Chapter 8: Diversity Techniques (Revision)

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Diversity Techniques

- In a fading environment reception errors occur when the channel attenuation is large (deep fades)
- One solution
  - Supply to the receiver *several replicas* of the same information signal
  - Transmission should be over independently fading channels
- Hence, the probability that all of them will simultaneously suffer from fading will be *greatly reduced*

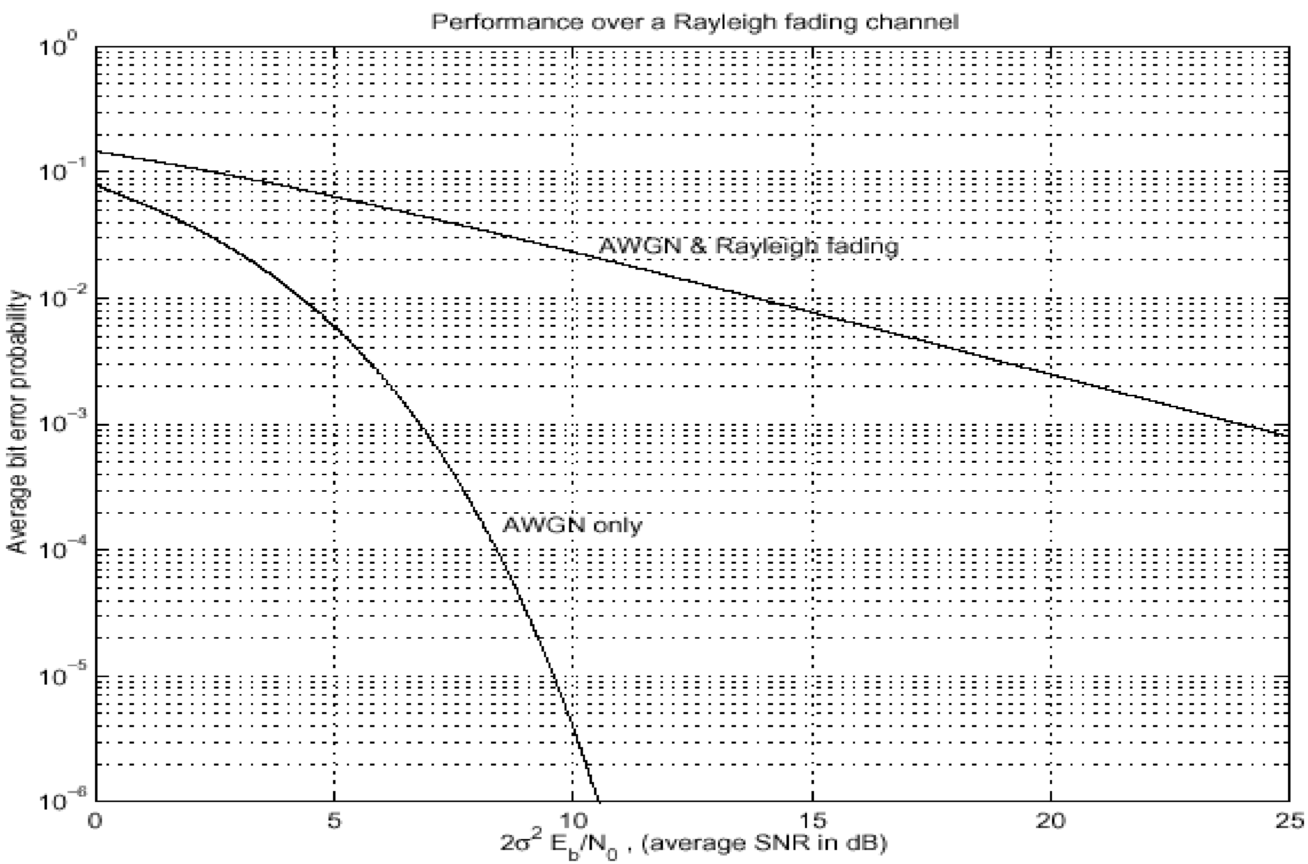
- Example
  - Given: p is the probability that any one of the signals will fade below a given threshold
  - The probability that L independent replicas will simultaneously fade to levels below the threshold will be p^L

Diversity Techniques …

- There are *different way* of providing the receiver with L independently fading replicas of the same information-bearing signals
- The above is in a way similar to *repetition coding* (block interleaving) that aids to convert a bursty channel to that that produce independent errors

Diversity Techniques …

- Performance of a Coherent BPSK AWGN and Flat Rayleigh Fading Channels



Different Ways to Deal with Fading

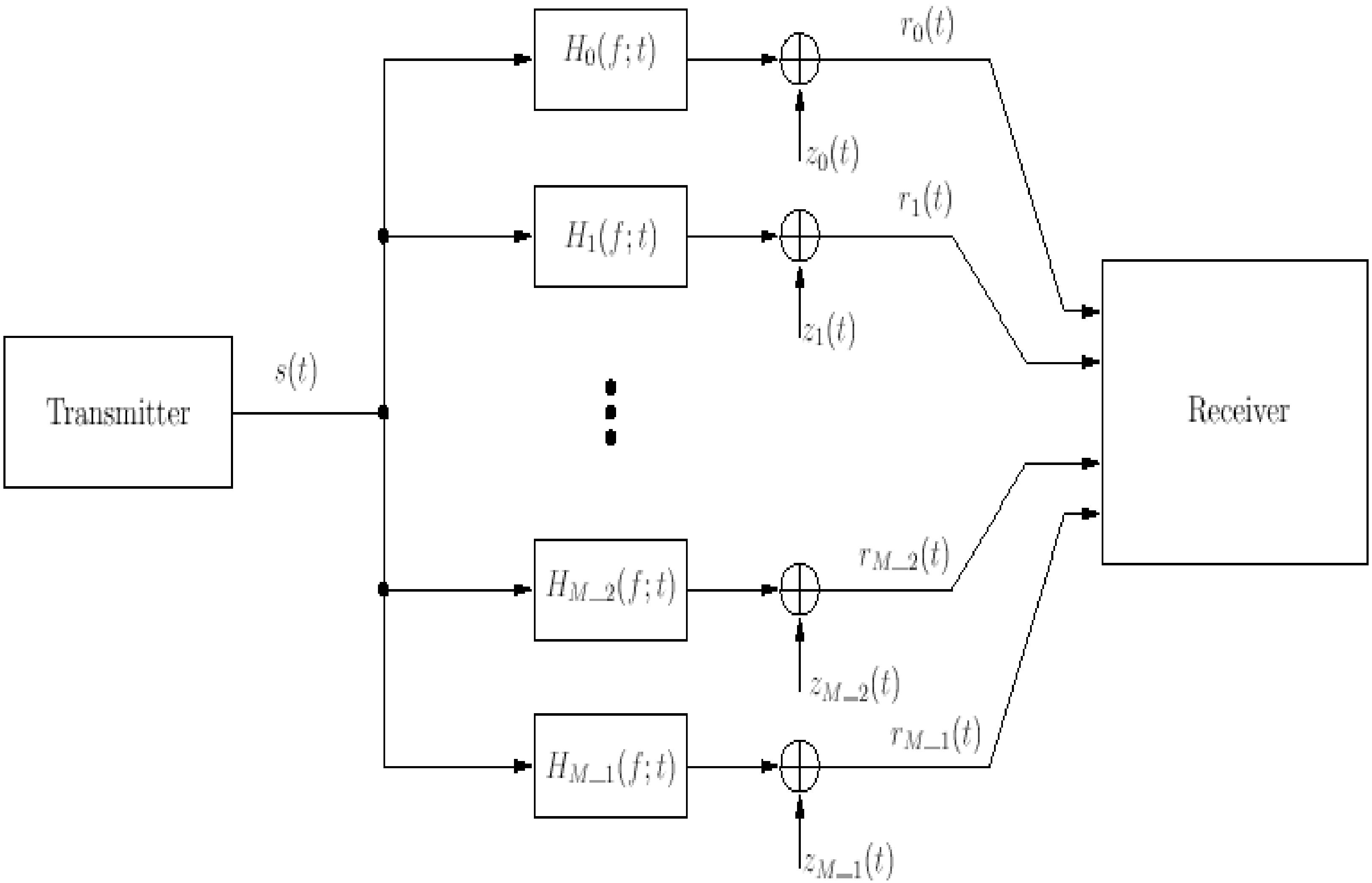
- Adding fading margin at the transmitter – Power inefficient
- Using *diversity* by taking advantage of the statistical behavior of the channel
- There are *three main methods* the diversity technique can be employed

1. *Frequency Diversity*
   - The same information bearing signal is transmitted on L carriers
   - The separation between successive carriers equals or exceeds the *coherent bandwidth* of the channel
2. *Time Diversity*
   - Transmit the same signal in L different time slots
   - The separation between the successive time slots equals or exceeds the *coherent time* of the channel

Different Ways to Deal with the Problem of Fading

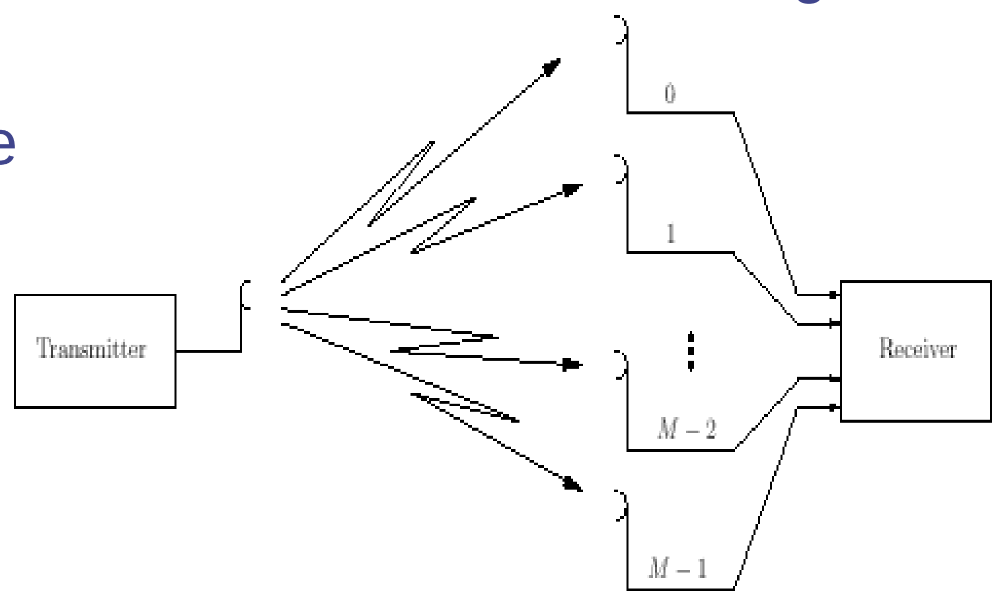
3. *Space diversity*
   - Use multiple antennas at the receiver or the transmitter or both to transmit the same signal
   - The antennas must be *spaced sufficiently far apart* such that the signal fade in each of the propagation paths are independent

Principle of Diversity



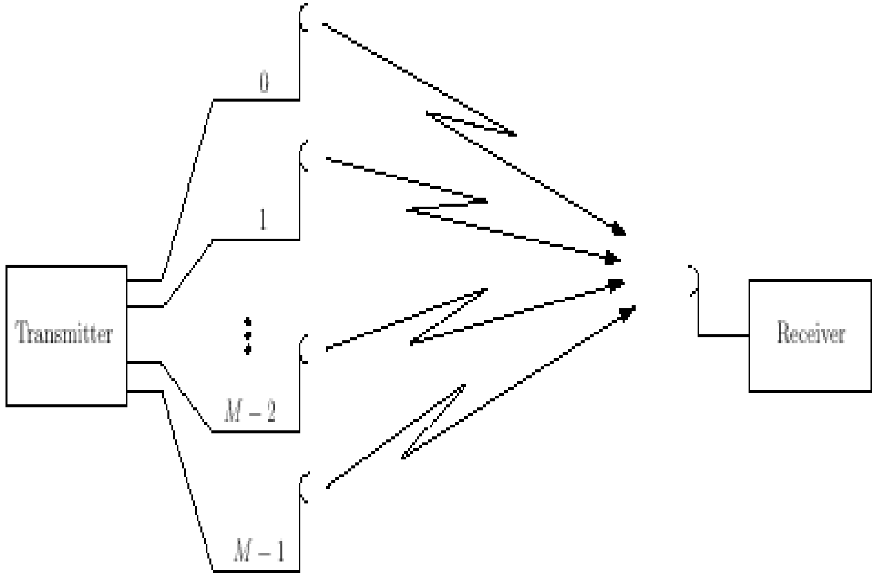
Space Diversity - Receiver Diversity

- The *space correlation properties* of the radio channel are used as mean of providing multiple uncorrelated copies of the same signal to the receiver
- M different antennas are used at the receiver to obtain independent fading signals
- Spacing distance between the antennas must be large enough >nλ
- No efficiency loss in use of transmitter power
- More hardware (antennas) at the receiver and added processing requirement



Space Diversity - Transmitter Diversity

- M different antennas are used at the transmitter to obtain uncorrelated fading signals at the receiver
- Spacing distance between the antennas must be sufficiently large
- The total transmitted power is split between the antennas
- Signal combining at the receiver is complicated
- More hardware (antennas) at the transmitter

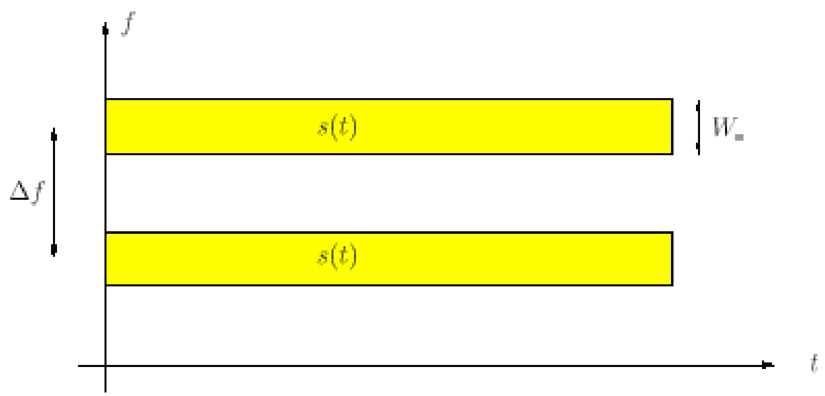


Frequency Diversity

- The *frequency correlation properties* of the radio channel are used as a means of providing multiple uncorrelated copies of the same signal to the receiver
- Optimum diversity is obtained when the carrier frequencies are far apart with

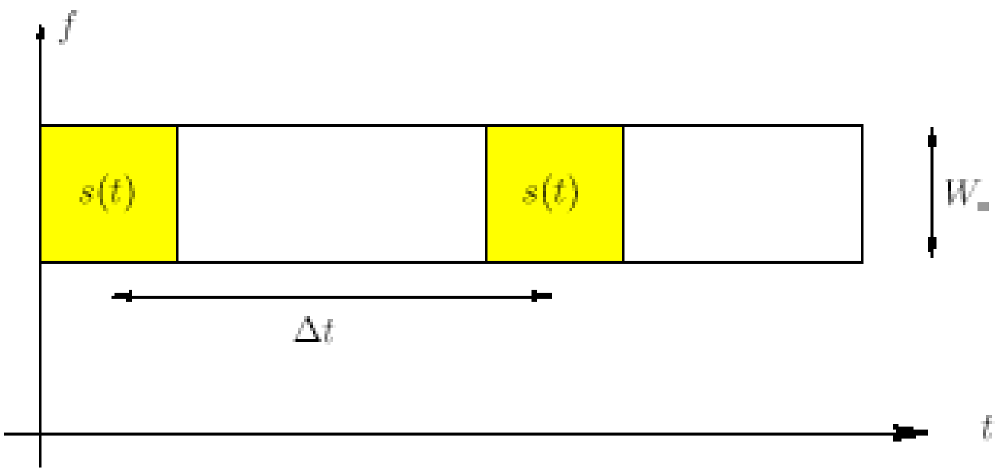
$$\Delta f > B_m = 1/T_{\mathrm{m}}$$

- Only one antenna is needed (both transmitter & receiver)
- More bandwidth is needed
- The total transmitted power is split between the carrier frequencies



Time Diversity

- The *time correlation properties* of the radio channel are used as a means of providing multiple uncorrelated copies of the same signal to the receiver
- Optimum diversity is achieved when *time slots* are far apart
- Only one antenna is needed (both transmitter & receiver)
- Reduction in efficiency (effective data rate < real data rate)
- Time separation depends on the velocity of the mobile v
- Slow moving mobiles require very long time separation $\Delta t \propto \frac{\lambda}{v}$



Combining Methods for Diversity Systems

- For a slowly fading channel, the equivalent lowpass of the received signal of branch i can be written as

$$r_i(t) = a_i e^{j\theta_i} s_l(t) + z_i(t); \quad i = 0, 1, \ldots\ldots, M-1$$

  - $s_l(t)$ is the equivalent lowpass of the transmitted signal
  - $a_i e^{j\theta_i}$ is the fading attenuation of branch I

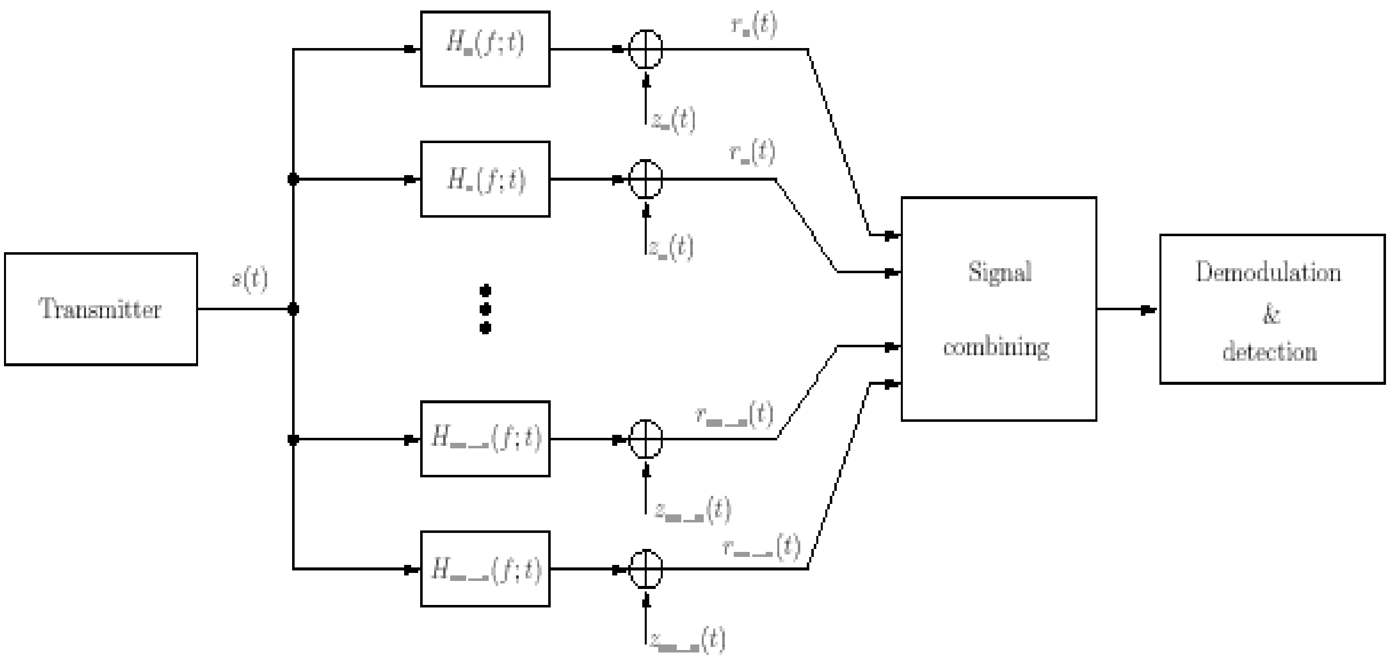
- Out of the M branches, M replicas of the transmitted signal are obtained

$$\underline{\mathbf{r}} = [r_0(t), r_1(t), \cdots, r_{M-1}(t)]$$

- For ideal diversity, all $a_i e^{j\theta_i}$ are i.i.d. random variables
- M is referred to as the *diversity order*

Combining Methods for Diversity Systems …

- *These M branches are then used by the receiver (in the best way possible) to extract the transmitted information*
- An important part of a diversity system is the manner in which the *M* branches are used before signal detection
- There are different techniques of combining the *M* received signals



Selection Diversity Combining

- Selection diversity combining uses *one branch* at a time
  - It always chooses the branch with the best signal quality
- The received vector of signals at the combiner input is given by

$$\underline{\mathbf{r}} = [r_0(t), r_1(t), \cdots, r_{M-1}(t)], \quad \textbf{with} \quad r_i(t) = a_i e^{j\theta_i} s_l(t) + z_i(t), \quad 0 \le t \le T$$

- With selection diversity combining, the combiner output is given by

$$y(t) = a e^{j\theta} s_l(t) + z(t), \quad \textbf{with} \quad a = \mathbf{max}\{a_0, a_1, \cdots, a_{M-1}\}$$

- *z(t)* is complex Gaussian with zero-mean and variance N_0

Selection Diversity Combining …

- The probability density function of *a* is obtained as

$$p_a(a) = M p_{a_i}(a) \left[P_{a_i}(a)\right]^{M-1}$$

  - $p_{a_i}(a)$ is the pdf of the fading amplitude a_i
  - $P_{a_i}(a)$ is the distribution function of the i_{a_i}

Selection Diversity in Rayleigh Fading

- In general, the average symbol error probability, in this case, is obtained as

$$P_s = \int_0^\infty P(s/a)\, p_a(a)\, da$$

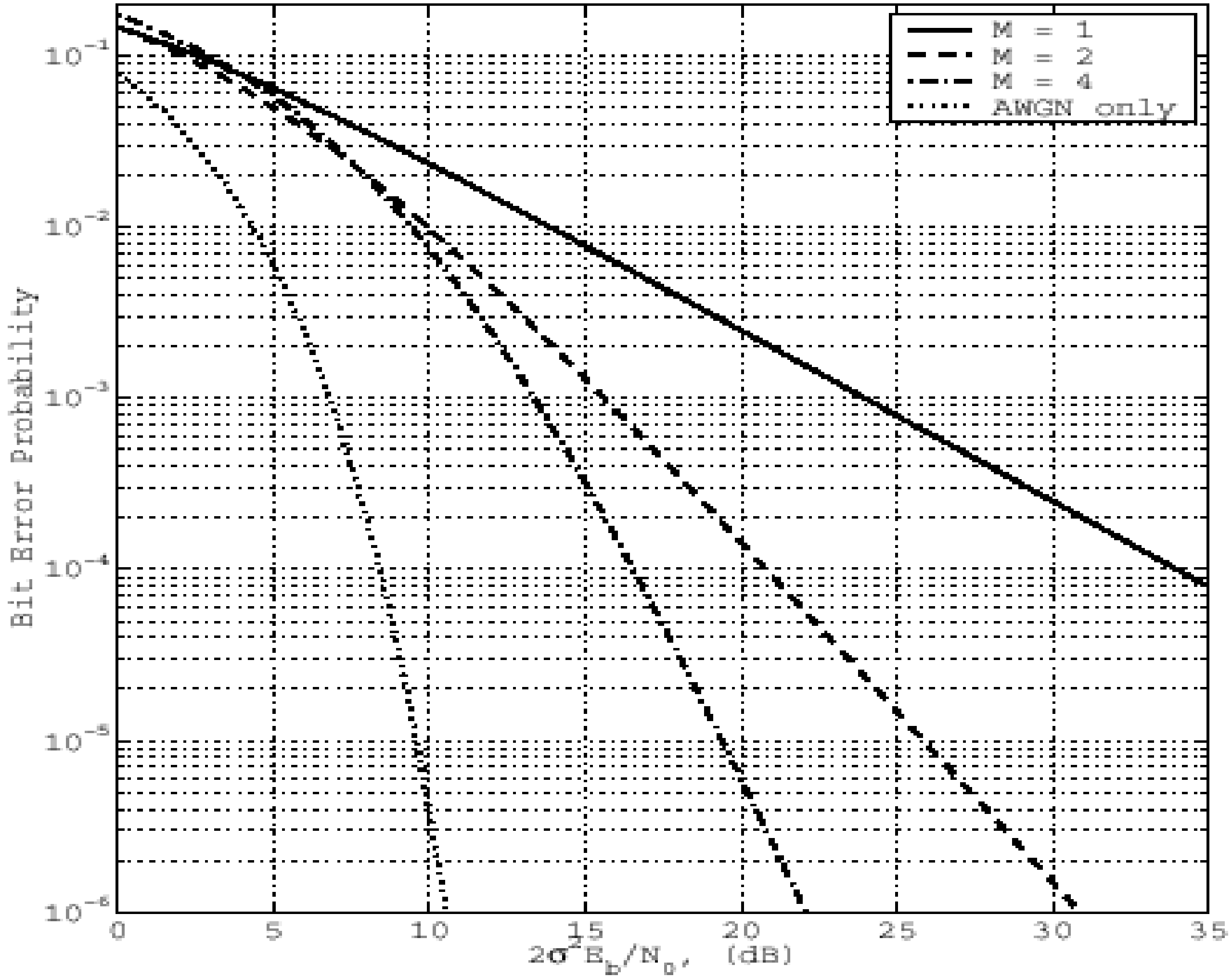
- Over the Rayleigh fading channel pdf of the fading amplitude *a* is given by

$$p_a = \frac{Ma}{\sigma^2} e^{-a^2/(2\sigma^2)} \left[1 - e^{-a^2/(2\sigma^2)}\right]^{M-1}$$

- Average bit error probability for binary PSK modulated signals with selection diversity under coherent detection

$$P_b = \frac{1}{2} \sum_{i=0}^{M} (-1)^i \binom{M}{i} \sqrt{\frac{\gamma_0}{i+\gamma_0}}; \quad \text{where } \gamma_0 = \frac{2\sigma^2 E_b}{N_0}$$

Selection Diversity Combining under Rayleigh Fading



Maximum Ratio Combining (MRC)

- Maximum-ratio combining *uses all* the branches simultaneously
- The combined signal is an optimized *linear combination* of the signal branches
  - Each signal is multiplied by the conjugate of its channel gain

$$y(t) = \sum_{i=0}^{M-1} a_i e^{-j\theta_i} r_i(t) = \left(\sum_{i=0}^{M-1} a_i^2 \right) s_l(t) + \sum_{i=0}^{M-1} a_i e^{-j\theta_i} z_i(t)$$

- The combined signal may now be rewritten as

$$y(t) = a s_l(t) + z(t); \quad With \quad a = \sqrt{\sum_{i=0}^{M-1} a_i^2}$$

  - Where z(t) is complex Gaussian with zero-mean and variance N_0

MRC in Rayleigh Fading

- The pdf of the fading amplitude *a*, in this case, is given by

$$p_a(a) = \frac{2\,a^{2M-1}}{(M-1)!(2\sigma^2)^M} e^{-a^2/(2\sigma^2)}$$

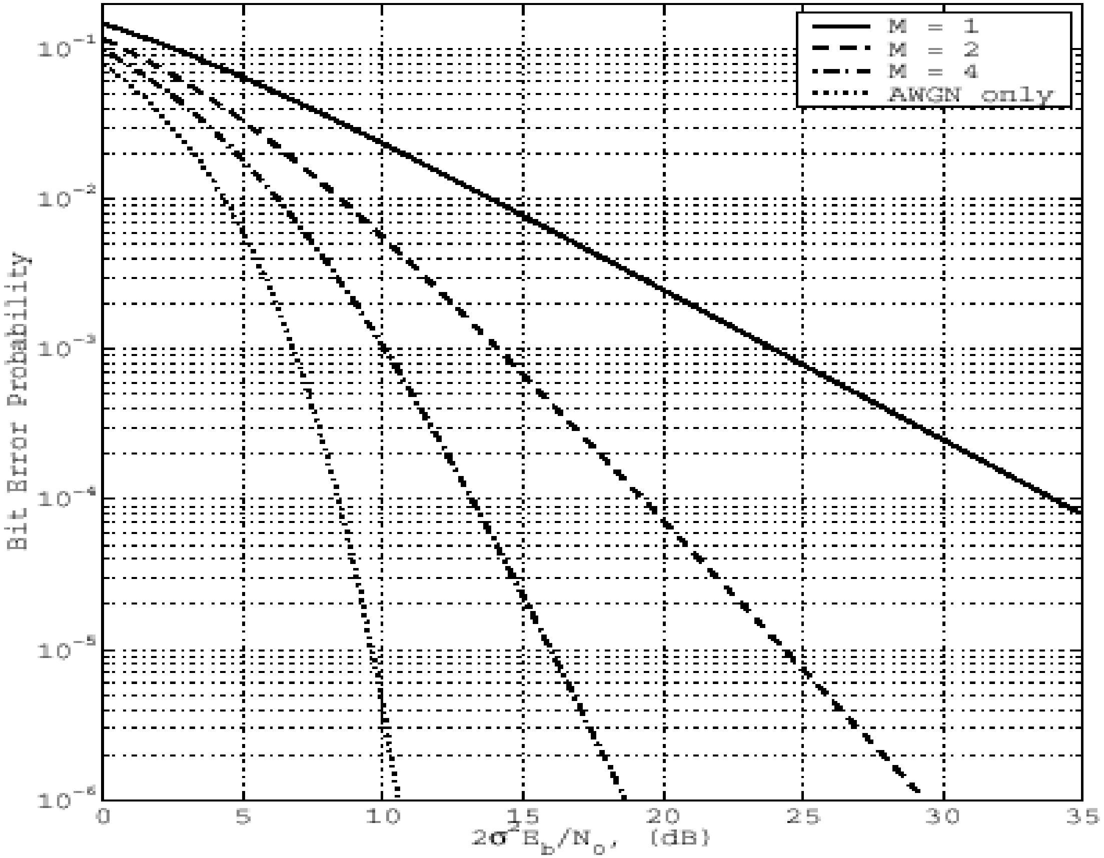
- Average bit error probability for binary PSK modulated signals with MRC under coherent detection

$$P_b \approx \binom{2M-1}{M}\left(\frac{1}{4\gamma_o}\right)^M \qquad \gamma_o = \frac{2\sigma^2 E_b}{N_o}$$

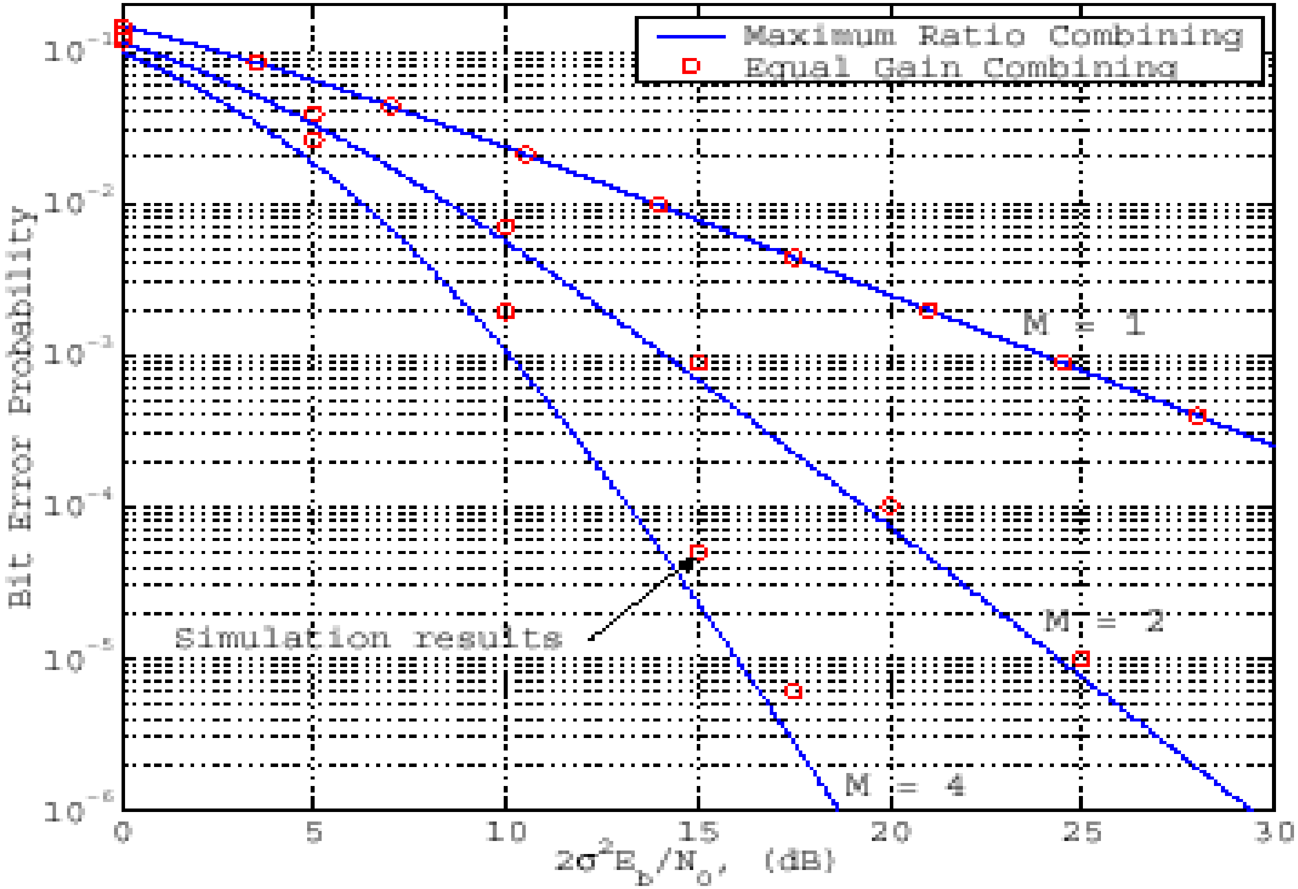
- A simplified version of maximum ratio combining is *Equal Gain Combining (EGC)*

MRC in Rayleigh Fading …

- Average bit error probability of coherent BPSK with MRC in Rayleigh fading channels



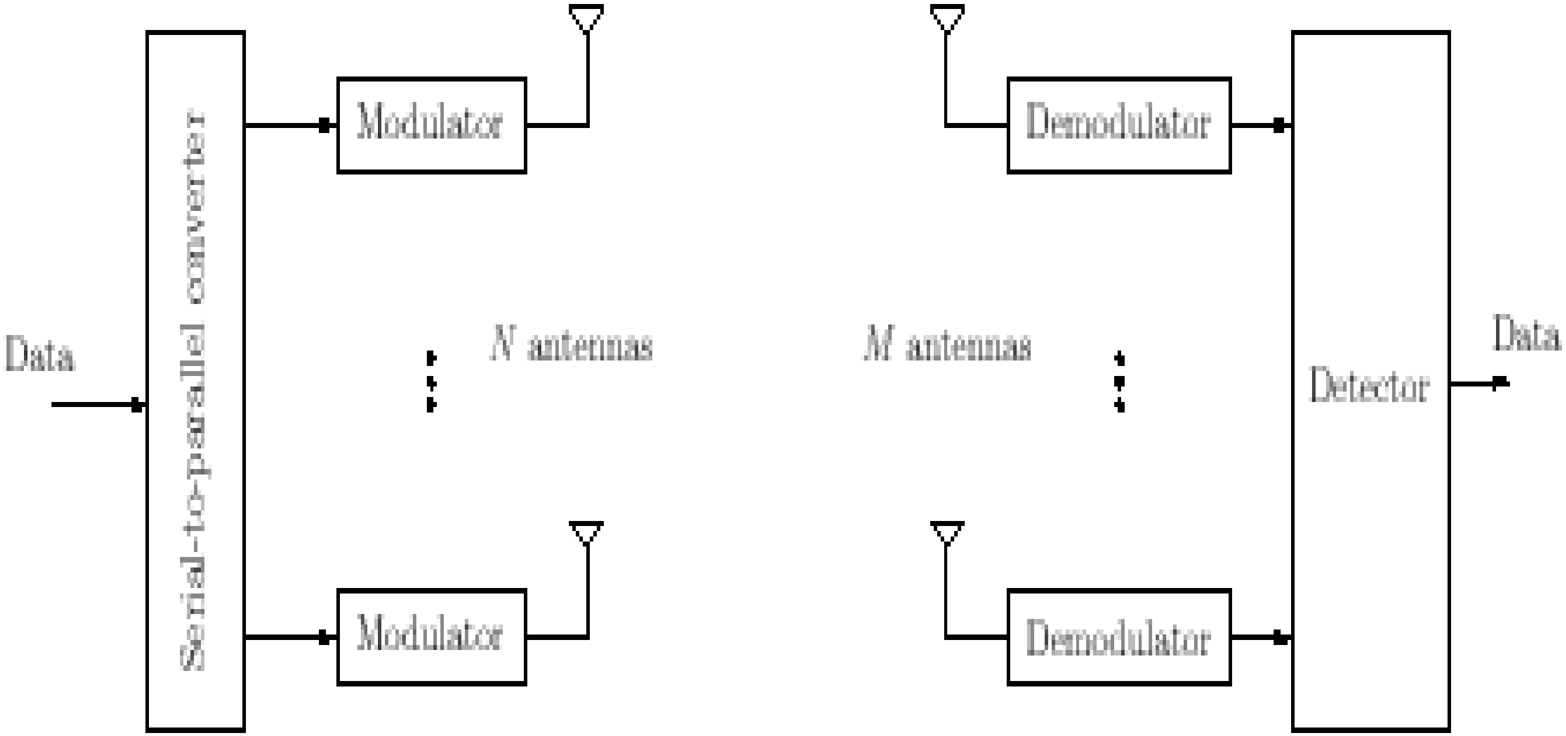
Equal Gain versus Maximum Ratio Combining



- Average bit error probability of coherent BPSK with EGC over Rayleigh fading channels

Multiple Antenna Systems

- Multiple antennas at both the transmitter and receiver increase the *diversity gain* (and/or the *data rate*)
- Multiple antennas at both the transmitter and receiver can be used as a *multiple access scheme*
- N-fold increase in data rate, Mth-order reception diversity



Multiple Antenna Systems …

- The equivalent lowpass of the received signal is written as

$$r_{l,m}(t) = \sum_{n=1}^{N} h_{mn}(t) s_n(t) + z_m(t); \quad m = 1,2,\ldots\ldots M$$

- After signal demodulation we get the k^{th} received sample as

$$r_{l,m}(k) = \sum_{n=1}^{N} h_{mn} I_n(k) + z_m(k)$$

  - h_{mn} is the complex Gaussian channel coefficient b/n antennas *n* & *m*
  - I_n is the modulated symbol transmitted at antenna n

- The received signal sample can also be written in a matrix form as

$$\boldsymbol{r}_l(k) = \boldsymbol{H}\boldsymbol{I}(k) + \boldsymbol{z}(k)$$

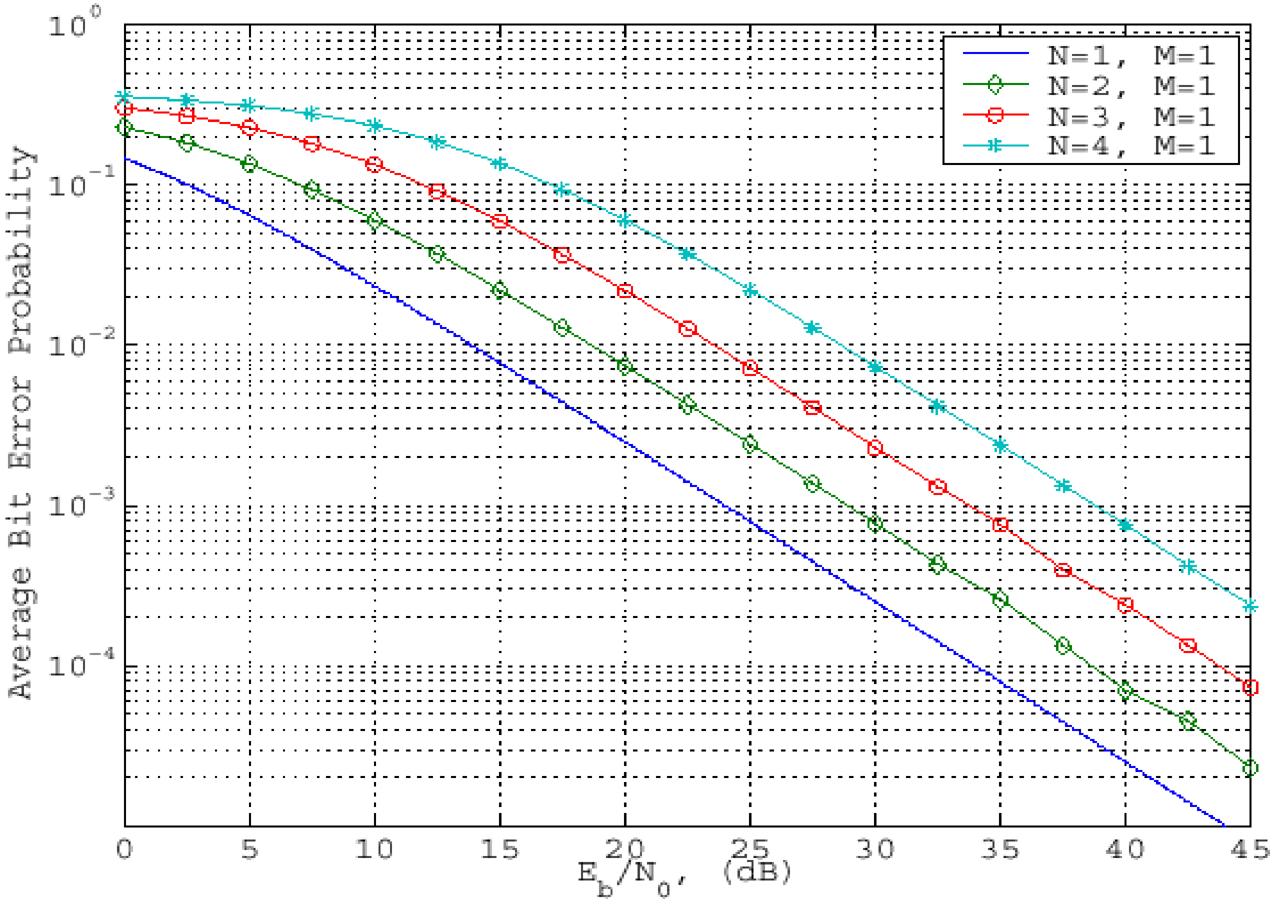
Multiple Antenna Systems …

- *Maximum Likelihood (ML) detection*: The receiver computes the following metric for all possible sequences and chooses the sequence that gives the minimum

$$\begin{aligned}\mathcal{C}(\boldsymbol{I}) &= \sum_{m=1}^{M}\left|y_{l,m}(k) - \sum_{n=1}^{N} h_{mn}\hat{I}_n(k)\right|^2 \\ &= \sum_{m=1}^{M}\left|\sum_{n=1}^{N} h_{mn}\left(I_n(k) - \hat{I}_n(k)\right) + z_m(k)\right|^2\end{aligned}$$

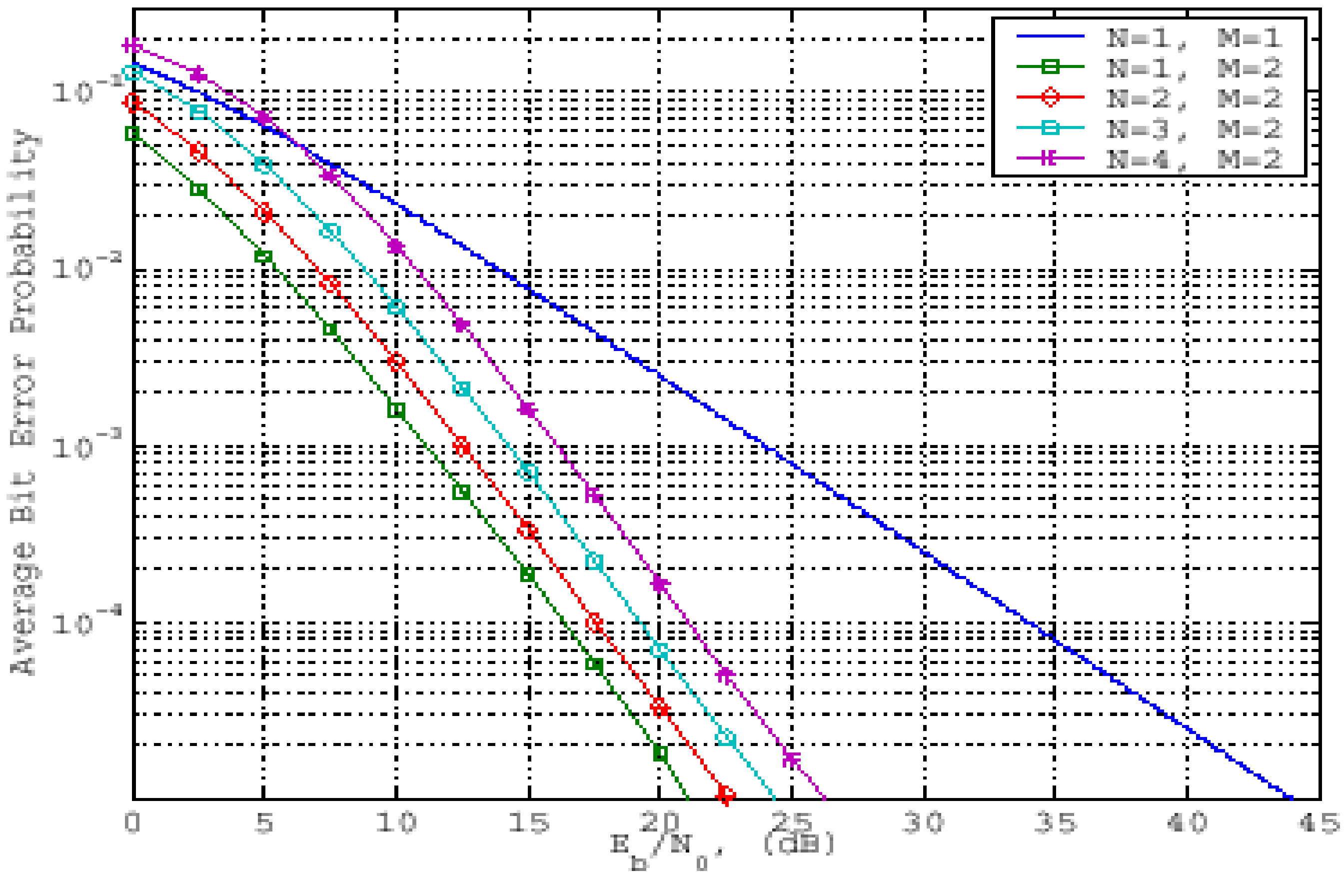
- It is also possible to employ *Minimum Mean-Square-Error* (MMSE) detection where the square of the difference between the expected data sequence and the actual received sequence is minimized

Multiple Antenna Systems - Performance of ML Detection



- Multiple transmit antennas and one receive antenna
- Coherent QPSK modulation is used

Multiple Antenna Systems - Performance of ML Detection



- Multiple transmit antennas and two receive antennas
- Coherent QPSK modulation is used