Method for Calculating Schedule Delay Considering Lost Productivity

Hyun-Soo Lee, M.ASCE[1]; Han-Guk Ryu[2]; Jung-Ho Yu[3]; and Jae-Jun Kim, M.ASCE[4]

Abstract: A delay claim often occurs when a difference between the actual completion date and the contract completion date exists. The duration of a delay is an essential piece of information required for determining the cause of a delay. However, it is difficult to analyze a delay claim due to the fact that numerous factors that cause this delay, thereby making it a very complex issue. One of such factors is the lost productivity or loss of productivity. Despite the fact that it is one of the major causes of delay, there have been only a few studies that focus on converting lost productivity into delay duration carried out to date. Claims for productivity losses are generally the result of tension between the contractor and the owner. This tension arises due to the great difficulty involved in quantifying disruption effects. Thus, to calculate accurately the delay duration, a logical method for analyzing schedule delay caused by lost productivity is necessary. Therefore, in this study, we propose a method for analyzing construction schedule delay where this lost productivity is taken into consideration. This methodology was implemented on a case project to ascertain its practicability, and to decide whether it can be utilized in the case of a delay claim related to lost productivity. The significance of this paper is twofold. One is the method to convert the lost productivity into the delay duration, which can be applied to reasonable delay claim settlement. The other is the process to analyze the construction schedule delay considering lost productivity.

Introduction

The duration of a construction project is an important factor to set forth when entering into a construction agreement. If a contractor works within a planned parameter, he/she should be able to finish the construction project in a timely manner. However, compared to other industries, it is difficult to complete a construction project in which many construction trades participate and numerous unknown variables exist. When such difficulties arise, construction schedules are delayed, and consequently delay claims occur.

Delays in construction may be caused by the owner, the contractor, acts of God, or a third party. They may occur early or late in the job, alone, or with other delays. In whatever cases, negotiating a fair and timely damage settlement is beneficial to all par-

ties (Bubshait and Cunningham 1998). Thus, the ascertainment of the period of project delay serves as basic information for the apportionment of responsibility, which may be a highly complex operation in cases with concurrent causes (Shi et al. 2001). Assigning responsibility for project delays is critical to the allocation of responsibility for time-related costs (Al-Saggaf 1998). In this respect, when a delay claim occurs, it is very important to assign responsibility and magnitude to the delay. However, many sources and causes of construction delays exist, and it is often difficult to analyze the ultimate liability in delay claims (Kraiem and Diekmann 1987). Lost productivity or loss of productivity is one of the most important causes of delay among the various causes of construction delays. Thus calculating the effect of lost productivity on delays is an intricate issue.

Although several studies have been directed toward the issue of analyzing delay and lost productivity (Kallo 1996; Al-Saggaf 1998; Bubshait and Cunningham 1998; Finke 1998; 1999; Kartam 1999; Reichard and Norwood 2001), but they have been mostly concerned about converting lost productivity into cost. There have been only a few studies conducted on concrete methods for converting lost productivity into delay duration.

The purpose of this study is to propose a practical method for converting lost productivity into delay duration. Several concepts pertinent to the lost productivity and its consequent delay analysis are introduced. The method is presented and implemented to a case project to ascertain its practicability.

Among the various factors that cause delays, this study focuses on the factors that cause loss of productivity. There are a few kinds of productivity, e.g., labor productivity, equipment productivity, etc. This study has focused on the labor productivity because labor productivity representatively shows all kinds of productivity. The effect of lost productivity on delay duration will

[1]Associate Professor, Dept. of Architecture, Seoul National Univ., Seoul 151-742, Korea (corresponding author). E-mail: hyunslee@snu.ac.kr

[2]PhD Candidate, Dept. of Architecture, Seoul National Univ., Seoul 151-742, Korea. E-mail: hglyu@cricmail.net

[3]PhD, Full–time Lecturer, Dept. of Architectural Engineering, Kwangwoon Univ., Seoul 139-701, Korea. E-mail: myazure@cricmail.net; formerly, Dept. of Architecture, Seoul National Univ., Seoul 151-742, Korea.

[4]Associate Professor, College of Architecture, Hanyang Univ., Seoul 133-791, Korea. E-mail: jjkim@hanyang.ac.kr
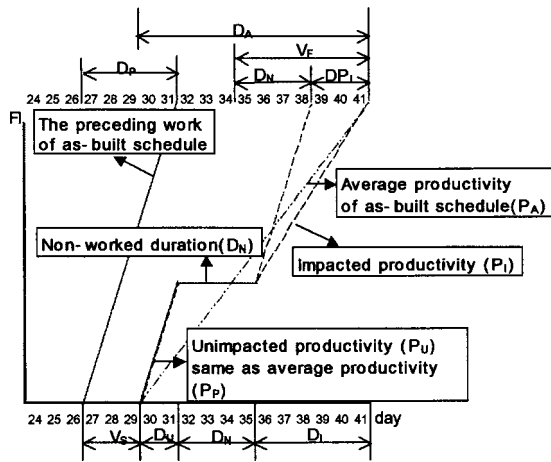
Fig. 1. Lost productivity

be calculated against critical works, which is represented as a line in the LOB form of a project schedule. The LOB form of a project schedule is practical in understanding the changes of productivity visually.

Schedule Delay Calculation Method Considering Lost Productivity

Review of Schedule Delay and Lost Productivity Calculation Methods

Many studies or methodologies directed at analyzing delay and lost productivity have been reported. The current status of delay calculating studies related to lost productivity can be summarized in three cases as follows. (1) It is assumed that lost productivity claim is a different type of impact claim than a delay claim. Claimants frequently confuse these two types of claims because the impacts of both occur simultaneously (Bramble et al. 1990). (2) Lost productivity claims are limited to studies of converting lost productivity into cost such as total labor cost method, measured mile analysis, expert estimate, industry factors, historical productivity data, etc. (Kallo 1996; Reichard and Norwood 2001). For example, the measured mile analysis is conducted by comparing productivity during an unimpacted period of time with the productivity during an impacted period of time. The claim amount is the difference between the two productivity measures multiplied by the costs associated with the lost productivity. Damages by lost productivity are calculated by subtracting the estimated costs from the actual costs incurred. Because the actual costs include the increased labor costs from the lost productivity, the cumulative effect of changes is addressed in the total cost overrun. (3) Delay causes are conceived as activities in a project schedule such as a method of "what-if" evaluation or "but-for" schedule (Al-Saggaf 1998; Bubshait and Cunningham 1998; Finke 1998, 1999, Kartam 1999). "But-for" schedule results from "pulling out" all owner delays that affected the as-built critical path. The amount of compensable delay is the difference in time between the actual completion date on the as-built schedule and the completion date on the "but-for" schedule.

As a result, a study concerning the methods of calculating schedule delay of lost productivity is not sufficient.

Basic Concepts for Considering Lost Productivity

Productivity may be defined as the quantity of work produced per man hour, equipment hour, or crew hour (Finke 1998). As shown in Fig. 1, the lost productivity or loss of productivity is represented by the productivity impacted by unexpected factors or impact factors. For example, a curtain wall crew consisting of five workers installing 34.65 m^2/h has a productivity rate of 6.93 $m^2/man\ h$ under ideal conditions without any other impact factor. However, if the work is impacted by another impact factor such as unexpected adverse weather, the work productivity will decline, and unexpected extra time will be required for the impacted work productivity to be unimpacted work productivity.

To introduce the basic concepts regarding delay and productivity, it is necessary to define the terminologies. Planned work duration (D_P) is the work duration with the planned work productivity (P_P) of as-planned schedule. Actual work duration (D_A) is the work duration with the average productivity of as-built schedule (P_A). As-built schedule is widely accepted for determining the impact of project delays. It is basically a comparison of what was planned to what actually occurred at the job site (Bramble and Callahan 1992). Start time variance (V_S) is the difference between the actual start time of work and the finish time of the preceding work of the as-built schedule. Finish time variance (V_F) as the difference between D_A and D_P is composed of D_N and D_P.

When lost productivity factors occur, the lost productivity (P_L) of work can be calculated using Eq. (1), where P_U and P_I denote the unimpacted productivity of the activity and the impacted productivity of a given activity, respectively. If the work is planned properly, then the P_U will be the same as the P_P

$$LP = P_u - P_i \qquad (1)$$

The lost productivity entails lost work, the amount of which could be completed without the lost productivity. The quantity of lost work due to the lost productivity (QP_L) can be calculated by the following equation, where L_I denotes the daily average labor during the impacted work duration (D_I)

$$QP_L = (P_U - P_I) \times L_I \times D_I \qquad (2)$$

The lost productivity also entails lost duration DP_L, which as an opportunity duration could be worked as much as QP_L with the L_I and P_U. The DP_L can be calculated by the following equation. All the variables in the equations hereafter are summarized in the Notation

$$DP_L = \frac{QP_L}{L_I P_U} = \frac{(P_U - P_I) \times L_I \times D_I}{L_I P_U} = D_I \left(1 - \frac{P_I}{P_U}\right) \qquad (3)$$

When the contractor's claim includes the "ripple effect," that is a request for compensation for activities whose productivity suffered indirectly due to the owner's actions, the situation is further complicated. The owner is usually reluctant to accept the existence of this ripple effect because it is not readily seen and because it may be used to cover up the inefficiency caused by the contractor's mismanagement (Abdul-Malak et al. 2002). Therefore, the characteristics of the impact factor must be known to fairly assign the DP_L.

Impact factors affecting the DP_L are categorized according to independence and impact on the next impact factor. An independent factor means that an impact factor does not have any effect on the duration of subsequent works (see Scenario 1 of Fig. 2). An impacting factor means that an impact factor adversely affects
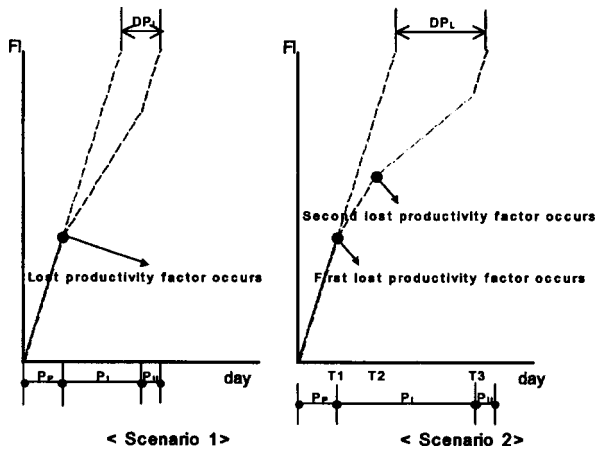
Fig. 2. Lost productivity factors' characteristics

the duration of subsequent works when the next impact factor occurs even before the previous impact factor's ripple effect has finished (see Scenario 2 of Figs. 2 and 3).

As shown in Fig. 3, if the $P_{L(i-1)}(F_{j-1})$ is impacted by F_j, the lost productivity will consist of the $P_{L(i-1)}(F_{j-1})\alpha$, the new $P_{Li}(F_j)$, and the intersection of $P_{L(i-1)}(F_j)$ and $P_{Li}(F_j)$, or $P_{L(i-1)}(F_{j-1})\beta$, where $P_{Li}(F_j)$ denotes the portion of ith productivity due to the ith factor. Usually the $P_{L(i-1)}(F_j)$ is not equal to $P_{L(i-1)}(F_{j-1})$ because $P_{L(i-1)}(F_{j-1})$ would naturally change through a mode of learning curve as times passes. Thus the $P_{L(i-1)}(F_j)$, which is $P_{L(i-1)}(F_{j-1})$ impacted by F_j, can be written as $P_{L(i-1)}(F_{j-1})\alpha$. The α is defined as the self ratio, the extent which $P_{L(i-1)}(F_{j-1})$ would naturally change from the initial point of F_{j-1} to the initial point of F_j without the F_j. The β is defined to explain the intersection as the impacting ratio, the extent which $P_{L(i-1)}(F_{j-1})$ affects $P_{Li}(F_j)$.

Process for Schedule Delay Analysis

A common method for calculating a schedule delay is by comparing the as-planned schedule and the as-built schedule prepared by the critical path method (CPM) (Kraiem and Diekmann 1987; Bubshait and Cunningham 1998). The critical works in the as-built schedule ultimately impact the delay duration. The critical works can be classified into two types. One type is the work impacting on the project completion date and the other type is the work that has no impact on the compeltion date. It is therefore necessary to analyze which works influence the completion date and to determine the degree of their impact. In other words, it is necessary to examine what the cause is, to determine who is ac-
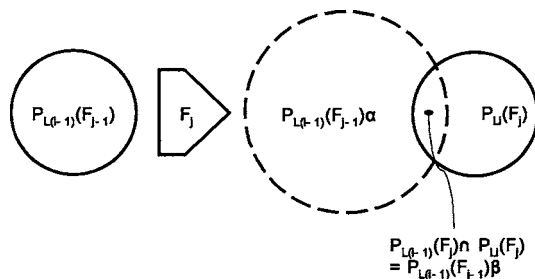

Fig. 3. Change of $P_{L(i-1)}(F_j)$ impacted by F_j
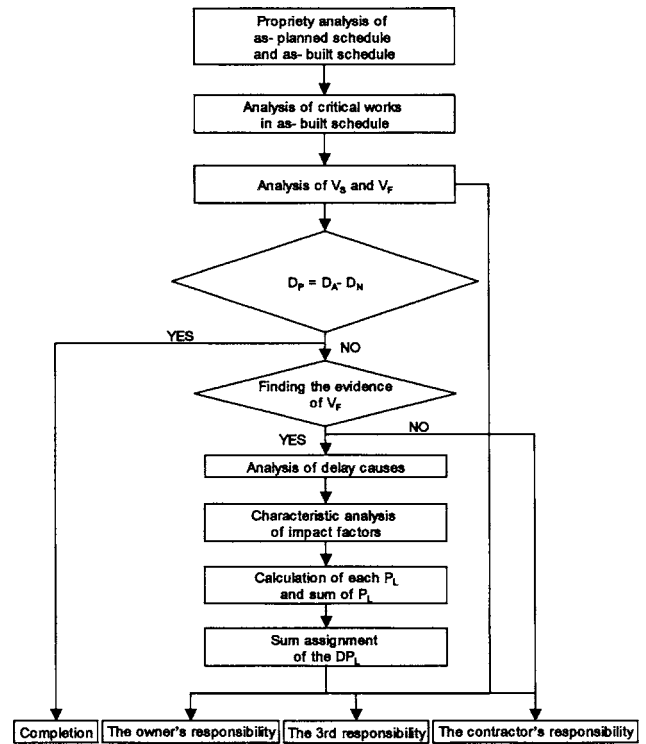

Fig. 4. Calculation process for schedule delay

countable for the work delay, and to calculate how much is impacted. The analysis of the delay duration can be carried out in the procedure depicted in Fig. 4.

1. Propriety analysis of as-planned schedule and as-built schedule. In this phase, propriety is analyzed to verify the reasonableness of the as-planned schedule and D_P considering the average labor productivity, appropriate resource allocation, labor usability, weather, appropriate materials, machine supply, and so forth. After verifying that the as-planned schedule and D_P are reasonable, the as-built schedule and D_A are examined based on the evidence (e.g., detailed work schedules, updated schedule, daily reports, correspondence, delay description, etc.). The as-built schedule reflects the actual progression of events that occurs during the execution of the project. As a result, the as-built schedule should be made by carefully studying the project reports and documents (Kraiem and Diekmann 1987) and based on this evidence, it can be justified whether the as-built schedule and D_A were appropriate.

2. Analysis of critical works in as-built schedule. A construction project normally proceeds at a pace that is usually different from the as-planned schedule. The criticality of individual activities in a CPM network changes due to delays and accelerations in construction (Arditi and Robinson 1995; Shi et al. 2001). A project delay is the accumulated effect of delays in individual activities (Shi et al. 2001). Because the delay of critical work in the as-built schedule ultimately affects the project completion date, delay causes within the critical works must be recorded and analyzed on the as-built schedule. It is important to know which work among the critical works influences the project delay and the extent of their influence on the as-built schedule. The extent of the impact on the project delay should be analyzed comprehensively by considering the project characteristics, field environment, the

impacted work characteristic, the completed work extent, and so forth.

3. Analysis of V_S and V_F. Through a comparison of the critical works in the as-built schedule and the works in the as-planned schedule, V_S and V_F can be calculated. After calculating V_S and V_F, the causes or impact factors of the V_S and V_F must be found and analyzed by investigating the evidence.

4. Finding the evidence of V_F. Once the claimant has been convinced that the construction could be completed if the works were carried out according to the as-planned schedule and that the as-planned schedule is sufficient to meet the work sequencing restraints, then he/she will question why the works were actually delayed. A construction claim is an assertion of and a demand for compensation by way of evidence produced and arguments advanced by a party in support of its case (Kululanga et al. 2001). So, the D_N of V_F can be found and assigned to the owner, the project contractor, or a third party according to their responsibility. If a D_A excluding the D_N is equal to the D_P, the delay calculation is complete. In this case, the actual productivity of the work is consistent with the P_P. However, if the D_A is not equal to the D_P, the calculating process proceeds to the next phase. If the project contractor has no evidence to prove the P_L, the calculating process should also be stopped. In other words, if ε is not verified, then ϵ is the contractor's responsibility [See Eq. (6)].

5. Analysis of delay causes. After the V_F evidence is confirmed, the impact factors can be analyzed by comparing the D_A with the D_P. The contractor can finish planned work quantity (Q_P) with the unimpacted daily labors (L_U) and the P_U within the duration (D_C). This can be calculated by the following equation, where Q_U denotes the quantities worked in the normal and realistic work conditions of an unimpacted work duration (D_U), and Q_I denotes the quantities worked in the impacted work duration (D_I)

$$D_C = \frac{Q_P}{L_U P_U} = \frac{Q_U + Q_I}{L_U P_U} \tag{4}$$

Time is of the essence in a construction contract. Typically, a time period is defined as the contract duration. The contractor is obliged under the contract to achieve substantial completion within the specified period (Shi et al. 2001).
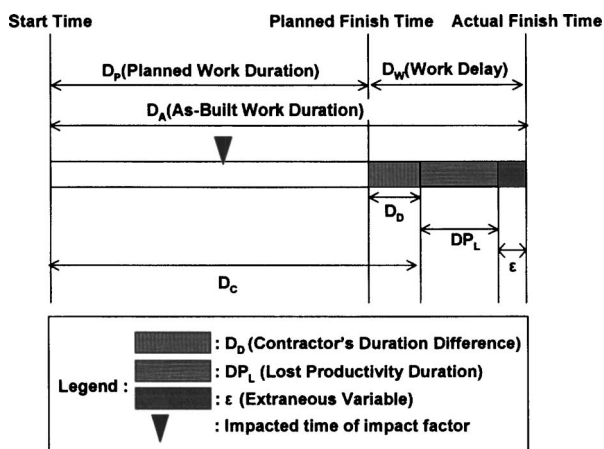

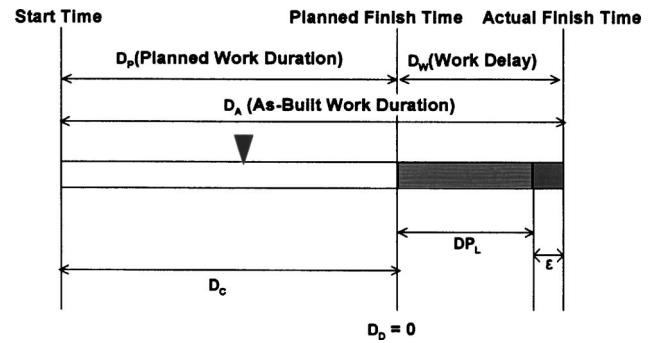Fig. 5. D_W including D_D and DP$_L$

Fig. 6. D_W including only DP$_L$

Therefore, contractor's difference (D_D) between D_C and D_P is a mistake made by the contractor as a result of a miscalculation of the work duration in planning. D_D can be calculated by the following equation:

$$D_D = D_C - D_P = \frac{Q_P}{L_U P_U} - D_P \tag{5}$$

Work delay (D_W) consists of D_D, DP$_L$, and ε as shown in the following equation, where D_D and DP$_L$ are independent variables and ε is an extraneous variable that accounts for any delays other than D_C and DP$_L$

$$D_W = D_D + DP_L + \varepsilon \tag{6}$$

D_W are classified into three cases: D_W including D_D and DP$_L$, D_W including only DP$_L$, and D_W including DP$_L$ less D_P.

1. D_W including D_D and DP$_L$. D_W includes all D_D and DP$_L$ during the impacted work duration as shown in Fig. 5. When D_D is greater than zero, as shown in the following equation, the delay is caused by the contractor's mistake such as the allocation of lower labor and productivities than the as-planned schedule. In this case, the contractor could not be compensated or, rather, should compensate the owner for the liquidated damage

$$D_D = D_C - D_P > 0 \tag{7}$$

2. D_W including only DP$_L$. The D_W includes only DP$_L$ in the case where the D_A of the as-built schedule is equal to the D_P of the as-planned schedule as shown in Fig. 6. When D_D is equal to zero, as shown in the following equation, the delay is caused only by P_L. As P_P is equal to P_A, the contractor shall not be held accountable for the delay
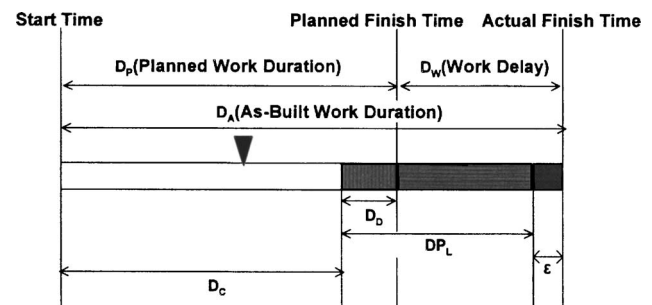
$$D_D = D_C - D_P = 0 \tag{8}$$

Fig. 7. D_W including DP$_L$ less D_D

		Time	
		Same	Different
S P A C E	Same	Integrated Impact	Combined Impact (Integrated Impact + Single Impact)
	Different	Combined Impact (Integrated Impact + Single Impact)	Single Impact

Fig. 8. Integrated, single, and combined impact

of the gondola. In this case, the lost productivities caused by the change order and the machine malfunction are different according to single impact.

The combined impact is the combination of integrated impact and single impact. For example, if 1 week later after the design change and the material items change of the main entrance occurred at the same time and at the same space (integrated impact factors), and there was also a malfunction in the gondola (single impact factor), the lost productivities of the integrated impact factors and single impact factor should both be calculated.

Sum of P_L(SP_L)

After analyzing the characteristics of impact factors by using the time-space 2×2 matrix, the sum of $P_L(SP_L)$ can be quantified according to the impact factor's characteristics. P_{Li} changes according to the characteristics of the impact factors from F_j to F_n. The SP_L from F_i to F_n can be written as the following equation:

$$SP_{L1\_}(F_1, F_2, \cdots, F_n) = \sum_{i=1}^{n} P_{L1}(F_i)$$

$$SP_{L2\_}(F_2, F_3, \cdots, F_n) = \sum_{i=2}^{n} P_{L2}(F_i)$$

$$\cdots \quad \cdots$$

$$SP_{Lk\_}(F_k, F_{k+1}, \cdots, F_n) = \sum_{i=k}^{n} P_{Lk}(F_i)$$

$$\cdots \quad \cdots$$

$$SP_{L(n-1)}(F_{n-1}, F_n) = \sum_{i=n-1}^{n} P_{L(n-1)}(F_i) \qquad (10)$$

Sum Assignment of DP_L

It is essential that distinctions of excusable/nonexcusable delays are made when analyzing delays. Excusable/compensable delays are due to some actions or omission of the owner, for example, lack of site access, or late arrival of owner-furnished material or

3. D_W including DP_L less D_D. When D_D is less than zero, as shown in the following equation, D_W is calculated by only DP_L. If it were not for the DP_L, the actual work could have been finished earlier than D_P, as demonstrated in Fig. 7. If the D_C is shorter than the D_P, the work might be performed by acceleration or the use of more labor than was originally planned, but the effort cannot be recognized as a compensable delay

$$D_D = D_C - D_P < 0 \qquad (9)$$

Assignment of Lost Productivity Duration

Calculation of Lost Productivity

After DP_L is verified, the DP_L must be assigned to the source by analyzing the impact factors. The responsibility for delays can fall on the owner, the contractor, or third parties. Compensation or liquidated damage can be determined in proportion to the extent of the impact: self ratio (α) and impacting ratio (β). Impact factors with α and impact factors with β can affect the productivity as a manner of integrated impact, single impact, and combined impact according to the time-space 2×2 matrix, as shown in Fig. 8.

When given impact factors affect the work productivity at the same time and at the same space, the extent of the integrated impact represents the extent to which proceeding impact factors equally affect the productivity of subsequent work. Integrated impact factor is analogous to the characteristics of concurrent delays. Concurrent delays are used to describe two or more delays that occur at the same time by one or more of the parties (Richter 1983; Kraiem and Diekmann 1987; Bramble and Callahan 1992; Rubin et al. 1992). For example, when the curtain wall design change of the main entrance and the material items' change of the curtain wall of the main entrance occurred by the owner at the same time, this would cause a delay in the work. In this case, the lost productivities caused by the design change and the material items' change of the main entrance occurred integrally according to integrated impact factors.

When given impact factors affect work productivity at a different time and at a different space, the extent of the single impact represents the extent which proceeding impact factors have different effects on the productivity of the subsequent work. For example, when a change order concerning the curtain wall work on the fifth floor is made, this would cause a delay in the work. Then, several days later after the change orders, the curtain wall work on the seventh floor was delayed because of a malfunction

Fig. 9. Master plan of project

NAME	Asplanned dur	Asbuilt dur	STV	FTV	ST_asplanned	FT_asplanned	ST_asbuilt	FT_asbuilt
Prj.Dur.	930	955	0	25	1997-04-15	1999-10-31	1997-04-15	1999-11-25
ACT1	38	38	0	0	1997-04-15	1997-05-22	1997-04-15	1997-05-22
ACT2	20	20	0	0	1997-05-23	1997-06-11	1997-05-23	1997-06-11
ACT3	145	145	0	0	1997-06-12	1997-11-03	1997-06-12	1997-11-03
ACT4	65	65	19	0	1997-11-04	1998-01-07	1997-11-23	1998-01-26
ACT5	76	64	19	-12	1998-01-08	1998-03-24	1998-01-27	1998-03-31
ACT6	54	54	19	0	1998-03-25	1998-05-17	1998-04-13	1998-06-05
ACT7	15	30	19	15	1998-05-18	1998-06-01	1998-06-06	1998-07-05
ACT8	46	46	19	0	1998-06-02	1998-07-06	1998-06-17	1998-08-20
ACT9	26	26	19	0	1998-08-02	1998-08-27	1998-08-21	1998-09-15
ACT10	34	34	19	0	1998-08-28	1998-09-30	1998-09-16	1998-10-19
ACT11	60	70	19	10	1998-10-01	1998-11-29	1998-10-20	1998-12-28
ACT12	69	62	29	-7	1998-11-30	1999-02-06	1998-12-29	1999-02-28
ACT13	83	83	22	0	1999-02-07	1999-04-30	1999-03-01	1999-05-22
ACT14	65	65	22	0	1999-05-01	1999-07-04	1999-05-23	1999-07-26
ACT15	76	73	22	-3	1999-07-05	1999-09-18	1999-07-28	1999-10-07
ACT16	43	45	19	2	1999-09-19	1999-10-31	1999-10-08	1999-11-21
ACT17	36	45	19	9	1998-01-08	1998-02-12	1998-01-27	1998-03-12
ACT18	66	49	28	-17	1998-02-13	1998-04-19	1998-03-13	1998-04-30
ACT19	31	31	11	0	1998-04-20	1998-05-20	1998-05-01	1998-05-31
ACT20	68	68	11	0	1998-05-21	1998-07-27	1998-06-01	1998-08-07
ACT21	65	54	11	-11	1998-07-28	1998-09-30	1998-08-08	1998-09-30
ACT22	65	65	0	0	1998-10-01	1998-12-04	1998-10-01	1998-12-04
ACT23	61	61	0	0	1998-12-05	1999-02-03	1998-12-05	1999-02-03
ACT24	50	50	0	0	1999-02-04	1999-03-25	1999-02-04	1999-03-25
ACT25	40	40	0	0	1999-03-26	1999-05-04	1999-03-26	1999-05-04
ACT26	43	44	5	1	1999-05-05	1999-06-16	1999-05-10	1999-06-22
ACT27	60	75	6	15	1999-06-17	1999-08-15	1999-06-23	1999-09-05
ACT28	40	54	21	14	1999-08-16	1999-09-24	1999-09-06	1999-10-29
ACT29	23	27	35	4	1999-09-25	1999-10-17	1999-10-30	1999-11-25

Fig. 10. Comparison of as-planned schedule and as-built schedule

Fig. 11. ACT27 schedule

equipment. In such cases, the contractor would be entitled to damages for extra costs incurred unless there is an enforceable contract clause barring such recovery. Excusable/noncompensable delays are delays for which neither party is at fault such as: acts of God, epidemics, etc., as set forth in the delay clause. Time extension is the only remedy for such delays. Nonexcusable delays are delays caused by the contractor. These could include failure in coordinating the work, insufficient manpower on the job, late delivery of equipment furnished by the contractor, low productivity, defective work that must be promptly corrected, etc. Such delays could be compensable to the owner in the form of liquidated or actual damages (Rubin et al. 1992).

After calculating P_L of each impact factor and SP_L, the responsibility for the SP_L can be assigned to the owner, the contractor, or the third party, based on the causes. The sum of the DP_L can be calculated as the following equation:

$$SDP_{Li}(F_j) = \frac{SP_{Li}(F_j) \times L_I \times D_I}{L_I P_U} \quad (11)$$

Case Study

The case project for this case study was the construction of a 20-story office building located in the business center o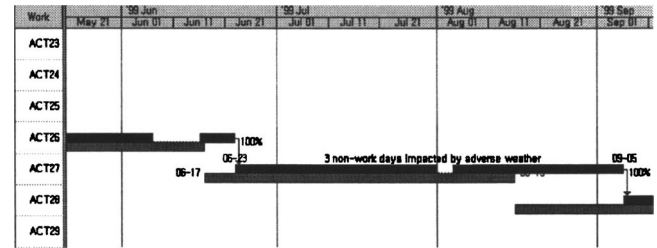f Seoul, Korea. The construction project was performed in the manner of a traditional delivery system or a single contract. One of the writers was actually involved in the project. Productivity data were collected daily in a database and used for the delay analysis in this case study. The following sections will explain the analysis conducted on the delay by using the process proposed in this paper.

Propriety Analysis of As-Planned Schedule and As-Built Schedule

Common work duration in the as-planned schedule was investigated thoroughly and the as-built schedule was reasonably compiled according to the evidence that was stored in the database. Therefore, the as-planned schedule and as-built schedule can be seen to be of acceptable level to be used in the proposed methodology.

Analysis of Critical Works in As-Built Schedule

The planned completion date of the master plan was October 31, 1999 but the actual completion date was November 25, 1999. The project was delayed for 25 days (See Figs. 9 and 10).

The planned critical works are from ACT1 to ACT16 (the planned project duration is 930 days) but the actual critical works are from ACT1 to ACT4 and from ACT17 to ACT29 (the actual project duration was 955 days). Thus, from ACT1 to ACT4 and from ACT17 to ACT29 were the impacting works of the actual delay of the project completion in this case study. This case study focuses particularly on the ACT27 (the exterior curtain wall activity) which was delayed the longest.

Table 1. Productivity Data Summary of ACT27

Duration	Work area	Manpower	Work quantity (m²)	Average work productivity (m²/man day)	Impacted duration	Impact factor
June 23–June 24	Gondola installation	40	—	—	—	—
June 25–July 8	2 step of 8th floor–6 step of 12th floor	249	1,725.75	6.93	—	—
July 9–July 10	Material delivery	19	—	—	—	July 9–first change order
July 11–July 27	1 step of 13th floor–5 step of 19th floor	456.5	2,880.35	6.31	*	—
July 28–July 29	Material delivery	9	—	—	—	July 29–second change order
July 30–July 31	6 step of 19th floor–4 step of 20th floor	52	221.62	4.26	*	—
August 1–August 3	Rainfall	—	—	—	—	Adverse weather
August 4–August 6	4 step of 20th floor–7 step of 20th floor	80	272.57	3.4	*	—
August 7–August 8	Parapet of roof	44	280.46	6.3	—	—
August 9–September 5	Finishing and gondola uninstallation	258	1,726.87	6.69	—	—

Table 2. SDP_L of ACT27

Classification	$Q(F_i)$	$P(F_i)$	$SP_L(F_i)$	$L(F_i)$	$D(F_i)$	$QP_L(F_i)$	$SDP_L(F_i)$
Unimpacted duration	1,725.75	6.93	—	17.8	14	—	—
First impacted duration	2,880.35	6.31	0.62	28.5	16	282.72	1.43
Second impacted duration	221.62	4.26	2.67	26	2	138.84	1.83
Third impacted duration	272.57	3.4	3.53	26.7	3	282.75	4.67

Analysis of V_S and V_F

The D_P of ACT27 was from June 17, 1999 to August 15, 1999 and the D_A of ACT27 was from June 23, 1999 to September 5, 1999 as shown in Fig. 11. It was proved that the 6 days of V_S was the cause of the proceeding work ACT26 performed by the same subcontractor and the V_F is 15 days.

Finding Evidence of V_F

The lost productivity of the ACT27 was impacted by three impact factors (F_S) : the owner's two change orders and adverse weather. The productivity data of the ACT27 is summarized in Table 1 based on the construction productivity database.

Cause Analysis of Work Delay

The total number of labor days was 249, the work quantity was 1,725.75 m^2, and no impact factor was found from June 25, 1999 to July 8, 1999. Thus L_U are 17.8 labor days and P_U are 6.93 m^2/man day

$$L_U = \frac{249 \text{ men}}{14 \text{ days}} = 17.8 \text{ man/day}$$

$$P_U = \frac{1,725.75 \text{ m}^2}{17.8 \text{ man/day} \times 14 \text{ days}} = 6.93 \text{ m}^2/\text{man day}$$

The D_D can be calculated using Eq. (5). The total work quantity of curtain wall less parapet work quantity (Q_P) is 6,827.16 m^2. The D_P except for the D_N is 54 days

$$D_D = D_C - D_P = \frac{Q_0}{L_U P_U} - D_P = \frac{6,827.16}{17.8 \times 6.93} - 54 = 1.34 > 0$$

Thus, D_W includes D_D and DP_L during D_I.

Characteristics of F_s and Calculation of SP_L

Because F_s occurred at a different time and at a different space, the F_s represent single impact factors. The consequence of analyzing the characteristics of the first, second, and third impact factor is independency ($\alpha=0\%$, $\beta=0\%$) in which $P_{L(i-1)}(F_{j-1})$ was dismissed during the following work duration. Thus, SP_{Li} caused by F_1, F_2, and F_3 can be calculated as follows:

$$SP_{L1\_}(F_1, F_2, F_3) = \sum_{i=1}^{3} P_{L1}(F_i) = P_{L1}(F_1) = 6.93 - 6.31 = 0.62$$

$$SP_{L2}(IF_2, IF_3) = \sum_{i=2}^{3} P_{L2}(F_{Li}) = P_{L2}(F_2) = 6.93 - 4.26 = 2.67$$

$$SLP_{3\_}IF_3 = LP_{3\_}IF_3 = 6.93 - 3.4 = 3.53$$

SDP_L Calculation of First, Second, and Third F

$SP_{L1}(F_1)$ is 0.62 (m^2/man day), L_1 is 28.5 (labor days), and D_1, except for a D_N (July 22) due to rain, is 16 days. QP_{L1} and SDP_{L1} are calculated as follows [See Eqs. (2) and (11)]. Other SDP_Ls are calculated in the same way as shown in Table 2.

$$QP_{L1} = (P_U - P_1) \times L_1 \times D_1 = 0.62 \times 28.5 \times 16 = 282.72 \text{ m}^2 \tag{12}$$

$$SDP_{L1} = \frac{QP_{L1}}{L_1 P_U} = \frac{282.72}{28.5 \times 6.93} = 1.43 \text{ days} \tag{13}$$

Assignment of SDP_L

According to the responsibility for the SDP_L, the SDP_L can be assigned to the owner, the contractor, or the third party. The first and second change orders issued by the owner are excusable/compensable delay and the adverse weather is excusable/noncompensable delay:

- nonexcusable delay=75−(60+7.67+3.26)=4.07 days,
- excusable/compensable delay=1.43+1.83=3.26 days, and
- excusable/noncompensable delay=3+4.67=7.67 days.

From this, we can conclude that the contractor has the responsibility for the 4.07 days among the total 15 delays of ACT27 (more accurately writing FTV of ACT27) of the liquidated damage (see Table 3).

Conclusion

An analysis of delays that occurred in a construction process and the allocation of the responsibilities for the delays always entail differences of opinion on the cause of the delay and on who should be held accountable. In particular, when delays caused by lost productivity are involved, the analysis becomes very complicated. There are several reasons that contribute to the delay of a project. Many studies or methodologies for analyzing the delay have focused on these reasons. The reasons for a delay are usually conceived of activities in a project schedule and the impacted activities are analyzed without considering the impacted productivity. However, if some variables impact the next sequence of the work in the construction project, the impacted work may become lost productivity work. As only a few studies have been reported on converting lost productivity into a delay, this paper presents a

Table 3. Variables Used in Equations

Work status	Work quantity	Work productivity	Daily average labors	Work duration (days)
Planned work	Q_0	P_P	L_P	D_P
As-built work	Q_A	P_A	L_A	D_A
Work of unimpacted duration	Q_U	P_U	L_U	D_U
Work of impacted duration	Q_I	P_I	L_I	D_I

method for calculating schedule delays considering lost productivity in an attempt to settle claims without litigation. The main conclusions from this study are as follows:

1. The methodology introduced several concepts regarding delay and productivity, such as planned and actual work duration, impact factors, lost productivity, the duration of lost productivity, start time variance, and finish time variance, etc.
2. Based on those concepts, a delay analysis process and equations for calculating the required values are developed with which schedule delays can be analyzed with greater accuracy.
3. The methodology was presented and implemented to a case project to show its practicality. The case study indicates that the method is a more logical process for analyzing of the complicated delay situations, and thus can provide more detailed analysis results on schedule delays.

We could also conclude that a properly designed database could aid in the accumulation of statistical data on self ratio (α) and impact ratio (β), and contribute in the application of the proposed methodology. As a result, further research on the utilization of such a database will be conducted in the future.

Acknowledgment

Notation

The following symbols are used in this paper:

D_C = duration within which planned work duration (D_P) could be finished with daily average labors (L_U) and unimpacted productivity (P_U);

D_D = contractor's difference between D_C and D_P;

D_i = portion of ith duration due to ith factor;

D_N = nonworked day;

D_W = work delay;

DP_L = lost productivity duration;

P_i = portion of ith productivity due to ith factor;

P_L = lost productivity;

$P_{Li}(F_j)$ = portion of ith lost productivity due to jth factor;

QP_L = lost productivity quantity;

V_F = finish time variance;

V_S = start time variance; and

ε_i = portion of ith lost productivity not due to factors detected.

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