TRAVELING WAVE

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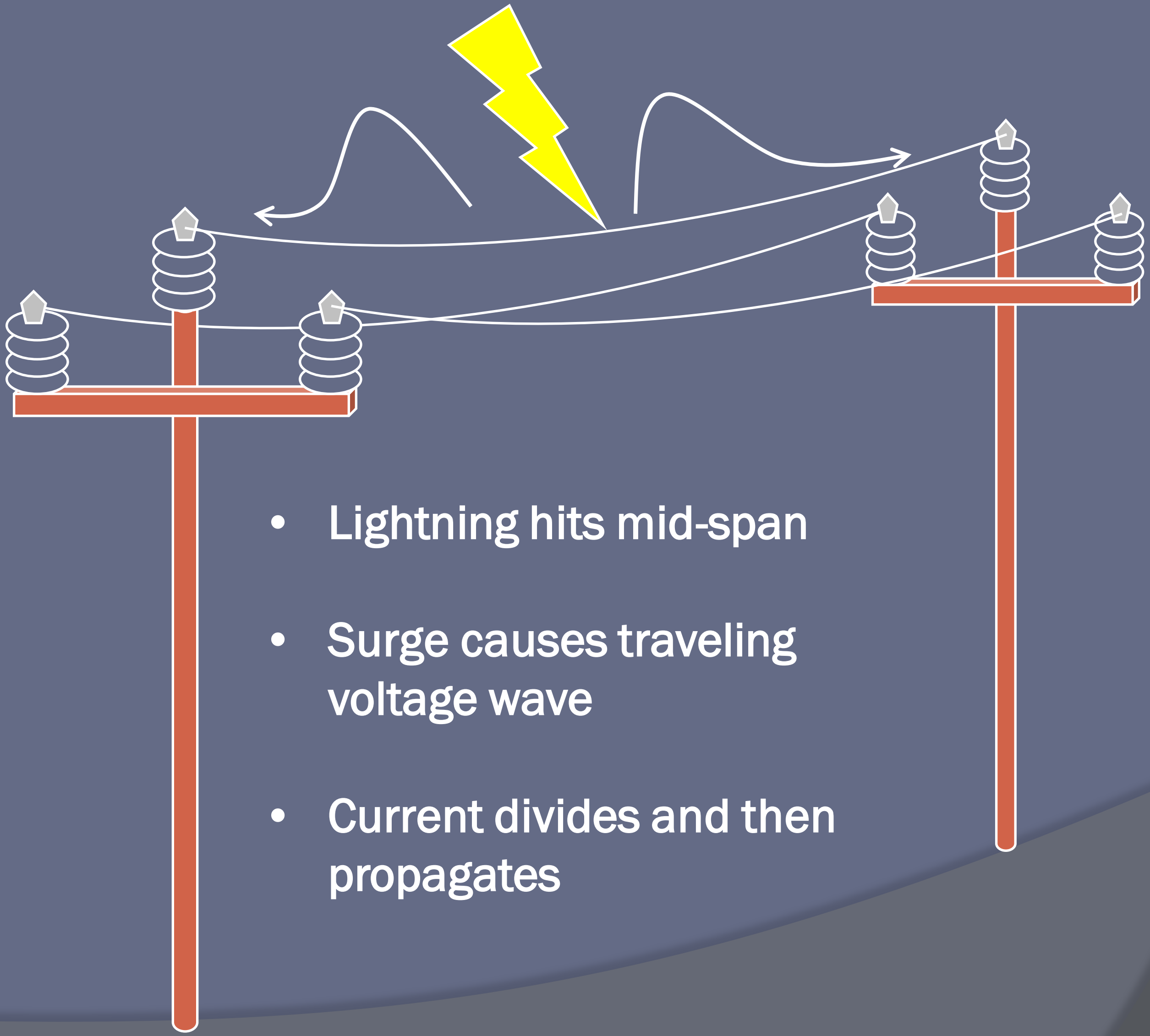
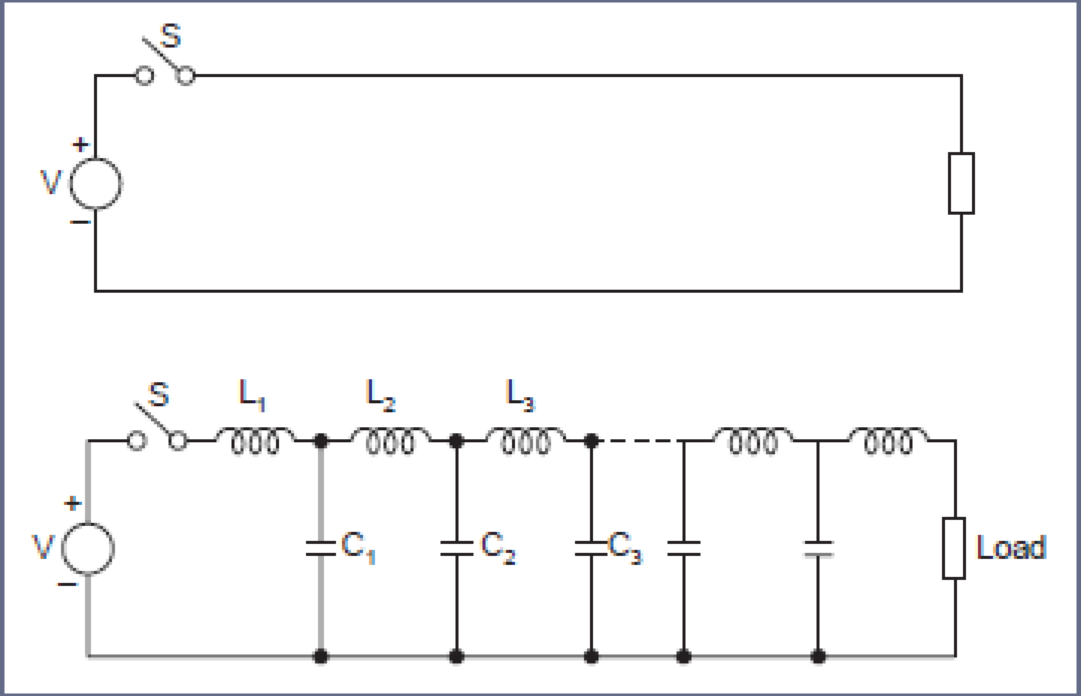
Traveling Wave



- Lightning hits mid-span
- Surge causes traveling voltage wave
- Current divides and then propagates

Traveling Wave

- **Surge**



$$\frac{\partial^2 v}{\partial t^2} - \frac{1}{LC}\frac{\partial^2 v}{\partial x^2} = 0$$

Wave Equation

Traveling Wave

- Disturbance represented by closing or opening the switch S.
- If Switch S closed, the line suddenly connected to the source. The first capacitor becomes charged immediately and the next inductor and so on.
- This gradual buildup of voltage over the line conductor can be regarded as a voltage wave is traveling from one end to the other end

Traveling Wave

- Suppose that the wave after time t has travelled through a distance x.
- Consider a distance *dx* which is travelled by the waves in time *dt*. The electrostatic flux which is equal to the charge between the conductors of the line upto a distance *x* is given by:

$$q = VCx$$

- The current in the conductor is determined by the rate at which the charge flows into and out of the line.

$$I = \frac{dq}{dt} = VC\frac{dx}{dt}$$

Traveling Wave

- Here *dx/dt* is the velocity of the travelling wave over the line conductor and let this be represented by *v*. Then

$$I = VCv$$

Traveling Wave

- Similarly the electromagnetic flux linkages created around the conductors due to the current flowing in them upto a distance of *x* is given by

$$\Phi = I\,L\,x$$

- The voltage is the rate at which the flux linkages link around the conductor

$$V = IL\frac{dx}{dt} = ILv$$

- Dividing equation (*) by (**), we get

$$V = ILx \qquad I = VCv$$

$$\frac{V}{I} = \frac{ILv}{VCv} = \frac{IL}{VC}$$

$$\frac{V^2}{I^2} = \frac{L}{C}$$

$$\frac{V}{I} = \sqrt{\frac{L}{C}} = Z_n$$

- Z_n=surge impedance of the line.

Traveling Wave

- It is also known as the natural impedance because this impedance has nothing to do with the load impedance. It is purely a characteristic of the transmission line.
- The value of this impedance is about 400 ohms for overhead transmission lines and 40 ohms for cables.

Traveling Wave

- Next, multiplying equations (*) with (**), we get

$$V = IL\frac{dx}{dt} = ILv$$

$$I = VCv$$

$$VI = VCv\, ILv = VILCv^2$$

$$v^2 = \frac{1}{LC}$$

$$v = \frac{1}{\sqrt{LC}}$$

- Now expressions for *L* and *C* for overhead lines are

$$L = 2 \times 10^{-7} \ln\frac{d}{r} \text{ H/metre}$$

$$C = \frac{2\pi\varepsilon}{\ln\frac{d}{r}} \text{ F/metre}$$

Traveling Wave

- Substituting these values in the above equation, the velocity of propagation of the wave

$$v = \frac{1}{\left(2\times 10^{-7}\ln\frac{d}{r}\frac{2\pi\varepsilon}{\ln d/r}\right)^{1/2}}$$

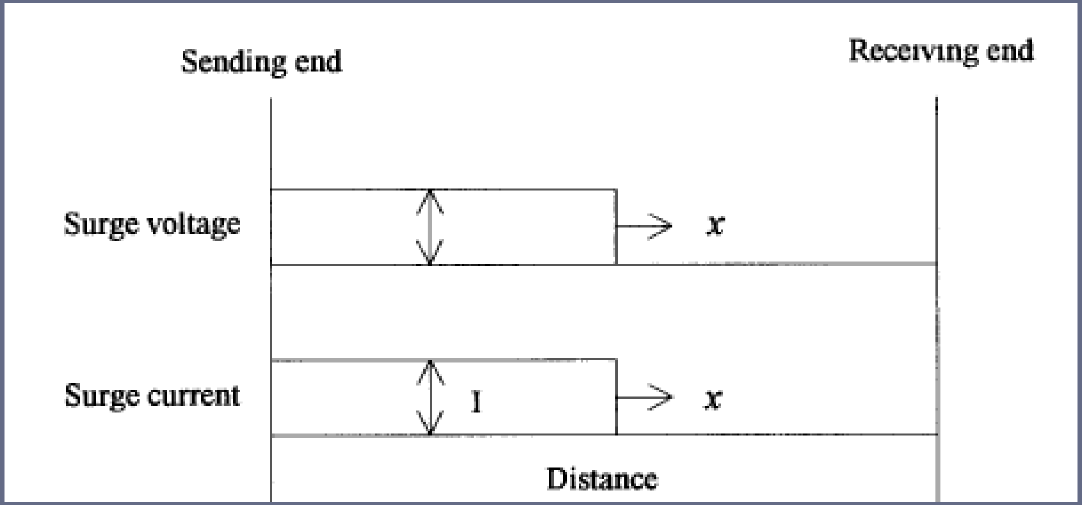
$$= \frac{1}{\sqrt{4\pi\varepsilon\,10^{-7}}} = \frac{1}{\sqrt{4\pi\frac{1}{36\pi}\times 10^{-9}\times 10^{-7}}}$$

$$= 3\times 10^{8}\text{ metres/sec.}$$

- It can be seen from the expression that the velocity of these waves over the cables will be smaller than over the overhead lines because of the permittivity term in the denominator.

Traveling Wave

- **Surge**

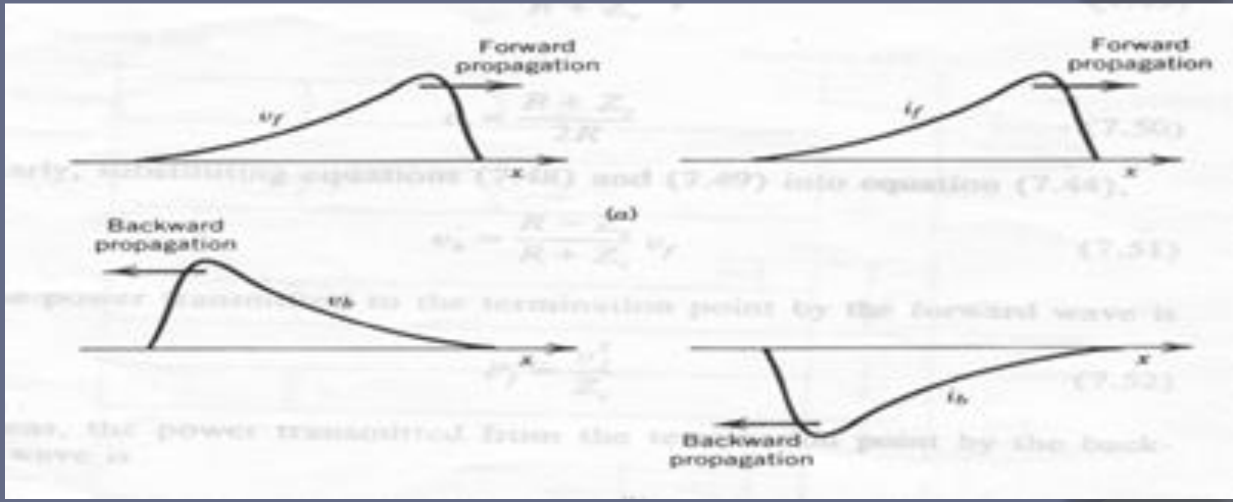


$$\frac{\partial^2 v}{\partial t^2} - \frac{1}{LC}\frac{\partial^2 v}{\partial x^2} = 0$$

Wave Equation

Traveling Wave

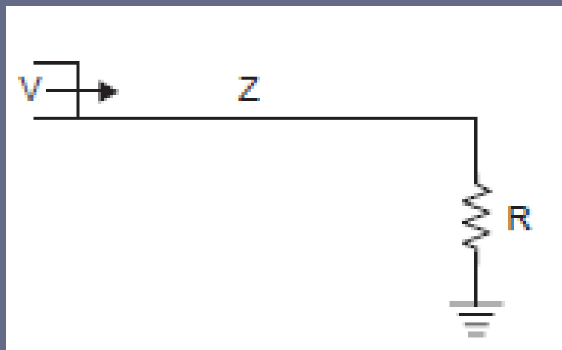
- $v(x,t)=v_f + v_b$
- $v_f=v_1(x-\upsilon t)$
- $v_b=v_2(x+\upsilon t)$
- $\upsilon = 1/ \sqrt{(LC)}$
- $v_f=Z_n i_f$
- $v_b=Z_n i_b$



$F(x)= x+1$
$F(x)= (x-2)+1$

Traveling Wave

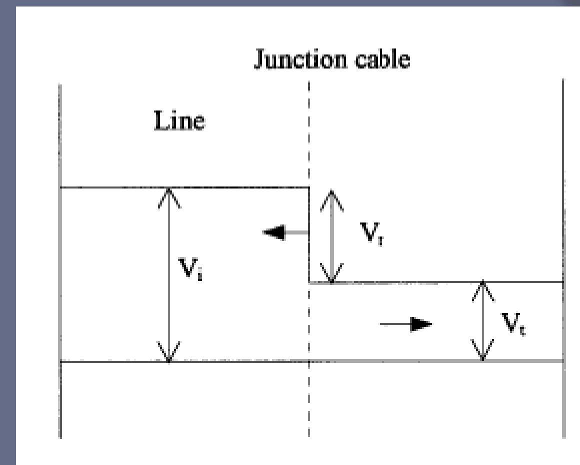
- **Surge reflection and refraction**

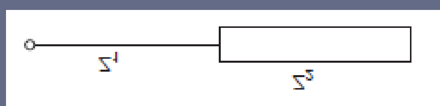


$$I = \frac{V}{Z}$$

$$I' = -\frac{V'}{Z}$$

$$I'' = \frac{V''}{Z}$$





Refracted or transmitted wave = Incident wave + Reflected wave

- Let *V''* and *I''* be the refracted voltage and current waves into the resistor *R* when the incident waves *V* and *I* reach the resistance *R*.

Traveling Wave

- **Surge reflection and refraction**

- Since $I'' = I + I'$ and $V'' = V + V'$, using these relations, we have:

$$\frac{V''}{R} = \frac{V}{Z} - \frac{V'}{Z} = \frac{V}{Z} - \frac{V'' - V}{Z} = \frac{2V}{Z} - \frac{V''}{Z}$$

$$V'' = \frac{2VR}{Z + R}$$

$$I'' = \frac{2V}{R + Z} = \frac{V}{Z} \cdot \frac{2Z}{R + Z} = I \cdot \frac{2Z}{R + Z}$$

- The coefficient of refraction b

$$= \frac{2Z}{R + Z}$$

Traveling Wave

- **Surge reflection and refraction**

- Similarly substituting for *V″* in terms of (*V* + *V′*), in the above equation:

$$\frac{V+V'}{R} = \frac{V}{Z} - \frac{V'}{Z}$$

$$V' = V\frac{R-Z}{R+Z}$$

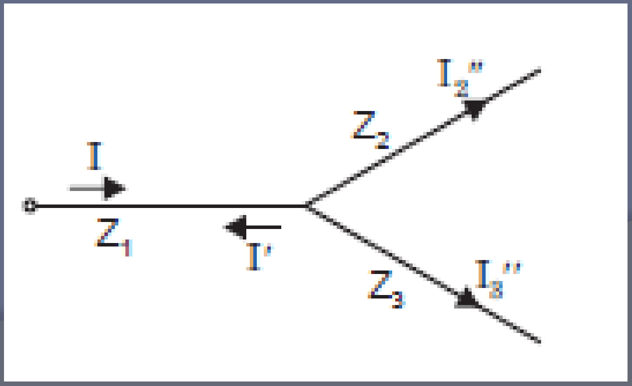
$$I' = -\frac{V'}{Z} = -\frac{V}{Z}\frac{(R-Z)}{R+Z}$$

- The coefficient of reflection for voltage a

$$= + \frac{R-Z}{R+Z}$$

Traveling Wave

- **Reflection and Refraction at a T-junction**
- A voltage wave V is travelling over the line with surge impedance Z_1. When it reaches the junction, it looks a change in impedance and, therefore, suffers reflection and refraction.
- Let V_2'', I_2'' and V_3'', I_3'' be the voltages and currents in the lines having surge impedances Z_2 and Z_3 respectively. Since Z_2 and Z_3 form a parallel path as far as the surge wave is concerned, $V_2'' = V_3'' = V''$.



Traveling Wave

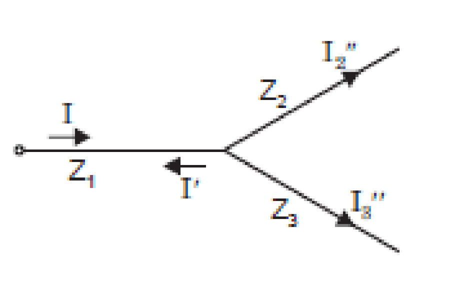
- **Reflection and Refraction at a T-junction**

$$V + V' = V''$$

$$I = \frac{V}{Z_1},\ I' = -\frac{V'}{Z_1}$$

$$I_2'' = \frac{V''}{Z_2} \quad \text{and} \quad I_3'' = \frac{V''}{Z_3}$$

$$I + I' = I_2'' + I_3''$$



- Substituting in equation (*) the values of currents

$$\frac{V}{Z_1} - \frac{V'}{Z_1} = \frac{V''}{Z_2} + \frac{V''}{Z_3}$$

Substituting for $V' = V'' - V$,

$$\frac{V}{Z_1} - \frac{V'' - V}{Z_1} = \frac{V''}{Z_2} + \frac{V''}{Z_3}$$

$$\frac{2V}{Z_1} = V''\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right)$$

$$V'' = \frac{2V/Z_1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}}$$

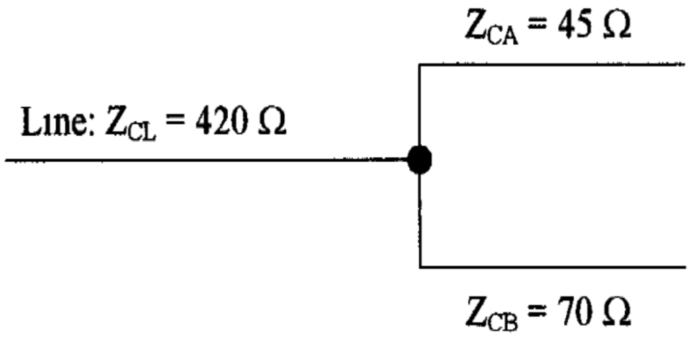
Traveling Wave

- **Surge reflection and refraction**
- **Exercise:** A *3-phase transmission line has conductors 1.5 cm in diameter spaced 1 meter apart in equilateral formation. The resistance and leakage are negligible. Calculate (i) the natural impedance of the line, (ii) the reflected and refracted voltage wave if 11 kV travels along the line travels in to a cable with the inductance and capacitance per phase per cm of* 0.5×10^{-8} *H and* 1×10^{-6} *F respectively.*

Traveling Wave

An overhead transmission line, which has a surge impedance of 420 Ω, is at one end connected to two cables which have surge impedances of 45 Ω and 70 Ω respectively A surge of 1 MV moves along the line to the junction. Calculate the magnitudes of the reflected and the transmitted voltages and currents at the junction.

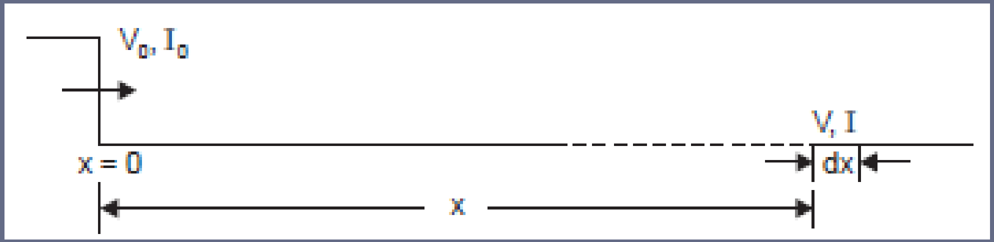
Solution:



Traveling Wave

Attenuation of Travelling Waves

- Let *R*, *L*, *C* and *G* be the resistance, inductance, capacitance and conductance respectively per unit length of a line.
- Let the value of voltage and current waves at $x = 0$ be V_0 and I_0.



Traveling Wave

Attenuation of Travelling Waves

- The power loss in the differential element is:

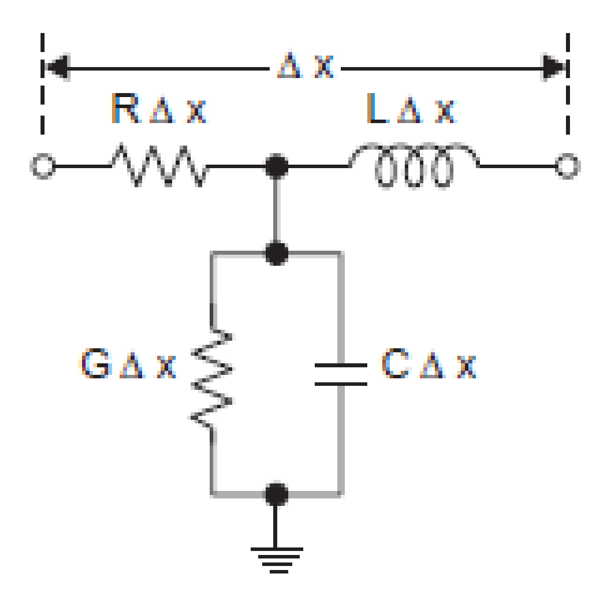
$$dp = I^2R\,dx + V^2G\,dx$$

- Also power at a distance *x*

$$VI = p = I^2Z_n$$

- Differential power

$$dp = -2IZ_n\,dI$$



Traveling Wave

Attenuation of Travelling Waves

- Equating the equation (*) and (**)

$$dp = -2IZ_n dI \qquad dp = I^2R\,dx + V^2G\,dx$$

$$\begin{aligned} -2IZ_n dI &= I^2Rdx + V^2Gdx \\ &= I^2Rdx + I^2Z_n{}^2Gdx \end{aligned}$$

$$dI = -\frac{I(R + GZ_n^2)}{2Z_n}dx$$

$$\frac{dI}{I} = -\frac{(R + GZ_n^2)}{2Z_n}dx$$

$$\ln I = -\frac{(R + GZ_n^2)}{2Z_n}x + A$$

Traveling Wave

Attenuation of Travelling Waves

At $x = 0$, $I = I_0$, $\therefore \quad A = \ln I_0$

$$\ln \frac{I}{I_0} = -\frac{R + GZ_n^2}{2Z_n} x = -\alpha x \text{ (say)}$$

$$\alpha = \frac{R + GZ_n^2}{2Z_n}.$$

$$I = I_0 e^{-\alpha x}.$$

- Similarly it can be proved that $V = V_0 e^{-\alpha x}.$