CENG 6108 Construction Economics

Dealing with Uncertainty and Risk in Economic Analysis

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TO DO

1. Uncertainty and Risk
2. Dealing with Uncertainty:
   - Sensitivity Graphs
   - Break-Even Analysis
   - Scenario Analysis
3. Dealing with Risk:
   - Decision Trees
   - Monte Carlo Simulation

Introduction:

- Economic analyses are not complete unless we try to asses the potential effects of project uncertainties on the outcomes of the evaluations.
- We have so far assumed, the following are known with certainty:
  - Prices,
  - Interest rates,
  - Magnitude and timing of cash flows, etc.
- Uncertainty stems from: Probability or Possibility
- Uncertainty for economic analysis deals with a situation characterized by a range of possible outcomes, however, the probability assessment of each outcome is not known; while in the case of risk, the probability of each outcome is known.

TO DO

- Dealing with Uncertainty:
  - Sensitivity Graphs
  - Break-Even Analysis
  - Scenario Analysis

Sensitivity Analysis:

- Sensitivity analysis is an approach to project evaluation that can be used to gain better understanding of how uncertainty or error affects the outcome of the economic evaluations by examining how sensitive the outcome is to changes in the uncertain parameters.
- Three basic methods are commonly used:
  - Sensitivity graphs
  - Break-even analysis
  - Scenario analysis

Sensitivity Graphs:

- Sensitivity graphs illustrate the sensitivity of a particular measures (e.g., present worth or annual worth) to one-at-time changes in the uncertain parameters of a project.
- Steps:
  - 1. Develop Base Case: All estimated parameter values are used to evaluate the performance measures of the project (present worth, annual worth, or IRR)
  - 2. Vary the key parameters values above or below the base case one at time, while holding all other parameters fixed
  - 3. Plot the changes in the performance measures of the project brought by these one-at-a-time changes
  - 4. Evaluate which parameters have the greatest effect of the performance measures

Sensitivity Graphs:

- Example: Cogenesis Corporation is replacing their current steam plant with a 6-megawatt cogeneration plant that will produce both steam and electric power for their operation:
  - Estimated first cost of equipment and installation is $3 000 000
  - Plant will have 20-year life and no scrap value
  - The turbo-generator will require an overhaul with an estimated cost of $35 000 at the end of years 4, 8, 12, and 16
  - The cooling tower will need an overhaul at the end of 10 years, and the expected cost is $17 000
  - The additional operating and maintenance costs is $65 000 per year
  - The additional annual cost for wood fuel is $375 000
  - The cogeneration plant will save Cogenesis from purchase of 40 000 000 KW hours of electricity per year at $0.025 per KW-hour
  - If Cogenesis uses a MARR of 12%, What is the PW of the incremental investment? What is the impact of of a 5% and 10% increase and decrease in each parameters of the problem?

Sensitivity Graphs:

- The base case solution is:
- $PW_{Cogeneration\ plant} = -\$3\,000\,000 - (\$65\,000 + \$375\,000 - \$1\,000\,000)(\mathrm{P/A}, 12\%, 20) - \$17\,000\,(\mathrm{P/F}, 12\%, 10) - \$35\,000\,[(\mathrm{P/F}, 12\%, 4) + (\mathrm{P/F}, 12\%, 8) + (\mathrm{P/F}, 12\%, 12) + (\mathrm{P/F}, 12\%, 16)] = \$\,1\,126\,343$

- The following parameters are evaluated for sensitivity:
  - Initial Investment
  - Annual operating and maintenance costs
  - Cooling tower overhaul (after 10 years)
  - Turbogenerator overhauls (after 4, 8, 12, and 16 years)
  - Annual wood costs
  - Annual savings in electricity costs
  - MARR

Sensitivity Graphs:

- Summary Data:

Cost Category	- 10%	- 5%	Base Case	5%	10%
Initial Investment	2 700 000	2 850 000	3 000 000	3 150 000	3 300 000
Annual operating and maintenance costs	58 500	61 750	65 000	68 250	71 500
Cooling tower overhaul (after 10 years)	15 300	16 150	17 000	17 850	18 700
Turbogenerator overhauls (after 4, 8, 12, and 16 years)	31 500	33 250	35 000	36 750	38 500
Annual wood costs	337 500	356 250	375 000	393 750	412 500
Annual savings in electricity costs	900 000	950 000	1 000 000	1 050 000	1 100 000
MARR	0.108	0.114	0.12	0.126	0.132

Sensitivity Graphs:

- Present Worth Variations from Base Case: As the PW in ±10% range is all positive, the investment is viable.

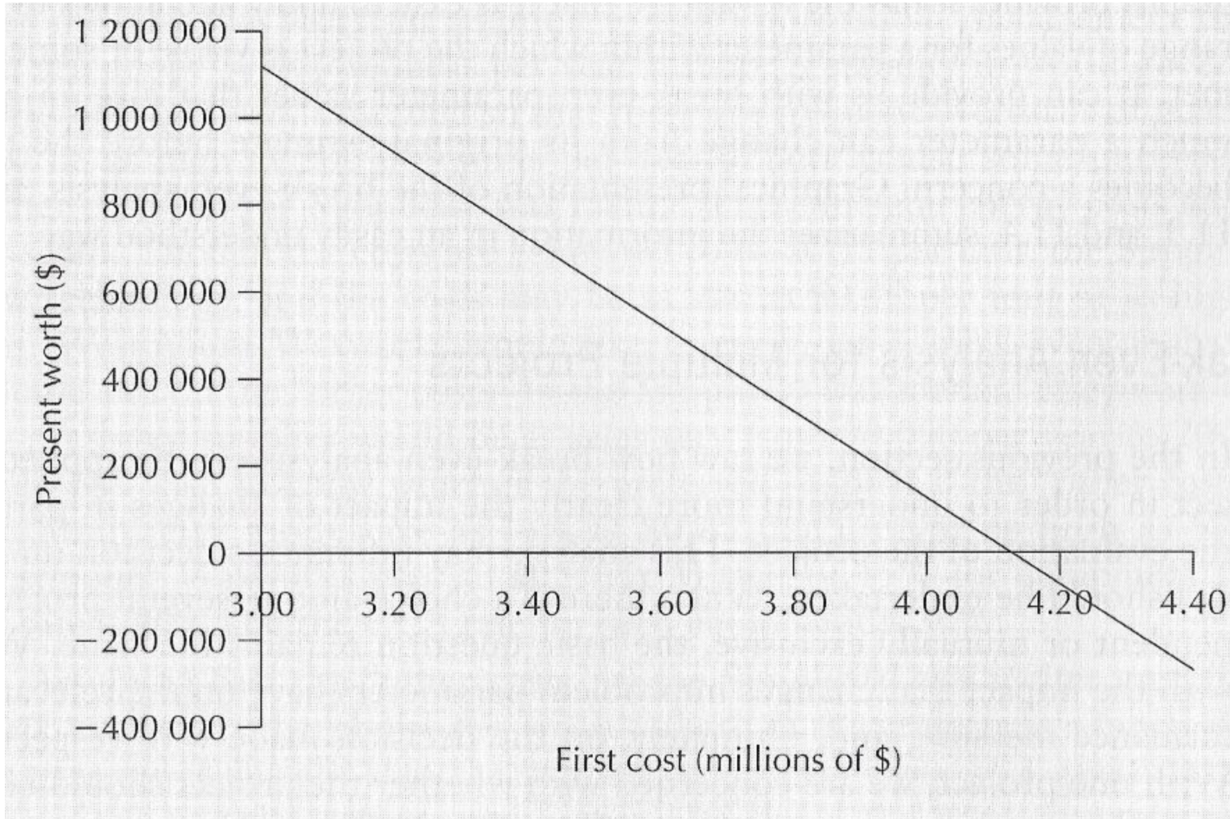
Cost Category	- 10%	- 5%	Base Case	5%	10%
Initial Investment	1 426 343	1 276 343	1 126 343	976 343	826 343
Annual operating and maintenance costs	1 174 894	1 150 619	1 126 343	1 102 067	1 077 792
Cooling tower overhaul (after 10 years)	1 126 890	1 126 617	1 126 343	1 126 069	1 125 796
Turbogenerator overhauls (after 4, 8, 12, and 16 years)	1 131 450	1 128 897	1 126 343	1 123 789	1 121 236
Annual wood costs	1 406 447	1 266 395	1 126 343	986 291	846 239
Annual savings in electricity costs	379 399	752 871	1 126 343	1 499 815	1 873 287
MARR	1 456 693	1 286 224	1 126 343	976 224	835 115

Sensitivity Graphs:

- Example: Cogenesis Corporation is replacing their current steam plant with a 6-megawatt cogeneration plant that will produce both steam and electric power for their operation:
  - The management recognizes that the PW of the cogeneration plant is quite sensitive to the savings in electricity costs, the MARR, and the initial costs.

- Taking the Initial cost parameter:
  - Carryout break-even analysis using the PW equation and calculate the initial cost value that will set PW = 0
  - Construct a break-even graph showing the PW as a function of the initial cost

Break-even Chart: Initial Cost



- Assuming that all other cost estimates are accurate (base case), the project will be viable as long as the initial cost is below $4 126 350. Similarly,
- $\text{MARR}_{\text{break-even}}$ = 17.73%
- $\text{Electricity Savings}_{\text{break-even}}$ = $849 207

Scenario Analysis:

- Both sensitivity graphs and break-even analyses can look at parameter changes only one at a time.
- Scenario Analysis allows us to look at the overall impact of different sets of parameter values, referred as ”scenarios,” on project evaluation.
- Scenarios are developed based on “what if” cases, and commonly used scenarios are:
  - Optimistic (best case)
  - Pessimistic (worst case)
  - Expected (most likely case)

Scenario Analysis:

- Example: Cogenesis Corporation is examining the following scenarios:

Cost Category	Pessimistic Scenario	Expected Scenario	Optimistic Scenario
Initial Investment	3 300 000	3 000 000	2 700 000
Annual operating and maintenance costs	75 000	65 000	60 000
Cooling tower overhaul (after 10 years)	21 000	17 000	13 000
Turbogenerator overhauls (after 4, 8, 12, and 16 years)	40 000	35 000	30 000
Annual wood costs	400 000	375 000	350 000
Annual savings in electricity costs	920 000	1 000 000	1 080 000
MARR	0.13	0.12	0.11
Present Worth of plant	-234 639	1 126 343	2 583 848

TO DO

- Dealing with Risk:
  - Decision Trees
  - Monte Carlo Simulation

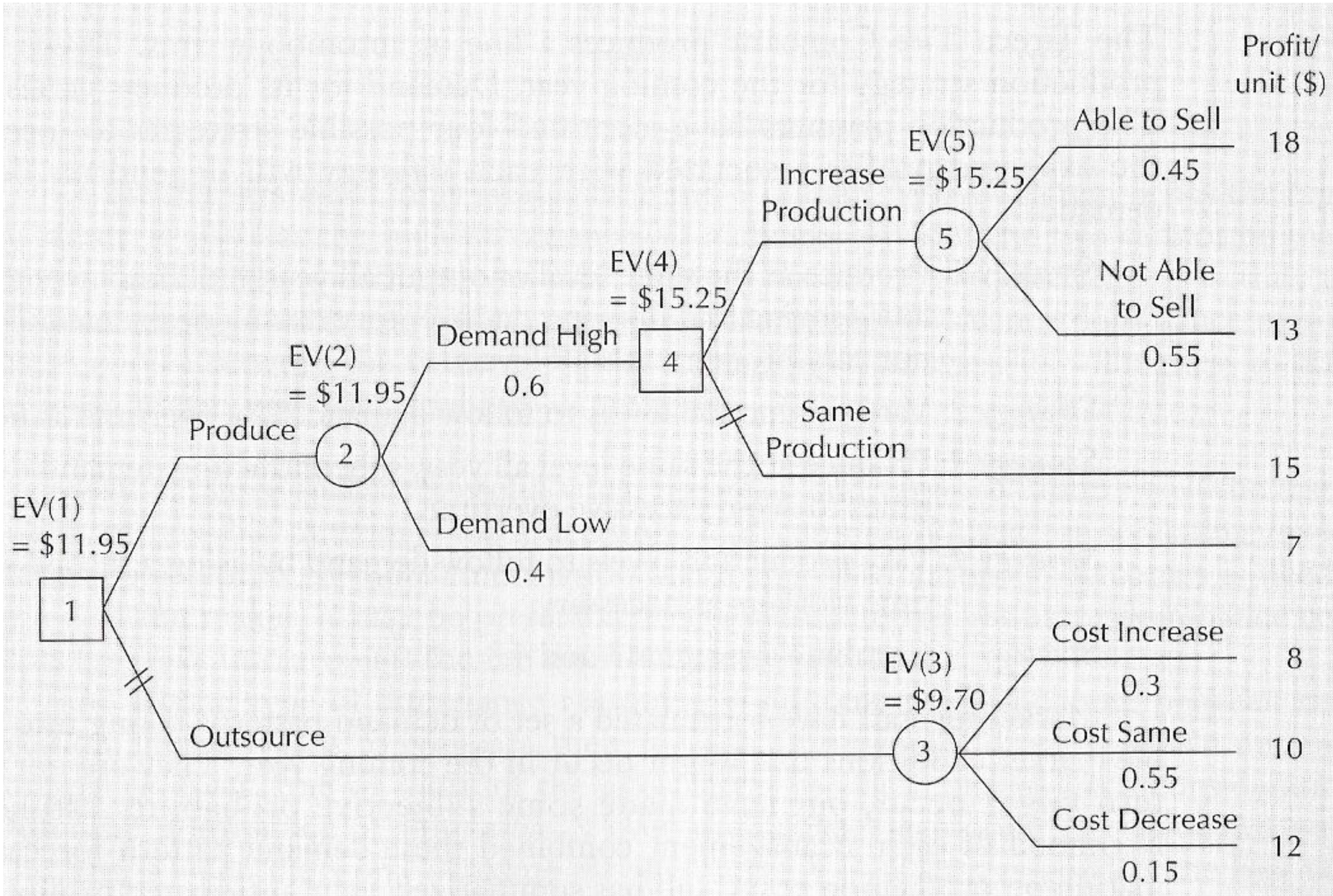
Expected Value Analysis:

- Decisions made under risk are those where the analyst can characterize a possible range of future outcomes and has available an estimate of the probability of each outcome.
- Decision tree is a graphical representation of the logical structure of a decision problem in terms of the sequence of decisions to be made and outcomes of chance events.
- Main elements:
  - Decision nodes ◻ – depict decision to made
  - Chance nodes ○ – depicts an event whose outcome is unknown
  - Branches – depict the sequence of possible decisions and chance events
  - Leaves – values or payoffs associated with each branch

Decision Criteria:

- Expected Value: $E(x) = \sum_{i=m}^{m} x_i \, p(x_i)$, where x_i is value or payoff of an event and $p(x_i)$ is probability of the event
  - Steps:
    - 1. Develop decision tree
    - 2. At each chance node, computed expected value of the possible outcomes, by rolling back from right to left
    - 3. At each decision node, select the option with the best expected value
    - 4. For the option(s) not selected at this time, cancel the corresponding branch, using double-slash (//)
    - 5. Continue rolling back until the leftmost node is reached
- Example:
- Carryout a decision tree analysis for the decision problem shown below:

Expected Value



- Decision: Produce TV screens in-house, and if demand is high, the production level should be increased.

Decision Criteria:

- Expected value is only a summary measure (based on mean) and does not consider the dispersion of the outcomes associated with a decision.
- Dominance: focuses on quantifying the risk on the basis of the probability distribution (mean and variance) of the outcomes, rather than simple mean
- $E(x) = \sum_{i=m}^{m} x_i \, p(x_i)$,
- $Var(x) = \sum_{i=m}^{m} p(x_i)(x_i - E(x))^2$,
- Where $E(x)$ is mean, $Var(x)$ is variance, x_i is value or payoff of an event, and $p(x_i)$ is probability of the event
- Dominance reasoning types:
  - Mean-variance dominance
  - Outcome dominance
  - Stochastics dominance

Expected Value Analysis:

- Mean-variance dominance: Alternative X is said to have mean-dominance over alternative Y if:
  - $EV(X) \geq EV(Y) and\ Var(x) < Var(Y)$
  - $EV(X) > EV(Y) and\ Var(x) \leq Var(Y)$

- Outcome dominance: Alternative X is said to have outcome dominance over alternative Y if:
  - Worst outcome of alternative X is at least as good as the best outcome of alternative Y.
  - Alternative X is as least as preferred to another alternative for each outcome, and is better for at least one outcome.

- Is used to screening decision alternatives that are clearly worse than other among the set of choices.

Expected Value Analysis:

- Stochastic dominance: If two decision alternatives a and b have outcome cumulative distribution functions $\mathrm{F}(x)$ and $\mathrm{G}(x)$, respectively, then alternative a is said to have first order stochastic dominance over alternative b if $\mathrm{F}(x) \geq \mathrm{G}(x)$ for all x.

- It means, alternative a is more likely to give higher (better) outcome that alternative b for all possible outcomes.

- However, first-order stochastic dominance and outcome dominance can be used to screen alternatives, it is often the case that they are not able to provide a definitive "best" alternative.

Monte Carlo Simulation:

- Monte Carlo simulation is used to analyze risk in complex decisions, such as decisions with large and complex decision tree where the input parameters are random.
- It allows the analysis of the combined impact of multiple sources of uncertainty in order to develop an overall picture of overall risk.
- It evaluates the decision strategies by randomly sampling braches of the decision tree, and assemble probability distributions (risk profiles) for relevant performance measures.

Monte Carlo Simulation:

- Steps:
  - 1. Analytical Model: Develop analytical model by identifying all input random variables that affect the outcome performance measure
  - 2. Probability Distribution: Establish an appropriate probability distribution for each input variable
  - 3. Random sampling: Sample value for each input variable from the associated probability distribution
    a. For each discrete random variable create a random number assignment ranges table

i	Outcome x_i	Probability $p(x_i)$	Random Number (Z) Assignment Number
1	x_1	$p(x_1)$	$0 \leq Z \leq p(x_1)$
2	x_2	$p(x_2)$	$p(x_1) \leq Z \leq p(x_1) + p(x_2)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$m-1$	x_{m-1}	$p(x_{m-1})$	$p(x_1) + \cdots + p(x_{m-2}) \leq Z \leq p(x_1) + \cdots + p(x_{m-1})$
m	x_m	$p(x_m)$	$p(x_1) + \cdots + p(x_{m-1}) \leq Z \leq 1$

Monte Carlo Simulation:

- Random Number Generation:
- Linear Congruential Scheme (LCS)
  - Initialize $Z_n = Z_0$ (called Seed)
  - For the n^{th} iteration:
  - $R_n = \frac{Z_n}{m}$
  - $Z_n = a\ x\ Z_{n-1} \bmod m$
  - m is a modulus, set to large integer value (e.g. $2^{31}-1$)
  - a is a multiplier usually set to be 7^5

Monte Carlo Simulation:

- Initial values: a = 5, m = 7 and Z_0= 9 (Seed)
- LCS recursive equation: $Z_n = 5 \times Z_{n-1} \bmod 7$
- Random number equation: $R_n = \frac{Z_n}{7}$
- Results (generated random numbers)

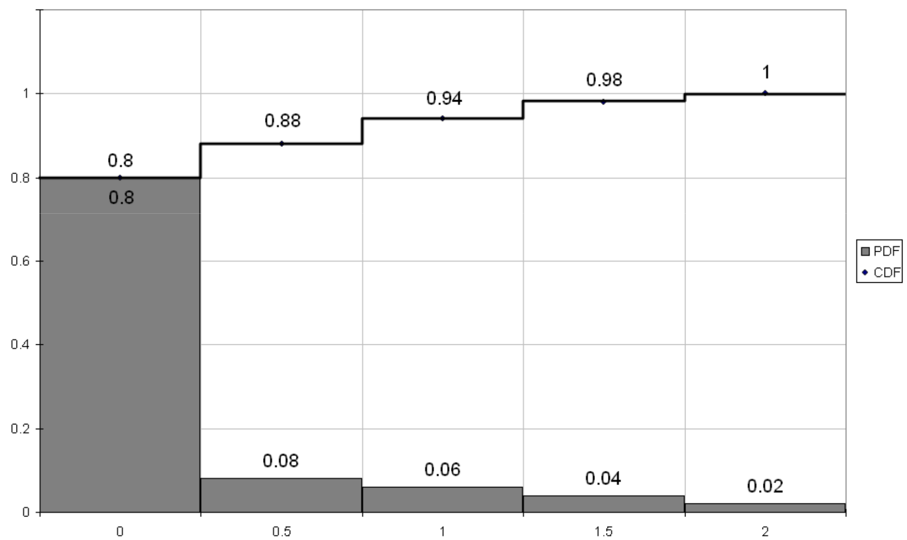
n	Z_n	R_n
0	9	–
1	3	0.4285714
2	1	0.1428571
3	5	0.7142857
4	4	0.5714286
5	6	0.8571429
6	2	0.2857143
7	3	0.4285714

Note that random numbers repeat at the 7th iteration, i.e., the mth iteration hence the need for a large m value.

Monte Carlo Simulation:

- Steps:
  - 3. Random sampling: Sample value for each input variable from the associated probability distribution

    b. For each input random variable, generate a random number. Find the range to which Z belongs from in the above Table and assign appropriate outcome.



Crushing plant	Probability	Cumulative Probability	R.N.
No break down	0.8	0.8	00-79
Break down which requires 0.5 hour repair	0.08	0.88	80-87
Break down which requires 1.0 hour repair	0.06	0.94	88-93
Break down which requires 1.5 hour repair	0.04	0.98	94-97
Break down which requires 2.0 hour repair	0.02	1	98-99
	1		

Monte Carlo Simulation:

- Steps:
  - 3. Random sampling: Sample value for each input variable from the associated probability distribution
    b. For each input random variable, generate a random number. Find the range to which Z belongs from in the above Table and assign appropriate outcome.
    c. Substitute the sample values of the random variables into the expression for the outcome measure, Y, and compute the value of Y. This forms one sample point in the procedure.
  - 4. Repeat sampling: Continue sampling until a sufficient sample size $(n \cong 100)$ is obtained for the value of Y.
  - 5. Summarize the frequency distribution of the sample outcomes using a histogram. Summary statistics, like the range of possible outcomes and expected value, can be calculated from the sample outcomes.

Monte Carlo Simulation:

- Example:
- An insurance company has consulted several energy experts in order to further understand the implications of electricity and natural gas price changes on their two energy efficiency projects.
  - They use MARR of 10%.
  - Each project has a service life of 10 years and zero scarp value.
  - Project 1 – Installing high-efficiency motors on HVAC system:
    - Use 7% less electricity than the current motors, leading to savings of 70 000 kilowatt-hours
    - Cost $28 000 to purchase and install and will require maintenance costs of $700 annually
  - Project 2 – Installing a heat exchange unit on on HVAC system:
    - During winter, the heat exchange unit transfers heat from warm room air to the cold ventilation air before the air is sent back to the building, savings of 2 250 000 cubic feet of natural gas per year

Monte Carlo Simulation:

- Example:
  - Project 2 – Installing a heat exchange unit on on HVAC system:
    - In the summer, the heat exchange unit removes heat from the hot ventilation air before it is added to the cooler room air for recirculation, leading to a savings of 29 000 kilowatt-hours of electricity annually
    - Each heat exchange unit costs $40 000 to purchase and install and annual maintenance costs are $3200.
  - Current prices are 0.07 per kilowatt-hour of electricity and $3.50 per 1000 cubic feet of natural gas
  - The cost electricity can range from $0.063 per kilowatt-hour to $0.077 per kilowatt-hour, and the price of natural gas can range from $3.55 to $3.66 per 1000 cubic feet.
  - Carryout a Monte Carlo simulation to determine the probability distribution of the PW of the two energy efficiency projects, based on the probability distributions shown in the next Table.

Monte Carlo Simulation:

- Example:

Electricity Cost (per kWh)	Probability	Natural Gas Price (per 1000 cubic feet)	Probability
$0.063	0.125	$3.35	1/7
0.065	0.125	3.40	1/7
0.067	0.125	3.45	1/7
0.069	0.125	3.50	1/7
0.071	0.125	3.55	1/7
0.073	0.125	3.60	1/7
0.075	0.125	3.65	1/7
0.077	0.125		

Monte Carlo Simulation:

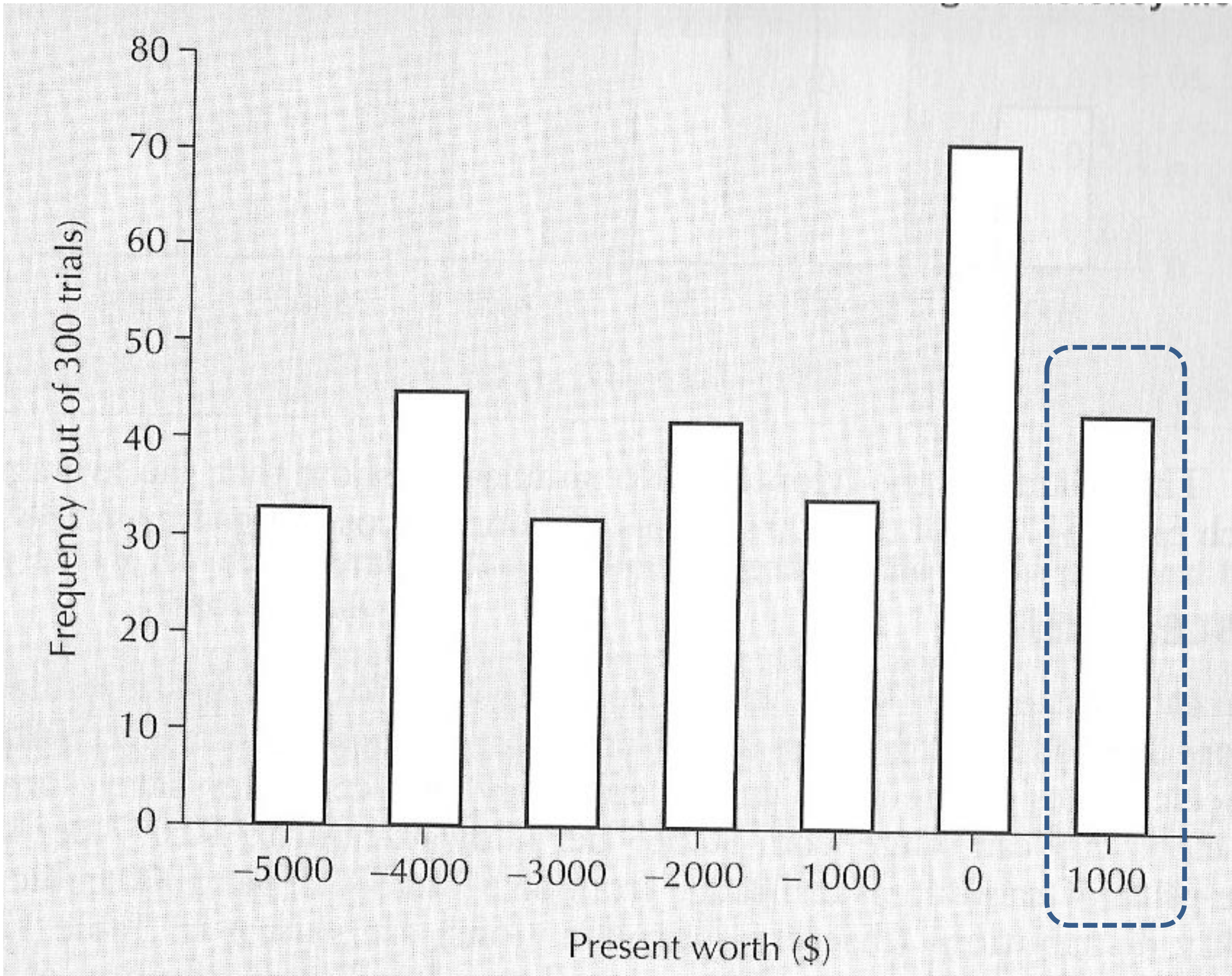
- Solution:
- Analytical Model:
  - $PW_{(High\ efficency\ motor)} = -\$28{,}000 + (P/A, 10\%, 10) * [\$70\,000\ (Electricity) - \$700]$
  - $PW_{(High\ efficency\ motor)} = -\$28{,}000 + 6.1446 * [\$70\,000\ (Electricity) - \$700]$
  - $PW_{(Heat\ exchanger)} = -\$40{,}000 + (P/A, 10\%, 10) * [\$29\,000\ (Electricity) + \$2250\ (Natural\ Gas) - \$32000]$
  - $PW_{(Heat\ exchanger)} = -\$40{,}000 + 6.1446 * [\$29\,000\ (Electricity) + \$2250\ (Natural\ Gas) - \$32000]$
- Taking 300 samples for the electricity and natural gas cost distributions and computing the PW:

Monte Carlo Simulation:

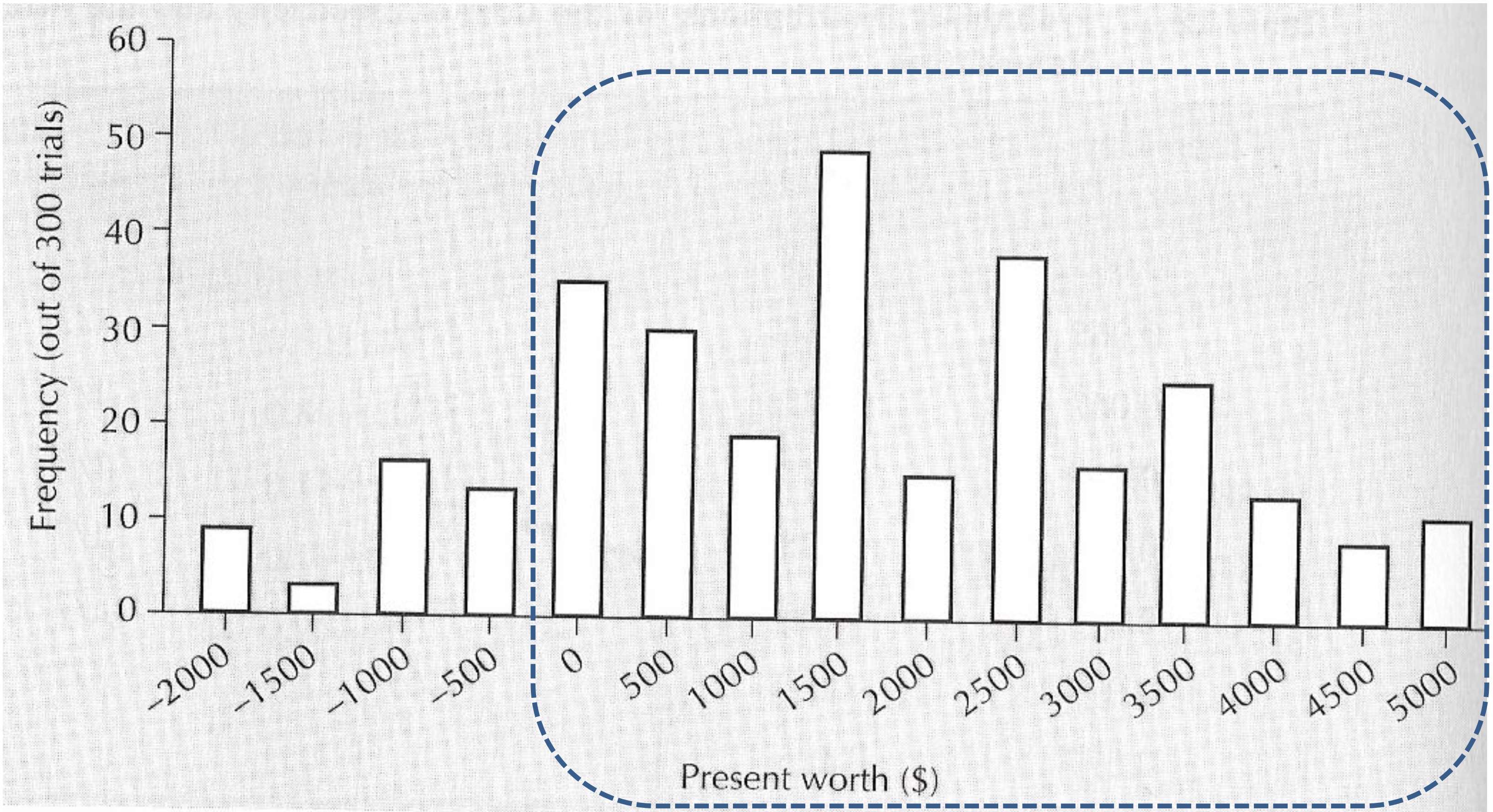
- Example:

Electricity Cost (per kWh)	Probability	Cumulative Probability	R.N.
$0.063	0.125	0.125	0.000 – 0.124
0.065	0.125	0.25	0.125 – 0.240
0.067	0.125	0.375	0.250 – 0.374
0.069	0.125	0.5	0.375 – 0.490
0.071	0.125	0.625	0.500 – 0.624
0.073	0.125	0.75	0.625 – 0.740
0.075	0.125	0.875	0.750 – 0.874
0.077	0.125	1	0.875 – 0.999

Monte-Carlo Simulation Results for the High-Efficiency Motor



Monte-Carlo Simulation Results for the Heat Exchange Unit



- Decision: Based on the chance of having positive PW, Heat Exchange Unit is the better choice.

References:

- Fraser, N.M., Jewkes, E., Bernhardt, I., Tajima, M. (2006). *Engineering Economics in Canada*. 3rd edition, Prentice Hall.