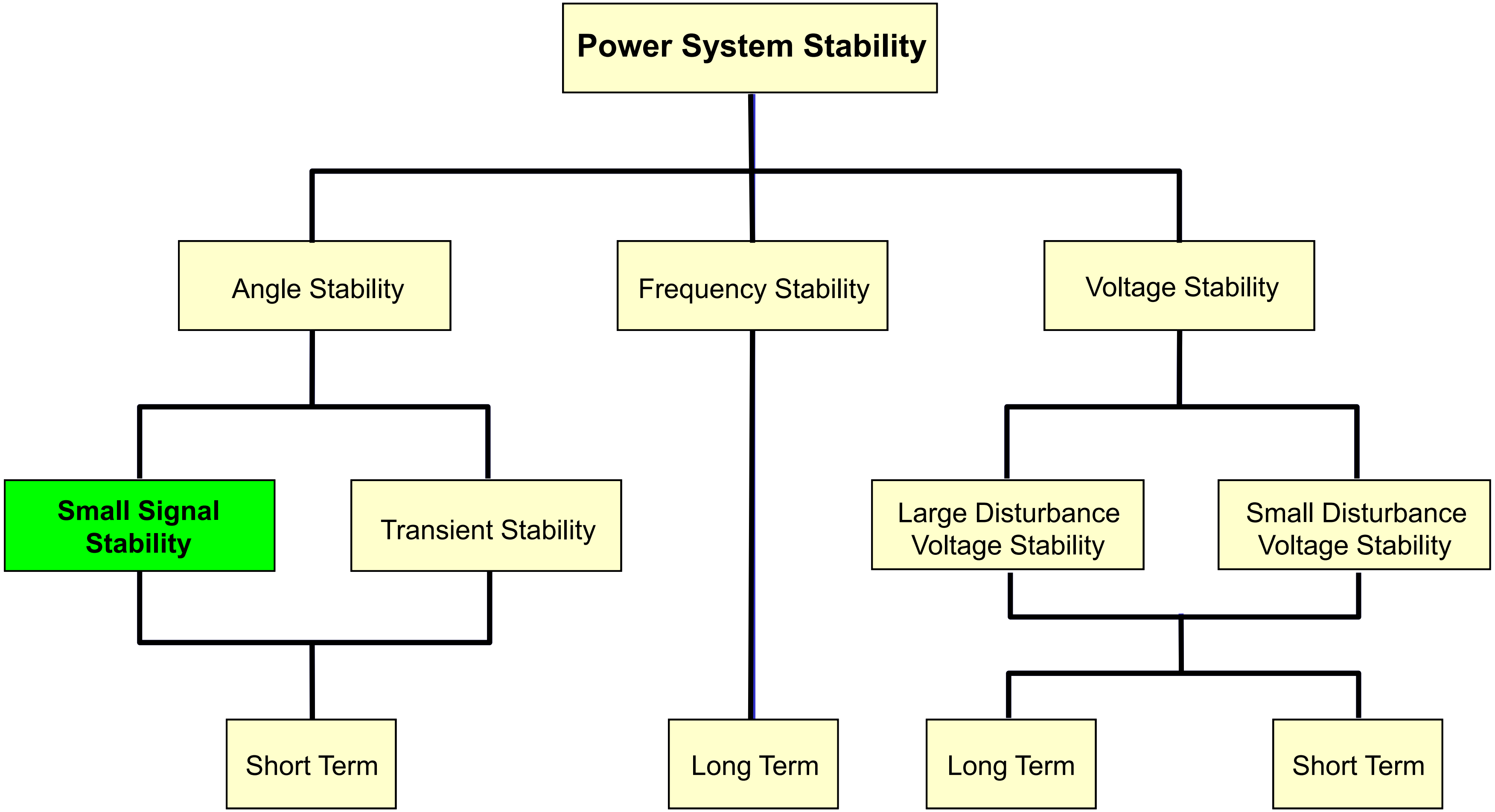
Small Signal Stability

Outline

- Description of Small Signal Stability Problems
  - Local problems
  - Global problems
- Methods of analysis
  - Time domain analysis and its limitations
  - Modal analysis using linearized model
- Characteristics of local plant mode oscillations
- Characteristics of inter-area oscillations
- Enhancement of Small Signal Stability

Classification Of Power System Stability



Small Signal Stability

- Small-Signal (or Small Disturbance) Stability is the ability of a power system to maintain synchronism when subjected to small disturbances
  - Such disturbances occur continually on the system due to small variations in loads and generation
  - A disturbance is considered sufficiently small if linearization of system equations is permissible for analysis
- Small-signal analysis using linear analysis techniques provides valuable information about the inherent dynamic characteristics of the power system and assists in its robust design

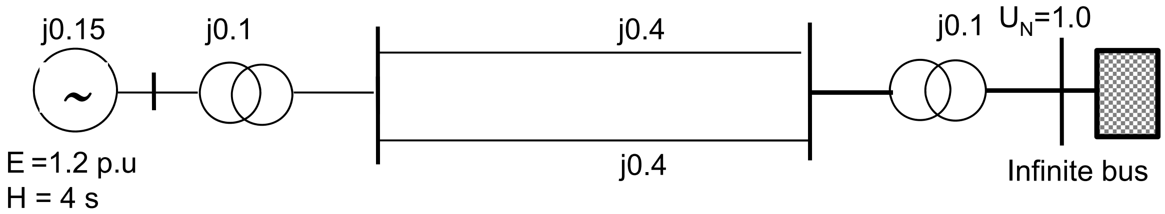
Instability Forms

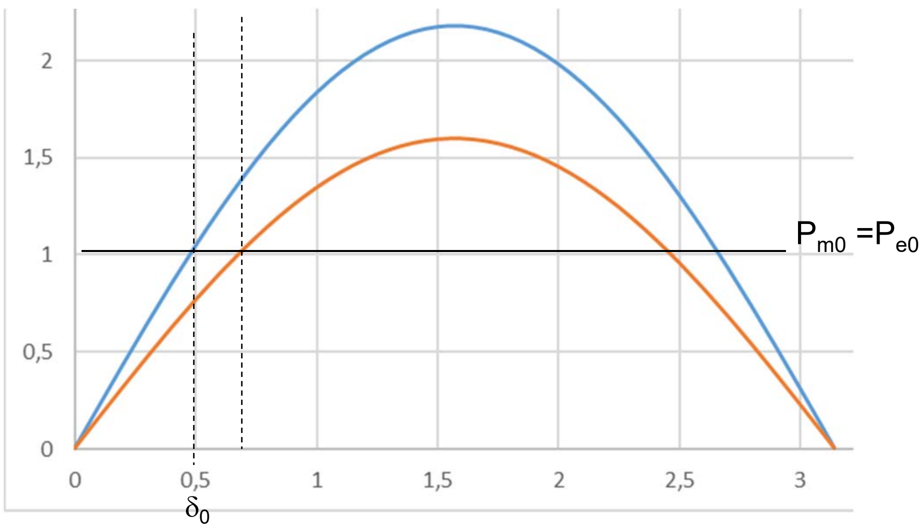
- Small signal instability may take two forms:
  - aperiodic increase in rotor angle due to lack of sufficient synchronizing torque
  - rotor oscillations of increasing amplitude due to lack of sufficient damping torque

Damping and Synchronising Torques

Example:

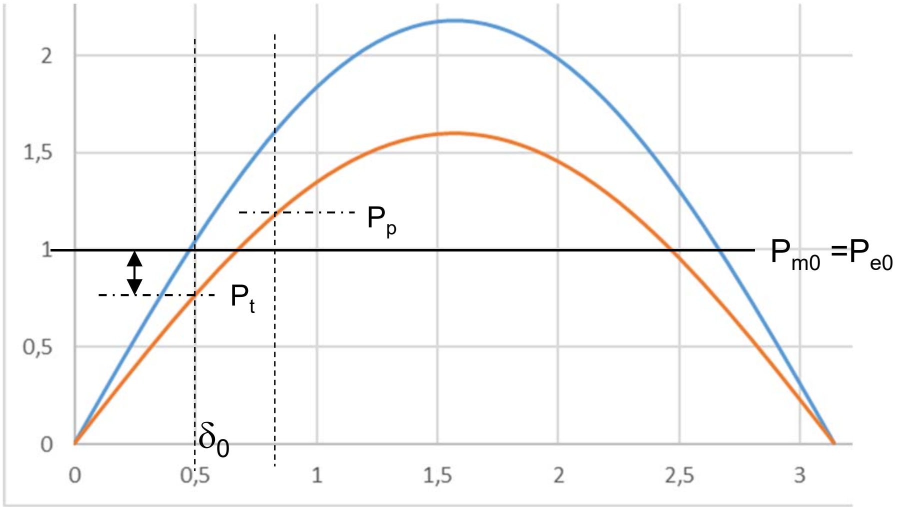
Assume a synchronous generator is connected through a transformer by two parallel transmission lines to a receiving-end transformer and a large system





- when two lines are in service an equilibrium or steady-state condition exists, in which the power output of the prime-mover P_{m0} is equal to the electrical power output of the generator, P_{e0} , at synchronous speed and the rotor angle is δ_0 ;
- at time t, one of the two lines is opened
- the power output of the prime-mover assumed to remain constant during a disturbance on the electrical system

Synchronising and Damping Torques



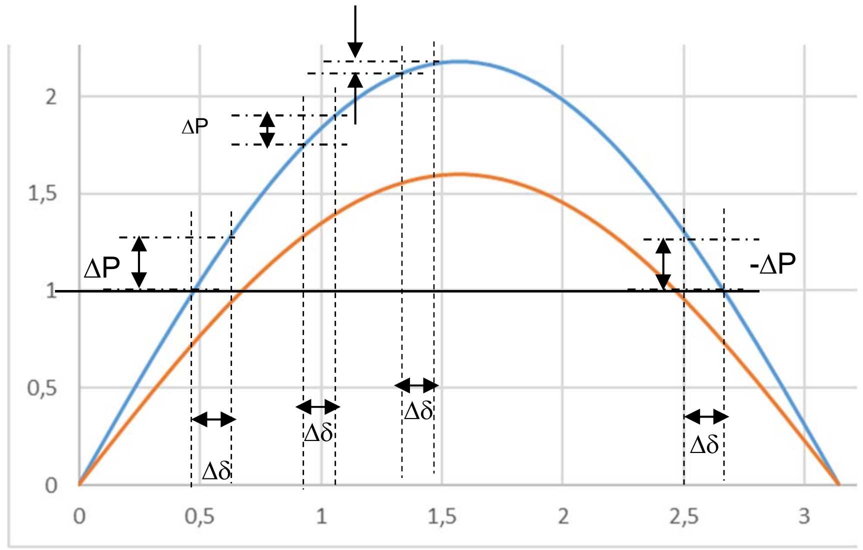
- Synchronism in this scenario is maintained by the electrical power flow

$$P = P_{max} \sin \delta$$

between the generator and the system, resulting in a synchronizing torque being produced on the shaft of the generator.

- Immediately after the disturbance the electric power output of the generator falls to P_t.
- The net torque acting on the shaft of the generator will cause it to accelerate with respect to the system.
- The rotor angle of the generator, δ_0, immediately starts to increase.
- Once the electrical power output exceeds the prime-mover power output P_{m0} the generator decelerates but, due to the inertia of the rotor, the rotor angle continues to increase until the speed falls to synchronous speed.
- At this time the electric power output and the rotor angle are at their peak values, P_p
- However, the net decelerating torque continues acting on the shaft to reduce the electrical power flow until zero net accelerating torque once more
- Due to inertia, the electric power output and rotor angle continue to decrease and reach their minimum values at P_t and at synchronous speed.
- Thereafter the process repeats itself with the electric power output and rotor angle oscillating about P_{m0}, between peak and trough values Pp, and Pt, respectively.
- In the absence of damping, these oscillations will continue indefinitely.

Synchronising torque as a function of operating point



- The level of the synchronising torque depends on the operating point

Nature of small signal stability problem

- In today's practical systems, small signal stability is usually one of insufficient damping of system oscillations
  - Local problems / global problems
- Local problems involve a small part of the system. They may be associated with
  - rotor angle modes
  - local plant modes
  - inter-machine modes
  - control modes
  - torsional modes
- Global problems have widespread effects
  - They are associated with inter-area oscillations

- **Local plant mode oscillations**
  - **oscillation of a single generator or plant against the rest of the power system**
- **Inter-machine or inter-plant mode oscillations**
  - **oscillation between the rotors of a few generators close to each other**

Local Rotor Angle Stability Problems

- **Associated with either local plant mode oscillations or inter-machine oscillations**
  - **frequency of oscillation in the range of 0.7 to 2.0 Hz**
- **Stability of the local plant mode oscillations is determined by the strength of the transmission as seen by the plant excitation control, plant output and voltage**
- **Instability may also be associated with a non-oscillatory mode**
  - **encountered with manual excitation control**

Global Rotor Angle Stability Problems

Large interconnected systems usually have two distinct forms of inter-area oscillations:

- **A very low frequency mode involving all the generators in the system**
  - **system is essentially split into two parts**
  - **generators in one part swing against generators in the other part**
  - **frequency in the order of 0.1 to 0.3 Hz**
- **Higher frequency modes involving sub-group of generators swinging against each other**
  - **frequency typically in the range of 0.4 to 0.7 Hz**

Methods of Small-Signal Stability Analysis

- State Space Representation of the Dynamic System
- Linearization

State space representation

- **The behaviour of a dynamic system can be described by a set of first order differential equations in the state-space form:**

$$\dot{x} = f(x, u)$$

  - **x is an n-dimensional state vector**
  - **f is an n-dimensional nonlinear function**
  - **u is an r-dimensional input vector**

- **The outputs of the system are nonlinear functions of the state and input vectors:**

$$\boldsymbol{y} = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u})$$

  - **y is an m-dimensional output vector**
  - **g is an m-dimensional nonlinear function**

- **In steady state, the system is at an equilibrium point x_0 satisfying:**

$$\boldsymbol{f}(\boldsymbol{x_0}, \boldsymbol{u_0}) = \boldsymbol{0}$$

Linearization

- **For small perturbation about equilibrium point:**

$$x = x_0 + \Delta x, \quad u = u_0 + \Delta u$$

- **New state equation:**

$$\dot{\boldsymbol{x}} = \dot{\boldsymbol{x}}_{\mathbf{0}} + \Delta\dot{\boldsymbol{x}} = \boldsymbol{f}((\boldsymbol{x}_{\mathbf{0}} + \Delta\boldsymbol{x}), (\boldsymbol{u}_{\mathbf{0}} + \Delta\boldsymbol{u}))$$

- **Since perturbations are small:**
  - **f(x,u) can be expressed in terms of Taylor's series expansion**
  - **terms involving second and higher order powers of Δx and Δu may be neglected**

Linearization

$$\Delta\dot{x} = A\,\Delta x + B\,\Delta u$$
$$\Delta y = C\,\Delta x + D\,\Delta u$$

- **A, B, C, D are the Jacobians of the system. A is also referred to as the state matrix or the plant matrix.**

$$A = \begin{bmatrix} \frac{\partial f_1}{x_1} & \cdots & \frac{\partial f_1}{x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{x_1} & \cdots & \frac{\partial n}{x_n} \end{bmatrix} \qquad B = \begin{bmatrix} \frac{\partial f_1}{u_1} & \cdots & \frac{\partial f_1}{u_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{u_1} & \cdots & \frac{\partial n}{u_n} \end{bmatrix}$$

$$C = \begin{bmatrix} \frac{\partial g_1}{x_1} & \cdots & \frac{\partial g_1}{x_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_m}{x_1} & \cdots & \frac{\partial g_m}{x_m} \end{bmatrix} \qquad D = \begin{bmatrix} \frac{\partial g_1}{u_1} & \cdots & \frac{\partial g_1}{u_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_m}{u_1} & \cdots & \frac{\partial g_m}{u_m} \end{bmatrix}$$

Stability

- Stability is concerned with determination of conditions of an equilibrium point
  - what will happen if the system is perturbed at an equilibrium condition
- Stability of a linear system is independent of the input
- Stability of a nonlinear system depends on
  - the type and magnitude of input
  - the initial state
- In control system theory, it is common practice to classify stability of nonlinear systems into the following categories, depending on the region of state space in which the state vector ranges:
  - local stability or stability in the small
  - finite stability
  - global stability or stability in the large

Stability categories

- **Local stability**
  - The system is said to be locally stable about an equilibrium point, if when subjected to a small perturbation, it remains within a small region surrounding the equilibrium point
  - If, as time increases, the system returns to the original state, it is said to be asymptotically stable in the small
- **Finite stability**
  - If the state of a system remains within a finite region R, the system is said to be stable within R
  - If, further, the state returns to the original equilibrium point from any point within R, it is said to be asymptotically stable within the finite region R
- **Global stability**
  - The system is said to be globally stable if R includes the entire finite space

Analysis of Stability in the Small (Small Signal Stability)

Nonlinear Time Domain Analysis

Using nonlinear time domain simulations to analyze small signal stability problems has the following limitations:

- Results can be deceptive
  - critical mode may not be sufficiently excited by the chosen disturbance
  - poorly damped modes may not be dominant in the observed response
- This approach does not give insight into the nature of the problem
  - difficult to identify sources of the problem
  - mode shapes not clearly identified
  - corrective measures are not readily indicated
- Computational burden high ; massive amount of data has to be analyzed

Analysis of Stability in the Small (Small Signal Stability)

Modal analysis

- **The theoretical foundation for the analysis of stability in the small is based on Liapunov's first method:**
  - **The stability in the small of a nonlinear system is given by the roots of the characteristic equation of the system of first approximation, i.e., by the eigenvalues of the state matrix A**
  - **If the eigenvalues have negative real parts, then the original system is asymptotically stable**
  - **When at least one of the eigenvalues has a positive real part, the original system is unstable**

Modal Analysis Approach

- Modal analysis using eigenvalue approach has proven to be the most practical way to analyze small signal stability problems
- Advantages are:
  - individual modes of oscillations are clearly identified
  - relationships between modes and system variables/parameters can be easily determined by computing eigenvectors
- Frequency response, poles, zeros, and residues can be easily computed. Such information is useful in control system design

Eigenproperties of the State Matrix

- **Eigenvalues and eigenvectors**

$$\boldsymbol{A} . \qquad = \boldsymbol{\lambda} . \phi$$

$$\boldsymbol{\psi} . \boldsymbol{A} = \boldsymbol{\lambda} . \boldsymbol{\psi}$$

  - A is an n x n matrix (real for a physical system)
  - λ is the eigenvalue
  - ϕ is the right eigenvector associated with λ
  - ψ is the left eigenvector associated with λ

- **Modal matrices**

$$\boldsymbol{\Phi} = [\phi_{\boldsymbol{1}} \; \phi_{\boldsymbol{2}} \; \cdots \phi_{\boldsymbol{n}} \,]$$

$$\boldsymbol{\psi} = \left[\boldsymbol{\psi}_{\boldsymbol{1}}^{\boldsymbol{t}} \; \boldsymbol{\psi}_{2}^{\boldsymbol{t}} \cdots \boldsymbol{\psi}_{\boldsymbol{n}}^{\boldsymbol{t}}\right]$$

  » **Φ is the right eigenvector matrix**

  » **ψ is the left eigenvector matrix**

Eigenproperties of the State Matrix

- **Relationships**

$$\boldsymbol{A}\,.\,\boldsymbol{\Phi} = \boldsymbol{\Phi}.\,\boldsymbol{\Lambda} \qquad\qquad \boldsymbol{\psi}\,.\,\boldsymbol{\Phi} = \boldsymbol{I}$$

$$\boldsymbol{\Phi}^{-1}.\,\boldsymbol{A}\,.\,\boldsymbol{\Phi} = \boldsymbol{\Lambda}$$

  - **I is the unit matrix**
  - **Λ is a diagonal matrix: Λ = diag [$\lambda_1 \ldots \lambda_n$]**

Free Motion of a Linear Dynamic System

- **Free motion of a linear dynamic system is described by**

$$\dot{x} = A.x$$

- **In order to eliminate the cross coupling between the state variables consider the <u>state transformation</u>**

$$x = \phi.z$$

- **State space equations in z is a set of <u>decoupled</u>**

$$\dot{z} = \phi^{-1}.A.\phi.z = \Lambda.z$$

- **The above represents uncoupled first order (scalar) differential equations:**

$$\dot{z}_i = \lambda_i.z_i \text{ , i = 1,2,…,n}$$

- **Time domain response**

$$z_i(t) = z_i(0).e^{\lambda_i t}$$

Where $z_i(0) = \psi_i.x(0)$ is the initial condition

Time Response of System Variables

- **Response in terms of the original state vector**

$$x(t) = \phi . z(t)$$

- **The time response of the state variable x_i is given by**

$$x_i(t) = \phi_{i1}.C_1.e^{\lambda_1 t} + \phi_{i2}.C_2.e^{\lambda_2 t} + \ldots + \phi_{in}.C_n.e^{\lambda_n t}$$

  - **a linear combination of n dynamic modes corresponding to the n eigenvalues of the state matrix**
  - **$c_i = \psi_I x(0)$ represents the magnitude of the excitation of the i^{th} mode due to the initial conditions**
  - **if the initial condition lies along the j^{th} eigenvector, only the j^{th} mode will be excited (since $\psi_I \phi_j = 0$ for all $i \neq j$)**

- **if the vector representing the initial condition is not an eigenvector, it can be represented by a linear combination of the n eigenvectors. The response of the system will be the sum of n responses**
  - **if a component along an eigenvector of the initial conditions is zero, the corresponding mode will not be excited.**

Eigenvalue and stability

- A **real eigenvalue** corresponds to a **non-oscillatory** mode
- A **pair of complex eigenvalues** $\lambda = \sigma \pm j\,\omega$ correspond to an **oscillatory** mode
- Frequency of the mode:

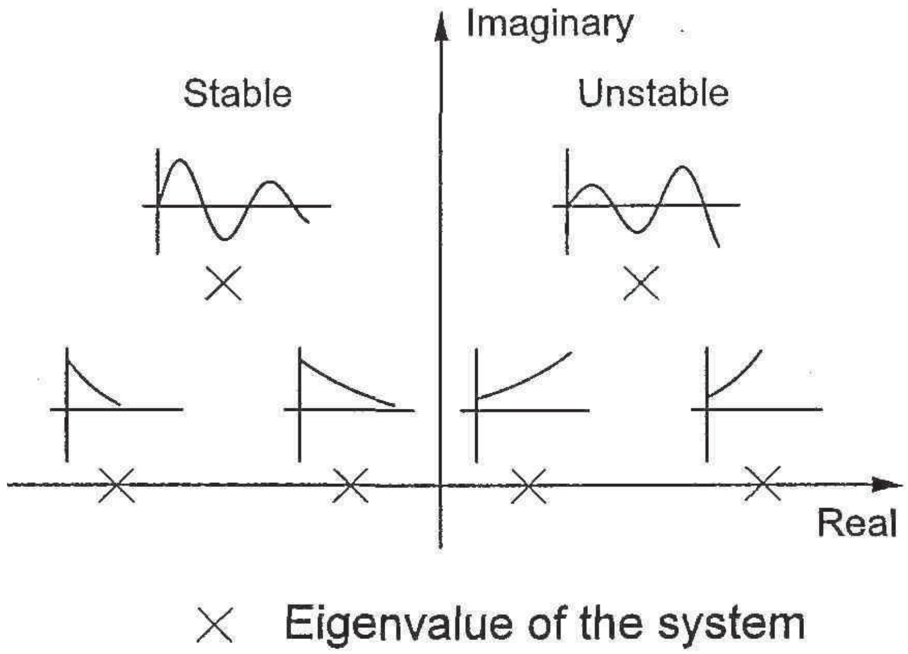
$$f = \frac{\omega}{2\pi}$$

- Damping ratio of the mode:

$$\varsigma = \frac{-\sigma}{\sqrt{\sigma^2 + \omega^2}}$$

A real eigenvalue, or a pair of complex eigenvalues, is usually referred to as a mode

To ensure an acceptable performance, a **damping margin** ς in the range of 3% - 5% is normally required



Modal characteristics

- While an eigenvalue indicates the stability, its right and left eigenvectors give much more information on the characteristics of the mode
- The right eigenvector shows the **mode shape**, i.e., the observability of the mode
- A mode should be observable from generator rotor oscillations if the generator is high in its mode shape
- A weighted left eigenvector shows the **participation factors**, i.e., the controllability of the mode
- A mode should be controllable from a generator if the generator is high in its participation factors
- A generator which is high in the mode shape of a mode **is not necessarily** high in the participation factor of the same mode

Controllability and Observability

- **For a linear dynamic system**

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\,\boldsymbol{x} + \boldsymbol{B}\,\boldsymbol{u}$$
$$\boldsymbol{y} = \boldsymbol{C}\,\boldsymbol{x} + \boldsymbol{D}\,\boldsymbol{u}$$

- **Apply state transformation x = ϕ z**

$$\dot{z} = \phi^{-1}.A.\phi.z = \Lambda.z$$
$$y = C\,\phi\,z + D\,u$$

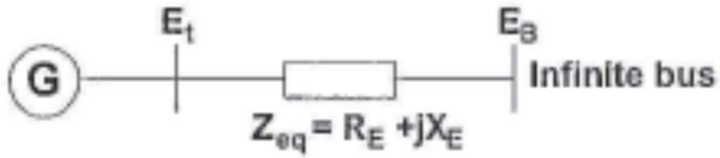
- **If the ith row of matrix $\boldsymbol{\phi}^{-1}\,\boldsymbol{B}$ is zero, the ith mode is said to be uncontrollable**
- **If the ith column of matrix C ϕ is zero, the ith mode is said to be unobservable**

Characteristics of Local Plant Mode Oscillations

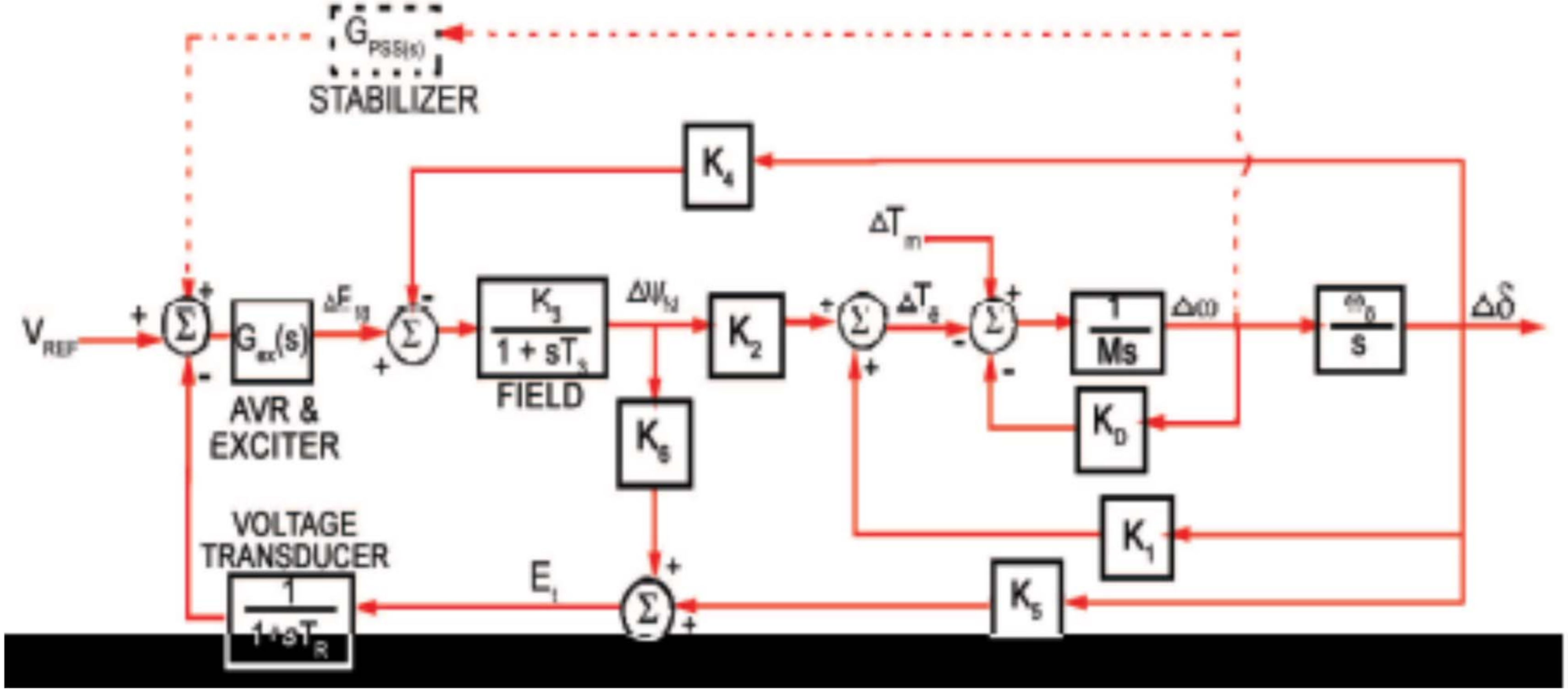
- **Local mode oscillation problems most commonly encountered**
  - dates back to the 1950s and 1960s
  - associated with units of a plant swinging against the rest of the system
- **Characteristics well understood**
  - analysis using block diagram approach (K-constants); gives physical insight
- **Encountered by a plant with high output feeding into weak transmission network (K_5 negative)**
  - more pronounced with high response exciters/AVR
- **Adequate damping readily achieved using Power System Stabilizers (PSS)**
  - excitation control

Block Diagram Approach to the Analysis of Local Mode Problems

- **First published by Heffron and Phillips to analyze a single machine (or a plant) connected to a large system (represented by an infinite bus) through a transmission network**



- **System is represented by a block diagram (the following slide):**



δ	= ROTOR ANGLE (rads)	G_{ex}	= EXCITER TRANSFER FUNCTION
ω	= ROTOR SPEED (p.u.)	G_{PSS}	= PSS TRANSFER FUNCTION
Ψ_{fd}	= FIELD FLUX LINKAGE	M	= INERTIA CONSTANT (2H)

$$K_1 = \left.\frac{\Delta T_e}{\Delta \delta}\right|_{E'_q = E'_{q0}} \qquad K_2 = \left.\frac{\Delta T_e}{\Delta E'_q}\right|_{\delta = \delta_0} \qquad K_4 = \left.\frac{-1}{K_3}\frac{\Delta E'_q}{\Delta \delta}\right|_{E_{FD} = cons\tan t}$$

$$K_5 = \left.\frac{\Delta V_t}{\Delta \delta}\right|_{E'_q = E'_{q0}} \qquad K_6 = \left.\frac{\Delta V_t}{\Delta E'_q}\right|_{\delta = \delta_0}$$

Interpretation of the block diagram

- **Rotor acceleration**

$$\frac{d\Delta\omega}{dt} = \frac{1}{M}(\Delta T_m - \Delta T_e)$$

$$\frac{d\Delta\delta}{dt} = \omega_0 \Delta\omega$$

- **Electrical torque**

$$\Delta T_e = K_1(\Delta\delta) + K_2(\Delta\Psi_{fd})$$

- **Field flux linkage**

$$\Delta\Psi_{fd} = (\Delta E_{fd} - K_4 \Delta\delta)\frac{K_3}{1 + sT_3}$$

- **Terminal voltage**

$$\Delta E_t = K_5(\Delta\delta) + K_6(\Delta\Psi_{fd})$$

Power System Stabilizers

- Small signal stability problem is usually one of insufficient damping of system oscillations
- Power system stabilizers (PSS) are the most cost effective means of solving SSS problems
- The purpose is to add damping to the generator rotor oscillations
- This is achieved by modulating the generator excitation so as to develop a component of electrical torque in phase with rotor speed deviations
- Common input signals include: shaft speed, integral of power and generator terminal frequency

Characteristics of Low Frequency Interarea Oscillations (LFIO)

- Oscillations between two groups of generators

- Two distinct forms:

  a) A very low frequency mode involving all generators

     → entire system split into two parts, with generators in one part swinging against generators in the other part

     → frequency in the range: 0.1 to 0.3 Hz

  b) Higher frequency modes involving a subgroup of generators swinging against another subgroup

     → frequency in the range: 0.4 to 0.7 Hz

Fundamental Nature of Low Frequency Interarea Oscillations (LFIO)

- Characteristics (mode shape, damping) of LFIO are a complex function of:
  - network configuration/strength
  - load characteristics
  - types of excitation systems and their locations
- Load characteristics, in particular, have a major effect
  - more pronounced with slow exciters
- In a stressed system, motor or constant power load at
  - receiving end has adverse effect on damping
  - sending end has slightly beneficial effect
- A mode of oscillation in one part of system can excite units in a remote part due to mode coupling
- Analysis requires detailed and same level of representation throughout the system

Damping of Low Frequency Interarea Oscillations (LFIO) with PSS

- The controllability of LFIO with PSS is a function of:
  - location of units with PSS
  - characteristics and locations of loads
  - types of exciters on other units
- Damping of LFIO wit PSS achieved primarily by modulating loads
- Identification of units on which PSS most effective:
  - a high *participation factor* is a necessary but not sufficient condition
  - initial screening by participation factors
  - residues and frequency responses can supplement screening

Enhancement of small signal stability

1. Excitation Control: Power System Stabilizers

2. Supplementary Control of HVDC Links, SVCs, and other FACTS devices