Simplified Program Evaluation and Review Technique (PERT)

By Wayne D. Cottrell,[1] P.E.

Abstract: A simplified version of the program evaluation and review technique (PERT) for project planning is developed and tested. The simplification is to reduce the number of estimates required for activity durations from three, as in conventional PERT, to two. This is accomplished by applying the normal distribution, rather than the beta, to an activity duration. The two required duration estimates are the ''most likely'' and the ''pessimistic.'' These modifications reduce the level of effort needed to apply PERT. Simplified PERT durations are subject to errors of greater than 10% when the skewness of the actual distribution is greater than 0.28 or less than −0.48. In analyzing 12 project networks, though, the simplified PERT produced values similar to those of conventional PERT for activity and project durations and variances and project duration probabilities. Hence, when activity duration distributions are not highly skewed, results similar to those of conventional PERT can be obtained using the simpler technique. Two suggestions for future research are to survey practitioners on the usefulness of the simplified PERT and to find a fixed, skewed distribution that can approximate activity durations having long tails.

INTRODUCTION

The current paper presents and evaluates a simplified program evaluation and review technique (PERT), a planning tool for application to construction and other projects. The outline of the paper is as follows: First, PERT is described; second, criticisms of PERT in the literature are reviewed; third, the simplified PERT is presented; fourth, the simplified technique is tested and evaluated, both on activity duration means and on a set of project networks. The results are compared with those of conventional PERT. Finally, the conclusions are stated. The objective of the present research is to present a simplified technique that might be more easily and readily applied in industry, particularly in construction project planning, than conventional PERT. The simplified PERT algorithm could be embedded into construction scheduling software to facilitate its application.

Description of PERT

PERT was originated by the U.S. Navy in 1958 as a tool for scheduling the development of a complete weapons system (Malcolm et al. 1959). The technique considers a project to be an acyclic network of events and activities. The duration of a project is determined by a system flow plan in which the duration of each task has an expected value and a variance. The critical path includes a sequence of activities that cannot be delayed without jeopardy to the entire project. PERT can be used to estimate the probability of completing either a project or individual activities by any specified time. It is also possible to determine the time duration corresponding to a given probability (Callahan et al. 1992).

The first step in applying PERT is to diagram the project network, where each arc represents an activity and each node symbolizes an event (such as the beginning or completion of a task), as in Fig. 1. Alternatively, each node can symbolize an activity. The second step is to designate three time estimates for each task: optimistic (a), pessimistic (b), and most likely (m). Small probabilities are associated with a and b. In the original PERT, a is the minimum duration of an activity; the probability of a shorter duration is zero. Similarly, b is the maximum duration; the probability that the duration will be less than or equal to b is 100%. No assumption is made about the position of m relative to a and b. In statistical terms, a and b are the extreme ends of a hypothetical distribution of duration times. The mode of the distribution is m. To accommodate flexibility in the positions of these parameters, the beta distribution is used, as shown in Fig. 2 (Malcolm et al. 1959; Clark 1962). The beta distribution is useful for describing empirical data and can be either symmetric or skew (Benjamin and Cornell 1970).

The third step is to compute the expected value and variance of the duration of each activity in the project network. The mean of a beta distribution is a cubic equation. The PERT equation for the mean [(1)] is a linear approximation to this

$$\tau_e = (a + 4m + b)/6 \tag{1}$$

where τ_e = expected duration of an activity. Badiru (1991) shows that (1) is exact when m is equal to the mode, which occurs when a and b are symmetrical about m.

In unimodal probability distributions, the standard deviation of the distribution is equal to approximately one-sixth of the range (Watson et al. 1993). With 100% of the possible durations bound by a and b, the estimated variance of the duration is as follows:

$$\sigma^2_{100}(\tau_e) = [(b - a)/6]^2 \tag{2}$$

where σ^2 = variance of the activity duration. Moder and Rodgers (1968) argue that the exact endpoints of the range of the duration are impossible to define. Their alternative is to define a and b as the 5% and 95% thresholds of the range, respectively. Then, the variance is as follows:

$$\sigma^2_{90}(\tau_e) = [(b - a)/3.2]^2 \tag{3}$$

Perry and Greig (1975), alternatively, use 3.25 in the denominator of (3), rather than 3.2. They argue that subjective probability distributions tend to be rounded (platykurtic) rather than peaked. The denominator of 3.25 is more appropriate for platykurtic, bell-shaped curves (Perry and Greig 1975). Moder and Rodgers' result seems to be cited more frequently in the literature, though.

The fourth step is to order the activities sequentially, from the beginning to the end of the project, in a tabular format, listing the optimistic, pessimistic, most likely, and expected durations and the variances. Fifth, forward and backward passes through the network are performed to identify the critical path, just as in the widely used critical path method. The

[1]Independent Res., 3009 West Hyde Ave., Visalia, CA 93291.

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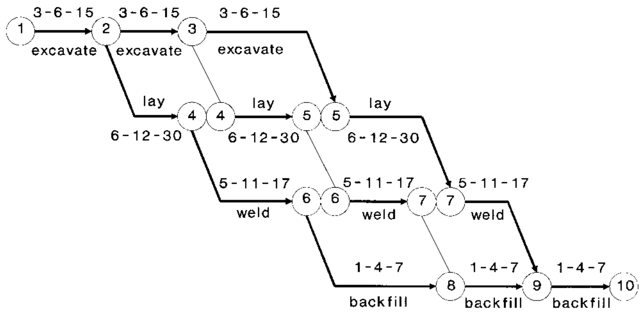

FIG. 1. Project Network for Gas Pipeline Operation: *a-m-b* Activity Durations

central limit theorem is then applied as follows: The distribution of the sum of the expected durations of the activities along the critical path is approximately normal, particularly as the number of activities increases. The expected duration of each sum is equal to the sum of the expected durations. Similarly, the variance of each sum is the sum of the variances.

These applications of the central limit theorem enable the computation of project duration probabilities using the deviations from a zero mean of the standard normal variable (Z). These probabilities can be critical in making financial decisions about the viability of a project (Callahan et al. 1992).

EVALUATION OF PERT IN LITERATURE

Problems with PERT

Various criticisms and proposed modifications to PERT have appeared in the literature since the early 1960s. There are five recognized problems with PERT. First, it is difficult for project engineers and planners to accurately estimate the optimistic, most likely, and pessimistic durations of an activity. Grubbs (1962) and Moder et al. (1983) note that subjective estimates of a, m, and b are based on judgment and may not be closely related to statistical sampling of the actual times. The latter authors note that the subjectivity is compounded by the fact that the activity duration distribution is purely hypothetical, as well, as discussed later. MacCrimmon and Ryavec (1964) calculate the sensitivity of (1) and (2) to incorrect estimates of a, m, and b. Swanson and Pazer (1971), along with Lau et al. (1996), indicate that ''optimistic'' and ''pessimistic'' are ambiguous and are subject to interpretation. For example, b has been described as having a ''small chance'' or a ''one in 100'' chance of being exceeded (Swanson and Pazer 1971). Littlefield and Randolph (1987) state that, based on past research, ''people are not very good estimators of the extreme values.'' Moder and Rodgers (1968) relax this requirement, as shown in (3). Lau et al. (1996) further state that little is known about modal estimation. The simplification of PERT proposed in the present paper reduces the dependence on subjective estimation of activity durations by decreasing the number of time estimates from three to two.

Second, the mean and variance of an activity duration, as calculated using (1) and (2) or (1) and (3), are estimates of the actual mean and variance of a beta distribution. Badiru (1991) summarizes and discusses both the estimated and actual equations. MacCrimmon and Ryavec (1964) compute the maximum possible errors between the estimated and actual means and variances. If (2) is assumed to be accurate, then the maximum possible error in the mean is about 33%; if (1) is presumed accurate, then the maximum possible error in the standard deviation is about 17%. McBride and McClelland (1967), alternatively, calculate a maximum possible error of 18.8% in the mean when (2) is held as accurate. (The writer verified MacCrimmon and Ryavec's result.) Sasieni (1986) finds no basis for the values of the beta distribution parameters assumed in the derivation of (1). Littlefield and Randolph (1987) refute Sasieni, restating the rationale given by the developers and early reviewers of PERT.

Third, the beta distribution is presumed to be applicable to all project activities. Grubbs (1962) and MacCrimmon and Ryavec (1964) criticize this aspect of PERT. At the time of PERT's development, no empirical study had been done to determine the typical distribution of activity times of representative projects. MacCrimmon and Ryavec (1964) estimate the level of error introduced by an incorrect activity distribution assumption. AbouRizk and Halpin (1992) show, through their analysis of empirical construction activity duration data, that the beta distribution is appropriate. AbouRizk and Halpin (1994) develop a procedure to fit beta distributions to construction operations.

Fourth, PERT considers only the critical path in computing project completion time probabilities. The method ignores

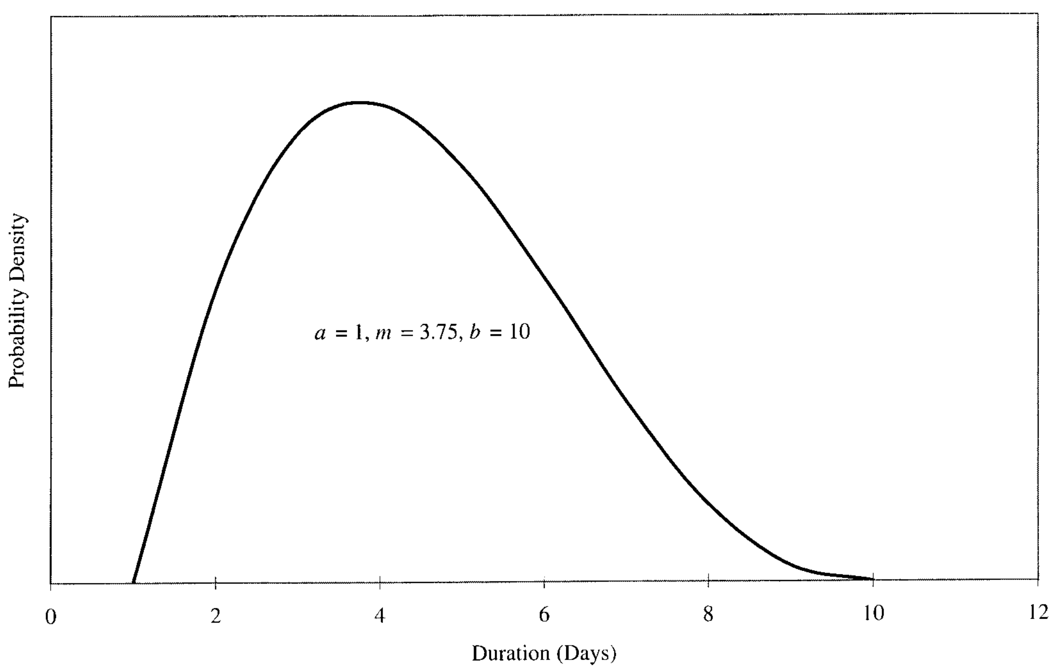

FIG. 2. Beta Distributed Activity Duration Example

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near-critical paths that possess a not in significant probability of becoming critical (Callahan et al. 1992). Numerous authors have described this problem and have proposed solutions, including time-probability computations (Fulkerson 1962), criticality indexes (Van Slyke 1963; Dodin 1984; Bandopadhyay and Sundararajan 1987), activity time adjustments (Lindsey 1972), and total project duration probability distributions (Hartley and Wortham 1966; Robillard and Trahan 1977; Kamburowski 1985; Anklesaria and Drezner 1986; Sculli and Shum 1991). One of the results of missing near-critical paths in PERT is a ''merge event bias.'' The magnitude of this problem, which worsens as the number of parallel paths in a network increases, has been evaluated by a number of authors (Crandall 1976; Moder et al. 1983; Sculli 1983). The simplified PERT proposed in the present paper does not attempt to tackle the near-critical paths or the merge event bias problem. This would be a subject for further research.

Finally, Callahan et al. (1992) state that ''one of the major drawbacks of PERT in construction applications is that it requires multiple time estimates, which can be time-consuming to develop.'' The authors indicate that PERT is rarely used on construction projects. Moder et al. (1983) propose that PERT is not used either because ''top project managers do not understand the basic principles of probability and statistics'' or, regardless of their understanding of the subject, ''they have not learned how to use PERT.'' One of the objectives of the present paper is to enhance PERT's applicability by reducing the number of time estimates needed for its use. Project managers would still need to know a few basics about probability and statistics; however, the availability of scheduling software would make knowledge of the mathematics and of the intricacies of the algorithm less essential.

Modifications to PERT

Numerous authors have developed modifications to PERT, including the adjustments to (2) (Moder and Rodgers 1968; Perry and Greig 1975) and criticality indexes (see prior text). Some authors have, as is done in the present paper, modified PERT's time estimates. Troutt (1989) replaces the mode with the median in (1). He states that this produces a good estimate of the mean regardless of the probability distribution assumed. Three time estimates are still required, however. Izuchukwu (1990) eliminates *m* and uses only *a* and *b*. He argues that *m* is ''practically useless,'' but offers no evaluation of his new procedure. Contrarily, other research has indicated that *a* and *b* are more difficult to estimate than is *m* (Littlefield and Randolph 1987; Lau et al. 1996). Finally, Lau et al. (1996) state that the estimation of all three of PERT's time estimates is subject to ambiguity. They replace *a*, *m*, and *b* with sets of either five or seven quantiles (the 0.25 quantile, for example, is greater than 25% of the numbers in the distribution). The estimation of quantiles is argued as being more straightforward than the estimation of modes and extreme values. Their method has merit, but the number of time estimates required is increased from three to five or seven.

SIMPLIFIED PERT PROCEDURE

The proposed simplification of PERT is to reduce the number of time estimates required for each task from three to two. This reduction decreases both the level of effort needed to apply PERT and the required knowledge of activity durations. To retain a probabilistic procedure, the time estimates must be inputs to determining the expected value and variance of an activity duration. The only choice is to assume that the distribution of a duration is symmetric, i.e., normal, as in Fig. 3, rather than beta. A unique normal distribution is defined by any given pair of mean and standard deviation values (Watson et al. 1993). Thus, a unique normal distribution can be defined by any two points on one side of the curve. The other elements of the PERT technique remain the same.

Given that Izuchukwu (1990) uses *a* and *b* in his procedure, the question remains as to which of *a*, *m*, and *b* to use in the new, simplified procedure. Moder et al. (1983) report that most time estimates are on the optimistic side, resulting in actual project durations being longer than those forecast. This finding is congruous with Izuchukwu's on the closeness of *m* to *a*. Hence, relying on *a* and *m* only may result in optimistic time estimates. The more conservative approach is to use *m* and *b*. Here, *m*, the mode, is equal to the mean, since the distribution is symmetric.

The expected duration of an activity in simplified PERT can be determined as follows:

$$T_e = m \quad (4)$$

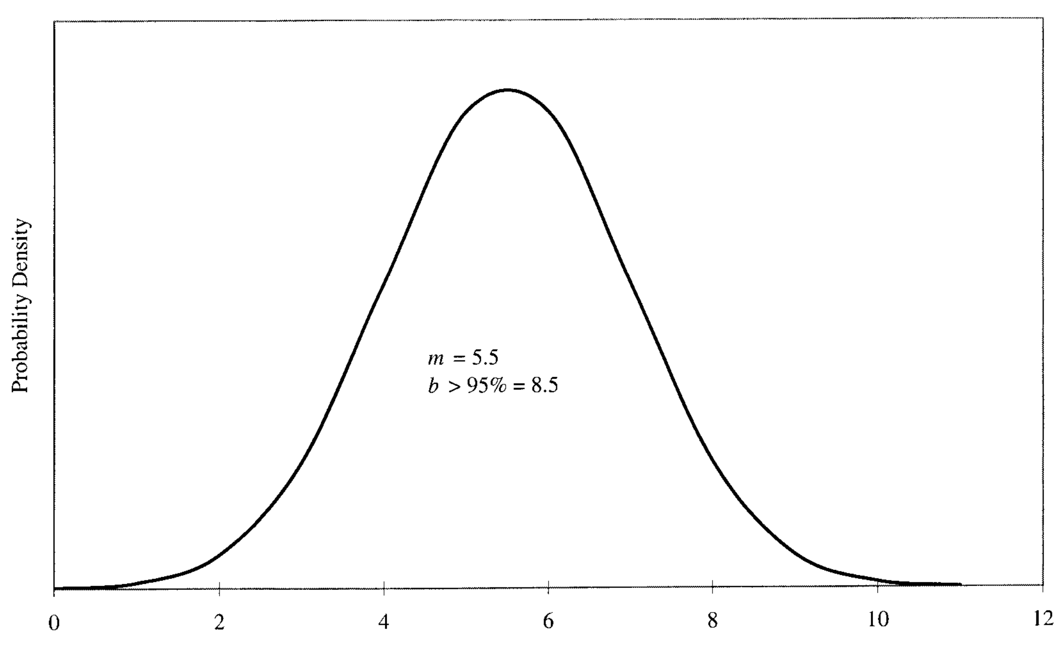

FIG. 3. Normally Distributed Activity Duration Example

where T_e = expected duration. The two variances can be computed as follows: The standard normal variable, Z, is equal to 3.44 when b is the upper bound on 100% of all durations, and is 1.645 when b is greater than 95% of the durations. $Z = (b - m)/\sigma$, so

$$\sigma^2_{100}(T_e) = [(b - m)/3.44]^2 \quad (5)$$

and

$$\sigma^2_{90}(T_e) = [(b - m)/1.645]^2 \quad (6)$$

In choosing between (5) and (6) for the variance, it is recognized that the normal distribution is not bound on either end. Hence, b as the upper limit on 100% of the durations would be, by definition, infinite. Eq. (6), therefore, is preferable. The remainder of the simplified PERT procedure is the same as that of conventional PERT. Hence, the merge event bias problem is not explicitly corrected.

TESTING AND EVALUATION

Normal Distribution Assumption for Activity Durations

The first step in evaluating the simplified PERT procedure is to review the normal distribution assumption for individual activity durations. AbouRizk and Halpin (1992) perform a statistical analysis of construction activity duration data. The authors developed a database of 71 construction activities, including various forms of loading, dumping, bulldozing, pipe jacking, and hauling. The data are not described in detail, but the authors state that at least 20 observations of each activity were available, with over 100 observations in some samples. Most of the sample duration distributions are associated with some degree of skewness ($-1.207 \le g_1 \le 4.869$) and kurtosis ($1.008 \le g_2 \le 31.346$). The beta distribution is found to cover most of the density shapes.

The normal distribution is associated with a skewness of 0 and a kurtosis of 3 (AbouRizk and Halpin 1992). Demenais et al. (1986), in discussing how major genes are analyzed in family studies, identify three ranges of skewness for sample sizes of 100 nuclear families: low, intermediate, and high. When there is low skewness ($-0.2 \le g_1 \le 0.2$), the sampled distribution can be considered normal. For intermediate skewness ($-0.4 \le g_1 \le -0.2$ and $0.2 \le g_1 \le 0.4$), the distribution is nearly normal. In cases of high skewness ($g_1 < -0.4$ or $g_1 > 0.4$), the normal distribution assumption is questionable. Of the 71 construction activities sampled by AbouRizk and Halpin (1992), 13 have low skewness, 10 have intermediate skewness, and 48 have high skewness. There does not appear to be any pattern in the degree of skewness by type of activity. Hence, in about one-third of the cases, a normal distribution assumption is reasonable.

The indication is that the normal distribution is viable only part of the time. The questions, then, are: How much error might be introduced into the mean of an activity duration by assuming that its distribution is normal, when, in fact, it is skewed? What cumulative effect might such errors have on project duration probabilities? To answer these questions, a theoretical evaluation of activity durations and a practical evaluation of some project durations were conducted.

Evaluation of Activity Duration Means

The beta distribution has four parameters that enable its flexibility. For a standardized (0, 1) beta distribution, the mode (m_β), mean (μ_β), and variance (σ^2_β) are as shown in (7), (8), and (9), respectively (Benjamin and Cornell 1970; Swanson and Pazer 1971)

$$m_\beta = \alpha/(\alpha + \beta) \quad (7)$$

$$\mu_\beta = (\alpha + 1)/(\alpha + \beta + 2) \quad (8)$$

$$\sigma^2_\beta = [(\alpha + 1)(\beta + 1)]/[(\alpha + \beta + 2)^2(\alpha + \beta + 3)] \quad (9)$$

In the preceding equations, α and β = parameters of the beta distribution. The conventional PERT equations for the mean and variance of an activity duration are approximations. For example, (1) is exact only when $\alpha + \beta = 4$ (Gallagher 1987). This restriction was established to simplify the equation for the mean. Badiru (1991) indicates that, regardless of the values of α and β, (1) is exact when the PERT estimates are symmetric about m.

Another approach, based on stronger assumptions, is to let the standard deviation of the beta distribution be equal to one-sixth of the range. Then, the variance, shown in (9), is equal to 1/36. Solving this equation, and using (7) and (8) to substitute terms, produces a cubic equation in terms of α and m_β, as shown in (10) (Littlefield and Randolph 1987)

$$\alpha^3 + (36m^3_\beta - 36m^2_\beta + 7m_\beta)\alpha^2 - 20m^2_\beta\alpha - 24m^3_\beta = 0 \quad (10)$$

Varying m_β between 0 and 1 produces (α, β) pairs that facilitate the computation of the actual mean μ_β. Then, μ_β can be compared to the estimated mean τ_e. To evaluate simplified PERT's time estimate, μ_β can also be compared to T_e from (4).

The comparison between the actual mean of the beta distribution, the estimated mean of conventional PERT, and the mean of simplified PERT, over a 0–1 range [i.e., in (1), a = 0 and b = 1], is shown in Table 1. Note that this approach requires that the original formula for the standard deviation be used—the variance is shown in (2)—rather than Moder and Rodgers' (1968) modification. Since a different definition for the variance is used in simplified PERT, the evaluation focused on comparing the means. For values of m_β (=T_e) ranging from 0.01 to 0.99 (column 1), both τ_e and μ_β were computed, as shown in columns 4 and 5, along with the percent differences between T_e and μ_β and between τ_e and μ_β, as shown in columns 6 and 7. The difference between τ_e and μ_β ranges from 16.8% at m_β = 0.01, to 0.0% at m_β = 0.50, to −4.4% at m_β = 0.99. By comparison, the difference between T_e and μ_β ranges from 95.2% at m_β = 0.01, to 0.0% at m_β = 0.50, to −25.1% at m_β = 0.99. The skewness of the beta distribution, computed from (14) in Appendix I, ranges from 1.01 at m_β = 0.01, to 0.00 at m_β = 0.50, to −1.01 at m_β = 0.99. The indication is that the simplified PERT's T_e is subject to greater error than the conventional PERT's τ_e. The error is particularly large at small m_β values, which occur when the beta distribution has a large positive skewness.

TABLE 1. Means of Simplified and Conventional PERT Activity Durations

$m_\beta = T_e$ (1)	α (2)	β (3)	τ_e (4)	μ_β (5)	% Difference $\mu_\beta - T_e$ (6)	% Difference $\mu_\beta - \tau_e$ (7)	$g_{1\beta}$ (8)
0.01	0.03	2.91	0.173	0.208	95.2	16.8	1.01
0.10	0.36	3.25	0.233	0.243	58.8	3.8	0.81
0.20	0.89	3.57	0.300	0.293	31.7	−2.4	0.59
0.30	1.57	3.67	0.367	0.355	15.5	−3.2	0.38
0.40	2.32	3.48	0.433	0.426	6.0	−1.8	0.18
0.50	3.00	3.00	0.500	0.500	0.0	0.0	0.00
0.60	3.48	2.32	0.567	0.574	−4.5	1.3	−0.18
0.70	3.67	1.57	0.633	0.645	−8.6	1.8	−0.38
0.80	3.57	0.89	0.700	0.707	−13.1	1.0	−0.59
0.90	3.25	0.36	0.767	0.758	−18.8	−1.2	−0.81
0.99	2.91	0.03	0.827	0.792	−25.1	−4.4	−1.01

Note: m_β and α were determined from (10); β was deduced using (7); τ_e was calculated using (1) with a = 0 and b = 1; μ_β was computed from (8). Skewness $g_{1\beta}$ is that of beta distribution, as computed using (14).

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It is apparent that the potential for an error in estimating the mean of an activity duration is greater with simplified PERT than with conventional PERT. An error of ±10% in the mean time of a simplified PERT activity occurs at standardized T_e values of 0.35 and 0.75, respectively. Standardized durations of between 0.35 and 0.75 occur at degrees of skewness between 0.28 and −0.48. Thus, when the skewness of the distribution is greater than 0.28 or less than −0.48, the error is greater than 10%. In AbouRizk and Halpin (1992), 51 of the 71 construction activities studied have such skewed distributions. Hence, applying the normal distribution to the mean of an activity duration distribution would be adequate in 30%, but inaccurate in about 70%, of all construction activities. This conclusion assumes that the activities surveyed by AbouRizk and Halpin (1992) are common and would be critical in a typical project.

Practical Evaluation of Conventional and Simplified PERT Project Durations

The second phase in evaluating the simplified PERT is to compare its results for entire projects with those obtained using the conventional PERT. Comparisons of multiple networks have been used to assess the computing speed of probabilistic scheduling methods (Diaz and Hadipriono 1993). To apply a similar approach, expected durations and variances were computed for a set of 12 project networks, each featuring a, m, and b estimates for the activities. One of these networks is shown in Fig. 1. The duration data and network layouts were obtained from a host of sources (Van Slyke 1963; MacCrimmon and Ryavec 1964; Moder et al. 1983; Dodin 1984; Callahan et al. 1992; Diaz and Hadipriono 1993).

In each of the 12 project networks, the critical path is the same for both PERT procedures; this does not always have to be the case. The results are shown in Table 2. Column 1 distinguishes the projects; project ''G'' is the network shown in Fig. 1. Column 2 lists the number of activities on the critical paths; the range is from 3 to 16. Column 3 lists the number of merge events on the critical paths. The probability that another path is critical increases as the number of merge events increases (Moder et al. 1983).

Column 4 lists the expected project durations using conventional PERT. These values can be compared with those in column 6, which are the expected durations using simplified PERT. In all 12 networks, $T_e \le \tau_e$. The differences between the expected durations range from −10.9% to zero. There are three possibilities for the expected durations computed by the two procedures, each of which can be demonstrated mathematically. By definition, $b \ge m \ge a$

$$(a + 4m + b)/6 = m, \text{ so } a + b = 2m = m + m;$$

$$\text{thus } b - m = m - a \tag{11}$$

$$(a + 4m + b)/6 > m, \text{ so } a + b > 2m \quad \text{or} \quad m + m;$$

$$\text{thus } b - m > m - a \tag{12}$$

$$(a + 4m + b)/6 < m, \text{ so } a + b < 2m \quad \text{or} \quad m + m;$$

$$\text{thus } b - m < m - a \tag{13}$$

When (11) holds, a and b are symmetric about m. When (12) holds, the duration distribution is skewed left. When (13) holds, the distribution is skewed right. The a, m, and b values of the 172 activities included in the 12 project networks were studied. Izuchukwu (1990) states that a and m are typically closer together than are m and b. This actually occurs in only 67 of the 172 activities. However, the overall average values of $m - a$ and $b - m$ are 3.05 and 4.16, respectively, with variances of 10.06 and 21.67. A hypothesis test on the equivalence of the two means was conducted using the Satterthwaite test (Watson et al. 1993). This test is appropriate for two samples with unequal variances. The test is described in Appendix II. The null hypothesis that the mean values of $m - a$ and $b - m$ are equal was rejected. Hence, even though the condition shown in (13) occurs with the greatest frequency, $b - m > m - a$ in enough cases to offset this.

TABLE 2. Critical Path PERT Duration Data

Project (1)	Activities (2)	Merge events (3)	Conventional τ_e (4)	Conventional $\sigma^2_{90}(\tau_e)$ (5)	Simplified T_e (6)	Simplified $\sigma^2_{90}(T_e)$ (7)
A	5	1	89.0	36.7	89.0	48.0
B	7	4	55.0	16.2	55.0	15.2
C	5	2	41.7	13.3	41.0	19.2
D	10	3	58.3	29.3	58.0	29.6
E	16	4	286.2	396.6	285.0	385.1
F	3	2	12.2	4.4	12.0	5.2
G	6	4	64.0	200.4	57.0	405.8
H	3	1	13.0	3.1	13.0	3.0
I	4	1	12.0	2.7	12.0	2.6
J	10	4	47.7	38.3	45.1	92.2
K	3	1	7.2	2.8	7.0	3.3
L	4	3	66.0	211.9	63.0	290.1

The most extreme example of the difference between T_e and τ_e occurs in the network shown in Fig. 1; the simplified PERT expected duration is 10.9% less than that of conventional PERT. An examination reveals that, for four of the six activities on the critical path, $(b - m) = 3^*(m - a)$. In the other 11 project networks, these ratios are not nearly as great.

Column 5 of Table 2 lists the project variances computed using (3). These can be compared with the simplified PERT variances listed in column 7. The value of $\sigma^2_{90}(\tau_e)$ ranges from 2.7 to 396.6 while $2.6 \le \sigma^2_{90}(T_e) \le 405.8$. The simplified PERT variances are greater than the conventional PERT variances in eight of the 12 networks. This indicates that, in general, the simplified PERT variances are greater than those computed using conventional PERT.

Project duration probabilities are shown in Table 3. Column 3 lists the project durations expected to occur with 80% probability using conventional PERT (τ_{80}). Column 5 lists T_{80} values for simplified PERT. Columns 4 and 6 list the probabilities that the expected project durations will be exceeded by 10% (P_{10}). Column 4 lists P_{10} values for conventional PERT, while column 6 lists P_{10} values for simplified PERT. The probabilities were calculated by applying the central limit theorem to the sums of the individual project activities. Then, the standard normal variable (Z) along with cumulative normal distribution

TABLE 3. PERT Project Duration Probabilities

Project (1)	Activities (2)	Conventional, b > 95% τ_{80} (3)	Conventional, b > 95% P_{10} (4)	Simplified, b > 95% T_{80} (5)	Simplified, b > 95% P_{10} (6)
A	5	94.1	7.1%	94.8	10.0%
B	7	58.4	8.6%	58.3	7.9%
C	5	44.7	12.7%	44.7	17.5%
D	10	62.9	14.1%	62.6	14.3%
E	16	302.9	7.5%	301.5	7.3%
F	3	13.9	28.1%	13.9	29.9%
G	6	75.9	32.6%	74.0	38.9%
H	3	14.5	23.1%	14.4	22.4%
I	4	13.4	23.4%	13.4	22.8%
J	10	52.8	22.1%	53.2	31.9%
K	3	8.6	33.5%	8.5	35.1%
L	4	78.3	32.5%	77.3	35.6%

Note: τ_{80}, T_{80} = project duration estimated to occur with 80% probability. P_{10} = probability that project's duration is 10% longer than expected duration.

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TABLE 4. Hypothesis Tests on Activity Durations and Variances

Statistic (1)	X_1 (2)	X_2 (3)	s_1^2 (4)	s_2^2 (5)	s_{X1-X2} (6)	t_{calc} (7)	ν (8)	$t_{\alpha/2,\nu}$ (9)
Expected duration	9.10	8.91	86.93	86.21	1.10	0.185	171	1.98
Variance, $b > 95\%$	14.42	10.19	1,379.24	651.87	3.44	1.229	171	1.98

Note: Symbols and equations are detailed in Appendices II and IV.

tables were used to compute probabilities. The T_{80} values range from 2.5% less than to 0.8% greater than τ_{80}. Seven of the τ_{80} values are greater than, three are equal to, and two are less than the T_{80} values. Eight of the simplified PERT P_{10} values are greater than those of conventional PERT, while the remaining four are less.

It is difficult to draw firm conclusions from this analysis of the variances and probabilities of simplified and conventional PERT durations in the 12 project networks. The indication is that simplified PERT produces shorter expected project durations, but greater project duration variances, than does conventional PERT. The effects of these cancel each other, such that the project duration probabilities of simplified PERT are not much different from those of conventional PERT. Thus, in these and similar project networks, results at least as reliable as those of conventional PERT can be obtained using the simpler technique.

To extend the analysis, the durations of the 172 activities comprising the 12 networks were examined. Two null hypotheses were tested: that the means of the 172 expected durations computed using conventional and simplified PERT are equal, and that the means of the 172 variances with $b > 95\%$ of the durations are equal. The Satterthwaite test was used (see Appendix II).

The results are shown in Table 4. The null hypotheses that the means of both the expected durations and the variances are equal are not rejected in any case. The overall conclusion is that the conventional and simplified PERTs produce similar values for activity durations and variances.

CONCLUSIONS

A simplified version of PERT has been developed. The new technique reduces the level of effort required by conventional PERT because only two time estimates, rather than three, are required for each activity. The remainder of the procedure is identical to the conventional method. The reduced effort may result in a significant time savings for large projects in which there are many tasks. Two evaluations of the new method were conducted. In the first phase, the estimated means of a range of modal duration values were computed for both simplified and conventional PERT. The estimated means were compared with actual means obtained from the standardized beta distribution. The activity duration means computed using simplified PERT are subject to greater error than are those computed using conventional PERT, especially when the distribution is highly skewed. At degrees of skewness between 0.28 and −0.48, the error in the mean would be less than or equal to 10% of the actual value. Based on the AbouRizk and Halpin (1992) study, about 30% of all construction activities have skewness levels within this range.

This second phase of the evaluation featured the computation of project durations for 12 networks using both simplified and conventional PERT. The expected durations and variances of individual project activities, computed using simplified PERT, are essentially equal to those computed using the conventional procedure. Simplified PERT produces shorter project durations, but greater project duration variances, than does conventional PERT. The combination of these two effects results in similar project duration probabilities. These conclusions are drawn from the analysis of the networks and via hypothesis tests on 172 project activities. These results are in contrast to those obtained in the first phase of the evaluation. The implication is that the 12 networks tested did not include activities with highly skewed duration distributions. If the 12 project networks are truly representative, then the simplified PERT produces results similar to those obtained with conventional PERT, but with less effort.

Further analysis of the simplified PERT procedure is recommended, possibly on other networks, and via simulation of activity durations on a single network. The new procedure does not explicitly improve upon PERT's merge event bias problem. Further research is suggested toward developing a probabilistic procedure that is easy to apply and that eliminates merge event bias and the ignorance of near-critical paths. Further analysis of construction activity duration distributions is also suggested. If the critical activities in a given project feature highly skewed duration distributions, then simplified PERT may be inappropriate. An alternative might be to use a fixed, positively skewed distribution, defined by two parameters, for all activities. A survey of practitioners is suggested to ascertain the usefulness of the new procedure. To facilitate its usage, the simplified PERT algorithm could be embedded into construction scheduling software.

APPENDIX I. DEGREE OF SKEWNESS OF BETA DISTRIBUTION

The degree of skewness of a standardized (0, 1) beta distribution can be computed as follows:

$$g_{1\beta} = \frac{1}{\sigma^3}\frac{(\alpha+1)}{(\alpha+\beta+2)}\left[\frac{(\alpha+3)(\alpha+2)}{(\alpha+\beta+4)(\alpha+\beta+3)} - \frac{3(\alpha+1)(\alpha+2)}{(\alpha+\beta+2)(\alpha+\beta+3)} + \frac{2(\alpha+1)^2}{(\alpha+\beta+2)^2}\right] \quad (14)$$

The parameters of the beta distribution are α and β, while σ = standard deviation.

APPENDIX II. SATTERTHWAITE TEST

The Satterthwaite test (Aspin test; Welch test) is applied to a hypothesis test on two means when the samples have unequal or separate variances (Watson et al. 1993). The test is as follows:

$$H_0\text{: } \mu_1 = \mu_2 \quad \text{and} \quad H_a\text{: } \mu_1 \neq \mu_2 \quad (15)$$

$$\text{Reject } H_0 \text{ if } t_{calc} < -t_{\alpha/2,\nu} \quad \text{or} \quad t_{calc} > t_{\alpha/2,\nu}, \text{ where} \quad (16)$$

$$t_{calc} = (\bar{X}_1 - \bar{X}_2)/s_{\bar{X}1-\bar{X}2} \quad (17)$$

$$s_{\bar{X}1-\bar{X}2} = [(s_1^2 + s_2^2)/n]^{0.5}, \text{ and} \quad (18)$$

$$\nu = n - 1 \quad (19)$$

In (15)–(19), H_0 = null hypothesis; H_a = alternative hypothesis; μ = population mean; t = value of the statistic from Student's t distribution; α indicates the confidence level; ν = degrees of freedom; $\bar{X}$ = sample mean; s^2 = sample variance; and $s_{\overline{X1}-\overline{X2}}$ = pooled sample standard deviation. Eqs. (18) and (19) are valid when the number of observations in each sample is the same.

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APPENDIX III. REFERENCES

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APPENDIX IV. NOTATION

The following symbols are used in this paper:

a = optimistic activity duration; first shape parameter of beta distribution;
b = pessimistic activity duration; third shape parameter of beta distribution;
g_1 = sample skewness coefficient;
$g_{1\beta}$ = skewness coefficient of standardized beta distribution;
g_2 = sample kurtosis coefficient;
m = most likely or modal activity duration;
m_β = mode of beta distribution;
n = number of observations in sample;
P_{10} = probability that project's duration is 10% longer than expected duration;
s^2 = variance of sample of values;
$s_{\overline{X1}-\overline{X2}}$ = pooled sample standard deviation;
T_e = expected duration time of activity using simplified PERT procedure;
T_{80} = simplified PERT project duration estimated to occur with 80% probability;
t, t_{calc} = value of statistic in Student's t distribution;
$\bar{X}$ = mean of sample of values;
Z = standard normal variable;
α = parameter of beta distribution; 1 − (desired statistical confidence level/100);
β = parameter of beta distribution;
μ = population mean;
μ_β = mean of beta distribution;
ν = degrees of freedom;
σ^2_β = variance of beta distribution;
σ^2_{90} = variance of activity's duration when a and b are greater than 5% and 95%, respectively, of activity's duration;
σ^2_{100} = variance of activity's duration when a and b are lower and upper bounds, respectively, on activity's possible durations;
τ_e = expected duration time of activity using conventional PERT procedure; and
τ_{80} = conventional PERT duration estimated to occur with 80% probability.

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