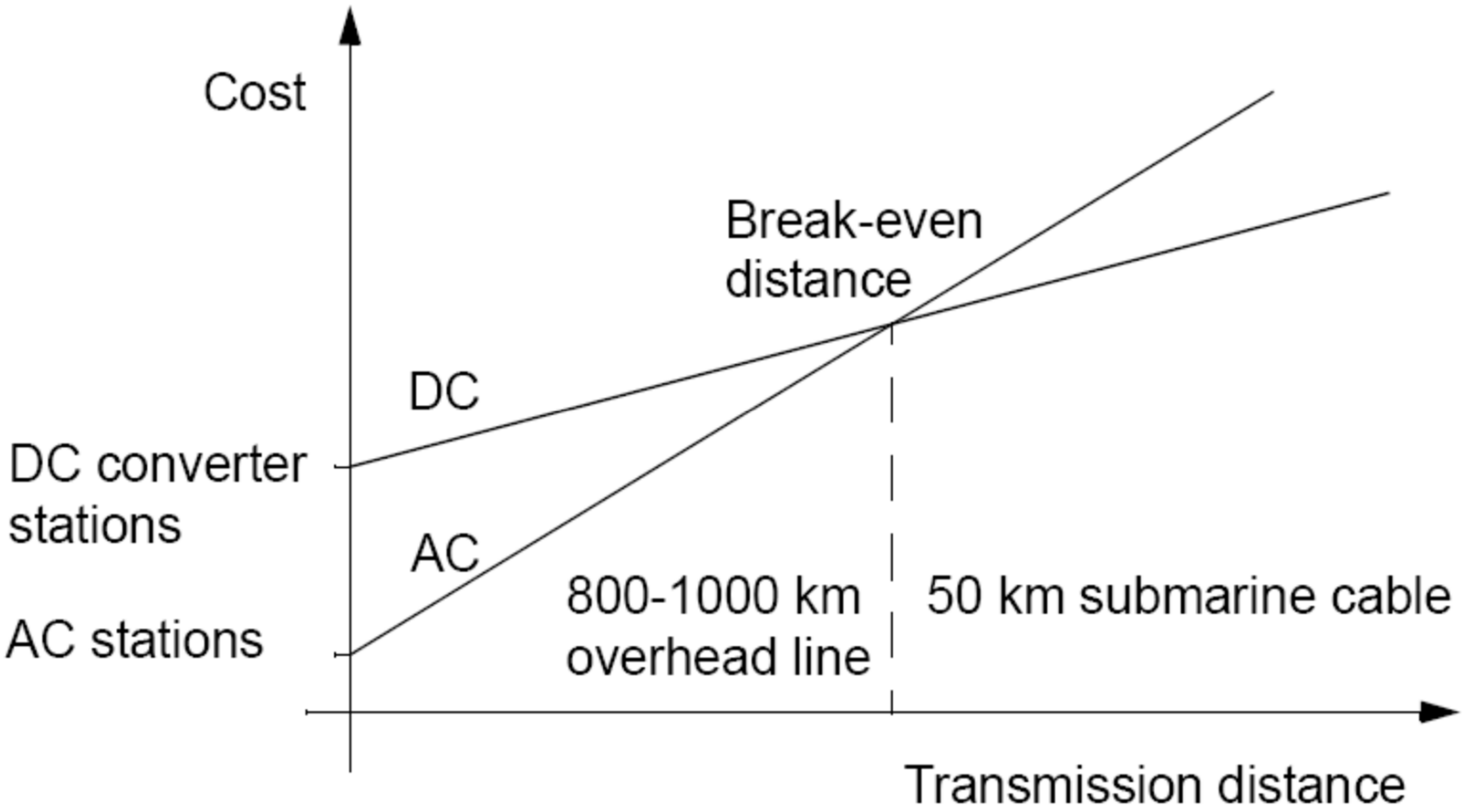
Introduction to power systems

AC/DC, 50

- Why AC, and not DC?
- Why 50 Hz (or 60 Hz), and not 25/100Hz?
- Why three-phase system, not two/four phase?

Why AC, and not DC ?



The choice of frequency (1)

- Between 1885 and 1890 in the U.S.A.:
  - 140, 133⅓, 125, 83 ⅓, 66 ⅔, 50, 40, 33 ⅓, 30, 25 and 16⅔ Hz
- Now:
  - 60 Hz in North America, Brazil and Japan (has also 50 Hz!)
  - 50 Hz in most other countries
  - 25 Hz Railways
  - 16⅔ Hz Railways (Germany, Austria and Switzerland)
  - 400 Hz Oil rigs, ships and airplanes

The choice of Frequency (2)

- too low a frequency, like 10 or 20 Hz causes flicker
- too high a frequency
  - Increases the hysteresis losses:

$$P_{hys} \succ f\psi^{1.5-2.5}$$

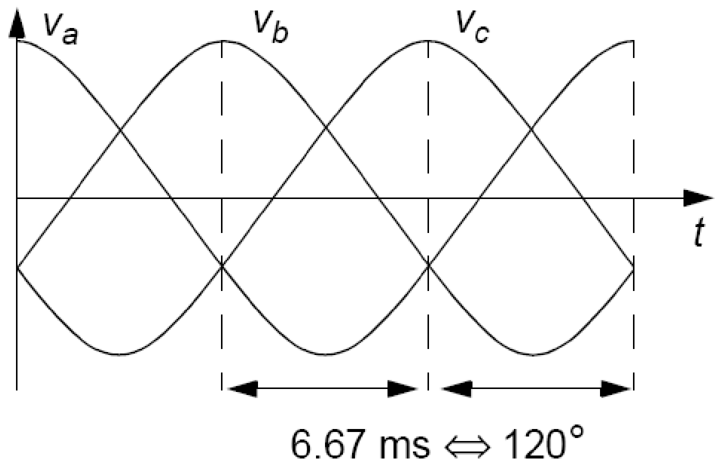
  - Increases the eddy current losses:

$$P_{eddy} \succ f^2\psi^2$$

  - Increases the cable and line impedance

Three Phase Systems (1)

Phase voltages in a balanced three-phase system (50 Hz)



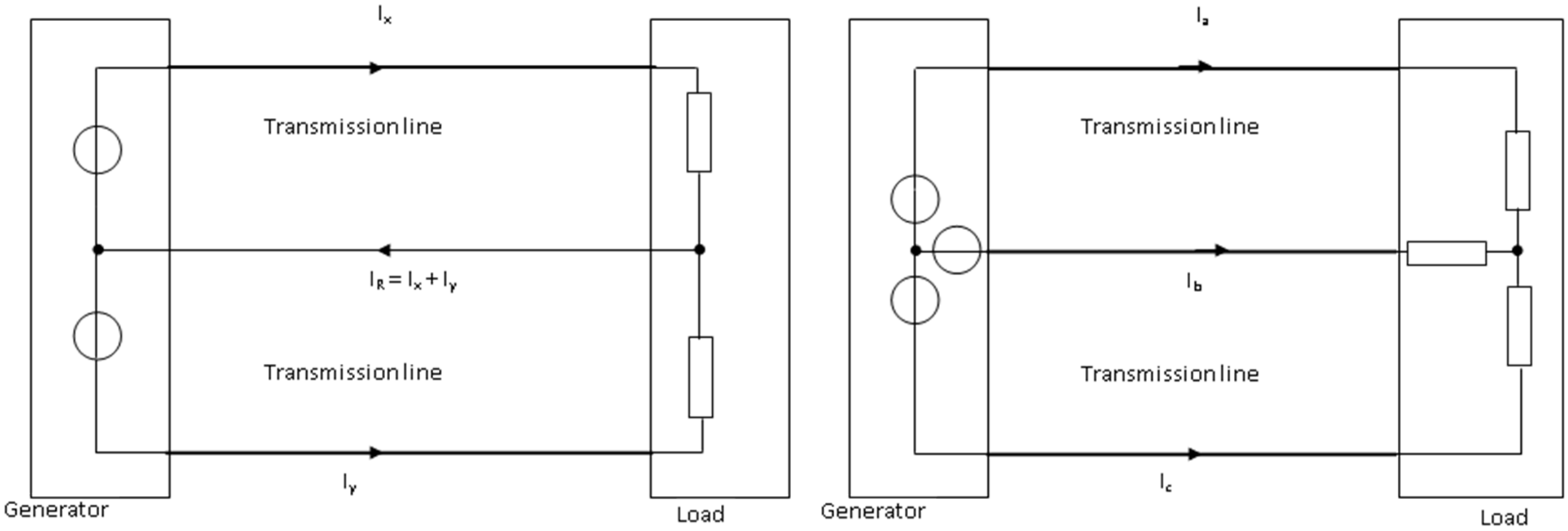
$$v_a = \sqrt{2}|V|\cos(\omega t)$$

$$v_b = \sqrt{2}|V|\cos\left(\omega t - \frac{2\pi}{3}\right)$$

$$v_c = \sqrt{2}|V|\cos\left(\omega t - \frac{4\pi}{3}\right)$$

Three Phase Systems (2)

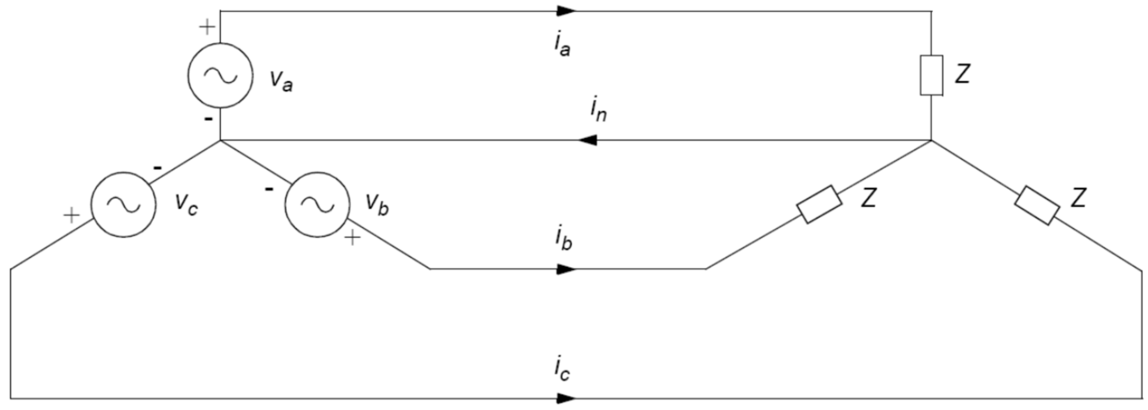
The magnetic field generated by a three-phase system is a rotating field



Background

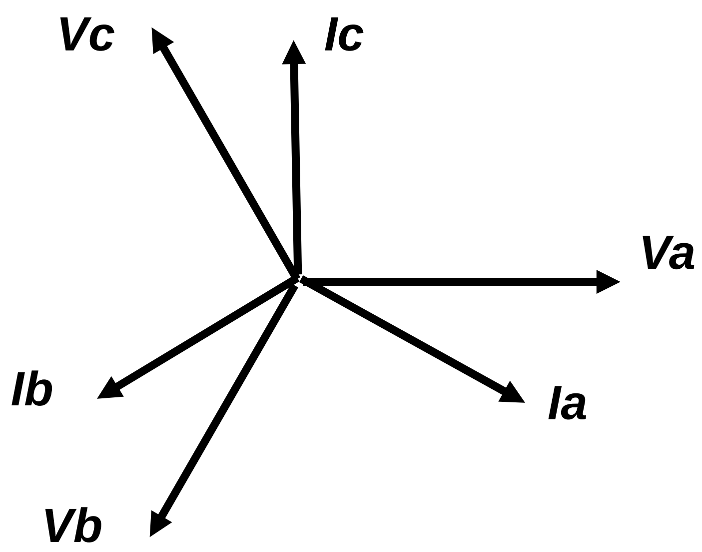
- 3 phase systems
- Power
- Voltage levels
- Phasors
- Per unit calculation
- Power system structure

Balanced Three Phase System

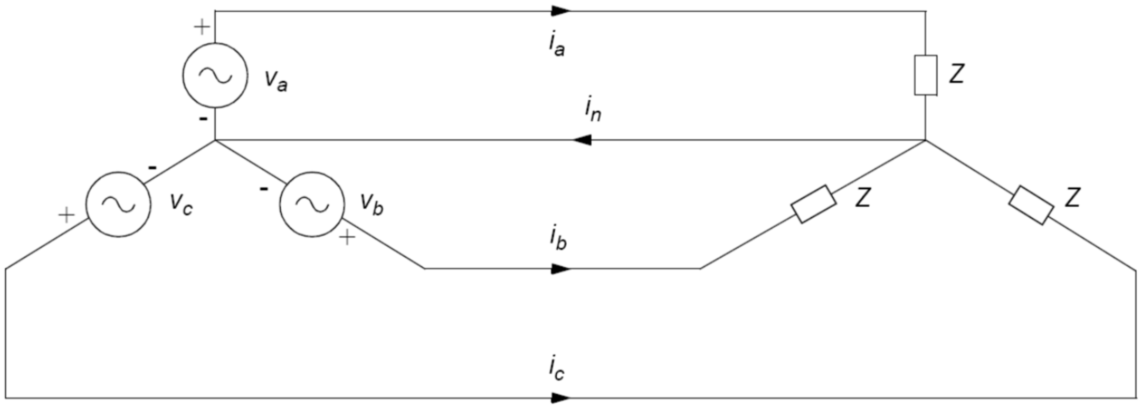


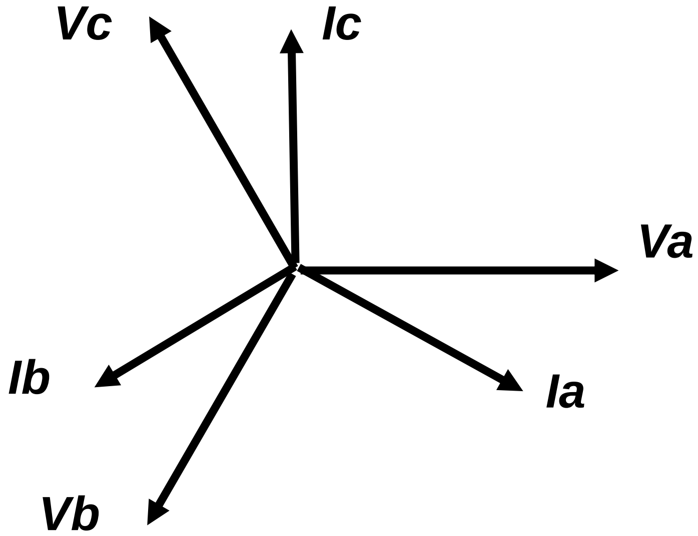
- Voltages in the 3 phases have the same amplitude, but differ by 120 electrical degrees in phase

- Equal impedances in the 3 phases results in symmetrical currents

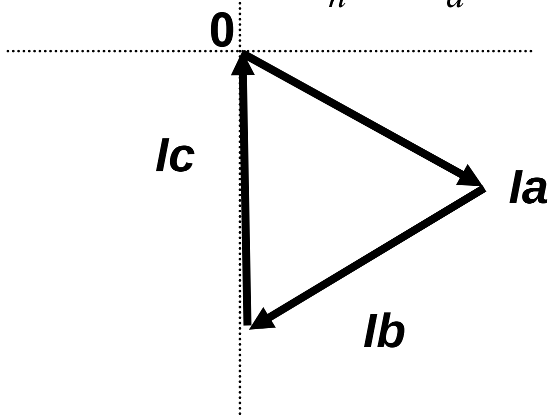


Balanced Three Phase System

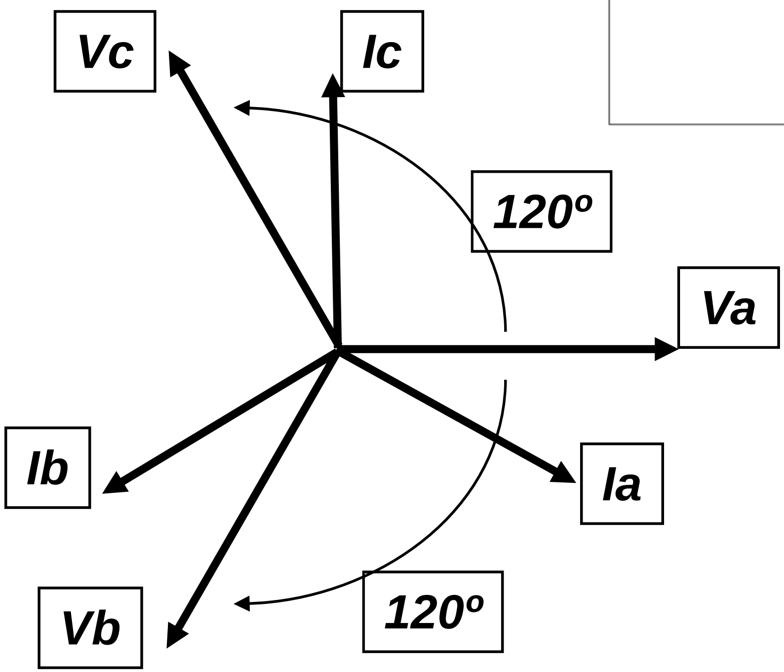


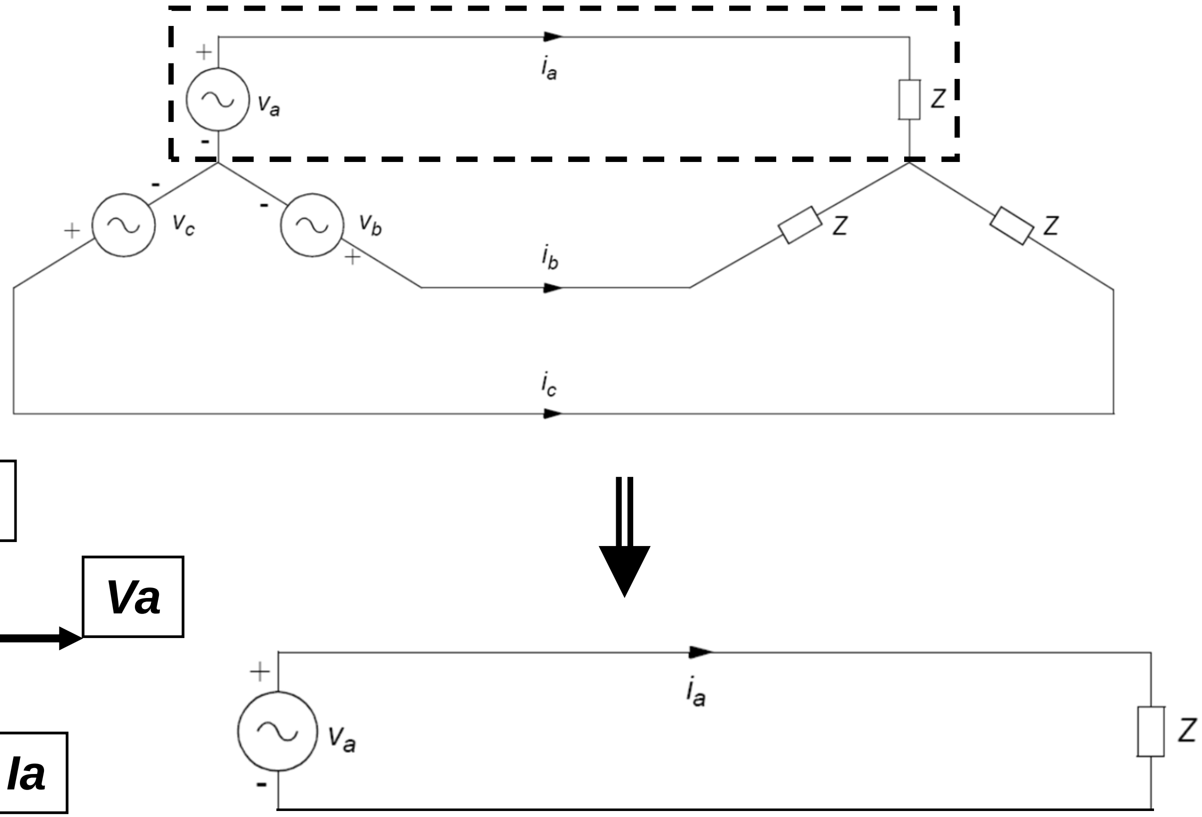


$$I_n = I_a + I_b + I_c = 0$$

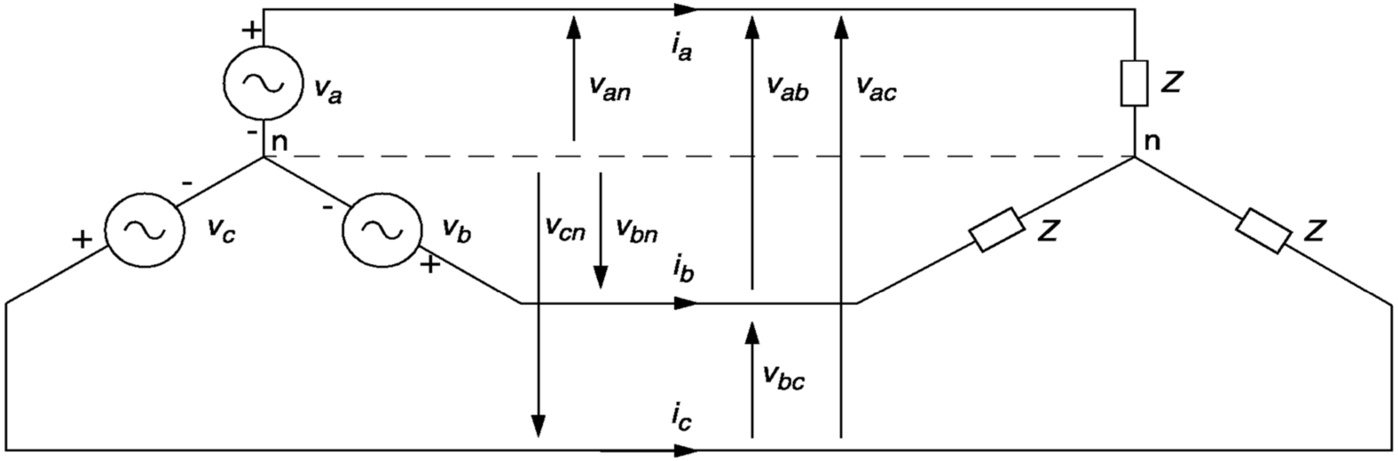


Per phase analysis

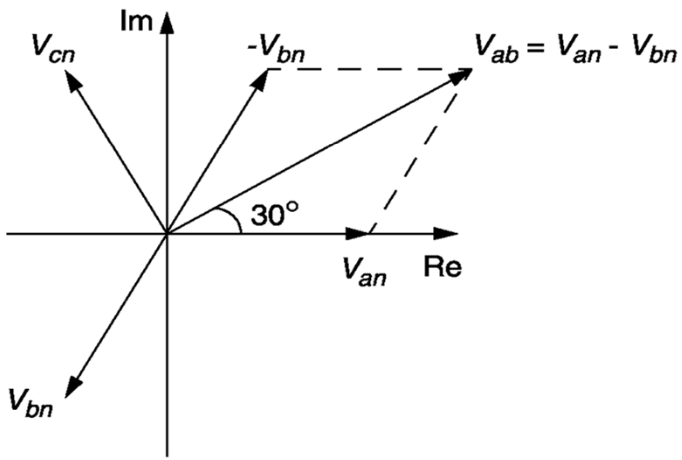




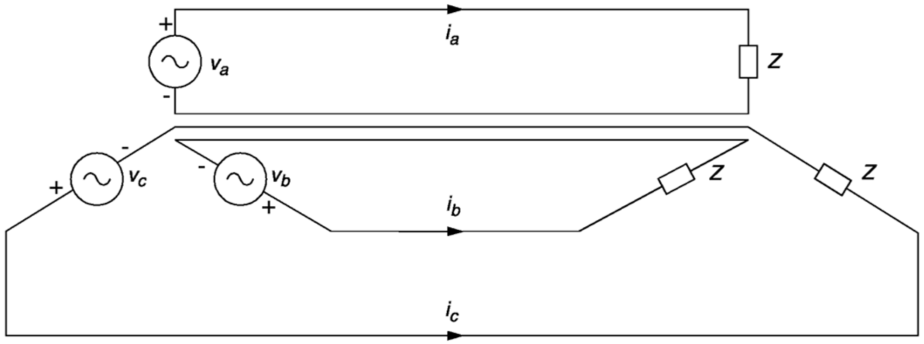
Line-to-Line Voltage

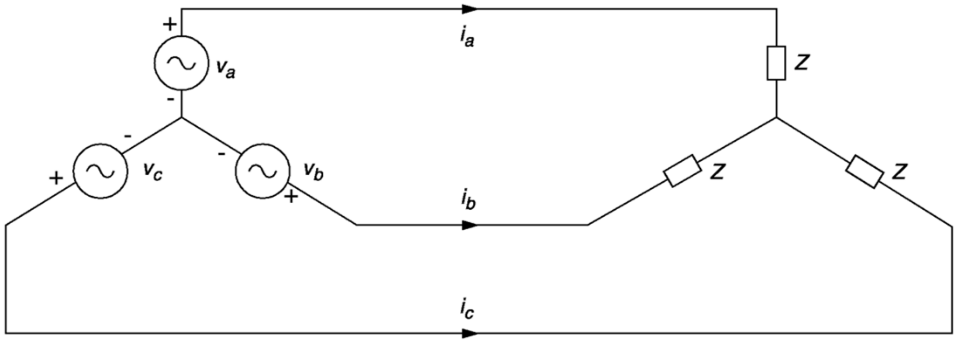


$$V_{ab} = V_{an} - V_{bn} = \frac{3}{2}|V| + j\frac{1}{2}\sqrt{3}\,|V| =$$
$$= \sqrt{3}\,|V|\angle 30 = \sqrt{3}V_{an}\angle 30$$



Three Phase Complex Power





- 3 x 1-phase complex power

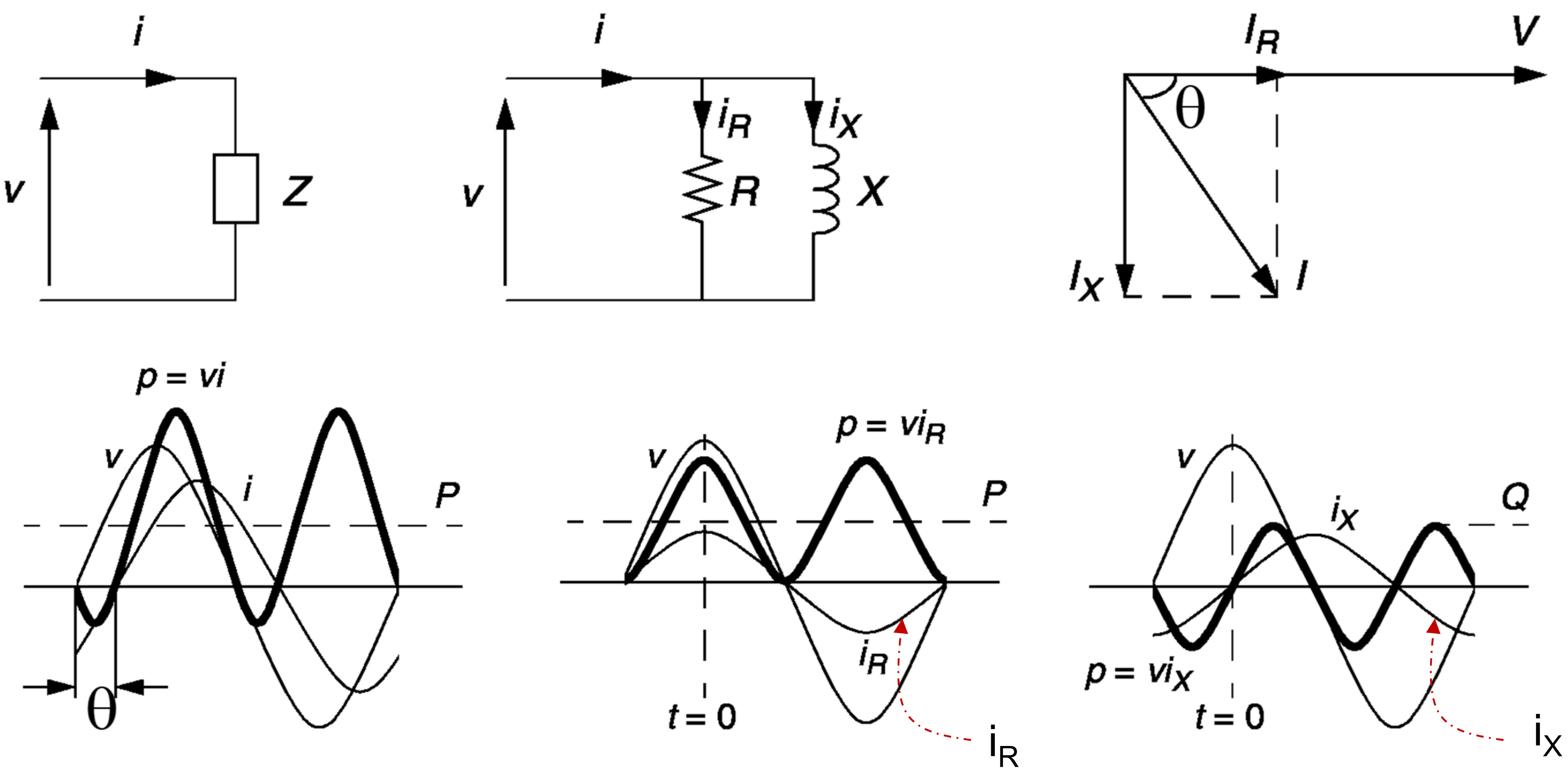
$$S_{3ph} = 3S_{1ph} = 3V_{ph}I_{ph}^* = P_{3ph} + jQ_{3ph}$$

$$|S_{3ph}| = 3|S_{1ph}| = 3|V_{ph}||I_{ph}| = \sqrt{3}|V_{LL}||I_{ph}|$$

$$P_{3ph} = 3P_{1ph} = 3|V_{ph}||I_{ph}|\cos\theta = \sqrt{3}|V_{LL}||I_{ph}|\cos\theta$$

$$Q_{3ph} = 3Q_{1ph} = 3|V_{ph}||I_{ph}|\sin\theta = \sqrt{3}|V_{LL}||I_{ph}|\sin\theta$$

Power (1)

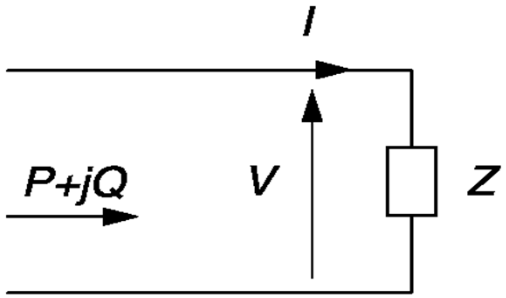


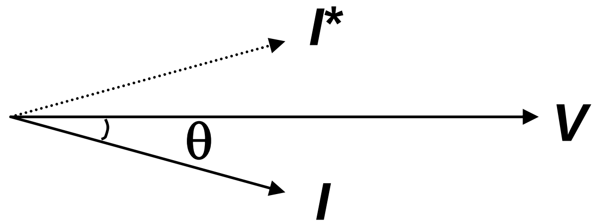
P: Active power (average value vi_R)

Q: Reactive power (average value vi_X)

Power (2)

How to calculate P and Q from the voltage and current phasor ?





- Inductive load consumes reactive power (Q>0)
  - Current lags the supply voltage

$$V = |V|\angle 0 \quad I = |I|\angle -\theta$$

$$S = VI^* = |V||I|\angle\theta = |V||I|(\cos\theta + j\sin\theta) = P + jQ$$

Positive

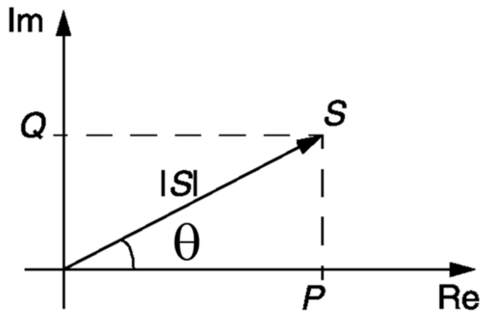
- Capacitive load generates reactive power (Q<0)
  - Current leads the voltage

$$V = |V|\angle 0 \quad I = |I|\angle\theta$$

$$S = VI^* = |V||I|\angle -\theta = |V||I|(\cos\theta - j\sin\theta) = P + jQ$$

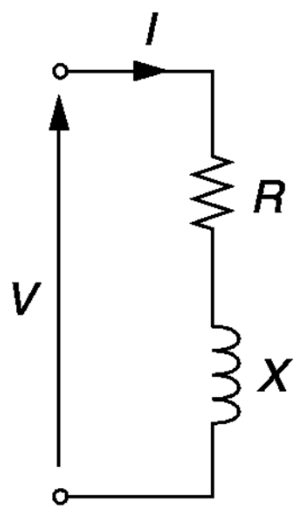
Negative

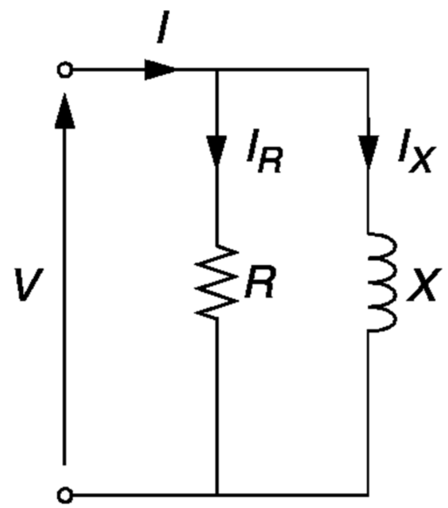
Power (3)



S	Complex power	VA	$S = \lvert S\rvert\angle\theta = P + jQ = VI^* =$ $= \lvert V\rvert\lvert I\rvert\angle\theta = \lvert V\rvert\lvert I\rvert(\cos\theta + j\sin\theta)$
$\lvert S\rvert$	Apparent power	VA	$\lvert S\rvert = \lvert V\rvert\lvert I\rvert = \sqrt{P^2 + Q^2}$
P	Active power Average power	W	$P = \mathrm{Re}(S) = \lvert S\rvert\cos(\theta) = \lvert V\rvert\lvert I\rvert\cos(\theta)$
Q	Reactive power	var	$Q = \mathrm{Im}(S) = \lvert S\rvert\sin(\theta) = \lvert V\rvert\lvert I\rvert\sin(\theta)$

Series / Parallel





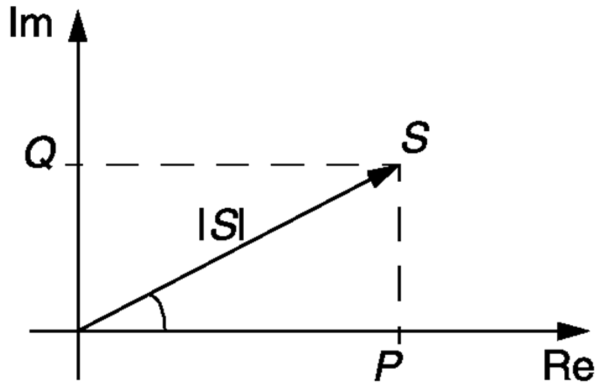
$$P = |I|^2 R$$

$$Q = |I|^2 X$$

$$P = |V|^2 / R$$

$$Q = |V|^2 / X$$

Power Factor



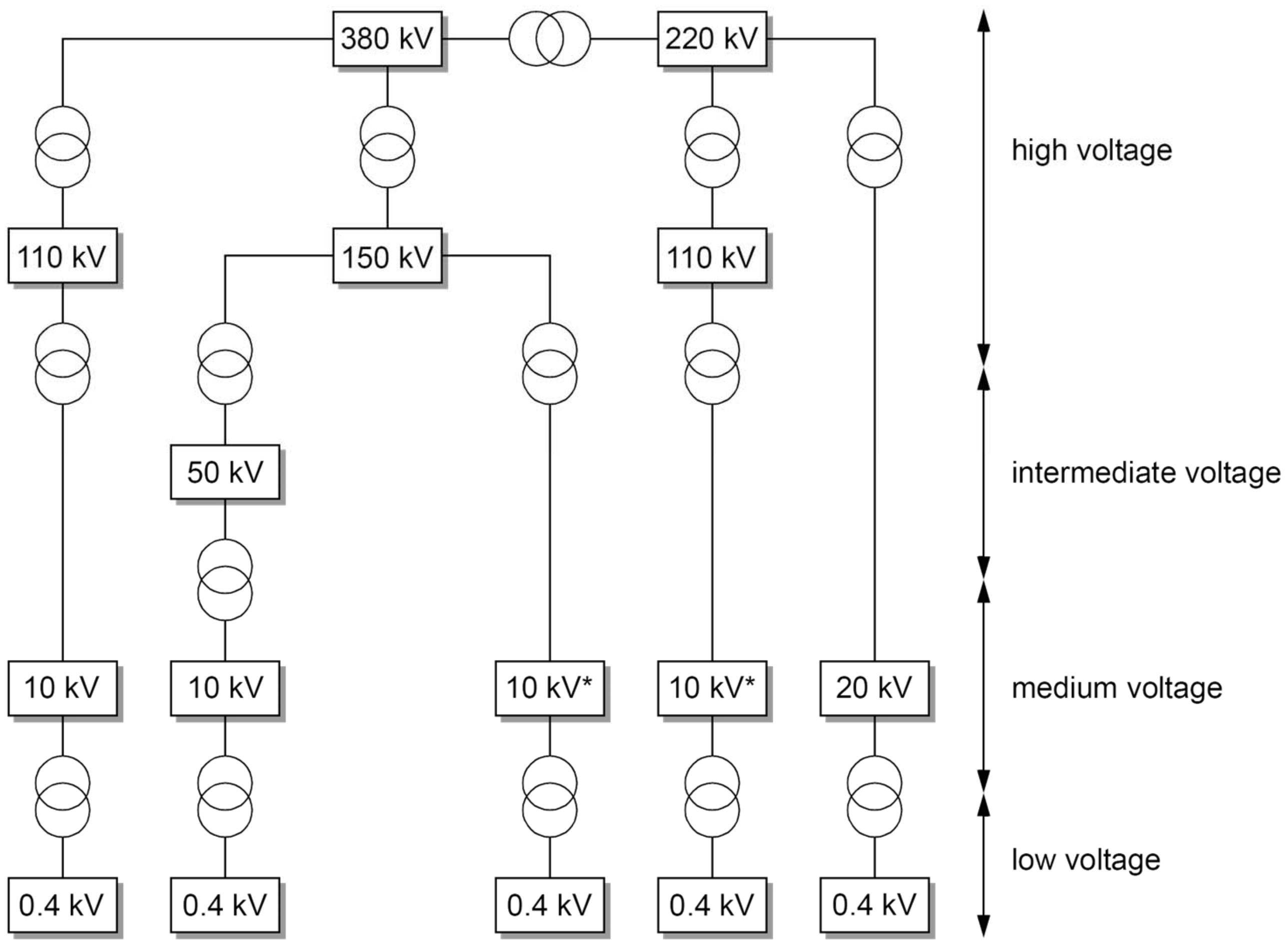
$$P = |S|\cos(\theta) = |V||I|\cos(\theta)$$

Power factor ➔ That part of the apparent power that is related to the mean energy flow

$$\cos(\theta) = \frac{P}{|S|} = \frac{P}{\sqrt{P^2 + Q^2}}$$

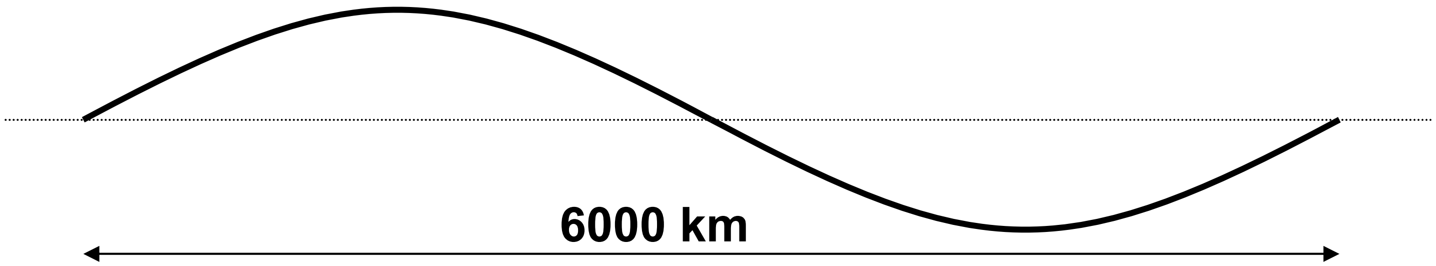
$$\cos(\theta) = \cos\left(\tan^{-1}\left(\frac{Q}{P}\right)\right)$$

System Voltage Levels

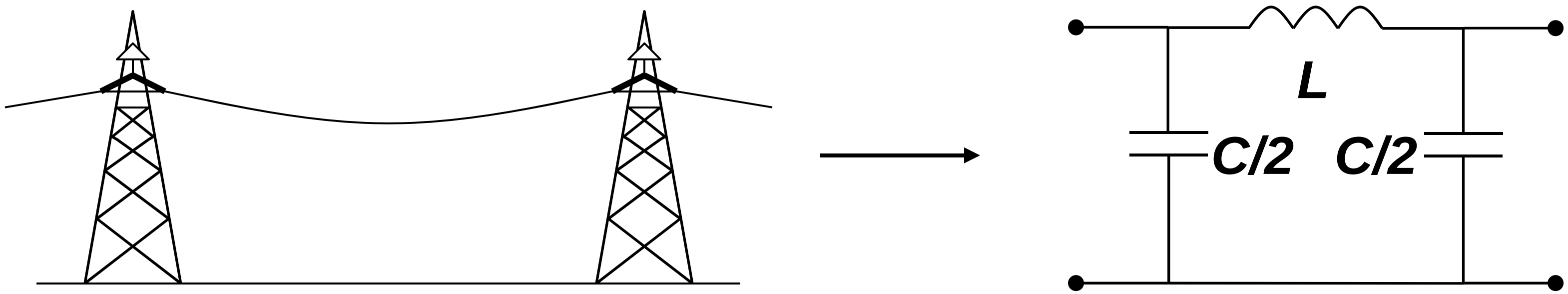


Steady State Analysis: *f* = 50 Hz

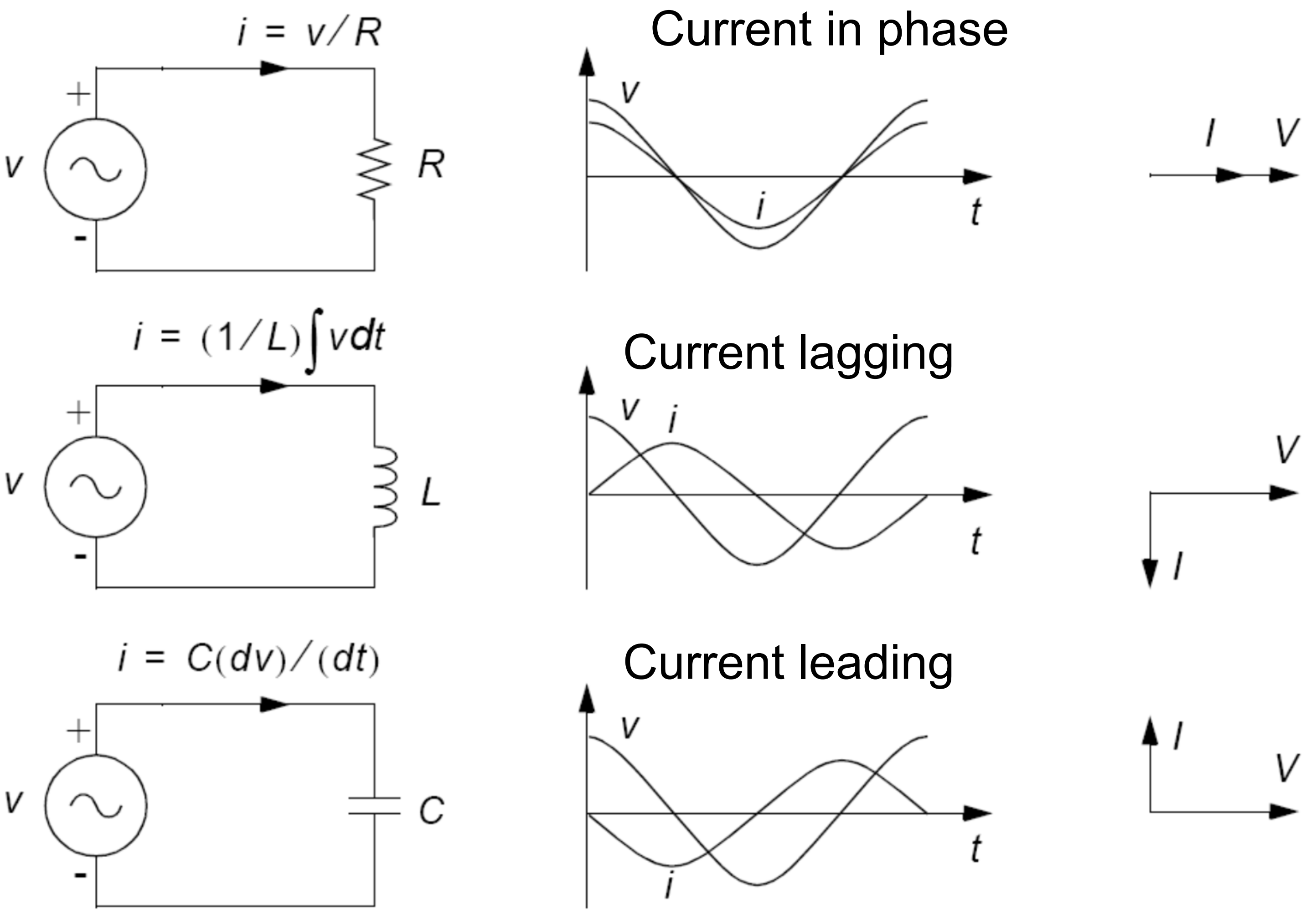
- f = 50Hz ➔ $\lambda = v/f$ = 3e8/50 = 6000km



- Modelling with R, G, L and C



Time ⇔ Phasor



$U = IR$

$U = j\omega LI$

$I = j\omega CU$

Per-Unit Representation

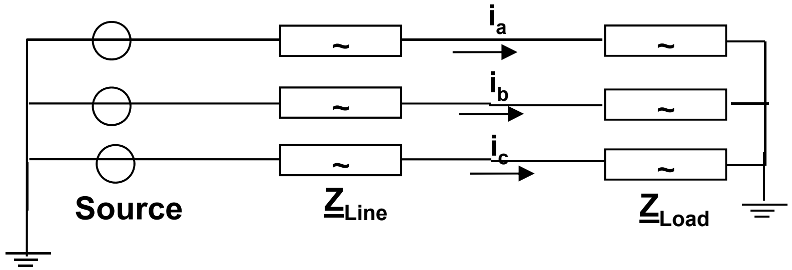
Base quantity	*Value*
Voltage	$\|V_b\| = 100$ kV (line-to-line)
(apparent) Power	$\|S_b\| = 100$ MVA (three-phase)
Current	$\|I_b\| = \|S_b\| / \left(\sqrt{3} \cdot \|V_b\|\right) = 577.3$ A
Impedance	$\|Z_b\| = \|V_b\| / \left(\sqrt{3} \cdot \|I_b\|\right) = \|V_b\|^2 / \|S_b\| = 100\ \Omega$

Transformations

- Symmetrical Components
- $\alpha\beta$ (Clarke) Transformation
- dq (Park) Transformation

Symmetrical components

Symmetrical three-phase system



$$i_a = 5\cos(\omega t + \varphi_i)$$
$$i_b = 5\cos(\omega t + \varphi_i - 2\pi/3)$$
$$i_c = 5\cos(\omega t + \varphi_i + 2\pi/3)$$

$$i_a = 5\cos(\omega t + \varphi_i) = Re\{5\cos(\omega t + \varphi_i) + j5\cos(\omega t + \varphi_i)\}$$
$$= Re\{5\, e^{j(\omega t + \varphi_i)}\} = Re\{5\, e^{j\varphi_i} e^{j\omega t}\} = \sqrt{2} Re\left\{\left[\frac{5}{\sqrt{2}}\, e^{j\varphi_i}\right] e^{j\omega t}\right\}$$

Time Phasor
Zeitzeiger

$$\underline{I}_a = \frac{5}{\sqrt{2}}\, e^{j\varphi_i} \quad \Longleftrightarrow \quad i_a = \sqrt{2}\, Re\{\underline{I}_a e^{j\omega t}\}$$

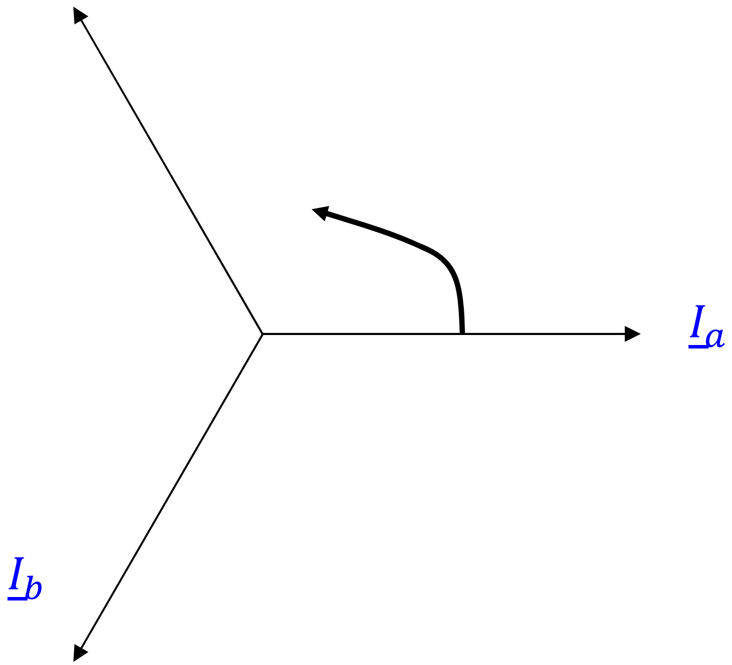
$$\underline{I}_b = \underline{I}_a\, e^{-j2\pi/3} \qquad i_b = \sqrt{2}\, Re\{\underline{I}_b e^{j\omega t}\}$$

$$\underline{I}_c = \underline{I}_a\, e^{+j2\pi/3} \qquad i_c = \sqrt{2}\, Re\{\underline{I}_c e^{j\omega t}\}$$

The operator „a“

$$\underline{a} = e^{j\frac{2\pi}{3}} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

- The analysis of a balanced three-phase circuit is simple
  - phase voltages and currents have equal magnitude and displaced 120° from each other
  - all circuit elements in each phase are equal
- analysis in a single-phase leads directly to the three-phase solution



$\underline{I}_a = \underline{I}_a$ $\quad$ $\underline{I}_b = \underline{a}^2\, \underline{I}_a$ $\quad$ $\underline{I}_c = \underline{a}\, \underline{I}_a$

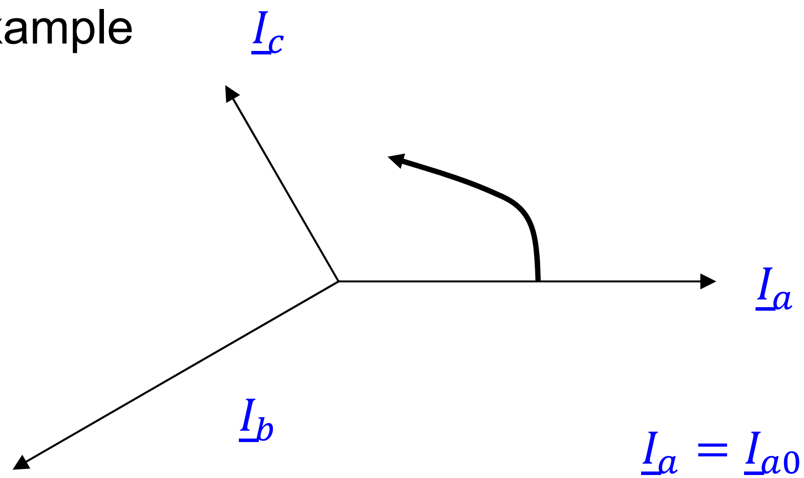
$\underline{I}_c$

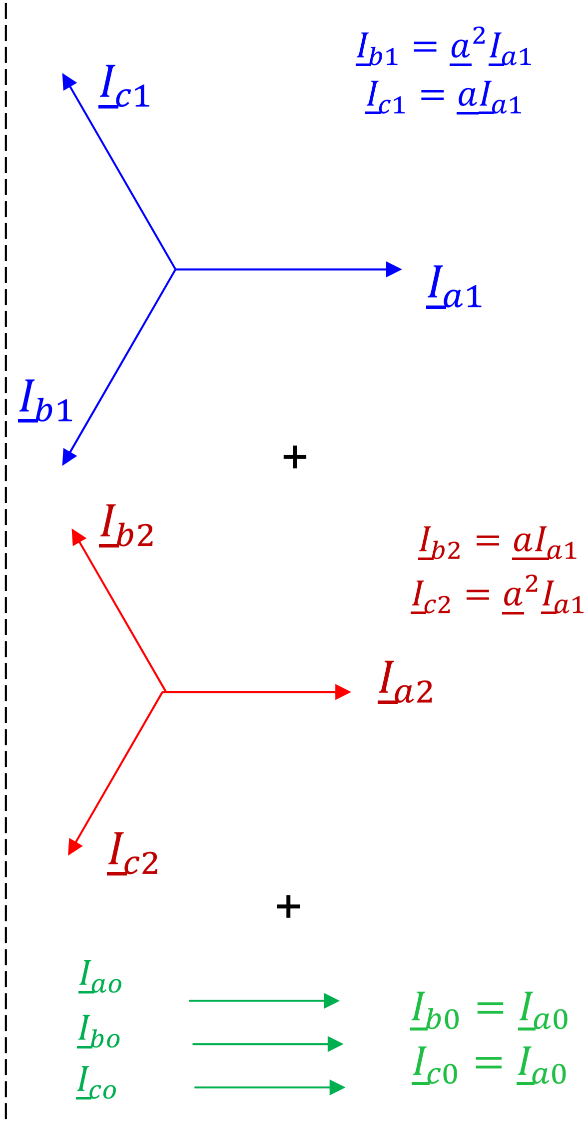
A general three-phase system:

$$\underline{I}_a = \underline{I}_a \qquad \underline{I}_b \neq \underline{a}^2\,\underline{I}_a \qquad \underline{I}_c \neq \underline{a}\,\underline{I}_a$$

A general three-phase system decomposed into symmetrical compoents:

Example





$$\underline{I}_a = \underline{I}_{a0} + \underline{I}_{a1} + \underline{I}_{a2}$$
$$\underline{I}_b = \underline{I}_{b0} + \underline{I}_{b1} + \underline{I}_{b2}$$
$$\underline{I}_c = \underline{I}_{c0} + \underline{I}_{c1} + \underline{I}_{c2}$$

$$\underline{I}_a = \underline{I}_{a0} + \underline{I}_{a1} + \underline{I}_{a2}$$
$$\underline{I}_b = \underline{I}_{a0} + \underline{a}^2\underline{I}_{a1} + \underline{a}\,\underline{I}_{a2}$$
$$\underline{I}_c = \underline{I}_{a0} + \underline{a}\,\underline{I}_{a1} + \underline{a}^2\underline{I}_{a2}$$

$$\rightarrow \begin{pmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{pmatrix} \begin{pmatrix} \underline{I}_{a0} \\ \underline{I}_{a1} \\ \underline{I}_{a2} \end{pmatrix}$$

abc and symmetrical components

Current transformation

$$\begin{pmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{pmatrix} \begin{pmatrix} \underline{I}_{a0} \\ \underline{I}_{a1} \\ \underline{I}_{a2} \end{pmatrix} \leftrightarrow \begin{pmatrix} \underline{I}_{a0} \\ \underline{I}_{a1} \\ \underline{I}_{a2} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{pmatrix} \begin{pmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{pmatrix}$$

Voltage transformation

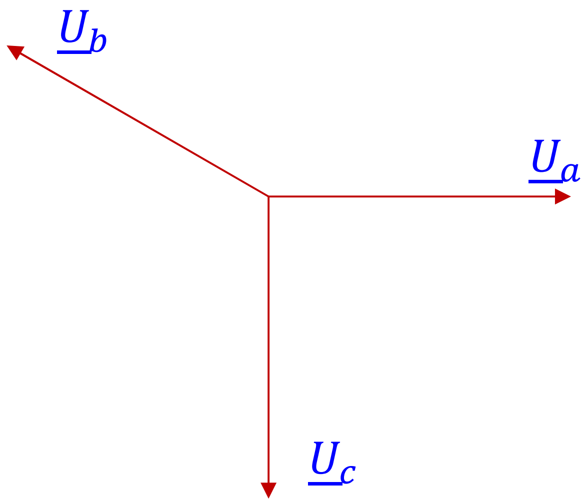
$$\begin{pmatrix} \underline{U}_a \\ \underline{U}_b \\ \underline{U}_c \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{pmatrix} \begin{pmatrix} \underline{U}_{a0} \\ \underline{U}_{a1} \\ \underline{U}_{a2} \end{pmatrix} \leftrightarrow \begin{pmatrix} \underline{U}_{a0} \\ \underline{U}_{a1} \\ \underline{U}_{a2} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{pmatrix} \begin{pmatrix} \underline{U}_a \\ \underline{U}_b \\ \underline{U}_c \end{pmatrix}$$

Example

$\underline{U}_a$ = 100 V∠30^0

$\underline{U}_b$= 100 V∠150^0

$\underline{U}_c = $ 100 V∠−90^0



$$\begin{pmatrix}\underline{U}_{a0}\\ \underline{U}_{a1}\\ \underline{U}_{a2}\end{pmatrix} = \frac{1}{3}\begin{pmatrix}1 & 1 & 1\\ 1 & \underline{a} & \underline{a}^2\\ 1 & \underline{a}^2 & \underline{a}\end{pmatrix}\begin{pmatrix}100\text{ V}\angle 30^0\\ 100\text{ V}\angle 150^0\\ 100\text{ V}\angle -90^0\end{pmatrix}$$

$$\begin{pmatrix}\underline{U}_{a0}\\ \underline{U}_{a1}\\ \underline{U}_{a2}\end{pmatrix} = \begin{pmatrix}0\\ 0\\ 100\text{ V}\angle 30^0\end{pmatrix}$$

$\underline{U}_{a2} =$ 100 V∠30^0 $\underline{U}_{b2} =$ 100 V∠150^0 $\underline{U}_{c2} =$ 100 V∠−90^0

Check

$$\underline{U}_a = \underline{U}_{a0} + \underline{U}_{a1} + \underline{U}_{a2}$$
$$\underline{U}_b = \underline{U}_{b0} + \underline{U}_{b1} + \underline{U}_{b2}$$
$$\underline{U}_c = \underline{U}_{c0} + \underline{U}_{c1} + \underline{U}_{c2}$$

Power in symmetrical components

$$\underline{S} = \underline{U}_a \underline{I}_a^* + \underline{U}_b \underline{I}_b^* + \underline{U}_c \underline{I}_c^* \rightarrow \underline{S} = \begin{pmatrix} \underline{U}_a & \underline{U}_b & \underline{U}_c \end{pmatrix} \begin{pmatrix} \underline{I}_a^* \\ \underline{I}_b^* \\ \underline{I}_c^* \end{pmatrix}; \quad \begin{pmatrix} \underline{I}_a^* \\ \underline{I}_b^* \\ \underline{I}_c^* \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{pmatrix} \begin{pmatrix} \underline{I}_{a0}^* \\ \underline{I}_{a1}^* \\ \underline{I}_{a2}^* \end{pmatrix}$$

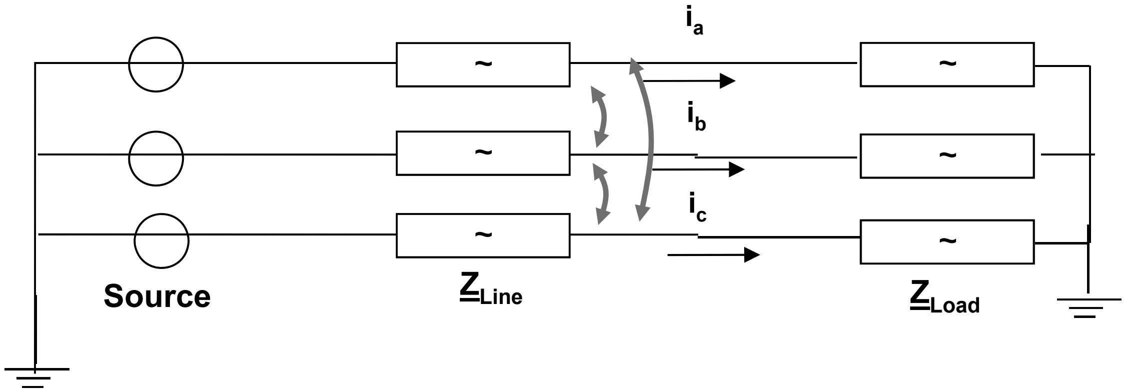
$$\begin{pmatrix} \underline{U}_a \\ \underline{U}_b \\ \underline{U}_c \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{pmatrix} \begin{pmatrix} \underline{U}_{a0} \\ \underline{U}_{a1} \\ \underline{U}_{a2} \end{pmatrix}$$

$$\begin{pmatrix} \underline{U}_a & \underline{U}_b & \underline{U}_c \end{pmatrix} = \begin{pmatrix} \underline{U}_a \\ \underline{U}_b \\ \underline{U}_c \end{pmatrix}^t = \begin{pmatrix} \underline{U}_{a0} & \underline{U}_{a1} & \underline{U}_{a2} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{pmatrix} \rightarrow$$

$$\underline{S} = \begin{pmatrix} \underline{U}_{a0} & \underline{U}_{a1} & \underline{U}_{a2} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{pmatrix} \begin{pmatrix} \underline{I}_{a0}^* \\ \underline{I}_{a1}^* \\ \underline{I}_{a2}^* \end{pmatrix}$$

$$\underline{S} = \begin{pmatrix} \underline{U}_{a0} & \underline{U}_{a1} & \underline{U}_{a2} \end{pmatrix} \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} \underline{I}_{a0}^* \\ \underline{I}_{a1}^* \\ \underline{I}_{a2}^* \end{pmatrix} = 3\left(\underline{U}_{a0} \underline{I}_{a0}^* + \underline{U}_{a1} \underline{I}_{a1}^* + \underline{U}_{a2} \underline{I}_{a2}^*\right)$$

$$\begin{pmatrix} \underline{U}_{aS} \\ \underline{U}_{bS} \\ \underline{U}_{cS} \end{pmatrix}$$



$$\begin{pmatrix} \underline{U}_{a0} \\ \underline{U}_{a1} \\ \underline{U}_{a2} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{pmatrix} \begin{pmatrix} \underline{U}_{aS} \\ \underline{U}_{bS} \\ \underline{U}_{cS} \end{pmatrix}$$

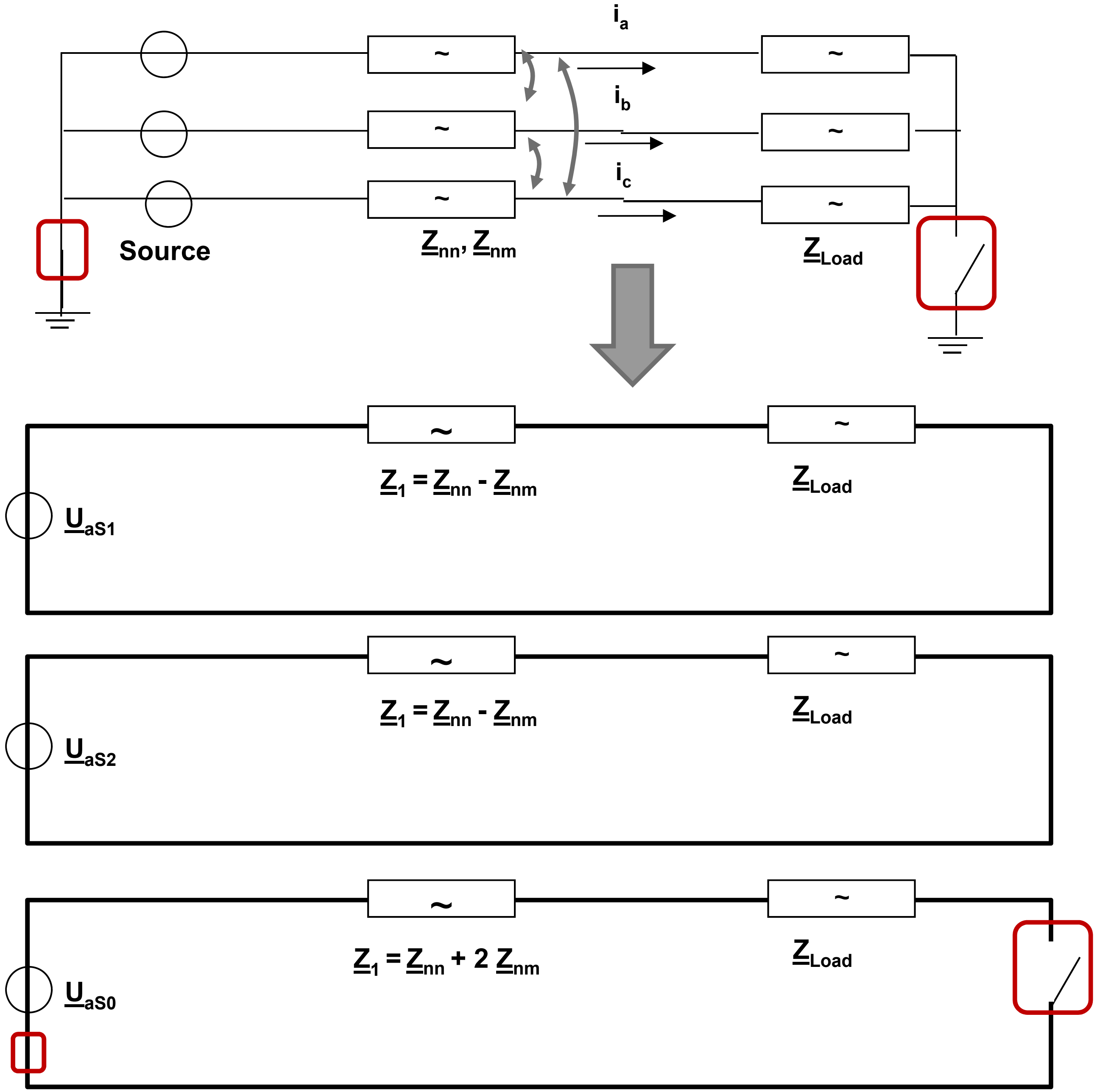
$$\begin{pmatrix} \underline{U}_{a} \\ \underline{U}_{b} \\ \underline{U}_{c} \end{pmatrix} = \begin{pmatrix} \underline{Z}_{aa} & \underline{Z}_{ab} & \underline{Z}_{ac} \\ \underline{Z}_{ba} & \underline{Z}_{bb} & \underline{Z}_{bc} \\ \underline{Z}_{ca} & \underline{Z}_{cb} & \underline{Z}_{cc} \end{pmatrix} \begin{pmatrix} \underline{I}_{a} \\ \underline{I}_{b} \\ \underline{I}_{c} \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{pmatrix} \begin{pmatrix} \underline{U}_{a0} \\ \underline{U}_{a1} \\ \underline{U}_{a2} \end{pmatrix} = \begin{pmatrix} \underline{Z}_{aa} & \underline{Z}_{ab} & \underline{Z}_{ac} \\ \underline{Z}_{ba} & \underline{Z}_{bb} & \underline{Z}_{bc} \\ \underline{Z}_{ca} & \underline{Z}_{cb} & \underline{Z}_{cc} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{pmatrix} \begin{pmatrix} \underline{I}_{a0} \\ \underline{I}_{a1} \\ \underline{I}_{a2} \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} \underline{U}_{a0} \\ \underline{U}_{a1} \\ \underline{U}_{a2} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{pmatrix} \begin{pmatrix} \underline{Z}_{aa} & \underline{Z}_{ab} & \underline{Z}_{ac} \\ \underline{Z}_{ba} & \underline{Z}_{bb} & \underline{Z}_{bc} \\ \underline{Z}_{ca} & \underline{Z}_{cb} & \underline{Z}_{cc} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{pmatrix} \begin{pmatrix} \underline{I}_{a0} \\ \underline{I}_{a1} \\ \underline{I}_{a2} \end{pmatrix}$$

Assuming:

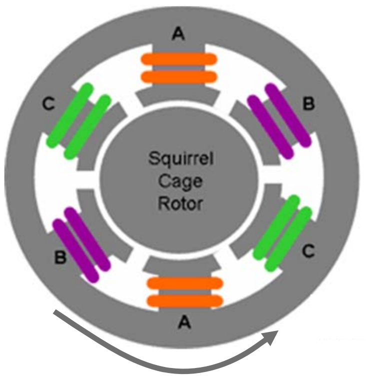
$$\underline{Z}_{aa} = \underline{Z}_{bb} = \underline{Z}_{cc} = \underline{Z}_{nn}$$
$$\underline{Z}_{ab} = \underline{Z}_{bc} = \underline{Z}_{ca} = \underline{Z}_{nm}$$

$$\begin{pmatrix} \underline{U}_{a0} \\ \underline{U}_{a1} \\ \underline{U}_{a2} \end{pmatrix} = \begin{pmatrix} \underline{Z}_{nn} + 2\underline{Z}_{nm} & 0 & 0 \\ 0 & \underline{Z}_{nn} - \underline{Z}_{nm} & 0 \\ 0 & 0 & \underline{Z}_{nn} - \underline{Z}_{nm} \end{pmatrix} \begin{pmatrix} \underline{I}_{a0} \\ \underline{I}_{a1} \\ \underline{I}_{a2} \end{pmatrix}$$

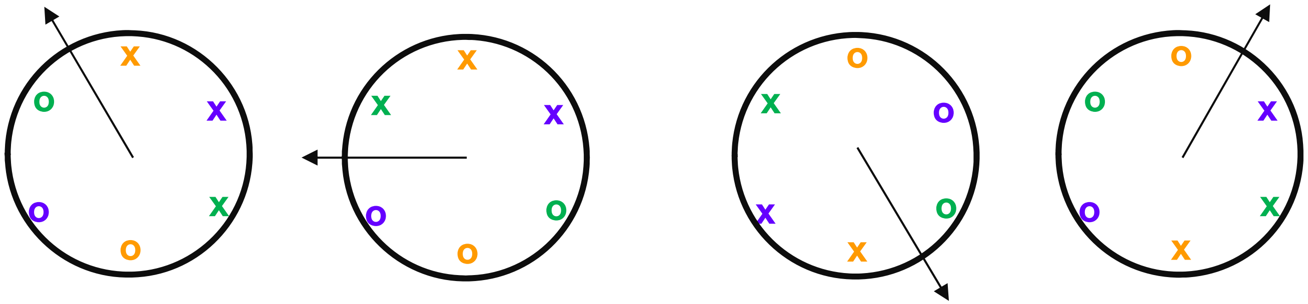


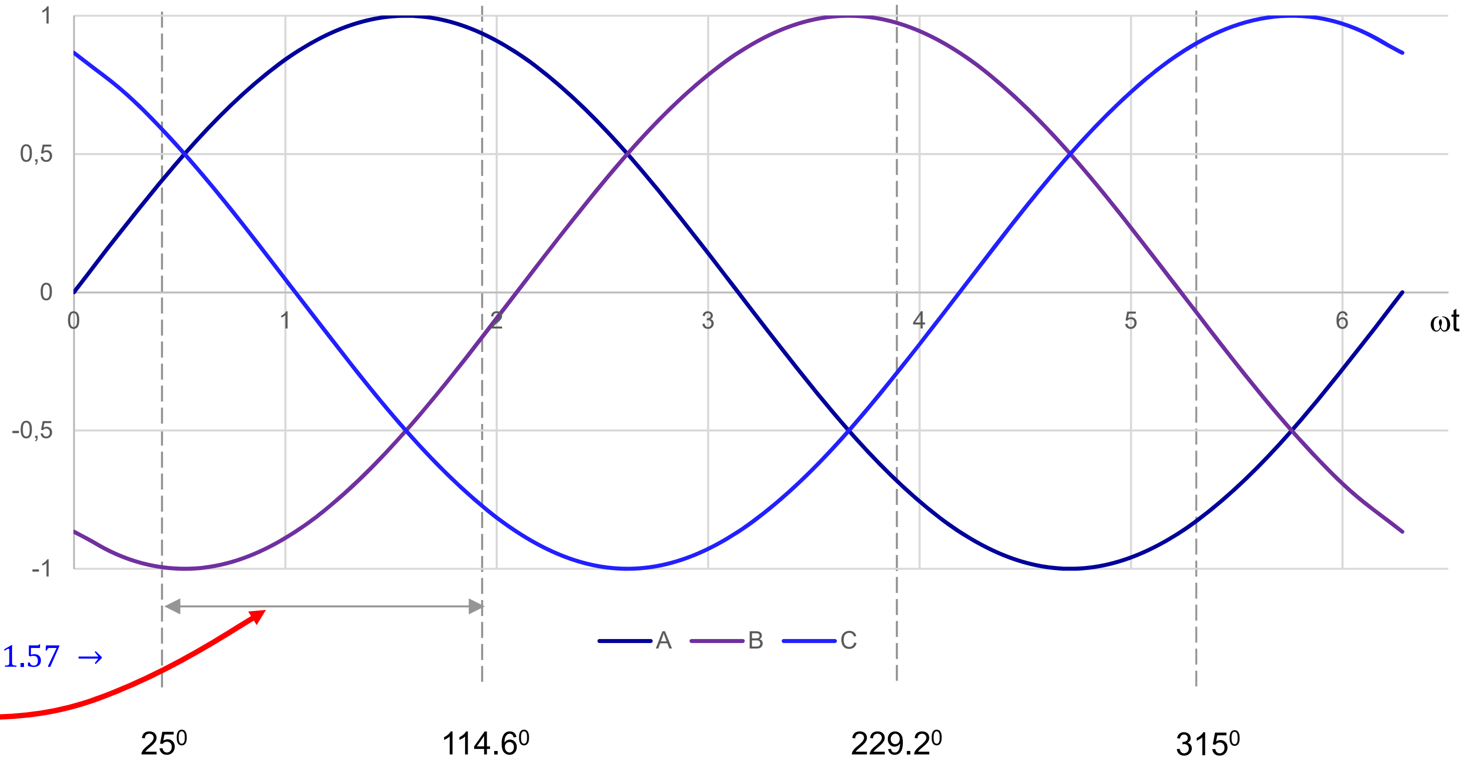
i_a
i_b
i_c
Source
$\underline{Z}_{nn}, \underline{Z}_{nm}$
$\underline{Z}_{Load}$
$\underline{Z}_1 = \underline{Z}_{nn} - \underline{Z}_{nm}$
$\underline{Z}_{Load}$
$\underline{U}_{aS1}$
$\underline{Z}_1 = \underline{Z}_{nn} - \underline{Z}_{nm}$
$\underline{Z}_{Load}$
$\underline{U}_{aS2}$
$\underline{Z}_1 = \underline{Z}_{nn} + 2\ \underline{Z}_{nm}$
$\underline{Z}_{Load}$
$\underline{U}_{aS0}$

The Concept of Rotating Magnetic Field



$$i_A = sin\,\omega t$$
$$i_B = \sin(\omega t - 2\pi/3)$$
$$i_C = \sin(\omega t + 2\pi/3)$$





$\Delta\vartheta = \Delta t\ \omega = \frac{114.6 - 25}{180}\pi = 1.57 \rightarrow$

$\Delta t = \frac{1.57}{100\,\pi} \approx 5\ ms$

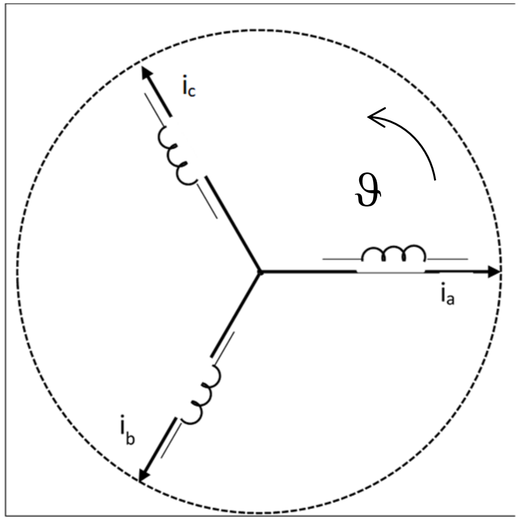
Rotating magnetic field – mathematical formulation

$$F_a(\theta, t) = F_m \cos\omega t \cos\theta$$

$$F_b(\theta, t) = F_m \cos(\omega t - 120°)\cos(\theta - 120°)$$

$$F_c(\theta, t) = F_m \cos(\omega t - 240°)\cos(\theta - 240°)$$

F: magnetomotoive force (MMF); unit: Amp - turn



Adding the three MMFs above:

$$F(\theta, t) = F_a(\theta, t) + F_b(\theta, t) + F_c(\theta, t) =$$

$$F_m \cos\omega t \cos\theta +$$

$$F_m \cos(\omega t - 120°)\cos(\theta - 120°) +$$

$$F_m \cos(\omega t - 240°)\cos(\theta - 240°)$$

$\cos\alpha \cos\beta = \frac{1}{2}(\cos(\alpha-\beta)+\cos(\alpha+\beta))$

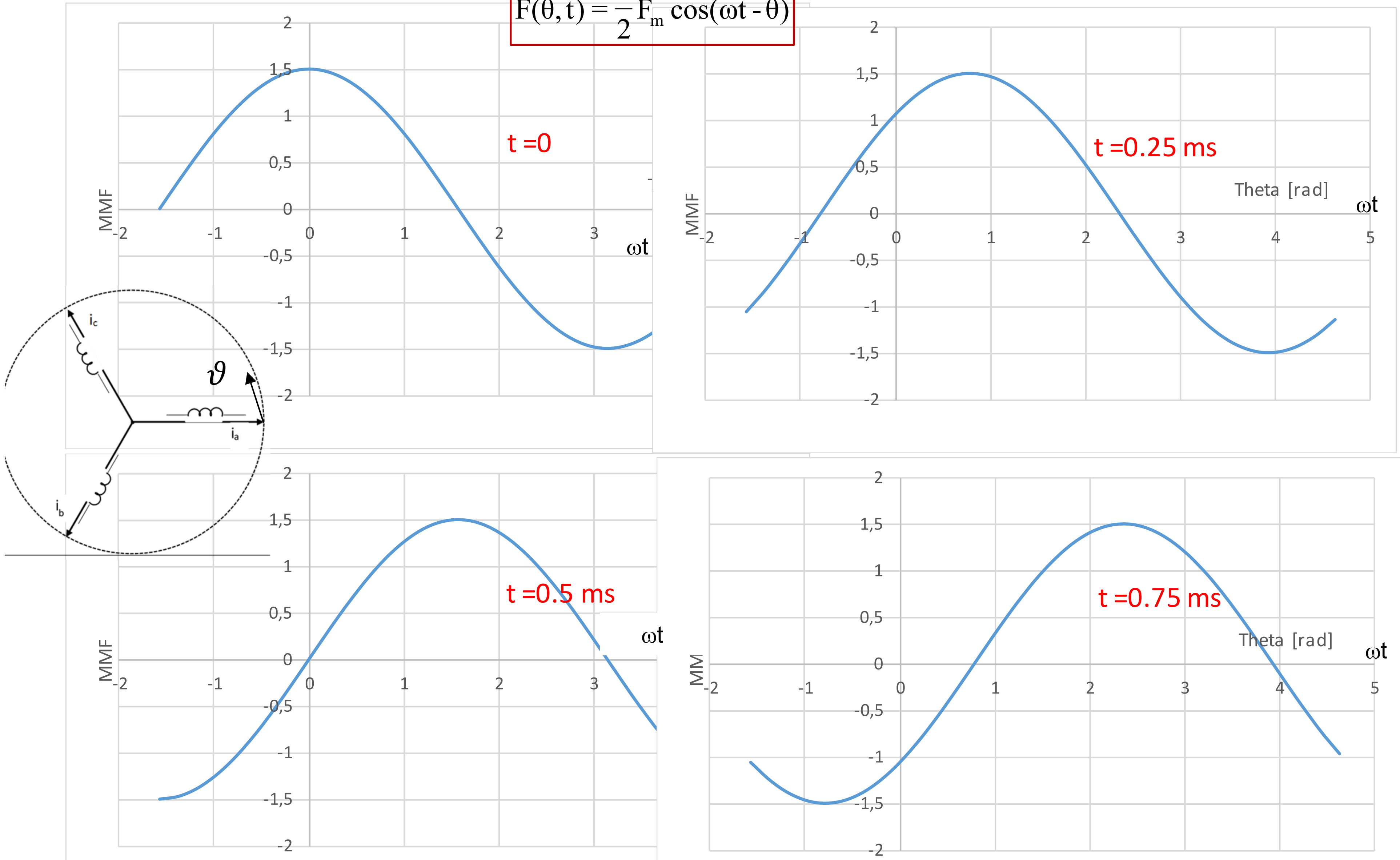
$$F(\theta, t) = \frac{3}{2} F_m \cos(\omega t - \theta)$$

rotating magnetic field

F Function of location (along the air gap) and time

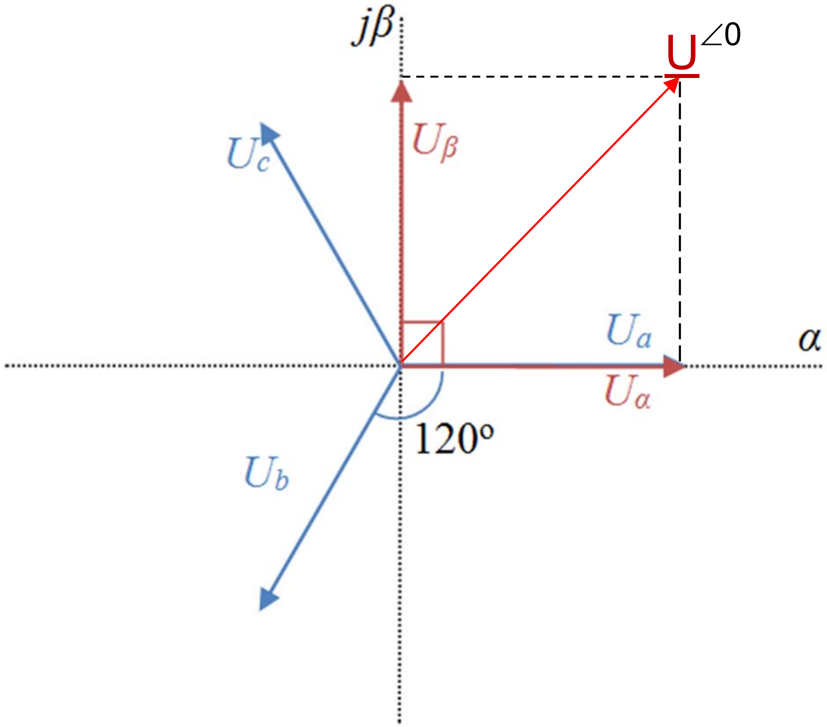
Rotating magnetic field

$$F(\theta, t) = \frac{3}{2} F_m \cos(\omega t - \theta)$$



αβ (Clarke)Transformation

αβTransformation and Definition of Space Vector



The transformation relationship from abc → αβ0

$$\begin{bmatrix} g_\alpha(t) \\ g_\beta(t) \\ g_0(t) \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} g_a(t) \\ g_b(t) \\ g_c(t) \end{bmatrix}$$

$$\underline{g}^{\angle 0} = g_\alpha + jg_\beta.$$

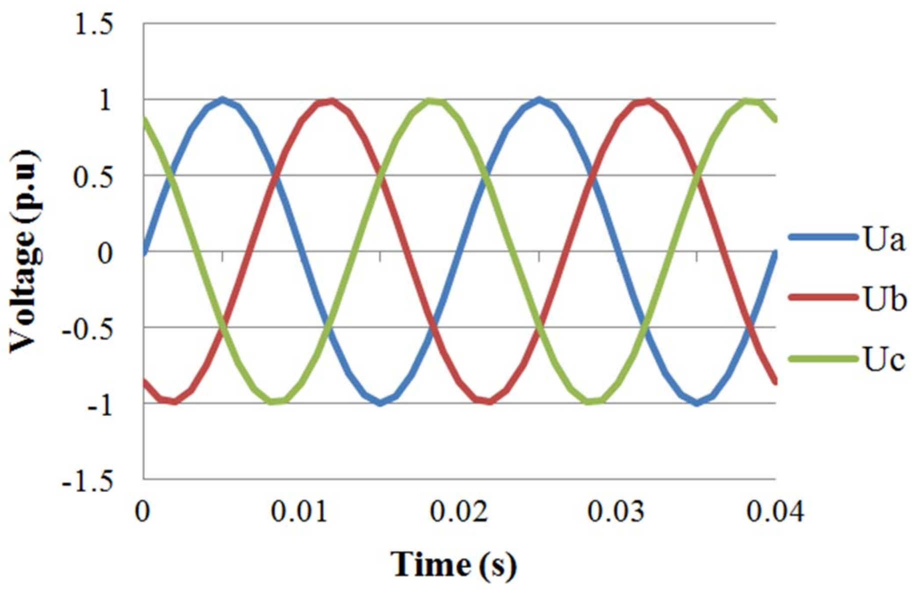
$$\underline{a} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}, \qquad \underline{a}^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2},$$

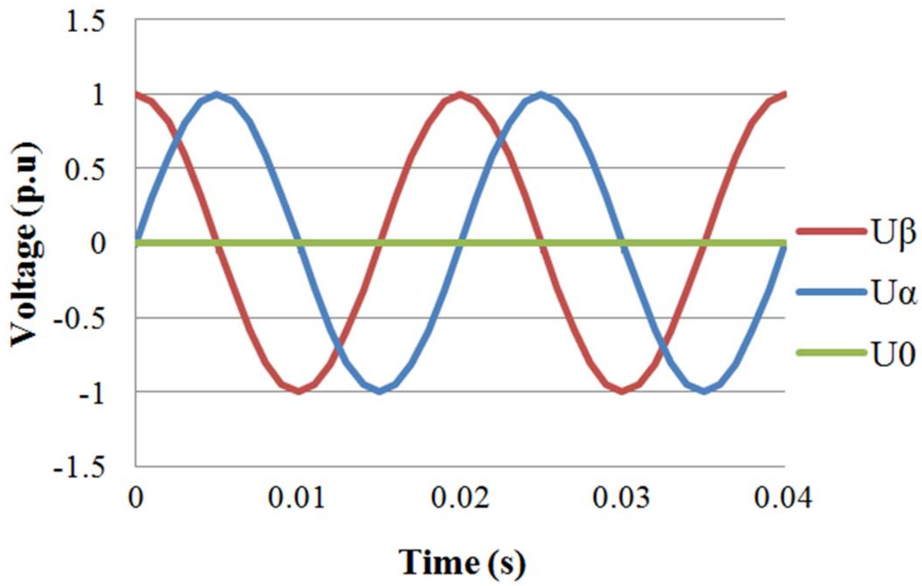
$$\underline{g}^{\angle 0} = \frac{2}{3}\left(g_a + \underline{a}\, g_b + \underline{a}^2\, g_c\right).$$

$\underline{g}^{\angle 0}$ **Space vector**

rotates at the speed corresponding to the system radian frequency with constant amplitude

αβTransformation and Definition of Space Vector





Clarke Transformation

Transformation of line quantities (voltage, current, flux) to a, b, 0 components

$$\begin{bmatrix} g_\alpha(t) \\ g_\beta(t) \\ \hat{g}_0(t) \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ 0 & \sqrt{3} & -\sqrt{3} \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} g_a(t) \\ g_b(t) \\ g_c(t) \end{bmatrix}$$

Edith Clarke
10.02. 1883 – 29.10.1959

AT&T, MIT, GE, U-Texas

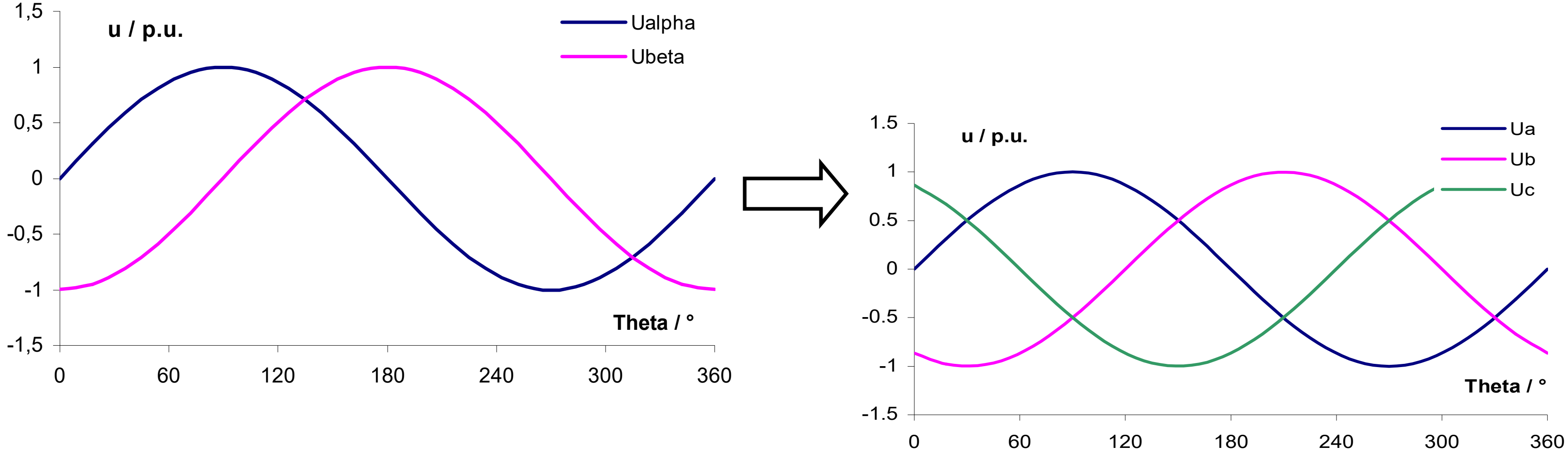
Space vector in stationary coordinates

$$\underline{g}^{\angle 0} = g_\alpha + \mathrm{j}g_\beta. \quad \text{and with} \quad \underline{a} = -\frac{1}{2} + \mathrm{j}\frac{\sqrt{3}}{2}, \qquad \underline{a}^2 = -\frac{1}{2} - \mathrm{j}\frac{\sqrt{3}}{2},$$

$$\underline{g}^{\angle 0} = \frac{2}{3}\left(g_a + \underline{a}\, g_b + \underline{a}^2\, g_c\right)$$

Inverse Clarke Transformation

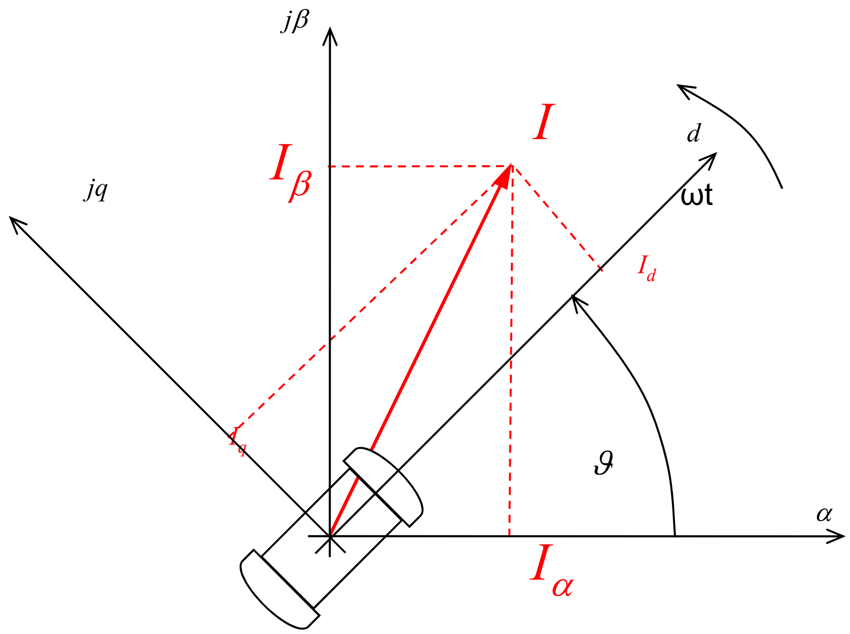
$$\begin{pmatrix} u_a \\ u_b \\ u_c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{2} & \sqrt{\frac{3}{4}} \\ -\frac{1}{2} & -\sqrt{\frac{3}{4}} \end{pmatrix} \cdot \begin{pmatrix} u_\alpha \\ u_\beta \end{pmatrix}$$



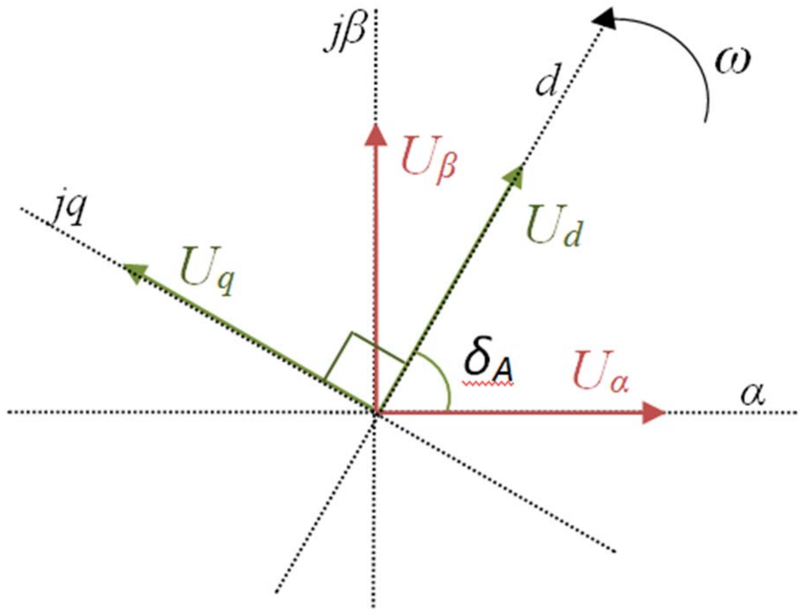
Park Transformation relationship

$$\begin{bmatrix} g_\alpha(t) \\ g_\beta(t) \\ \hat{g}_0(t) \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ 0 & \sqrt{3} & -\sqrt{3} \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} g_a(t) \\ g_b(t) \\ g_c(t) \end{bmatrix}.$$

$$\begin{bmatrix} g_d \\ g_q \\ g_0 \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\vartheta & \cos(\vartheta - 2\pi/3) & \cos(\vartheta + 2\pi/3) \\ -\sin\vartheta & -\sin(\vartheta - 2\pi/3) & -\sin(\vartheta + 2\pi/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} g_a \\ g_b \\ g_c \end{bmatrix},$$



αβTransformation and definition of space vector



The constants k_0, k_q, and k_d are chosen differently:

- $k_0=1/3$, $k_d=k_q= 2/3$ causes the magnitude of the d-q quantities to be equal to that of the phase quantities; but a 3/2 multiplier is required in front of the power expression
- $k_0=1/\sqrt{3}$, $k_d=k_q=\sqrt{2/3}$ power invariant

The transformation relationship from **αβ** to dq0 components

$$\begin{bmatrix} u_d \\ u_q \\ u_0 \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ \cos\theta & -\sin\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_\alpha \\ u_\beta \\ u_0 \end{bmatrix}$$

The transformation relationship from abc to dq0 components

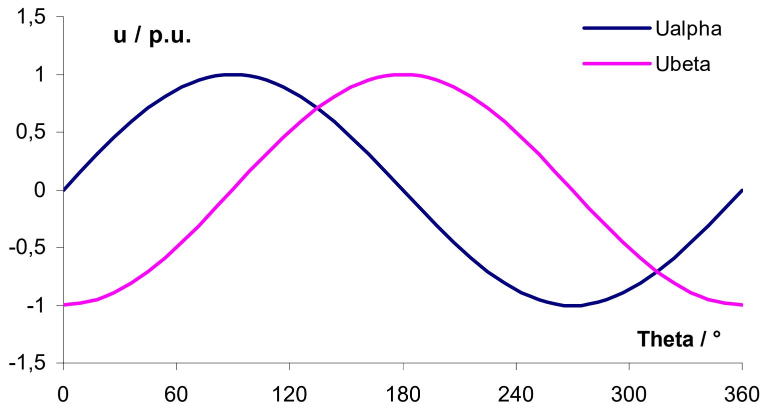
$$\begin{bmatrix} g_d \\ g_q \\ g_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos\vartheta & \cos\left(\vartheta - 2\pi/3\right) & \cos\left(\vartheta + 2\pi/3\right) \\ -\sin\vartheta & -\sin\left(\vartheta - 2\pi/3\right) & -\sin\left(\vartheta + 2\pi/3\right) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} g_a \\ g_b \\ g_c \end{bmatrix},$$

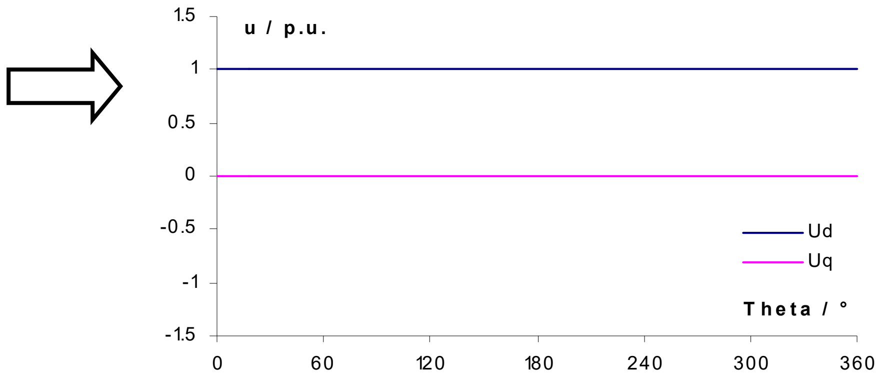
dq (Park) Transformation

Park Transformation

$$\begin{pmatrix} u_d \\ u_q \end{pmatrix} = \begin{pmatrix} \cos(\vartheta) & \sin(\vartheta) \\ -\sin(\vartheta) & \cos(\vartheta) \end{pmatrix} \cdot \begin{pmatrix} u_\alpha(t) \\ u_\beta(t) \end{pmatrix}$$

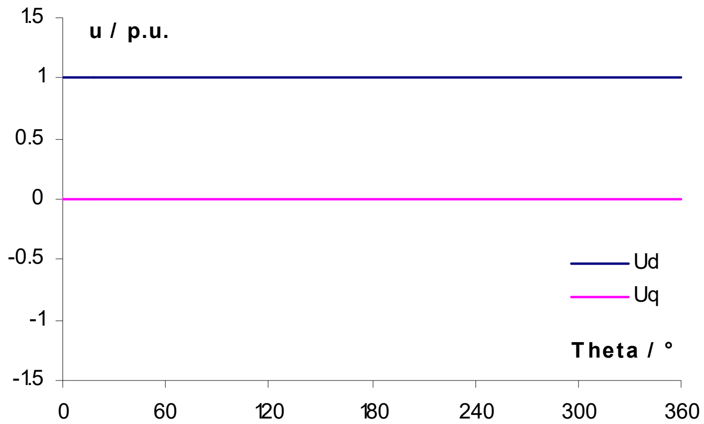
$$\underline{u} = u_d + ju_q$$

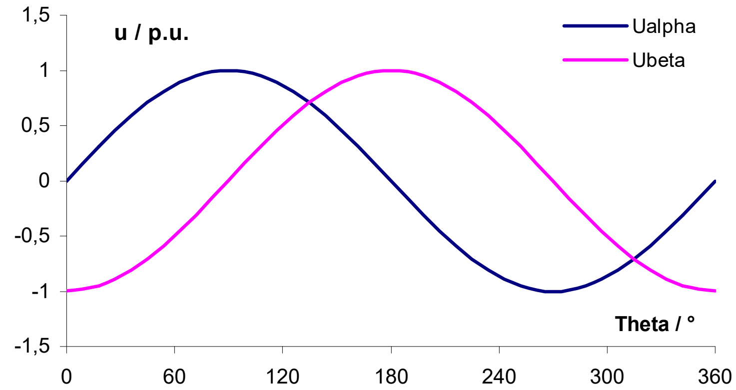




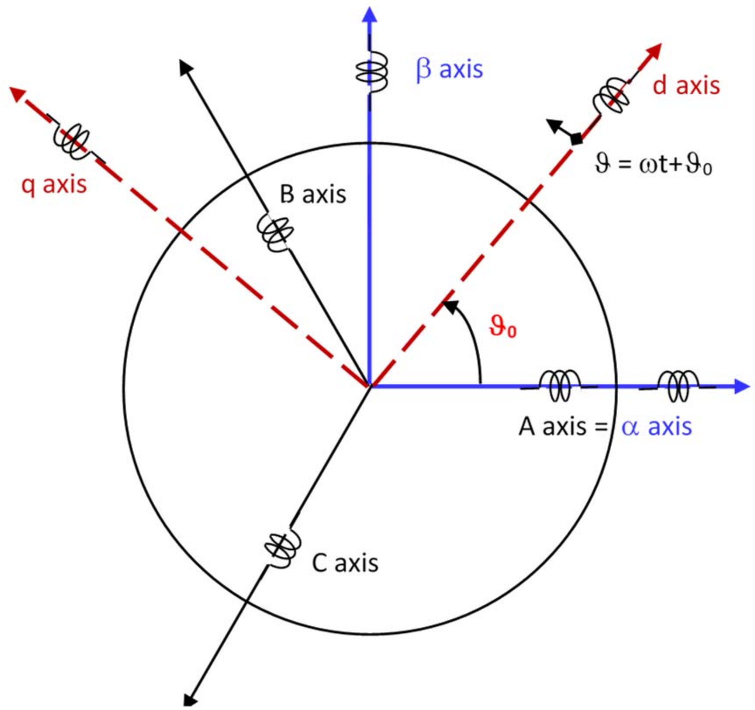
Inverse Park Transformation

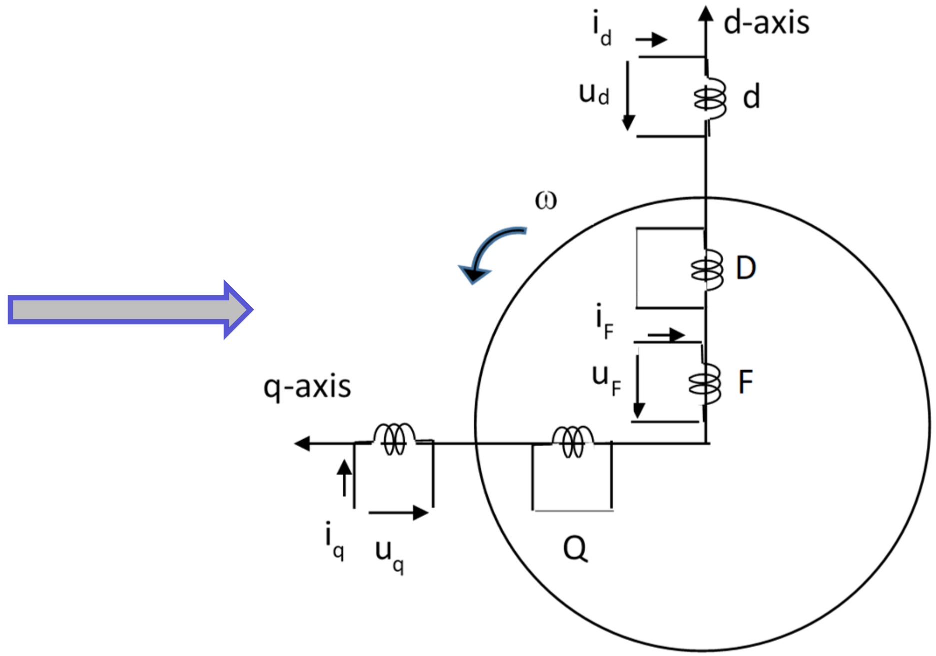
$$\begin{pmatrix} u_\alpha \\ u_\beta \end{pmatrix} = \begin{pmatrix} \cos(\vartheta) & -\sin(\vartheta) \\ \sin(\vartheta) & \cos(\vartheta) \end{pmatrix} \cdot \begin{pmatrix} u_d \\ u_q \end{pmatrix}$$





Voltage, current, and instantaneous power in abc, αβ0 and dq0 coordinates





$$i_d = \frac{2}{3}\left(i_a \cos\vartheta + i_b \cos(\vartheta - 2\pi/3) + i_c \cos(\vartheta + 2\pi/3)\right)$$

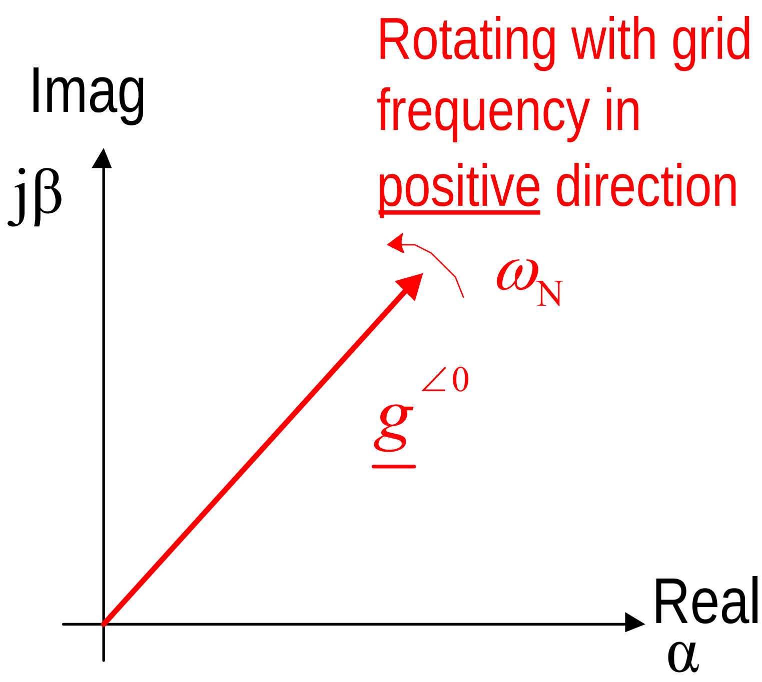
$$i_q = \frac{2}{3}\left(-i_a \sin\vartheta - i_b \sin(\vartheta - 2\pi/3) - i_c \sin(\vartheta + 2\pi/3)\right)$$

$$i_0 = \frac{1}{3}\left(i_a + i_b + i_c\right)$$

$$\begin{pmatrix} i_d \\ i_q \\ i_0 \end{pmatrix} = \frac{2}{3}\begin{pmatrix} \cos\vartheta & \cos(\vartheta - 2\pi/3) & \cos(\vartheta + 2\pi/3) \\ -\sin\vartheta & -\sin(\vartheta - 2\pi/3) & -\sin(\vartheta + 2\pi/3) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix}$$

Space Vector

For a symmetrical 3phase system:



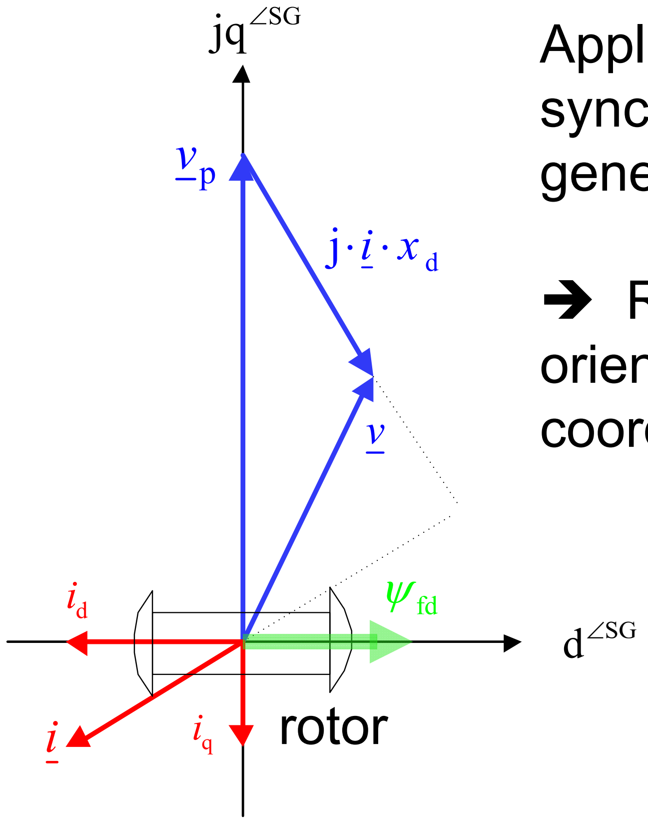
dq Components

Generalized definition of dq coordinate systems:

$$\underline{g}^{\angle \omega_X} = \underline{g}^{\angle 0} \cdot e^{-j(\omega_X t + \varphi)}$$

ω_X defines the rotational speed of the coordinate system

φ can be used to define the proper position of the d axis

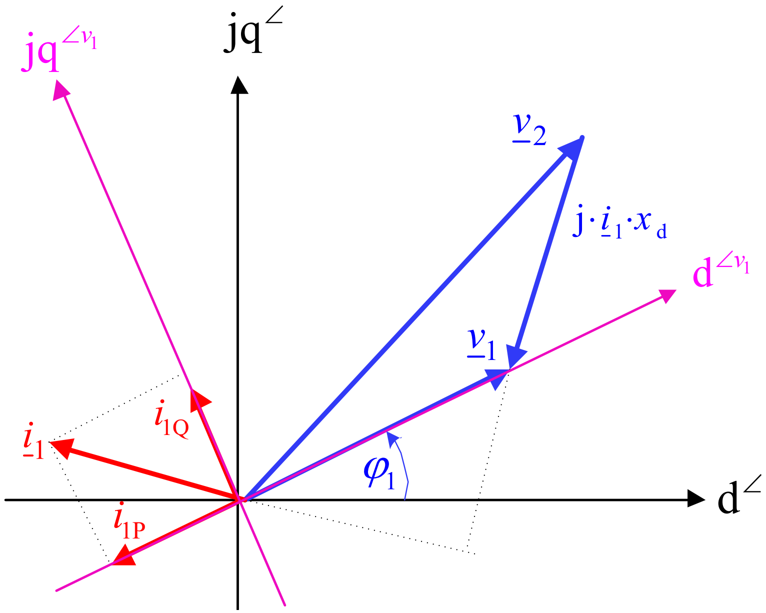


Application to synchronous generators

➔ Rotor flux oriented dq coordinates system

dq Components (2)

Definition of d axis along the voltage ➔ voltage oriented coordinates



Advantages:

$i_d \triangleq i_P$ active current

$-i_q \triangleq i_Q$ reactive current

$P_1 = |\underline{v}_1| \cdot i_{1P} \qquad Q_1 = |\underline{v}_1| \cdot i_{1Q}$

$\underline{i}_1^{\angle v_1} = \underline{i}_1^{\angle} \cdot e^{-j\varphi_1}$

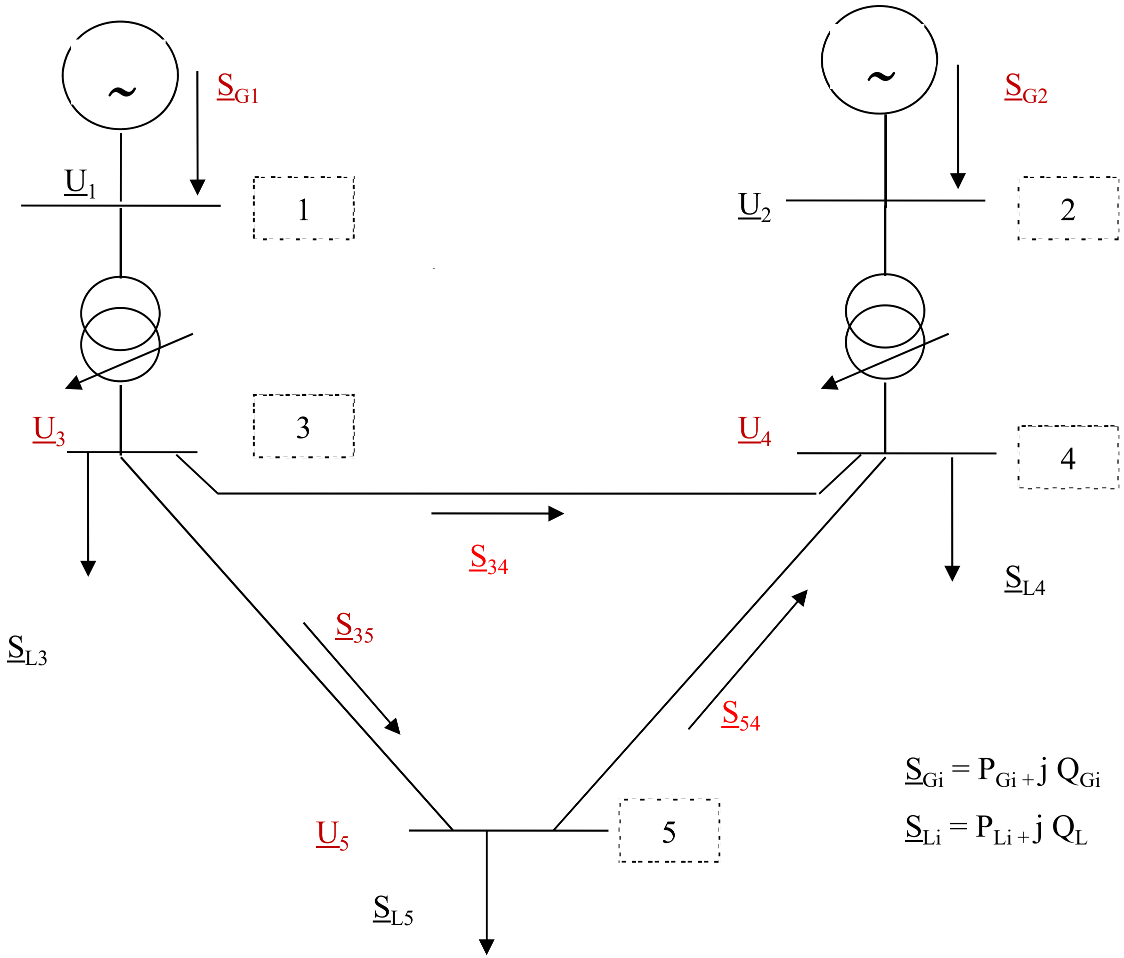
In WT converters the voltage oriented coordinates provide the advantage of controlling

➔ P by id - real components of the current

➔ Q by -iq - imaginary components of the current

Power flow analysis

Power flow



$$\underline{S}_{Gi} = P_{Gi} + j\,Q_{Gi}$$

$$\underline{S}_{Li} = P_{Li} + j\,Q_{L}$$

Objectives of the power flow calculation

Power flow calculation makes it possible enables the testing and evaluation the operation of the power supply system.

- Are the voltages at each busbar in the system acceptable?
- What is the level of utilization of the various resources in the system (transformers, transmission lines, generators, etc.)?
- How can the best possible operational configuration of the system be achieved?
- Does the system have one or more vulnerabilities?
  - If so, where are these and what countermeasures can be taken?

During system operation

- Important information about current state of the network
- Possible behavior of the network in different configurations of the network (eg switches open, closed, etc.)

For planning engineer

- Forecast on behavior of the system after expansion measures
- To determine the most suitable operating configurations
- To recognize possible weaknesses

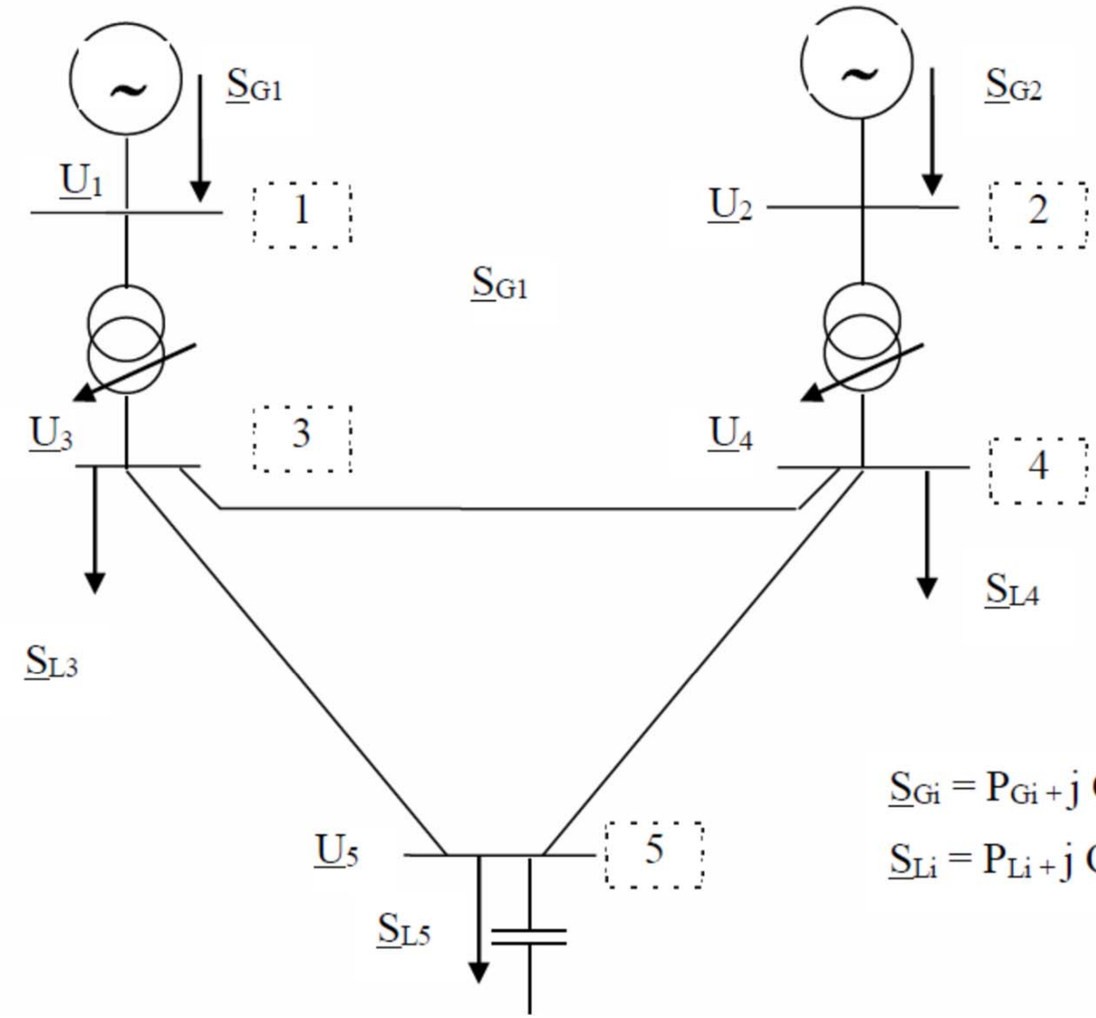
Power flow calculation analyzes the power supply system in steady state.

- The steady state is a state in which all variables and parameters are constant quantities (at least) during the observation period.
  - If we need to determine the behavior of the system for each hour of the day, we need to perform 24 power flows;
  - if the behavior is needed for every second, the number of required power flow calculations would be 86,400.

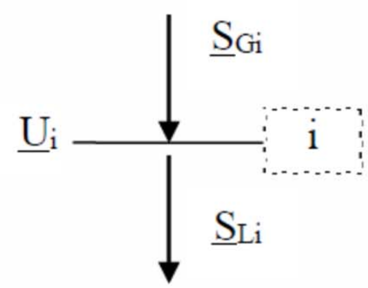
The following aspects are important for the assessment of a solution:

- The power flow configuration must not lead to overloading of equipment (lines, transformers, etc.).
- The voltage profile of the network must be within predefined limits through the correct distribution of the reactive power generation resources and suitable transformer tap settings.
- After failure of a transmission line or another major element in the system, the (n-1) principle must be fulfilled.

Formulation of the power flow equation (PFE)



At bus i:



$\underline{S}_{Gi} = P_{Gi} + j\,Q_{Gi}$

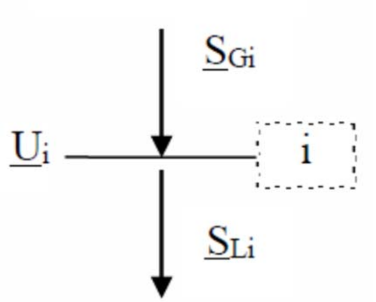
$\underline{S}_{Li} = P_{Li} + j\,Q_{L}$

Generation: $\underline{\mathbf{S}}_{Gi}$
Load: $\underline{\mathbf{S}}_{Li}$
Voltage: U_i, δ_i

Classification of buses

Six variables can be associated to a general bus i :

Generation	P_{Gi}	Q_{Gi}
Load	P_{Li}	Q_{Li}
Voltage	U_i	δ_i



On the other hand, for a network consisting of n nodes:
we have n complex equations (KCL) (i.e. 2n real equations)

As a general rule, $P_{Li} + jQ_{Li}$ at each bus assumed to be known.

Classification

1. **Load bus – PQ buses**
2. **Voltage controlled buses**
3. **Slack bus**

1. Load (PQ) bus

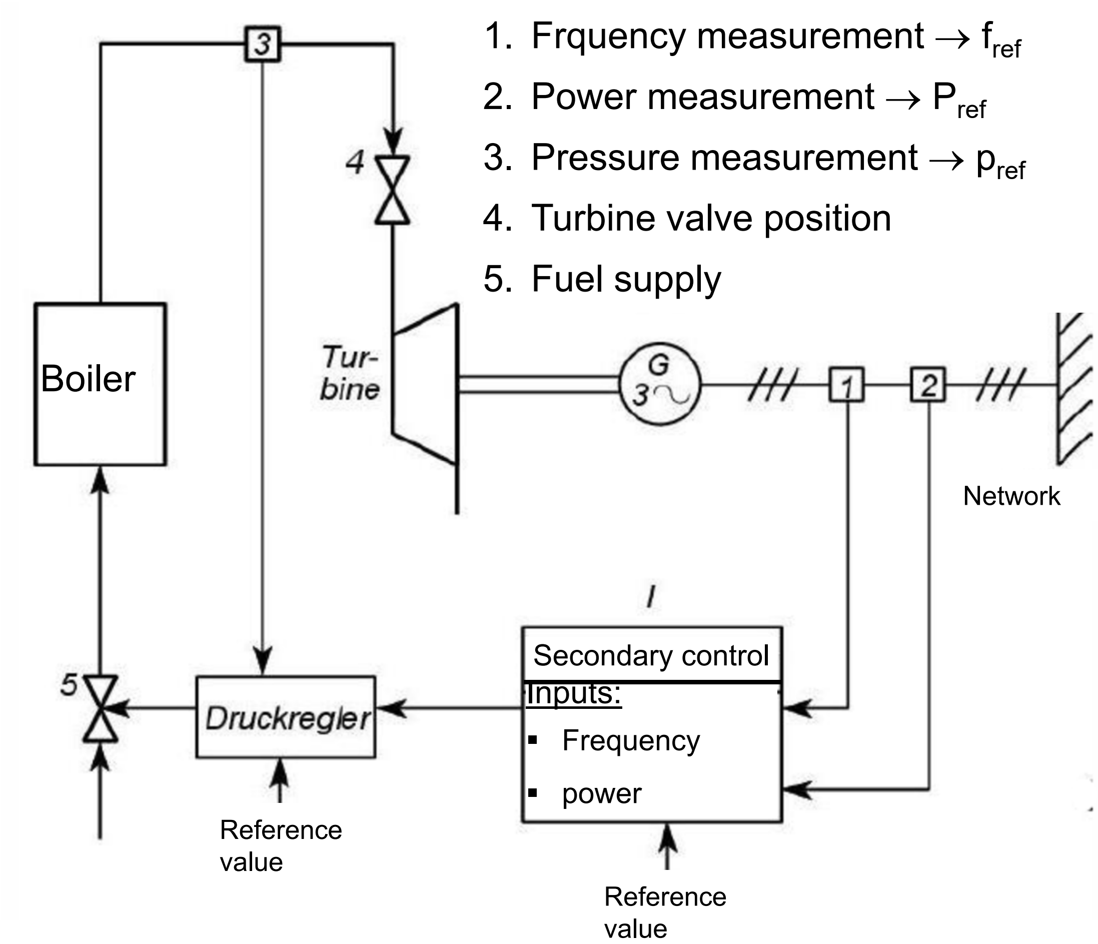
- At the load bus, due to the fact that no generator is attached to the bus:
  $P_{Gi} = Q_{Gi} = 0$
- The load flow calculation will yield:
  U_i , δ_i

Generation	$\mathbf{P_{Gi} = 0}$	**given**
	$Q_{Gi} = 0$	
Load	P_{Li}	known
	P_{Li}	
Voltage	U_i	To be calculated
	δ_i	

2. Voltage controlled (PV) bus

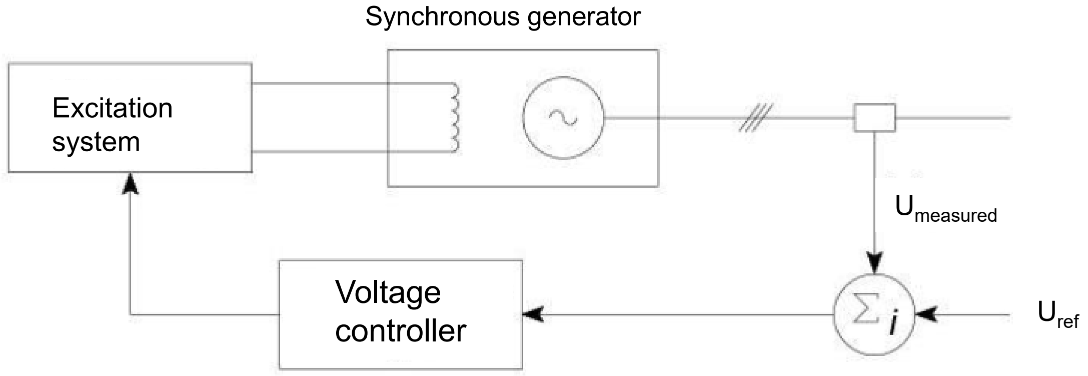
At generator bus

1. Frquency measurement → f_{ref}
2. Power measurement → P_{ref}
3. Pressure measurement → p_{ref}
4. Turbine valve position
5. Fuel supply



2. Voltage controlled (PV) bus

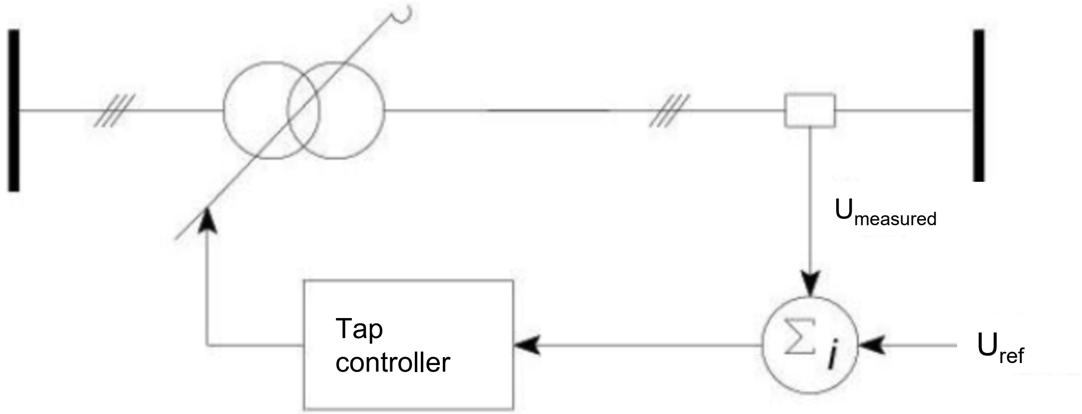
At a generator bus



At generator buses P_{Gi} and U_i known.

2. Voltage controlled (PV) bus

Bus with tap changing traformer

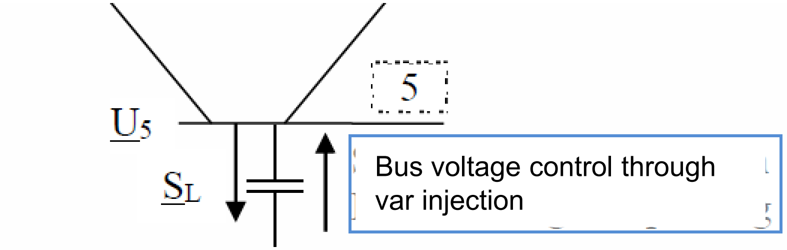


The voltage U_i controlled by transformer tap controller

P_{Gi} (=0) and U_i known.

2. Voltage controlled - PV bus

Bus connected to a var source



P_{Gi} (=0) and U_i known.

All three buses belong to the category „voltage controlled (PV) bus „

3. Reference – Slack bus

Objective during power system operation:

"The entire required active power must be delivered to the consumers by dividing the required power to the various plants in an optimal manner (economic dispatch / optimum power flow)."

This goal can not be exactly realized a priori for the following reasons:

- The power to be divided between the various generation power plants is based on estimation is not known exactly
- The transmission losses are not yet known

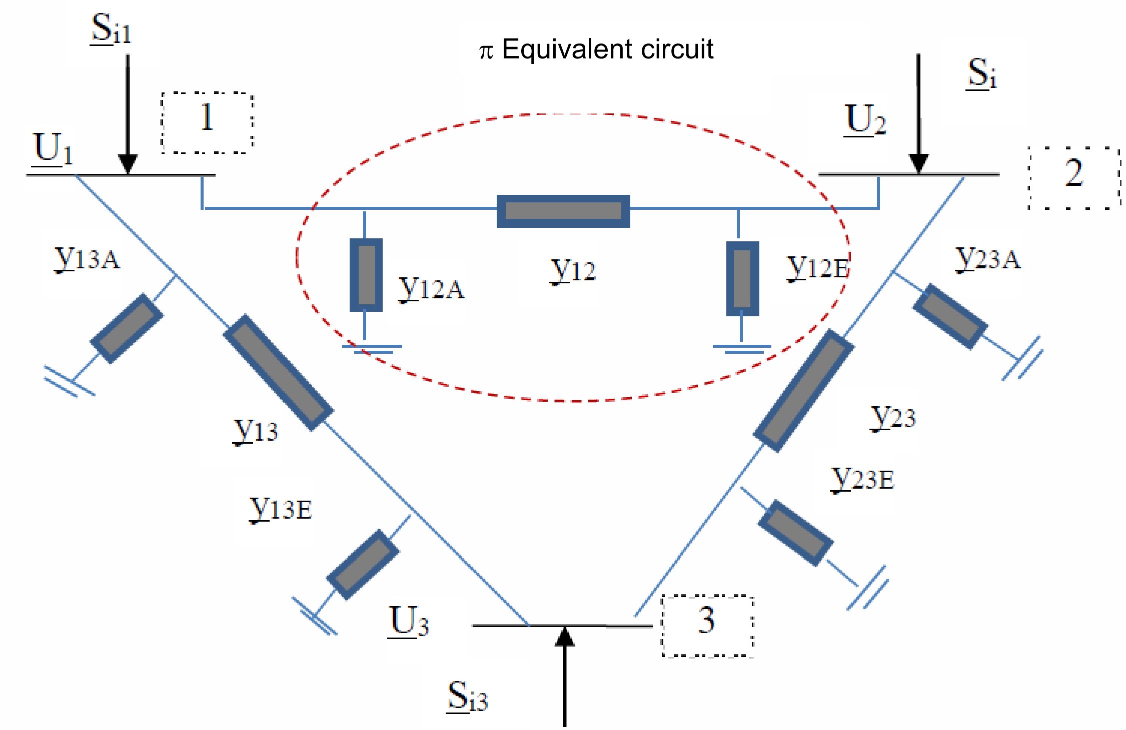
Thus:

- ✓ At least one generator must be able to feed variably.
- ✓ The corresponding node is called "reference node" – or the slack bus

Summary

	Load bus	Voltage controlled bus	Slack bus
Generation **P_{Gi} + Q_{Gi}**	known P_{Gi}= 0 Q_{Gi}= 0	P_{Gi}: known/specified Q_{Gi}: to be calculated Bus with tap changer: ▪ % tap change to be calculated ▪ Q_{Gi} = 0	P_{Gi}: to be calculated Q_{Gi}: to be calculated
Load **P_{Li} + Q_{Li}**	given	given	given
Voltage **$U_{i,}$ δ_i**	U_i: to be calculated δ_i : to be calculated	U_i: given/specified δ_i = to be calculated	U_i: given/specified δ_i = 0

$$[I] = [Y]\,[U]$$



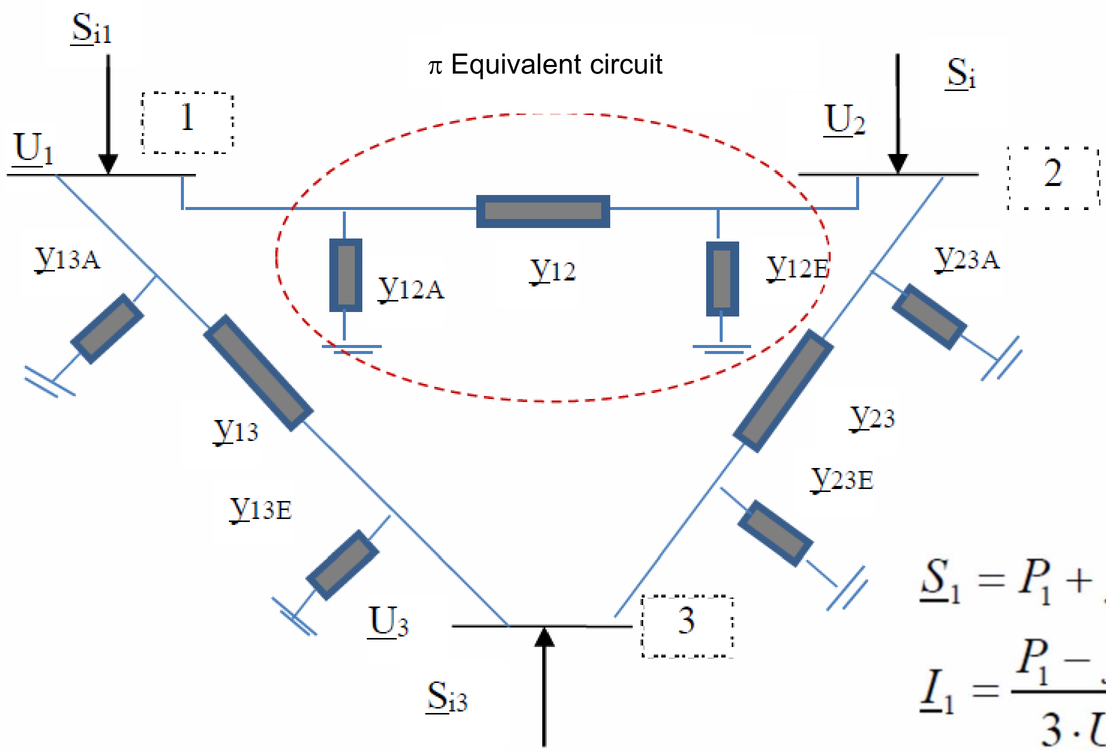
$$[I] = bus\ current\ vector$$

$$[U] = bus\ voltage\ vector$$

$$[Y] = nodal\ bus\ admittance\ matrix$$

$$I_1 = \underline{U}_1 \cdot \underline{y}_{12A} + (\underline{U}_1 - \underline{U}_2) \cdot \underline{y}_{12} + \underline{U}_1 \cdot \underline{y}_{13A} + (\underline{U}_1 - \underline{U}_3) \cdot \underline{y}_{13}$$

$$\begin{pmatrix} \underline{I}_1 \\ \underline{I}_2 \\ \underline{I}_3 \end{pmatrix} = \begin{pmatrix} \underline{y}_{12A} + \underline{y}_{12} + \underline{y}_{13A} + \underline{y}_{13} & -\underline{y}_{12} & -\underline{y}_{13} \\ -\underline{y}_{12} & \underline{y}_{12E} + \underline{y}_{12} + \underline{y}_{23A} + \underline{y}_{23} & -\underline{y}_{23} \\ -\underline{y}_{13} & -\underline{y}_{23} & \underline{y}_{13E} + \underline{y}_{13} + \underline{y}_{23E} + \underline{y}_{23} \end{pmatrix} \begin{pmatrix} \underline{U}_1 \\ \underline{U}_2 \\ \underline{U}_3 \end{pmatrix}$$



$$\underline{S}_1 = P_1 + jQ_1 = 3 \cdot \underline{U}_1 \cdot \underline{I}_1 \rightarrow$$

$$\underline{I}_1 = \frac{P_1 - jQ_1}{3 \cdot \underline{U}_1^*} = (\underline{U}_1 \cdot \underline{Y}_{11} + \underline{U}_2 \cdot \underline{Y}_{12} + \underline{U}_3 \cdot \underline{Y}_{13})$$

$$P_1 - jQ_1 = 3 \cdot \underline{U}_1^* \cdot \ (\underline{U}_1 \cdot \underline{Y}_{11} + \underline{U}_2 \cdot \underline{Y}_{12} + \underline{U}_3 \cdot \underline{Y}_{13})$$

$$P_2 - jQ_2 = 3 \cdot \underline{U}_2^* \cdot \ (\underline{U}_1 \cdot \underline{Y}_{21} + \underline{U}_2 \cdot \underline{Y}_{22} + \underline{U}_3 \cdot \underline{Y}_{23})$$

$$P_3 - jQ_3 = 3 \cdot \underline{U}_3^* \cdot \ (\underline{U}_1 \cdot \underline{Y}_{31} + \underline{U}_2 \cdot \underline{Y}_{32} + \underline{U}_3 \cdot \underline{Y}_{33})$$

Nonlinear system of equations →
only iterative solution possible!!

Example: Gaus –Seidel Method (without acceleration factor)

2x + xy = 1

2y - xy = -1

$$x^{(i+1)} = \frac{1}{2}\cdot\left(1 - x^{(i)}\cdot y^{(i)}\right)$$

$$y^{(i+1)} = \frac{1}{2}\cdot\left(-1 + x^{(i)}\cdot y^{(i)}\right)$$

step	x	y
0	0	0
1	0.5	-0.5
2	0.625	-0.625
3	0.695	-0.695
4	0.742	-0.742
5	0.775	-0.775
6	0.8	-0.8
:	:	:
	1	-1

Newton – Raphson Method

$$F_1(x, y) = 4y \sin x + 0.6 = 0$$
$$F_2(x, y) = 4y^2 - 4y \cos x + 0.3 = 0$$

Solution procedure

$$\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}^{(i)} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}^{(i)-1} \cdot \left(\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} - \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}^{(i)} \right)$$

$$x^{(i+1)} = x^{(i)} + \Delta x^{(i)}$$
$$y^{(i+1)} = y^{(i)} + \Delta y^{(i)}$$

Initial values: $x^{(0)} = 0$, $y^{(0)} = 1$

$$J = \begin{pmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \end{pmatrix} = \begin{pmatrix} 4y \cdot \cos x & 4 \cdot \sin x \\ 4y \cdot \sin x & 8y - 4 \cdot \cos x \end{pmatrix}$$

$$J^{(0)} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \qquad J^{(0)-1} = \begin{pmatrix} 0.25 & 0 \\ 0 & 0.25 \end{pmatrix}$$

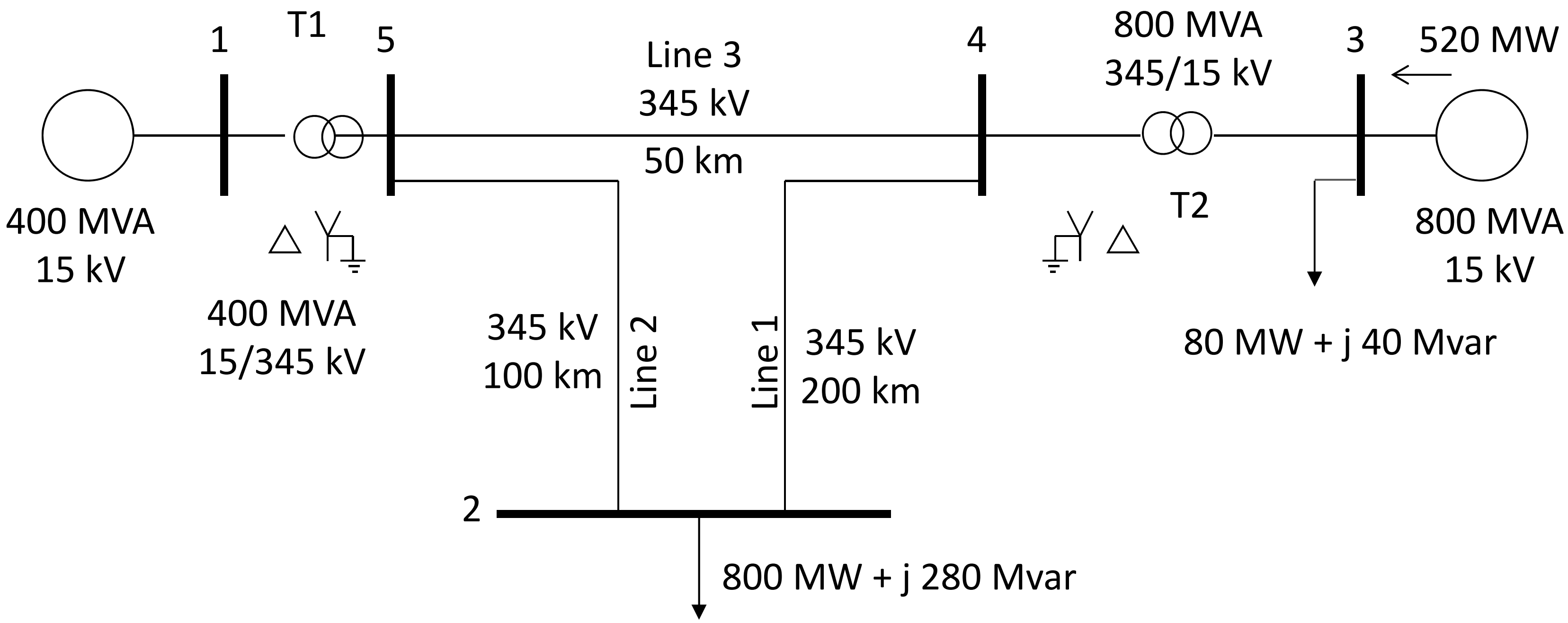
$$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}^{(0)} = \begin{pmatrix} 4y \cdot \sin x + 0.6 \\ 4y^2 - 4y \cdot \cos x + 0.3 \end{pmatrix} = \begin{pmatrix} 0.6 \\ 0.3 \end{pmatrix} \qquad \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \Delta F_1 \\ \Delta F_2 \end{pmatrix}^{(0)} = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} - \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}^{(0)} = \begin{pmatrix} -0.6 \\ -0.3 \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}^{(0)} = \begin{pmatrix} 0.25 & 0 \\ 0 & 0.25 \end{pmatrix} \cdot \begin{pmatrix} -0.6 \\ -0.3 \end{pmatrix} = \begin{pmatrix} -0.150 \\ -0.075 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix}^{(1)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} -0.15 \\ -0.075 \end{pmatrix} = \begin{pmatrix} -0.150 \\ 0.925 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix}^{(2)} = \begin{pmatrix} -0.15 \\ 0.925 \end{pmatrix} + \begin{pmatrix} -0.0163 \\ -0.0212 \end{pmatrix} = \begin{pmatrix} -0.1663 \\ 0.9038 \end{pmatrix}$$

The N-R Power Flow: 5-bus Example



Single-line diagram

The N-R Power Flow: 5-bus Example

Table 1. Bus input data

Bus	Type	|V| per unit	θ degrees	P_G per unit	Q_G per unit	P_L per unit	Q_L per unit	Q_{Gmax} per unit	Q_{Gmin} per unit
1	Slack	1.0	0	—	—	0	0	—	—
2	Load	—	—	0	0	8.0	2.8	—	—
3	Constant voltage	1.05	—	5.2	—	0.8	0.4	4.0	-2.8
4	Load	—	—	0	0	0	0	—	—
5	Load	—	—	0	0	0	0	—	—

Table 2. Line input data

Bus-to-Bus	R per unit	X per unit	G per unit	B per unit	Maximum MVA per unit
2-4	0.0090	0.100	0	1.72	12.0
2-5	0.0045	0.050	0	0.88	12.0
4-5	0.00225	0.025	0	0.44	12.0

Base: 100 MVA

The N-R Power Flow: 5-bus Example

Table 3. Transformer input data

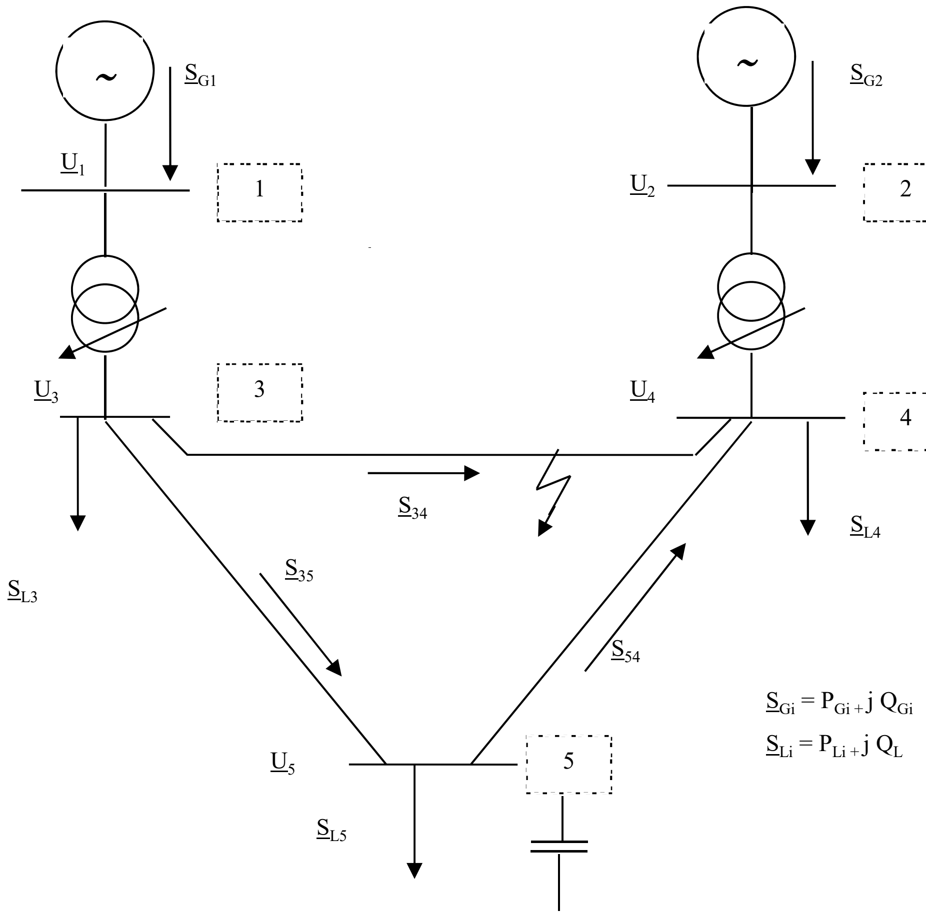
Bus-to-Bus	R per unit	X per unit	G_c per unit	B_m per unit	Maximum MVA per unit	Maximum TAP Setting per unit
1-5	0.00150	0.02	0	0	6.0	—
3-4	0.00075	0.01	0	0	10.0	—

Table 4. Input data and unknowns

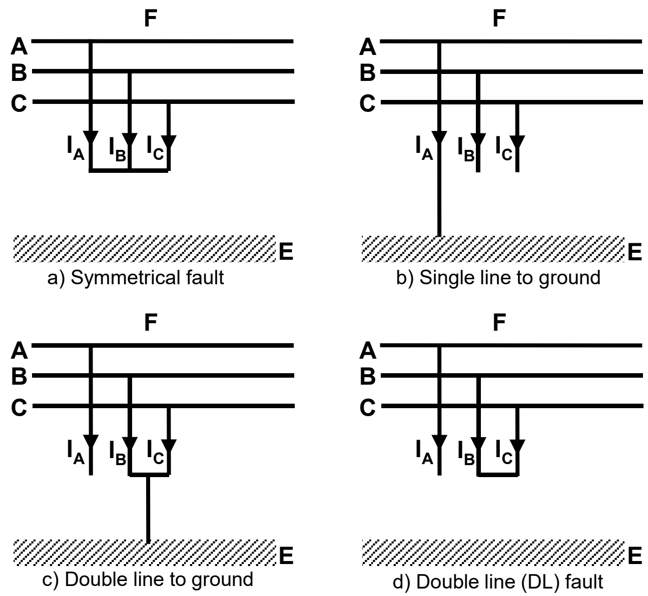
Bus	Input Data	Unknowns
1	$\|V_1\| = 1.0$, $\theta_1 = 0$	P_1, Q_1
2	$P_2 = P_{G2} - P_{L2} = -8$ $Q_2 = Q_{G2} - Q_{L2} = -2.8$	$\|V_2\|$, θ_2
3	$\|V_3\| = 1.05$ $P_3 = P_{G3} - P_{L3} = 4.4$	Q_3, θ_3
4	$P_4 = 0$, $Q_4 = 0$	$\|V_4\|$, θ_4
5	$P_5 = 0$, $Q_5 = 0$	$\|V_5\|$, θ_5

Short circuit analysis

Fault types



$\underline{S}_{G1}$
$\underline{S}_{G2}$
$\underline{U}_1$
$\underline{U}_2$
1
2
$\underline{U}_3$
$\underline{U}_4$
3
4
$\underline{S}_{34}$
$\underline{S}_{L3}$
$\underline{S}_{L4}$
$\underline{S}_{35}$
$\underline{S}_{54}$
$\underline{U}_5$
5
$\underline{S}_{L5}$
$\underline{S}_{Gi} = P_{Gi} + j\, Q_{Gi}$
$\underline{S}_{Li} = P_{Li} + j\, Q_{L}$



F
A
B
C
I_A I_B I_C
E
a) Symmetrical fault
b) Single line to ground
c) Double line to ground
d) Double line (DL) fault

The need for short-circuit current calculation

Short circuits in power supply networks are caused by Overvoltages, lightning, thunderstorms, switching overvoltages, mechanical errors, etc.

- For the safe operation of the network, knowledge of the currents and voltages occurring during the short circuit (i.e. the stress to which the equipment is subjected to) at the fault location and at all other places in the network is of utmost importance.
- The equipment in the network (lines, transformers, generators, cables, switches, bubars, etc.) must be able to cope with the mechanical and thermal stresses associated with the short circuit current, at least for the duration of the short circuit (about 150 ms).
  - Must be designed accordingly

The need for short-circuit current …

- The short-circuit current can reach values of up to 100 kA.
- The most heavily stressed (mechanically and thermally) equipment is the circuit breaker, which must interrupt the full short-circuit current near the short-circuit location.
- Before the equipment can be dimensioned to withstand the mechanical and thermal stress caused by the short-circuit current:
  - A short-circuit current calculation must be carried out for multiple fault locations.
- Only then can the maximum conceivable short-circuit current be determined and the equipment be suitably dimensioned.
- As a general rule, the maximum currents and thus the highest stresses (mechanical and thermal) are caused by a 3-phase short circuit

Short-circuit current calculation methods

Standards are used to calculate short-circuit currents

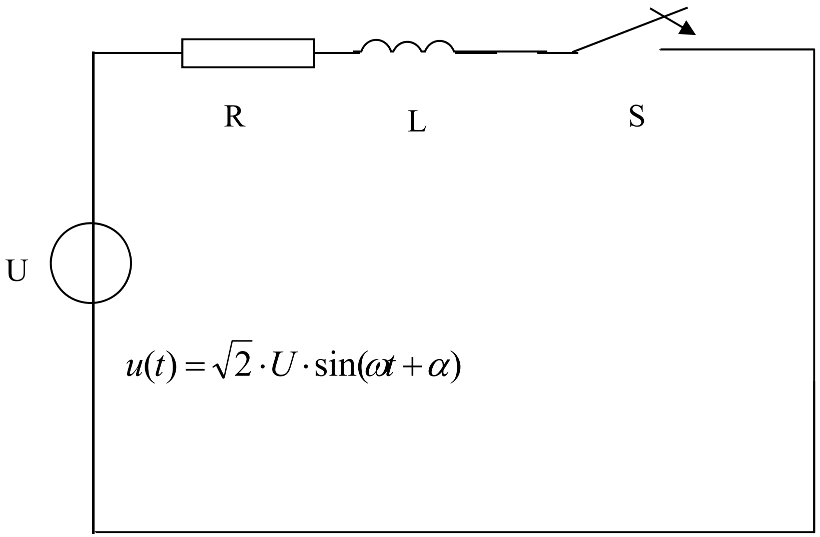
„The aim of the standards is to establish a general (uniform) and simple short-circuit current calculation method that can lead to results that are on the safe side with sufficient accuracy

- IEEE Std 551™-2006 (recognised as American National Standard (ANSI))
- IEC 60909
- IEC 61363
- German Standard DIN VDE 0102. (the use of this method is manatory in Germany)

Short-circuit current calculation methods

- IEC 60909 classify the fault current according to their magnitude (maximum or minimum) and fault distance from the source (far or near).
- Far from source and near to source classification will determine whether to include the decay of the AC component of the fault current.
- In IEC 60909 (also in VDE 0102), an equivalent voltage source at the fault location will replace ALL VOLTAGE SOURCES.
- A voltage factor will adjust the value of the equivalent voltage source for the maximum and minimum fault current calculations.
- IEC 61363 calculation methods are for use in unmeshed three-phase alternating current systems, but will take into consideration all source independently.
- IEC61363 only calculates the symmetrical short circuit current.

Short circuit current in an RL circuit



$$\sqrt{2} \cdot U \cdot \sin(\omega t + \alpha) = R \cdot i + L \frac{di}{dt} \rightarrow$$

$$i(t) = \sqrt{2} \cdot I \cdot \sin(\omega t + \alpha - \theta) - \sqrt{2} \cdot I \cdot \sin(\alpha - \theta) \cdot e^{-\frac{t}{\tau}}$$

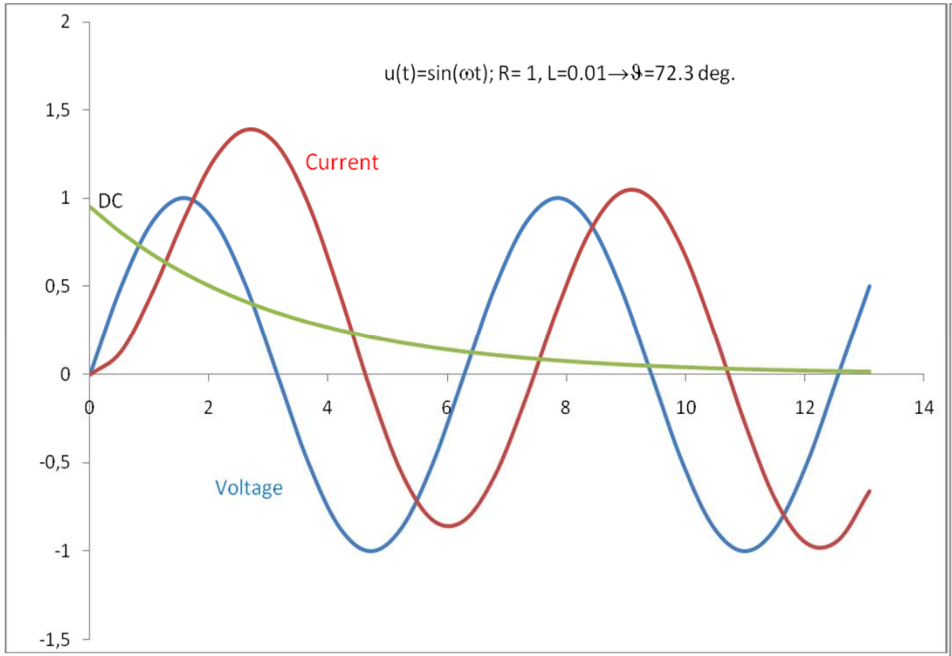
$$i(t) = i_{a.c.} + i_{d.c.}$$

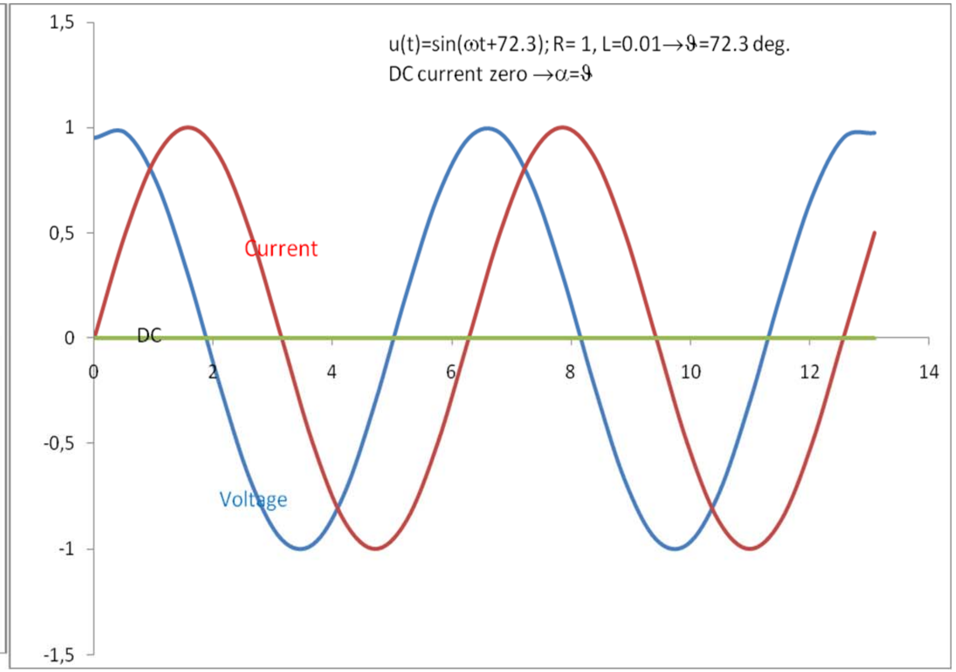
$$i(t) = \sqrt{2} \cdot I \cdot \left(\sin(\omega t + \alpha - \theta) - \sin(\alpha - \theta) \cdot e^{-\frac{t}{\tau}} \right)$$

$$\theta = tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\tau = \frac{L}{R}$$

Short circuit current computation

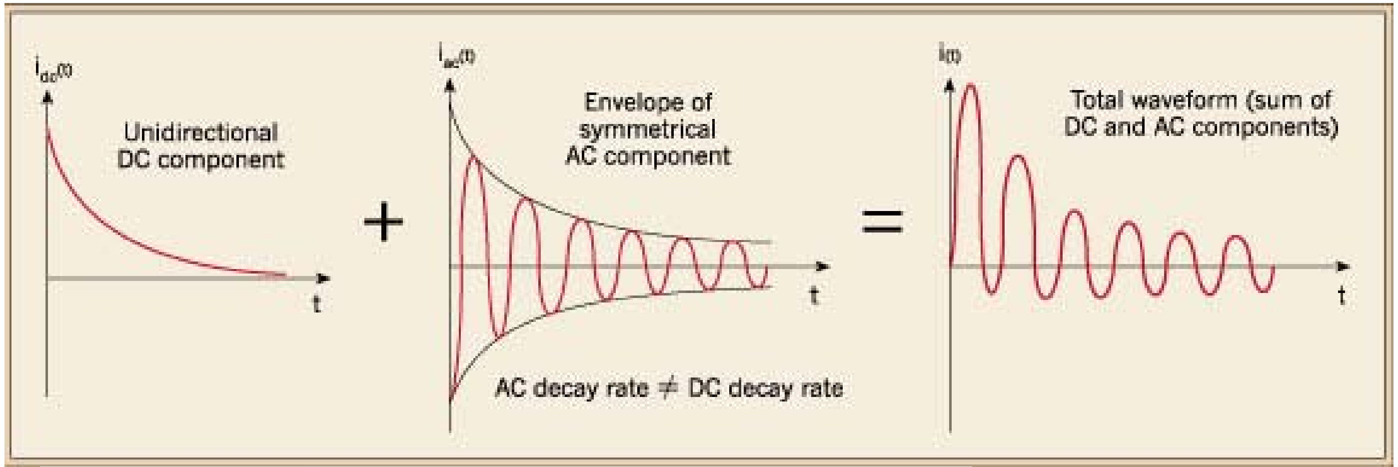


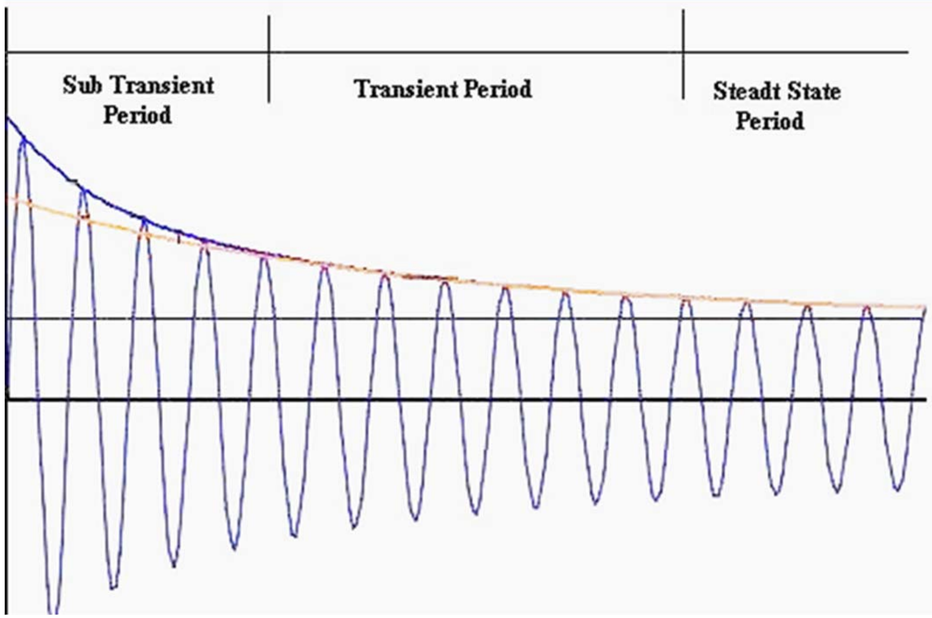


$$i(t) = \sqrt{2} \cdot I \cdot \left(sin(\omega t + \alpha - \theta) - sin(\alpha - \theta) \cdot e^{-\frac{t}{\tau}} \right)$$

Short circuit current - generator

Generator: $$i_k(t) = \sqrt{2}\left[\left((I_k^{"} - I_k^{\prime})e^{-\frac{t}{T_d^{"}}} + (I_k^{\prime} - I_k)e^{-\frac{t}{T_d^{\prime}}} + I_k\right)\sin(\omega t - \alpha) + I_k^{"}\, e^{-\frac{t}{T_g}} \sin\alpha\right]$$





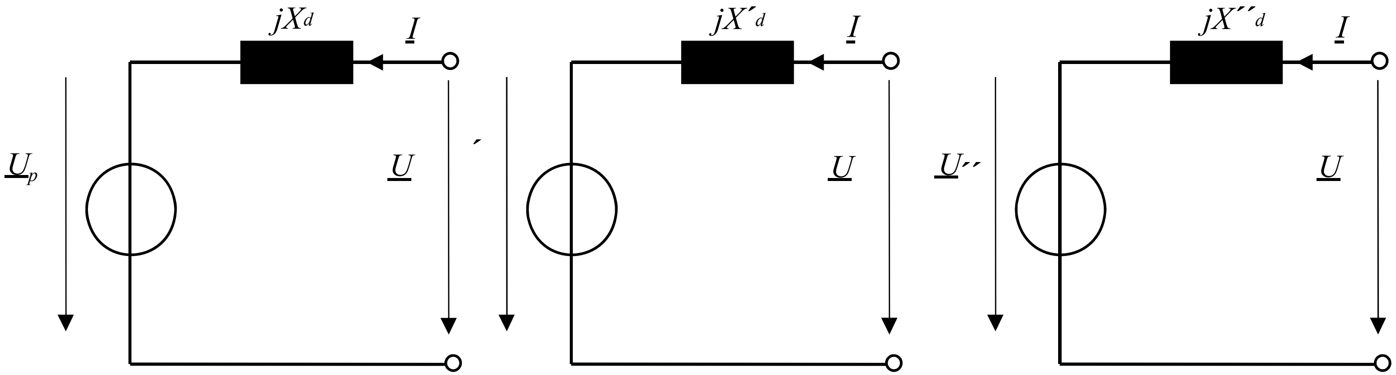
Short circuit current - generator

Short circuit current - RL circuit :

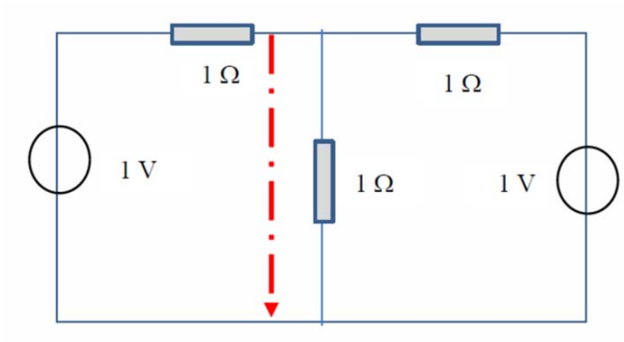
$$i(t) = \sqrt{2} \cdot I \cdot \left(sin(\omega t + \alpha - \theta) - sin(\alpha - \theta) \cdot e^{-\frac{t}{\tau}} \right)$$

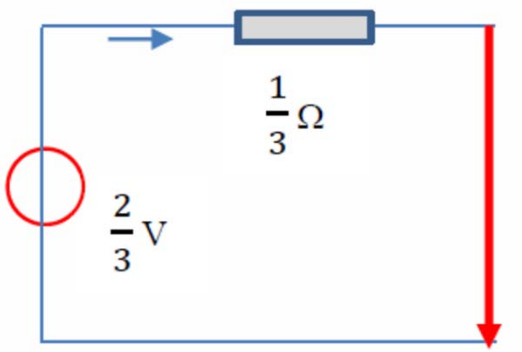
Short circuit current - generator :

$$i_k(t) = \sqrt{2}\left[\left(\left(I_k'' - I_k'\right)e^{-\frac{t}{T_d''}} + \left(I_k' - I_k\right)e^{-\frac{t}{T_d'}} + I_k\right)\sin(\omega t - \alpha) + I_k'' \, e^{-\frac{t}{T_g}} \sin\alpha\right]$$

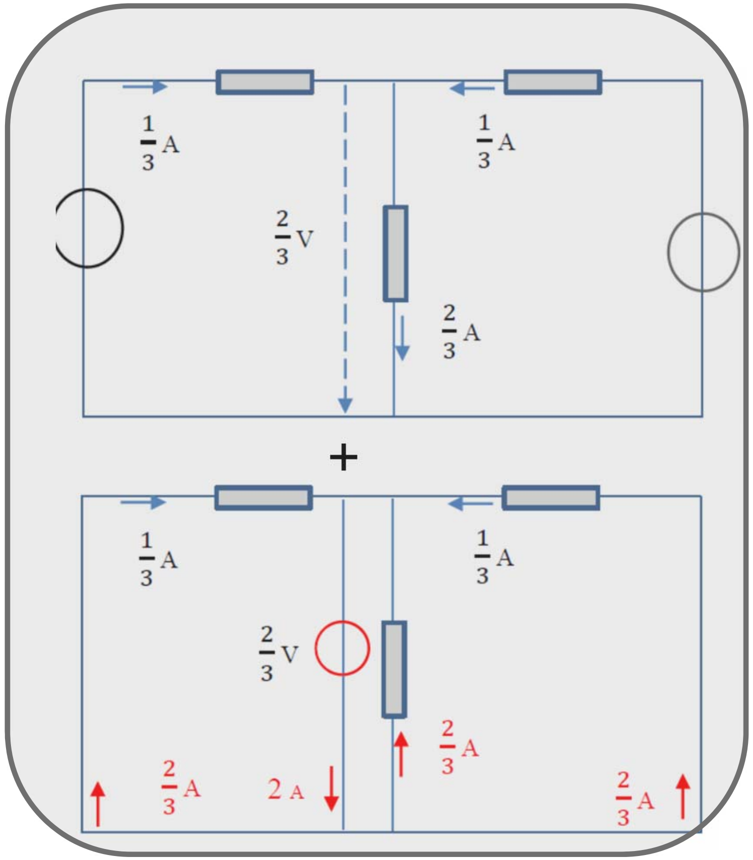


Short-circuit current calculation according to DIN VDE 0102





Equivalent voltage source
at the fault location

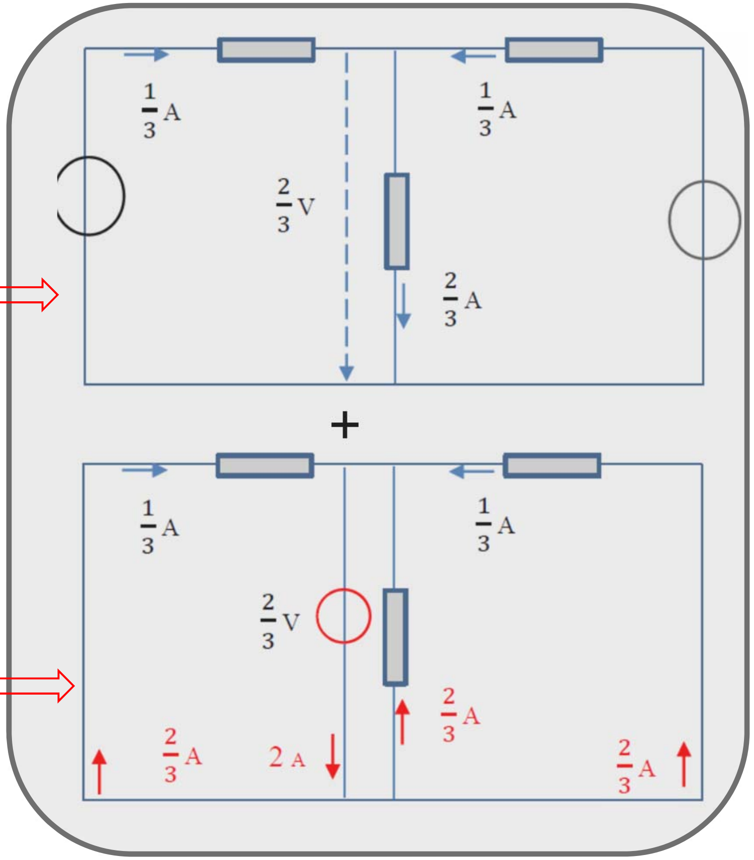


General approach

1. Perform load flow calculation and determine the initial bus voltages and line flows in the network including the voltage at the fault location

2. Determine current and voltage distribution in the network with the voltage at the fault location as the only voltage source

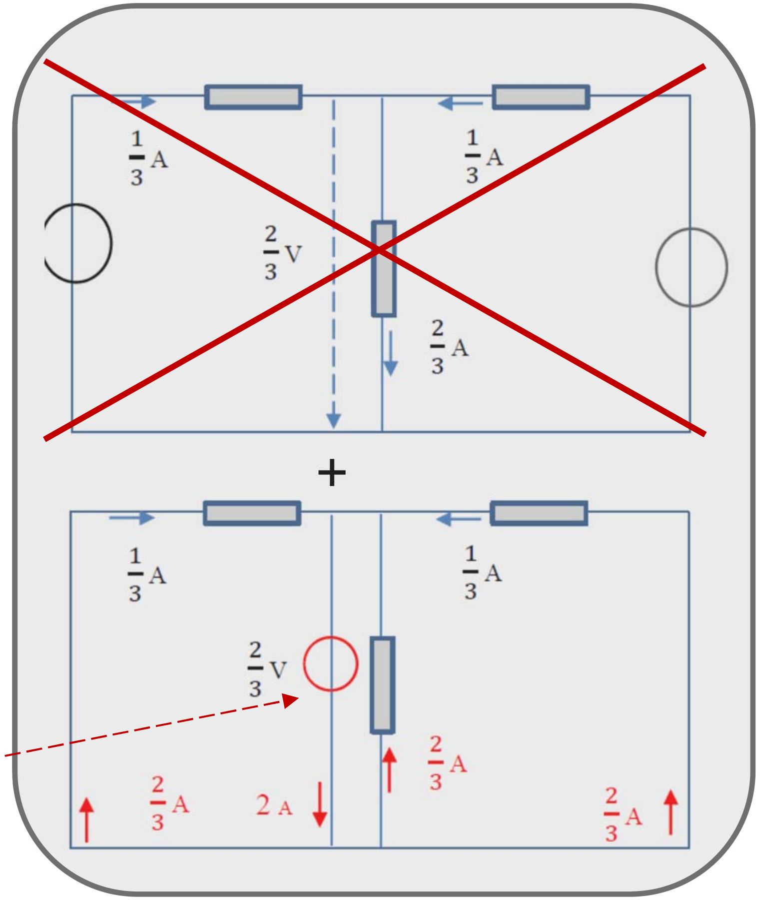
3. The superimposition of the two distributions yields the state in the network after the fault.



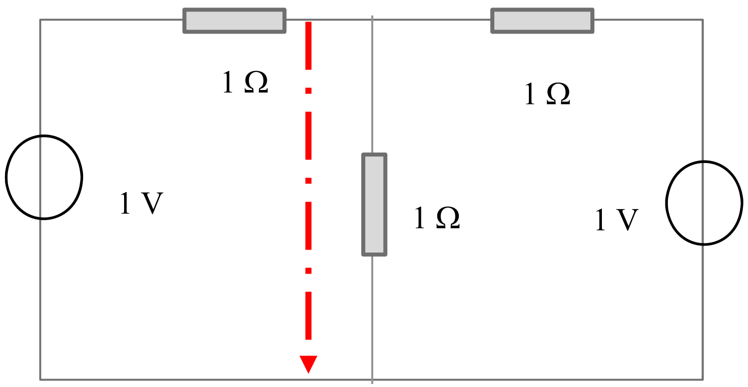
Simplified approacch (according to DIN VDE 0102):

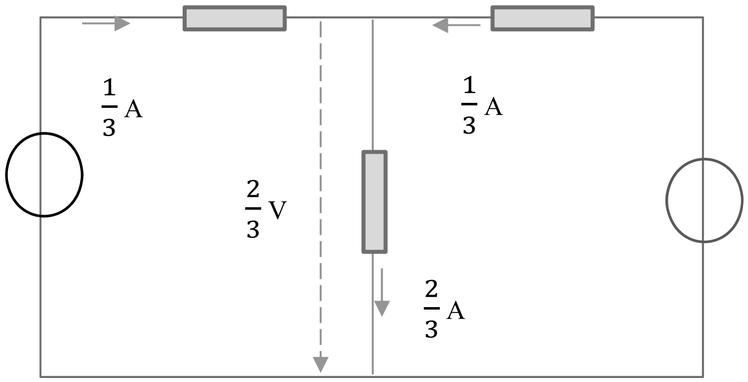
- All line capacitances and impedances of non-motoric loads (e.g., lighting, heaters, etc) are neglected.
- All shunt admittances are omitted.
- All generator and external network voltages are short-circuited.
- The only voltage source in the network is the backup voltage source at the fault location with:

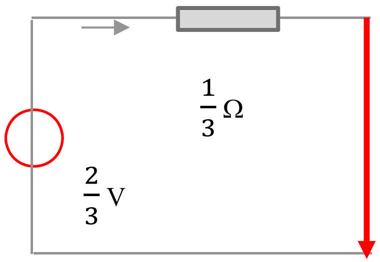
$$U^{"} = \frac{c\,U_n}{\sqrt{3}}$$

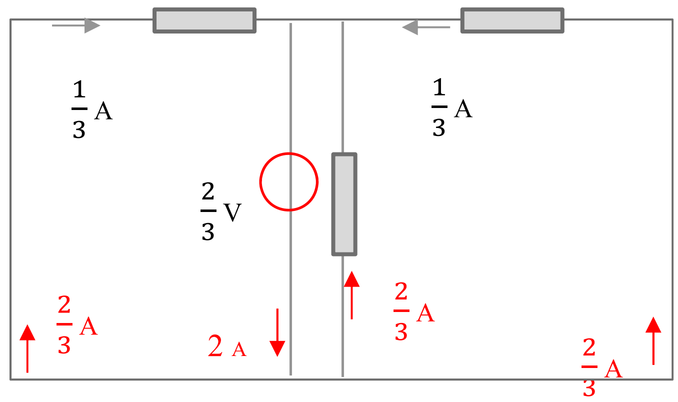


Equivalent voltage source method







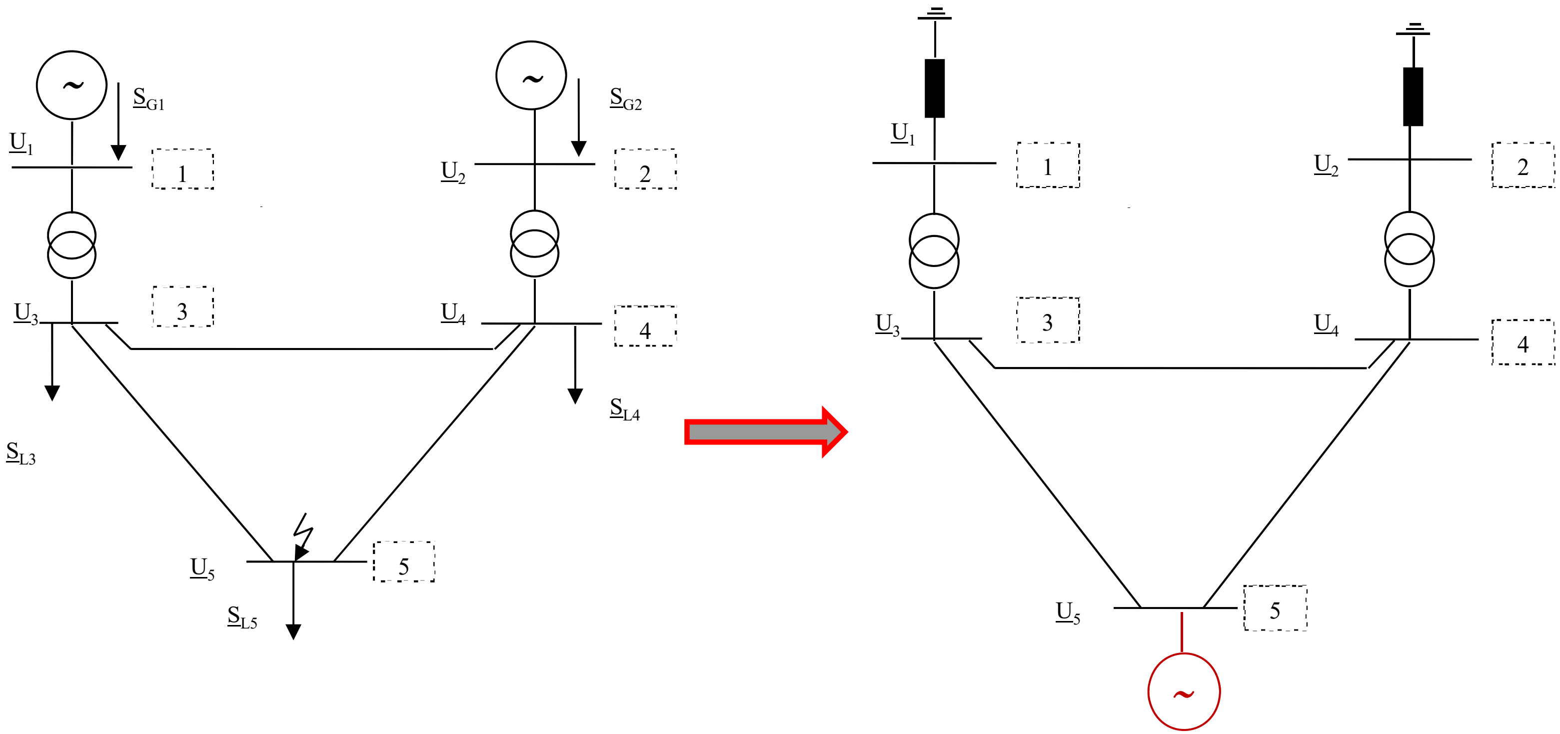


Equivalent voltage source method

•Equivalent voltage source at the fault location:

$$U'' = \frac{c \cdot Un}{\sqrt{3}}$$

- Voltage factor c;
  - ✓ High- and medium voltage: c_{max}=1.1; c_{min}=1.0
  - ✓ Low voltage: c_{max}= 1.05; c_{min}=0.95



$$U'' = \frac{c\,U_n}{\sqrt{3}}$$

Other values of interest

- The first amplitude of the short-circuit current (i_p) and the surge factor (κ)

$$i_p = \kappa \cdot \sqrt{2} \cdot I_k''$$
$$\kappa = 1.02 + 0.98 \cdot e^{-3R/X}$$

- DC component of the fault current ($i_{d.c.}$)

$$i_{d.c.} = \sqrt{2} \cdot I_k'' \cdot e^{-2\pi \cdot f \cdot R \cdot t / X}$$

- Short-circuit interrupting current (I_b) and the decay factor (μ)

$$I_b = \mu \cdot I_k''$$

Short-circuit far from source: $\mu = 1$

Short-circuit near a generator:

$$\mu = 0.84 + 0.26 \cdot e^{-0.26 \cdot I_{kG}'' / I_{rG}} \quad for \; t_{min} = 0.02s$$
$$\mu = 0.71 + 0.51 \cdot e^{-0.30 \cdot I_{kG}'' / I_{rG}} \quad for \; t_{min} = 0.05s$$
$$\mu = 0.62 + 0.72 \cdot e^{-0.32 \cdot I_{kG}'' / I_{rG}} \quad for \; t_{min} = 0.10s$$
$$\mu = 0.56 + 0.94 \cdot e^{-0.38 \cdot I_{kG}'' / I_{rG}} \quad for \; t_{min} \geq 0.25s$$

$t_{min} = Minimum\ delay$; $I_{kG}'' = Generator\ short\ circuit\ current$;

$I_{rG} = Genertator\ rated\ current$

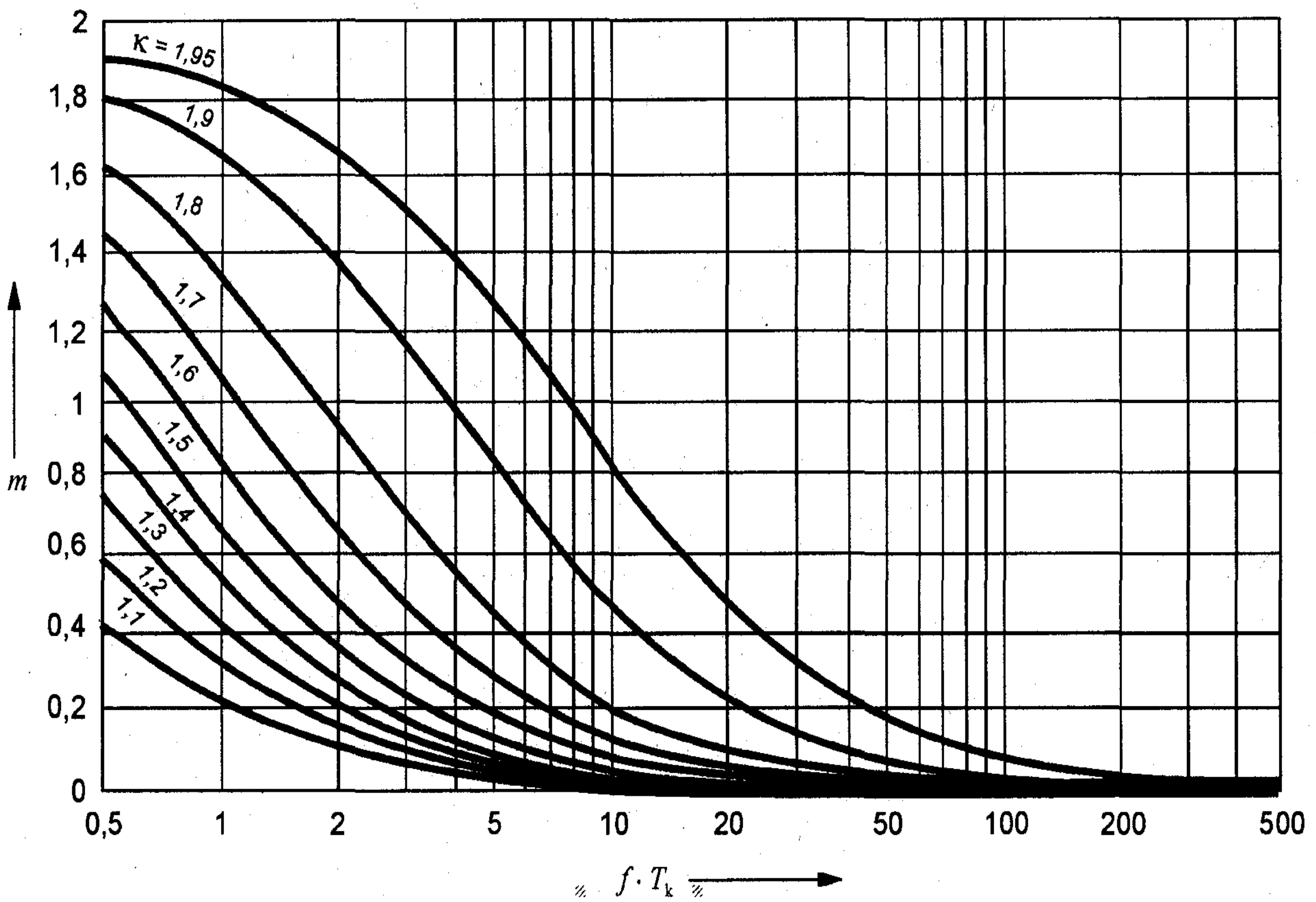
Thermal equivalent of the short-circuit current

- Thermal equivalent of the short-circuit current (I_{th}) and the parameters m and n
  - m: thermal component of the dc current; f(κ, T_k)
  - n: thermal component of the ac current; f(I_k"/$I_{k,}$ T_k)

$$I_{th} = I_k^{"} \cdot \sqrt{m+n}$$

- I_k : steady-state short-circuit current
- T_k : short-circuit duration
- κ : the surge factor

Thermal equivalent of the short-circuit current



Thermal equivalent short-circuit current