CHAPTER 10

STABILITY OF SLOPES

10.1 INTRODUCTION

Slopes of earth are of two types

1. Natural slopes
2. Man made slopes

Natural slopes are those that exist in nature and are formed by natural causes. Such slopes exist in hilly areas. The sides of cuttings, the slopes of embankments constructed for roads, railway lines, canals etc. and the slopes of earth dams constructed for storing water are examples of man made slopes. The slopes whether natural or artificial may be

1. Infinite slopes
2. Finite slopes

The term infinite slope is used to designate a constant slope of infinite extent. The long slope of the face of a mountain is an example of this type, whereas finite slopes are limited in extent. The slopes of embankments and earth dams are examples of finite slopes. The slope length depends on the height of the dam or embankment.

Slope Stability: Slope stability is an extremely important consideration in the design and construction of earth dams. The stability of a natural slope is also important. The results of a slope failure can often be catastrophic, involving the loss of considerable property and many lives.

Causes of Failure of Slopes: The important factors that cause instability in a slope and lead to failure are

1. Gravitational force
2. Force due to seepage water
3. Erosion of the surface of slopes due to flowing water

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4. The sudden lowering of water adjacent to a slope
5. Forces due to earthquakes

The effect of all the forces listed above is to cause movement of soil from high points to low points. The most important of such forces is the component of gravity that acts in the direction of probable motion. The various effects of flowing or seeping water are generally recognized as very important in stability problems, but often these effects have not been properly identified. It is a fact that the seepage occurring within a soil mass causes seepage forces, which have much greater effect than is commonly realized.

Erosion on the surface of a slope may be the cause of the removal of a certain weight of soil, and may thus lead to an increased stability as far as mass movement is concerned. On the other hand, erosion in the form of undercutting at the toe may increase the height of the slope, or decrease the length of the incipient failure surface, thus decreasing the stability.

When there is a lowering of the ground water or of a freewater surface adjacent to the slope, for example in a sudden drawdown of the water surface in a reservoir there is a decrease in the buoyancy of the soil which is in effect an increase in the weight. This increase in weight causes increase in the shearing stresses that may or may not be in part counteracted by the increase in

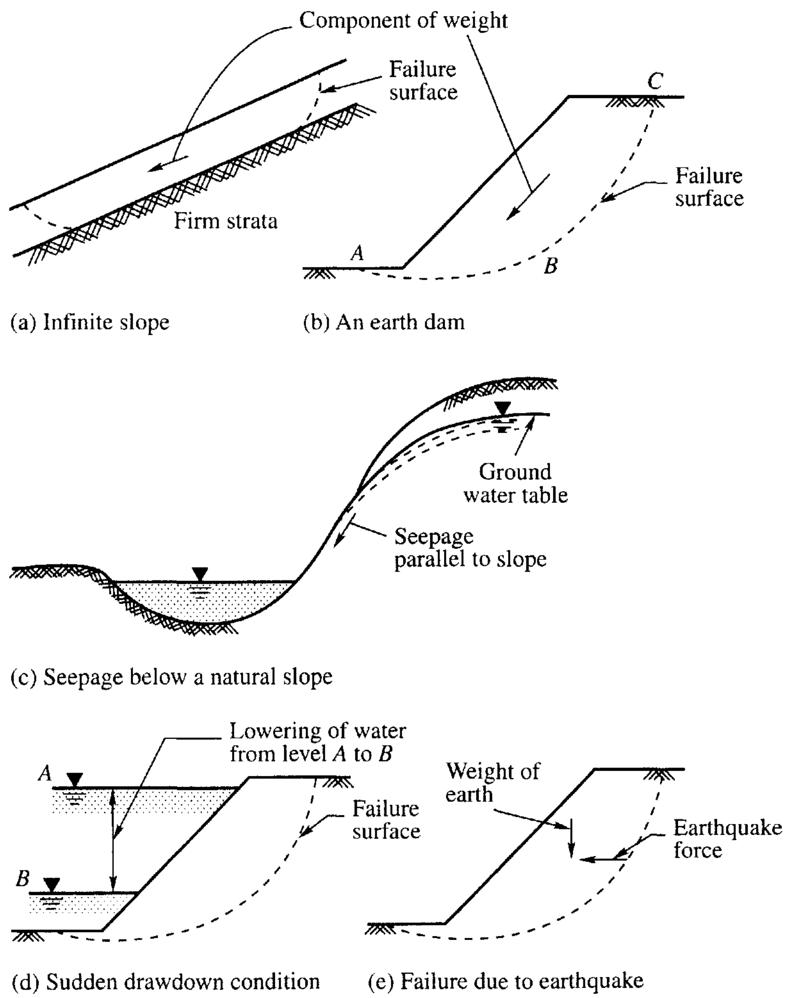

Figure 10.1 Forces that act on earth slopes

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shearing strength, depending upon whether or not the soil is able to undergo compression which the load increase tends to cause. If a large mass of soil is saturated and is of low permeability, practically no volume changes will be able to occur except at a slow rate, and in spite of the increase of load the strength increase may be inappreciable.

Shear at constant volume may be accompanied by a decrease in the intergranular pressure and an increase in the neutral pressure. A failure may be caused by such a condition in which the entire soil mass passes into a state of liquefaction and flows like a liquid. A condition of this type may be developed if the mass of soil is subject to vibration, for example, due to earthquake forces.

The various forces that act on slopes are illustrated in Fig. 10.1.

10.2 GENERAL CONSIDERATIONS AND ASSUMPTIONS IN THE ANALYSIS

There are three distinct parts to an analysis of the stability of a slope. They are:

1. Testing of samples to determine the cohesion and angle of internal friction

If the analysis is for a natural slope, it is essential that the sample be undisturbed. In such important respects as rate of shear application and state of initial consolidation, the condition of testing must represent as closely as possible the most unfavorable conditions ever likely to occur in the actual slope.

2. The study of items which are known to enter but which cannot be accounted for in the computations

The most important of such items is progressive cracking which will start at the top of the slope where the soil is in tension, and aided by water pressure, may progress to considerable depth. In addition, there are the effects of the non-homogeneous nature of the typical soil and other variations from the ideal conditions which must be assumed.

3. Computation

If a slope is to fail along a surface, all the shearing strength must be overcome along that surface which then becomes a surface of rupture. Any one such as *ABC* in Fig. 10.1(b) represents one of an infinite number of possible traces on which failure might occur.

It is assumed that the problem is two dimensional, which theoretically requires a long length of slope normal to the section. However, if the cross section investigated holds for a running length of roughly two or more times the trace of the rupture, it is probable that the two dimensional case holds within the required accuracy.

The shear strength of soil is assumed to follow Coulomb's law

$$s = c' + \sigma' \tan \phi'$$

where,

c' = effective unit cohesion

σ' = effective normal stress on the surface of rupture = $(\sigma - u)$

σ = total normal stress on the surface of rupture

u = pore water pressure on the surface of rupture

ϕ' = effective angle of internal friction.

The item of great importance is the loss of shearing strength which many clays show when subjected to a large shearing strain. The stress-strain curves for such clays show the stress rising with increasing strain to a maximum value, after which it decreases and approaches an ultimate value which may be much less than the maximum. Since a rupture surface tends to develop progressively rather than with all the points at the same state of strain, it is generally the ultimate value that should be used for the shearing strength rather than the maximum value.

10.3 FACTOR OF SAFETY

In stability analysis, two types of factors of safety are normally used. They are

1. Factor of safety with respect to shearing strength.
2. Factor of safety with respect to cohesion. This is termed the factor of safety with respect to height.

Let,

F_s = factor of safety with respect to strength

F_c = factor of safety with respect to cohesion

F_H = factor of safety with respect to height

F_ϕ = factor of safety with respect to friction

c'_m = mobilized cohesion

ϕ'_m = mobilized angle of friction

τ = average value of mobilized shearing strength

s = maximum shearing strength.

The factor of safety with respect to shearing strength, F_s, may be written as

$$F_s = \frac{s}{\tau} = \frac{c' + \sigma' \tan\phi'}{\tau}$$

The shearing strength mobilized at each point on a failure surface may be written as

$$\tau = \frac{c'}{F_s} + \sigma' \frac{\tan\phi'}{F_s}$$

or $\tau = c'_m + \sigma' \tan\phi'_m$ (10.2)

where $c'_m = \dfrac{c'}{F_s}$, $\tan\phi'_m = \dfrac{\tan\phi'}{F_s}$

Actually the shearing resistance (mobilized value of shearing strength) does not develop to a like degree at all points on an incipient failure surface. The shearing strains vary considerably and the shearing stress may be far from constant. However the above expression is correct on the basis of average conditions.

If the factors of safety with respect to cohesion and friction are different, we may write the equation of the mobilized shearing resistance as

$$\tau = \frac{c'}{F_c} + \sigma' \frac{\tan\phi'}{F_\phi} \tag{10.3}$$

It will be shown later on that F_c depends on the height of the slope. From this it may be concluded that the factor of safety with respect to cohesion may be designated as the *factor of safety with respect to height.* This factor is denoted by F_H and it is the ratio between the critical height and the actual height, the critical height being the maximum height at which it is possible for a slope to be stable. We may write from Eq. (10.3)

$$\tau = \frac{c'}{F_H} + \sigma' \tan\phi' \tag{10.4}$$

where F_ϕ is arbitrarily taken equal to unity.

Example 10.1

The shearing strength parameters of a soil are

$c' = 26.7$ kN/m^2

$\phi' = 15°$

$c'_m = 17.8$ kN/m^2

$\phi'_m = 12°$

Calculate the factor of safety (a) with respect to strength, (b) with respect to cohesion and (c) with respect to friction. The average intergranular pressure σ' on the failure surface is 102.5 kN/m^2.

Solution

On the basis of the given data, the average shearing strength on the failure surface is

$s = 26.7 + 102.5 \tan 15°$

$= 26.7 + 102.5 \times 0.268 = 54.2$ kN/m^2

and the average value of mobilized shearing resistance is

$\tau = 17.8 + 102.5 \tan 12°$

$= 17.8 + 102.5 \text{ x } 0.212 = 39.6$ kN/m^2

$$F_s = \frac{54.2}{39.6} = 1.37; \quad F_c = \frac{26.70}{17.80} = 1.50; \quad F_\phi = \frac{\tan\phi'}{\tan\phi_m} = \frac{0.268}{0.212} = 1.26$$

The above example shows the factor of safety with respect to shear strength, F_s is 1.37, whereas the factors of safety with respect to cohesion and friction are different. Consider two extreme cases:

1. When the factor of safety with respect to cohesion is unity.
2. When the factor of safety with respect to friction is unity.

Case 1

$$\tau = 39.60 = 26.70 + \frac{102.50}{F_\phi}\tan 15° = 26.70 + \frac{102.50 \times 0.268}{F_\phi} = 26.70 + \frac{27.50}{F_\phi}$$

$$F_\phi = \frac{27.50}{12.90} = 2.13$$

Case 2

$$\tau = 39.60 = \frac{26.70}{F_c} + 102.50 \tan 15°$$

$$= \frac{26.70}{F_c} + 27.50$$

$$F_c = \frac{26.70}{12.10} = 2.20$$

We can have any combination of F_c and F_ϕ between these two extremes cited above to give the same mobilized shearing resistance of 39.6 kN/m^2. Some of the combinations of F_c and F_ϕ are given below.

Combination of F_c and F_ϕ

F_c	1.00	1.26	1.37	1.50	2.20
F_ϕ	2.12	1.50	1.37	1.26	1.00

Under Case 2, the value of $F_c = 2.20$ when $F_\phi = 1.0$. The factor of safety $F_c = 2.20$ is defined as the *factor of safety with respect to cohesion.*

Example 10.2

What will be the factors of safety with respect to average shearing strength, cohesion and internal friction of a soil, for which the shear strength parameters obtained from the laboratory tests are $c' = 32$ kN/m^2 and $\phi' = 18°$; the expected parameters of mobilized shearing resistance are $c'_m = 21$ kN/m^2 and $\phi'_m = 13°$ and the average effective pressure on the failure plane is 110 kN/m^2. For the same value of mobilized shearing resistance determine the following:

1. Factor of safety with respect to height;
2. Factor of safety with respect to friction when that with respect to cohesion is unity; and
3. Factor of safety with respect to strength.

Solution

The available shear strength of the soil is

$s = 32 + 110 \tan 18° = 32 + 35.8 = 67.8$ kN/m^2

The mobilized shearing resistance of the soil is

$\tau = 21 + 110 \tan 13° = 21 + 25.4 = 46.4$ kN/m^2

Factor of safety with respect to average strength, $F_s = \dfrac{67.8}{46.4} = 1.46$

Factor of safety with respect to cohesion, $F_c = \dfrac{32}{21} = 1.52$

Factor of safety with respect to friction, $F_\phi = \dfrac{\tan\phi'}{\tan\phi'_m} = \dfrac{\tan 18°}{\tan 13} = \dfrac{0.3249}{0.2309} = 1.41$

Factor of safety with respect to height, F_H $(= F_c)$ will be at $F_\phi = 1.0$

$\tau = 46.4 = \dfrac{32}{F_c} + \dfrac{110 \tan 18°}{1.0}$, therefore, $F_c = \dfrac{32}{46.4 - 35.8} = 3.0$

Factor of safety with respect to friction at $F_c = 1.0$ is

$$\tau = 46.4 = \frac{32}{1.0} + \frac{110 \tan 18^\circ}{F_\phi}, \text{ therefore, } F_\phi = \frac{35.8}{46.4 - 32} = 2.49$$

Factor of safety with respect to strength F_s is obtained when $F_c = F_\phi$. We may write

$$\tau = 46.4 = \frac{32}{F_s} + \frac{110 \tan 18^\circ}{F_s} \text{ or } F_s = 1.46$$

10.4 STABILITY ANALYSIS OF INFINITE SLOPES IN SAND

As an introduction to slope analysis, the problem of a slope of infinite extent is of interest. Imagine an infinite slope, as shown in Fig. 10.2, making an angle β with the horizontal. The soil is cohesionless and completely homogeneous throughout. Then the stresses acting on any vertical plane in the soil are the same as those on any other vertical plane. The stress at any point on a plane *EF* parallel to the surface at depth z will be the same as at every point on this plane.

Now consider a vertical slice of material *ABCD* having a unit dimension normal to the page. The forces acting on this slice are its weight W, a vertical reaction R on the base of the slice, and two lateral forces P_1 acting on the sides. Since the slice is in equilibrium, the weight and reaction are equal in magnitude and opposite in direction. They have a common line of action which passes through the center of the base *AB*. The lateral forces must be equal and opposite and their line of action must be parallel to the sloped surface.

The normal and shear stresses on plane *AB* are

$$\sigma'_n = \gamma z \cos^2 \beta$$

$$\tau = \gamma z \cos \beta \sin \beta$$

where σ'_n = effective normal stress,

γ = effective unit weight of the sand.

If full resistance is mobilized on plane *AB*, the shear strength, s, of the soil per Coulomb's law is

$$s = \sigma'_n \tan \phi'$$

when $\tau = s$, substituting for s and σ'_n, we have

$$\gamma z \cos \beta \sin \beta = \gamma z \cos^2 \beta \tan \phi'$$

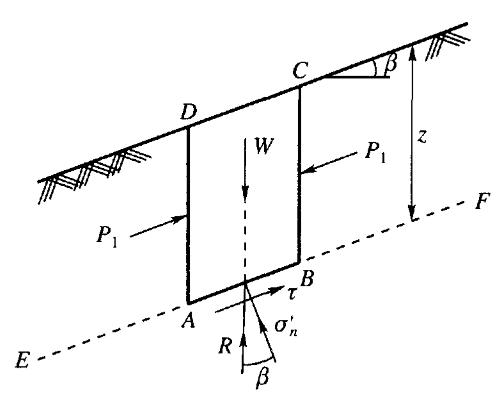

Figure 10.2 Stability analysis of infinite slope in sand

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or $\tan\beta = \tan\phi'$ (10.5a)

Equation (10.5a) indicates that the maximum value of β is limited to ϕ' if the slope is to be stable. This condition holds true for cohesionless soils whether the slope is completely dry or completely submerged under water.

The factor of safety of infinite slopes in sand may be written as

$$F_s = \frac{\tan\phi'}{\tan\beta} \tag{10.5b}$$

10.5 STABILITY ANALYSIS OF INFINITE SLOPES IN CLAY

The vertical stress σ_v acting on plane AB (Fig. 10.3) where

$$\sigma_v = \gamma z \cos\beta$$

is represented by OC in Fig. 10.3 in the stress diagram. The normal stress on this plane is OE and the shearing stress is EC. The line OC makes an angle β with the σ-axis.

The Mohr strength envelope is represented by line FA whose equation is

$$s = c' + \sigma' \tan\phi'$$

According to the envelope, the shearing strength is ED where the normal stress is OE.

When β is greater than ϕ' the lines OC and FD meet. In this case the two lines meet at A. As long as the shearing stress on a plane is less than the shearing strength on the plane, there is no danger of failure. Figure 10.3 indicates that at all depths at which the direct stress is less than OB, there is no possibility of failure. However at a particular depth at which the direct stress is OB, the

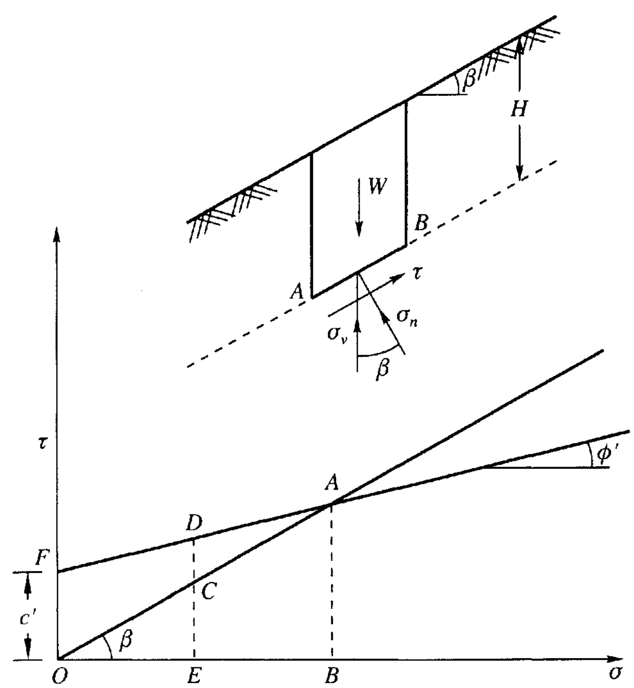

Figure 10.3 Stability analysis of infinite slopes in clay soils

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shearing strength and shearing stress values are equal as represented by *AB*, failure is imminent. This depth at which the shearing stress and shearing strength are equal is called the *critical depth.* At depths greater than this critical value, Fig. 10.3 indicates that the shearing stress is greater than the shearing strength but this is not possible. Therefore it may be concluded that the slope may be steeper than ϕ' as long as the depth of the slope is less than the critical depth.

Expression for the Stability of an Infinite Slope of Clay of Depth *H*

Equation (10.2) gives the developed shearing stress as

$$\tau = c'_m + \sigma' \tan \phi'_m \tag{10.6}$$

Under conditions of no seepage and no pore pressure, the stress components on a plane at depth H and parallel to the surface of the slope are

$$\tau = \gamma H \sin \beta \cos \beta$$

$$\sigma' = \gamma H \cos^2 \beta$$

Substituting these stress expressions in the equation above and simplifying, we have

$$c'_m = \gamma H \cos^2 \beta (\tan \beta - \tan \phi'_m)$$

$$\text{or } N_s = \frac{c'_m}{\gamma H} = \cos^2 \beta (\tan \beta - \tan \phi'_m) \tag{10.7}$$

where H is the allowable height and the term $c'_m/\gamma H$ is a dimensionless expression called the *stability number* and is designated as N_s. This dimensionless number is proportional to the required cohesion and is inversely proportional to the allowable height. The solution is for the case when no seepage is occurring. If in Eq. (10.7) the factor of safety with respect to friction is unity, the stability number with respect to cohesion may be written as

$$N_s = \frac{c'}{F_c \gamma H} = \cos^2 \beta (\tan \beta - \tan \phi') \tag{10.8}$$

where $c'_m = \dfrac{c'}{F_c}$

The stability number in Eq. (10.8) may be written as

$$N_s = \frac{c'}{F_c \gamma H} = \frac{c'}{\gamma H_c} \tag{10.9}$$

where H_c = critical height. From Eq. (10.9), we have

$$F_c = \frac{H_c}{H} = F_H \tag{10.10}$$

Eq. (10.10) indicates that the factor of safety with respect to cohesion, F_c, is the same as the factor of safety with respect to height F_H.

If there is seepage parallel to the ground surface throughout the entire mass, with the free water surface coinciding with the ground surface, the components of effective stresses on planes parallel to the surface of slopes at depth H are given as [Fig. 10.4(a)].

Normal stress

$$\sigma'_n = (\gamma_{sat} - \gamma_w) H \cos^2 \beta = \gamma_b H \cos^2 \beta \tag{10.11a}$$

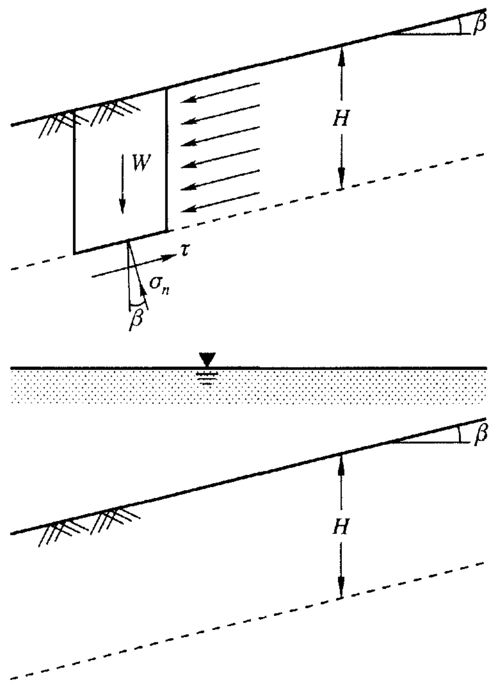

Figure 10.4 Analysis of infinite slope (a) with seepage flow through the entire mass, and (b) with completely submerged slope.

the shearing stress

$$\tau = \gamma_{sat} H \sin\beta \cos\beta \tag{10.11b}$$

Now substituting Eqs (10.11a) and (10.11b) into equation

$$\tau = c'_m + \sigma'_n \tan\phi'_m$$

and simplifying, the stability expression obtained is

$$\frac{c'_m}{\gamma_{sat} H} = \cos^2\beta \ \tan\beta - \frac{\gamma_b}{\gamma_{sat}} \tan\phi'_m \tag{10.12}$$

As before, if the factor of safety with respect to friction is unity, the stability number which represents the cohesion may be written as

$$N_s = \frac{c'}{F_c \gamma_{sat} H} = \frac{c'}{\gamma_{sat} H_c} = \cos^2\beta \ \tan\beta - \frac{\gamma_b}{\gamma_{sat}} \tan\phi' \tag{10.13}$$

If the slope is completely submerged, and if there is no seepage as in Fig. 10.4(b), then Eq. (10.13) becomes

$$N_s = \frac{c'}{F_c \gamma_b H} = \frac{c'}{\gamma_b H_c} = \cos^2\beta(\tan\beta - \tan\phi') \tag{10.14}$$

where γ_b = submerged unit weight of the soil.

Example 10.3

Find the factor of safety of a slope of infinite extent having a slope angle = 25°. The slope is made of cohesionless soil with $\phi = 30°$.

Solution

Factor of safety

$$F_s = \frac{\tan\phi'}{\tan\beta} = \frac{\tan 30°}{\tan 25°} = \frac{0.5774}{0.4663} = 1.238$$

Example 10.4

Analyze the slope of Example 10.3 if it is made of clay having $c' = 30\ \text{kN/m}^2$, $\phi' = 20°$, $e = 0.65$ and $G_s = 2.7$ and under the following conditions: (i) when the soil is dry, (ii) when water seeps parallel to the surface of the slope, and (iii) when the slope is submerged.

Solution

For $e = 0.65$ and $G_s = 2.7$

$$\gamma_d = \frac{2.7 \times 9.81}{1 + 0.65} = 16.05\ \text{kN/m}^3, \quad \gamma_{\text{sat}} = \frac{(2.7 + 0.65) \times 9.81}{1 + 0.65} = 19.9\ \text{kN/m}^3,$$

$\gamma_b = 10.09\ \text{kN/m}^3$

(i) For dry soil the stability number N_s is

$$N_s = \frac{c'}{\gamma_d H_c} = \cos^2\beta(\tan\beta - \tan\phi') \quad \text{when } F_\phi = 1$$

$$= (\cos 25°)^2(\tan 25° - \tan 20°) = 0.084.$$

$$\text{Therefore, the critical height } H_c = \frac{c'}{\gamma_d \times N_s} = \frac{30}{16.05 \times 0.084} = 22.25\ \text{m}$$

(ii) For seepage parallel to the surface of the slope [Eq. (10.13)]

$$N_s = \frac{c'}{\gamma_t H_c} = \cos^2 25° \ \tan 25° - \frac{10.09}{19.9}\tan 20° \ = 0.2315$$

$$H_c = \frac{c'}{\gamma_t N_s} = \frac{30}{19.9 \times 0.2315} = 6.51\ \text{m}$$

(iii) For the submerged slope [Eq. (10.14)]

$N_s = \cos^2 25°\ (\tan 25° - \tan 20°) = 0.084$

$$H_c = \frac{c'}{\gamma_b N_s} = \frac{30}{10.09 \times 0.084} = 35.4\ \text{m}$$

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10.6 METHODS OF STABILITY ANALYSIS OF SLOPES OF FINITE HEIGHT

The stability of slopes of infinite extent has been discussed in previous sections. A more common problem is the one in which the failure occurs on curved surfaces. The most widely used method of analysis of homogeneous, isotropic, finite slopes is the *Swedish method* based on circular failure surfaces. Petterson (1955) first applied the circle method to the analysis of a soil failure in connection with the failure of a quarry wall in Goeteberg, Sweden. A Swedish National Commission, after studying a large number of failures, published a report in 1922 showing that the lines of failure of most such slides roughly approached the circumference of a circle. The failure circle might pass above the toe, through the toe or below it. By investigating the strength along the arc of a large number of such circles, it was possible to locate the circle which gave the lowest resistance to shear. This general method has been quite widely accepted as offering an approximately correct solution for the determination of the factor of safety of a slope of an embankment and of its foundation. Developments in the method of analysis have been made by Fellenius (1947), Terzaghi (1943), Gilboy (1934), Taylor (1937), Bishop (1955), and others, with the result that a satisfactory analysis of the stability of slopes, embankments and foundations by means of the circle method is no longer an unduly tedious procedure.

There are other methods of historic interest such as the *Culmann method* (1875) and the *logarithmic spiral method.* The Culmann method assumes that rupture will occur along a plane. It is of interest only as a classical solution, since actual failure surfaces are invariably curved. This method is approximately correct for steep slopes. The logarithmic spiral method was recommended by Rendulic (1935) with the rupture surface assuming the shape of logarithmic spiral. Though this method makes the problem statically determinate and gives more accurate results, the greater length of time required for computation overbalances this accuracy.

There are several methods of stability analysis based on the circular arc surface of failure. A few of the methods are described below

Methods of Analysis

The majority of the methods of analysis may be categorized as limit equilibrium methods. The basic assumption of the limit equilibrium approach is that Coulomb's failure criterion is satisfied along the assumed failure surface. A free body is taken from the slope and starting from known or assumed values of the forces acting upon the free body, the shear resistance of the soil necessary for equilibrium is calculated. This calculated shear resistance is then compared to the estimated or available shear strength of the soil to give an indication of the factor of safety.

Methods that consider only the whole free body are the (a) slope failure under undrained conditions, (b) friction-circle method (Taylor, 1937, 1948) and (c) Taylor's stability number (1948).

Methods that divide the free body into many vertical slices and consider the equilibrium of each slice are the Swedish circle method (Fellenius, 1927), Bishop method (1955), Bishop and Morgenstern method (1960) and Spencer method (1967). The majority of these methods are in chart form and cover a wide variety of conditions.

10.7 PLANE SURFACE OF FAILURE

Culmann (1875) assumed a plane surface of failure for the analysis of slopes which is mainly of interest because it serves as a test of the validity of the assumption of plane failure. In some cases this assumption is reasonable and in others it is questionable.

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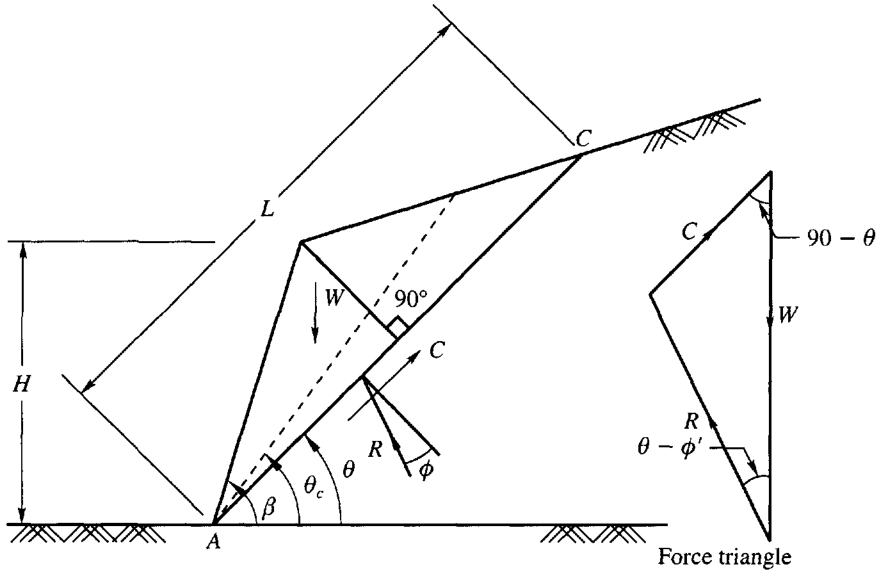

Figure 10.5 Stability of slopes by Culmann method

The method as indicated above assumes that the critical surface of failure is a plane surface passing through the toe of the dam as shown in Fig. 10.5.

The forces that act on the mass above trial failure plane AC inclined at angle θ with the horizontal are shown in the figure. The expression for the weight, W, and the total cohesion C are respectively,

$$W = \frac{1}{2}\gamma L H \operatorname{cosec}\beta \sin(\beta - \theta)$$

and $C = c'L$

The use of the law of sines in the force triangle, shown in the figure, gives

$$\frac{C}{W} = \frac{\sin(\theta - \phi')}{\cos\phi'}$$

Substituting herein for C and W, and rearranging we have

$$\left.\frac{c'}{\gamma H}\right|_{\theta} = \frac{1}{2}\operatorname{cosec}\beta \sin(\beta - \theta)\sin(\theta - \phi')\sec\phi'$$

in which the subscript θ indicates that the stability number is for the trial plane at inclination θ.

The most dangerous plane is obtained by setting the first derivative of the above equation with respect to θ equal to zero. This operation gives

$$\theta_c' = \frac{1}{2}(\beta + \phi')$$

where θ_c' is the critical angle for limiting equilibrium and the stability number for limiting equilibrium may be written as

$$\frac{c'}{\gamma H_c} = \frac{1 - \cos(\beta - \phi')}{4\sin\beta\cos\phi'} \tag{10.15}$$

where H_c is the critical height of the slope.

If we write

$$F_c = \frac{c'}{c'_m}, \quad F_\phi = \frac{\tan\phi'}{\tan\phi'_m}$$

where F_c and F_ϕ are safety factors with respect to cohesion and friction respectively, Eq. (10.15) may be modified for chosen values of c'_m and ϕ'_m as

$$\frac{c'_m}{\gamma H} = \frac{1-\cos(\beta-\phi'_m)}{4\sin\beta\cos\phi'_m} \tag{10.16}$$

The critical angle for any assumed values of c'_m and ϕ'_m is

$$\theta_c = \frac{1}{2}(\beta+\phi'_m) \tag{10.17}$$

From Eq. (10.16), the allowable height of a slope is

$$H = \frac{4c'_m \sin\beta\cos\phi'_m}{\gamma[1-\cos(\beta-\phi'_m)]} \tag{10.18}$$

Example 10.5

Determine by Culmann's method the critical height of an embankment having a slope angle of 40° and the constructed soil having $c' = 630$ psf, $\phi' = 20°$ and effective unit weight = 114 lb/ft^3. Find the allowable height of the embankment if $F_c = F_\phi = 1.25$.

Solution

$$H_c = \frac{4c'\sin\beta\cos\phi'}{\gamma[1-\cos(\beta-\phi')]} = \frac{4\times 630\times\sin 40°\cos 20°}{114(1-\cos 20°)} = 221 \text{ ft}$$

For $F_c = F_\phi = 1.25, \quad c'_m = \dfrac{c'}{F_c} = \dfrac{630}{1.25} = 504 \text{ lb/ft}^2$

and $\tan\phi'_m = \dfrac{\tan\phi'}{F_\phi} = \dfrac{\tan 20°}{1.25} = 0.291, \quad \phi'_m = 16.23°$

Allowable height, $H = \dfrac{4\times 504\sin 40°\cos 16.23°}{114[1-\cos(40-16.23°)]} = 128.7$ ft.

10.8 CIRCULAR SURFACES OF FAILURE

The investigations carried out in Sweden at the beginning of this century have clearly confirmed that the surfaces of failure of earth slopes resemble the shape of a circular arc. When soil slips along a circular surface, such a slide may be termed as a rotational slide. It involves downward and outward movement of a slice of earth as shown in Fig. 10.6(a) and sliding occurs along the entire surface of contact between the slice and its base. The types of failure that normally occur may be classified as

1. Slope failure

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2. Toe failure
3. Base failure

In slope failure, the arc of the rupture surface meets the slope above the toe. This can happen when the slope angle β is quite high and the soil close to the toe possesses high strength. Toe failure occurs when the soil mass of the dam above the base and below the base is homogeneous. The base failure occurs particularly when the base angle β is low and the soil below the base is softer and more plastic than the soil above the base. The various modes of failure are shown in Fig. 10.6.

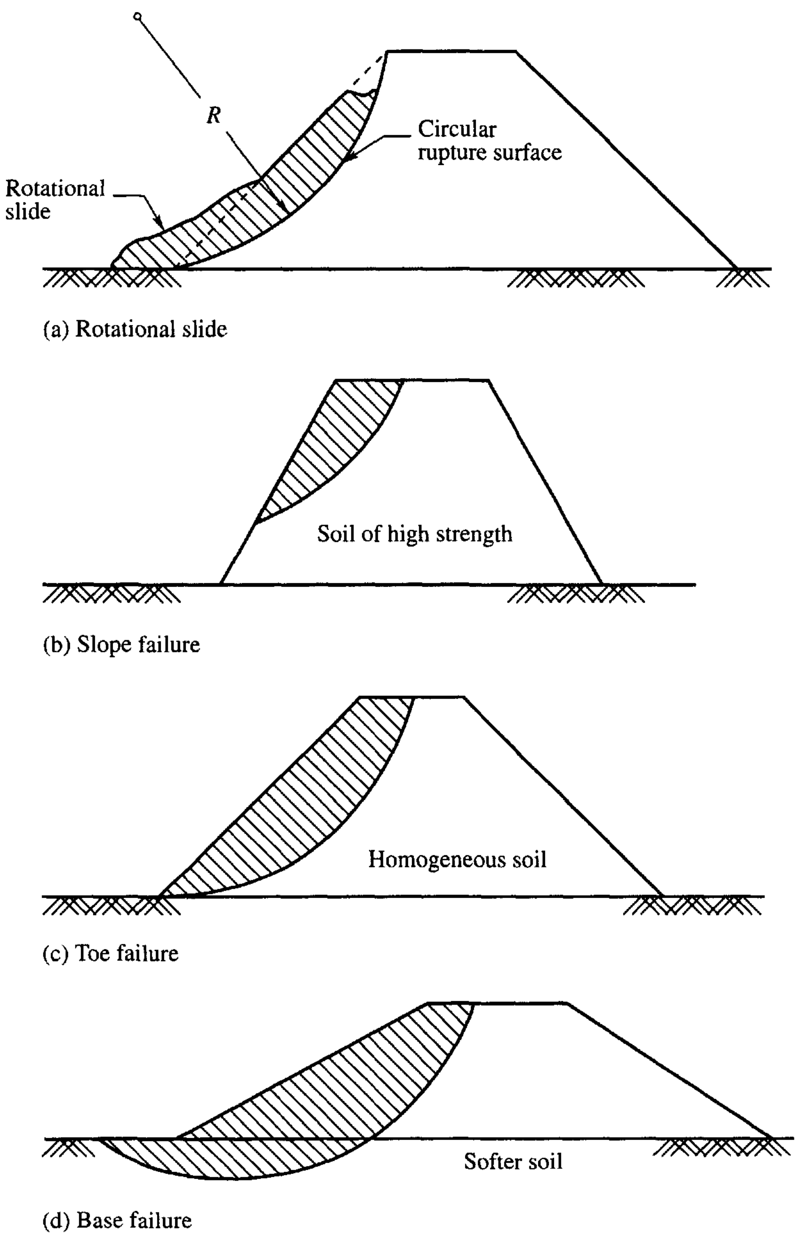

Figure 10.6 Types of failure of earth dams

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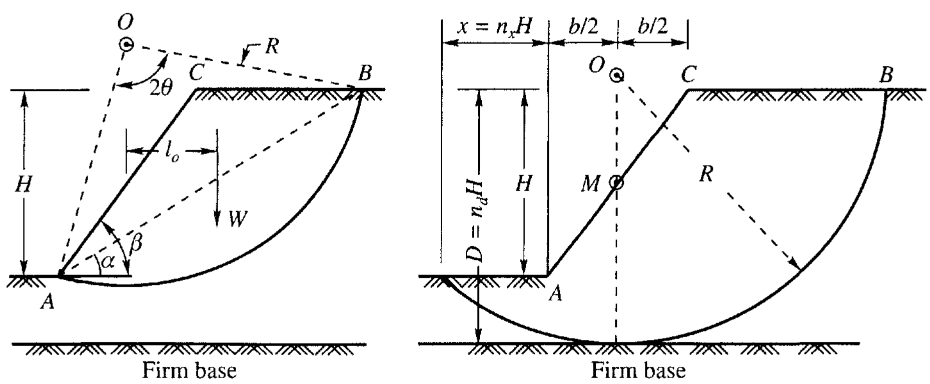
10.9 FAILURE UNDER UNDRAINED CONDITIONS (ϕ_u = 0)

A fully saturated clay slope may fail under undrained conditions (ϕ_u = 0) immediately after construction. The stability analysis is based on the assumption that the soil is homogeneous and the potential failure surface is a circular arc. Two types of failures considered are

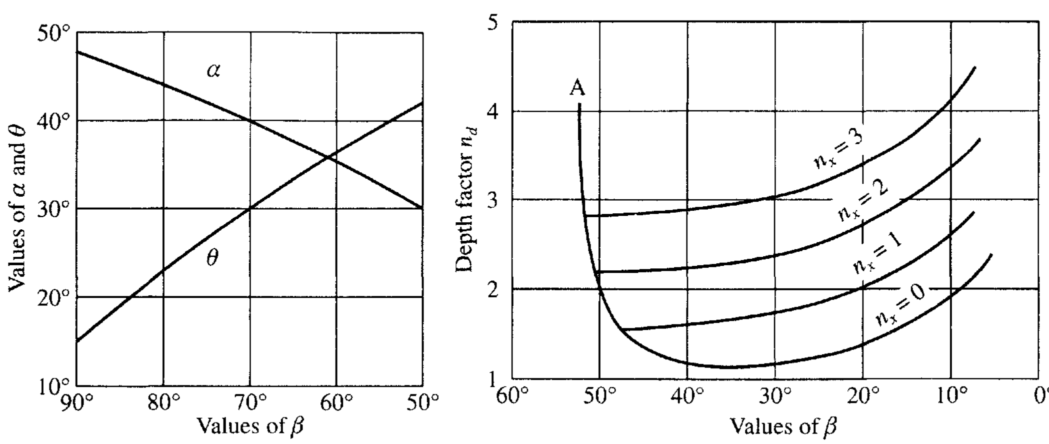
1. Slope failure
2. Base failure

The undrained shear strength c_u of soil is assumed to be constant with depth. A trial failure circular surface *AB* with center at *O* and radius *R* is shown in Fig. 10.7(a) for a toe failure. The slope *AC* and the chord *AB* make angles β and α with the horizontal respectively. *W* is the weight per unit



(a) Toe failure (b) Base failure

Figure 10.7 Critical circle positions for (a) slope failure (after Fellenius, 1927), (b) base failure



(a) (b)

Figure 10.8 (a) Relation between slope angle β and parameters α and θ for location of critical toe circle when β is greater than 53°; (b) relation between slope angle β and depth factor n_d for various values of parameter n_x (after Fellenius, 1927)

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length of the soil lying above the trial surface acting through the center of gravity of the mass. l_o is the lever arm, L_a is the length of the arc, L_c the length of the chord AB and c_m the mobilized cohesion for any assumed surface of failure.

We may express the factor of safety F_s as

$$F_s = \frac{c_u}{c_m} \tag{10.19}$$

For equilibrium of the soil mass lying above the assumed failure surface, we may write

resisting moment M_r = actuating moment M_a

The resisting moment $M_r = L_a c_m R$

Actuating moment, $M_a = Wl_o$

Equation for the mobilized c_m is

$$c_m = \frac{Wl_o}{L_a R} \tag{10.20}$$

Now the factor of safety F for the assumed trial arc of failure may be determined from Eq. (10.19). This is for one trial arc. The procedure has to be repeated for several trial arcs and the one that gives the least value is the critical circle.

If failure occurs along a toe circle, the center of the critical circle can be located by laying off the angles α and 2θ as shown in Fig. 10.7(a). Values of α and θ for different slope angles β can be obtained from Fig. 10.8(a).

If there is a base failure as shown in Fig. 10.7(b), the trial circle will be tangential to the firm base and as such the center of the critical circle lies on the vertical line passing through midpoint M on slope AC. The following equations may be written with reference to Fig. 10.7(b).

$$\text{Depth factor, } n_d = \frac{D}{H}, \quad \text{Distance factor, } n_x = \frac{x}{H} \tag{10.21}$$

Values of n_x can be estimated for different values of n_d and β by means of the chart Fig. 10.8(b).

Example 10.6

Calculate the factor of safety against shear failure along the slip circle shown in Fig. Ex. 10.6 Assume cohesion = 40 kN/m^2, angle of internal friction = zero and the total unit weight of the soil = 20.0 kN/m^3.

Solution

Draw the given slope $ABCD$ as shown in Fig. Ex. 10.6. To locate the center of rotation, extend the bisector of line BC to cut the vertical line drawn from C at point O. With O as center and OC as radius, draw the desired slip circle.

Radius $OC = R = 36.5$ m, Area $BECFB = \frac{2}{3} \times EF \times BC$

$$= \frac{2}{3} \times 4 \times 32.5 = 86.7 \text{ m}^2$$

Therefore $W = 86.7 \times 1 \times 20 = 1734$ kN

W acts through point G which may be taken as the middle of FE.

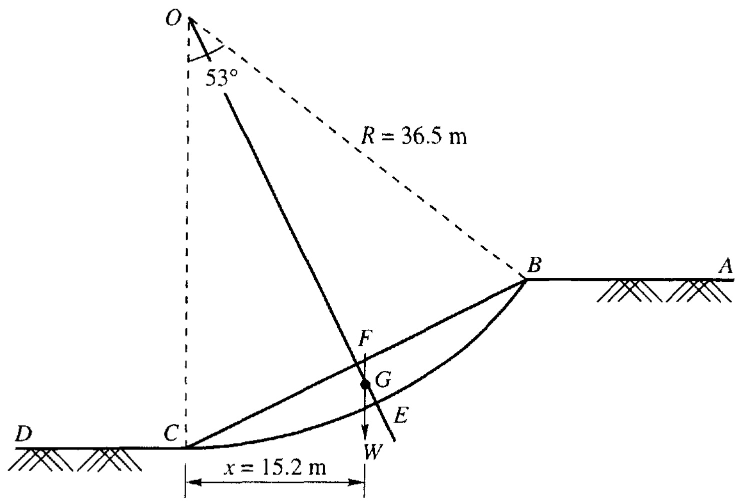

Figure. Ex. 10.6

From the figure we have, $x = 15.2$ m, and $\theta = 53°$

$$\text{Length of arc } BEC = R\theta = 36.5 \times 53° \times \frac{3.14}{180} = 33.8 \text{ m}$$

$$F_s = \frac{\text{length of arc} \times \text{cohesion} \times \text{radius}}{Wx} = \frac{33.8 \times 40 \times 36.5}{1734 \times 15.2} = 1.87$$

10.10 FRICTION-CIRCLE METHOD

Physical Concept of the Method

The principle of the method is explained with reference to the section through a dam shown in Fig. 10.9. A trial circle with center of rotation O is shown in the figure. With center O and radius

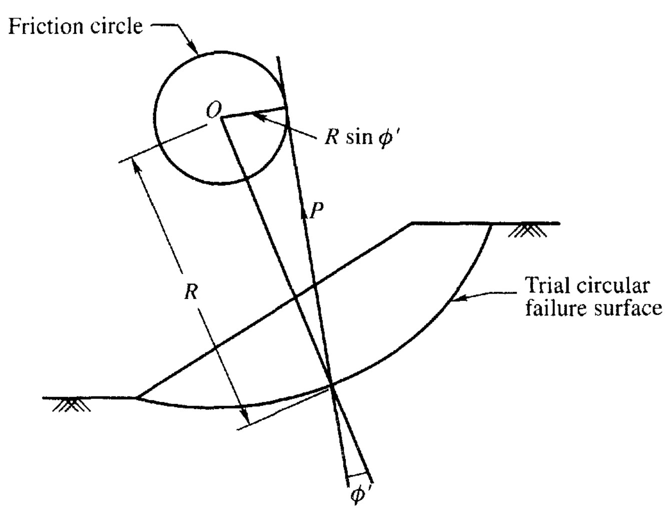

Figure 10.9 Principle of friction circle method

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sin ϕ', where R is the radius of the trial circle, a circle is drawn. Any line tangent to the inner circle must intersect the trial circle at an angle ϕ' with R. Therefore, any vector representing an intergranular pressure at obliquity ϕ' to an element of the rupture arc must be tangent to the inner circle. This inner circle is called the *friction circle* or *ϕ-circle*. The friction circle method of slope analysis is a convenient approach for both graphical and mathematical solutions. It is given this name because the characteristic assumption of the method refers to the ϕ-circle.

The forces considered in the analysis are

1. The total weight W of the mass above the trial circle acting through the center of mass. The center of mass may be determined by any one of the known methods.
2. The resultant boundary neutral force U. The vector U may be determined by a graphical method from flownet construction.
3. The resultant intergranular force, P, acting on the boundary.
4. The resultant cohesive force C.

Actuating Forces

The actuating forces may be considered to be the total weight W and the resultant boundary force U as shown in Fig. 10.10.

The boundary neutral force always passes through the center of rotation O. The resultant of W and U, designated as Q, is shown in the figure.

Resultant Cohesive Force

Let the length of arc AB be designated as L_a, the length of chord AB by L_c. Let the arc length L_a be divided into a number of small elements and let the mobilized cohesive force on these elements be designated as C_1, C_2, C_3, etc. as shown in Fig. 10.11. The resultant of all these forces is shown by the force polygon in the figure. The resultant is $A'B'$ which is parallel and equal to the chord length AB. The resultant of all the mobilized cohesional forces along the arc is therefore

$$C = c'_m L_c$$

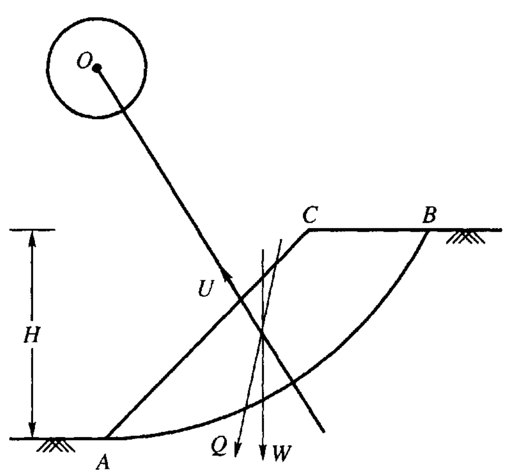

Figure 10.10 Actuating forces

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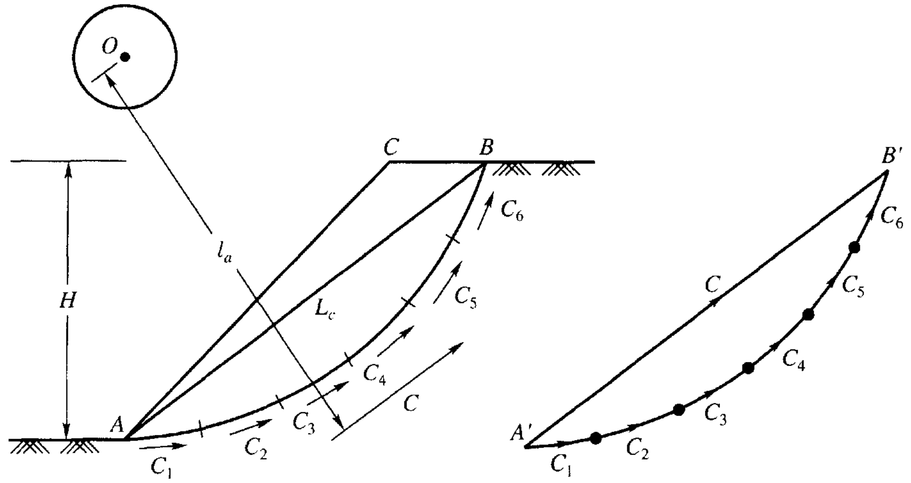

(a) Cohesive forces on a trial arc (b) Polygon of forces

Figure 10.11 Resistant cohesive forces

We may write $c'_m = \dfrac{c'}{F_c}$

wherein c' = unit cohesion, F_c = factor of safety with respect to cohesion.

The line of action of C may be determined by moment consideration. The moment of the total cohesion is expressed as

$$c'_m L_a R = c'_m L_c l_a$$

where l_a = moment arm. Therefore,

$$l_a = R\frac{L_a}{L_c} \tag{10.22}$$

It is seen that the line of action of vector C is independent of the magnitude of c'_m.

Resultant of Boundary Intergranular Forces

The trial arc of the circle is divided into a number of small elements. Let P_1, P_2, P_3, etc. be the intergranular forces acting on these elements as shown in Fig. 10.12. The friction circle is drawn with a radius of $R \sin \phi'_m$

where $\tan \phi'_m = \dfrac{\tan \phi'}{F_\phi}$

The lines of action of the intergranular forces P_1, P_2, P_3, etc. are tangential to the friction circle and make an angle of ϕ'_m at the boundary. However, the vector sum of any two small forces has a line of action through point D, missing tangency to the ϕ'_m-circle by a small amount. The resultant of all granular forces must therefore miss tangency to the ϕ'_m-circle by an amount which is not considerable. Let the distance of the resultant of the granular force P from the center of the circle be designated as $KR \sin \phi'_m$ (as shown in Fig. 10.12). The

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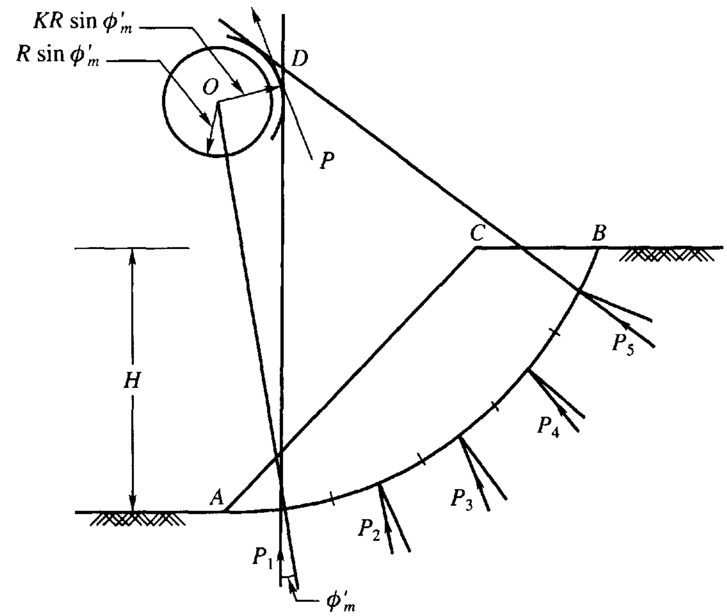

Figure 10.12 Resultant of intergranular forces

magnitude of K depends upon the type of intergranular pressure distribution along the arc. The most probable form of distribution is the sinusoidal distribution.

The variation of K with respect to the central angle α' is shown in Fig. 10.13. The figure also gives relationships between α' and K for a uniform stress distribution of effective normal stress along the arc of failure.

The graphical solution based on the concepts explained above is simple in principle. For the three forces Q, C and P of Fig. 10.14 to be in equilibrium, P must pass through the intersection of

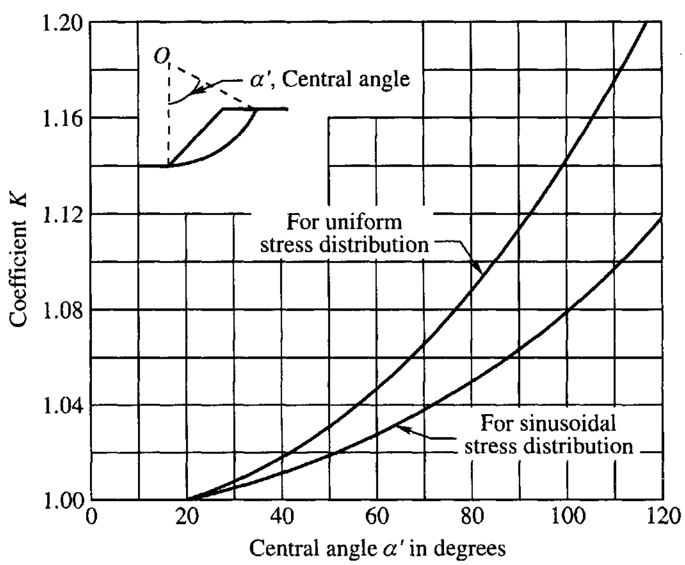

Figure 10.13 Relationship between K and central angle α'

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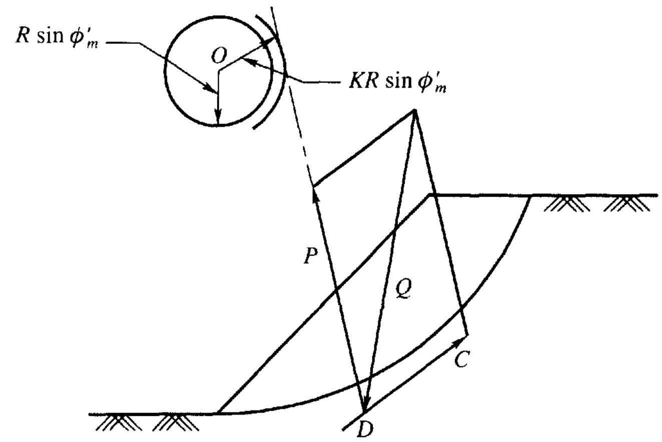

Figure 10.14 Force triangle for the friction-circle method

the known lines of action of vectors Q and C. The line of action of vector P must also be tangent to the circle of radius $KR \sin \phi'_m$. The value of K may be estimated by the use of curves given in Fig. 10.13, and the line of action of force P may be drawn as shown in Fig. 10.14. Since the lines of action of all three forces and the magnitude of force Q are known, the magnitude of P and C may be obtained by the force parallelogram construction that is indicated in the figure. The circle of radius of $KR \sin \phi'_m$ is called the *modified friction circle*.

Determination of Factor of Safety With Respect to Strength

Figure 10.15(a) is a section of a dam. AB is the trial failure arc. The force Q, the resultant of W and U is drawn as explained earlier. The line of action of C is also drawn. Let the forces Q and C

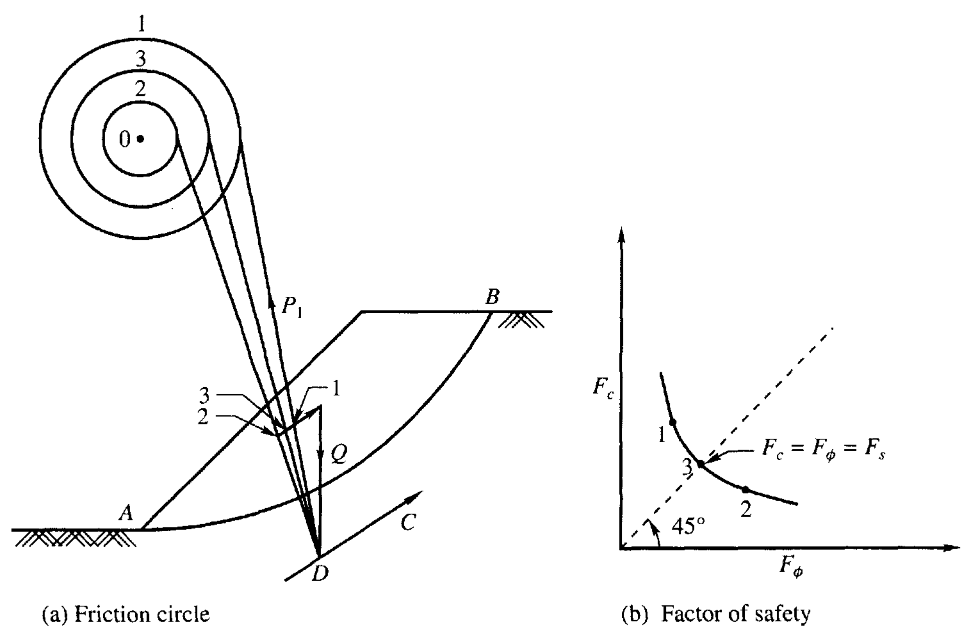

Figure 10.15 Graphical method of determining factor of safety with respect to strength

meet at point D. An arbitrary first trial using any reasonable ϕ'_m value, which will be designated by ϕ'_{m1} is given by the use of circle 1 or radius $KR \sin \phi'_{m1}$. Subscript 1 is used for all other quantities of the first trial. The force P_1 is then drawn through D tangent to circle 1. C_1 is parallel to chord and point 1 is the intersection of forces C_1 and P_1. The mobilized cohesion is equal $c'_{m1}L_c$. From this the mobilized cohesion c'_{m1} is evaluated. The factors of safety with respect to cohesion and friction are determined from the expressions

$$F'_c = \frac{c'}{c'_{m1}}, \text{ and } F_{\phi 1} = \frac{\tan \phi'}{\tan \phi'_{m1}}$$

These factors are the values used to plot point 1 in the graph in Fig. 10.15(b). Similarly other friction circles with radii $KR \sin \phi'_{m2}$, $KR \sin \phi'_{m3}$. etc. may be drawn and the procedure repeated. Points 2, 3 etc. are obtained as shown in Fig. 10.15(b). The 45° line, representing $F_c = F_\phi$, intersects the curve to give the factor of safety F_s for this trial circle.

Several trial circles must be investigated in order to locate the critical circle, which is the one having the minimum value of F_s.

Example 10.7

An embankment has a slope of 2 (horizontal) to 1 (vertical) with a height of 10 m. It is made of a soil having a cohesion of 30 kN/m^2, an angle of internal friction of 5° and a unit weight of 20 kN/m^3. Consider any slip circle passing through the toe. Use the friction circle method to find the factor of safety with respect to cohesion.

Solution

Refer to Fig. Ex. 10.7. Let EFB be the slope and AKB be the slip circle drawn with center O and radius R = 20 m.

Length of chord $AB = L_c$ = 27 m

Take J as the midpoint of AB, then

Area $AKBFEA$ = area $AKBJA$ + area $ABEA$

$$= \frac{2}{3} AB \times JK + \frac{1}{2} AB \times EL$$

$$= \frac{2}{3} \times 27 \times 5.3 + \frac{1}{2} \times 27 \times 2.0 = 122.4 \text{ m}^2$$

Therefore the weight of the soil mass = 122.4 × 1 × 20 = 2448 kN

It will act through point G, the centroid of the mass which can be taken as the mid point of FK.

Now, $\theta = 85°$,

$$\text{Length of arc } AKB = L = R\theta = 20 \times 85 \times \frac{3.14}{180} = 29.7 \text{ m}$$

$$\text{Moment arm of cohesion, } l_a = R\frac{L}{L_c} = 20 \times \frac{29.7}{27} = 22 \text{ m}$$

From center O, at a distance l_a, draw the cohesive force vector C, which is parallel to the chord AB. Now from the point of intersection of C and W, draw a line tangent to the friction circle

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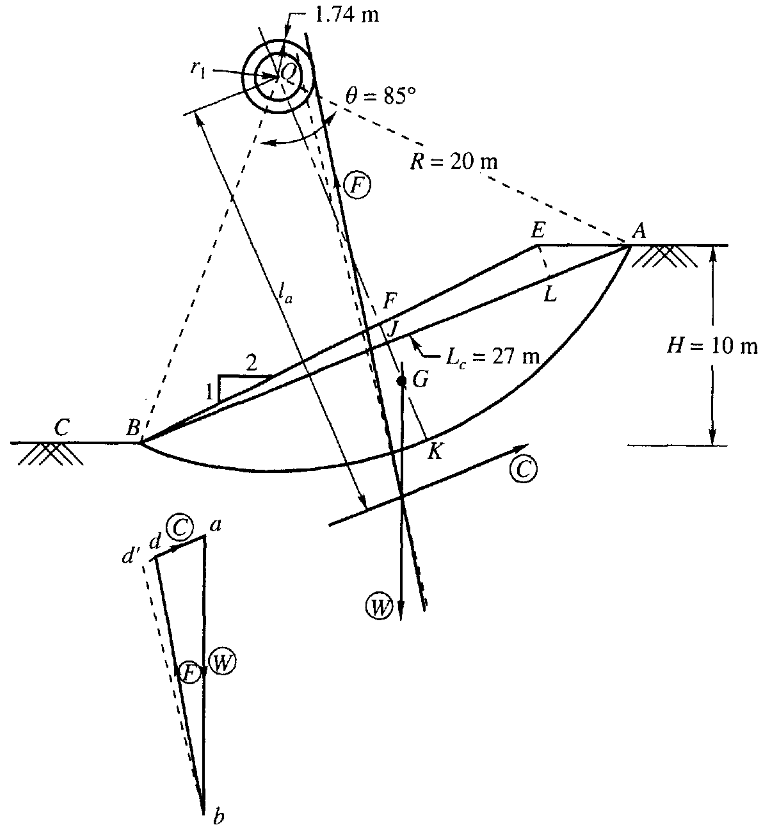

Figure Ex. 10.7

drawn at O with a radius of $R \sin \phi' = 20 \sin 5° = 1.74$ m. This line is the line of action of the third force F.

Draw a triangle of forces in which the magnitude and the direction for W is known and only the directions of the other two forces C and F are known.

Length ad gives the cohesive force $C = 520$ kN

Mobilized cohesion,

$$c'_m = \frac{C}{L} = \frac{520}{29.7} = 17.51 \text{ kN/m}^2$$

Therefore the factor of safety with respect to cohesion, F_c, is

$$F_c = \frac{c'}{c'_m} = \frac{30}{17.51} = 1.713$$

F_c will be 1.713 if the factor of safety with respect to friction, $F_\phi = 1.0$

$$\text{If, } F_s = 1.5, \text{ then } \phi'_m = \frac{\tan 5°}{F_s} = 0.058 \text{ rad; or } \phi'_m = 3.34°$$

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The new radius of the friction circle is

$r_1 = R \sin \phi'_m = 20 \times \sin 3.3° = 1.16$ m.

The direction of F changes and the modified triangle of force abd' gives,

cohesive force = C = length ad' = 600 kN

Mobilised cohesino, $c'_m = \frac{C}{L} = \frac{600}{29.7} = 20.2 \text{ kN/m}^2$

Therefore, $F_c = \frac{c'}{c'_m} = \frac{30}{20.2} \approx 1.5$

10.11 TAYLOR'S STABILITY NUMBER

If the slope angle β, height of embankment H, the effective unit weight of material γ, angle of internal friction ϕ', and unit cohesion c' are known, the factor of safety may be determined. In order to make unnecessary the more or less tedious stability determinations, Taylor (1937) conceived the idea of analyzing the stability of a large number of slopes through a wide range of slope angles and angles of internal friction, and then representing the results by an abstract number which he called the "*stability number*". This number is designated as N_s. The expression used is

$$N_s = \frac{c'}{F_c \gamma H} \tag{10.23}$$

From this the factor of safety with respect to cohesion may be expressed as

$$F_c = \frac{c'}{N_s \gamma H} \tag{10.24}$$

Taylor published his results in the form of curves which give the relationship between N_s and the slope angles β for various values of ϕ' as shown in Fig. 10.16. These curves are for circles passing through the toe, although for values of β less than 53°, it has been found that the most dangerous circle passes below the toe. However, these curves may be used without serious error for slopes down to β = 14°. The stability numbers are obtained for factors of safety with respect to cohesion by keeping the factor of safety with respect to friction (F_ϕ) equal to unity.

In slopes encountered in practical problems, the depth to which the rupture circle may extend is usually limited by ledge or other underlying strong material as shown in Fig. 10.17. The stability number N_s for the case when $\phi' = 0$ is greatly dependent on the position of the ledge. The depth at which the ledge or strong material occurs may be expressed in terms of a depth factor n_d which is defined as

$$n_d = \frac{D}{H} \tag{10.25}$$

where D = depth of ledge below the top of the embankment, H = height of slope above the toe.

For various values of n_d and for the $\phi = 0$ case the chart in Fig. 10.17 gives the stability number N_s for various values of slope angle β. In this case the rupture circle may pass through the toe or below the toe. The distance x of the rupture circle from the toe at the toe level may be expressed by a distance factor n_x which is defined as

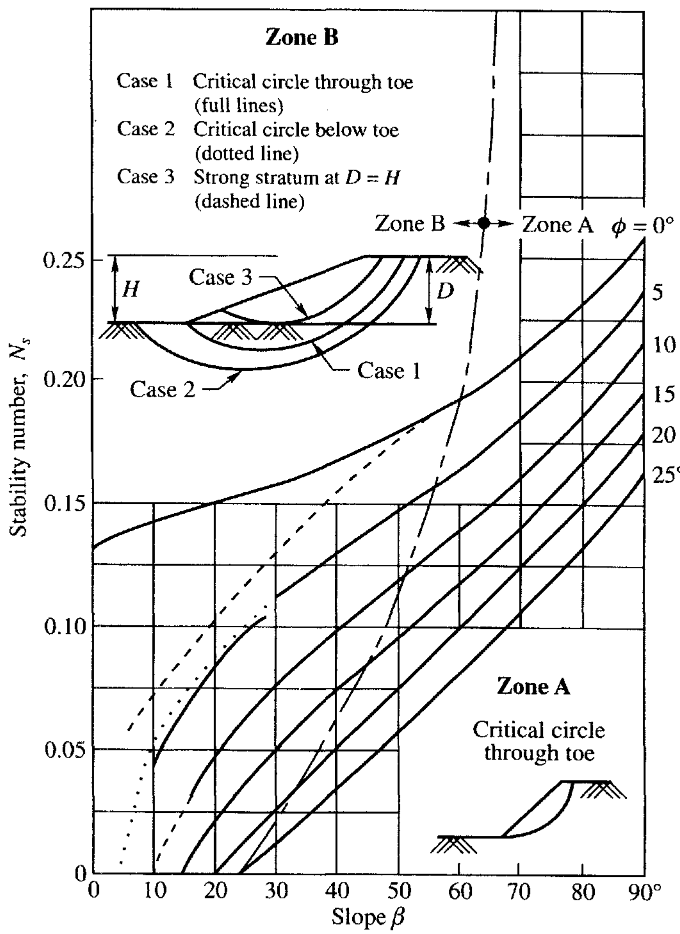

Figure 10.16 Taylor's stability numbers for circles passing through the toe and below or above the toe (after Taylor, 1937)

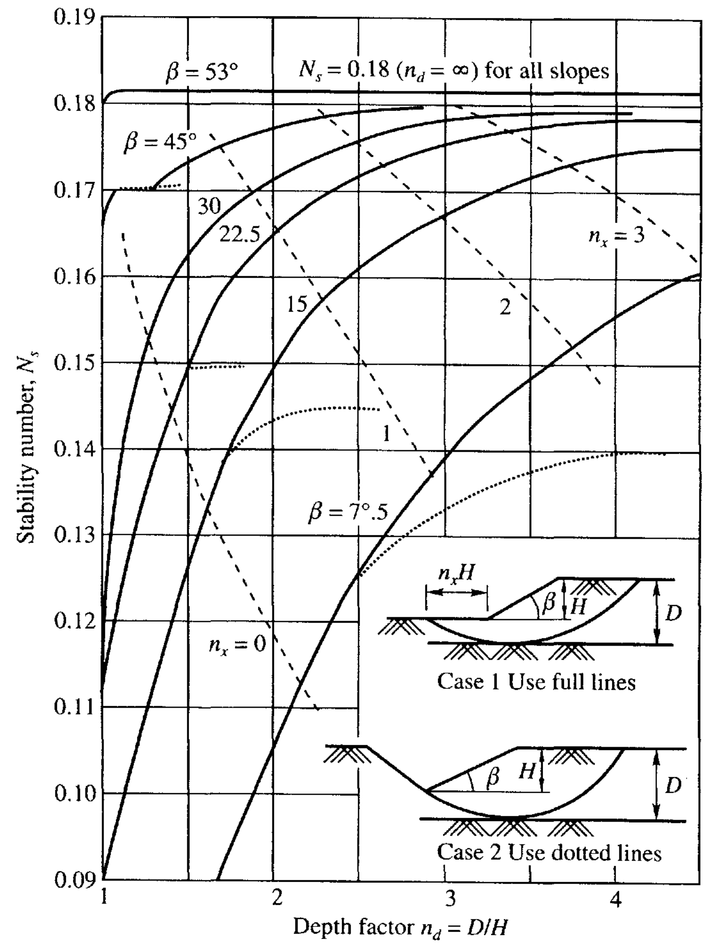

Figure 10.17 Taylor's stability numbers for $\phi' = 0$ case (after Taylor, 1937)

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$$n_x = \frac{x}{H} \tag{10.26}$$

The chart in Fig. 10.17 shows the relationship between n_d and n_x. If there is a ledge or other stronger material at the elevation of the toe, the depth factor n_d for this case is unity.

Factor of Safety with Respect to Strength

The development of the stability number is based on the assumption that the factor of safety with respect to friction F_{ϕ}, is unity. The curves give directly the factor of safety F_c with respect to cohesion only. If a true factor of safety F_s with respect to strength is required, this factor should apply equally to both cohesion and friction. The mobilized shear strength may therefore be expressed as

$$s_m = \frac{s}{F_s} = \frac{c'}{F_s} + \frac{\sigma' \tan\phi'}{F_s}$$

In the above expression, we may write

$$\frac{c'}{F_s} = c'_m, \quad \tan\phi'_m = \frac{\tan\phi'}{F_s}, \quad \text{or } \phi'_m = \frac{\phi'}{F_s} \text{ (approx.)} \tag{10.27}$$

c'_m and ϕ'_m may be described as average values of mobilized cohesion and friction respectively.

Example 10.8

The following particulars are given for an earth dam of height 39 ft. The slope is submerged and the slope angle $\beta = 45°$.

$\gamma_b = 69$ lb/ft^3

$c' = 550$ lb/ft^2

$\phi' = 20°$

Determine the factor of safety F_s.

Solution

Assume as a first trial $F_s = 2.0$

$$\phi'_m = \frac{20}{2} = 10° \text{ (approx.)}$$

For $\phi'_m = 10°$, and $\beta = 45°$ the value of N_s from Fig. 10.16 is 0.11, we may write

From Eq. (10.23) $N_s = \dfrac{c'}{F_c \gamma H}$, substituting

$$0.11 = \frac{550}{2 \times 69 \times H}$$

$$\text{or } H = \frac{550}{2 \times 69 \times 0.11} = 36.23 \text{ ft}$$

$$\text{If } F_s = 1.9, \phi'_m = \frac{20}{1.9} = 10.53° \text{ and } N_s = 0.105$$

$$H = \frac{550}{1.9 \times 69 \times 0.105} = 40 \text{ ft}$$

The computed height 40 ft is almost equal to the given height 39 ft. The computed factor of safety is therefore 1.9.

Example 10.9

An excavation is to be made in a soil deposit with a slope of 25° to the horizontal and to a depth of 25 meters. The soil has the following properties:

$c' = 35\text{kN/m}^2$, $\phi' = 15°$ and $\gamma = 20$ kN/m^3

1. Determine the factor of safety of the slope assuming full friction is mobilized.
2. If the factor of safety with respect to cohesion is 1.5, what would be the factor of safety with respect to friction?

Solution

1. For $\phi' = 15°$ and $\beta = 25°$, Taylor's stability number chart gives stability number $N_s = 0.03$.

$$F_c = \frac{c'}{N_s \gamma H} = \frac{35}{0.03 \times 20 \times 25} = 2.33$$

2. For $F_c = 1.5$, $N_s = \dfrac{c'}{F_c \times \gamma \times H} = \dfrac{35}{1.5 \times 20 \times 25} = 0.047$

For $N_s = 0.047$ and $\beta = 25°$, we have from Fig. 10.16, $\phi'_m = 13°$

$$\text{Therefore, } F_\phi = \frac{\tan \phi'}{\tan \phi'_m} = \frac{\tan 15°}{\tan 13°} = \frac{0.268}{0.231} = 1.16$$

Example 10.10

An embankment is to be made from a soil having $c' = 420$ lb/ft^2, $\phi' = 18°$ and $\gamma = 121$ lb/ft^3. The desired factor of safety with respect to cohesion as well as that with respect to friction is 1.5. Determine

1. The safe height if the desired slope is 2 horizontal to 1 vertical.
2. The safe slope angle if the desired height is 50 ft.

Solution

$$\tan \phi' = \tan 18° = 0.325, \ \phi'_m = \tan^{-1} \frac{0.325}{1.5} = 12.23°$$

1. For $\phi' = 12.23°$ and $\beta = 26.6°$ (i.e., 2 horizontal and 1 vertical) the chart gives $N_s = 0.055$

$$\text{Therefore, } 0.055 = \frac{c'}{F_c \gamma H} = \frac{420}{1.5 \times 121 \times H}$$

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$$\text{Therefore, } H_{safe} = \frac{420}{1.5 \times 121 \times 0.055} = 42 \text{ ft}$$

2. $$\text{Now, } N_s = \frac{c'}{F_c \gamma H} = \frac{420}{1.5 \times 121 \times 50} = 0.046$$

For $N_s = 0.046$ and $\phi'_m = 12.23°$, slope angle $\beta = 23.5°$

10.12 TENSION CRACKS

If a dam is built of cohesive soil, tension cracks are usually present at the crest. The depth of such cracks may be computed from the equation

$$z_0 = \frac{2c'}{\gamma} \tag{10.28}$$

where z_0 = depth of crack, c' = unit cohesion, γ = unit weight of soil.

The effective length of any trial arc of failure is the difference between the total length of arc minus the depth of crack as shown in Fig. 10.18.

10.13 STABILITY ANALYSIS BY METHOD OF SLICES FOR STEADY SEEPAGE

The stability analysis with steady seepage involves the development of the pore pressure head diagram along the chosen trial circle of failure. The simplest of the methods for knowing the pore pressure head at any point on the trial circle is by the use of flownets which is described below.

Determination of Pore Pressure with Seepage

Figure 10.19 shows the section of a homogeneous dam with an arbitrarily chosen trial arc. There is steady seepage flow through the dam as represented by flow and equipotential lines. From the equipotential lines the pore pressure may be obtained at any point on the section. For example at point a in Fig. 10.19 the pressure head is h. Point c is determined by setting the radial distance ac

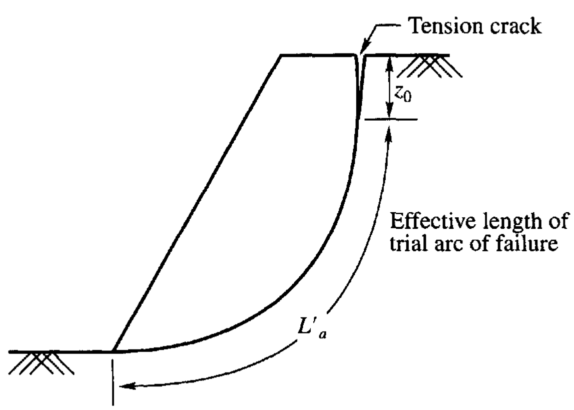

Figure 10.18 Tension crack in dams built of cohesive soils

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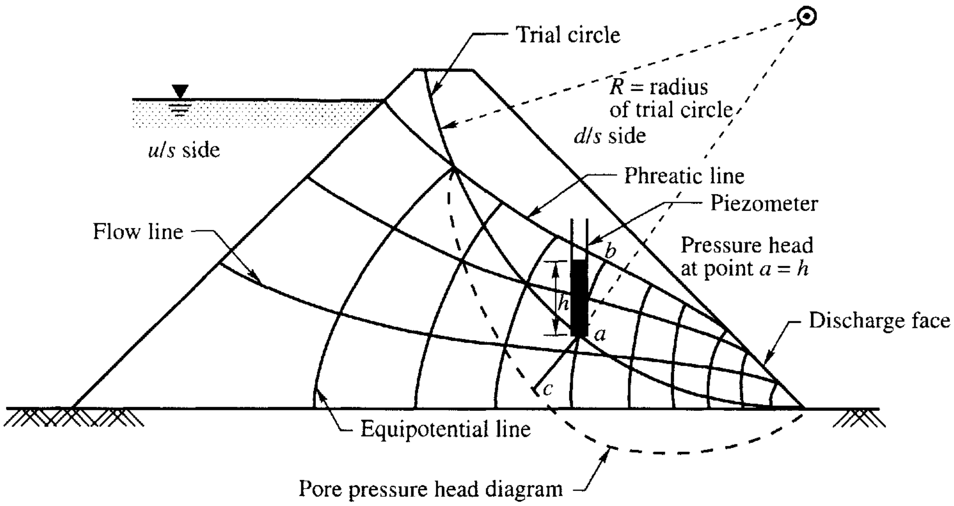

Figure 10.19 Determination of pore pressure with steady seepage

equal to h. A number of points obtained in the same manner as c give the curved line through c which is a pore pressure head diagram.

Method of Analysis (graphical method)

Figure 10.20(a) shows the section of a dam with an arbitrarily chosen trial arc. The center of rotation of the arc is 0. The pore pressure acting on the base of the arc as obtained from flow nets is shown in Fig. 10.20(b).

When the soil forming the slope has to be analyzed under a condition where full or partial drainage takes place the analysis must take into account both cohesive and frictional soil properties based on *effective stresses*. Since the effective stress acting across each elemental length of the assumed circular arc failure surface must be computed in this case, the method of slices is one of the convenient methods for this purpose. The method of analysis is as follows.

The soil mass above the assumed slip circle is divided into a number of vertical slices of equal width. The number of slices may be limited to a maximum of eight to ten to facilitate computation. The forces used in the analysis acting on the slices are shown in Figs. 10.20(a) and (c). The forces are:

1. The weight W of the slice.
2. The normal and tangential components of the weight W acting on the base of the slice. They are designated respectively as N and T.
3. The pore water pressure U acting on the base of the slice.
4. The effective frictional and cohesive resistances acting on the base of the slice which is designated as S.

The forces acting on the sides of the slices are statically indeterminate as they depend on the stress deformation properties of the material, and we can make only gross assumptions about their relative magnitudes.

In the conventional slice method of analysis the lateral forces are assumed equal on both sides of the slice. This assumption is not strictly correct. The error due to this assumption on the mass as a whole is about 15 percent (Bishop, 1955).

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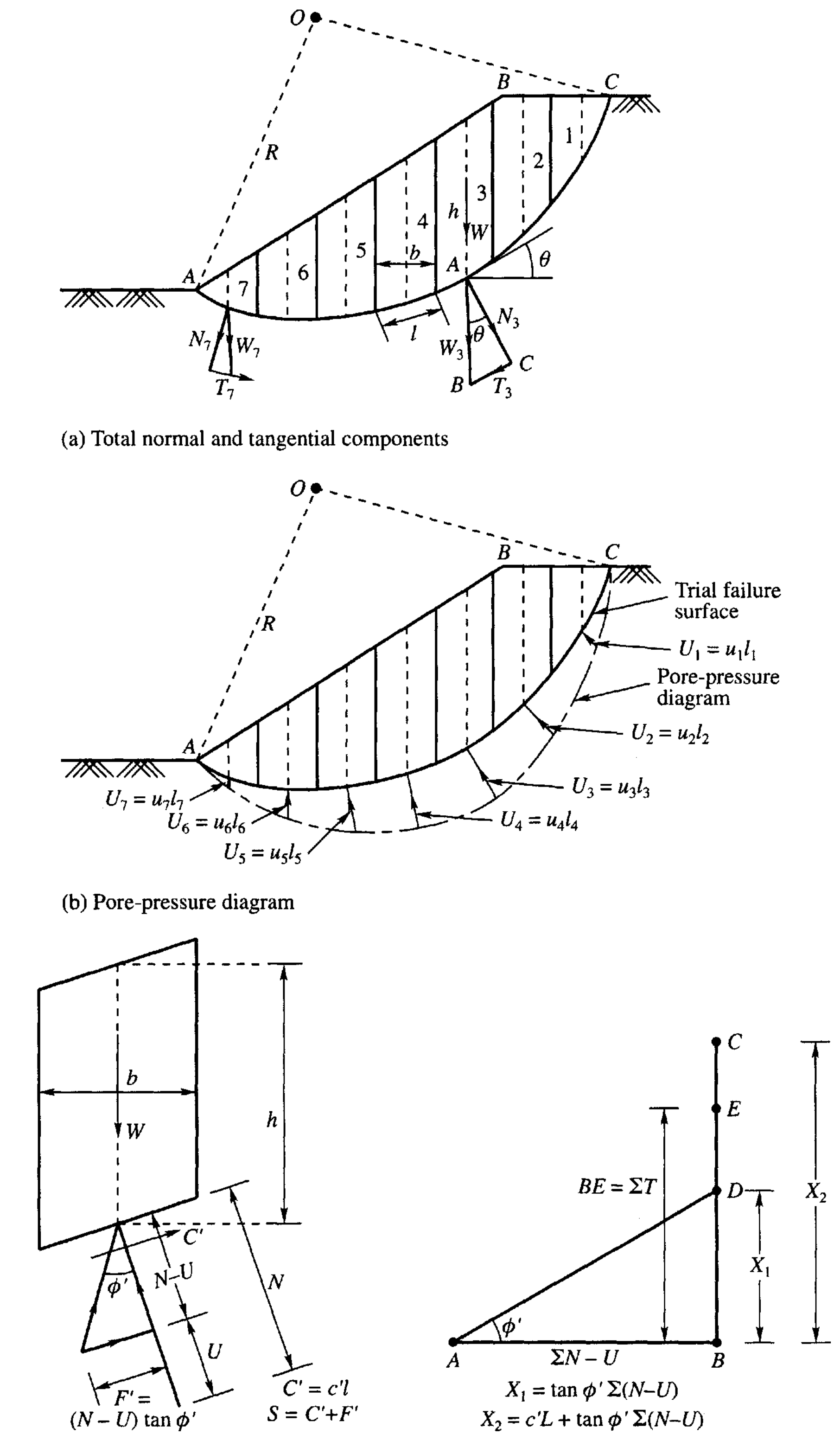

(c) Resisting forces on the base of slice (d) Graphical representation of all the forces

Figure 10.20 Stability analysis of slope by the method of slices

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The forces that are actually considered in the analysis are shown in Fig. 10.20(c). The various components may be determined as follows:

1. The weight, W, of a slice per unit length of dam may be computed from

   $W = \gamma hb$

   where, γ = total unit weight of soil, h = average height of slice, b = width of slice.

   If the widths of all slices are equal, and if the whole mass is homogeneous, the weight W can be plotted as a vector AB passing through the center of a slice as in Fig. 10.20(a). AB may be made equal to the height of the slice.
2. By constructing triangle ABC, the weight can be resolved into a normal component N and a tangential component T. Similar triangles can be constructed for all slices. The tangential components of the weights cause the mass to slide downward. The sum of all the weights cause the mass to slide downward. The sum of all the tangential components may be expressed as $\bar{T} = \Sigma T$. If the trial surface is curved upward near its lower end, the tangential component of the weight of the slice will act in the opposite direction along the curve. The algebraic sum of T should be considered.
3. The average pore pressure u acting on the base of any slice of length l may be found from the pore pressure diagram shown in Fig. 10.20(b). The total pore pressure, U, on the base of any slice is

   $U = ul$
4. The effective normal pressure N' acting on the base of any slice is

   $N' = N - U$ [Fig. 10.20(c)]
5. The frictional force F' acting on the base of any slice resisting the tendency of the slice to move downward is

   $F = (N - U) \tan \phi'$

   where ϕ' is the effective angle of friction. Similarly the cohesive force C' opposing the movement of the slice and acting at the base of the slice is

   $C' = c'l$

   where c' is the effective unit cohesion. The total resisting force S acting on the base of the slice is

   $S = C' + F' = c'l + (N - U) \tan \phi'$

   Figure 10.20(c) shows the resisting forces acting on the base of a slice.

   The sum of all the resisting forces acting on the base of each slice may be expressed as

   $S_s = c' \Sigma l + \tan \phi' \Sigma(N - U) = c'L + \tan \phi' \Sigma(N - U)$

   where $\Sigma l = L$ = length of the curved surface.

   The moments of the actuating and resisting forces about the point of rotation may be written as follows:

   Actuating moment = $R \Sigma T$

   Resisting moment = $R[c'L + \tan \phi' \Sigma(N - U)]$

   The factor of safety F_s may now be written as

$$F_s = \frac{[c'L + \tan \phi' \Sigma(N - U)]}{\Sigma T} \tag{10.29}$$

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The various components shown in Eq. (10.29) can easily be represented graphically as shown in Fig. 10.20(d). The line AB represents to a suitable scale $\Sigma(N - U)$. BC is drawn normal to AB at B and equal to $c'L + \tan\phi' \Sigma(N - U)$. The line AD drawn at an angle ϕ' to AB gives the intercept BD on BC equal to $\tan\phi'\Sigma(N - U)$. The length BE on BC is equal to ΣT. Now

$$F_s = \frac{BC}{BE} \tag{10.30}$$

Centers for Trial Circles Through Toe

The factor of safety F_s as computed and represented by Eq. (10.29) applies to one trial circle. This procedure is followed for a number of trial circles until one finds the one for which the factor of safety is the lowest. This circle that gives the least F_s is the one most likely to fail. The procedure is quite laborious. The number of trial circles may be minimized if one follows the following method.

For any given slope angle β (Fig. 10.21), the center of the first trial circle center O may be determined as proposed by Fellenius (1927). The direction angles α_A and α_B may be taken from Table 10.1. For the centers of additional trial circles, the procedure is as follows:

Mark point C whose position is as shown in Fig. 10.21. Join CO. The centers of additional circles lie on the line CO extended. This method is applicable for a homogeneous $(c - \phi)$ soil. When the soil is purely cohesive and homogeneous the direction angles given in Table 10.1 directly give the center for the critical circle.

Centers for Trial Circles Below Toe

Theoretically if the materials of the dam and foundation are entirely homogeneous, any practicable earth dam slope may have its critical failure surface below the toe of the slope. Fellenius found that the angle intersected at 0 in Fig. 10.22 for this case is about 133.5°. To find the center for the critical circle below the toe, the following procedure is suggested.

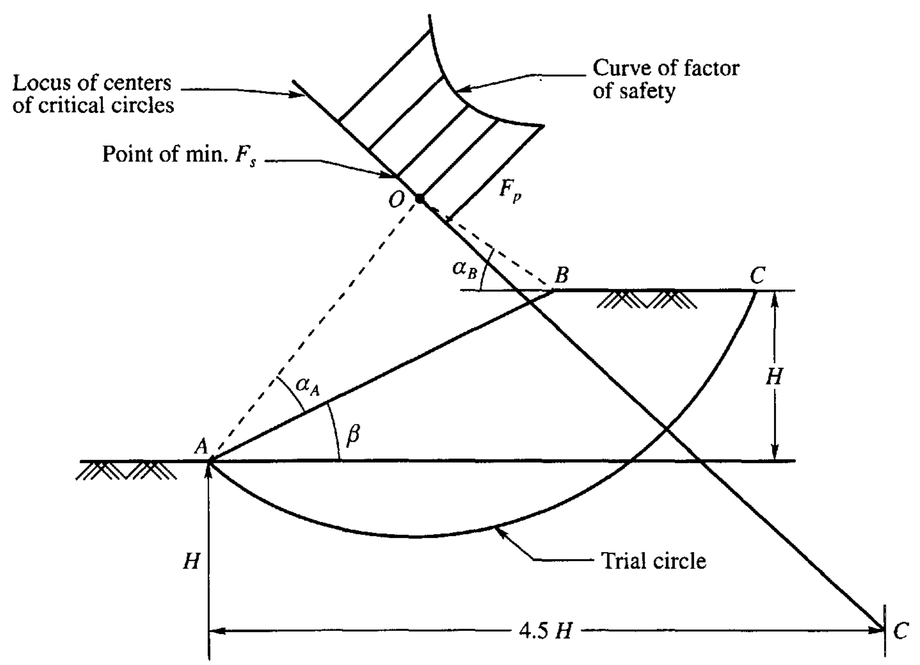

Figure 10.21 Location of centers of critical circle passing through toe of dam

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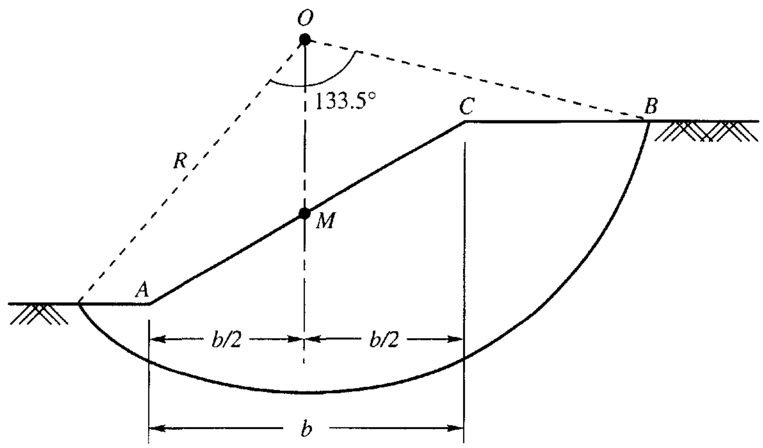

Figure 10.22 Centers of trial circles for base failure

Table 10.1 Direction angles α°_A and α°_B for centers of critical circles

Slope	Slope angle β	Direction angles α°_A	α°_B
0.6 : 1	60	29	40
1 : 1	45	28	37
1.5 : 1	33.8	26	35
2 : 1	26.6	25	35
3 : 1	18.3	25	35
5 : 1	11.3	25	37

Erect a vertical at the midpoint M of the slope. On this vertical will be the center O of the first trial circle. In locating the trial circle use an angle (133.5°) between the two radii at which the circle intersects the surface of the embankment and the foundation. After the first trial circle has been analyzed the center is some what moved to the left, the radius shortened and a new trial circle drawn and analyzed. Additional centers for the circles are spotted and analyzed.

Example 10.11

An embankment is to be made of a sandy clay having a cohesion of 30 kN/m^2, angle of internal friction of 20° and a unit weight of 18 kN/m^3. The slope and height of the embankment are 1.6 : 1 and 10 m respectively. Determine the factor of safety by using the trial circle given in Fig. Ex. 10.11 by the method of slices.

Solution

Consider the embankment as shown in Fig. Ex.10.11. The center of the trial circle O is selected by taking $\alpha_A = 26°$ and $\alpha_B = 35°$ from Table 10.1. The soil mass above the slip circle is divided into 13 slices of 2 m width each. The weight of each slice per unit length of embankment is given by $W = h_a b \gamma_t$, where h_a = average height of the slice, b = width of the slice, γ_t = unit weight of the soil.

The weight of each slice may be represented by a vector of height h_a if b and γ_t remain the same for the whole embankment. The vectors values were obtained graphically. The height vectors

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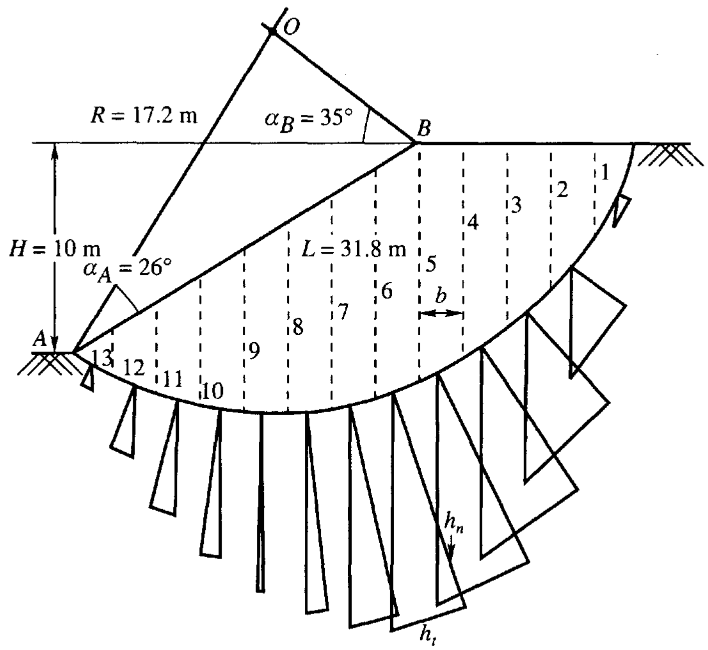

Figure Ex. 10.11

may be resolved into normal components h_n and tangential components h_t. The values of h_a, h_n and h_t for the various slices are given below in a tabular form.

Values of h_a, h_n and h_t

Slice No.	$h_a(m)$	$h_n(m)$	$h_t(m)$	Slice No.	$h_a(m)$	$h_n(m)$	$h_t(m)$
1	1.8	0.80	1.72	8	9.3	9.25	1.00
2	5.5	3.21	4.50	9	8.2	8.20	–0.20
3	7.8	5.75	5.30	10	6.8	6.82	–0.80
4	9.5	7.82	5.50	11	5.2	5.26	–1.30
5	10.6	9.62	4.82	12	3.3	3.21	–1.20
6	11.0	10.43	3.72	13	1.1	1.0	–0.50
7	10.2	10.20	2.31				

The sum of these components h_n and h_t may be converted into forces ΣN and ΣT respectively by multiplying them as given below

$$\Sigma h_n = 81.57 \text{ m}, \quad \Sigma h_t = 24.87 \text{ m}$$

Therefore, $\Sigma N = 81.57 \times 2 \times 18 = 2937$ kN

$$\Sigma T = 24.87 \times 2 \times 18 = 895 \text{ kN}$$

Length of arc $= L = 31.8$ m

$$\text{Factor of safety} = \frac{c'L + \tan\phi'\Sigma N}{\Sigma T} = \frac{30 \times 31.8 + 0.364 \times 2937}{895} = 2.26$$

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10.14 BISHOP'S SIMPLIFIED METHOD OF SLICES

Bishop's method of slices (1955) is useful if a slope consists of several types of soil with different values of c and ϕ and if the pore pressures u in the slope are known or can be estimated. The method of analysis is as follows:

Figure 10.23 gives a section of an earth dam having a sloping surface AB. ADC is an assumed trial circular failure surface with its center at O. The soil mass above the failure surface is divided into a number of slices. The forces acting on each slice are evaluated from limit equilibrium of the slices. The equilibrium of the entire mass is determined by summation of the forces on each of the slices.

Consider for analysis a single slice *abcd* (Fig. 10.23a) which is drawn to a larger scale in Fig. 10.23(b). The forces acting on this slice are

W = weight of the slice
N = total normal force on the failure surface *cd*
U = pore water pressure = ul on the failure surface *cd*
F_R = shear resistance acting on the base of the slice
E_1, E_2 = normal forces on the vertical faces *bc* and *ad*
T_1, T_2 = shear forces on the vertical faces *bc* and *ad*
θ = the inclination of the failure surface *cd* to the horizontal

The system is statically indeterminate. An approximate solution may be obtained by assuming that the resultant of E_1 and T_1 is equal to that of E_2 and T_2, and their lines of action coincide. For equilibrium of the system, the following equations hold true.

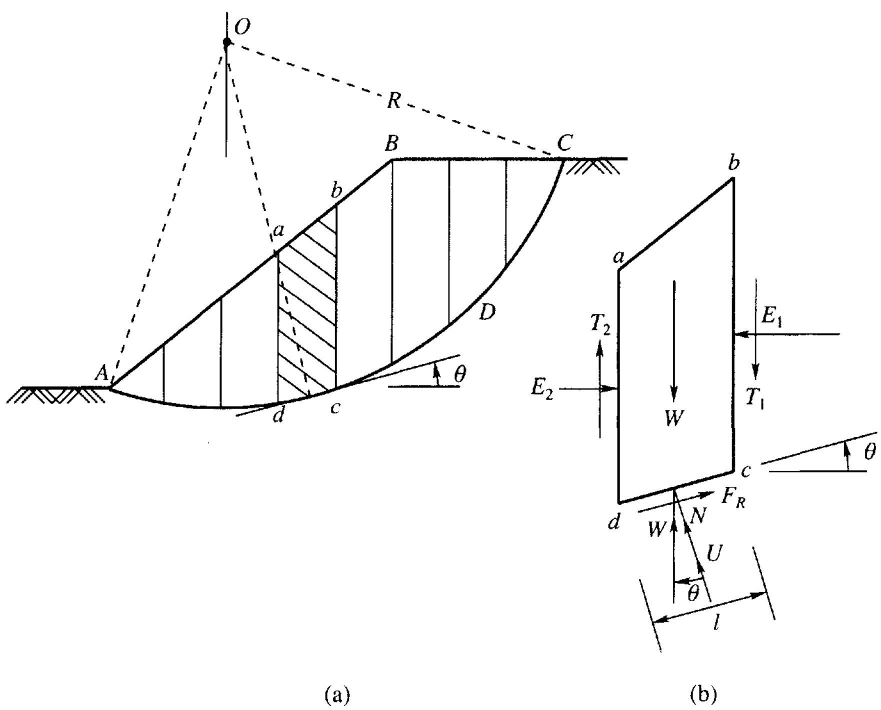

Figure 10.23 Bishop's simplified method of analysis

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$$N = W\cos\theta$$
$$F_t = W\sin\theta \tag{10.31}$$

where F_t = tangential component of W

The unit stresses on the failure surface of length, l, may be expressed as

$$\text{normal stress, } \sigma_n = \frac{W\cos\theta}{l}$$
$$\text{shear stress, } \tau_n = \frac{W\sin\theta}{l} \tag{10.32}$$

The equation for shear strength, s, is

$$s = c' + \sigma'\tan\phi' = c' + (\sigma - u)\tan\phi'$$

where σ' = effective normal stress
c' = effective cohesion
ϕ' = effective angle of friction
u = unit pore pressure

The shearing resistance to sliding on the base of the slice is

$$sl = c'l + (W\cos\theta - ul)\tan\phi'$$

where ul = U, the total pore pressure on the base of the slice (Fig 10.23b)
$sl = F_R$

The total resisting force and the actuating force on the failure surface ADC may be expressed as

Total resisting force F_R is

$$F_R = [c'l + (W\cos\theta - ul)\tan\phi'] \tag{10.33}$$

Total actuating force F_t is

$$F_t = W\sin\theta \tag{10.34}$$

The factor of safety F_s is then given as

$$F_s = \frac{F_R}{F_t} = \frac{[c'l + (W\cos\theta - ul)\tan\phi']}{W\sin\theta} \tag{10.35}$$

Eq. (10.35) is the same as Eq. (10.29) obtained by the conventional method of analysis.

Bishop (1955) suggests that the accuracy of the analysis can be improved by taking into account the forces E and T on the vertical faces of each slice. For the element in Fig. 10.23(b), we may write an expression for all the forces acting in the vertical direction for the equilibrium condition as

$$N'\cos\theta = W + (T_1 - T_2) - ul\cos\theta - F_R\sin\theta \tag{10.36}$$

If the slope is not on the verge of failure ($F_s > 1$), the tangential force F_t is equal to the shearing resistance F_R on cd divided by F_s.

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$$F_R = \frac{c'l}{F_s} + N'\frac{\tan\phi'}{F_s} \tag{10.37}$$

where, $N' = N - U$, and $U = ul$.

Substituting Eq. (10.37) into Eq. (10.36) and solving for N', we obtain

$$N' = \frac{W + \Delta T - U\cos\theta - \dfrac{c'l}{F_s}\sin\theta}{\cos\theta + \dfrac{\tan\phi'\sin\theta}{F_s}} \tag{10.38}$$

where, $\Delta T = T_1 - T_2$.

For equilibrium of the mass above the failure surface, we have by taking moments about O

$$W\sin\theta R = F_R R \tag{10.39}$$

By substituting Eqs. (10.37) and (10.38) into Eq. (10.39) and solving we obtain an expression for F_s as

$$F_s = \frac{\sum\left\{c'l\cos\theta + [(W - U\cos\theta) + \Delta T]\tan\phi'\right\}\frac{1}{m_\theta}}{\sum W\sin\theta} \tag{10.40}$$

where, $$m_\theta = \cos\theta + \frac{\tan\phi'\sin\theta}{F_s} \tag{10.41}$$

The factor of safety F_s is present in Eq. (10.40) on both sides. The quantity $\Delta T = T_1 - T_2$ has to be evaluated by means of successive approximation . Trial values of E_1 and T_1 that satisfy the equilibrium of each slice, and the conditions

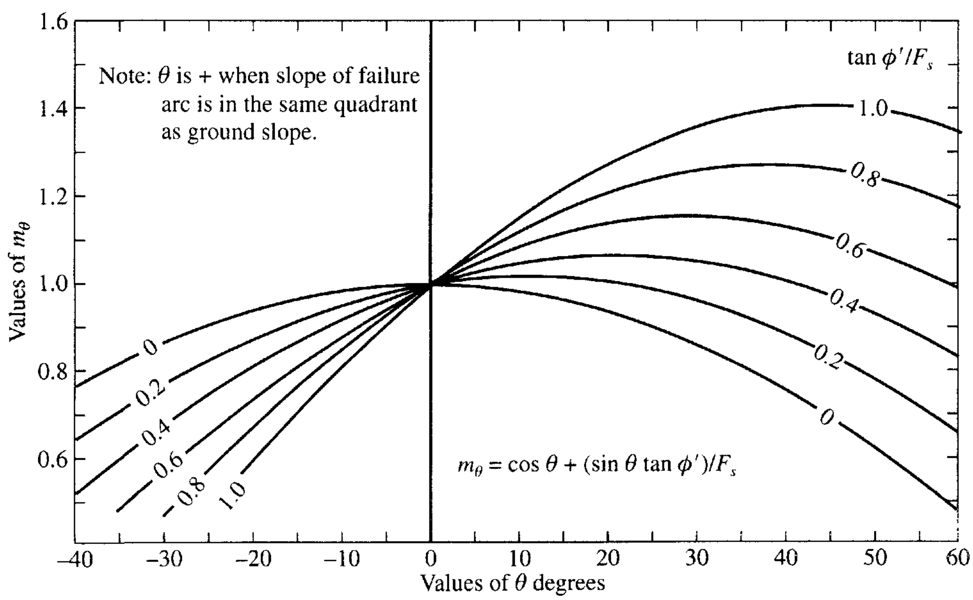

Figure 10.24 Values of m_θ (after Janbu et al., 1956)

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$$(E_1 - E_2) = 0 \quad \text{and} \quad (T_1 - T_2) = 0$$

are used. The value of F_s may then be computed by first assuming an arbitrary value for F_s. The value of F_s may then be calculated by making use of Eq. (10.40). If the calculated value of F_s differs appreciably from the assumed value, a second trial is made and the computation is repeated. Figure 10.24 developed by Janbu et al. (1956) helps to simplify the computation procedure.

It is reported that an error of about 1 percent will occur if we assume $\Sigma(T_1 - T_2)\tan\phi' = 0$. But if we use the conventional method of analysis using Eq. (10.35) the error introduced is about 15 percent (Bishop, 1955).

10.15 BISHOP AND MORGENSTERN METHOD FOR SLOPE ANALYSIS

Equation (10.40) developed based on Bishop's analysis of slopes, contains the term pore pressure u. The Bishop and Morgenstern method (1960) proposes the following equation for the evaluation of u

$$r_u = \frac{u}{\gamma h} \tag{10.42}$$

where, u = pore water pressure at any point on the assumed failure surface

γ = unit weight of the soil

h = the depth of the point in the soil mass below the ground surface

The pore pressure ratio r_u is assumed to be constant throughout the cross-section, which is called a *homogeneous pore pressure distribution.* Figure 10.25 shows the various parameters used in the analysis.

The factor of safety F_s is defined as

$$F_s = m - nr_u \tag{10.43}$$

where, m, n = stability coefficients.

The m and n values may be obtained either from charts in Figs. B.1 to B.6 or Tables B1 to B6 in Appendix B. The depth factor given in the charts or tables is as per Eq. (10.25), that is $n_d = D/H$, where H = height of slope, and D = depth of firm stratum from the top of the slope. Bishop and Morgenstern (1960) limited their charts (or tables) to values of $c'/\gamma H$ equal to 0.000, 0.025, and 0.050.

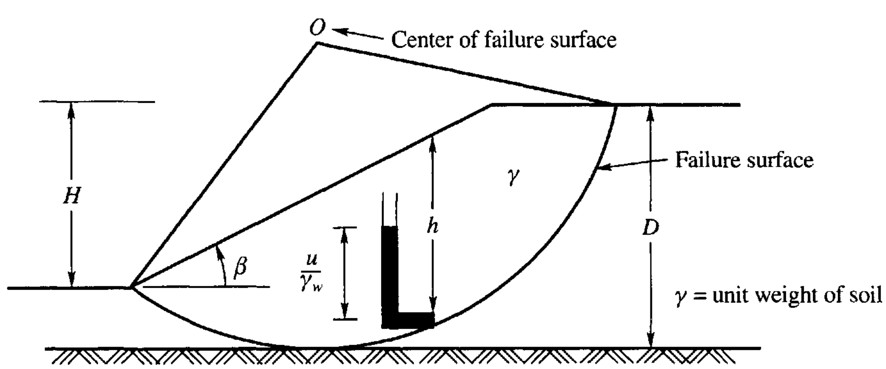

Figure 10.25 Specifications of parameters for Bishop-Morgenstern method of analysis

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Extension of the Bishop and Morgenstern Slope Stability Charts

As stated earlier, Bishop and Morgenstern (1960) charts or tables cover values of $c'/\gamma H$ equal to 0.000, 0.025, and 0.050 only. These charts do not cover the values that are normally encountered in natural slopes. O' Connor and Mitchell (1977) extended the work of Bishop and Morgenstern to cover values of $c'/\gamma H$ equal to 0.075 and 0.100 for various values of depth factors n_d. The method employed is essentially the same as that adopted by the earlier authors. The extended values are given in the form of charts and tables from Figs. B.7 to B.14 and Tables B7 to B14 respectively in Appendix B.

Method of Determining F_s

1. Obtain the values of r_u and $c/\gamma H$
2. From the tables in Appendix B, obtain the values of m and n for the known values of $c/\gamma H$, ϕ and β, and for n_d = 0, 1, 1.25 and 1.5.
3. Using Eq. (10.43), determine F_s for each value of n_d.
4. The required value of F_s is the lowest of the values obtained in step 3.

Example 10.12

Figure Ex. 10.12 gives a typical section of a homogeneous earth dam. The soil parameters are: $\phi' = 30°$, $c' = 590$ lb/ft^2, and $\gamma = 120$ lb/ft^3. The dam has a slope 4:1 and a pore pressure ratio $r_u = 0.5$. Estimate the factor of safety F_s by Bishop and Morgenstern method for a height of dam $H = 140$ ft.

Solution

Height of dam $H = 140$ ft

$$\frac{c'}{\gamma H} = \frac{590}{120 \times 140} = 0.035$$

Given: $\phi' = 30°$, slope 4:1 and $r_u = 0.5$.

Since $c'/\gamma H = 0.035$, and $n_d = 1.43$ for $H = 140$ ft, the F_s for the dam lies between $c'/\gamma H$ = 0.025 and 0.05 and n_d between 1.0 and 1.5. The equation for F_s is

$$F_s = m - nr_u$$

Using the Tables in Appendix B, the following table can be prepared for the given values of $c'/\gamma H$, ϕ, and β.

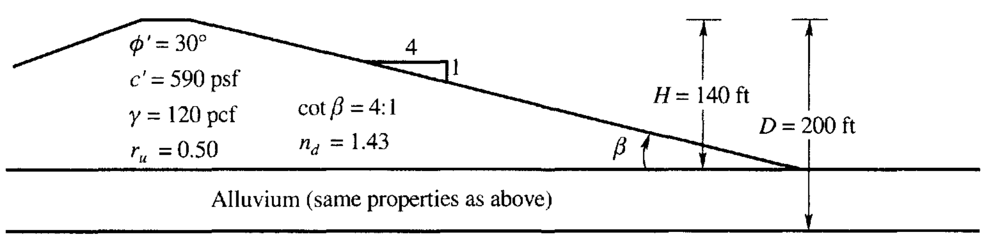

Figure Ex. 10.12

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From Tables B2 and B3 for $c'/\gamma H$ =0.025

n_d	m	n	F_s	
1.0	2.873	2.622	1.562	
1.25	**2.953**	**2.806**	**1.55**	Lowest

From Table B4, B5 and B6 for $c'/\gamma H$ = 0.05

n_d	m	n	F_s	
1.0	3.261	2.693	1.915	
1.25	**3.221**	**2.819**	**1.812**	Lowest
1.50	3.443	3.120	1.883	

Hence n_d = 1.25 is the more critical depth factor. The value of F_s for $c'/\gamma H$ = 0.035 lies between 1.55 (for $c'/\gamma H$ = 0.025) and 1.812 (for $c'/\gamma H$ = 0.05). By proportion F_s = 1.655.

10.16 MORGENSTERN METHOD OF ANALYSIS FOR RAPID DRAWDOWN CONDITION

Rapid drawdown of reservoir water level is one of the critical states in the design of earth dams. Morgenstern (1963) developed the method of analysis for rapid drawdown conditions based on the Bishop and Morgenstern method of slices. The purpose of this method is to compute the factor of safety during rapid drawdown, which is reduced under no dissipation of pore water pressure. The assumptions made in the analysis are

1. Simple slope of homogeneous material
2. The dam rests on an impermeable base
3. The slope is completely submerged initially
4. The pore pressure does not dissipate during drawdown

Morgenstern used the pore pressure parameter $\overline{B}$ as developed by Skempton (1954) which states

$$\overline{B} = \frac{u}{\sigma_1} \tag{10.45}$$

where $\sigma_1 = \gamma h$

γ = total unit weight of soil or equal to twice the unit weight of water

h = height of soil above the lower level of water after drawdown

The charts developed take into account the drawdown ratio which is defined as

$$R_d = \frac{\overline{H}}{H} \tag{10.46}$$

where R_d = drawdown ratio

$\overline{H}$ = height of drawdown

H = height of dam (Fig. 10.26)

All the potential sliding circles must be tangent to the base of the section.

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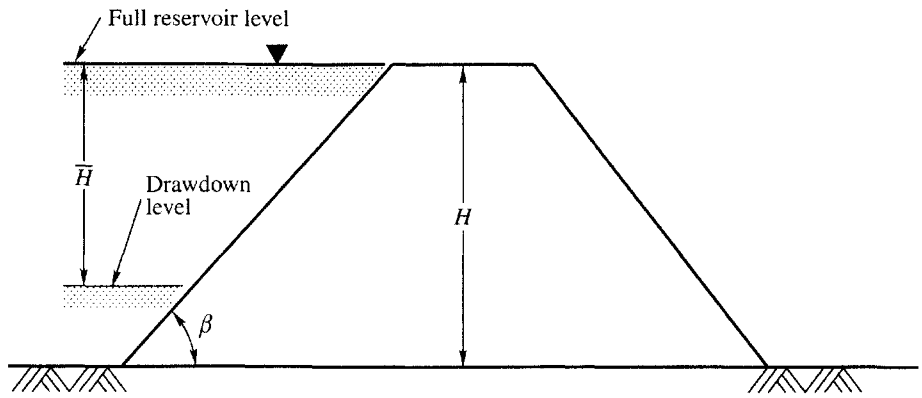

Figure 10.26 Dam section for drawdown conditions

The stability charts are given in Figs. 10.27 to 10.29 covering a range of stability numbers $c'/\gamma H$ from 0.0125 to 0.050. The curves developed are for the values of ϕ' of 20°, 30°, and 40° for different values of β.

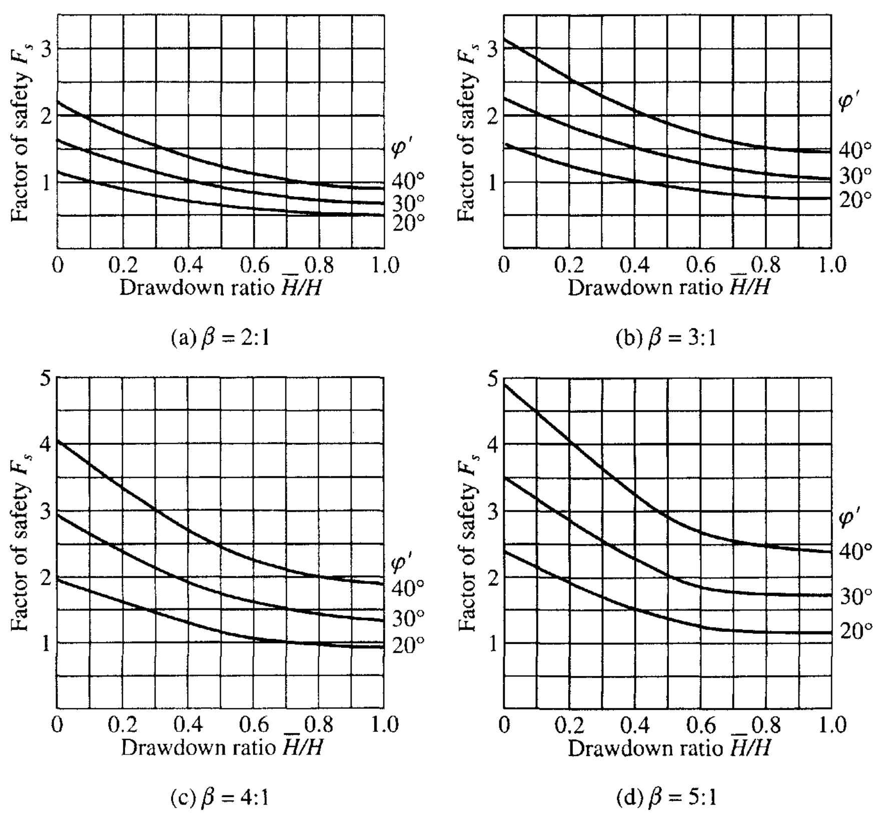

Figure 10.27 Drawdown stability chart for $c'/\gamma H$ = 0.0125 (after Morgenstern, 1963)

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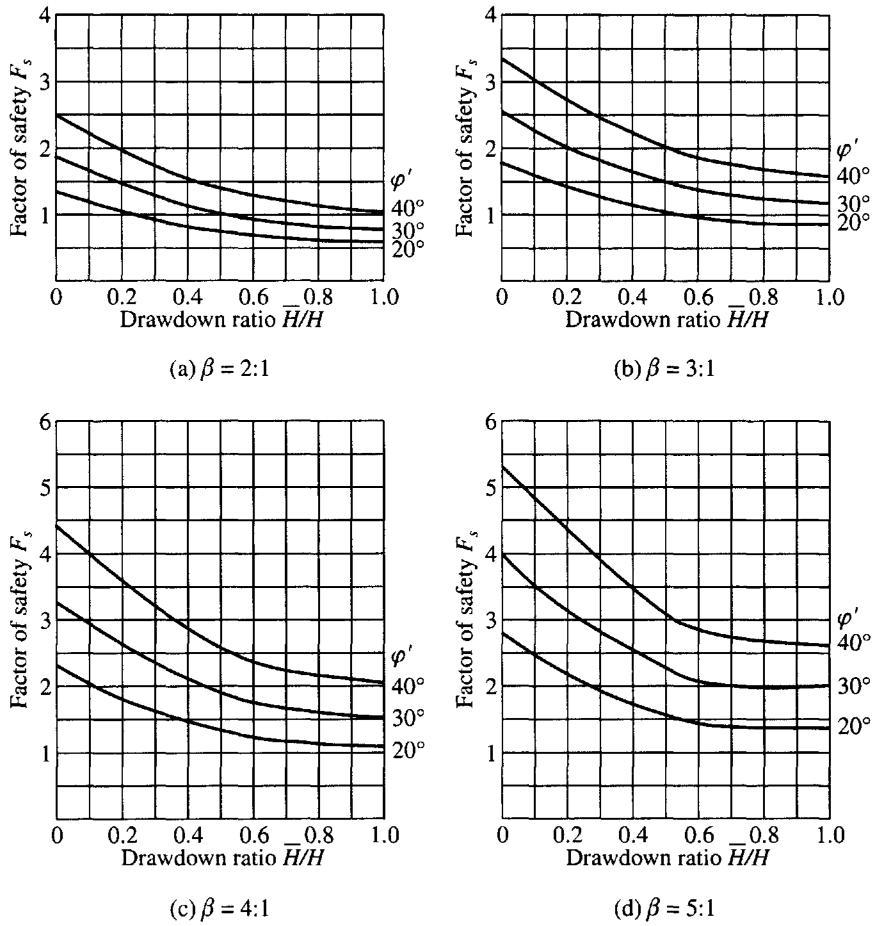

Figure 10.28 Drawdown stability chart for $c'/\gamma H$ = 0.025 (after Morgenstern, 1963)

Example 10.13

It is required to estimate the minimum factor of safety for the complete drawdown of the section shown in Fig. Ex. 10.13 (Morgenstern, 1963)

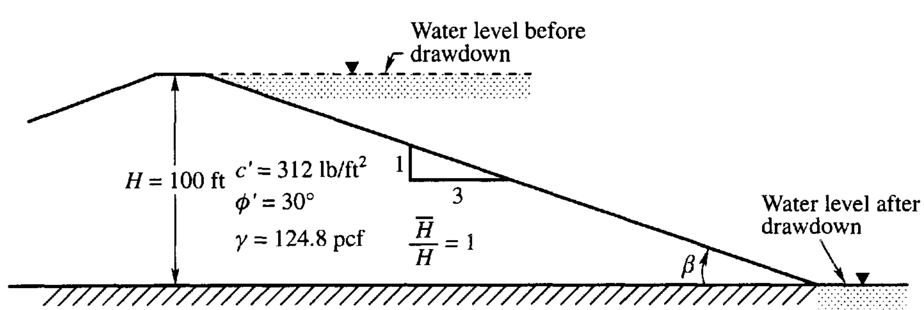

Figure Ex. 10.13

Solution

From the data given in the Fig. Ex. 10.13

$$N_s = \frac{c'}{\gamma H} = \frac{312}{124.8 \times 100} = 0.025$$

From Fig. 10.28, for $N_s = 0.025$, $\beta = 3:1$, $\phi' = 30°$, and $\bar{H}/H = 1$,

$F_s = 1.20$

It is evident than the critical circle is tangent to the base of the dam and no other level need be investigated since this would only raise the effective value of N_s resulting in a higher factor of safety.

10.17 SPENCER METHOD OF ANALYSIS

Spencer (1967) developed his analysis based on the method of slices of Fellenius (1927) and Bishop (1955). The analysis is in terms of effective stress and satisfies two equations of

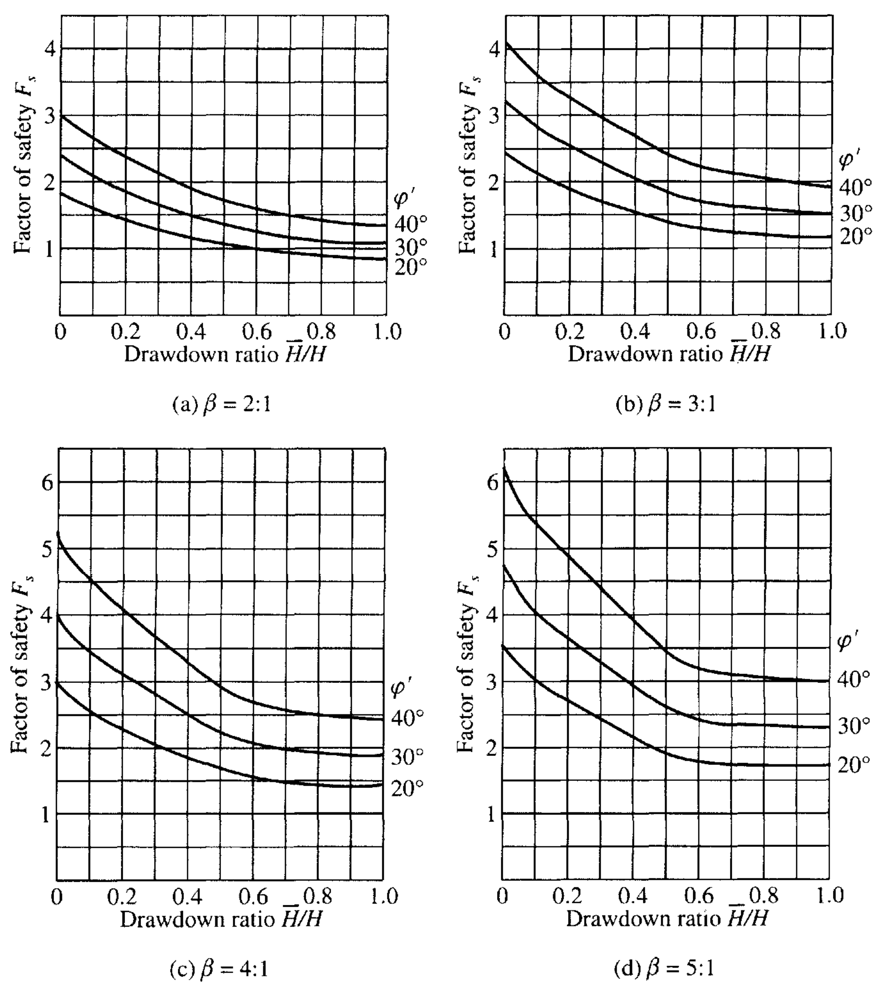

Figure 10.29 Drawdown stability chart for $c'/\gamma H = 0.05$ (after Morgenstern, 1963)

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equilibrium, the first with respect to forces and the second with respect to moments. The interslice forces are assumed to be parallel as in Fig. 10.23. The factor of safety F_s is expressed as

$$F_s = \frac{\text{Shear strength available}}{\text{Shear strength mobilized}} \tag{10.47}$$

The mobilized angle of shear resistance and other factors are expressed as

$$\tan\phi'_m = \frac{\tan\phi'}{F_s} \tag{10.48}$$

$$\text{pore pressure ratio,} \quad r_u = \frac{u}{\gamma h} \tag{10.49}$$

$$\text{Stability factor,} \quad N_s = \frac{c'}{F_s \gamma H} \tag{10.50}$$

The charts developed by Spencer for different values of N_s, ϕ'_m and r_u are given in Fig. 10.30. The use of these charts will be explained with worked out examples.

Example 10.14

Find the slope corresponding to a factor of safety of 1.5 for an embankment 100 ft high in a soil whose properties are as follows:

$c' = 870$ lb/sq ft, $\gamma = 120$ lb/ft3, $\phi' = 26°$, $r_u = 0.5$

Solution (by Spencer's Method)

$$N_s = \frac{c'}{F_s \gamma H} = \frac{870}{1.5 \times 120 \times 100} = 0.048$$

$$\tan\phi'_m = \frac{\tan\phi'}{F_s} = \frac{0.488}{1.5} = 0.325$$

$$\phi'_m = 18°$$

Referring to Fig. 10.30c, for which $r_u = 0.5$, the slope corresponding to a stability number of 0.048 is 3:1.

Example 10.15

What would be the change in strength on sudden drawdown for a soil element at point P which is shown in Fig. Ex. 10.15? The equipotential line passing through this element represents loss of water head of 1.2 m. The saturated unit weight of the fill is 21 kN/m^3.

Solution

The data given are shown in Fig. Ex. 10.15. Before drawdown,

The stresses at point P are:

$$\sigma_0 = \gamma_w h_w + \gamma_{sat} h_c = 9.81 \times 3 + 21 \times 4 = 113 \text{ kN/m}^2$$

$$u_0 = \gamma_w (h_w + h_c - h') = 9.81(3 + 4 - 1.2) = 57 \text{ kN/m}^2$$

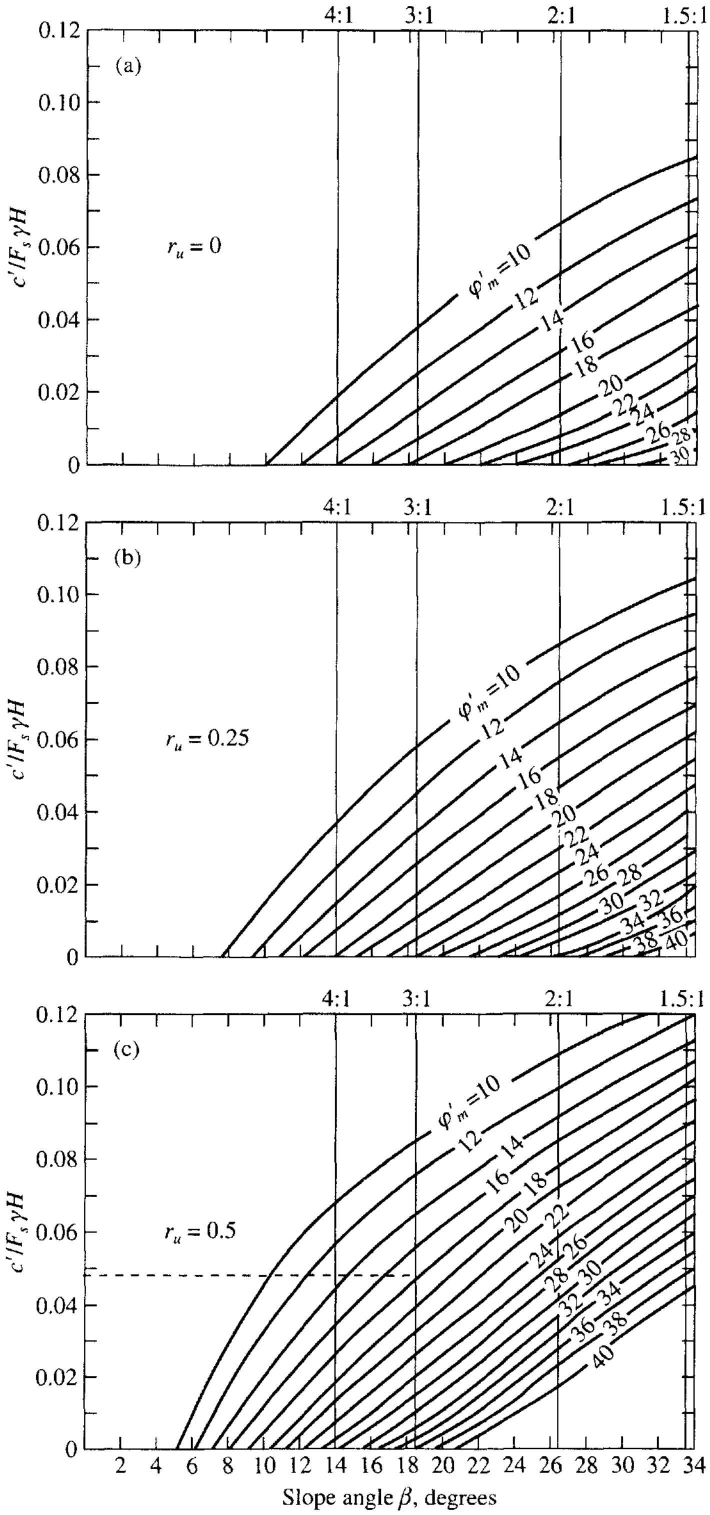

Figure 10.30 Stability charts (after Spencer, 1967)

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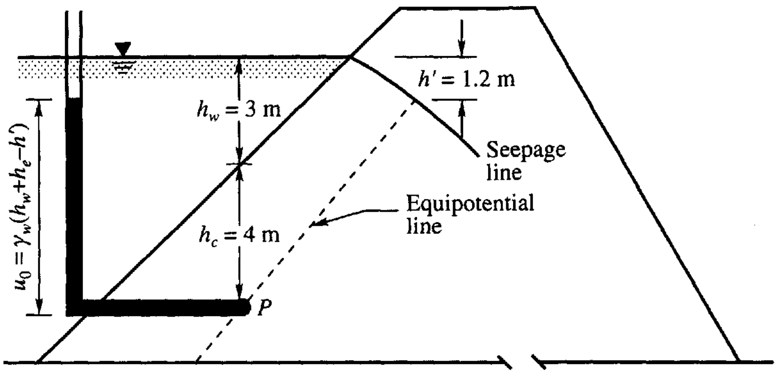

Figure Ex. 10.15

Therefore $\sigma'_0 = \sigma_0 - u_0 = 113 - 57 = 56$ kN/m^2

After drawdown,

$\sigma = \gamma_{sat} h_c = 21 \times 4 = 84$ kN/m^2

$u = \gamma_w (h_c - h') = 9.81(4 - 1.2) = 27.5$ kN/m^2

$\sigma' = \sigma - u = 84 - 27.5 = 56.5$ kN/m^2

The change in strength is zero since the effective vertical stress does not change.

Note: There is no change in strength due to sudden drawdown but the direction of forces of the seepage water changes from an inward direction before drawdown to an outward direction after drawdown and this is the main cause for the reduction in stability.

10.18 PROBLEMS

10.1 Find the critical height of an infinite slope having a slope angle of 30°. The slope is made of stiff clay having a cohesion 20 kN/m^2, angle of internal friction 20°, void ratio 0.7 and specific gravity 2.7. Consider the following cases for the analysis.

(a) the soil is dry.

(b) the water seeps parallel to the surface of the slope.

(c) the slope is submerged.

10.2 An infinite slope has an inclination of 26° with the horizontal. It is underlain by a firm cohesive soil having $G_s = 2.72$ and $e = 0.52$. There is a thin weak layer 20 ft below and parallel to the slope ($c' = 525$ lb/ft^2, $\phi' = 16°$). Compute the factors of safety when (a) the slope is dry, and (b) ground water flows parallel to the slope at the slope level.

10.3 An infinite slope is underlain with an overconsolidated clay having $c' = 210$ lb/ft^2, $\phi' = 8°$ and $\gamma_{sat} = 120$ lb/ft^3. The slope is inclined at an angle of 10° to the horizontal. Seepage is parallel to the surface and the ground water coincides with the surface. If the slope fails parallel to the surface along a plane at a depth of 12 ft below the slope, determine the factor of safety.

10.4 A deep cut of 10 m depth is made in sandy clay for a road. The sides of the cut make an angle of 60° with the horizontal. The shear strength parameters of the soil are $c' = 20$ kN/m^2, $\phi' = 25°$, and $\gamma = 18.5$ kN/m^3. If AC is the failure plane (Fig Prob. 10.4), estimate the factor of safety of the slope.

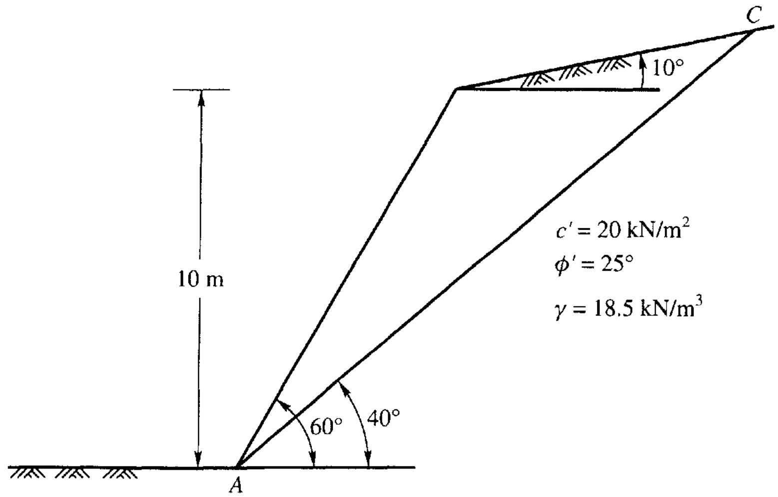

Figure Prob. 10.4

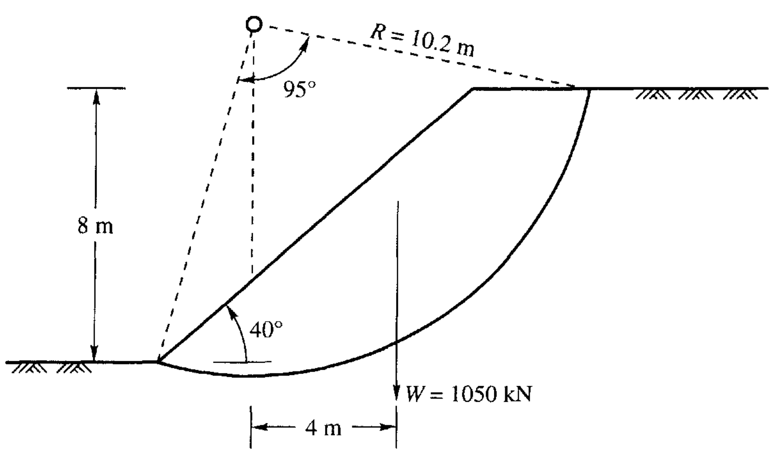

Figure Prob. 10.5

10.5 A 40° slope is excavated to a depth of 8 m in a deep layer of saturated clay having strength parameters $c = 60$ kN/m^2, $\phi = 0$, and $\gamma = 19$ kN/m^3. Determine the factor of safety for the trial failure surface shown in Fig. Prob. 10.5.

10.6 An excavation to a depth of 8 m with a slope of 1:1 was made in a deep layer of saturated clay having $c_u = 65$ kN/m^2 and $\phi_u = 0$. Determine the factor of safety for a trial slip circle passing through the toe of the cut and having a center as shown in Fig. Prob. 10.6. The unit weight of the saturated clay is 19 kN/m^3. No tension crack correction is required.

10.7 A 45° cut was made in a clayey silt with $c = 15$ kN/m^2, $\phi = 0$ and $\gamma = 19.5$ kN/m^3. Site exploration revealed the presence of a soft clay stratum of 2 m thick having $c = 25$ kN/m^2 and $\phi = 0$ as shown in Fig. Prob. 10.7. Estimate the factor of safety of the slope for the assumed failure surface.

10.8 A cut was made in a homogeneous clay soil to a depth of 8 m as shown in Fig. Prob. 10.8. The total unit weight of the soil is 18 kN/m^3, and its cohesive strength is 25 kN/m^2.

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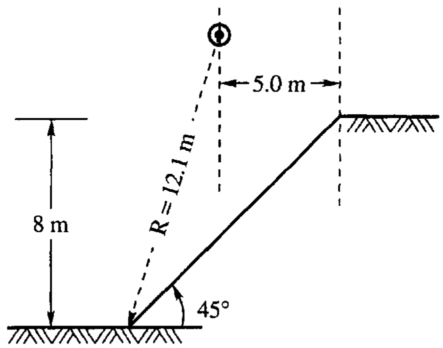

Figure Prob. 10.6

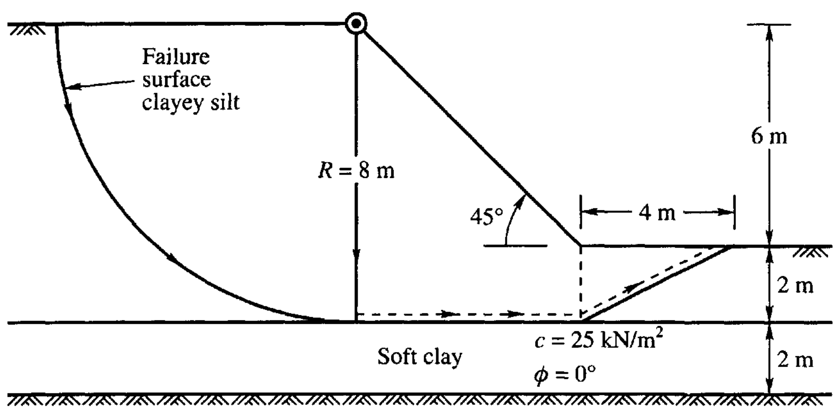

Figure Prob. 10.7

Assuming a $\phi = 0$ condition, determine the factor of safety with respect to a slip circle passing through the toe. Consider a tension crack at the end of the slip circle on the top of the cut.

10.9 A deep cut of 10 m depth is made in natural soil for the construction of a road. The soil parameters are: $c' = 35$ kN/m^2, $\phi' = 15°$ and $\gamma = 20$ kN/m^3.

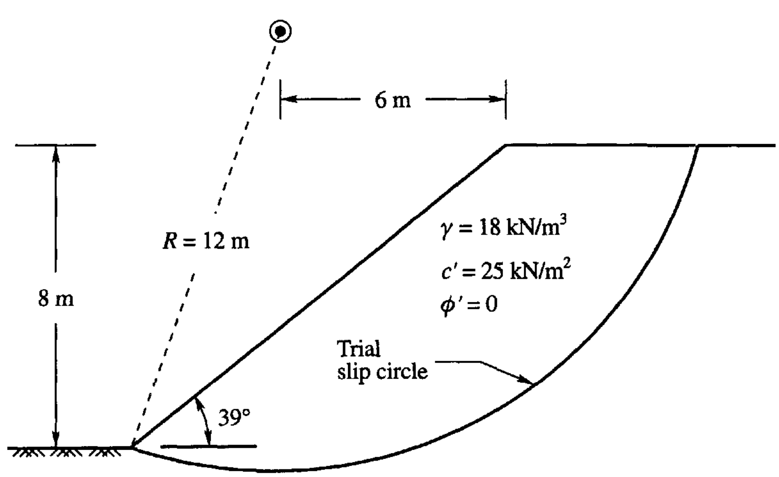

Figure Prob. 10.8

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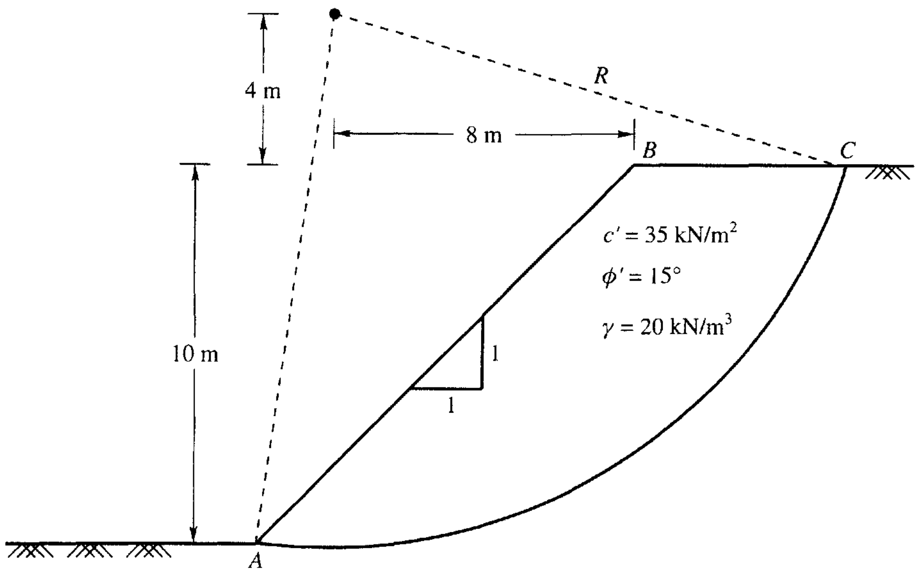

Figure Prob. 10.9

The sides of the cut make angles of 45° with the horizontal. Compute the factor of safety using friction circle method for the failure surface AC shown in Fig. Prob. 10.9.

10.10 An embankment is to be built to a height of 50 ft at an angle of 20° with the horizontal. The soil parameters are: $c' = 630$ lb/ft^2, $\phi' = 18°$ and $\gamma = 115$ lb/ft^3.
Estimate the following;
1. Factor of safety of the slope assuming full friction is mobilized.
2. Factor of safety with respect to friction if the factor of safety with respect to cohesion is 1.5.

Use Taylor's stability chart.

10.11 A cut was made in natural soil for the construction of a railway line. The soil parameters are: $c' = 700$ lb/ft^2, $\phi' = 20°$ and $\gamma = 110$ lb/ft^3.
Determine the critical height of the cut for a slope of 30° with the horizontal by making use of Taylor's stability chart.

10.12 An embankment is to be constructed by making use of sandy clay having the following properties: $c' = 35$ kN/m^2, $\phi' = 25°$ and $\gamma = 19.5$ kN/m^3.
The height of the embankment is 20 m with a slope of 30° with the horizontal as shown in Fig. Prob. 10.12. Estimate the factor of safety by the method of slices for the trial circle shown in the figure.

10.13 If an embankment of 10 m height is to be made from a soil having $c' = 25$ kN/m^2, $\phi' = 15°$, and $\gamma = 18$ kN/m^3, what will be the safe angle of slope for a factor of safety of 1.5?

10.14 An embarkment is constructed for an earth dam of 80 ft high at a slope of 3:1. The properties of the soil used for the construction are: $c' = 770$ lb/ft^2, $\phi' = 30°$, and $\gamma = 110$ lb/ft^3. The estimated pore pressuer ratio $r_u = 0.5$. Determine the factor of safety by Bishop and Morgenstern method.

10.15 For the Prob. 10.14, estimate the factor of safety for $\phi' = 20°$. All the other data remain the same.

10.16 For the Prob. 10.14, estimate the factor of safety for a slope of 2:1 with all the oother data remain the same.

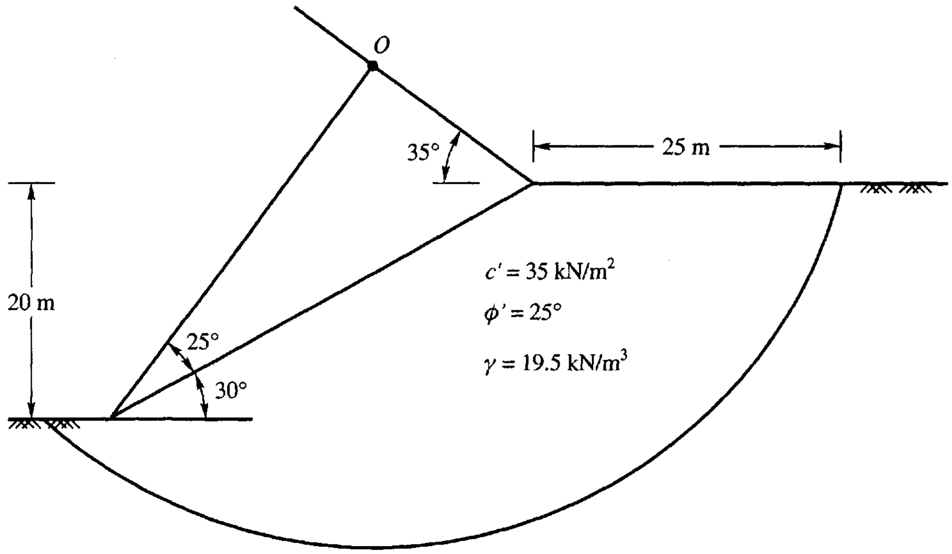

Figure Prob. 10.12

10.17 A cut of 25 m depth is made in a compacted fill having shear strength parameters of $c' = 25$ kN/m^2, and $\phi' = 20°$. The total unit weight of the material is 19 kN/m^3. The pore pressuer ratio has an average value of 0.3. The slope of the sides is 3:1. Estimate the factor of safety using the Bishop and Morgenstern method.

10.18 For the Prob. 10.17, estimate the factor of safety for $\phi' = 30°$, with all the other data remain the same.

10.19 For the Prob. 10.17, esatimate the factor of safety for a slope of 2:1 with all the other data remaining the same.

10.20 Estimate the minimum factor of safety for a complete drawdown condition for the section of dam in Fig. Prob. 10.20. The full reservoir level of 15 m depth is reduced to zero after drawdown.

10.21 What is the safety factor if the reservoir level is brought down from 15 m to 5 m depth in the Prob. 10.20?

10.22 An earth dam to be constructed at a site has the following soil parameters: $c' = 600$ lb/ft^2, $\gamma = 110$ lb/ft^3, and $\phi' = 20°$. The height of of dam $H = 50$ ft.

The pore pressure ratio $r_u = 0.5$. Determine the slope of the dam for a factor of safety of 1.5 using Spencer's method (1967).

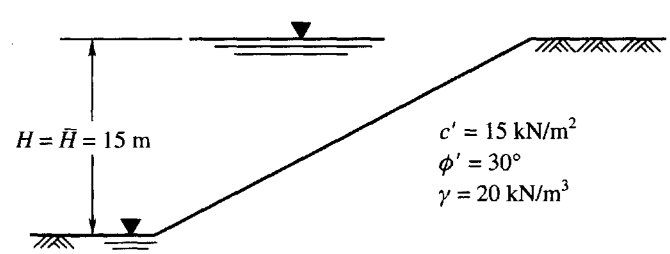

Figure Prob. 10.20

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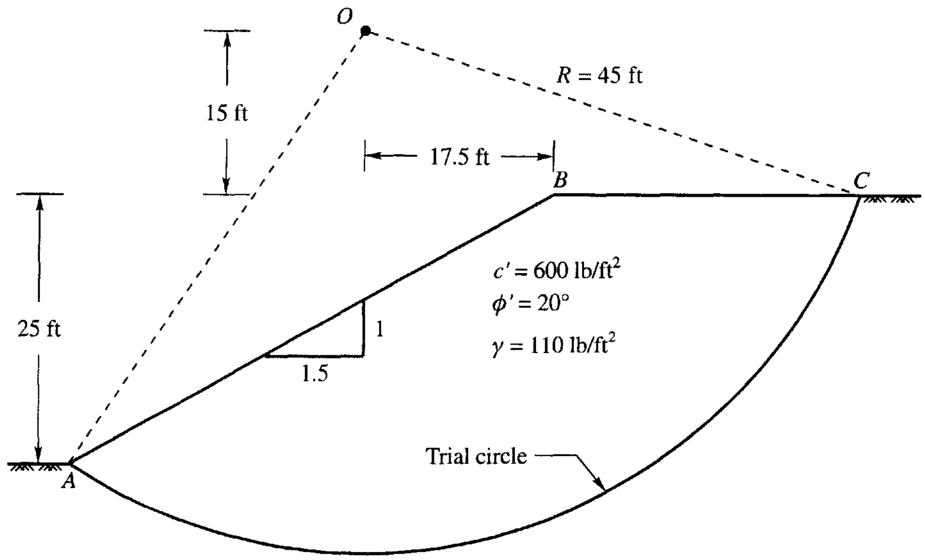

Figure Prob. 10.24

10.23 If the given pore pressure ratio is 0.25 in Prob. 10.22, what will be the slope of the dam?

10.24 An embankment has a slope of 1.5 horizontal to 1 vertical with a height of 25 feet. The soil parameters are:

$c' = 600$ lb/ft^2, $\phi' = 20°$, and $\gamma = 110$ lb/ft^3.

Determine the factor of safety using friction circle method for the failure surface AC shown in Fig. Prob. 10.24.

10.25 It is required to construct an embankment for a reservoir to a height of 20 m at a slope of 2 horizontal to 1 vertical. The soil parameters are:

$c' = 40$ kN/m^2, $\phi' = 18°$, and $\gamma = 17.5$ kN/m^3.

Estimate the following:

1. Factor of safety of the slope assuming full friction is mobilized.

2. Factor of safety with respect to friction if the factor of safety with respect to cohesion is 1.5.

Use Taylor's stability chart.

10.26 A cutting of 40 ft depth is to be made for a road as shown in Fig. Prob. 10.26. The soil properties are:

$c' = 500$ lb/ft^2, $\phi' = 15°$, and $\gamma = 115$ lb/ft^3.

Estimate the factor of safety by the method of slices for the trial circle shown in the figure.

10.27 An earth dam is to be constructed for a reservior. The height of the dam is 60 ft. The properties of the soil used in the construction are:

$c' = 400$ lb/ft^2, $\phi° = 20°$, and $\gamma = 115$ lb/ft^3, and $\beta = 2{:}1$.

Estimate the minimum factor of safety for the complete drawn from the full reservior level as shown in Fig. Prob. 10.27 by Morgenstern method.

10.28 What is the factor of safety if the water level is brought down from 60 ft to 20 ft above the bed level of reservoir in Prob. 10.27?

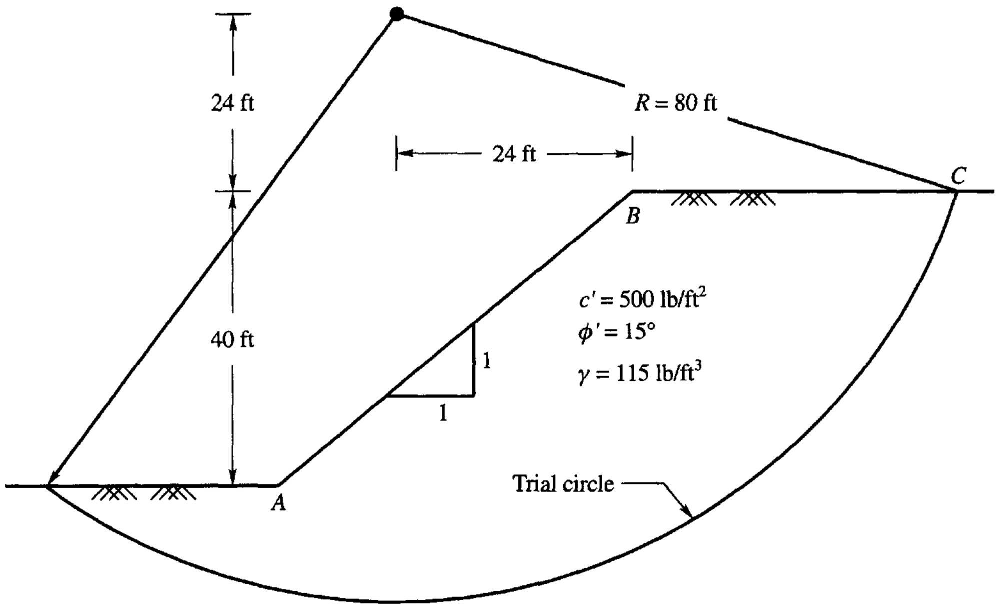

Figure Prob. 10.26

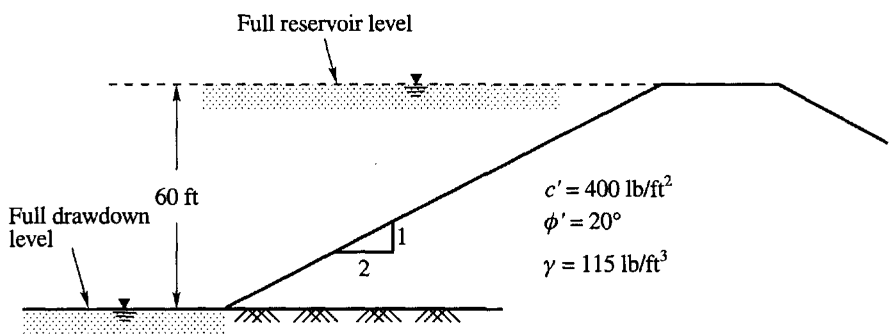

Figure Prob. 10.27

10.29 For the dam given in Prob. 10.27, determine the factor of safety for $r_u = 0.5$ by Spencer's method.

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