State Estimation

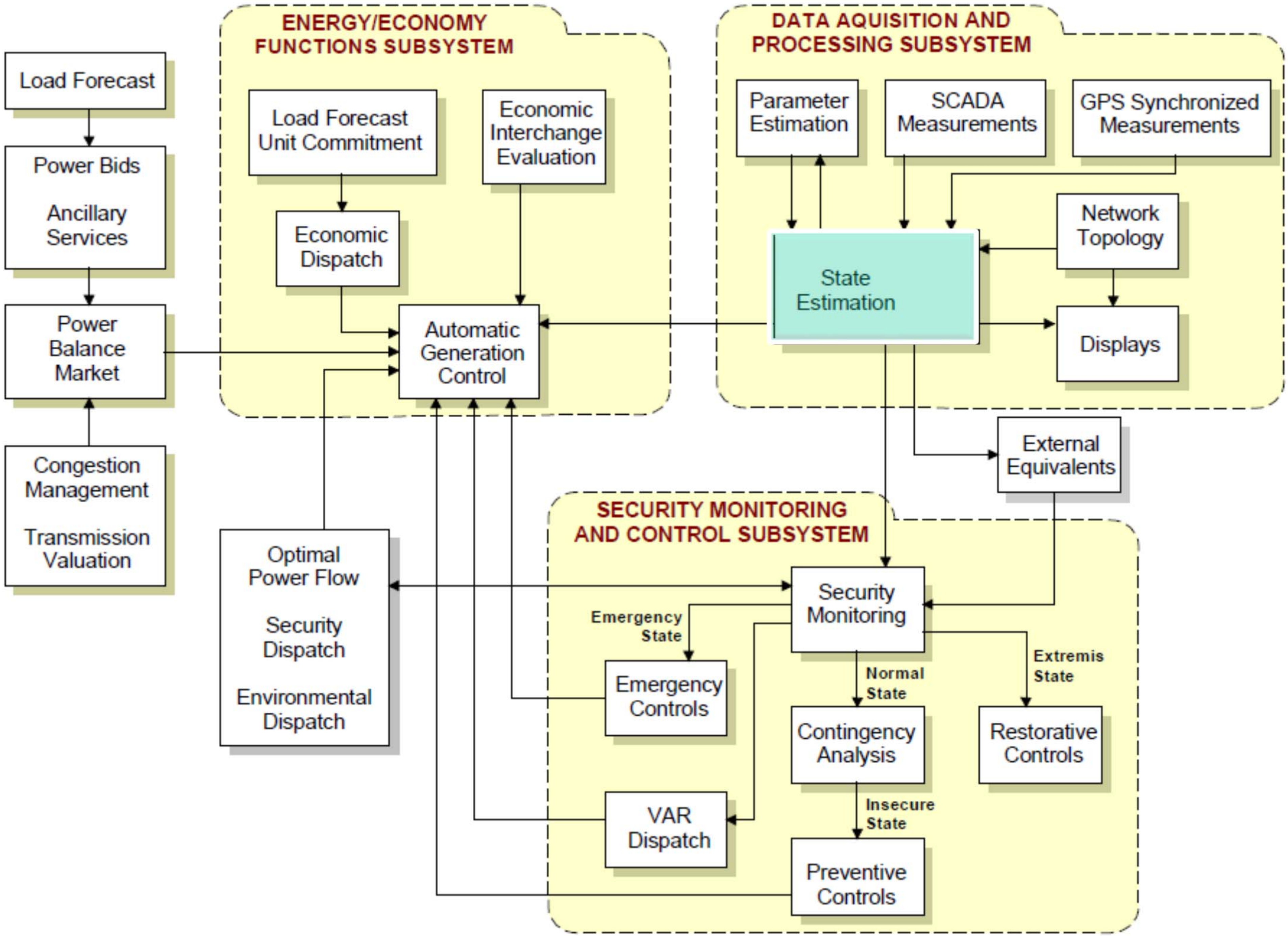
State estimation (SE)

- What is a "state"?
  - All variables in a power network can be calculated if voltage magnitudes and angles at all buses are known.
    - These quantities provide the unique description of the state of the system at this operating point
      - ✓ are the "state variables" of the system.
- Why "estimate" the state?
  - Meters aren't everywhere.
  - Meters aren't perfect.
  - Voltage phase angle measurement difficult

SE as part of EMS functions

- The state estimator is a central part of every control center.
- Out of all energy management system (EMS) functions, SE is the most important, because
  - Other EMS functions will work only when SE is running normally.
  - SE gives the base case for further analysis.

✓ SE result is the starting point for other applications dealing with contingency analysis and system optimization

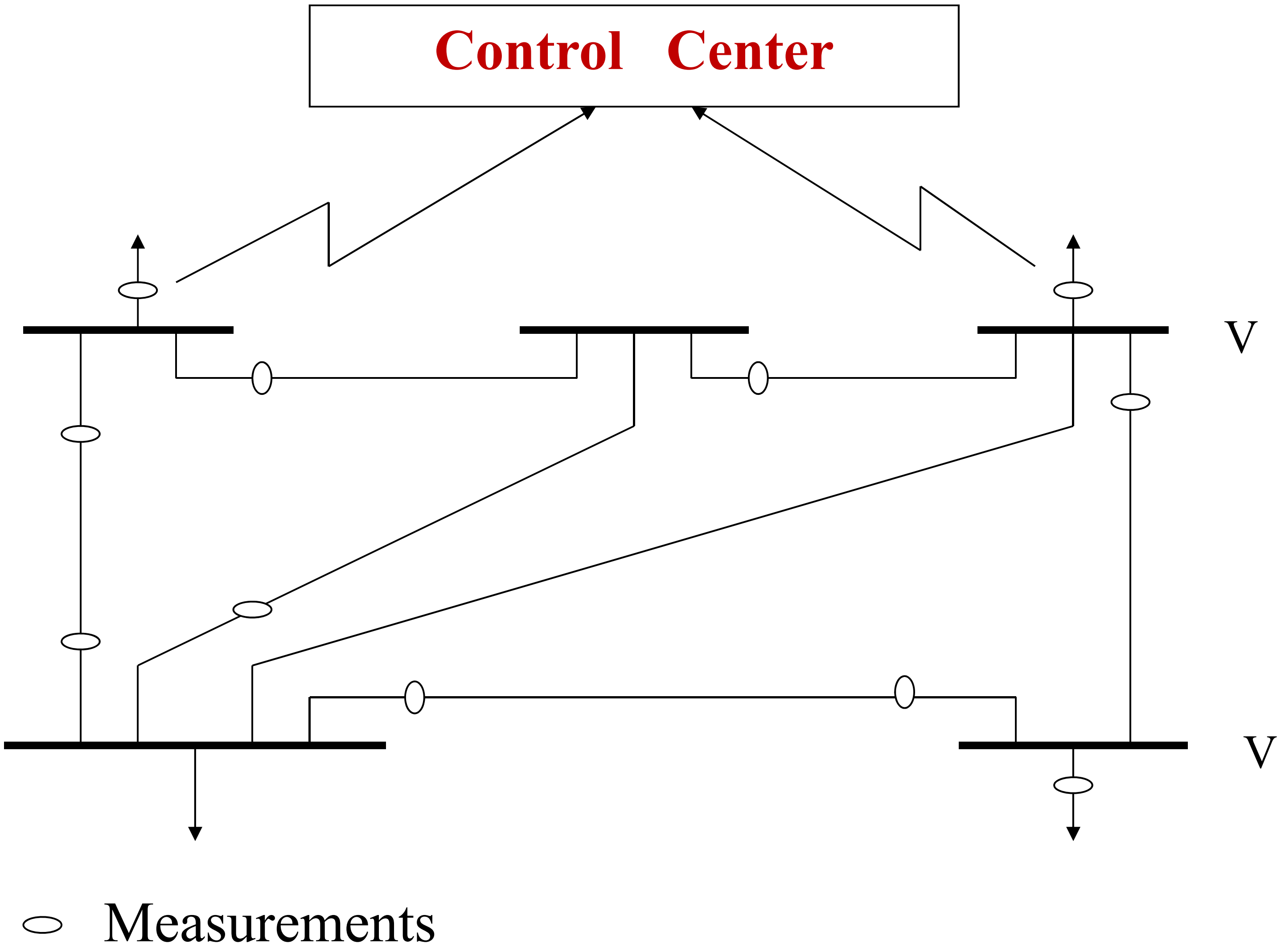


Load Forecast
Power Bids
Ancillary Services
Power Balance Market
Congestion Management
Transmission Valuation
ENERGY/ECONOMY FUNCTIONS SUBSYSTEM
Load Forecast Unit Commitment
Economic Interchange Evaluation
Economic Dispatch
Automatic Generation Control
Optimal Power Flow
Security Dispatch
Environmental Dispatch
DATA AQUISITION AND PROCESSING SUBSYSTEM
Parameter Estimation
SCADA Measurements
GPS Synchronized Measurements
Network Topology
State Estimation
Displays
External Equivalents
SECURITY MONITORING AND CONTROL SUBSYSTEM
Security Monitoring
Emergency State
Emergency Controls
Normal State
Extremis State
Contingency Analysis
Restorative Controls
Insecure State
VAR Dispatch
Preventive Controls

State estimation - definition

- Definition:
  - ✓ State Estimation is the process of assigning values to unknown system state variables based on limited measurements from that system.
  - – SE provides an estimate for all metered and unmetered quantities;
  - – Filters out small errors due to model approximations and measurement inaccuracies;
  - – Detects and identifies discordant measurements, the so-called bad data.
  - – Detects topology error

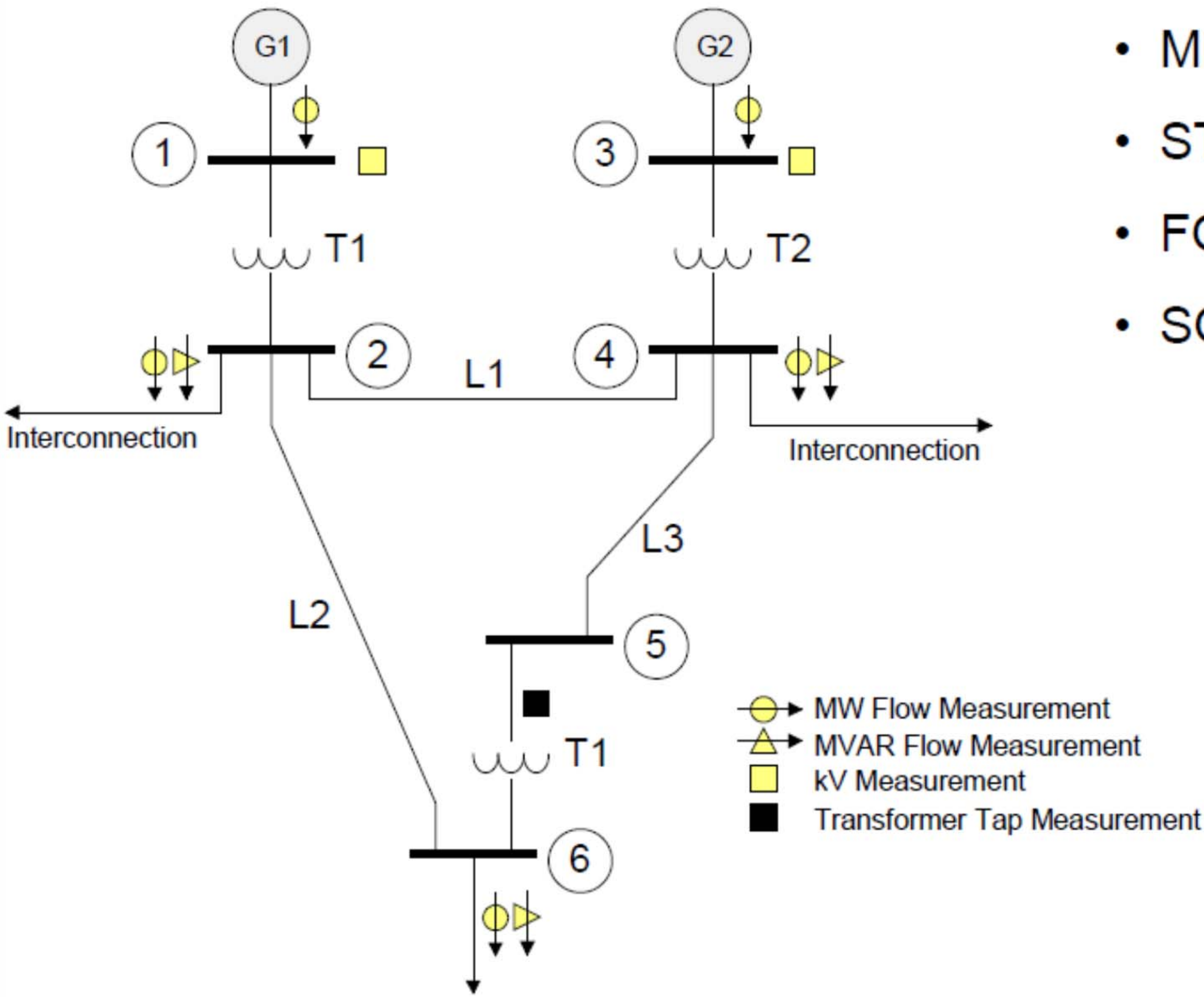
State Estimation



Measurement for use in SE

- Measurements that can be used
  - Bus voltage magnitudes.
  - Active, reactive and current injections.
  - Active, reactive and current branch flows.
  - Bus voltage magnitude and angle differences.
  - Transformer tap/phase settings.
  - Sums of real and reactive power flows.
  - Active and reactive zone interchanges.

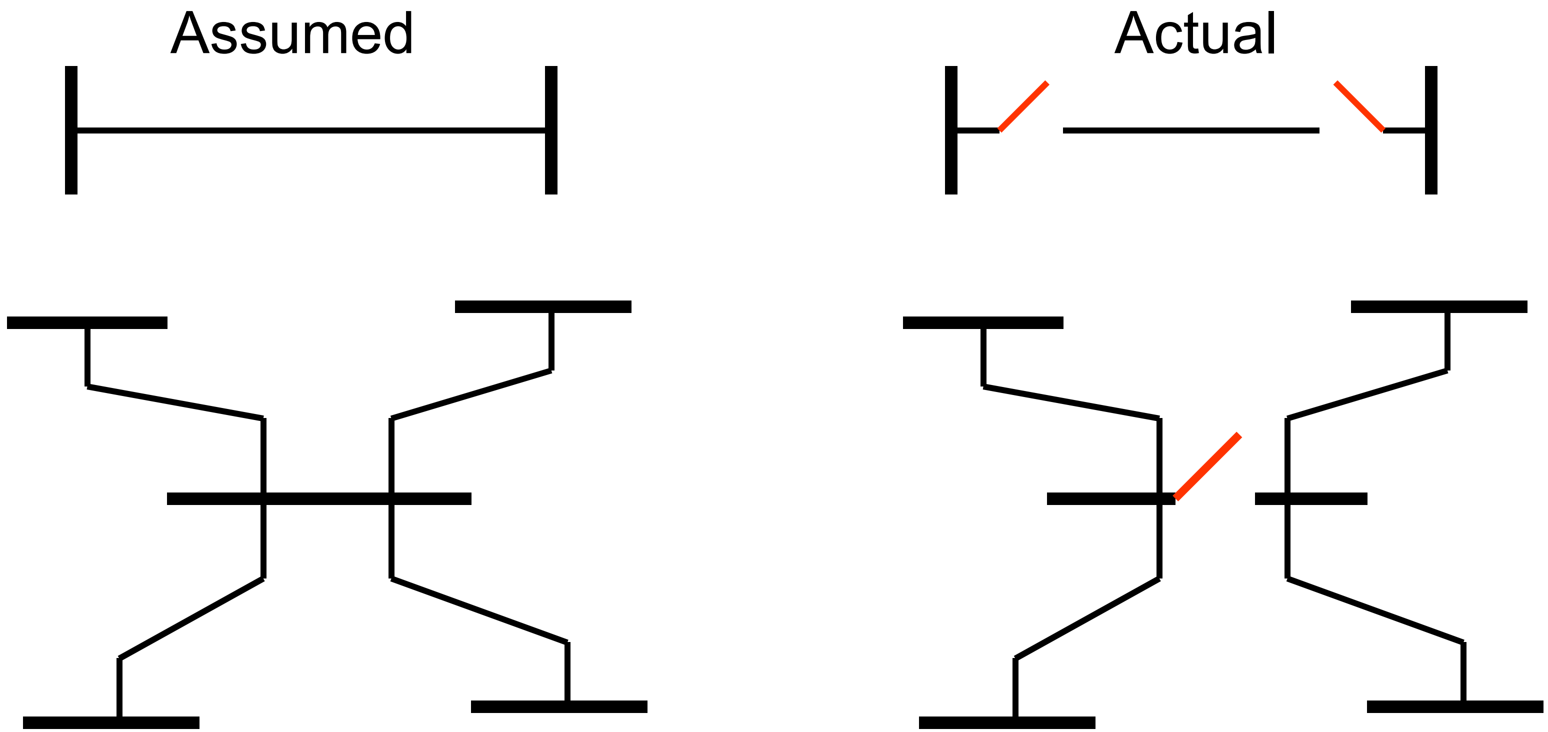
State Estimator



- MEASUREMENTS:
- STATE:
- FORMULATION:
- SOLUTION:

Topology Error Identification

- A topology error is caused by errors in the status of the circuit breakers of a line, a transformer, a shunt capacitor or a bus coupler.



State estimation - process

- The process involves imperfect, redundant measurements
  - the process of estimating the system states is based on a statistical criterion that estimates the true value of the state variables by minimizing the error.
  - ✓ Most Commonly used method: minimizing **Weighted Least Squares**

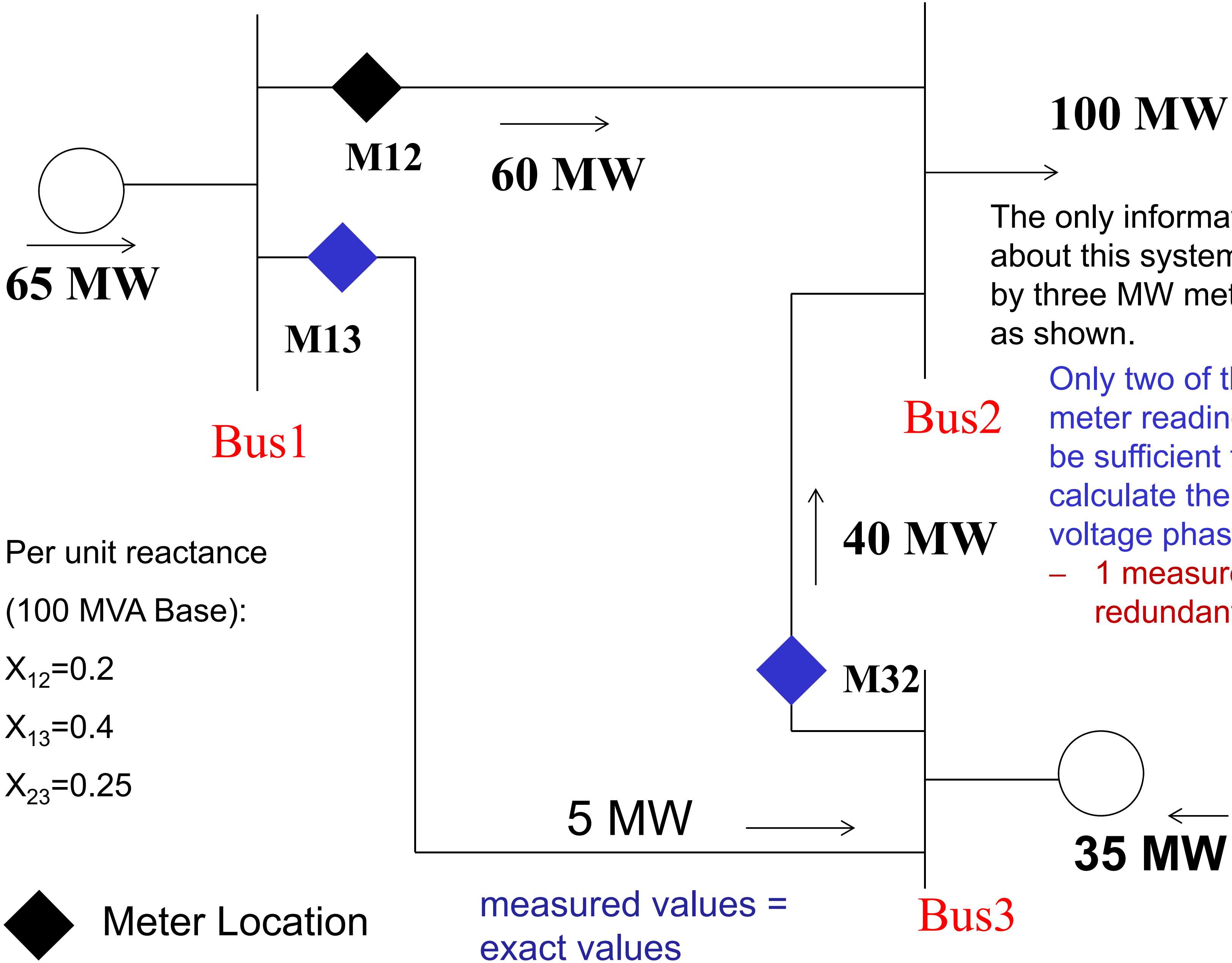
General assumptions:

- The system is balanced.
- The line parameters are known.
- The topology is known.
- No time-skew between measurements.

State Estimation - process

- Inputs to the estimator:
  - measurements (voltage magnitude, P, Q, or I flows).
- The estimator algorithm:
  - is designed to produce the "best estimate" of the system voltage and phase angles, recognizing that there can be errors in the measured quantities and that there may be redundant measurements
- Output:
  - State Variables (voltage magnitudes and relative phase angles at all network nodes).

Definition of the problem - example



The only information we have about this system is provided by three MW meters located as shown.

Only two of these meter readings would be sufficient to calculate the bus voltage phase angles

- 1 measurement is redundant

Per unit reactance

(100 MVA Base):

X_{12}=0.2

X_{13}=0.4

X_{23}=0.25

measured values = exact values

Solution - M_{13} and M_{32} are chosen

$$P_{12} = \frac{U_1 U_2}{X_{12}} sin\theta_{12}$$

$U_1 \cong U_2 \cong 1$ p.u.; $sin\theta_{12} = \sin(\theta_1 - \theta_2) \approx \theta_1 - \theta_2$ **(DC Power Flow)**

Measurement: M_{13} = 5 MW = 0.05 p.u. M_{32} = 40 MW = 0.40 p.u.

Functions: $P_{13} = f_{13} = 1/x_{13} * (\theta_1 - \theta_3) = M_{13} = 0.05$ p.u.

$P_{32} = f_{32} = 1/x_{32} * (\theta_3 - \theta_2) = M_{32} = 0.40$ p.u.

Solution:

$1/0.4 * (\theta_1 - 0) = 0.05$ ($\theta_3 = 0$ (chosen to be reference bus))

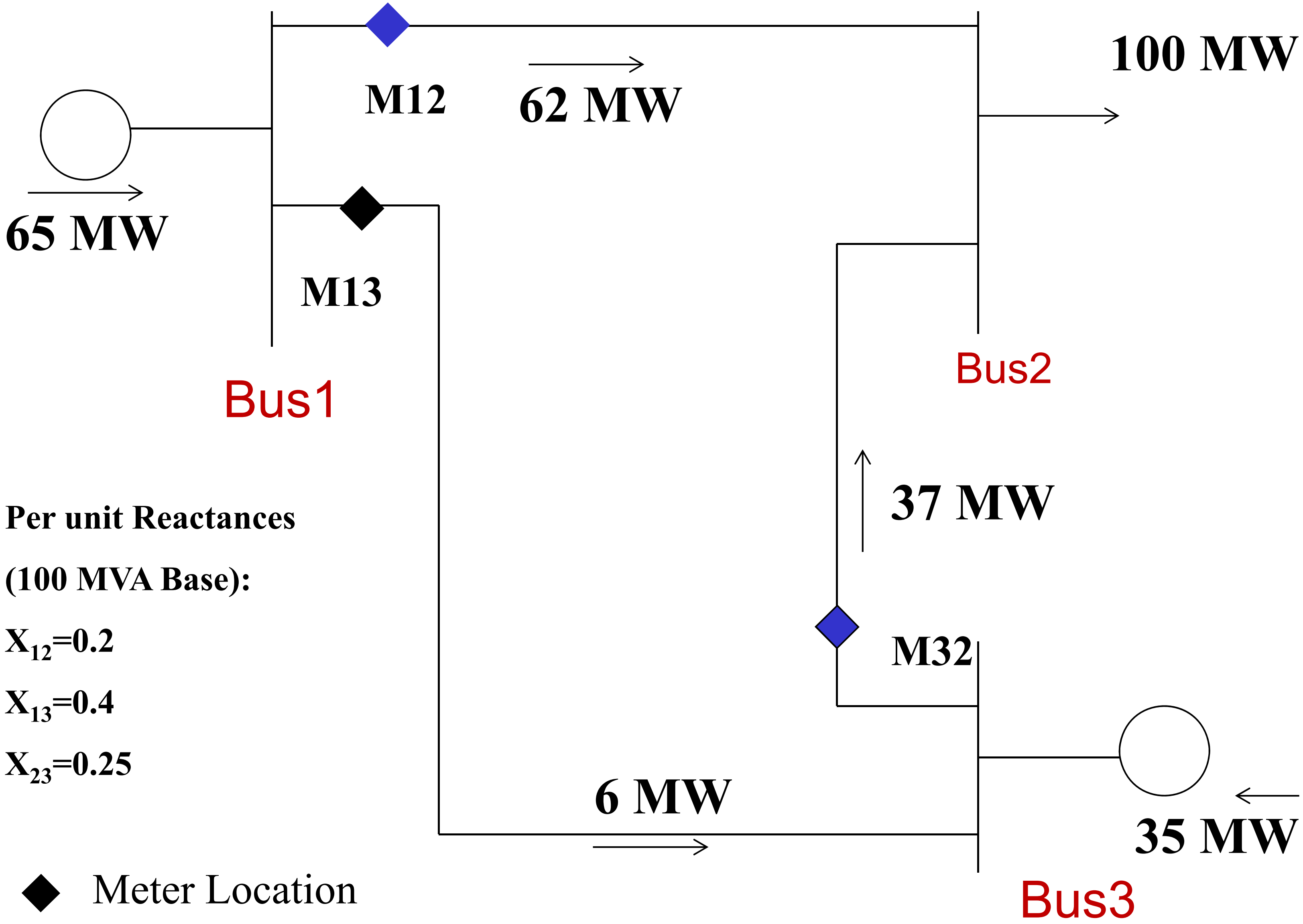
$1/0.25 * (0 - \theta_2) = 0.40$

$\theta_3 = 0.0$

$\theta_1 = 0.02$ rad

$\theta_2 = -0.10$ rad

Case with measurement error



Per unit Reactances

(100 MVA Base):

X_{12}**=0.2**

X_{13}**=0.4**

X_{23}**=0.25**

Solution using different measurements

Solution using M_{13} and M_{32}

M_{13}=6 MW=0.06 p.u.

M_{32} =37 MW=0.37 p.u.

$f_{13}=1/x_{13}*(\theta_1-\theta_3)=M_{13}=0.06$

$f_{32}=1/x_{32}*(\theta_3-\theta_2)=M_{32}=0.37$

$\theta_3=0$

$1/0.4*(\theta_1-0)=0.06$

$1/0.25*(0-\theta_2)=0.37$

θ_1 = 0.024 rad

θ_2 = -0.0925 rad

Solution using M_{12} and M_{32}

M_{12}=62 MW=0.62 p.u.

M_{32} =37 MW=0.37 p.u.

$f_{12}=1/x_{12}*(\theta_1-\theta_2)=M_{12}=0.62$

$f_{32}=1/x_{32}*(\theta_3-\theta_2)=M_{32}=0.37$

$\theta_3=0$

$1/0.2*(\theta_1-\theta_2)=0.62$

$1/0.25*(0-\theta_2)=0.37$

θ_1 = 0.0315 rad

θ_2 = -0.0925 rad

SE procedure

- To estimate the two states θ_1 and θ_2,only two measurements would be enough

  SE uses information available from all three meters to produce the best estimate

  - ✓ the redundant measurement is utilized to improve estimation accuracy, detect bad data and topology error

Solution Algorithms

- Objective… Weighted Least Squares:

Minimize: $J(x) = 0.5\, [Z - h(x)]^t R^{-1} [Z - h(x)]$

$$Z = [\, h(x) + \eta \,]$$

where,

J = Weighted least squares matrix
R = Error covariance matrix
Z = Measurement vector
h (x) = System model relating state vector to the measurement set
x = State vector (voltage magnitudes and angles)
η = Error vector associated with the measurement set

Mathematical formulation

In the previous example:

$f_{12}=1/x_{12}*(\theta_1- \theta_2)$
$f_{13}=1/x_{13}*(\theta_1- \theta_3)$
$f_{32}=1/x_{32}*(\theta_3- \theta_2)$

$$h(x) = \begin{pmatrix} 1/x_{12} & -1/x_{12} & 0 \\ 1/x_{13} & 0 & -1/x_{13} \\ 0 & 1/x_{32} & -1/x_{32} \end{pmatrix} . \underline{x}$$

$\underline{x} = (\theta_1 \ \theta_2 \ \theta_3)^T$

$Z = (M_{12} \ M_{13} \ M_{32})^T$

η : unknown

General solution of the SE problem

- True values : $\underline{\hat{z}} = \begin{pmatrix} \hat{z}_1 \\ \hat{z}_2 \\ \vdots \\ \hat{z}_n \end{pmatrix}$

- Errors: $\underline{\eta} = \begin{pmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_n \end{pmatrix}$

- Measured $\underline{z} = \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{pmatrix} = \underline{h}(\underline{x})$

$$\underline{z} = \underline{\hat{z}} + \underline{\eta} = \underline{h}(\underline{x}) \rightarrow \underline{\eta} = \underline{z} - \underline{h}(\underline{x})$$

General solution of the SE problem

$\underline{\eta} = \underline{z} - \underline{h}(\underline{x})$

The sum of the squared errors:

$$J = \frac{1}{2}\sum_{i=1}^{m} {\eta_i}^2 = \frac{1}{2}\underline{\eta}^T\underline{\eta} = \frac{1}{2}\left(\underline{z} - \underline{h}(\underline{x})\right)^T\left(\underline{z} - \underline{h}(\underline{x})\right)$$

Solution of the SE problem→
Determining $\underline{x}$ that minimizes J

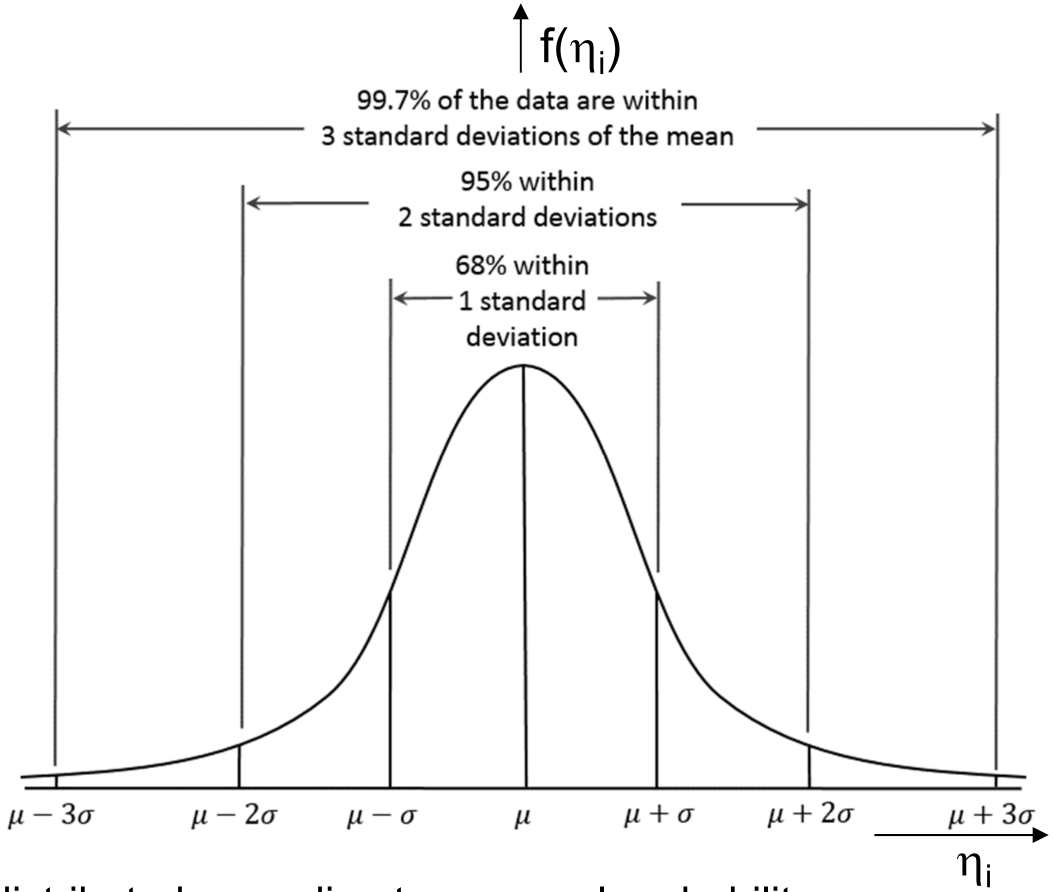
- Some measurement devices are more precise than others →
  - It is therefore reasonable to place more weight on the better measuring devices.

Calibration curve

η_i assumed to be a *random variable* with a normal (Gaussian) distribution having zero mean

$$f(x \mid \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Where:
μ : the mean value
σ : the standard deviation
σ^2 : the variance



- Measurement errors distributed according to a normal probability density function with the standard deviation σ
  - Meter reading will be within +/- 3σ of the true value for 99.7 % of the time.
- Example: meter full scale value = 100 MW; accuracy +/- 3 MW
  - σ = 1MW/100 MW = 0.01

Weighted least squares solution

- Classical Approach: Weighted Least Squares

Minimize: $J(x) = \frac{1}{2}\left(\underline{z} - \underline{h}(\underline{x})\right)^T . W . \left(\underline{z} - \underline{h}(\underline{x})\right)$

where,

J = Weighted least squares matrix

W = Weighting matrix

Place more weight on better measuring devices:

Good device → small variance ${\sigma_i}^2$ → large covariance $\frac{1}{{\sigma_i}^2}$

Conversely, bad device → small covariance $\frac{1}{{\sigma_i}^2}$

Weighting matrix W → Covariance matrix

SE problem in final form

Find $\underline{x}$ which minimises:

$$J = \frac{1}{2}\left(\underline{z} - \underline{h}(\underline{x})\right)^T \underline{R}^{-1}\left(\underline{z} - \underline{h}(\underline{x})\right)$$

with

$$W = \underline{R}^{-1} = \begin{pmatrix} \frac{1}{{\sigma_1}^2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ o & \cdots & \frac{1}{{\sigma_m}^2} \end{pmatrix}$$

Problem formulation

Minimize:

$$J = \frac{1}{2}\underbrace{\left(\underline{z} - \underline{h}(\underline{x})\right)}_{\underline{\eta}}^{T} \underline{R}^{-1}\left(\underline{z} - \underline{h}(\underline{x})\right) = \frac{1}{2}\sum_{i=1}^{m} \frac{\left(\underline{z} - \underline{h}(\underline{x})\right)^{2}}{\sigma_i^{2}}$$

m: number of measurements

→ At minimum error, all first order derivatives with respect to decision variables must be zero, i.e.

$$\nabla_{\underline{x}} J = \underline{0} \rightarrow$$

$$\nabla_{\underline{x}} J = \frac{\partial J}{\partial \underline{x}} = \begin{pmatrix} \frac{\partial J}{\partial x_1} \\ \frac{\partial J}{\partial x_2} \\ \vdots \\ \frac{\partial J}{\partial x_n} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

n: number of state variables

Problem formulation

Example for a single element:

$$\frac{\partial J}{\partial x_1} = \frac{1}{2} \cdot \frac{\partial}{\partial x_1}\left(\sum_{i=1}^{m} \frac{\left(\underline{z} - \underline{h}(\underline{x})\right)^2}{\sigma_i{}^2}\right) = \sum_{i=1}^{m} - \frac{\left(\underline{z} - h_i(\underline{x})\right)}{\sigma_i{}^2} \frac{\partial h_i(\underline{x})}{\partial x_1}$$

$$\frac{\partial J}{\partial x_1} = \begin{pmatrix} \frac{\partial h_1(\underline{x})}{\partial x_1} & \frac{\partial h_2(\underline{x})}{\partial x_1} & \cdots & \frac{\partial h_m(\underline{x})}{\partial x_1} \end{pmatrix} \underline{R}^{-1} \begin{pmatrix} z_1 - h_1(\underline{x}) \\ z_2 - h_2(\underline{x}) \\ \vdots \\ z_m - h_m(\underline{x}) \end{pmatrix}$$

For all elements:

$$\frac{\partial J}{\partial \underline{x}} = - \begin{pmatrix} \frac{\partial h_1(\underline{x})}{\partial x_1} & \frac{\partial h_2(\underline{x})}{\partial x_1} & \cdots & \frac{\partial h_m(\underline{x})}{\partial x_1} \\ \frac{\partial h_1(\underline{x})}{\partial x_2} & \frac{\partial h_2(\underline{x})}{\partial x_2} & \cdots & \frac{\partial h_m(\underline{x})}{\partial x_2} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial h_1(\underline{x})}{\partial x_m} & \frac{\partial h_2(\underline{x})}{\partial x_m} & \cdots & \frac{\partial h_m(\underline{x})}{\partial x_m} \end{pmatrix} \underline{R}^{-1} \begin{pmatrix} z_1 - h_1(\underline{x}) \\ z_2 - h_2(\underline{x}) \\ \vdots \\ z_m - h_m(\underline{x}) \end{pmatrix}$$

Problem formulation

$$\frac{\partial J}{\partial \underline{x}} = -\begin{pmatrix} \frac{\partial h_1(\underline{x})}{\partial x_1} & \frac{\partial h_2(\underline{x})}{\partial x_1} & \cdots & \frac{\partial h_m(\underline{x})}{\partial x_1} \\ \frac{\partial h_1(\underline{x})}{\partial x_2} & \frac{\partial h_2(\underline{x})}{\partial x_2} & \cdots & \frac{\partial h_m(\underline{x})}{\partial x_2} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial h_1(\underline{x})}{\partial x_n} & \frac{\partial h_2(\underline{x})}{\partial x_n} & \cdots & \frac{\partial h_m(\underline{x})}{\partial x_n} \end{pmatrix} \underline{R}^{-1} \begin{pmatrix} z_1 - h_1(\underline{x}) \\ z_2 - h_2(\underline{x}) \\ \vdots \\ z_n - h_m(\underline{x}) \end{pmatrix}$$

The matrix of the partial derivatives resembles the Jacobian matrix, but :

- It is an n x m matrix (and not a square matrix)
- Unlike the standard Jacobian, the rows vary with variables (x_1, x_2, …), and not the functions (h_1, h_2, …)

Problem formulation

Define:

$$\underline{H} = -\begin{pmatrix} \frac{\partial h_1(\underline{x})}{\partial x_1} & \frac{\partial h_1(\underline{x})}{\partial x_2} & \cdots & \frac{\partial h_1(\underline{x})}{\partial x_m} \\ \frac{\partial h_2(\underline{x})}{\partial x_1} & \frac{\partial h_2(\underline{x})}{\partial x_2} & \cdots & \frac{\partial h_2(\underline{x})}{\partial x_m} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial h_m(\underline{x})}{\partial x_1} & \frac{\partial h_m(\underline{x})}{\partial x_2} & \cdots & \frac{\partial h_m(\underline{x})}{\partial x_m} \end{pmatrix}$$

$\underline{H}$ is an m x n matrix and the transpose of the previous matrix

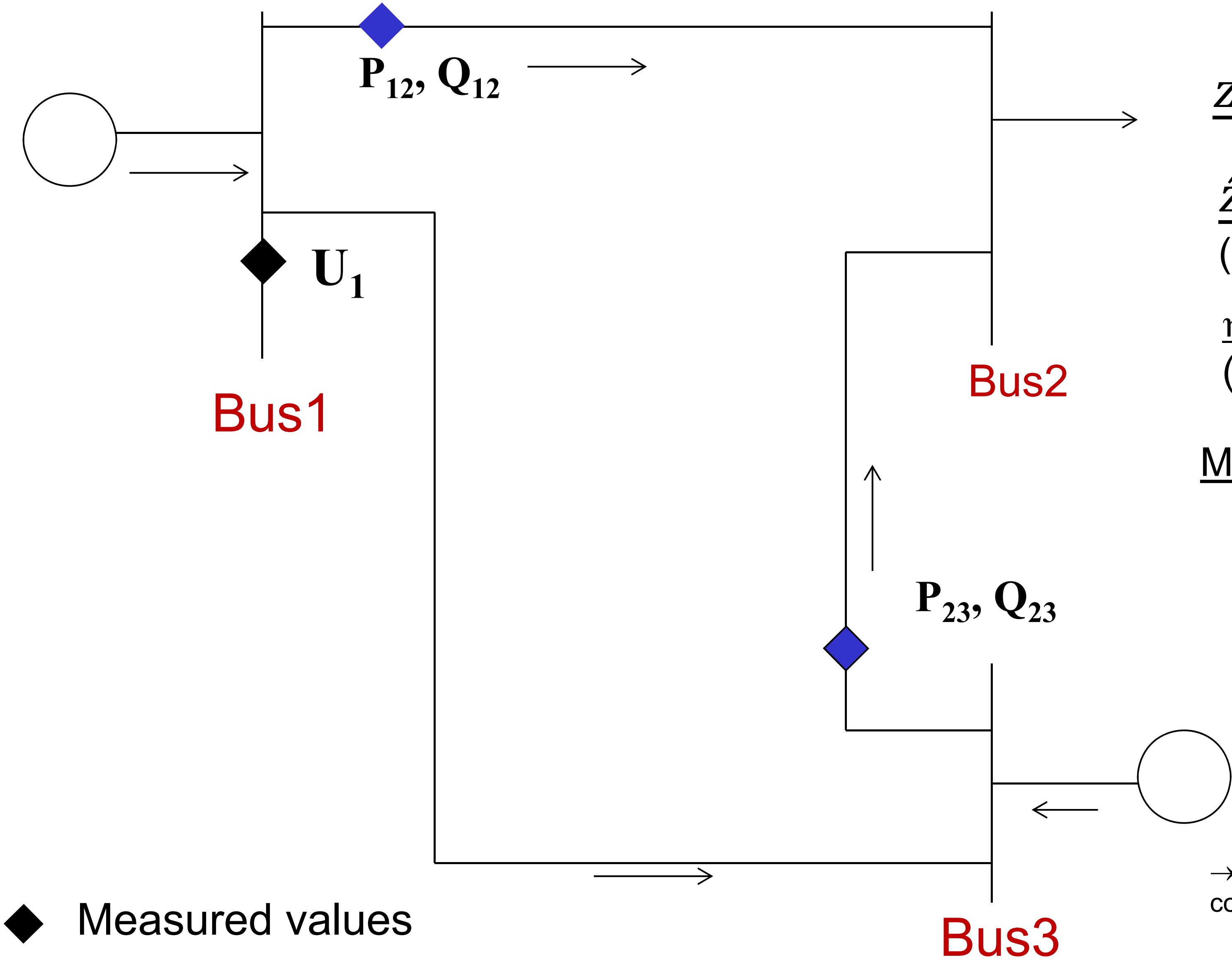
– Thus we have as the optimality condition:

$$\nabla_{\underline{x}} J = -\underline{H}^T(\underline{x})\underline{R}^{-1}\left(\underline{z} - \underline{h}(\underline{x})\right) = \underline{0}$$

n nonlinear equations

The solution will yield the estmated state vector x
→ which minimizes the squared error.

EXAMPLE



$$\underline{z} = \underline{\hat{z}} + \underline{\eta}$$

$\underline{\hat{z}}$ = true value (unknown)

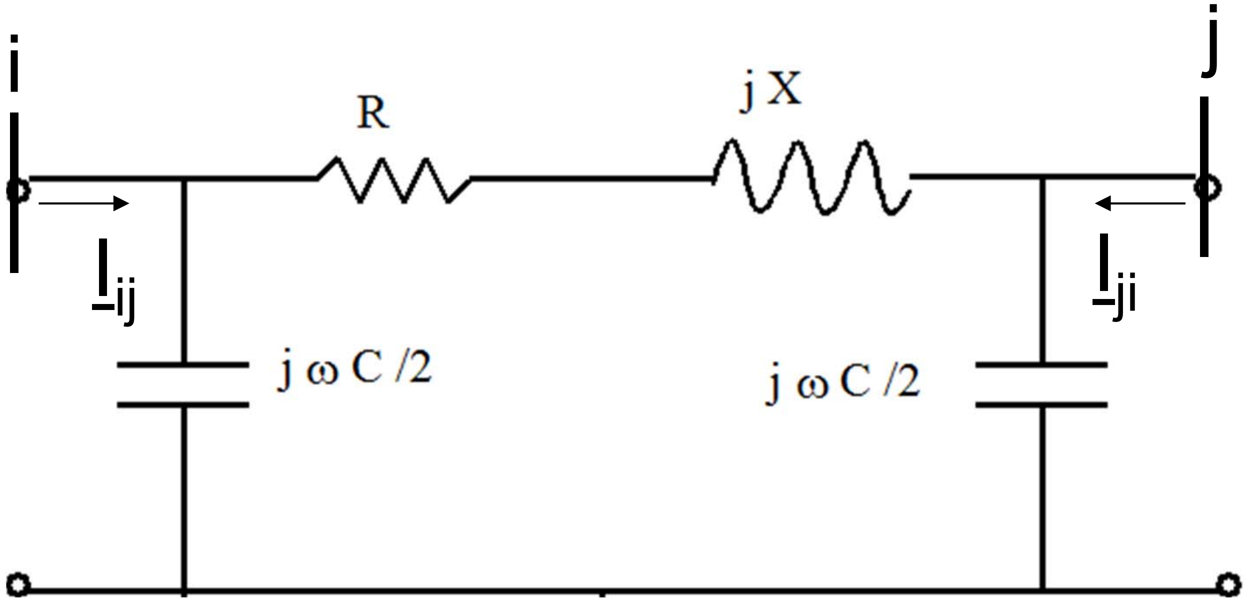
$\underline{\eta}$= error (unkown)

<u>Measured values</u>

$$\underline{z} = \begin{pmatrix} U_1 \\ P_{12} \\ Q_{12} \\ P_{23} \\ Q_{23} \end{pmatrix}$$

→ Write the corresponding equations

Power flow equation



$$\underline{S}_{ij} = \underline{U}_i \underline{I}_{ij}^* = P_{ij} + Q_{ij}$$

$$\mathrm{j}B_i = \frac{j\omega C_i}{2} \qquad \frac{1}{R+jX} = \frac{R-jX}{R^2+X^2} \qquad G_{ij} = \frac{R}{R^2+X^2} \qquad \text{-j}B_{ij} = \frac{-jX}{R^2+X^2}$$

Where:

R +jX	: line series impedance
C	: line capacitance

Power flow equations (cont'd)

$$\underline{S}_{ij} = \underline{U}_i \underline{I}_{ij}^* = P_{ij} + Q_{ij}$$

$$\underline{I}_{ij} = \underline{U}_i \frac{j\omega C}{2} + \frac{\underline{U}_i - \underline{U}_j}{R + jX}$$

$$P_{ij} = \frac{R.\left({U_i}^2 - U_i.U_j\, cos\theta_{ij}\right) + X.U_i.U_j sin\theta_{ij}}{R^2 + X^2}$$

$$= G_{ij}.\left({U_i}^2 - U_i.U_j\, cos\theta_{ij}\right) + B_{ij}.U_i.U_j sin\theta_{ij}$$

$$Q_{ij} = -{U_i}^2 \frac{\omega C}{2} + \frac{X.\left({U_i}^2 - U_i.U_j cos\theta_{ij}\right) - R.U_i.U_j sin\theta_{ij}}{R^2 + X^2}$$

$$= -B_i.{U_i}^2 + B_{ij}.\left({U_i}^2 - U_i.U_j.cos\theta_{ij}\right) - G_{ij}.U_i.U_j.sin\theta_{ij}$$

θ_{ij} : the voltage phase angle difference θ_i-θ_j

Problem formulation

- For each parameter for which we have a measurement, we write an equation in terms of the states

$$\underline{\hat{z}} = h(\underline{x})$$

  - For a voltage measurement (U_k):

$$\underline{\hat{z}}_i = U_k$$

  - For power flow across the line from bus i - bus j:

$$P_{12} = G_{12}.\left(U_1{}^2 - U_1.U_2\,cos\theta_{12}\right) + B_{12}.\,U_1.U_2 sin\theta_{12}$$

$$P_{23} = G_{23}.\left(U_2{}^2 - U_2.U_3\,cos\theta_{23}\right) + B_{23}.\,U_2.U_3 sin\theta_{23}$$

$$Q_{12} = -B_1.U_1{}^2 + B_{12}.\left(U_1{}^2 - U_1.U_2.cos\theta_{12}\right) - G_{12}.U_1.U_2.sin\theta_{12}$$

$$Q_{23} = -B_2.U_2{}^2 + B_{23}.\left(U_2{}^2 - U_2.U_3.cos\theta_{23}\right) - G_{23}.U_2.U_3.sin\theta_{23}$$

Problem formulation

$$\underline{z} = \begin{pmatrix} U_1 \\ P_{12} \\ Q_{12} \\ P_{23} \\ Q_{23} \end{pmatrix} = \begin{pmatrix} U_1 \\ G_{12}.\left(U_1{}^2 - U_1.U_2\, cos\theta_{12}\right)+B_{12}.\,U_1.U_2 sin\theta_{12} \\ -B_1.U_1{}^2 + B_{12}.\left(U_1{}^2 - U_1.U_2.cos\theta_{12}\right)-G_{12}.U_1.U_2.sin\theta_{12} \\ G_{23}.\left(U_2{}^2 - U_2.U_3\, cos\theta_{23}\right)+B_{23}.\,U_2.U_3 sin\theta_{23} \\ -B_2.U_2{}^2 + B_{23}.\left(U_2{}^2 - U_2.U_3.cos\theta_{23}\right)-G_{23}.U_2.U_3.sin\theta_{23} \end{pmatrix}$$

$$\underline{x} = \begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix}$$

$$\underline{H} = -\begin{pmatrix} \frac{\partial h_1(\underline{x})}{\partial x_1} & \frac{\partial h_1(\underline{x})}{\partial x_2} & \cdots & \frac{\partial h_1(\underline{x})}{\partial x_n} \\ \frac{\partial h_2(\underline{x})}{\partial x_1} & \frac{\partial h_2(\underline{x})}{\partial x_2} & \cdots & \frac{\partial h_2(\underline{x})}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial h_m(\underline{x})}{\partial x_1} & \frac{\partial h_m(\underline{x})}{\partial x_2} & \cdots & \frac{\partial h_m(\underline{x})}{\partial x_n} \end{pmatrix}$$

Problem formulation

$$\underline{z} = \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{pmatrix} = \begin{pmatrix} U_1 \\ G_{12}.\left(U_1{}^2 - U_1.U_2\, cos\theta_{12}\right) + B_{12}.\, U_1.U_2 sin\theta_{12} \\ -B_1.U_1{}^2 + B_{12}.\left(U_1{}^2 - U_1.U_2.cos\theta_{12}\right) - G_{12}.U_1.U_2.sin\theta_{12} \\ G_{23}.\left(U_2{}^2 - U_2.U_3\, cos\theta_{23}\right) + B_{23}.\, U_2.U_3 sin\theta_{23} \\ -B_2.U_2{}^2 + B_{23}.\left(U_2{}^2 - U_2.U_3.cos\theta_{23}\right) - G_{23}.U_2.U_3.sin\theta_{23} \end{pmatrix}$$

$$\underline{x} = \begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ \theta_2 \\ \theta_3 \end{pmatrix}$$

$$\underline{H} = -\begin{pmatrix} \frac{\partial h_1(\underline{x})}{\partial U_1} & \frac{\partial h_1(\underline{x})}{\partial U_2} & \frac{\partial h_1(\underline{x})}{\partial U_3} & \frac{\partial h_1(\underline{x})}{\partial \theta_2} & \frac{\partial h_1(\underline{x})}{\partial \theta_3} \\ \frac{\partial h_2(\underline{x})}{\partial U_1} & \frac{\partial h_2(\underline{x})}{\partial U_2} & \frac{\partial h_2(\underline{x})}{\partial U_3} & \frac{\partial h_2(\underline{x})}{\partial \theta_2} & \frac{\partial h_2(\underline{x})}{\partial \theta_3} \\ \frac{\partial h_3(\underline{x})}{\partial U_1} & \frac{\partial h_3(\underline{x})}{\partial U_2} & \frac{\partial h_3(\underline{x})}{\partial U_3} & \frac{\partial h_3(\underline{x})}{\partial \theta_2} & \frac{\partial h_3(\underline{x})}{\partial \theta_3} \\ \frac{\partial h_4(\underline{x})}{\partial U_1} & \frac{\partial h_4(\underline{x})}{\partial U_2} & \frac{\partial h_4(\underline{x})}{\partial U_3} & \frac{\partial h_4(\underline{x})}{\partial \theta_2} & \frac{\partial h_4(\underline{x})}{\partial \theta_3} \\ \frac{\partial h_5(\underline{x})}{\partial U_1} & \frac{\partial h_5(\underline{x})}{\partial U_2} & \frac{\partial h_5(\underline{x})}{\partial U_3} & \frac{\partial h_5(\underline{x})}{\partial \theta_2} & \frac{\partial h_5(\underline{x})}{\partial \theta_3} \end{pmatrix}$$

$$-\underline{H}^T(\underline{x})\underline{R}^{-1}\left(\underline{z} - \underline{h}(\underline{x})\right) = \underline{0} \rightarrow \underline{x} = ?$$

Solution procedure

$$\underline{G}(\underline{x}) = -\underline{H}^T(\underline{x})\ \underline{R}^{-1}\left(\underline{z} - \underline{h}(\underline{x})\right) = \underline{0}$$

$\underline{G}(\underline{x})$: n x 1; $\underline{H}^T(\underline{x})$: n x m; $\underline{R}^{-1}$: m x m; $\left(\underline{z} - \underline{h}(\underline{x})\right)$: m x 1

n : number of state variables
m : number of measured values

Performing a Taylor series expansion of $\underline{G}(\underline{x})$ around an initial estimate $\underline{x}_0$

$$\underline{x} = \begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ \theta_2 \\ \theta_3 \end{pmatrix}$$

$$\underline{H} = -\begin{pmatrix}
\frac{\partial h_1(\underline{x})}{\partial U_1} & \frac{\partial h_1(\underline{x})}{\partial U_2} & \frac{\partial h_1(\underline{x})}{\partial U_3} & \frac{\partial h_1(\underline{x})}{\partial \theta_2} & \frac{\partial h_1(\underline{x})}{\partial \theta_3} \\
\frac{\partial h_2(\underline{x})}{\partial U_1} & \frac{\partial h_2(\underline{x})}{\partial U_2} & \frac{\partial h_2(\underline{x})}{\partial U_3} & \frac{\partial h_2(\underline{x})}{\partial \theta_2} & \frac{\partial h_2(\underline{x})}{\partial \theta_3} \\
\frac{\partial h_3(\underline{x})}{\partial U_1} & \frac{\partial h_3(\underline{x})}{\partial U_2} & \frac{\partial h_3(\underline{x})}{\partial U_3} & \frac{\partial h_3(\underline{x})}{\partial \theta_2} & \frac{\partial h_3(\underline{x})}{\partial \theta_3} \\
\frac{\partial h_4(\underline{x})}{\partial U_1} & \frac{\partial h_4(\underline{x})}{\partial U_2} & \frac{\partial h_4(\underline{x})}{\partial U_3} & \frac{\partial h_4(\underline{x})}{\partial \theta_2} & \frac{\partial h_4(\underline{x})}{\partial \theta_3} \\
\frac{\partial h_5(\underline{x})}{\partial U_1} & \frac{\partial h_5(\underline{x})}{\partial U_2} & \frac{\partial h_5(\underline{x})}{\partial U_3} & \frac{\partial h_5(\underline{x})}{\partial \theta_2} & \frac{\partial h_5(\underline{x})}{\partial \theta_3}
\end{pmatrix}$$

Bad Data Suppression

- Bad Data Detection
  - Mulit-level process.
  - “Bad data pockets” identified.
  - Zoom in on “bad data pocket’ for rigorous topological analysis.
  - Status estimation in the event of topological errors.

Final Measurement Statuses

- Used… The measurement was found to be “good” and was used in determining the final SE solution.
- Not Used… Not enough information was available to use this information in the SE solution.
- Suppressed… The measurement was initially used, but found to be inconsistent (or “bad”).
- Smeared… At some point in the solution process, the measurement was removed. Later it was determined that the measurement was “smeared” by another bad measurement.

State Estimation...
Measurements and Estimates

- SE Measurement Summary Display
  - Standard Deviations… Indicates the relative confidence placed on an individual measurement.
  - Measurement Status… Each measurement may be determined as “used”, “not used”, or “suppressed”.
  - Meter Bias… Accumulates residual to help identify metering that is consistently poor. The bias value should “hover” about zero.

State Estimation... Measurements and Estimates (Cont.)

- Observable System
  - Portions of the system that can be completely solved based on real-time telemetry are called "observable".
  - Observable buses and devices are not color-coded (white).
- Unobservable System
  - Portions of the network that cannot be solved completely based on real-time telemetry are called "unobservable" and are color-coded yellow.