TRANSIENT STABILITY

OUTLINE

- Description of Transient Stability (TS)
- An elementary view of TS
- Methods of TS analysis
  - ☞ Time-domain simulation
  - ☞ Structure of power system model
  - ☞ Representation of faults

What is Transient (Angle) Stability?

- **The ability of the power system to maintain synchronous operation when subjected to a severe transient disturbance**
  - ☞ **faults on transmission circuits, transformers, buses**
  - ☞ **loss of generation**
  - ☞ **loss of loads**

- **Response involves large excursions of generator rotor angles influenced by nonlinear power-angle relationship**

- **Stability depends on both the initial operating state of the system and the severity of the disturbance**

- **Post-disturbance steady-state operating conditions usually differ from pre-disturbance conditions**

The equation of motion

An elementary principle of dynamics states that:

$$J\frac{d^2\delta_m}{dt^2} = T_m - T_e = T_a$$

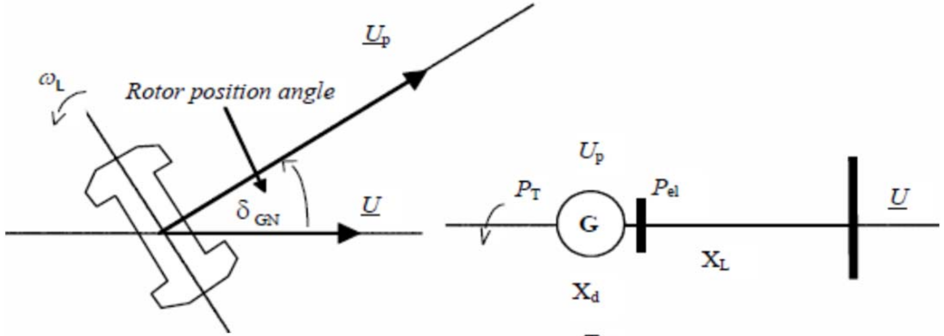
where:

- J = the total moment of inertia of the rotating masses in $kg.m^2$
- δ_m = the angular displacement of the rotor with respect to a synchronously rotating reference in mechanical radians
- T_m = the mechanical torque in N.m.
- T_e = the electrical torque in N.m.
- T_a = accelerating torque in N.m.

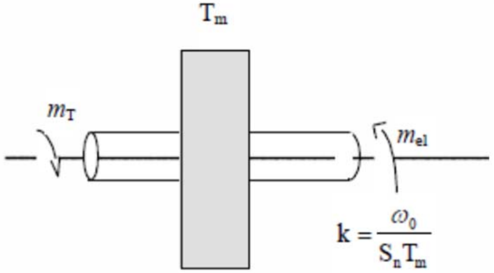
Example

$$\frac{d^2\delta}{dt^2} = k\,(P_T - P_{el}) \rightarrow \dot{\omega} = k\,(P_m - P_e)$$

$$\dot{\delta}_{GN} = \omega - \omega_0$$



$$P_e = \frac{3\,U\,U_p}{X_d + X_L}\sin\delta_{GN}$$



The state equation

$$\dot{\omega} = k\left(P_T - \frac{3\,U\,U_p}{X_d + X_L}\sin\delta_{GN}\right)$$

$$\dot{\delta}_{GN} = \omega - \omega_0$$

The equation of motion in per unit

Multiply both sides of the equation by ω_m

$$\omega_m J \frac{d^2\delta_m}{dt^2} = \omega_m (T_m - T_e) = P_m - P_e$$

where:

ω_m = angular speed in mech. rad/s

P_m = mechanical power in W

P_e = electrical power in W

Basic relationships and definitions:

$$\omega_m = \frac{\omega}{p} \qquad \delta_m = \frac{\delta}{p} \qquad H = \frac{1}{2}\frac{J\,\omega_{m0}^2}{S_n}$$

$$angular\ speed\ in\ mech\ rad/s = \frac{angular\ speed\ in\ el\ rad/s}{number\ of\ pole\ pairs}$$

H = inertia constant in seconds

S_n = machine rated power

S_r = machine rated power in MVA

p = pole pair

ω_{m0} = rated angular speed in mech. rad/s

ω/ δ = angular speed (in el rad/s)/
displacement angle (in el. rad)

$$H = \frac{Kinetic\ energy\ at\ the\ rated\ speed\ (in\ mech.\frac{rad}{s})}{Machine\ rated\ power}$$

The equation of motion in per unit

$$\omega_m J \frac{d^2\delta_m}{dt^2} = \omega_m (T_m - T_e) = P_m - P_e \rightarrow$$

$$\frac{\omega}{p^2} J \frac{d^2\delta}{dt^2} = \omega_m (T_m - T_e) = P_m - P_e \rightarrow$$

$$H = \frac{1}{2} \frac{J\,\omega_{m0}^2}{S_n} \rightarrow J = \frac{2\,H\,S_n\,p^2}{\omega^2}$$

Assuming $\omega_{m0} \cong \omega / p = 2\,\pi\,f\,/p$

Equation of motion / swing equation in pu

$$\frac{d^2\delta}{dt^2} = \frac{\pi\,f\,(p_m - p_e)}{H}$$

p_m = mechanical power in pu

p_e = electrical power in pu

$$\omega_m = \frac{\omega}{p}$$

$$\delta_m = \frac{\delta}{p}$$

$$H = \frac{1}{2} \frac{J\,\omega_{m0}^2}{S_n}$$

Equal area criterion for determination of stability

Equation of motion / swing equation in pu

$$\frac{d^2\delta}{dt^2} = \frac{\pi\, f\, (p_m - p_e)}{H}$$

Multiply both sides by $\left(2\,\frac{d\delta}{dt}\right) \rightarrow$

$$2\,\frac{d\delta}{dt}\,\frac{d^2\delta}{dt^2} = 2\,\frac{d\delta}{dt}\,\frac{\pi\, f\, (p_m - p_e)}{H}$$

$$\frac{d}{\cancel{dt}}\left(\frac{d\delta}{dt}\right)^2 = 2\,\frac{d\delta}{\cancel{dt}}\,\frac{\pi\, f\, (p_m - p_e)}{H} \rightarrow$$

$$d\left(\frac{d\delta}{dt}\right)^2 = \frac{2\,\pi\, f\, (p_m - p_e)}{H}\,d\delta \rightarrow$$

$$\frac{d\delta}{dt} = \sqrt{\frac{2\,\pi\, f}{H}\int_{\delta_0}^{\delta}(p_m - p_e)\,d\delta}$$

Assumption:

$$2\,\frac{d\delta}{dt}\,\frac{d^2\delta}{dt^2} = \frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2$$

Proof:

$$\frac{d\delta}{dt} = x \rightarrow \frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = \frac{d}{dt}\left(x^2\right)$$

$$= 2\,x\,\frac{dx}{dt} = 2\,\frac{d\delta}{dt}\,\frac{d^2\delta}{dt^2}$$

Equal area criterion for estimation of stability

If the generator (after a disturbance) settles at a stable equilibrium point, then

$$\frac{d\delta}{dt} = 0$$

i.e. rate of change of angle with respect to time should become zero.

Hence:

$$\frac{d\delta}{dt} = \sqrt{\frac{2\,\pi\,f}{H}\int_{\delta_0}^{\delta}(p_m - p_e)d\delta} = 0 \;\rightarrow \int_{\delta_0}^{\delta}(p_m - p_e)d\delta = 0$$

We see from the swing equation $\dfrac{d^2\delta}{dt^2} = \dfrac{\pi\,f\,(p_m - p_e)}{H}$ $that \rightarrow \begin{cases}(p_m - p_e) > 0\text{: acceleration}\\(p_m - p_e) < 0\text{: deceleration}\end{cases}$

We can divide the interval into two parts, i.e

$$\int_{\delta_0}^{\delta}(p_m - p_e)d\delta = 0 \rightarrow \int_{\delta_0}^{\delta_c}(p_m - p_e)d\delta + \int_{\delta_c}^{\delta_{max}}(p_m - p_e)d\delta = 0$$

A^+ : accelerating area

A^- : decelerating area

Critical fault clearing angle (δ_c) is the angle at which:

$A^+ = A^-$

Example

A three-phase, 50 Hz, synchronous generator is connected to an infinite bus through a transformer and two parallel transmission lines. The input mechanical power to the synchronous generator is given as 0.8 pu. The generator supplies to the grid a complex power of 0.8+ j0.6 pu at the rated voltage.

Find the critical clearing angle for

a) A three-phase fault at the generator terminal, where system returns to its pre-fault topology after fault clearing.

b) A three-phase fault in the middle on the second transmission line and after the fault the second transmission line is disconnected from the system.

Illustration using an example

- Demonstrate the phenomenon using a very simple system and simple models
- System shown in Fig. 1
- All resistances are neglected
- Generator is represented by the classical model (voltage source behind a transient reactance)

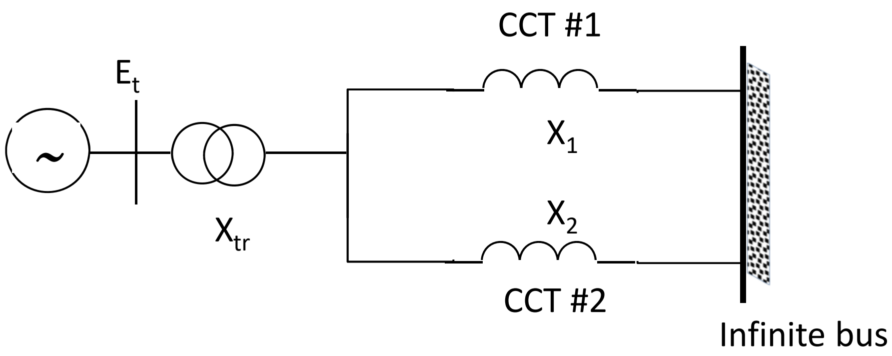
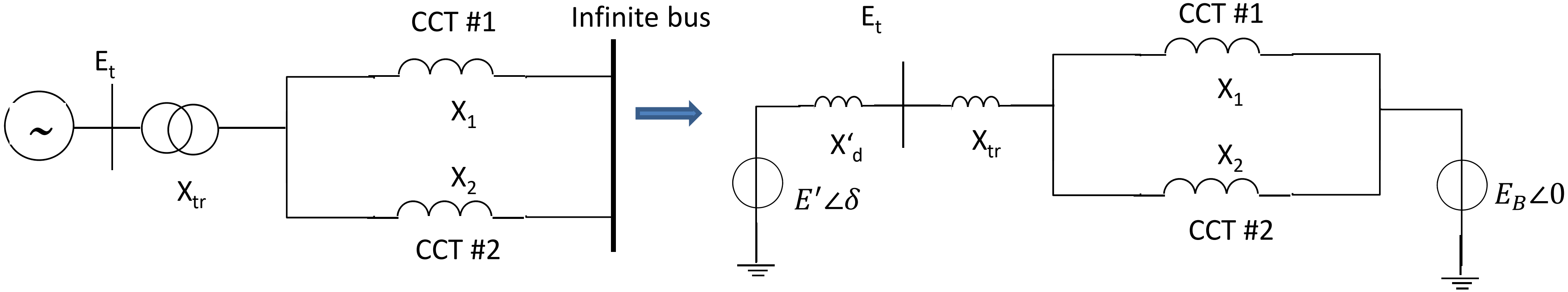


Fig. 1 Single machine - infinite bus system

Pre-fault condition



$$\underline{E}' = \underline{E}_{G0} + j\, X'_d\, \underline{I}_{G0}$$

$\underline{E}_{G0}$: initial voltage at the generator terminals

$\underline{I}_{G0}$: initial generator current

$X_T = X'_d + X_{tr} + (X_1 \,//\, X_2)$

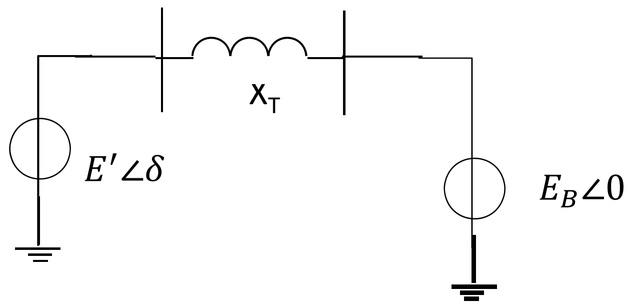


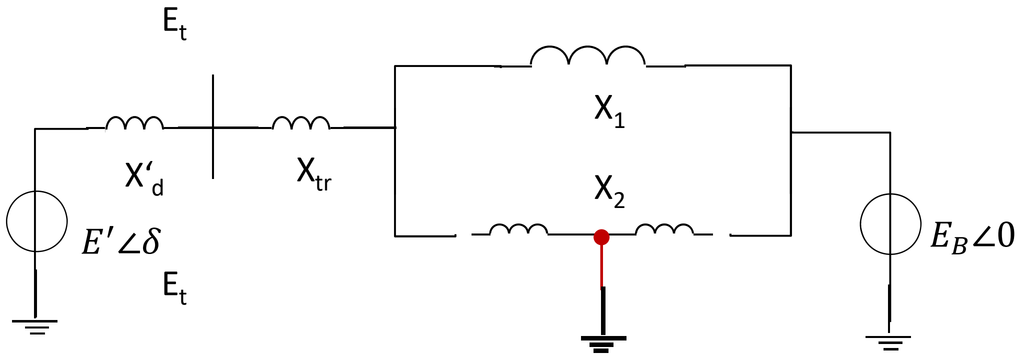
Fig. 2 System representation with generator represented by classical model

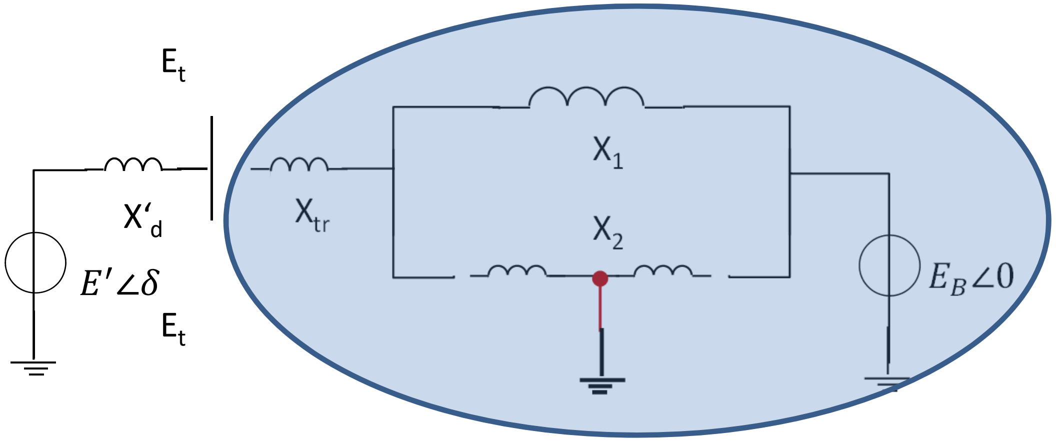
- **The generator's electrical power output is:**

$$P_e = \frac{E' \cdot E_B}{X_T} \cdot \sin\delta = P_{\max} \cdot \sin\delta$$

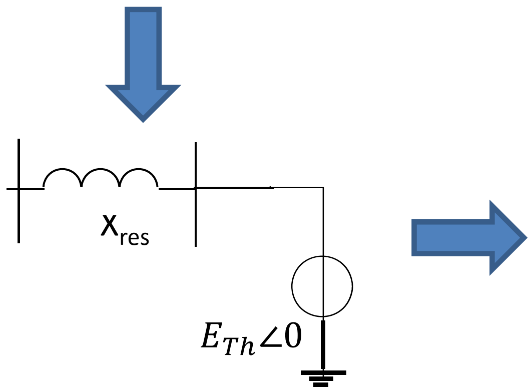
- **With the stator resistance neglected, Pe represents the air-gap power as well as the terminal power**

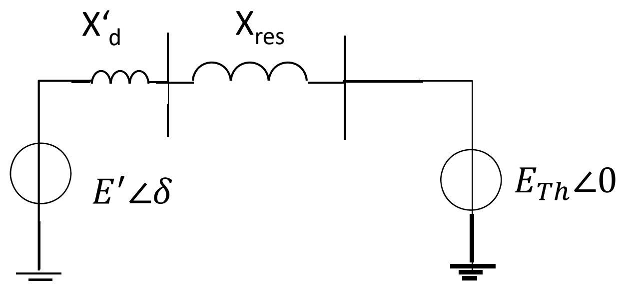
Condition during fault



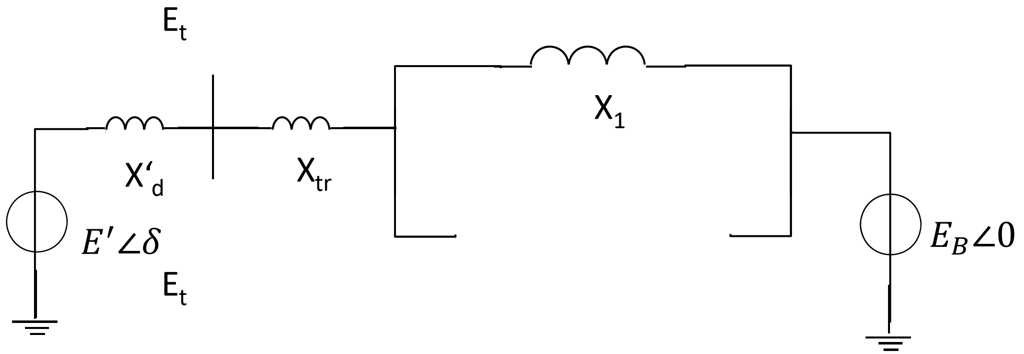


$$P_e = \frac{E' E_{Th}}{X'_d + X_{res}} \sin\delta$$

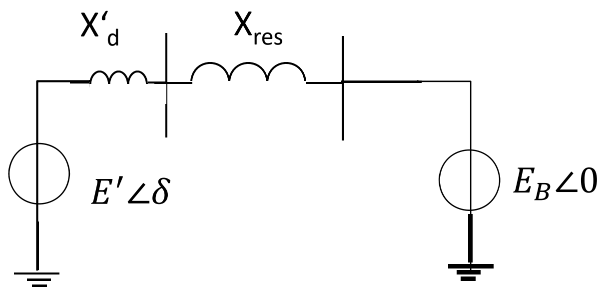




Postfault



$X_{res} = X_{tr} + X_1$



$$P_e = \frac{E' E_{Th}}{X'_d + X_{res}} \sin\delta$$

Power Angle Relationship

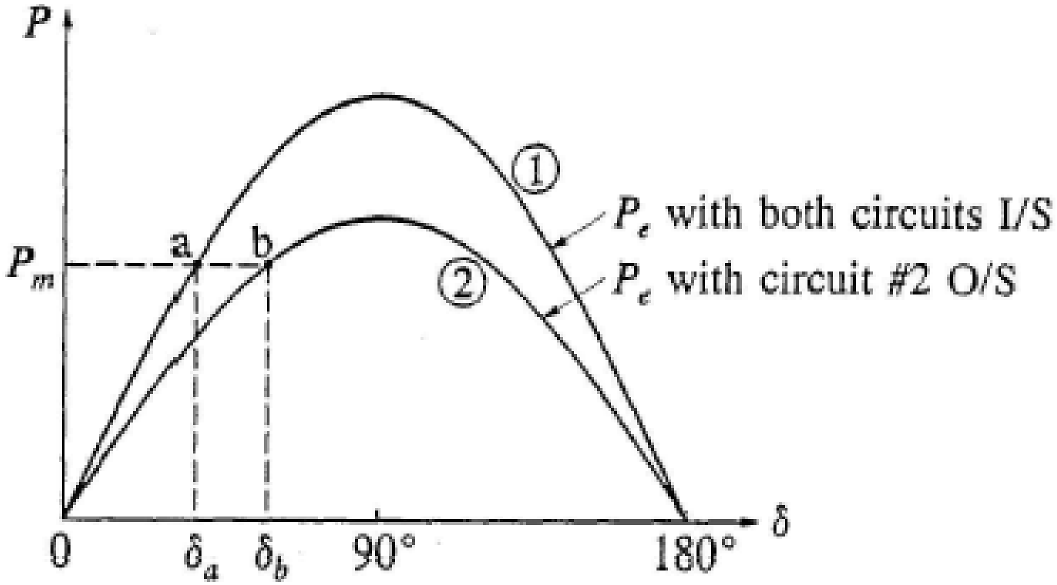


Fig. 3 Power versus power angle relationship

- **Both transmission circuits in-service: Curve 1**
  - ☞ **operate at point "a" (*Pe = Pm*)**
- **One circuit out-of-service: Curve 2**
  - ☞ **lower *Pmax***
  - ☞ **operate at point "b"**
- **higher reactance → higher δ to transmit same power**

Effects of Disturbance

- **The oscillation of δ is superimposed on the synchronous speed ω_0**
- **Speed deviation ($\Delta\omega = d\delta/dt$) << ω_0**
  - ☞ **the generator speed is practically equal to ω_0, and the per unit (pu) air-gap torque may be considered to be equal to the pu air-gap power**
  - ☞ **torque and power are used interchangeably when referring to the swing equation.**

<u>Equation of Motion or Swing Equation:</u>

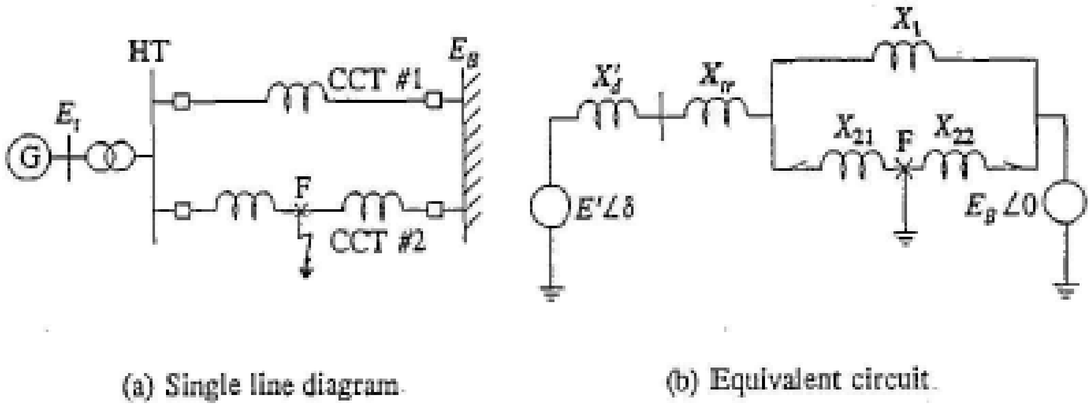
$$\frac{2H}{\omega_0}\frac{d^2\delta_m}{dt^2} = P_m - P_{\max}.\sin\delta$$

where:

Pm **= mechanical power input (pu)**
Pmax **= maximum electrical power output (pm)**
H **= inertia constant (MWs/MVA)**
δ **= rotor angle (elec. radians)**
t **= time (s)**

Response to a Fault

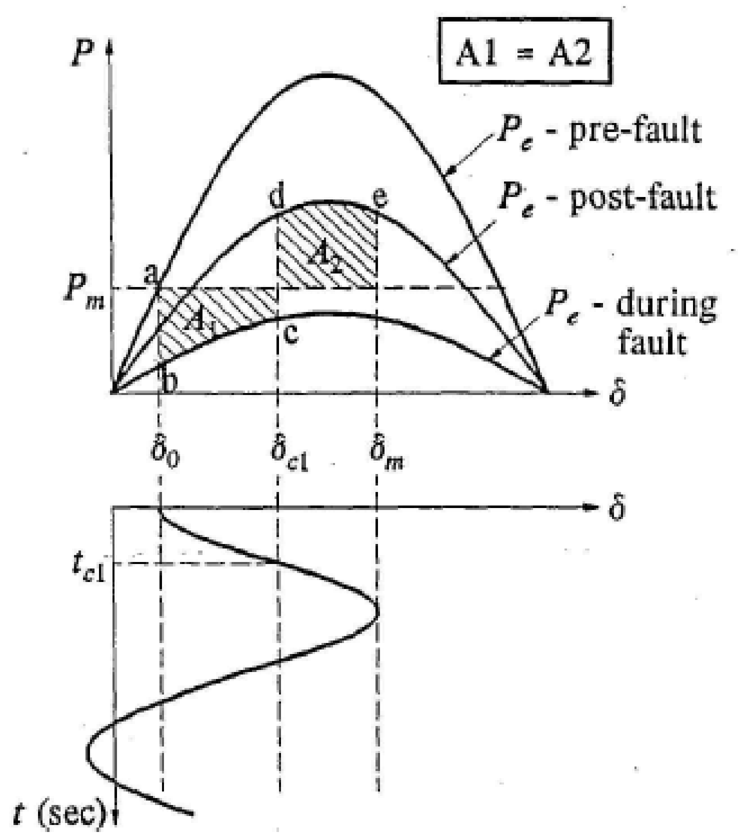
- **Illustrate the equal area criterion using the following system:**



- **Examine the impact on stability of different fault clearing times**

Stable Case

- **Illustrate the equal area criterion using the following system:**



Response to a fault cleared in t_{cl} seconds - stable case

Stable Case

Pre-disturbance:

- both circuits in service : Pe = Pm, $\delta = \delta 0$
- operating point a

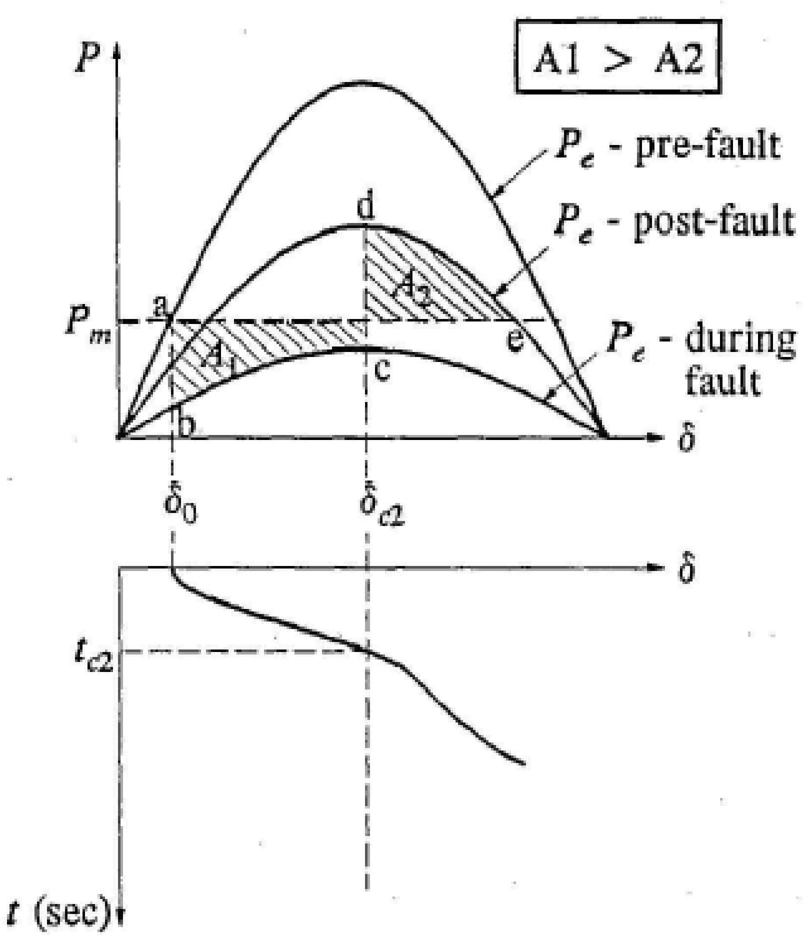
During fault:

- operating point moves from a to b
- inertia prevents δ from changing instantaneously
- Pm > Pe → rotor accelerates to operating point c

Post Fault:

- faulted circuit is tripped, operating point shifts to d
- Pe > Pm → rotor decelerates
- rotor speed > ω_0 → δ increases
- operating point moves from d to e such that A1 = A2
- at e, speed = ω_0, and $\delta = \delta_m$
- Pe > Pm → rotor decelerates; speed below ω_0
- δ decreases and operating point retraces e to d
- with no damping, rotor continues to oscillate

Unstable Case



Response to a fault cleared in t_{C2} seconds - unstable case

Unstable Case

- **Area A2 above Pm is less than A1**
- **When the operating point reaches e, the kinetic energy gained during the accelerating period has not yet been completely expended**
  ☞ **the speed is still greater than ω_0 and δ continues to increase**
- **Beyond point e, Pe<Pm, → rotor begins to accelerate again**
- **The rotor speed and angle continue to increase leading to loss of synchronism**

Factors Influencing Transient Stability

a. How heavily the generator is initially loaded.

b. The generator output during the fault. This depends on the fault location and type.

c. The fault clearing time.

d. The post-fault transmission system reactance.

e. The generator reactance. A lower reactance increases peak power and reduces initial rotor angle.

f. The generator inertia. The higher the inertia, the slower the rate of change angle. This reduces the kinetic energy gained during fault, i.e. area A1 is reduced.

g. The generator internal voltage magnitude (E'). This depends on the field excitation.

h. The infinite bus voltage magnitude E_B.

Practical Method of Transient Stability Analysis

- Practical power systems have complex network structures

- Accurate analysis of transient stability requires detailed models for:
  - ☞ generating unit and controls
  - ☞ voltage dependent load characteristics
  - ☞ HVDC converters, FACTs devices, etc.

- At present, the most practical available method of transient stability analysis is <u>time domain simulation</u>:
  - ☞ solution of nonlinear differential equations and algebraic equations
  - ☞ step-by-step <u>numerical integration techniques</u>
  - ☞ complimented by efficient techniques for solving non-linear highly sparse algebraic equations

Numerical Integration Methods

- **Differential equations to be solved are nonlinear ordinary differential equations with known initial values:**

$$\frac{dx}{dt} = f(x, t)$$

x is the state vector of n dependent variables,
t is the independent variable (time)

Objective: solve x **as a function of t, with the initial values of** x **and t equal to** x_0 **and** t_0**, respectively.**

Methods:

Euler's Method
Modified Euler's Method
Runge-Kutta (R-K) Methods
Trapezoidal Rule

Simulation of Power System Dynamic Response

Structure of the Power System Model:

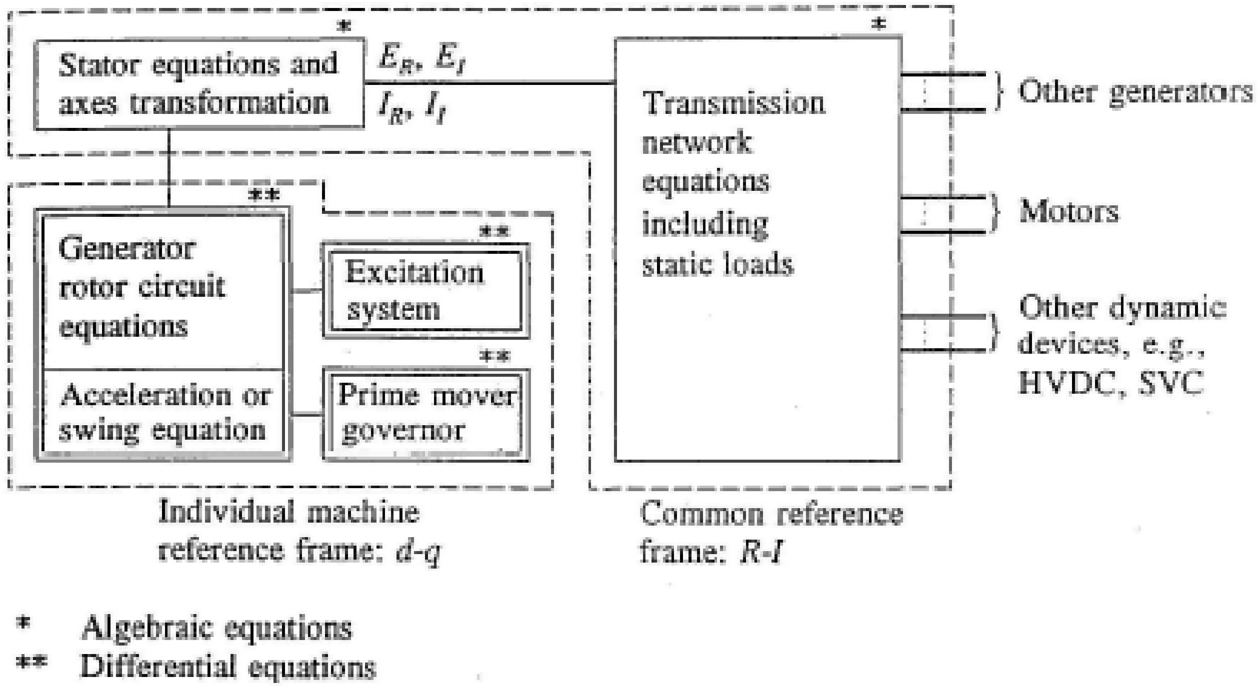
Components:

- Synchronous generators, and the associated excitation systems and prime movers
- Interconnecting transmission network including static loads
- Induction and synchronous motor loads
- Other devices such as HVDC converters and SVCs

Monitored Information:

- Basic stability information
- Bus voltages
- Line flows
- Performance of protective relaying, particularly transmission line protection

Simulation of Power System Dynamic Response



Structure of the complete power system model for transient stability analysis

Simulation of Power System Dynamic Response

- Models used must be appropriate for transient stability analysis transmission network and machine stator transients are neglected
  - ☞ dynamics of machine rotors and rotor circuits, excitation systems, prime movers and other devices such as HVDC converters are represented
  - ☞ Equations must be organized in a form suitable for numerical integration

- Large set of ordinary differential equations and large sparse algebraic equations
  - ☞ differential-algebraic initial value problem

Overall System Equations

- **Equations for each dynamic device:**

$$\dot{x}_d = f_d(x_d, V_d)$$
$$I_d = g_d(x_d, V_d)$$

where

x_d = state vector of individual device

I_d = *R* and *I* components of current injection from the device into the network

V_d = *R* and *I* components of bus voltage

Network equation:

$$\boldsymbol{I} = \boldsymbol{Y}_N \boldsymbol{V}$$

where

Y_N = network mode admittance matrix

I = node current vector

V = node voltage vector

Overall System Equations

Overall system equations:
comprises a set of first order differentials

$$\dot{x} = f(x, V)$$

and a set of algebraic equations

$$I(x, V) = Y_N V$$

where

x = state vector of the system
V = bus voltage vector
I = current injection vector

Time *t* does not appear explicitly in the above equations explicitly

Many approaches for solving these equations characterized by:

a. **The manner of interface between the differential an algebraic equations: partitioned or simultaneous**
b. **Integration method used**
c. **Method used for solving the algebraic equations:**
   - **Gauss-Seidel method based on admittance matrix**
   - **direct solution using sparsity oriented triangular factorization**
   - **iterative solution using Newton-Raphson method**