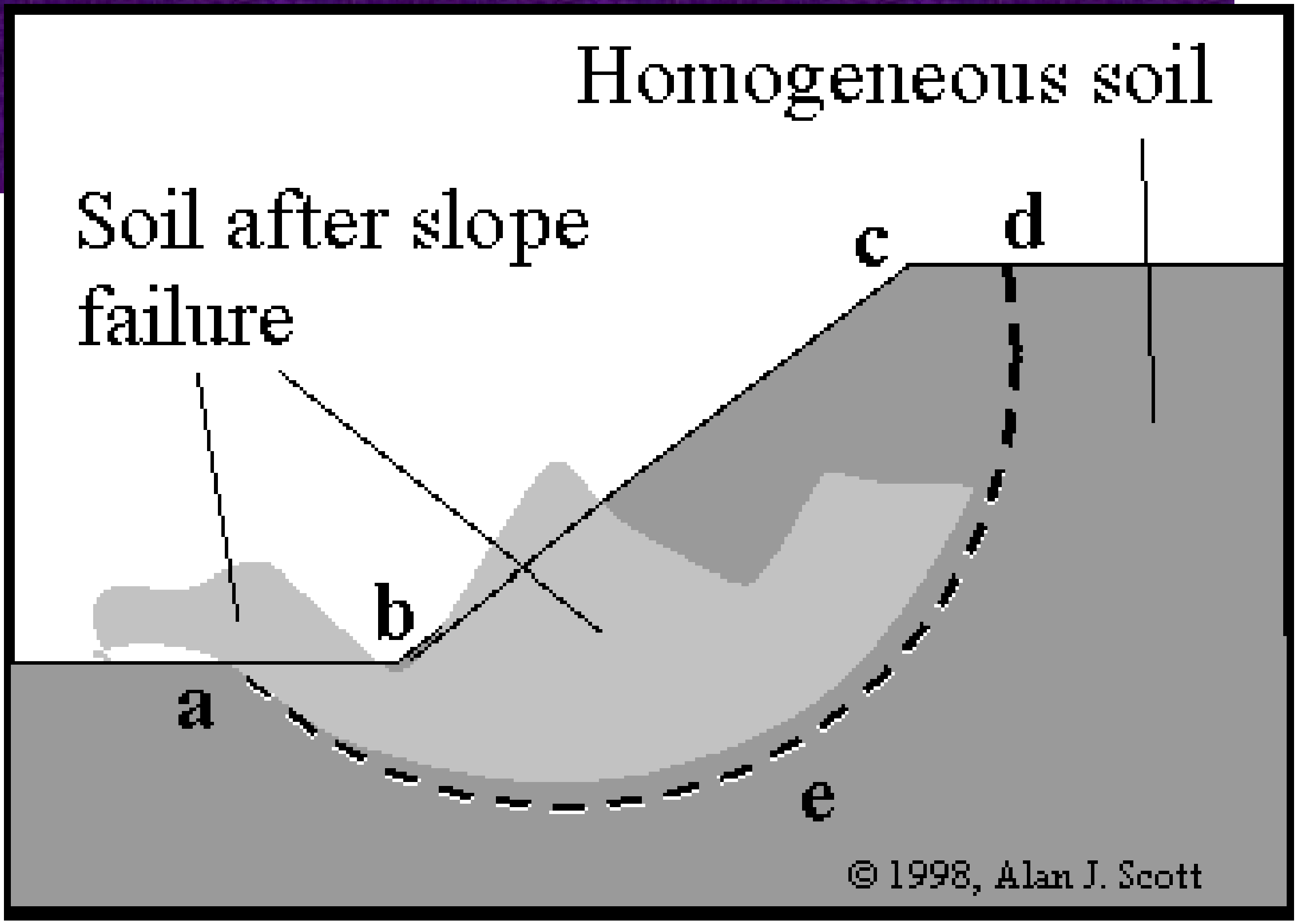
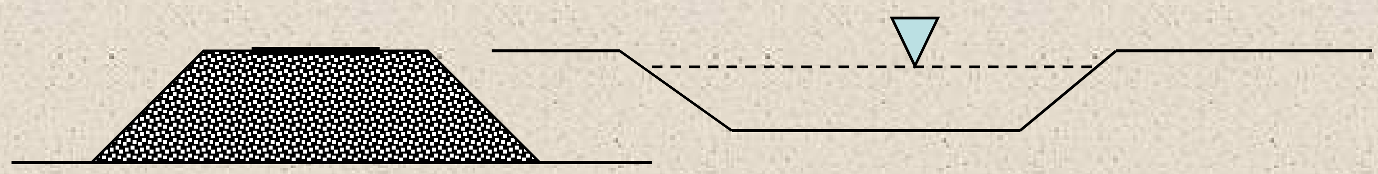
5. SLOPE STABILITY



•**An earth slope is an unsupported, inclined surface of a soil mass.**

Earth slopes are formed for

- ❖ Railway formation
- ❖ Highway embankments
- ❖ Earth dams
- ❖ Canal banks



The slope of earth are of two types

1. Natural slopes :- are those that exist in nature and are formed by natural causes.

- Example:- slopes exist in hilly sides

2. Man made slopes :- are the artificial slopes formed by man.

- Examples:- the slopes of
  - embankments,
  - earth dams

The slopes whether natural or artificial may be

- **Infinite slopes** :- This term is used to designate a constant slope of infinite extent.

  Examples:- The long slope of the face of a mountain.
- **Finite slopes** :- This term used to designate a slope of limited in extent.

  Examples:- The slopes of
    - Embankments,
    - Earth dams

- Infinite slope

- finite slope

Causes of failure of slope.

The important factors that cause instability in a slope and lead to failure are;

- » **Gravitational force**
- » **Force due to seepage of water**
- » **Erosion of the surface of slopes due to flowing water**
- » **The sudden lowering of water adjecent to a slope**
- » **Force due to earthquake**

- The above factors may be classified into two categories.

1. The factors which cause an increase in the shear stresses.

Stresses may increases due to

- Weight of water causing saturation of soils
- Surcharge loads
- Seepage pressure
- Steepening of slopes either by excavation or by natural erosion

This landslide was triggered by heavy rainfall

2. The factors which cause a decrease in the shear strength of the soil.

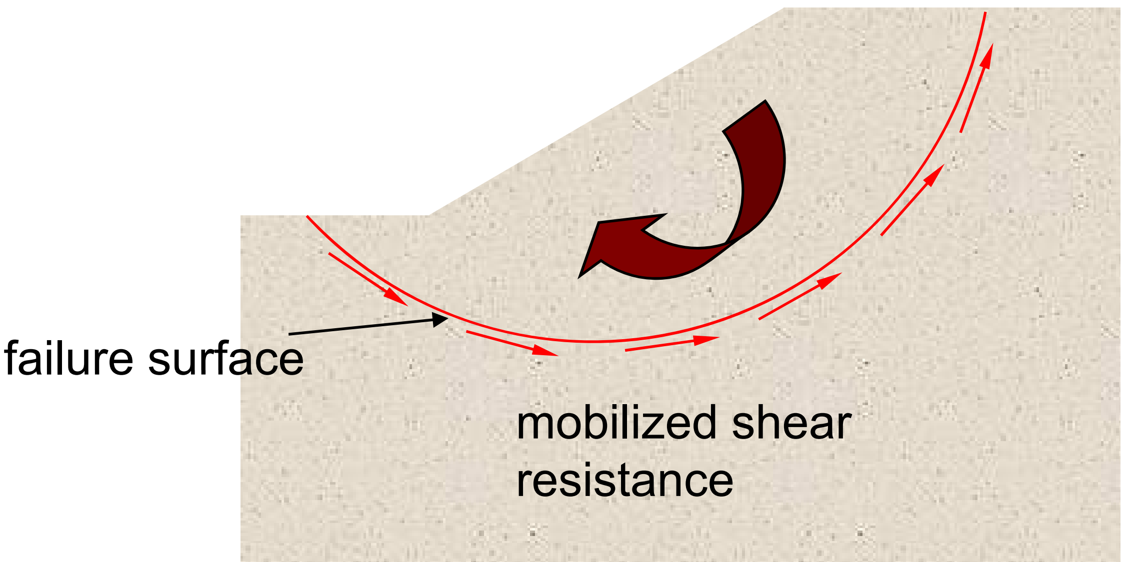
Loss of shear may occur due to

- **An increase in water content**
- **Increase in pore water pressure**
- **Shock or cyclic load**

- **Most of the natural slope failure occur during rainy seasons, as the presence of water causes both increased stresses and the loss of strength**

- Sloping ground can become unstable if the gravity forces acting on a mass of soil exceed the shear strength available at the base of the mass and within it.

- Movement of the mass of soil down the slope will then occur.



At failure, shear stress along the failure surface (τ) reaches the shear strength (τ_f).

- The result of slope failure can often be catastrophic, involving the loss of considerable property and life.

- It is, therefore, essential to check the stability of proposed slopes.

- Civil engineers are often expected to make calculations to check the safety of natural slopes, slopes of excavations, and compacted embankments.

- This check involves the determination and comparison of the shear stress developed along the most likely rupture surface to the shear strength of the soil and is called slop stability analysis.

Factor of safety

-The task of the engineer charged with a slope stability analysis is to determine the factor of safety.

-Factor of safety with respect to strength , F_s

$$F_s = \frac{\tau_f}{\tau_d} \qquad F_S = \frac{C + \sigma \tan \phi}{C_d + \sigma \tan \phi_d}$$

-Factor of safety with respect to cohesion and the factor of safety with respect to friction

$$F_C = \frac{C}{C_d} \qquad F_\phi = \frac{\tan \phi}{\tan \phi_d}$$

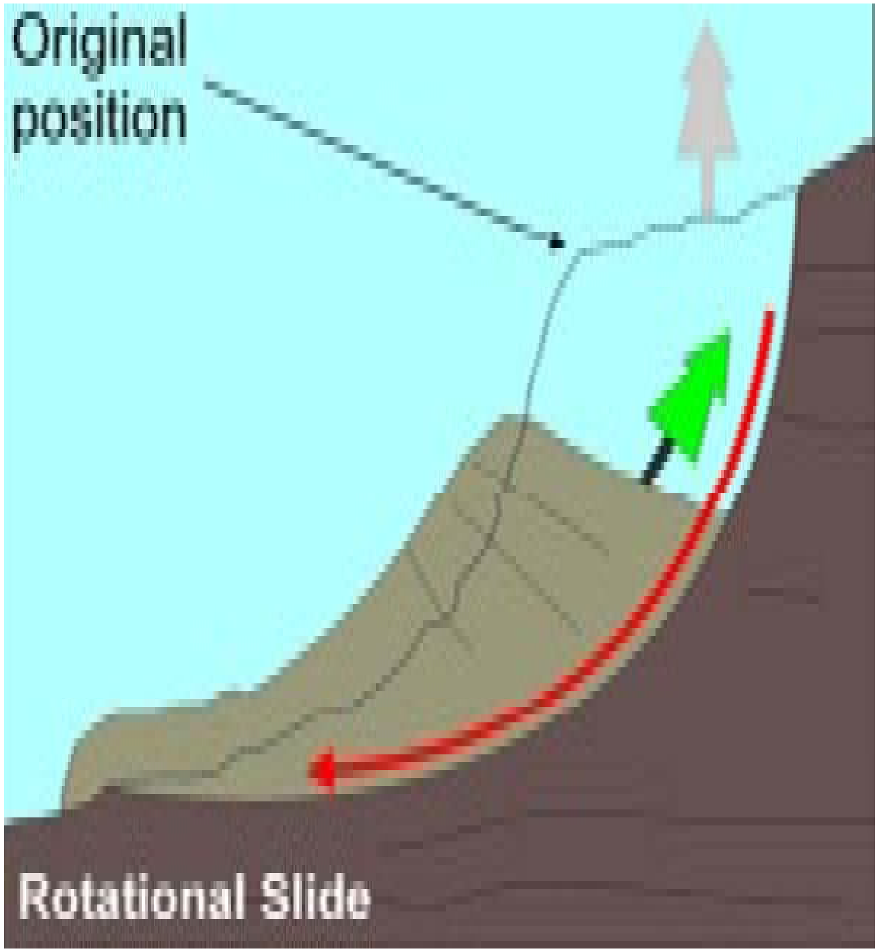
-In the analysis of slope stability , generally the three factors of safety are taken equal. i.e, Fs = Fc= Fϕ

TYPES OF SLOPE FAILURE

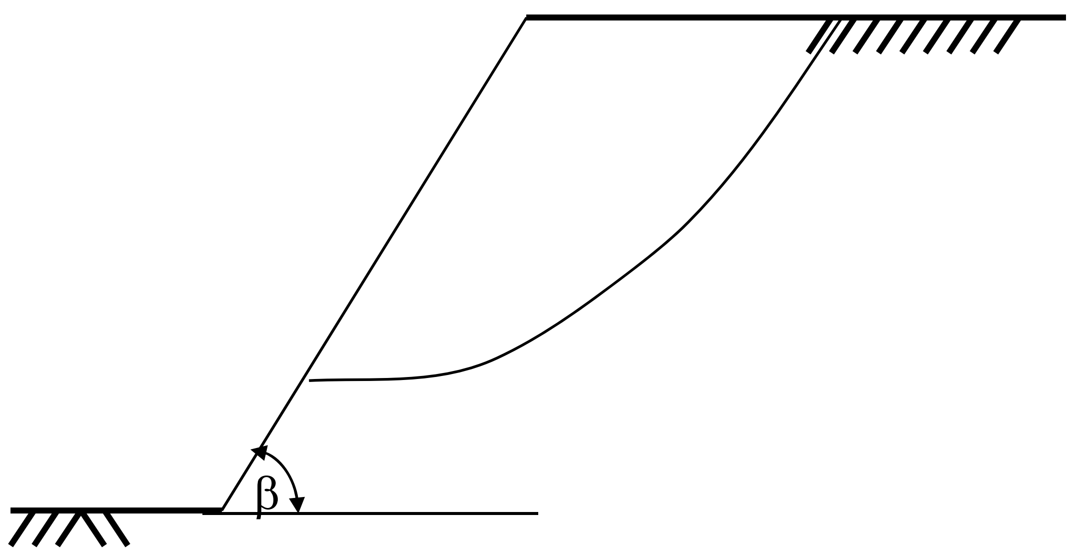
- The type of failures that normally occur may be classified as

- **Rotational Failure**

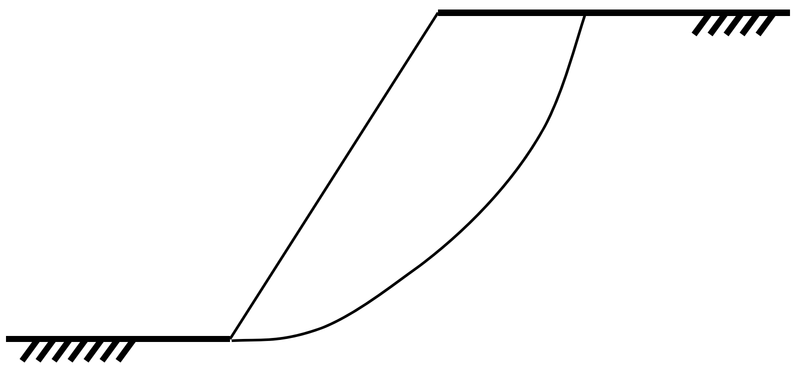
- This type failure occurs by rotation along a slip surface by downward and outward movement of the soil mass
- The slip surface in generally circular for homogeneous soil conditions and non circular in case of non-homogeneous conditions.
- Rotational slips are further divided into 3 types


Original
position
Rotational Slide

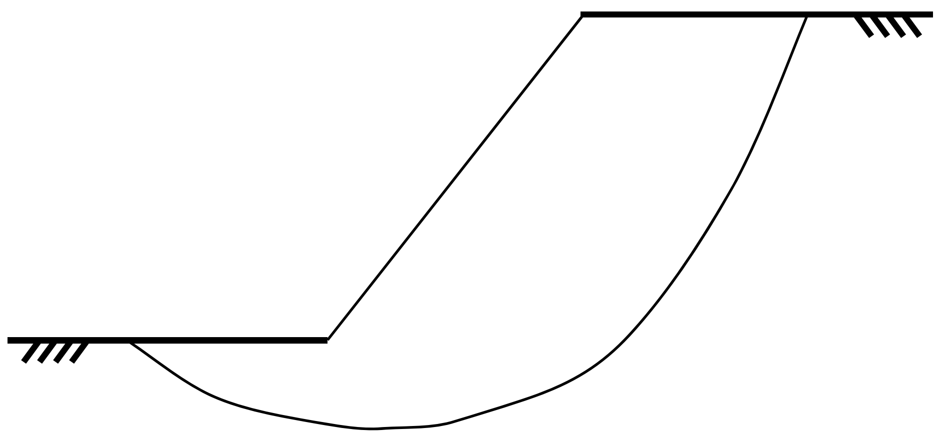
1. **Slope failure: -** In slope failure, the arc of the rupture surface meets the slope above the toe. This can happen when the slope angle β is quite high and the soil close to the toe possesses high strength.



2. <u>Toe failure</u>: - occurs when the soil mass above and below the base is homogenous.



3. <u>Base failure</u>: - takes particularly when the base angle β is low and the soil below the base is softer and more plastic than the soil above the base.



- **Translational Failure**
- Translational failure occurs in an infinite slope along a long failure surface parallel to the slope.
- The shape of the failure surface is influenced by the presence of any hard stratum at a shallow depth below the slope surface. These failures may also occur along slopes of layered materials
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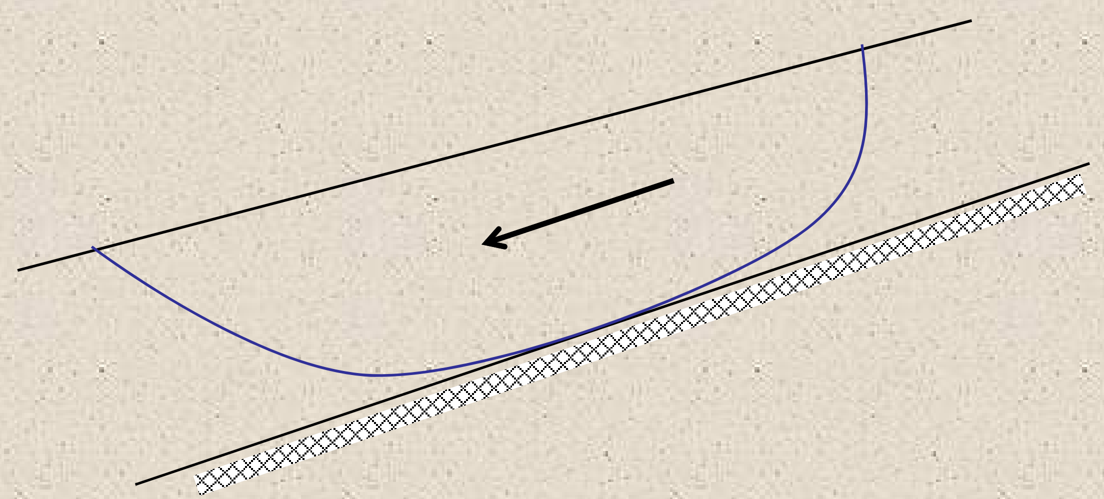
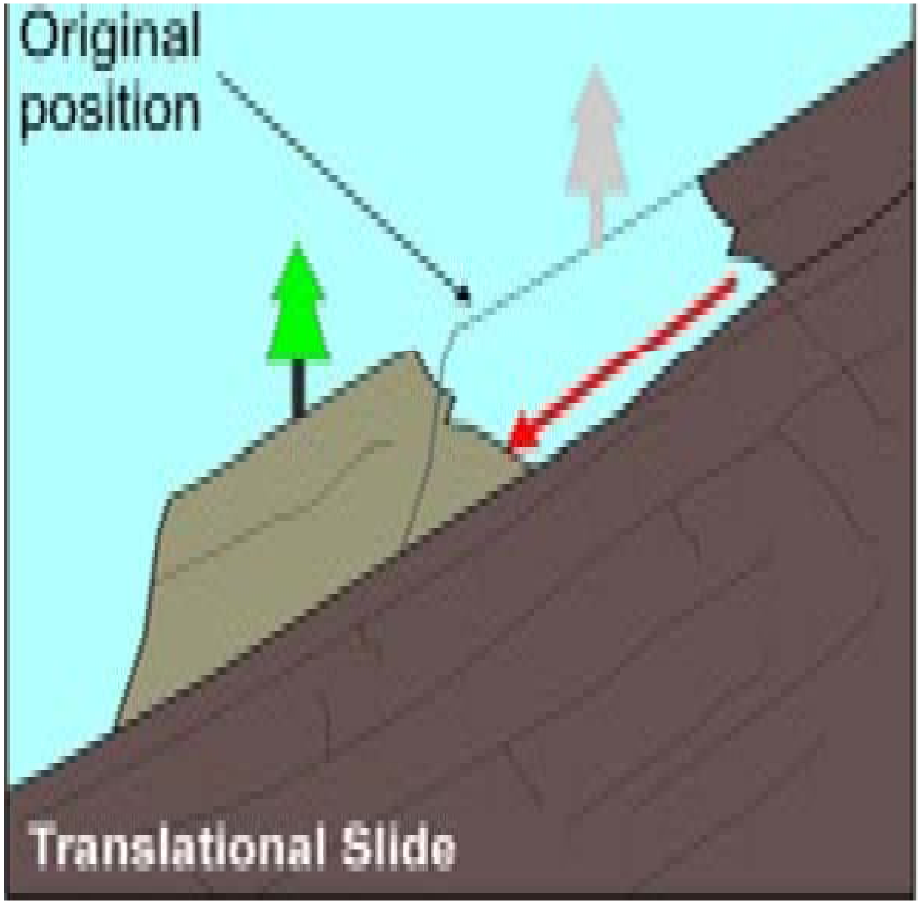
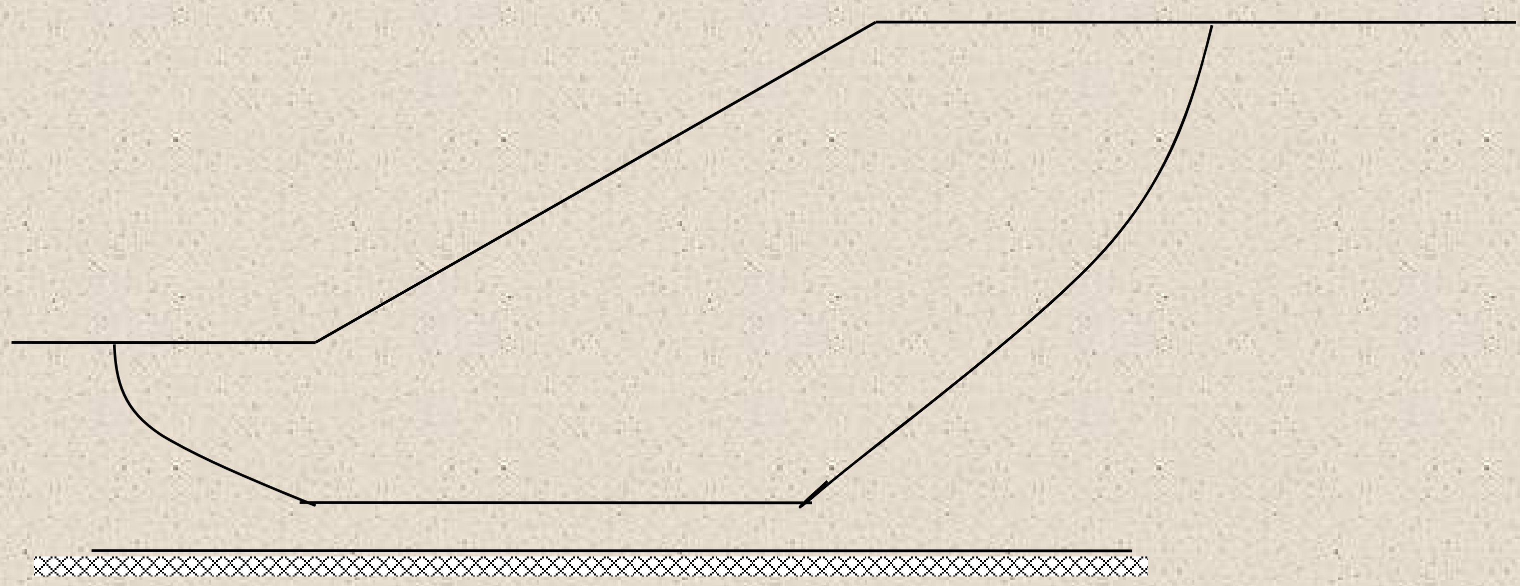



Original
position
Translational Slide

- **Compound Failure**
- A compound failure is a combination of the rotational slips and the translational slips. A compound failure surface is curved at the two ends and plane in the middle portion.
- A compound failure generally occurs when a hard stratum exists at considerable depth below the toe.



- **Wedge Failure**
- A failure along an inclined plane is known as plane failure or wedge failure or block failure.
- A wedge failure is similar to transitional failure in many respects. However, unlike transitional failure which occurs in an infinite slope, a wedge failure may occur both in infinite and finite slope consisting of two different materials or in a homogeneous slope having cracks, fissures, joints or any other specific plane of weakness.

- **Miscellaneous Failure**
- In addition to above four types of failures, some complex type of failures in the form of spreads and flows may also occur.

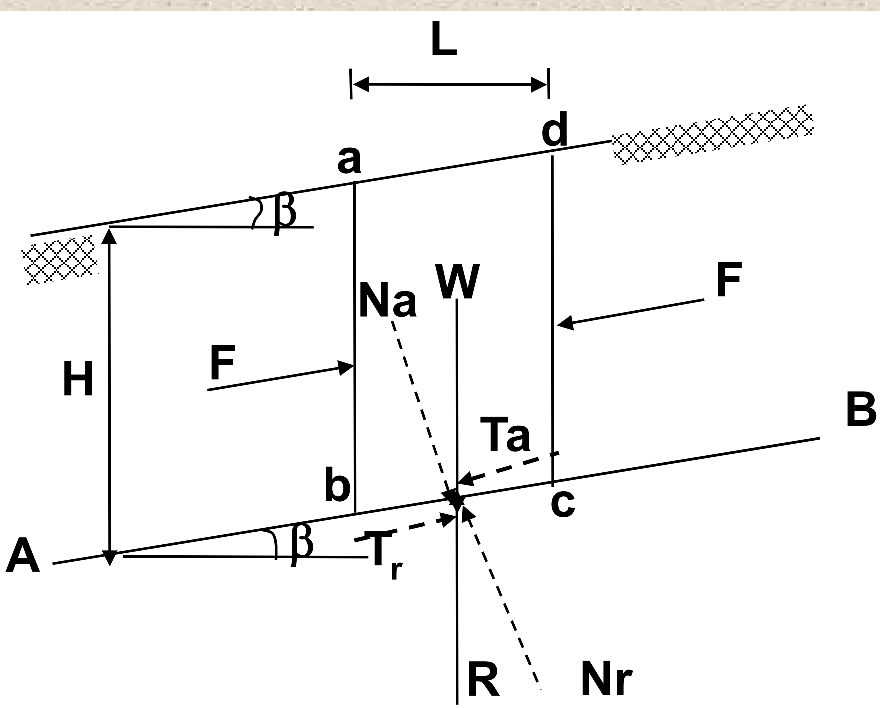
FLOWS

Mark Reid, *US Geological Survey*

STABILITY OF INFINITE SLOPES

Infinite slopes without Seepage

- Consider an infinite slope shown in below
- Assuming that the pore water pressure is zero
- We will evaluate the factor of safety against a possible slope failure along a plane AB located at a depth H below the ground surface.



- The weight of the soil element
  - » $W = \gamma HL$.
- The weight **W** can be resolved into two components

1.Force perpendicular to the plane

» $Na = W\cos\beta = \gamma LH \cos\beta$

2. Force parallel to the plane

» $Ta = W\sin\beta = \gamma LH \sin\beta$

❖note that this force tends to cause the slip along the plane.

- This normal stress σ and the shear stress τ at the base of the slope element are given by

$$\sigma = \frac{Na}{area\ of\ base} = \frac{\gamma LH \cos\beta}{(L/\cos\beta)} = \gamma H \cos^2\beta$$

$$\tau = \frac{T_a}{area\ of\ base} = \frac{\gamma LH \sin\beta}{(L/\cos\beta)} = \gamma H \cos\beta \sin\beta$$

- The reaction to the weight W is an equal and opposite force R. The normal and tangential components of R with respect to the plane AB are Nr and Tr.
  - N_r = Rcosβ = Wcos β
  - T_r = Rsinβ = W sin β

- For equilibrium, the resistive shear stress developed at the base of the element is

$$\frac{T_r}{(area\ \ of\ \ base)} = \gamma H \sin\beta \cos\beta$$

It may also be written in the form

$$\tau_d = C_d + \sigma \tan\phi_d$$

But $\sigma = \gamma H \cos^2\beta$ **and** $\tau_d = \gamma H \sin\beta\cos\beta$

Then $\gamma H \sin\beta\cos\beta = C_d + \gamma H \cos^2\beta \tan\phi_d$

$$\frac{Cd}{\gamma H} = Sin\beta Cos\beta - Cos^2\beta \tan\phi_d$$

$$= \cos^2\beta[\tan\beta - \tan\phi_d] \qquad \text{.......................(1)}$$

- The factor of safety with respect to strength

$$\tan \phi_d = \frac{\tan \phi}{F_s}, \qquad C_d = \frac{C}{F_s}$$

- Substituting the above relations in Eq(1), we obtain

$$F_s = \frac{C}{\gamma H \cos^2 \beta \tan \beta} + \frac{\tan \phi}{\tan \beta} \qquad \text{..........................} \qquad \textbf{(2)}$$

- For granular soils, C=0 and the factor of safety

$$F_S = \frac{\tan \phi}{\tan \beta}$$

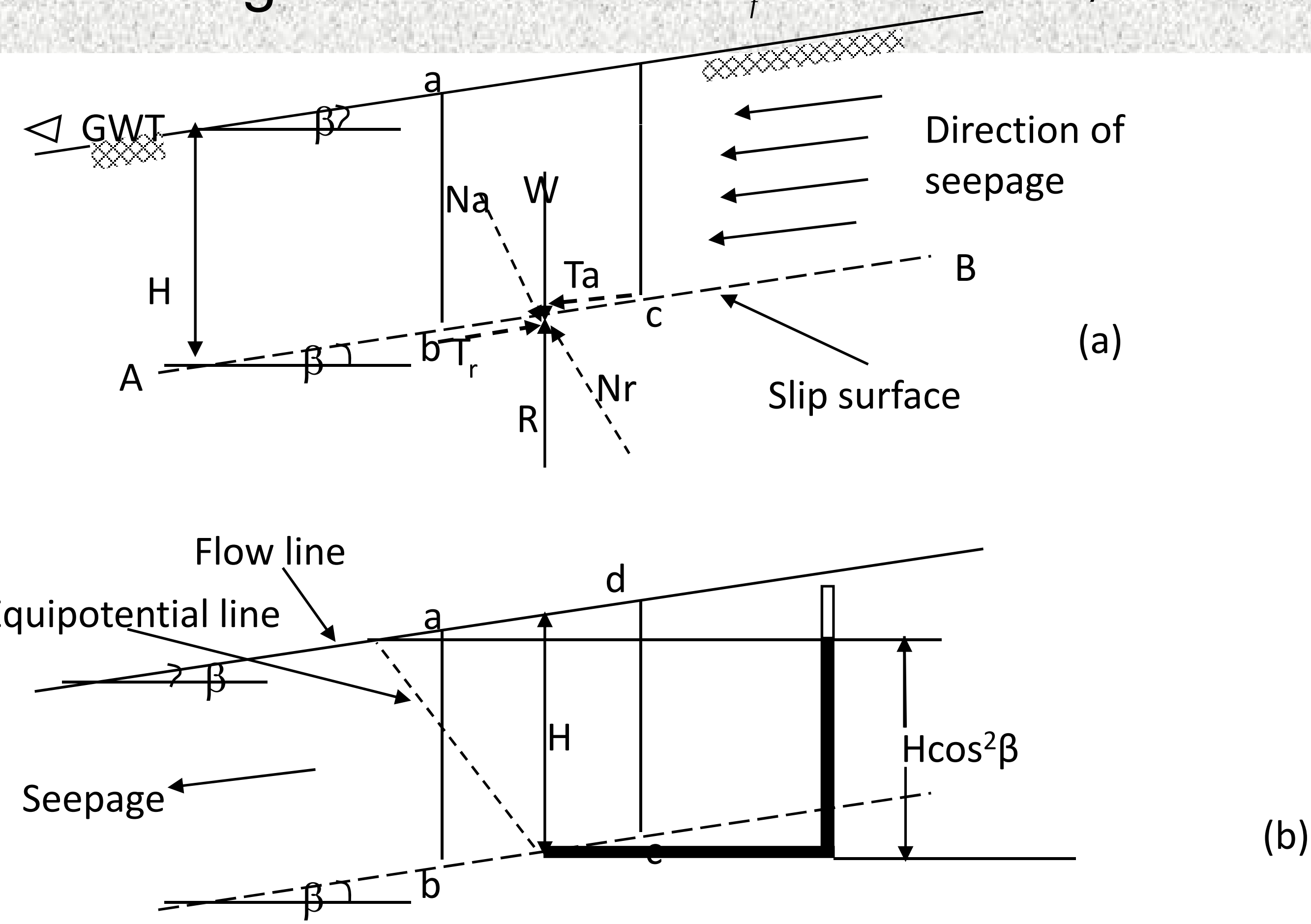
- **This indicates that in an infinite slope in sand, F_s is independent of height H and the slope is stable as long as $\beta < \phi$.**

- **If a soil possesses cohesion and friction, the depth of the plane along which critical equilibrium occurs may be determined by substituting F_s=1 and $H = H_{cr}$ in Eq.(2).**

$$H_{cr} = \frac{C}{\gamma} \frac{1}{\cos^2 \beta(\tan \beta - \tan \phi)}$$

Infinite Slopes with Seepage

- Figure shown below shows an infinite slope.
- Seepage through the soil is assumed and the groundwater level coincides with the ground surface.
- The shear strength of the soil is $\tau_f = C + \sigma' \tan\phi$



- The total normal stress and the shear stress at the base of the element are respectively

$$\sigma = \frac{Nr}{area\ of\ base} = \frac{\gamma_{sat} LH \cos\beta}{(L/\cos\beta)} = \gamma_{sat} H \cos^2\beta$$

$$\tau = \frac{T_r}{area\ of\ base} = \frac{\gamma_{sat} LH \sin\beta}{(L/\cos\beta)} = \gamma_{sat} H \cos\beta \sin\beta$$

- The resistive shear stress developed at the base of the element is also given by

» $\tau_d = C_d + \sigma' \tan\phi_d = C_d + (\sigma - u)\tan\phi_d$

- Where the pore water pressure $u = \gamma_w H \cos^2\beta$
- $\tau_d = C_d + (\gamma_{sat} H \cos^2\beta - \gamma_w H \cos^2\beta)\tan\phi_d = C_d + (\gamma_b H \cos^2\beta)\tan\phi_d$

- $\tau = \tau_d$
- $\gamma_{Sat}\ H \cos\beta \sin\beta = C_d + \gamma_b\ H \cos^2 \beta \tan\phi_d$
- $C_d / (\gamma_{sat}) = \cos^2 \beta(\ \tan\beta - (\gamma_b/\gamma_{sat})\tan\phi_d)$

- The factor of safety with respect to strength can be found by substituting $\tan\phi_d = \tan\phi / F_s$ and $C_d = C/F_s$

$$F_s = \frac{C}{\gamma_{sat} H \cos^2 \beta \tan \beta} + \frac{\gamma_b}{\gamma_{sat}} \frac{\tan \phi}{\tan \beta}$$

- **Different cases**

1. When the ground water table is at a depth of h_w below the natural ground surface

$$F_S = \frac{C}{\gamma_{sat}\ H\cos^2\beta\tan\beta} + \frac{1}{H\,\gamma_{sat}}\left[H\gamma_{sat} - H\gamma_w - h_w\ \gamma_w\right]\frac{\tan\emptyset}{\tan\beta}$$

2. Wet (submerged) cohesionless slope $C=0$, $h_w = 0$

$$F_S = \frac{\gamma_b \tan\phi}{\gamma_{sat} \tan\beta}$$

3. Cohesionless slope with pore pressure ratio r_u

- The pore pressure ratio, r_u can be used to represent overall or local pore pressure conditions in a slope.
- It is given by the ratio of the pore water pressure to the total stress:

$$r_u = \frac{u}{\gamma h}$$

$$\tau = \tau_d$$

- $\gamma_{sat} H \cos\beta\sin\beta = (\cos^2 \beta H\gamma_{sat} - r_u\gamma_{sat}H)\tan\phi_d$
- $\cos\beta\sin\beta = \cos^2 \beta\,(1- r_u/\cos^2\beta)\tan\phi/F_S$

$$F_S = [1 - r_u sec^2\beta]\frac{tan\emptyset}{tan\beta}$$

4. Undrained condition

$$\tau = \tau_d$$

$$\gamma_{Sat} H \cos\beta\sin\beta = C_d$$

$$\gamma_{Sat} H \cos\beta\sin\beta = C_u/Fs$$

$$F_S = \frac{C_u}{\gamma H cos\beta sin\beta}$$

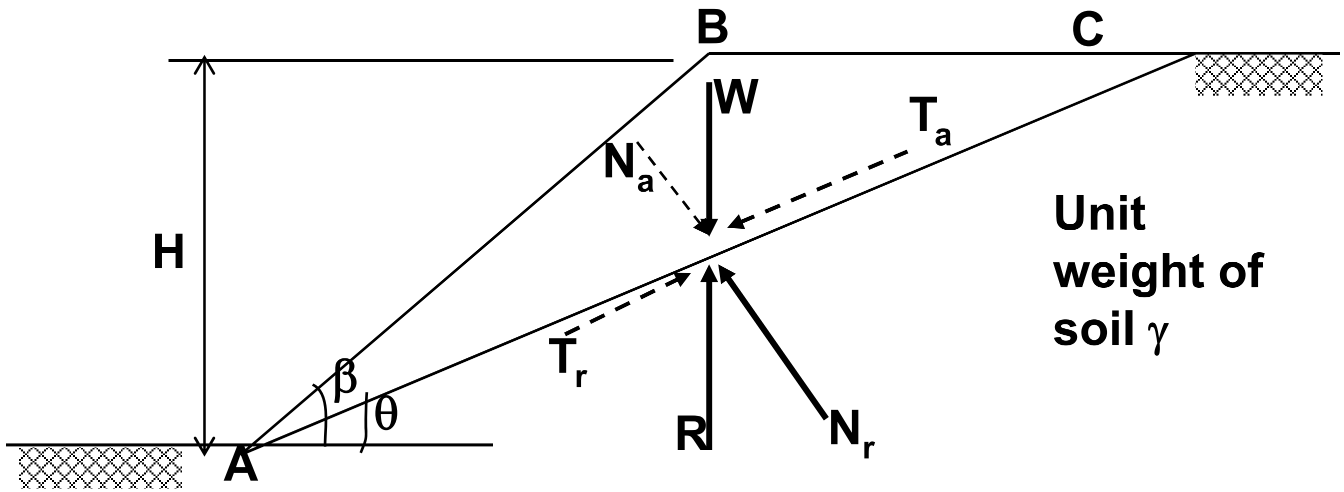
Stability of Finite Slopes

- Stability analysis of a finite slope in a homogeneous soil needs to assume the general shape of the surface of potential failure.

    - » Plane failure surface
    - » Curved failure surfaces

Wedge Method (Culmann's Method)

- Assumption
  - slope failure occurs along a plane when the average shearing stress tending to cause the slip is greater than the soil shear strength.
- The most critical plane is the one which the ratio of the average shearing stress tending to cause failure to the shear strength of soil is a minimum.
- The factor of safety F_s calculated by Culmann's approximation gives fairly good results for near vertical slopes only
- Fig. shown below shows a slope of height H. The slope rises at an angle β with the horizontal. Plane AC is a trial failure plane. Considering a unit thickness perpendicular to the section of the slope.



-The weight W of wedge ABC is

$$W = \tfrac{1}{2}(H)\ \overline{BC}(1)(\gamma)$$

$$= \tfrac{1}{2}(H)\,[H\cot\theta - H\cot\beta]\,\gamma = \tfrac{1}{2}\,\gamma H^2\left[\frac{\sin(\beta-\theta)}{\sin\beta\,\sin\theta}\right]$$

•**The normal component N_a and tangential component T_a of W with respect to the plane**

$$N_a = W\cos\theta = \tfrac{1}{2}\gamma H^2 \left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\cos\theta$$

$$T_a = W\sin\theta = \tfrac{1}{2}\gamma H^2 \left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\sin\theta$$

$$\sigma = \frac{N_a}{(\overline{AC})(1)} = \frac{N_a}{H/Sin\theta} = \frac{1}{2}\gamma H\left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\cos\theta\sin\theta$$

$$\tau = \frac{T_a}{(\overline{Ac})(1)} = \frac{T_a}{(H/\sin\theta)} = \frac{1}{2}\gamma H\left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\sin^2\theta$$

- **The average resistive shearing stress developed along the plane AC is**

$$\tau_d = C_d + \sigma \tan\phi_d$$

$$= C_d + \tfrac{1}{2}\gamma H \left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\cos\theta\sin\theta\tan\phi_d$$

$$\tfrac{1}{2}\gamma H \left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\sin^2\theta = C_d + \tfrac{1}{2}\gamma H \left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\cos\theta\sin\theta\tan\phi_d$$

$$C_d = \tfrac{1}{2}\gamma H \left[\frac{\sin(\beta-\theta)(\sin\theta-\cos\theta\tan\phi_d)}{\sin\beta}\right]$$

- **The above equation is derived for the trial failure plane. To determine the critical plane, the principle of maxima or minima is used (for a given value of ϕ_d) to find the angle θ where the developed cohesion would be maximum. Thus, the first derivative of C_d with respect to θ is zero**

$$\frac{dC_d}{d\theta} = 0 \Rightarrow \frac{d}{d\theta}\left[\sin(\beta-\theta)(\sin\theta - \cos\theta\tan\phi_d)\right] = 0$$

•**Solution of this equation gives the critical value of θ**

$$\theta_{cr} = \frac{(\beta + \phi_d)}{2}$$

- Substituting the value of $\theta = \theta_{cr}$ yields

$$C_d = \frac{\gamma H}{4}\left[\frac{1-\cos(\beta-\phi_d)}{\sin\beta\cos\phi_d}\right]$$

•The maximum height of the slope for which critical equilibrium occurs (H_{cr}) can be obtained by substituting C_d=C and ϕ_d=ϕ into the above equation. Thus

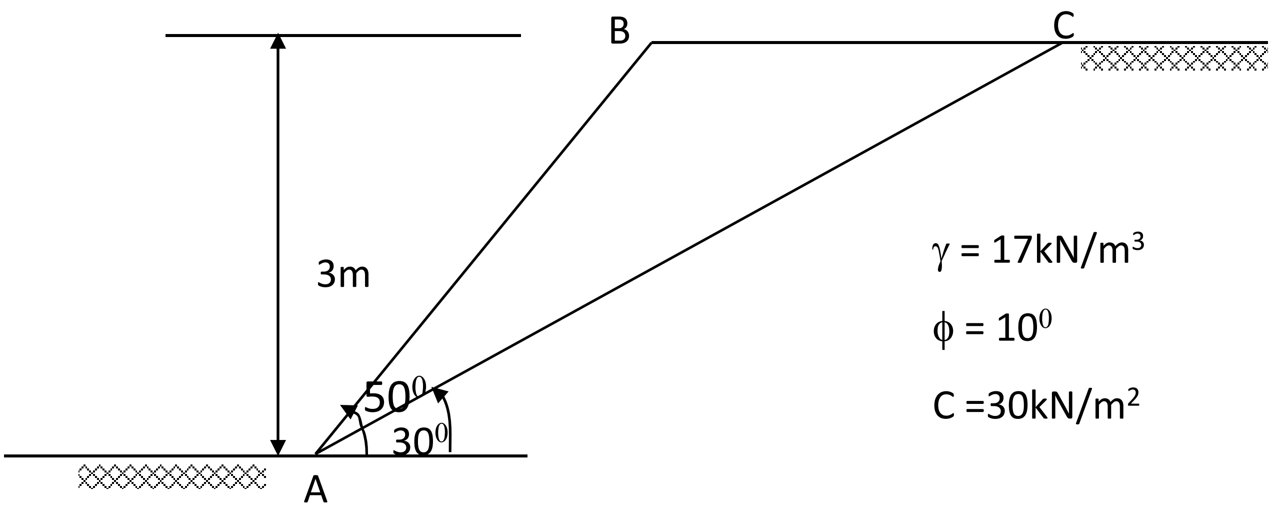
$$H_{cr} = \frac{4C}{\gamma}\left[\frac{\sin\beta\cos\phi}{1-\cos(\beta-\phi)}\right]$$

- For purely cohesive soil where $\phi = 0$

$$H_{cr} = \frac{4C}{\gamma}\left[\frac{\sin\beta}{1-\cos\beta}\right]$$

ASSIGNMENT No. 3

1. A cut is to be made in a soil having γ = 18kN/m^3, C=30kN/m^2, and ϕ=15^0. The side of the cut slope makes an angle of 45^0 with the horizontal. What should be the depth of the cut slope for a factor of safety F_S of 3?
2. A slope is shown in Fig below; represents a trail failure plane. For the wedge , find the factor of safety, F_S, against sliding.



Curved failure surfaces

- **Finite slope with circular cylindrical Failure Surface**
- After extensive investigation of slope failures, *a Swedish Geotechnical Commission* recommended that the actual surface of sliding may be approximated to be circularly cylindrical.
- This method has been quite widely accepted as offering an approximately correct solution for the determination of factor of safety of the slope of an embankment and of its foundation.
- When soil slips along a circular surface, such a slide may be termed as a rotational slide

- **Various procedures of stability analysis may, in general, be divided into two major classes**

- **a) *Mass procedure*: - in this method the mass of the soil above the surface of sliding is taken as a unit. This procedure is useful when the soil forming the slope is assumed to be homogenous**

- **b) *Method of slices: -* in this method the soil above the surface of sliding is divided into a number of vertical parallel slices and the stability of each slice is separately calculated.**
- **This technique is versatile; non-homogeneity of the soils and pore water pressure can be taken into consideration.**

- A) ***Mass procedure***
- **Swedish Circle Method of Analysis**

i, Undained condition or $\phi_u = 0$ method of analysis.

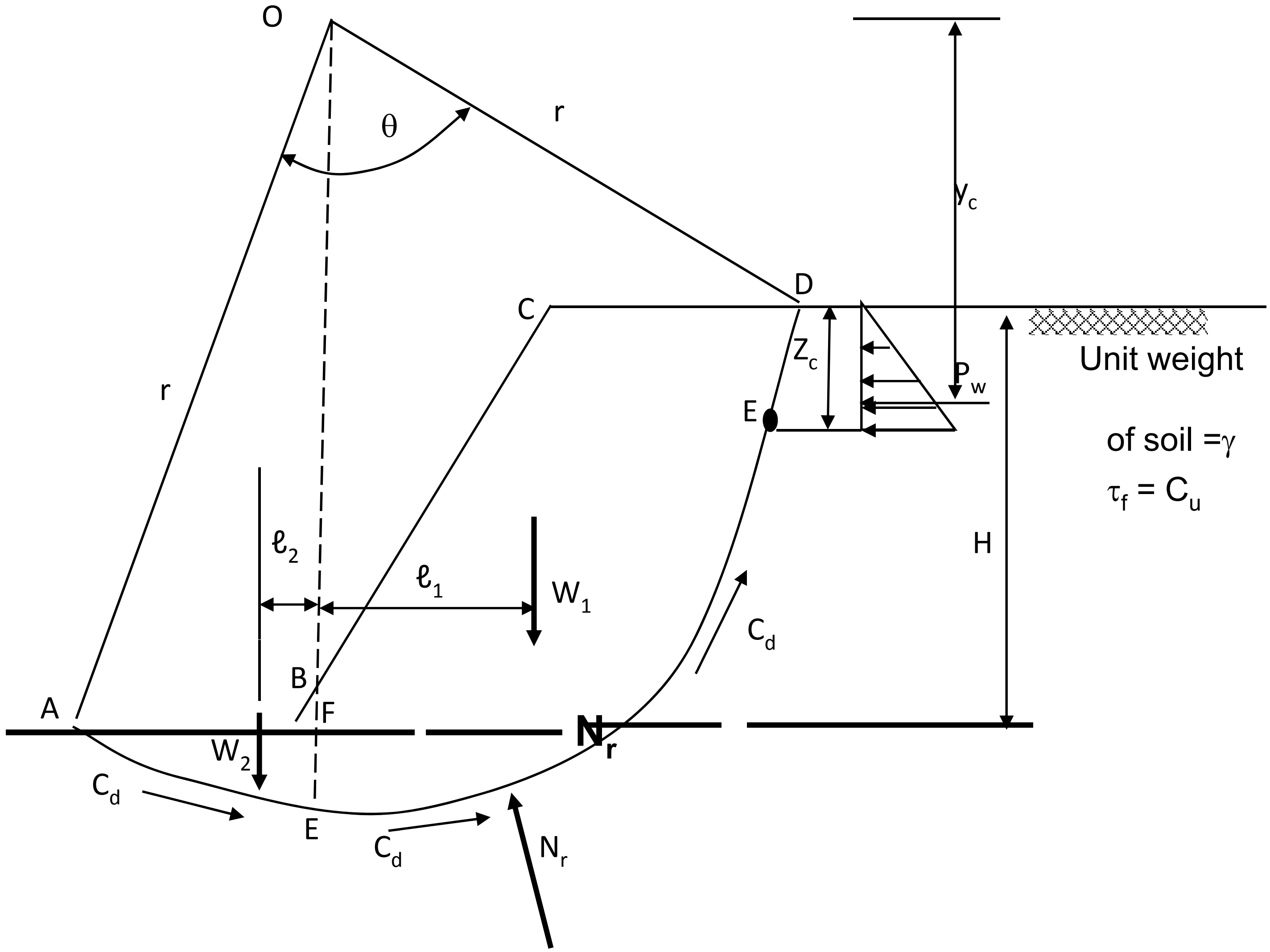
- The simplest circular analysis used to analyze the short-term stability for homogeneous slopes based on the assumptions that a rigid, cylindrical block will fail by rotation about its center and the friction angle is zero so the shear strength is assumed to be due to cohesion only.

- The factor of safety, defined as the ratio of the shear strength to mobilized shear strength, can be calculated by summing moments about the center of the circular surface:

- The undrained shear strength of the soil is assumed to be constant with depth and is given by $\tau_f = C_u$.

- choose a trial potential curve of sliding AED, which is an arc of a circle having a radius r.

- Considering unit thickness perpendicular to the section of the slope, the weight of the soil above the curve AED may be given by $W = W_1+W_2$, where W_1=(area of FCDEF) (γ) and W_2 = (area of ABFEA) (γ)



O
r
θ
r
y_c
D
C
Z_c
P_w
Unit weight
of soil = γ
$\tau_f = C_u$
E
r
ℓ_2
ℓ_1
W_1
H
C_d
B
A
F
N_r
W_2
C_d
E
C_d
N_r

- The moment of the driving force about O to cause slope instability is

$$M_d = W_1 \ell_1 - W_2 \ell_2$$

  - Where ℓ_1 and ℓ_2 are the moment arms

- If C_d is the cohesion that needs to be developed to prevent slope failure, the moment of the resisting forces about O is
- $M_R = (C_d)$ (arc AED) r , but arcAED = $r\,\theta$

  $= C_d r^2 \theta$

- For equilibrium, $M_R = M_d$; thus $C_d r^2 \theta = W_1 \ell_1 - W_2 \ell_2$.

Or

$$C_d = \frac{(W_1\ell_1 - W_2\ell_{2})}{(r^2\theta)}$$

The factor of safety against sliding is

$$F_S = \frac{\tau_f}{C_d} = \frac{C_u}{C_d}$$

- The critical surface is the one for which the ratio of C_u to C_d is a minimum, or in other words C_d is maximum.

- To find the critical surface for sliding, a number of trials are made by different trial circles. The minimum value of the factor of safety thus obtained is the factor of safety against sliding for the slope, and the corresponding circle is the critical circle.

- **Tension crack**
- **As the condition of limiting equilibrium develops with the factor of safety close to 1, a tension crack may form near the top of the slope through which no shear strength can be developed, and if it fills with water a horizontal hydrostatic force P_w will increase the disturbing moment by $P_w y_c$.**

- **The factor of safety will be further reduced because of the shorter length of circular arc along which shearing resistance can be mobilized.**

- **The depth of a tension crack can be taken as:**

$$Z_c = \frac{2C_u}{\gamma}$$

- and the hydrostatic force P_w is: $$P_w = \frac{1}{2}\gamma_w z_c^2$$

- ii, **Friction Circle Method for c, ϕ soil**

- This method is suitable for total or effective stress types of analysis in homogeneous soils with ø > 0.

- Arc AC is a trial circular arc passing through the toe of the slope and O is the center of the circle.
- The shear strength of the soil is given by
  - $\tau_f = C + \sigma \tan\phi$.

- Considering unit thickness perpendicular to the section of the slope,

  weight of soil = W = (area of ABC) (γ).

- For equilibrium, other forces acting on the wedge are

1. C_d, resultant of cohesive force

- C_d = (unit cohesion developed) x (length of cord AC)
  - » $C_d = C_d$ cord AC

- The resultant C_d acts in a direction parallel to chord AC and at a distance, a, from the center of the circle O such that:
- C_d cord AC(a) = C_d (arcAc)r let arc AC = L_a
- $C_d L_C(a) = C_d L_a r$ and chord Ac = L_C

$$a = \frac{C_d L_a r}{L_c C_d}$$

$$a = \frac{L_a}{L_c} r$$

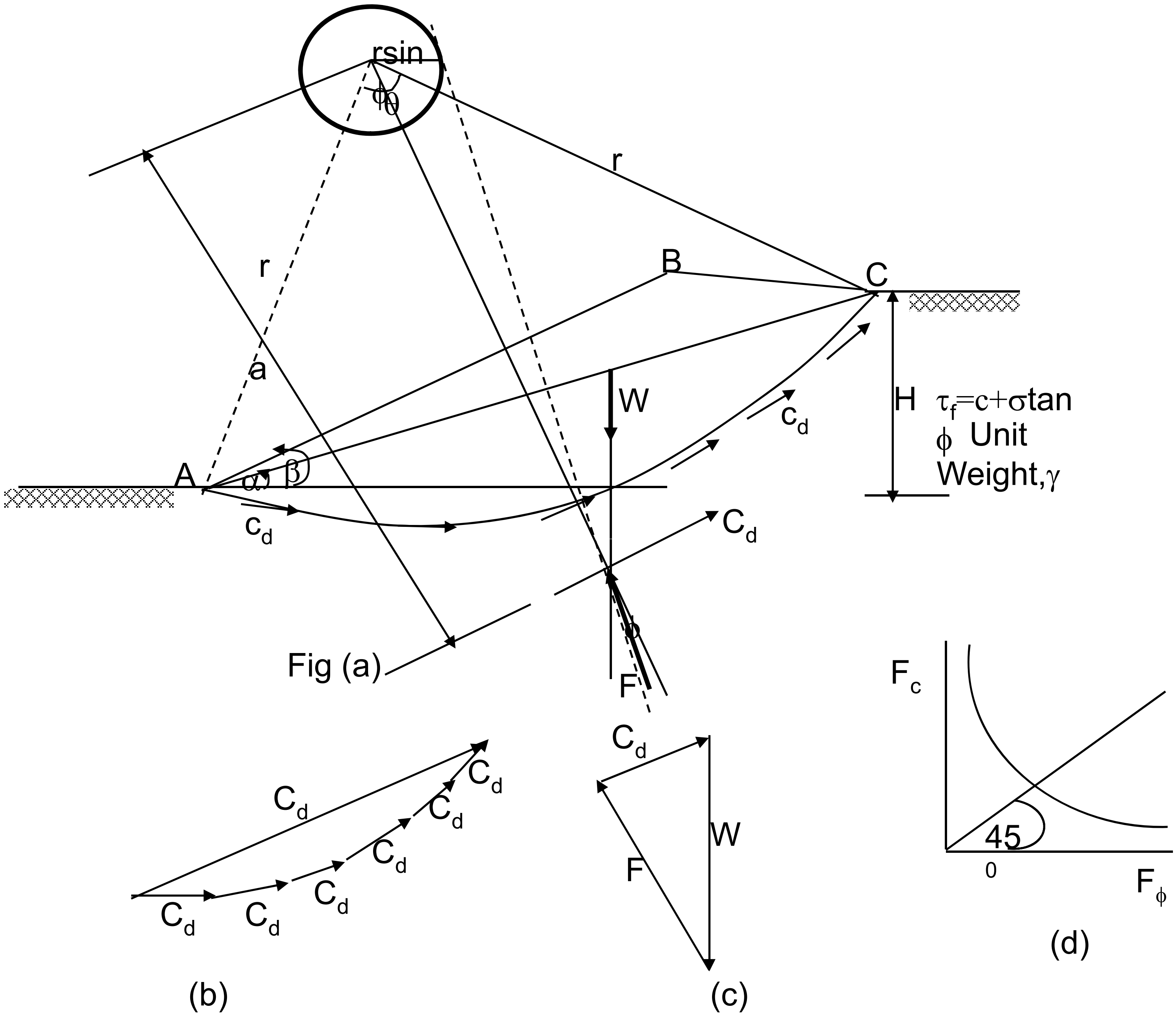


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Fig (a)
Cd
Cd
Cd
Cd
Cd
Cd
Cd
Cd
(b)
Cd
W
F
(c)
Fc
45
0
Fφ
(d)

2. F, resultant of the normal and frictional forces along the surface of sliding. For equilibrium, the line of action of F will pass through the point of intersection of lines of action of W and C_d.

•The value of ϕ_d is obtained from the following equation, after choosing a value of $F\phi$.

$$\tan \phi_d = \frac{\tan \phi}{F_\phi}$$

•The line of action of F will make an angle of ϕ_d with a normal to the arc; and thus it will be a tangent to a circle with its center at O and a radius of $r\sin\phi_d$. This circle is called the friction circle.

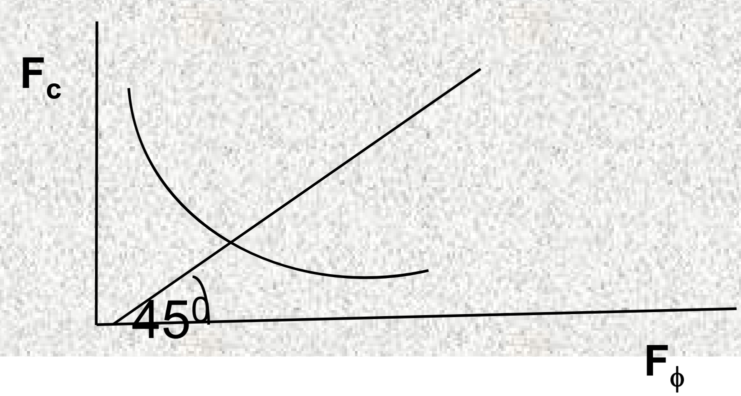
•Since the directions of W, C_d, and F and the magnitude of W are known, a force polygon can be plotted. The magnitude of C_d can be determined from the force polygon.

- The mobilized cohesion is equal to the cohesion force C_d divided by the length of the chord L_c. Thus

  » $C_d = C_d$ /cord AC

$$F_c = \frac{C}{C_d}$$

- If the value of F_c obtained from the above equation is not equal to the assumed value of F_ϕ, the analysis is repeated.
- The procedure is repeated after taking another trial surface. The slip surface which gives the minimum factor of safety (F_s) is the most critical circle.
- Generally, the analysis is repeated 3-4 times to obtain a curve between the assumed value of F_ϕ and the computed value of F_c as shown below
- The factor of safety with respect to shear strength F_s is obtained by drawing a line at 45^0, which gives $F_c = F_\phi = F_s$

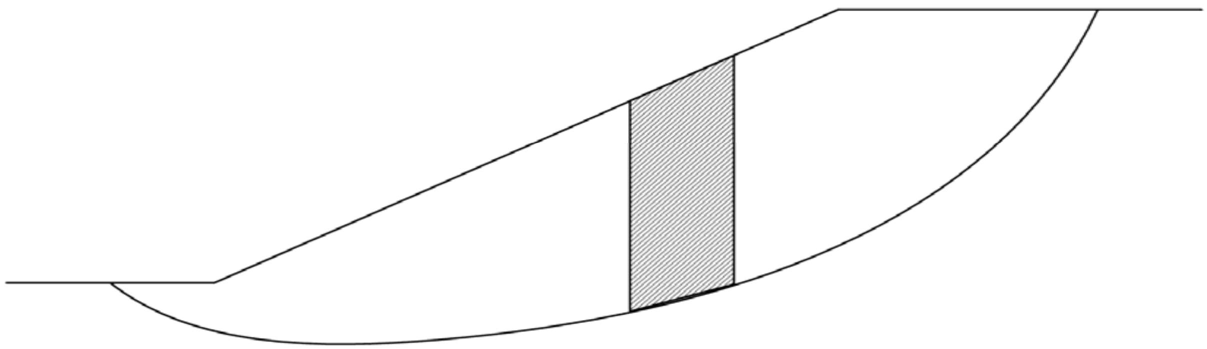


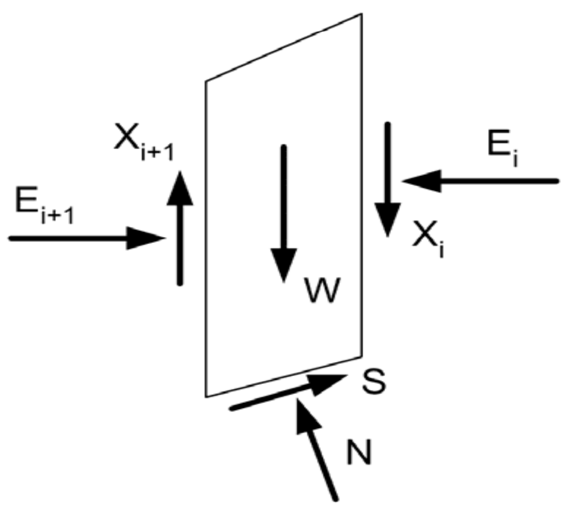
Methods of Slices

- In the method of slices, the soil mass above the slip surface is divided into a number of vertical slices and the equilibrium of each of these slices is considered.

- The actual number of the slices depends on the slope geometry and soil profile. However, breaking the mass up into a series of vertical slices does not make the problem statically determinate.

- In order to get the factor of safety by using method of slices, it is necessary to make assumptions to remove the extra unknowns and these assumptions are the key roles of distinguishing the methods.





X_{i+1}
E_{i+1}
W
E_i
X_i
S
N

Methods of Analysis

- Many different solution techniques for the method of slice have been developed over the years. Basically, all are very similar.
- The differences between the methods are
  - What equations statics are included and satisfied
  - Which interslice forces are included and
  - What is the assumed relationship between the interslice shear and normal forces
- The following are some of the methods used:
  1. Swedish circular method
  2. Bishop's simplified method
  3. Janbu's simplified method
  4. Bishop's rigorous method
  5. Janbu's general method
  6. Spencer's method
  7. Morgenstern-Price method

  etc

Equation of statics satisfied

Method Force	Moment Equilibrium	Force equilibrium
Ordinary or Fellenius	yes	No
Bishop’s simplified	yes	No
Janbu’s simplified	No	Yes
Janbu generalized	Yes	Yes
Bishop’s Rigorous	yes	yes
Spencer	Yes	Yes
Morgenstern-Price	Yes	Yes

- Interslice force characteristics and relationships

Method	Interslice Normal (E)	Interslice shear(X)	Inclination of X/E resultant and X-E relationships
Ordinary or Fellenius	No	No	Interslice forces are neglected.
Bishop’s simplified	Yes	No	X ignored, E considered
Janbu’s simplified	Yes	No	X ignored, E considered
Janbu generalized	Yes	Yes	Location of the interslice normal force is defined by an assumed line of thrust.
Bishop’s Rigorous	yes	yes	Interslice shear force distribution is assumed.
Spencer	Yes	Yes	Resultant interslice forces are of constant slope throughout the sliding mass.
Morgenstern-Price	Yes	Yes	Direction of the resultant interslice forces is determined using an arbitrary function.

• Ordinary method of slices

- This method is also referred to as "Fellenius' Method" and the "Swedish Circle Method"; it is the simplest method of slices to use.

- In this method, all the interslice forces are ignored.

- The factor of safety is the total available shear strength along the slip surface divided by the summation of the gravitational driving force (moblized shear)

- Only moment equilibrium is satisfied.

- The following procedure may be used for ordinary method of slices
- **A circular arc failure is assumed.**
- **A possible slip circle is drawn and a sector of unit thickness is divided into a number of vertical parallel slices.**
- **The weight of each slice is represented by a vector drawn vertically through the center of gravity of the area.**
- **This vector is then resolved into components normal and tangential to the sliding surface.**
- **The normal component passes through the assumed center of rotation of sliding mass and hence does not contribute to the tendency of slice to move.**

- On the other hand, the tangential component of the weight does affect stability as it tends to cause movement along the sliding surface.

- Tangential components of the weight of slice to the right to center of rotation constitute driving force, but those to the left provide resistance.

- In the slices method it is considered that both the resisting and driving forces act along the failure surface.

- The moment of the actuating and resisting forces about the point of rotation may be written as

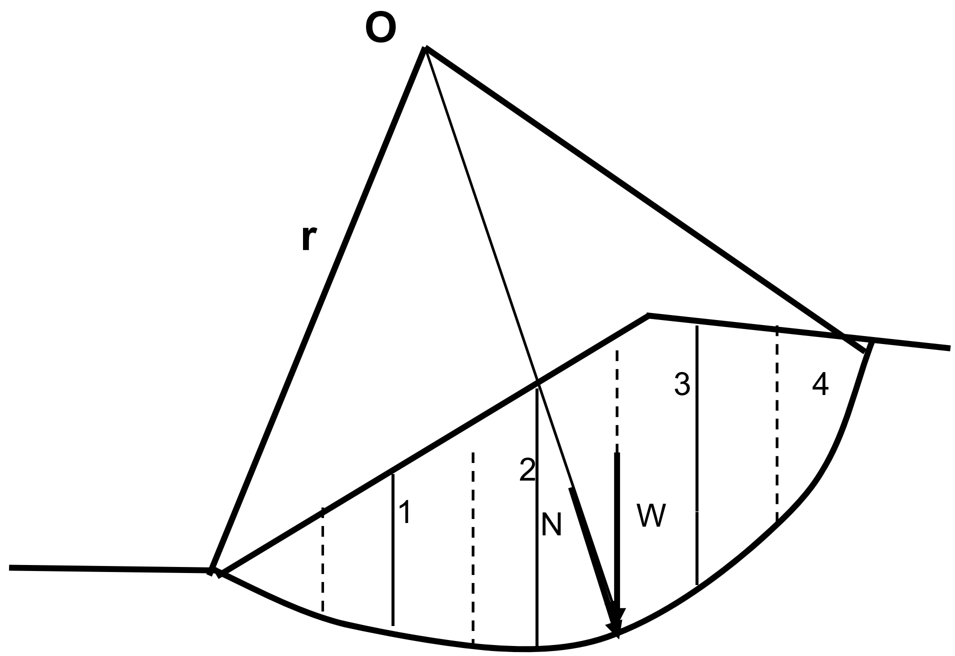
- **Actuating moment = r $\sum(T_D - T_R)$**
- **Resisting moment = r($\sum C\Delta L + \sum N\tan\phi$)**
- **The factor of safety Fs may now be written as:**

$$F_s = \frac{r\sum C\Delta L + \sum N\tan\phi}{r(\sum T_D - \sum T_R)}$$

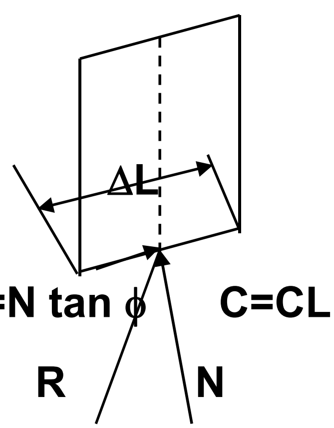
$$= \frac{\sum C\Delta L + \sum N\tan\phi}{\sum T_D - \sum T_R}$$

- **If C and ϕ are constant,**

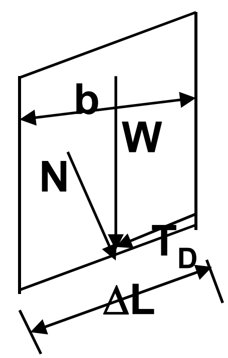
$$F_s = \frac{CL_a + \tan\phi \sum N}{\sum T_D - \sum T_R}$$



Resisting moment



Driving Force



- The procedure can be summarized as under:

1. Take a trial wedge and divide it into 6 to 12 vertical slices.
2. Determine the weight of each slice and its line of action.

(For convenience, the weight is generally taken proportional to the middle ordinate of the slice and it is assumed to have line of its action through the middle of the slice).

3. The weight is resolved (analytically or graphically) into normal and tangential components.
4. The curved length ΔL of each slice is measured or computed.
5. Determine the factor of safety.

The stability analysis is repeated for a number of trial surfaces. The circle which gives the minimum factor of safety is the most critical circle.

Slope Stability

This analysis is very conducive to a tabular solution

1	2	3	4	5	6	7	8	9	10
Wedge	c	ϕ	α	L	W	$W \sin \alpha$	$c L$	$W \cos \alpha \tan\phi$	$8 + 9$

$$F_s = \Sigma\,(10) \,/\, \Sigma\,(7)$$

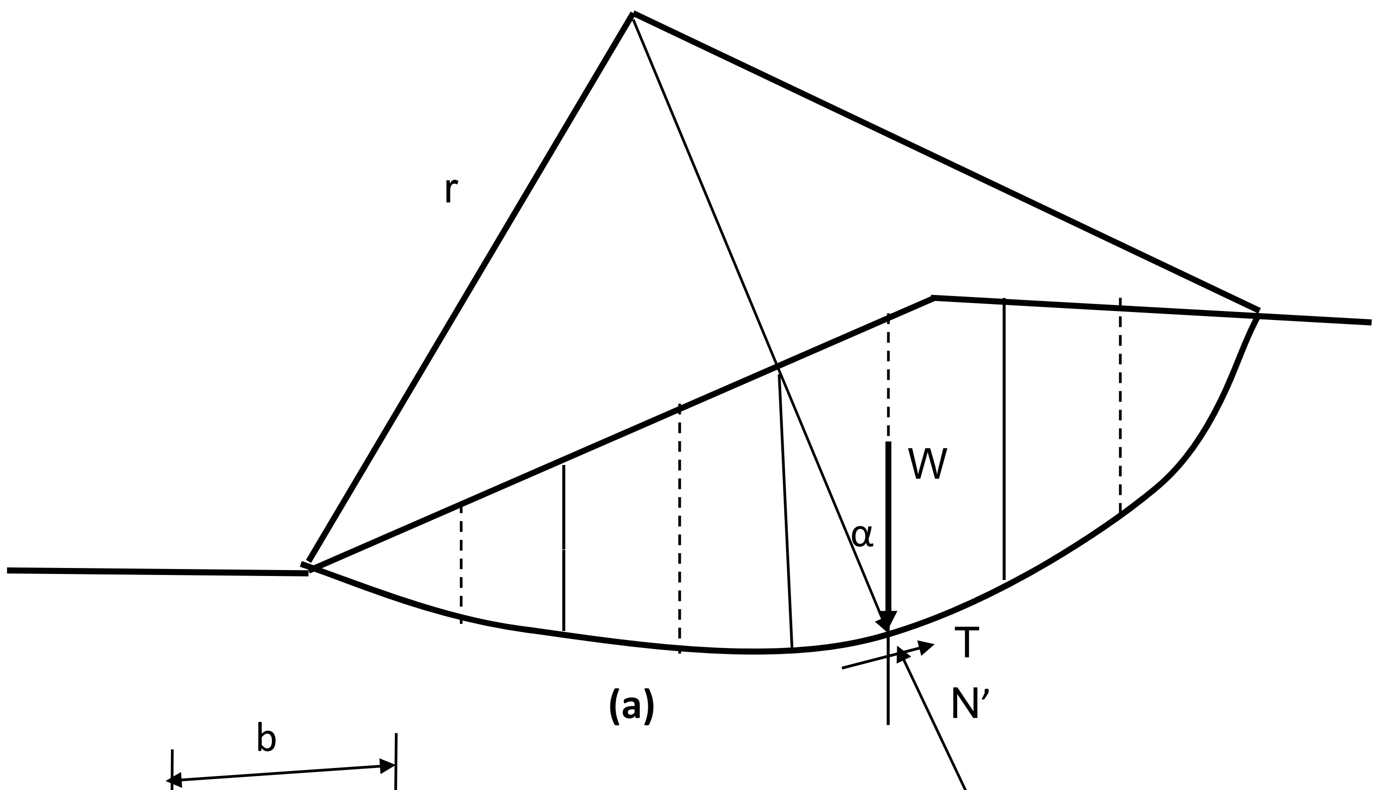
• Bishop's Simplified Method

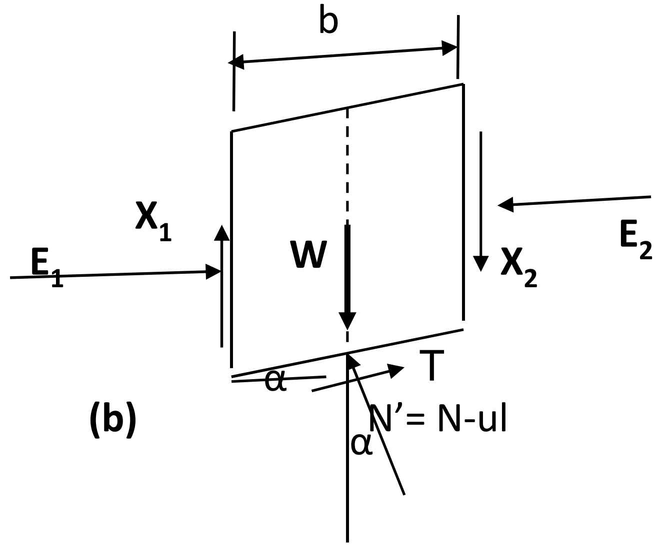
- Bishop (1955) gave a simplified method of analysis which considers the forces on the sides of each slice.
- In this method it is assumed that the tangential inter-slice forces are equal and opposite, I.e.,
- $X_1 = X_2$
- But the normal inter-slice forces are not equal, $E_1 \neq E_2$.

- However, resolving forces in the vertical direction gives

$$W = N' \cos\alpha + ul\cos\alpha + \frac{C'l}{F_s}\sin\alpha + \frac{N'}{F_s}\tan\phi'\sin\alpha$$

- Therefore, $$N' = \frac{W - ul\cos\alpha - \frac{C'l}{F_s}\sin\alpha}{\cos\alpha + \frac{\tan\phi'}{F_s}\sin\alpha}$$





l = b secα
N’ = N-ul

- Taking moments about the center of the circle derives the factor of safety:

$$F_s = \frac{1}{\sum W \sin\alpha} \sum \left[\frac{\{C'b + (W - ub)\tan\emptyset'\}\sec\alpha}{1 + \frac{\tan\emptyset' \tan\alpha}{F_s}} \right]$$

- As the factor of safety appears on both sides of the expression a trial value for F_s must chosen on the right hand side to obtain a value of F_s on the left hand side. By successive iteration convergence on the true value of F_s is obtained.
- Although the simplified Bishop method does not satisfy complete static equilibrium, the procedure gives relatively accurate values for the factor of safety.
- Bishop (1955) showed that the Simplified Bishop method is more accurate than the Ordinary Method of Slices, especially for effective stress analysis with high pore water pressure.

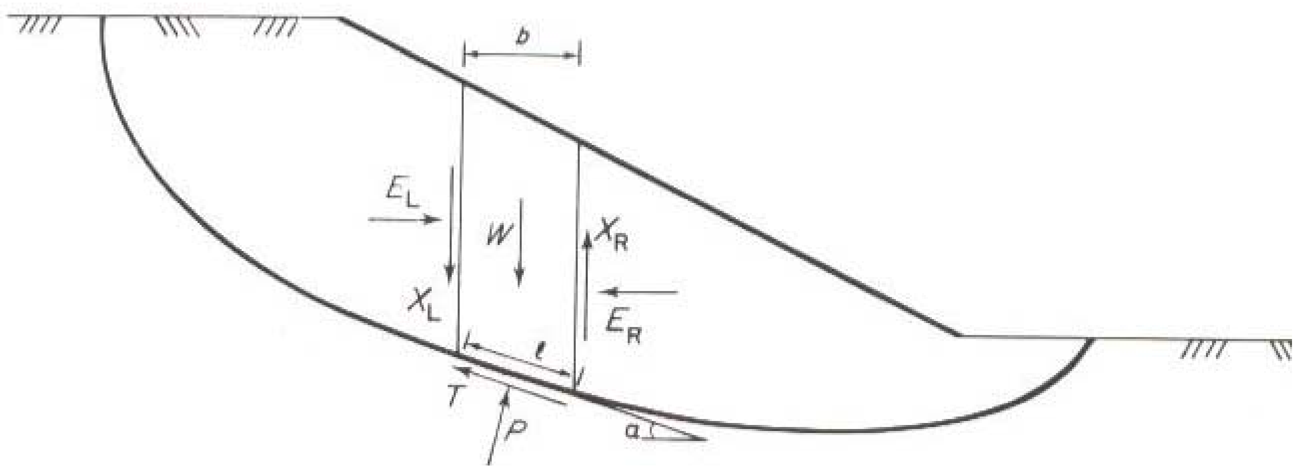
- **Pore pressure ratio, r_u**

$$r_u = \frac{u}{\gamma h}$$

$$F_S = \frac{1}{\sum W sin\alpha} \sum \left[\frac{\{C'b + W(1 - r_u) tan\emptyset'\} sec\alpha}{1 + \frac{\tan\emptyset' \tan\alpha}{F_s}} \right]$$

• Janbu's Simplified Method

- The Janbu's simplified method (1956) is applicable to non-circular slip surfaces as shown in figure below.
- In this method, the interslice forces are assumed to be horizontal and thus the shear forces are zero. Therefore, the expression obtained for the total normal force on the base of each slice is the same as that obtained by Bishop's method



- By examining overall horizontal force equilibrium, a value of the factor of safety F_0 is obtained:

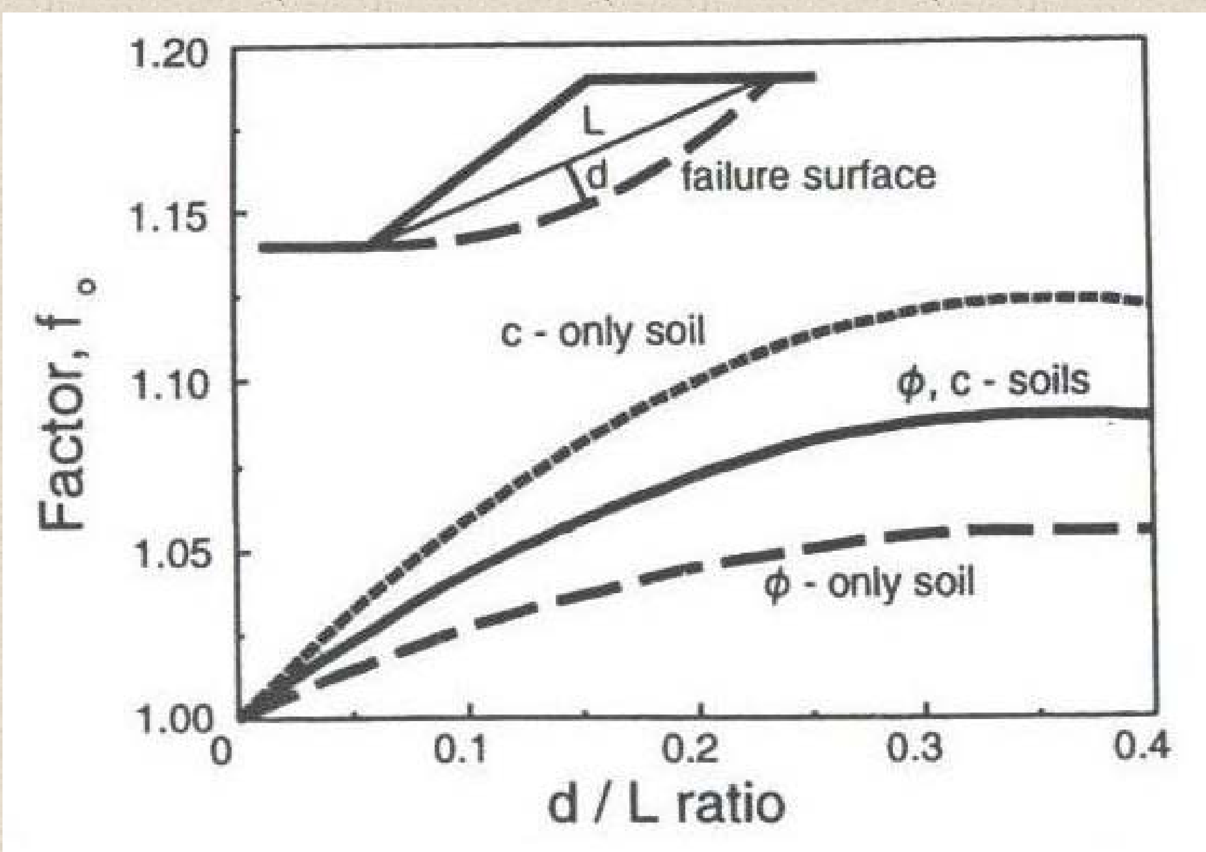
$$F_o = \frac{\sum (C'b + (N - ub)\tan\phi')m_\alpha}{\sum W \tan\alpha}$$

$$where \quad m_\alpha = \frac{\sec^2\alpha}{1 + \dfrac{\tan\phi'\tan\alpha}{F_o}}$$

- To take account of the interslice shear forces, Janbu proposed the correction factor $\mathbf{f_0}$ shown in Figure below:

$$F = f_o . F_o$$

- This correction factor is a function of the slide geometry and the strength parameters of the soil. The correction factor was presented by Janbu based on a number of slope stability computations using both the simplified and rigorous methods for the same slopes



- For convenience, this correction factor can also be calculated according to the following formula

$$f_o = 1 + b_1\left[\frac{d}{L} - 1.4(d/L)^2\right]$$

- where b_1 varies according to the soil types
  - » ϕ =0 soils: b_1 = 0.69
  - » C= 0 soils: b_1 = 0.31
  - » C> 0, ϕ > 0 soils: b_1 = 0.5

• Spencer's Method

- Spencer's method was presented originally for the analysis of circular slip surfaces, but it is easily extended to non-circular slip surfaces by adopting a frictional center of rotation.
- This method based on the assumption that the interslice forces are parallel so they have the same inclination:

$$\tan\theta = X_L / E_L = X_R / E_R$$

- where θ is the angle of resultant interslice force from the horizontal.
- Spencer summed forces perpendicular to the interslice forces to derive the normal force on the base of the slice:

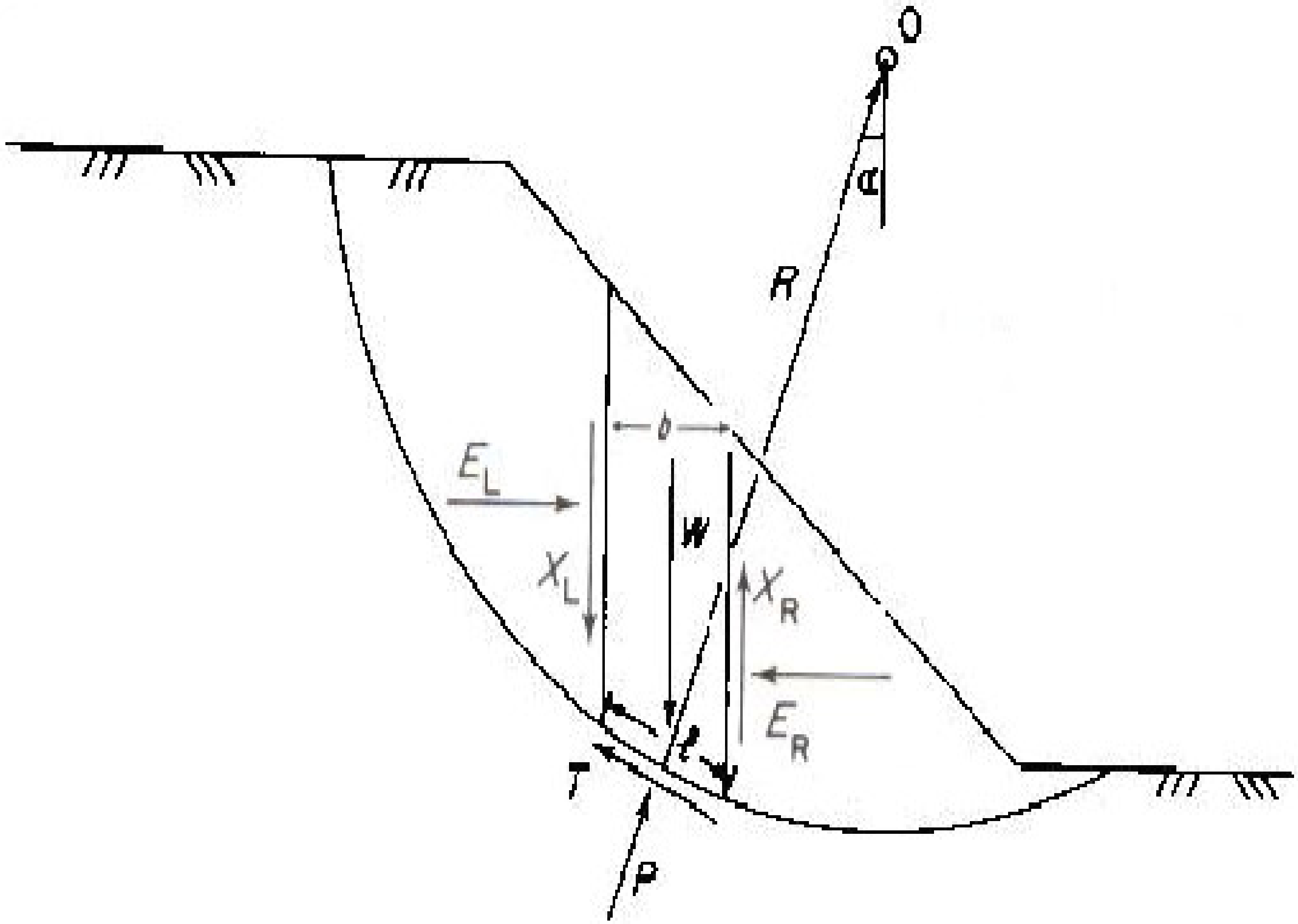
$$P = \frac{\left[W - (ER - EL)\tan\theta - \frac{1}{F} + (c'l\sin\alpha - ul\tan\phi'\sin\alpha \right]}{\cos\alpha(+\tan\alpha\frac{\tan\phi'}{F})}$$

- By considering overall force equilibrium and overall moment, two values of factor of safety Ff and Fm are obtained.
- The factor of safety (F_m) can be derived based on the overall moment equilibrium about a common point (O):

$$\sum WR\sin\alpha = \sum TR$$

$$T = \frac{1}{F}(C'l + (P - ul)\tan\phi')$$

$$F_m = \frac{\sum\left(C'l + (P - ul)\tan\phi'\right)}{\sum W\sin\alpha}$$



O
α
R
b
E_L
W
X_L
X_R
E_R
ℓ
T
P

- The factor of safety (Ff) can be derived based on the overall force equilibrium:

$$\sum F_H = 0 \Rightarrow T\cos\alpha - P\sin\alpha + E_R - E_L = 0$$

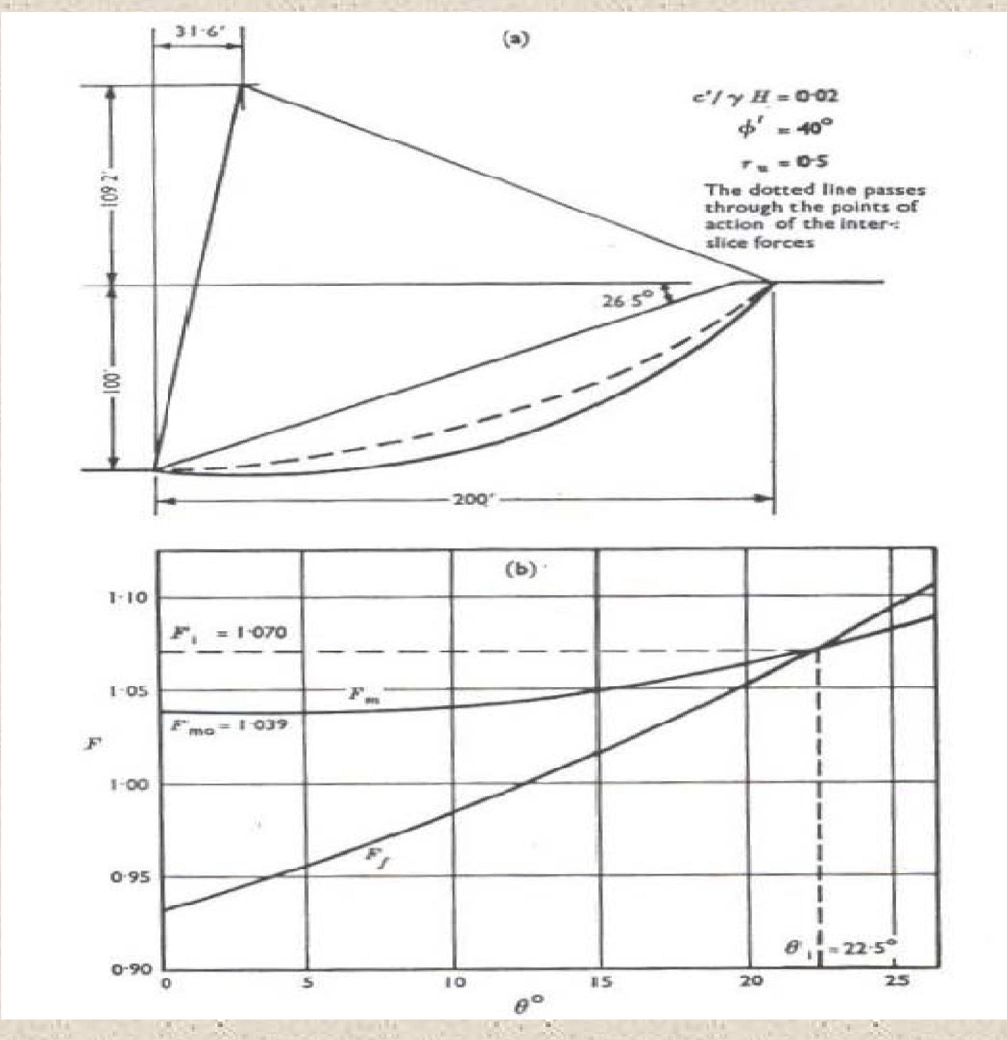
- By rearranging and substituting for T:

$$\sum(E_R - E_L) = \sum P\sin\alpha - \frac{1}{F_f}\sum\left[C'l + (P - ul)\tan\phi'\right]\cos\alpha$$

- Using Spencer's assumption, $\tan\theta$ =E/X constant throughout the slope, and $\sum(X_R - X_L) = 0$ *in absence of surface loading:*

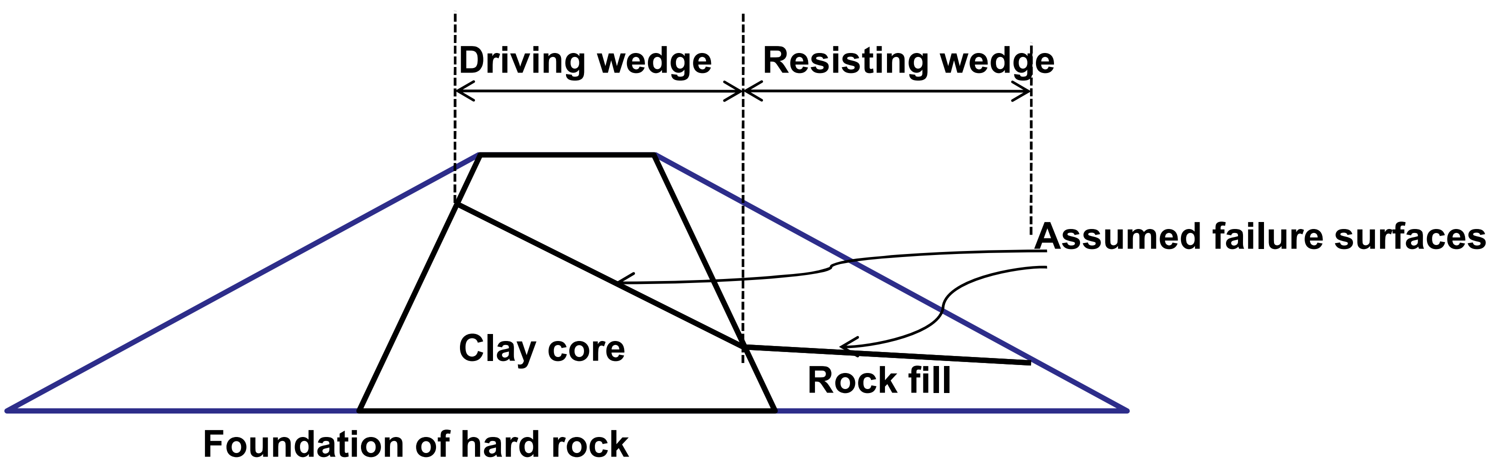
$$F_f = \frac{\sum(C'l + (P - ul)\tan\phi')\sec\alpha}{\sum(W - (X_R - X_L))\tan\alpha}$$

- Trial and error procedure is used to solve the equation. At some angle of the interslice forces, the two factor of safety are equal and both moment and force equilibrium are satisfied.



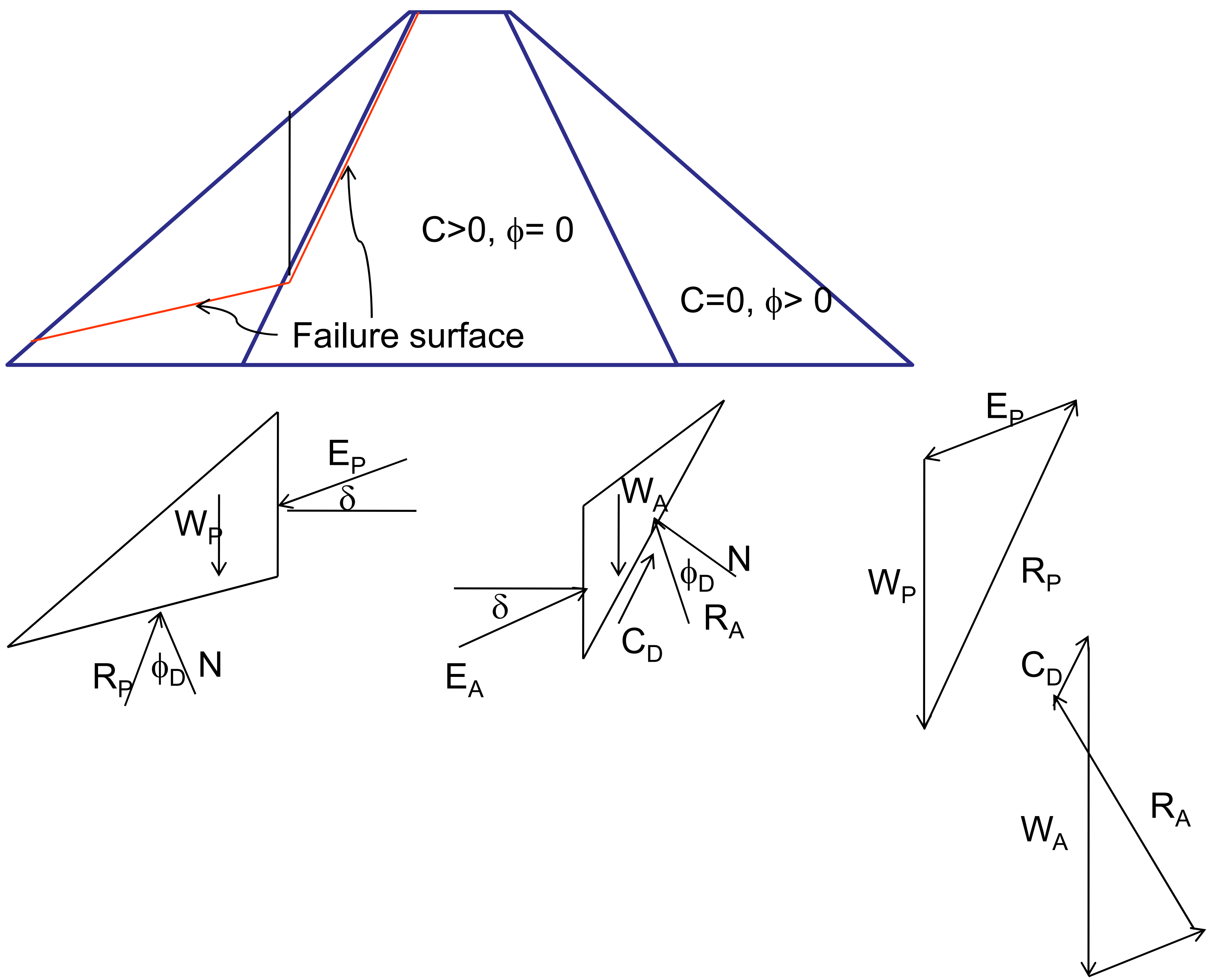
• Wedge (Sliding Block) Method

- This method is similar to the slices method of stability analysis except that only two or three slices, called blocks or wedges, are used and the surface of sliding consists of two or three planer surfaces.
- The upper wedge is called the driving or the active wedge, and the lower wedge is called the resisting or the passive wedge.
- Where there are three wedges, the middle one is called the neutral block.



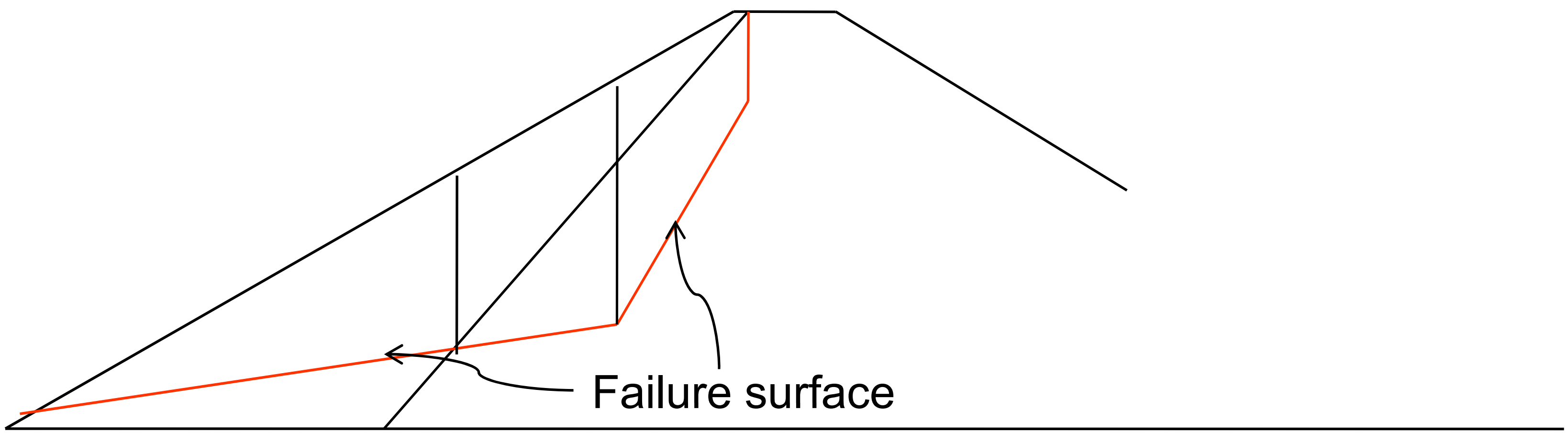
• Two Wedge method

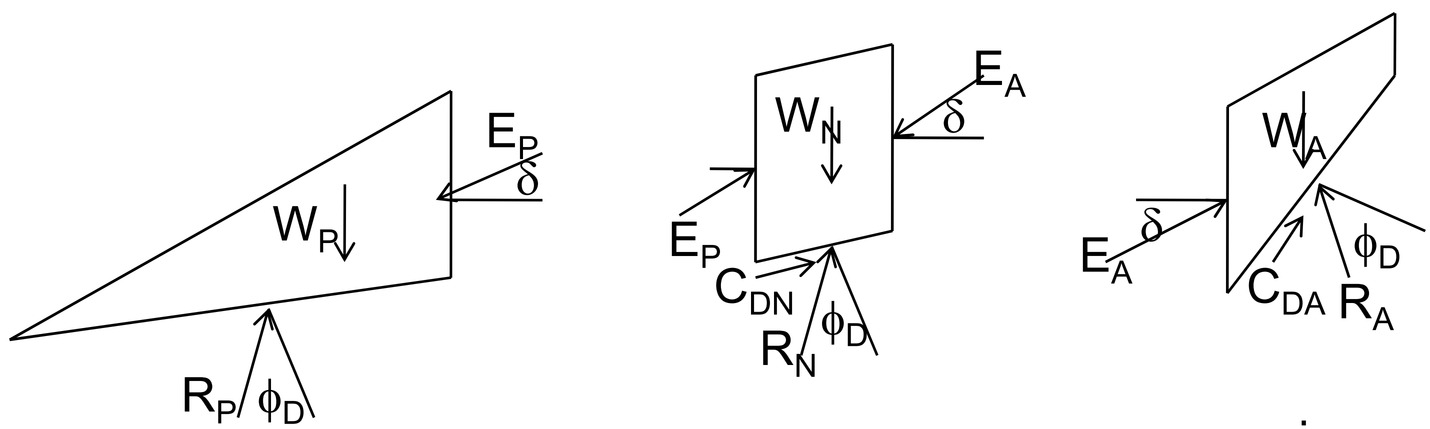
- The two wedge method briefly consists in computing the inter-slice forces acting on the surface between the active and passive wedges by considering their equilibrium individually under the developed sliding and resisting forces.
- For static equilibrium of the active and passive wedges, the inclined inter-slice forces obtained for the two wedges should be equal and opposite.
- Computations are done with assumed values of factor of safety and the value which gives equal values of the inter-slice forces is taken as the desired value of factor of safety.
  - » $\tan\phi_D = \tan\phi / FS$
  - » $C_D = C/FS$
  - » $\delta = 1/2\ \phi_D$ to $2/3\ \phi_D$



C>0, ϕ= 0
C=0, ϕ> 0
Failure surface
E_P
δ
W_P
R_P
ϕ_D
N
W_A
N
ϕ_D
R_A
C_D
δ
E_A
E_P
W_P
R_P
C_D
W_A
R_A

- Three wedge method





Passive wedge

Neutral wedge

passive wedge

• IMPROVING STABILITY OF SLOPES

- The slopes which are susceptible to failure by sliding can be improved and made usable and safe.
- Various methods are used to stabilize the slopes.
- The methods generally involve one or more of the following measures which either reduce the mass which may cause sliding or improve the shear strength of the soil in the failure zone.

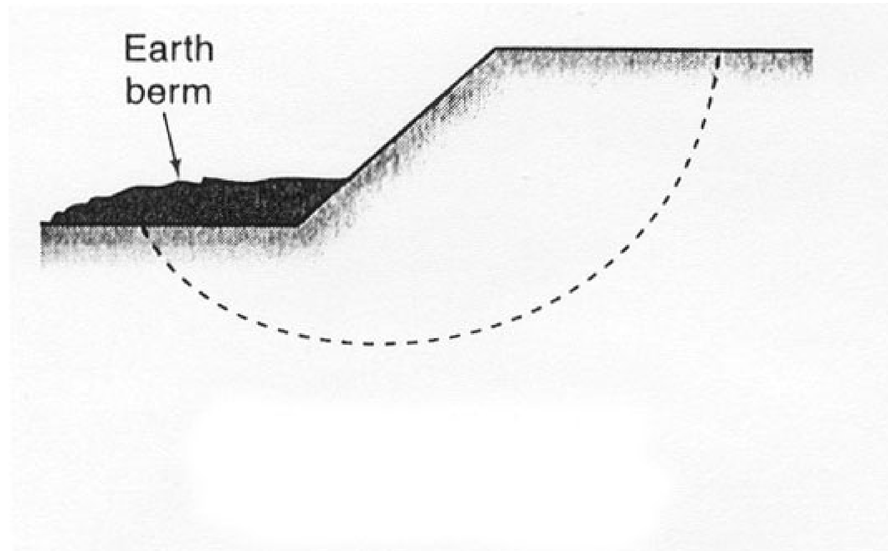
1. **Slope flattening reduces the weight of the mass tending to slide. It can be used wherever possible.**
2. **Providing a berm below t he toe of the slope increases the resistance to movement. It is especially useful when there is a possibility of a base failure.**
3. **Drainage helps in reducing the seepage forces and hence increases the stability. The zone of subsurface water is lowered and infiltration of the surface water is prevented**

4. Densification by use of different techniques helps in increasing the shear strength of cohesionless soils and thus increasing the stability.
5. Consolidation by surcharging or other method helps in increasing the stability of slopes in cohesive soils.
6. Grouting and injection of cement or other compounds into specific zone help in increasing the stability of slopes.
7. Sheet piles and retaining walls can be installed to provide lateral support and to increase the stability. However, the method is quite expensive.
8. Stabilization of the soil helps in increasing the stability of the slopes.

- In the interest of economy, relatively inexpensive methods, such as slope flattening and drainage control are generally preferred.

Vegetation Work

Removal Soil Work



Counterweight Fill Work

Avoid increasing PWP
Draw down groundwater table

Surface Drainage Work

Horizontal Drainage Work

RC Retaining Wall

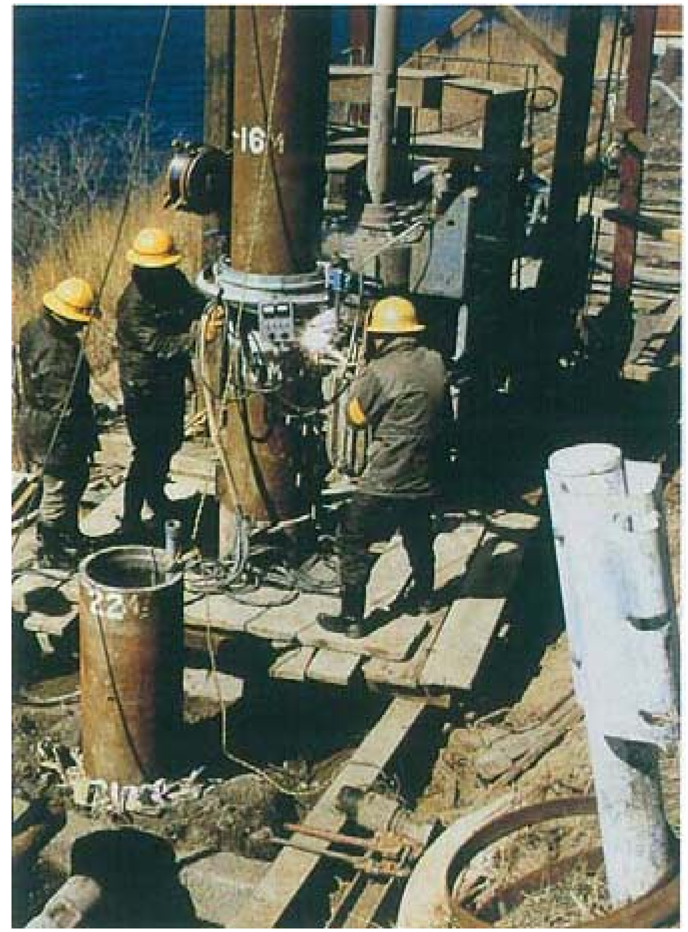
Retaining Wall+ Rock Fence

Gabion Work

Rock fall protection fence

Ground Anchor Work

Rock Bolt Work + Frame Work



Pile Work