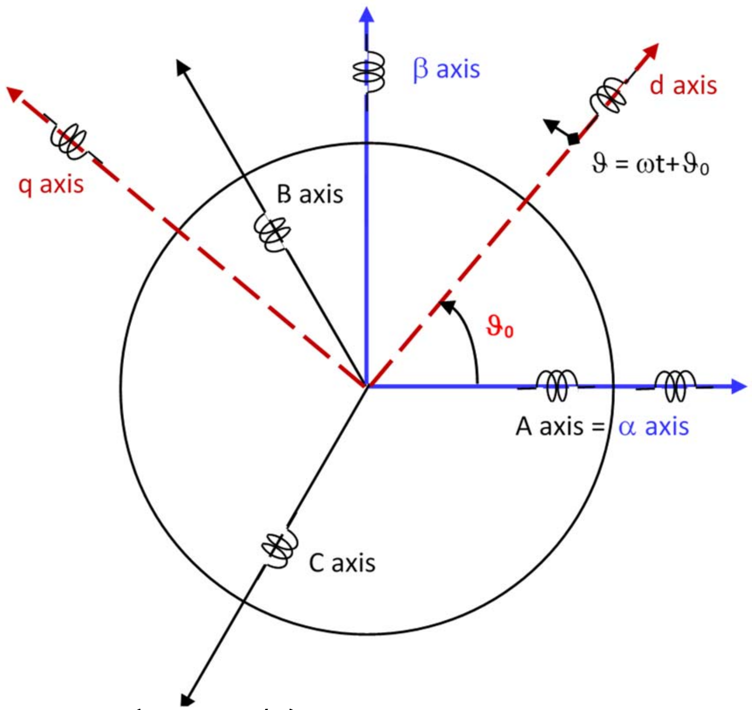
Synchronous Machine Model

Voltage, current, and instantaneous power in abc, αβ0 and dq0 coordinates



$$MMF_{abc}(\vartheta) = i_a \cos\vartheta + i_b \cos(\vartheta + 2\pi/3) + i_c \cos(\vartheta - 2\pi/3)$$

$$MMF_{\alpha\beta}(\vartheta) = i_\alpha \cos\vartheta + i_\beta \cos(\vartheta + \pi/2)$$

$$MMF_{\alpha\beta}(\vartheta) = MMF_{abc}(\vartheta) \rightarrow$$
$$i_\alpha \cos\vartheta + i_\beta \cos(\vartheta + \pi/2) = i_a \cos\vartheta + i_b \cos(\vartheta + 2\pi/3) + i_c \cos(\vartheta - 2\pi/3)$$

$$\cos(\vartheta + \pi/2) = -\sin\vartheta$$

$$i_\alpha \cos\vartheta - i_\beta \sin\vartheta = i_a \cos\vartheta + i_b \left(-\frac{1}{2}\cos\vartheta - \frac{\sqrt{3}}{2}\sin\vartheta\right) + i_c \left(-\frac{1}{2}\cos\vartheta + \frac{\sqrt{3}}{2}\sin\vartheta\right)$$

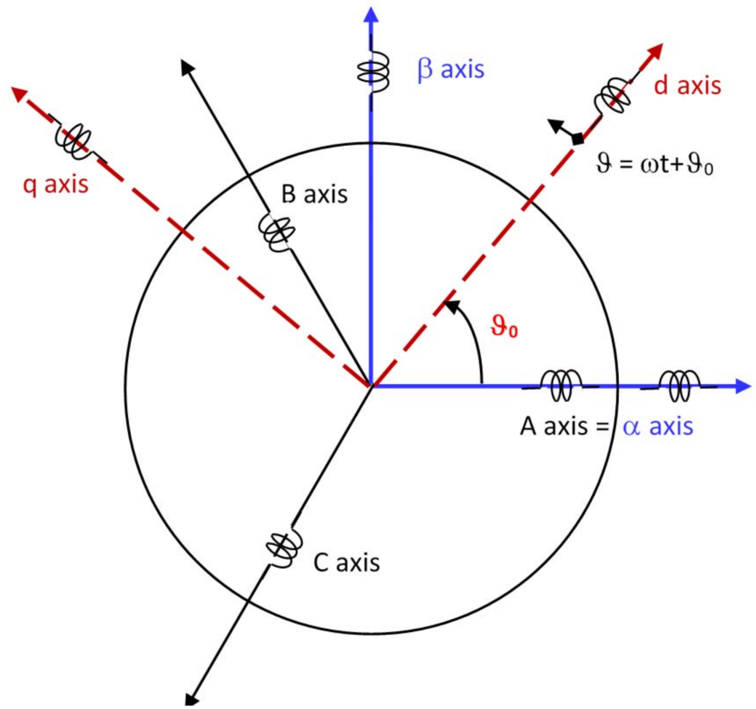
Voltage, current, and instantaneous power in abc, αβ0 and dq0 coordinates

Example:

$i_a = i_m \cos \omega t$

$i_b = i_m \cos(\omega t - 2\pi/3)$

$i_c = i_m \cos(\omega t + 2\pi/3)$



At location $\vartheta = 0$ and time $t = 0$:

$$i_\alpha = i_a + i_b\left(-\frac{1}{2}\right) + i_c\left(-\frac{1}{2}\right) = \frac{3}{2} i_m$$

At location $\vartheta = \pi/2$ and time $\omega t = \pi/2$:

$$i_\beta = -i_b\frac{\sqrt{3}}{2} + i_c\frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2} = -\frac{3}{2} i_m$$

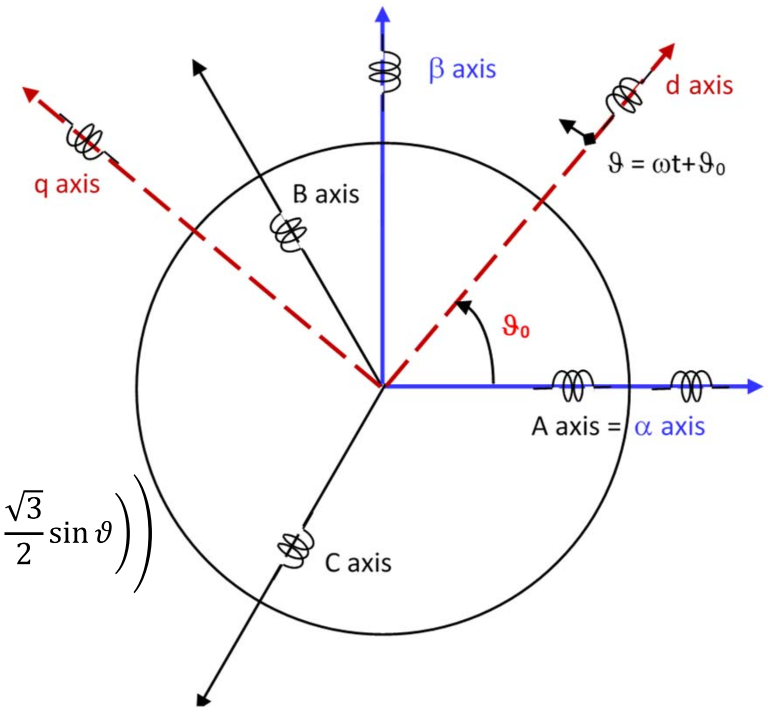
Amplitude of $i_\alpha : \frac{3}{2} i_m$

Amplitude of $i_\beta : \frac{3}{2} i_m$

$$\longrightarrow i_\alpha \cos\vartheta - i_\beta \sin\vartheta = \frac{2}{3}\left(i_a \cos\vartheta + i_b\left(-\frac{1}{2}\cos\vartheta - \frac{\sqrt{3}}{2}\sin\vartheta\right) + i_c\left(-\frac{1}{2}\cos\vartheta + \frac{\sqrt{3}}{2}\sin\vartheta\right)\right)$$

Amplitudes of both $i_\alpha; i_\beta = i_m$

Voltage, current, and instantaneous power in abc and αβ0 coordinates



$$i_\alpha \cos\vartheta - i_\beta \sin\vartheta$$

$$= \frac{2}{3}\left(i_a \cos\vartheta + i_b\left(-\frac{1}{2}\cos\vartheta - \frac{\sqrt{3}}{2}\sin\vartheta\right) + i_c\left(-\frac{1}{2}\cos\vartheta + \frac{\sqrt{3}}{2}\sin\vartheta\right)\right)$$

$$i_\alpha = \frac{2}{3}\left(i_a - \frac{1}{2}\, i_b - \frac{1}{2}\, i_c\right)$$

$$i_\beta = \frac{2}{3}\left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\, i_c\right)$$

If the star point is grounded:

$$i_0 = \frac{1}{3}\,(i_a + i_b + i_c)$$

$$\begin{pmatrix} i_\alpha \\ i_\beta \\ i_0 \end{pmatrix} = \frac{2}{3}\begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 & -1 & -1 \\ 0 & \sqrt{3} & -\sqrt{3} \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix}$$

Space phasor

$$i_\alpha = \frac{2}{3}\left(i_a - \frac{1}{2}\, i_b - \frac{1}{2}\, i_c\right)$$

$$i_\beta = \frac{2}{3}\left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\, i_c\right)$$

Space phasor

$$\underline{I}_{\alpha\beta}^{\angle 0} = i_\alpha + j i_\beta = \frac{2}{3}\left(i_a + \, i_b\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + \, i_c\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\right) \quad = \frac{2}{3}\left(i_a + \, i_b\, \underline{a} + \, i_c\, \underline{a}^2\right)$$

Time phasor

$$u(t) = \sqrt{2}U cos(\omega t + \varphi_u) = Re\{\sqrt{2}U cos(\omega t + \varphi_u) + j\sqrt{2}U sin(\omega t + \varphi_u)\}$$
$$= Re\{\sqrt{2}U.e^{j\varphi_u} e^{j\omega t}\}$$

Time phasor

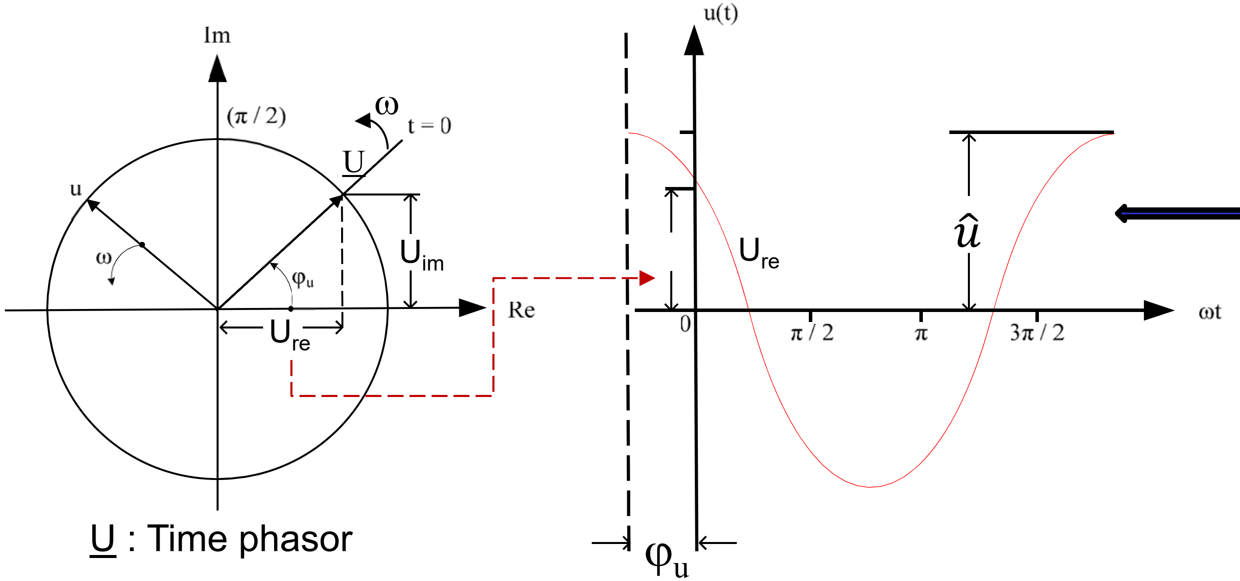
$$\underline{U} = U.e^{j\varphi_u}$$

Space phasor

$$\underline{U}^{\angle 0} = \frac{2}{3}\left(u_a + \, u_b\, \underline{a} + u\, \underline{a}^2\right)$$

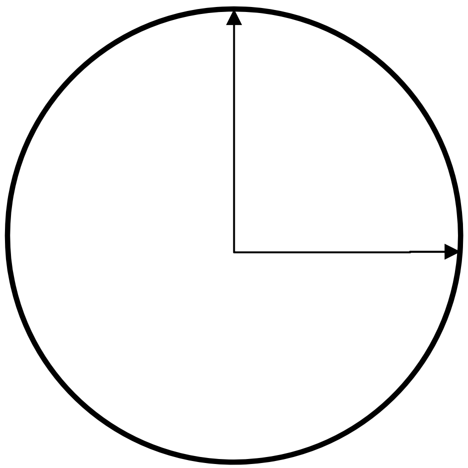
Diference between time & space phasor

Time phasor is function of time



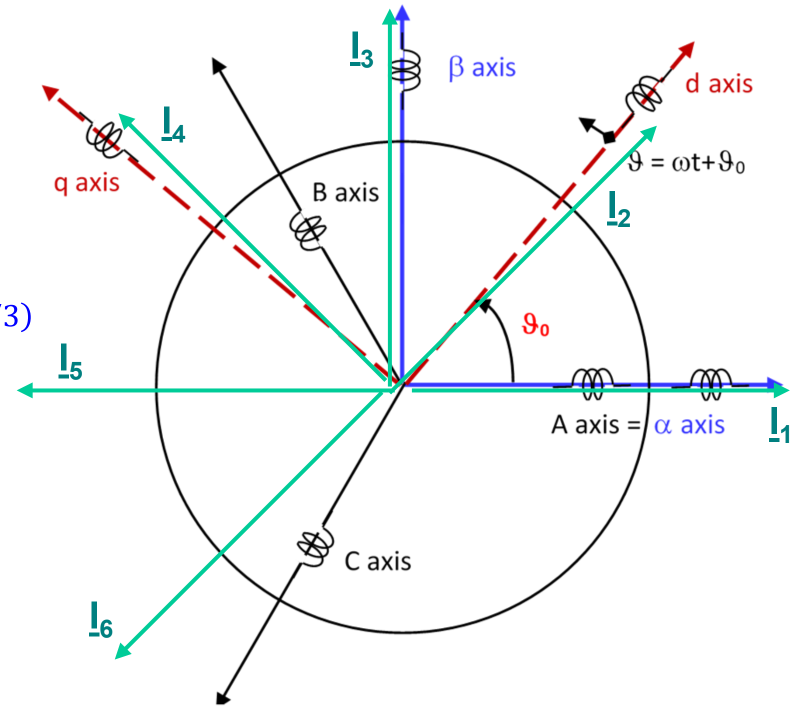
$$u(t) = \sqrt{2}\,\mathrm{Re}\{\underline{U}e^{j\omega t}\}$$

$\underline{U}$: Time phasor



Space phasor w.r.t stationary referecne frame

Space phasor $\underline{I}_{\alpha\beta}^{\angle 0} = i_\alpha + j i_\beta = \frac{2}{3}\left(i_a + i_b\,\underline{a} + i_c\,\underline{a}^2\right)$



Example

$i_a = 5\cos\omega t \quad i_b = 5\cos(\omega t - 2\pi/3) \qquad i_c = 5\cos(\omega t + 2\pi/3)$

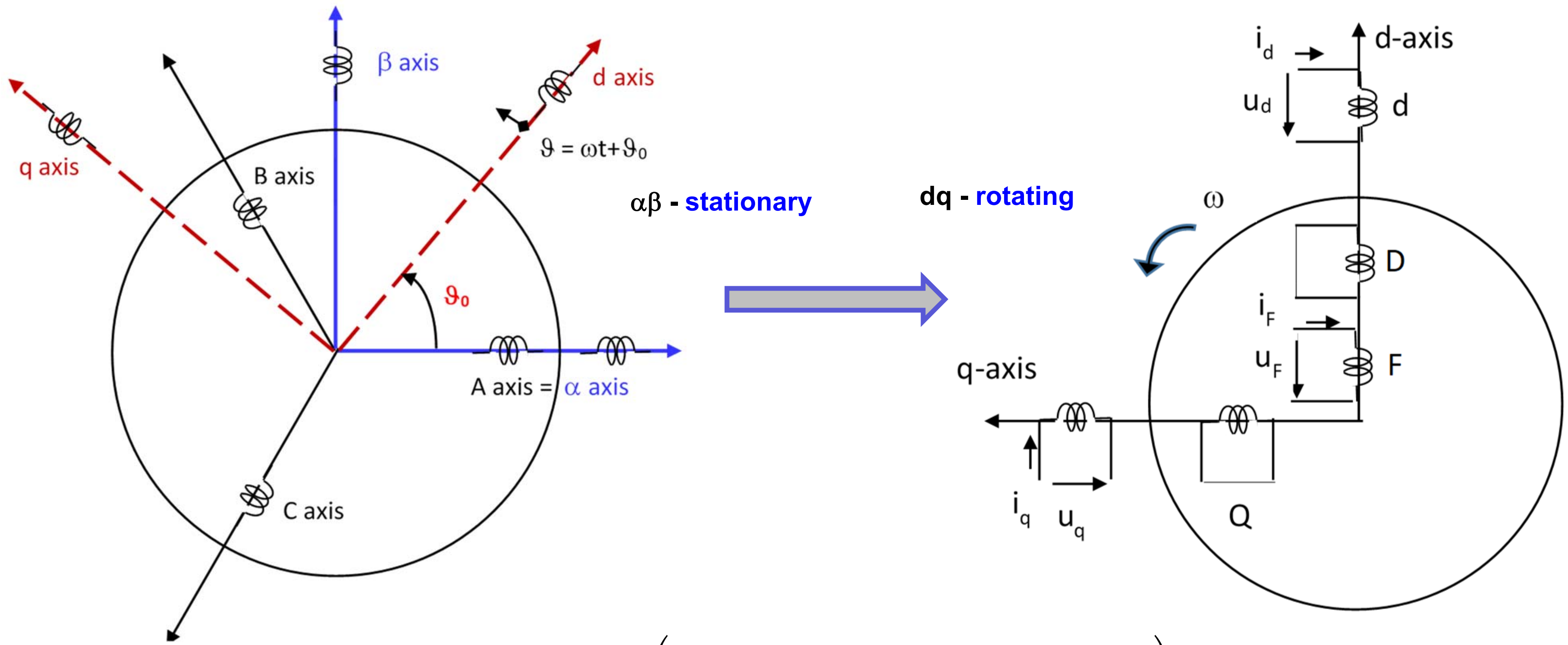
$$\begin{pmatrix} i_\alpha \\ i_\beta \\ i_0 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 & -1 & -1 \\ 0 & \sqrt{3} & -\sqrt{3} \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 5\cos\omega t \\ 5\cos(\omega t - 2\pi/3) \\ 5\cos(\omega t + 2\pi/3) \end{pmatrix}$$

$$\begin{pmatrix} i_a \\ i_b \\ c \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \end{pmatrix}\begin{pmatrix} i_\alpha \\ i_\beta \\ i_0 \end{pmatrix}$$

ωt	**0**	**π/4**	**π/2**	**3π/4**	**π**	**5π/4**
i_α	5	3.54	0	-3.54	-5	-3.54
i_β	0	3.54	5	3.54	0	-3.54
$\underline{I}_{\alpha\beta}^{\angle 0}$	$5\angle 0^0$	$5\angle 45^0$	$5\angle 90^0$	$5\angle 135^0$	$5\angle 180^0$	$5\angle 225^0$
	$\underline{\mathbf{I}}_1$	$\underline{\mathbf{I}}_2$	$\underline{\mathbf{I}}_3$	$\underline{\mathbf{I}}_4$	$\underline{\mathbf{I}}_5$	$\underline{\mathbf{I}}_6$

→ **Rotates in space with respect to a stationary reference frame**

Voltage, current, and instantaneous power in a dq0 coordinates



Space phasor -

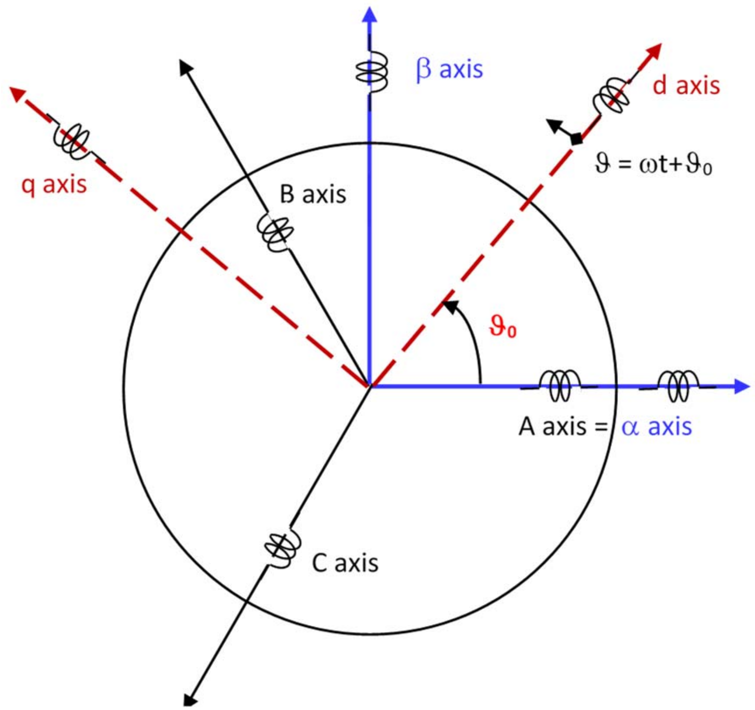
$$\underline{I}_{\alpha\beta}^{\angle 0} = i_\alpha + j i_\beta = \frac{2}{3}\left(i_a + i_b\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + i_c\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\right) = \frac{2}{3}\left(i_a + i_b\,\underline{a} + i_c\,\underline{a}^2\right)$$

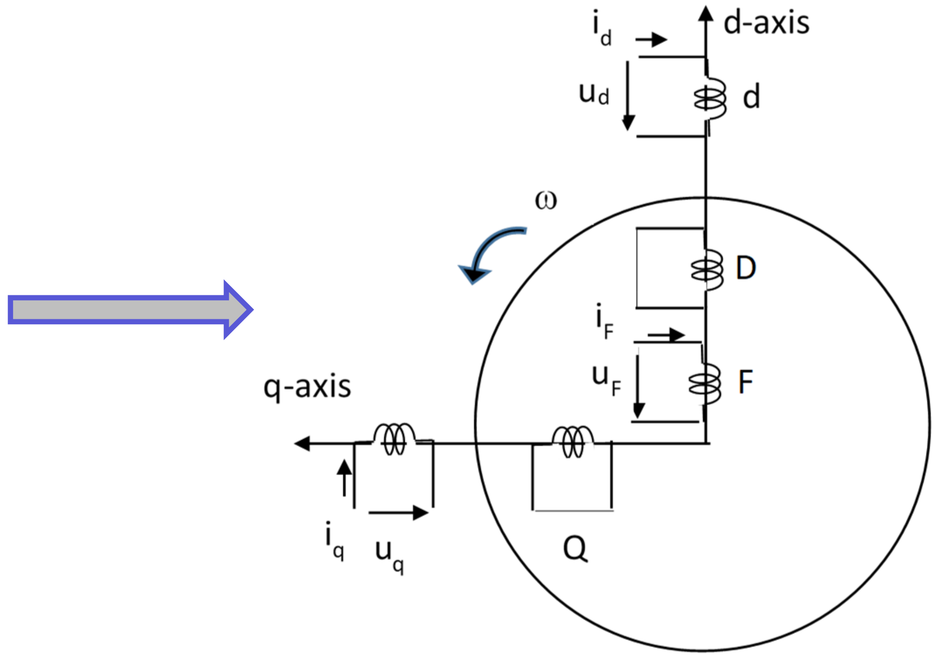
$$\underline{I}_{dq}^{\angle \omega} = \underline{I}_{\alpha\beta}^{\angle 0}.e^{-j\vartheta} = i_d + j i_q = \frac{2}{3}\left(i_a + i_b\,\underline{a} + i_c\,\underline{a}^2\right).e^{-j\vartheta} = \frac{2}{3}\left(i_a + i_b e^{-j\left(\vartheta - \frac{2\pi}{3}\right)} + i_c e^{-j\left(\vartheta + \frac{2\pi}{3}\right)}\right)$$

$$i_d = \frac{2}{3}\left(i_a \cos\vartheta + i_b \cos\left(\vartheta - \frac{2\pi}{3}\right) + i_c \cos\left(\vartheta + \frac{2\pi}{3}\right)\right)$$

$$i_q = \frac{2}{3}\left(-i_a \sin\vartheta - i_b \sin\left(\vartheta - \frac{2\pi}{3}\right) - i_c \sin\left(\vartheta + \frac{2\pi}{3}\right)\right)$$

Voltage, current, and instantaneous power in abc, αβ0 and dq0 coordinates





$$i_d = \frac{2}{3}(i_a \cos\vartheta + i_b \cos(\vartheta - 2\pi/3) + i_c \cos(\vartheta + 2\pi/3))$$

$$i_q = \frac{2}{3}(-i_a \sin\vartheta - i_b \sin(\vartheta - 2\pi/3) - i_c \sin(\vartheta + 2\pi/3))$$

$$i_0 = \frac{1}{3}(i_a + i_b + i_c)$$

$$\begin{pmatrix} i_d \\ i_q \\ i_0 \end{pmatrix} = \frac{2}{3}\begin{pmatrix} \cos\vartheta & \cos(\vartheta - 2\pi/3) & \cos(\vartheta + 2\pi/3) \\ -\sin\vartheta & -\sin(\vartheta - 2\pi/3) & -\sin(\vartheta + 2\pi/3) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix}$$

Space phasor w.r.t rotating reference frame

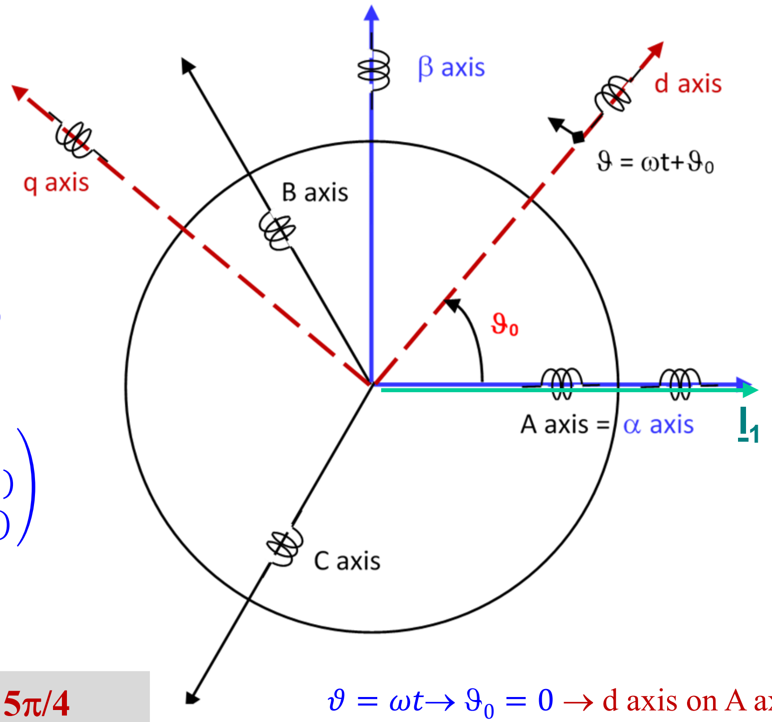
Space phasor $\underline{I}_{dq}^{\angle\omega} = i_d + j i_q$

Example

$i_a = 5\cos\omega t \quad i_b = 5\cos(\omega t - 2\pi/3) \quad i_c = 5\cos(\omega t + 2\pi/3)$

$$\begin{pmatrix} i_d \\ i_q \\ i_0 \end{pmatrix} = \frac{2}{3}\begin{pmatrix} \cos\vartheta & \cos(\vartheta - 2\pi/3) & \cos(\vartheta + 2\pi/3) \\ -\sin\vartheta & -\sin(\vartheta - 2\pi/3) & -\sin(\vartheta + 2\pi/3) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 5\cos\omega t \\ 5\cos(\omega t - 2\pi/3) \\ 5\cos(\omega t + 2\pi/3) \end{pmatrix}$$

$\vartheta = \omega t$



$\vartheta = \omega t \rightarrow \vartheta_0 = 0 \rightarrow$ d axis on A axis

ωt	**0**	**π/4**	**π/2**	**3π/4**	**π**	**5π/4**
i_d	5	5	5	5	5	5
i_q	0	0	0	0	0	0
$\underline{I}_{dq}^{\angle\omega}$	$5\angle 0^0$	$5\angle 0^0$	$5\angle 0^0$	$5\angle 0^0$	$5\angle 0^0$	$5\angle 0^0$
	$\underline{I}_1$					

→ **Stationary in space with respect to a rotating reference frame**

abc → αβ0 and abc → dq0

abc → αβ0	abc → dq0
$\begin{pmatrix} g_\alpha \\ g_\beta \\ g_0 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 & -1 & -1 \\ 0 & \sqrt{3} & -\sqrt{3} \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} g_a \\ g_b \\ g_c \end{pmatrix}$	$\begin{pmatrix} g_d \\ g_q \\ g_0 \end{pmatrix} = \frac{2}{3}\begin{pmatrix} \cos\vartheta & \cos(\vartheta - 2\pi/3) & \cos(\vartheta + 2\pi/3) \\ -\sin\vartheta & -\sin(\vartheta - 2\pi/3) & -\sin(\vartheta + 2\pi/3) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} g_a \\ g_b \\ g_c \end{pmatrix}$ $\vartheta = \omega t + \vartheta_0$
$p = \frac{3}{2}\left(u_\alpha i_\alpha + u_\beta i_\beta + 2u_0 i_0\right)$	$p = \frac{3}{2}\left(u_d i_d + u_q i_q + 2u_0 i_0\right)$

g stands for current, voltage, flux (i, u, ψ)

Examples

$$\begin{pmatrix} g_\alpha \\ g_\beta \\ g_0 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 & -1 & -1 \\ 0 & \sqrt{3} & -\sqrt{3} \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} g_a \\ g_b \\ g_c \end{pmatrix}$$

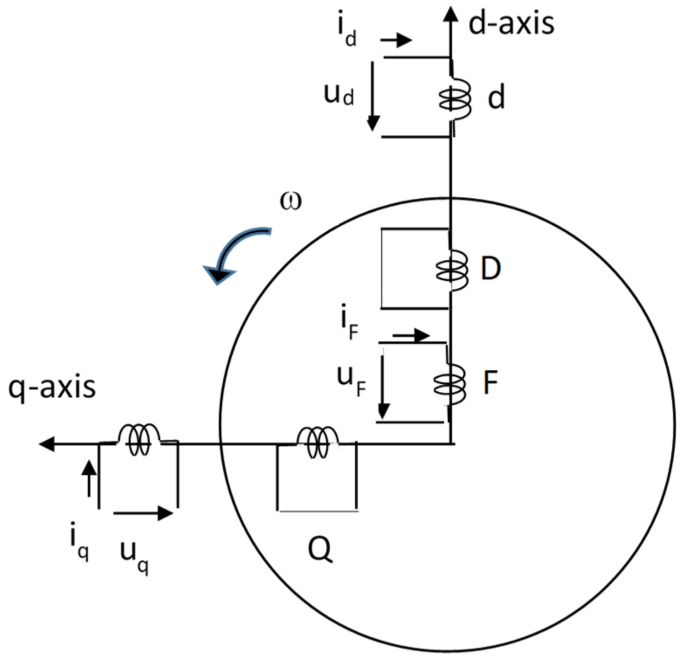
$$\begin{pmatrix} g_d \\ g_q \\ g_0 \end{pmatrix} = \frac{2}{3}\begin{pmatrix} \cos\vartheta & \cos(\vartheta - 2\pi/3) & \cos(\vartheta + 2\pi/3) \\ -\sin\vartheta & -\sin(\vartheta - 2\pi/3) & -\sin(\vartheta + 2\pi/3) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} g_a \\ g_b \\ g_c \end{pmatrix}$$

$i_a = 5\cos\omega t$ $\quad i_b = 5\cos(\omega t - 2\pi/3)$ $\quad i_c = 5\cos(\omega t + 2\pi/3)$ $\quad \vartheta = \vartheta_0 + \omega t$

$$\begin{pmatrix} i_\alpha \\ i_\beta \\ i_0 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 & -1 & -1 \\ 0 & \sqrt{3} & -\sqrt{3} \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix} = \begin{pmatrix} 5\cos\omega t \\ -5\sin\omega t \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} i_d \\ i_q \\ i_0 \end{pmatrix} = \frac{2}{3}\begin{pmatrix} \cos\vartheta & \cos(\vartheta - 2\pi/3) & \cos(\vartheta + 2\pi/3) \\ -\sin\vartheta & -\sin(\vartheta - 2\pi/3) & -\sin(\vartheta + 2\pi/3) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix}$$

System of equations in per unit

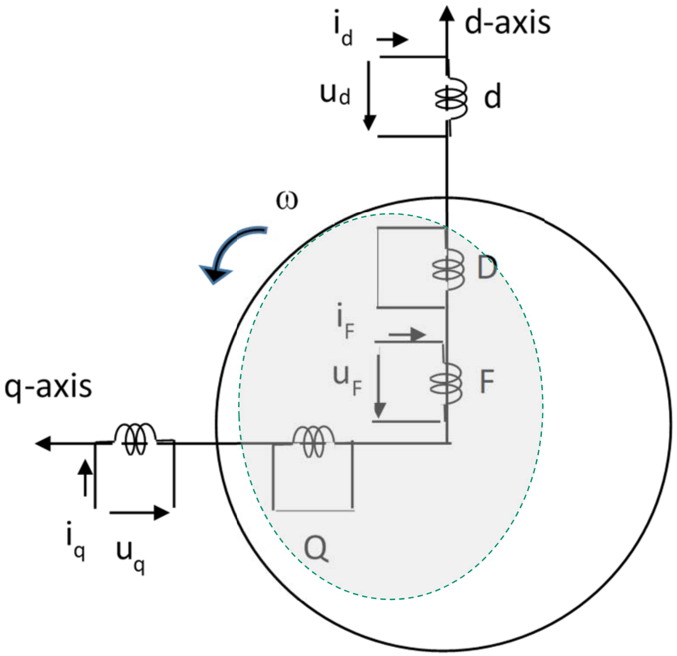


Voltage equations		Flux equations	
Stator:		Stator	
$u_d = r_a i_d + \frac{d\psi_d}{dt} - \omega\psi_q$	(1)	$\psi_d = L_d i_d + L_{AD}(i_F + i_D)$	(6)
$u_q = r_a i_q + \frac{d\psi_q}{dt} + \omega\psi_d$	(2)	$\psi_q = L_q i_q + L_{AQ}\, i_Q$	(7)
Rotor		Rotor	
$u_F = r_F i_F + \frac{d\psi_F}{dt}$	(3)	$\psi_F = L_F i_F + L_{AD}(i_d + i_D)$	(8)
$0 = r_D i_D + \frac{d\psi_D}{dt}$	(4)	$\psi_D = L_D i_D + L_{AD}(i_d + i_F)$	(9)
$0 = r_Q i_Q + \frac{d\psi_Q}{dt}$	(5)	$\psi_Q = L_Q i_Q + L_{AQ} i_q$	(10)

- The stator base quantities are based on the machine rating
- The rotor base quantities are chosen so that:
  - the mutual inductances between different circuits are reciprocal (e.g. $L_{Fd} = L_{dF}$) and that the mutual inductances between the rotor and stator circuits in each axis are equal (e.g., $L_{Fd} = L_{Dd}$)

    → The p.u. system is referred to as the "L_{AD} base reciprocal p.u. system"

Synchronous machine sub-transient inductance - Ld”



Assume all rotor windings are short circuited and symmetrical three phase voltage applied such that:

$$\begin{pmatrix} u_a(t) \\ u_b(t) \\ u_c(t) \end{pmatrix} = \sqrt{2}\,U \begin{pmatrix} \cos(\omega t) \\ \cos(\omega t + 2\pi/3) \\ \cos(\omega t - 2\pi/3) \end{pmatrix} c(t)$$

c(t) = 0 for t < 0 and c(t) = 1 for t ≥ 0.

$$\begin{pmatrix} u_o \\ u_d \\ u_q \end{pmatrix} = \frac{2}{3} \begin{pmatrix} 1/2 & 1/2 & 1/2 \\ \cos(\omega t) & \cos(\omega t - 2\pi/3) & \cos(\omega t + 2\pi/3) \\ -\sin(\omega t) & -\sin(\omega t - 2\pi/3) & -\sin(\omega t + 2\pi/3) \end{pmatrix} \begin{pmatrix} u_a(t) \\ u_b(t) \\ u_c(t) \end{pmatrix}$$

$$= \sqrt{2}U \begin{pmatrix} 0 \\ c(t) \\ 0 \end{pmatrix} \rightarrow$$ DC voltage applied across coil d.

Eq. (8) & (9):

$$0 = L_F i_F + L_{AD}(i_d + i_D)$$

$$0 = L_D i_D + L_{AD}(i_d + i_F)$$

$$i_D = -\frac{L_F L_{AD} - L_{AD}^2}{L_F L_D - L_{AD}^2}\, i_d$$

$$i_F = -\frac{L_D L_{AD} - L_{AD}^2}{L_F L_D - L_{AD}^2}\, i_d$$

Substituted in (6):

$$\psi_d = L_d i_d + L_{AD}(i_F + i_D) = \left(L_d - \frac{L_D\, L_{AD}^2 + L_F\, L_{AD}^2 - 2L_{AD}^3}{L_F L_D - L_{AD}^2}\right) . i_d$$

$\psi_d = L_d^{"} . i_d$ $L_d^{"}$: the d-axis **sub-transient** inductance

Synchronous machine transient inductance - Ld'

If there is no damper winding in the d-axis, or if the current in the damper winding has decayed to zero, i.e. $i_D = 0$:

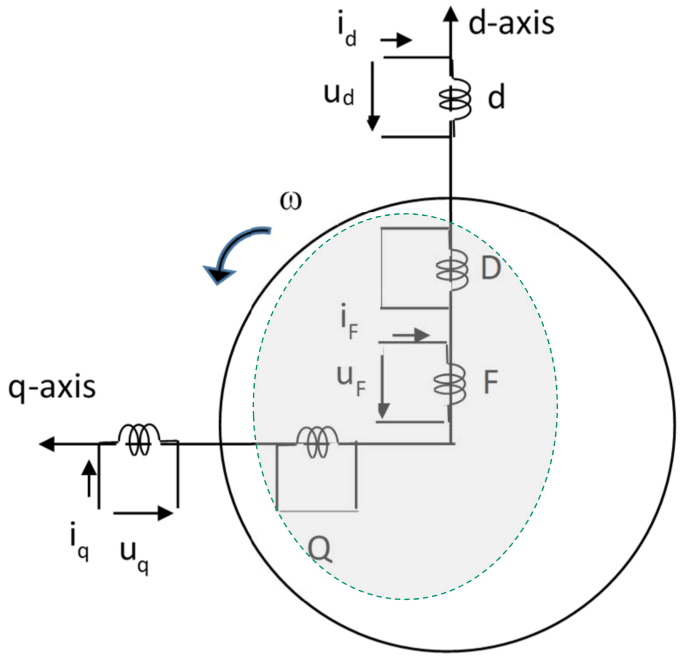
$$0 = L_F i_F + L_{AD} i_d \rightarrow i_F = -L_{AD} i_d / L_F$$

$$\psi_d = L_d i_d + L_{AD}(i_F + i_D) = \left(L_d - \frac{{L_{AD}}^2}{L_F}\right) . i_d$$

$$L'_d = \left(L_d - \frac{{L_{AD}}^2}{L_F}\right)$$

the d-axis **transient** inductance

Synchronous machine transient inductance - Lq'



Assume all rotor windings are short circuited and symmetrical three phase voltage applied such that:

$$\begin{pmatrix} u_a(t) \\ u_b(t) \\ u_c(t) \end{pmatrix} = \sqrt{2}\,U \begin{pmatrix} \sin(\omega t) \\ \sin(\omega t + 2\pi/3) \\ \sin(\omega t - 2\pi/3) \end{pmatrix} c(t)$$

c(t) = 0 for t < 0 and c(t) = 1 for t ≥ 0.

$$\begin{pmatrix} u_o \\ u_d \\ u_q \end{pmatrix} = \frac{2}{3} \begin{pmatrix} 1/2 & 1/2 & 1/2 \\ \cos(\omega t) & \cos(\omega t - 2\pi/3) & \cos(\omega t + 2\pi/3) \\ -\sin(\omega t) & -\sin(\omega t - 2\pi/3) & -\sin(\omega t + 2\pi/3) \end{pmatrix} \begin{pmatrix} u_a(t) \\ u_b(t) \\ u_c(t) \end{pmatrix}$$

$$= \sqrt{2}U \begin{pmatrix} 0 \\ 0 \\ c(t) \end{pmatrix} \rightarrow$$ DC voltage applied across q coil.

Eq. (10) :

$$0 = L_Q i_Q + L_{AQ} i_q \quad \Longrightarrow \quad i_Q = -\frac{L_{AQ}}{L_Q}\, i_q$$

Substituted in (7):

$$\psi_q = L_q i_q + L_{AQ}\, i_Q = \left(L_q - \frac{{L_{AQ}}^2}{L_Q} \right) . i_d$$

$\psi_d = L_q^{''} \; . i_q$ $\qquad L_q^{''}$: the q-axis **sub-transient** inductance

Synchronous machine inductances

- There is no field winding in the q-axis.
  - For a salient-pole machine with damper winding in the q-axis, the effective inductance after the current in the damper winding has decayed is practically equal to the synchronous inductance
    - ✓ $L'_q = L_q$ (The transient and synchronous inductances in the q-axis are equal)
- General relationship:

$$L''_d < L'_d < L_d$$
$$L''_q < L'_q = L_q$$

Synchronous machine open-circuit time constants (Tdo', Tdo", Tqo')

Assume:

all three stator windings are open-circuited and a step voltage is applied on the field winding, i.e. at t = 0, $u_F = U_F\, c(t)$

The result:

$$u_F = r_F i_F + \frac{d\psi_F}{dt} = U_F\, c(t)$$

$$0 = r_D i_D + \frac{d\psi_D}{dt}$$

Since $i_d = 0$ (open-circuit):

$$\psi_F = L_F i_F + L_{AD}(i_d + i_D) = L_F i_F + L_{AD} i_D$$

$$\psi_D = L_D i_D + L_{AD}(i_d + i_F) = L_D i_D + L_{AD} i_F$$

Synchronous machine open-circuit time constants (Tdo', Tdo'', Tqo')

- Full model (3) – (5) plus the mechanical equations (swing equation):

$$u_F = r_F i_F + \frac{d\psi_F}{dt} \quad (3)$$

$$0 = r_D i_D + \frac{d\psi_D}{dt} \quad (4)$$

$$0 = r_Q i_Q + \frac{d\psi_Q}{dt} \quad (5)$$

- 4th Order model (3), (5) plus the mechanical equations
- Eliminate all parameters from the differential equations except T''_{d0}, T'_{d0}, T''_{q0}, x''_q, x''_d, x'_d

Synchronous machine open-circuit time constants (Tdo', Tdo'', Tqo')

After the transients in the damper winding D have died down, i.e. i_D = 0, the flux linkage is solely determined by the field current. Thus,

$$T'_{do} = \frac{L_F}{r_F}$$

For a synchronous machine with a damper winding in the q-axis:

$$T''_{q0} = \frac{L_Q}{r_Q}$$

4^{th} Order model of the synchronous machine

q-axis

$$\psi_q = L_q i_q + L_{AQ}\, i_Q \qquad (7)$$

$$\psi_Q = L_Q i_Q + L_{AQ} i_q \qquad (10)$$

$$\psi_Q = L_Q i_Q + L_{AQ} i_q \rightarrow \quad i_Q = \frac{\psi_Q - L_{AQ} i_q}{L_Q} \rightarrow \qquad \psi_q = L_q i_q + L_{AQ}\, i_Q \rightarrow \psi_q = L_q i_q + L_{AQ} \frac{\psi_Q - L_{AQ} i_q}{L_Q} \rightarrow$$

$$\psi_q - \frac{L_{AQ}}{L_Q}\psi_Q = i_q\left(L_q - \frac{{L_{AQ}}^2}{L_Q}\right) = i_q L_q''$$

$$L_q'' = \left(L_q - \frac{{L_{AQ}}^2}{L_Q}\right)$$

Define: $e_d' = \omega \frac{L_{AQ}}{L_Q}\psi_Q \rightarrow$

$$\psi_q - \frac{e_d'}{\omega} = i_q L_q'' \rightarrow \omega\, \psi_q = e_d' + i_q\, x_q''$$

$$\frac{de_d'}{dt} = \omega \frac{L_{AQ}}{L_Q}\frac{d\psi_Q}{dt} \qquad \text{(assuming } \frac{d\omega}{dt} = 0\text{)}$$

Re-writing (7):

$$x_q i_q + \omega\, L_{AQ} i_Q - e_d' = i_q\, x_q'' \rightarrow \quad i_Q = \frac{e_d' - (x_q - x_q'')\, i_q}{\omega L_{AQ}}$$

$$0 = r_Q i_Q + \frac{d\psi_Q}{dt} \qquad (5)$$

$$r_Q i_Q + \frac{d\psi_Q}{dt} = 0 \rightarrow r_Q \frac{e_d' - (x_q - x_q'')\, i_q}{\omega L_{AQ}} + \frac{L_Q}{\omega L_{AQ}}\frac{de_d'}{dt} = 0$$

$$T_{q0}''\cdot\frac{de_d'}{dt} + e_d' - (x_q - x_q'')\, i_q = 0$$

$$T_{q0}''\cdot\frac{de_d'}{dt} + e_d' - (x_q - x_q'')\, i_q = 0$$

$$T_{q0}'' = \frac{L_Q}{r_Q}$$

d-axis

$$\psi_d = L_d i_d + L_{AD}(i_F + i_D) \quad \textbf{(6)}$$

$$\psi_F = L_F i_F + L_{AD}(i_d + i_D) \quad \textbf{(8)}$$

$$\psi_F = L_F i_F + L_{AD} i_d \quad \rightarrow \quad i_F = \frac{\psi_F - L_{AD} i_d}{L_F}$$

(i_D= 0 in transient phase)

$$\psi_d = L_d i_d + L_{AD} i_F = L_{AD} \frac{\psi_F - L_{AD} i_d}{L_F}$$

$$\psi_d - \frac{L_{AD}\psi_F}{L_F} = \left(L_d - \frac{L_{AD}^2}{L_F}\right) i_d = L'_d\, i_d$$

With $L'_d = \left(L_d - \frac{L_{AD}^2}{L_F}\right)$

Define: $e'_q = -\omega \frac{L_{AD}}{L_F}\psi_F \rightarrow \quad \psi_d - \frac{L_{AD}\psi_F}{L_F} = L'_d\, i_d \quad \rightarrow$

$$\boldsymbol{\psi_d + \frac{e'_q}{\omega} = i_d L'_d \rightarrow \omega\psi_d = -e'_q + i_d x'_d}$$

Substitute in (6): $x_d i_d + \omega L_{AD} i_F + e'_q = i_d x'_d$

$$\boldsymbol{i_F = \frac{-e'_q - (x_d - x'_d)\, i_d}{\omega L_{AD}}}$$

$$\frac{de'_q}{dt} = -\omega \frac{L_{AD}}{L_F} \frac{d\psi_F}{dt} \quad \textbf{(assuming } \frac{d\omega}{dt} = 0\textbf{)}$$

$$u_F = r_F i_F + \frac{d\psi_F}{dt} \quad (3)$$

$$u_F = -r_F \frac{e'_q + (x_d - x'_d)\, i_d}{\omega L_{AD}} - \frac{L_F}{\omega L_{AD}} \frac{de'_q}{dt} \rightarrow \quad \frac{\omega L_{AD} u_F}{r_F} = -e'_q - (x_d - x'_d)\, i_d - T'_{do} \frac{de'_q}{dt}$$

$$T'_{do} = \frac{L_F}{r_F}$$

With $e_F = \frac{\omega L_{AD} u_F}{r_F}$

$$\boldsymbol{T'_{do} \frac{de'_q}{dt} + e'_q + (x_d - x'_d)\, i_d = -e_F}$$

Synchronous machine capability curve

Operational limits of synchronous generator

- The loading limits of a synchronous generator are determined by its size and construction, i. e, by:
  - Cooling of the rotor and stator windings,
  - Iron saturation limit,
  - The capacity of the excitation system, etc.
- These technical limits must be observed during operation at all times.
- The operational chart of the synchronous machine shows these boundaries

Operational chart of a synchronous machine

1. Limit: Maximum armature current

- The maximum apparent power S is the limit with regard to stator heating
  - This limit is given by the equation:

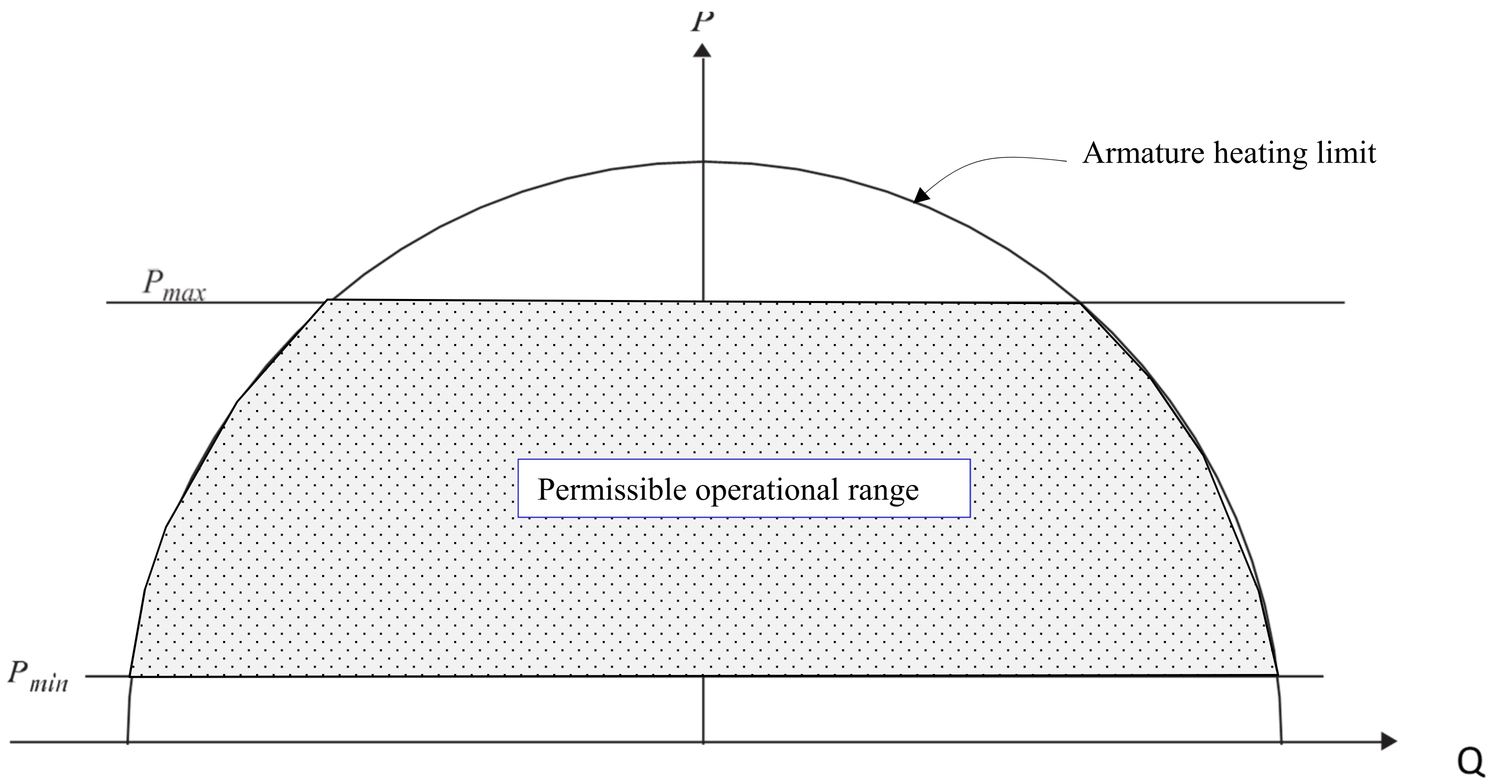
$$S^2 = P^2 + Q^2 = konst.$$

Minimum/maximum active power limit

2. **<u>Limit: minimum and maximum active power loading</u>**

- For thermal power plants, these limits (Pmin, Pmax) are determined by the minimum and maximum steam flow through the turbine
- For hydropower generators: Pmin = 0, and Pmax determined by the maximum flow rate of the water.
    - The minimum and maximum deliverable active power, Pmin and Pmax, are shown as horizontal lines in the operational chart

Active power / armature heating limits



Under excitation limit

3. Limit: Steady state under-excitation limit

$$\underline{I}_a = \frac{\underline{U}_p - U}{jX_d} = \frac{U_p\angle\delta - U}{jX_d} = \frac{U_p\angle\delta - \pi/2 + jU}{X_d}$$

Up: excitation voltage
U: terminal voltage

$$\underline{S} = 3\cdot U\cdot \underline{I}_a^* = 3\cdot U\cdot \frac{U_p\angle -\delta + (\pi/2) - jU}{X_d} = \frac{3\cdot U\cdot U_p\cdot\sin\delta}{X_d} + j\frac{3\cdot U\cdot U_p\cdot\cos\delta - U^2}{X_d}$$

$$P = \frac{3\cdot U\cdot U_p\cdot\sin\delta}{X_d} = \omega_m\cdot M \qquad Q = \frac{3\cdot U\cdot U_p\cdot\cos\delta - U^2}{X_d}$$

- The theoretical stability limit is δ = 90°.
  - To allow a saftey margin, however, a maximum power angle is limited to δ_{max} = 70° (for example).

Stability limit

$$\sin\delta = \frac{P \cdot X_d}{3 \cdot U \cdot U_p}$$

$$\cos\delta = \frac{Q \cdot X_d + U^2}{3 \cdot U \cdot U_p}$$

$$\tan\delta = \frac{P \cdot X_d}{Q \cdot X_d + U^2} \rightarrow P = Q \cdot \tan\delta + \frac{U^2}{X_d} \cdot \tan\delta$$

The stability limit in a Q – P plane (assuming δ_{max} = 70°) is:

- a straight line with the slope: $\tan 70° = 2.75$
- X-intercept: $P = 0 \rightarrow Q = -\frac{U^2}{X_d}$

Minimum/maximum excitation

4. Limit: Maximum allowable rotor heating ($U_{P,max}$)

- Most generators are equipped with AVR
- By changing the excitation current, the excitation voltage and thus also the terminal voltage of the generator can be controlled.
- The voltage regulation specifies a minimum excitation voltage $U_{p,min}$.
- The maximum excitation voltage $U_{p,max}$ is predetermined by the maximum permissible heating of the rotor winding.
  - These two limits can be represented in the operating diagram by two circles.

$$\sin\delta = \frac{P \cdot X_d}{3 \cdot U \cdot U_p} \qquad \cos\delta = \frac{Q \cdot X_d + U^2}{3 \cdot U \cdot U_p}$$

$$\left(\frac{3 \cdot U \cdot U_p}{X_d}\right)^2 = P^2 + \left(Q + \frac{U^2}{X_d}\right)^2$$

Maximum/maximum excitation

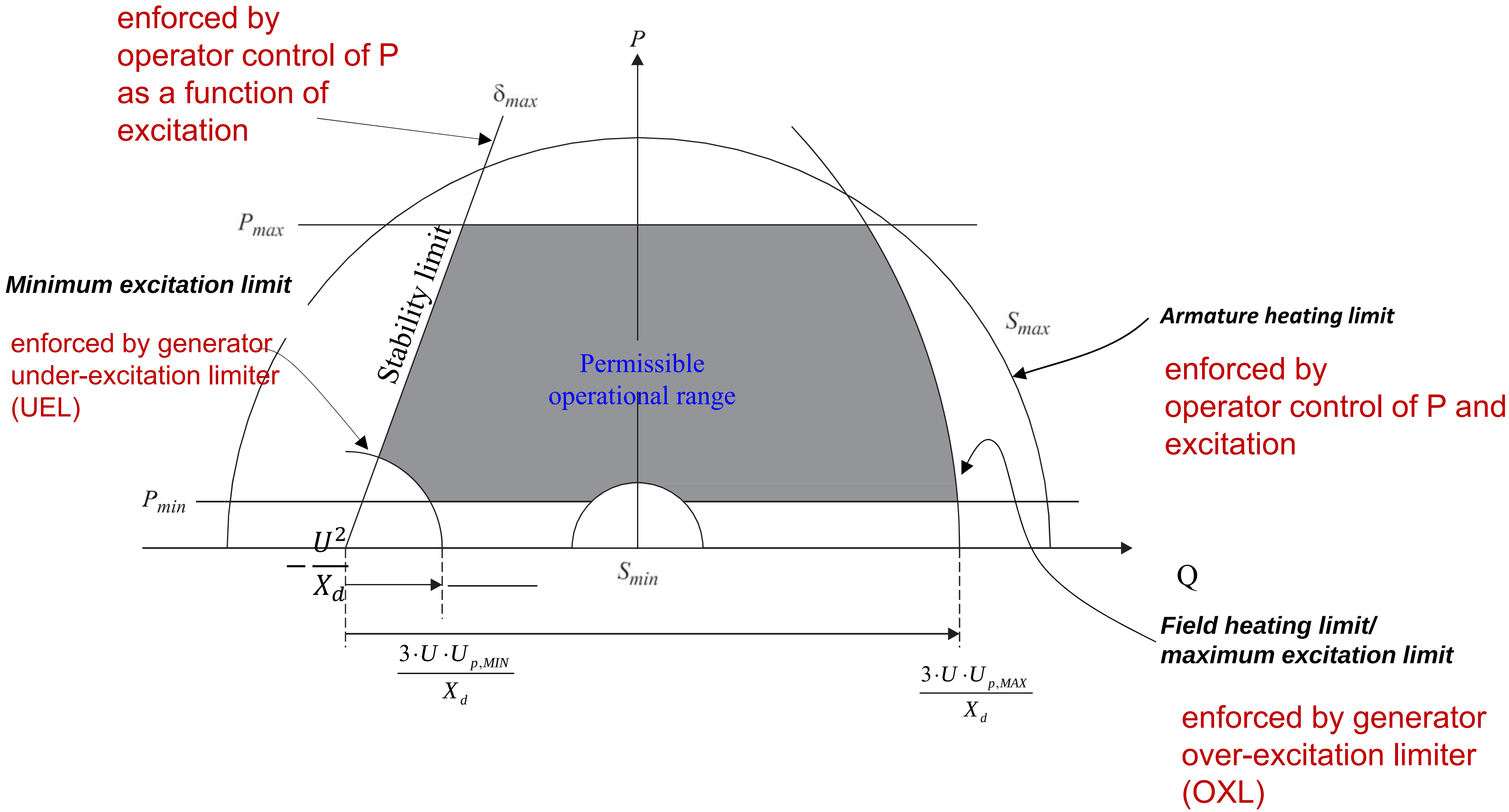
Locus:

$$\left(\frac{3\cdot U\cdot U_p}{X_d}\right)^2 = P^2 + \left(Q+\frac{U^2}{X_d}\right)^2$$

Equation of a circle with origin: $\left(-\frac{U^2}{X_d},0\right)$ and radius: $\frac{3\cdot U\cdot U_p}{X_d}$

Minimum excitation $\frac{3\cdot U\cdot U_{p,MIN}}{X_d}$ Maximum excitation: $\frac{3\cdot U\cdot U_{p,MAX}}{X_d}$

Operational diagram



Effect of generator reactive power limit

- Voltage instability is typically preceded by generators hitting their upper reactive limit (OXL)
  - Accurate modeling of Q_{max} is very important for the analysis of voltage instability

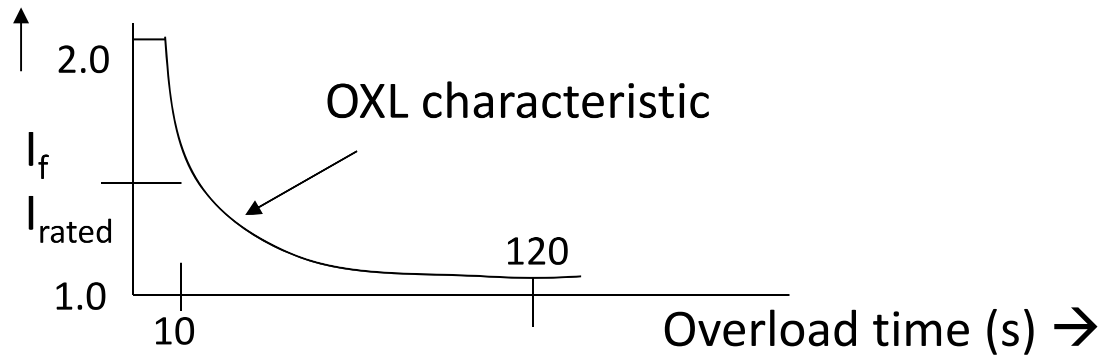
- Most power flow programs represent generator Q_{max} as fixed.
  - However, this is an approximation. In reality, Q_{max} is not fixed. The generator capability diagram shows quite clearly that Q_{max} is a function of P and becomes more restrictive as P increases.
  - A first-order approximation(instead of using fixed Q_{max}) is to model Q_{max} as a function of P.

Effect of generator reactive power limit

- Q_{max} is limited by the Over-eXcitation Limiter (OXL). The field circuit has a rated steady-state field current $I_{f\text{-}max}$, set by field circuit heating limitations. Since heating is proportional to:

$$\int_{\text{overload time}} I_f^{\,2}\,dt$$

- Small overloads can be tolerated for longer times. Therefore, most modern OXLs are set with a time-inverse characteristic:



- As soon as the OXL acts to limit I_f, then no further increase in reactive power is possible.
- When drawing PV or QV curves, the action of a generator hitting $Q_{max,}$ will manifest itself as a sharp discontinuity in the curve.

Basic countermeasures

- Basic strategy:
  - Apply shunt capacitor banks, mainly in distribution and load area transmission substations to minimize reactive power transmission, allowing automatically controlled reactive power reserve at generators
  - Design and operate transmission network for high, flat voltage profile to minimize I^2X losses
- Switched shunt capacitor banks:
  - Local or wide-area control
- Series capacitor banks
- Static var compensators or STATCOMs for short-term voltage stability:
- Load shedding:
  - Local undervoltage or wide-area load shedding

Torque

$$t_e \approx p_e = u_d i_d + u_q i_q = -e'_d i_d - e'_q i_q + i_q i_d (x'_d - x''_q) = -e'_q i_q + i_q i_d (x'_d - x''_q)$$

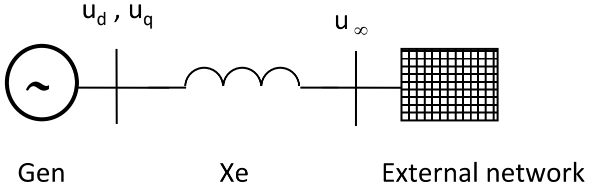
$$x''_q \rightarrow x_q$$

$$\begin{pmatrix} u_d \\ u_q \end{pmatrix} = -\begin{pmatrix} 0 \\ e'_q \end{pmatrix} + \begin{pmatrix} r_a & -x_q \\ x'_d & r_a \end{pmatrix} \cdot \begin{pmatrix} i_d \\ i_q \end{pmatrix} \qquad \begin{pmatrix} u_{dN} \\ u_{qN} \end{pmatrix} = -\begin{pmatrix} 0 \\ e'_q \end{pmatrix} + \begin{pmatrix} r_a & -(x_q + x_e) \\ x'_d + x_e & r_a \end{pmatrix} \cdot \begin{pmatrix} i_d \\ i_q \end{pmatrix}$$

Incorporating the network

$t_e \approx p_e = u_d i_d + u_q i_q$ = $-e'_d i_d$ - $e'_q i_q + \boldsymbol{i_q i_d}(\boldsymbol{x'_d} - \boldsymbol{x''_q})$ = - $\boldsymbol{e'_q} i_q + \boldsymbol{i_q i_d}(\boldsymbol{x'_d} - \boldsymbol{x''_q})$

$\boldsymbol{x''_q} \rightarrow x_q$



$$\begin{pmatrix} u_d \\ u_q \end{pmatrix} = -\begin{pmatrix} 0 \\ e'_q \end{pmatrix} + \begin{pmatrix} r_a & -x_q \\ x'_d & r_a \end{pmatrix} \cdot \begin{pmatrix} i_d \\ i_q \end{pmatrix}$$

$$\begin{pmatrix} u_{dN} \\ u_{qN} \end{pmatrix} = -\begin{pmatrix} 0 \\ e'_q \end{pmatrix} + \begin{pmatrix} r_a & -(x_q + x_e) \\ x'_d + x_e & r_a \end{pmatrix} \cdot \begin{pmatrix} i_d \\ i_q \end{pmatrix}$$

Fourth Order Model

$$T''_{q0} \cdot \frac{de'_d}{dt} + e'_d - (x_q - x''_q)\, i_q = 0$$

$$T'_{do}\ \frac{de'_q}{dt} + e'_q + (x_d - x'_d)\, i_d = \ -e_F$$

$$\frac{d\delta}{dt} = \omega - \ \omega_0$$

$$\frac{d\omega}{dt} = \frac{\pi\, f}{H}(p_m - D\,\omega - p_e)$$

$$t_e \approx p_e = u_d i_d + u_q i_q = -e'_d i_d - e'_q i_q + i_q i_d (x'_d - x''_q)$$