Chapter 4: Optimum Receivers for Additive Gaussian Noise Channel

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Graduate Program
School of Electrical and Computer Engineering

Overview

- Regenerative repeaters
- Link budget

Regenerative Repeaters

- The performance of communication systems that we have seen up to now over a given channel depends solely on the received SNR, ε_b/N_0
- Hence, performance is limited by the ***additive noise***
- Another factor is the channel attenuation, where the received signal is

$$r(t) = \alpha s(t) + n(t); \qquad 0 \le \alpha \le 1$$

- Such that, if the transmitted energy is ε_b, then the received energy will be $\alpha^2\varepsilon_b$ and the received SNR will be $\alpha^2\varepsilon_b/N_0$
- This makes the system more vulnerable to the additive noise

Regenerative Repeaters …

- For analog systems repeaters (amplifiers) are used to boost the signal strength along the transmission path
  - Note that the noise is also amplified, i.e, noise propagation
- In digital systems, the "repeater" detects and regenerates a clean signal (noise free) along the transmission channel
  - Called regenerative repeaters and are used on wire line, fiber and wireless transmission systems
- Regenerative repeaters consist of demodulators/detectors and transmitters
- If an error occurs in the detector, this error is propagated forward

Regenerative Repeaters …

- To evaluate the effect of errors on overall performance of regenerative repeaters, consider a binary PAM where the probability of error for a single hop is

$$P_b = Q\left(\sqrt{\frac{2\varepsilon_b}{N_0}}\right)$$

- Since P_b is very small we ignore the possibility that any one bit will be detected incorrectly more than once in transmission through the channel with k repeaters
- Then, the probability of error will be

$$P_b \approx KQ\left(\sqrt{\frac{2\varepsilon_b}{N_0}}\right)$$

Regenerative Repeaters …

- For k analog repeaters in the channel the received SNR is reduced by a factor of k such that the probability of error becomes

$$P_{bA} = Q\left(\sqrt{\frac{2\varepsilon_b}{kN_0}}\right)$$

- Example: k = 100 (1000km with repeaters every 10km) and an overall error probability of 10^{-5}
  - For regenerative repeaters $\frac{\varepsilon_b}{N_o}$ = 11.3 dB
  - For analog repeaters $\frac{\varepsilon_b}{N_o}$ = 18.3 dB which is *70 times transmitter power compared to the digital system*

Overview

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Communication Link Budget

- For line of sight (LOS) microwave and satellite channels the following parameters are under the control of the system designer
  - *Size of the transmitter and receiver antennas*
  - *Transmitter power*
  - *SNR required for a specified level of performance at some desired data rate*
- Factors that influence system design procedure are
- Transmitter antenna isotropically radiating with P_T in free space produces a power density $P_T/4\pi d^2$ at a distance d from the antenna
- If the antenna has directivity (gain) G_T, the power density is $G_T P_T/4\pi d^2$
  - Note: $G_T P_T$ is the effective radiated power (ERP)

Regenerative Repeaters …

- Receiving antenna directed in the direction of the radiated power gathers a portion of the power proportional to the cross-sectional area
- Thus the received power is: $P_R = \frac{P_T G_T}{4\pi d^2} A_R$
- Where $A_R = \frac{G_R \lambda^2}{4\pi}$ is the effective area of the antenna and

  G_R is the gain of the receiver antenna
- Substitution gives:

$$P_R = \frac{P_T G_T G_R \lambda^2}{(4\pi d)^2} = \frac{P_T G_T G_R}{\left(4\pi d / \lambda\right)^2}$$

- The free-space pathloss L_S is given as $L_s = \left(\lambda / 4\pi d\right)^2$

Regenerative Repeaters …

- Other losses such as atmospheric loss can be accounted for by introducing an additional loss factor whose total effect is represented by L_a
- Then the received power level is given by

$$P_R = P_T G_T G_R L_S L_a$$

- OR in dB

$$(P_R)_{dB} = (P_T)_{dB} + (G_T)_{dB} + (G_R)_{dB} + (L_S)_{dB} + (L_a)_{dB}$$

Regenerative Repeaters …

- Antenna gain and effective area depend on the wavelength of the radiation and the physical dimensions of the antenna
- Example 1: Parabolic antenna (dish) of diameter D has

$$A_R = \frac{1}{4}\pi D^2 \eta$$

- where $\frac{1}{4}\pi D^2$ is the physical area and η is the *illumination efficiency* which is normally $0.5 \le \eta \le 0.6$
- Thus the receiver antenna gain can be expressed as

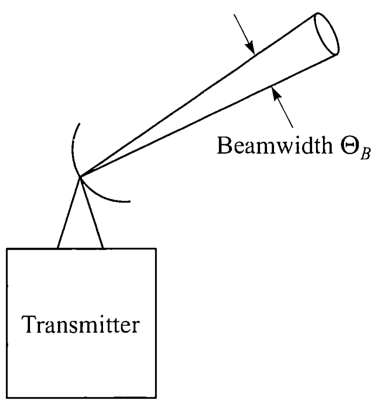
$$G_R = \eta\left(\frac{\pi D}{\lambda}\right)^2$$

Regenerative Repeaters …

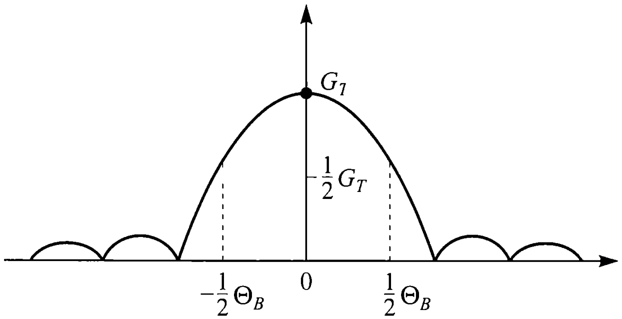
- Example 2: Horn antenna having a physical area A and efficiency factor of 0.8, an effective area of A_R=0.8A and

$$G_R \approx \frac{10\,A}{\lambda^2}$$

- Note that the directivity or gain of an antenna depends on the -3 dB beamwidth of the antenna, Θ_B
- For a parabolic antenna $\Theta_B = 70\lambda / D$ (degrees) and the gain is inversely proportional to square of Θ_B
- I.e., reducing the beam width by a factor of ½ increases the gain by a factor of 4



(a) Beamwidth of antenna



(b) Antenna pattern

Regenerative Repeaters …

- Example: The following are parameters given for a geosynchronous satellite in an orbit that is 36,000 Km above the surface of the earth
  - Transmitter power P_T = 100W (20 dB above 1W)
  - Transmitter antenna gain = 17 dB → ERP +37 dB
  - Receiver antenna is parabolic with diameter of 3m and an efficiency factor of 0.5
  - Down link frequency = 4GHz
- From the given data the wavelength $\lambda=3/40$ m

$$G_R = 0.5\,(3 \times 40\pi / 3)^2 = 7895.68 \approx 39 \text{ dB}$$

$$L_S = [(3/40) \times 1/(4\pi \times 3.6 \times 10^7)]^2 = 1.66 \times 10^{-10} = -195.6 \text{ dB}$$

- If there are no other transmission losses ($L_a = 0$) the received power in dB will be

$$P_R = 20 + 17 + 39 - 195.6 = -119.6 \text{ dB} \approx 1.1 \times 10^{-12} \text{ W}$$

Regenerative Repeaters …

- Consider thermal noise that arises at the front end of the receiver which has a relatively flat power density spectrum up to 10^{12} Hz
- i.e., $N_0 = k_B T_0$ W/ Hz; where $k_B = 1.38 \times 10^{-23}$ W-S /^oK (Boltzman constant) and T_0 is the absolute temperature
- Total power for a band width W = WN_0; and

$$\frac{\varepsilon_b}{N_0} = \frac{T_b P_R}{N_0} = \frac{1}{R}\frac{P_R}{N_0}$$

$$\frac{P_R}{N_0} = R\left(\frac{\varepsilon_b}{N_0}\right)_{req}$$

Regenerative Repeaters …

- Let T_0=300 K, then N_0 = 4.1X10^{-21} W/Hz=-203.9 W/Hz
- If the required SNR per bit =10dB, then

$$\frac{P_R}{N_0} = -119.6 + 203.9 = 84.3\ dB;$$

$$R_{dB} = 84.3 - 10 = 74.3\ dB$$ (with respect to 1 bit/s)

$$R = 26.9\ Megabit/s$$ (420 PCM channels each operating at 64 Kbps)