AAiT
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SCHOOL OF CIVIL AND ENVIROMENTAL ENGINEERING

Surveying II
CENG 2092

Chapter 3
Curves

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2012EC (2019/20GC)
2nd Sem

Engineering Surveying -- Def

The ASCE defines engineering surveying as those activities involved in the planning and execution of surveys for the location, design, construction, operation, and maintenance of civil and other engineered projects. Such activities include:

- The preparation of survey and related mapping specifications;
- Execution of photogrammetric and field surveys for the collection of required data
- Calculation, reduction and plotting of survey data for use in engineering design including use for geographic information systems (GIS);
- Design and provision of horizontal and vertical control survey networks;
- Monitoring of ground and structural stability, including alignment observations, settlement levels, and related reports and certifications; and
- Analysis of errors and tolerances associated with the measurement, field layout and mapping.

Engineering surveying

- Curve setting out
- Earth work computation
- Staking of structures like building, bridge, culvert and tunnel
- Monitoring of ground and structural stability
- Route surveying

Measurement and setting out

- Accurate large-scale plan → planning and design of a construction project

- The plan →set out on the ground in the correct absolute and relative position and to its correct dimensions

Curve

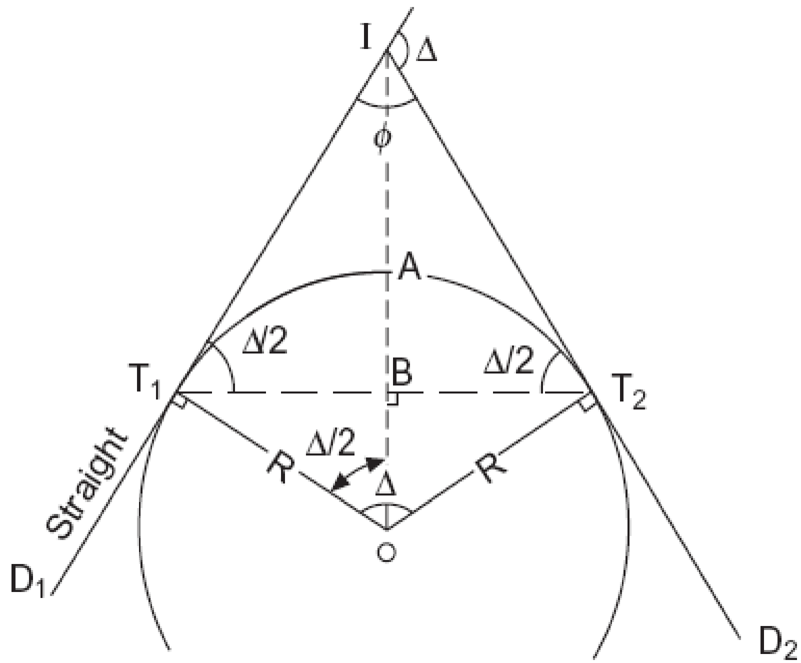
- The center line of a road consists of series of straight lines interconnected by curves that are used to change the alignment, direction, or slope of the road.
- Those curves that change the alignment or direction are known as **horizontal curves**, and
- Those that change the slope are **vertical curves.**

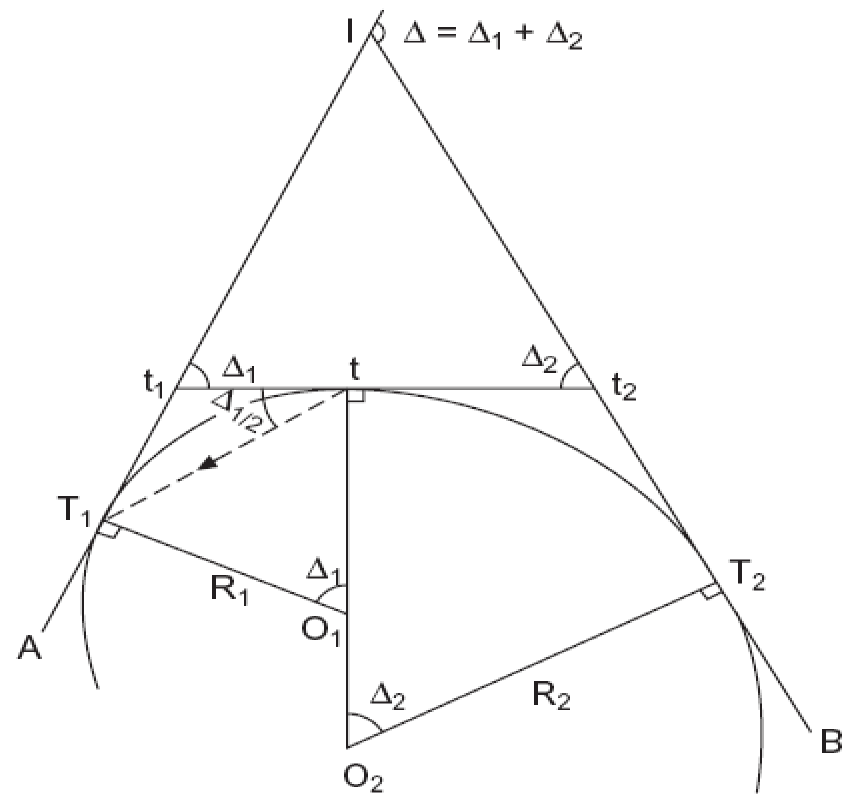
Curve Setting out

- **Horizontal Curve**
    - Circular Curve
    - Compound Curve
    - Reverse Curve
    - Transition Curve

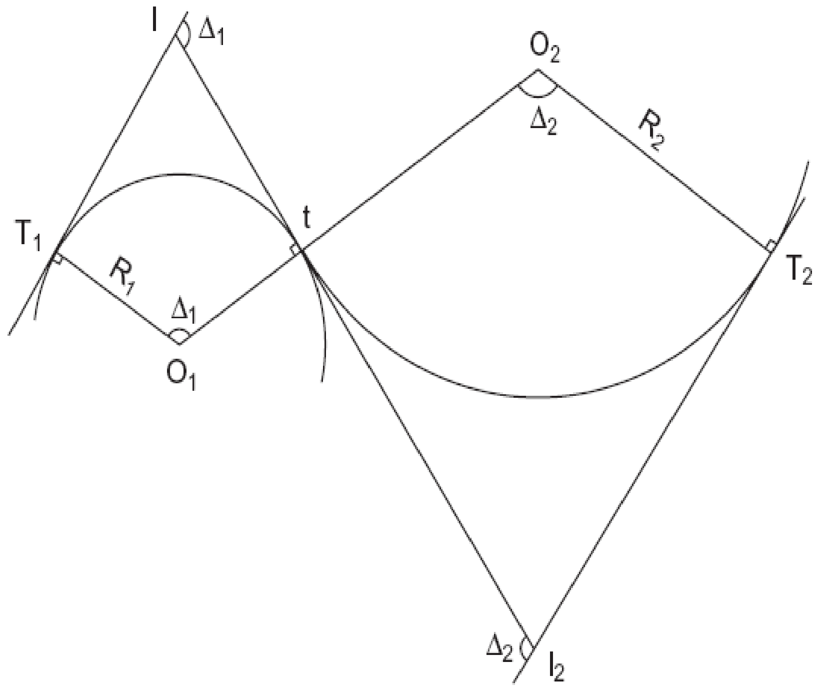
- **Vertical Curve**
    - Parabolic curve

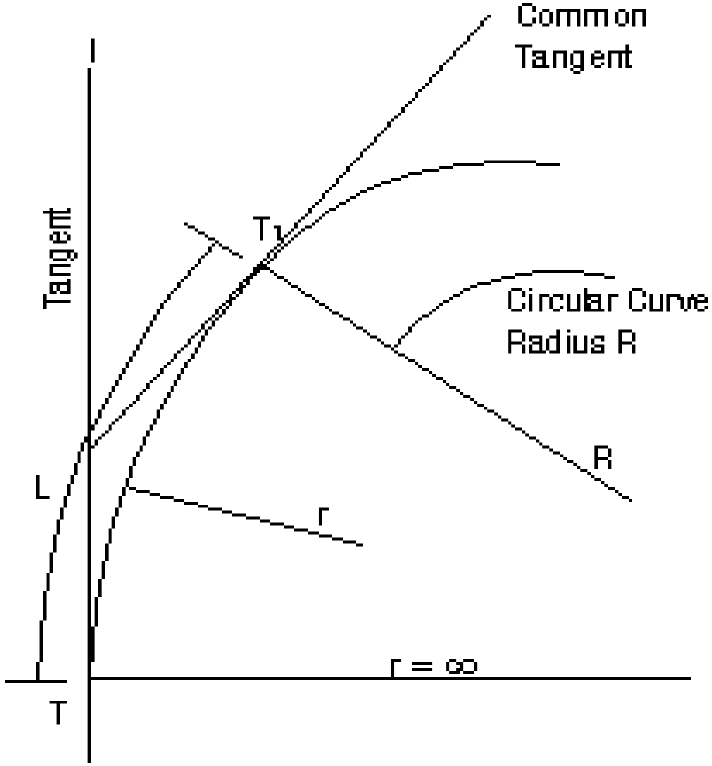
Simple circular and Compound



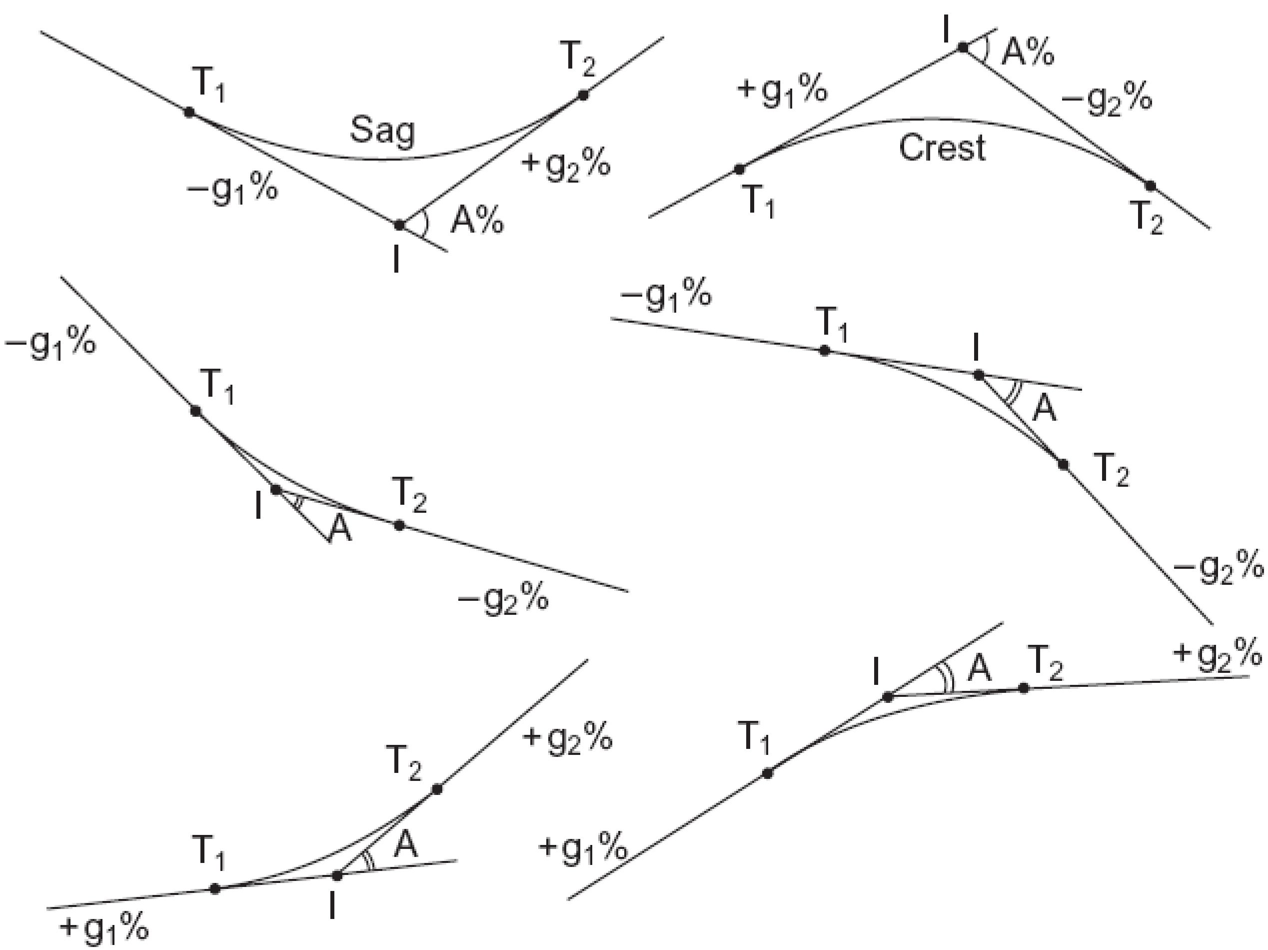


Reverse Curve and Transition curve





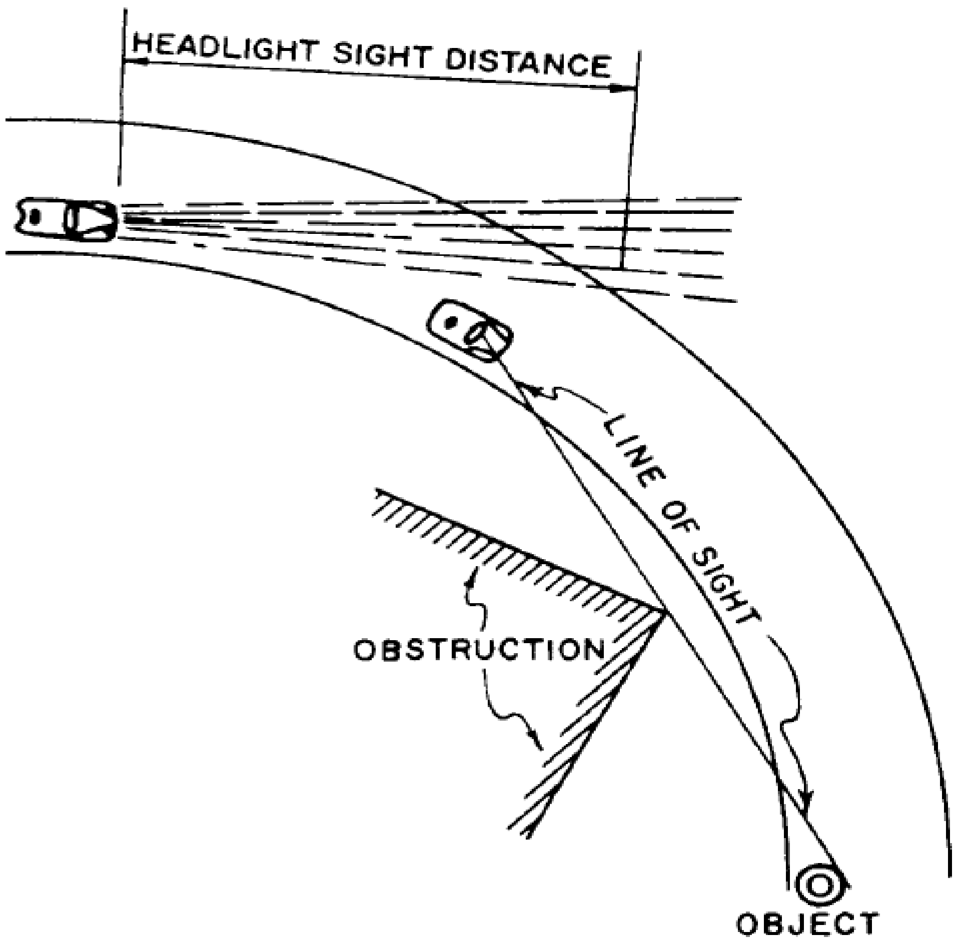
Vertical Curves



Horizontal Curves

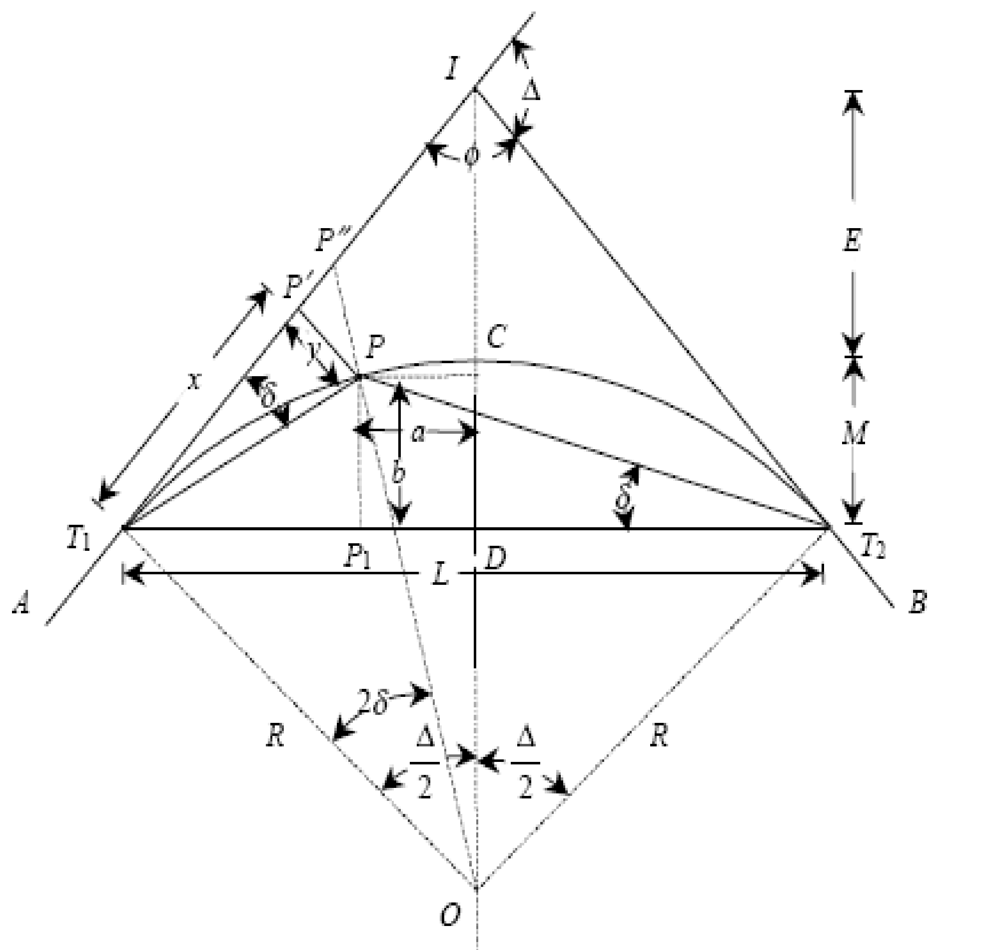
- The principal consideration in the design of a curve is the selection of the length of the radius or the degree of curvature

- This selection is based on such considerations as the **design speed** of the highway and the **sight distance** as limited by headlights or obstructions



Elements of Horizontal Curves

- I – Point of intersection
- T1 – Point of commencement
- T2 – Point of tangency
- E – External distance
- T1T2 – Length of the curve
- T1I = IT2 = tangent length
- AI – Back Tangent
- BI – Forward Tangent
- T1T2 – Chord length
- Δ - Deflection angle
- R – Radius of the curve
- ϕ - Angle of intersection
- M – Mid-ordinate



Formula to calculate the various elements of a circular curve for use in design and setting out

$$\text{Tangent length } (T) = R \tan \frac{\Delta}{2}$$

$$\text{Length of curve } (l) = \frac{\pi R \Delta}{180}$$

$$\text{Long chord } (L) = 2R \sin \frac{\Delta}{2}$$

$$\text{External distance } (E) = R\left(\sec \frac{\Delta}{2} - 1\right)$$

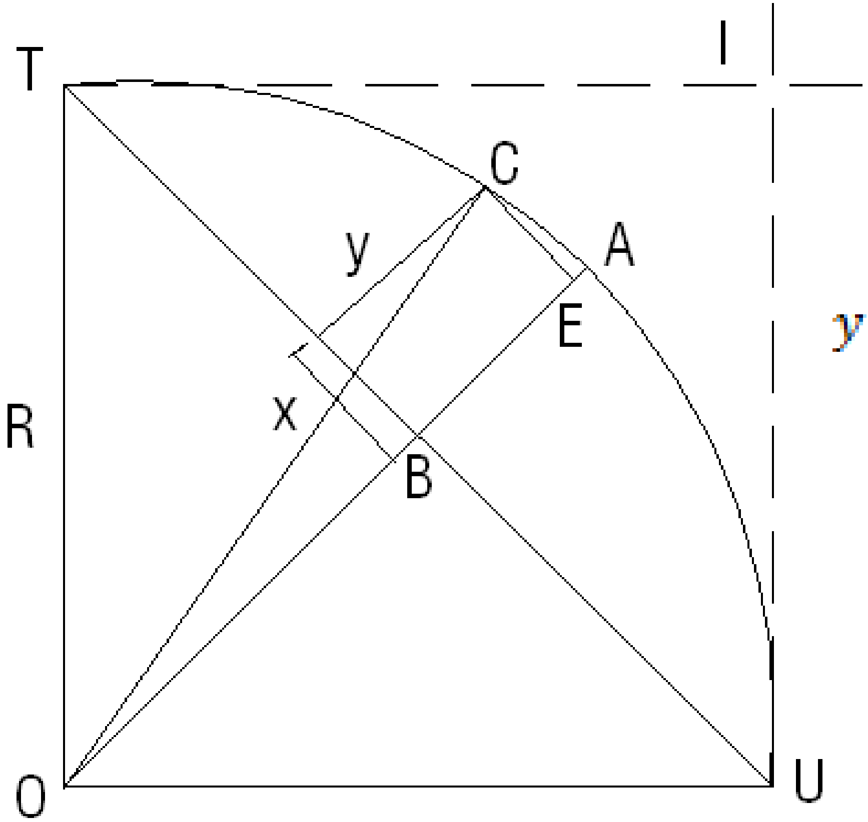
$$\text{Mid-ordinate } (M) = R\left(1 - \cos \frac{\Delta}{2}\right)$$

$$\text{Chainage of } T_1 = \text{Chainage of P.I.} - T$$

$$\text{Chainage of } T_2 = \text{Chainage of } T_1 + l.$$

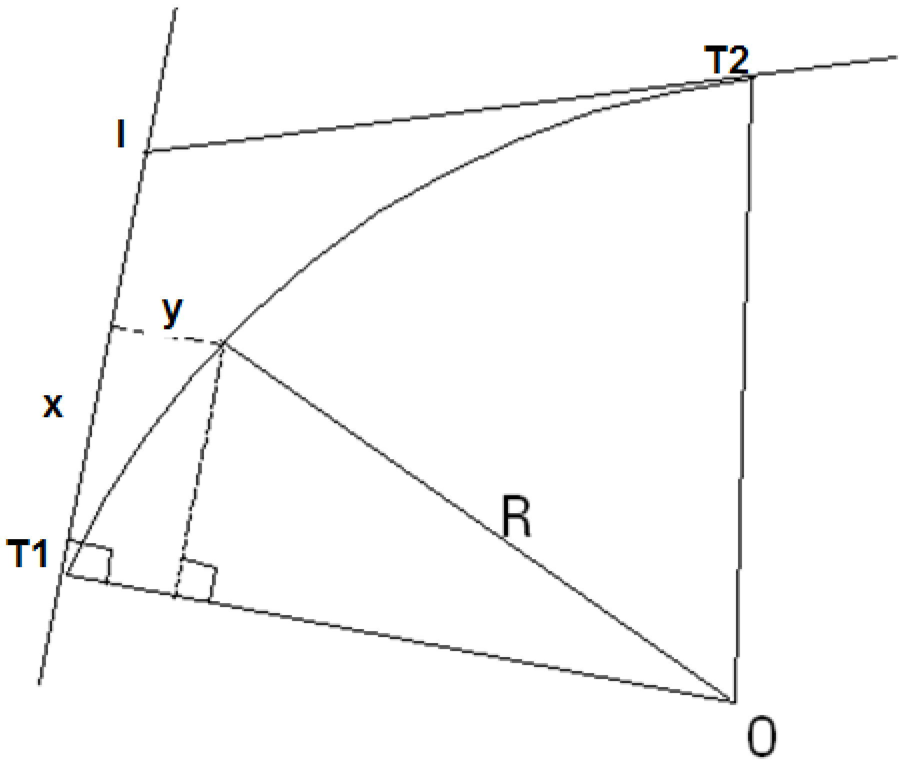
Setting out of circular curves

- Offset from long chord



$$y = \sqrt{(R^2 - x^2)} - \sqrt{(R^2 - \frac{L^2}{4})}$$

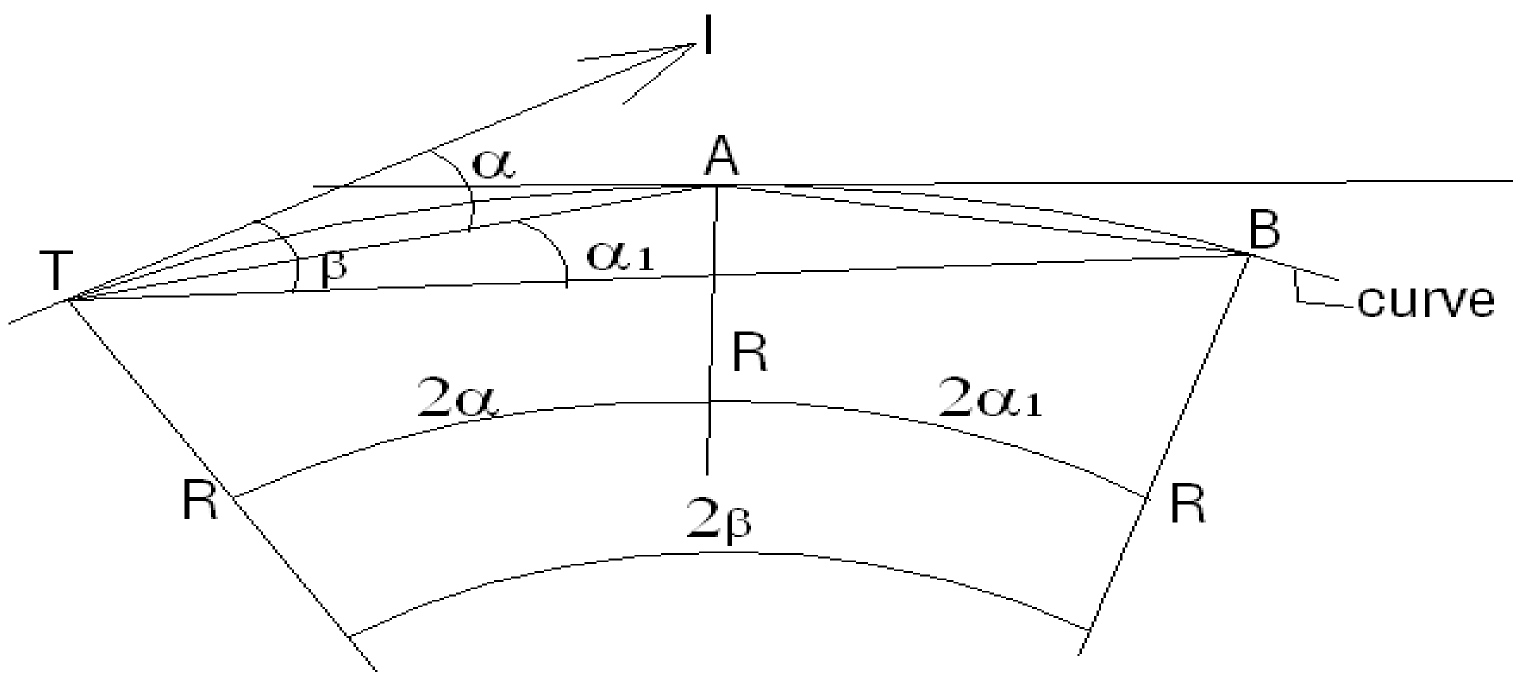
Offset from tangent



$$y = R - \sqrt{(R^2 - x^2)} \quad (exact)$$

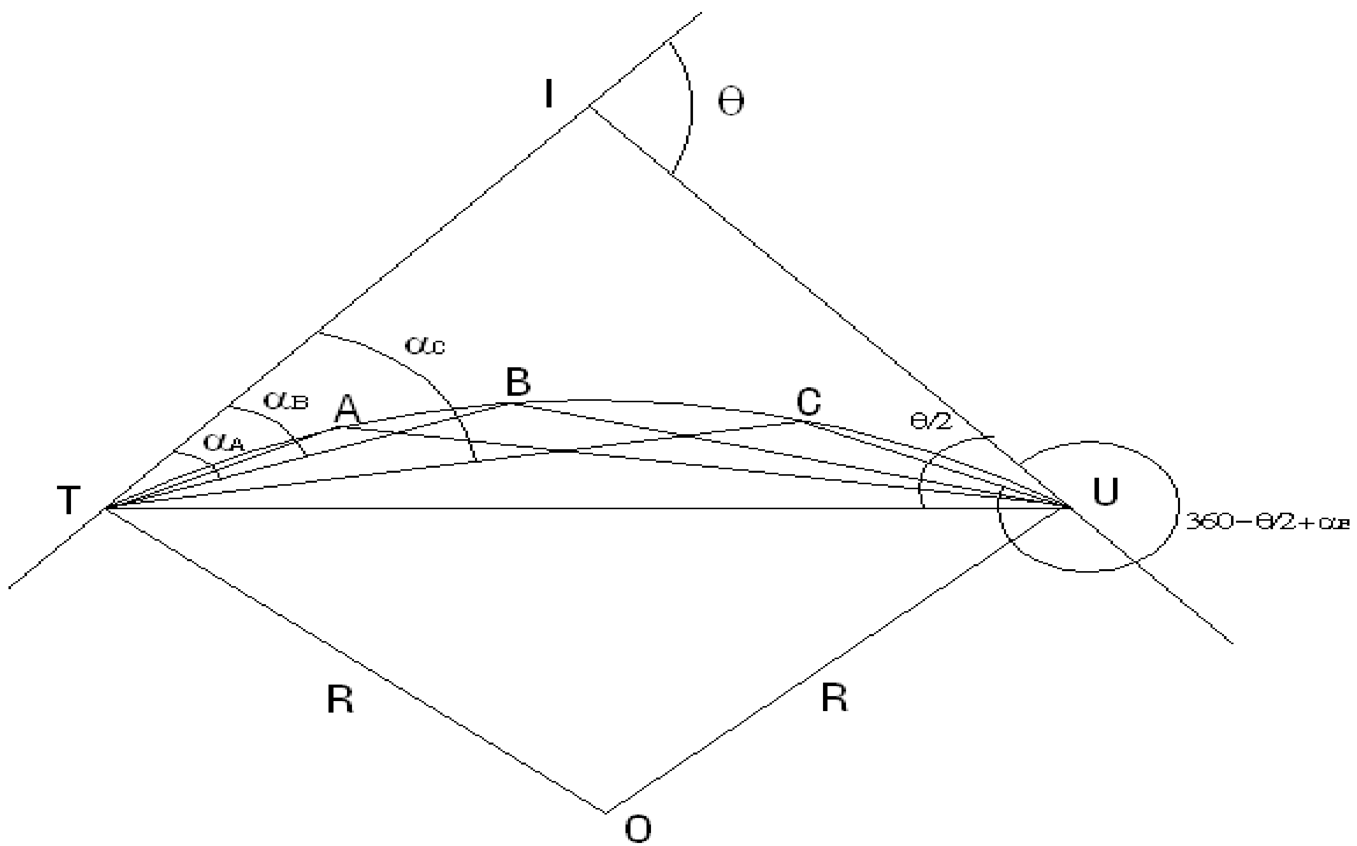
$$= \frac{x^2}{2R} \quad (approximate)$$

- **Setting out using one theodolite and tape**

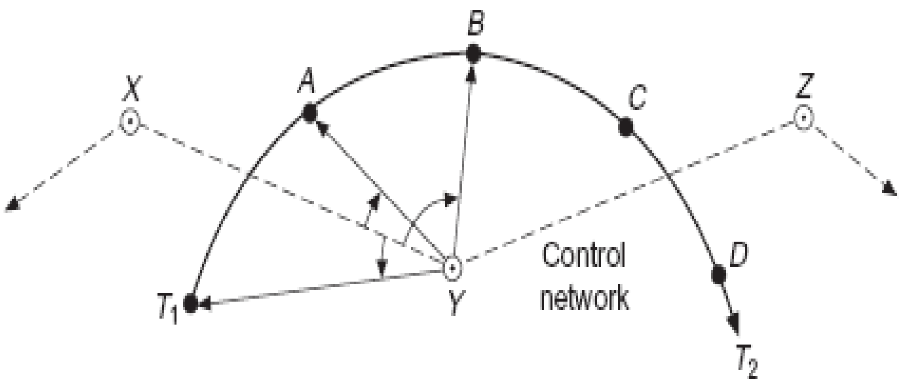


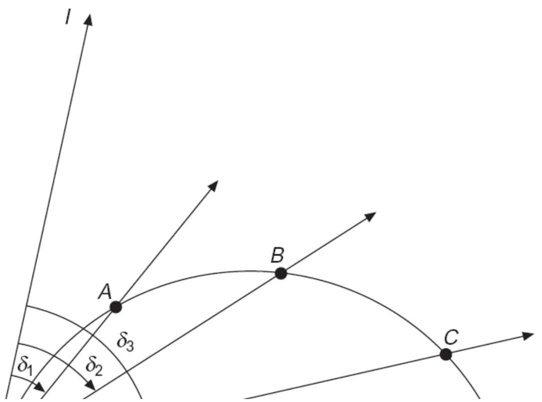
$$\text{Tangential angle } \delta_n = 1718.9\frac{c_n}{R} \text{ minutes}$$

- **Setting out using two theodolite**



Setting out using Coordinates



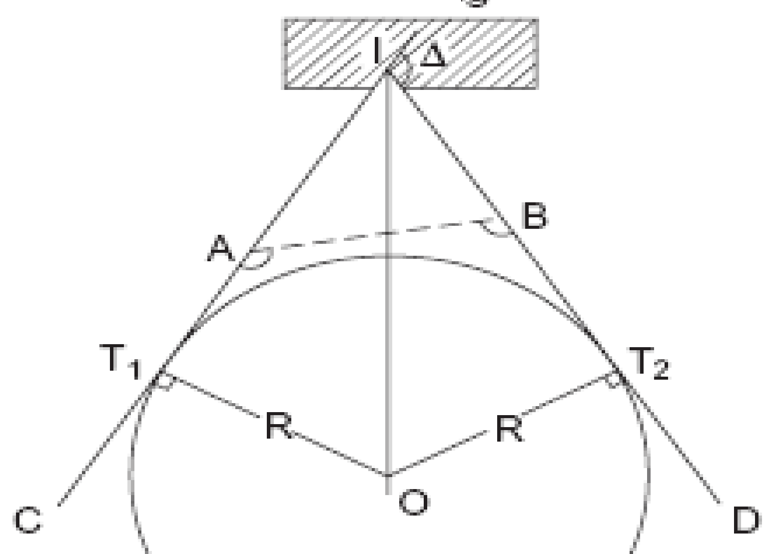


- Coordinates of T1 and I known form design
- δ known from deflection angle method
- Bearing of T1I is computed
- Using δs and bearing of T1I bearings of T1A, T1B ..
- Distance T1A, T1B .. can be computed as 2Rsin(δ1) …
- using the bearing and distances coordinates of A, B, C .. are obtained
- These points can now be set out from the nearest control points by polar or intersection methods
- By computing bearing and distance

Setting out with inaccessible intersection point

Project the straights forward as far as possible and establish two points A and B on them. Measure distance AB and angles BAC and DBA then:

angle $IAB = 180° - B\hat{A}C$ and angle $IBA = 180° - D\hat{B}A$, from which angle BIA is deduced and angle Δ. The triangle AIB can now be solved for lengths IA and IB. These lengths, when subtracted from the computed tangent lengths ($R \tan \Delta/2$), give AT_1 and BT_2, which are set off along the straight to give positions T_1 and T_2 respectively.



Example 1

- A circular curve of 500 m radius to be set out joining the two straights with deflection angle of 38°. Calculate the necessary data for setting out the curve by the method of

- Offset from long chord
- Offset from tangent
- Deflection angle method

Take peg interval of 30 m length and the chainage of I (60 + 13.385)

Solution

Calculations of elements of the Curve

- Tangent length = Rtan(Δ/2) = 500tan19° = 172.164 m
- Length of the curve (l) = (πRΔ/180) = 331.613 m
- Chainage of T1 = Chainage of I – T1I = 60x30 + 13.385 – 172. 164
- = 1641.221 = 54 + 21.22
- Chainage of T2 = Chainage of I + l = 1641.221 + 331.613
- = 1972.834 = 65 + 22.83
- Long chord (L)= 2Rsin(Δ/2) = 325.57 m
- L/2 = 162.785 m
- To locate the points on the curve, for distances
- X = 0, 30, 60, 90, 120, 150, 162.785 m

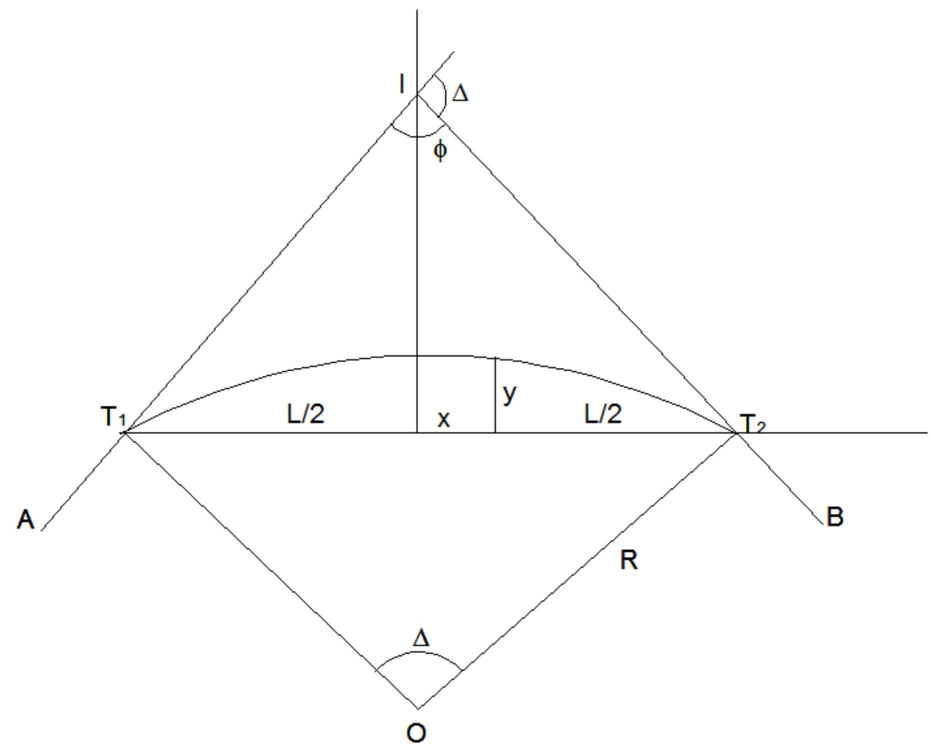
The offset from long chord is calculated as:

$$y = \sqrt{\left(R^2 - x^2\right)} - \sqrt{\left(R^2 - \frac{L}{4}\right)^2}$$

$$= \sqrt{\left(500^2 - x^2\right)} - 472.759$$

The calculated value of y,

Distance x (m)	0	30	60	90	120	150	162.79
Offset y (m)	27.24	26.34	23.63	19.07	12.63	4.21	0.00



Offset from tangent

$$y = R - \sqrt{(R^2 - x^2)}$$

x	0 (T1)	30	60	90	120	150	172.164 (I)
y	0	0.901	3.613	8.167	14.614	23.030	30.575

Deflection angle method

Length of first sub-chord = (54 + 30) – (54 + 21.22) = 8.78 m
Length of last sub-chord = (65 + 22.83) – (65 + 0) = 22.83 m
Number of normal chords N = 65 – 55 = 10
Total number of chords n = 10 + 2 = 12

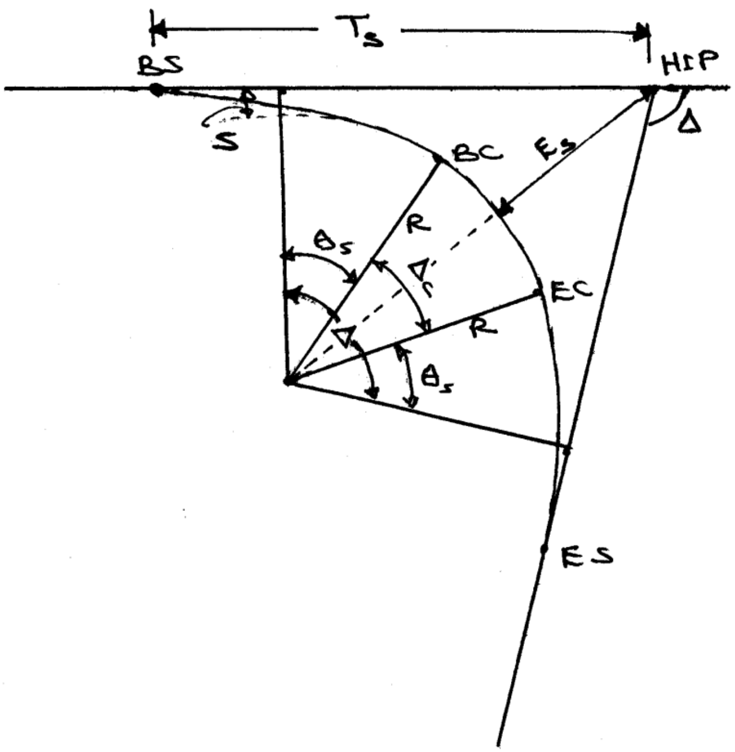
$$\text{Tangential angle } \delta_n = 1718.9 \frac{c_n}{R} \text{ minutes}$$

Tangential angle for the first chord = 1718.9*8.78/500 = 30.184 min
Tangential angle for the normal chord = 1718.9*30/500 = 103.184 min
Tangential angle for the last chord = 1718.9*22.83/500 = 78.485 min

Point	Chainage	Chord Length (m)	Tangential angle (‘)	Deflection Angle (‘)	Angle set on 1”
0(T_1)	54 + 21.22	-	0.0	0.0	
1	55 + 00	8.78	30.184	30.184	**00°30’11”**
2	56 + 00	30	103.134	133.318	**02°13’19”**
3	57 + 00	30	103.134	236.452	**03°56’27”**
4	58 + 00	30	103.134	339.586	**05°39’35’**
5	59 + 00	30	103.134	442.720	**07°22’43”**
6	60 + 00	30	103.134	546.104	**09°06’06”**
7	61 + 00	30	103.134	648.988	**10°48’59”**
8	62 + 00	30	103.134	752.122	**12°32’07”**
9	63 + 00	30	103.134	855.256	**14°15’15”**
10	64 + 00	30	103.134	958.839	**15°58’50”**
11	65 + 00	30	103.134	1061.524	**17°41’31”**
12(T_2)	65 + 22.83	22.83	78.485	1140.009	**19°00’01”**

Transition curves

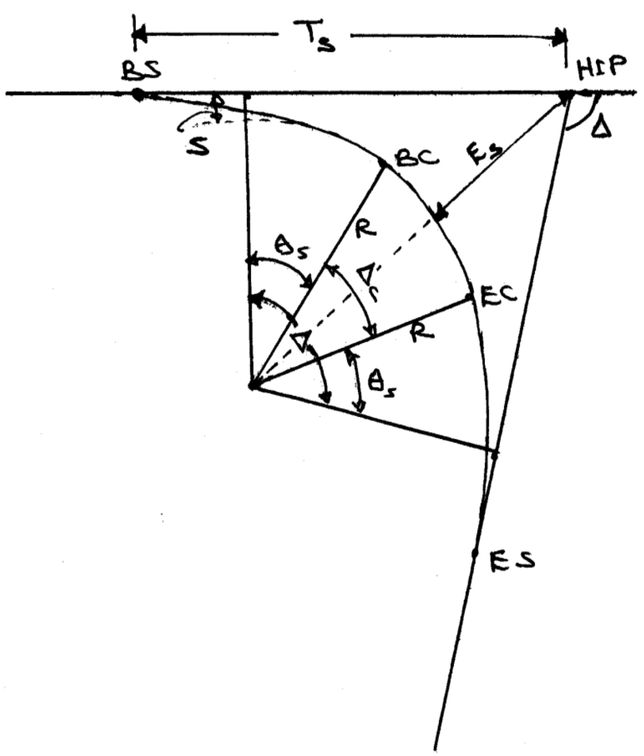
- The *transition curve* is a curve of constantly changing radius



Some of the important properties of the spirals are given below:

- $L = 2R\theta$
- $\theta = (L / L_s)^2 * \theta_s$
- $\theta_s = L_s / 2R_c$ (in radians) $= 28.65L_s / R_c$ (in degrees)
- $T_s = L_s /2 + (R_c + S)*\tan(\Delta/2)$
- $S = L_s^2 / 24R_c$
- $E_s = (R_c + S)*\sec(\Delta/2) - R_c$

Transition curves



Note:
θ_s = spiral angle
Δ = total central angle
Δ_c = central angle of the circular arc extending fro BC to EC = Δ - 2 θ_s
R_c = radius of circular curve
L = length of spiral from starting point to any point
R = radius of curvature of the spiral at a point L distant from starting point.
T_s = tangent distance
E_s = external distance
S = shift
HIP = horizontal intersection point
BS = beginning of spiral
BC = beginning of circular curve
EC = end of circular curve
ES = end of spiral curve

Shift, S

- Where transition curves are introduced between the tangents and a circular curve of radius R, the circular curve is “shifted” inwards from its original position by an amount BP = S so that the curves can meet tangentially.

Referring the figure below <NMO = <RTO = 90°

BM = NT_1 = Maximum offset on transition curve

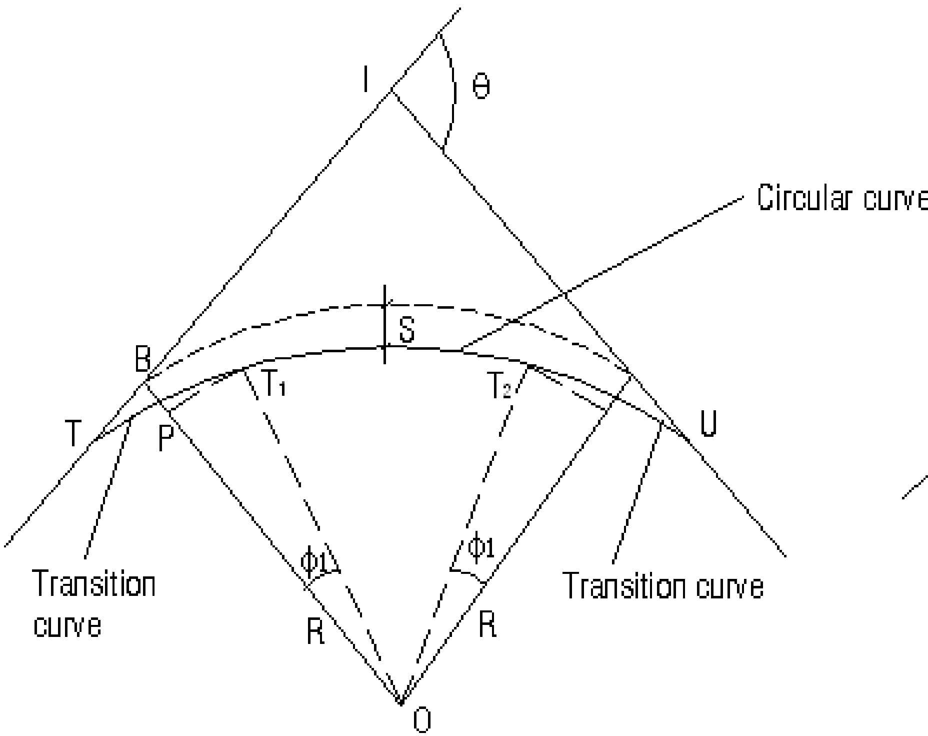
Also shift = S = $BP = BM - MO = NT_1 - (PO - MO)$

$$= \frac{L^3}{6LR} - (R - R\cos\phi_1) = \frac{L^3}{6LR} - \left\{R - R(1 - \frac{\phi_1^{\,2}}{2!} + \frac{\phi_1^{\,4}}{4!} - \ldots)\right\} = \frac{L^3}{6LR} - R\frac{\phi_1^{\,2}}{2}$$

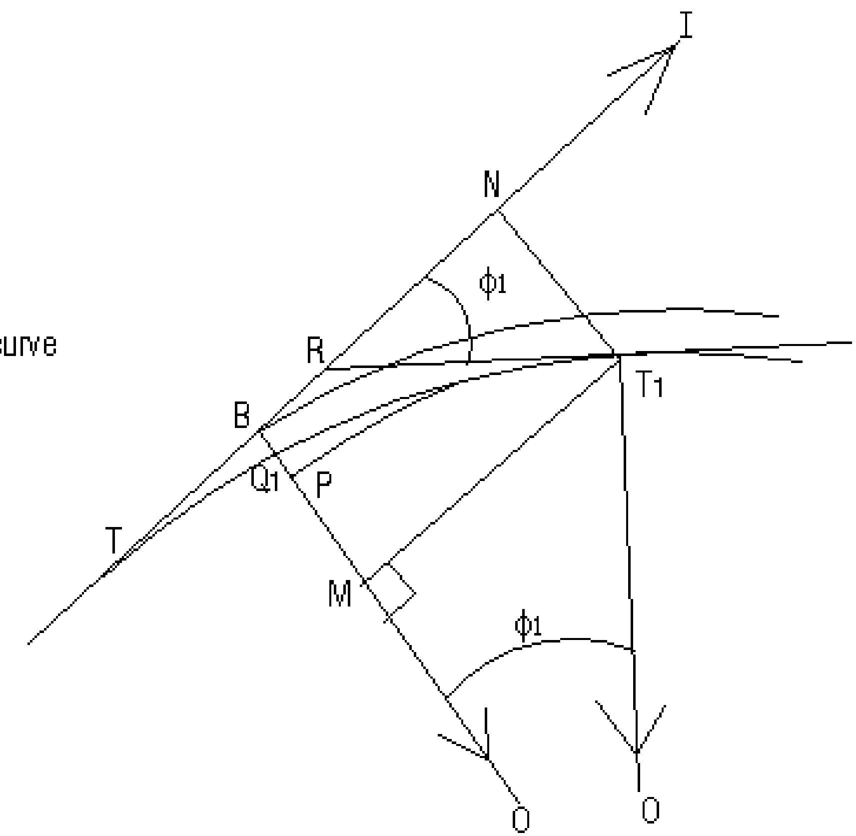
$$= \frac{L^3}{6LR} - \frac{R}{2}\left(\frac{L^2}{2LR}\right)^2$$

$$= \frac{L^2}{24R}$$

Also $Q_1T_1 \approx PT_1 \approx R\phi_1 = \frac{RL^2}{2LR} = \frac{L}{2}$



a)



b)

Setting out of transition curve

Step 1

Calculate the shift S from the expression $S = \frac{L^2}{24R}$

Step 2

Calculate IB = $(R+S)\tan\frac{\theta}{2}$

Step 3

BT = L/2 Then IT = $(R+S)\tan\frac{\theta}{2} + \frac{L}{2}$

Step 4

Either calculate offset from

$x = \frac{l^3}{6K}$ or $x = \frac{y^3}{6K}$

OR

Calculate the deflection angles δ from particular distances l from T using the fact that

$$\delta = \phi/3 = \frac{l^2}{6RL}\text{ rad}$$

$$= \frac{1800}{\pi}\frac{l^2}{RL}\text{ min} \quad \delta_{max} = L/6R \text{ rad OR} = \frac{1800L}{\pi R}$$

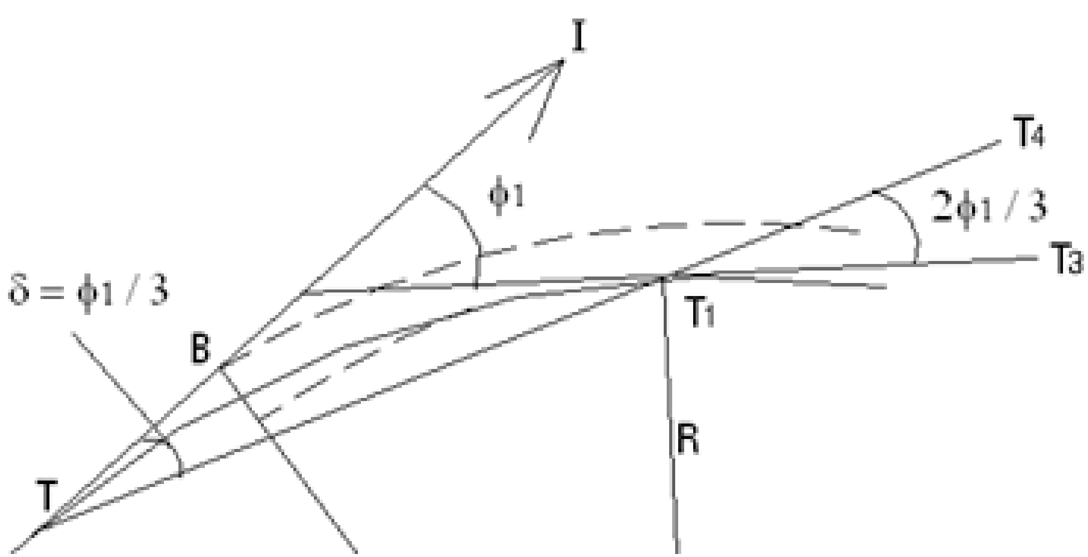
Step 5
Setting out the circular curve
Calculate the deflection angles for the circular curve from

$$\delta_{Circular} = 1718.9\frac{c}{R}\min$$

Where c is the chord length for the circular arc
The angle subtended by the circular arc T_1T_2 at the center of that arc is $(\theta - 2\phi_1)$

Length of the arc T_1T_2 = $R(\theta - 2\phi_1)\frac{\pi}{180}$

Step 6
Set up the theodolite at T_1 and sight back on T; then , transit the telescope and locate T_1T_3 by setting off an angle 2/3 ϕ_1. Set out the circular curve by the deflection angle method, from this tangent.



Example 2

- Two straights *AB and BC intersect at chainage 1530.685 m, the total deflection* angle being 33° 08′. It is proposed to insert a circular curve of 1000 m radius and the transition curves for a rate of change of radial acceleration of 0.3 m/s3, and a velocity of 108 km/h. Determine setting out data using theodolite and tape for the transition curve at 20 m intervals and the circular curve at 50 m intervals.

Solution

- α = 0.3 m/cub sec
- V = 108 km/hr = 30 m/s
- Δ = 33°08’ → Δ/2 = 16° 34’
- Radius of circular curve = 1000 m
- Chainage of I = 1530.686 m
- Peg interval for transition curve = 20 m
- Peg interval for circular curve = 50 m

$$\alpha = \frac{V^3}{LR} \rightarrow L = \frac{V^3}{\alpha R} = \frac{30^3}{0.3x1000} = 90.0m$$

$$Shift, S = \frac{L^2}{24R} = \frac{90^2}{24x1000} = 0.338\ m$$

- Tangent length IT= (R+S)tan(Δ/2) + (L/2)
- = 342.580
- $\phi 1$= L/2R rad = 90/(2x1000) x(180/π)= 2°34'42''
- Angle subtended by T1- T2 = θ= Δ - 2$\phi 1$ = 27°58'36''
- Length of the curve, l = (πRθ/180) = 488.285 m
- Chainage of T = Chainage of I – IT = 1188.105 m
- = 59 + 8.105 chains for peg interval 20 m
- Chainage of T1 = Chainage of T + L = 1278,105 m
- = 25 + 28.105 chains for peg interval 50 m
- Chainage of T2 = Chainage of T1 + l = 1766.390 m
- = 35 + 16.390 chains for peg interval 50 m
- = 88 + 6.390 chains for peg interval 20 m
- Chainage of U = Chainage of T2 + L = 1856.390 m
- = 92 + 16.390 chains for peg interval 20 m

- Length of the first sub chord for transition
- (59 + 20) – (59 + 8.105) = 11.895 m
- Length of first sub chord for circular curve
- (25 + 50) – (25 + 28.105) = 21.895 m
- Length of last sub chord for circular curve
- (35 + 16.390) – (35 + 0) = 16.390 m

Deflection angle for transition curve

$$\delta = \frac{1800 l^2}{\pi R L} = 0.006366\, l^2 \ \text{min}$$

- ## Deflection angle for circular curve

$$\delta = 1718.9 \frac{c}{R} \ \text{min}$$

Setting out data for 1st transition curve

Point	Chainage (m)	Chord length (m)	l (m)	Deflection angle (δ)
0 (**T**)	1188.105	0.0	0.0	0.0
1	1200.000	11.895	11.895	0°00′54″
2	1220.000	20	31.895	0°06′29″
3	1240.000	20	51.895	0°17′09″
4	1260.000	20	71.895	0°32′54″
5 (**T1**)	1278.105	18.105	90.000	0°51′34″

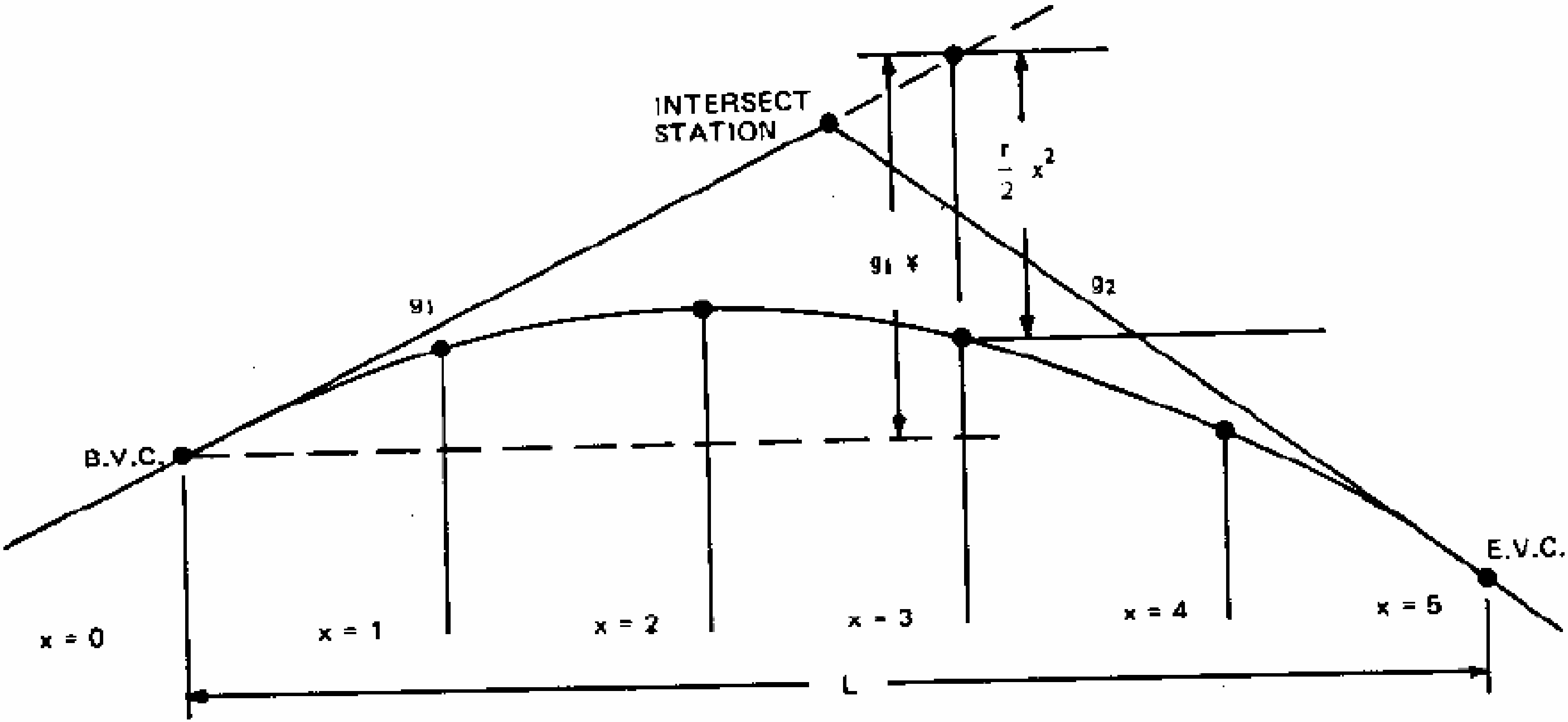
Setting out data for circular curve

Point	Chainage (m)	Chord length (m)	Tangential angle (δ)	Deflection angle (Δ)
5 **T1**	1278.105	0.0	0.0	0.0
6	1300.000	21.895	0°37′38.1″	0°37′38″
7	1350.000	50	1°25′56.7″	2°03′35″
8	1400.000	50	1°25′56.7″	3°29′32″
9	1450.000	50	1°25′56.7″	4°55′28″
10	1500.000	50	1°25′56.7″	6°21′25″
11	1550.000	50	1°25′56.7″	7°47′22″
12	1600.000	50	1°25′56.7″	9°13′18″
13	1650.000	50	1°25′56.7″	10°39′15″
14	1700.000	50	1°25′56.7″	12°05′12″
15	1750.766	50	1°25′56.7″	13°31′08″
16 **T2**	1766.390	16.390	0°28′10.4″	13°59′19″

Setting out data for 2nd Transition curve

Point	Chainage (m)	Chord length (m)	*l* (m)	Deflection angle (δ)
16 **T2**	1766.390	13.610	90.000	0°51′34″
17	1780.000	20.000	76.390	0°37′09″
18	1800.000	20.000	56.390	0°20′15″
19	1820.000	20.000	36.390	0°08′26″
20	1840.000	16.390	16.390	0°01′43″
21 **U**	1856.390	0.0	0.0	0.0

Vertical Curve



Vertical curves (VC) are used to connect intersecting gradients in the vertical plane.

They should be of sufficiently large curvature to provide comfort to the driver, that is, they should have a low ‘rate of change of grade’.

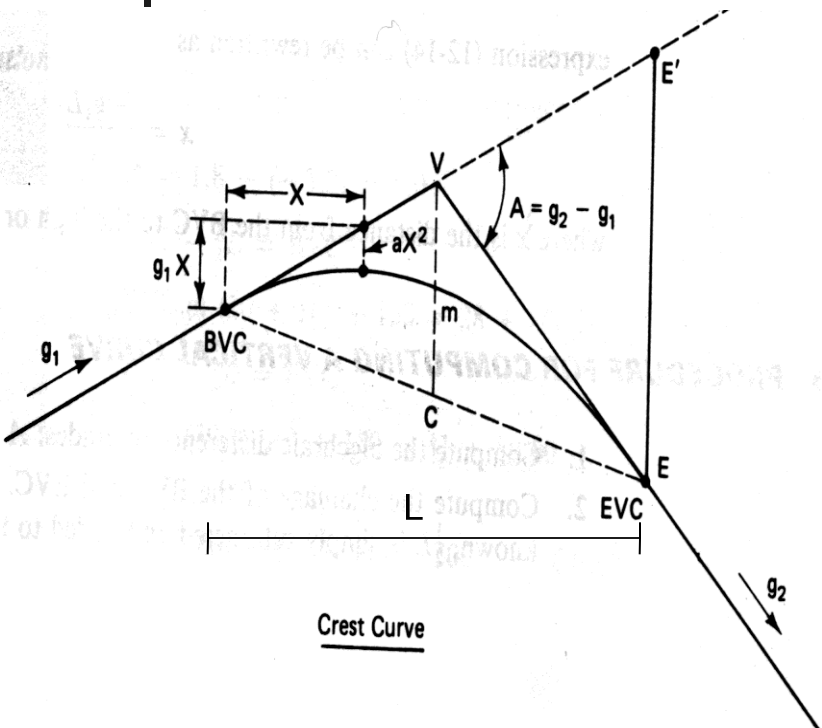
Vertical curve formula

$$r = \frac{g_2 - g_1}{L} \qquad y = \frac{rx^2}{2} + g_1 x + \text{ elevation of BVC}$$

Where

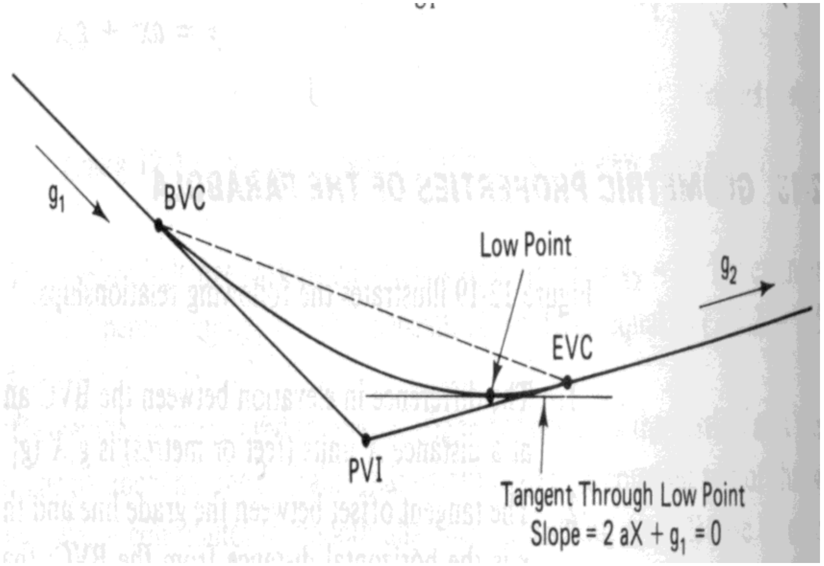
r	=	rate of change of grade per section (%)
g1	=	starting grade (%)
g2	=	ending grade (%)
L	=	length of curve (horizontal distance m)
y	=	elevation of a point on the curve
x	=	distance in stations from the BVC (meters/100)
BVC	=	beginning of the vertical curve
EVC	=	end of the vertical curve

Properties of a Vertical Curve



- The difference in elevation between the BVC and a point on the g_1 grade line at a distance X units (feet or meters) is g_1X (g_1 is expressed as a decimal).
  The tangent offset between the grade line and the curve is given by ax^2, where x is the horizontal distance from the BVC; (that is, tangent offsets are proportional to the squares of the horizontal distances).
  The elevation of the curve at distance X from the BVC is given (on a crest curve) by:
  $$BVC + g_1x - ax^2$$
  (the signs would be reversed in a sag curve).
  The grade lines (g1 and g2) intersect midway between the BVC and the EVC. That is, BVC to V = 1/2L = V to EVC.
- Offsets from the two grade lines are symmetrical with respect to the PVI.
- The curve lies midway between the PVI and the midpoint of the chord; that is, Cm = mV.

Computation of Low or High Point on Curve



The locations of curve high and low points are important for drainage and bridge considerations. For example, on curbed streets catch basins must be installed precisely at the drainage low point.

It was noted earlier that the slope was given by the expression

Slope = 2ax + g1

The figure above shows a sag vertical curve with a tangent drawn through the low point; it is obvious that the tangent line is horizontal with a slope of zero; that is,

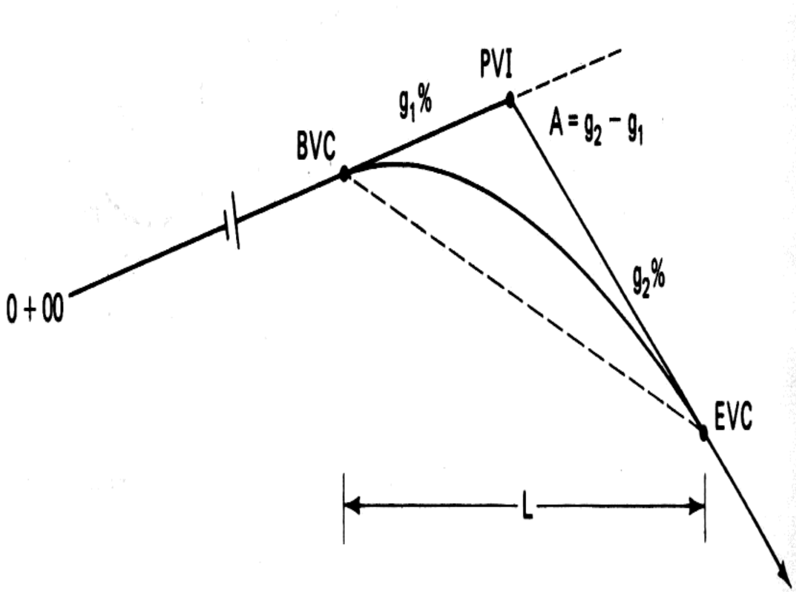
$$2ax + g1 = 0$$

Since 2a = A/L

$$x = -g1L/A$$

where x is the distance from the BVC to the high or low point.

Procedure for Computing a Vertical Curve



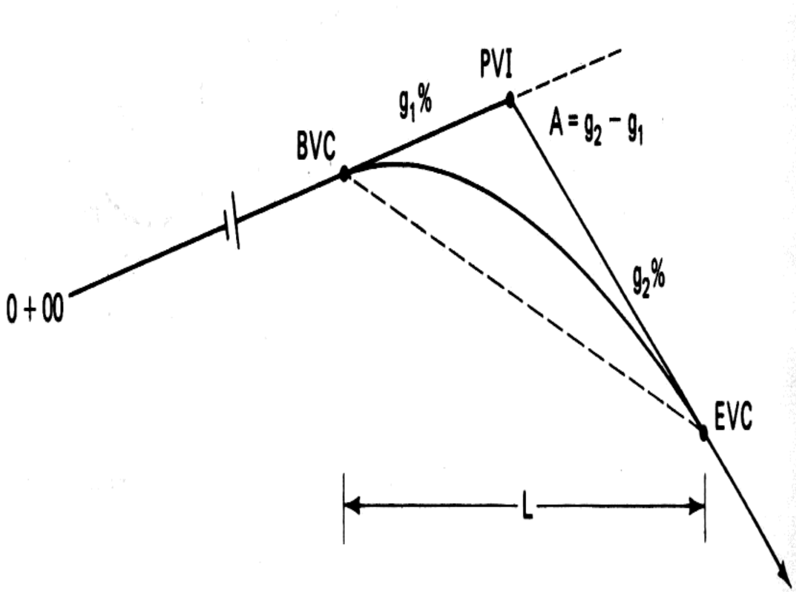
1. Compute the algebraic difference in grades: A
2. Compute the chainage of the BVC and EVC. If the chainage of the PVI is known,
   1/2L is simply subtracted and added to the PVI chainage.
3. Compute the distance from the BVC to the high or low point (if applicable):

   x = -g1L/A

   and determine the station of the high/low point.
4. Compute the tangent grade line elevation of the BVC and the EVC.
5. Compute the tangent grade line elevation for each required station.
6. Compute the midpoint of chord elevation

   {elevation of BVC + elevation of EVC}/2

Procedure for Computing a Vertical Curve



7. Compute the tangent offset (d) at the PVI (i.e., distance Vm):

d = {elevation of PVI - elevation of midpoint of chord}/2

8. Compute the tangent offset for each individual station.

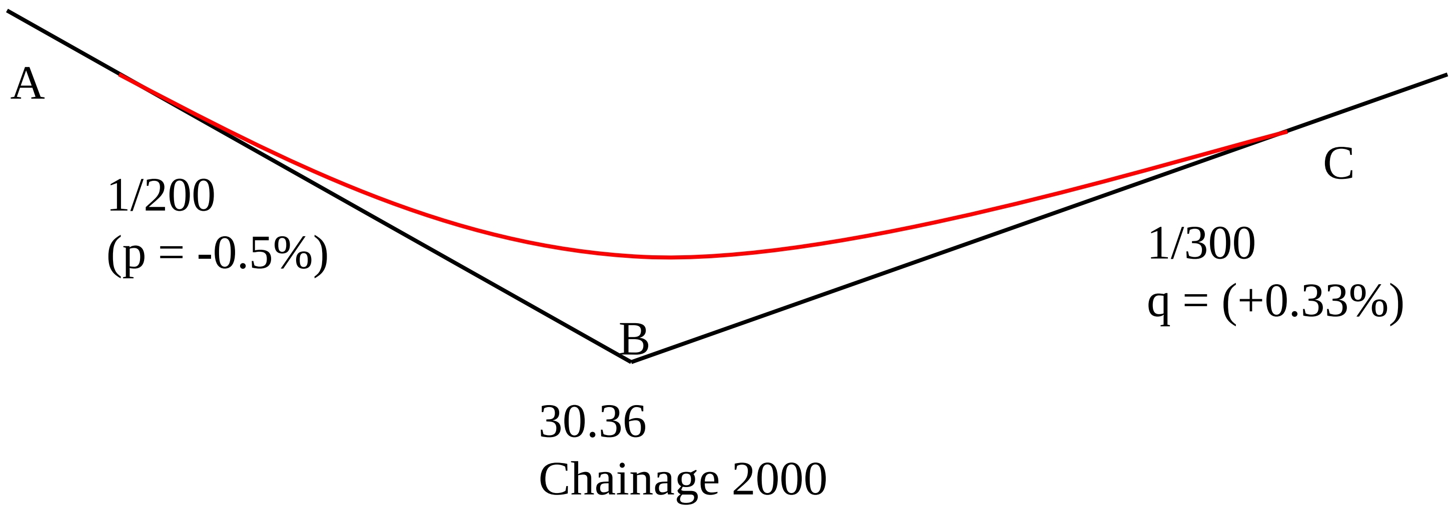
Tangent offset = $\{x/(L/2)\}^2 d$

where x is the distance from the BVC or EVC (whichever is closer) to the required station.

9. Compute the elevation on the curve at each required station by combining the tangent offsets with the appropriate tangent grade line elevations. Add for sag curves and subtract for crest curves.

Example 3

- A vertical curve **120 m** long of the parabola type is to join a falling gradient of **1 in 200** to a rising gradient of **1 in 300**. If the level of the intersection of the two gradients is **30.36 m.** Compute the levels at **15-m** intervals along the curve.



Solution

Chainage of A = Chainage of B – L/2

= 2000 – 60 = 1940 m

Chainage of C = Chainage of B + L/2

= 2000 + 60 = 2060 m

Elevation of A = Elev. B + (pL/200)

= 30.36 + (0.5*120/200) =30.660

Elevation of C = Elev. B + (qL/200)

= 30.36 + (0.33*120/200) = 30.56 m

$$r = \frac{g_2 - g_1}{L} \qquad y = \frac{rx^2}{2} + g_1 x + \text{ elevation of BVC}$$

Station	Chainage (m)	x(m)	y_i
A (BVC)	1940	0	30.660
1	1955	15	30.593
2	1970	30	30.541
3	1985	45	30.505
4 (above B)	2000	60	30.485
5	2015	75	30.479
6	2030	90	30.490
7	2045	105	30.516
C (EVC)	2060	120	30.558≈ 30.56

Example 4

Given that L = 300 m, g_1= -3.2%, g_2= +1.8%, PVI at 30 + 030, and elevation = 465.92. Determine the location and elevation of the low point and elevations on the curve at 50m interval starting from BVC. 1 st =100