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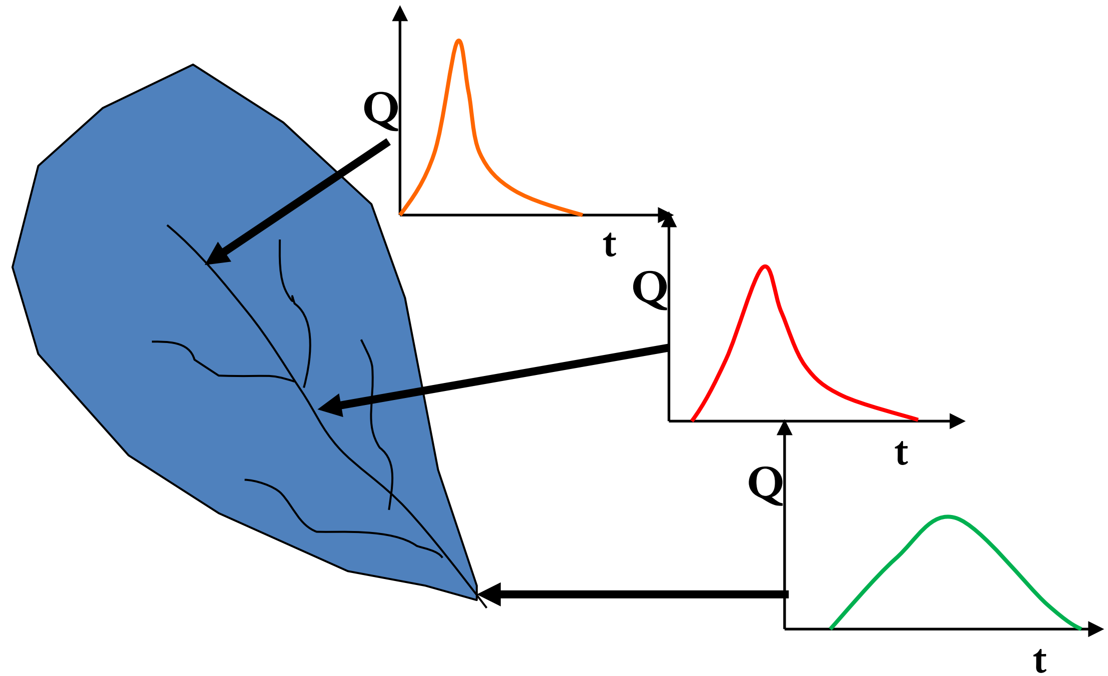
Chapter Three
Rainfall-Runoff Analysis

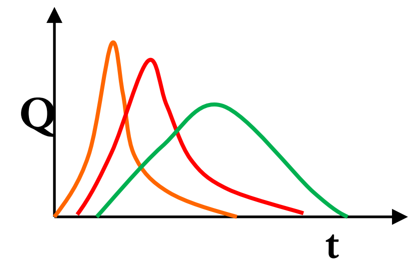
- **Linear System Theory and Rainfall-Runoff Analysis**
  - Unit hydrograph theory
    - From Stream flow Data
    - Synthetically
    - “Fitted” Distributions
  - Instantaneous unit hydrograph (IUH)
  - IUH analysis methods:
    - S-Hydrograph
    - Conceptual model
    - Fitting Harmonic analysis Fourier transforms
    - Theoretically from Laplace transforms
- **River and Reservoir Flood Routing**
  - Flood Routing
  - Reservoir flood routing methods:
  - Linear Muskingum method:
  - Multiple reach Muskingum method
  - Nonlinear Muskingum method:

Flow Routing

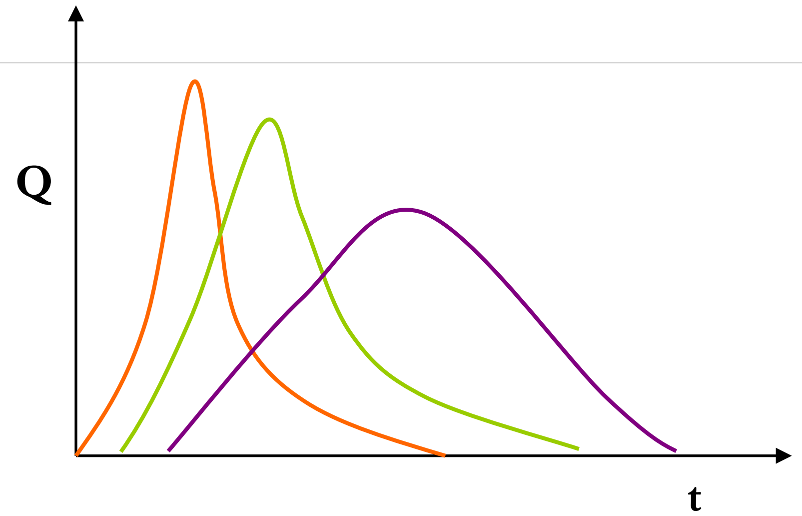
Procedure to determine the flow hydrograph at a point on a watershed from a known hydrograph upstream



As the hydrograph travels, it
- attenuates
- gets delayed



Why route flows?



- Account for changes in flow hydrograph as a flood wave passes downstream
- This helps in
  - Calculate for storages
  - Studying the attenuation of flood peaks

Types of flow routing

- Lumped/hydrologic
  - Flow is calculated as a function of time alone at a particular location
  - Governed by continuity equation and flow/storage relationship
  - Methods combine the continuity equation with some relationship between storage, outflow, and possibly inflow.
  - These relationships are usually assumed, **empirical, or analytical** in nature
- Distributed/hydraulic
  - Flow is calculated as a function of space and time throughout the system
  - Governed by continuity and momentum equations
  - Methods combine the continuity equation with some **more physical relationship to describe** actual physics of the movement of water.
  - In hydraulic routing analysis, it is intended that the dynamics of the water or flood wave movement be more accurately described

BASIC EQUATIONS

- Continuity Equation
  - The change in storage (dS) equals the difference between inflow (I) and outflow (O) or

$$I - Q = \frac{ds}{dt}$$

where I = inflow rate ,
Q outflow rate and
S= storage

$$\overline{I}\Delta t - \overline{Q}\Delta t = \Delta S$$

- Momentum Equation
  - Expressed by considering the external forces acting on a control section of water as it moves down a channel

$$\frac{\partial v}{\partial t} + V\frac{\partial v}{\partial x} + \frac{g}{A}\frac{\partial(\overline{y}A)}{2x} + \frac{vg}{A} = g(S_o - S_f)$$

$$S_f = S_o - \frac{\partial y}{\partial x} - \frac{v}{g}\frac{\partial v}{\partial x} - \frac{1}{g}\frac{\partial v}{\partial t}$$

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Routing Methods

- Modified Puls
  - Level pool routing
  - Storage indication method
- Mass curve
- Goodrich method
- Coefficient method
- Woodward method
- Muskingum
- Muskingum-Cunge
- Kinematic Wave
- Dynamic

Modified Puls

- The modified puls routing method is probably most often applied to reservoir routing
- The method may also be applied to river routing for certain channel situations.
- The modified puls method is also referred to as the storage-indication method.
- The heart of the modified puls equation is found by considering the finite difference form of the continuity equation.

$$\frac{I_1 + I_2}{2} - \frac{(O_1 + O_2}{2} = \frac{S_2 - S_1}{\Delta t}$$

Continuity Equation

$$I_1 + I_2 + \left(\frac{2S_1}{\Delta t} - O_1 \right) = \frac{2S_2}{\Delta t} + O_2$$

Rewritten

The solution to the modified puls method is accomplished by developing a graph (or table) of Q -vs- [2S/Δt + Q].

In order to do this, a stage-discharge-storage relationship must be known, assumed, or derived.

EXAMPLE

A reservoir has elevation, discharge and storage relationship of table 1, when the reservoir water level was 100.5m the flood its hydrograph presented in table-2 entered the reservoir. Route the flood and obtain

- *Outflow hydrograph*
- *Reservoir elevation-time curve during the passage of the flood wave*

E(m)	100.0	100.5	101.1	101.5	102.0	102.5	102.75	103.0
S (10^6 m^3)	3.350	3.472	3.880	4.383	4.882	5.370	5.5527	5.856
Q (m^3/s)	0	10	26	46	72	100	116	130

Time (hr)	0	6	12	18	24	30	36	42	48	54	60	66	72
Q (m^3/s)	10	20	55	80	73	58	46	36	55	20	15	13	11

Mass Curve Method

- The mass-curve method of reservoir routing is very versatile.
- It can be applied numerically or graphically.
- The numerical routing operation is a trial and error procedure while the graphical approach is a direct solution.
- A mass flow curve is a plotting of accumulated volume of flow and time.
- At any point, (i.e, at any time), the slope of the mass flow curve is equal to the rate of flow.
- The mass flow curve is the integral of the hydrograph since its ordinates measure accumulated volume at any time.

$$MI_2 - (MO_1 + \bar{O}\,\Delta t) = S_2$$

where

MI_2 = mass inflow at time 2

MO_1 = mass outflow at time 1

$\bar{O}$ = average discharge during the routing interval

Δt = routing interval = time 2 minus time 1

S_2 = storage at time 2

Muskingum Method

- Our storage discharge equation is written in a finite difference form:

$$1/2\ (I_1 + I_2) - 1/2\ (Q_1 + Q_2) = (S_2 - S_1)/\Delta t$$

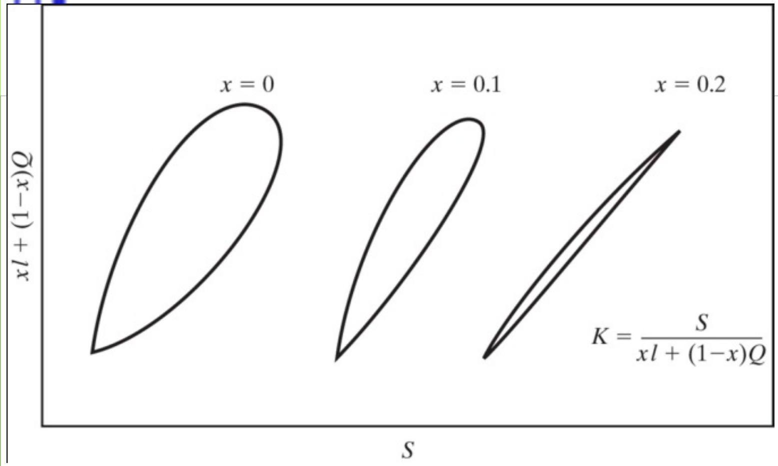
- The Muskingum routing procedure itself uses this form combined with in the form
- To produce the Muskingum outflow equation

$$S = K[xI + (1-x)Q]$$

$$S_2 - S_1 = K[x(I_2 - I_1) + (1-x)(Q_2 - Q_1)]$$

$$Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1$$

Estimating K and x



- The Muskingum K is usually estimated from the travel time for a flood wave through the reach.

- This requires two flow gages with frequent data collection, one at the top and one at the bottom of single channel reaches, and a big flood.

- If the two hydrographs are available , K and x can be better estimated.
  - Storage S is plotted against the weighted discharge xI + (1-x)Q for several values of x.
  - Since Muskingum method assumes this is a straight line, the straightest is x.
  - Then K can be calculated from

$$K = \frac{S}{xI + (1-x)Q}$$

Routing Procedures

$$Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1$$

must be calculated

$$C_0 = \frac{-Kx + 0.5\Delta t}{D}$$

$$C_1 = \frac{Kx + 0.5\Delta t}{D}$$

$$C_2 = \frac{K - Kx - 0.5\Delta t}{D}$$

$$D = K - Kx + 0.5\Delta t$$

Note

- K and Δt must have the same units, and that $2Kx < \Delta t \leq K$ is needed for numerical accuracy.
- Also $C_0+C_1+C_2 = 1$, because they are proportions.
- The routing procedure is accomplished successively, with Q_2 from Q_1 of the previous calculation.

Example

A. The inflow and outflow hydrographs for a river reach are given below Determine Muskingum's coefficients K and x for the reach

Time (h)	0	12	24	36	48	60	72	84	96	108	120
Inflow (m^3/s)	15	195	255	170	115	80	65	50	35	30	20
Outflow ($m^3.s$)	10	28	115	175	165	140	120	90	70	50	30

B. Find the outflow hydrograph section B, if the inflow hydrograph at section A, which is upstream of B, is given below? Using the values of K and x determine above and take the outflow at the beginning of routing step equal to inflow.

Time(h)	12	24	36	48	60	72	84	96	108	120	132	144
Inflow (m3/s)	14	22	36	93	141	102	86	73	61	50	38	26

Muskingum-Cunge

- Muskingum-Cunge formulation is similar to the Muskingum type formulation
- The Muskingum-Cunge derivation begins with the continuity equation and includes the diffusion form of the momentum equation.
- These equations are combined and linearized

$$\frac{\partial Q}{\partial t} + \frac{\partial Q}{\partial x} = \mu \frac{\partial^2 Q}{\partial x^2} + c q_{Lat}$$

working equation

where :

Q = discharge
t = time
x = distance along channel
q_{lat} = lateral inflow
c = wave celerity
m = hydraulic diffusivity

- Method attempts to account for diffusion by taking into account channel and flow characteristics.
- Hydraulic diffusivity is found to be :

$$\mu = \frac{Q}{2BS_o}$$

- The Wave celerity in the x-direction is :

$$C = \frac{dQ}{dA}$$

Calculation of K & X

$$k = \frac{\Delta x}{c}$$

$$X = \frac{1}{2}\left(1 - \frac{Q}{BS_O c \Delta x}\right)$$

Estimation of K & X is more "physically based" and should be able to reflect the "changing" conditions - better.

Solution of Muskingum-Cunge

Solution of the Muskingum is accomplished by discretizing the equations on an x-t plane.

$$Q_{j+1}^{n+1} = C_1 Q_j^n + C_2 Q_j^{n+1} + C_3 Q_{j+1}^n + C_4 Q_L$$

$$C_1 = \frac{\frac{\Delta t}{k} + 2x}{\frac{\Delta t}{k} + 2(1-x)} \qquad C_2 = \frac{\frac{\Delta t}{k} - 2x}{\frac{\Delta t}{k} + 2(1-x)} \qquad C_3 = \frac{2(1-x) - \frac{\Delta t}{k}}{\frac{\Delta t}{k} + 2(1-x)} \qquad C_4 = \frac{2\left(\frac{\Delta t}{k}\right)}{\frac{\Delta t}{k} + 2(1-x)}$$

Example

The hydrograph at the upstream end of a river is given in the following table. The reach of interest is 18 km long. Using a sub-reach length Dx of 6 km, determine the hydrograph at the end of the reach using the Muskingum-Cunge method. Assume c = 2m/s, B = 25.3 m, S_o = 0.001m and no lateral flow.

Time (hr)	Flow (m^3/s)
0	10
1	12
2	18
3	28.5
4	50
5	78
6	107
7	134.5
8	147
9	150
10	146
11	129
12	105
13	78
14	59
15	45
16	33
17	24
18	17
19	12
20	10

Muskingum-Cunge Answer

This is repeated for the rest of the columns and the subsequent columns to produce the following table. Note that when you change rows, “n” changes. When you change columns, “j” changes.

Time (hr)	0 km	6 km	12 km	18 km
0	10	10	10	10
2	18	13.89	11.89	10.92
4	50	34.51	24.38	18.19
6	107	81.32	59.63	42.96
8	147	132.44	111.23	88.60
10	146	149.91	145.88	133.35
12	105	125.16	138.82	145.37
14	59	77.93	99.01	117.94
16	33	41.94	55.52	73.45
18	17	23.14	29.63	38.75
20	10	12.17	16.29	21.02
22	10	9.49	9.91	12.09
24	10	10.12	9.70	9.30
26	10	9.97	10.15	10.01
28	10	10.01	9.95	10.08

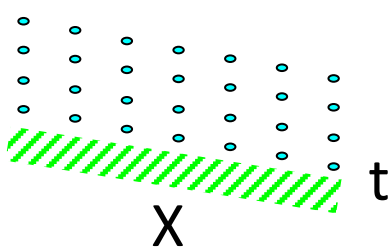
Kinematic Wave

- Kinematic wave channel routing is probably the most basic form of hydraulic routing.
- This method combines the continuity equation with a very simplified form of the St. Venant equations.
- Kinematic wave routing assumes that the friction slope is equal to the bed slope.
- Additionally, the kinematic wave form of the momentum equation assumes a simple stage-discharge relationship.

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_L$$

$$Q = \alpha A^m$$

•An explicit finite difference scheme in a space-time grid domain is often used for the solution of the kinematic wave procedure.



When the average celerity, c, is greater than the ratio Δx/Δt, a conservative form of these equations is applied. In this conservative form, the spatial and temporal derivatives are only estimated at the previous time step and previous location.

$$\frac{Q_{(i,j)} - Q_{(i-1,j)}}{\Delta x} + \frac{A_{(i-1,j)} - A_{(i-1,j-1)}}{\Delta t} = \bar{q}$$

Kinematic Wave Assumptions

- The method does not explicitly allow for separation of the main channel and the overbanks.
- Strictly speaking, the kinematic method does not allow for attenuation of a flood wave. Only translation is accomplished.
- The hydrostatic pressure distribution is assumed to be applicable, thus neglecting any vertical accelerations.
- No lateral, secondary circulations may be present, i.e. - the channel is represented by a straight line.
- Channel slopes should be 10% or less.
- The channel is stable with no lateral migration, degradation, and aggradation.
- Flow resistance may be estimated via Manning's equation or the Chezy equation.

Project work

- Take your selected catchment
- Model it with HEC-HMS
- Evaluate the results your model with
  - different loss rate methods
  - different transform methods
  - different base flow separation methods
  - Different routing methods