Chapter 8: Communication Through Fading Multipath Channels (Revision)

Addis Ababa Institute of Technology
Addis Ababa University

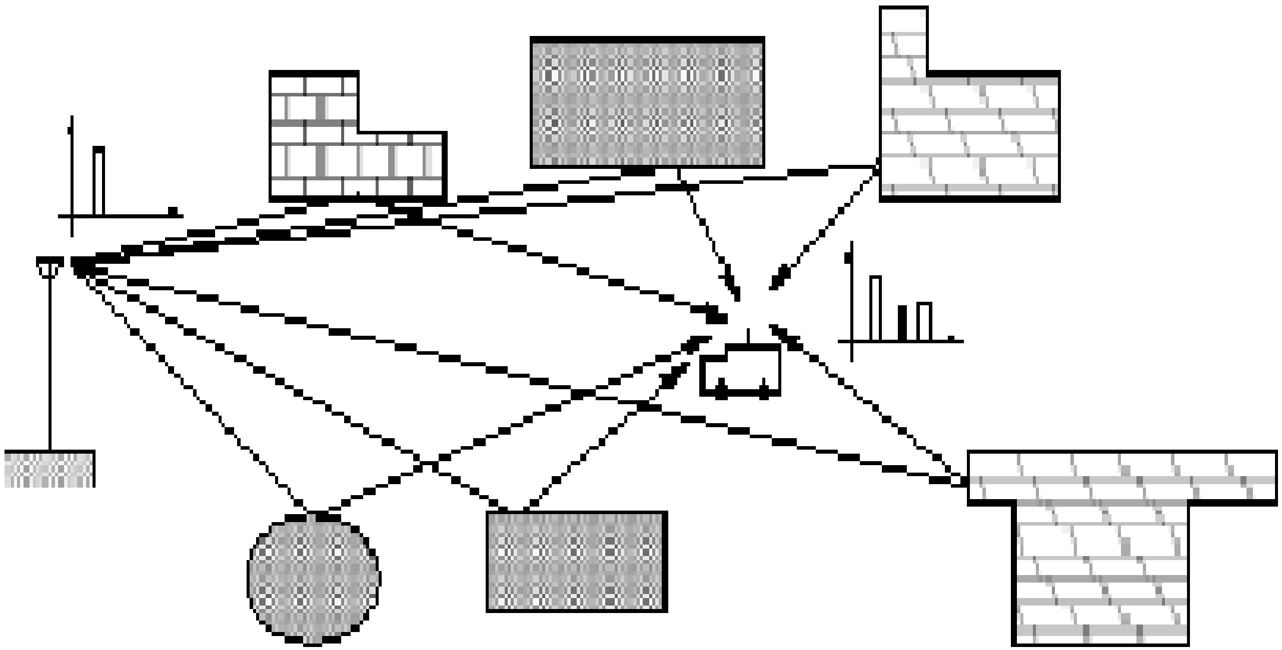
Graduate Program
Department of Electrical and Computer Engineering

Communication Through Fading Multipath Channels

- Earlier we studied design and performance of digital communication systems for transmission over
  - Either an AWGN channel
  - Or a linear filter channel with AWGN
- Now consider signal design, receiver structure and performance for channels having *randomly time variant impulse responses*
- Such channels arise in transmissions over many radio communication channels such as the following
  - Shortwave ionospheric radio communication (3 – 30 MHz)
  - VHF ionospheric forward scatter (30 – 300MHz)
  - Tropsopheric scatter in the 300 - 3000MHz frequency band (UHF) and the 3000 – 30,000MHz band (SHF)

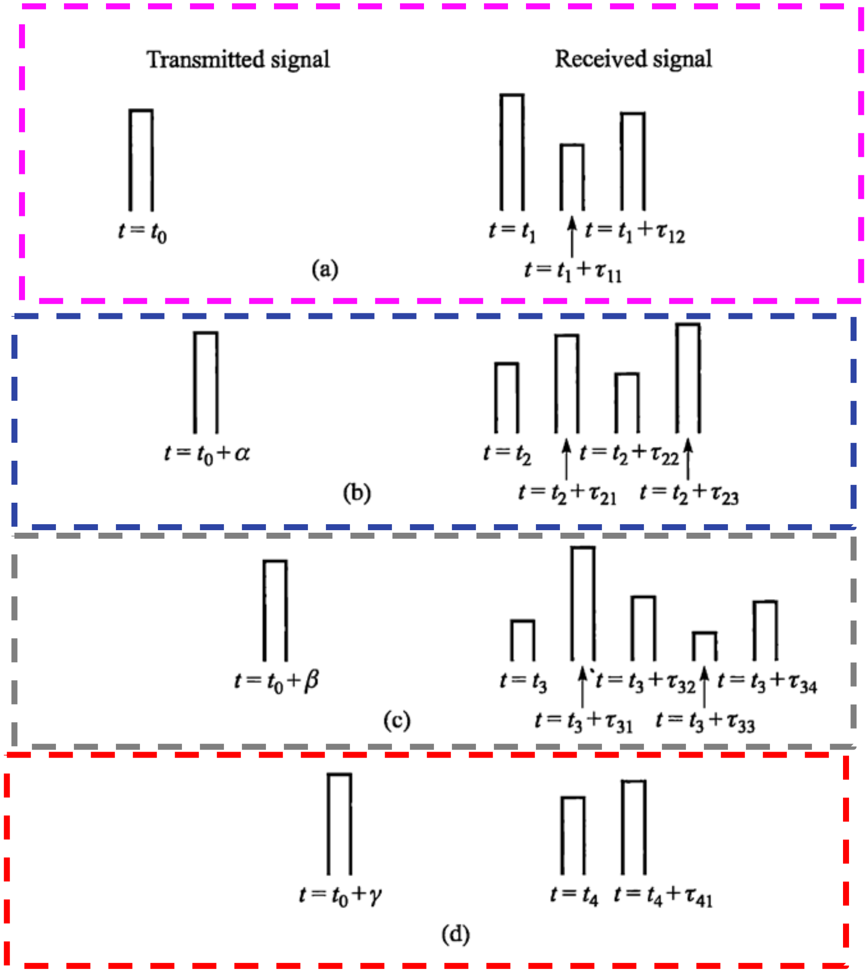
Communication Through Fading Multipath …

- The time-variant impulse responses of these channels are due to the constantly changing physical characteristics of the medium in which the signal propagates



A Multipath Fading Environment

Characterization of Fading Multipath ….



- Observations
  - Time spreading
  - Time variation

E.g., response of a time-variant multipath channel to a very narrow pulse

Characterization of Fading Multipath Channels

- Time Spread: A short pulse transmitted over a time-varying multiple channel is received as a *train of pulses* of varying magnitude
- The time variations appear to be *unpredictable* to the user of the channel
- Reasonable to characterize the time variant multipath channel *statistically*
- Consider the transmission of the signal *s(t)* over the radio channel

$$s(t) = Re\{s_l(t)e^{j2\pi f_c t}\}$$

Characterization of Fading Multipath Channels …

- The received signal can be written as

$$r(t) = \sum_i \alpha_i(t) s[t - \tau_i(t)] + n(t)$$
$$= Re\left\{\left(\sum_i \alpha_i(t) e^{-j2\pi f_c \tau_i(t)} s_l[t - \tau_i(t)]\right) e^{j2\pi f_c t}\right\} + n(t)$$

- The equivalent lowpass of the received signal can be expressed as

$$r_l(t) = \sum_i \alpha_i(t) e^{-j\theta_i(t)} s_l[t - \tau_i(t)] + z(t)$$

- $\alpha_i(t)$ is the weight coefficient of path i
- $\tau_i(t)$ is the time delay of path i
- $z(t)$ is lowpass equivalent complex additive channel noise

Characterization of Fading Multipath Channels …

- The received signal can also be written as

$$r_l(t) = \int_{-\infty}^{\infty} h(\tau;\mathrm{t})\, s_l(t-\tau)\, d\tau + z(t)$$

- $h(\tau;\mathrm{t})$ is the channel impulse response

$$h(\tau;\mathrm{t}) = \sum_{i=-\infty}^{\infty} \alpha_i(t) e^{-j\theta_i(t)} \delta\left(\tau - \tau_i(t)\right)$$

- The channel transfer function can be written as

$$H(f;t) = \int_{-\infty}^{\infty} h(\tau;\mathrm{t}) e^{-j2\pi f\tau} d\tau$$

$$= \sum_{-\infty}^{\infty} \alpha_i(t) e^{-j\theta_i(t)} e^{-j2\pi f\tau_i(t)}$$

Characterization of Fading Multipath Channels …

- The signal transmitted over such a communication channel will experience
  - A time dependent amplitude distortion
  - A time dependent envelope delay distortion
- The *time-varying* channel is often referred to as *doubly spread channel*
  - The time variation of the channel spreads the signal in frequency
  - The frequency variation of the channel spreads the signal in time
- We will explore these characteristics further in what follows through characterizing the communication channel using correlation properties of the of the fading channel

Correlation Properties of Fading Multipath Channel

- The channel impulse response is modeled as *wide-sense stationary* process with uncorrelated scattering
- The autocorrelation function of the process *H(f;t)* is

$$\begin{aligned}\phi_H(f, f+\Delta f; t, t+\Delta t) &= E\left\{H^*(f,t)\,H(f+\Delta f; t+\Delta t\right\} \\ &= \phi_H(\Delta \mathrm{f}\,; \Delta t)\end{aligned}$$

- The function $\phi_H(\Delta f;\Delta t)$ is called the *spaced-frequency, spaced-time* correlation function of the channel
- With $\Delta t = 0$, we obtain the *frequency correlation* function of the channel

$$\phi_H(\Delta f) \equiv \phi_H(\Delta f; 0)$$

- With $\Delta f = 0$, we obtain the *time correlation* function

$$\phi_H(\Delta t) \equiv \phi_H(0; \Delta t)$$

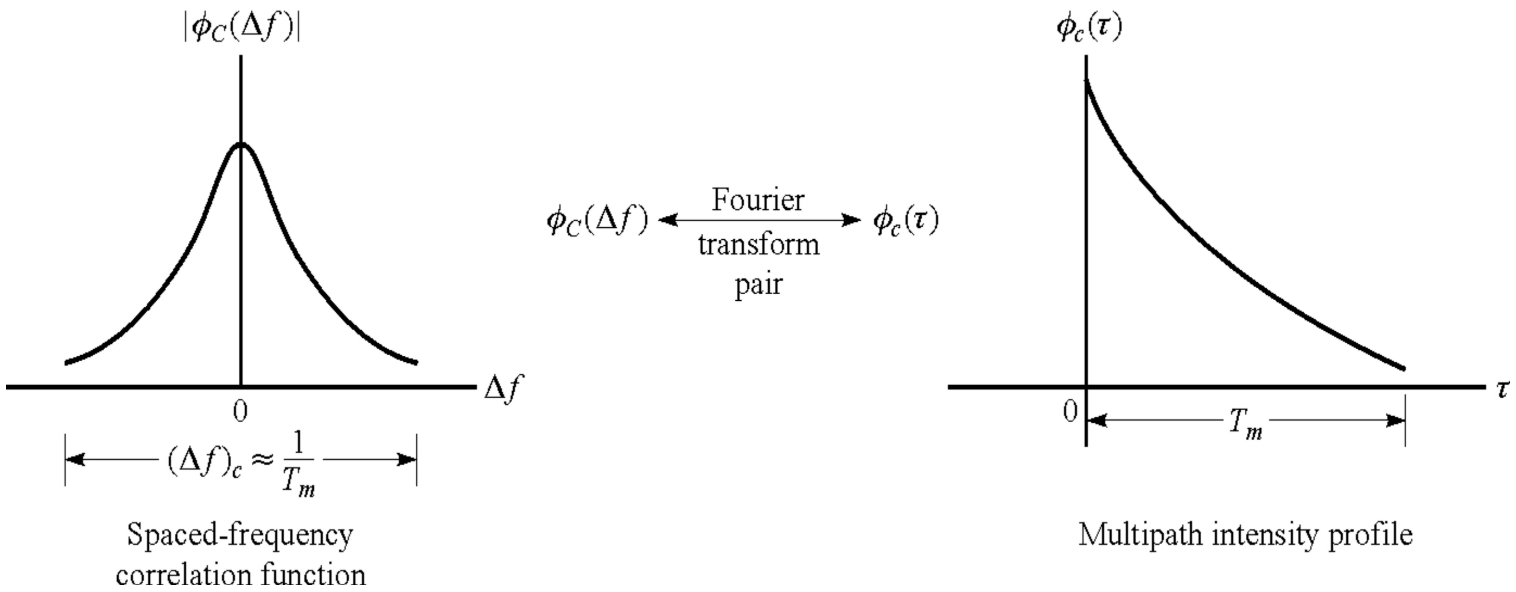
Frequency Correlation of Fading Multipath Channels

- The *intensity profile* of the channel is defined as the inverse transform of $\phi_H(\Delta f)$

$$\phi_h(\tau) = F^{-1}\left\{\phi_H(\Delta f)\right\} = \int_{-\infty}^{\infty} \phi_H(\Delta f) e^{j2\pi\Delta f\tau} d\Delta f$$

- The width of the region where $\phi_h(\tau)$ is non-zero is called the *maximum delay spread* of the channel, denoted by T_m
- The width of the region where $\phi_H(\Delta f)$ is non-zero is called the *coherence bandwidth* of the channel, denoted by B_m ($B_m \approx 1/T_m$)
- Two signals separated in frequency by more than B_m are affected differently by the channel

Frequency Correlation of Fading Multipath …



$$(\Delta f)_c = B_m$$

Relationship between $\phi_c(\Delta f)$ and $\phi_c(\tau)$

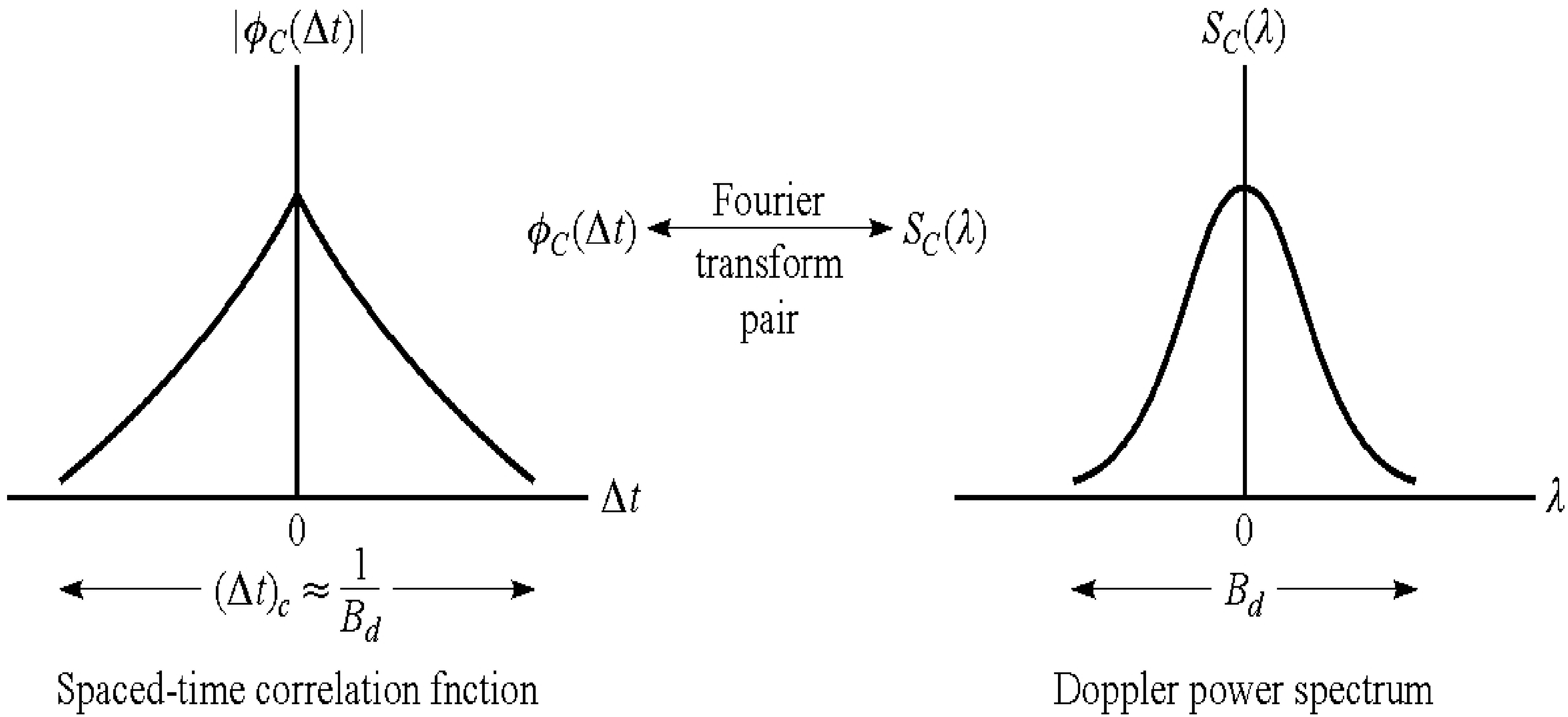
Time Correlation of Fading Multipath Channel

- The Doppler power spectrum of the channel is defined as

$$S_H(\psi) = \int_{-\infty}^{\infty} \phi_H(\Delta t) e^{j2\pi\psi\Delta t} d\Delta t$$

- The width of the region where $S_H(\psi)$ is non-zero is called the *maximum Doppler spread* of channel, denoted by B_d
- The width of the region where $S_H(\Delta t)$ is non-zero is called the *coherence time* of the channel, denoted by T_d with $T_d \approx 1/B_d$
- Two signals separated in time by more than T_d are affected differently by the channel

Time Correlation of Fading Multipath Channel …



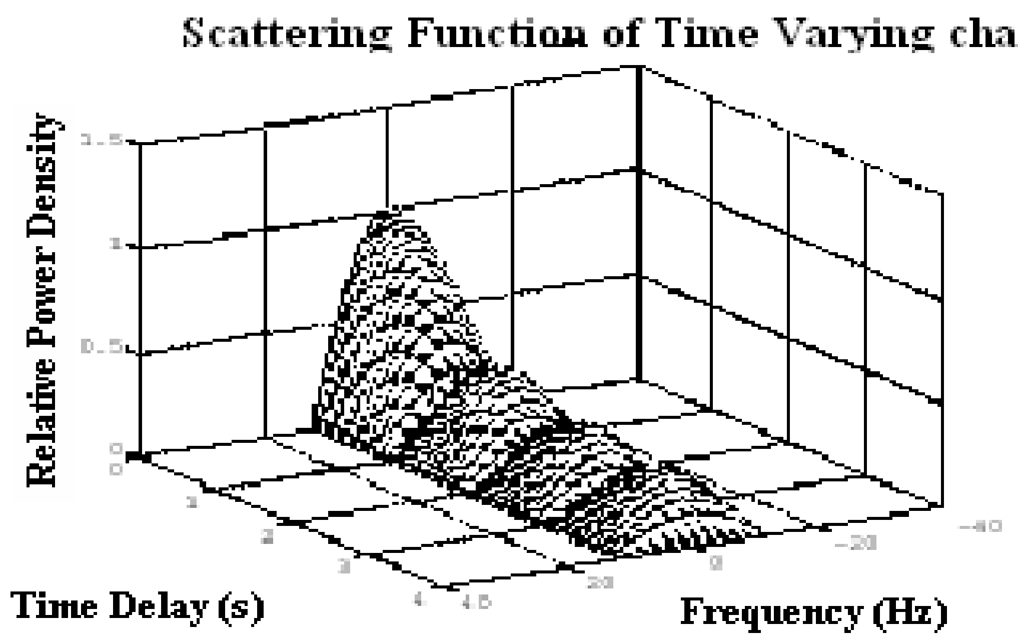
$(\Delta t)_c = T_d$

Relationship between $\phi_c(\Delta t)$ and $S_c(\lambda)$

The Scattering Function of the Channel

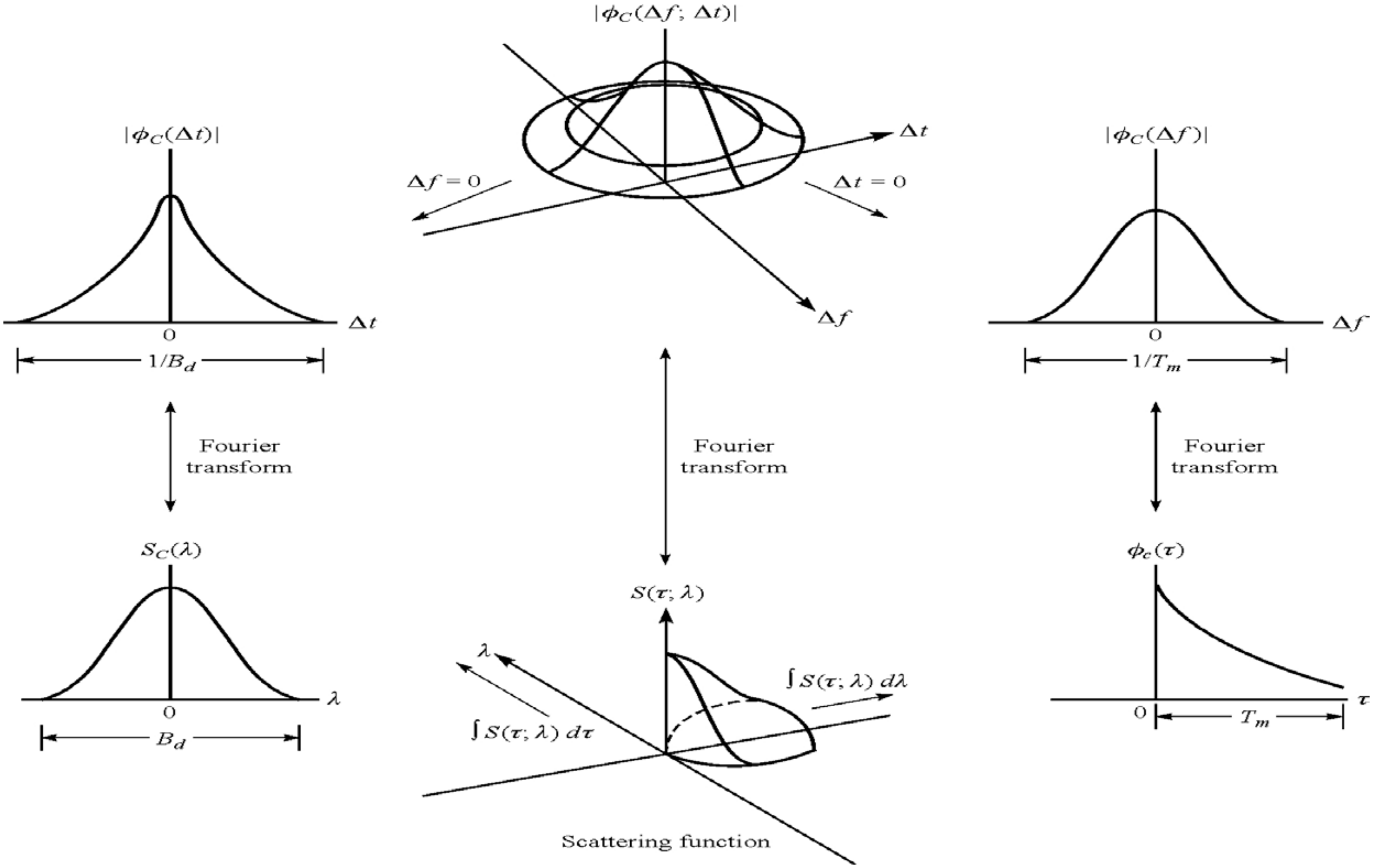
- The scattering function of the Channel is defined as

$$S_h(\tau,\psi)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi_H(\Delta f,\Delta t)e^{j2\pi\psi\Delta t}e^{j2\pi\tau\Delta f}\,d\Delta t\,d\Delta f$$



Example of Scattering function of Time Varying Channel

The Scattering Function of the Channel …



Relationships among the channel correlation functions and power spectra

Frequency-Nonselective (Flat) Fading Channels

- The baseband equivalent of the received signal is given by

$$r_l(t) = \sum_{\mathrm{i}} \alpha_{\mathrm{i}}(\mathrm{t})\mathrm{e}^{-\mathrm{j}\theta_i(\mathrm{t})} s_l\left[t - \tau_i(t)\right] + z(t)$$

- When $W_s << B_m$, the signal $r_l(t)$ can be written as

$$r_l(t) = \left(\sum_i \alpha_i(t) e^{-j\theta_i(t)} \right) s_l(t - \tau_0) + z(t) = H(0;t) s_l(t - \tau_0) + z(t)$$

- For a medium with a lot of scatterers, the central limit theorem applies such the $H(0;t)$ is a complex Gaussian process

$$\mathrm{H}(0,\mathrm{t}) = \mathrm{x}_\alpha(\mathrm{t}) + \mathrm{j}\mathrm{y}_\alpha(\mathrm{t}) = \alpha(\mathrm{t})\mathrm{e}^{-\mathrm{j}\theta_i(\mathrm{t})}$$

and x_α(t) and y_α(t) are uncorrelated Gaussian processes

Frequency-Nonselective (Flat) Fading Channels …

- The received signal then becomes

$$r_l(t) = \alpha(t)e^{-j\theta(t)}s_l(t-\tau_0) + z(t)$$

  - The channel causes amplitude attenuation → flat fading
  - The channel does not cause any delay → No ISI

Rayleigh Fading Channel

- For channels with only a diffused multipath signal, the multiplicative distortion

$$\alpha(t)\, e^{-j\theta(t)} = x_\alpha(t) + j y_\alpha(t)$$

- Is a zero-mean complex Gaussian process
- x_α(t) and y_α(t) are uncorrelated zero-mean Gaussian process
- The amplitude α(t) is *Rayleigh distributed* with pdf

$$p(\alpha) = \frac{\alpha}{\sigma^2} e^{-\alpha^2/2\sigma^2} \quad \alpha \geq 0 \text{ with } 2\sigma^2 = \sum_i \overline{\left|\alpha_i(t)\right|^2}$$

- The phase θ(t) is *uniformly distributed* over the interval (0,2π)

Rayleigh Fading Channel …

- The channel is said to be *slowly varying* over the interval *T* *iff* $T \ll T_d$

$$\alpha(t)e^{-j\theta(t)} = \alpha e^{-j\theta} \quad 0 \leq t \leq T$$

- α and θ are now random variables

Rician Fading Channel

- In addition to the diffused fading multipath process, a *dominant line-of-sight* signal may also arrive at the receiver
- The received signal will thus be

$$\begin{aligned} r_l(t) &= \alpha_i(t)e^{-j\theta(t)}s_l(t-\tau_0)+z(t) \\ &= \left[\alpha_0 + x_\alpha(t) + jy_\alpha(t)\right]s_l(t-\tau_0)+z(t) \end{aligned}$$

  - α_0 is constant and is the line-of-sight component
  - *x(t)* and *y(t)* are zero-mean Gaussian processes
- The fading amplitude α(t) is Rician distributed with pdf

$$p(\alpha) = \frac{\alpha}{\sigma^2} e^{-\frac{\alpha^2+\alpha_0}{2\sigma^2}} I_0\left(\frac{\alpha\alpha_0}{\sigma^2}\right) \quad \alpha \geq 0$$

Rician Fading Channel …

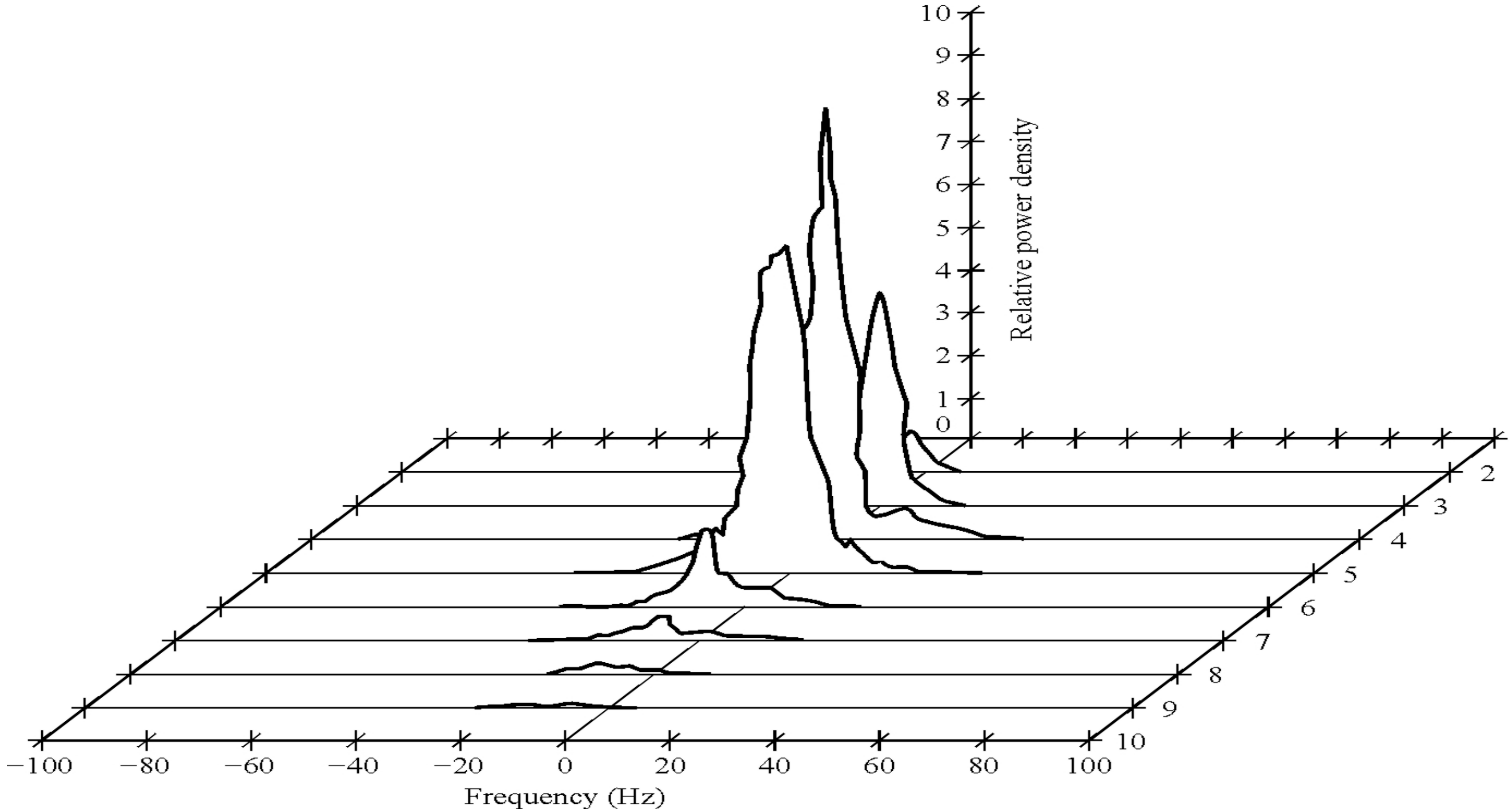
- The strength of the line-of-sight component is defined by the Ricean factor *K*

$$K = \frac{\alpha_0^2}{2\sigma^2}$$

- *K* = +∞ corresponds to the case of ideal channel (perfect line of sight)
- *K* = 0 corresponds to the case of Rayleigh fading channel

Rician Fading Channel …

- Scattering function of a medium-range tropsopheric scatter channel
- The taps delay increment is 0.1 μs.



Mobile radio Channel – Doppler Frequency Shift

- When the mobile moves the delay of each path will vary with time

$$\tau_i(t) = \tau_i(t_0) \pm \frac{v}{c} \cos\varphi_i t \quad \Rightarrow \quad \theta_i(t) = \theta_i(t_0) \pm 2\pi(f_m \cos\varphi_i)t$$

  - φ_i is the angle of arrival of the path *i*
  - f_m is the maximum Doppler frequency shift given as $f_m = \frac{v}{c} f_0$
- The channel impulse response becomes

$$h(\tau;t) = \sum_{i=\infty}^{\infty} \alpha_i(t) e^{-j[2\pi f_i t + \theta_i(t_0)]} \delta[\tau - \tau_i(t)]$$

- $f_i = \pm f_m \cos\varphi_i$ is the Doppler frequency shift of path *i*

Mobile radio Channel – Doppler Frequency Shift …

- The received signal corresponding to the signal *s(t)* will be

$$r(t) = \mathrm{Re}\left\{ \sum_{i=\infty}^{\infty} \alpha_i(t) e^{-j\theta_i(t)} s_l\left[t - \tau_i(t)\right] e^{j2\pi(f_0+f_i)} \right\} + n(t)$$

- Each path *i* shifts the signal carrier frequency by f_i
- The time correlation of the multiplicative distortion for the signal *s(t)* can be shown to be

$$\phi_H(\Delta t) = E\left\{H^*(f;t)H(f,t+\Delta t)\right\} = 2\sigma^2 J_0\left(2\pi f_D \Delta t\right)$$

  - Where J_0( ) is the zero order Bessel function of the first kind
- The Doppler spectrum is then given by

$$S(\psi) = \mathcal{F}\left\{\phi_H(\Delta t)\right\} = \begin{cases} \frac{k}{\sqrt{1-(\psi/f_D)^2}}, & |\psi| < f_D \\ 0, & |\psi| > f_D \end{cases}$$

Frequency Selective Fading Channels

- Consider a digital signal with equivalent lowpass

$$s_l(t) = \sum_{n=-\infty}^{+\infty} I_n g_t(t - nT_s)$$

in a multipath environment with coherent bandwidth B_m

- The received sample can be expressed as

$$r_k = r_l(t) * g_r(t)\,|_{t=kT_s} = \sum_{n=-\infty}^{+\infty} I_n \left[\sum_{i=-\infty}^{+\infty} \alpha_i(kT_s) e^{-\theta_i(kT_s)} g\left[(k-n)T_s - \tau_i\right]\right] + z_k$$

$$= \sum_{n=-\infty}^{+\infty} c_{k-n}(kT_s) I_n + z_k \approx \sum_{n=0}^{L-1} c_n(kT_s) I_{k-n} + z_k$$

$$g(t) = g_t(t) * g_r(t)$$

Frequency Selective Fading Channels …

- The received signal sample can then be written as (Truncated)

$$r_k = \sum_{n=0}^{L-1} c_n(t) I_{k-n} + z_k = c_0(t) I_k + \sum_{n=1}^{L-1} c_n(t) I_{k-n} + z_k$$

- The *c(t)'s* are uncorrelated complex Gaussian processes

Frequency Selective Fading Channels …

- The channel can now be modeled as a tapped delay-line *(See section 14.5.1 in text for details)*