PRINCIPLES OF FOUNDATION ENGINEERING, 4th ED.
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CHAPTER EIGHT

SHEET PILE STRUCTURES

8.1 INTRODUCTION

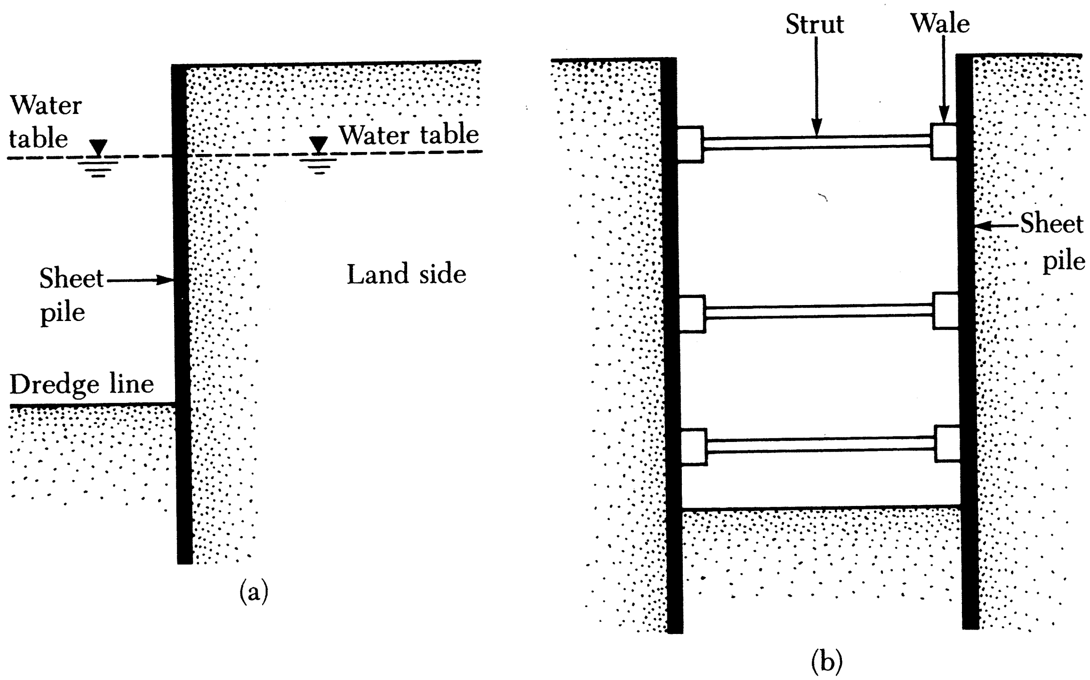
Connected or semiconnected sheet piles are often used to build continuous walls for waterfront structures that may range from small waterfront pleasure boat launching facilities to large dock facilities (Figure 8.1a). In contrast to the construction of other types of retaining wall, the building of sheet pile walls does not usually require dewatering of the site. Sheet piles are also used for some temporary structures, such as braced cuts (Figure 8.1b). The principles of sheet pile wall design and the design used in braced cuts are discussed in this chapter.

Several types of sheet pile are commonly used in construction: (a) wooden sheet piles, (b) precast concrete sheet piles, and (c) steel sheet piles. Aluminum sheet piles are also marketed.

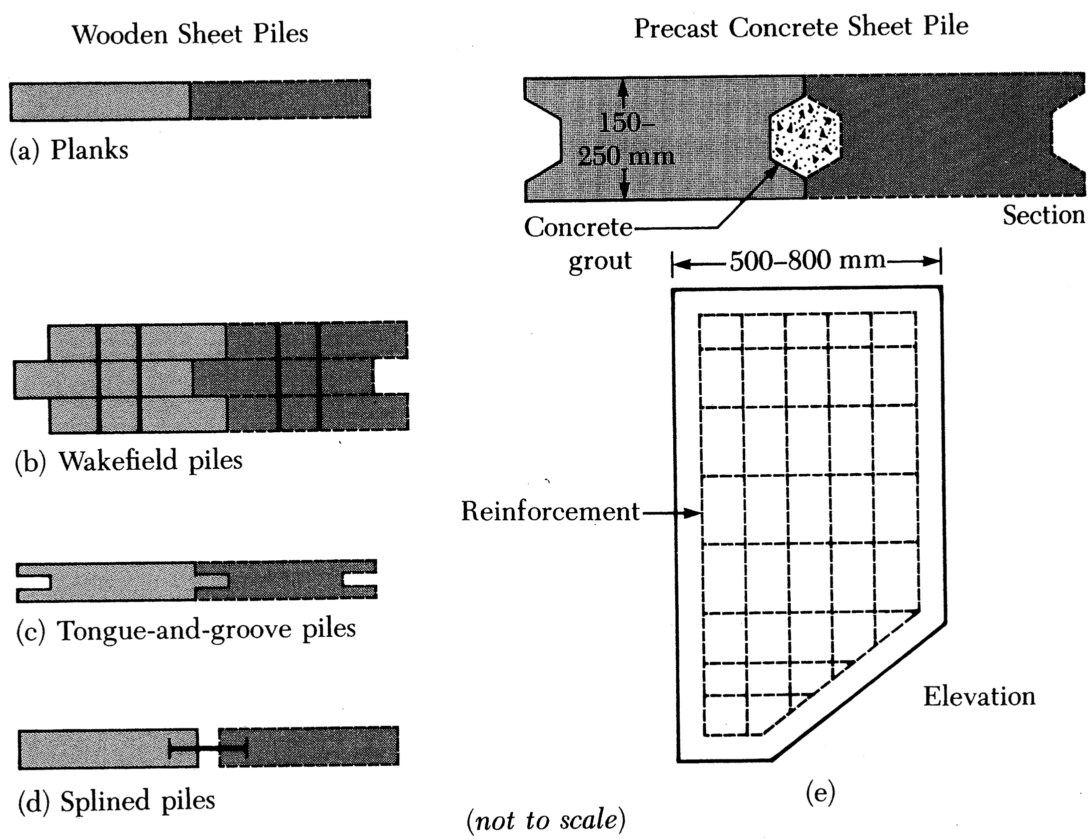
Wooden sheet piles are used only for temporary light structures that are above the water table. The most common types are ordinary wooden planks and *Wakefield piles.* The wooden planks are about 2 in. × 12 in. (50 mm × 300 mm) in cross section and are driven edge to edge (Figure 8.2a). Wakefield piles are made by nailing three planks together with the middle plank offset by 2–3 in. (50–75 mm) (Figure 8.2b). Wooden planks can also be milled to form *tongue-and-groove piles,* as shown in Figure 8.2c. Figure 8.2d shows another type of wooden sheet pile that has precut grooves. Metal *splines* are driven into the grooves of the adjacent sheetings to hold them together after they are driven into the ground.

Precast concrete sheet piles are heavy and are designed with reinforcements to withstand the permanent stresses to which the structure will be subjected after construction and also to handle the stresses produced during construction. In cross section, these piles are about 20–32 in. (500–800 mm) wide and 6–10 in. (150–250 mm) thick. Figure 8.2e shows schematic diagrams of the elevation and the cross section of a reinforced concrete sheet pile.

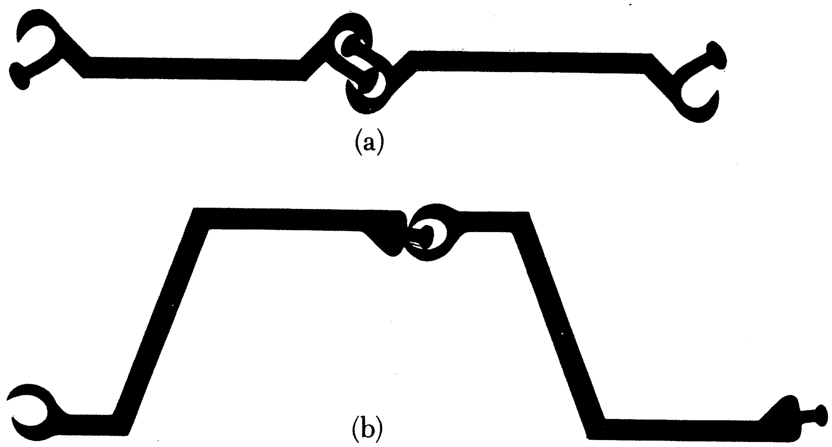
Steel sheet piles in the United States are about 0.4–0.5 in. (10–13 mm) thick. European sections may be thinner and wider. Sheet pile sections may be *Z, deep arch, low arch,* or *straight web* sections. The interlocks of the sheet pile sections are shaped like a *thumb-and-finger* or a *ball-and-socket* for watertight connections. Figure 8.3a shows schematic diagrams of the thumb-and-finger type of interlocking for straight web sections. The ball-and-socket type of interlocking for *Z* section piles is



▼ **FIGURE 8.1** Examples of uses of sheet piles: (a) waterfront sheet pile wall; (b) braced cut



▼ **FIGURE 8.2** Various types of wooden and concrete sheet pile



▼ **FIGURE 8.3** Nature of sheet pile connections: (a) thumb-and-finger type; (b) ball-and-socket type

shown in Figure 8.3b. Table C-1 (Appendix C) shows the properties of the sheet pile sections produced by the Bethlehem Steel Corporation. The allowable design flexural stress for the steel sheet piles is as follows:

Type of steel	Allowable stress (lb/in^2)
ASTM A-328	25,000 lb/in^2 (170 MN/m^2)
ASTM A-572	30,000 lb/in^2 (210 MN/m^2)
ASTM A-690	30,000 lb/in^2 (210 MN/m^2)

Steel sheet piles are convenient to use because of their resistance to high driving stress developed when being driven into hard soils. They are also lightweight and reusable.

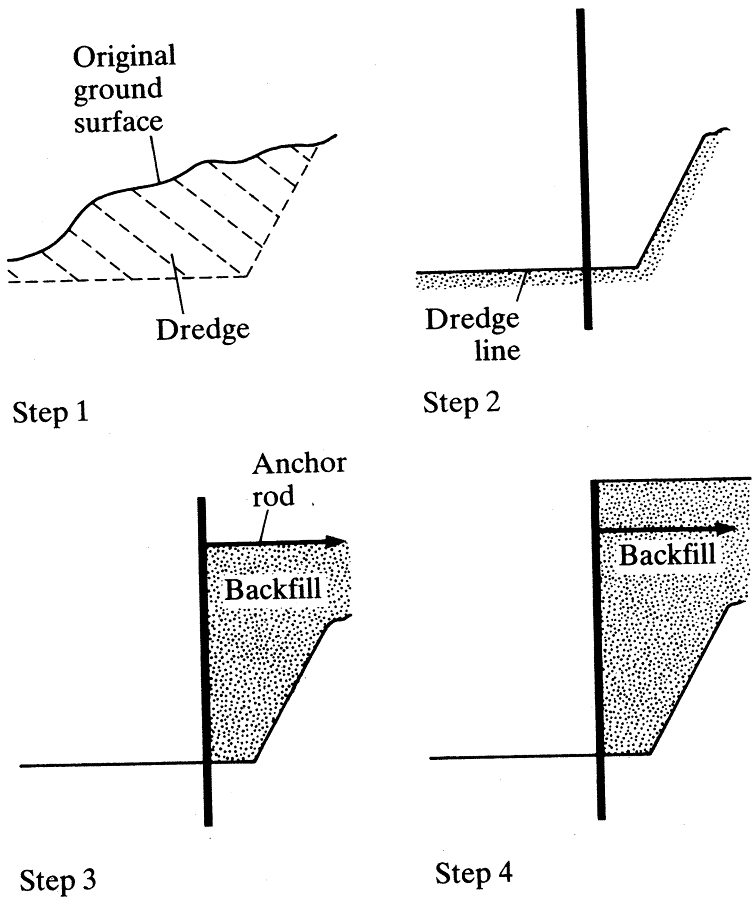
SHEET PILE WALLS

8.2 CONSTRUCTION METHODS

Sheet pile walls may be divided into two basic categories: (a) cantilever and (b) anchored.

In the construction of sheet pile walls, sheet piles may be driven into the ground and then the backfill is placed on the land side, or the sheet pile may first be driven into the ground and the soil in front of the sheet pile dredged. In any case, the soil used for backfill behind the sheet pile wall is usually granular. The soil below the dredge line may be sandy or clayey soil. The surface of soil on the water side is referred to as the *mud line* or *dredge line.*

Thus construction methods generally can be divided into two categories (Tsinker, 1983):



▼ **FIGURE 8.4** Sequence of construction for a backfilled structure

1. Backfilled structure
2. Dredged structure

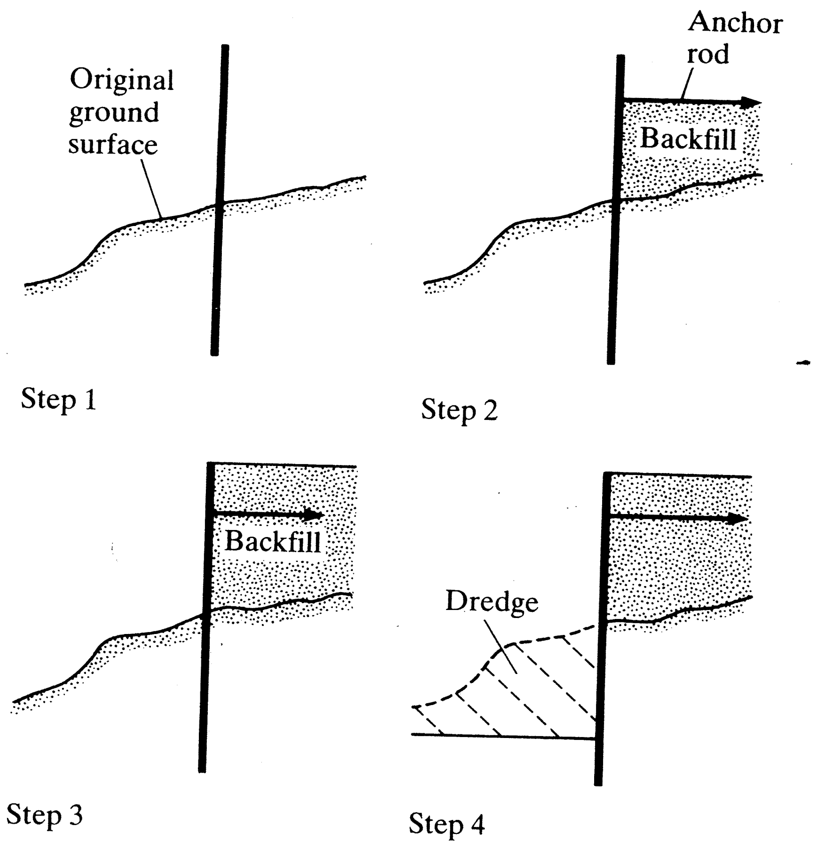
The sequence of construction for a *backfilled structure* is as follows (Figure 8.4):

Step 1. Dredge the *in situ* soil in front and back of the proposed structure.
Step 2. Drive the sheet piles.
Step 3. Backfill up to the level of the anchor and place the anchor system.
Step 4. Backfill up to the top of the wall.

For a cantilever type of wall, only Steps 1, 2, and 4 apply. The sequence of construction for a *dredged structure* is as follows (Figure 8.5):

Step 1. Drive the sheet piles.
Step 2. Backfill up to the anchor level and place the anchor system.
Step 3. Backfill up to the top of the wall.
Step 4. Dredge the front side of the wall.

For cantilever sheet pile walls, Step 2 is not required.

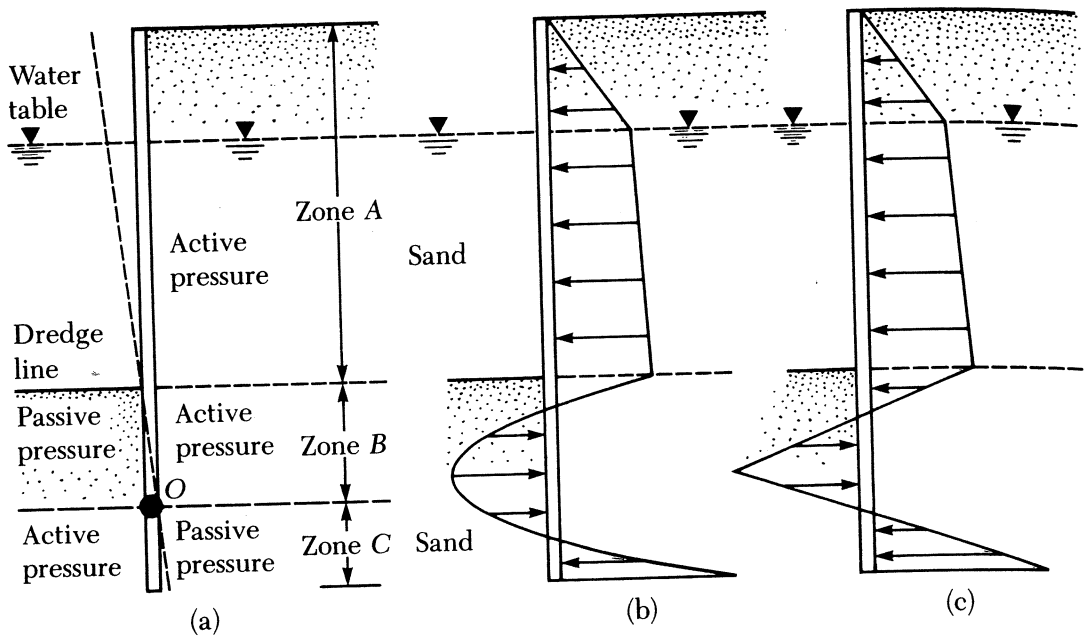


▼ **FIGURE 8.5** Sequence of construction for a dredged structure

8.3 CANTILEVER SHEET PILE WALLS—GENERAL

Cantilever sheet pile walls are usually recommended for walls of moderate height—about 20 ft ($\approx$6 m) or less, measured above the dredge line. In such walls, the sheet piles act as a wide cantilever beam above the dredge line. The basic principles for estimating net lateral pressure distribution on a cantilever sheet pile wall can be explained with the aid of Figure 8.6. It shows the nature of lateral yielding of a cantilever wall penetrating a sand layer below the dredge line. The wall rotates about point O. Because the hydrostatic pressures at any depth from both sides of the wall will cancel each other, we consider only the effective lateral soil pressures. In zone A, the lateral pressure is only the active pressure from the land side. In zone B, because of the nature of yielding of the wall, there will be active pressure from the land side and passive pressure from the water side. The condition is reversed in zone C— that is, below the point of rotation, O. The net actual pressure distribution on the wall is like that shown in Figure 8.6b. However, for design purposes, Figure 8.6c shows a simplified version.

Sections 8.4–8.7 present the mathematical formulation of the analysis of cantilever sheet pile walls. Note that, in some waterfront structures, the water level may



▼ **FIGURE 8.6** Cantilever sheet pile penetrating sand

fluctuate as the result of tidal effects. Care should be taken in determining the water level that will affect the net pressure diagram.

8.4 CANTILEVER SHEET PILING PENETRATING SANDY SOILS

To develop the relationships for the proper depth of embedment of sheet piles driven into a granular soil, we refer to Figure 8.7a. The soil retained by the sheet piling above the dredge line also is sand. The water table is at depth L_1 below the top of the wall. Let the angle of friction of the sand be ϕ. The intensity of the active pressure at a depth $z = L_1$ is

$$p_1 = \gamma L_1 K_a \tag{8.1}$$

where K_a = Rankine active pressure coefficient = $\tan^2(45 - \phi/2)$
γ = unit weight of soil above the water table

Similarly, the active pressure at depth $z = L_1 + L_2$ (that is, at the level of the dredge line) is

$$p_2 = (\gamma L_1 + \gamma' L_2)K_a \tag{8.2}$$

where γ' = effective unit weight of soil = $\gamma_{sat} - \gamma_w$

Note that, at the level of the dredge line, the hydrostatic pressures from both sides of the wall are the same magnitude and cancel each other.

To determine the net lateral pressure below the dredge line up to the point of rotation O, as shown in Figure 8.6a, an engineer has to consider the passive pressure acting from the left side (water side) toward the right side (land side) and also the

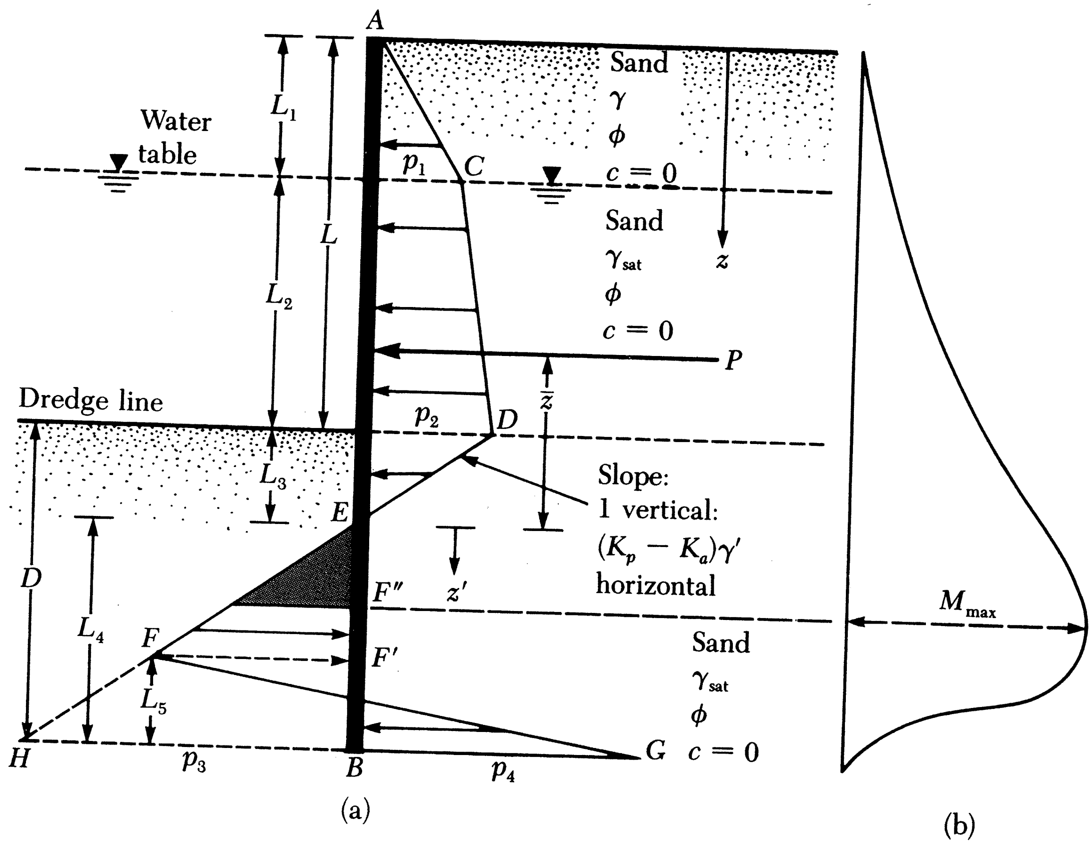


FIGURE 8.7 Cantilever sheet pile penetrating sand: (a) variation of net pressure diagram; (b) variation of moment

active pressure acting from the right side toward the left side of the wall. For such cases, ignoring the hydrostatic pressure from both sides of the wall, the active pressure at depth z is

$$p_a = [\gamma L_1 + \gamma' L_2 + \gamma'(z - L_1 - L_2)]K_a \tag{8.3}$$

Also, the passive pressure at depth z is

$$p_p = \gamma'(z - L_1 - L_2)K_p \tag{8.4}$$

where K_p = Rankine passive pressure coefficient = $\tan^2(45 + \phi/2)$

Hence, combining Eqs. (8.3) and (8.4) yields the net lateral pressure:

$$\begin{aligned} p = p_a - p_p &= (\gamma L_1 + \gamma' L_2)K_a - \gamma'(z - L_1 - L_2)(K_p - K_a) \\ &= p_2 - \gamma'(z - L)(K_p - K_a) \end{aligned} \tag{8.5}$$

where $L = L_1 + L_2$

The net pressure, p, equals zero at depth L_3 below the dredge line, so

$$p_2 - \gamma'(z - L)(K_p - K_a) = 0$$

or

$$(z - L) = L_3 = \frac{p_2}{\gamma'(K_p - K_a)} \tag{8.6}$$

Equation (8.6) indicates that the slope of the net pressure distribution line DEF is 1 vertical to $(K_p - K_a)\gamma'$ horizontal, so, in the pressure diagram

$$\overline{HB} = p_3 = L_4(K_p - K_a)\gamma' \tag{8.7}$$

At the bottom of the sheet pile, passive pressure, p_p, acts from the right toward the left side and active pressure acts from the left toward the right side of the sheet pile, so, at $z = L + D$,

$$p_p = (\gamma L_1 + \gamma' L_2 + \gamma' D)K_p \tag{8.8}$$

At the same depth

$$p_a = \gamma' D K_a \tag{8.9}$$

Hence the net lateral pressure at the bottom of the sheet pile is

$$\begin{aligned} p_p - p_a = p_4 &= (\gamma L_1 + \gamma' L_2)K_p + \gamma' D(K_p - K_a) \\ &= (\gamma L_1 + \gamma' L_2)K_p + \gamma' L_3(K_p - K_a) + \gamma' L_4(K_p - K_a) \\ &= p_5 + \gamma' L_4(K_p - K_a) \end{aligned} \tag{8.10}$$

where $p_5 = (\gamma L_1 + \gamma' L_2)K_p + \gamma' L_3(K_p - K_a)$ (8.11)

$D = L_3 + L_4$ (8.12)

For the stability of the wall, the principles of statics can now be applied:

Σ horizontal forces per unit length of wall $= 0$

and

Σ moment of the forces per unit length of wall about point $B = 0$

For summation of the horizontal forces,

Area of the pressure diagram $ACDE$ $-$ area of $EFHB$ $+$ area of $FHBG$ $= 0$

or

$$P - \tfrac{1}{2}p_3L_4 + \tfrac{1}{2}L_5(p_3 + p_4) = 0 \tag{8.13}$$

where P = area of the pressure diagram $ACDE$

Summing the moment of all the forces about point B yields

$$P(L_4 + \bar{z}) - \left(\frac{1}{2}L_4p_3\right)\left(\frac{L_4}{3}\right) + \frac{1}{2}L_5(p_3 + p_4)\left(\frac{L_5}{3}\right) = 0 \tag{8.14}$$

From Eq. (8.13),

$$L_5 = \frac{p_3L_4 - 2P}{p_3 + p_4} \tag{8.15}$$

Combining Eqs. (8.7), (8.10), (8.14), and (8.15) and simplifying them further, we obtain the following fourth-degree equation in terms of L_4:

$$L_4^4 + A_1L_4^3 - A_2L_4^2 - A_3L_4 - A_4 = 0 \tag{8.16}$$

where

$$A_1 = \frac{p_5}{\gamma'(K_p - K_a)} \tag{8.17}$$

$$A_2 = \frac{8P}{\gamma'(K_p - K_a)} \tag{8.18}$$

$$A_3 = \frac{6P[2\bar{z}\gamma'(K_p - K_a) + p_5]}{\gamma'^2(K_p - K_a)^2} \tag{8.19}$$

$$A_4 = \frac{P(6\bar{z}p_5 + 4P)}{\gamma'^2(K_p - K_a)^2} \tag{8.20}$$

Step-by-Step Procedure for Obtaining the Pressure Diagram

Based on the preceding theory, the step-by-step procedure for obtaining the pressure diagram for a cantilever sheet pile wall penetrating a granular soil is as follows:

1. Calculate K_a and K_p.
2. Calculate p_1 [Eq. (8.1)] and p_2 [Eq. (8.2)]. *Note:* L_1 and L_2 will be given.
3. Calculate L_3 [Eq. (8.6)].
4. Calculate P.
5. Calculate $\bar{z}$ (that is, the center of pressure for the area *ACDE*) by taking the moment about *E*.
6. Calculate p_5 [Eq. (8.11)].
7. Calculate A_1, A_2, A_3, and A_4 [Eqs. (8.17) to (8.20)].
8. Solve Eq. (8.16) by trial and error to determine L_4.
9. Calculate p_4 [Eq. (8.10)].
10. Calculate p_3 [Eq. (8.7)].
11. Obtain L_5 from Eq. (8.15).
12. Draw the pressure distribution diagram like the one shown in Figure 8.7a.
13. Obtain the theoretical depth [Eq. (8.12)] of penetration as $L_3 + L_4$. The actual depth of penetration is increased by about 20%–30%.

Note: Some designers prefer to use a factor of safety on the passive earth pressure coefficient at the beginning. In that case, in Step 1

$$K_{p(\text{design})} = \frac{K_p}{FS}$$

where FS = factor of safety (usually between 1.5 to 2)

For this type of analysis, follow Steps 1–12 with the value of $K_a = \tan^2(45 - \phi/2)$ and $K_{p(\text{design})}$ (instead of K_p). The actual depth of penetration can now be determined by adding L_3, obtained from Step 3, and L_4, obtained from Step 8.

Calculation of Maximum Bending Moment

The nature of variation of the moment diagram for a cantilever sheet pile wall is shown in Figure 8.7b. The maximum moment will occur between points E and F'. To obtain the maximum moment ($M_{\max}$) per unit length of the wall requires determining the point of zero shear. For a new axis z' (with origin at point E) for zero shear,

$$P = \tfrac{1}{2}(z')^2(K_p - K_a)\gamma'$$

or

$$z' = \sqrt{\frac{2P}{(K_p - K_a)\gamma'}} \qquad (8.21)$$

Once the point of zero shear force is determined (point F'' in Figure 8.7a), the magnitude of the maximum moment can be obtained as

$$M_{\max} = P(\bar{z} + z') - [\tfrac{1}{2}\gamma' z'^2(K_p - K_a)](\tfrac{1}{3})z' \qquad (8.22)$$

The necessary profile of the sheet piling is then sized according to the allowable flexural stress of the sheet pile material, or

$$S = \frac{M_{\max}}{\sigma_{\text{all}}} \qquad (8.23)$$

where S = section modulus of the sheet pile required per unit length of the structure

σ_{all} = allowable flexural stress of the sheet pile

▼ EXAMPLE 8.1

Figure 8.8 shows a cantilever sheet pile wall penetrating a granular soil. Here, $L_1 = 2$ m, $L_2 = 3$ m, $\gamma = 15.9$ kN/m^3, $\gamma_{\text{sat}} = 19.33$ kN/m^3, and $\phi = 32°$.

a. What is the theoretical depth of embedment, D?
b. For a 30% increase in D, what should be the total length of the sheet piles?
c. What should be the minimum section modulus of the sheet piles? Use $\sigma_{\text{all}} = 172$ MN/m^2.

Solution

Part a

The following table can now be prepared for a step-by-step calculation. Refer to Figure 8.7a for the pressure distribution diagram.

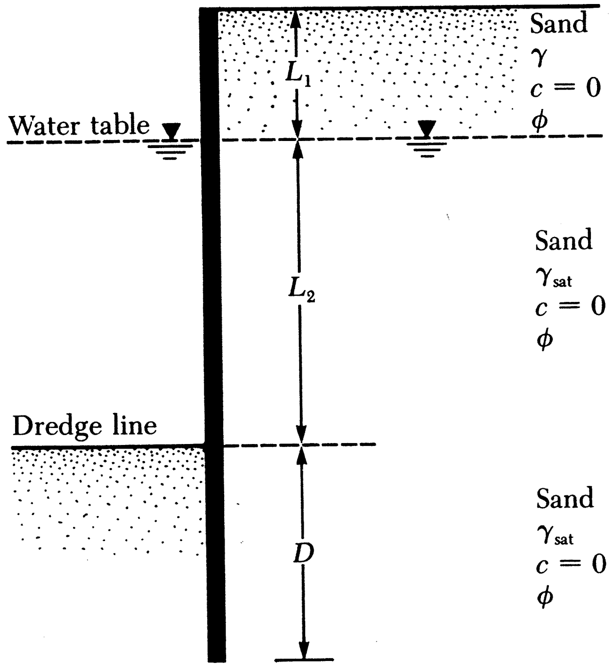
Quantity required	Eq. no.	Equation and calculation
K_a	—	$\tan^2\left(45-\frac{\phi}{2}\right)=\tan^2\left(45-\frac{32}{2}\right)=0.307$
K_p	—	$\tan^2\left(45+\frac{\phi}{2}\right)=\tan^2\left(45+\frac{32}{2}\right)=3.25$
p_1	8.1	$\gamma L_1 K_a = (15.9)(2)(0.307) = 9.763\ \text{kN/m}^2$
p_2	8.2	$(\gamma L_1 + \gamma' L_2)K_a = [(15.9)(2) + (19.33 - 9.81)(3)](0.307) = 18.53\ \text{kN/m}^2$
L_3	8.6	$\dfrac{p_2}{\gamma'(K_p-K_a)}=\dfrac{18.53}{(19.33-9.81)(3.25-0.307)}=0.66\ \text{m}$
P	—	$\frac{1}{2}p_1L_1+p_1L_2+\frac{1}{2}(p_2-p_1)L_2+\frac{1}{2}p_2L_3$ $=(\frac{1}{2})(9.763)(2)+(9.763)(3)+(\frac{1}{2})(18.53-9.763)(3)+(\frac{1}{2})(18.53)(0.66)$ $=9.763+29.289+13.151+6.115=58.32\ \text{kN/m}$
$\bar{z}$	—	$\dfrac{\Sigma M_E}{P}=\dfrac{1}{58.32}\left[\begin{array}{l}9.763(0.66+3+\frac{2}{3})+29.289(0.66+(\frac{3}{2}))\\+13.151(0.66+\frac{3}{3})+6.115(0.66\times\frac{2}{3})\end{array}\right]=2.23\ \text{m}$
p_5	8.11	$(\gamma L_1+\gamma' L_2)K_p+\gamma' L_3(K_p-K_a)=[(15.9)(2)+(19.33-9.81)(3)](3.25)$ $+(19.33-9.81)(0.66)(3.25-0.307)=214.66\ \text{kN/m}^2$
A_1	8.17	$\dfrac{p_5}{\gamma'(K_p-K_a)}=\dfrac{214.66}{(19.33-9.81)(3.25-0.307)}=7.66$
A_2	8.18	$\dfrac{8P}{\gamma'(K_p-K_a)}=\dfrac{(8)(58.32)}{(19.33-9.81)(3.25-0.307)}=16.65$
A_3	8.19	$\dfrac{6P[2\bar{z}\gamma'(K_p-K_a)+p_5]}{\gamma'^2(K_p-K_a)^2}$ $=\dfrac{(6)(58.32)[(2)(2.23)(19.33-9.81)(3.25-0.307)+214.66]}{(19.33-9.81)^2(3.25-0.307)^2}=151.93$
A_4	8.20	$\dfrac{P(6\bar{z}p_5+4P)}{\gamma'^2(K_p-K_a)^2}=\dfrac{58.32[(6)(2.23)(214.66)+(4)(58.32)]}{(19.33-9.81)^2(3.25-0.307)^2}=230.72$
L_4	8.16	$L_4^4+A_1L_4^3-A_2L_4^2-A_3L_4-A_4=0$ $L_4^4+7.66L_4^3-16.55L_4^2-151.93L_4-230.72=0;\quad L_4\approx 4.8\ \text{m}$

$$D_{\text{theory}} = L_3 + L_4 = 0.66 + 4.8 = \mathbf{5.46\ m}$$

Part b

Total length of sheet piles,

$$L_1 + L_2 + 1.3(L_3 + L_4) = 2 + 3 + 1.3(5.46) = \mathbf{12.1\ m}$$



▼ FIGURE 8.8

Part c

Quantity required	Eq. no.	Equation and calculation
z'	8.21	$\sqrt{\dfrac{2P}{(K_p - K_a)\gamma'}} = \sqrt{\dfrac{(2)(58.32)}{(3.25 - 0.307)(19.33 - 9.81)}} = 2.04\ \text{m}$
M_{max}	8.22	$P(\bar{z} + z') - \left[\frac{1}{2}\gamma' z'^2 (K_p - K_a)\right]\frac{z'}{3} = (58.32)(2.23 + 2.04) - \left[\left(\frac{1}{2}\right)(19.33 - 9.81)(2.04)^2(3.25 - 0.307)\right]\frac{2.04}{3} = 209.39\ \text{kN}\cdot\text{m/m}$
S	8.29	$\dfrac{M_{max}}{\sigma_{all}} = \dfrac{209.39\ \text{kN}\cdot\text{m}}{172 \times 10^3\ \text{kN/m}^2} = \mathbf{1.217 \times 10^{-3}\ m^3/m\ of\ wall}$

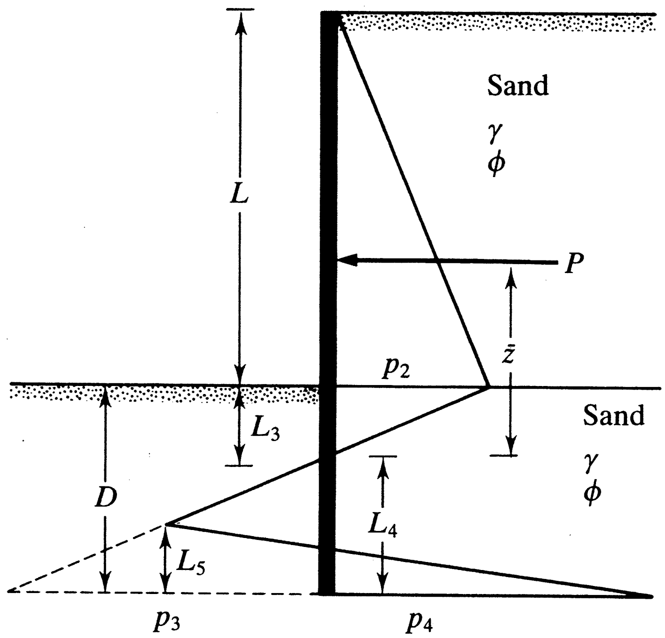
▲

8.5 SPECIAL CASES FOR CANTILEVER WALLS (PENETRATING A SANDY SOIL)

Following are two special cases of the mathematical formulation shown in Section 8.4.

Case 1: Sheet Pile Wall in the Absence of Water Table

In the absence of the water table, the net pressure diagram on the cantilever sheet pile wall will be as shown in Figure 8.9, which is a modified version of Figure 8.7. In this case,



▼ **FIGURE 8.9** Sheet piling penetrating a sandy soil in the absence of the water table

$$p_2 = \gamma L K_a \tag{8.24}$$

$$p_3 = L_4(K_p - K_a)\gamma \tag{8.25}$$

$$p_4 = p_5 + \gamma L_4(K_p - K_a) \tag{8.26}$$

$$p_5 = \gamma L K_p + \gamma L_3(K_p - K_a) \tag{8.27}$$

$$L_3 = \frac{p_2}{\gamma(K_p - K_a)} = \frac{LK_a}{(K_p - K_a)} \tag{8.28}$$

$$P = \tfrac{1}{2}p_2 L + \tfrac{1}{2}p_2 L_3 \tag{8.29}$$

$$\bar{z} = L_3 + \frac{L}{3} = \frac{LK_a}{K_p - K_a} + \frac{L}{3} = \frac{L(2K_a + K_p)}{3(K_p - K_a)} \tag{8.30}$$

and Eq. (8.16) transforms to

$$\boxed{L_4^4 + A_1' L_4^3 - A_2' L_4^2 - A_3' L_4 - A_4' = 0} \tag{8.31}$$

where

$$A_1' = \frac{p_5}{\gamma(K_p - K_a)} \tag{8.32}$$

$$A_2' = \frac{8P}{\gamma(K_p - K_a)} \tag{8.33}$$

$$A_3' = \frac{6P[2\bar{z}\gamma(K_p - K_a) + p_5]}{\gamma^2(K_p - K_a)^2} \tag{8.34}$$

$$A_4' = \frac{P(6\bar{z}p_5 + 4P)}{\gamma^2(K_p - K_a)^2} \tag{8.35}$$

Case 2: Free Cantilever Sheet Piling

Figure 8.10 shows a free cantilever sheet pile wall penetrating a sandy soil and subjected to a line load of P per unit length of the wall. For this case,

$$D^4 - \left[\frac{8P}{\gamma(K_p - K_a)}\right]D^2 - \left[\frac{12PL}{\gamma(K_p - K_a)}\right]D - \left[\frac{2P}{\gamma(K_p - K_a)}\right]^2 = 0 \tag{8.36}$$

and

$$L_5 = \frac{\gamma(K_p - K_a)D^2 - 2P}{2D(K_p - K_a)\gamma} \tag{8.37}$$

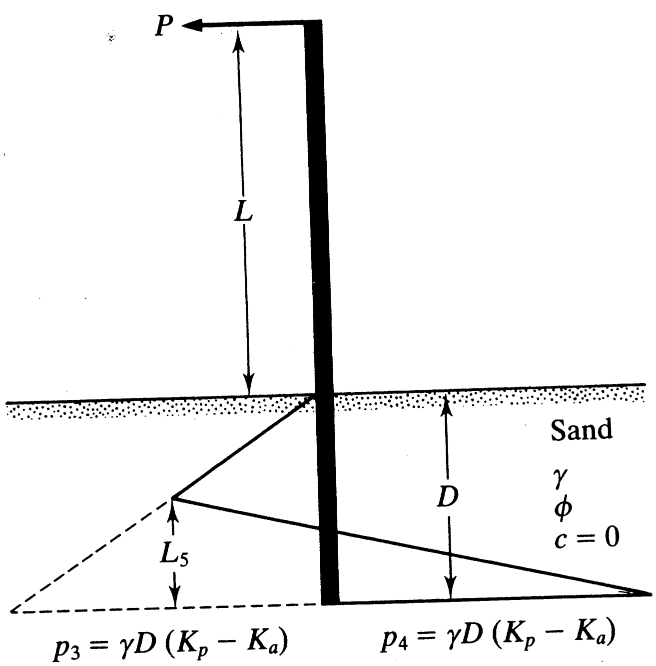


FIGURE 8.10 Free cantilever sheet piling penetrating a sand layer

$$M_{max} = P(L + z') - \frac{\gamma z'^3(K_p - K_a)}{6} \tag{8.38}$$

$$z' = \sqrt{\frac{2P}{\gamma'(K_p - K_a)}} \tag{8.39}$$

EXAMPLE 8.2

Redo Parts a and b of Example 8.1 assuming the absence of the water table. Use $\gamma = 15.9\ \text{kN/m}^3$ and $\phi = 32°$. Note: $L = 5$ m.

Solution

Part a

Quantity required	Eq. no.	Equation and calculation
K_a	—	$\tan^2\left(45 - \frac{\phi}{2}\right) = \tan^2\left(45 - \frac{32}{2}\right) = 0.307$
K_p	—	$\tan^2\left(45 + \frac{\phi}{2}\right) = \tan^2\left(45 + \frac{32}{2}\right) = 3.25$
p_2	8.24	$\gamma LK_a = (15.9)(5)(0.307) = 24.41\ \text{kN/m}^2$
L_3	8.28	$\frac{LK_a}{K_p - K_a} = \frac{(5)(0.307)}{3.25 - 0.307} = 0.521\ \text{m}$
p_5	8.27	$\gamma LK_p + \gamma L_3(K_p - K_a) = (15.9)(5)(3.25) + (15.9)(0.521)(3.25 - 0.307) = 282.76\ \text{kN/m}^2$
P	8.29	$\frac{1}{2}p_2L + \frac{1}{2}p_2L_3 = \frac{1}{2}p_2(L + L_3) = (\frac{1}{2})(24.41)(5 + 0.521) = 67.38\ \text{kN/m}$
$\bar{z}$	8.30	$\frac{L(2K_a + K_p)}{3(K_p - K_a)} = \frac{5[(2)(0.307) + 3.25]}{3(3.25 - 0.307)} = 2.188\ \text{m}$
A_1'	8.32	$\frac{p_5}{\gamma(K_p - K_a)} = \frac{282.76}{(15.9)(3.25 - 0.307)} = 6.04$
A_2'	8.33	$\frac{8P}{\gamma(K_p - K_a)} = \frac{(8)(67.38)}{(15.9)(3.25 - 0.307)} = 11.52$
A_3'	8.34	$\frac{6P[2\bar{z}\gamma(K_p - K_a) + p_5]}{\gamma^2(K_p - K_a)^2} = \frac{(6)(67.38)[(2)(2.188)(15.9)(3.25 - 0.307) + 282.76]}{(15.9)^2(3.25 - 0.307)^2} = 90.01$
A_4'	8.35	$\frac{P(6\bar{z}p_5 + 4P)}{\gamma^2(K_p - K_a)^2} = \frac{(67.38)[(6)(2.188)(282.76) + (4)(67.38)]}{(15.9)^2(3.25 - 0.307)^2} = 122.52$
L_4	8.31	$L_4^4 + A_1'L_4^3 - A_2'L_4^2 - A_3'L_4 - A_4' = 0$; $L_4^4 + 6.04L_4^3 - 11.52L_4^2 - 90.01L_4 - 122.52 = 0$; $L_4 \approx 4.1\ \text{m}$

$$D_{\text{theory}} = L_3 + L_4 = 0.521 + 4.1 = \mathbf{4.7\ m}$$

Part b

$$\text{Total length, } L + 1.3(D_{\text{theory}}) = 5 + 1.3(4.7) = \mathbf{11.11\ m}$$ ▲

▼ EXAMPLE 8.3

Refer to Figure 8.10. For $L = 15$ ft, $\gamma = 110$ lb/ft^3, $\phi = 30°$, and $P = 2000$ lb/ft, determine:

a. The theoretical depth of penetration, D
b. The maximum moment, M_{max} (lb-ft/ft)

Solution

$$K_p = \tan^2\left(45 + \frac{\phi}{2}\right) = \tan^2\left(45 + \frac{30}{2}\right) = 3$$

$$K_a = \tan^2\left(45 - \frac{\phi}{2}\right) = \tan^2\left(45 - \frac{30}{2}\right) = \frac{1}{3}$$

$$K_p - K_a = 3 - 0.333 = 2.667$$

Part a

From Eq. (8.36),

$$D^4 - \left[\frac{8P}{\gamma(K_p - K_a)}\right]D^2 - \left[\frac{12PL}{\gamma(K_p - K_a)}\right]D - \left[\frac{2P}{\gamma(K_p - K_a)}\right]^2 = 0$$

and

$$\frac{8P}{\gamma(K_p - K_a)} = \frac{(8)(2000)}{(110)(2.667)} = 54.54$$

$$\frac{12PL}{\gamma(K_p - K_a)} = \frac{(12)(2000)(15)}{(110)(2.667)} = 1227.1$$

$$\frac{2P}{\gamma(K_p - K_a)} = \frac{(2)(2000)}{(110)(2.667)} = 13.63$$

so

$$D^4 - 54.54D^2 - 1227.1D - (13.63)^2 = 0$$

From the preceding equation, $D \approx$ **12.5 ft**

Part b

From Eq. (8.39),

$$z' = \sqrt{\frac{2P}{\gamma(K_p - K_a)}} = \sqrt{\frac{(2)(2000)}{(110)(2.667)}} = 3.69 \text{ ft}$$

From Eq. (8.38),

$$M_{max} = P(L + z') - \frac{\gamma z'^3(K_p - K_a)}{6}$$

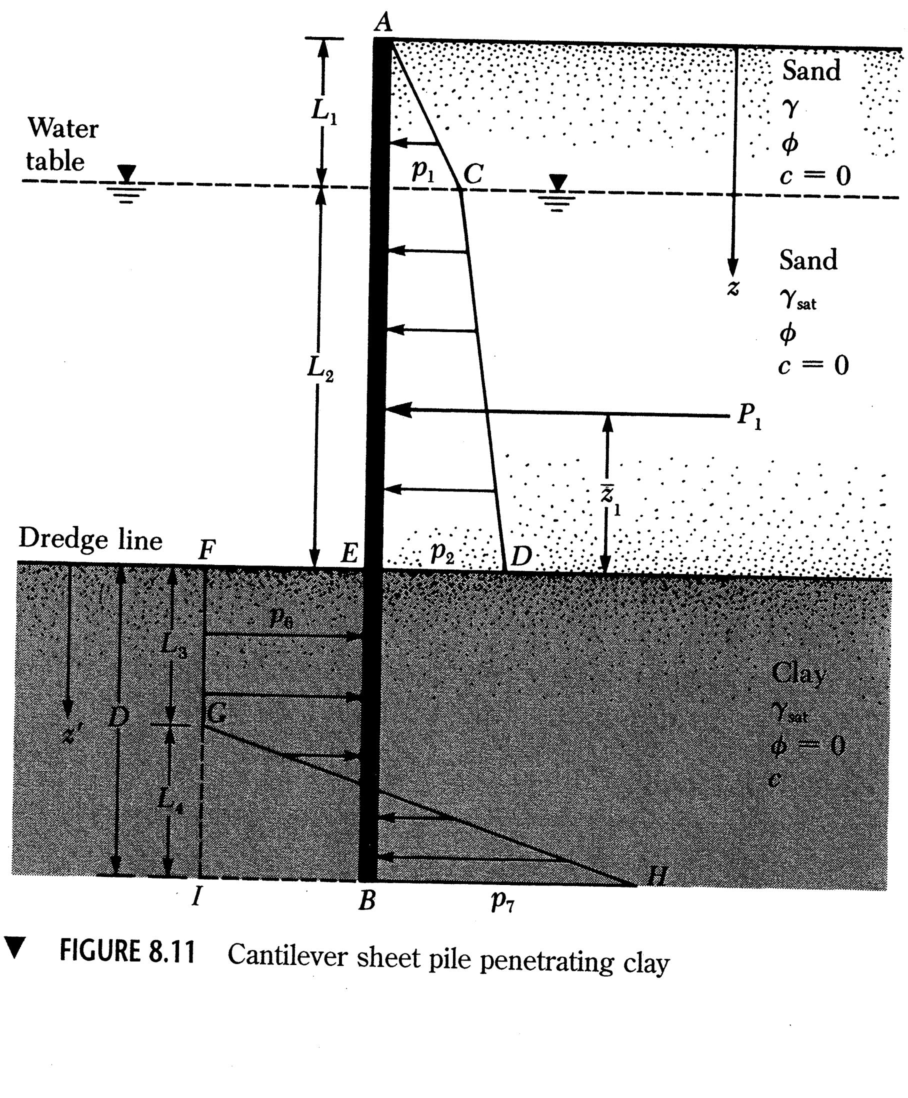
$$= (2000)(15 + 3.69) - \frac{(110)(3.69)^3(2.667)}{6}$$

$$= 37{,}380 - 2456.65 \approx \mathbf{34{,}923\ lb\text{-}ft/ft}$$

▲

8.6 CANTILEVER SHEET PILING PENETRATING CLAY

At times, cantilever sheet piles must be driven into a clay layer possessing an undrained cohesion, c ($\phi = 0$ concept). The net pressure diagram will be somewhat different from that shown in Figure 8.7a. Figure 8.11 shows a cantilever sheet pile wall driven into clay with a backfill of granular soil above the level of the dredge line. The water table is at depth L_1 below the top of the wall. As before, Eqs. (8.1) and (8.2) give the intensity of the net pressures p_1 and p_2, and the diagram for pressure distribution above the level of the dredge line can be drawn. The diagram for net pressure distribution below the dredge line can now be determined as follows.



▼ **FIGURE 8.11** Cantilever sheet pile penetrating clay

At any depth greater than $L_1 + L_2$, for $\phi = 0$ condition, the Rankine active earth pressure coefficient $K_a = 1$. Similarly, for $\phi = 0$ condition, the Rankine passive earth pressure (K_p) is equal to 1. Thus, above the point of rotation (point O in Figure 8.6a), the active pressure, p_a, from right to left is

$$p_a = [\gamma L_1 + \gamma' L_2 + \gamma_{sat}(z - L_1 - L_2)] - 2c \tag{8.40}$$

Similarly, the passive pressure, p_p, from left to right may be expressed as

$$p_p = \gamma_{sat}(z - L_1 - L_2) + 2c \tag{8.41}$$

Thus the net pressure is

$$\begin{aligned} p_6 = p_p - p_a &= [\gamma_{sat}(z - L_1 - L_2) + 2c] \\ &\quad - [\gamma L_1 + \gamma' L_2 + \gamma_{sat}(z - L_1 - L_2)] + 2c \\ &= 4c - (\gamma L_1 + \gamma' L_2) \end{aligned} \tag{8.42}$$

At the bottom of the sheet pile, the passive pressure from right to left is

$$p_p = (\gamma L_1 + \gamma' L_2 + \gamma_{sat} D) + 2c \tag{8.43}$$

Similarly, the active pressure from left to right is

$$p_a = \gamma_{sat} D - 2c \tag{8.44}$$

Hence the net pressure is

$$p_7 = p_p - p_a = 4c + (\gamma L_1 + \gamma' L_2) \tag{8.45}$$

For equilibrium analysis, $\Sigma F_H = 0$—that is, area of pressure diagram $ACDE$ − area of $EFIB$ + area of GIH = 0, or

$$P_1 - [4c - (\gamma L_1 + \gamma' L_2)]D + \tfrac{1}{2} L_4 [4c - (\gamma L_1 + \gamma' L_2) + 4c + (\gamma L_1 + \gamma' L_2)] = 0$$

where P_1 = area of the pressure diagram $ACDE$

Simplifying the preceding equation produces

$$L_4 = \frac{D[4c - (\gamma L_1 + \gamma' L_2)] - P_1}{4c} \tag{8.46}$$

Now, taking the moment about point B, $\Sigma M_B = 0$, yields

$$P_1(D + \bar{z}_1) - [4c - (\gamma L_1 + \gamma' L_2)]\frac{D^2}{2} + \frac{1}{2} L_4 (8c)\left(\frac{L_4}{3}\right) = 0 \tag{8.47}$$

where $\bar{z}_1$ = distance of the center of pressure of the pressure diagram $ACDE$ measured from the level of the dredge line

Combining Eqs. (8.46) and (8.47) yields

$$D^2[4c - (\gamma L_1 + \gamma' L_2)] - 2DP_1 - \frac{P_1(P_1 + 12c\bar{z}_1)}{(\gamma L_1 + \gamma' L_2) + 2c} = 0 \tag{8.48}$$

Equation (8.48) may be solved to obtain D, the theoretical depth of penetration of the clay layer by the sheet pile.

Step-by-Step Procedure to Obtain the Pressure Diagram

1. Calculate $K_a = \tan^2(45 - \phi/2)$ for the granular soil (backfill).
2. Obtain p_1 and p_2 [Eqs. (8.1) and (8.2)].
3. Calculate P_1 and $\bar{z}_1$.
4. Use Eq. (8.48) to obtain the theoretical value of D.
5. Using Eq. (8.46), calculate L_4.
6. Calculate p_6 and p_7 [Eqs. (8.42) and (8.45)].
7. Draw the pressure distribution diagram as shown in Figure 8.11.
8. The actual depth of penetration is

$$D_{\text{actual}} = 1.4 \text{ to } 1.6(D_{\text{theoretical}})$$

Maximum Bending Moment

According to Figure 8.11, the maximum moment (zero shear) will occur between $L_1 + L_2 < z < L_1 + L_2 + L_3$. Using a new coordinate system z' ($z' = 0$ at dredge line) for zero shear gives

$$P_1 - p_6 z' = 0$$

or

$$z' = \frac{P_1}{p_6} \tag{8.49}$$

The magnitude of the maximum moment may now be obtained:

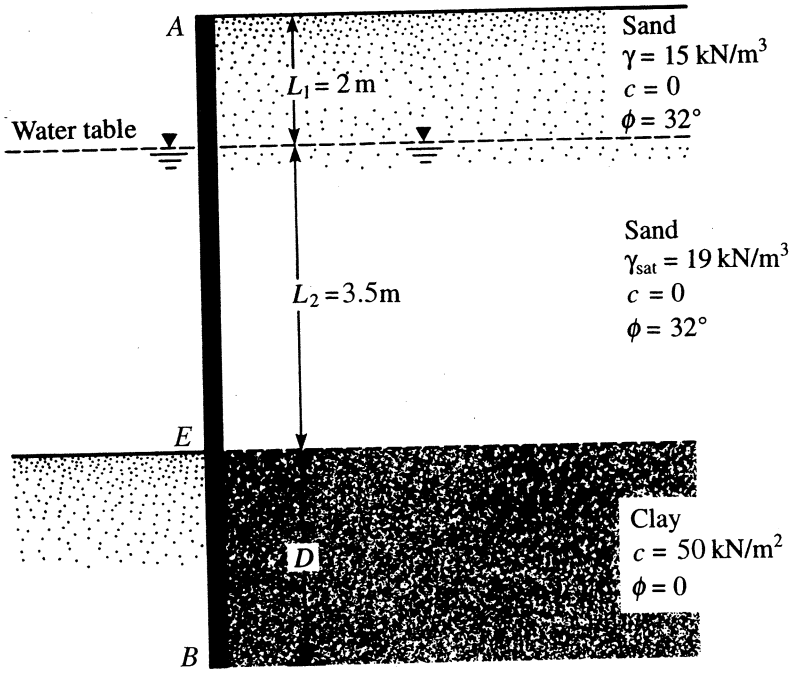
$$M_{\max} = P_1(z' + \bar{z}_1) - \frac{p_6 z'^2}{2} \tag{8.50}$$

Knowing the maximum bending moment, we determine the section modulus of the sheet pile section from Eq. (8.23).

▼ EXAMPLE 8.4

Refer to Figure 8.12. For the sheet pile wall, determine the

a. Theoretical and actual depth of penetration. Use $D_{\text{actual}} = 1.5D_{\text{theory}}$.
b. Minimum size of sheet pile section necessary. Use $\sigma_{\text{all}} = 172 \text{ MN/m}^2$.



▼ **FIGURE 8.12**

Solution

Part a

Refer to the pressure diagram shown in Figure 8.11.

Quantity required	Eq. no.	Equation and calculation
K_a	—	$\tan^2\left(45 - \frac{\phi}{2}\right) = \tan^2\left(45 - \frac{32}{2}\right) = 0.307$
p_1	8.1	$\gamma L_1 K_a = (15)(2)(0.307) = 9.21\text{ kN/m}^2$
p_2	8.2	$(\gamma L_1 + \gamma' L_2)K_a = [(15)(2) + (19 - 9.81)(3.5)](0.307) = 19.08\text{ kN/m}^2$
P_1	—	$\frac{1}{2}p_1 L_1 + p_1 L_2 + \frac{1}{2}(p_2 - p_1)L_2$ $= (\frac{1}{2})(9.21)(2) + (9.21)(3.5) + (\frac{1}{2})(19.08 - 9.21)(3.5)$ $= 9.21 + 32.24 + 17.27 = 58.72\text{ kN/m}$
$\bar{z}_1$	—	$\frac{\Sigma M_E}{P_1} = \frac{1}{58.72}\left[9.21\left(3.5 + \frac{2}{3}\right) + 32.24\left(\frac{3.5}{2}\right) + (17.27)\left(\frac{3.5}{3}\right)\right] = 1.957\text{ m}$
D_{theory}	8.48	$D^2[4c - (\gamma L_1 + \gamma' L_2)] - 2DP_1 - \frac{P_1(P_1 + 12c\bar{z}_1)}{(\gamma L_1 + \gamma' L_2) + 2c}$ $D^2\{(4)(50) - [(15)(2) + (19 - 9.81)(3.5)]\} - (2)(D)(58.72)$ $- \frac{58.72[58.72 + (12)(50)(1.957)]}{[(15)(2) + (19 - 9.81)(3.5)] + (2)(50)} = 0$ $137.84D^2 - 117.44D - 446.44 = 0;\quad D \approx$ **2.3 m**
D_{actual}	—	$1.5D_{\text{theory}} = (1.5)(2.3) =$ **3.45 m**

Part b

Quantity required	Eq. no.	Equation and calculation
z'	8.49	$\frac{P_1}{p_6} = \frac{P_1}{4c - (\gamma L_1 + \gamma' L_2)} = \frac{58.72}{(4)(50) - [(15)(2) + (19 - 9.81)(3.5)]} = 0.426 \text{ m}$
M_{max}	8.50	$P_1(z' + \bar{z}_1) - \frac{p_6 z'^2}{2} = P_1(z' + \bar{z}_1) - \frac{[4c - (\gamma L_1 + \gamma' L_2)]z'^2}{2}$
		$= (58.72)(0.426 + 1.957)$
		$- \frac{\{(4)(50) - [(15)(2) + (19 - 9.81)(3.5)]\}(0.426)^2}{2}$
		$= 127.42 \text{ kN} \cdot \text{m/m}$
S	—	$\frac{M_{max}}{\sigma_{all}} = \frac{127.42}{172 \times 10^3} = \mathbf{0.741 \times 10^{-3}}$ **m³/m of wall**

▲

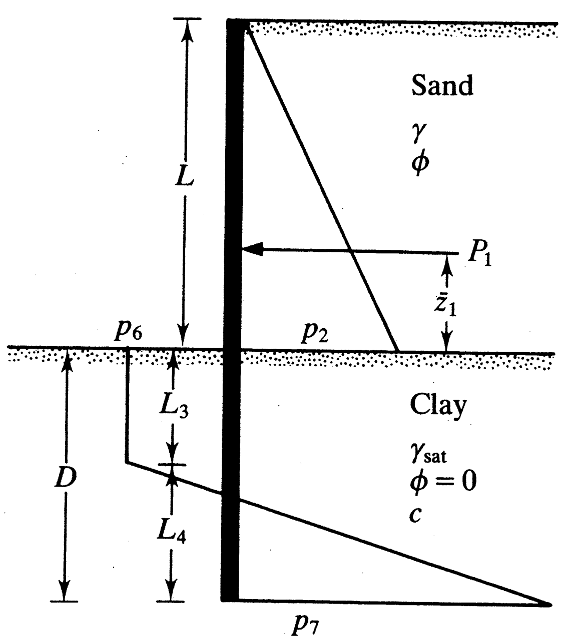
8.7 SPECIAL CASES FOR CANTILEVER WALLS (PENETRATING CLAY)

As in Section 8.5, relationships for special cases for cantilever walls penetrating clay may also be derived.

Case 1: Sheet Pile Wall in the Absence of Water Table

Referring to Figure 8.13, we can write

$$p_2 = \gamma L K_a \tag{8.51}$$



▼ **FIGURE 8.13** Sheet pile wall penetrating clay

$$p_6 = 4c - \gamma L \tag{8.52}$$

$$p_7 = 4c + \gamma L \tag{8.53}$$

$$P_1 = \tfrac{1}{2} L p_2 = \tfrac{1}{2} \gamma L^2 K_a \tag{8.54}$$

$$L_4 = \frac{D(4c - \gamma L) - \frac{1}{2}\gamma L^2 K_a}{4c} \tag{8.55}$$

The theoretical depth of penetration, D, can be calculated [similar to Eq. (8.48)] as

$$D^2(4c - \gamma L) - 2DP_1 - \frac{P_1(P_1 + 12c\bar{z}_1)}{\gamma L + 2c} = 0 \tag{8.56}$$

where $\bar{z}_1 = \dfrac{L}{3}$ (8.57)

The magnitude of the maximum moment in the wall is

$$M_{\max} = P_1(z' + \bar{z}_1) - \frac{p_6 z'^2}{2} \tag{8.58}$$

where $z' = \dfrac{P_1}{p_6} = \dfrac{\frac{1}{2}\gamma L^2 K_a}{4c - \gamma L}$ (8.59)

Case 2: Free Cantilever Sheet Pile Wall Penetrating Clay

Figure 8.14 shows a free cantilever sheet pile wall penetrating a clay layer. The wall is being subjected to a line load of P per unit length. For this case,

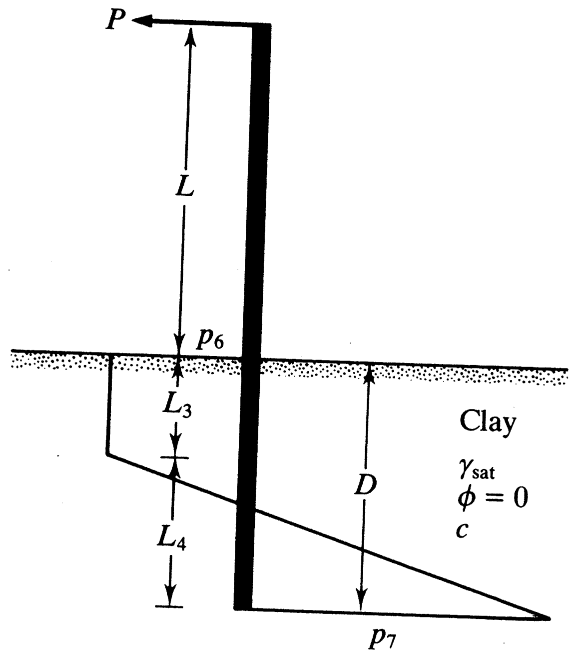
$$p_6 = p_7 = 4c \tag{8.60}$$

The depth of penetration, D, may be obtained from

$$4D^2c - 2PD - \frac{P(P + 12cL)}{2c} = 0 \tag{8.61}$$

Also note that, for pressure diagram construction,

$$L_4 = \frac{4cD - P}{4c} \tag{8.62}$$



▼ **FIGURE 8.14** Free cantilever sheet piling penetrating clay

The maximum moment in the wall is

$$M_{max} = P(L + z') - \frac{4cz'^2}{2} \tag{8.63}$$

where $z' = \dfrac{P}{4c}$ (8.64)

▼ EXAMPLE 8.5

Refer to the free cantilever sheet pile wall shown in Figure 8.14, for which $P = 32$ kN/m, $L = 3.5$ m, and $c = 12$ kN/m². Calculate the theoretical depth of penetration.

Solution From Eq. (8.61),

$$4D^2c - 2PD - \frac{P(P + 12cL)}{2c} = 0$$

$$(4)(D^2)(12) - (2)(32)(D) - \frac{32[32 + (12)(12)(3.5)]}{(2)(12)} = 0$$

$$48D^2 - 64D - 714.7 = 0$$

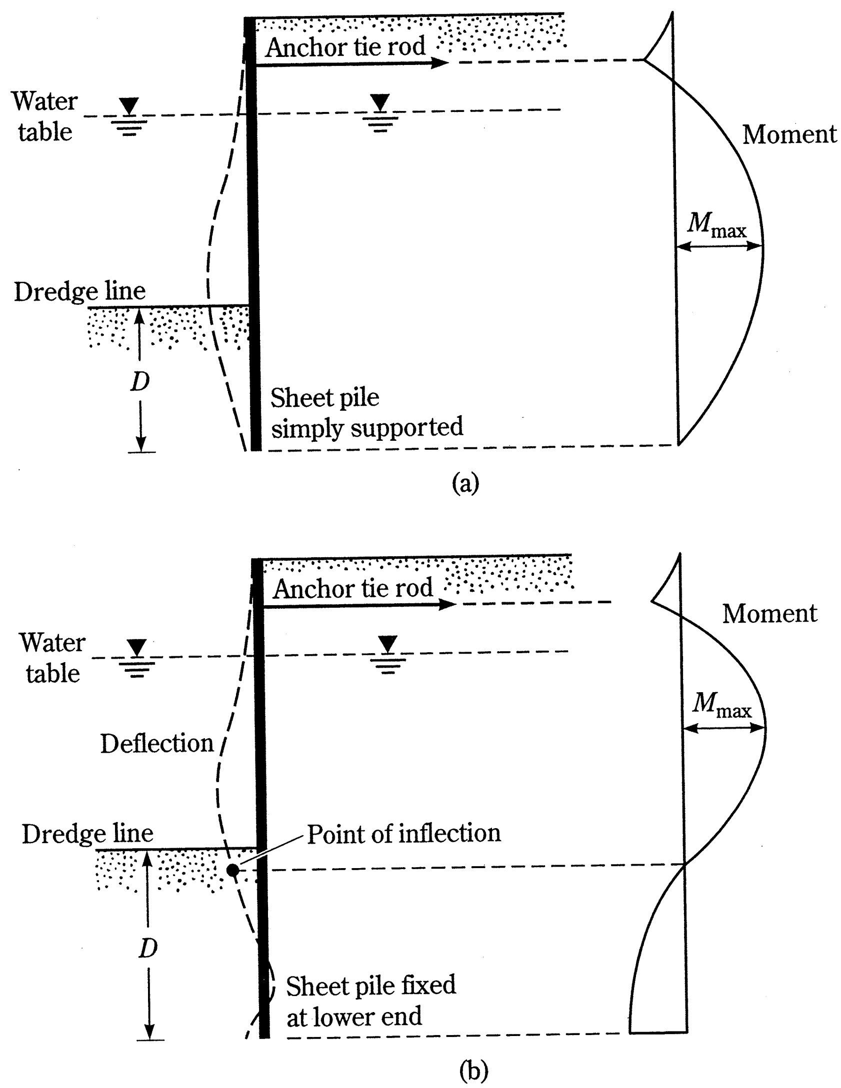
Hence $D \approx$ **4.6 m.**

▲

8.8 ANCHORED SHEET PILE WALL—GENERAL

When the height of the backfill material behind a cantilever sheet pile wall exceeds about 20 ft (≈6 m), tying the sheet pile wall near the top to anchor plates, anchor walls, or anchor piles becomes more economical. This type of construction is referred to as *anchored sheet pile wall* or an *anchored bulkhead.* Anchors minimize the depth of required penetration by the sheet piles and also reduce the cross-sectional area and weight of the sheet piles needed for construction. However, the tie rods and anchors must be carefully designed.

The two basic methods of designing anchored sheet pile walls are (a) the *free earth support* method and (b) the *fixed earth support* method. Figure 8.15 shows the assumed nature of deflection of the sheet piles for the two methods.



▼ **FIGURE 8.15** Nature of variation of deflection and moment for anchored sheet piles: (a) free earth support method; (b) fixed earth support method

The free earth support method involves minimum penetration depth. Below the dredge line, no pivot point exists for the static system. The nature of variation of the bending moment with depth for both methods is also shown in Figure 8.15. Note that

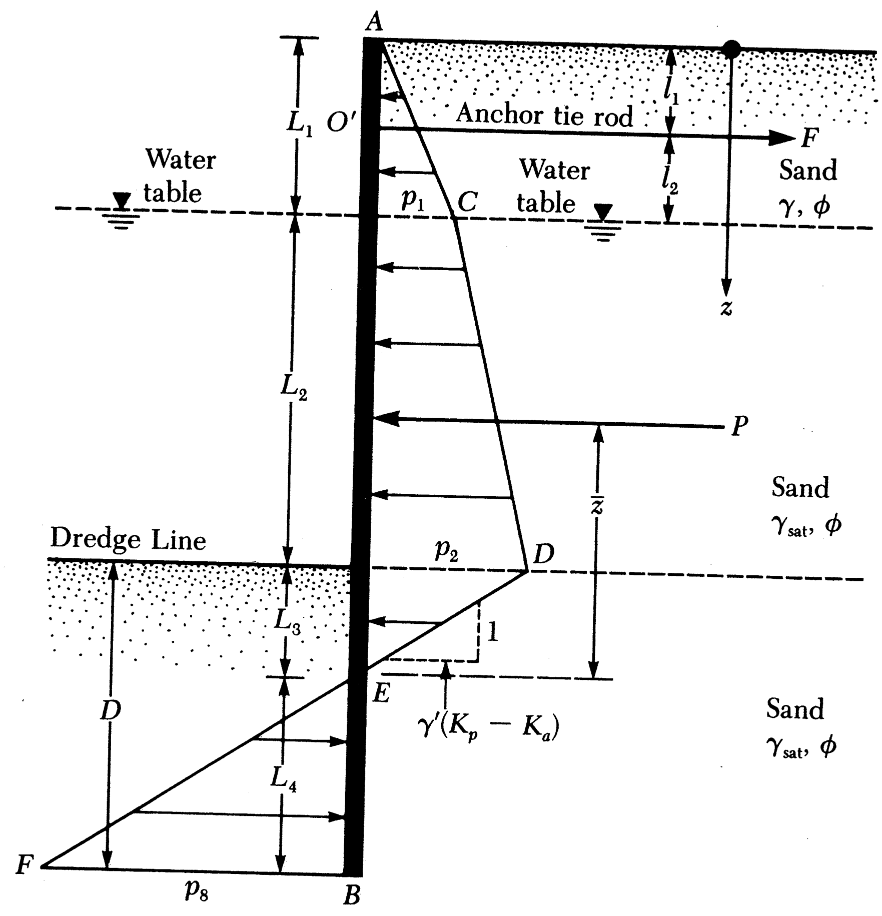
$$D_{\text{free earth}} < D_{\text{fixed earth}}$$

8.9 FREE EARTH SUPPORT METHOD FOR PENETRATION OF SANDY SOIL

Figure 8.16 shows an anchor sheet pile wall with a granular soil backfill; the wall has been driven into a granular soil. The tie rod connecting the sheet pile and the anchor is located at depth l_1 below the top of the sheet pile wall.

The diagram of net pressure distribution above the dredge line is similar to that shown in Figure 8.7. At depth $z = L_1$, $p_1 = \gamma L_1 K_a$; and, at $z = L_1 + L_2$, $p_2 = (\gamma L_1 + \gamma' L_2)K_a$. Below the dredge line, the net pressure will be zero at $z = L_1 + L_2 + L_3$. The relation for L_3 is given by Eq. (8.6), or

$$L_3 = \frac{p_2}{\gamma'(K_p - K_a)}$$



▼ **FIGURE 8.16** Anchored sheet pile wall penetrating sand

At $z = (L_1 + L_2 + L_3 + L_4)$, the net pressure is given by

$$p_8 = \gamma'(K_p - K_a)L_4 \tag{8.65}$$

Note that the slope of the line DEF is 1 vertical to $\gamma'(K_p - K_a)$ horizontal.

For equilibrium of the sheet pile, Σ horizontal forces = 0, and Σ moment about $O' = 0$. (*Note:* Point O' is located at the level of the tie rod.)

Summing the forces in the horizontal direction (per unit length of the wall) gives

Area of the pressure diagram $ACDE$ − area of EBF − $F = 0$

where F = tension in the tie rod/unit length of the wall, or

$$P - \tfrac{1}{2}p_8L_4 - F = 0$$

or

$$F = P - \tfrac{1}{2}[\gamma'(K_p - K_a)]L_4^2 \tag{8.66}$$

where P = area of the pressure diagram $ACDE$

Now, taking the moment about point O' gives

$$-P[(L_1 + L_2 + L_3) - (\bar{z} + l_1)] + \tfrac{1}{2}[\gamma'(K_p - K_a)]L_4^2(l_2 + L_2 + L_3 + \tfrac{2}{3}L_4) = 0$$

or

$$L_4^3 + 1.5L_4^2(l_2 + L_2 + L_3) - \frac{3P[(L_1 + L_2 + L_3) - (\bar{z} + l_1)]}{\gamma'(K_p - K_a)} = 0 \tag{8.67}$$

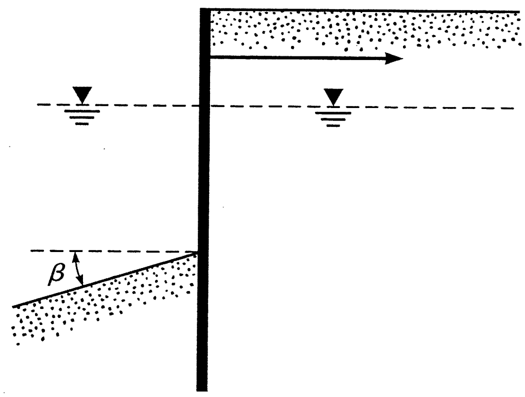
Equation (8.67) may be solved by trial and error to determine the theoretical depth, L_4:

$$D_{\text{theoretical}} = L_3 + L_4$$

The theoretical depth is increased by about 30%–40% for actual construction, or

$$D_{\text{actual}} = 1.3 \text{ to } 1.4D_{\text{theoretical}} \tag{8.68}$$

The step-by-step procedure in Section 8.4 indicated that a factor of safety can be applied to K_p at the beginning [that is, $K_{p(\text{design})} = K_p/FS$]. If done, there is no



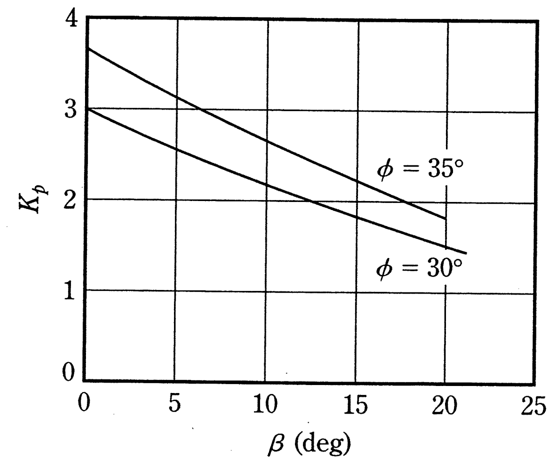


FIGURE 8.17 (a) Anchored sheet pile wall with sloping dredge line; (b) variation K_p with β and ϕ

need to increase the theoretical depth by 30%–40%. This approach is often more conservative.

The maximum theoretical moment to which the sheet pile will be subjected occurs at a depth between $z = L_1$ and $z = L_1 + L_2$. The depth, z, for zero shear and hence maximum moment, may be evaluated from

$$\tfrac{1}{2}p_1L_1 - F + p_1(z - L_1) + \tfrac{1}{2}K_a\gamma'(z - L_1)^2 = 0 \tag{8.69}$$

Once the value of z is determined, the magnitude of the maximum moment is easily obtained. The procedure for determining the holding capacity of anchors is treated in Sections 8.16 and 8.17.

In some cases, the dredge line may be sloping at an angle β with respect to the horizontal, as shown in Figure 8.17a. In that case, the passive pressure coefficient will not be equal to $\tan^2(45 + \phi/2)$. The variations of K_p (Coulomb—for wall friction angle of zero) with β for $\phi = 30°$ and $35°$ are shown in Figure 8.17b. With these values of K_p, the procedure described above may be used to determine the depth of penetration, D.

EXAMPLE 8.6

Refer to Figure 8.16. Here, $L_1 = 3.05$ m, $L_2 = 6.1$ m, $l_1 = 1.53$ m, $l_2 = 1.52$ m, $c = 0$, $\phi = 30°$, $\gamma = 16$ kN/m³, and $\gamma_{sat} = 19.5$ kN/m³.

a. Determine the theoretical and actual depths of penetration. Note: $D_{actual} = 1.3D_{theory}$.
b. Find the anchor force per unit length of the wall.

Solution

Part a

Quantity required	Eq. no.	Equation and calculation
K_a	—	$\tan^2\left(45-\frac{\phi}{2}\right)=\tan^2\left(45-\frac{30}{2}\right)=\frac{1}{3}$
K_p	—	$\tan^2\left(45+\frac{\phi}{2}\right)=\tan^2\left(45+\frac{30}{2}\right)=3$
$K_a - K_p$	—	$3 - 0.333 = 2.667$
γ'	—	$\gamma_{sat} - \gamma_w = 19.5 - 9.81 = 9.69\ \text{kN/m}^3$
p_1	8.1	$\gamma L_1 K_a = (16)(3.05)(\frac{1}{3}) = 16.27\ \text{kN/m}^2$
p_2	8.2	$(\gamma L_1 + \gamma' L_2)K_a = [(16)(3.05) + (9.69)(6.1)]\frac{1}{3} = 35.97\ \text{kN/m}^2$
L_3	8.6	$\frac{p_2}{\gamma'(K_p - K_a)} = \frac{35.97}{(9.69)(2.667)} = 1.39\ \text{m}$
P	—	$\frac{1}{2}p_1L_1 + p_1L_2 + \frac{1}{2}(p_2 - p_1)L_2 + \frac{1}{2}p_2L_3 = (\frac{1}{2})(16.27)(3.05) + (16.27)(6.1) + (\frac{1}{2})(35.97 - 16.27)(6.1) + (\frac{1}{2})(35.97)(1.39) = 24.81 + 99.25 + 60.01 + 25.0 = 209.07\ \text{kN/m}$
$\bar{z}$	—	$\frac{\Sigma M_E}{P} = \left[\begin{array}{l}(24.81)\left(1.39+6.1+\frac{3.05}{3}\right)+(99.25)\left(1.39+\frac{6.1}{2}\right)\\ +(60.01)\left(1.39+\frac{6.1}{3}\right)+(25.0)\left(\frac{2\times1.39}{3}\right)\end{array}\right]\frac{1}{209.07} = 4.21\ \text{m}$
L_4	8.67	$L_4^3 + 1.5L_4^2(l_2 + L_2 + L_3) - \frac{3P[(L_1 + L_2 + L_3) - (\bar{z} + l_1)]}{\gamma'(K_p - K_a)} = 0$
		$L_4^3 + 1.5L_4^2(1.52 + 6.1 + 1.39) - \frac{(3)(209.07)[(3.05 + 6.1 + 1.39) - (4.21 + 1.53)]}{(9.69)(2.667)} = 0$
		$L_4 = 2.7\ \text{m}$
D_{theory}	—	$L_3 + L_4 = 1.39 + 2.7 = 4.09 \approx \mathbf{4.1\ m}$
D_{actual}	—	$1.3D_{theory} = (1.3)(4.1) = \mathbf{5.33\ m}$

Part b

$$F = P - \tfrac{1}{2}\gamma'(K_p - K_a)L_4^2$$
$$= 209.07 - (\tfrac{1}{2})(9.69)(2.667)(2.7)^2 = 114.87\ \text{kN/m} \approx \mathbf{115\ kN/m}$$

▲

8.10 DESIGN CHARTS FOR FREE EARTH SUPPORT METHOD (PENETRATION INTO SANDY SOIL)

Using the free earth support method, Hagerty and Nofal (1992) provided simplified design charts for quick estimation of the depth of penetration, D, anchor force, F and maximum moment, M_{max}, for anchored sheet pile walls penetrating into sandy soil, as shown in Figure 8.16. They made the following assumptions for their analysis.

a. The soil friction angle, ϕ, above and below the dredge line is the same.
b. The angle of friction between the sheet pile wall and the soil is $\phi/2$.
c. The passive earth pressure below the dredge line has a logarithmic spiral failure surface.
d. For active earth pressure calculation, Coulomb's theory is valid.

The magnitudes of D, F, and M_{max} may be calculated from the following relationships:

$$\frac{D}{L_1 + L_2} = (GD)(CDL_1) \tag{8.70}$$

$$\frac{F}{\gamma_a (L_1 + L_2)^2} = (GF)(CFL_1) \tag{8.71}$$

$$\frac{M_{max}}{\gamma_a (L_1 + L_2)^3} = (GM)(CML_1) \tag{8.72}$$

where γ_a = average unit weight of soil

$$= \frac{\gamma L_1^2 + (\gamma_{sat} - \gamma_w) L_2^2 + 2\gamma L_1 L_2}{(L_1 + L_2)^2} \tag{8.73}$$

GD = generalized nondimensional embedment

$$= \frac{D}{L_1 + L_2} \quad (\text{for } L_1 = 0 \text{ and } L_2 = L_1 + L_2)$$

GF = generalized nondimensional anchor force

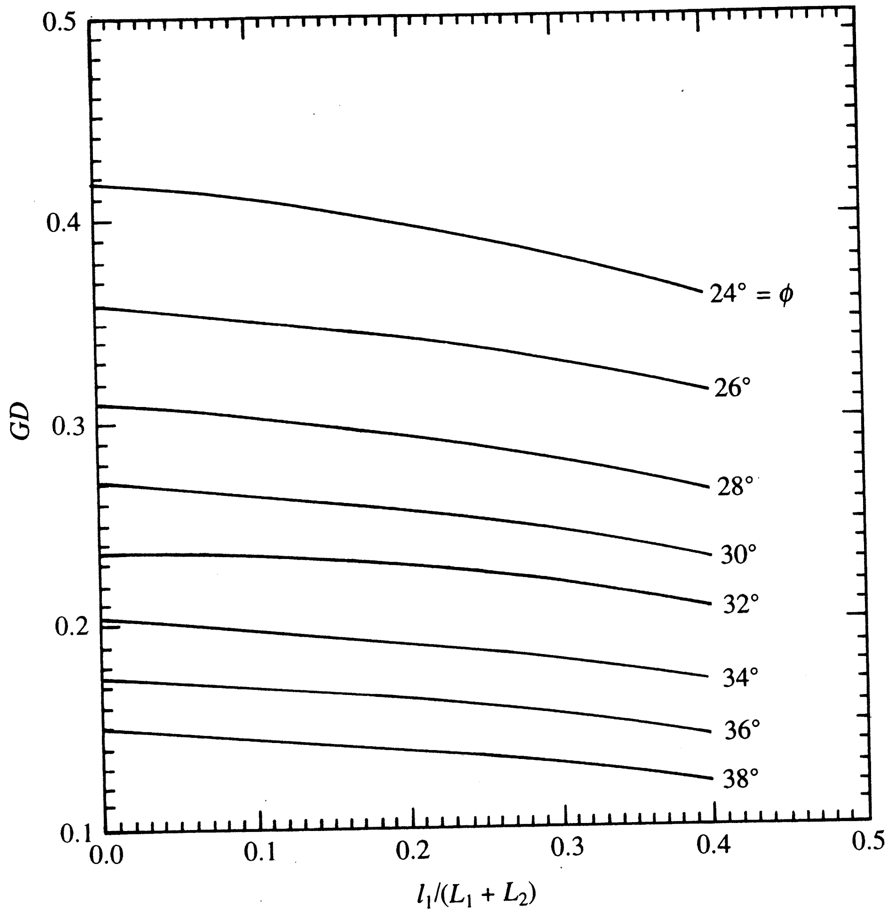
$$= \frac{F}{\gamma_a (L_1 + L_2)^2} \quad (\text{for } L_1 = 0 \text{ and } L_2 = L_1 + L_2)$$

GM = generalized nondimensional moment

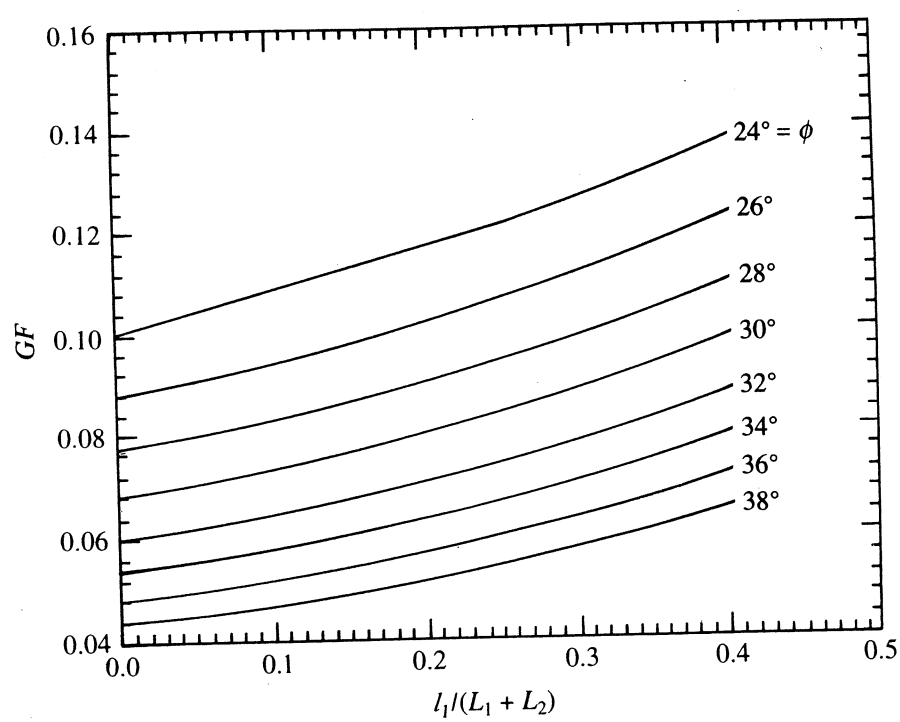
$$= \frac{M_{max}}{\gamma_a (L_1 + L_2)^3} \quad (\text{for } L_1 = 0 \text{ and } L_2 = L_1 + L_2)$$

CDL_1, CFL_1, CML_1 = correction factors for $L_1 \neq 0$

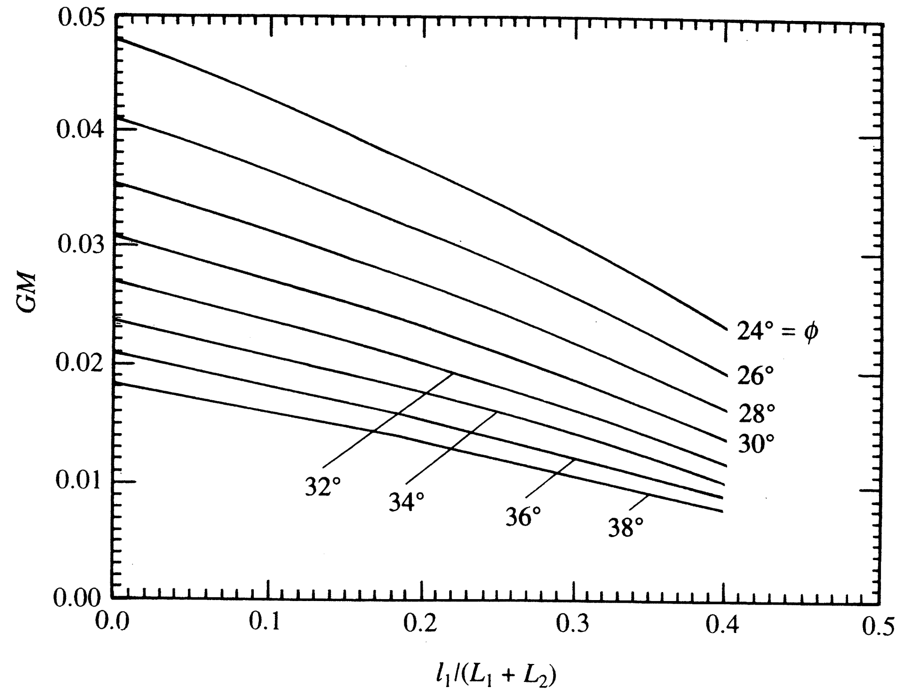
The variations of GD, GF, GM, CDL_1, CFL_1, and CML_1 are shown in Figures 8.18, 8.19, 8.20, 8.21, 8.22, and 8.23, respectively.



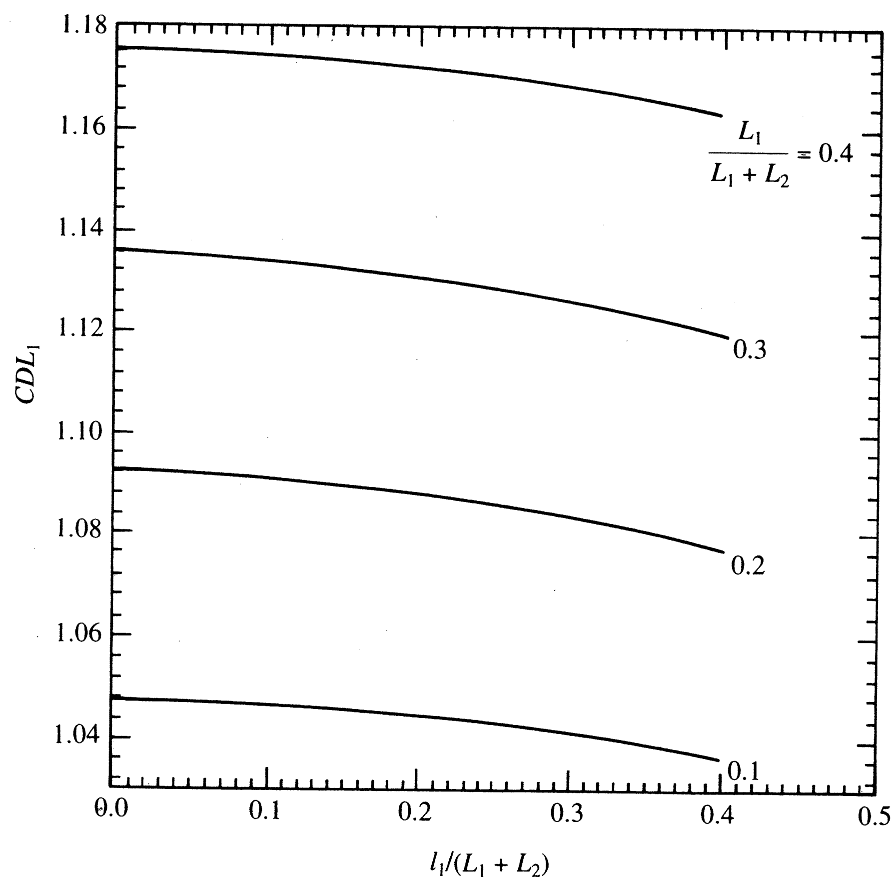
▼ **FIGURE 8.18** Variation of *GD* with $l_1/(L_1 + L_2)$ and ϕ (after Hagerty and Nofal, 1992)



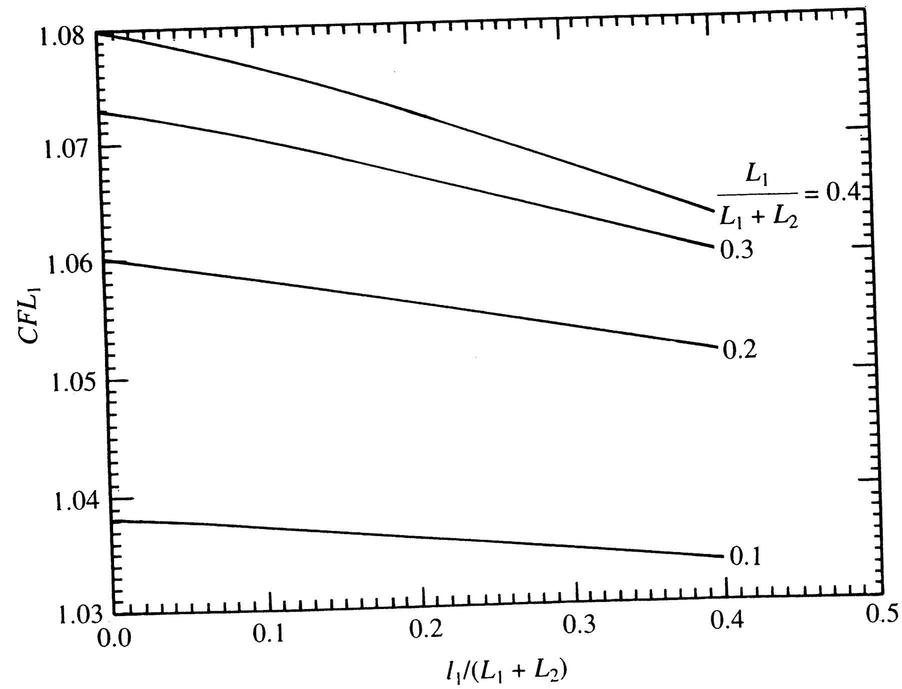
▼ **FIGURE 8.19** Variation of *GF* with $l_1/(L_1 + L_2)$ and ϕ (after Hagerty and Nofal, 1992)



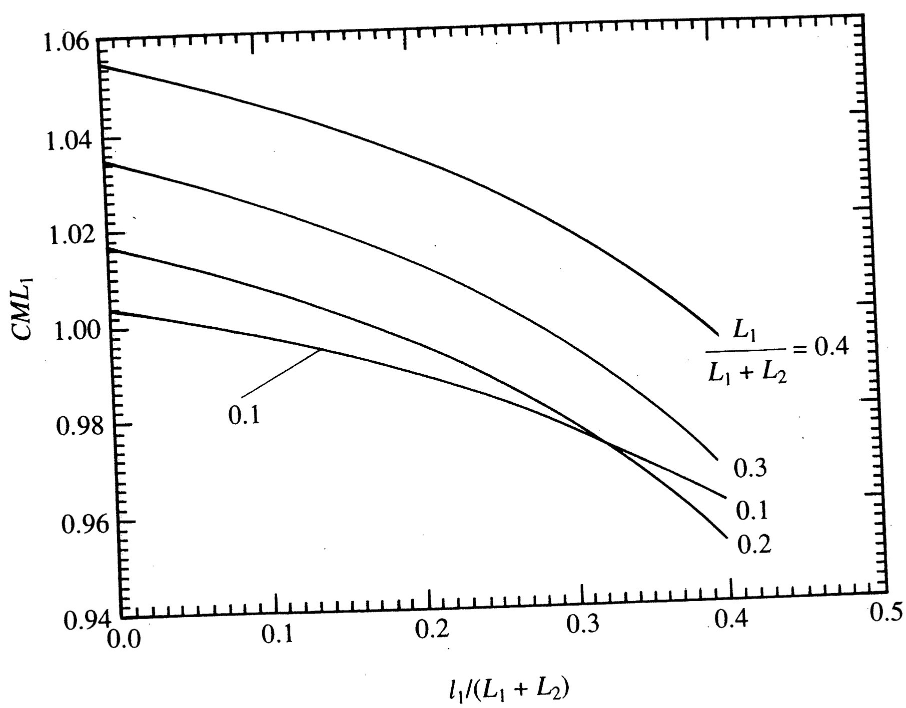
▼ **FIGURE 8.20** Variation of GM with $l_1/(L_1 + L_2)$ and ϕ (after Hagerty and Nofal, 1992)



▼ **FIGURE 8.21** Variation of CDL_1 with $L_1/(L_1 + L_2)$ and $l_1/(L_1 + L_2)$ (after Hagerty and Nofal, 1992)



▼ **FIGURE 8.22** Variation of CFL_1 with $L_1/(L_1 + L_2)$ and $l_1/(L_1 + L_2)$ (after Hagerty and Nofal, 1992)



▼ **FIGURE 8.23** Variation of CML_1 with $L_1/(L_1 + L_2)$ and $l_1/(L_1 + L_2)$ (after Hagerty and Nofal, 1992)

▼ EXAMPLE 8.7

Refer to Figure 8.16. Given: $L_1 = 2$ m, $L_2 = 3$ m, $l_1 = l_2 = 1$ m, $c = 0$, $\phi = 32°$, $\gamma = 15.9$ kN/m³, $\gamma_{sat} = 19.33$ kN/m³. Determine:

a. Theoretical and actual depth of penetration. Note: $D_{actual} = 1.4D_{theory}$.
b. Anchor force per unit length of wall.
c. Maximum moment, M_{max}.

Use the charts presented in Section 8.10.

Solution

Part a

From Eq. (8.70),

$$\frac{D}{L_1 + L_2} = (GD)(CDL_1)$$

$$\frac{l_1}{L_1 + L_2} = \frac{1}{2 + 3} = 0.2$$

From Figure 8.18 for $l_1/(L_1 + L_2) = 0.2$ and $\phi = 32°$, $GD = 0.22$. From Figure 8.21, for

$$\frac{L_1}{L_1 + L_2} = \frac{2}{2 + 3} = 0.4 \qquad \text{and} \qquad \frac{l_1}{L_1 + L_2} = 0.2$$

$CDL_1 \approx 1.172$. So

$$D_{theory} = (L_1 + L_2)(GD)(CDL_1) = (5)(0.22)(1.172) \approx 1.3$$

$$D_{actual} \approx (1.4)(1.3) = 1.82 \approx \mathbf{2\ m}$$

Part b

From Figure 8.19 for $l_1/(L_1 + L_2) = 0.2$ and $\phi = 32°$, $GF \approx 0.074$. Also, from Figure 8.22, for

$$\frac{L_1}{L_1 + L_2} = \frac{2}{2 + 3} = 0.4, \qquad \frac{l_1}{L_1 + L_2} = 0.2, \qquad \text{and} \qquad \phi = 32°$$

$CFL_1 = 1.073$. From Eq. (8.73),

$$\gamma_a = \frac{\gamma L_1^2 + \gamma' L_2^2 + 2\gamma L_1 L_2}{(L_1 + L_2)^2}$$

$$= \frac{(15.9)(2)^2 + (19.33 - 9.81)(3)^2 + (2)(15.9)(2)(3)}{(2 + 3)^2} = 13.6\ \text{kN/m}^3$$

Using Eq. (8.71) yields

$$F = \gamma_a (L_1 + L_2)^2 (GF)(CFL_1) = (13.6)(5)^2(0.074)(1.073) \approx \mathbf{27\ kN/m}$$

Part c

From Figure 8.20, for $l_1/(L_1 + L_2) = 0.2$ and $\phi = 32°$, $GM = 0.021$. Also, from Figure 8.23, for

$$\frac{L_1}{L_1 + L_2} = \frac{2}{2 + 3} = 0.4, \qquad \frac{l_1}{L_1 + L_2} = 0.2, \qquad \text{and} \qquad \phi = 32°$$

$CML_1 = 1.036$. Hence from Eq. (8.72),

$$M_{max} = \gamma_a(L_1 + L_2)^3(GM)(CML_1) = (13.6)(5)^3(0.021)(1.036) = \mathbf{36.99\ kN \cdot m/m}$$

Note: If this problem had been solved by the procedure described in Section 8.9, the following answers would have been obtained:

$$D_{actual} = 2.9\text{ m}$$

$$F = 30.86\text{ kN/m}$$

$$M_{max} = 43.72\text{ kN/m}$$

The difference between the results is primarily due to the assumed wall friction angle and the method used to calculate passive earth pressure. ▲

8.11 MOMENT REDUCTION FOR ANCHORED SHEET PILE WALLS

Sheet piles are flexible and hence sheet pile walls yield (that is, displace laterally), which redistributes the lateral earth pressure. This change tends to reduce the maximum bending moment, M_{max}, as calculated by the procedure outlined in Sections 8.9 and 8.10. For that reason, Rowe (1952, 1957) suggested a procedure to reduce the maximum design moment on the sheet pile walls ***obtained from the free earth support method.*** This section discusses the procedure of moment reduction for sheet piles ***penetrating into sand.***

In Figure 8.24, which is valid for the case of a sheet pile penetrating sand, the following notations are used.

1. H' = total height of pile driven (that is, $L_1 + L_2 + D_{actual}$)
2. Relative flexibility of pile $= \rho = 10.91 \times 10^{-7}\left(\frac{H'^4}{EI}\right)$ (8.74)

where H' is in meters
E = modulus of elasticity of the pile material (MN/m²)
I = moment of inertia of the pile section per meter of the wall (m⁴/m of wall)

3. M_d = design moment
4. M_{max} = maximum theoretical moment

In English units, Eq. (8.74) takes the form

$$\rho = \frac{H'^4}{EI} \tag{8.75}$$

where H' is in ft, E is in lb/in², and I is in in⁴/ft of the wall

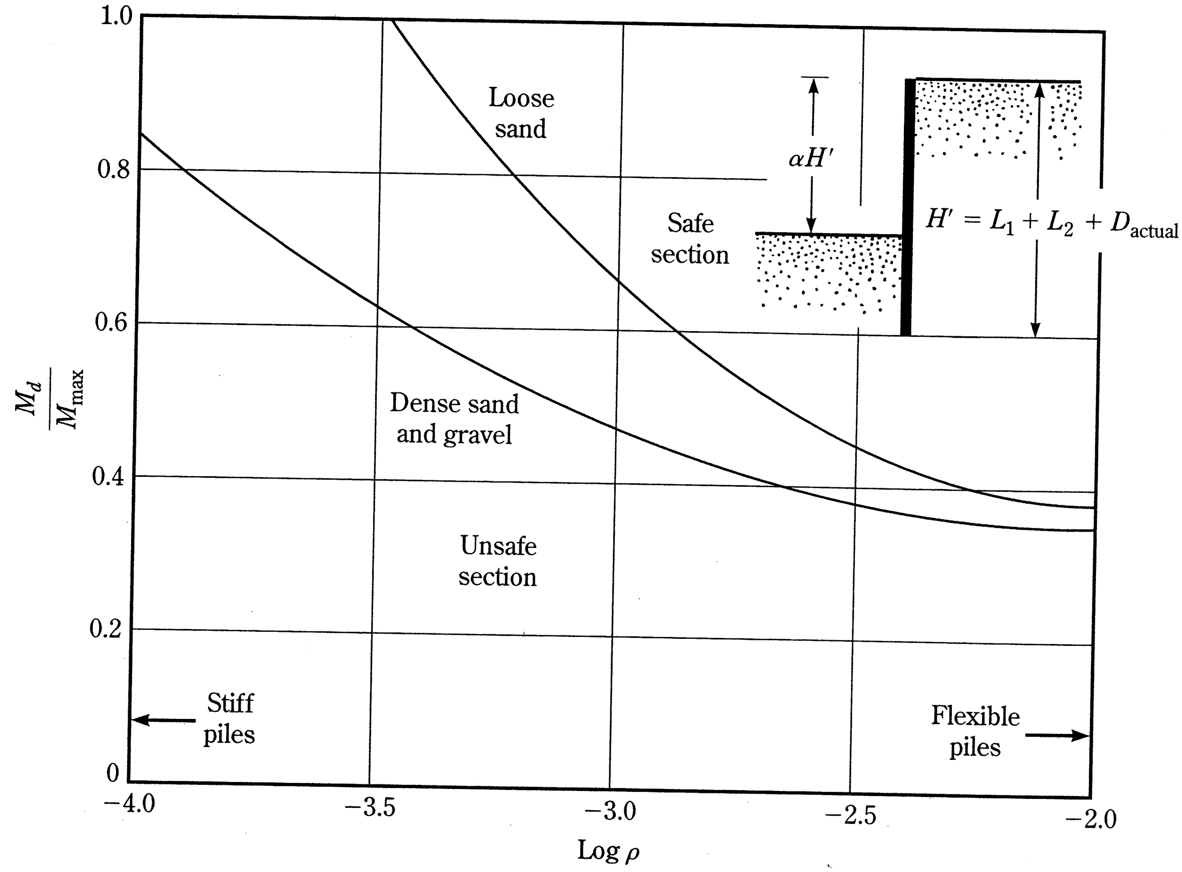


FIGURE 8.24 Plot of log ρ against M_d/M_{max} for sheet pile walls penetrating sand (after Rowe, 1952)

The procedure for the use of the moment reduction diagram (Figure 8.24) is as follows:

Step 1. Choose a sheet pile section (such as those given in Table B-1 in Appendix B).

Step 2. Find the section modulus, S, of the selected section (Step 1) per unit length of the wall.

Step 3. Determine the moment of inertia of the section (Step 1) per unit length of the wall.

Step 4. Obtain H' and calculate ρ [Eq. (8.74) or Eq. (8.75)].

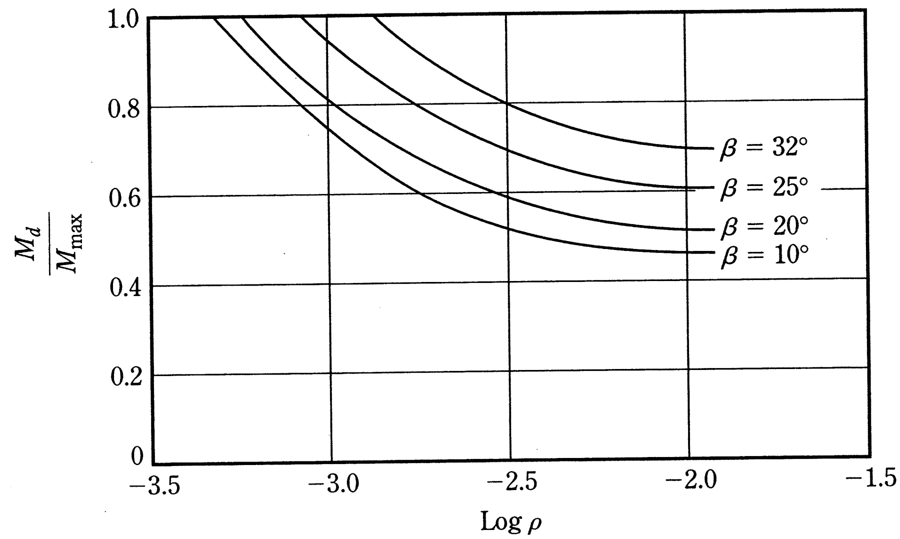
Step 5. Find log ρ.

Step 6. Find the moment capacity of the pile section chosen in Step 1 as $M_d = \sigma_{all}S$.

Step 7. Determine M_d/M_{max}. Note that M_{max} is the maximum theoretical moment determined before.

Step 8. Plot log ρ (Step 5) and M_d/M_{max} in Figure 8.24.

Step 9. Repeat Steps 1–8 for several sections. The points that fall above the curve (loose sand or dense sand, as the case may be) are *safe sections.*



▼ **FIGURE 8.25** Plot of ρ against M_d/M_{max} for sheet pile walls penetrating into sand with a sloping dredge line (after Schroeder and Roumillac, 1983)

Those points that fall below the curve are ***unsafe sections***. The cheapest section may now be chosen from those points that fall above the proper curve. Note that the section chosen will have an $M_d < M_{max}$.

For anchor sheet pile walls penetrating into sand with a sloping dredge line (Figure 8.17), a moment reduction procedure similar to that outlined above may be adopted. For this procedure, Figure 8.25 (which was developed by Schroeder and Roumillac, 1983) should be used.

▼ EXAMPLE 8.8

Refer to Example 8.6.

a. Determine the maximum moment, M_{max}.
b. Use Rowe's moment reduction technique and find a suitable sheet pile section. Use $\sigma_{all} = 172{,}500$ kN/m².

Given: $E = 207 \times 10^3$ MN/m².

Solution

Part a

From Eq. (8.69), for zero shear,

$$\tfrac{1}{2}p_1L_1 - F + p_1(z - L_1) + \tfrac{1}{2}K_a\gamma'(z - L_1)^2 = 0$$

Let $z - L_1 = x$, so

$$\tfrac{1}{2}p_1l_1 - F + p_1x + \tfrac{1}{2}K_a\gamma'x^2 = 0$$

$$(\tfrac{1}{2})(16.27)(3.05) - 115 + (16.27)(x) + (\tfrac{1}{2})(\tfrac{1}{3})(9.69)x^2 = 0$$

$$x^2 + 10.07x - 55.84 = 0$$

$x = 4$ m; $z = x + L_1 = 4 + 3.05 = 7.05$ m. Taking the moment about the point of zero shear,

$$M_{max} = -\frac{1}{2}p_1L_1\left(x + \frac{3.05}{3}\right) + F(x + 1.52) - p_1\frac{x^2}{2} - \frac{1}{2}K_a\gamma' x^2\left(\frac{x}{3}\right)$$

or

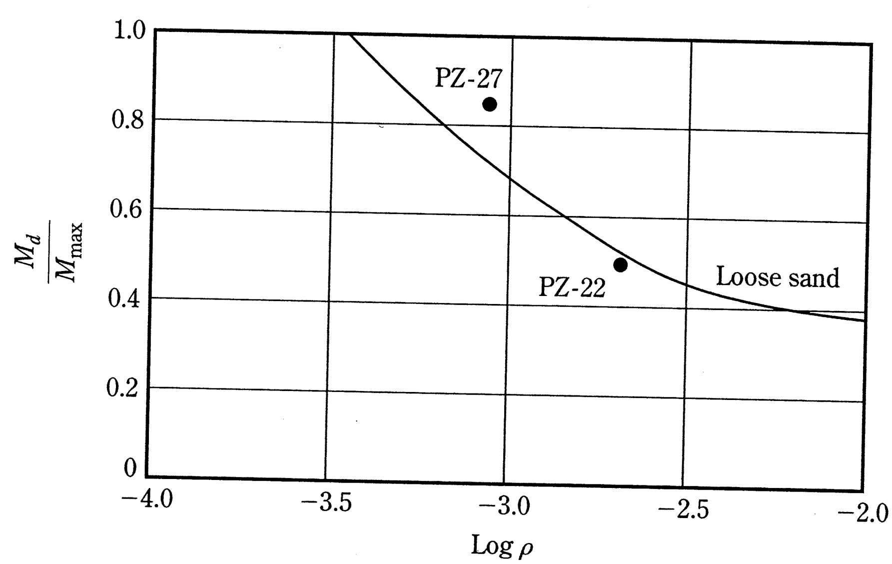
$$M_{max} = -\left(\frac{1}{2}\right)(16.27)(3.05)\left(4 + \frac{3.05}{3}\right) + (115)(4 + 1.52) - (16.27)\left(\frac{4^2}{2}\right)$$
$$-\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)(9.69)(4)^2\left(\frac{4}{3}\right) = \mathbf{344.9\ kN \cdot m/m}$$

Part b

$$H' = L_1 + L_2 + D_{actual} = 3.05 + 6.1 + 5.33 = 14.48\text{ m}$$

Section	I (m^4/m)	H' (m)	$\rho = 10.91 \times 10^{-7}\left(\frac{H'^4}{EI}\right)$	log ρ	S (m^3/m)	$M_d = S\sigma_{all}$ (kN · m/m)	$\frac{M_d}{M_{max}}$
PZ-22	115.2×10^{-6}	14.48	20.11×10^{-4}	−2.7	97×10^{-5}	167.33	0.485
PZ-27	251.5×10^{-6}	14.48	9.21×10^{-4}	−3.04	162.3×10^{-5}	284.84	0.826

Figure 8.26 shows the plot of M_d/M_{max} versus ρ. It can be seen that **PZ-27** will be sufficient.



▼ FIGURE 8.26 ▲

8.12 FREE EARTH SUPPORT METHOD FOR PENETRATION OF CLAY

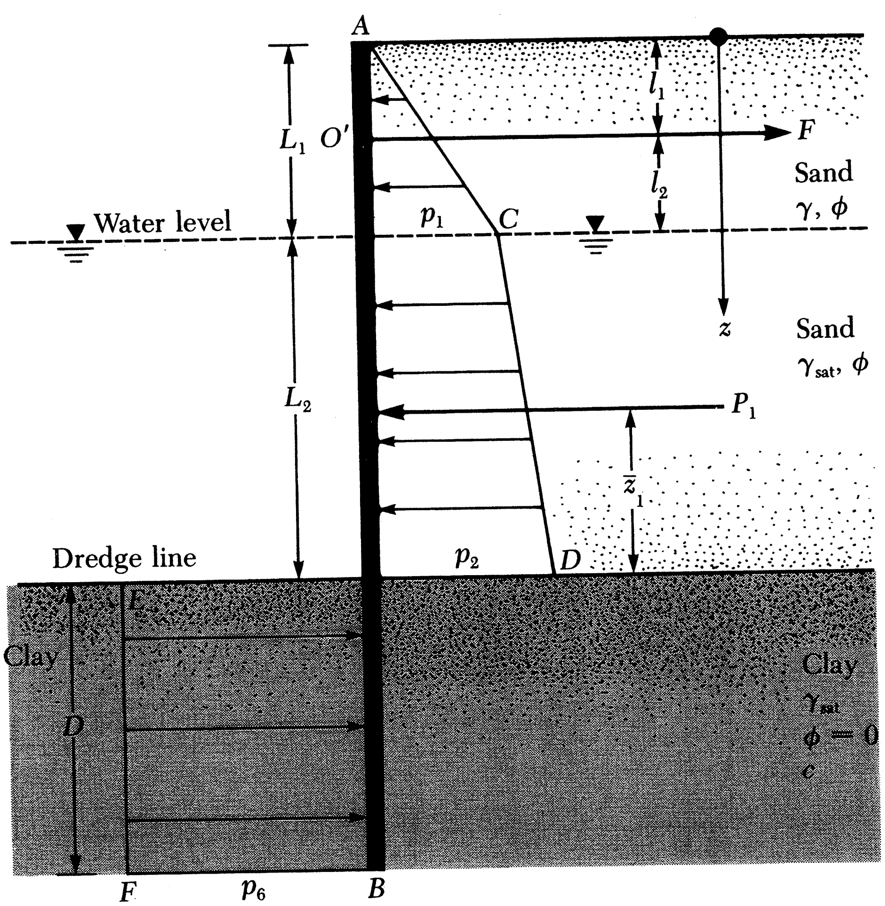
Figure 8.27 shows an anchored sheet pile wall penetrating a clay soil and having a granular soil backfill. The diagram of pressure distribution above the dredge line is similar to that shown in Figure 8.11. From Eq. (8.42), the net pressure distribution below the dredge line (from $z = L_1 + L_2$ to $z = L_1 + L_2 + D$) is

$$p_6 = 4c - (\gamma L_1 + \gamma' L_2)$$

For static equilibrium, the sum of the forces in the horizontal direction is

$$\boxed{P_1 - p_6 D = F} \tag{8.76}$$

where P_1 = area of the pressure diagram ACD
F = anchor force per unit length of the sheet pile wall



▼ **FIGURE 8.27** Anchored sheet pile wall penetrating clay

Again, taking the moment about O' produces

$$P_1(L_1 + L_2 - l_1 - \bar{z}_1) - p_6 D\left(l_2 + L_2 + \frac{D}{2}\right) = 0$$

Simplification yields

$$\boxed{p_6 D^2 + 2p_6 D(L_1 + L_2 - l_1) - 2P_1(L_1 + L_2 - l_1 - \bar{z}_1) = 0} \quad (8.77)$$

Equation (8.77) gives the theoretical depth of penetration, D.

As in Section 8.9, the maximum moment in this case occurs at depth $L_1 < z < L_1 + L_2$. The depth of zero shear (and thus the maximum moment) may be determined from Eq. (8.69).

A moment reduction technique similar to that in Section 8.11 for anchored sheet piles penetrating into clay has also been developed by Rowe (1952, 1957). This technique is presented in Figure 8.28. In this figure, the notations are as follows:

1. The stability number is

$$S_n = 1.25 \frac{c}{(\gamma L_1 + \gamma' L_2)} \quad (8.78)$$

   where c = undrained cohesion ($\phi = 0$)

   For the definition of γ, γ', L_1, and L_2, see Figure 8.27.
2. The nondimensional wall height is

$$\alpha = \frac{L_1 + L_2}{L_1 + L_2 + D_{\text{actual}}} \quad (8.79)$$

3. Flexibility number, ρ [see Eqs. (8.74) or Eq. (8.75)]
4. M_d = design moment
   $M_{\max}$ = maximum theoretical moment

The procedure for moment reduction using Figure 8.28 is as follows:

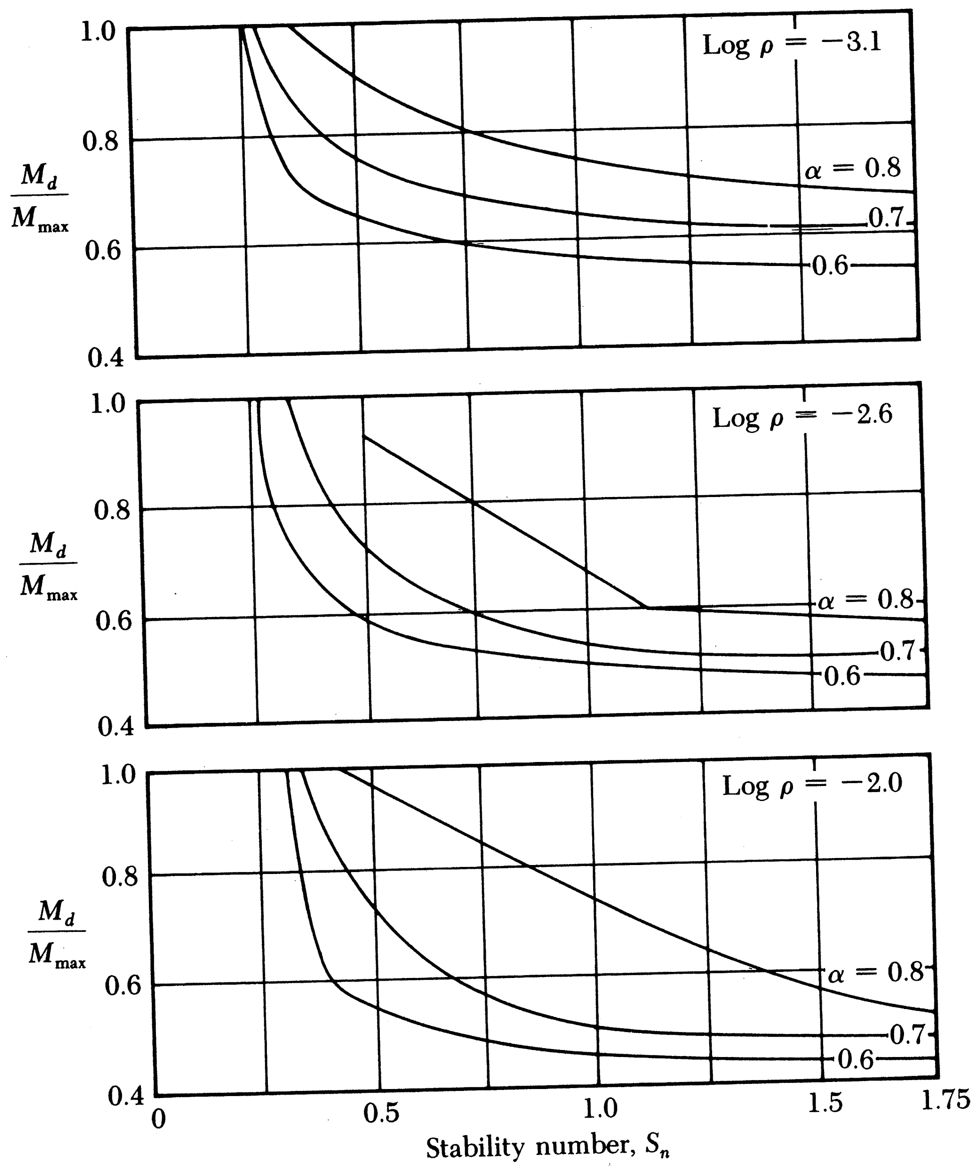
Step 1. Obtain $H' = L_1 + L_2 + D_{\text{actual}}$

Step 2. Determine $\alpha = (L_1 + L_2)/H'$.

Step 3. Determine S_n [Eq. (8.78)].

Step 4. For the magnitudes of α and S_n obtained (Steps 2 and 3), determine $M_d/M_{\max}$ for various values of $\log \rho$ from Figure 8.28 and plot $M_d/M_{\max}$ against $\log \rho$.

Step 5. Follow Steps 1–9 as outlined for the case of moment reduction of sheet pile walls penetrating granular soil (Section 8.11).



▼ **FIGURE 8.28** Plot of M_d/M_{max} against stability number for sheet pile wall penetrating clay (after Rowe, 1957)

▼ EXAMPLE 8.9

Refer to Figure 8.27, which shows that $L_1 = 10.8$ ft, $L_2 = 21.6$ ft, and $l_1 = 5.4$ ft. Also, $\gamma = 108$ lb/ft^3, $\gamma_{sat} = 127.2$ lb/ft^3, $\phi = 35°$, and $c = 850$ lb/ft^2.

a. Determine the theoretical depth of embedment.
b. Calculate the anchor force per unit length of the sheet pile wall.

Solution

Part a

Quantity to be determined	Eq. no.	Equation and calculation
K_a	—	$\tan^2\left(45 - \frac{\phi}{2}\right) = \tan^2\left(45 - \frac{35}{2}\right) = 0.271$
γ'	—	$\gamma_{sat} - \gamma_w = 127.2 - 62.4 = 64.8 \text{ lb/ft}^3$
p_1	8.1	$\gamma L_1 K_a = (0.108)(10.8)(0.271) = 0.316 \text{ kip/ft}^2$
p_2	8.2	$(\gamma L_1 + \gamma' L_2)K_a = [(0.108)(10.8) + (0.0648)(21.6)](0.271) = 0.695 \text{ kip/ft}^2$
P_1	—	$\frac{1}{2}p_1L_1 + p_1L_2 + \frac{1}{2}(p_2 - p_1)L_2$ $= (\frac{1}{2})(0.316)(10.8) + (0.316)(21.6) + (\frac{1}{2})(0.695 - 0.316)(21.6)$ $= 1.706 + 6.826 + 4.093 = 12.625 \text{ kip/ft}$
$\bar{z}_1$	—	$\frac{\Sigma M_{\text{about dredge line}}}{P_1} = \frac{(1.706)\left(21.6 + \frac{10.8}{3}\right) + (6.826)(10.8) + (4.093)\left(\frac{21.6}{3}\right)}{12.625}$ $= 11.58 \text{ ft}$
p_6	8.42	$4c - (\gamma L_1 + \gamma' L_2) = (4)(0.850) - [(0.108)(10.8) + (0.0648)(21.6)]$ $= 0.834 \text{ kip/ft}^2$
D	8.77	$p_6D^2 + 2p_6D(L_1 + L_2 - l_1) - 2P_1(L_1 + L_2 - l_1 - \bar{z}_1) = 0$ $0.834D^2 + (2)(0.834)(D)(27) - (2)(12.625)(15.42) = 0$ $D^2 + 54D - 466.85 = 0;\ D = \mathbf{7.6\ ft}$

Part b

From Eq. (8.76)

$$F = P_1 - p_6D = 12.625 - (0.834)(7.6) = \mathbf{6.29\ kip/ft}$$

▲

8.13 COMPUTATIONAL PRESSURE DIAGRAM METHOD (FOR PENETRATION INTO SANDY SOIL)

The computational pressure diagram method (CPD method) for sheet pile penetrating a sandy soil is a simplified method of design and an alternative to the free earth method described in Sections 8.9, 8.10, and 8.11 (Nataraj and Hoadley, 1984). In this method, the net pressure diagram shown in Figure 8.16 is replaced by rectangular pressure diagrams, as shown in Figure 8.29. Note that $\bar{p}_a$ is the width of the net active pressure diagram above the dredge line and $\bar{p}_p$ is the width of the net passive pressure diagram below the dredge line. The magnitudes of $\bar{p}_a$ and $\bar{p}_p$ may be

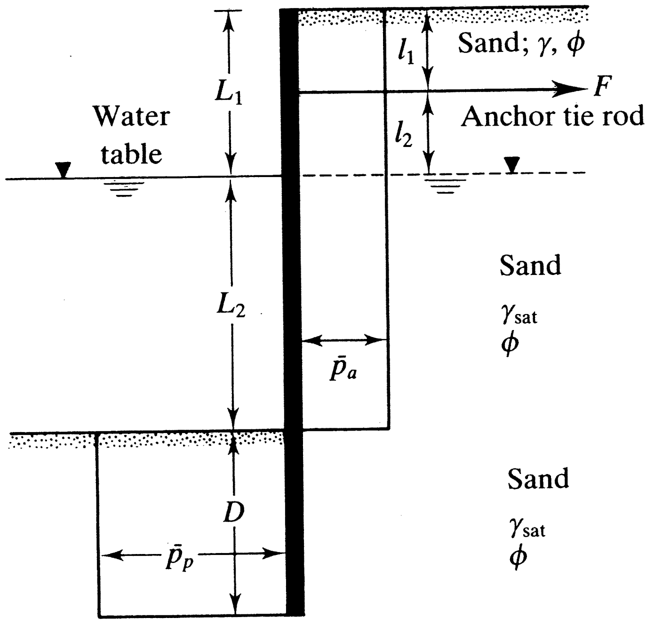


FIGURE 8.29 Computational pressure diagram method (*note:* $L_1 + L_2 = L$)

expressed as

$$\bar{p}_a = CK_a\gamma_{av}L \tag{8.80}$$

$$\bar{p}_p = RCK_a\gamma_{av}L = R\bar{p}_a \tag{8.81}$$

where γ_{av} = average effective unit weight of sand

$$\approx \frac{\gamma L_1 + \gamma' L_2}{L_1 + L_2} \tag{8.82}$$

C = coefficient

$$R = \text{coefficient} = \frac{L(L - 2l_1)}{D(2L + D - 2l_1)} \tag{8.83}$$

The range of values for C and R is given in Table 8.1.

The depth of penetration, D, anchor force per unit length of the wall, F, and maximum moment in the wall, M_{max}, are obtained from the following relationships.

TABLE 8.1 Range of Values for C and R [Eqs. (8.80) and (8.81)]

Soil type	C[a]	R
Loose sand	0.8–0.85	0.3–0.5
Medium sand	0.7–0.75	0.55–0.65
Dense sand	0.55–0.65	0.60–0.75

[a] Valid for the case in which there is no surcharge above the granular backfill (that is, on the right side of the wall as shown in Figure 8.29)

Depth of Penetration

$$D^2 + 2DL\left[1 - \left(\frac{l_1}{L}\right)\right] - \left(\frac{L^2}{R}\right)\left[1 - 2\left(\frac{l_1}{L}\right)\right] = 0 \tag{8.84}$$

Anchor Force

$$F = \bar{p}_a(L - RD) \tag{8.85}$$

Maximum Moment

$$M_{max} = 0.5\bar{p}_a L^2\left[\left(1 - \frac{RD}{L}\right)^2 - \left(\frac{2l_1}{L}\right)\left(1 - \frac{RD}{L}\right)\right] \tag{8.86}$$

Note the following qualifications.

1. The magnitude of D obtained from Eq. (8.84) is about 1.25 to 1.5 times the value of D_{theory} obtained by the conventional free earth support method (Section 8.9), so

   $$\underset{\text{Eq. (8.84)}}{\underset{\uparrow}{D}} \approx \underset{\text{Eq. (8.68)}}{\underset{\uparrow}{D_{actual}}}$$

2. The magnitude of F obtained by using Eq. (8.85) is about 1.2 to 1.6 times the value obtained by using Eq. (8.66). Thus an additional factor of safety for actual design of anchors need not be used.
3. The magnitude of M_{max} obtained from Eq. (8.86) is about 0.6 to 0.75 times the value of M_{max} obtained by the conventional free earth support method. Hence this value of M_{max} can be used as the actual design value, and Rowe's moment reduction need not be applied.

▼ EXAMPLE 8.10

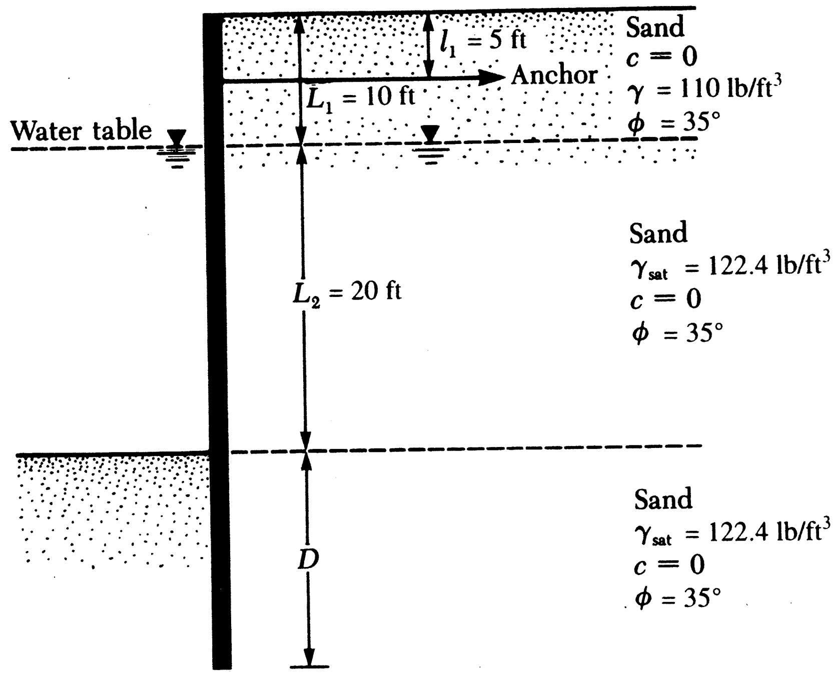
For the anchored sheet pile wall shown in Figure 8.30, determine (a) D, (b) F, and (c) M_{max}. Use the CPD method; assume that $C = 0.68$ and $R = 0.6$.

Solution

Part a

$$\gamma' = \gamma_{sat} - \gamma_w = 122.4 - 62.4 = 60 \text{ lb/ft}^3$$

From Eq. (8.82),



▼ FIGURE 8.30

$$\gamma_{av} = \frac{\gamma L_1 + \gamma' L_2}{L_1 + L_2} = \frac{(110)(10) + (60)(20)}{10 + 20} = 76.67 \text{ lb/ft}^3$$

$$K_a = \tan^2\left(45 - \frac{\phi}{2}\right) = \tan^2\left(45 - \frac{35}{2}\right) = 0.271$$

$$\bar{p}_a = CK_a\gamma_{av}L = (0.68)(0.271)(76.67)(30) = 423.9 \text{ lb/ft}^2$$

$$\bar{p}_p = R\bar{p}_a = (0.6)(423.9) = 254.3 \text{ lb/ft}^2$$

From Eq. (8.84):

$$D^2 + 2DL\left[1 - \left(\frac{l_1}{L}\right)\right] - \frac{L^2}{R}\left[1 - 2\left(\frac{l_1}{L}\right)\right] = 0$$

or

$$D^2 + 2(D)(30)\left[1 - \left(\frac{5}{30}\right)\right] - \frac{(30)^2}{0.6}\left[1 - 2\left(\frac{5}{30}\right)\right] = D^2 + 50D - 1000 = 0$$

Hence $D \approx$ **15.3 ft.**

Check for the assumption of R:

$$R = \frac{L(L - 2l_1)}{D(2L + D - 2l_1)} = \frac{30[30 - (2)(5)]}{15.3[(2)(30) + 15.3 - (2)(5)]} \approx \mathbf{0.6—OK}$$

Part b

From Eq. (8.85),

$$F = \bar{p}_a(L - RD) = 423.9[30 - (0.6)(15.3)] = \mathbf{8826\ lb/ft}$$

Part c

From Eq. (8.86),

$$M_{max} = 0.5\bar{p}_a L^2 \left[\left(1 - \frac{RD}{L}\right)^2 - \left(\frac{2l_1}{L}\right)\left(1 - \frac{RD}{L}\right)\right]$$

$$1 - \frac{RD}{L} = 1 - \frac{(0.6)(15.3)}{30} = 0.694$$

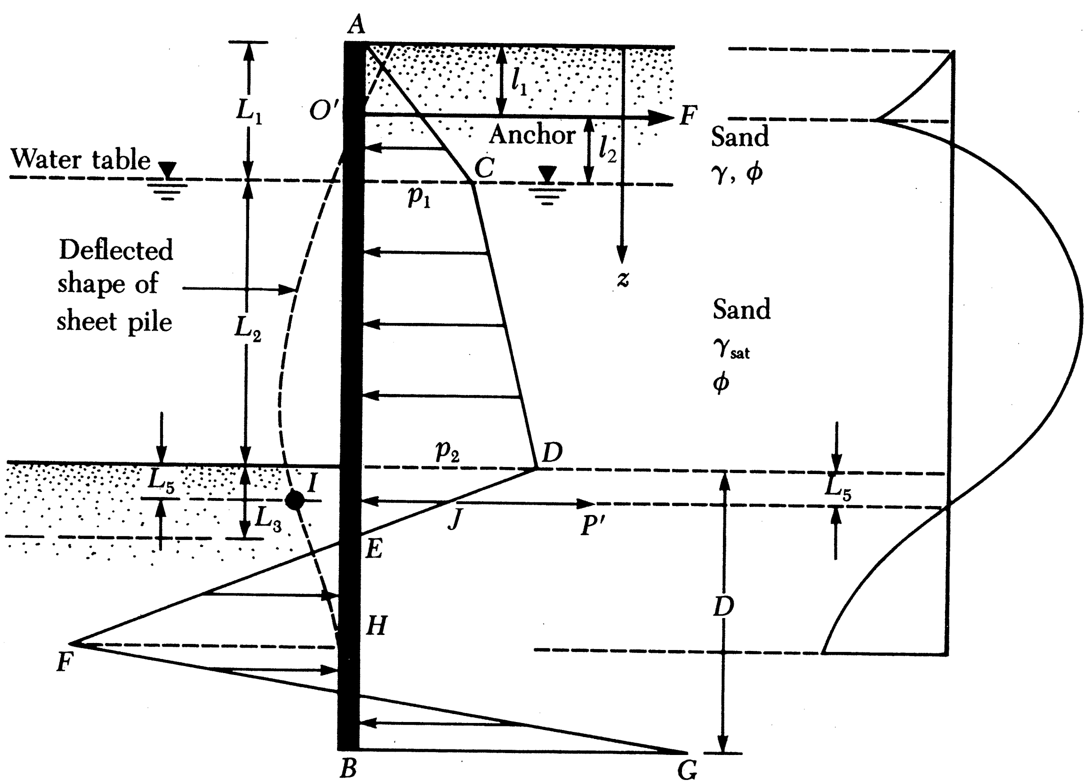
So

$$M_{max} = (0.5)(423.9)(30)^2\left[(0.694)^2 - \frac{(2)(5)(0.694)}{30}\right] = \mathbf{47{,}810\ lb\text{-}ft/ft}$$

▲

8.14 FIXED EARTH SUPPORT METHOD FOR PENETRATION INTO SANDY SOIL

When using the fixed earth support method we assume that the toe of the pile is restrained from rotating as shown in Figure 8.31a. In the fixed earth support solution,



(a) Pressure diagram (b) Moment diagram

▼ FIGURE 8.31 Fixed earth support method for penetration of sandy soil a simplified method called the ***equivalent beam solution*** is generally used to calculate L_3 and, thus, D. The development of the equivalent beam method is generally attributed to Blum (1931).

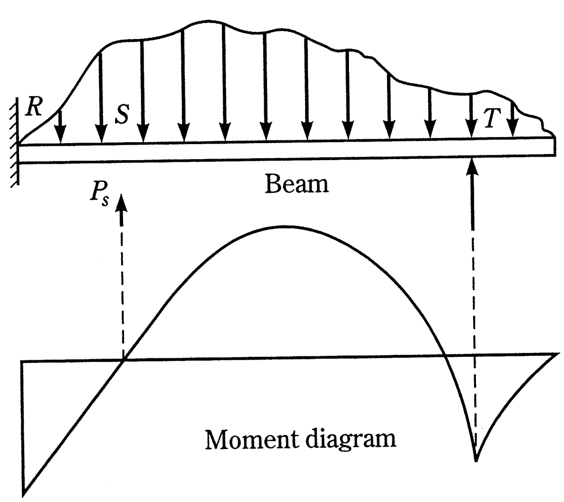
In order to understand this method, compare the sheet pile to a loaded cantilever beam *RSTU,* as shown in Figure 8.32. Note that the support at T for the beam is equivalent to the *anchor load reaction* (F) on the sheet pile (Figure 8.31). It can be seen that the point S of the beam *RSTU* is the inflection point of the elastic line of the beam, which is equivalent to point I in Figure 8.31. If the beam is cut at S and a free support (reaction P_s) is provided at that point, the bending moment diagram for portion *STU* of the beam will remain unchanged. This beam *STU* will be equivalent to the section *STU* of the beam *RSTU.* The force P' shown in Figure 8.31a at I will be equivalent to the reaction P_s on the beam (Figure 8.32).

Following is an approximate procedure for the design of an anchored sheet pile wall (Cornfield, 1975). Refer to Figure 8.31.

1. Determine L_5, which is a function of the soil friction angle ϕ below the dredge line, from the following:

ϕ (deg)	$\frac{L_5}{L_1 + L_2}$
30	0.08
35	0.03
40	0

2. Calculate the span of the equivalent beam as $l_2 + L_2 + L_5 = L'$.
3. Calculate the total load of the span, W. This is the area of the pressure diagram between O' and I.



▼ **FIGURE 8.32** Equivalent cantilever beam concept

4. Calculate the maximum moment, M_{max}, as $WL'/8$.
5. Calculate P' by taking the moment about O', or

$$P' = \frac{1}{L'}\text{(moment of area } ACDJI \text{ about } O')$$

6. Calculate D as

$$D = L_5 + 1.2\sqrt{\frac{6P'}{(K_p - K_a)\gamma'}} \tag{8.87}$$

7. Calculate the anchor force per unit length, F, by taking the moment about I, or

$$F = \frac{1}{L'}\text{(moment of area } ACDJI \text{ about } I)$$

▼ EXAMPLE 8.11

Consider the anchored sheet pile structure described in Example 8.6. Using the equivalent beam method described in Section 8.14, determine

a. Maximum moment
b. Theoretical depth of penetration
c. Anchor force per unit length of the structure

Solution

Part a

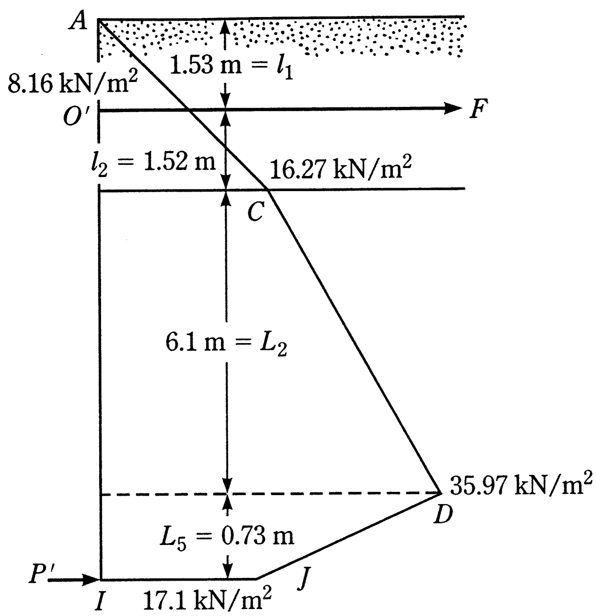
Determination of L_5: For $\phi = 30°$,

$$\frac{L_5}{L_1 + L_2} = 0.08$$

$$\frac{L_5}{3.05 + 6.1} = 0.08$$

$$L_5 = 0.73$$

Net Pressure Diagram: From Example 8.6, $K_a = \frac{1}{3}$, $K_p = 3$, $\gamma = 16\ \text{kN/m}^3$, $\gamma' = 9.69\ \text{kN/m}^3$, $p_1 = 16.27\ \text{kN/m}^2$, $p_2 = 35.97\ \text{kN/m}^2$. The net active pressure at a



▼ FIGURE 8.33

depth L_5 below the dredge line can be calculated as

$$p_2 - \gamma'(K_p - K_a)L_5 = 35.97 - (9.69)(3 - 0.333)(0.73) = 17.1 \text{ kN/m}^2$$

The net pressure diagram from $z = 0$ to $z = L_1 + L_2 + L_5$ is shown in Figure 8.33.

Maximum Moment:

$$W = \left(\frac{1}{2}\right)(8.16 + 16.27)(1.52) + \left(\frac{1}{2}\right)(6.1)(16.27 + 35.97)$$
$$+ \left(\frac{1}{2}\right)(0.73)(35.97 + 17.1)$$
$$= 197.2 \text{ kN/m}$$

$$L' = l_2 + L_2 + L_5 = 1.52 + 6.1 + 0.73 = 8.35 \text{ m}$$

$$M_{max} = \frac{WL'}{8} = \frac{(197.2)(8.35)}{8} = \mathbf{205.8\ kN \cdot m/m}$$

Part b

$$P' = \frac{1}{L'} \text{(moment of area } ACDJI \text{ about } O')$$

$$P' = \frac{1}{8.35}\left[\begin{array}{l}\left(\frac{1}{2}\right)(16.27)(3.05)\left(\frac{2}{3}\times 3.05 - 1.53\right) + (16.27)(6.1)\left(1.52 + \frac{6.1}{2}\right) \\ + \left(\frac{1}{2}\right)(6.1)(35.97 - 16.27)\left(1.52 + \frac{2}{3}\times 6.1\right) + \left(\frac{1}{2}\right)(35.97 + 17.1) \\ \times (0.73)\left(1.52 + 6.1 + \frac{0.73}{2}\right)\end{array}\right]$$

↑
Approximate

$$= 114.48 \text{ kN/m}$$

From Eq. (8.87)

$$D = L_5 + 1.2\sqrt{\frac{6P'}{(K_p - K_a)\gamma'}} = 0.73 + 1.2\sqrt{\frac{(6)(114.48)}{(3 - 0.333)(9.69)}} = \mathbf{6.92\ m}$$

Part c

Taking the moment about I (Figure 8.33)

$$F = \frac{1}{8.35}\left[\begin{array}{l}\left(\frac{1}{2}(16.27)(3.05)\left(0.73 + 6.1 + \frac{3.05}{3}\right) + (16.27)(6.1)\left(0.73 + \frac{6.1}{2}\right)\right. \\ + \left(\frac{1}{2}\right)(6.1)(35.97 - 16.27)\left(0.73 + \frac{6.1}{3}\right) + \left(\frac{1}{2}\right)(35.97 + 17.1)(0.73)\left(\frac{0.73}{2}\right)\end{array}\right]$$

↑
Approximate

$$= \mathbf{88.95\ kN/m}$$

▲

8.15 FIELD OBSERVATIONS FOR ANCHORED SHEET PILE WALLS

In the preceding sections, large factors of safety were used for the depth of penetration, D. In most cases, designers use smaller magnitudes of soil friction angle, ϕ, thereby ensuring a built-in factor of safety for the active earth pressure. This procedure is followed primarily because of the uncertainties involved in predicting the actual earth pressure a sheet pile wall will be subjected to in the field. In addition, Casagrande (1973) observed that if the soil behind the sheet pile wall has grain sizes that are predominantly smaller than those of coarse sand, the active earth pressure after construction sometimes increases to an at-rest earth pressure condition. Such an increase causes a large increase in the anchor force, F. The following two case histories are given by Casagrande (1973).

Equation (8.112) gives

$$\gamma_{av} = \frac{1}{H}[\gamma_1 H_1 + \gamma_2 H_2 + \cdots + \gamma_n H_n)$$

$$= \frac{1}{70}[(110)(13) + (127)(33) + (130)(11) + (135)(6) + (135)(7)]$$

$$= 125.8 \text{ lb/ft}^3$$

For the equivalent clay layer of 70 ft,

$$\frac{\gamma_{av} H}{c_{av}} = \frac{(125.8)(70)}{730} = 12.06 > 4$$

Hence the apparent pressure envelope will be of the type shown in Figure 8.51. From Eq. (8.107)

$$p_a = \gamma H\left[1 - \left(\frac{4c_{av}}{\gamma_{av} H}\right)\right] = (125.8)(70)\left[1 - \frac{(4)(730)}{(125.8)(70)}\right] = 5886 \text{ lb/ft}^2$$

The pressure envelope is shown in Figure 8.80. The area of this pressure diagram is 201 kip/ft. Thus Peck's pressure envelope gives a lateral earth pressure of about 1.8 times that actually observed. This result is not surprising because the pressure envelope provided by Figure 8.51 is an envelope developed considering several cuts made at different locations. Under actual field conditions, past experience with the behavior of similar soils can help reduce overdesigning substantially.

PROBLEMS

8.1 Figure P8.1 shows a cantilever sheet pile wall penetrating a granular soil. Here, L_1 = 8 ft, L_2 = 15 ft, γ = 100 lb/ft^3, γ_{sat} = 110 lb/ft^3, ϕ = 35°.
- **a.** What is the theoretical depth of embedment, D?
- **b.** For a 30% increase in D, what should be the total length of the sheet piles?
- **c.** Determine the theoretical maximum moment of the sheet pile.

8.2 Solve Problem 8.1 with the following: L_1 = 4 m, L_2 = 8 m, γ = 16.1 kN/m^3, γ_{sat} = 18.2 kN/m^3, ϕ = 32°.

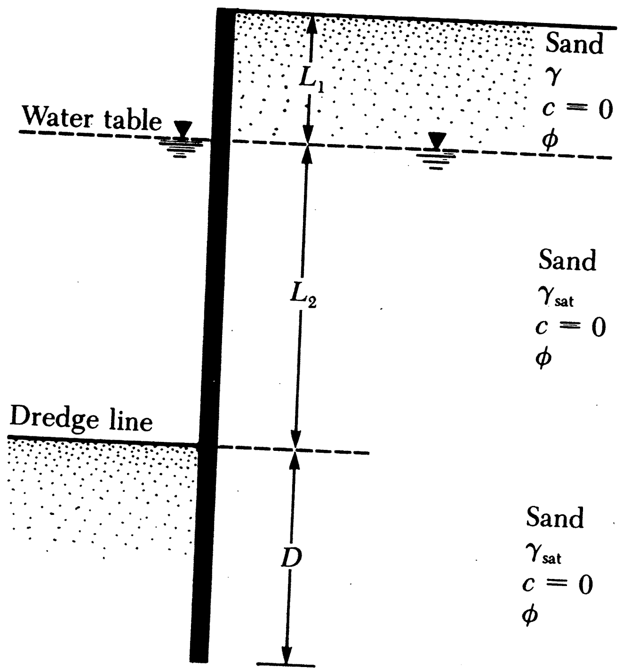
8.3 Redo Problem 8.1 with the following: L_1 = 3 m, L_2 = 6 m, γ = 17.3 kN/m^3, γ_{sat} = 19.4 kN/m^3, ϕ = 30°.

8.4 Refer to Figure 8.9. Given: L = 15 ft, γ = 108 lb/ft^3, and ϕ = 35°. Calculate the theoretical depth of penetration, D, and the maximum moment.

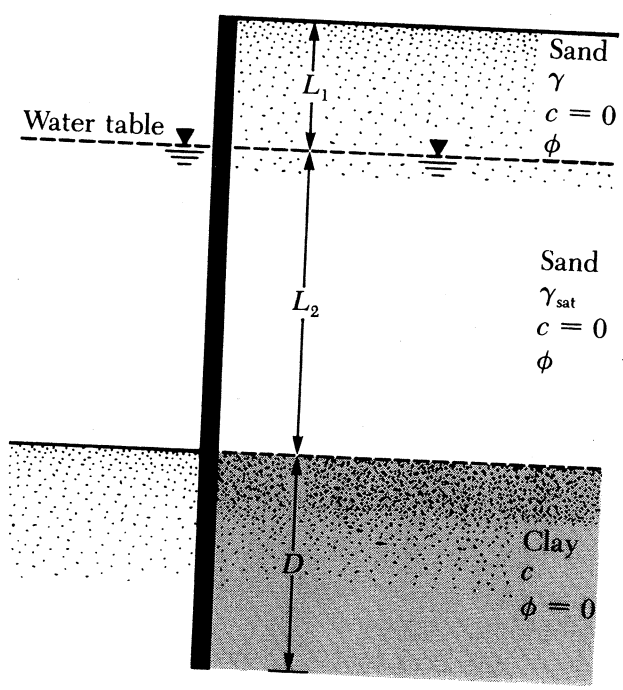
8.5 Repeat Problem 8.4 with the following: L = 3 m, γ = 16.7 kN/m^3, ϕ = 30°.

8.6 Refer to Figure 8.10, which shows a free cantilever sheet pile wall penetrating a sand layer. Determine the theoretical depth of penetration and the maximum moment. Given: γ = 17 kN/m^3, ϕ = 36°, L = 4 m, and P = 15 kN/m.

8.7 Refer to Figure P8.7, for which L_1 = 2.4 m, L_2 = 4.6 m, γ = 15.7 kN/m^3, γ_{sat} = 17.3 kN/m^3, c = 29 kN/m^2, ϕ = 30°.
- **a.** Find the theoretical depth of penetration, D.
- **b.** Increase D by 40%. What length of sheet piles is needed?
- **c.** Determine the theoretical maximum moment in the sheet pile.



▼ FIGURE P8.1

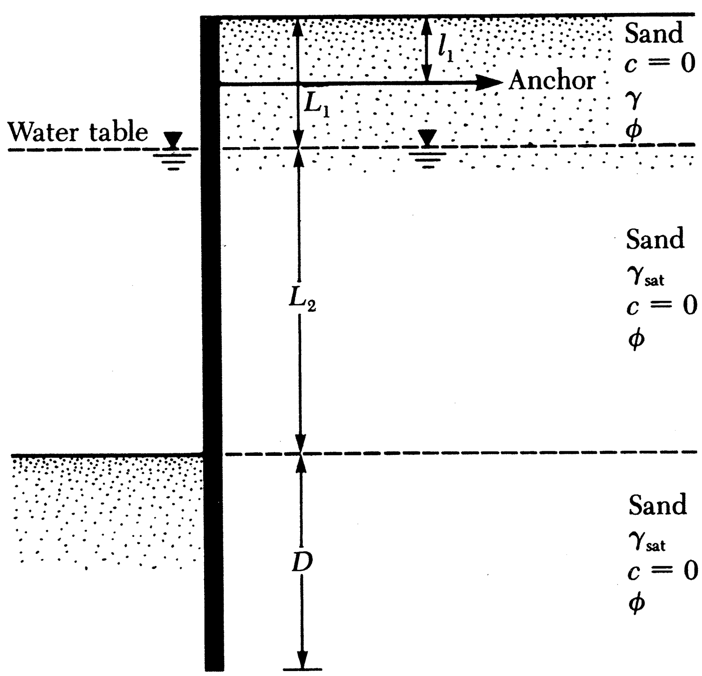


▼ FIGURE P8.7

8.8 Solve Problem 8.7 with the following: $L_1 = 5$ ft, $L_2 = 10$ ft, $\gamma = 108$ lb/ft^3, $\gamma_{sat} = 122.4$ lb/ft^3, $\phi = 36°$, and $c = 800$ lb/ft^2.

8.9 Refer to Figure 8.14. Here, $L = 4$ m, $P = 8$ kN/m, $c = 45$ kN/m^2, and $\gamma_{sat} = 19.2$ kN/m^3. Find the theoretical value of D and the maximum moment.

8.10 An anchored sheet pile bulkhead is shown in Figure P8.10. Given: $L_1 = 5$ m, $L_2 = 8$ m, $l_1 = 2$ m, $\gamma = 16.5$ kN/m^3, $\gamma_{sat} = 18.5$ kN/m^3, and $\phi = 32°$.



▼ FIGURE P8.10

a. Calculate the theoretical value of the depth of embedment, D.
b. Draw the pressure distribution diagram.
c. Determine the anchor force per unit length of the wall.

Use the free earth support method.

8.11 Refer to Problem 8.10. Assume that $D_{actual} = 1.25 D_{theory}$.
a. Determine the theoretical maximum moment.
b. Choose a sheet pile section by using Rowe's moment reduction technique. Use $E = 207 \times 10^3$ MN/m^2, $\sigma_{all} = 210{,}000$ kN/m^2.

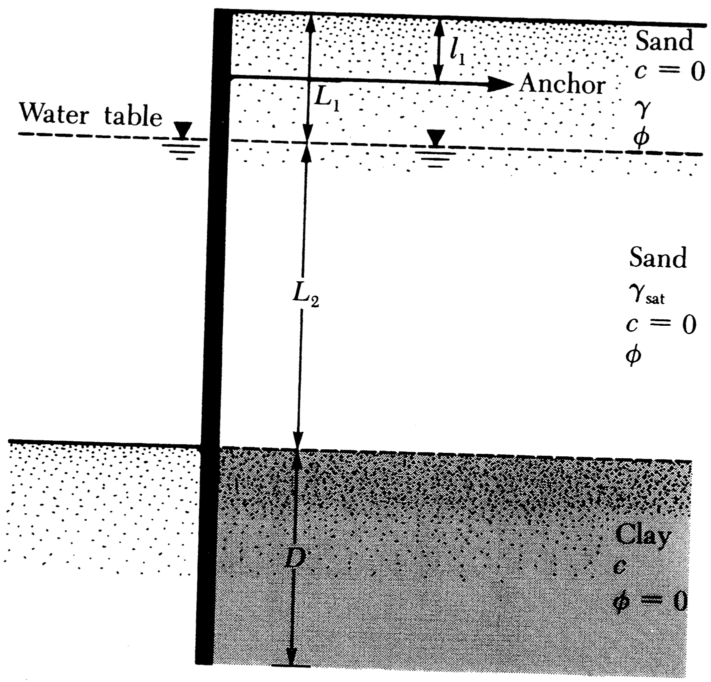
8.12 Redo Problem 8.10 with the following: $L_1 = 10$ ft, $L_2 = 25$ ft, $l_1 = 4$ ft, $\gamma = 120$ lb/ft^3, $\gamma_{sat} = 129.4$ lb/ft^3, and $\phi = 40°$.

8.13 Refer to Problem 8.12. Assume that $D_{actual} = 1.3\, D_{theoretical}$. Using Rowe's moment reduction method, choose a sheet pile section. Use $E = 29 \times 10^6$ lb/in^2 and $\sigma_{all} = 25$ kip/in^2.

8.14 Refer to Figure P8.10, for which $L_1 = 3$ m, $L_2 = 6$ m, $l_1 = 1.5$ m, $\gamma = 17.5$ kN/m^3, $\gamma_{sat} = 19.5$ kN/m^3, and $\phi = 35°$. Use the computational diagram method (Section 8.13) to determine D, F, and M_{max}. Assume that $C = 0.68$ and $R = 0.6$.

8.15 Refer to Figure P8.10. Given: $L_1 = 4$ m, $L_2 = 8$ m, $l_1 = l_2 = 2$ m, $\gamma = 16$ kN/m^3, $\gamma_{sat} = 18.5$ kN/m^3, and $\phi = 35°$.
Determine:
a. Theoretical depth of penetration
b. Anchor force per unit length
c. Maximum moment in the sheet pile. Use the charts presented in Section 8.10.

8.16 An anchored sheet pile bulkhead is shown in Figure P8.16. Given: $L_1 = 2$ m, $L_2 = 6$ m, $l_1 = 1$ m, $\gamma = 16$ kN/m^3, $\gamma_{sat} = 18.86$ kN/m^3, $\phi = 32°$, and $c = 27$ kN/m^2.



▼ FIGURE P8.16

a. Determine the theoretical depth of embedment, D.
b. Calculate the anchor force per unit length of the sheet pile wall.

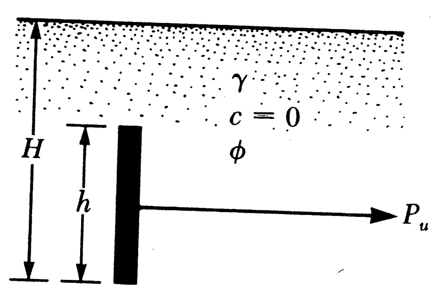
Use the free earth support method.

8.17 Repeat Problem 8.16 with the following: $L_1 = 8$ ft, $L_2 = 20$ ft, $l_1 = 4$ ft, $c = 1500$ lb/ft^2, $\gamma = 115$ lb/ft^3, $\gamma_{sat} = 128$ lb/ft^3, and $\phi = 40°$.

8.18 Refer to Figure P8.10. Given: $L_1 = 9$ ft, $L_2 = 26$ ft, $l_1 = 5$ ft, $\gamma = 108.5$ lb/ft^3, $\gamma_{sat} = 128.5$ lb/ft^3, and $\phi = 35°$. Use the fixed earth support method to determine the following:
a. Maximum moment
b. Theoretical depth of penetration
c. Anchor force per unit length of sheet pile wall

8.19 Refer to Figure 8.42a. For the anchor slab in sand, $H = 5$ ft, $h = 3$ ft, $B = 4$ ft, $S' = 7$ ft, $\phi = 30°$, and $\gamma = 110$ lb/ft^3. Calculate the ultimate holding capacity of each anchor. The anchor plates are made of concrete and have a thickness of 3 in. Use $\gamma_{concrete} = 150$ lb/ft^3. Use Ovesen and Stromann's method.

8.20 A single anchor slab is shown in Figure P8.20. Here, $H = 0.9$ m, $h = 0.3$ m, $\gamma = 17$ kN/m^3, and $\phi = 32°$. Calculate the ultimate holding capacity of the anchor slab if the width B is (a) 0.3 m, (b) 0.6 m, and (c) 0.9 m. (*Note:* center-to-center spacing, $S' = \infty$.) Use the empirical correlation given in Section 8.17.

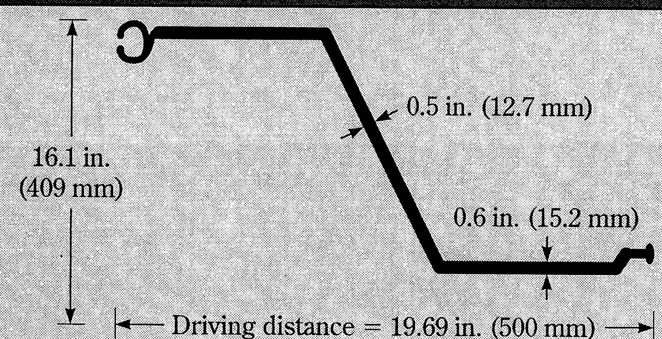
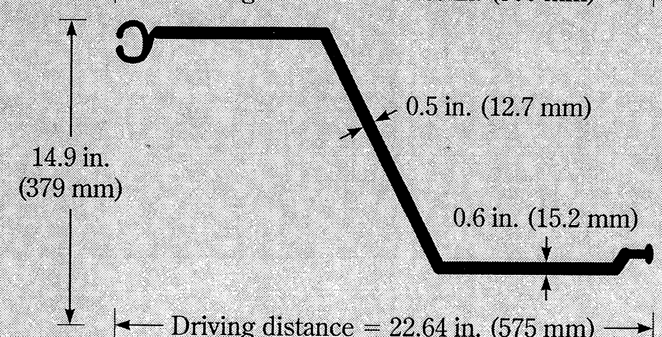
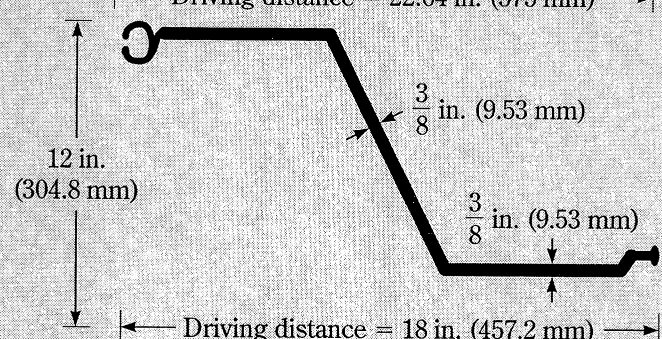


▼ FIGURE P8.20

APPENDIX C

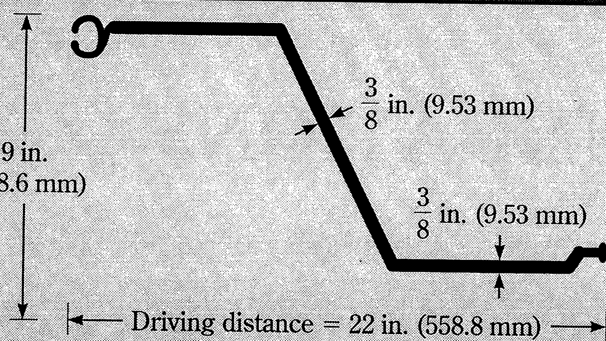
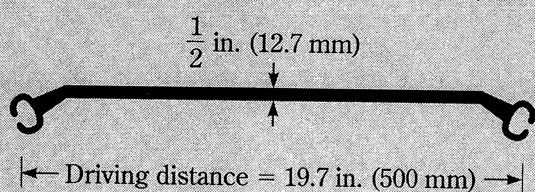
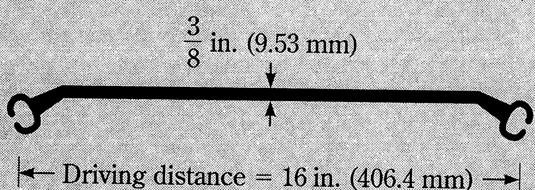
SHEET PILE SECTIONS

TABLE C.1 Properties of Some Sheet Pile Sections (Produced by Bethlehem Steel Corporation)

Section designation	Sketch of section	Section modulus in³/ft of wall	Section modulus m³/m of wall	Moment of inertia in⁴/ft of wall	Moment of inertia m⁴/m of wall
PZ-40		60.7	326.4×10^{-5}	490.8	670.5×10^{-6}
PZ-35		48.5	260.5×10^{-5}	361.2	493.4×10^{-6}
PZ-27		30.2	162.3×10^{-5}	184.2	251.5×10^{-6}

(continued)

▼ **TABLE C.1** (Continued)

Section designation	Sketch of section	Section modulus		Moment of inertia	
		in^3/ft of wall	m^3/m of wall	in^4/ft of wall	m^4/m of wall
PZ-22		18.1	97×10^{-5}	84.4	115.2×10^{-6}
PSA-31		2.01	10.8×10^{-5}	3.23	4.41×10^{-6}
PSA-23		2.4	12.8×10^{-5}	4.13	5.63×10^{-6}