CHAPTER 13

Radiation Exchange Between Surfaces

Having thus far restricted our attention to radiative processes that occur at a *single surface*, we now consider the problem of radiative exchange between two or more surfaces. This exchange depends strongly on the surface geometries and orientations, as well as on their radiative properties and temperatures. Initially, we assume that the surfaces are separated by a *nonparticipating medium.* Since such a medium neither emits, absorbs, nor scatters, it has no effect on the transfer of radiation between surfaces. A vacuum meets these requirements exactly, and most gases meet them to an excellent approximation.

Our first objective is to establish geometrical features of the radiation exchange problem by developing the notion of a *view factor*. Our second objective is to develop procedures for predicting radiative exchange between surfaces that form an *enclosure*. We conclude our consideration of radiation exchange between surfaces by considering the effects of a *participating medium*, namely, an intervening gas that emits and absorbs radiation.

13.1 *The View Factor*

To compute radiation exchange between any two surfaces, we must first introduce the concept of a *view factor* (also called a *configuration* or *shape factor*).

13.1.1 The View Factor Integral

The view factor F_{ij} is defined as the *fraction of the radiation leaving surface i that is intercepted by surface j*. To develop a general expression for F_{ij}, we consider the arbitrarily oriented surfaces A_i and A_j of Figure 13.1. Elemental areas on each surface, dA_i and dA_j, are connected by a line of length R, which forms the polar angles θ_i and θ_j, respectively, with the surface normals $\mathbf{n}_i$ and $\mathbf{n}_j$. The values of R, θ_i, and θ_j vary with the position of the elemental areas on A_i and A_j.

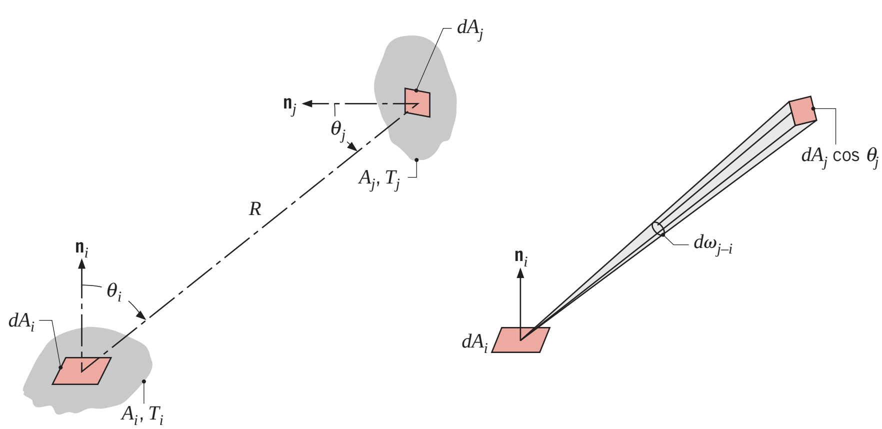


FIGURE 13.1 View factor associated with radiation exchange between elemental surfaces of area dA_i and dA_j.

From the definition of the radiation intensity, Section 12.2.2, and Equation 12.6, the rate at which radiation *leaves* dA_i and is *intercepted* by dA_j may be expressed as

$$dq_{i \to j} = I_{e+r,i} \cos \theta_i \, dA_i \, d\omega_{j-i}$$

where $I_{e+r,i}$ is the intensity of radiation leaving surface i by emission and reflection and $d\omega_{j-i}$ is the solid angle subtended by dA_j when viewed from dA_i. With $d\omega_{j-i} = (\cos \theta_j \, dA_j)/R^2$ from Equation 12.2, it follows that

$$dq_{i \to j} = I_{e+r,i} \frac{\cos \theta_i \cos \theta_j}{R^2} dA_i \, dA_j$$

Assuming that surface i *emits* and *reflects diffusely* and substituting from Equation 12.22, we then obtain

$$dq_{i \to j} = J_i \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_i \, dA_j$$

The total rate at which radiation leaves surface i and is intercepted by j may then be obtained by integrating over the two surfaces. That is,

$$q_{i \to j} = J_i \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_i \, dA_j$$

where it is assumed that the radiosity J_i is uniform over the surface A_i. From the definition of the view factor as the fraction of the radiation that leaves A_i and is intercepted by A_j,

$$F_{ij} = \frac{q_{i \to j}}{A_i J_i}$$

it follows that

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_i \, dA_j \qquad (13.1)$$

Similarly, the view factor F_{ji} is defined as the fraction of the radiation that leaves A_j and is intercepted by A_i. The same development then yields

$$F_{ji} = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_i \, dA_j \qquad (13.2)$$

Either Equation 13.1 or 13.2 may be used to determine the view factor associated with any two surfaces that are *diffuse emitters* and *reflectors* and have *uniform radiosity*.

13.1.2 View Factor Relations

An important view factor relation is suggested by Equations 13.1 and 13.2. In particular, equating the integrals appearing in these equations, it follows that

$$A_i F_{ij} = A_j F_{ji} \qquad (13.3)$$

This expression, termed the *reciprocity relation*, is useful in determining one view factor from knowledge of the other.

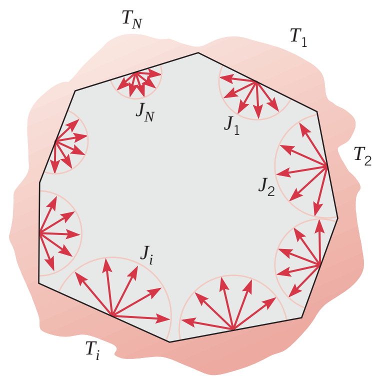


FIGURE 13.2
Radiation exchange in an enclosure.

Another important view factor relation pertains to the surfaces of an *enclosure* (Figure 13.2). From the definition of the view factor, the *summation rule*

$$\sum_{j=1}^{N} F_{ij} = 1 \tag{13.4}$$

may be applied to each of the N surfaces in the enclosure. This rule follows from the conservation requirement that all radiation leaving surface i must be intercepted by the enclosure surfaces. The term F_{ii} appearing in this summation represents the fraction of the radiation that leaves surface i and is directly intercepted by i. If the surface is concave, it *sees itself* and F_{ii} is nonzero. However, for a plane or convex surface, $F_{ii} = 0$.

To calculate radiation exchange in an enclosure of N surfaces, a total of N^2 view factors is needed. This requirement becomes evident when the view factors are arranged in the matrix form:

$$\begin{bmatrix} F_{11} & F_{12} & \cdots & F_{1N} \\ F_{21} & F_{22} & \cdots & F_{2N} \\ \vdots & \vdots & & \vdots \\ F_{N1} & F_{N2} & \cdots & F_{NN} \end{bmatrix}$$

However, all the view factors need not be calculated *directly*. A total of N view factors may be obtained from the N equations associated with application of the

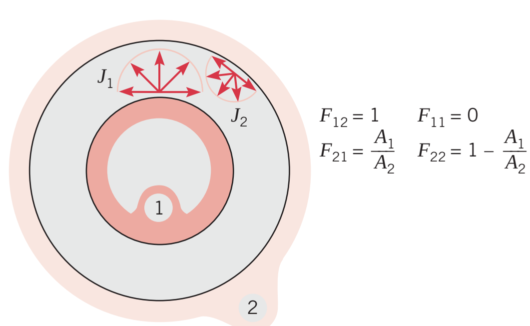


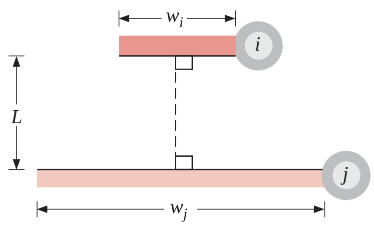
FIGURE 13.3
View factors for the enclosure formed by two spheres.

summation rule, Equation 13.4, to each of the surfaces in the enclosure. In addition, $N(N-1)/2$ view factors may be obtained from the $N(N-1)/2$ applications of the reciprocity relation, Equation 13.3, which are possible for the enclosure. Accordingly, only $[N^2 - N - N(N-1)/2] = N(N-1)/2$ view factors need be determined directly. For example, in a three-surface enclosure this requirement corresponds to

TABLE 13.1 View Factors for Two-Dimensional Geometries [4]

Geometry | **Relation**

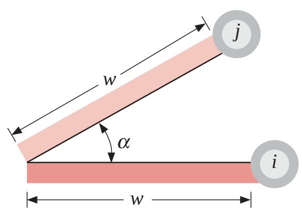
Parallel Plates with Midlines Connected by Perpendicular



$$F_{ij} = \frac{[(W_i + W_j)^2 + 4]^{1/2} - [(W_j - W_i)^2 + 4]^{1/2}}{2W_i}$$

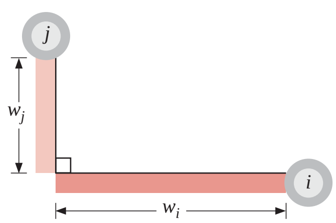
$$W_i = w_i/L, \; W_j = w_j/L$$

Inclined Parallel Plates of Equal Width and a Common Edge



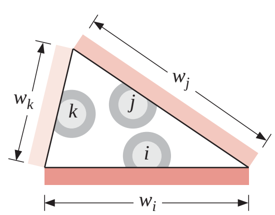
$$F_{ij} = 1 - \sin\left(\frac{\alpha}{2}\right)$$

Perpendicular Plates with a Common Edge



$$F_{ij} = \frac{1 + (w_j/w_i) - [1 + (w_j/w_i)^2]^{1/2}}{2}$$

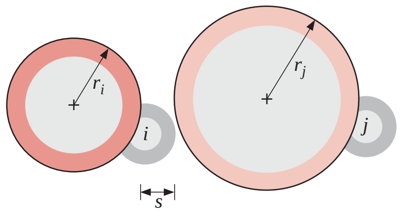
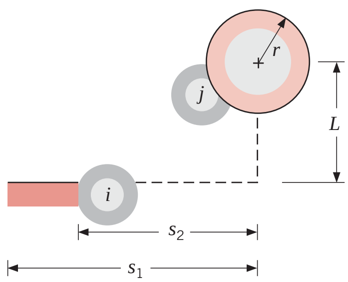
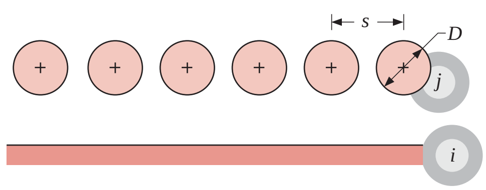
Three-Sided Enclosure



$$F_{ij} = \frac{w_i + w_j - w_k}{2w_i}$$

(*continues*)

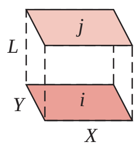
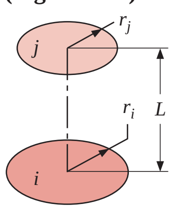
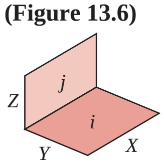
TABLE 13.1 *Continued*

Geometry	Relation
Parallel Cylinders of Different Radii 	$F_{ij} = \frac{1}{2\pi}\Big\{\pi + [C^2 - (R+1)^2]^{1/2} - [C^2 - (R-1)^2]^{1/2} + (R-1)\cos^{-1}\Big[\Big(\frac{R}{C}\Big) - \Big(\frac{1}{C}\Big)\Big] - (R+1)\cos^{-1}\Big[\Big(\frac{R}{C}\Big) + \Big(\frac{1}{C}\Big)\Big]\Big\}$ $R = r_j/r_i,\ S = s/r_i$ $C = 1 + R + S$
Cylinder and Parallel Rectangle 	$F_{ij} = \frac{r}{s_1 - s_2}\Big[\tan^{-1}\frac{s_1}{L} - \tan^{-1}\frac{s_2}{L}\Big]$
Infinite Plane and Row of Cylinders 	$F_{ij} = 1 - \Big[1 - \Big(\frac{D}{s}\Big)^2\Big]^{1/2} + \Big(\frac{D}{s}\Big)\tan^{-1}\Big[\Big(\frac{s^2 - D^2}{D^2}\Big)^{1/2}\Big]$

only $3(3 - 1)/2 = 3$ view factors. The remaining six view factors may be obtained by solving the six equations that result from use of Equations 13.3 and 13.4.

To illustrate the foregoing procedure, consider a simple, two-surface enclosure involving the spherical surfaces of Figure 13.3. Although the enclosure is characterized by $N^2 = 4$ view factors (F_{11}, F_{12}, F_{21}, F_{22}), only $N(N - 1)/2 = 1$ view factor need be determined directly. In this case such a determination may be made by *inspection*. In particular, since all radiation leaving the inner surface must reach the outer surface, it follows that $F_{12} = 1$. The same may not be said of radiation leaving

TABLE 13.2 View Factors for Three-Dimensional Geometries [4]

Geometry	Relation
Aligned Parallel Rectangles **(Figure 13.4)** 	$\bar{X} = X/L,\ \bar{Y} = Y/L$ $F_{ij} = \frac{2}{\pi \bar{X}\bar{Y}}\left\{\ln\left[\frac{(1+\bar{X}^2)(1+\bar{Y}^2)}{1+\bar{X}^2+\bar{Y}^2}\right]^{1/2} + \bar{X}(1+\bar{Y}^2)^{1/2}\tan^{-1}\frac{\bar{X}}{(1+\bar{Y}^2)^{1/2}} + \bar{Y}(1+\bar{X}^2)^{1/2}\tan^{-1}\frac{\bar{Y}}{(1+\bar{X}^2)^{1/2}} - \bar{X}\tan^{-1}\bar{X} - \bar{Y}\tan^{-1}\bar{Y}\right\}$
Coaxial Parallel Disks **(Figure 13.5)** 	$R_i = r_i/L,\ R_j = r_j/L$ $S = 1 + \frac{1+R_j^2}{R_i^2}$ $F_{ij} = \frac{1}{2}\{S - [S^2 - 4(r_j/r_i)^2]^{1/2}\}$
Perpendicular Rectangles with a Common Edge **(Figure 13.6)** 	$H = Z/X,\ W = Y/X$ $F_{ij} = \frac{1}{\pi W}\left(W\tan^{-1}\frac{1}{W} + H\tan^{-1}\frac{1}{H} - (H^2+W^2)^{1/2}\tan^{-1}\frac{1}{(H^2+W^2)^{1/2}} + \frac{1}{4}\ln\left\{\frac{(1+W^2)(1+H^2)}{1+W^2+H^2}\left[\frac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)}\right]^{W^2} \times \left[\frac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)}\right]^{H^2}\right\}\right)$

the outer surface, since this surface sees itself. However, from the reciprocity relation, Equation 13.3, we obtain

$$F_{21} = \left(\frac{A_1}{A_2}\right)F_{12} = \left(\frac{A_1}{A_2}\right)$$

From the summation rule, we also obtain

$$F_{11} + F_{12} = 1$$

in which case $F_{11} = 0$, and

$$F_{21} + F_{22} = 1$$

in which case

$$F_{22} = 1 - \left(\frac{A_1}{A_2}\right)$$

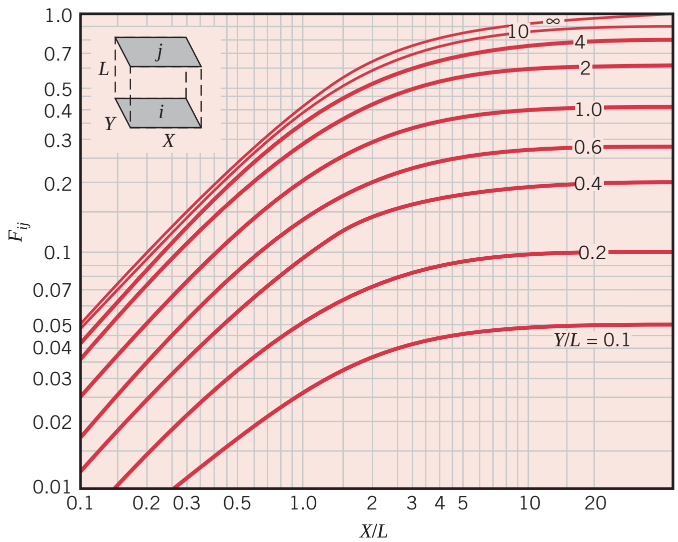


FIGURE 13.4 View factor for aligned parallel rectangles.

For more complicated geometries, the view factor may be determined by solving the double integral of Equation 13.1. Such solutions have been obtained for many different surface arrangements and are available in equation, graphical, and tabular form [1–4]. Results for several common geometries are presented in Tables 13.1 and 13.2 and Figures 13.4 through 13.6. The configurations of Table 13.1 are assumed to be infinitely long (in a direction perpendicular to the page) and are hence two-dimensional. The configurations of Table 13.2 and Figures 13.4 through 13.6 are three-dimensional.

It is useful to note that the results of Figures 13.4 through 13.6 may be used to determine other view factors. For example, the view factor for an end surface of a

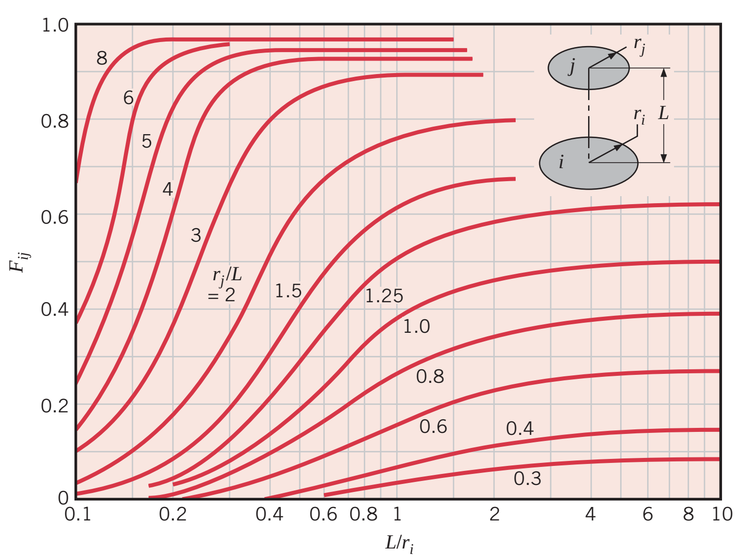


FIGURE 13.5 View factor for coaxial parallel disks.

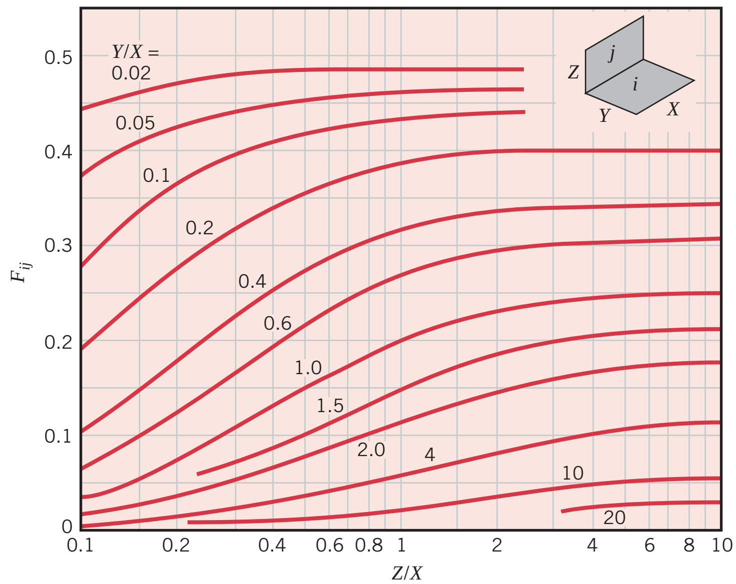


FIGURE 13.6 View factor for perpendicular rectangles with a common edge.

cylinder (or a truncated cone) relative to the lateral surface may be obtained by using the results of Figure 13.5 with the summation rule, Equation 13.4. Moreover, Figures 13.4 and 13.6 may be used to obtain other useful results if two additional view factor relations are developed.

The first relation concerns the additive nature of the view factor for a subdivided surface and may be inferred from Figure 13.7. Considering radiation from surface i to surface j, which is divided into n components, it is evident that

$$F_{i(j)} = \sum_{k=1}^{n} F_{ik} \tag{13.5}$$

where the parentheses around a subscript indicate that it is a composite surface, in which case (j) is equivalent to $(1, 2, \ldots, k, \ldots, n)$. This expression simply states that radiation reaching a composite surface is the sum of the radiation reaching its parts. Although it pertains to subdivision of the receiving surface, it may also be used to obtain the second view factor relation, which pertains to subdivision of the originating

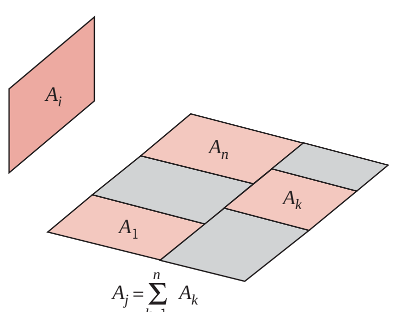


FIGURE 13.7 Areas used to illustrate view factor relations.

surface. Multiplying Equation 13.5 by A_i and applying the reciprocity relation, Equation 13.3, to each of the resulting terms, it follows that

$$A_j F_{(j)i} = \sum_{k=1}^{n} A_k F_{ki} \tag{13.6}$$

or

$$F_{(j)i} = \frac{\sum_{k=1}^{n} A_k F_{ki}}{\sum_{k=1}^{n} A_k} \tag{13.7}$$

Equations 13.6 and 13.7 may be applied when the originating surface is composed of several parts.

For problems involving complicated geometries, analytical solutions to Equation 13.1 may not be obtainable, in which case values of the view factors must be estimated using numerical methods. In situations involving extremely complex structures that may have hundreds or thousands of radiative surfaces, considerable error may be associated with the numerically calculated view factors. In such situations, Equation 13.3 should be used to check the accuracy of individual view factors, and Equation 13.4 should be used to determine whether the conservation of energy principle is satisfied [5].

Example 13.1

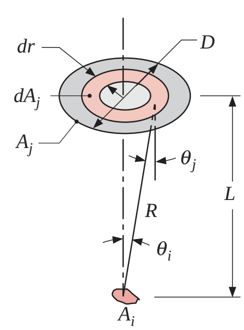
Consider a diffuse circular disk of diameter D and area A_j and a plane diffuse surface of area $A_i \ll A_j$. The surfaces are parallel, and A_i is located at a distance L from the center of A_j. Obtain an expression for the view factor F_{ij}.

Solution

Known: Orientation of small surface relative to large circular disk.

Find: View factor of small surface with respect to disk, F_{ij}.

Schematic:



Assumptions:

1. Diffuse surfaces.
2. $A_i \ll A_j$.

Analysis: The desired view factor may be obtained from Equation 13.1.

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} \, dA_i \, dA_j$$

Recognizing that θ_i, θ_j, and R are approximately independent of position on A_i, this expression reduces to

$$F_{ij} = \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} \, dA_j$$

or, with $\theta_i = \theta_j \equiv \theta$,

$$F_{ij} = \int_{A_j} \frac{\cos^2\theta}{\pi R^2} \, dA_j$$

With $R^2 = r^2 + L^2$, $\cos\theta = (L/R)$, and $dA_j = 2\pi r \, dr$, it follows that

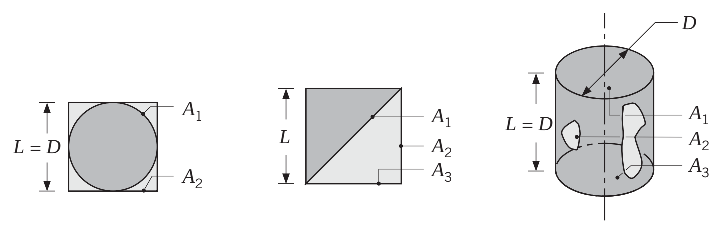
$$F_{ij} = 2L^2 \int_0^{D/2} \frac{r \, dr}{(r^2 + L^2)^2} = \frac{D^2}{D^2 + 4L^2} \qquad \triangleleft \qquad (13.8)$$

Comments:

1. Equation 13.8 may be used to quantify the asymptotic behavior of the curves in Figure 13.5 as the radius of the lower circle, r_i, approaches zero.
2. The preceding geometry is one of the simplest cases for which the view factor may be obtained from Equation 13.1. Geometries involving more detailed integrations are considered in the literature [1, 3].

EXAMPLE 13.2

Determine the view factors F_{12} and F_{21} for the following geometries:



(1) (2) (3)

1. Sphere of diameter D inside a cubical box of length $L = D$.
2. One side of a diagonal partition within a long square duct.
3. End and side of a circular tube of equal length and diameter.

SOLUTION

Known: Surface geometries.

Find: View factors.

Assumptions: Diffuse surfaces with uniform radiosities.

Analysis: The desired view factors may be obtained from inspection, the reciprocity rule, the summation rule, and/or use of the charts.

1. Sphere within a cube:
   By inspection, $F_{12} = 1$ ◁

   By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\pi D^2}{6L^2} \times 1 = \frac{\pi}{6}$ ◁

2. Partition within a square duct:

   From summation rule, $F_{11} + F_{12} + F_{13} = 1$

   where $F_{11} = 0$

   By symmetry, $F_{12} = F_{13}$

   Hence $F_{12} = 0.50$ ◁

   By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\sqrt{2}\,L}{L} \times 0.5 = 0.71$ ◁

3. Circular tube:

   From Table 13.2 or Figure 13.5, with $(r_3/L) = 0.5$ and $(L/r_1) = 2$, $F_{13} = 0.172$

   From summation rule, $F_{11} + F_{12} + F_{13} = 1$

   or, with $F_{11} = 0$, $F_{12} = 1 - F_{13} = 0.828$ ◁

   From reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\pi D^2/4}{\pi DL} \times 0.828 = 0.207$ ◁

13.2 *Radiation Exchange Between Opaque, Diffuse, Gray Surfaces in an Enclosure*

In general, radiation may leave an opaque surface due to both reflection and emission, and on reaching a second opaque surface, experience reflection as well as absorption. In an enclosure, such as that of Figure 13.8*a*, radiation may experience multiple reflections off all surfaces, with partial absorption occurring at each.

Analyzing radiation exchange in an enclosure may be simplified by making certain assumptions. Each surface of the enclosure is assumed to be *isothermal* and to be characterized by a *uniform radiosity* and a *uniform irradiation*. *Opaque, diffuse, gray* surface behavior is also assumed, and the medium within the enclosure is taken to be *nonparticipating*. The problem is generally one in which either the temperature T_i or the net radiative heat flux q_i'' associated with each of the surfaces is known. The objective is to use this information to determine the unknown radiative heat fluxes and temperatures associated with each of the surfaces.

13.2.1 Net Radiation Exchange at a Surface

The term q_i, which is the *net* rate at which radiation *leaves* surface i, represents the net effect of radiative interactions occurring at the surface (Figure 13.8*b*). It is the rate at which energy would have to be transferred to the surface by other means to maintain it at a constant temperature. It is equal to the difference between the surface radiosity and irradiation and may be expressed as

$$q_i = A_i(J_i - G_i) \tag{13.9}$$

From Equation 13.9 and the definition of the radiosity J_i,

$$J_i \equiv E_i + \rho_i G_i \tag{13.10}$$

The net radiative transfer from the surface may also be expressed in terms of the surface emissive power and the absorbed irradiation:

$$q_i = A_i(E_i - \alpha_i G_i) \tag{13.11}$$

where use has been made of the relationship $\alpha_i = 1 - \rho_i$ for an opaque surface. This relationship is illustrated in Figure 13.8*c*. Substituting from Equation 12.35 and recognizing that $\rho_i = 1 - \alpha_i = 1 - \varepsilon_i$ for an opaque, diffuse, gray surface, the radiosity may also be expressed as

$$J_i = \varepsilon_i E_{bi} + (1 - \varepsilon_i)G_i \tag{13.12}$$

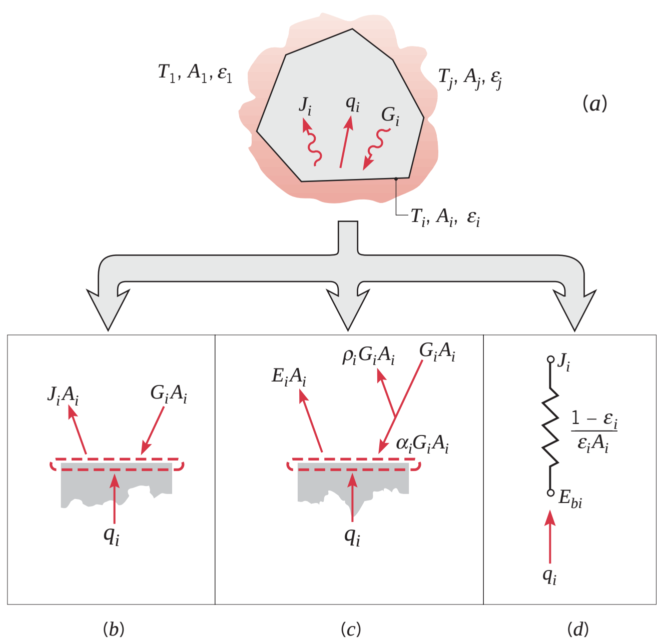


FIGURE 13.8 Radiation exchange in an enclosure of diffuse, gray surfaces with a nonparticipating medium. (*a*) Schematic of the enclosure. (*b*) Radiative balance according to Equation 13.9. (*c*) Radiative balance according to Equation 13.11. (*d*) Network element representing the net radiation transfer from a surface.

Solving for G_i and substituting into Equation 13.9, it follows that

$$q_i = A_i\left(J_i - \frac{J_i - \varepsilon_i E_{bi}}{1 - \varepsilon_i}\right)$$

or

$$q_i = \frac{E_{bi} - J_i}{(1 - \varepsilon_i)/\varepsilon_i A_i} \tag{13.13}$$

Equation 13.13 provides a convenient representation for the net radiative heat transfer rate from a surface. This transfer, which may be represented by the network element of Figure 13.8*d*, is associated with the driving potential $(E_{bi} - J_i)$ and a *surface radiative resistance* of the form $(1 - \varepsilon_i)/\varepsilon_i A_i$. Hence, if the emissive power that the surface would have if it were black exceeds its radiosity, there is net radiation heat transfer from the surface; if the inverse is true, the net transfer is to the surface.

It is sometimes the case that one of the surfaces is very large relative to the other surfaces under consideration. For example, the system might consist of multiple small surfaces in a large room. In this case, the area of the large surface is effectively infinite $(A_i \rightarrow \infty)$, and we see that its surface radiative resistance, $(1 - \varepsilon_i)/\varepsilon_i A_i$, is effectively zero, just as it would be for a black surface $(\varepsilon_i = 1)$. Hence, $J_i = E_{bi}$, and *a surface which is large relative to all other surfaces under consideration can be treated as if it were a blackbody*. This important conclusion was reached in Section 12.6 based on a physical argument and has now been confirmed from our treatment of gray surface radiation exchange. Again, the physical explanation is that, even though the large surface may reflect some of the irradiation incident upon it, it is so big that there is a high probability that the reflected radiation reaches another point on the same large surface. After many such reflections, all the radiation that was originally incident on the large surface is absorbed by the large surface, and none ever reaches any of the smaller surfaces.

13.2.2 Radiation Exchange Between Surfaces

To use Equation 13.13, the surface radiosity J_i must be known. To determine this quantity, it is necessary to consider radiation exchange between the surfaces of the enclosure.

The irradiation of surface *i* can be evaluated from the radiosities of all the surfaces in the enclosure. In particular, from the definition of the view factor, it follows that the total rate at which radiation reaches surface *i* from all surfaces, including *i*, is

$$A_i G_i = \sum_{j=1}^{N} F_{ji} A_j J_j$$

or from the reciprocity relation, Equation 13.3,

$$A_i G_i = \sum_{j=1}^{N} A_i F_{ij} J_j$$

Canceling the area A_i and substituting into Equation 13.9 for G_i,

$$q_i = A_i\left(J_i - \sum_{j=1}^{N} F_{ij} J_j\right)$$

or, from the summation rule, Equation 13.4,

$$q_i = A_i\left(\sum_{j=1}^{N} F_{ij} J_i - \sum_{j=1}^{N} F_{ij} J_j\right)$$

Hence

$$q_i = \sum_{j=1}^{N} A_i F_{ij}(J_i - J_j) = \sum_{j=1}^{N} q_{ij} \qquad (13.14)$$

This result equates the net rate of radiation transfer from surface i, q_i, to the sum of components q_{ij} related to radiative exchange with the other surfaces. Each component may be represented by a network element for which $(J_i - J_j)$ is the driving potential and $(A_iF_{ij})^{-1}$ is a *space* or *geometrical resistance* (Figure 13.9).

Combining Equations 13.13 and 13.14, we then obtain

$$\frac{E_{bi} - J_i}{(1 - \varepsilon_i)/\varepsilon_i A_i} = \sum_{j=1}^{N} \frac{J_i - J_j}{(A_i F_{ij})^{-1}} \qquad (13.15)$$

As shown in Figure 13.9, this expression represents a radiation balance for the radiosity *node* associated with surface i. The rate of radiation transfer (current flow) to i through its surface resistance must equal the net rate of radiation transfer (current flows) from i to all other surfaces through the corresponding geometrical resistances.

Note that Equation 13.15 is especially useful when the surface temperature T_i (hence E_{bi}) is known. Although this situation is typical, it does not always apply. In particular, situations may arise for which the net radiation transfer rate at the surface q_i, rather than the temperature T_i, is known. In such cases the preferred form of the radiation balance is Equation 13.14, rearranged as

$$q_i = \sum_{j=1}^{N} \frac{J_i - J_j}{(A_i F_{ij})^{-1}} \qquad (13.16)$$

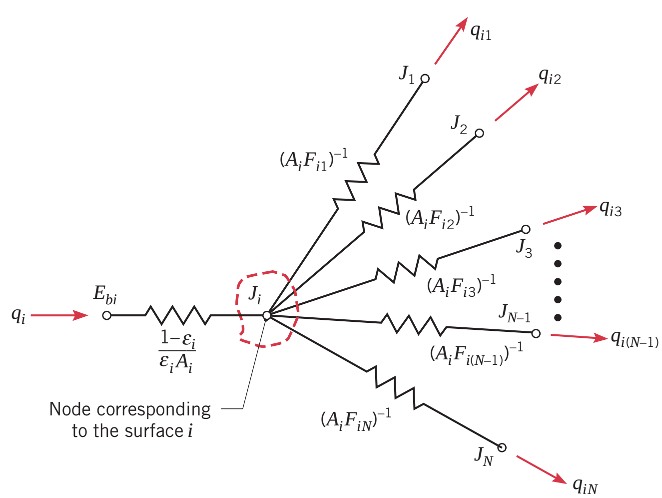


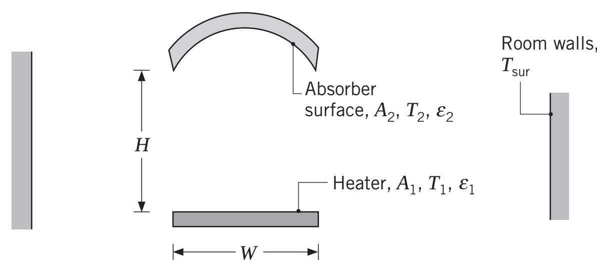
FIGURE 13.9 Network representation of radiative exchange between surface i and the remaining surfaces of an enclosure.

Use of network representations was first suggested by Oppenheim [6]. The network is built by first identifying nodes associated with the radiosities of each of the N surfaces of the enclosure. The method provides a useful tool for visualizing radiation exchange in the enclosure and, at least for simple enclosures, may be used as the basis for predicting this exchange.

An alternative *direct approach* to solving radiation enclosure problems involves writing Equation 13.15 for each surface at which T_i is known, and writing Equation 13.16 for each surface at which q_i is known. The resulting set of N linear, algebraic equations is solved for $J_1, J_2, \ldots, J_N$. With knowledge of the J_i, Equation 13.13 may then be used to determine the net radiation heat transfer rate q_i at each surface of known T_i or the value of T_i at each surface of known q_i. For any number N of surfaces in the enclosure, the foregoing problem may readily be solved by the iteration or matrix inversion methods of Chapter 4.

Example 13.3

In manufacturing, the special coating on a curved solar absorber surface of area $A_2 = 15\ \text{m}^2$ is cured by exposing it to an infrared heater of width $W = 1$ m. The absorber and heater are each of length $L = 10$ m and are separated by a distance of $H = 1$ m. The upper surface of the absorber and the lower surface of the heater are insulated.



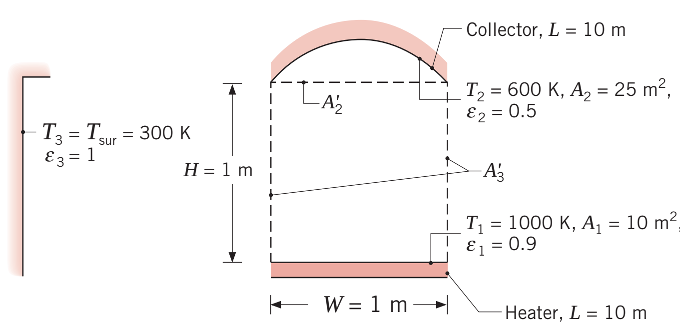
The heater is at $T_1 = 1000$ K and has an emissivity of $\varepsilon_1 = 0.9$, while the absorber is at $T_2 = 600$ K and has an emissivity of $\varepsilon_2 = 0.5$. The system is in a large room whose walls are at 300 K. What is the net rate of heat transfer to the absorber surface?

Solution

Known: A curved, solar absorber surface with a special coating is being cured by use of an infrared heater in a large room.

Find: Net rate of heat transfer to the absorber surface.

Schematic:

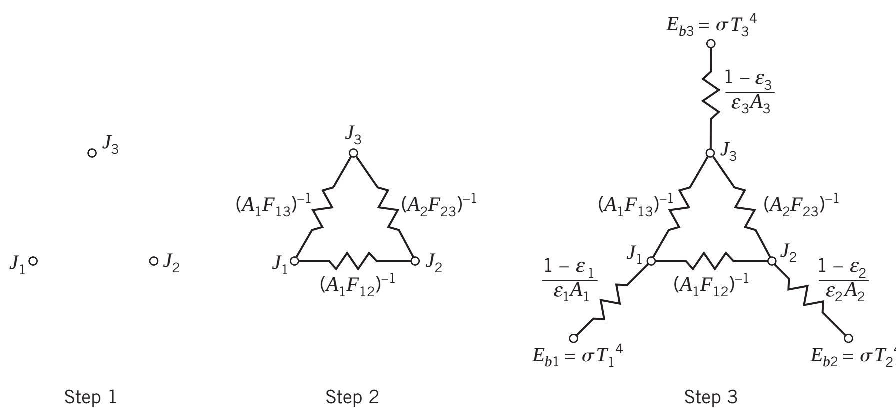


Assumptions:

1. Steady-state conditions exist.
2. Convection effects are negligible.
3. Absorber and heater surfaces are diffuse and gray.
4. The surrounding room is large and therefore behaves as a blackbody.

Analysis: The system may be viewed as a three-surface enclosure, with the third surface being the large surrounding room, which behaves as a blackbody. We are interested in obtaining the net rate of radiation transfer to surface 2. We solve the problem using both the radiation network and direct approaches.

Radiation Network Approach The radiation network is constructed by first identifying nodes associated with the radiosities of each surface, as shown in step 1 in the schematic below. Then each radiosity node is connected to each of the other radiosity nodes through the appropriate space resistance, as shown in step 2. We will treat the surroundings as having a large but unspecified area, which introduces difficulty in expressing the space resistances $(A_3F_{31})^{-1}$ and $(A_3F_{32})^{-1}$. Fortunately, from the reciprocity relation (Equation 13.3), we can replace A_3F_{31} with A_1F_{13} and A_3F_{32} with A_2F_{23}, which are more readily obtained. The final step is to connect the blackbody emissive powers associated with the temperature of each surface to the radiosity nodes, using the appropriate form of the surface resistance.



In this problem, the surface resistance associated with surface 3 is zero according to assumption 4; therefore, $J_3 = E_{b3} = \sigma T_3^4 = 459\ \text{W/m}^2$.

Summing currents at the J_1 node yields

$$\frac{\sigma T_1^4 - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{1/A_1F_{12}} + \frac{J_1 - \sigma T_3^4}{1/A_1F_{13}} \qquad (1)$$

while summing the currents at the J_2 node results in

$$\frac{\sigma T_2^4 - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} = \frac{J_2 - J_1}{1/A_1F_{12}} + \frac{J_2 - \sigma T_3^4}{1/A_2F_{23}} \qquad (2)$$

The view factor F_{12} may be obtained by recognizing that $F_{12} = F_{12'}$, where A_2' is shown in the schematic as the rectangular base of the absorber surface. Then, from

Figure 13.4 or Table 13.2, with $Y/L = 10/1 = 10$ and $X/L = 1/1 = 1$,

$$F_{12} = 0.39$$

From the summation rule, and recognizing that $F_{11} = 0$, it also follows that

$$F_{13} = 1 - F_{12} = 1 - 0.39 = 0.61$$

The last needed view factor is F_{23}. We recognize that, since radiation propagating from surface 2 to surface 3 must pass through the hypothetical surface A_2',

$$A_2 F_{23} = A_2' F_{2'3}$$

and from symmetry $F_{2'3} = F_{13}$. Thus

$$F_{23} = \frac{A_2'}{A_2} F_{13} = \frac{10\ \text{m}^2}{15\ \text{m}^2} \times 0.61 = 0.41$$

We may now solve Equations 1 and 2 for J_1 and J_2. Recognizing that $E_{b1} = \sigma T_1^4 = 56{,}700\ \text{W/m}^2$ and canceling the area A_1, we can express Equation 1 as

$$\frac{56{,}700 - J_1}{(1 - 0.9)/0.9} = \frac{J_1 - J_2}{1/0.39} + \frac{J_1 - 459}{1/0.61}$$

or

$$-10J_1 + 0.39J_2 = -510{,}582 \tag{3}$$

Noting that $E_{b2} = \sigma T_2^4 = 7348\ \text{W/m}^2$ and dividing by the area A_2, we can express Equation 2 as

$$\frac{7348 - J_2}{(1 - 0.5)/0.5} = \frac{J_2 - J_1}{15\ \text{m}^2/(10\ \text{m}^2 \times 0.39)} + \frac{J_2 - 459}{1/0.41}$$

or

$$0.26J_1 - 1.67J_2 = -7536 \tag{4}$$

Solving Equations 3 and 4 simultaneously yields $J_2 = 12{,}487\ \text{W/m}^2$.

An expression for the net rate of heat transfer *from* the absorber surface, q_2, may be written upon inspection of the radiation network and is

$$q_2 = \frac{\sigma T_2^4 - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2}$$

resulting in

$$q_2 = \frac{(7348 - 12{,}487)\text{W/m}^2}{(1 - 0.5)/(0.5 \times 15\ \text{m}^2)} = -77.1\ \text{kW}$$

Hence, the net heat transfer rate *to* the absorber is $q_{\text{net}} = -q_2 = 77.1$ kW. ◁

Direct Approach Using the direct approach, we write Equation 13.15 for each of the three surfaces. We use reciprocity to rewrite the space resistances in terms of the known view factors from above and to eliminate A_3.

Surface 1

$$\frac{\sigma T_1^4 - J_1}{(1 - \varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{1/A_1 F_{12}} + \frac{J_1 - J_3}{1/A_1 F_{13}} \tag{5}$$

Surface 2

$$\frac{\sigma T_2^4 - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} = \frac{J_2 - J_1}{1/A_2F_{21}} + \frac{J_2 - J_3}{1/A_2F_{23}} = \frac{J_2 - J_1}{1/A_1F_{12}} + \frac{J_2 - J_3}{1/A_2F_{23}} \quad (6)$$

Surface 3

$$\frac{\sigma T_3^4 - J_3}{(1-\varepsilon_3)/\varepsilon_3 A_3} = \frac{J_3 - J_1}{1/A_3F_{31}} + \frac{J_3 - J_2}{1/A_3F_{32}} = \frac{J_3 - J_1}{1/A_1F_{13}} + \frac{J_3 - J_2}{1/A_2F_{23}} \quad (7)$$

Substituting values of the areas, temperatures, emissivities, and view factors into Equations 5 through 7 and solving them simultaneously, we obtain $J_1 = 51{,}541$ W/m^2, $J_2 = 12{,}487$ W/m^2, and $J_3 = 459$ W/m^2. Equation 13.13 may then be written for surface 2 as

$$q_2 = \frac{\sigma T_2^4 - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2}$$

This expression is identical to the expression that was developed using the radiation network. Hence, $q_2 = -77.1$ kW.

Comments:

1. In order to solve Equations 5 through 7 simultaneously, we must first multiply both sides of Equation 7 by $(1 - \varepsilon_3)/\varepsilon_3 A_3 = 0$ to avoid division by zero, resulting in the simplified form of Equation 7, which is $J_3 = \sigma T_3^4$.
2. If we substitute $J_3 = \sigma T_3^4$ into Equations 5 and 6, it is evident that Equations 5 and 6 are identical to Equations 1 and 2, respectively.
3. The direct approach is recommended for problems involving $N \geq 4$ surfaces, since radiation networks become quite complex as the number of surfaces increases.
4. As will be seen in Section 13.3, the radiation network approach is particularly useful when thermal energy is transferred to or from surfaces by additional means, that is, by conduction and/or convection. In these *multimode* heat transfer situations, the additional energy delivered to or taken from the surface can be represented by additional current into or out of a node.
5. Recognize the utility of using a hypothetical surface (A_2') to simplify the evaluation of view factors.
6. We could have approached the solution in a slightly different manner. Radiation leaving surface 1 must pass through the openings (hypothetical surface 3′) in order to reach the surroundings. Thus, we can write.

$$F_{13} = F_{13'}$$
$$A_1F_{13} = A_1F_{13'} = A_3'F_{3'1}$$

   A similar relationship can be written for exchange between surface 2 and the surroundings, that is, $A_2F_{23} = A_3'F_{3'2}$. Thus, the space resistances which connect to radiosity node 3 in the radiation network above can be replaced by space resistances pertaining to surface 3′. The resistance network would be unchanged, and the space resistances would have the same values as those determined in the foregoing solution. However, it may be more convenient to calculate the view factors by utilizing the hypothetical surfaces 3′. With the surface resistance for surface 3 equal to zero, we see that *openings of enclosures that exchange radiation with large surroundings may be treated as hypothetical, nonreflecting black surfaces* ($\varepsilon_3 = 1$) *whose temperature is equal to that of the surroundings* ($T_3 = T_{sur}$).

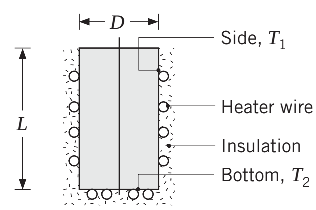
13.2.3 Blackbody Radiation Exchange

In Example 13.3, we saw that large surroundings may be treated as hypothetical black surfaces. Real surfaces, such as those that are charred or coated with high-emissivity finishes, also exhibit near-black behavior. Matters are simplified significantly when all the surfaces of the enclosure are black. Since the absorptivity of a black surface is unity, there is no reflection and the radiosity is composed solely of the emitted energy. Hence, Equation 13.14 reduces to

$$q_i = \sum_{j=1}^{N} A_i F_{ij} \sigma (T_i^4 - T_j^4) \tag{13.17}$$

EXAMPLE 13.4

A furnace cavity, which is in the form of a cylinder of 75-mm diameter and 150-mm length, is open at one end to large surroundings that are at 27°C. The sides and bottom may be approximated as blackbodies, are heated electrically, are well insulated, and are maintained at temperatures of 1350 and 1650°C, respectively.



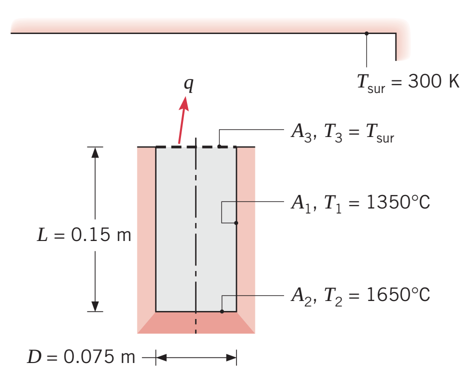
How much power is required to maintain the furnace conditions?

SOLUTION

Known: Surface temperatures of cylindrical furnace and surroundings.

Find: Power required to maintain prescribed temperatures.

Schematic:



Assumptions:

1. Interior surfaces behave as blackbodies.
2. Heat transfer by convection is negligible.
3. Outer surface of furnace is adiabatic.
4. Treat opening as a hypothetical black surface at T_{sur}.

Analysis: The power needed to operate the furnace at the prescribed conditions must balance heat losses from the furnace. The heat loss may then be expressed as

$$q = q_{13} + q_{23}$$

or, from Equation 13.17,

$$q = A_1 F_{13} \sigma (T_1^4 - T_3^4) + A_2 F_{23} \sigma (T_2^4 - T_3^4)$$

From Table 13.2 (or Figure 13.5), with $(r_i/L) = (r_j/L) = (0.0375 \text{ m}/0.15 \text{ m}) = 0.25$, $F_{23} = 0.056$. From the summation rule

$$F_{21} = 1 - F_{23} = 1 - 0.056 = 0.944$$

and from reciprocity

$$F_{12} = \frac{A_2}{A_1} F_{21} = \frac{\pi (0.075 \text{ m})^2/4}{\pi (0.075 \text{ m})(0.15 \text{ m})} \times 0.944 = 0.118$$

Hence, since $F_{13} = F_{12}$ from symmetry,

$$q = (\pi \times 0.075 \text{ m} \times 0.15 \text{ m})\, 0.118 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$$

$$\times [(1623 \text{ K})^4 - (300 \text{ K})^4] + \left(\frac{\pi}{4}\right)(0.075 \text{ m})^2 \times 0.056$$

$$\times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 [(1923 \text{ K})^4 - (300 \text{ K})^4]$$

$$q = 1639 \text{ W} + 191 \text{ W} = 1830 \text{ W}$$

13.2.4 The Two-Surface Enclosure

The simplest example of an enclosure is one involving two surfaces that exchange radiation only with each other. Such a two-surface enclosure is shown schematically in Figure 13.10*a*. Since there are only two surfaces, the net rate of radiation transfer *from* surface 1, q_1, must equal the net rate of radiation transfer *to* surface 2, $-q_2$, and both quantities must equal the net rate at which radiation is exchanged between 1 and 2. Accordingly,

$$q_1 = -q_2 = q_{12}$$

The radiation transfer rate may be determined by applying Equation 13.15 to surfaces 1 and 2 and solving the resulting two equations for J_1 and J_2. The results could then be used with Equation 13.13 to determine q_1 (or q_2). However, in this case the desired result is more readily obtained by working with the network representation of the enclosure shown in Figure 13.10*b*.

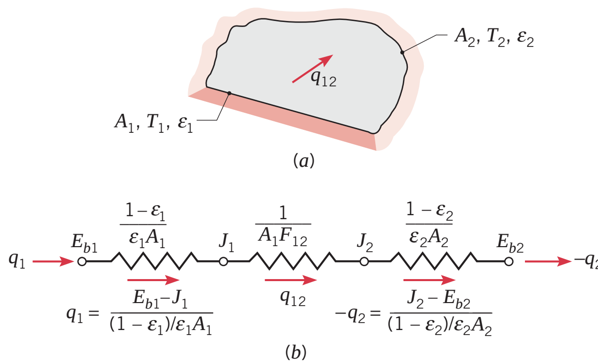


FIGURE 13.10 The two-surface enclosure. (*a*) Schematic. (*b*) Network representation.

From Figure 13.10*b* we see that the total resistance to radiation exchange between surfaces 1 and 2 is comprised of the two surface resistances and the geometrical resistance. Hence, substituting from Equation 12.26, the net radiation exchange between surfaces may be expressed as

$$q_{12} = q_1 = -q_2 = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}} \tag{13.18}$$

The foregoing result may be used for any two diffuse, gray surfaces *that form an enclosure.* Important special cases are summarized in Table 13.3.

13.2.5 Radiation Shields

Radiation shields constructed from low emissivity (high reflectivity) materials can be used to reduce the net radiation transfer between two surfaces. Consider placing a radiation shield, surface 3, between the two large, parallel planes of Figure 13.11*a*. Without the radiation shield, the net rate of radiation transfer between surfaces 1 and 2 is given by Equation 13.19. However, with the radiation shield, additional resistances are present, as shown in Figure 13.11*b*, and the heat transfer rate is reduced. Note that the emissivity associated with one side of the shield ($\varepsilon_{3,1}$) may differ from that associated with the opposite side ($\varepsilon_{3,2}$) and the radiosities will always differ. Summing the resistances and recognizing that $F_{13} = F_{32} = 1$, it follows that

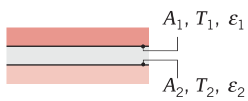
$$q_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} + \dfrac{1-\varepsilon_{3,1}}{\varepsilon_{3,1}} + \dfrac{1-\varepsilon_{3,2}}{\varepsilon_{3,2}}} \tag{13.23}$$

Note that the resistances associated with the radiation shield become very large when the emissivities $\varepsilon_{3,1}$ and $\varepsilon_{3,2}$ are very small.

Equation 13.23 may be used to determine the net heat transfer rate if T_1 and T_2 are known. From knowledge of q_{12} and the fact that $q_{12} = q_{13} = q_{32}$, the value of T_3 may then be determined by expressing Equation 13.19 for q_{13} or q_{32}.

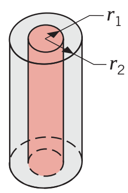
TABLE 13.3 Special Diffuse, Gray, Two-Surface Enclosures

Large (Infinite) Parallel Planes



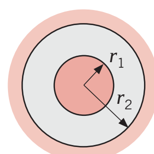
$$A_1 = A_2 = A \qquad F_{12} = 1 \qquad q_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} \tag{13.19}$$

Long (Infinite) Concentric Cylinders



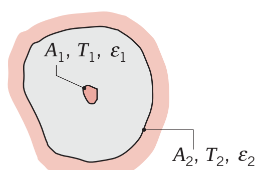
$$\frac{A_1}{A_2} = \frac{r_1}{r_2} \qquad F_{12} = 1 \qquad q_{12} = \frac{\sigma A_1(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1 - \varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)} \tag{13.20}$$

Concentric Spheres



$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} \qquad F_{12} = 1 \qquad q_{12} = \frac{\sigma A_1(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1 - \varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)^2} \tag{13.21}$$

Small Convex Object in a Large Cavity



$$\frac{A_1}{A_2} \approx 0 \qquad F_{12} = 1 \qquad q_{12} = \sigma A_1 \varepsilon_1 (T_1^4 - T_2^4) \tag{13.22}$$

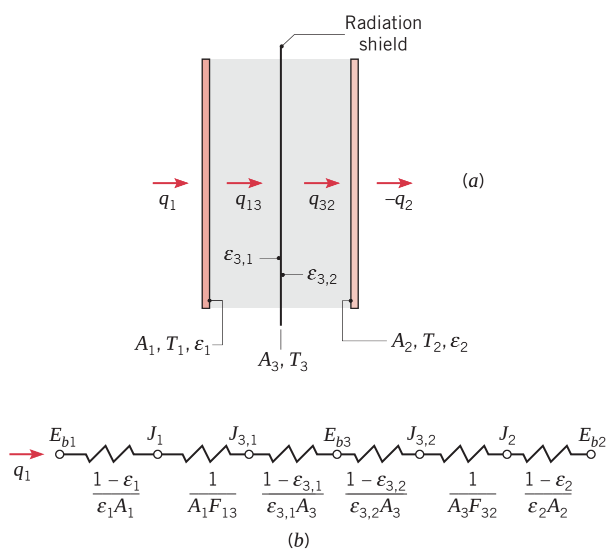


FIGURE 13.11 Radiation exchange between large parallel planes with a radiation shield. (*a*) Schematic. (*b*) Network representation.

The foregoing procedure may readily be extended to problems involving multiple radiation shields. In the special case for which all the emissivities are equal, it may be shown that, with N shields,

$$(q_{12})_N = \frac{1}{N+1}(q_{12})_0 \tag{13.24}$$

where $(q_{12})_0$ is the radiation transfer rate with no shields ($N = 0$).

Example 13.5

A cryogenic fluid flows through a long tube of 20-mm diameter, the outer surface of which is diffuse and gray with $\varepsilon_1 = 0.02$ and $T_1 = 77$ K. This tube is concentric with a larger tube of 50-mm diameter, the inner surface of which is diffuse and gray with $\varepsilon_2 = 0.05$ and $T_2 = 300$ K. The space between the surfaces is evacuated. Calculate the heat gain by the cryogenic fluid per unit length of tubes. If a thin radiation shield of 35-mm diameter and $\varepsilon_3 = 0.02$ (both sides) is inserted midway between the inner and outer surfaces, calculate the change (percentage) in heat gain per unit length of the tubes.

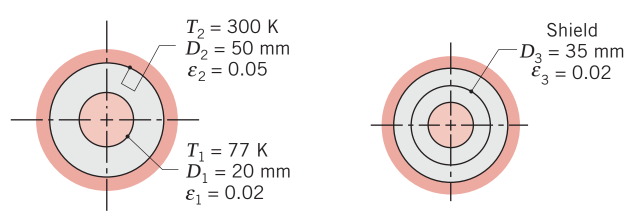
Solution

Known: Concentric tube arrangement with diffuse, gray surfaces of different emissivities and temperatures.

Find:

1. Heat gain by the cryogenic fluid passing through the inner tube.
2. Percentage change in heat gain with radiation shield inserted midway between inner and outer tubes.

Schematic:



Assumptions:

1. Surfaces are diffuse and gray.
2. Space between tubes is evacuated.
3. Conduction resistance for radiation shield is negligible.
4. Concentric tubes form a two-surface enclosure (end effects are negligible).

Analysis:

1. The network representation of the system without the shield is shown in Figure 13.10, and the desired heat rate may be obtained from Equation 13.20, where

$$q = \frac{\sigma(\pi D_1 L)(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)}$$

Hence

$$q' = \frac{q}{L} = \frac{5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4\,(\pi \times 0.02\ \text{m})[(77\ \text{K})^4 - (300\ \text{K})^4]}{\dfrac{1}{0.02} + \dfrac{1-0.05}{0.05}\left(\dfrac{0.02\ \text{m}}{0.05\ \text{m}}\right)}$$

$$q' = -0.50\ \text{W/m} \quad \triangleleft$$

2. The network representation of the system with the shield is shown in Figure 13.11, and the desired heat rate is now

$$q = \frac{E_{b1} - E_{b2}}{R_{\text{tot}}} = \frac{\sigma(T_1^4 - T_2^4)}{R_{\text{tot}}}$$

where

$$R_{\text{tot}} = \frac{1-\varepsilon_1}{\varepsilon_1(\pi D_1 L)} + \frac{1}{(\pi D_1 L)F_{13}} + 2\left[\frac{1-\varepsilon_3}{\varepsilon_3(\pi D_3 L)}\right] + \frac{1}{(\pi D_3 L)F_{32}} + \frac{1-\varepsilon_2}{\varepsilon_2(\pi D_2 L)}$$

or

$$R_{\text{tot}} = \frac{1}{L}\left\{\frac{1-0.02}{0.02(\pi \times 0.02\ \text{m})} + \frac{1}{(\pi \times 0.02\ \text{m})1} + 2\left[\frac{1-0.02}{0.02(\pi \times 0.035\ \text{m})}\right] + \frac{1}{(\pi \times 0.035\ \text{m})1} + \frac{1-0.05}{0.05(\pi \times 0.05\ \text{m})}\right\}$$

$$R_{\text{tot}} = \frac{1}{L}(779.9 + 15.9 + 891.3 + 9.1 + 121.0) = \frac{1817}{L}\left(\frac{1}{\text{m}^2}\right)$$

Hence

$$q' = \frac{q}{L} = \frac{5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4\,[(77\ \text{K})^4 - (300\ \text{K})^4]}{1817\ (1/\text{m})} = -0.25\ \text{W/m} \quad \triangleleft$$

The percentage change in the heat gain is then

$$\frac{q'_w - q'_{wo}}{q'_{wo}} \times 100 = \frac{(-0.25\ \text{W/m}) - (-0.50\ \text{W/m})}{-0.50\ \text{W/m}} \times 100 = -50\%$$

13.2.6 The Reradiating Surface

The assumption of a *reradiating surface* is common to many industrial applications. This idealized surface is characterized by *zero* net radiation transfer ($q_i = 0$). It is closely approached by real surfaces that are well insulated on one side *and* for

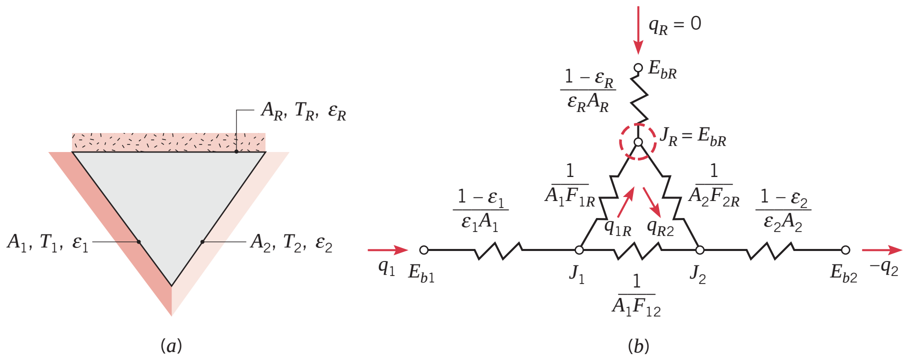


FIGURE 13.12 A three-surface enclosure with one surface reradiating. (*a*) Schematic. (*b*) Network representation.

which convection effects may be neglected on the opposite (radiating) side. With $q_i = 0$, it follows from Equations 13.9 and 13.13 that $G_i = J_i = E_{bi}$. Hence, if the radiosity of a reradiating surface is known, its temperature is readily determined. In an enclosure, the equilibrium temperature of a reradiating surface is determined by its interaction with the other surfaces, and it is *independent of the emissivity of the reradiating surface.*

A three-surface enclosure, for which the third surface, surface R, is reradiating, is shown in Figure 13.12*a*, and the corresponding network is shown in Figure 13.12*b*. Surface R is presumed to be well insulated, and convection effects are assumed to be negligible. Hence, with $q_R = 0$, the net radiation *transfer* from surface 1 must equal the net radiation transfer to surface 2. The network is a simple series–parallel arrangement, and from its analysis it is readily shown that

$$q_1 = -q_2 = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1F_{12} + [(1/A_1F_{1R}) + (1/A_2F_{2R})]^{-1}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}} \qquad (13.25)$$

Knowing $q_1 = -q_2$, Equation 13.13 may be applied to surfaces 1 and 2 to determine their radiosities J_1 and J_2. Knowing J_1, J_2, and the geometrical resistances, the radiosity of the reradiating surface J_R may be determined from the radiation balance

$$\frac{J_1 - J_R}{(1/A_1F_{1R})} - \frac{J_R - J_2}{(1/A_2F_{2R})} = 0 \qquad (13.26)$$

The temperature of the reradiating surface may then be determined from the requirement that $\sigma T_R^4 = J_R$.

Note that the general procedure described in Section 13.2.2 may be applied to enclosures with reradiating surfaces. For each such surface, it is appropriate to use Equation 13.16 with $q_i = 0$.

EXAMPLE 13.6

A paint baking oven consists of a long, triangular duct in which a heated surface is maintained at 1200 K and another surface is insulated. Painted panels, which are maintained at 500 K, occupy the third surface. The triangle is of width $W = 1$ m on a side, and the heated and insulated surfaces have an emissivity of 0.8. The emissivity of the panels is 0.4. During steady-state operation, at what rate must energy be supplied to the heated side per unit length of the duct to maintain its temperature at 1200 K? What is the temperature of the insulated surface?

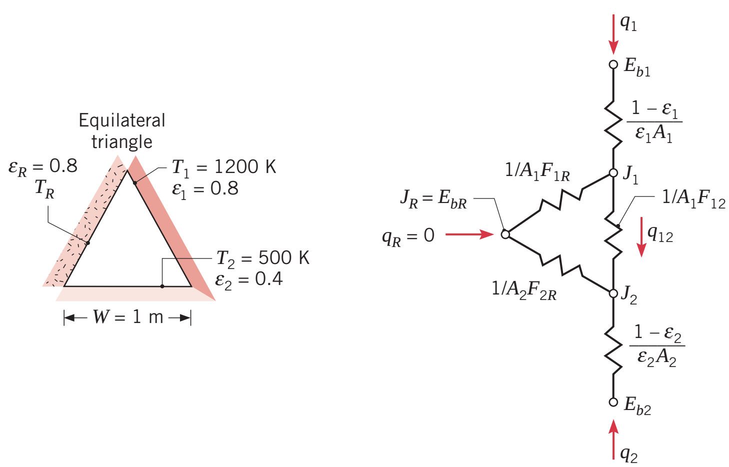
Solution

Known: Surface properties of a long triangular duct that is insulated on one side and heated and cooled on the other sides.

Find:

1. Rate at which heat must be supplied per unit length of duct.
2. Temperature of the insulated surface.

Schematic:



Assumptions:

1. Steady-state conditions exist.
2. All surfaces are opaque, diffuse, gray, and of uniform radiosity.
3. Convection effects are negligible.
4. Surface R is reradiating.
5. End effects are negligible.

Analysis:

1. The system may be modeled as a three-surface enclosure with one surface reradiating. The rate at which energy must be supplied to the heated surface may then be obtained from Equation 13.25:

$$q_1 = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1F_{12} + [(1/A_1F_{1R}) + (1/A_2F_{2R})]^{-1}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

From symmetry, $F_{12} = F_{1R} = F_{2R} = 0.5$. Also, $A_1 = A_2 = W \cdot L$, where L is the duct length. Hence

$$q_1' = \frac{q_1}{L} = \frac{5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4\ (1200^4 - 500^4)\ \text{K}^4}{\dfrac{1-0.8}{0.8 \times 1\ \text{m}} + \dfrac{1}{1\ \text{m} \times 0.5 + (2+2)^{-1}\ \text{m}} + \dfrac{1-0.4}{0.4 \times 1\ \text{m}}}$$

or

$$q_1' = 37\ \text{kW/m} = -q_2' \qquad \triangleleft$$

2. The temperature of the insulated surface may be obtained from the requirement that $J_R = E_{bR}$, where J_R may be obtained from Equation 13.26. However, to use this expression J_1 and J_2 must be known. Applying the surface energy balance, Equation 13.13, to surfaces 1 and 2, it follows that

$$J_1 = E_{b1} - \frac{1-\varepsilon_1}{\varepsilon_1 W} q_1' = 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4\ (1200\ \text{K})^4 - \frac{1-0.8}{0.8 \times 1\ \text{m}} \times 37{,}000\ \text{W/m} = 108{,}323\ \text{W/m}^2$$

$$J_2 = E_{b2} - \frac{1-\varepsilon_2}{\varepsilon_2 W} q_2' = 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4\ (500\ \text{K})^4 - \frac{1-0.4}{0.4 \times 1\ \text{m}} (-37{,}000\ \text{W/m}) = 59{,}043\ \text{W/m}^2$$

From the energy balance for the reradiating surface, Equation 13.26, it follows that

$$\frac{108{,}323 - J_R}{\dfrac{1}{W \times L \times 0.5}} - \frac{J_R - 59{,}043}{\dfrac{1}{W \times L \times 0.5}} = 0$$

Hence

$$J_R = 83{,}683\ \text{W/m}^2 = E_{bR} = \sigma T_R^4$$

$$T_R = \left(\frac{83{,}683\ \text{W/m}^2}{5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4}\right)^{1/4} = 1102\ \text{K} \qquad \triangleleft$$

Comments:

1. Note that temperature and radiosity discontinuities cannot exist at the corners, and the assumptions of uniform temperature and radiosity are weakest in these regions.
2. The results are independent of the value of ε_R.
3. This problem may also be solved using the direct approach. The solution involves first determining the three unknown radiosities J_1, J_2, and J_R. The governing equations are obtained by writing Equation 13.15 for the two surfaces of known temperature, 1 and 2, and Equation 13.16 for surface R. The three equations are

$$\frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{(A_1 F_{12})^{-1}} + \frac{J_1 - J_R}{(A_1 F_{1R})^{-1}}$$

$$\frac{E_{b2} - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} = \frac{J_2 - J_1}{(A_2 F_{21})^{-1}} + \frac{J_2 - J_R}{(A_2 F_{2R})^{-1}}$$

$$0 = \frac{J_R - J_1}{(A_R F_{R1})^{-1}} + \frac{J_R - J_2}{(A_R F_{R2})^{-1}}$$

Canceling the area A_1, the first equation reduces to

$$\frac{117{,}573 - J_1}{0.25} = \frac{J_1 - J_2}{2} + \frac{J_1 - J_R}{2}$$

or

$$10J_1 - J_2 - J_R = 940{,}584 \qquad (1)$$

Similarly, for surface 2,

$$\frac{3544 - J_2}{1.50} = \frac{J_2 - J_1}{2} + \frac{J_2 - J_R}{2}$$

or

$$-J_1 + 3.33J_2 - J_R = 4725 \qquad (2)$$

and for the reradiating surface

$$0 = \frac{J_R - J_1}{2} + \frac{J_R - J_2}{2}$$

or

$$-J_1 - J_2 + 2J_R = 0 \qquad (3)$$

Solving Equations 1, 2, and 3 simultaneously yields

$J_1 = 108{,}328\ \text{W/m}^2 \qquad J_2 = 59{,}018\ \text{W/m}^2 \qquad$ and $\qquad J_R = 83{,}673\ \text{W/m}^2$

Recognizing that $J_R = \sigma T_R^4$, it follows that

$$T_R = \left(\frac{J_R}{\sigma}\right)^{1/4} = \left(\frac{83{,}673\ \text{W/m}^2}{5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4}\right)^{1/4} = 1102\ \text{K}$$

13.3 Multimode Heat Transfer

Thus far, radiation exchange in an enclosure has been considered under conditions for which conduction and convection could be neglected. However, in many applications, convection and/or conduction are comparable to radiation and must be considered in the heat transfer analysis.

Consider the general surface condition of Figure 13.13*a*. In addition to exchanging energy by radiation with other surfaces of the enclosure, there may be external heat addition to the surface, as, for example, by electric heating, and heat transfer from the surface by convection and conduction. From a surface energy balance, it follows that

$$q_{i,\text{ext}} = q_{i,\text{rad}} + q_{i,\text{conv}} + q_{i,\text{cond}} \qquad (13.27)$$

where $q_{i,\text{rad}}$, the net rate of radiation transfer from the surface, is determined by standard procedures for an enclosure. Hence, in general, $q_{i,\text{rad}}$ may be determined from Equation 13.13 or 13.14, while for special cases such as a two-surface enclosure and a three-surface enclosure with one reradiating surface, it may be determined from Equations 13.18 and 13.25, respectively. The surface network element of the

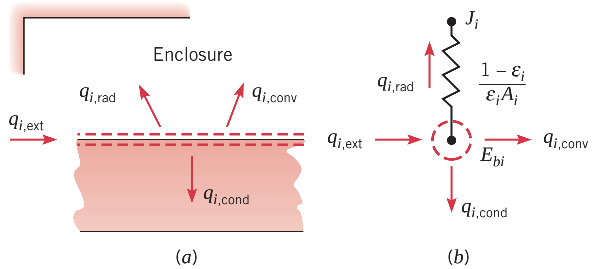


FIGURE 13.13 Multimode heat transfer from a surface in an enclosure. (*a*) Surface energy balance. (*b*) Circuit representation.

radiation circuit is modified according to Figure 13.13*b*, where $q_{i,\text{ext}}$, $q_{i,\text{cond}}$, and $q_{i,\text{conv}}$ represent current flows to or from the surface node. Note, however, that while $q_{i,\text{cond}}$ and $q_{i,\text{conv}}$ are proportional to temperature differences, $q_{i,\text{rad}}$ is proportional to the difference between temperatures raised to the fourth power. Conditions are simplified if the back of the surface is insulated, in which case $q_{i,\text{cond}} = 0$. Moreover, if there is no external heating and convection is negligible, the surface is reradiating.

EXAMPLE 13.7

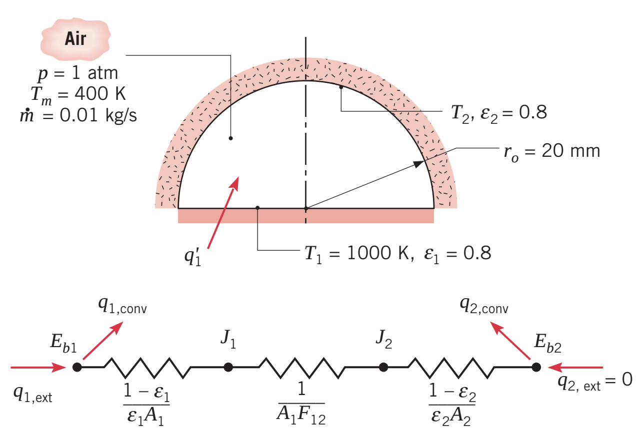
Consider an air heater consisting of a semicircular tube for which the plane surface is maintained at 1000 K and the other surface is well insulated. The tube radius is 20 mm, and both surfaces have an emissivity of 0.8. If atmospheric air flows through the tube at 0.01 kg/s and $T_m = 400$ K, what is the rate at which heat must be supplied per unit length to maintain the plane surface at 1000 K? What is the temperature of the insulated surface?

SOLUTION

Known: Airflow conditions in tubular heater and heater surface conditions.

Find: Rate at which heat must be supplied and temperature of insulated surface.

Schematic:



Assumptions:

1. Steady-state conditions.
2. Diffuse, gray surfaces.
3. Negligible tube end effects and axial variations in gas temperature.
4. Fully developed flow.

Properties: Table A.4, air (1 atm, 400 K): $k = 0.0338\ \text{W/m}\cdot\text{K}$, $\mu = 230 \times 10^{-7}\ \text{kg/s}\cdot\text{m}$, $c_p = 1014\ \text{J/kg}\cdot\text{K}$, $Pr = 0.69$.

Analysis: Since the semicircular surface is well insulated and there is no external heat addition, a surface energy balance yields

$$-q_{2,\text{rad}} = q_{2,\text{conv}}$$

Since the tube constitutes a two-surface enclosure, the net radiation transfer to surface 2 may be evaluated from Equation 13.18. Hence

$$\frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}} = hA_2(T_2 - T_m)$$

where the view factor is $F_{12} = 1$ and, per unit length, the surface areas are $A_1 = 2r_o$ and $A_2 = \pi r_o$. With

$$Re_D = \frac{\rho u_m D_h}{\mu} = \frac{\dot{m} D_h}{A_c \mu} = \frac{\dot{m} D_h}{(\pi r_o^2/2)\mu}$$

the hydraulic diameter is

$$D_h = \frac{4A_c}{P} = \frac{2\pi r_o}{\pi + 2} = \frac{0.04\pi\ \text{m}}{\pi + 2} = 0.0244\ \text{m}$$

Hence

$$Re_D = \frac{0.01\ \text{kg/s} \times 0.0244\ \text{m}}{(\pi/2)\,(0.02\ \text{m})^2 \times 230 \times 10^{-7}\ \text{kg/s}\cdot\text{m}} = 16{,}900$$

From the Dittus–Boelter equation,

$$Nu_D = 0.023 Re_D^{4/5}\, Pr^{0.4}$$

$$Nu_D = 0.023(16{,}900)^{4/5}(0.69)^{0.4} = 47.8$$

$$h = \frac{k}{D_h} Nu_D = \frac{0.0338\ \text{W/m}\cdot\text{K}}{0.0244\ \text{m}}\, 47.8 = 66.2\ \text{W/m}^2\cdot\text{K}$$

Dividing both sides of the energy balance by A_1, it follows that

$$\frac{5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\,[(1000)^4 - T_2^4]\ \text{K}^4}{\dfrac{1-0.8}{0.8} + 1 + \dfrac{1-0.8}{0.8}\dfrac{2}{\pi}} = 66.2\,\frac{\pi}{2}\,(T_2 - 400)\ \text{W/m}^2$$

or

$$5.67 \times 10^{-8}\, T_2^4 + 146.5 T_2 - 115{,}313 = 0$$

A trial-and-error solution yields

$$T_2 = 696\ \text{K} \qquad \triangleleft$$

From an energy balance at the heated surface,

$$q_{1,\text{ext}} = q_{1,\text{rad}} + q_{1,\text{conv}} = q_{2,\text{conv}} + q_{1,\text{conv}}$$

Hence, on a unit length basis,

$$q'_{1,\text{ext}} = h\pi r_o(T_2 - T_m) + h2r_o(T_1 - T_m)$$

$$q'_{1,\text{ext}} = 66.2 \times 0.02[\pi(696 - 400) + 2(1000 - 400)]\ \text{W/m}$$

$$q'_{1,\text{ext}} = (1231 + 1589)\ \text{W/m} = 2820\ \text{W/m} \qquad \triangleleft$$

Comments: Applying an energy balance to a differential control volume about the air, it follows that

$$\frac{dT_m}{dx} = \frac{q'_1}{\dot{m}c_p} = \frac{2820\ \text{W/m}}{0.01\ \text{kg/s}\ (1014\ \text{J/kg}\cdot\text{K})} = 278\ \text{K/m}$$

Hence the air temperature change is significant, and a more representative analysis would subdivide the tube into axial zones and would allow for variations in air and insulated surface temperatures between zones. Moreover, a two-surface analysis of radiation exchange would no longer be appropriate.

13.4 *Radiation Exchange with Participating Media*

Although we have developed means for predicting radiation exchange between surfaces, it is important to be cognizant of the inherent limitations. Recall that we have considered *isothermal*, *opaque*, *gray* surfaces that *emit* and *reflect diffusely* and that are characterized by *uniform* surface *radiosity* and *irradiation*. For enclosures we have also considered the medium that separates the surfaces to be *nonparticipating*; that is, it neither absorbs nor scatters the surface radiation, and it emits no radiation.

The foregoing conditions and the related equations may often be used to obtain reliable first estimates and, in most cases, highly accurate results for radiation transfer in an enclosure. Sometimes, however, the assumptions are grossly inappropriate and more refined prediction methods are needed. Although beyond the scope of this text, the methods are discussed in more advanced treatments of radiation transfer [3, 7–12].

We have said little about gaseous radiation, having confined our attention to radiation exchange at the surface of an opaque solid or liquid. For *nonpolar* gases, such as O_2 or N_2, such neglect is justified, since the gases do not emit radiation and are essentially transparent to incident thermal radiation. However, the same may not be said for polar molecules, such as CO_2, H_2O (vapor), NH_3, and hydrocarbon gases, which emit and absorb over a wide temperature range. For such gases matters are complicated by the fact that, unlike radiation from a solid or a liquid, which is distributed continuously with wavelength, gaseous radiation is concentrated in specific *wavelength intervals* (called bands). Moreover, gaseous radiation is not a surface phenomenon, but is instead a *volumetric* phenomenon.

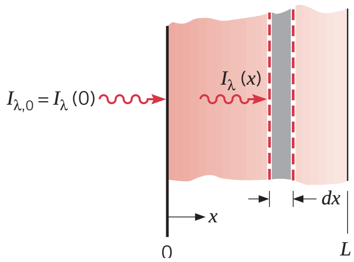


Figure 13.14 Absorption in a gas or liquid layer.

13.4.1 Volumetric Absorption

Spectral radiation absorption in a gas (or in a semitransparent liquid or solid) is a function of the absorption coefficient κ_λ (1/m) and the thickness L of the medium (Figure 13.14). If a monochromatic beam of intensity $I_{\lambda,0}$ is incident on the medium, the intensity is reduced due to absorption, and the reduction occurring in an infinitesimal layer of thickness dx may be expressed as

$$dI_\lambda(x) = -\kappa_\lambda I_\lambda(x)\, dx \tag{13.28}$$

Separating variables and integrating over the entire layer, we obtain

$$\int_{I_{\lambda,0}}^{I_{\lambda,L}} \frac{dI_\lambda(x)}{I_\lambda(x)} = -\kappa_\lambda \int_0^L dx$$

where κ_λ is assumed to be independent of x. It follows that

$$\frac{I_{\lambda,L}}{I_{\lambda,0}} = e^{-\kappa_\lambda L} \tag{13.29}$$

This exponential decay, termed *Beer's law*, is a useful tool in approximate radiation analysis. It may, for example, be used to infer the overall spectral absorptivity of the medium. In particular, with the transmissivity defined as

$$\tau_\lambda = \frac{I_{\lambda,L}}{I_{\lambda,0}} = e^{-\kappa_\lambda L} \tag{13.30}$$

the absorptivity is

$$\alpha_\lambda = 1 - \tau_\lambda = 1 - e^{-\kappa_\lambda L} \tag{13.31}$$

If Kirchhoff's law is assumed to be valid, $\alpha_\lambda = \varepsilon_\lambda$, Equation 13.31 also provides the spectral emissivity of the medium.

13.4.2 Gaseous Emission and Absorption

A common engineering calculation is one that requires determination of the radiant heat flux from a gas to an adjoining surface. Despite the complicated spectral and directional effects inherent in such calculations, a simplified procedure may be used. The method was developed by Hottel [13] and involves determining radiation emission from a hemispherical gas mass of temperature T_g to a surface element dA_1, which is located at the center of the hemisphere's base. Emission from the gas per unit area of the surface is expressed as

$$E_g = \varepsilon_g \sigma T_g^4 \tag{13.32}$$

where the gas emissivity ε_g was determined by correlating available data. In particular, ε_g was correlated in terms of the temperature T_g and total pressure p of the gas, the partial pressure p_g of the radiating species, and the radius L of the hemisphere.

Results for the emissivity of water vapor are plotted in Figure 13.15 as a function of the gas temperature, for a total pressure of 1 atm, and for different values of the product of the vapor partial pressure and the hemisphere radius. To evaluate the emissivity for total pressures other than 1 atm, the emissivity from Figure 13.15 must be multiplied by the correction factor C_w from Figure 13.16. Similar results were obtained for carbon dioxide and are presented in Figures 13.17 and 13.18.

The foregoing results apply when water vapor or carbon dioxide appear *separately* in a mixture with other species that are nonradiating. However, the results may readily be extended to situations in which water vapor and carbon dioxide appear *together* in a mixture with other nonradiating gases. In particular, the total gas emissivity may be expressed as

$$\varepsilon_g = \varepsilon_w + \varepsilon_c - \Delta\varepsilon \tag{13.33}$$

where the correction factor $\Delta\varepsilon$ is presented in Figure 13.19 for different values of the gas temperature. This factor accounts for the reduction in emission associated with the mutual absorption of radiation between the two species.

Recall that the foregoing results provide the emissivity of a hemispherical gas mass of radius L radiating to an element of area at the center of its base. However,

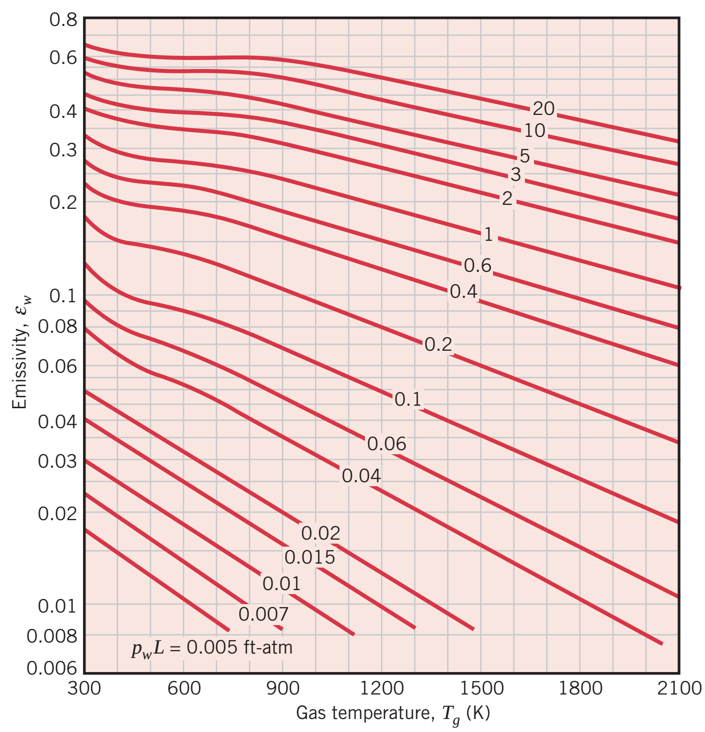


FIGURE 13.15 Emissivity of water vapor in a mixture with nonradiating gases at 1-atm total pressure and of hemispherical shape [13]. Used with permission.

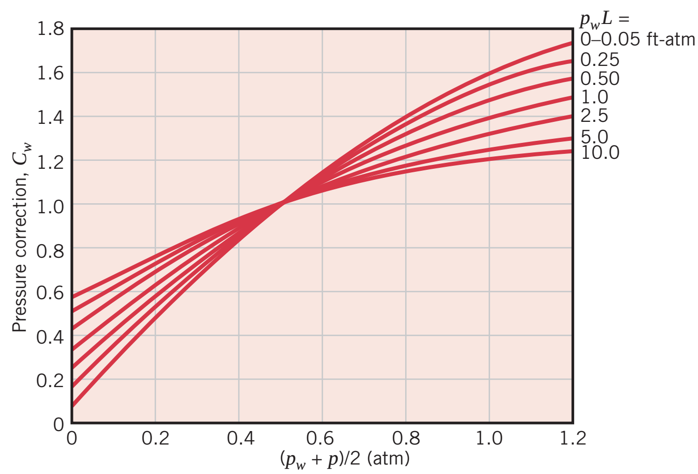


FIGURE 13.16 Correction factor for obtaining water vapor emissivities at pressures other than 1 atm ($\varepsilon_{w,p\neq 1\text{atm}} = C_w \varepsilon_{w,p=1\text{atm}}$) [13]. Used with permission.

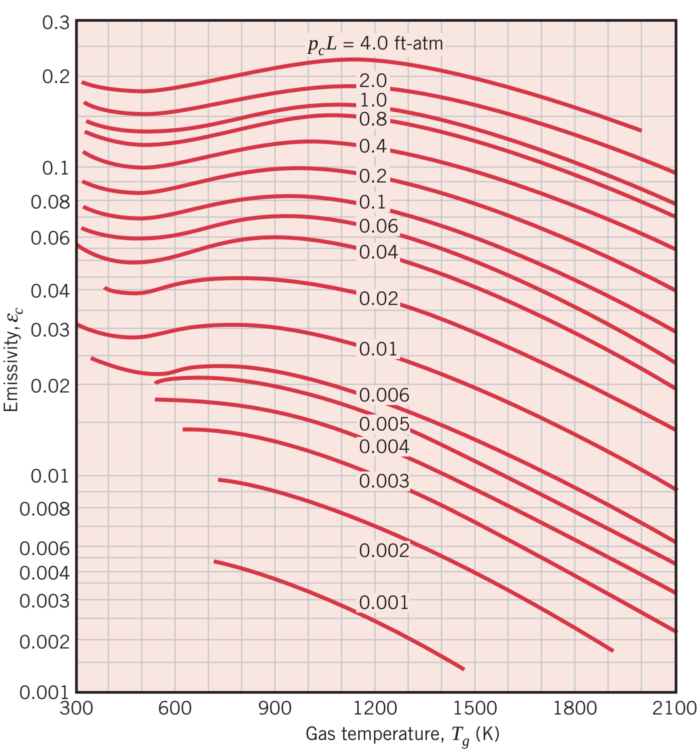


FIGURE 13.17 Emissivity of carbon dioxide in a mixture with nonradiating gases at 1-atm total pressure and of hemispherical shape [13]. Used with permission.

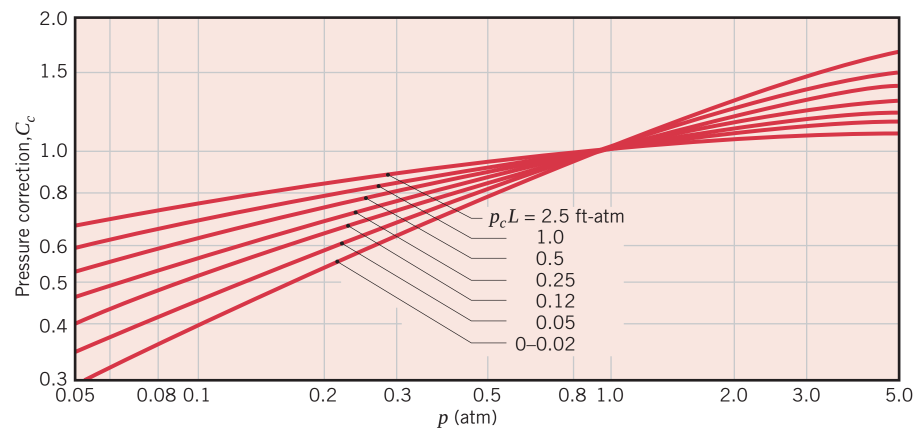


FIGURE 13.18 Correction factor for obtaining carbon dioxide emissivities at pressures other than 1 atm ($\varepsilon_{c,p\neq 1\,\text{atm}} = C_c \varepsilon_{c,p=1\,\text{atm}}$) [13]. Used with permission.

the results may be extended to other gas geometries by introducing the concept of a *mean beam length*, L_e. The quantity was introduced to correlate, in terms of a single parameter, the dependence of gas emissivity on both the size and the shape of the gas geometry. It may be interpreted as the radius of a hemispherical gas mass whose emissivity is equivalent to that for the geometry of interest. Its value has been determined for numerous gas shapes [13], and representative results are listed in Table 13.4. Replacing L by L_e in Figures 13.15 through 13.19, the emissivity associated with the geometry of interest may then be determined.

Using the results of Table 13.4 with Figures 13.15 through 13.19, it is possible to determine the rate of radiant heat transfer to a surface due to emission from an adjoining gas. This heat rate may be expressed as

$$q = \varepsilon_g A_s \sigma T_g^4 \tag{13.34}$$

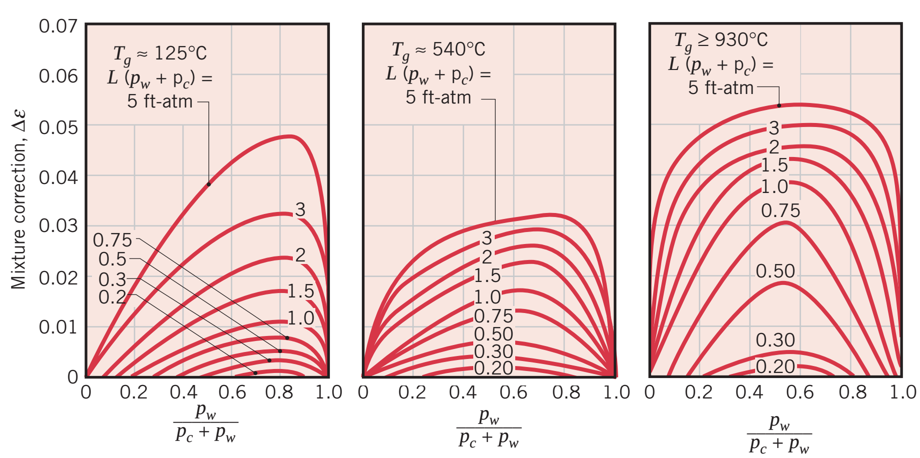


FIGURE 13.19 Correction factor associated with mixtures of water vapor and carbon dioxide [13]. Used with permission.

TABLE 13.4 Mean Beam Lengths L_e for Various Gas Geometries

Geometry	Characteristic Length	L_e
Sphere (radiation to surface)	Diameter (D)	$0.65D$
Infinite circular cylinder (radiation to curved surface)	Diameter (D)	$0.95D$
Semi-infinite circular cylinder (radiation to base)	Diameter (D)	$0.65D$
Circular cylinder of equal height and diameter (radiation to entire surface)	Diameter (D)	$0.60D$
Infinite parallel planes (radiation to planes)	Spacing between planes (L)	$1.80L$
Cube (radiation to any surface)	Side (L)	$0.66L$
Arbitrary shape of volume V (radiation to surface of area A)	Volume to area ratio (V/A)	$3.6V/A$

where A_s is the surface area. If the surface is black, it will, of course, absorb all this radiation. A black surface will also emit radiation, and the net rate at which radiation is exchanged between the surface at T_s and the gas at T_g is

$$q_{\text{net}} = A_s\sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4) \tag{13.35}$$

For water vapor and carbon dioxide the required gas absorptivity α_g may be evaluated from the emissivity by expressions of the form [13]

Water:

$$\alpha_w = C_w\left(\frac{T_g}{T_s}\right)^{0.45} \times \varepsilon_w\left(T_s, p_w L_e \frac{T_s}{T_g}\right) \tag{13.36}$$

Carbon dioxide:

$$\alpha_c = C_c\left(\frac{T_g}{T_s}\right)^{0.65} \times \varepsilon_c\left(T_s, p_c L_e \frac{T_s}{T_g}\right) \tag{13.37}$$

where ε_w and ε_c are evaluated from Figures 13.15 and 13.17, respectively, and C_w and C_c are evaluated from Figures 13.16 and 13.18, respectively. Note, however, that in using Figures 13.15 and 13.17, T_g is replaced by T_s and $p_w L_e$ or $p_c L_e$ is replaced by $p_w L_e(T_s/T_g)$ or $p_c L_e(T_s/T_g)$, respectively. Note also that, in the presence of both water vapor and carbon dioxide, the total gas absorptivity may be expressed as

$$\alpha_g = \alpha_w + \alpha_c - \Delta\alpha \tag{13.38}$$

where $\Delta\alpha = \Delta\varepsilon$ is obtained from Figure 13.19.

13.5 *Summary*

In this chapter we focused on the analysis of radiation exchange between the surfaces of an enclosure, and in treating this exchange we introduced the concept of a *view factor*. Because knowledge of this geometrical quantity is essential to determining radiation exchange between any two diffuse surfaces, you should be familiar