1
LATERAL EARTH PRESSURE

1.1 INTRODUCTION

Retaining structures commonly used in foundation engineering, such as retaining walls, basement walls and bulkheads to support almost vertical slopes of earth masses. Proper design and construction of these structures require a thorough knowledge of the lateral forces interacting between the retaining structures and the soil masses retained. These lateral forces are due to lateral earth pressure. This chapter discusses lateral earth pressure for different backfill materials.

1.2 DIFFERENT TYPES OF LATERAL EARTH PRESSURES

Soil is neither a solid nor a liquid, but it has some of the characteristics of both of these states of mater. One of its characteristics which is similar to that of a liquid is the tendency to exert a lateral pressure against an object with which it comes in contact. This property of soil is highly important in engineering practice, since it influences the design of retaining walls, sheet pile bulk heads, basement walls of buildings, abutments, and many other structures of a similar nature.

There are two distinct kinds of lateral soil pressure, and a clear understanding of the nature of each is essential. First, consider a retaining wall, which holds back a mass of clean, dry, cohesionless sand. The sand exerts a push against the wall by virtue of its tendency to slip laterally and seek its natural slope or angle of repose, thus making the wall to move slightly away from the backfilled soil mass. This kind of pressure is known as the active lateral pressure of the soil. In this case the soil is the actuating element, and if stability is maintained, the structure must be able to withstand the pressure exerted by the soil (Fig. 1.1). Next, imagine that in some manner the retaining wall is caused to move toward the soil. When this situation develops, the retaining wall or other type of structure is the actuating element, and the soil provides the resistance for maintaining stability. The pressure, which the soil develops in response to movement toward it, is called the passive earth pressure, or more appropriately passive earth resistance (Fig. 1.2). It may be very much greater than the active pressure. The surface over which the sheared off soil wedge tends to slide is referred to as the surface of sliding or rupture.

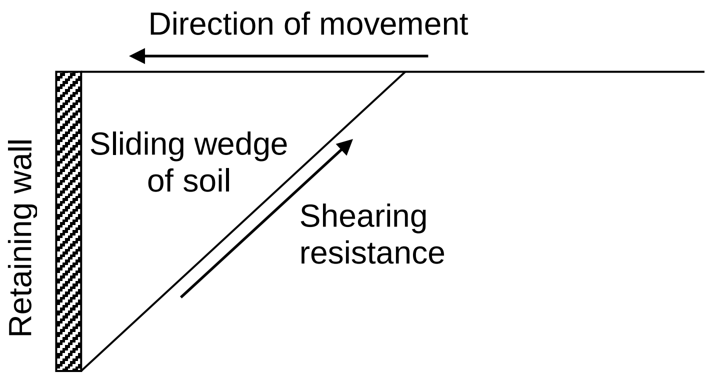


Fig. 1.1 Direction of movement and shearing resistance; active pressure case

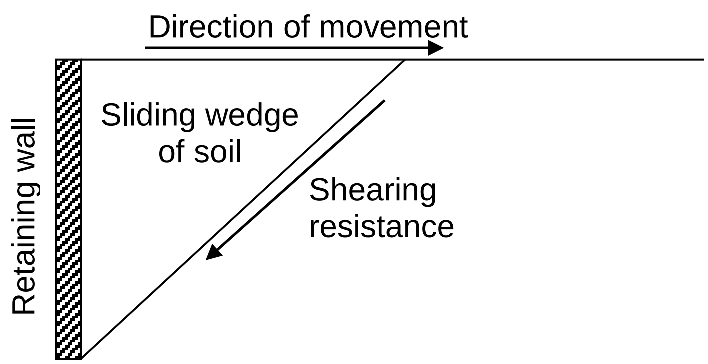


Fig. 1.2 Direction of movement and shearing resistance: passive pressure case.

Earth pressure at Rest;- Active pressures are accompanied by movements directed away from the soil, and passive pressures, which are much larger, are accompanied by movements toward the soil. There must be therefore an intermediate pressure situation when the retaining structure is perfectly stationary and does not move in either direction. The pressure, which develops at zero movement, is called earth pressure at rest. Its value somewhat larger than the value of active pressure but it is considerably less than the passive pressure.

1.3 VARIATION OF LATERAL EARTH PRESSURE WITH WALL MOVEMENT

Consider a rigid retaining wall with a plane vertical face, as shown in Fig. 1.3 (a), backfilled with cohesionless soil. If the wall does not move even after filling the materials, the pressure exerted on the wall is termed as pressure for the at rest condition of the wall. If suppose the wall gradually rotates about the point **A** and moves away from the backfill, the unit pressure on the wall gets gradually reduced and after a particular displacement of the wall at the top, the pressure reaches a constant value. This pressure is the minimum possible value and is termed as active pressure. The pressure is called as active because the weight of the backfill is responsible for the movement of the wall.

If the wall is now rotated about **A** towards the backfill, the unit pressure on the wall increases from the value of at rest condition to the maximum possible value. The maximum pressure that developed is termed as passive earth pressure. The pressure is called as passive because the weight of the backfill opposes the move of the wall. The gradual decrease or increase of pressure on the wall with the movement of the wall from at rest condition may be depicted as shown in Fig. 1.3 (b).

The movement Δp required to develop passive state is considerably larger than Δa required for active state.

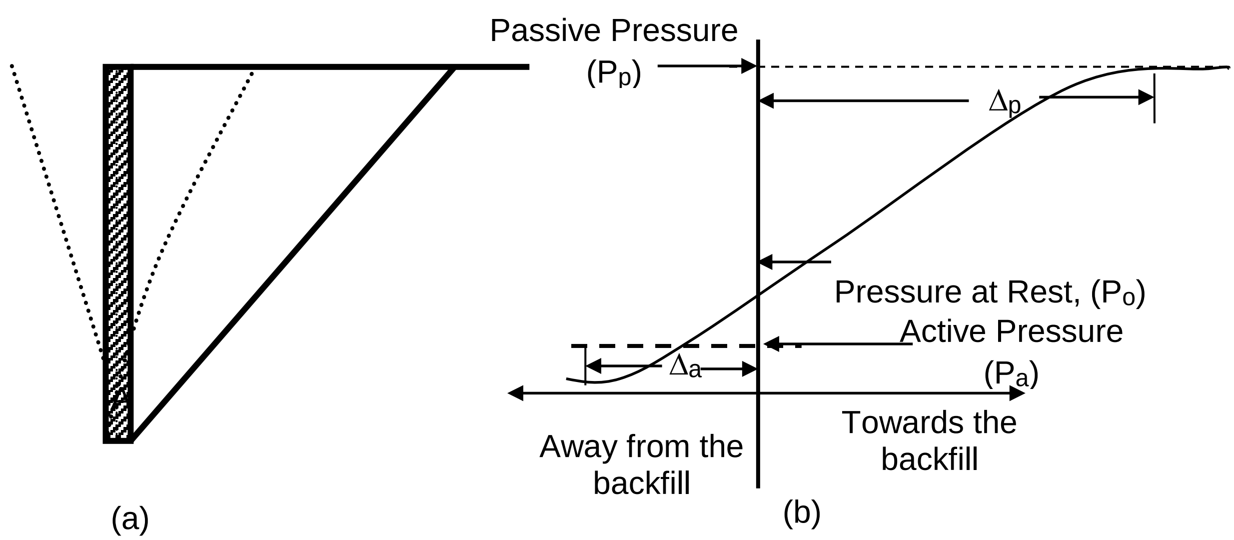


Fig. 1.3 Variation of lateral earth pressure with wall movement

1.4 RANKINE'S EARTH PRESSURE THEORY

There are two well-known classical earth pressure theories, the Rankine theory and the coulomb theory. These theories propose to estimate the magnitudes of two pressures called active earth pressure and passive earth pressure. As originally proposed, the Rankine's theory covered the uniform cohesionless soils only, although latter on it was extended to stratified, partially immersed cohesionless masses and cohesive soils too.

Rankine (1857) considered the equilibrium of a soil element within a soil mass bounded by a plane surface. The following assumptions were made by Rankine for the derivation of earth pressure.

a. The soil mass is homogeneous and semi-infinite
b. The back of the retaining wall is smooth and vertical
c. The backfill slope must be less than the backfill friction angle.
d. The failure wedge is a plane surface and is a function of soil's friction and the backfill slope
e. The soil element is in the state of plastic equilibrium.

A mass of soil is said to be in a state of plastic equilibrium if failure is about to occur simultaneously at all points within the mass.

1.4.1 Relationships between Vertical Pressure and Active and Passive Earth Pressures at State of Plastic Equilibrium.

Expressions for the relationships between the vertical pressure and the active earth pressure and the passive earth pressure are developed as explained below:

Active Pressure

Consider a semi-infinite mass of homogeneous and isotropic soil with a horizontal surface and having a vertical boundary formed by a smooth wall surface extending to semi-infinite depth, as shown in Fig 1.4 (a). A soil element at any depth h is subjected to a vertical stress σ_v (=γh) and a horizontal pressure σ_h and, since there can be no lateral transfer of weight if the surface is horizontal; no shear stresses exist on horizontal and vertical planes. The vertical and horizontal planes would therefore, act as the principal planes, and their corresponding stresses would be the principal stresses.

If there is now a movement of the wall away from the soil, the value of σ_h decreases as the soil dilates or expands outwards, the decrease in σ_h being an unknown function of the lateral strain in the soil. If the expansion is large enough the value of σ_h decreases to a minimum value such that a state of plastic equilibrium develops. Since this state is developed by a decrease in the horizontal pressure σ_h, this must be the minor principal stress (σ_3). The vertical pressure σ_v is then the major principal stress (σ_1).

The stress σ_1 (= γh) is the overburden pressure at depth h and is fixed value for any depth. The horizontal stress σ_3=P_a is defined as the active pressure, being due directly to the self-weight of the soil. The value of this active earth pressure is determined when a Mohr circle through the point representing σ_v= γh touches the failure envelope for the soil as shown in Fig. 1.4 (b). The relationship between σ_1=γh and P_a when the soil reaches a state of plastic equilibrium can be derived from the geometry of this Mohr circle as follows:

$$\sin \phi = \frac{\gamma h - P_a}{\gamma h + P_a}$$

$$(\gamma h + P_a) \sin \phi = \gamma h - P_a$$

$$P_a \sin \phi + P_a = \gamma h - \gamma h \sin \phi$$

$$P_a = \gamma h \frac{1 - \sin \phi}{1 + \sin \phi} \quad \ldots\ldots\ldots\ldots\ldots \quad (1.1)$$

$$P_a = \gamma h \, k_a \quad \ldots\ldots\ldots\ldots\ldots \quad (1.2)$$

where $$k_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \tan^2 \left(45 - \frac{\phi}{2}\right)$$

= Coefficient of active earth pressure

Passive Pressure

In the above derivation a movement of the wall away from the soil was considered. If, on the other hand, the wall is moved against the soil mass, there will be lateral compression of the soil and the value of σ_h will increase until a state of plastic equilibrium is reached. For this condition σ_h becomes a maximum value and is the major principal stress (σ_h=σ_1). The pressure σ_v, equal to the overburden pressure, is then the minor principal stress, i.e. σ_v= σ_3 = γh.

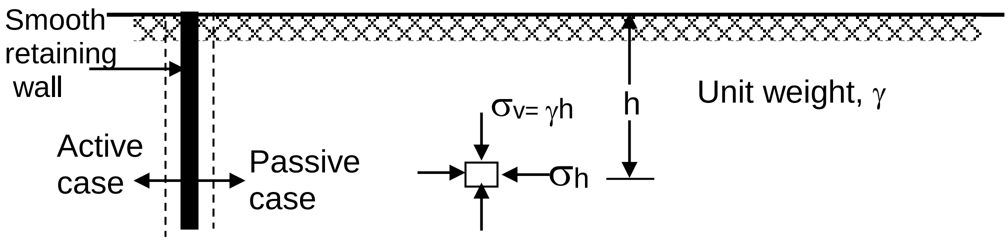
The maximum value σ_1 is reached when the Mohr circle through the point representing the fixed value σ_3 touches the failure envelope for the soil (Fig 1.4 (b)). In this case the horizontal pressure is defined as passive pressure (P_p) representing the maximum inherent resistance of the soil to lateral compression. By a similar procedure the following expression for the passive pressure may be developed.

$$P_p = \gamma h \frac{1+\sin\phi}{1-\sin\phi} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1.3)$$

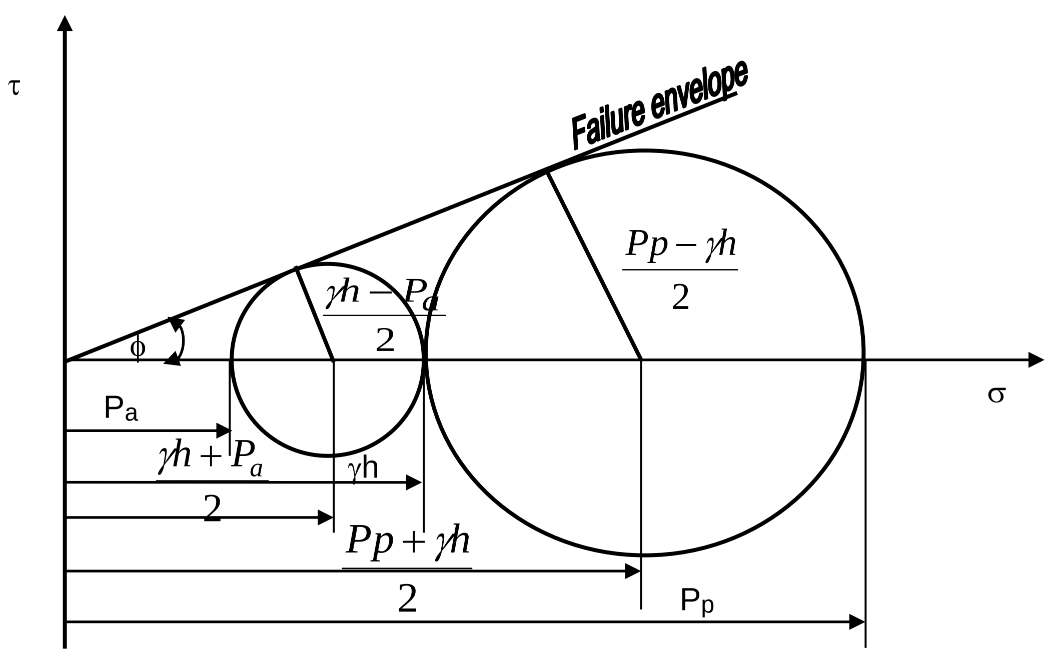
$$P_p = \gamma h K_p$$

where $K_p = \frac{1+\sin\phi}{1-\sin\phi} = \tan^2\left(45+\frac{\phi}{2}\right)$

= Coefficient of passive earth pressure



(a) Semi-infinite mass, cohesionless soil



(b) Mohr stress diagram

Fig. 1.4 Rankine's condition for active and passive failures in a semi-infinite mass of cohesionless soil

1.5 TOTAL LATERAL EARTH PRESSURE AGAINST A VERTICAL WALL WITH THE SURFACE HORIZONTAL.

1.5.1 Active Earth Pressure

The active earth pressure at a depth H against a retaining wall with cohesionless backfill, which is horizontal, is,

$$P_a = k_a \gamma H$$

where $$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \tan^2\left(45 - \frac{\phi}{2}\right)$$

The pressure for the totally active case acts parallel to the face of the backfill and varies linearly with depth as shown in Fig. 1.5. The resultant thrust on the wall acts at ⅓ H from the base and its magnitude is given by, $P_A = \frac{1}{2} k_a \gamma H^2$.

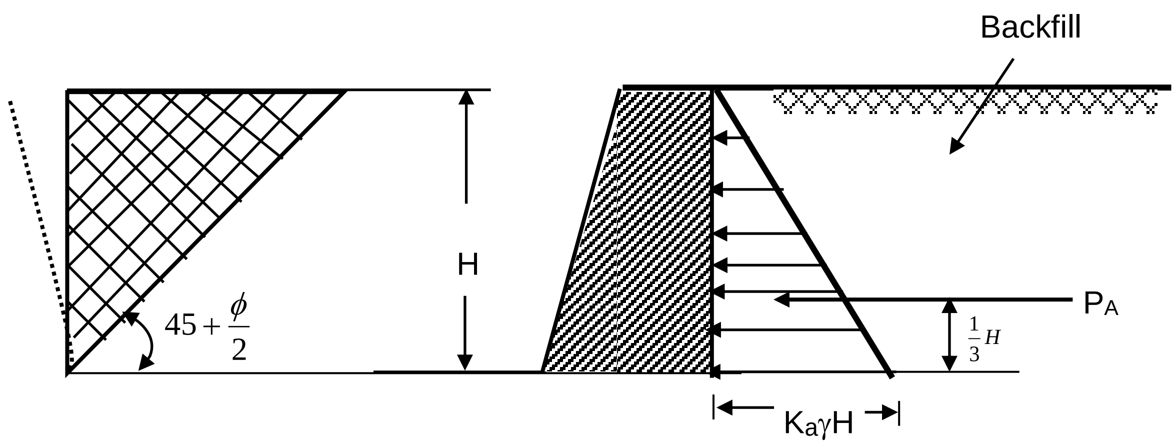


Fig 1.5 Lateral active earth pressure (cohesionless backfill)

1.5.2 Passive Earth Pressure

The passive earth pressure P_p, at a depth H is given by, $P_p = k_p \gamma H$.

where $$k_p = \frac{1+\sin\phi}{1-\sin\phi} = \tan^2\left(45 + \frac{\phi}{2}\right)$$

The resultant thrust, $P_p = \frac{1}{2} k_p \gamma H^2$

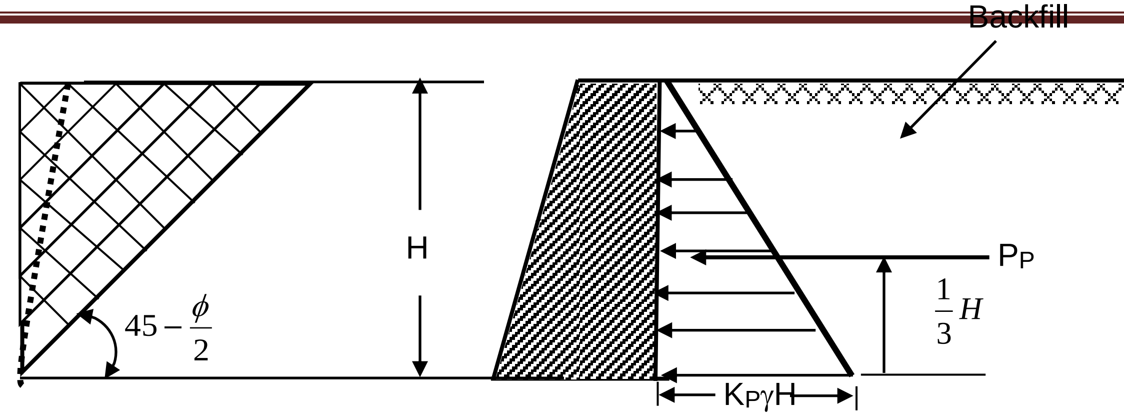


Fig. 1.6 Lateral passive earth pressure (cohesionless backfill)

1.5.3 Lateral Earth Pressure at Rest

As defined earlier the lateral earth pressure at rest exists when there is no lateral yielding. In this condition the soil is in the elastic equilibrium. The lateral pressure is given by

$$P_o = k_o \gamma H$$

where k_o = Coefficient of lateral earth pressure at rest condition

Since the at rest condition does not involve failure of the soil as in the active and passive cases, and it represents a state of elastic equilibrium, the Mohr circle representing σ_z and P_o will not touch the failure envelop (as is done by σ_z and P_a and σ_z and P_p). The horizontal stress P_o cannot, therefore, be evaluated from the Mohr envelop, as is done for evaluating P_a and P_p.

The most accurate way to evaluate, k_o would be to measure P_o in-situ using pressuremeter, or some other test. However, these in-situ tests are not often used in engineering practice, so we usually must rely on empirical correlations with other soil properties. Several such correlations have been developed, including the followings.

For sands and normally consolidated clays, (Jaky 1944),

$$k_o = 1 - \sin\phi$$

For normally consolidated soils and vertical walls, the coefficient of at-rest lateral earth pressure may be taken as,

$$k_o = (1=\sin\phi)(1+\sin\beta)$$

Variables β and $\boldsymbol{\phi}$ are the slope angle of the ground surface behind the wall, and the internal friction angle of the soil, respectively.

For overconsolidated clays, (Mayne and Kulhawy 1982),

$$K_o = (1-\sin\phi)\ OCR^{\sin\phi}$$

This formula is applicable only when the ground surface is level.

where

k_o = coefficient of lateral earth pressure at rest

ϕ = friction angle of soil

OCR = overconsolidated of soil

For fine-grained, normally consolidated soils, Massarsch (1979) suggested the following equation for k**o**

$$k_o = 0.44 + 0.42\left(\frac{PI\%}{100}\right)$$

For overconsolidated clays, the coefficient of earth pressure at rest can be approximated as

$$k_{o(overconsolidated)} = k_{o(normally\,consolidated)}\sqrt{OCR}$$

Table 1.1 Typical Values of K_o

Soil Type	**K_o**
Loose sand, gravel	0.4
Dense sand, gravel	0.6
Sand, tamped in layers	0.8
Soft clay	0.6
Hard clay	0.5

The coefficient of lateral earth pressure at rest can also be calculated using theory of elasticity, assuming the soil to the semi-infinite, homogeneous, elastic and isotropic. Consider an element of soil at a depth z, being acted upon by vertical stress σ_z and horizontal stress σ_h. The lateral strain ε_h in the horizontal direction is given by:

$$\varepsilon_h = \frac{1}{E}\left[\sigma_h - \mu(\sigma_v + \sigma_h)\right]$$

The earth pressure at rest corresponding to the conditions of zero lateral strain (ε_h=0). Hence

$$\frac{1}{E}\left[\sigma_h - \mu(\sigma_v + \sigma_h)\right] = 0$$

$$\sigma_h - \mu\sigma_v - \mu\sigma_h = 0$$

$$\sigma_h(1-\mu) = \mu\sigma_v$$

$$\frac{\sigma_h}{\sigma_v} = k_o = \frac{\mu}{1-\mu}$$

1.6 RANKINE'S LATERAL EARTH PRESSURE THEORY FOR DIFFERENT BACKFILL CASES

1.6.1 Lateral Earth Pressure of Partially Submerged Cohesionless Soil.

In Fig. 1.7(a), the water table is located at a depth of H_1 below the top of the wall. The unit weight of the backfill is γ in the moist state above the water table and γ_b is the effective unit weight below the water table.

The unit weight of water is γ_ω

At any section below the top of the wall, up to depth H_1,

$$P_a = K_a \gamma z$$

Effective vertical pressure, below H_1, (i.e.z> H_1)

$$P_v = \gamma H_1 + \gamma_b (z-H_1)$$

$$\therefore \quad P_a = K_a \gamma H_1 + K_a\gamma_b (z-H_1)$$

Total $\quad P_a = K_a \gamma H_1 + K_a \gamma_b (z-H_1) + \gamma_\omega(z-H_1)$

The total lateral earth pressure diagram is shown Fig.1.7 (a)

1.6.2 Lateral Earth Pressure of Cohesionless Soil Carrying Uniform surcharge

Loading imposed on the backfill is commonly referred to as a surcharge. It may be either live or dead loading and may be distributed or concentrated. When a uniformly distributed surcharge is applied to the backfill, as shown in Fig. 1.7(b), the vertical pressures at all depths in the backfill are increased equally. Without the surcharge the vertical pressure at any depth would be γh. When a surcharge with an intensity of Δp is added, the vertical pressures become $\gamma h+\Delta p$ for this particular form of surcharge. Utilizing the previously developed relationship between stresses at a point, the lateral pressure on the back of the wall at any depth h is given by the expression.

$$P_a = (\gamma h+\Delta p)\,\frac{1-\sin\phi}{1+\sin\phi}$$

As shown in Fig.1.7(b), the pressure distribution for this case is trapezoidal rather than triangular. At the top of the wall the lateral pressure for the active case is

$$P_a = \Delta P\,\frac{1-\sin\phi}{1+\sin\phi} = \Delta P\,K_a$$

At the bottom of the wall the lateral pressure is

$$P_a = (\gamma H+\Delta p)\,\frac{1-\sin\phi}{1+\sin\phi} = (\gamma H+\Delta p)\,K_a$$

Thus an expression for the total resultant pressure for the active case may be developed as follows: -

$$P_A = H\left(\frac{\gamma H + 2\Delta P}{2}\right) K_a$$

or

$$P_A = \left(\frac{\gamma H^2}{2} + H\Delta P\right) K_a$$

The total resultant pressure under these conditions may be considered as being made up of two parts, one representing the effect of backfill alone, the other the effect of the surcharge. Pressure due to the backfill alone is given by the expression.

$$P_1 = \frac{\gamma H^2}{2} K_a$$

This resultant acts at the distance $\frac{H}{3}$ above the base of the wall. Pressure due to the surcharge alone is given by the expression.

$P_2 = H\Delta P\,K_a$

This resultant acts at mid-height on the wall.

In analyzing the stability of the wall, the effect of these two resultants may be considered respectively if desired. If it is preferred to deal with the total resultant only, it may be shown for either the active or passive case that the line of action of P is at a distance Y above the base, which is given by the following equation:-

$$Y=\left(\frac{H}{3}\right)\frac{\gamma H+3\Delta P}{\gamma H+2\Delta P}$$

1.6.3 Lateral Earth Pressure of Cohesionless Soil on Sloping Back wall

If the back of the wall slopes as shown in Fig. 1.7 (c), it is assumed that the lateral pressure acts horizontally on a vertical plane in the backfill and the wedge of the earth with weight W_1 acts as part of the wall.

1.6.4 Lateral Earth Pressure of Cohesionless Soil with Inclined Surface

The lateral pressure distribution on a vertical wall that retains a cohesionless backfill with surface at inclination i, is assumed to act parallel to the surface of the fill as shown in Fig.1.7(d), The resultant thrusts for active and passive cases are given by the following expressions.

Active thrust,

$$P_A = \frac{\gamma H^2}{2}\cos i\,\frac{\cos i-\sqrt{\cos^2 i-\cos^2\phi}}{\cos i+\sqrt{\cos^2 i-\cos^2\phi}}$$

Passive thrust,

$$P_p = \frac{\gamma H^2}{2}\cos i\,\frac{\cos i+\sqrt{\cos^2 i-\cos^2\phi}}{\cos i-\sqrt{\cos^2 i-\cos^2\phi}}$$

Where γ = Unit weight of soil

H = Wall height

i = Inclination of backfill slope

ϕ = Angle of internal friction

1.6.5 Lateral Earth Pressure of Cohesionless Soil for a Sloping Backfill and a Sloping Back Wall Face

Rankine (1857) derived expressions for k_a and k_p for a soil mass with a sloping surface that were later extended to include a sloping wall face by Chu (1991). You can refer to Rankine's paper and Chu's paper for the mathematical details.

With reference to Fig. 1,7(e), the lateral earth pressure (σ'_a) at a depth z can be given as

$$\sigma_a' = \frac{\gamma z \cos\alpha \sqrt{1+\sin^2\phi' - 2\sin\phi'\cos\psi_a}}{\cos\alpha + \sqrt{\sin^2\phi' - \sin^2\alpha}}$$

$$where \quad \psi_a = \sin^{-}\left(\frac{\sin\alpha}{\sin\phi'}\right) - \alpha + 2\theta$$

The pressure σ'_a will be inclined at angle β' with the plane drawn at right angle to the wall, and

$$\beta' = \tan^{-}\left[\frac{\sin\phi' \sin\psi_a}{1-\sin\phi'\cos\psi_a}\right]$$

The resultant active thrust P_A for unit length of the wall then can be calculated as

$$P_A = \frac{1}{2}\gamma H^2 k_a$$

where

$$k_a = \frac{\cos(\alpha-\theta)\sqrt{1+\sin^2\phi' - 2\sin\phi'\cos\psi_a}}{\cos^2\theta(\cos\alpha + \sqrt{\sin^2\phi' - \sin^2\alpha}}$$

= Rankine active earth pressure coefficient for generalized case

As a special case, for a vertical back face of the wall (that is, $\theta = 0$), the above equation simplify to the following

$$k_a = \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi'}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi'}}$$

The failure wedge inclined at angle η with the horizontal, or

$$\eta = \frac{\pi}{4} + \frac{\phi'}{2} + \frac{\alpha}{2} - \frac{1}{2}\sin^{-}\left(\frac{\sin\alpha}{\sin\phi'}\right)$$

Similar to the active case, for the Rankine passive case, one can obtain the following relationships.

$$\sigma_p' = \frac{\gamma z\cos\alpha\sqrt{1+\sin^2\phi' + 2\sin\phi'\cos\psi_p}}{\cos\alpha - \sqrt{\sin^2\phi' - \sin^2\alpha}}$$

$$where \quad \psi_p = \sin^{-}\left(\frac{\sin\alpha}{\sin\phi'}\right) + \alpha - 2\theta$$

The pressure σ'_p will be inclined at angle β' with the plane drawn at right angle to the wall, and

$$\beta' = \tan^{-}\left[\frac{\sin\phi'\sin\psi_p}{1+\sin\phi'\cos\psi_p}\right]$$

The passive force per unit length of the wall is

$$P_p = \frac{1}{2}\gamma H^2 k_p$$

where

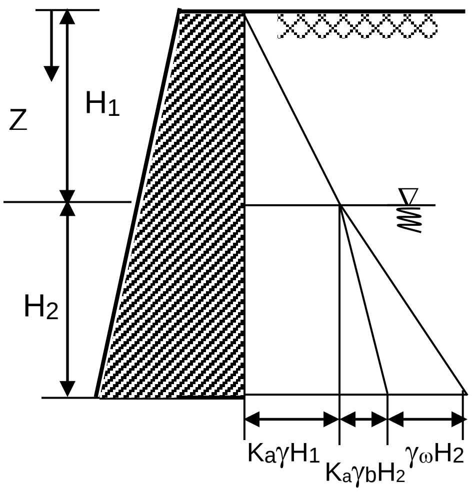
$$k_p = \frac{\cos(\alpha-\theta)\sqrt{1+\sin^2\phi' + 2\sin\phi'\cos\psi_p}}{\cos^2\theta(\cos\alpha - \sqrt{\sin^2\phi' - \sin^2\alpha}}$$

For walls with vertical back face, $\theta = 0$,

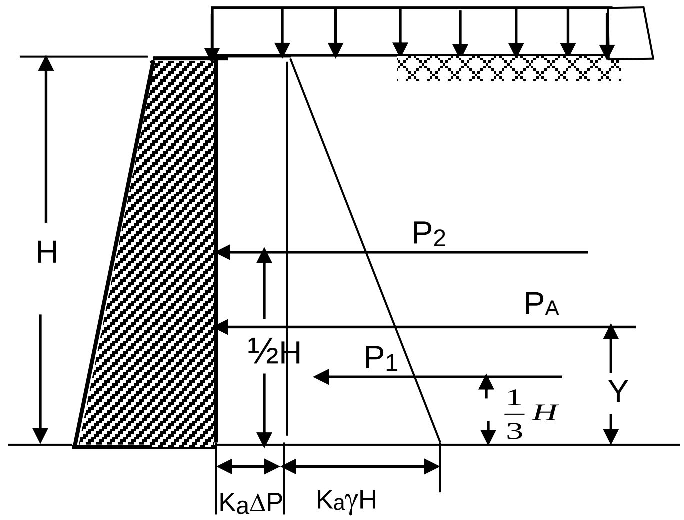
$$k_p = \cos\alpha\frac{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi'}}{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi'}}$$

The inclination of the failure wedge, η, from the horizontal may be expressed as

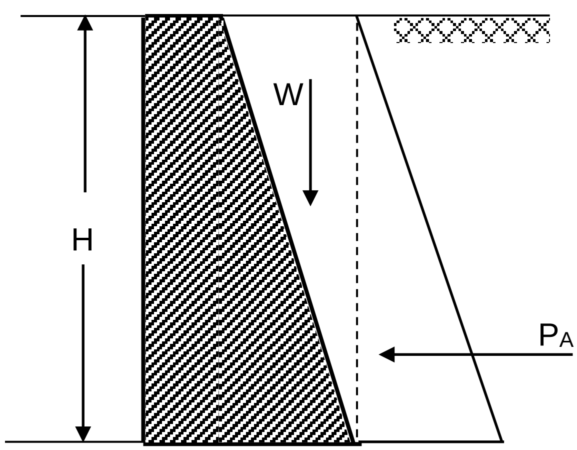
$$\eta = \frac{\pi}{4} - \frac{\phi'}{2} + \frac{\alpha}{2} + \frac{1}{2}\sin^{-}\left(\frac{\sin\alpha}{\sin\phi'}\right)$$



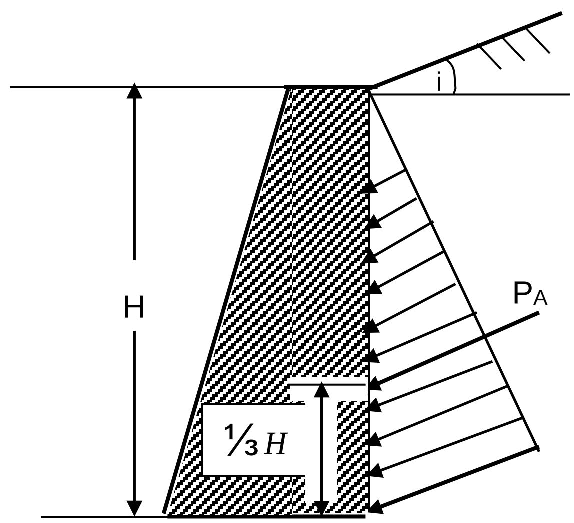
a) Partially submerged cohesionless soil



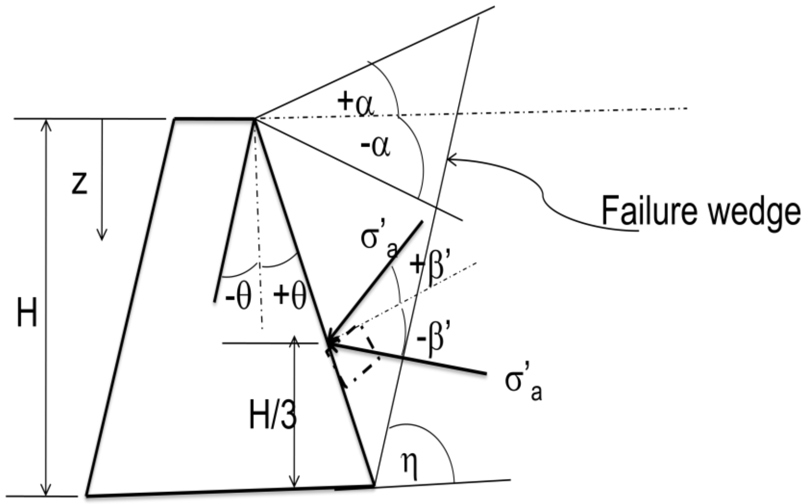
b) Cohesionless soil carrying uniform surcharge



c) Cohesionless soil on sloping back wall



d) Cohesionless soil with inclined surface



e) Cohesionless soil on sloping back wall face and with sloping backfill

Fig 1.7 Rankine active earth pressure diagram for different backfill cases

1.7 RANKINE'S EARTH PRESSURE THEORY FOR COHESIVE SOILS

Rankine's original theory was for cohesionless soils. It was extended by Resal (1910) and Bell (1915) for cohesive soils. The treatment is similar to that for cohesionless soils with one basic difference that the failure envelope has a cohesion intercept **c**, whereas for cohesionless soils is zero.

1.7.1 Active and Passive Earth Pressures of Pure Cohesive Soils.

Relationships Between Vertical and Lateral Pressures: - For a pure cohesive soil, which derives strength solely from cohesion, the Mohr stress diagram would be as represented in Fig. 1.8(a). General relationships between the vertical and lateral pressures indicated by this diagram may be expressed as follows;

$$P_a = P_v - 2C = \gamma H - 2C \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1.4)$$

$$P_p = P_v + 2C = \gamma H + 2C \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1.5)$$

These stress relationships are represented graphically in Figs. 1.8 (b) and 1.8 (c).

Active Pressure:- The active lateral earth pressure for a backfill containing pure clay is determined as illustrated in Fig. 1.8 (b). Due to the effect of cohesion, P_a is negative in the upper part of the retaining wall. The depth Z_o at which $P_a = 0$ is, from the above equation,

$$\gamma H - 2C = 0$$

or $$Z_o = \frac{2C}{\gamma} \quad \text{.................................} \quad (1.6)$$

If the wall has a height $2Zo = H_{cr}$ the total earth pressure is equal to zero.

i.e., $$\tfrac{1}{2}\gamma H_{cr}^2 - 2CH_{cr} = 0$$

$$H_{cr} = \frac{4C}{\gamma} = 2Z_O \quad \text{...........} \quad (1.7)$$

This indicates that a vertical bank of height smaller than H_{cr} can stand without lateral support. H_{cr} is called as critical depth.

Since there is no contact between the soil and the wall to a depth of Z_o after the development of tensile crack, only the active pressure distribution against the wall between Z_o to H is considered. In that case

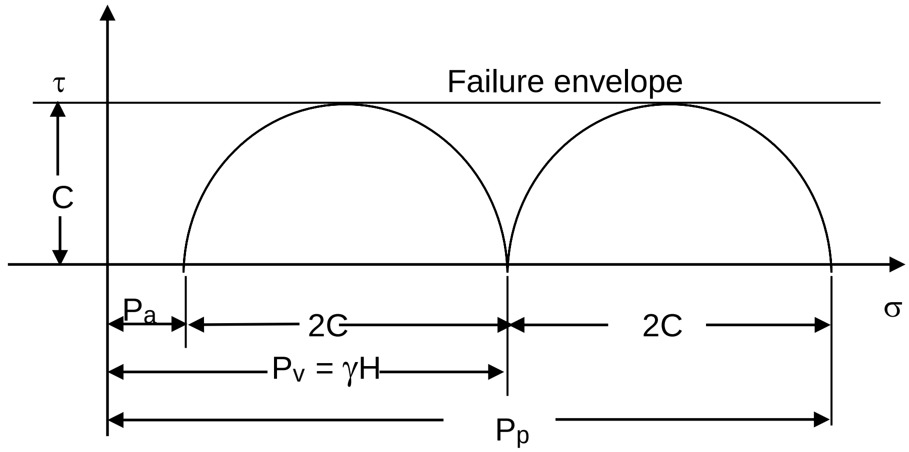
$$P_A = \tfrac{1}{2}(\gamma H - 2C)(H - Z_o)$$

but $$Z_o = \frac{2C}{\gamma}$$

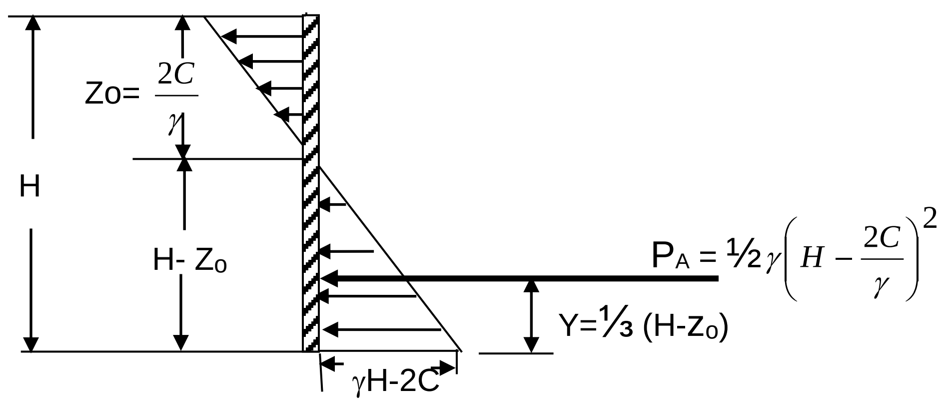
$$P_A = \tfrac{1}{2}(\gamma H - 2C)\left(H - \frac{2C}{\gamma}\right) = \frac{\gamma}{2}\left(H - \frac{2C}{\gamma}\right)\left(H - \frac{2C}{\gamma}\right)$$

$$P_A = \frac{\gamma}{2}\left(H - \frac{2C}{\gamma}\right)^2 \quad \text{........................} \quad (1.8)$$

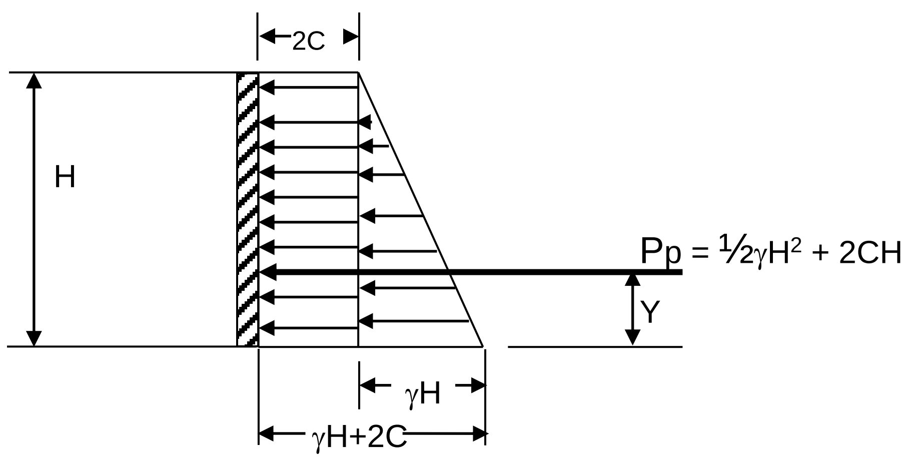
Passive Pressure:- The passive lateral pressure due to backfill containing pure cohesive soil is determined as shown in Fig.1.8(c).



(a) General case



b) Active pressure case



c) Passive pressure case

Fig. 1.8 Relation of lateral to vertical pressure, pure cohesive soil

1.7.2 Active and Passive Earth Pressures of Mixed Soils (or C-ϕ Soils)

Like the previous one, relationships between vertical and lateral pressures may be developed with reference to Mohr stress diagram as indicated in Fig. 1.9 (a).

Active Pressure: - The active lateral earth pressure due to C-ϕ soil is determined as illustrated in Fig. 1.9(b).

$$P_a = \gamma H k_a - 2C\sqrt{K_a} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (1.9)$$

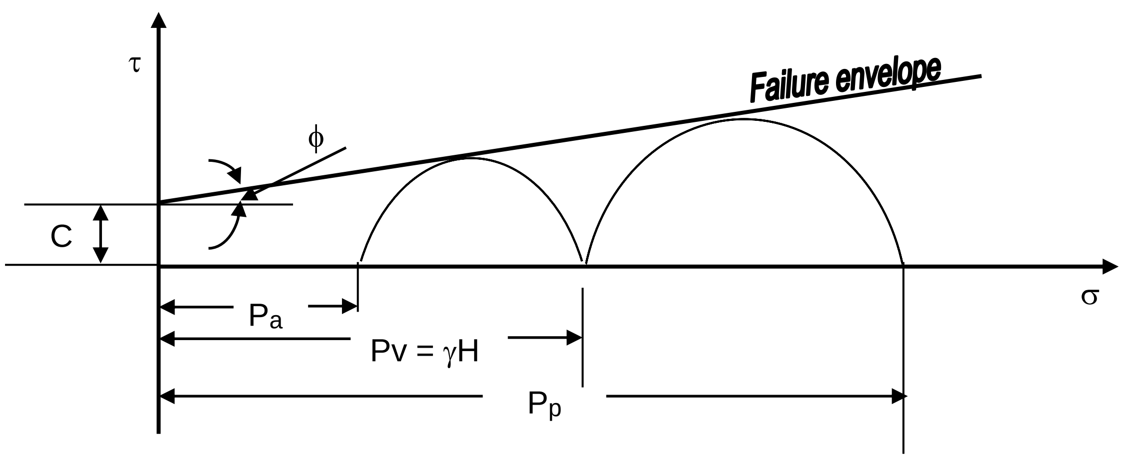
The resultant thrust

$$P_A = \tfrac{1}{2}\left(\gamma H k_a - 2C\sqrt{K_a}\right)\left(H - \frac{2C}{\gamma\sqrt{K_a}}\right)$$

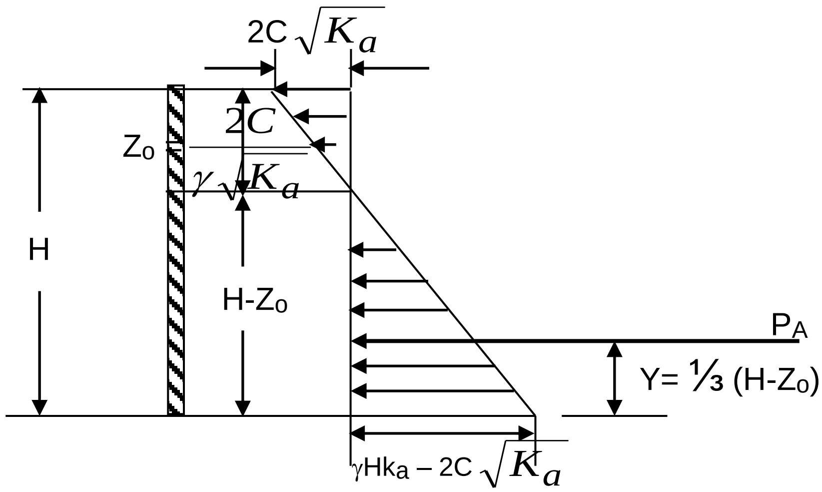
$$P_A = \tfrac{1}{2}\gamma H^2 k_a - 2CH\sqrt{K_a} + \frac{2C^2}{\gamma} \qquad \ldots\ldots\ldots \qquad (1.10)$$

Passive Pressure: - Referring to the pressure distribution diagram given in Fig.1.9(c), the total passive pressure on a vertical plane in a mixed soil formation with height H would have the value,

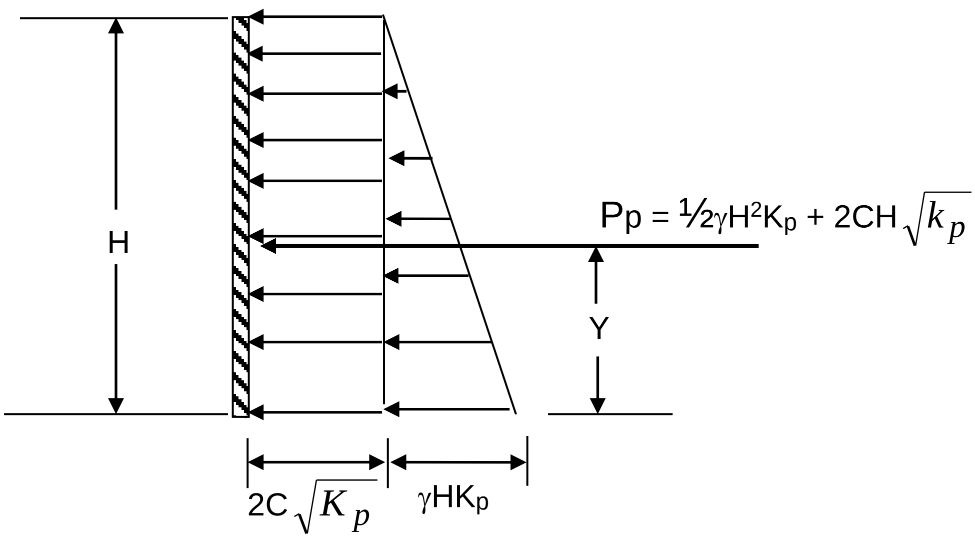
$$P_p = \tfrac{1}{2}\gamma H^2 k_p + 2CH\sqrt{K_p} \qquad \ldots\ldots\ldots \qquad (1.11)$$



a) General case



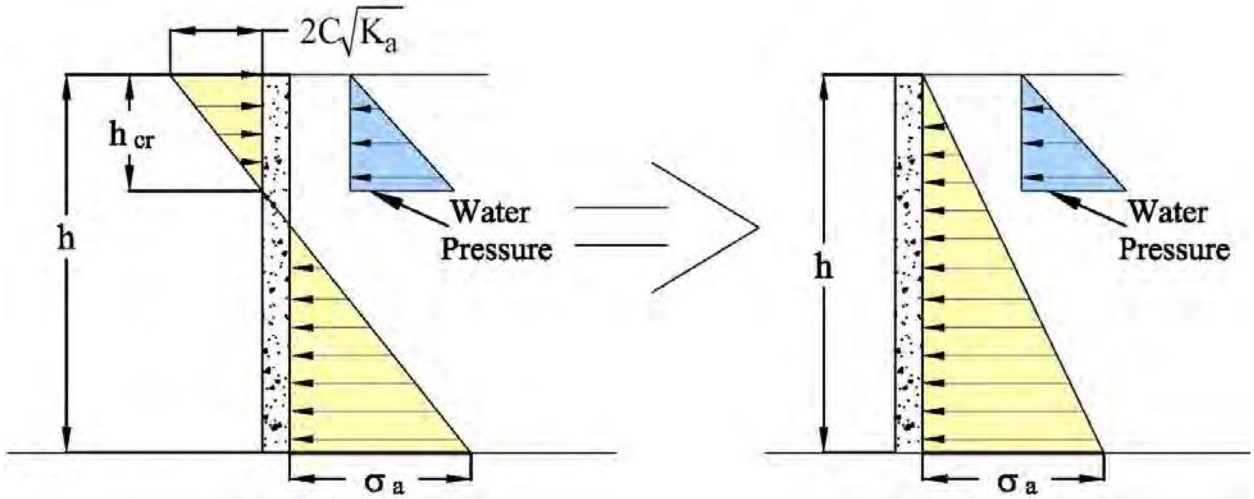
b) Active Pressure case



c) Passive pressure case

Fig. 1.9 Relation of lateral to vertical pressure, mixed soil

When designing earth retaining systems that support cohesive backfills, the tensile stress distributed over the tension crack zone should be ignored, and the simplified lateral earth pressure distribution acting along the entire wall height *h*—including the pore water pressure—should be used, as shown in Figure 1.10.



(a) Tension Crack with Water (b) Recommended Pressure Diagram for Design

Fig. 1.10 Stress Distribution for Cohesive Backfill Considered in Design

The apparent active earth pressure coefficient, *Kap*, is defined as

$$K_{ap} = \frac{\sigma_a}{\gamma\ h} \geq 0.25$$

This equation indicates that the active lateral earth pressure (σ_a) acting over the wall height (h) in a cohesive soil should be taken no less than 0.25 times the effective overburden pressure at any depth.

1.8 Retaining walls with frictions

So far in our study of earth pressures, we have considered the case of frictionless walls. However, retaining walls are rough and shear force develop between the face of the wall and the backfill. To understand the effect of wall friction on the failure surface, let us consider a rough retaining wall AB with a horizontal ground backfill as shown in Fig.1.11

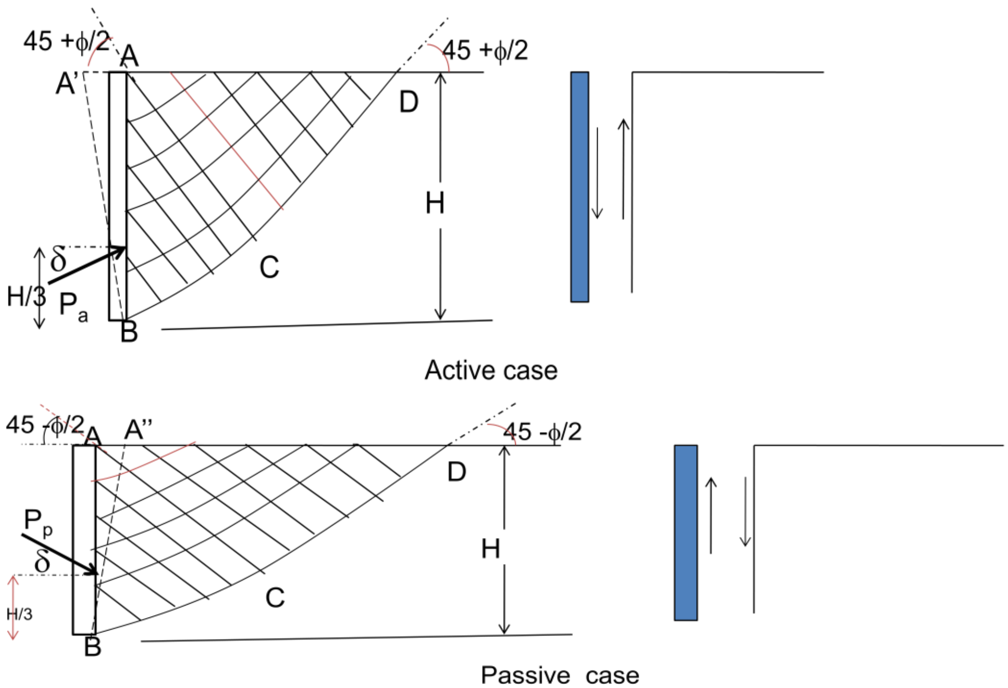


Fig . 1.11 Effect of wall friction on failure surface.

Active case: - In this case, when the wall AB moves to a position A'B, the soil mass in the active zone stretched outward, giving rise to a downward motion of the soil relative to the wall. This motion causes a downward shear on the wall, which is called a positive wall friction in active case. If δ is the angle of friction between the wall and the backfill, the resultant active force P_a is inclined at an angle δ to the normal drawn to the back face of the retaining wall**.** Advanced studies show that the failure surface in the backfill can be represented by BCD. Portion BC is curved and CD is a straight line, and Rankine's active state exists in the zone ACD.

Passive case: - When wall AB is pushed into a position A''B, the soil in the passive zone will be compressed, resulting in an upward motion relative to the wall. This upward motion causes an upward shear on the retaining wall, which is referred to as positive wall friction in passive case. The resultant passive force P_p is inclined at angle δ to the normal drawn to the back face of the wall. The failure surface in the soil has a curved lower portion BC and a straight upper portion CD. Rankine's passive state exists in the zone ACD. In the design actual design of retaining walls, the value of the wall friction, δ, is assumed to be between range of $\phi/2$ and $2/3\phi$

1.9 COULOMB'S EARTH PRESSURE THEORY

Coulomb (1776) developed a method for the determination of the earth pressure in which he considered the equilibrium of the sliding wedge which is formed when the movement of the retaining wall takes place.

Assumptions

1. The backfill is dry, cohesionless, homogeneous, isotropic and ideally plastic material.
2. The slip surface is a plane surface which passes through the heel of the wall.
3. The wall surface is rough. The resultant earth pressure on the wall is inclined at angle of δ to the normal to the wall.
4. The sliding wedge itself acts as a rigid body.

- The magnitude of earth pressure obtained by considering the equilibrium of the sliding wedge as a whole.

In Coulomb's theory, a plane failure surface is assumed and the lateral force required to maintain the equilibrium of the wedge is found using the principles of statics. The procedure is repeated for several trial surfaces. The trial surface which gives the largest force for the active case, and the smallest force for the passive case, is the actual failure surface. The method readily accommodates the friction between the wall and the backfill, irregular backfill, sloping wall, surcharge loads, etc. Although the initial theory was for dry, cohesionless soils, it has now been extended to wet soils and cohesive soils as well. Thus Coulomb's theory is more general than the Rankine theory.

Active Case

Let AB (Fig. 1.12) be the back face of a retaining wall supporting a cohesionless soil, the surface of which is consistently sloping at angle i with the horizontal. The BC plane is a trial failure surface

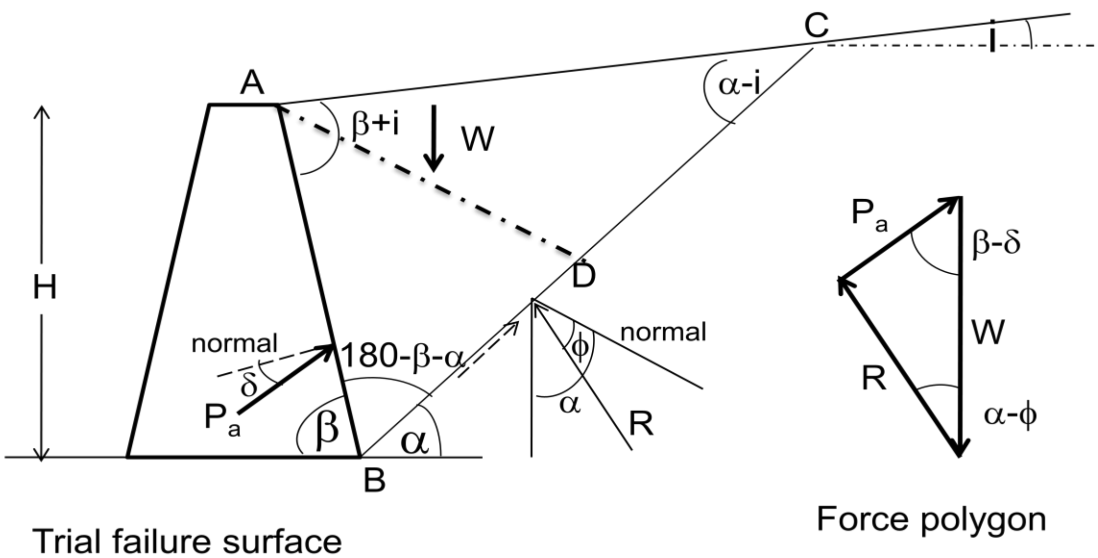


Fig . 1.12 Coulomb's active pressure.

In the stability consideration of the probable failure wedge ABC, the following forces are involved (per unit width of the wall)

1. W, the weight of the soil wedge
2. R, the resultant of the shear and normal forces on the surface of failure, BC, which is inclined at angle of ϕ to the normal drawn to the plane BC
3. P_a, the active force per unit width of the wall, which is inclined at angle δ to the normal drawn to the face of the wall supporting the soil.

From the force polygon, using the law of sines,

$$\frac{P_a}{\sin(\alpha-\phi)} = \frac{W}{\sin(180-(\beta-\delta)-(\alpha-\phi))}$$

$$P_a = \frac{W\sin(\alpha-\phi)}{\sin(180-(\beta-\delta)-(\alpha-\phi))} \quad \text{.......... ....(1.12)}$$

The weight of the wedge is

$$W = \tfrac{1}{2}(AD)(BC)\gamma \quad \text{..(1.13)}$$

$$AD = AB\sin(180-\beta-\alpha) = \frac{H}{\sin\beta}\sin(180-\beta-\alpha) = H\frac{\sin(\beta+\alpha)}{\sin\beta} \quad (1.14)$$

Again, from the law of sines,

$$\frac{AB}{\sin(\alpha-i)} = \frac{BC}{\sin(\beta+i)}$$

$$BC = AB\frac{\sin(\beta+i)}{\sin(\alpha-i)} = H\frac{\sin(\beta+i)}{\sin\beta\sin(\alpha-i)} \quad (1.15)$$

Substituting Eqs.(1.14) and (1.15) into Eqn..1.13

$$W = \frac{1}{2}\gamma H^2\frac{\sin(\beta+\alpha)\sin(\beta+i)}{\sin^2\beta\sin(\alpha-i)} \quad (1.16)$$

Substituting the expression for W in Eq.(1.12),

$$P_a = \frac{1}{2}\gamma H^2\left\{\frac{\sin(\beta+\alpha)\sin(\beta+i)\sin(\alpha-\phi)}{\sin^2\beta\sin(\alpha-i)\sin(180-(\beta-\delta)-(\alpha-\phi))}\right\} \quad (1.17)$$

In Eqn. (1.17), γ, H, β, i, ϕ, and δ are constants; α is the only variable . To determine the critical value of α for maximum P_a, we have

$$\frac{dP_a}{d\alpha} = 0 \quad (1.18)$$

After solving Eqn. (1.18), the relation of α is substituted into Eqn. (1.17) to obtain Coulomb's active earth pressure P_a as

$$P_a = \frac{1}{2}K_a\gamma H^2$$

Where K_a is Coulomb's active earth pressure coefficient given by

$$K_a = \frac{\sin^2(\beta+\phi)}{\sin^2\beta\sin(\beta-\delta)\left\{1+\left[\frac{\sin(\phi+\delta)\sin(\phi-i)}{\sin(\beta-\delta)\sin(\beta+i)}\right]^{1/2}\right\}^2}$$

Note that when i =0, β=90^0, and δ =0, Coulomb's active earth pressure coefficient becomes (1-sinϕ)/(1+sinϕ), which is the same as the Rankine's earth pressure coefficient

The shear strength and wall friction act in support of the wedge of soil so the active thrust transmitted to the wall is smaller for stronger soil and greater wall friction.

Passive Case

Figure 1.13 shows a retaining wall with a sloping cohesionless backfill similar to that considered in Fig. 1.12. The procedure for computing Coulomb's passive earth pressure is similar to one for the active case. However, there is one difference. In this case, the critical surface is that which gives the minimum value of P_p.

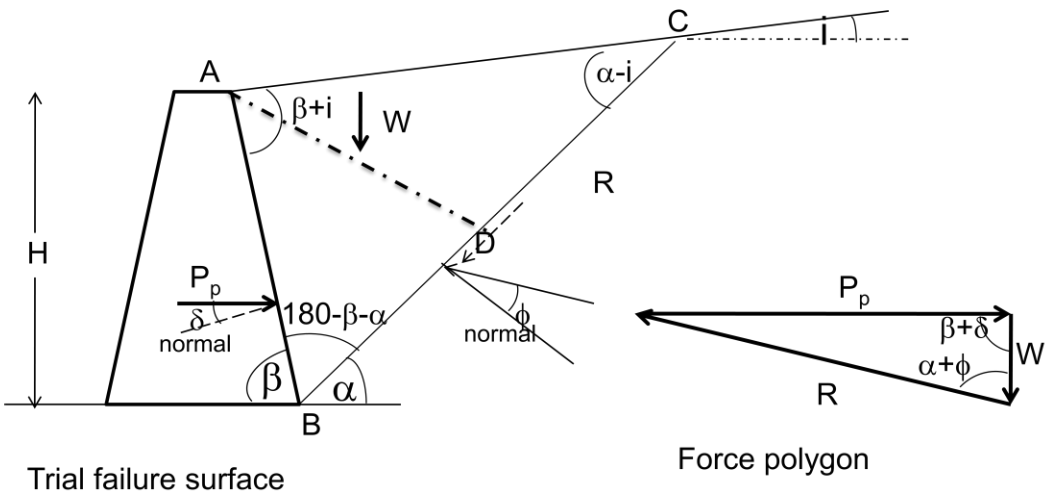


Fig. 1.13 Coulomb's passive pressure

The value of P_p is determined from the force polygon. The procedure is repeated after assuming a new trial failure surface. The minimum value of P_p is the Coulomb passive pressure. Using the procedure similar to that for active case, it can be shown that the passive pressure is given by

$$P_p = \frac{1}{2} K_p \gamma H^2$$

Where K_p is Coulomb's passive earth pressure coefficient given by

$$K_p = \frac{\sin^2(\beta - \phi)}{\sin^2 \beta \sin(\beta + \delta)\left\{1 - \left[\frac{\sin(\phi + \delta)\sin(\phi + i)}{\sin(\beta + \delta)\sin(\beta + i)}\right]^{1/2}\right\}^2}$$

The resultant passive pressure P_p acts at height of H/3 measured from the bottom of the wall. It would be inclined at angle δ to the normal. However, when the retaining wall moves up relative to the soil, the friction angle δ is measured below the normal and δ is said to be negative. The negative wall friction produces a value of passive pressure lower than that for the usual positive wall friction. It is worth noting that the wall friction decreases the active pressure, but it increases the passive pressure.

The shear strength and wall friction resist upward movement of the wedge so the passive thrust transmitted to the wall be larger for stronger soil and greater wall friction.

1.10 GRAPHICAL SOLUTIONS FOR LATERAL EARTH PRESSURES

The methods that are described here are

1. Culmann's solution
2. Trial wedge solution
3. Logarithm spiral trial wedge solution

1.10.1 Culmann's Solution for Coulomb's Active Earth pressure

An expedient method for graphic solution of Coulomb's earth pressure theory described in the preceding section has been given by Culmann (1875). Culmann's solution can be used for any wall friction and irregularity of backfill, for layered backfill and surcharges; hence it provides a very powerful technique for lateral earth pressure estimation.

The procedure consists of the following steps:

1. Draw the features of the retaining wall and the backfill to a convenient scale.
2. Determine the value of Ψ (degrees) $=\beta-\delta$, where β is the inclination of the back face of the wall with the horizontal and δ is the angle of wall friction.
3. Draw line BD that makes an angle ϕ with the horizontal.
4. Draw line BE that makes an angle Ψ with line BD.
5. To consider some trial failure wedges, draw line BC_1, BC_2, BC_3…..
6. Find the areas of ABC_1, ABC_2, ABC_3…..
7. Determine the weight of soil, W, per unit width of the retaining wall in each trial failure wedge:

   W_1 = (area ABC_1) (γ) (1)

   W_2= (area ABC_2) (γ) (1)

   W_3 = area ABC_3) (γ) (1)

8. Adopt a convenient load scale and plot the weights W_1 , W_2 , W_3… determined from step 7 on line BD . (Note; $Bc_1 =W_1$, $Bc_2 = W_2$, $Bc_3 =W_3$,….)

9. Draw $c_1c'_1$, $c_2c'_2$, $c_3c'_3$…. parallel to line BE. (Note; c'_1, c'_2, c'_3… are located on lines BC_1, BC_2, BC_3, …. respectively).
10. Draw a smooth curve through points c'_1, c'_2, c'_3…It is called the Culmann line.
11. Draw a tangent B'D' to the smooth curve drawn in step 10; B'D' is parallel to BD. Let c'_a be the point of tangency.
12. Draw a line c'_ac_a parallel to line BE.
13. Determine the active force per unit width of wall: Pa = (length of c'_ac_a) (load scale).
14. Draw line Bc'_ac_a ; ABCa is the desired failure surface.

Note that the construction procedure essentially is to draw a number of force polygons for a number of trial wedges and find the maximum value of active force that the wall can be subjected t.

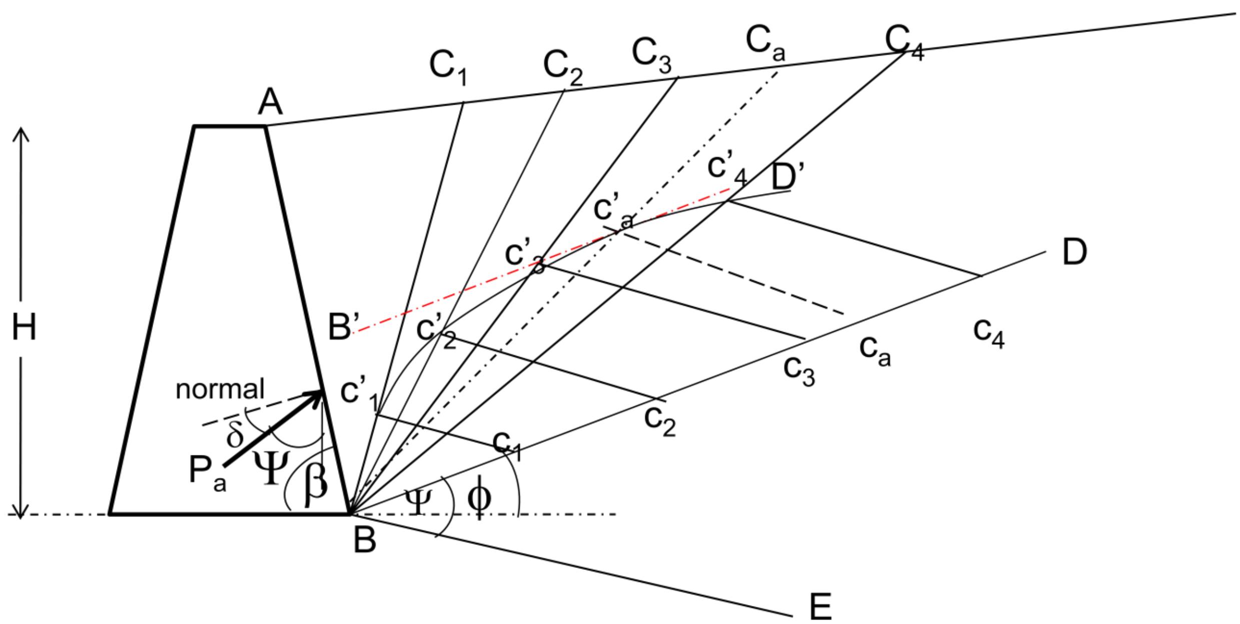


Fig. 1.14 Culmann's solution of active earth pressure

1.10.2 Culmann's Solution for Coulomb's Passive Earth pressure

Fig. 1.14 gives Culmann's construction procedure. The various steps in the construction are:

1. Draw the features of the retaining wall and the backfill to a convenient scale.
2. Determine the value of Ψ (degrees)=$\beta+\delta$, where β is the inclination of the back face of the wall with the horizontal and δ is the angle of wall friction.
3. Draw line BD an angle ϕ below the horizontal.
4. Draw line BE that makes an angle Ψ with line BD.

5. To consider some trial failure wedges, draw line BC_1, BC_2, BC_3…..
6. Find the areas of ABC_1, ABC_2, ABC_3…..
7. Determine the weight of soil, W, per unit width of the retaining wall in each trial failure wedge:
   - W_1 = (area ABC_1) (γ) (1)
   - W_2= (area ABC_2) (γ) (1)
   - W_3 = area ABC_3) (γ) (1)
8. Adopt a convenient load scale and plot the weights W_1 , W_2 , W_3… determined from step 7 on line BD . (Note; $Bc_1 = W_1$, $Bc_2 = W_2$, $Bc_3 = W_3$,….)
9. Draw $c_1c'_1$, $c_2c'_2$, $c_3c'_3$…. parallel to line BE. (Note; c'_1, c'_2, c'_3… are located on lines BC_1, BC_2, BC_3, …. respectively).
10. Draw a smooth curve through points c'1, c'_2, c'_3…It is called the Culmann line.
11. Draw a tangent B'D' to the smooth curve drawn in step 10; B'D' is parallel to BD. Let c'_a be the point of tangency.
12. Draw a line c'_ac_a parallel to line BE.
13. Determine the passive force per unit width of wall: P_p = (length of c'_ac_a) (load scale).
14. Draw line Bc_a ; ABCa is the desired failure surface.

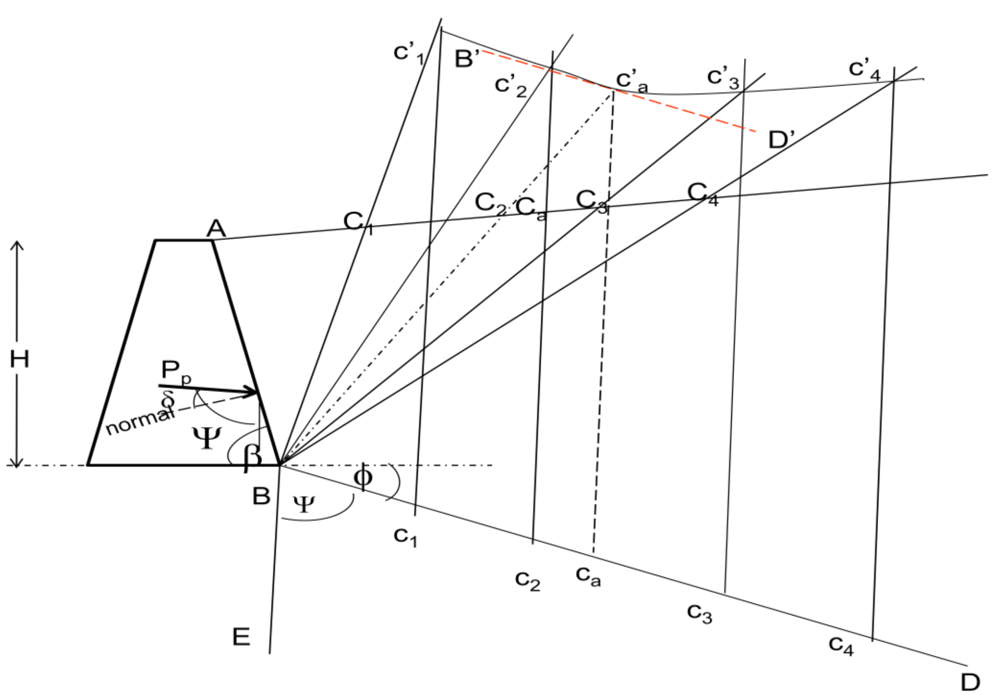


Fig. 1.15 Culmann's solution of passive earth pressure

1.10.3 Trial Wedge Solution

This is graphical solution, very similar to Culmann's solution, and is used for backfill with cohesion. There are two approaches to this problem, one using plane failure surfaces and the other using logarithmic spiral.

The procedure is based on force polygon for the forces which act on any failure wedge.

The forces which act on a failure surface wedge include any or all of the following forces

Wall adhesion, friction and cohesion forces on the failure surfaces and the weight of the failure wedge.

The shearing strength of the backfill is given by

- $S = C + \sigma\tan\phi$
- The shearing resistance between the wall and the soil is given as $S = Ca + \sigma\tan\delta$,

Where, Ca = adhesion between the soil and the wall

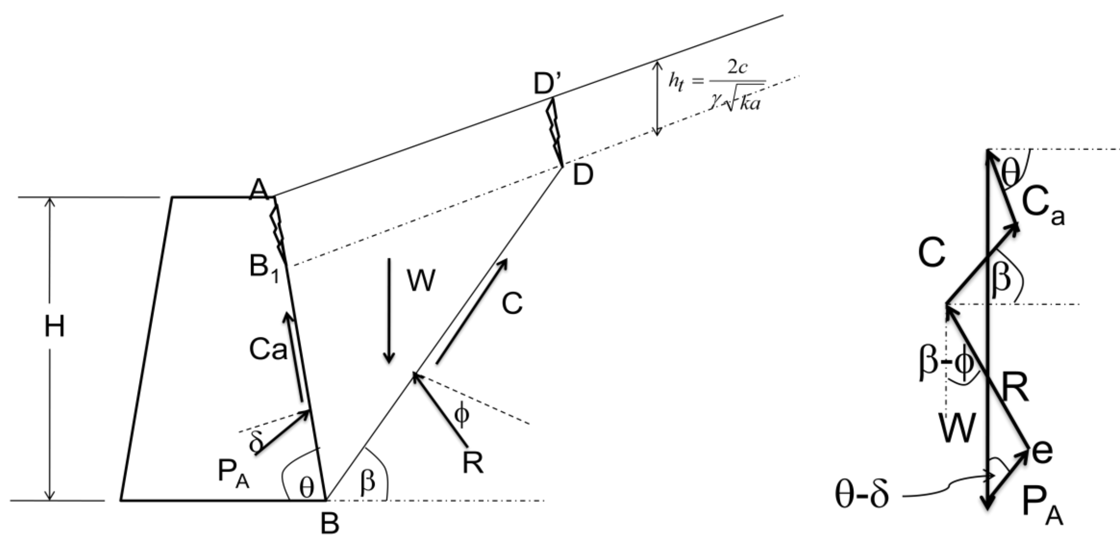


Fig. 1.16 Force polygon for trial wedge to find Pa due to cohesive soil backfill

1. Draw the wall and the ground surface to a convenient scale and compute the depth of the tension crack as $h_t = {2c}/{\gamma\sqrt{k_a}}$
2. Lay off the trial wedge $AB_1BD'D$, compute the weight W
3. Compute $C_a = c_a(\overline{BB_1})$ = adhesive force along backfill side of the wall
4. Compute $C = c(\overline{BD})$ = cohesive force along failure surface
5. Draw the weight vector W to a convenient scale

6. From the tail of the weight vector draw Ca parallel to AB and from the end of Ca draw C parallel to failure surface BD
7. From the end of C, R is laid off at a slope of (θ-ϕ) as shown
8. From the end of the weight vector P_A is drawn at a slope of (θ-δ) intersecting R at e
9. With several trial wedges the above procedures are repeated, establishing finally intersection points of P_A and R. The intersection of P_A and R from a locus points through which a smooth curve is drawn
10. Draw a tangent to the curve parallel to the weight vector and draw the vector P_A through the point of tangency. This gives the max. possible P_A.

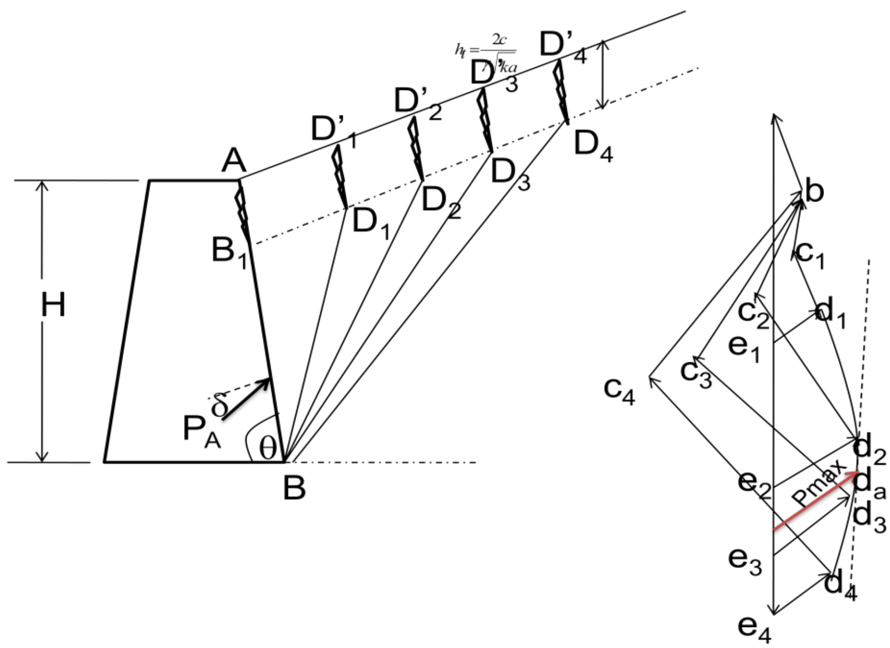


Fig. 1.17 Trial wedge solution for determining Pa

1.10.4 Logarithm spiral trial wedge solution

In Rankine's and Coulomb's earth pressure theories, the failure surface is assumed to be planar. It has been long recognized that when there is a significant friction at the wall-soil interface, the assumption of a planar failure surface becomes unrealistic. Instead, a logarithmic failure surface develops, as illustrated in Figure 1.18

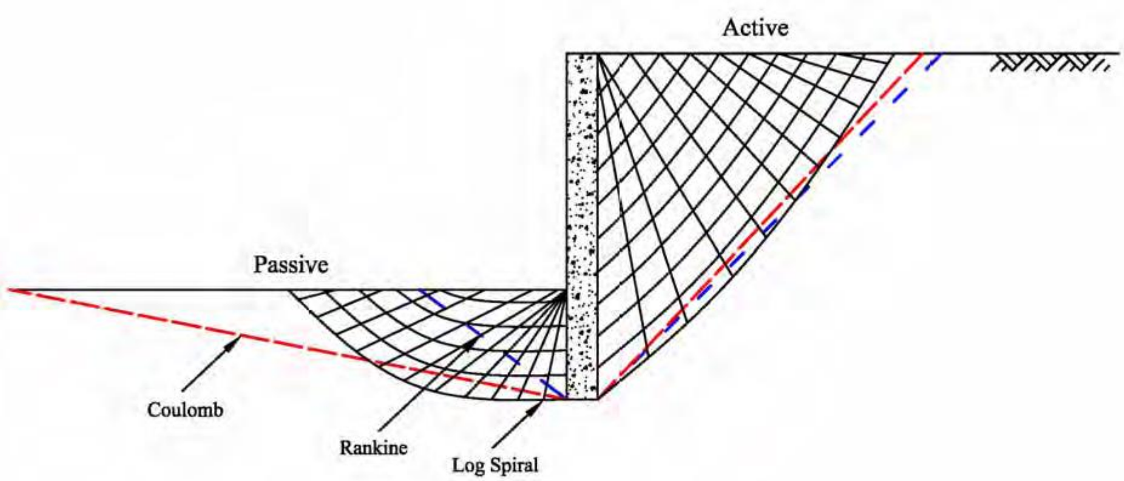


Figure 1.18: Illustration of the Logarithmic Spiral Failure Surface

Figure 1.18 provides a comparison between the potential failure surfaces using Rankine or Coulomb methods versus the log-spiral method for both the active and the passive conditions. For the active case, the failure surfaces determined via the Rankine and Coulomb methods appear to be reasonably close to the log-spiral failure surface. However, for the passive case, the planar failure surfaces determined using the Rankine and Coulomb methods are very different than that determined using the log-spiral method, if the wall-interface friction angle δ is larger than 1/3 of the backfill friction angle, ϕ . The active and passive earth pressures are functions of the soil mass within the failure surface. The mobilized soil mass within the Coulomb and Rankine active zone is about the same as that of a log-spiral active zone. In contrast, the mobilized soil mass within the Coulomb passive zone is much higher than the log-spiral passive zone, and the mobilized soil mass within Rankine passive zone is much lower than log-spiral passive zone. Thus, it is reasonable state that the Coulomb theory overestimates the magnitude of the passive earth pressure, and the Rankine theory underestimates the magnitude of the passive earth pressure. Therefore, Rankine's earth pressure theory is conservative, Coulomb's theory is non-conservative, and the log-spiral result is the most realistic estimate of the passive earth pressure.

Passive earth pressure with curved failure surface

Assuming the failure surface as a plane in Coulomb's theory grossly overestimates the passive resistance of walls, particularly when $\delta > \phi/3$. This approximation is unsafe for all design purposes. The actual rupture surface resembles more closely a log-spiral in such a case.

The equation of logarithmic spiral used in solving the problems of passive resistance against a wall is given by

$$r = r_o e^{\theta \tan \phi}$$

Where r = radius of the spiral

r_o = starting radius at $\theta = 0$

ϕ = angle of internal friction of soil

θ = angle between r and r_o

The basic parameters of a logarithmic spiral are shown in figure below in which O is the center of the spiral.

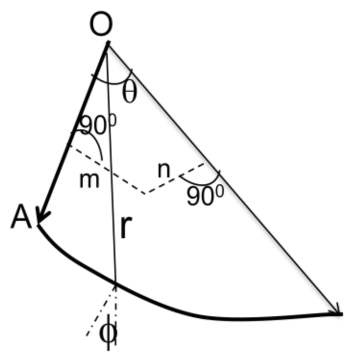


Fig. 1.19 General parameters of logarithmic spiral

r is a radius that makes an angle ϕ with the normal to the curve drawn at the point of intersection of the radius and the spiral. The area A of the sector OAB is given by

$$A = \int_0^{\theta_1} (1/2) r (r d\theta), but\, r = r_o e^{\theta \tan \phi}$$

$$then \quad A = \int_0^{\theta_1} (1/2) r_o^2 e^{2\theta \tan \phi} d\theta = (r_1^2 - r_o^2)/(4 \tan \phi)$$

The location of the centroid can be defined by the intersection of the distance m and n measured from OA and OB, respectively, and given by (Hijab 1957),

$$m = \tfrac{4}{3} r_o \frac{\tan\phi}{\left(9\tan^2\phi + 1\right)} \left[\frac{\left(r_1/r_o\right)^3 \left(3\tan\phi\sin\theta - \cos\theta\right) + 1}{\left(r_1/r_o\right)^2 - 1} \right]$$

$$n = \tfrac{4}{3} r_o \frac{\tan\phi}{\left(9\tan^2\phi + 1\right)} \left[\frac{\left(r_1/r_o\right)^3 - 3\tan\phi\sin\theta - \cos\theta}{\left(r_1/r_o\right)^2 - 1} \right]$$

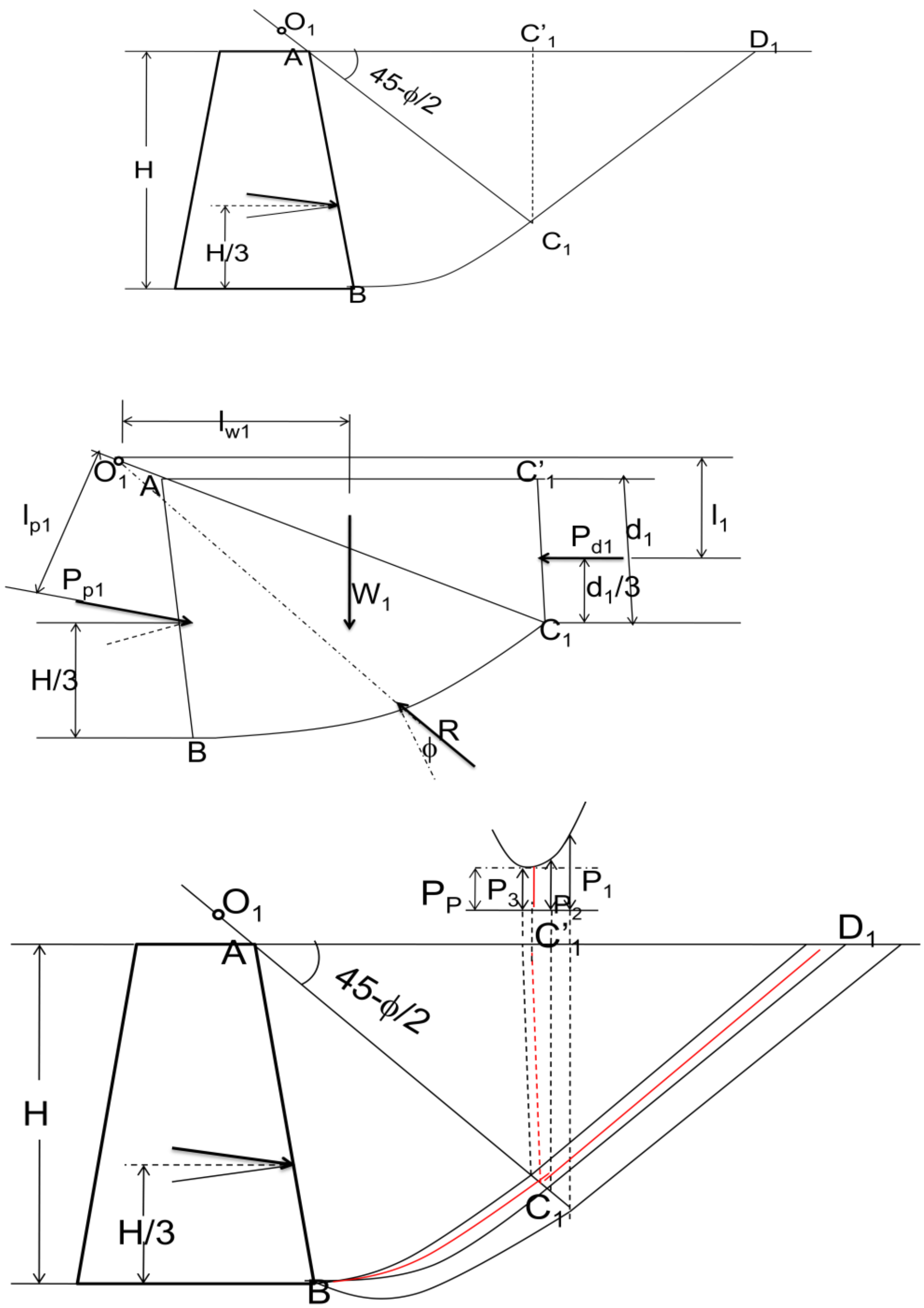


Fig. 1.20 Passive earth pressure against retaining wall with curved failure surface

Refer to the sketch shown above. The retaining wall is first drawn to scale.

The line C_1A is drawn so that it makes an angle of (45-ϕ/2) with surface of the backfill. Wedge $ABC_1D_1$is a trial wedge in which $BC_1$is the arc of a logarithmic spiral $r_1 = r_o e^{\theta \tan\phi}$;O_1 is the center of the spiral for the trial

Arc BC1 can be traced by using trial and error procedure and superimposing the worksheet on another sheet of paper on which a logarithmic spiral is drawn

Consider the stability of the soil mass $ABC_1C'_1$.

For equilibrium the following forces per unit width of the wall are considered.

1. Weight of soil in zone $ABC_1C'_1$ = W1 = γ * (area $ABC_1C'_1$)
2. Vertical face $C_1C'_1$ is in the zone of Rankine passive state, hence the force,

$$P_{d1=}\frac{1}{2}\gamma d_1^2 \tan^2(45+\phi/2)$$

3. R_1 is the resultant of shear and normal forces acting along the surface of sliding BC_1. It makes angle ϕ with the normal to the spiral at its point of application and also passes through the center of the spiral.
4. P_{p1} is the passive force per unit width of the wall and acts at H/3 from the base and is inclined at angle δ with the normal to the back face of the wall.

Taking moment about O1 for equilibrium condition yields the following;

$$W_1 l_{w1} + P_{d1} l_1 - P_{p1} l_{p1} = 0$$

$$P_{p1} = \frac{W_1 l_{w1} + P_{d1} l_1}{l_{p1}}$$

Values of l_{w1}, l_1, l_{p1} and d_1 are obtained from graphical construction.

The above procedure for finding the passive force per unit width of the wall is repeated for several trial wedges. The forces are plotted to scale on the upper side of the figure. A smooth curve is drawn through the points. The lowest point of the curve defines the actual passive force Pp per unit width of the wall.

1.11 Surcharge Loads

Loads due to stockpiled material, machinery, roadways, and other influences resting on the soil surface in the vicinity of the wall increase the lateral pressures on the wall. When a wedge method is used for calculating the earth pressures, the resultant of the surcharge acting on the top surface of the failure wedge is included in the equilibrium of the wedge. If the soil system admits to application of the coefficient method, the effects of surcharges, other than a uniform surcharge, are evaluated from the theory of elasticity solutions presented in the following paragraphs.

a. Uniform surcharge. A uniform surcharge is assumed to be applied at all points on the soil surface. The effect of the uniform surcharge is to increase the effective vertical soil pressure by an amount equal to the magnitude of the surcharge.

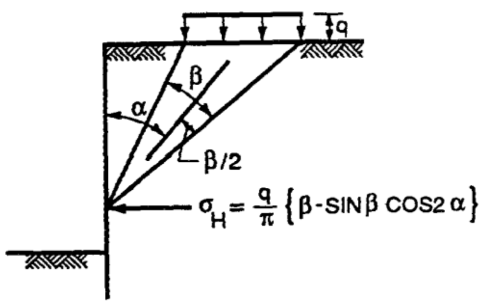
b. Strips loads. A strip load is continuous parallel to the longitudinal axis of the wall but is of finite extent perpendicular to the wall as illustrated in Figure1.11. The additional pressure on the wall is given by the equations in Figure 1.11. Any negative pressures calculated for strips loads are to be ignored.

c. Line loads. A continuous load parallel to the wall but of narrow dimension perpendicular to the wall may be treated as a line load as shown in Figure 1.22. The lateral pressure on the wall is given by the equation in Figure 1.22.

d. Ramp load. A ramp load, Figure 1.23, increases linearly from zero to a maximum which subsequently remains uniform away from the wall. The ramp load is assumed to be continuous parallel to the wall. The equation for lateral pressure is given by the equation in Figure 1.23.

e. Triangular loads. A triangular load varies perpendicular to the wall as shown in Figure 1.24 and is assumed to be continuous parallel to the wall. The equation for lateral pressure is given in Figure 1.24

g. Point loads. A surcharge load distributed over a small area may be treated as a point load. The equations for evaluating lateral pressures are given in Figure 1.25. Because the pressures vary horizontally parallel to the wall; it may be necessary to consider several unit slices of the wall/soil system for design.



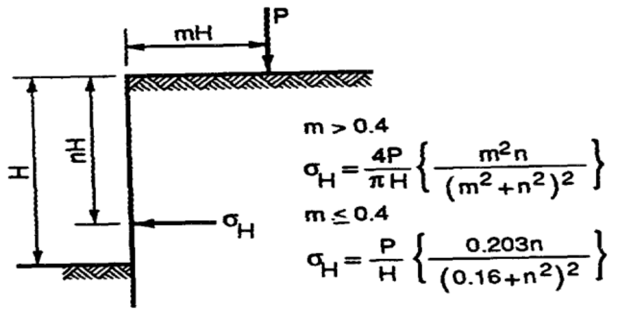
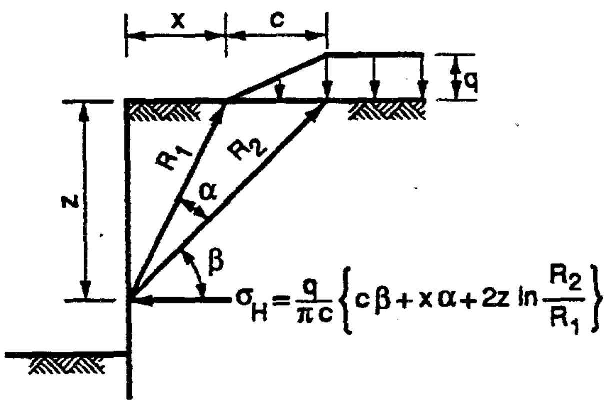


Fig 1.21 Strip load

Fig. 1.22 Line load



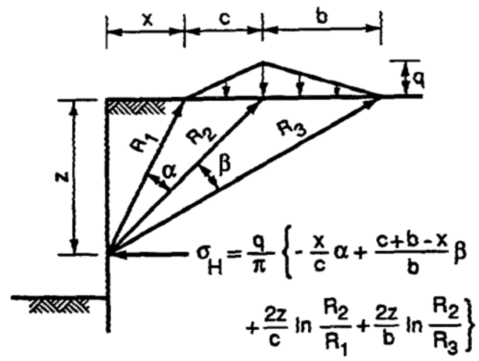


Fig. 1.23 Ramp load

Fig 1.24. Triangular load

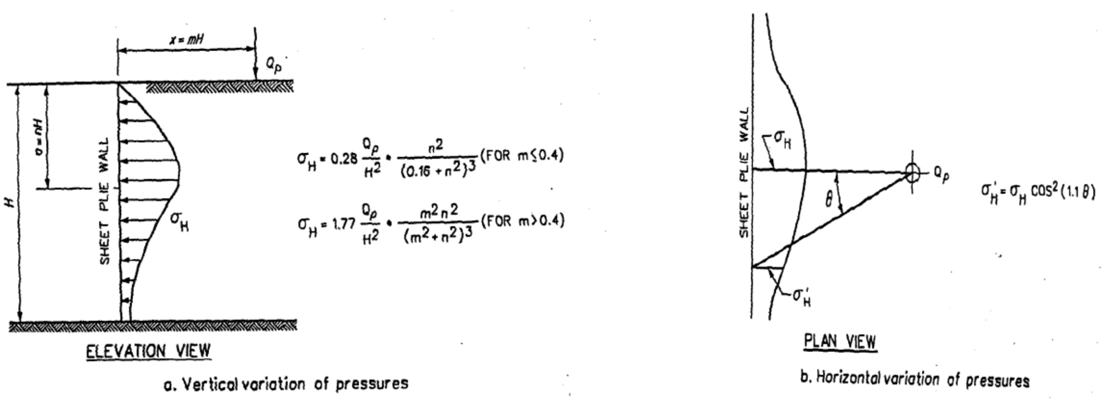


Fig 1.25 Point load (after Terzaghi 1954)

Example 1.1

A retaining wall of 7m high supports sand. The properties of the sand are e=0.5, G_s=2.70 and ϕ =30^0. Using Rankine's theory determine active earth pressure at the base of the retaining wall when the backfill is

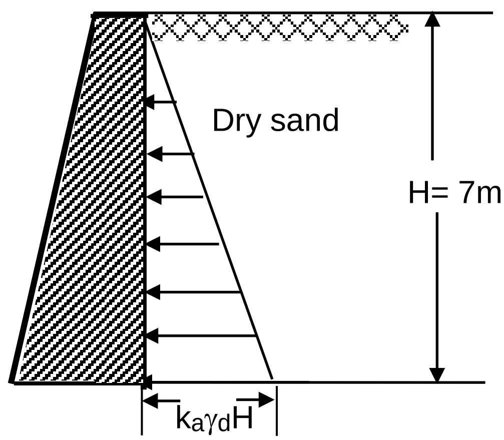
i) Dry ii) saturated and iii) Submerged

Solution

i) When the sand is dry

The dry unit weight of the sand is;

$$\gamma_d = \frac{G_s \gamma_\omega}{1+e} = \frac{2.70 \times 10}{1+0.5}$$



Coefficient of active earth pressure

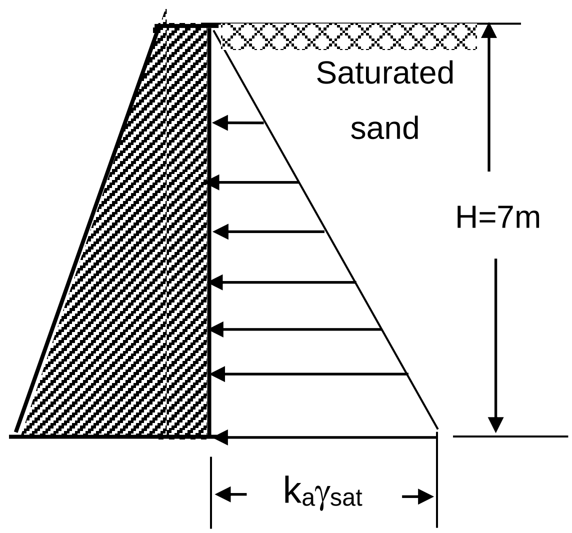
$$K_a = \frac{1-\sin\phi}{1+\sin\phi}$$

$$K_a = \frac{1-\sin 30^0}{1+\sin 30^0} = ⅓$$

Active earth pressure at the base of the wall

$P_a = k_a\gamma_d H$ = ⅓ (18) (17) = 41 kN/m^2

ii) When the sand is saturated



The saturated unit weight of the sand is,

$$\gamma_{sat} = \frac{\gamma_\omega (G_s + e)}{1+e} = \frac{10(2.70+0.5)}{1+0.5}$$

= 21.3 kN/m^3

k_a = ⅓

Active earth pressure at the base of the wall

$P_a = k_a \gamma_{sat} H$ = ⅓ x 21.3x7 = 49.7 kN/m^2

iii, When the sand is submerged

The submerged unit weight of the sand is

$$\gamma_b = \frac{\gamma_\omega (G_s - 1)}{1+e} = \frac{10(2.70-1)}{1+0.5} = 11.3kN/m^3$$

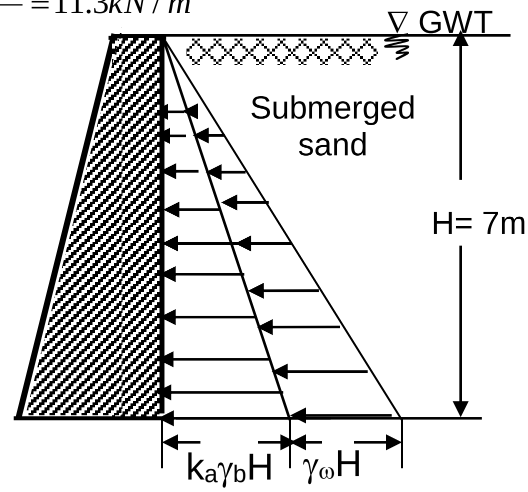
$K_a = ⅓$

Active pressure at the base of the wall

$P_a = K_a \gamma_b H + \gamma_\omega H$

$= (⅓ \times 11.3 \times 7) + (10 \times 7)$

$= 96.37 kN/m^2$

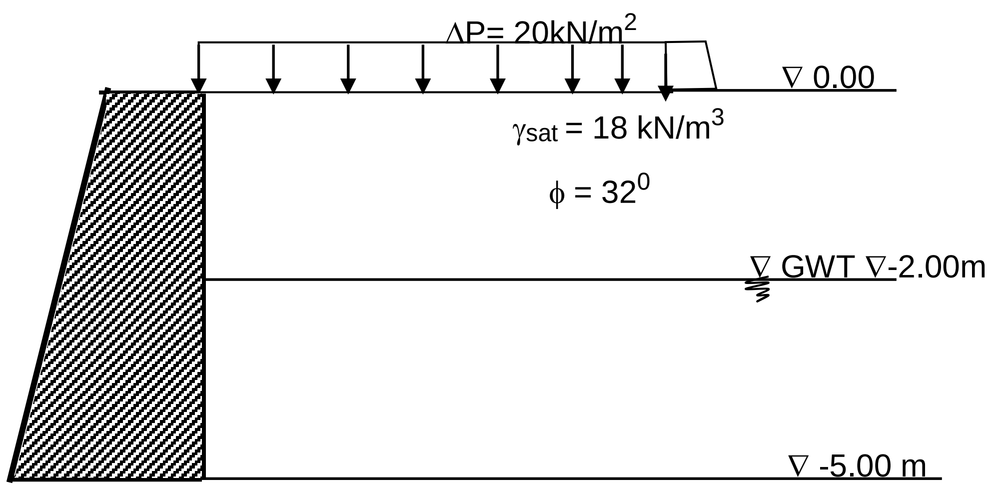


Example 1.2

Given:- A smooth vertical wall with the loading condition shown in Fig. below,

Required:- The lateral force per unit width of the wall

a) If the wall prevented from yielding
b) If the wall yield to satisfy the active Rankine state.



Solution

a) If the wall is prevented from yielding the backfill and the surcharge exert at rest earth pressure.

According to Jaky , $K_o = 1 - \sin\phi$

$= 1 - \sin 32^0 = 0.47$

From Surcharge

$K_o\Delta p = 0.47x20 = 9.40\ kN/m^2$

From backfill

at depth = -2.00 m

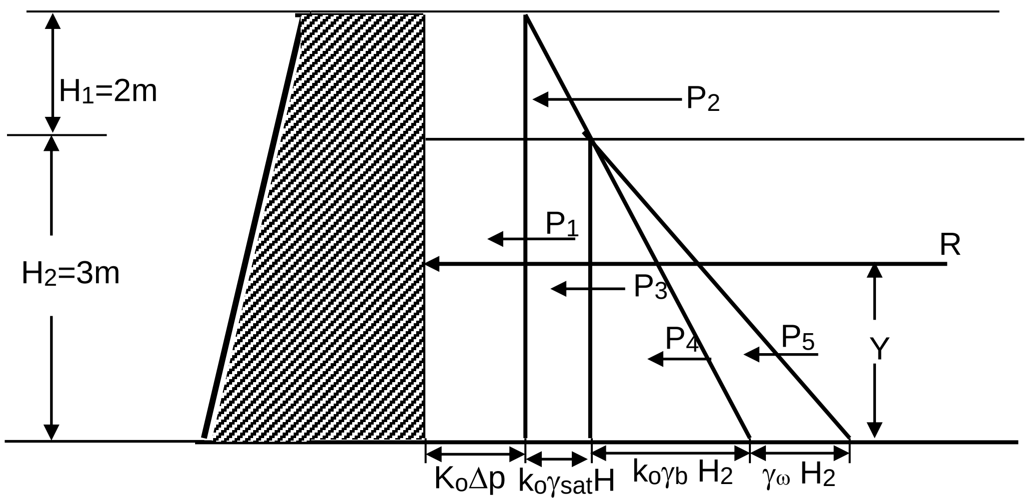
$k_o\gamma_{sat}\ H_1 = 0.47\ x\ 18x2 = 16.92\ kN/m^2$

at depth – 5.00 m

$k_o\gamma_{sat}\ H_1 + k_o\gamma_b H_2 = 16.92 + 0.47x8x3 = 28.20\ kN/m^2$

From hydrostatic

$\gamma_\omega H_2 = 10x3 = 30\ kN/m^2$



$P_1 = \Delta p K_o H = 9.40\ x5 \qquad = 47\ kN/m$

$P_2 = ½\ k_o\ \gamma_{sat}\ H_1^2 = ½\ x\ 16.92x2 \qquad = 16.92\ kN/m$

$P_3 = k_o\gamma_{sat}\ H_1\ H_2 = 16.92\ x\ 3 \qquad = 50.76\ kN/m$

$P_4 = ½\ k_o\gamma_b\ H_2^{\ 2} = ½\ x\ 11.28x3 \qquad = 16.92\ kN/m$

$P_5 = ½\ \gamma_\omega\ H_2^2 = ½\ x\ 30\ x\ 3 \qquad = 45\ kN/m$

Resultant, $R = \sum_{i=1}^{5} P_i$

$= 47+16.92+50.76+16.92+45$

$= 176.60 kN/m$

Point of application of the resultant

This is determined by taking moment about the base of the retaining wall

$$RY = P_1 x2.5 + P_2 x\left(\frac{1}{3}(2)+3\right) + P_3 x\frac{3}{2} + P_4 x\frac{1}{3}(3) + P_5 x\frac{1}{3}(3)$$

176.60Y = 47x2.5+16.92x3.67+50.76x1.5+16.92x1+45x1

$$Y = \frac{317.60 kN/m.m}{176.60 kN/m} = 1.80 \text{ from the base}$$

b) For the active Rankine state

$$K_a = \tan^2\left(45 - \frac{\phi}{2}\right) = \tan^2\left(45 - \frac{32}{2}\right)$$

$$= 0.31$$

From surcharge

$$K_a \Delta p = 0.31x20 = 6.20 \text{ kN/m}^2$$

From backfill

at depth = 2.00 m

$$K_a \gamma_{sat} H_1 = 0.31x18x2 = 11.16 \text{ kN/m}^2$$

at depth – 5.00 m

$$K_a \gamma_{sat} H_1 + k_a \gamma_b H_2 = 11.16 + 0.31x8x3 = 18.50 \text{ kN/m}^2$$

From hydrostatic

$$\gamma_\omega H_2 = 10 \text{ x } 3 = 30 \text{ kN/m}^2$$

P_1 = 6.20 x 5 = 31kN/m

P_2 = ½ x 11.16 x 2 = 11.16 kN/m

P_3 = 11.16 x 3 = 33.48 kN/m

P_4 = ½ x 7.44 x 3 = 11.16kN/m

P_5 = ½ x 30 x 3 = 45kN/m

Resultant, $R = \sum_{i=1}^{5} P_i$

= 31+11.16+33.48+11.16+45

= 131.80kN/m

Point of application

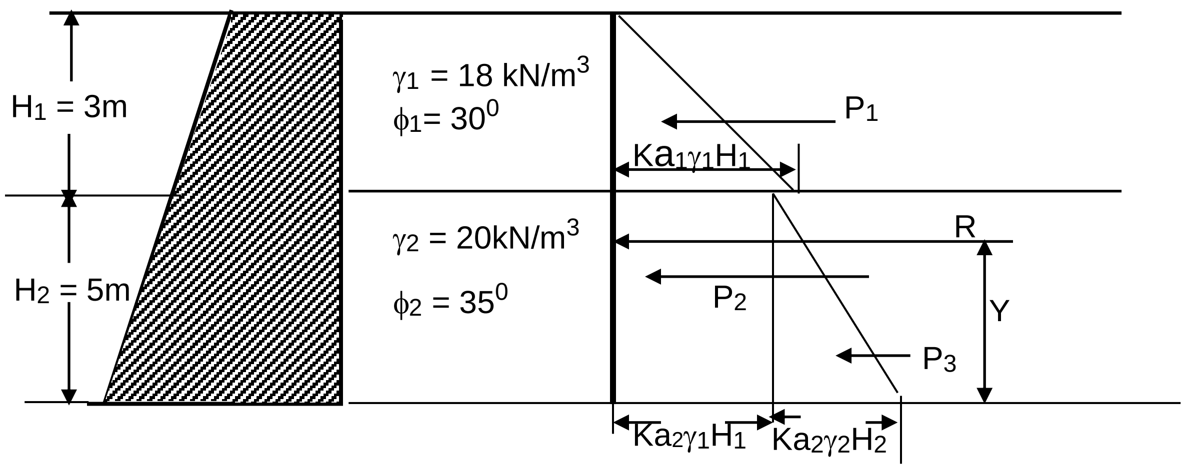
31.80 y=31x2.5+11.16 x 33.48+ 3.48 x 1.5+11.16 x1 + 45x1

$$Y = \frac{224.48}{131.80} = 1.71\text{m from the base}$$

Example 1.3

A retaining wall with vertical back has 8m heights. The unit weight of the top 3m of fill is 18kN/m^3 and the angle of internal friction 30^0, for the lower 5m the value of γ is 20 kN/m^3 and the angle of internal friction is 35^0. Find the magnitude and point of application of the active thrust on the wall per linear meter.

Solution



Active earth pressure coefficient

$$K_{a1} = \tan^2\left(45 - \frac{\phi}{2}\right) = \tan^2\left(45 - \frac{30}{2}\right) = ⅓$$

$$K_{a2} = \tan^2\left(45 - \frac{\phi}{2}\right) = \tan^2\left(45 - \frac{35}{2}\right) = 0.27$$

Active lateral earth pressure

at depth – 3.00 m

$$K_{a1}\gamma_1 H_1 = ⅓\,(18)(3) = 18 \text{ kN/m}^2$$

at depth – 5.00 m

$$K_{a2}\gamma_1 H_1 + k_{a2}\gamma_2 H_2 = 0.27x18x3+0.27x20x5$$

$$= 14.58 + 27 = 41.58 \text{ kN/m}^2$$

$$P_1 = ½ \text{ x } 18 \text{ x } 3 = 27 \text{ kN/m}$$

$P_2 = 14.58 \times 5 \quad = 72.9$ kN/m

$P_3 = ½ \times 27 \times 5 = 67.5$ kN/m

Resultant, $R = \sum_{i=1}^{3} P_i = 167.40 kN/m$

Point of application of the resultant

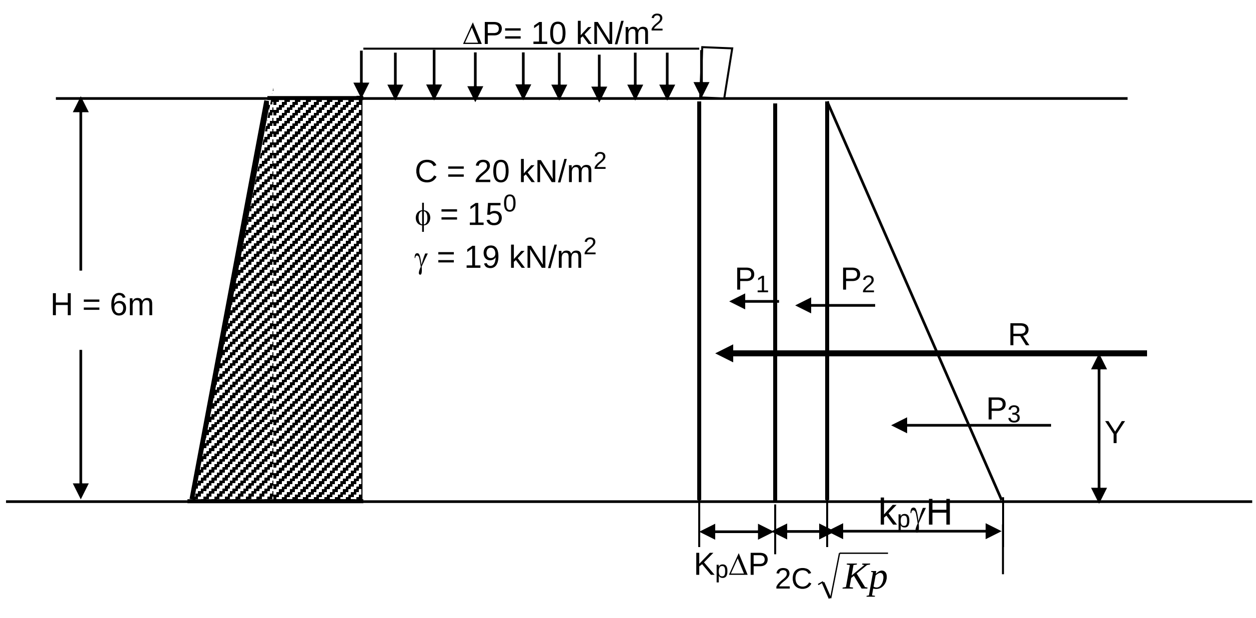
$167.40Y = 27 \times 6 + 72.9 \times 2.5 + 67.5 \times 1.67$

$Y = \frac{456.75}{167.40} = 2.73$ m from the base of the wall

Example 1.4

A smooth vertical wall 6 m high is pushed against a mass of soil having a horizontal surface and shearing resistance given by Coulomb's equation in which C= 20 kN/m^2 and ϕ = 15^0. The unit weight of the soil is 19 kN/m^3. Its surface carries a uniform load of 10kN/m^2. What is the total passive Rankine pressure? What is the distance from the base of the wall to the center of pressure?

Solution



$$K_p = \tan^2\left(45+\frac{\phi}{2}\right) = \tan^2\left(45+\frac{15}{2}\right) = 1.70$$

Passive earth pressure

From surcharge

$$K_p \Delta P = 1.70 x 10 = 17 \text{ kN/m}^2$$

From backfill

$$K_p \gamma H + 2C\sqrt{Kp} = 1.70 x 19 x 6 + 2x20x\sqrt{1.70}$$

$$= 193.80 + 52.15 \text{ kN/m}^2$$

$$= 245.95 \text{ kN/m}^2$$

$P_1 = k_P \Delta P H = 17x6 \qquad = 102$ kN/m

$P_2 = (2C\sqrt{Kp})\, H = 52.15x6 \qquad = 312.90$ kN/m

$P_3 = ½ (k_p \gamma H)\, H = ½ \times 193.80x6 \qquad = 581.40$ kN/m

Resultant, $\quad R = \sum_{i=1}^{3} P_i = 996.30 kN/m$

Location of the resultant

$$996.30\, Y = 102x3 + 312.90x3 + 581.40x2$$

$$Y = \frac{2407.5}{996.30} = 2.42 \text{ m from the base of the wall}$$

EXERCISES

4.1 A retaining wall 5 m high with vertical back face has a cohesionless soil for backfill. For the following cases, determine the total active force per unit length of the wall for Rankine's state and the location of the resultant.

a. γ = 17 kN/m^3, ϕ = 32^0

b. γ = 19kN/m^3, ϕ = 35^0

c. γ = 17.5 kN/m^3, ϕ = 33^0

4.2 A retaining wall is shown in Fig. E4.1. Determine the Rankine active force per unit length of the wall. Find also the location of the resultant for each case.

Given

a) H = 5.5 m, H_1= 2.75 m, γ_1 = 15.7 kN/m^3
γ_2 = 19.2 kN/m^3 ,ϕ_1 = 32^0, ϕ_2 = 34^0, ΔP = 15 kN/m^2

b) H = 5m, H_1 = 1.5m, γ_1 = 17.2 kN/m^3, γ_2= 20.4 kN/m^3
ϕ= 30^0, ϕ_2 = 32^0, ΔP= 19.2 kN/m^2

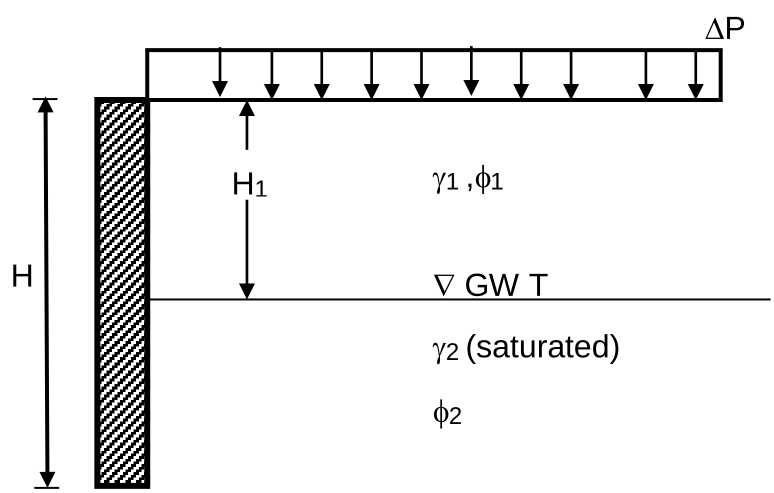


Fig E 4.1

4.3 Referring to Fig E4.1, determine the Rankine passive force per unit length of the wall for the following cases. Also find the location of the resultant for each case

a) H = 5.5 m, H_1 = 2.75 m, γ_1 = 15.7 kN/m^3

γ_2 = 19.2 kN/m^3, ϕ_1 =32^0, ϕ_2= 34^0, ΔP = 15 kN/m^2

b) H = 5m, H_1 = 1.5m, γ_1 = 17.2 kN/m^3, γ_2= 20.4 kN/m^3

ϕ_1= 30^0, ϕ_2 = 32^0, ΔP= 19.2 kN/m^2

4.4 A retaining wall 6 m high with vertical back face retains homogeneous saturated soft clay. The saturated unit weight of clay is 20.5 kN/m^3. Laboratory tests show that the undrained shear strength C_u of the clay is 50 kN/m^2.

a) Find the depth to which tensile crack may occur
b) Determine the total active force per unit length of the wall. Find also the location of the resultant.

4.5 A retaining wall 7 m high with vertical back face has a C-ϕ soil for backfill.

Given: For the backfill

γ = 18.6 kN/m^3, C = 25 kN/m^2, ϕ = 16^0.

Determine:-

a) The total active force per unit length of the wall.
b) The total passive force per unit length of the wall.