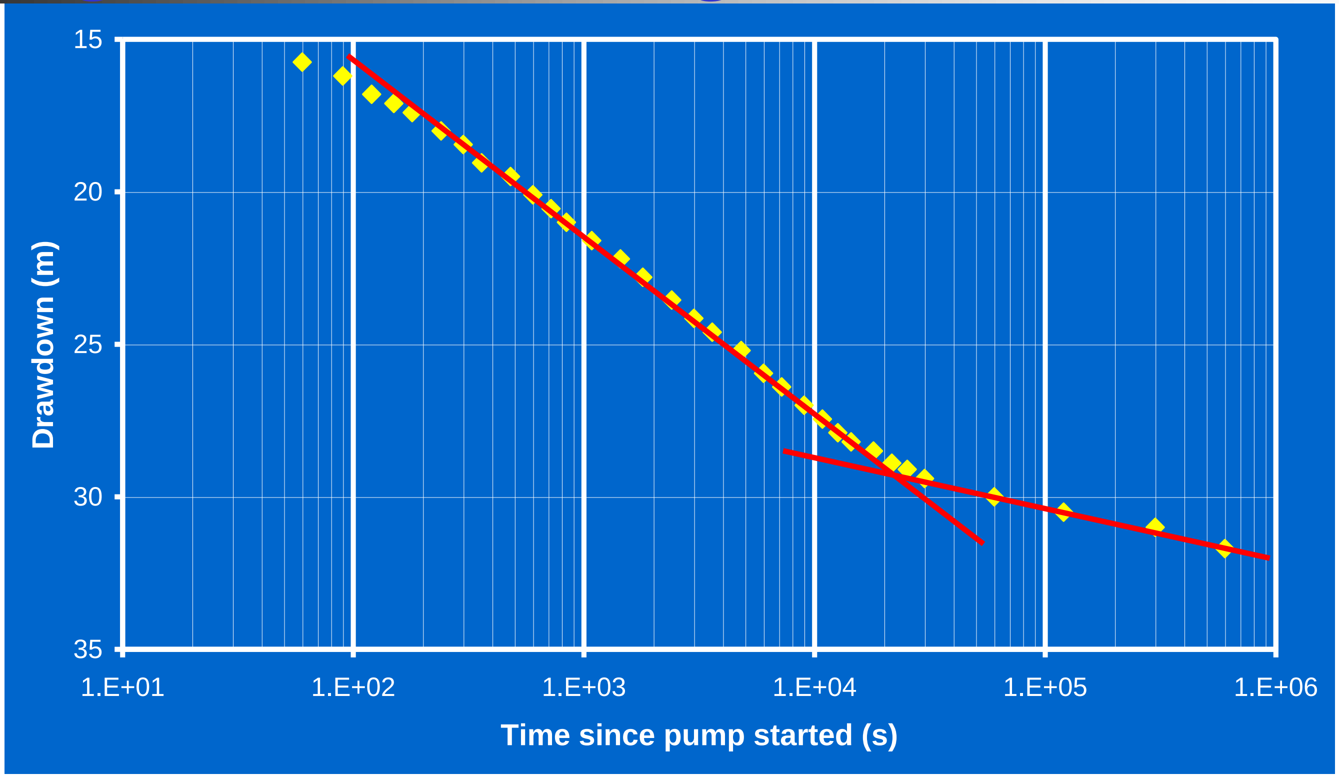
Recharge causes the slope of the log(time) vs drawdown curve to flatten as the recharge in the zone of influence of the well matches the discharge. The gradient and intercept can still be used to estimate the aquifer characteristics (T, S).

Recharge Effect : Leakage Rate



Recharge by vertical leakage from overlying (or underlying beds) can be quantified using analytical solutions developed by Jacob (1946). The analysis assumes a single uniform leaky bed.

Recharge Effect : Recharge < Well Yield



If the recharge is insufficient to match the discharge, the log(time) vs drawdown curve flattens but does not become horizontal and drawdown continues to increase at a reduced rate.

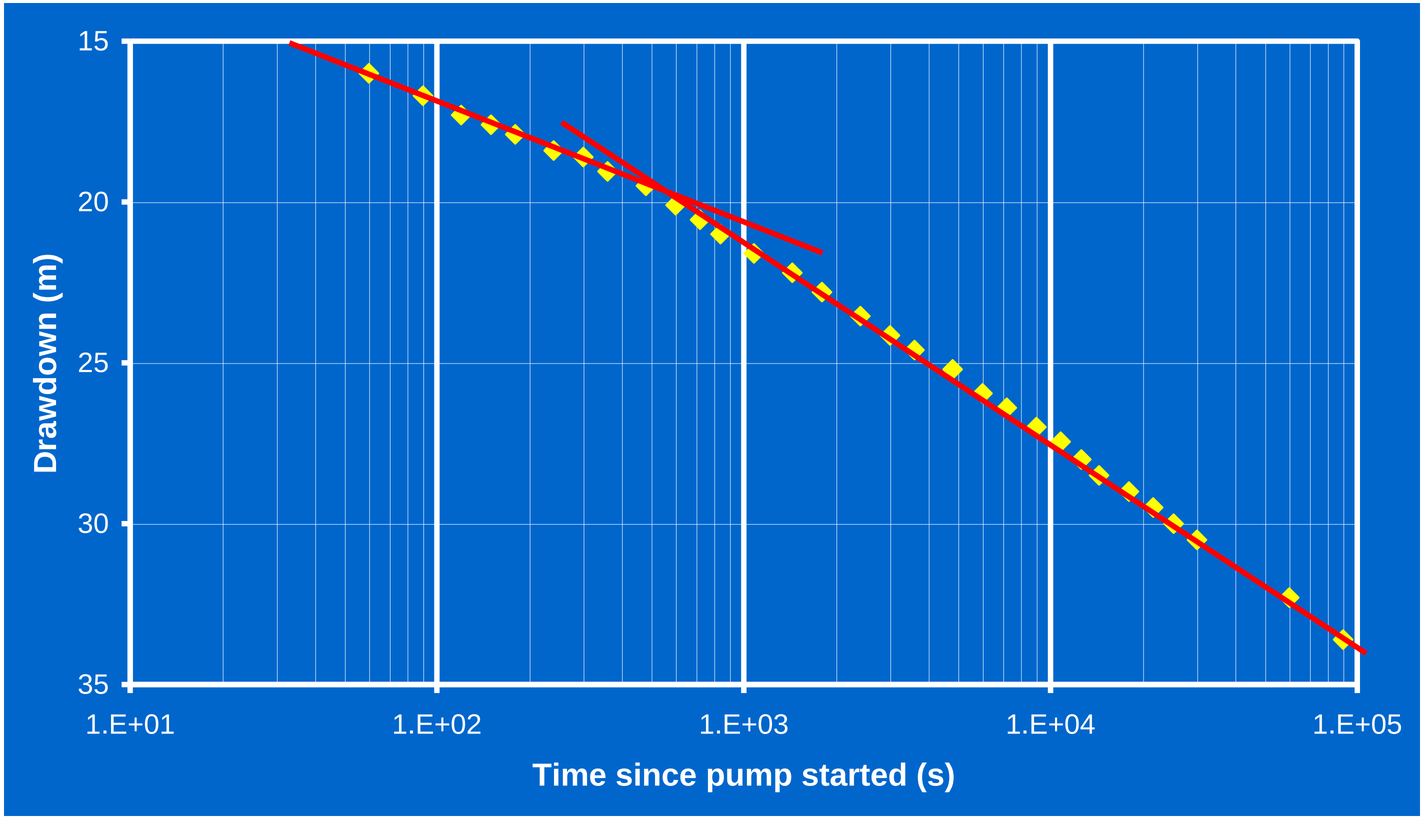
The same result will be obtained if the average T and/or S increased

T and S can be estimated from the first leg of the curve.

Sources of Recharge

- Various sources of recharge may cause deviation from the ideal Theis behavior.
- Surface water: river, stream or lake boundaries may provide a source of recharge, halting the expansion of the cone of depression.
- Vertical seepage from an overlying aquifer, through an intervening aquitard, as a result of vertical gradients created by pumping, can also provide a source of recharge.
- Where the cone of depression extends over large areas, leakage from aquitards may provide sufficient recharge.

Barrier Effect : No Flow Boundary



Steepening of the log(time) vs. drawdown curve indicates:

- An aquifer limited by a barrier boundary of some kind.
- The average transmissivity and/or storativity decreased

Aquifer characteristics (T,S) can be estimated from the first leg.

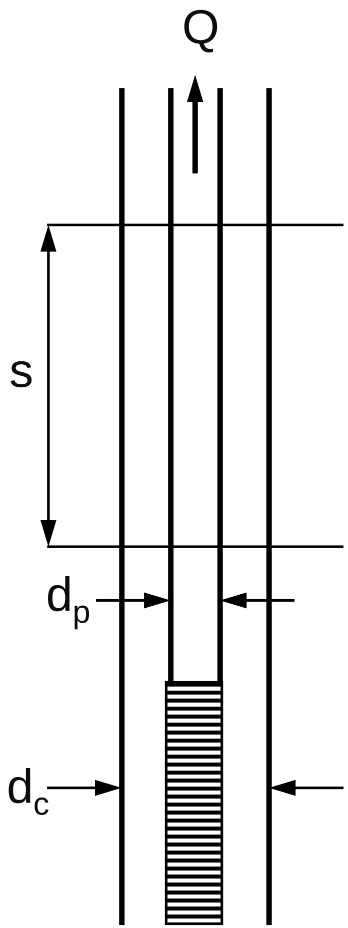
Potential Flow Barriers

- Various flow barriers may cause deviation from the ideal Theis behavior.
- Fault truncations against low permeability aquitards.
- Lenticular pinch outs and lateral facies changes associated with reduced permeability.
- Groundwater divides associated with scarp slopes.
- Spring lines with discharge captured by wells.
- Artificial barriers such as grout curtains and slurry walls.

Casing Storage

- It has been known for many decades that early time data can give erroneous results because of removal of water stored in the well casing.
- When pumping begins, this water is removed and the amount drawn from the aquifer is consequently reduced.
- The true aquifer response is masked until the casing storage is exhausted.
- Analytical solutions accounting for casing storage were developed by Papadopulos and Cooper (1967) and Ramey et al (1973)
- Unfortunately, these solutions require prior knowledge of well efficiencies and aquifer characteristics

Casing Storage



Schafer (1978) suggests that an estimate of the critical time to exhaust casing storage can be made more easily:

$$t_c = 3.75\pi(d_c^2 - d_p^2) / (Q/s) = 15\ V_a/Q$$

where: t_c is the critical time (d); d_c is the inside casing diameter (m); d_p is the outside diameter of the rising main (m); Q/s is the specific capacity of the well (m^3/d/m); V_a is the volume of water removed from the annulus between casing and rising main.

Note: It is safest to ignore data from pumped wells earlier than time t_c in wells in low-K region.

3. Distance-Drawdown

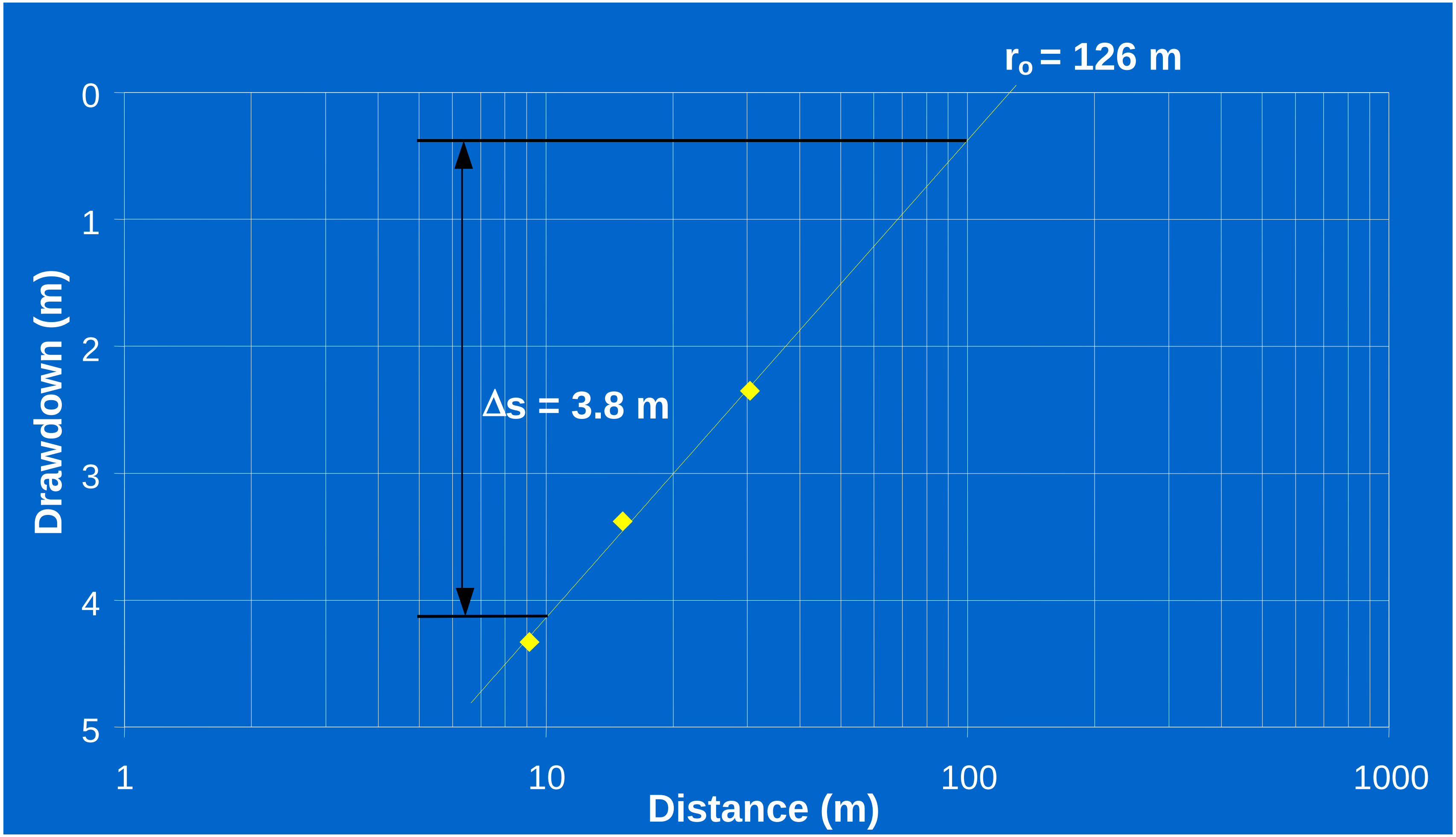
- Simultaneous drawdown data from at least three observation wells, each at different radial distances, can be used to plot a log(distance)-drawdown graph.
- The Cooper-Jacob equation, for fixed t, has the form:

$$s = \frac{2.3Q}{4\pi T}\log\left(\frac{2.25Tt}{r^2 S}\right) = \frac{2.3Q}{4\pi T}\log\left(\frac{2.25Tt}{S}\right) - \frac{4.6Q}{4\pi T}\log(r)$$

- So the log(distance)-drawdown curve can be used to estimate aquifer characteristics by measuring Δs for one log-cycle and the r_o intercept on the distance-axis.

$$T = \frac{4.6Q}{4\pi T(\Delta s)} \; and \; S = \frac{2.25Tt}{r_o^{\,2}}$$

Distance-Drawdown Graph



Distance drawdown Analysis

- For the example: t = 0.35 days and Q = 1100 m^3/d

$$T = 0.366 \times 1100 / 3.8 = 106 \ m^2/d$$

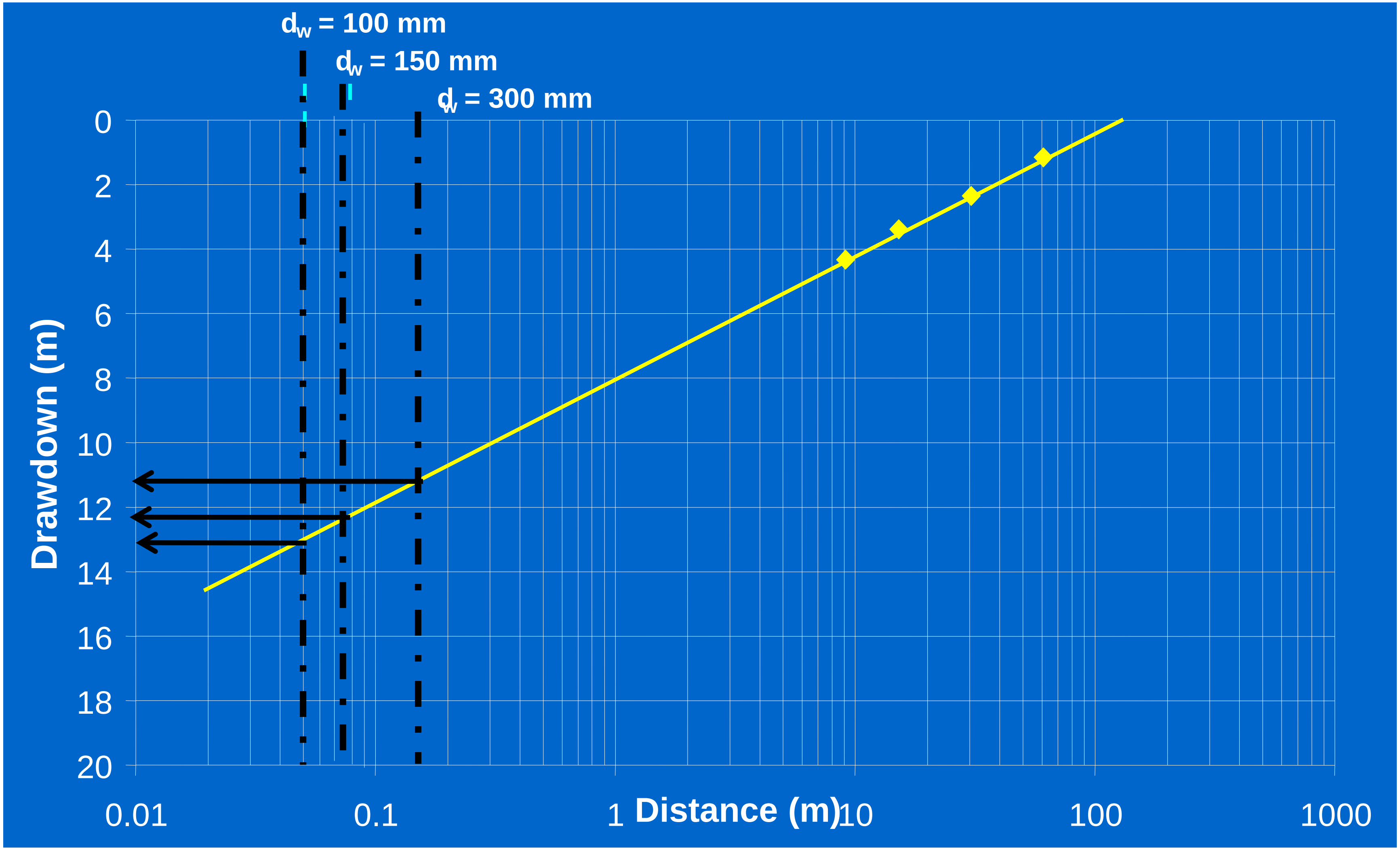
$$S = 2.25 \times 106 \times 0.35 / (126 \times 126) = 5.3 \times 10^{-3}$$

- The estimates of T and S from log(time)-drawdown and log(distance)-drawdown plots are independent of one another and so are recommended as a check for consistency in data derived from pump tests.
- Ideally 4 or 5 observation wells are needed for the distance-drawdown graph and it is recommended that T and S are computed for several different times.

Well Efficiency

- The efficiency of a pumped well can be evaluated using distance-drawdown graphs.
- The distance-drawdown graph is extended to the outer radius of the pumped well (including any filter pack) to estimate the theoretical drawdown for a 100% efficient well.
- This analysis assumes the well is fully-penetrating and the entire saturated thickness is screened.
- The theoretical drawdown (estimated) divided by the actual well drawdown (observed) is a measure of well efficiency.
- A correction is necessary for unconfined wells to allow for the reduction in saturated thickness as a result of drawdown.

Theoretical Pumped Well Drawdown



Unconfined Well Correction

- The adjusted drawdown for an unconfined well is given by:

$$s_c = s_a\left(1 - \frac{s_a}{2b}\right)$$

where b is the initial saturated thickness;
s_a is the measured drawdown; and
s_c is the corrected drawdown

- For example, if b = 20 m; s_a = 6 m; then the corrected drawdown $s_c = 0.85 s_a = 5.1$ m
- If the drawdown is not corrected, the Jacob and Theis analysis underestimates the true transmissivity under saturated conditions by a factor of s_c / s_a.

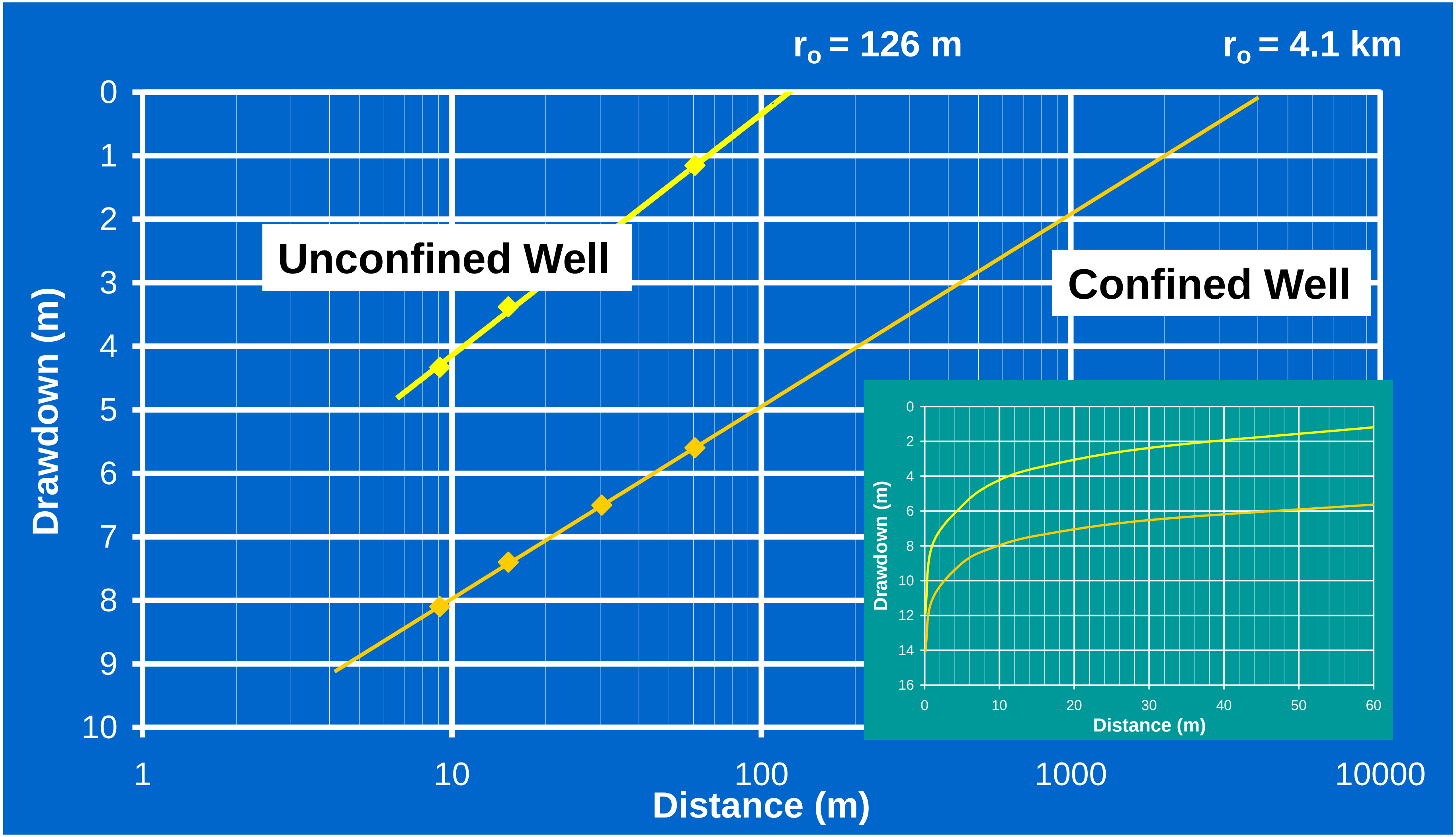
Causes of Well Inefficiency

- Factors contributing to well inefficiency (excess head loss) fall into two groups:
  - Design factors
    - Insufficient open area of screen
    - Poor distribution of open area
    - Insufficient length of screen
    - Improperly designed filter pack
  - Construction factor
    - Improper placement of screen relative to aquifer interval
    - Compaction of aquifers near by the well
    - Clogging of the aquifer by the drilling mud

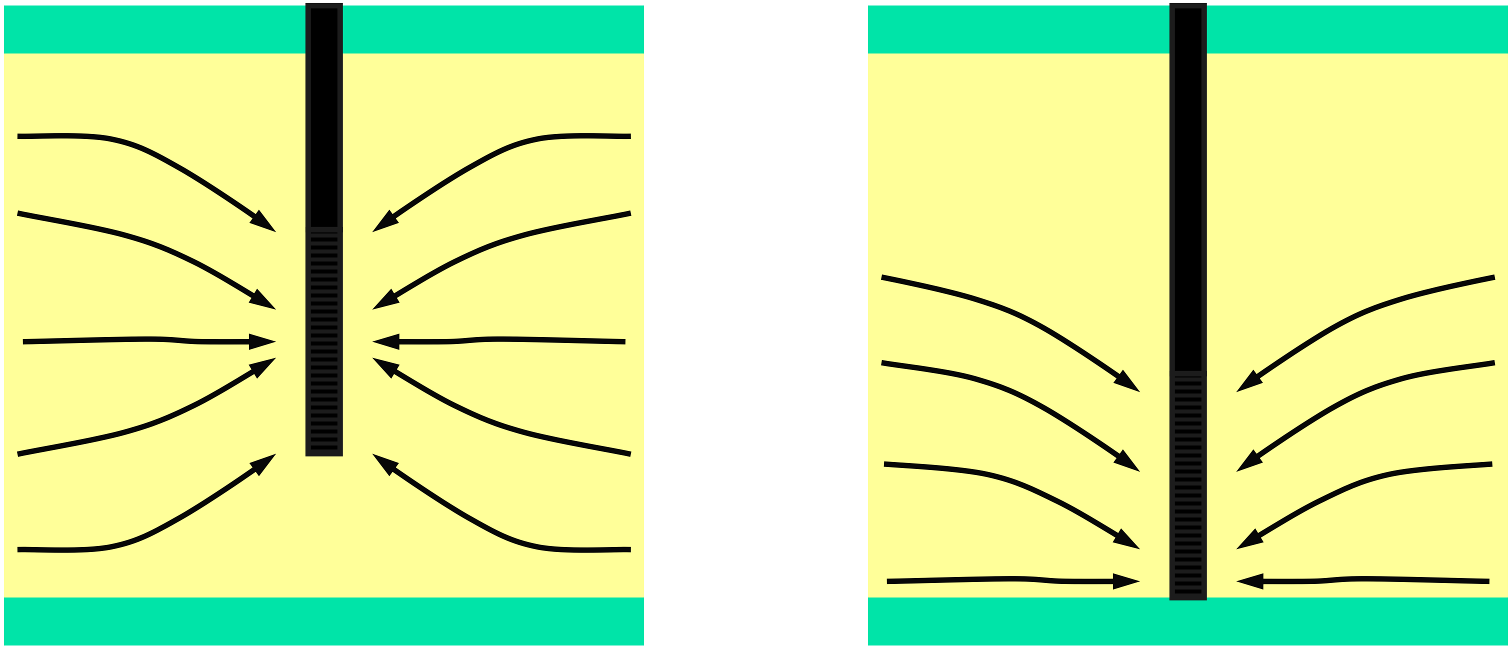
Radius of Influence

- The radius of influence of a well can be determined from a distance-drawdown plot.
- For all practical purposes, a useful comparative index is the intercept of the distance-drawdown graph on the distance axis.
- Radius of influence can be used as a guide for well spacing to avoid interference.
- Since radius of influence depends on the balance between aquifer recharge and well discharge, the radius may vary from year to year.
- For unconfined wells in productive aquifers, the radius of influence is typically a few hundred meters.
- For confined wells may have a radius of influence extending several kilometers.

Determining r_o



4. Partial Penetration



- Partial penetration effects occur when the intake of the well is less than the full thickness of the aquifer

Effects of Partial Penetration

- The flow is not strictly horizontal and radial.
- Flow-lines curve upwards and downwards as they approach the intake and flow-paths are consequently longer.
- The convergence of flow-lines and the longer flow-paths result in greater head-loss than predicted by the analytical equations.
- For a given yield (Q), the drawdown of a partially penetrating well is more than that for a fully penetrating well.
- The analysis of the partially penetrating case is difficult but Kozeny (1933) provides a practical method to estimate the change in specific capacity (Q/s).

Q/s Reduction Factors

- Kozeny (1933) gives the following approximate reduction factor to correct specific capacity (Q/s) for partial penetration effects:

$$F = \frac{L}{b}\left[1 + 7\cos\left(\pi L / 2b\right)\sqrt{r / 2L}\right]$$

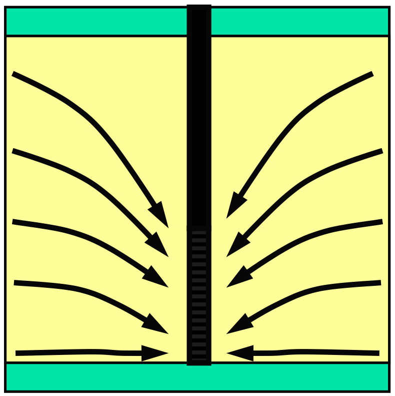
where b is the total aquifer thickness (m); r is the well radius (m); and L is screen length (m).

- The equation is valid for L/b < 0.5 and L/r > 30
- For a 300 mm dia. well with an aquifer thickness of 30 m and a screen length of 15 m, L/b = 0.5 and 2L/r = 200 the reduction factor is:

$$F = 0.5 \times \{1 + 7 \times 0.707 \sqrt{(1/200)}\} = 0.67$$

- Other factor are provided by Muskat (1937), Hantush (1964), Huisman (1964), Neumann (1974) but they are harder to use.

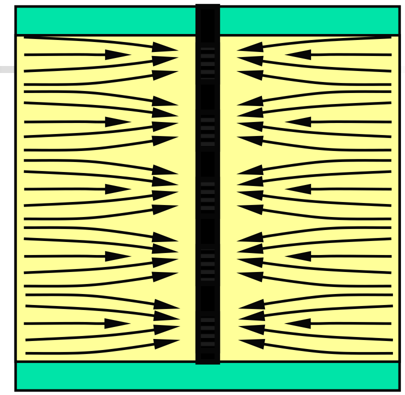
Screen Design



- 300 mm dia. well with single screened interval of 15 m in aquifer of thickness 30 m.

  L/b = 0.5 and 2L/r = 200

F = 0.5 x {1 + 7 x cos (0.5π/2) √(1/200)} = 0.67



- 300 mm dia. well with 5 x 3 m solid sections alternating with 5 x 3m screened sections, in an aquifer of thickness 30 m.

  There effectively are five aquifers.

  L/b = 0.5 and 2L/r = 40

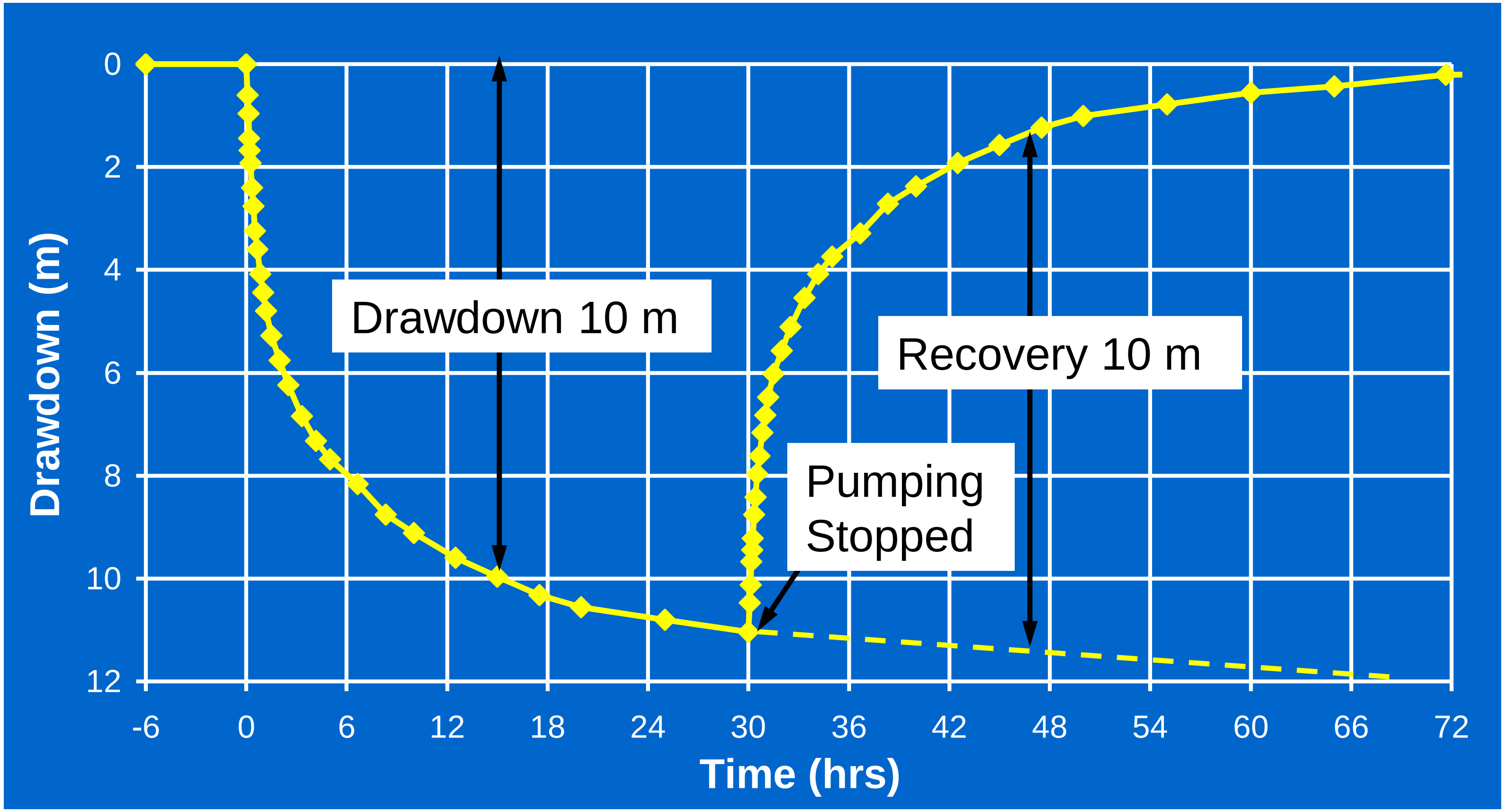
  F = 0.5 x {1 + 7 x cos(0.5π/2) √(1/40)} = 0.89

  This is clearly a much more efficient well completion.

5. Recovery Data

- When pumping is halted, water levels rise towards their pre-pumping levels.
- The rate of recovery provides a second method for calculating aquifer characteristics.
- Monitoring recovery heads is an important part of the well-testing process.
- Observation well data (from multiple wells) is preferable to that gathered from pumped wells.
- Pumped well recovery records are less useful but can be used in a more limited way to provide information on aquifer properties.

Recovery Curve



The recovery curve on a linear scale appears as an inverted image of the drawdown curve. The dotted line represent the continuation of the drawdown curve.

Superposition

- The drawdown (s) for a well pumping at a constant rate (Q) for a period (t) is given by:

$$s = h_o - h = \frac{QW(u)}{4\pi T}; \; where \; u = r^2 S / (4Tt)$$

- The effects of well recovery can be calculated by adding the effects of a pumping well to those of a recharge well using the superposition theorem.
- Applying this principle, it is assumed that, after the pump has been shut down, the well continues to be pumped at the same discharge as before, and that an imaginary recharge, equal to the discharge, is injected into the well. The recharge and discharge thus cancel each other, resulting in an idle well as is required for the recovery period.

- The drawdown (s_r) for a well recharged at a constant rate (-Q) for a period ($t' = t - t_r$) starting from time t_r (the time at which the pumping stopped) is given by:

$$s_r = -\frac{QW(u')}{4\pi T}; \; where \; u' = r^2S \big/ (4Tt')$$

- The total (Residual) drawdown according to Theis for $t > t_r$ is:

$$s' = s + s_r = \frac{Q(W(u) - W(u'))}{4\pi T}$$

Residual Drawdown and Recovery

- The Cooper-Jacob approximation can be applied giving:

$$s' = s + s_r = \frac{Q\left(\ln\left(2.25Tt/\left(r^2S\right)\right) - \ln\left(2.25Tt'/\left(r^2S\right)\right)\right)}{4\pi T}$$

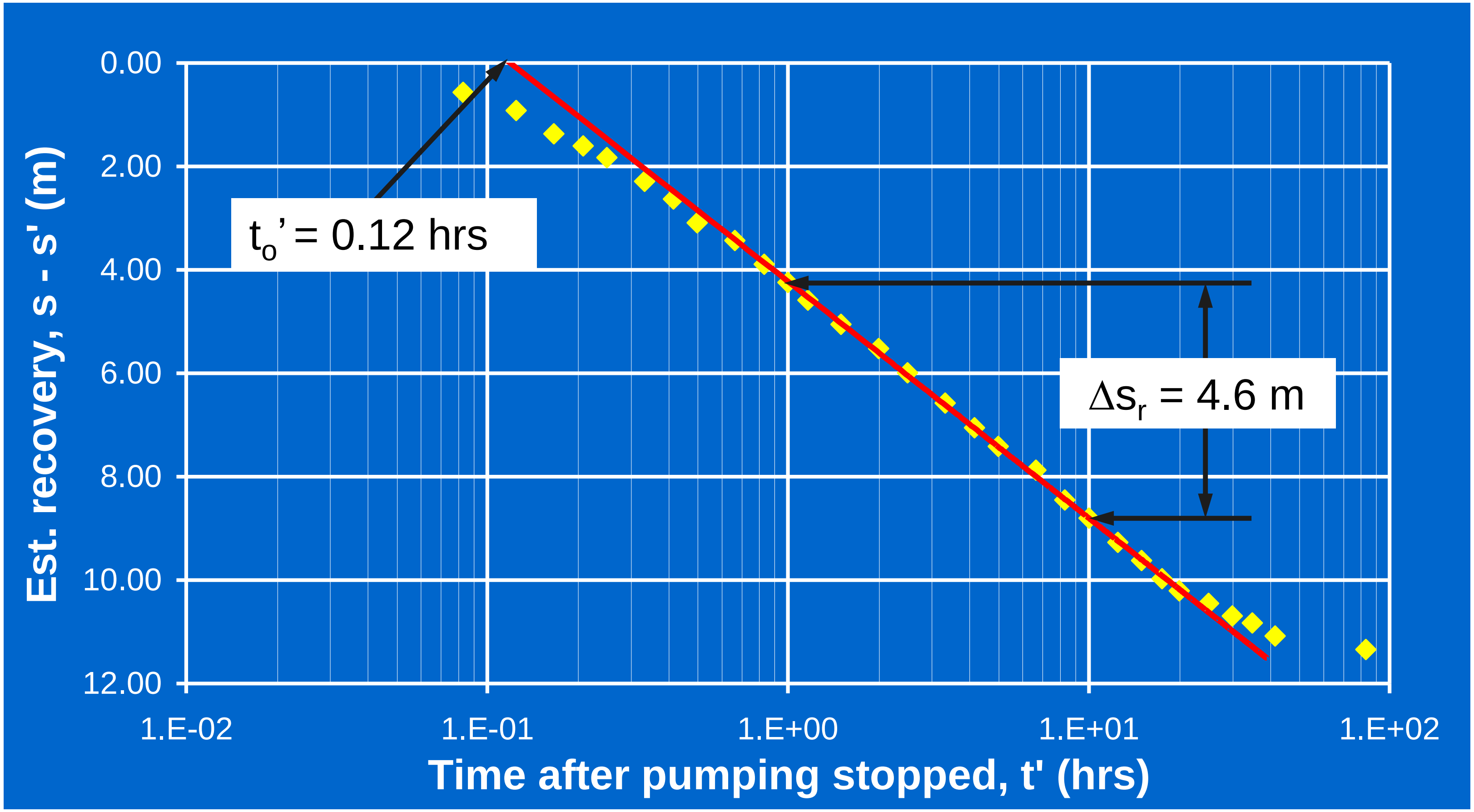
$$= \frac{Q\ln\left(t/t'\right)}{4\pi T}$$

- The equation predicting the **recovery** is:

$$s_r = \frac{-Q\left(\ln\left(2.25Tt'/\left(r^2S\right)\right)\right)}{4\pi T}$$

For $t > t_r$, the recovery s_r is the difference between the observed drawdown s' and the extrapolated pumping drawdown (s).

Time-Recovery Graph



Aquifer characteristics can be calculated from a log(time)-recovery plot but the drawdown (s) curve for the pumping phase must be extrapolated to estimate recovery (s - s')

Time-Recovery Analysis

- For a constant rate of pumping (Q), the recovery any time (t') after pumping stops:

$$T = \frac{Q}{4\pi\Delta(s - s')} = \frac{-Q}{-4\pi\Delta s_r} = \frac{Q}{4\pi\Delta s_r}$$

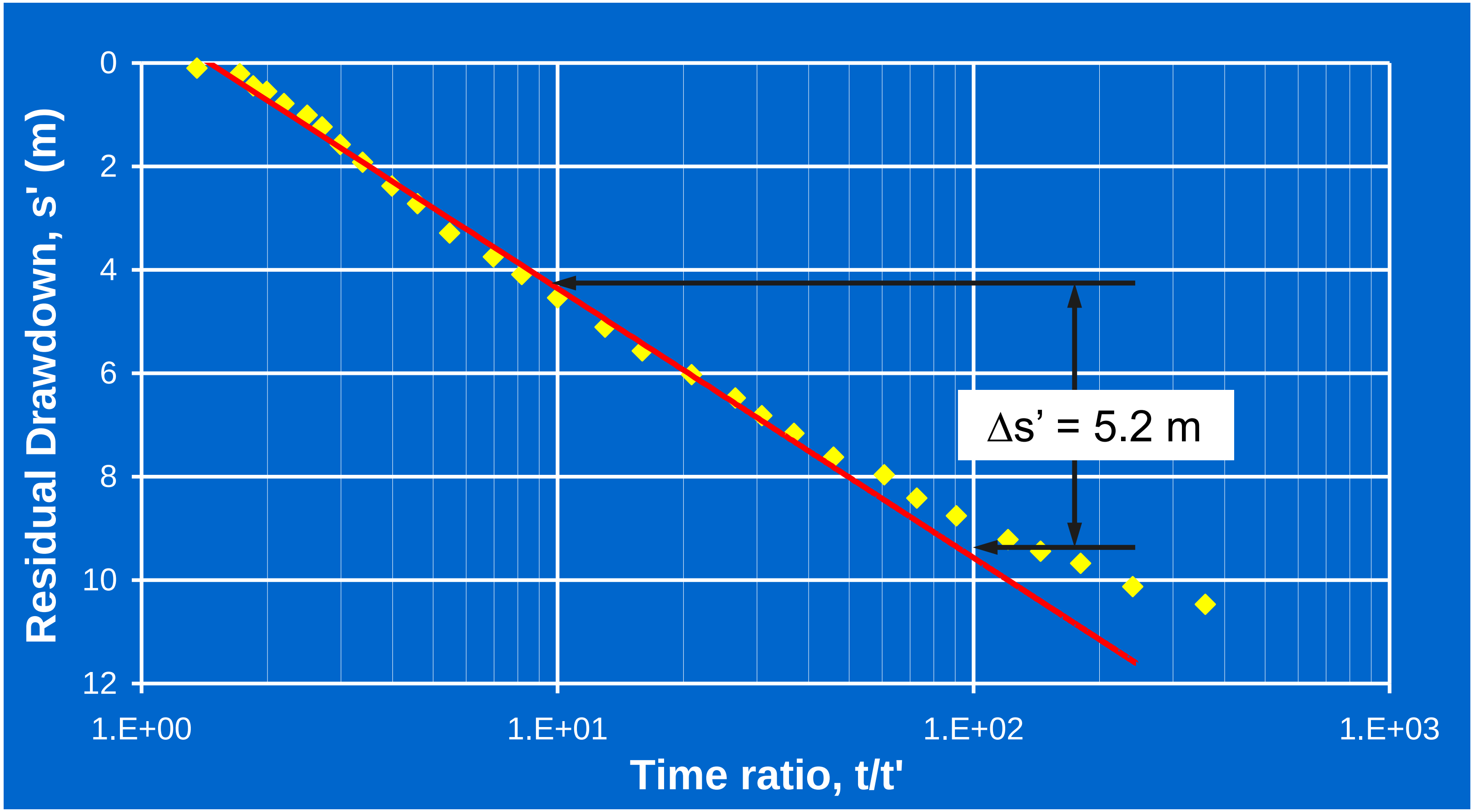
- For the example, Δs_r = 4.6 m and Q = 1100 m^3/d so:

  T = 1100 / (12.56 x 4.6) = 19 m^2/d

- The storage coefficient can be estimated for an observation well (r = 30 m) using: $S = 4Tt_o'/r^2$
- For the example, t_o' = 0.12 and Q = 1100 m^3/d so:

  S = 4 x 19 x 0.12 / (24 x 30 x 30) = 4.3 x 10^{-4}

Time-Residual Drawdown Graph



Transmissivity can be calculated from a log(time ratio)-residual drawdown (s') graph by determining the gradient. For such cases, the x-axis is log(t/t') and thus is a ratio.

Time-Residual Drawdown Analysis

- For a constant rate of pumping (Q), the recovery any time (t’) after pumping stops:

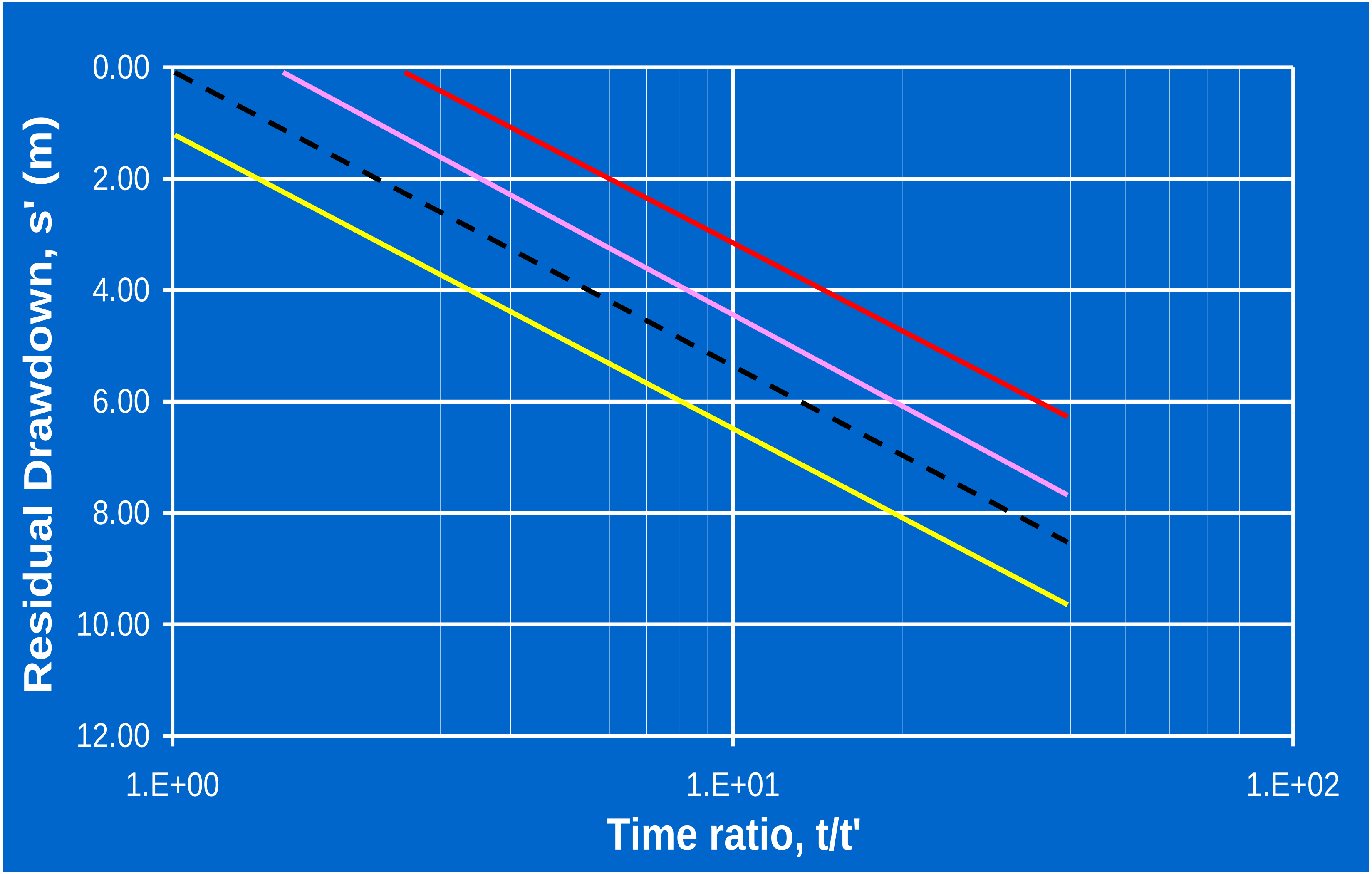
$$T = \frac{Q}{4\pi \Delta s'}$$

- For the example, Δs_r = 5.2 m and Q = 1100 m^3/d so:

  T = 1100 / (12.56 x 5.2) = 17 m^2/d

- Notice that the graph plots t/t’ so the points on the LHS represent long recovery times and those on the RHS short recovery times.
- The storage coefficient cannot be estimated for the residual drawdown plot because the intercept $t / t' \rightarrow 1$ as $t' \rightarrow \infty$.
  - Remembering $t' = t - t_r$ where t_r is the elapsed pumping time before recovery starts.

Residual Drawdown for Real Aquifers



Residual Drawdown for Real Aquifers

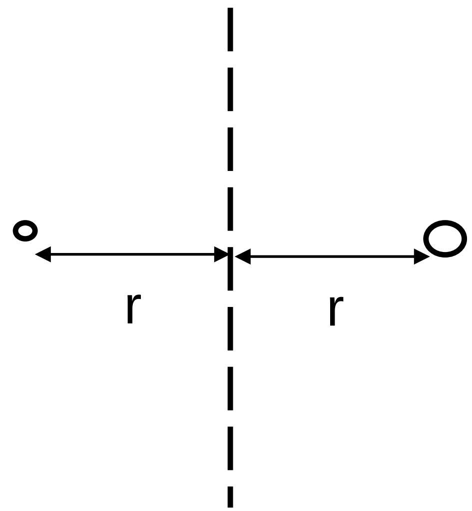
- Theoretical intercept is 1
- >> 1 indicates a recharge effect
- >1 may indicate greater S for pumping than recovery
- < 1 indicates incomplete recovery of initial head - finite aquifer volume
- << 1 indicates incomplete recovery of initial head - small aquifer volume

6. Bounded Aquifers

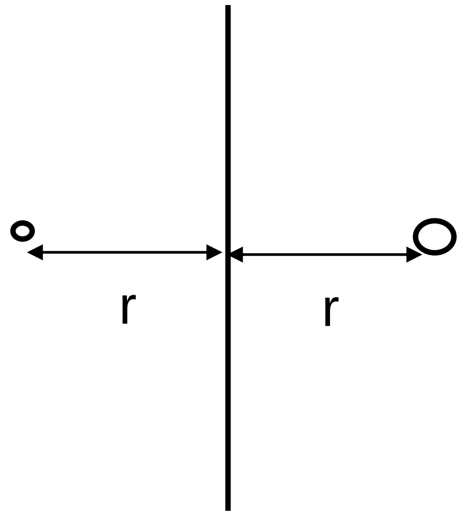
- Superposition was used to calculate well recovery by adding the effects of a pumping and recharge well starting at **different times**.
- Superposition can also be used to simulate the effects of aquifer boundaries by adding wells at **different positions**.
- For boundaries within the radius of influence, the wells that create the same effect as a boundary are called **image wells**.
- This, relatively simple application of superposition for analysis of aquifer boundaries, was described by Ferris (1959)

Image Wells

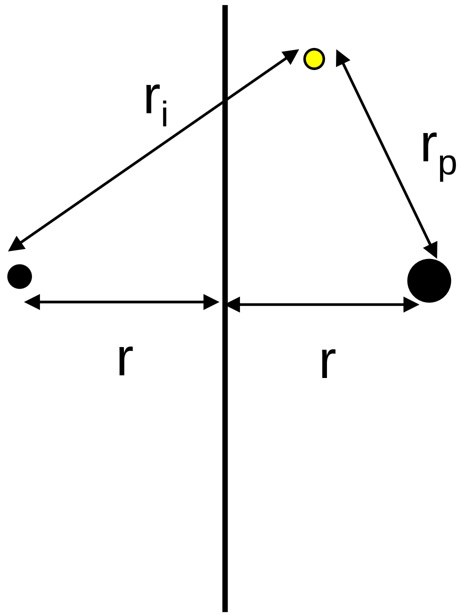
- Recharge boundaries at distance (r) are simulated by a recharge image well at an equal distance (r) across the boundary.



- Barrier boundaries at distance (r) are simulated by a pumping image well at an equal distance (r) across the boundary.



General Solution



The general solution for adding image wells to a real pumping well can be written:

$$s = s_p \pm s_i = \frac{Q}{4\pi T}\left[W(u_p) \pm W(u_i)\right]$$

$$u_p = \frac{r_p^2 S}{4Tt} \; and \; u_i = \frac{r_i^2 S}{4Tt}$$

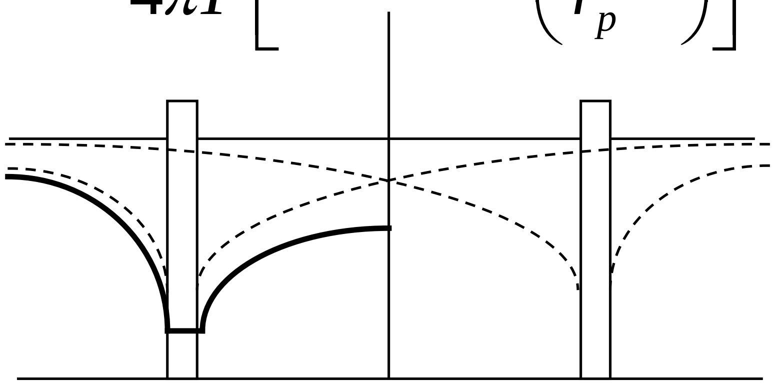
Where r_p, r_i are the distances from the pumping and image wells respectively.

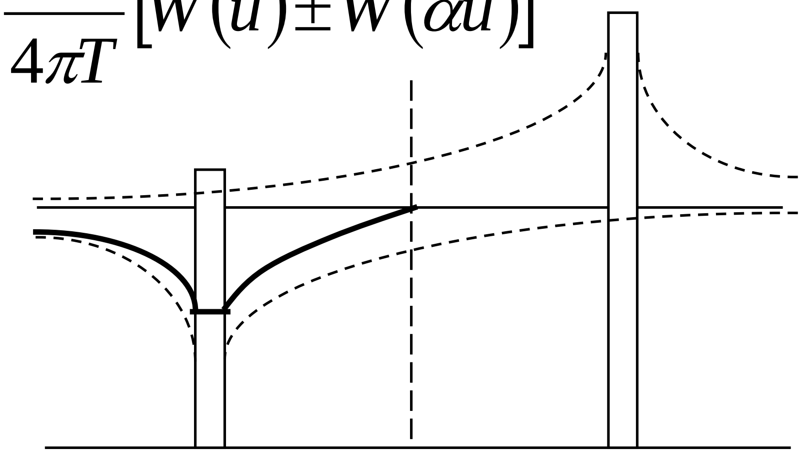
- For a barrier boundary, for all points on the boundary $r_p = r_i$ the drawdown is doubled.
- For a recharge boundary, for all points on the boundary $r_p = r_i$ the drawdown is zero.

Specific Solutions

- The use of Cooper-Jacob approximation is only possible for large values of 1/u i.e. u < 0.05 for all r so the Theis well function is used:

$$s=\frac{Q}{4\pi T}\left[W(u)\pm W\left(\frac{r_i^2}{r_p^2}u\right)\right]=\frac{Q}{4\pi T}\left[W(u)\pm W(\alpha u)\right]$$





- For the barrier boundary case:

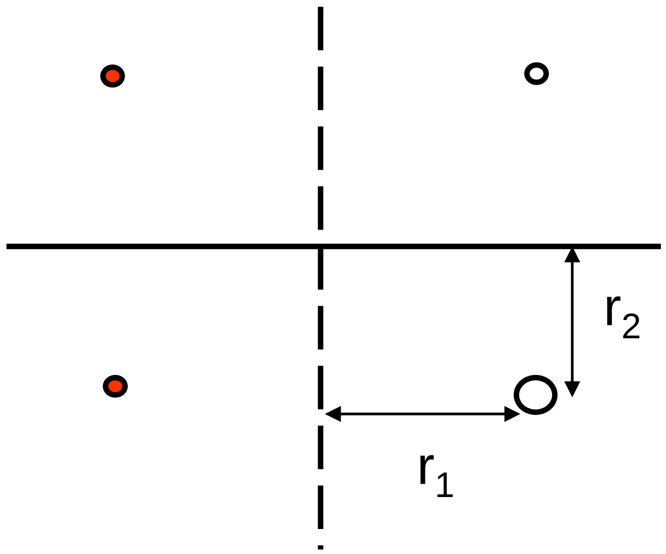
$$s=\frac{Q}{4\pi T}\left[W(u)+W(\alpha u)\right]$$

- For the recharge boundary case:

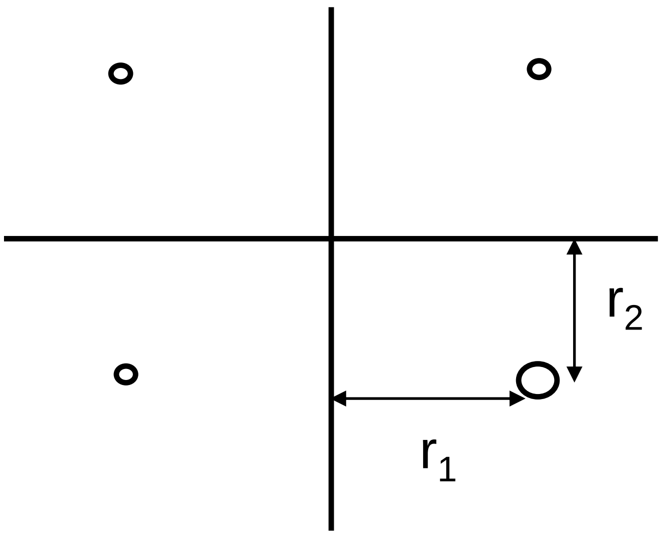
$$s=\frac{Q}{4\pi T}\left[W(u)-W(\alpha u)\right]$$

Multiple Boundaries

- A recharge boundary and a barrier boundary at right angles can be generated by two pairs of pumping and recharge wells.

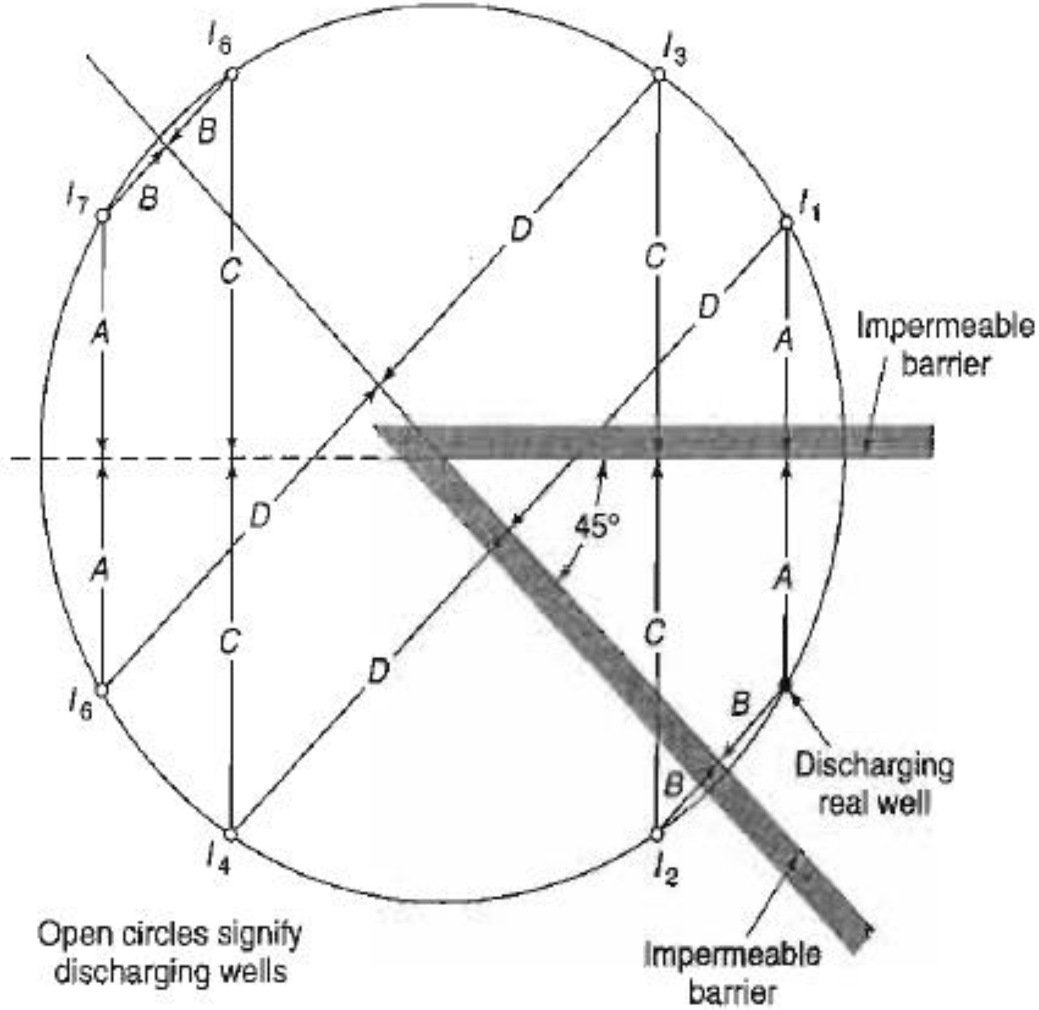


- Two barrier boundaries at right angles can be generated by superposition of an array of four pumping wells.



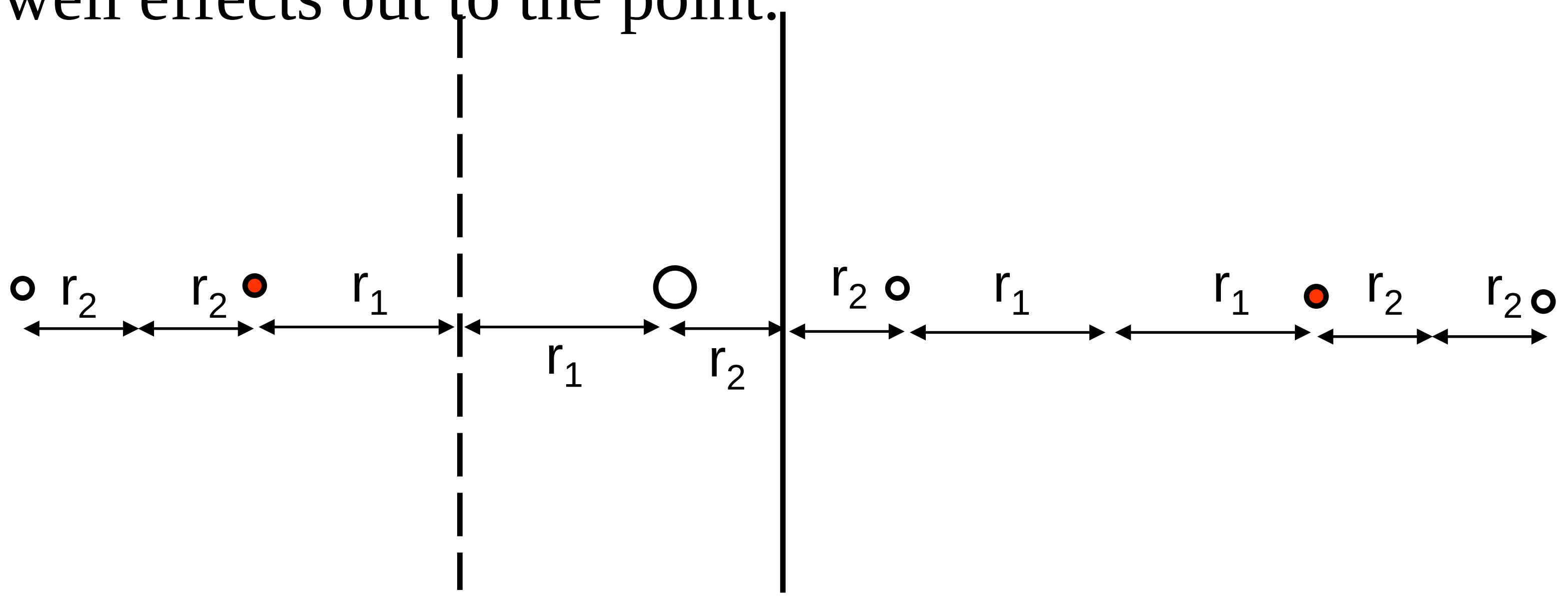
- The number of image wells, n, necessary for a wedge angle θ is given by: $n = 360/\theta - 1$.

- The image wells are usually lie on a circle centered at the apex of the wedge and radius equal to the distance between the pumping well and the apex.



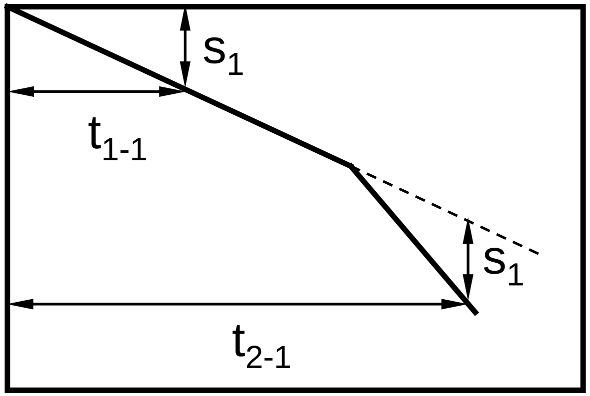
Parallel Boundaries

- A parallel recharge boundary and a barrier boundary (or any pattern with parallel boundaries) requires an infinite array of image wells.
- Each successively added secondary image well produces a residual effect at the opposite boundary.
- It is only necessary to add pairs of image wells until the next pair has negligible influence on the sum of all image well effects out to the point.

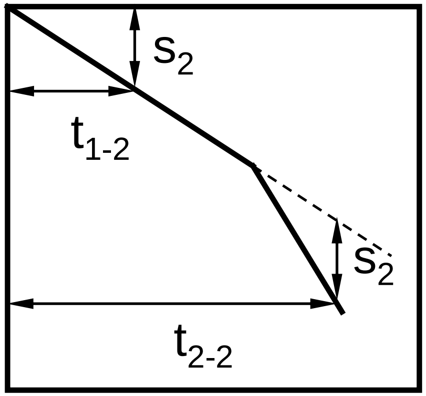


Boundary Location

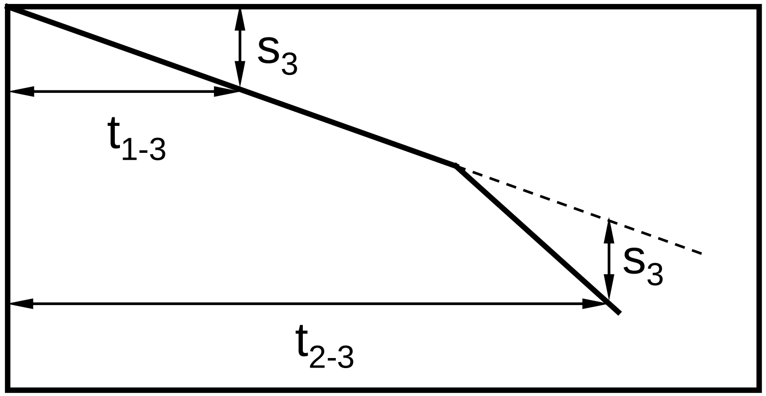
- For an observation well at distance r_1, measure off the same drawdown (s), before and after the “dog leg” on a log(time) vs. drawdown plot.



Observation well 1



Observation well 2

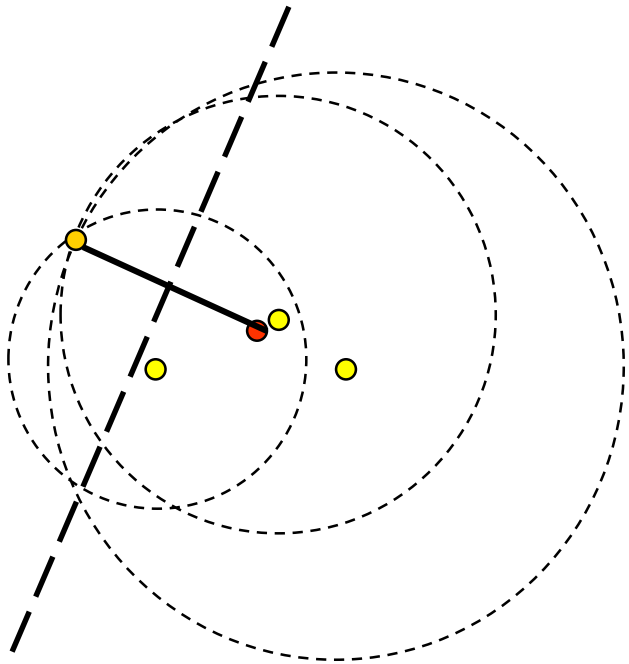


Observation well 3

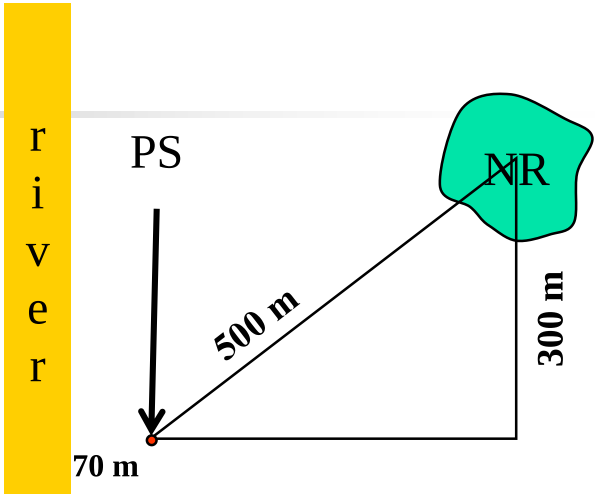
Boundary Location

- Assuming that the "dog leg" is created by an image well at distance r_2 , if the drawdown are identical then $W(u_1) = W(u_2)$ so $u_1 = u_2$.
- Thus: $$\frac{r_1^2 S}{4Tt} = \frac{r_2^2 S}{4Tt} \Rightarrow r_2 = r_1 \sqrt{\left(\frac{t_2}{t_1}\right)}$$

  - The distance r_2 the radial distance from the observation point to the boundary.
  - Repeating for additional observation wells may help locate the boundary.

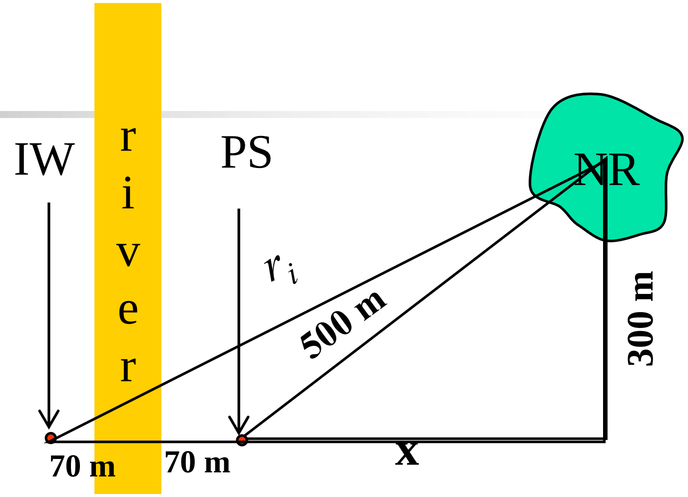


Example:



- Groundwater is pumped from a confined aquifer. The pumping discharge is 314.2 m^3 day^{-1}. The saturated depth of the aquifer is 50 m; the hydraulic conductivity of the aquifer equals 10 m day^{-1} and the storage coefficient of the aquifer is 0.0001. A river is located at a distance of 70 m of the pumping station (PS), and the midpoint of a nature reserve (NR) is located at a distance of 500 m from the pumping well.

Solution



- Pythagoras: $x^2 + 300^2 = 500^2 \Rightarrow x = 400\ m$
- *r of the image recharge well* $(r_i) = [(400 + 2x70)^2 + 300^2]^{1/2} = 619.14$ *m*
- The drawdown at NR could be estimated by Copper Jacob equation, since the image well is an injection well the drawdown at NR will be obtained by subtracting the drawdown due to the image (injection) well from the drawdown caused by pumping well.

$$s = \frac{2.3Q}{4\pi bK}\log\left(\frac{2.25Tt}{r_p^2 bSc}\right) - \frac{2.3Q}{4\pi bK}\log\left(\frac{2.25Tt}{r_i^2 bSc}\right) = \frac{2.3Q}{4\pi bK}\left(\log\left(\frac{2.25Tt}{r_p^2 bSc}\right) - \log\left(\frac{2.25Tt}{r_i^2 bSc}\right)\right) = \frac{2.3Q}{4\pi bK}\log\left(\frac{r_i^2}{r_p^2}\right)$$

$$= \frac{2.3 \times 314.2 m^3/d}{4\pi \times 50 \times 10 m/d}\log\left(\frac{(619.14)^2}{500^2}\right) = 0.021 m = 2.1 cm$$

7. Pumping Wells

- The drawdown observed in a pumping well has two component parts:
  - **Aquifer loss**
    - Drawdown due to laminar flow in the aquifer
  - **Well loss**
    - Drawdown due to turbulent flow in the immediate vicinity of the well through the screen and/or gravel pack
- Well loss is usually assumed to be proportional to the square of the pumping rate: $s_w = CQ^2$
- The total drawdown at a pumping well is given by:

$$s_t = s + s_w = \frac{Q}{4\pi T} W(u) + CQ^2 = BQ + CQ^2$$

Well Efficiency

- The ratio of the aquifer loss and total drawdown (s/s_t) is known as the well efficiency.

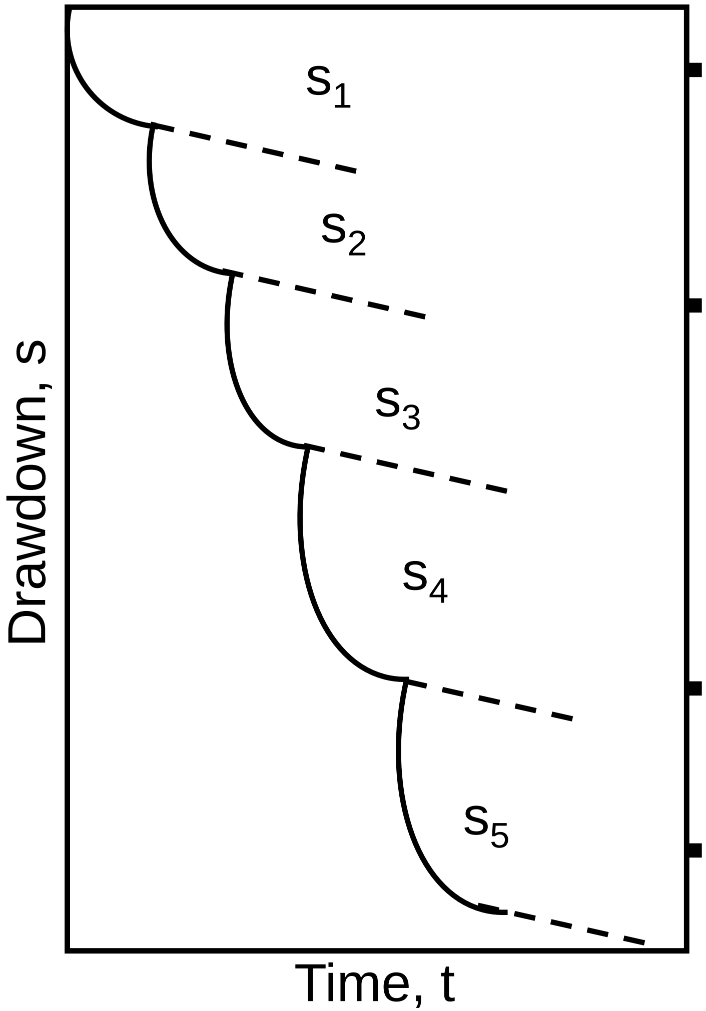
$$\frac{s}{s_t} = \frac{W(u)}{W(u) + 4\pi TCQ} = \frac{B}{B + CQ}$$

- Mogg (1968) defines well efficiency at a fixed time (t = 24 hrs). Thus, writing W(u) as the Cooper-Jacob approximation gives:

$$\frac{s}{s_t} = \frac{1}{1 + 4\pi TCQ / \left[\ln\left(2.25Tt / S\right) - 2\ln\left(r_w\right)\right]} = \frac{1}{1 + CQ / B(r_w)}$$

- Written in this form it is clear that well efficiency reduces with pumping rate (Q) and increases with well radius (r_w), where B is inversely related to well radius.
- The specific capacity is given by: $\frac{Q}{s_t} = \frac{1}{B + CQ}$

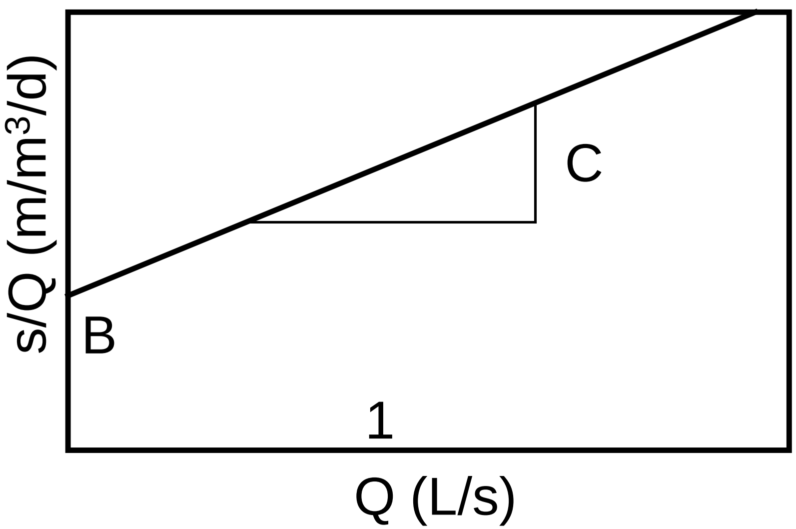
Step-Drawdown Test



- Step-drawdown tests are tests at different pumping rates (Q) designed to determine well efficiency.
- Normally pumping at each successively greater rate $Q_1 < Q_2 < Q_3 < Q_4 < Q_5$ takes place for 1-2 hours (Δt) and for 5 to 8 steps. The entire test usually takes place in one day.
- Equal pumping times (Δt) simplifies the analysis.
- At the end of each step, the pumping rate (Q) and drawdown (s) is recorded.

Step-Drawdown Test Analysis

- Step-drawdown tests are analyzed by plotting the reciprocal of specific capacity (s/Q) against the pumping rate (Q).



- The intercept of the graph at Q=0 is B = W(u)/(4πT) and the slope is the well loss coefficient, C.
- B can also be obtained independently from a Theis or Cooper-Jacob analysis of a pump test.
- For Q = 2700 m^3/d and s = 33.3 m the B = 0.012 m/m^3/d
- If C = 4 x 10^{-5}, then CQ^2 = 18.2 m
- The well efficiency is (33.3 – 18.2) /33.3 = 45.3%

- A well efficiency of 70% or more is usually acceptable.
- If a newly developed well has less than 65% efficiency, it should not be accepted.
- **A qualitative "Rule of Thumb" to recognize an inefficient well is:**
    - If the pump is shut off after 1 hour of pumping and 90% or more of the drawdown is recovered after 5 minutes, it can be concluded that the well is unacceptably inefficient.

Well Yield

Well yield			Nom. pump dia.		Opt. casing dia.		Min. casing dia.	
US gpm	**L/s**	**m³/d**	**in**	**mm**	**in**	**mm**	**in**	**mm**
< 100	< 6.4	550	4	100	6	150	5	130
< 170	< 11	950	5	130	8	200	6	150
< 350	< 22	1900	6	150	10	250	8	200
< 700	< 44	3800	8	200	12	300	10	250
< 1000	**< 64**	**5500**	**10**	**250**	**14**	**360**	**12**	**300**
< 1800	< 110	9800	12	300	16	410	14	360
< 3000	< 190	16000	14	360	20	510	16	410
< 3800	< 240	21000	16	410	24	610	20	510
< 6000	< 380	33000	20	510	30	760	24	610

- The chart is used to select casing sizes for a particular yield. The main constraint is pumping equipment.
- For example, if the well is designed to deliver 4,000 m^3/d, the optimum casing dia. is 360 mm (2 nom. sizes > pump dia.) and the minimum 300 mm.
- The drilled well diameter would have to be 410 to 510 mm to provide at least a 50 mm grout/cement annulus.

Pump Test Planning

- Several preliminary estimates are needed to design a successful test:
  - Estimate the maximum drawdown at the pumped well
  - Estimate the maximum pumping rate
  - Evaluate the best method to measure the pumped volumes
  - Plan discharge of pumped volumes distant from the well
  - Estimate drawdown at observation wells
  - Simulate the test before it is conducted
  - Measure all initial heads several times to ensure that steady-conditions prevail
  - Survey elevations of all well measurement reference points

Number of Observation Wells

- Number depends on test objectives and available resources for test program.
  - Single well can give aquifer characteristics (T and S). Reliability of estimates increases with additional observation points.
  - Three wells at different distances are needed for time-distance analysis
  - No maximum number; because anisotropy, homogeneity, and boundaries can be deduced from response

Pump Test Measurements

- The accuracy of drawdown data and the results of subsequent analysis depends on:
  - Maintaining a constant pumping rate
  - Measuring drawdown at several (>2) observation wells at different radial distances
  - Taking drawdown at appropriate time intervals at least every min (1-15 minutes); (every 5 minutes) 15-60 minutes; (every 30 minutes) 1-5 hrs; (every 60 minutes) 5-12 hrs; (every 8 hrs) >12 hrs
  - Measuring barometric pressure, stream levels, etc as necessary over the test period

Pump Test Measurements

- Measuring both pumping and recovery data
- Continuing tests for no less than 24 hours for a confined aquifers and 72 hours for unconfined aquifers in constant rate tests
- Collecting data over a 24 hour period for 5 or 6 pumping rates for step-drawdown tests

Measuring Pumping Rates

- Control of pumping is normally required as head and pump rpm changes. Frequent flow rate measurements are needed to maintain constant rate.
- Lower rates
  - Periodic measurements of time to fill a container of known volume
  - "v" notch weir - measure head (sensitive at low flows)
- Higher rates
  - Impellor driven water meter - measure velocity (insensitive)
  - Circular orifice weir - measure head $v=(2gh)^{½}$
  - Rectangular notch weir - measure head
  - Free-flow Parshall flume (drop in floor) - measure head
  - Cutthroat flume (flat floor) – measure head

Measuring Drawdown

- Pumped wells
  - Heads are hard to measure due to turbulence and pulsing.
  - Data cannot reliably estimate storage.
- Observation wells
  - Smallest possible diameter involves least time lag
  - Screens usually 1-2 m; longer is better but not critical. It should be at same depth as centre of production section
  - If too close (< 3 to 5 x aquifer thickness) can be strongly influenced by anisotropy (stratification)
  - If too far away (>200 m unconfined) $\Delta h(t)$ increases with time so a longer test is required – boundary and other effects can swamp aquifer response

Drawdown Instrumentation

- Dip meters
  - Let cable hang to remove kinks
  - Rely on light or buzzer, have spare batteries
- Steel tapes
  - Read wetted part for water level (chalking helps)
  - Hard to use where high-frequency readings are needed
- Pressure gauges
  - Measure head above reference point
  – need drawdown estimates to set gauge depth
- Pressure transducers/data loggers
  - Hang in well and record at predetermined interval
  - Remote sites (no personnel) and closest wells (frequency)

Groundwater Hydraulics

Chapter 4 – Groundwater Modelling

CENG 6606

AAU

Contents

1. Why model groundwater?
2. Simulating groundwater flow with software
3. Setting up a basic groundwater flow model: tutorial with the software

Why model?

- The effective way:
  - To test effects of groundwater management strategies
  - To make predictions about a ground-water system's response to a stress
  - To understand the system
  - To design field studies
  - Use as a thinking tool
- Processes we might be interested to model:
  - Groundwater flow
    - *Calculate both heads and flow*
  - Solute transport – requires information on flow (velocities)
    - *Calculate concentrations*

Types of models

- CONCEPTUAL MODEL QUALITATIVE DESCRIPTION OF SYSTEM
  - "a cartoon of the system in your mind”
- MATHEMATICAL MODEL MATHEMATICAL DESCRIPTION OF SYSTEM
  - SIMPLE - ANALYTICAL (provides a continuous solution over the model domain)
  - COMPLEX - NUMERICAL (provides a discrete solution - i.e. values are calculated at only a few points)
- ANALOG MODEL e.g. ELECTRICAL CURRENT FLOW through a circuit board with resistors to represent hydraulic conductivity and capacitors to represent storage coefficient
- PHYSICAL MODEL e.g. SAND TANK which poses scaling problems

1. Why Model Groundwater?

- Can be used for three general purposes:
- To predict or forecast expected artificial or natural changes in the system. Predictive is more applied to deterministic models since it carries higher degree of certainty, while forecasting is used with probabilistic (stochastic) models.
- To describe the system in order to analyse various assumptions
- To generate a hypothetical system that will be used to study principles of groundwater flow associated with various general or specific problems.

Why model groundwater?

- A model is any device that represents an approximation of a field situation. Two types:
    - *Physical models* such as laboratory sand tanks simulate groundwater flow directly.
    - A *mathematical* model simulates groundwater flow indirectly by means of a governing equation, together with equations that describe boundary conditions and/or initial conditions.
- The set of commands used to solve a mathematical model on a computer forms the computer program or code. The code is generic, whereas a model includes a set of boundary and initial conditions as well as a site-specific nodal grid and site-specific parameter values and hydrologic stresses.

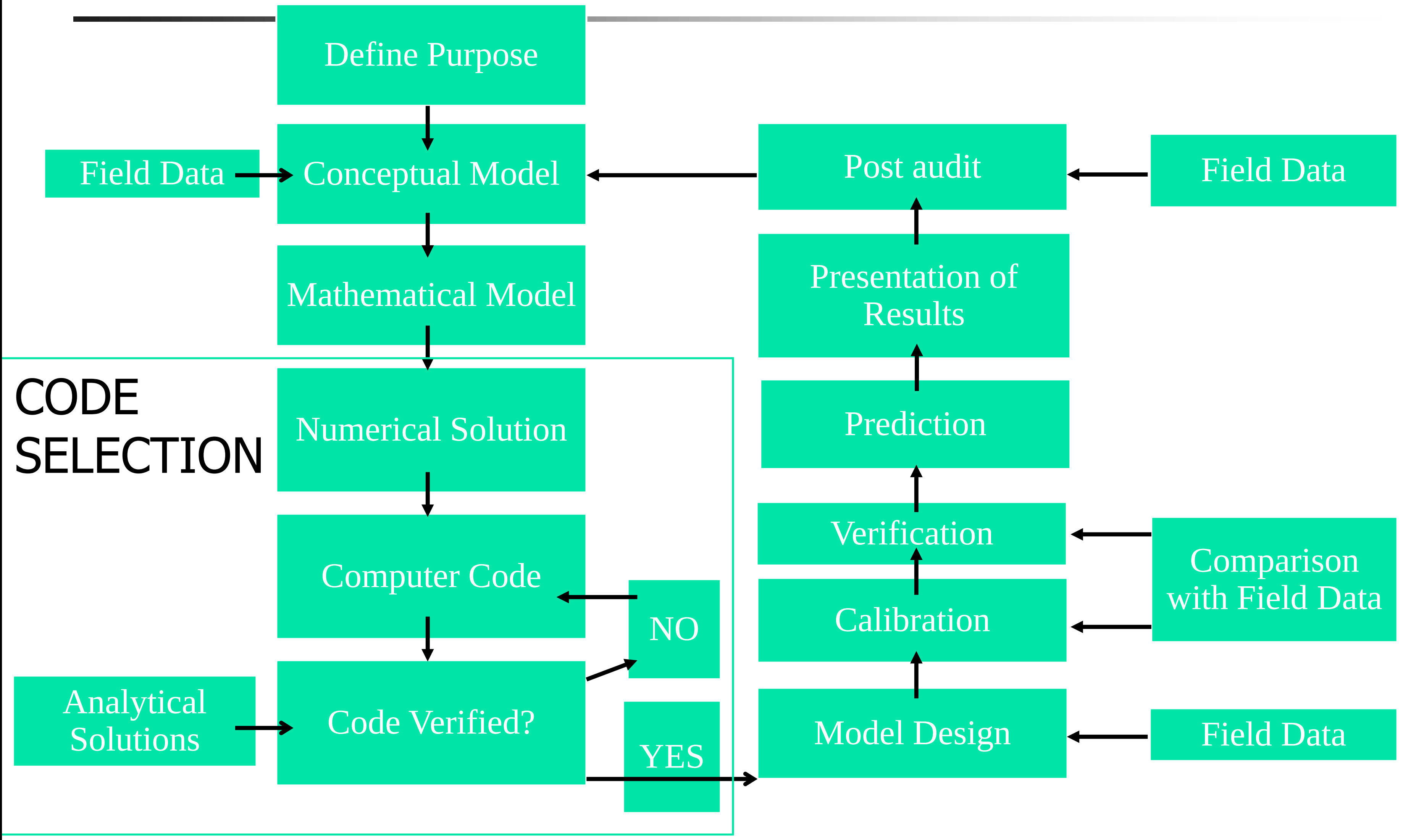
Why Model Groundwater?

- <u>**Predictive**</u>: Used to predict the future (**predicting the consequences of a proposed action);** requires calibration.
- <u>**Interpretive**</u>: Used as a framework for studying system dynamics (**to gain insight into the controlling parameters in a site-specific setting)** and/or organizing field data (**to improve understanding of regional flow systems)**; does not necessarily require calibration.
- <u>**Generic**</u>: Used to analyze flow in hypothetical hydrogeologic systems; may be useful to help frame regulatory guidelines for a specific region **screening tools to identify regions suitable or unsuitable for some proposed action)**; does not necessarily require calibration.

Basic components for model development

- Geologic model - lithologic facies, expert knowledge on hydraulic properties of materials, etc.
- Hydrologic model - conceptualization of boundary conditions and initial conditions - what type of features are present?
  - Clear description of where the model is confined, unconfined, or leaky
  - Choose one the three basic boundary conditions for all boundaries of the groundwater system
  - Define recharge and ET processes and determine if simulating the unsaturated zone is necessary or not
- Construct the numerical model using standard software such as, MODFLOW.

Modelling protocol



Purpose - What questions do you want the model to answer?

- Prediction;
- System Interpretation: **Inverse Modeling: Sensitivity Analysis;**
- Generic Modeling: **Used in a hypothetical sense, not necessarily for a real site;**
- What do you want to learn from the model?
- Is a modeling exercise the best way to answer the question?
- Can an analytical model provide the answer?

Conceptual Model

"Everything should be made as simple as possible, but not simpler." Albert Einstein

- Pictorial representation of the groundwater flow system
- Will set the dimensions of the model and the design of the grid
- "Parsimony"….conceptual model has been simplified as much as possible yet retains enough complexity so that it adequately reproduces system behavior.

- **Select Computer Model**
- **Code Verification**
    - Comparison to Analytical Solutions; Other Numerical Models
- **Model Design**
    - Design of Grid, selecting time steps, boundary and initial conditions, parameter data set

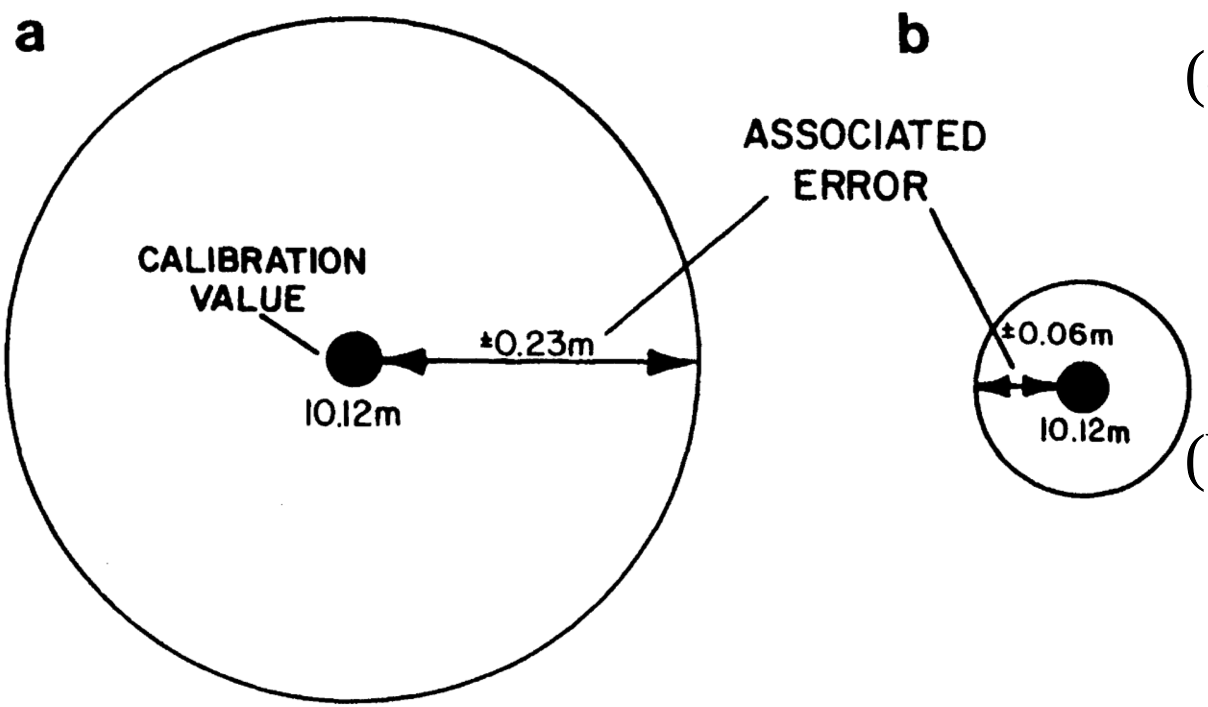
Steady/Unsteady..1, 2, or 3-D;
…Heterogeneous/Isotropic…..Instantaneous/Continuous

- **Calibration**:
  - Show that Model can reproduce field-measured heads and flow
  - Results in parameter data set that best represents field-measured conditions.
- **Calibration Sensitivity Analysis**
  - Uncertainty in Input Conditions
  - Determine Effect of Uncertainty on Calibrated Model
- **Model Verification**
  - Use Model to Reproduce a Second Set of Field Data
- **Prediction**
  - Desired Set of Conditions
  - Sensitivity Analysis: Effect of uncertainty in parameter values and future stresses on the predicted solution

- **Presentation of Modeling Design and Results**:
    - Effective Communication of Modeling Effort
        - Graphs, Tables, Text etc.
- **Post audit**:
    - New field data collected to determine if prediction was correct
    - Site-specific data needed to validate model for specific site application
- **Model Redesign**
    - Include new insights into system behavior

Inverse Modelling

- Calibration is accomplished by finding a set of parameters, boundary conditions, and stresses that produce simulated heads and fluxes that match field-measured values within a preestablished range of error



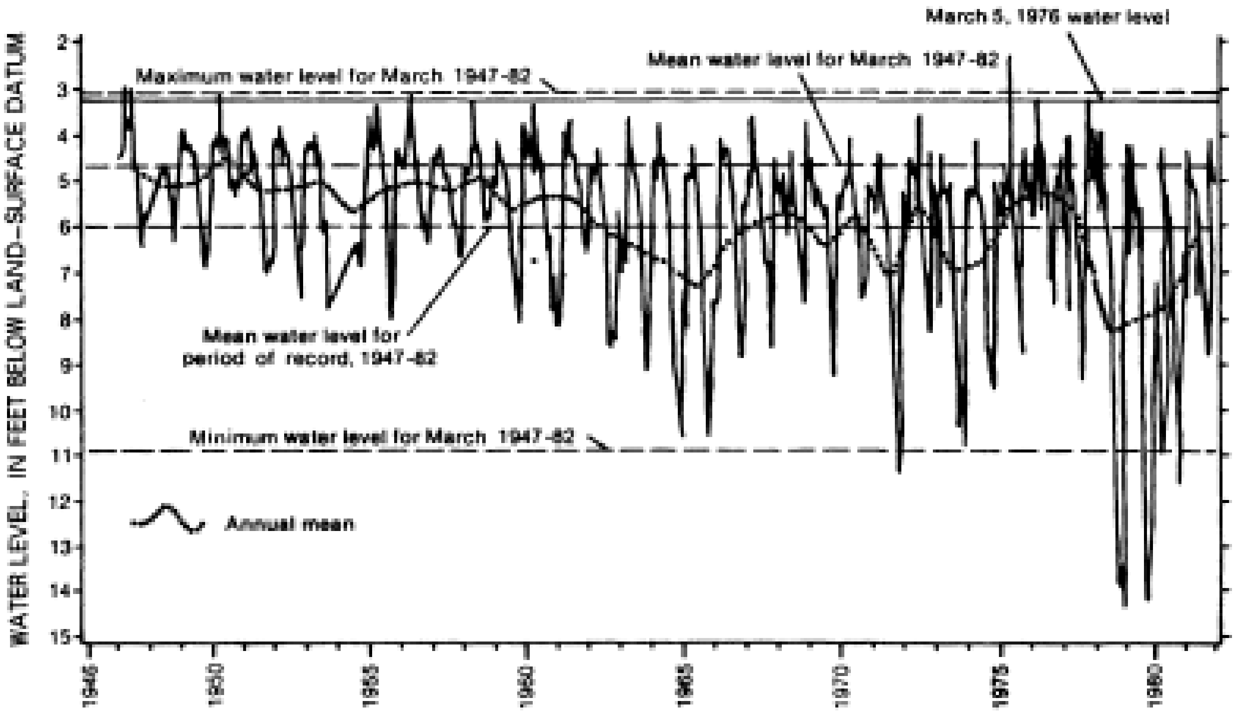
(a) A target for head with a large associated error, 10.12 m ± 0.23 m.

(b) A target for head with a small associated error, 10.12 m ± 0.06

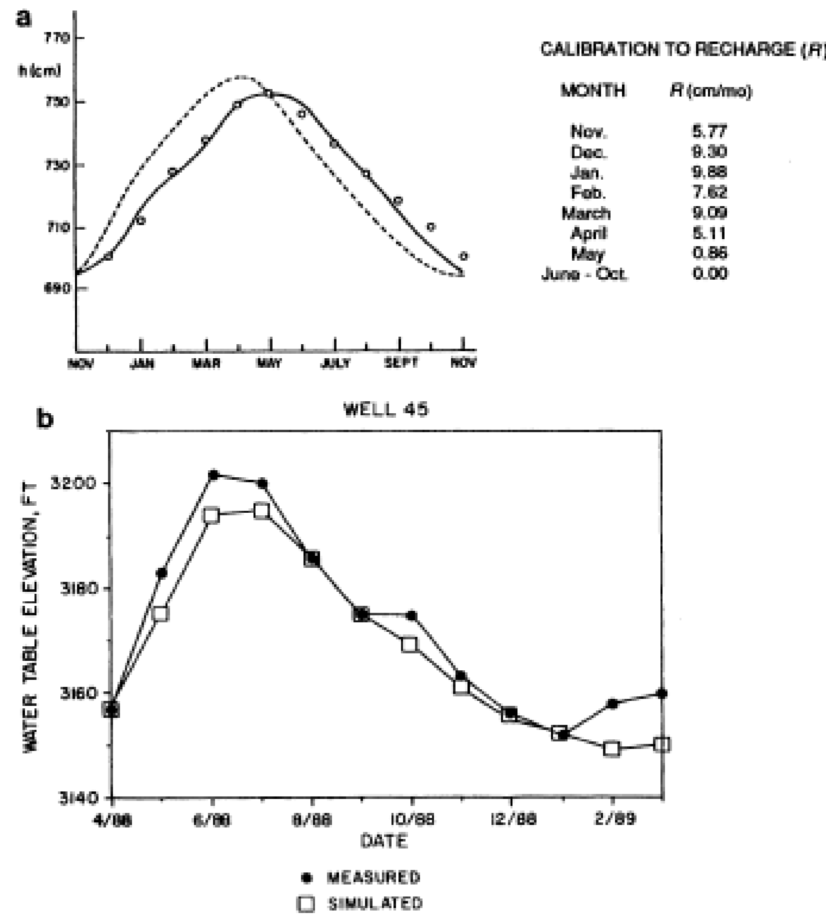
Inverse Modelling

- Finding set of parameter values amounts to solving what is known as the inverse problem.
- In an inverse problem the objective is to determine values of the parameters and hydrologic stresses from information about heads,
- whereas in the forward problem system parameters such as hydraulic conductivity, specific storage, and hydrologic stresses such as recharge rate are specified and the model calculates heads.
- Thus steady state models can be used to compute hydraulic conductivities while transient models must be used to estimate both specific storage and hydraulic conductivity values.
- Both models can be used to estimate recharge

- Definition of possible steady-state calibration values based on the mean water level for the period of record, the mean water level for a month for the period of record, and the mean water level for a year.



- Transient calibration.
- (a) Calibration to dynamic cyclic conditions as defined by a well hydrograph. Calibration is achieved by adjusting monthly recharge rates. The solid line (observed) against the Brocken line (simulated)
- (b) Transient calibration to a well hydrograph for an unconfined aquifer receiving recharge from adjacent highlands and leaking streams and irrigation ditches.



Calibration techniques

- Parameter estimation is essentially synonymous with model calibration, which is synonymous with solving the inverse problem.
- Kriging is a method of estimating the spatial distribution of parameters (or heads), but it is generally recognized that kriging should be combined with an inverse solution because the uncertainty associated with estimates of transmissivity can be greatly reduced when information about the head distribution is used to help estimate transmissivities.
- In other words, better estimates of aquifer parameters can be obtained when both prior information and sample information are used in the analysis.

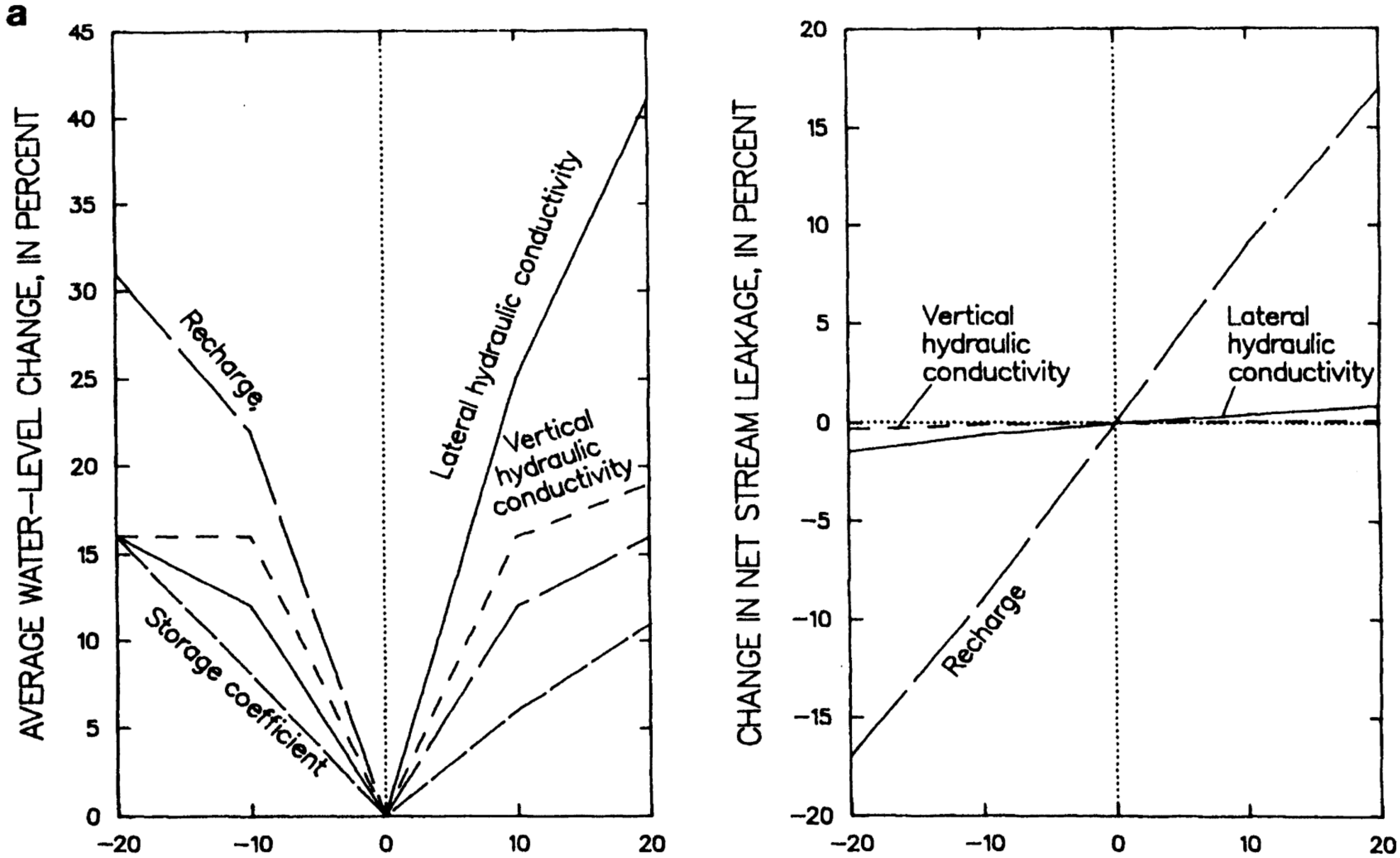
Two approaches

- Manual trial-and-error adjustment of parameters:
  - Does not give information on the degree of uncertainty in the final parameter selection, nor does it guarantee the statistically best solution.
- Automated statistically based solution
  - Quantifies the uncertainty in parameter estimates and gives the statistically most appropriate solution for the given input parameters provided it is based on an appropriate statistical model of errors.

Sensitivity Analysis

- The purpose is to quantify the uncertainty in the calibrated model caused by uncertainty in the estimates of aquifer parameters, stresses, and boundary conditions.
- Other uncertainty about the very geometry of the model area include: uncertainties of lithology, stratigraphy, and structure
- During a sensitivity analysis, calibrated values for hydraulic conductivity, storage parameters, recharge, and boundary conditions are systematically changed within the previously established plausible range. The magnitude of change in heads from the calibrated solution is a measure of the sensitivity of the solution to that particular parameter.
- The results of the sensitivity analysis are reported as the effects of the parameter change on the average measure of error selected as the calibration criterion.

- Sensitivity analysis is typically performed by changing one parameter value at a time.
- The effects of changing two or more parameters also might be examined to determine the widest range of plausible solutions.
- For example, hydraulic conductivity and recharge rate might be changed together so that low hydraulic conductivities are used with a high recharge rate and high hydraulic conductivities are used with a low recharge rate.
- The left-hand-side figure (next slide) shows the effect of varying the storage coefficient on the average water level change, plotted with other sensitivity analyses. The right-hand side figure shows the effect on stream leakage.

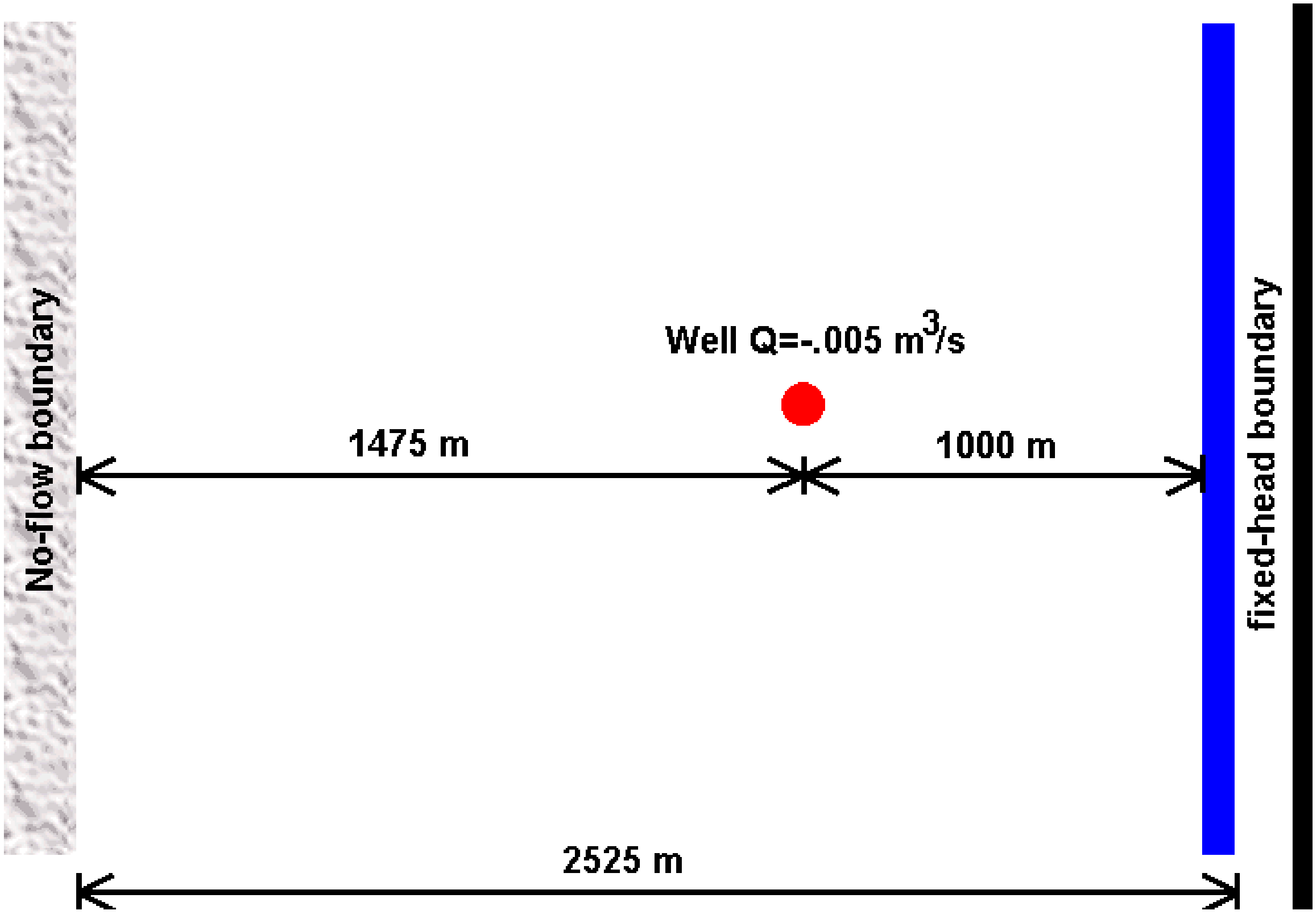

a
AVERAGE WATER-LEVEL CHANGE, IN PERCENT
45
40
35
30
25
20
15
10
5
0
-20
-10
0
10
20
Recharge,
Storage coefficient
Lateral hydraulic conductivity
Vertical hydraulic conductivity
CHANGE IN NET STREAM LEAKAGE, IN PERCENT
20
15
10
5
0
-5
-10
-15
-20
-20
-10
0
10
20
Vertical hydraulic conductivity
Lateral hydraulic conductivity
Recharge

Tutorial Handout

- Your First Groundwater model using PMWIN (30 Page Reading Assignment)

PMWIN Exercise

- Fig. below shows a part of an unconfined aquifer. The extent of the aquifer to the North and South is assumed to be unlimited. The aquifer is homogeneous and isotropic with a measured horizontal hydraulic conductivity of 0.0005 *m/s* and an effective porosity of 0.1. The elevations of the aquifer top and bottom are 15 *m* and 0 *m*, respectively.
- The aquifer is bounded by a no-flow zone to the west. To the east exists a river, which is in direct hydraulic connection with the aquifer and can be treated as fixed-head boundary. The river width is 50 *m* and stage is 10 *m*. The mean groundwater recharge rate is 8×10^{-9} *m/s*. A pumping well is located at a distance of 1000 *m* from the river.
- The task is to calculate the catchment area of the well and the 365-days-capture zone under steady-state flow conditions.


No-flow boundary
Well Q=-.005 m3/s
1475 m
1000 m
fixed-head boundary
2525 m