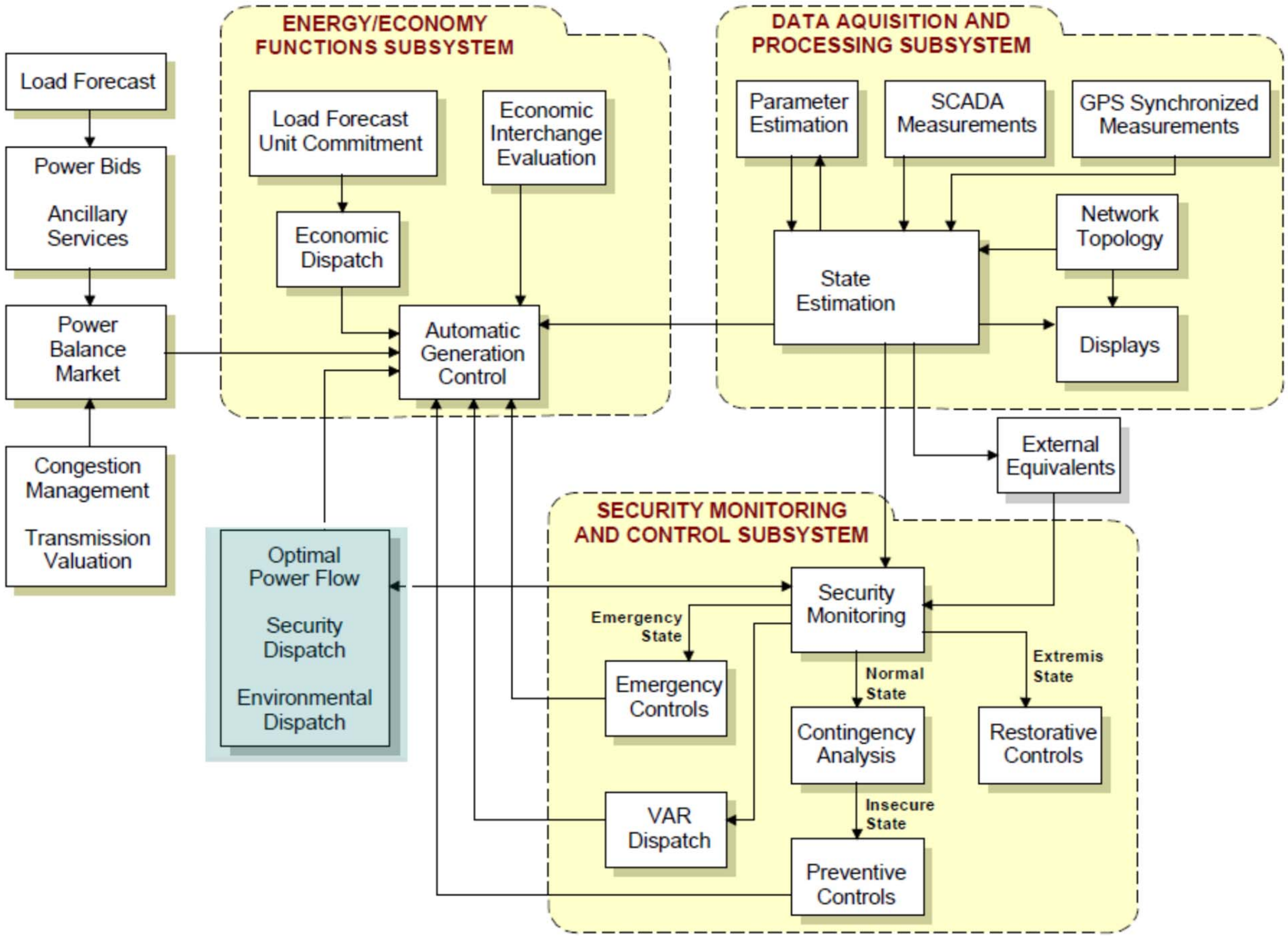
Optimal Power Flow
(OPF)



ENERGY/ECONOMY FUNCTIONS SUBSYSTEM
DATA AQUISITION AND PROCESSING SUBSYSTEM
Load Forecast
Power Bids
Ancillary Services
Power Balance Market
Congestion Management
Transmission Valuation
Load Forecast Unit Commitment
Economic Interchange Evaluation
Economic Dispatch
Automatic Generation Control
Parameter Estimation
SCADA Measurements
GPS Synchronized Measurements
Network Topology
State Estimation
Displays
External Equivalents
SECURITY MONITORING AND CONTROL SUBSYSTEM
Optimal Power Flow
Security Dispatch
Environmental Dispatch
Security Monitoring
Emergency State
Emergency Controls
Normal State
Extremis State
Contingency Analysis
Restorative Controls
VAR Dispatch
Insecure State
Preventive Controls

What is OPF?

- OPF is an optimization problem which seeks to optimize the operation of an electric power system
  - subject to the physical constraints imposed by the laws governing electrical circuits and engineering limits.
- OPF combines an objective function with the power flow equations to form the optimization problem.
- The presence of the power flow equations distinguishes OPF from other classes of power systems problems, such as:
  (1) Unit commitment (UC),
  (2) Market clearing problems,
  (3) Economic dispatch (ED).

(1) Unit commitment (UC)

- UC refers to the scheduling of generating units in such a way that total operating cost is minimized.
  - UC differs from economic dispatch (ED) in that it operates across multiple time intervals and schedules the on/off status of each generator in addition to its power output.
  - UC must address generator startup / shutdown time and costs, ramp rate limits, reserve margin requirements, and other scheduling constraints.
- The UC problem is a sequence of ED problems over a period of time plus some intra-temporal constraints.
- UC is a non linear mixed integer (states of generators 0/1), dynamic programming problem (a sequence of decisions)

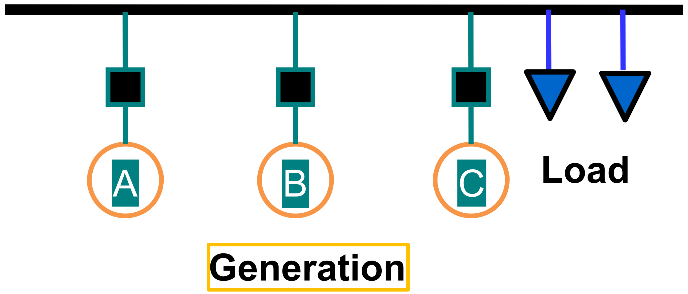
(2) Market clearing

- Market clearing is the process by which the supply of electricity traded is balanced to the demand.
- In its most basic form, the electricity market consists of a forward (typically a day-ahead) market and a balancing market.
  - The day-ahead market deals with generation from units that need advance planning in order to efficiently and reliably set their production levels.
    - The eventual energy adjustments needed to cope with the associated forecast errors and load uncertainties are left to the flexible units participating in the balancing market.
  - The balancing market clears the energy deployed to maintain the balance between supply and demand.
    - Balancing markets allow the trade of energy between flexible firms, which can adjust their output quickly, and stochastic producers, whose generation is predictable only with limited accuracy at the day-ahead stage.
- The day-ahead and the balancing markets are settled independently.

(3) Economic dispatch (ED)

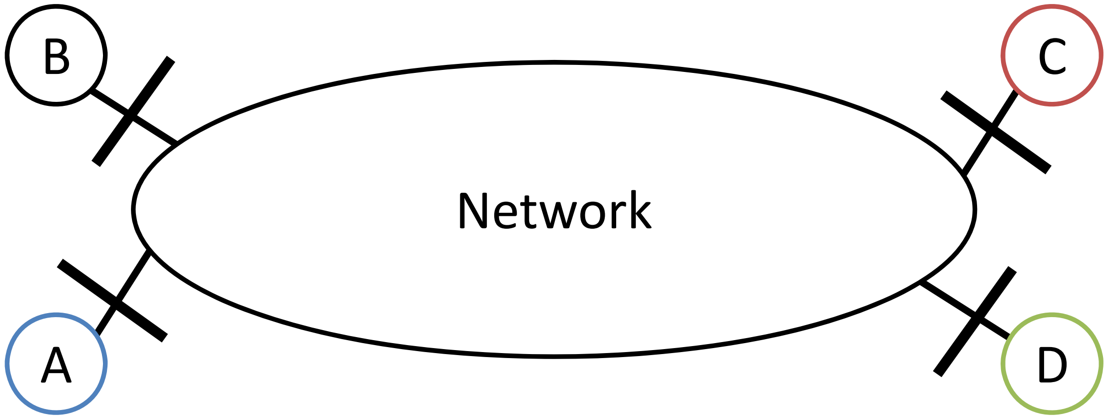
- ED is the process by which the short-term output of each of the electricity generation facilities required to meet the system load is determined in such a way that
  - the overall cost of generation is minimum, subject to transmission and operational constraints

Economic dispatch



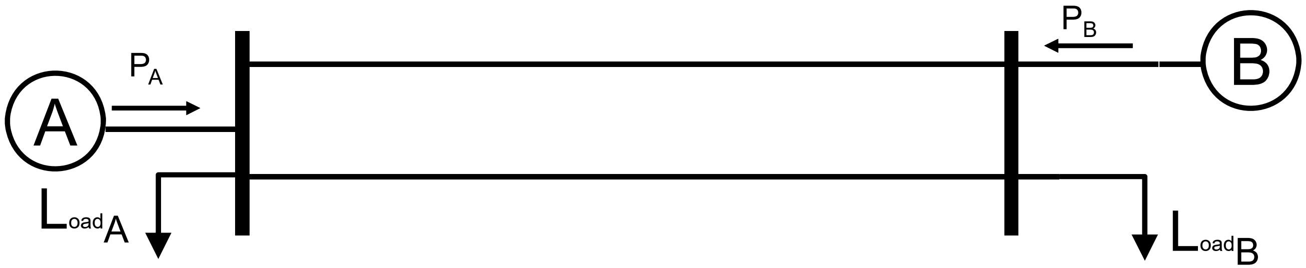
- Objective: minimize the cost of generation
- Constraints
  - Equality constraint: load / generation balance
  - Inequality constraints: upper and lower limits on generating unit outputs

Limitations of ED

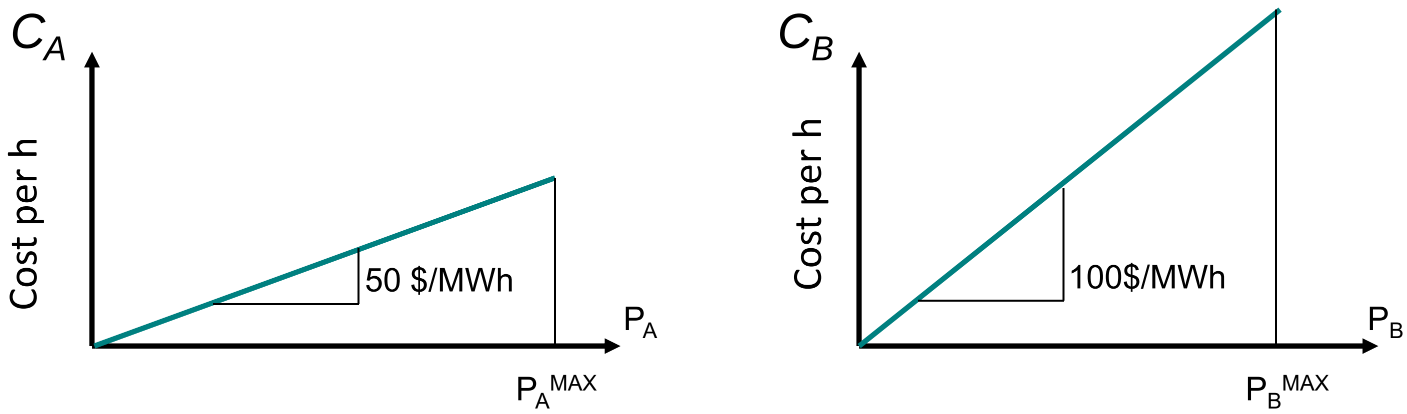


- Normally, generating units and loads are not all connected to the same bus
- ED may result in unacceptable flows, or lead to voltages outside the tolerance band in the network

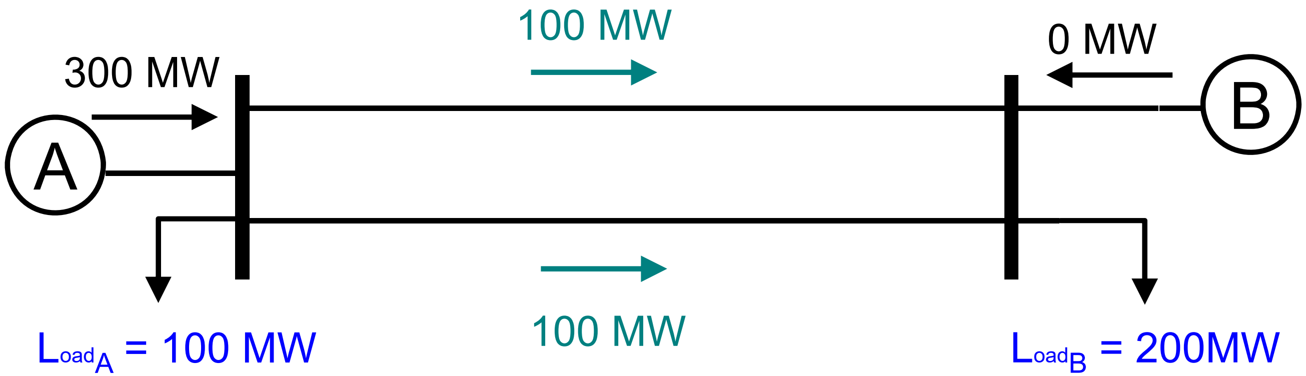
Example of network limitation



Assume maximum flow on each line: 100MW



Acceptable ED solution



The solution of this ED problem is:

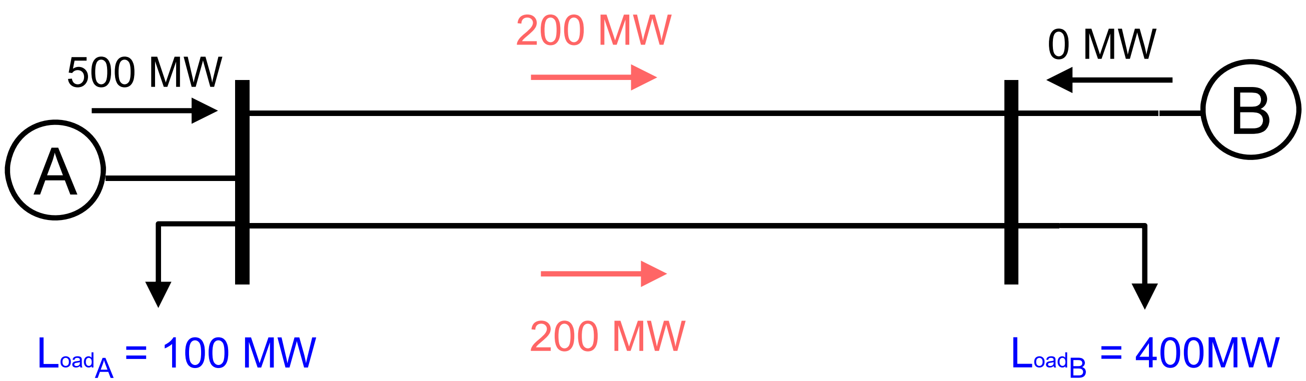
$$P_A = 300 \text{ MW}$$

$$P_B = 0 \text{ MW}$$

The flows on the lines are below the limit

- The ED solution is acceptable

Unacceptable ED solution



The solution of the ED problem now is:

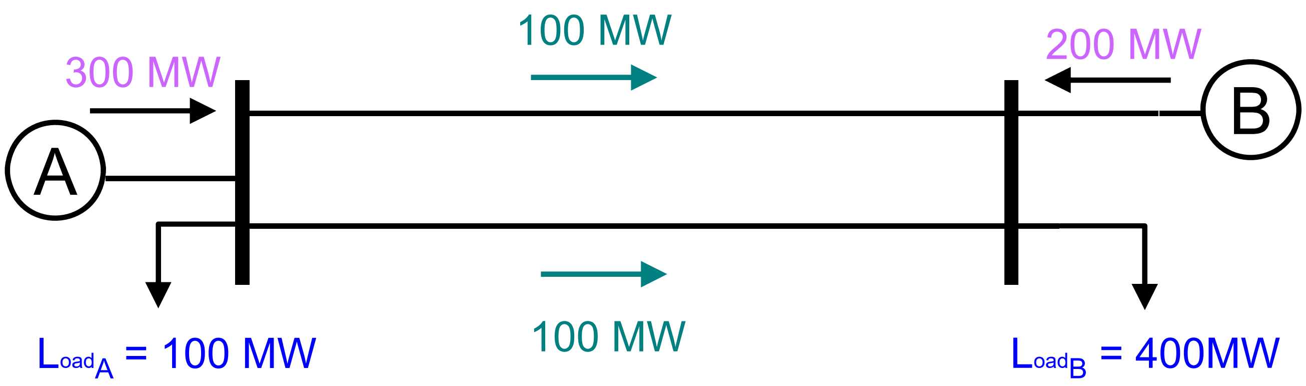
$$P_A = 500 \text{ MW}$$

$$P_B = 0 \text{ MW}$$

The resulting flows exceed line loading limits

- The ED solution is not acceptable

Modified ED solution



In this simple case, the solution of the economic dispatch can be modified easily to produce acceptable flows.

- For example, by adding the inequality constraint:

$$P_A - Load_A \leq 200 \text{ MW}$$

However, adding inequality constraints for each problem is not practical in more complex situations

- A general approach is needed

OPF - Problem formulation

- The classical formulation of OPF is an extension of the classical ED:
  - With the general objective of minimizing the total cost of electricity generation while maintaining the electric power system within permissible operating limits.
- Assume:
  - the power system consists of a set of N buses connected to one another by a set of L branches with controllable generators located at a subset $G \subseteq N$ of the system buses, and
  - the operating cost of each generator is a (typically quadratic) function of its active power output: $C_i = f(P_{Gi})$
    - ❖The objective is: minimize the total cost of generation

Mathematical formulation of OPF

$$\min \sum C_i\left(P_i^{\,G}\right)$$

cost function

$\underline{subject\ to}$:

$$P_i(U,\delta) = P_i^{\,G} - P_i^{\,L}$$
$$Q_i(U,\delta) = Q_i^{\,G} - Q_i^{\,L}$$

power flow equations (in polar form)

$$P_i^{\,G,min} \le P_i^{\,G} \le P_i^{\,G,max}$$
$$Q_i^{\,G,min} \le Q_i^{\,G} \le Q_i^{\,G,max}$$
$$U_i^{\,min} \le U_i \le U_i^{\,max}$$
$$\delta_i^{\,min} \le \delta_i \le \delta_i^{\,max}$$

system voltage and generator output power bounds

Additional constraints

- Modern power grids include additional control devices such as:
  - on-load tap changers,
  - phase shifters,
  - series and shunt capacitors,
  - FACTS devices
  - etc.
- Because such controls exert a big influence on power flow, it is necessary that they are incorporated into practical OPF formulations

OPF – a generalized overview

- OPF is an optimization problem
- Classical objective function
  – Minimize the cost of generation
- Equality constraints
  – Power balance at each node, i.e. power flow equations
- Inequality constraints
  – Network operating limits (line flows, voltage bounds)
  – Limits on control variables

Mathematical formulation of the OPF (1)

- Decision variables (control variables)
  - Active power output of the generating units
  - Voltage at the generating units
  - Position of the transformer taps
  - Position of the phase shifter (quadrature booster) taps
  - Status of the switched capacitors and reactors
  - Control settings of power electronic devices (HVDC, FACTS)
  - etc
- Vector of control variables: $\boldsymbol{u}$

Mathematical formulation of the OPF (2)

- State variables

  State variables describe the response of the system to changes in the control variables, viz.

  – Magnitude of voltage at each bus

    ❖ Except generator busses, which are control variables

  – Angle of voltage at each bus

    ❖ Except the slack bus

- Vector of state variables: $\boldsymbol{x}$

Mathematical formulation of the OPF (3)

- Parameters
  - Known values that describe the system and that remain constant, viz.
    - Network topology
    - Network parameters (R, X, B, flow and voltage limits)
    - Generator cost functions
    - Generator limits
    - …
- Vector of parameters: $\boldsymbol{y}$

Mathematical formulation of the OPF (4)

- Classical objective function:
  - Minimize total generating cost: $$\min \sum_{i=1}^{N_G} C_i(P_i)$$
- Many other objective functions are possible:
  - Minimize changes in controls: $$\min_{u} \sum_{i=1}^{N_U} C_i \left| u_i - u_i^0 \right|$$
  - Minimize system losses
  - …

Mathematical formulation of the OPF (5)

- Equality constraints:
  - Power balance at each node - power flow equations

$$P_k^G - P_k^L = \sum_{i=1}^{N} U_k U_i [G_{ki} \cos(\theta_k - \theta_i) + B_{ki} \sin(\theta_k - \theta_i)]$$

$$Q_k^G - Q_k^L = \sum_{i=1}^{N} U_k U_i [G_{ki} \sin(\theta_k - \theta_i) - B_{ki} \cos(\theta_k - \theta_i)]$$

- Compact expression:

$$\boldsymbol{G}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{y}) = 0$$

Mathematical formulation of the OPF (6)

- Inequality constraints:
  - Limits on the control variables: $\boldsymbol{u}_{min} \leq \boldsymbol{u} \leq \boldsymbol{u}_{max}$
  - Power flow limits: $F_{ij} \leq F_{ij,max}$
  - Voltage limits $U_{j,min} \leq U_j \leq U_{j,max}$
- Compact expression: $H(x, u, y) \geq 0$

Compact form of the OPF problem

$$\min_{\boldsymbol{u}} \boldsymbol{f}(\boldsymbol{u})$$

Subject to: $G(x, u, y) = 0$

$$H(x, u, y) \geq 0$$

OPF Challenges

- Size of the problem
  - thousands of lines, hundreds of controls
  - It is not always clear which inequality constraints are binding
- Problem is non-linear
- Problem is non-convex
- Some of the variables are discrete
  - Position of transformer and phase shifter taps
  - Status of switched capacitors or reactors

Solution methods

- Conventional methods
  - Gradient Methods
  - Newton’s Method
  - Linear Programming Method
  - Quadratic Programming Method
  - Interior Point Method

Solution methods (cont'd)

- Computational Intelligence based approaches
  - Artificial Neural Networks (ANN)
  - Genetic Algorithms
  - Particle Swarm Optimization (PSO)
  - Ant Colony Algorithm
  - etc.

Solving the OPF using gradient methods

- Build the Lagrangian function

$$\ell(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{y}, \lambda, \mu) = \boldsymbol{f}(\boldsymbol{u}) + \boldsymbol{\lambda}^T \boldsymbol{G}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{y}) + \boldsymbol{\mu}^T \boldsymbol{H}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{y})$$

- The gradient of the Lagrangian indicates the direction of the steepest ascent:

$$\nabla \ell(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{y}, \lambda, \mu)$$

- Move in the opposite direction to the point with the largest gradient
- Repeat until $\nabla \ell(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{y}, \lambda, \mu) \approx 0$

Problems associated with gradient methods

- Slow convergence
- Solution requires that objective function and constraints must be differentiable
- Difficulties in handling inequality constraints
  - Binding inequality constraints change as the solution progresses

Solving the OPF using interior point method

- Best technique when a full AC solution is needed
- Handle inequality constraints using barrier functions
- Start from a point in the "interior" of the solution space
- Efficient solution engines are available

Linearizing the OPF problem

- Use the power of linear programming
- Objective function
  - Use linear or piecewise linear cost functions
- Equality constraints
  - Use dc power flow instead of ac power flow
- Inequality constraints
  - dc power flow provides linear relations between injections (control variables) and MW line flows

Sequential LP OPF

- Consequence of linear approximation
  - The solution may be somewhat sub-optimal
  - The constraints may not be respected exactly
- Need to iterate the solution of the linearized problem
- Algorithm:
  1. Linearize the problem around an operating point
  2. Find the solution to this linearized optimization
  3. Perform a full AC power flow at that solution to find the new operating point
  4. Repeat

Advantages and disadvantages

- Advantages of LPOPF method
  - Convergence of linear optimization is guaranteed
  - Fast
  - Reliable optimization engines are available
  - Used to calculate nodal prices in electricity markets
- Disadvantages
  - Need to iterate the linearization
  - "Reactive power" aspects (VAr flows, voltages) are much harder to linearize than the "active power aspects" (MW flows)

DC Power Flow Approximation

Power Flow Equations

$$P_k^I - \sum_{i=1}^{N} V_k V_i [G_{ki} \cos(\theta_k - \theta_i) + B_{ki} \sin(\theta_k - \theta_i)] = 0$$

$$Q_k^I - \sum_{i=1}^{N} V_k V_i [G_{ki} \sin(\theta_k - \theta_i) - B_{ki} \cos(\theta_k - \theta_i)] = 0$$

- Set of non-linear simultaneous equations
- Need a simple linear relation for fast and intuitive analysis
- DC power flow provides such a relation but requires a number of approximations

Neglect Reactive Power

$$P_k^l - \sum_{i=1}^{N} V_k V_i [G_{ki} \cos(\theta_k - \theta_i) + B_{ki} \sin(\theta_k - \theta_i)] = 0$$

$$Q_k^l - \sum_{i=1}^{N} V_k V_i [G_{ki} \sin(\theta_k - \theta_i) - B_{ki} \cos(\theta_k - \theta_i)] = 0$$

$$\Downarrow$$

$$P_k^l - \sum_{i=1}^{N} V_k V_i [G_{ki} \cos(\theta_k - \theta_i) + B_{ki} \sin(\theta_k - \theta_i)] = 0$$

Neglect Resistance of the Branches

$$P_k^l - \sum_{i=1}^{N} V_k V_i [G_{ki} \cos(\theta_k - \theta_i) + B_{ki} \sin(\theta_k - \theta_i)] = 0$$

$$\Downarrow$$

$$P_k^l - \sum_{i=1}^{N} V_k V_i B_{ki} \sin(\theta_k - \theta_i) = 0$$

Assume All Voltage Magnitudes = 1.0 p.u.

$$P_k^l - \sum_{i=1}^{N} V_k V_i B_{ki} \sin(\theta_k - \theta_i) = 0$$

$$\Downarrow$$

$$P_k^l - \sum_{i=1}^{N} B_{ki} \sin(\theta_k - \theta_i) = 0$$

Assume all angles are small

$$P_k^l - \sum_{i=1}^{N} B_{ki} \sin(\theta_k - \theta_i) = 0$$

If α is small: $\sin\alpha \approx \alpha$ (α in radians)

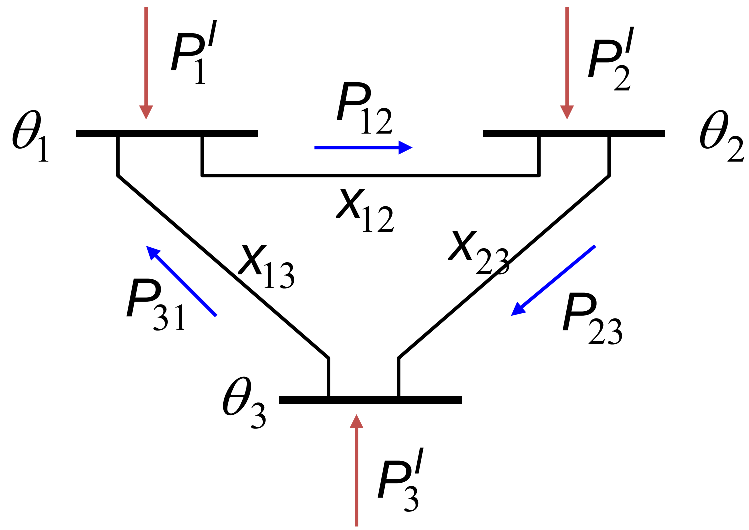
$$P_k^l - \sum_{i=1}^{N} B_{ki} (\theta_k - \theta_i) = 0 \quad \text{or} \quad P_k^l - \sum_{i=1}^{N} \frac{(\theta_k - \theta_i)}{x_{ki}} = 0$$

Interpretation

$$P_k^I - \sum_{i=1}^{N} \frac{(\theta_k - \theta_i)}{x_{ki}} = 0$$

$$P_k^I - \sum_{i=1}^{N} P_{ki} = 0$$

$$P_{ki} = \frac{(\theta_k - \theta_i)}{x_{ki}}$$



$$P_{12} = \frac{(\theta_1 - \theta_2)}{x_{12}}; \quad P_{23} = \frac{(\theta_2 - \theta_3)}{x_{23}}; \quad P_{31} = \frac{(\theta_3 - \theta_1)}{x_{13}}$$

Why is it called DC power flow?

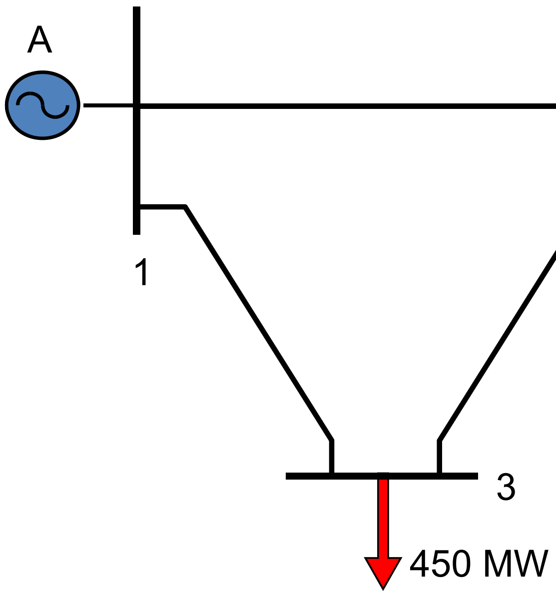
- Reactance plays the role of resistance in DC circuit
- Voltage angle plays the role of DC voltage
- Power plays the role of DC current

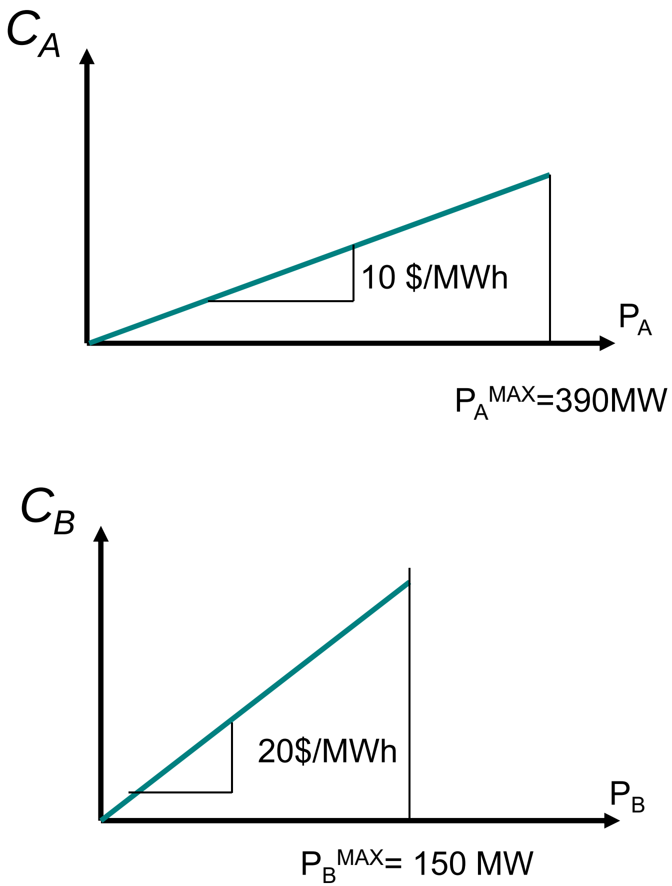
$$P_{ki} = \frac{(\theta_k - \theta_i)}{x_{ki}} \quad \Longleftrightarrow \quad I_{ki} = \frac{(V_k - V_i)}{R_{ki}}$$

Example of LPOPF

- Solving the full non-linear OPF problem by hand is too difficult, even for small systems
- We will solve linearized 3-bus examples by and

Example



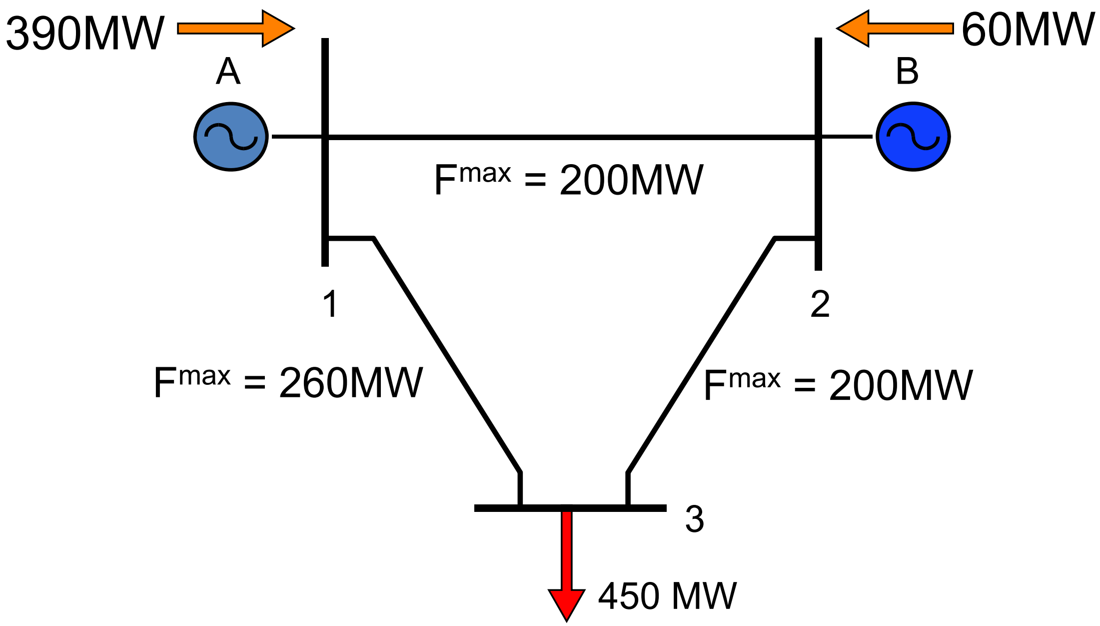


Economic dispatch:

$$P_A = P_A^{\max} = 390 \text{ MW}$$

$$P_B = 60 \text{ MW}$$

Flows resulting from the economic dispatch

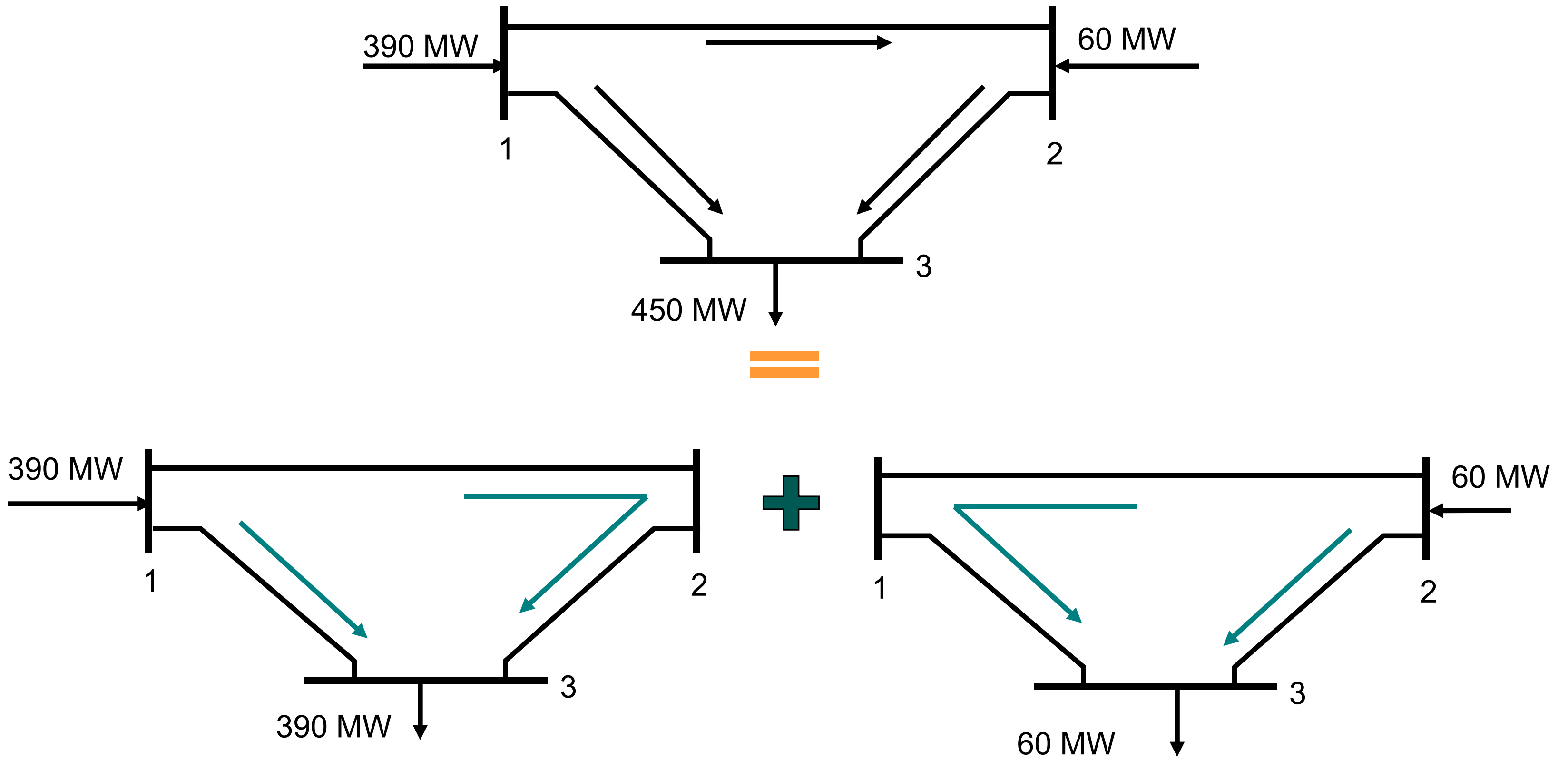


Assume that all the lines have the same reactance

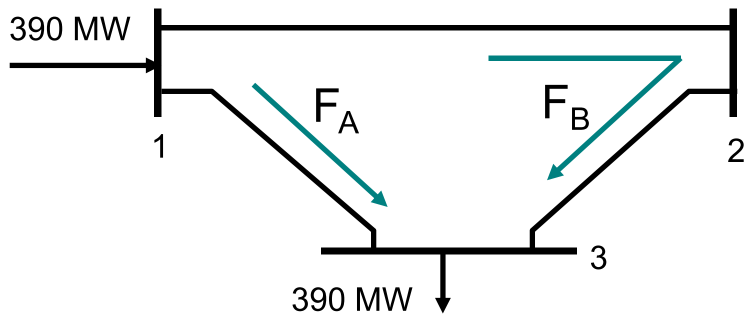
Do these injection result in acceptable flows?

Calculating the flows using superposition

Because we assume a linear model, superposition is applicable



Calculating the flows using superposition (1)

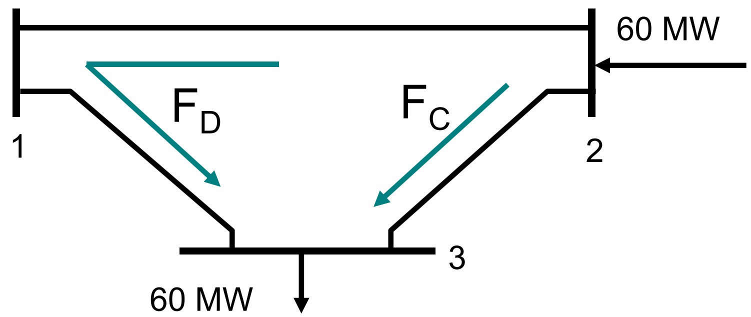


$F_A = 2 \times F_B$ because the path 1-2-3 has twice the reactance of the path 1-3

$F_A + F_B = 390$ MW

$F_A = 260$ MW
$F_B = 130$ MW

Calculating the flows using superposition (2)

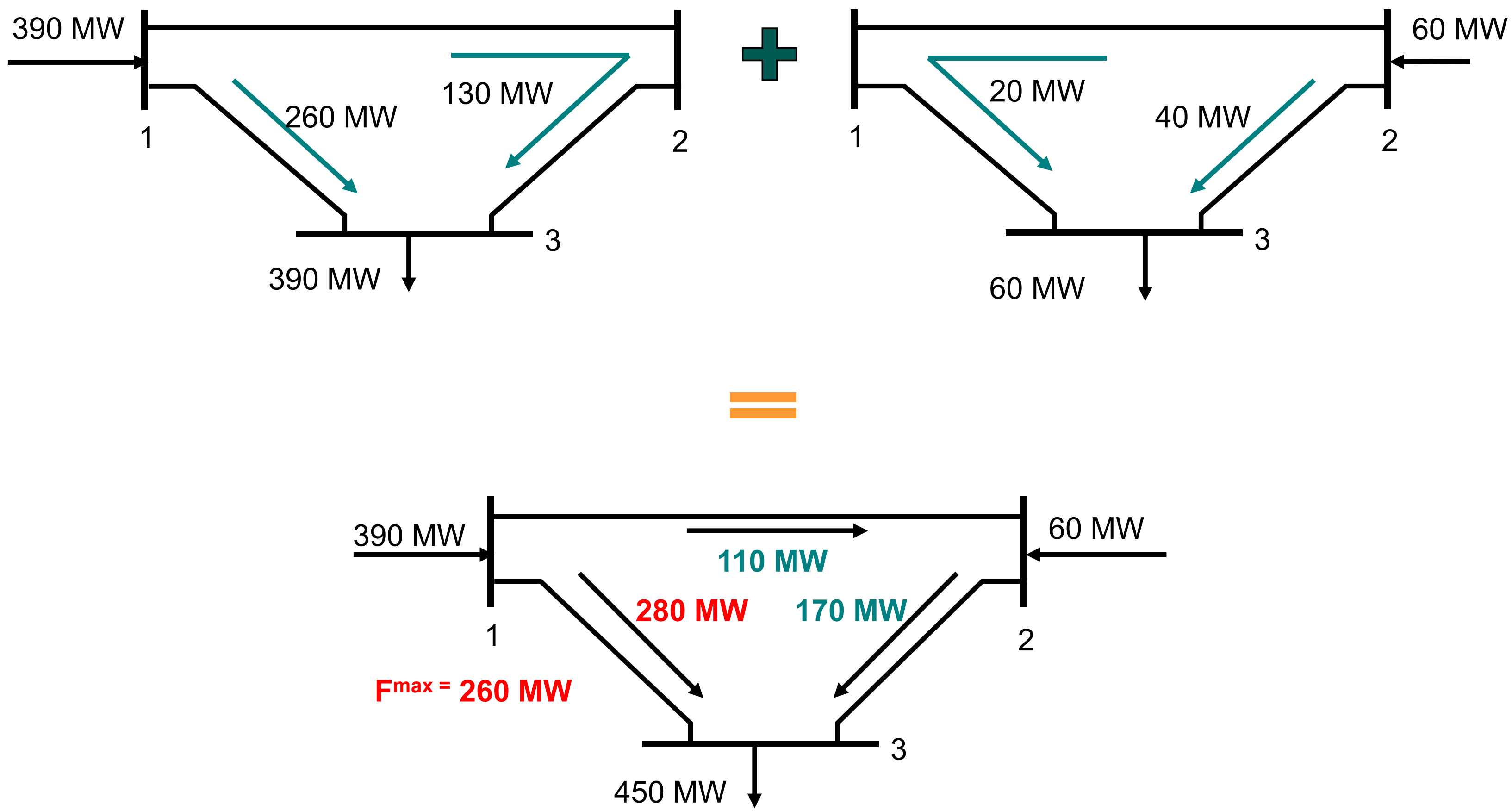


$F_C = 2 \times F_D$ because the path 2-1-3 has twice the reactance of the path 2-3

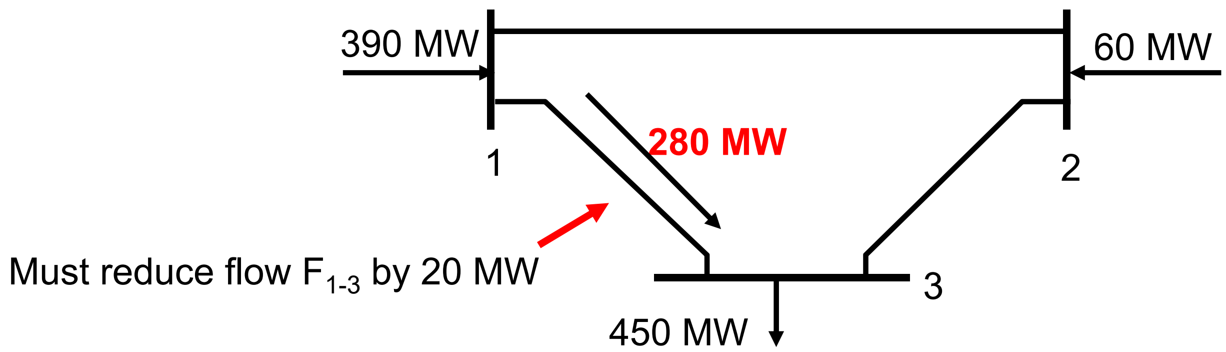
$F_C + F_D = 60$ MW

$F_C = 40$ MW
$F_D = 20$ MW

Calculating the flows using superposition (3)

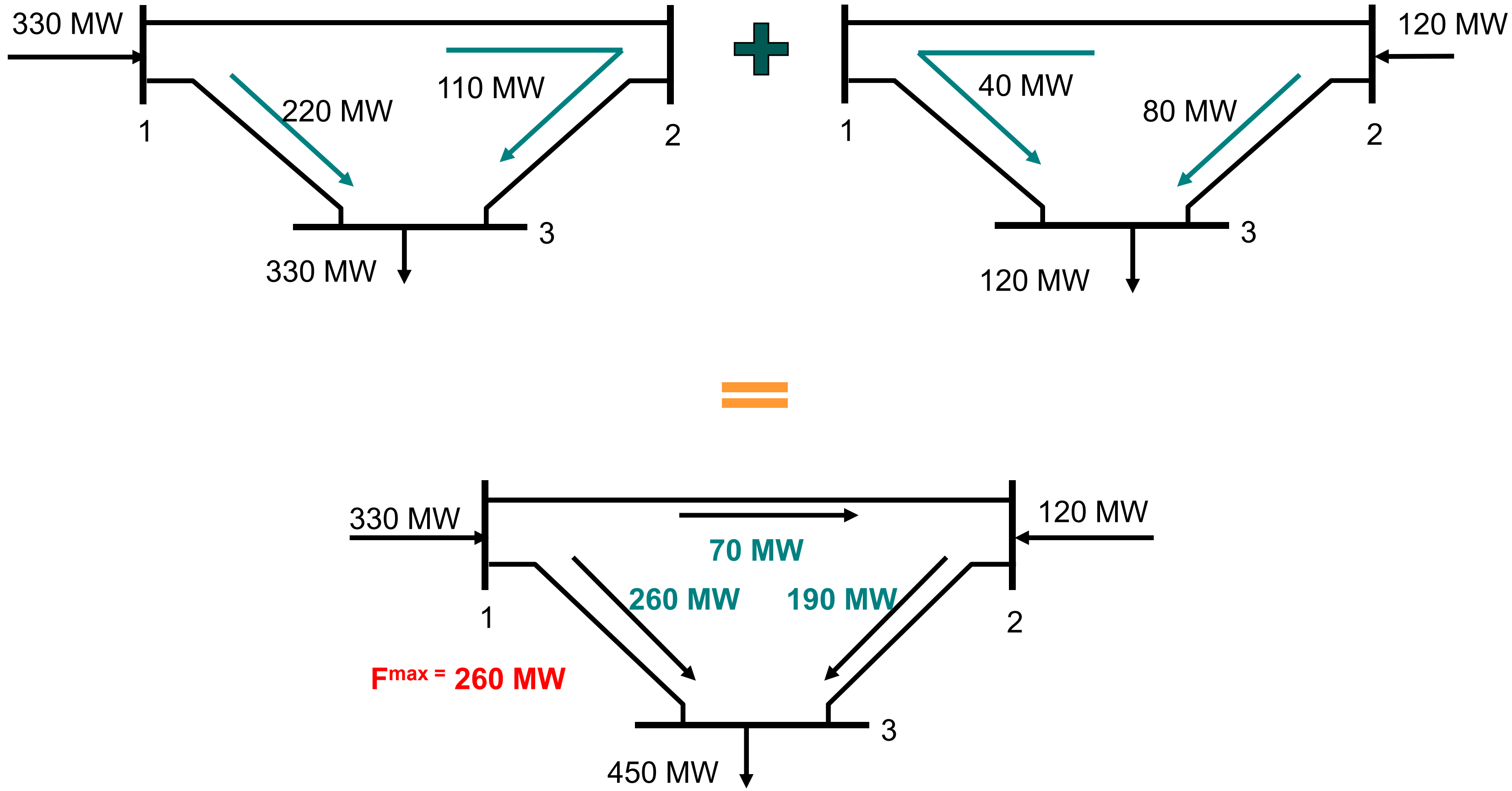


Correcting unacceptable flows



- Must use a combination of reducing the injection at bus 1 and increasing the injection at bus 2 to keep the load/generation balance
- Decreasing the injection at 1 by 3 MW reduces $F_{1\text{-}3}$ by 2 MW
- Increasing the injection at 2 by 3 MW increases $F_{1\text{-}3}$ by 1 MW
- A combination of a 3 MW decrease at 1 and 3 MW increase at 2 decreases $F_{1\text{-}3}$ by 1 MW
- To achieve a 20 MW reduction in $F_{1\text{-}3}$ we need to shift 60 MW of injection from bus 1 to bus 2

Check the solution using superposition



Comments (1)

- The OPF solution is more expensive than the ED solution
  - $C_{ED} = 10 \times 390 + 20 \times 60 = \$5,100$
  - $C_{OPF} = 10 \times 330 + 20 \times 120 = \$5,700$
- The difference is the cost of security
  - $C_{security} = C_{OPF} - C_{ED} = \$600$
- The constraint on the line flow is satisfied exactly
  - Reducing the flow below the limit would cost more

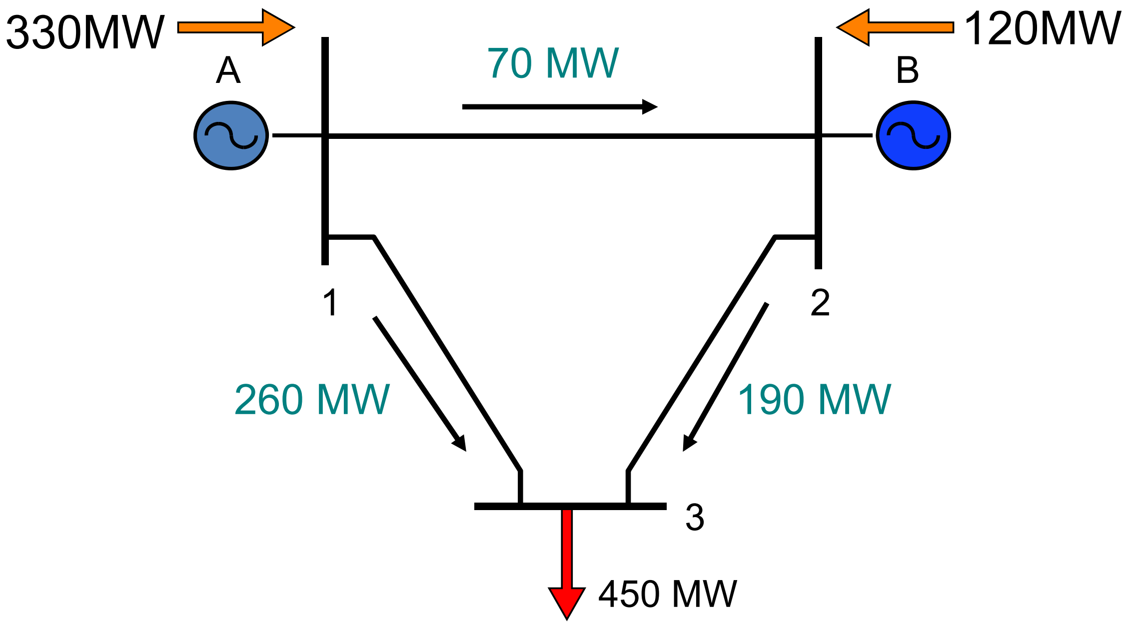
Comments (2)

- We have used an “ad hoc” method to solve this problem
- In practice, there are systematic techniques for calculating the sensitivities of line flows to injections
- These techniques are used to generate constraint equations that are added to the optimization problem

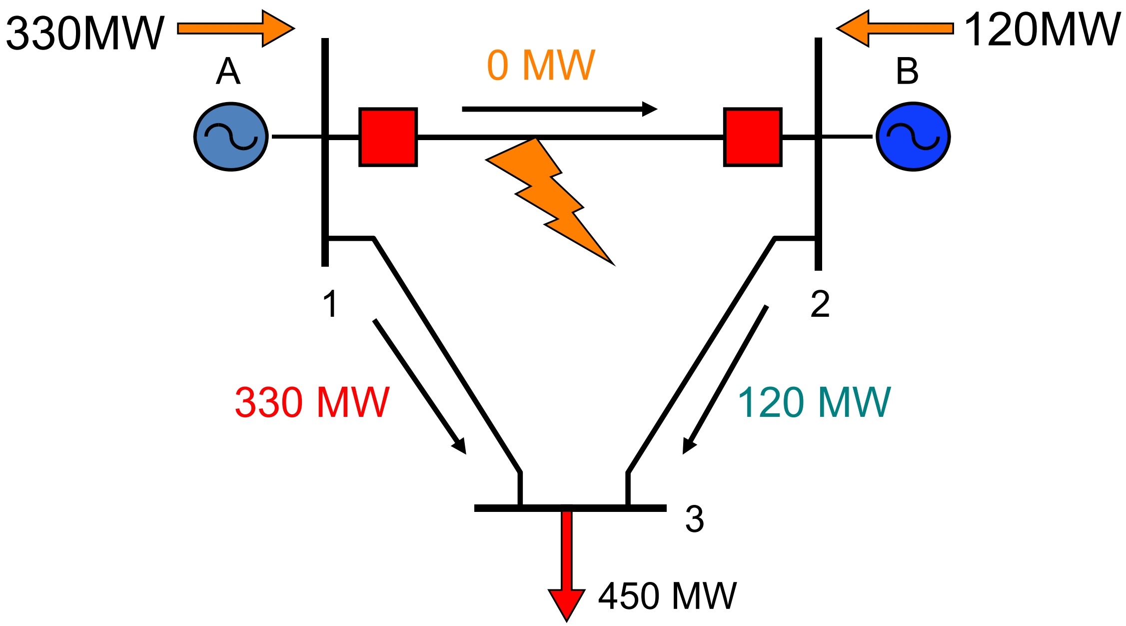
Security Constrained OPF (SCOPF)

- Conventional OPF only guarantees that the operating constraints are satisfied under normal operating conditions
  - All lines in service
- This does not guarantee security
  - Must consider N-1 contingencies

Example: base case solution of OPF



Example: contingency case



Unacceptable because overload of line 1-3 could lead to a cascade trip and a system collapse

Security-Constrained Economic Dispatch

- Security-constrained economic dispatch (SCED), sometimes referred to as security-constrained optimal power flow (SCOPF), is an OPF formulation, which includes power system contingency constraints.
  - A contingency is defined as an event, which removes one or more generators or transmission lines from the power system, increasing the stress on the remaining network.
- SCED seeks an optimal solution that remains feasible under any of a pre-specified set of likely contingency events. SCED is a restriction of the classic OPF formulation:
  - for the same objective function, the optimal solution to SCED will be no better than the optimal solution without considering contingencies.
- The justification for the restriction is that SCED mitigates the risk of a system failure (blackout) should one of the contingencies occur.

Formulation of the Security Constrained OPF

$$\min_{\boldsymbol{u}_0} \boldsymbol{f}(\boldsymbol{u}_0)$$

Subject to: $\boldsymbol{G}(\boldsymbol{x}_0, \boldsymbol{u}_0, \boldsymbol{y}_0) = 0$ ← Power flow equations for the base case

$\boldsymbol{H}(\boldsymbol{x}_0, \boldsymbol{u}_0, \boldsymbol{y}_0) \geq 0$ ← Operating limits for the base case

$\forall k$ $\left\{ \begin{array}{l} \boldsymbol{G}(\boldsymbol{x}_k, \boldsymbol{u}_0, \boldsymbol{y}_k) = 0 \\ \boldsymbol{H}(\boldsymbol{x}_k, \boldsymbol{u}_0, \boldsymbol{y}_k) \geq 0 \end{array} \right.$

$\boldsymbol{G}(\boldsymbol{x}_k, \boldsymbol{u}_0, \boldsymbol{y}_k) = 0$ ← Power flow equations for contingency *k*

$\boldsymbol{H}(\boldsymbol{x}_k, \boldsymbol{u}_0, \boldsymbol{y}_k) \geq 0$ ← Operating limits for contingency *k*

Subscript *0* indicates value of variables in the base case
Subscript *k* indicates value of variables for contingency *k*

Preventive security formulation

$$\min_{\boldsymbol{u}_0} \boldsymbol{f}(\boldsymbol{u}_0) \quad \text{subject to:} \quad \boldsymbol{G}(\boldsymbol{x}_0, \boldsymbol{u}_0, \boldsymbol{y}_0) = 0$$

$$\boldsymbol{H}(\boldsymbol{x}_0, \boldsymbol{u}_0, \boldsymbol{y}_0) \geq 0$$

$$\boldsymbol{G}(\boldsymbol{x}_k, \boldsymbol{u}_0, \boldsymbol{y}_k) = 0$$

$$\boldsymbol{H}(\boldsymbol{x}_k, \boldsymbol{u}_0, \boldsymbol{y}_k) \geq 0$$

This formulation implements preventive security because the control variables are not allowed to change after the contingency has occurred: $u_k = u_0 \quad \forall k$

Corrective security formulation

$$\min_{\boldsymbol{u}_0} \boldsymbol{f}(\boldsymbol{u}_0) \quad \text{subject to: } \begin{aligned} &\boldsymbol{G}(\boldsymbol{x}_0, \boldsymbol{u}_0, \boldsymbol{y}_0) = 0 \\ &\boldsymbol{H}(\boldsymbol{x}_0, \boldsymbol{u}_0, \boldsymbol{y}_0) \geq 0 \\ &\boldsymbol{G}(\boldsymbol{x}_k, \boldsymbol{u}_k, \boldsymbol{y}_k) = 0 \\ &\boldsymbol{H}(\boldsymbol{x}_k, \boldsymbol{u}_k, \boldsymbol{y}_k) \geq 0 \\ &\left| u_k - u_0 \right| \leq \Delta u^{\max} \end{aligned}$$

- This formulation implements corrective security because the control variables are allowed to change after the contingency has occurred
- The last equation limits the changes that can take place to what can be achieved in a reasonable amount of time
- The objective function considers only the value of the control variables in the base case

Size of the SCOPF problem

- Example - European transmission network:
  - 13,000 busses ⇔ 13,000 voltage constraints
  - 20,000 branches ⇔ 20,000 flow constraints
  - N-1 security ⇔ 20,000 contingencies
  - In theory, we must consider
    20,000 x (13,000 + 20,000) = 660 million inequality constraints…
- However:
  - Not all contingencies create limit violations
  - Some contingencies have only a local effect

Limitations of N-1 criterion

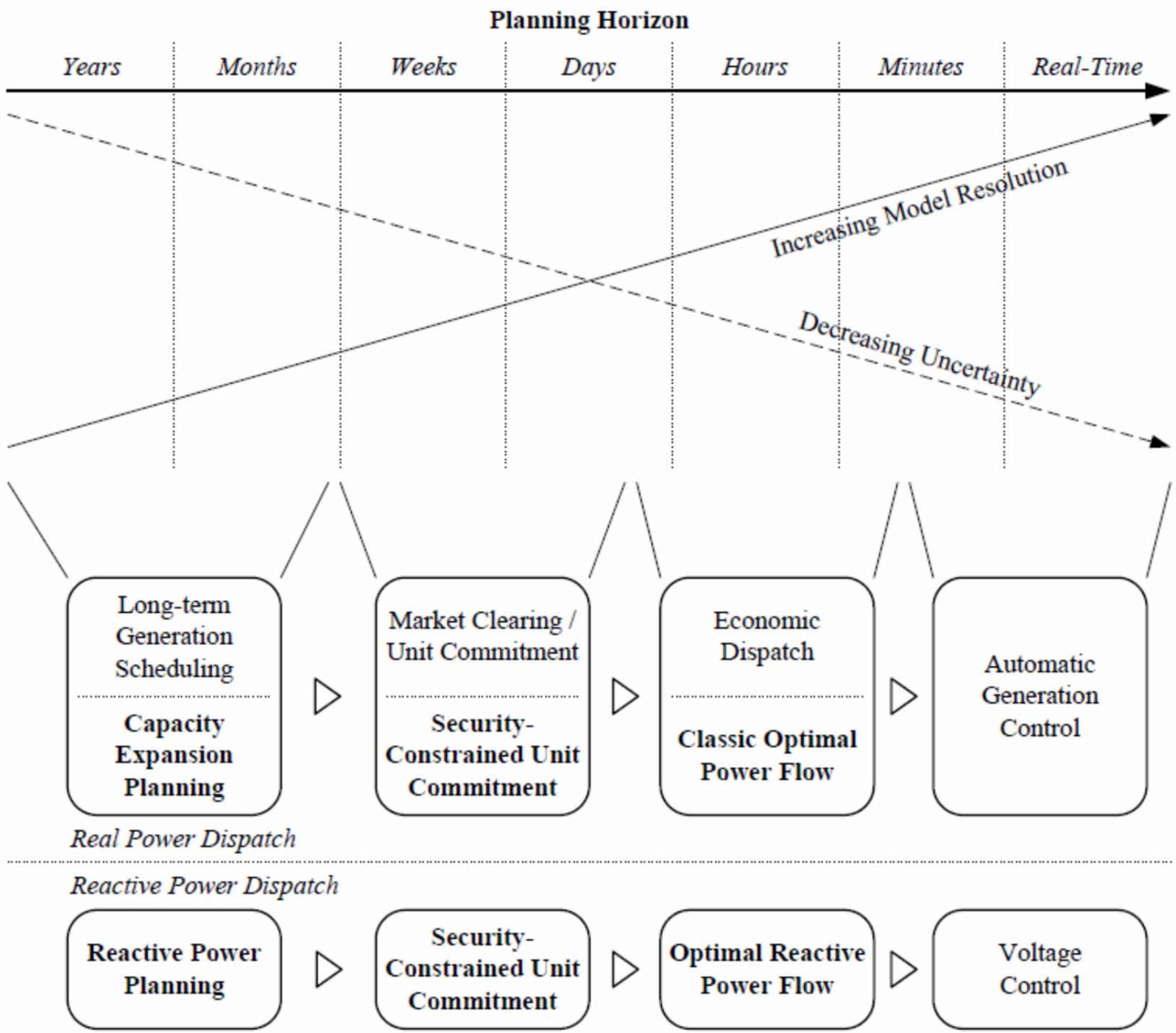
- Not all contingencies have the same probability
  - Long lines vs. short lines
  - Good weather vs. bad weather
- Not all contingencies have the same consequences
  - Local undervoltage vs. edge of stability limit
- N-2 conditions are not always "credible"
  - Non-independent events
- Does not ensure a consistent level of risk
  - Risk = probability x consequences

Probabilistic security analysis

- Goal: operate the system at a given risk level
- Challenges
  - Probabilities of non-independent events
    - “Electrical” failures compounded by IT failures
  - Estimating the consequences
    - What portion of the system would be blacked out?
  - What preventive measures should be taken?
    - Vast number of possibilities

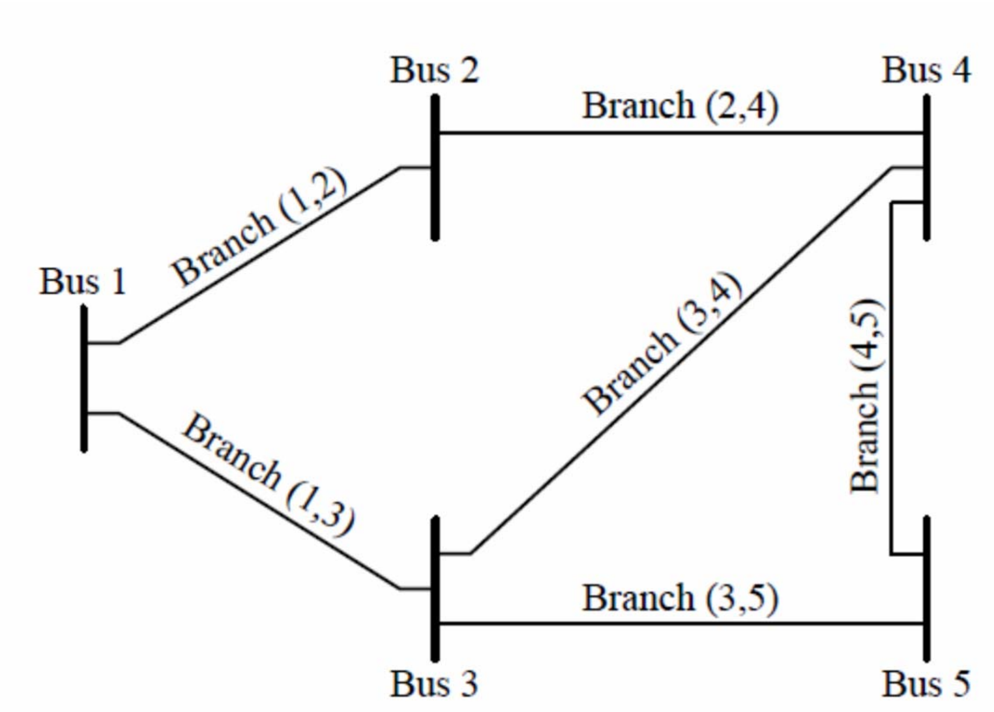
Optimal reactive power flow / planning

- Optimal Reactive Power Flow (ORPF)
  - ORPF, also known as reactive power dispatch or VAR control, seeks to optimize the system reactive power generation in order to minimize the total system losses. In ORPF, the system active power generation is determined a priori, from the outcome of, for example, a DC-OPF algorithm, UC, or another form of ED.
- Reactive Power Planning (RPP)
  - RPP extends the ORPF problem to the optimal allocation of new reactive power sources such as capacitor banks within a power system in order to minimize either system losses or total costs. RPP modifies ORPF to include a set of possible new reactive power sources; the presence or absence of each new source is modeled with a binary variable. The combinatorial nature of installing new reactive power sources has inspired many papers which apply heuristic methods to RPP.



Planning Horizon
Years
Months
Weeks
Days
Hours
Minutes
Real-Time
Increasing Model Resolution
Decreasing Uncertainty
Long-term Generation Scheduling
Capacity Expansion Planning
Market Clearing / Unit Commitment
Security-Constrained Unit Commitment
Economic Dispatch
Classic Optimal Power Flow
Automatic Generation Control
Real Power Dispatch
Reactive Power Dispatch
Reactive Power Planning
Security-Constrained Unit Commitment
Optimal Reactive Power Flow
Voltage Control

Example



Bus 2
Bus 4
Branch (2,4)
Branch (1,2)
Bus 1
Branch (3,4)
Branch (4,5)
Branch (1,3)
Branch (3,5)
Bus 3
Bus 5

Branch data

From i	To k	resistance R_{ik}	reactance X_{ik}	Susceptance B_{ik}	Voltage ratio	Phase angle
1	2	0	0.3			
1	3	0.023	0.145	0.04		
2	4	0.006	0.032	0.01		
3	4	0.02	0.26			-3.0^0
3	5	0	0.32		0.98	
4	5	0	0.5			

The admittance matrix:

$$\widetilde{Y} \approx \begin{pmatrix} 1.07 - j10.04 & 0.00 + j3.33 & -1.07 + j6.73 & 0 & 0 \\ 0.00 + j3.33 & 5.66 - j33.22 & 0 & -5.66 + j30.19 & 0 \\ -1.07 + j6.73 & 0 & 1.41 - j13.78 & -0.09 + j3.83 & 0.00 + j3.19 \\ 0 & -5.66 + j30.19 & -0.49 + j3.80 & 5.95 - j36.01 & 0.00 + j2.00 \\ 0 & 0 & 0.00 + j3.19 & 0.00 + j2.00 & 0.00 - j5.13 \end{pmatrix}$$

Power flow equations

Bus 1 as an example

$$P_1(U,\delta) = U_1 \sum_{k=1}^{5} U_k(G_{1k}\cos(\delta_1 - \delta_k)) + (B_{1k}\sin(\delta_1 - \delta_k))$$

$$= 1.07\, U_1{}^2 - 1.07 U_1 U_3 \cos(\delta_1 - \delta_3) + 3.33\, U_1 U_2 \sin(\delta_1 - \delta_2) + 6.73\, U_1 U_3 \sin(\delta_1 - \delta_3)$$

$$Q_1(U,\delta) = U_1 \sum_{k=1}^{5} U_k(G_{1k}\sin(\delta_1 - \delta_k)) - (B_{1k}\cos(\delta_1 - \delta_k))$$

$$= -1.07 U_1 U_3 \sin(\delta_1 - \delta_3) + 10.04\, U_1{}^2 - 3.33\, U_1 U_2 \cos(\delta_1 - \delta_2) - 6.73\, U_1 U_3 \cos(\delta_1 - \delta_3)$$

Formulation of the OPF algorithm

Nodal bus admittance matrix including the phase shifter and tap changer

$$-30^0 \leq \varphi_{34} \leq 30^0$$

$$0.95 \leq T_{34} \leq 1.05$$