Chapter 3: Characterization of Communication Signals and Systems

Addis Ababa Institute of Technology
Addis Ababa University

Graduate Program
School of Electrical and Computer Engineering

Overview

- Pulse amplitude modulation
- Phase modulation
- Quadrature amplitude modulation
- Multidimensional signals
- Biorthogonal signal

Linearly Modulated Digital Signals

- Linear digitally modulated signals are expanded in terms of two orthonormal basis functions of the form

$$f_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_c t \qquad and \qquad f_2(t) = \sqrt{\frac{2}{T}} \sin 2\pi f_c t$$

- If the low-frequency representation is desired

$$S_{lm} = x_l(t) + jy_l(t) \quad then \quad S_m(t) = x_l(t) f_1(t) + y_l(t) f_2(t)$$

where $x_l(t)$ and $y_l(t)$ represents the modulating signal

- Modulator maps blocks of $k=\log_2 M$ binary digits at a time from the information sequence $\{a_k\}$ and selecting one of $M=2^k$ deterministic and finite energy waveform $\{S_m(t), m=1,2,\ldots..M\}$ for transmission over the channel

Memoryless Modulation – PAM Signals

- Assume that the sequence of binary digits at the input of the modulator occurs at the rate of R bits/s
- In pulse-amplitude modulation (PAM) signals

$$S_m(t) = Re\left[A_m g(t) e^{j2\pi f_c t}\right] = A_m g(t) \cos 2\pi f_c t \quad m = 1,2,\ldots M$$

- $0 \le t \le T$ and $\{A_m,\ 1 \le m \le M\}$ denotes the M possible amplitudes corresponding to $M=2^k$ possible k-bit blocks or symbols
- A_m takes discrete values or levels, $A_m = (2m-1-M)d$ where 2d is the distance between two adjacent signal amplitudes
- g(t) is a real valued signal pulse whose shape influences the spectrum of the transmitted signal

Memoryless Modulation – PAM Signals …

- Symbol rate for PAM signals is R/k, the rate at which changes occur in the amplitude of the carrier
  - Bit interval $T_b = 1/R$ and Symbol interval $T = k/R = kT_b$
- PAM signals have energies

$$\varepsilon_m = \int_0^T S_m^2(t)\,dt = \frac{1}{2}A_m^2\int_0^T g^2(t)\,dt = \frac{1}{2}A_m^2\varepsilon_g$$

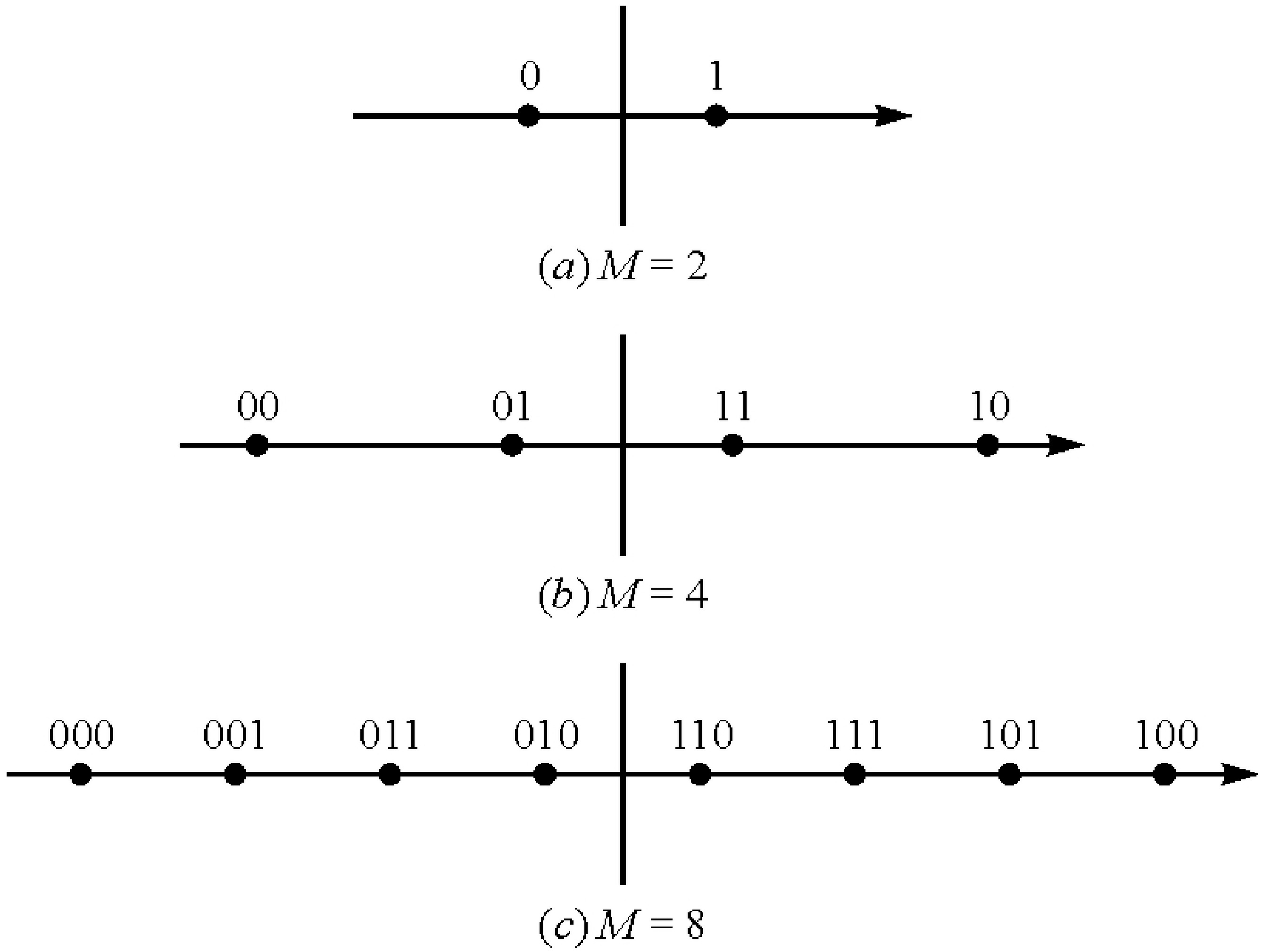
$$\varepsilon_g - \text{Energy of the pulse g(t)}$$

- Note that these signals are one dimensional (N=1) and hence can be represented by the general form

$$S(t) = S_m f(t)$$

- Where $f(t) = \sqrt{\frac{2}{\varepsilon_g}}\,g(t)\cos 2\pi f_c t$ and $S_m = A_m\sqrt{\frac{\varepsilon_g}{2}} \quad m = 1,2,\ldots\ldots.M$

Memoryless Modulation – PAM Signals …



Signal space diagram for digital PAM for M=2, 4, and 8

Memoryless Modulation – PAM Signals …

- The preferred assignment of k information bits to the $M=2^k$ possible signal amplitudes is one in which the adjacent signal amplitudes differ by only one binary digit (Gray Encoding)

- Note the Euclidean distance between any pair of signal points is

$$d_{mn}^{e} = \sqrt{(S_m - S_n)^2} = \sqrt{\frac{1}{2}\varepsilon_g}\,|A_m - A_n| = d\sqrt{2\varepsilon_g}\,|m-n|$$

- The minimum distance between a pair of adjacent signal point occurs when

$$|m-n| = 1; \qquad d_{\min}^{e} = d\sqrt{2\varepsilon_g}$$

Memoryless Modulation – PAM Signals …

- The above DSB signal requires a bandwidth $BW=2BW_{LP}$
- Alternatively, one may use SSB whose representation is

$$S_m = \mathrm{Re}\{A_m \left[g(t) \pm j\hat{g}(t) \right] e^{j2\pi f_c t}\}; \quad m = 1,2,\ldots.M$$

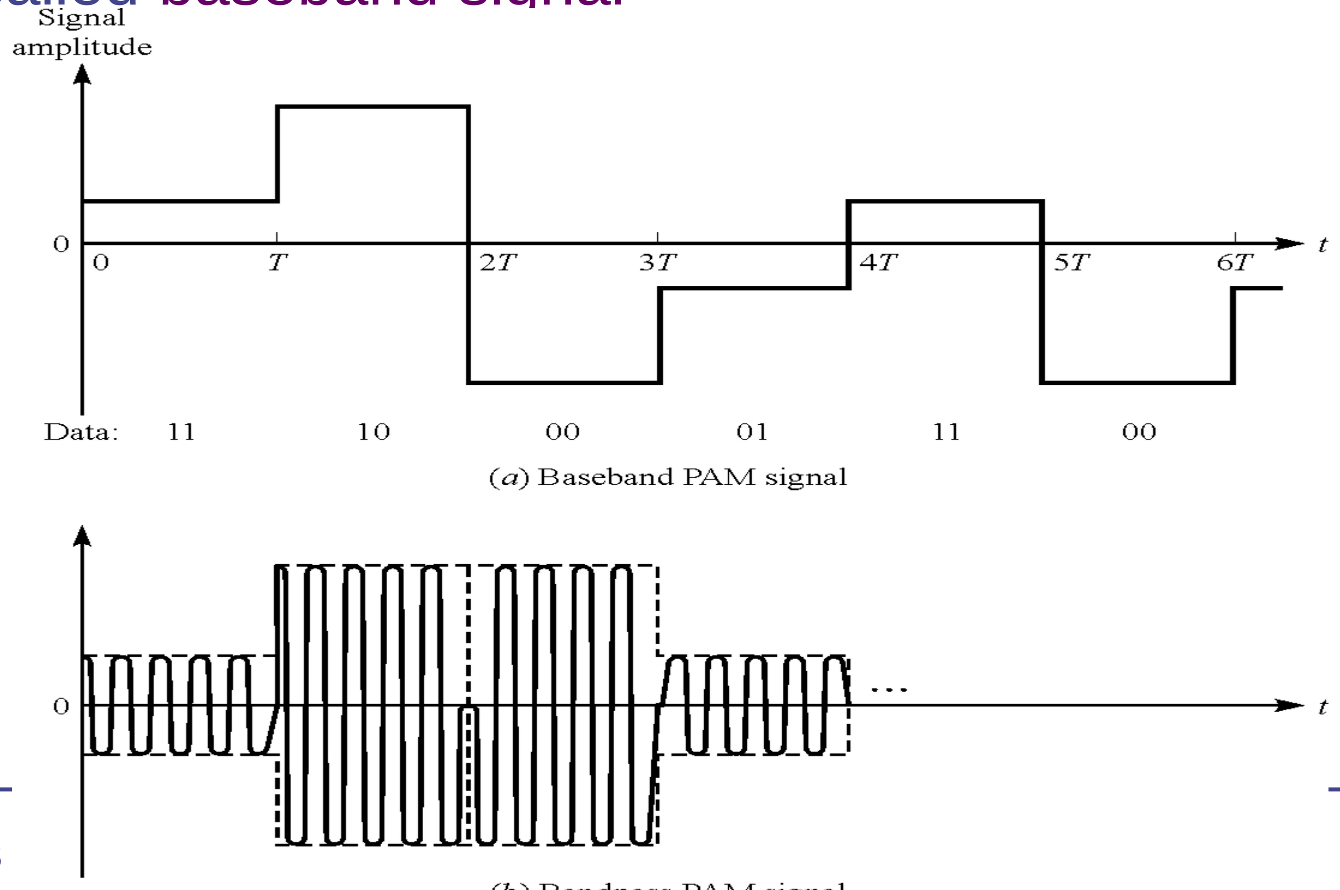
- Where $\hat{g}(t)$ is the Hilbert transform of $g(t)$
- The BW of SSB signal is half that of DSB signal

Memoryless Modulation – PAM Signals …

- For transmission over channels that does not require carrier modulation

$$S_m(t) = A_m g(t), \qquad m = 1,2,\ldots,M$$

- This is called baseband signal



(a) Baseband PAM signal

(b) Bandpass PAM signal

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Memoryless Modulation – Phase Modulated Signals

- In digital phase modulation, the M signal waveform are

$$S_m(t) = \mathrm{Re}\left[g(t) e^{j2\pi(\frac{m-1}{M})} e^{j2\pi f_c t} \right]; \quad m = 1,2,\ldots M.$$

$$= g(t)\cos\left[2\pi f_c t + \frac{2\pi}{M}(m-1) \right]; \quad 0 \le t \le T.$$

$$= g(t)\cos\frac{2\pi}{M}(m-1)\cos 2\pi f_c t - g(t)\sin\frac{2\pi}{M}(m-1)\sin 2\pi f_c t$$

- Where $g(t) - \text{signal pulse} \;\; \text{and} \;\; \theta_m = \frac{2\pi}{M}(m-1)$ are the M possible phases of the carrier T
- These signal waveforms have equal energy given by

$$\varepsilon = \int_0^T S_m^2(t)dt = \frac{1}{2}\int_0^T g^2(t)\,dt = \frac{1}{2}\varepsilon_g$$

Phase Modulated Signals …

- PM signal is also represented as a linear combination of two orthonormal waveforms $f_1(t)$ & $f_2(t)$ such that

$$S_m(t) = S_{m1} f_1(t) + S_{m2} f_2(t); \quad \bar{S}_m = \begin{bmatrix} S_{m1} & S_{m2} \end{bmatrix}$$

$$\text{Where} \quad f_1(t) = \sqrt{\frac{2}{\varepsilon_g}} g(t) \cos 2\pi f_c t \quad and$$

$$f_2(t) = \sqrt{\frac{2}{\varepsilon_g}} g(t) \sin 2\pi f_c t$$

- Alternatively these two dimensional vectors may be expressed as

$$\bar{S}_m = \begin{bmatrix} S_{m1} & S_{m2} \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{\varepsilon_g}{2}} \cos \frac{2\pi}{M}(m-1) & \sqrt{\frac{\varepsilon_g}{2}} \sin \frac{2\pi}{M}(m-1) \end{bmatrix}$$

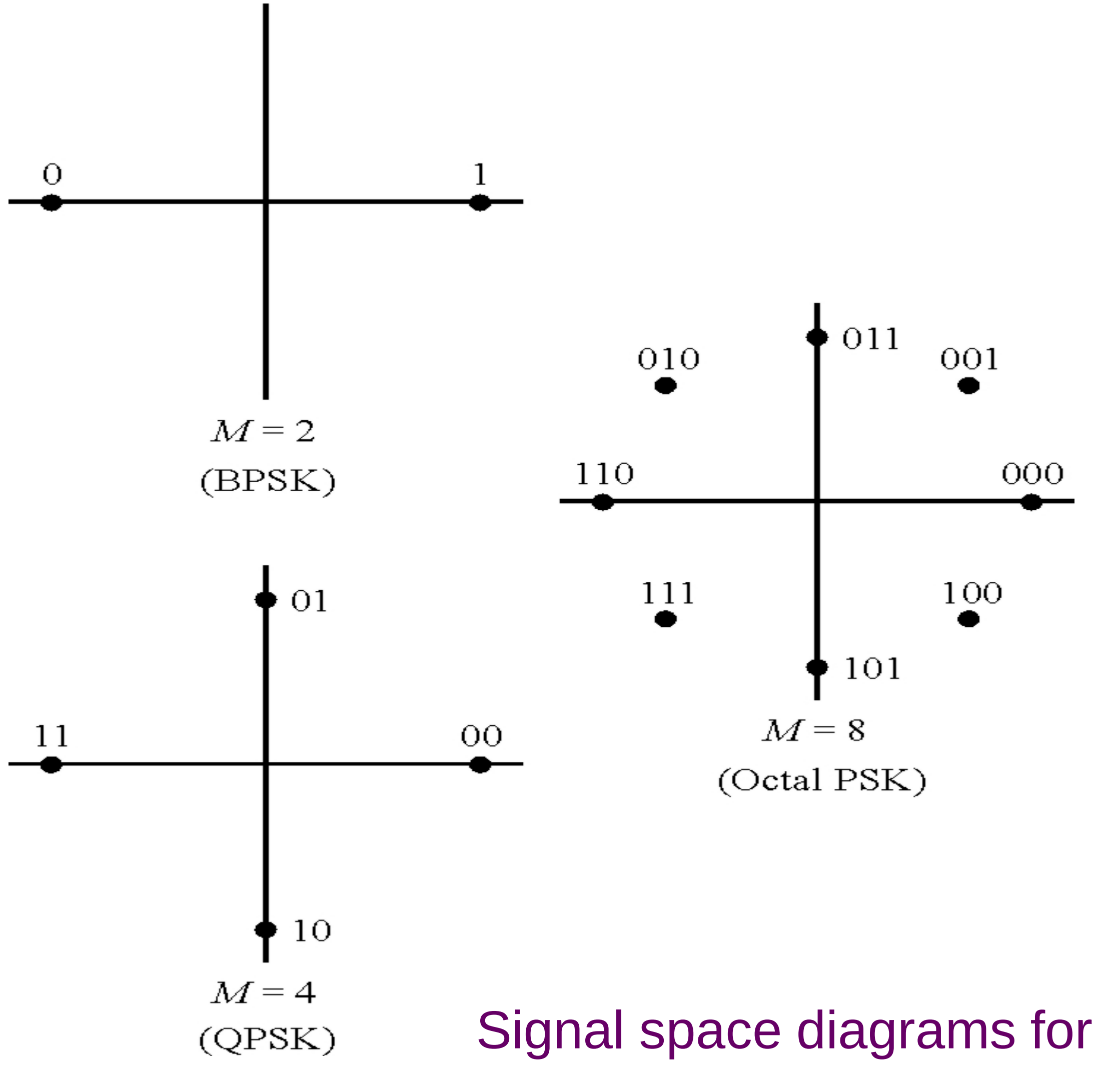
Phase Modulated Signals …

- The phase of the carrier signal is used for modulation (carrying information)
- Every symbol (k bits) is mapped into a given phase
- The total phase is divided equally among all possible symbols
- The signal space is two dimensional with signals having as coordinates

$$\bar{S}_m = [S_{m1} \quad S_{m2}] = \left[\sqrt{\frac{\varepsilon_g}{2}} \cos \frac{2\pi}{M}(m-1) \qquad \sqrt{\frac{\varepsilon_g}{2}} \sin \frac{2\pi}{M}(m-1) \right]$$

$$m = 1,2,\ldots\ldots\ldots\ldots\ldots\ldots M$$

Phase Modulated Signals …



Signal space diagrams for PSK signals

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Quadrature Amplitude Modulation (QAM)

- Bandwidth efficiency can be obtained by simultaneously impressing two separate k-bit symbols from the information sequence $\{a_n\}$ on the amplitude of the two quadrature carriers $\cos 2\pi f_c t$ *and* $\sin 2\pi f_c t$ such that

$$\begin{aligned} S_m(t) &= \operatorname{Re}\left[(A_{mc} + jA_{ms})\, g(t)\, e^{j2\pi f_c t}\right], \quad m = 1,2,\ldots\ldots, M \\ &= A_{mc}\, g(t) \cos 2\pi f_c t - A_{ms}\, g(t) \sin 2\pi f_c t \end{aligned}$$

- A_{mc} and A_{ms} are the information bearing signal amplitudes of the quadrature carriers and $g(t)$ is the signal pulse

Quadrature Amplitude Modulation …

- Alternatively, QAM signal waveform is represented by

$$S_m(t) = V_m g(t) \cos\left(2\pi f_c t + \theta_m\right)$$

- Where

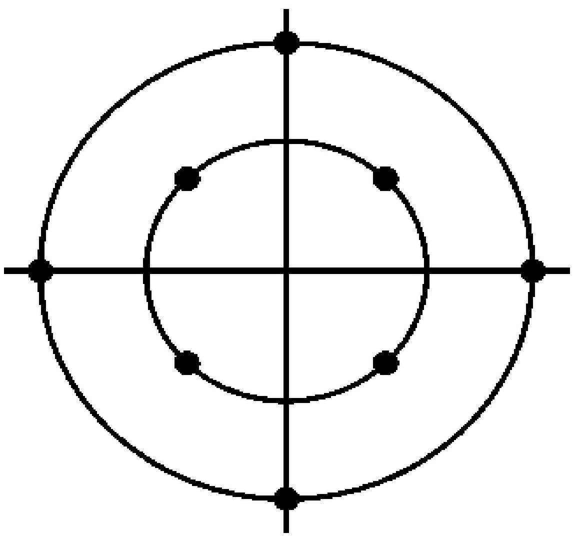
$$V_m = \sqrt{A_{mc}^2 + A_{ms}^2} \quad \text{and} \quad \theta_m = \tan^{-1}\left(\frac{A_{ms}}{A_{nc}}\right)$$

- Note: This may be viewed as a combined amplitude and phase modulation
- If we take M_1 as the PAM levels M_2 as the phases, we can construct an $M=M_1M_2$ combined PAM-PSK signal constellation

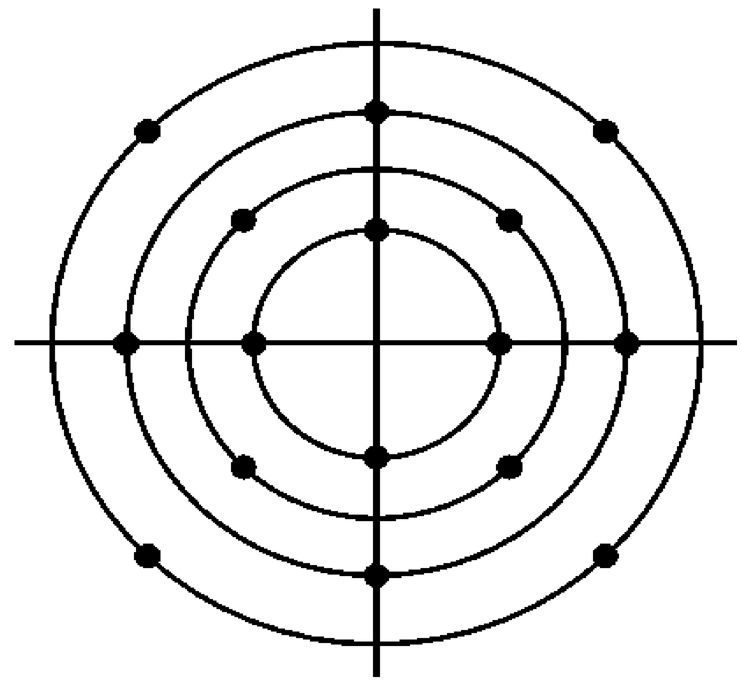
Quadrature Amplitude Modulation …

- $M_1 = 2^n$ and $M_2 = 2^m$ PAM-PSK signal constellation results in the simultaneous transmission of $m+n = \log_2 M_1 M_2$ binary digits occurring at the symbol rate of $R/(m+n)$

- Examples of combined PAM-PSK signal space diagrams



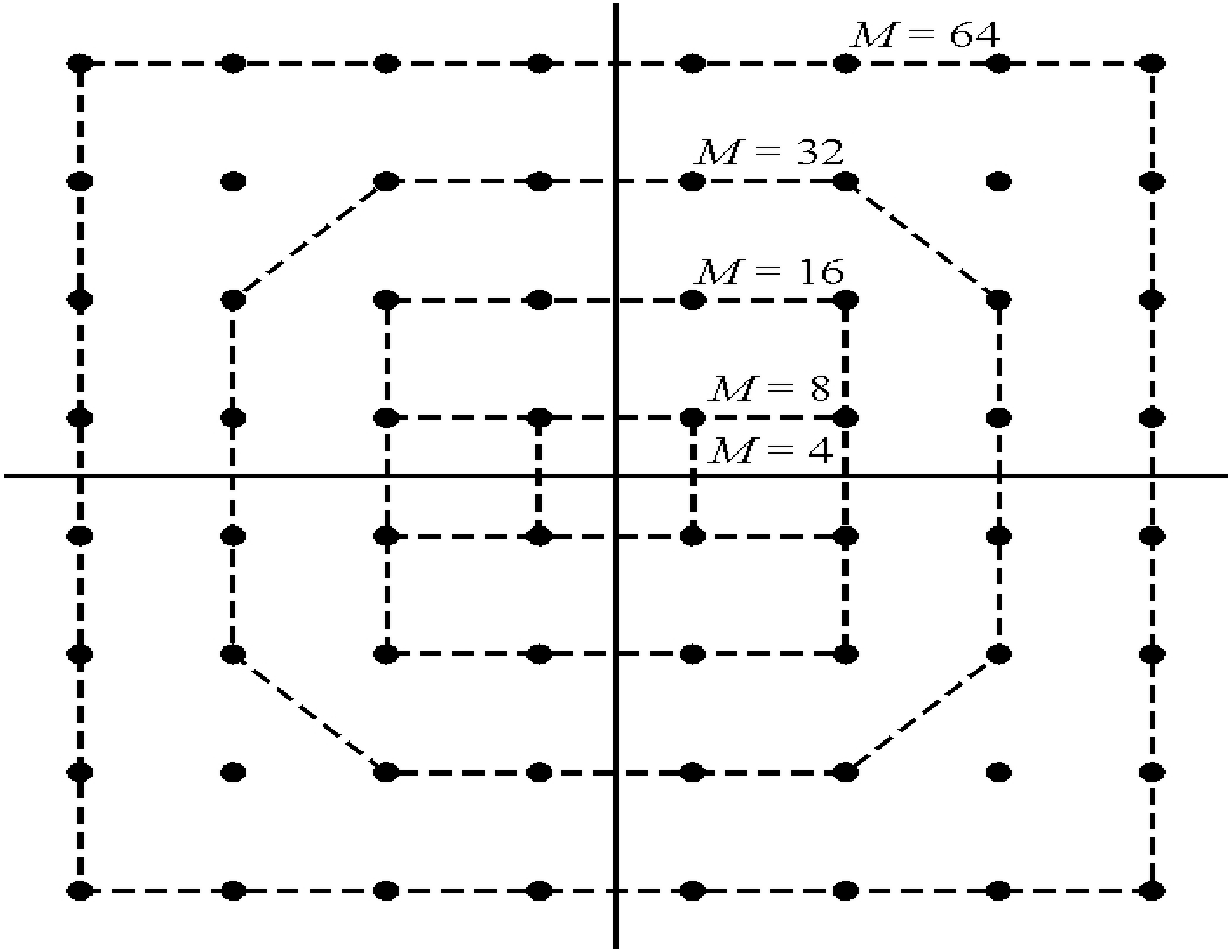
$M = 8$



$M = 16$

Quadrature Amplitude Modulation …

- Several Signal Space Diagrams for Rectangular QAM



Quadrature Amplitude Modulation …

- Like PSK signals, QAM signals may also be represented as a linear combination of two orthonormal signal waveforms $f_1(t)$ and $f_2(t)$ such that

$$S_m(t) = S_{m1} f_1(t) + S_{m2} f_2(t);$$

$$\text{Where} \quad f_1(t) = \sqrt{\frac{2}{\varepsilon_g}} g(t) \cos 2\pi f_c t \quad and \quad f_2(t) = \sqrt{\frac{2}{\varepsilon_g}} g(t) \sin 2\pi f_c t$$

$$\bar{S}_m = [S_{m1} \quad S_{m2}] = \left[A_{mc}\sqrt{\frac{\varepsilon_g}{2}} \qquad A_{ms}\sqrt{\frac{\varepsilon_g}{2}} \right];$$

- And the Euclidean distance between any pair of signal vectors is given by

$$d_{mn}^{(e)} = |S_m - S_n| = \left[\frac{\varepsilon_g}{2}((A_{mc} - A_{nc})^2 + (A_{ms} - A_{ns})^2) \right]^{1/2}$$

Quadrature Amplitude Modulation …

- Where the signal amplitudes take discrete values {2m-1-M}, m = 1,2,…M and the signal space diagram is rectangular
- Note that the minimum distance between adjacent pair of signal points is

$$d_{\min}^{(e)} = d\sqrt{2\varepsilon_g}$$

- Which is the same as for PAM

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Multidimensional Signals

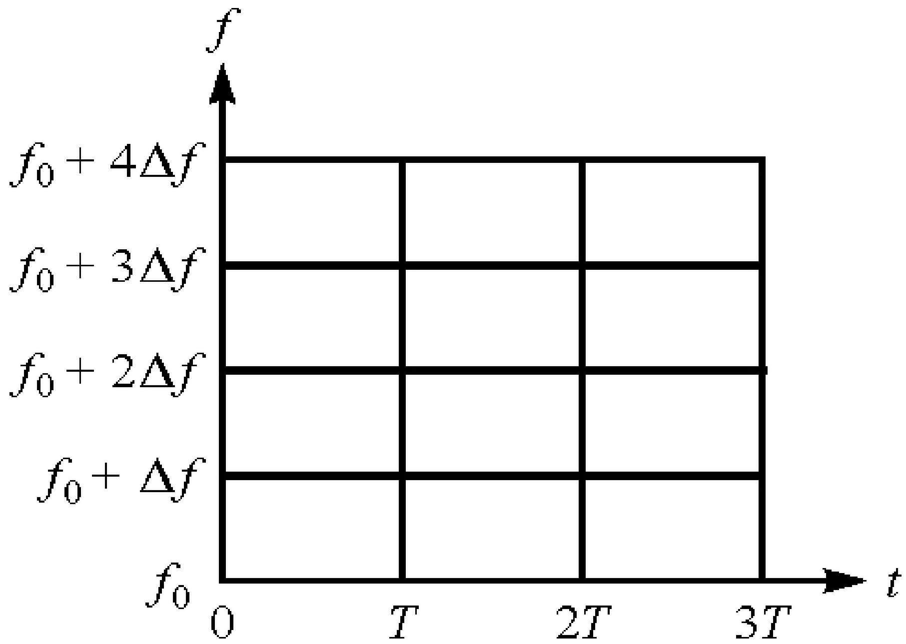
- Consider a set of N-dimensional signal vectors
- For any N subdivide a time interval $T_1 = NT$ into N subintervals of length $T = T_1 /N$
- In each subinterval we can use PAM to transmit an element of an N-dimensional signal vector
- For N even, we can use quadrature carriers to transmit two components independently
  - Thus, N-dimensional signal vectors are transmitted in NT/2 sec.

Multidimensional Signals …

- Alternatively, a frequency band of width $N\Delta f$ may be subdivided into N frequency slots each of width Δf
- N-dimensional signal vector can be transmitted by simultaneously modulating amplitudes of N carriers, one in each frequency slot
- Δf must be chosen such that there will not be cross-talk interference among the signals
- Using QAM, N-dimensional signal vectors (for N even) are transmitted in N/2 frequency slots
  - Thus reducing the channel bandwidth by a factor of two

Multidimensional Signals …

- In general one may use ***both time and frequency domains jointly*** to transmit an N-dimensional signal vector
- The figure below demonstrates this principle
  - A 24-dimensional signal can be transmitted using QAM
  - Or 12-dimensional signal can be transmitted using PAM



Orthogonal Multidimensional Signals

- As a special case of constructing a multidimensional signals, consider ***M equal-energy orthogonal*** signals that differ in frequency

$$S_m(t) = \mathrm{Re}\left[S_{lm}(t)\, e^{j2\pi f_c t}\right]; \quad m = 1,2,\ldots M \;\; and \quad 0 \le t \le T$$
$$= \sqrt{\frac{2\varepsilon}{T}} \cos(2\pi f_c t + 2\pi m \Delta f\, t)$$

- Where the equivalent low-pass signal waveforms are

$$S_{lm}(t) = \sqrt{\frac{2\varepsilon}{T}}\, e^{j2\pi m \Delta f\, t}; \quad m = 1,2,\ldots M \;\; and \;\; 0 \le t \le T$$

- This modulation is called ***frequency-shift keying*** (FSK)

Orthogonal Multidimensional Signals …

- These frequency modulated signals have equal energy and cross-correlation coefficients given by

$$\rho_{km} = \frac{2\varepsilon/2}{2\varepsilon}\int_0^T e^{j2\pi(m-k)\Delta ft}dt = \frac{\sin \pi T(m-k)\Delta f}{\pi T(m-k)\Delta f} e^{j\pi T(m-k)\Delta f}$$

SHOW!

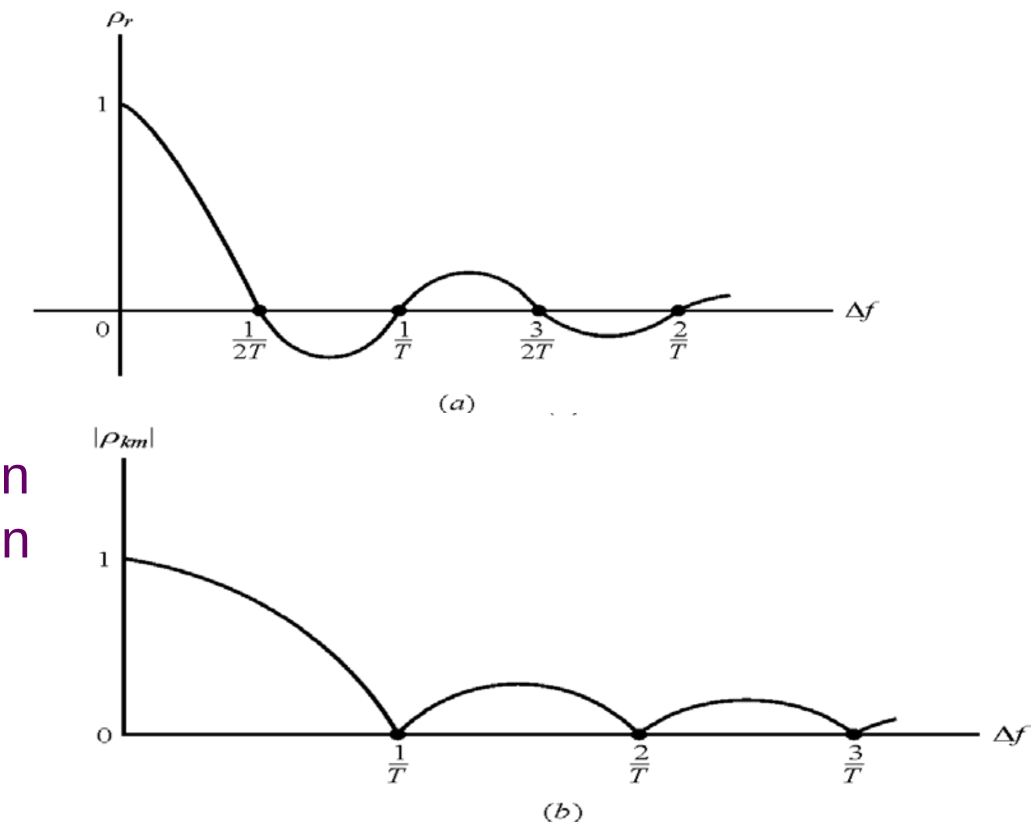
- Whose real part can be expressed as

$$\rho_r = Re(\ \rho_{km}\ ) = \frac{sin[2\pi T(m-k)\Delta f]}{2\pi T(m-k)\Delta f}$$

- Note that $\rho_r = 0$ when Δf =1/2T and m ≠ k
- Since |m - k |=1 corresponds to adjacent frequency slots, Δf =1/2T represents the minimum frequency separation between adjacent signal for orthogonality of the M signals

Orthogonal Multidimensional Signals …

- Plots of ρ_r and $|\rho_{km}|$ versus frequency are shown in the figure below
- Also note that $|\rho_{km}| = 0$ for multiples of 1/T whereas $\rho_r=0$ for multiples of 1/2T



Cross-correlation coefficient as a function of frequency separation for FSK signals

Orthogonal Multidimensional Signals …

- For the case in which Δf =1/2T, the M FSK signals are equivalent to the N-dimensional vectors

$$S_1 = [\sqrt{\varepsilon}\ \ 0\ \ 0\ \ \ldots\ldots\ldots\ldots\ldots 0\ \ 0]$$

$$S_2 = [\ 0\ \sqrt{\varepsilon}\ 0\ \ \ldots\ldots\ldots\ldots\ldots 0\ \ 0]$$

$$\cdot \qquad \cdot \qquad\qquad\qquad \cdot$$

$$\cdot \qquad \cdot \qquad\qquad\qquad \cdot$$

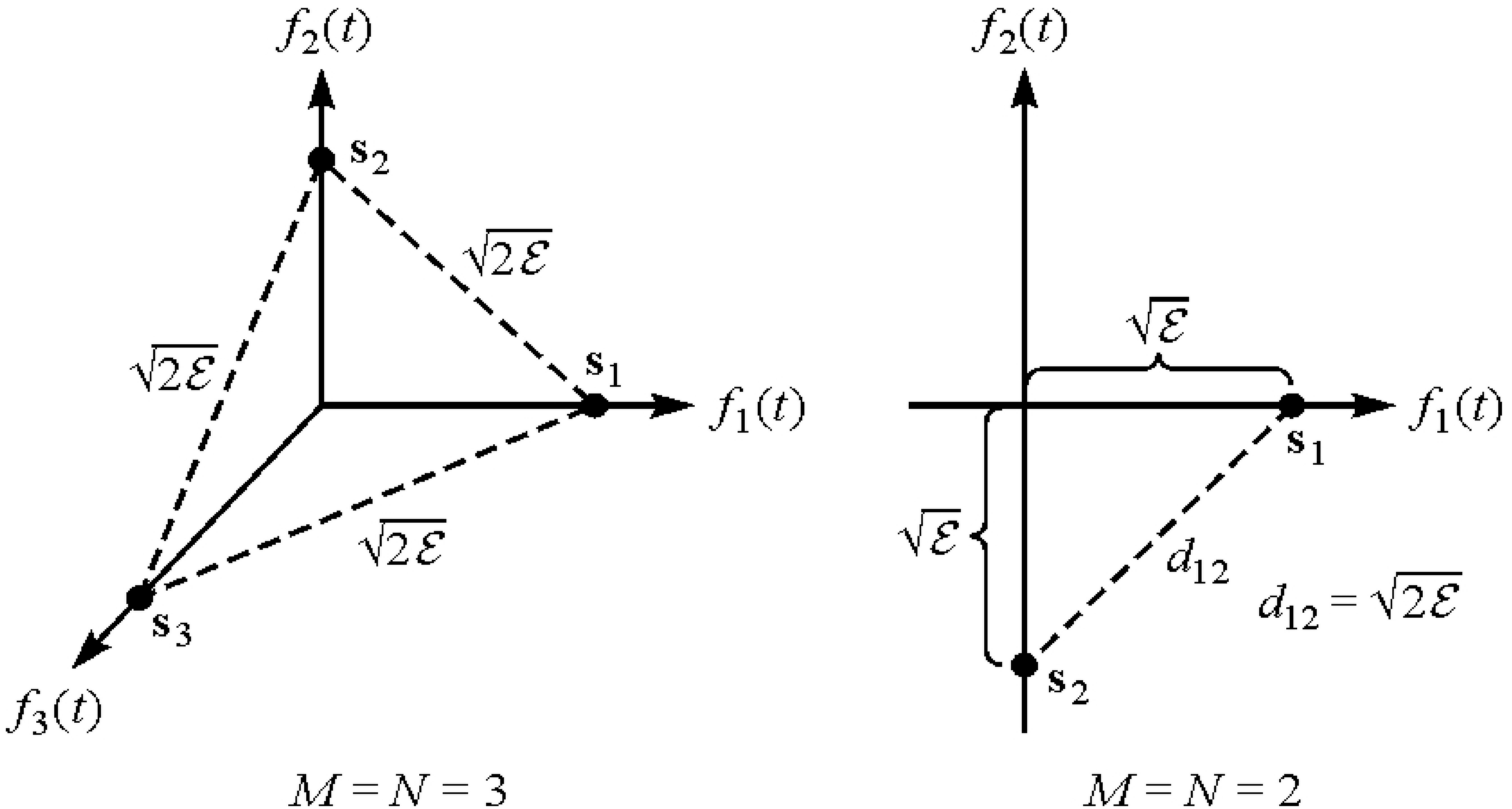
$$S_N = [\ 0\ \ 0\ \ 0\ \ldots\ldots\ldots\ldots\ldots \sqrt{\varepsilon}]$$

- Where N = M and the distance between pairs of signals is

$$d_{km}^{(e)} = \sqrt{2\varepsilon}$$

- for all m, k which is also the minimum distance

Orthogonal Signals for *M* = *N* = 3 and *M* = *N* = 2



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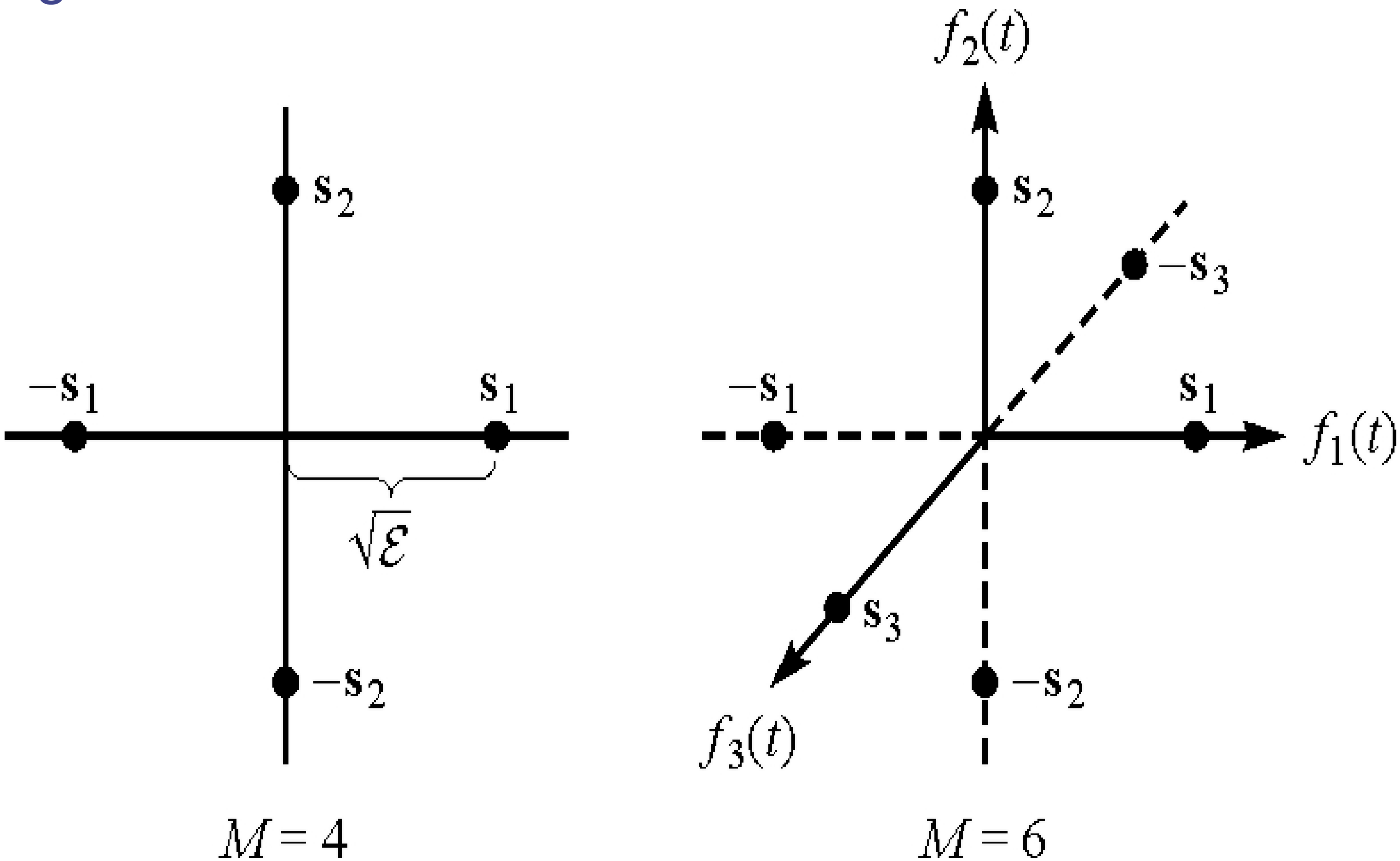
Biorthogonal signal

- A set M of biorthogonal signals can be constructed from 1/2M orthogonal signals by augmenting with negatives of the orthogonal signals
  - This requires N = ½ M dimensions
- The correlation between any pair of signals is either -1 or 0
- The corresponding distances are $d = 2\sqrt{\varepsilon}$ or $\sqrt{2\varepsilon}$, the latter being the minimum distance

Biorthogonal signal …

- Signal space diagrams for *M* = 4 and *M* = 6 biorthogonal signals



Simplex Signals

- Consider M orthogonal waveforms $\{S_m(t)\}$ or their vector representation $\{S_m\}$ whose mean is

$$\bar{S} = \frac{1}{M}\sum_{m=1}^{M} S_m$$

- Construct another set of signals by subtracting the mean from each of the M orthogonal signals; i,.e

$$\mathbf{S}'_{\mathbf{m}} = \mathbf{S}_{\mathbf{m}} - \bar{\mathbf{S}}; \quad m = 1,2,\ldots\ldots\ldots\ldots M$$

- The effect of this subtraction is to translate the origin of the m orthogonal signals to the point $\bar{\mathbf{S}}$
- The resulting signal waveform is called ***simplex signal***

Simplex Signals …

- It can be shown that, simplex signals have the following properties (*See text)*
- Energy per waveform is

$$\left\| S_m^{'} \right\| = \varepsilon\left(1 - \frac{1}{M}\right)$$

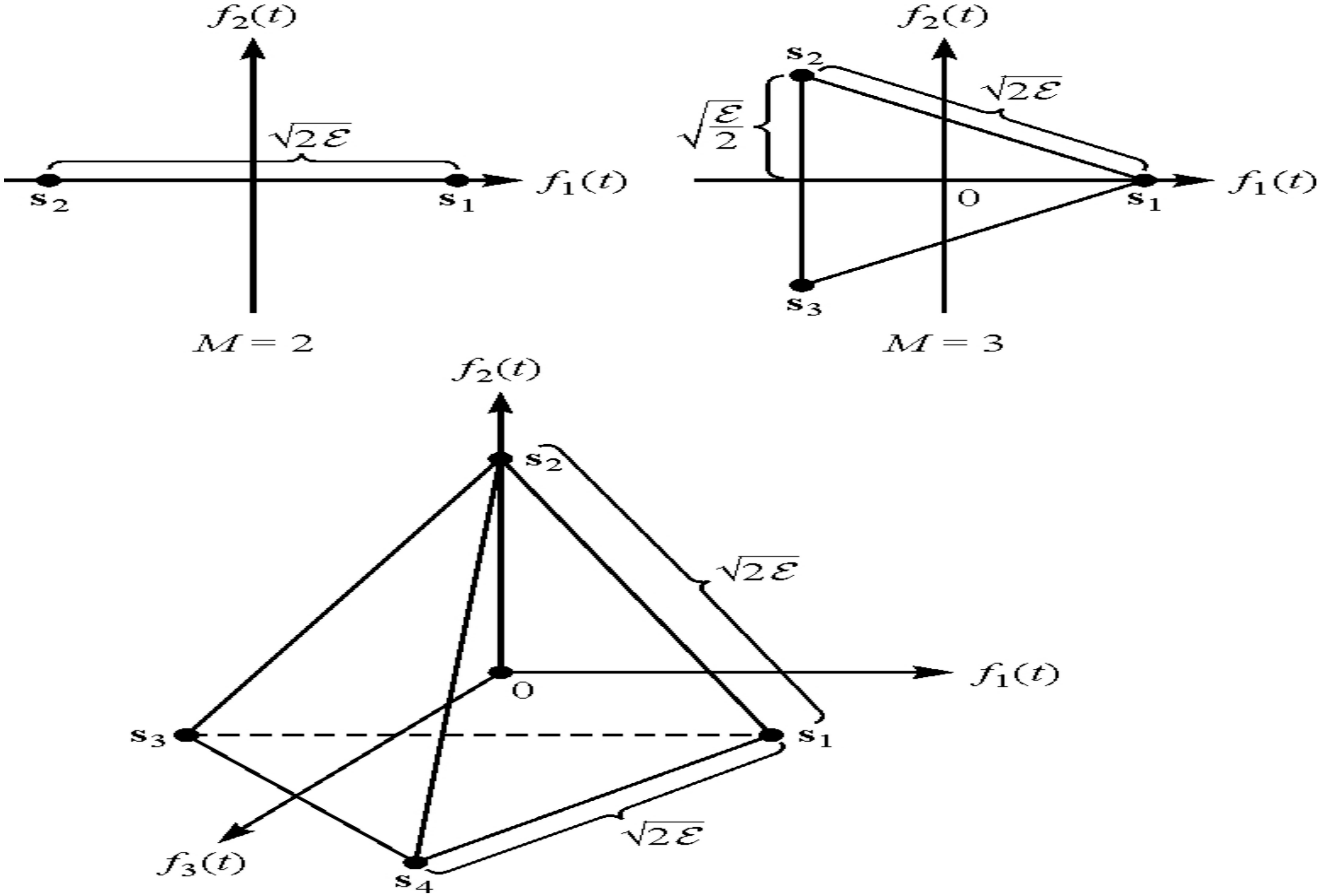
- Cross-correlation between any pair of signals in

$$\mathrm{Re}(\rho_{mn}) = \frac{-1/M}{1-1/M} = -\frac{1}{M-1} \quad for\ all\ m,n$$

- Hence, simplex waveforms are equally correlated & requires less energy (factor of 1-1/M) than orthogonal waveforms

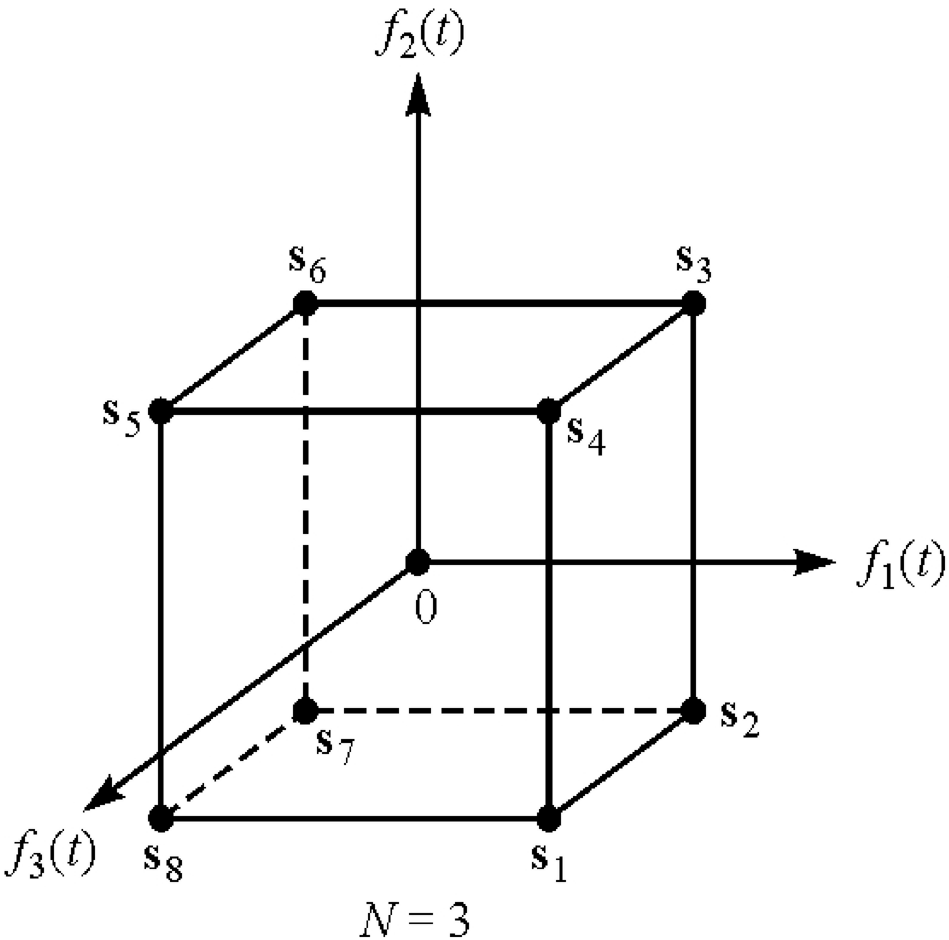
Simplex Signals …

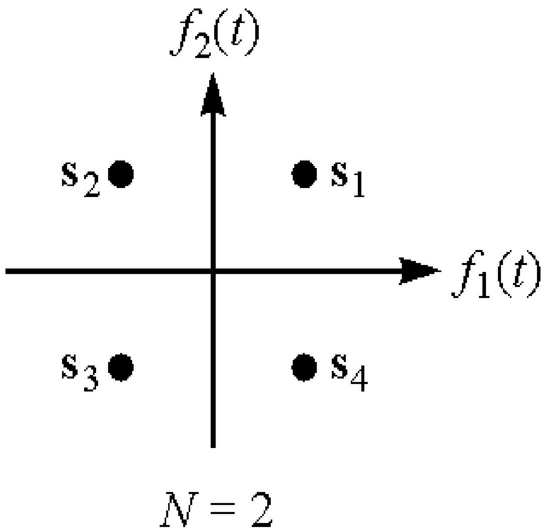
- Signal space diagrams for *M*-ary simplex signals



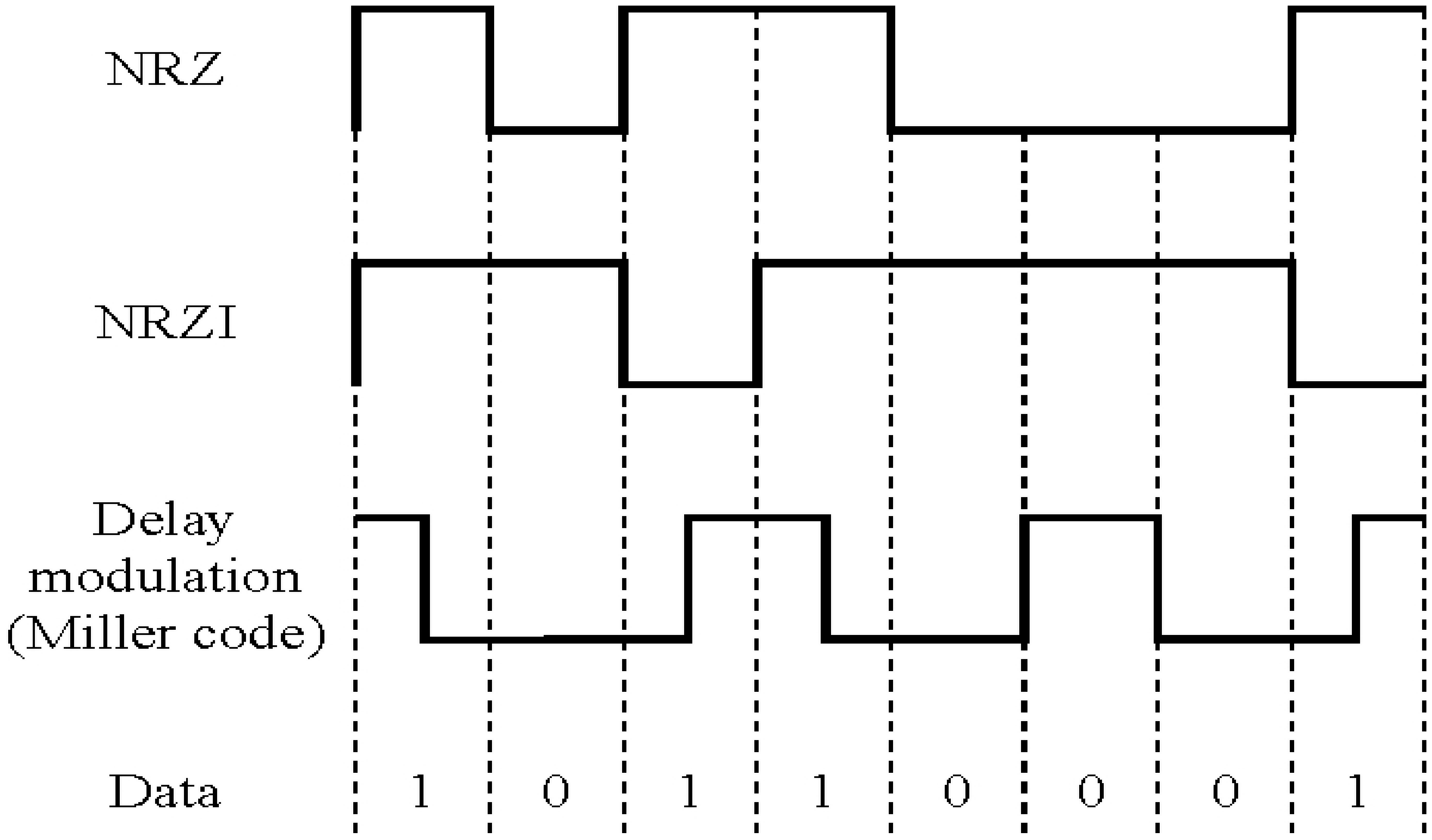
Signal Waveform from Binary Codes

- Signal space diagrams for signals generated from binary codes





Signal Waveform from Binary Codes …



Baseband signals