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Engineering Composite Materials

Micromechanics

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Composite micromechanics and mechanics theory

Micromechanics

Definition

- "Micromechanics" does **not** refer to mechanical behavior at the molecular level
- Study of composite behavior, about the interaction of constituent materials
- Volumetric composition, geometrical properties, and properties of reinforcement and matrix material

- Looks at components of a composite, the matrix and the fiber, and tries to predict the behavior of the assumed homogeneous composite material

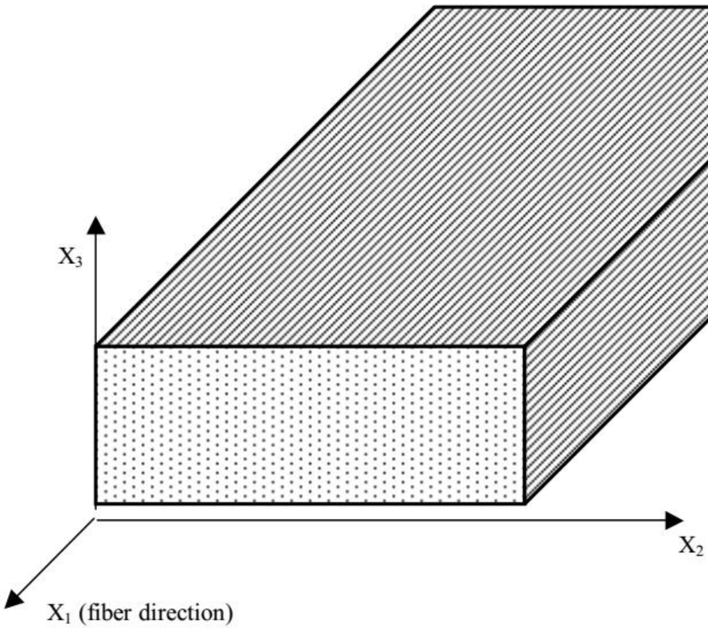
- Results of incorporating a reinforcement (fibers, whiskers, particles, etc.) in a matrix to make a composite→ to be able to predict the properties of a composite (given the properties and the geometric arrangement of the components in the composite)

Composite micromechanics and mechanics theory

Micromechanics

Concerned with modeling the mechanical interactions of the constituent materials in a basic (e.g., lamina) configuration

Example: unidirectional fiber reinforced composite



An idealized micromechanical view of a unidirectional fiber reinforced composite material

- The fibers have a very small diameter and a very high length-to-diameter ratio
- This geometry yields excellent stiffness and strength characteristics in the fiber,
- Since the crystals tend to align along the fiber axis and there are fewer internal and surface defects than in the bulk material.

Composite micromechanics and mechanics theory

Micromechanics

Volume fraction and weight fraction

Vi = volume, Wi = weight

vi=volume fraction = $\frac{V_i}{\sum V_i} = \frac{V_i}{V_c}$

wi=weight fraction = $\frac{W_i}{\sum W_i} = \frac{W_i}{W_c}$

Where,

subscripts i = c: composite

f: fiber

m: matrix

Conservation of mass:

$$W_c = W_f + W_m$$

$$\Rightarrow \frac{W_f}{W_c} + \frac{W_m}{W_c} = 1$$

$$\Rightarrow w_f + w_m = 1$$

Assuming composite is void free:

$$V_c = V_f + V_m$$

$$\Rightarrow \frac{V_f}{V_c} + \frac{V_m}{V_c} = 1$$

$$\Rightarrow v_f + v_m = 1$$

Composite micromechanics and mechanics theory

Micromechanics

Density

Consider a composite of mass m_c and volume v_c. The total mass of the composite is the sum total of the masses of fiber and matrix, that is,

$$m_c = m_f + m_m$$

The subscripts c, f, and m indicate composite, fiber, and matrix, respectively

The volume of the composite, however, must include the volume of voids, v_v. Thus,

$$v_c = v_f + v_m + v_v$$

The composite density $\rho_c = m_c/v_c$) is given by

$$\rho_c = m_c/v_c = (m_f + m_m)/v_c = (\rho_f v_f + \rho_m v_m)/v_c$$ or

$$\rho_c = \rho_f v_f + \rho_m v_m$$

$M_f + M_m = 1$ and
$V_f + V_m + V_v = 1$

M_f, M_m and V_f, V_m, V_v are mass and volume fractions

Micromechanics

Density

We can also derive an expression for ρ_c in terms of mass fractions.

Thus,

$$\rho_c = \frac{m_c}{v_c} = \frac{m_c}{v_f + v_m + v_v} = \frac{m_c}{m_f/\rho_f + m_m/\rho_m + v_v}$$

$$= \frac{1}{M_f/\rho_f + M_m/\rho_m + v_v/m_c}$$

$$= \frac{1}{M_f/\rho_f + M_m/\rho_m + v_v/\rho_c v_c}$$

$$= \frac{1}{M_f/\rho_f + M_m/\rho_m + V_v/\rho_c}.$$

to measure (indirectly) the volume fraction of voids in a composite

$$\rho_c = \frac{\rho_c}{\rho_c\left[M_f/\rho_f + M_m/\rho_m\right] + V_v}$$

or

$$V_v = 1 - \rho_c\left(\frac{M_f}{\rho_f} + \frac{M_m}{\rho_m}\right).$$

Krishan K. Chawla, Composite Materials Science and Engineering, Third Edition

Composite micromechanics and mechanics theory

Micromechanics

Void content determination

Experimental result (with voids): $\rho_{ce} = \rho_f v_f + \rho_m v_m + \rho_v v_v$

$$= \rho_f v_f + \rho_m v_m \ldots \quad (*)$$

Theoretical calculation (excluding voids):

$$\mathrm{m}_c = \mathrm{m}_f + \mathrm{m}_m$$

$$\rho_{ct}(1 - v_v) = \rho_f v_f + \rho_m v_m$$

$$\therefore \rho_{ct} = \rho_f v_f + \rho_m v_m + \rho_{c_t} v_v \ldots \quad (**)$$

$$void\ content : (**) - (*) \Rightarrow \quad v_v = \frac{\rho_{ct} - \rho_{ce}}{\rho_{ct}}$$

void content < 1% ➔ Good composite

> 5% ➔ Poor composite

Micromechanics

Void content determination

Exercise 1: Glass/epoxy composite

Weight of empty crucible = 47.6504 g

Weight of crucible +composite = 50.1817 g

Weight of crucible +glass fibers = 49.4476 g

Find v_v if $\rho_{ce} = 1.86\ {}^{g}/_{cm^3}$ $\rho_f = 2.5\ {}^{g}/_{cm^3}$, $\rho_m = 1.2\ {}^{g}/_{cm^3}$

Micromechanics

Rule of Mixture

To approximate estimation of composite material properties, based on an assumption that a composite property is the volume weighed average of the phases (matrix and dispersed phase) properties.

Since weight = (density)*(volume), we obtain immediately

$$\rho_c = \sum_{i=1}^{n} \rho_i v_i$$

Thus, a Rule of Mixtures, which for 2-phase composites becomes: $\rho_c = \rho_f v_f + \rho_m v_m$

And in terms of weight fractions instead of volume fractions:

$$\rho_c = \frac{1}{(w_f/\rho_f) + (w_m/\rho_m)}$$

The void fraction can be calculated from measured weights (**not** fractions) and densities:

$$v_{voids} = 1 - \frac{(W_f/\rho_f) + \dfrac{(W_{composite} - W_f)}{\rho_m}}{(W_{composite}/\rho_{composite})}$$

Micromechanics

Rule of Mixture

$$\rho_c = \rho_m \phi_m + \rho_f \phi_f$$

After fabrication the composite contains voids.

The amount of voids can be estimated as:

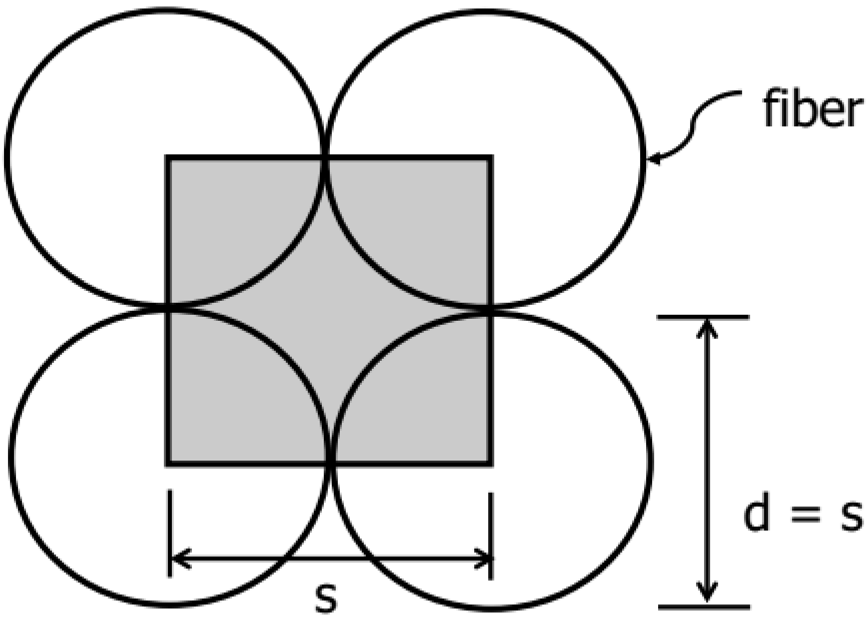
$$\Phi_v = (\rho_{theoretical} - \rho_{actual}) / \rho_{theoretical}$$

Micromechanics

Rule of Mixture

Maximum Fiber Volume Fraction - Square packing

The maximum volume fraction occurs when d = s



$$d = s$$

$$V_{comp} = s^2 L$$

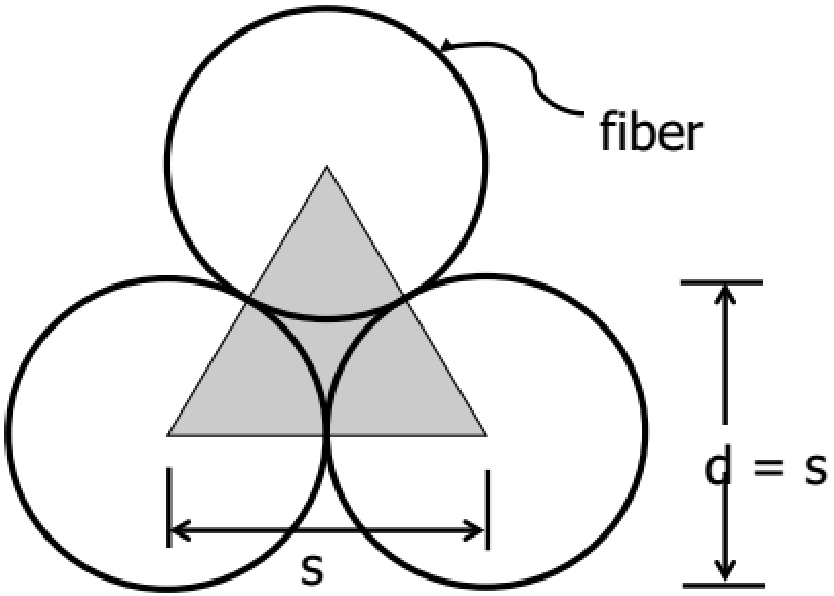
$$V_f = \frac{\pi s^2}{4} L$$

$$\varphi_f = \frac{V_f}{V} = \frac{\pi}{4} = 0.785$$

Composite micromechanics and mechanics theory

Micromechanics **Rule of Mixture**

Maximum Fiber Volume Fraction – Triangular packing



Representative Volume Elements

Fiber volume fraction

$$V = \frac{1}{2}s\left(s\sin 60^o\right) = \frac{1}{2}s^2\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4}s^2$$

$$V_f = \left(\frac{\pi d^2}{4}\right)\left(\frac{1}{6}+\frac{1}{6}+\frac{1}{6}\right) = \frac{\pi d^2}{8}$$

$$\phi_f = \frac{\pi}{2\sqrt{3}}\left(\frac{d}{s}\right)^2$$

$$v_f = \frac{\pi}{2\sqrt{3}}\left(\frac{d}{s}\right)^2$$

$$d = s$$

$$\phi_f = \frac{\pi}{2\sqrt{3}} = 0.907$$

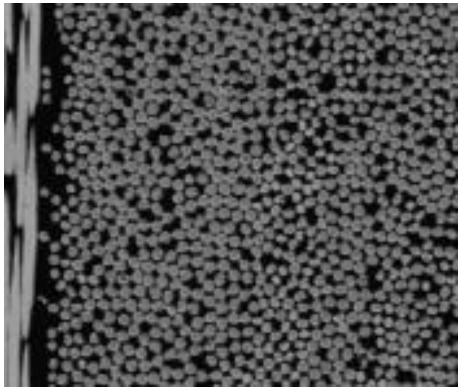
Micromechanics Maximum Fiber Volume Fraction

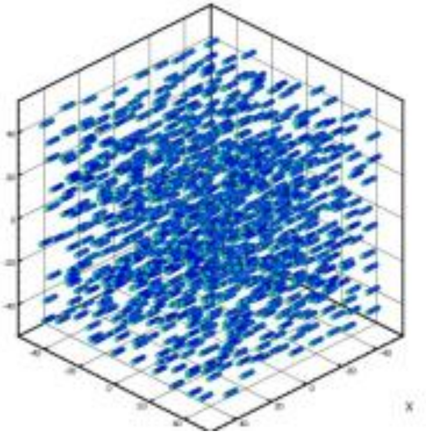
Comments

- Theoretical maximums are not achieved. They serve as guides.

- Actual values:

– Continuous Fibers = 0.5 – 0.8
– Chopped Fibers much lower





Composite micromechanics and mechanics theory

Micromechanics Mechanical Properties

Some of the methods for predicting

- **elastic constants,**
- **thermal properties, and**
- **transverse stresses in fibrous composites**
- **treat the mechanics of load transfer**

Mechanical Properties

- Mechanics of Materials Models
  - Elementary
  - Advanced
- Elasticity Models
- Finite Element Models
- Semi-empirical Models

Composite micromechanics and mechanics theory

Micromechanics **Mechanical Properties**

Elementary Mechanics of Materials Models

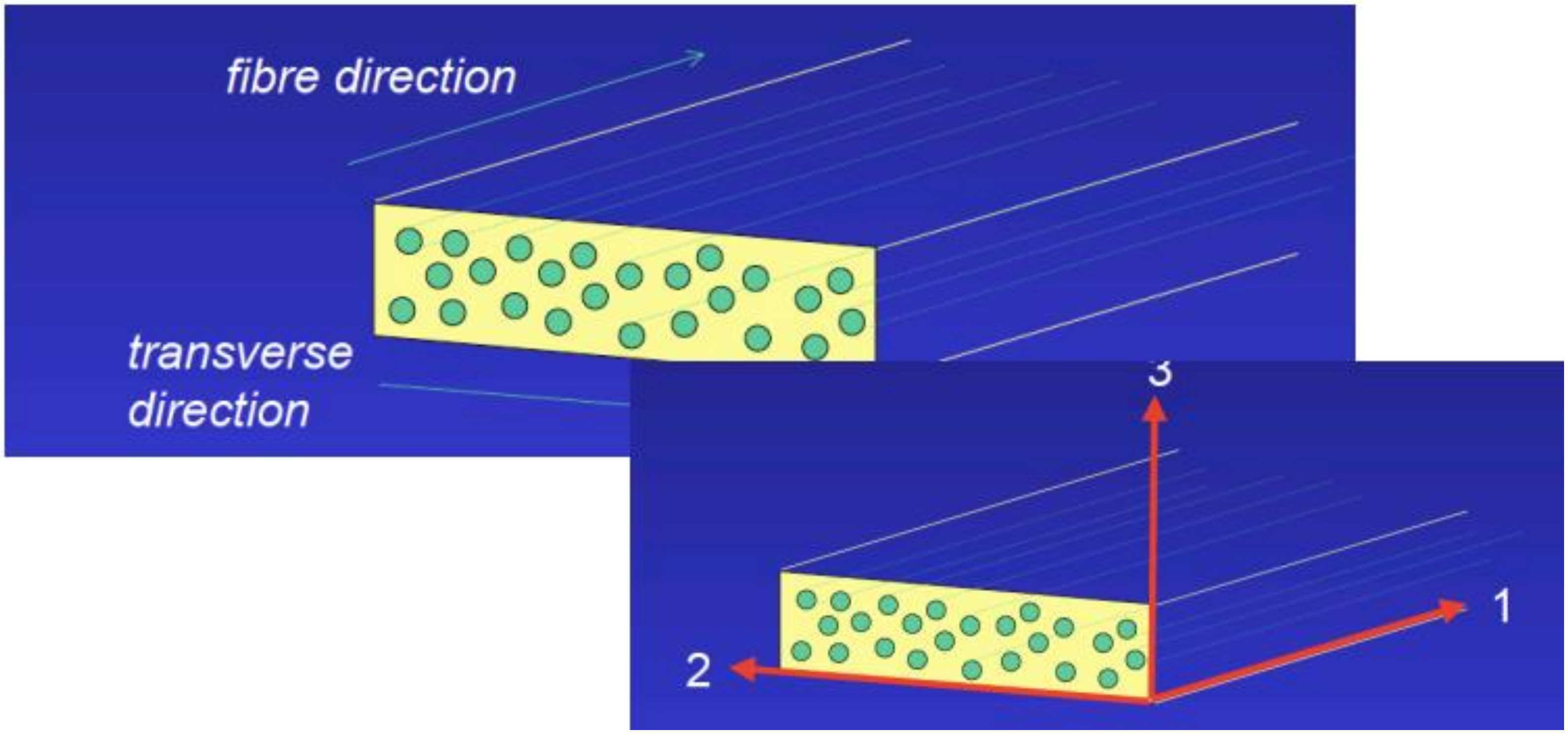
- Longitudinal Modulus E_1 or E_L
- Transverse Modulus E_2 or E_T
- Shear Modulus G_{12} or G_{LT}
- Main Poisson's Ratio ν_{12} or ν_{LT}
- Secondary Poisson's Ratio ν_{21} or ν_{TL}
- Interlaminar Shear Modulus

Composite micromechanics and mechanics theory

Micromechanics **Mechanical Properties**

Longitudinal Modulus of unidirectional composites

- Unidirectional fibers are the simplest arrangement of fibers to analyze
- They provide maximum properties in the fiber direction, but minimum properties in the transverse direction



Composite micromechanics and mechanics theory

Micromechanics Mechanical Properties

Longitudinal Modulus of unidirectional composites

Total Load = Fiber Load + Matrix Load

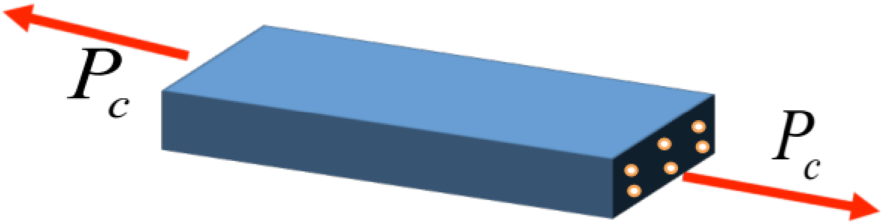
$P_c = P_f + P_m$

Let A_c *the* cross-section of the composite

A_m the cross-section of the matrix and

A_f *the* cross-section of the filler

$\sigma_c A_c = \sigma_m A_m + \sigma_f A_f$



Area Fractions

$$\frac{A_m}{A_c} = \varphi_m \quad \text{and} \quad \frac{A_f}{A_c} = \varphi_f$$

Since: $V = LA$ and *Lengths* are all equal

Equilibrium

$$\bar{\sigma}_c = \bar{\sigma}_m \varphi_m + \bar{\sigma}_f \varphi_f$$

The stress applied to the composite is shared between the fibers and the matrix proportionally to their volume content.

Mixture Rule

Composite micromechanics and mechanics theory

Micromechanics — Mechanical Properties

Assumptions

- Matrix is isotropic
- Fiber is orthotropic or isotropic
  - Graphite, Kevlar - orthotropic
  - Glass, Boron - isotropic
- One dimensional Hooke's law (ignore Poisson's effects)

Note: Most fibers are not isotropic, that means that the properties depend on the direction. When loaded with fibers, unless the fibers are randomly oriented, even if they are isotropic, the resulting composite becomes anisotropic.

For this reason the values referring to fibers and composite have a subscript that defines the direction along the indicated properties are evaluated.

$$\bar{\sigma}_{c1} = E_{c1}\bar{\varepsilon}_{c1}$$

$$\bar{\sigma}_{f1} = E_{f1}\bar{\varepsilon}_{f1}$$

$$\bar{\sigma}_{m} = E_{m}\bar{\varepsilon}_{m}$$

Composite micromechanics and mechanics theory

Micromechanics

Mechanical Properties

Equilibrium

$$\bar{\sigma}_{c1} = \bar{\sigma}_m \varphi_m + \bar{\sigma}_{f1} \varphi_f$$

becomes

$$E_1 \bar{\varepsilon}_{c1} = E_m \bar{\varepsilon}_{m1} \varphi_m + E_{f1} \bar{\varepsilon}_{f1} \varphi_f$$

Average strain in composite, fiber, and matrix in longitudinal direction are all equal

$$\bar{\boldsymbol{\varepsilon}}_{\mathbf{c1}} = \bar{\boldsymbol{\varepsilon}}_{\mathbf{m1}} = \bar{\boldsymbol{\varepsilon}}_{\mathbf{f1}}$$

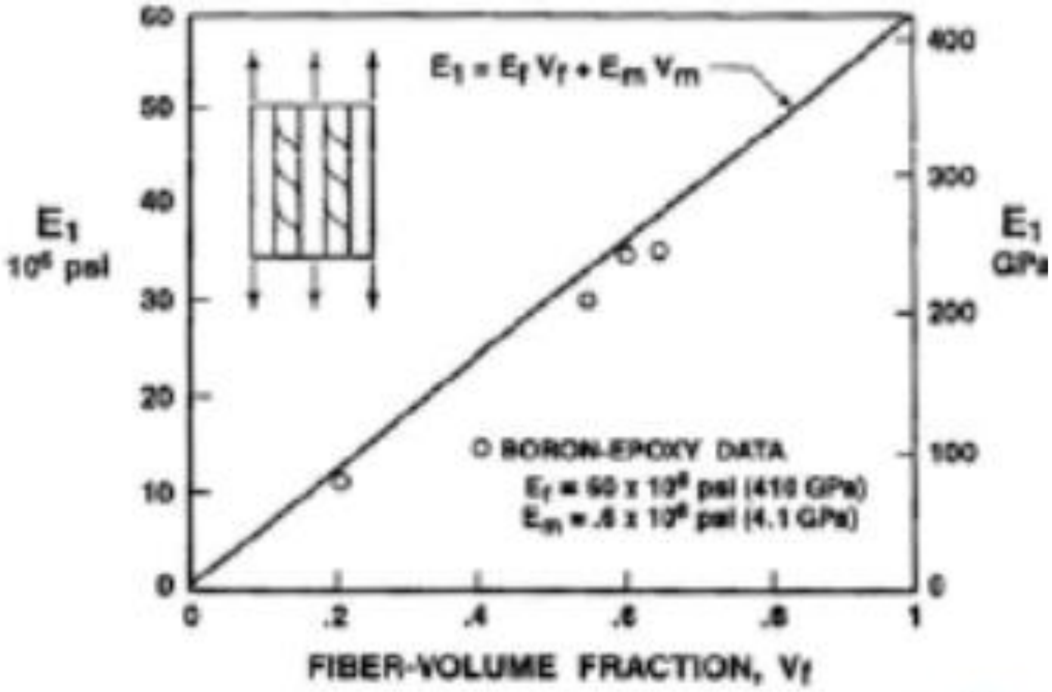
$$E_1 = E_m \phi_m + E_{f1} \phi_f$$

Rule of Mixture

Composite micromechanics and mechanics theory

Micromechanics Mechanical Properties

E_{c1} depends linearly on Φ_f, hence on the amount of fibers. In general, $E_f >> E_m$ (for glass/epoxy the ratio holds about 15, and can be much higher for carbon/epoxy). This means that the numeric contribution of the matrix to the elastic modulus of the composite is negligible. The plot of the modulus of the composite vs. the fiber volume fraction is linear, and the experimental values well agree with the theoretical prediction.



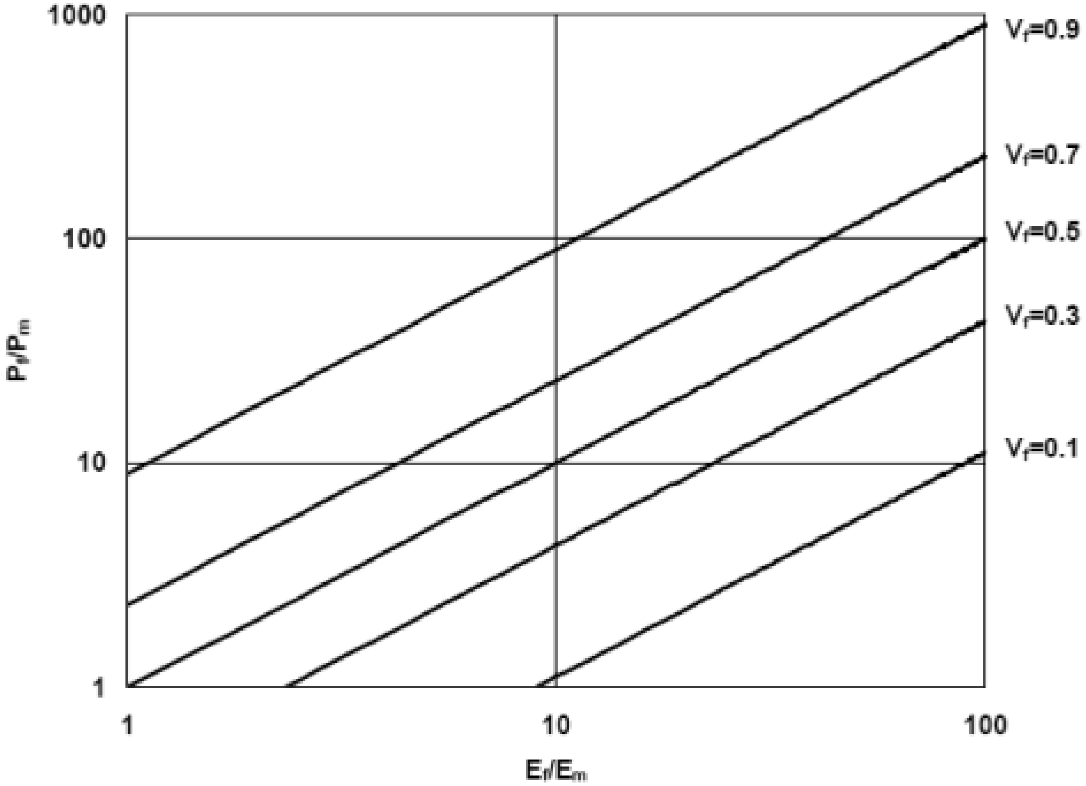
Composite micromechanics and mechanics theory

Micromechanics Mechanical Properties

The ratio of stresses in fiber and matrix is the same as the ratio of corresponding elastic moduli

$$\frac{P_f}{P_m} = \frac{\phi_f E_f}{\phi_m E_m}$$

$$\frac{P_f}{P_c} = \frac{\dfrac{E_f}{E_m}}{\dfrac{E_f}{E_m} + \dfrac{\phi_m}{\phi_f}}$$

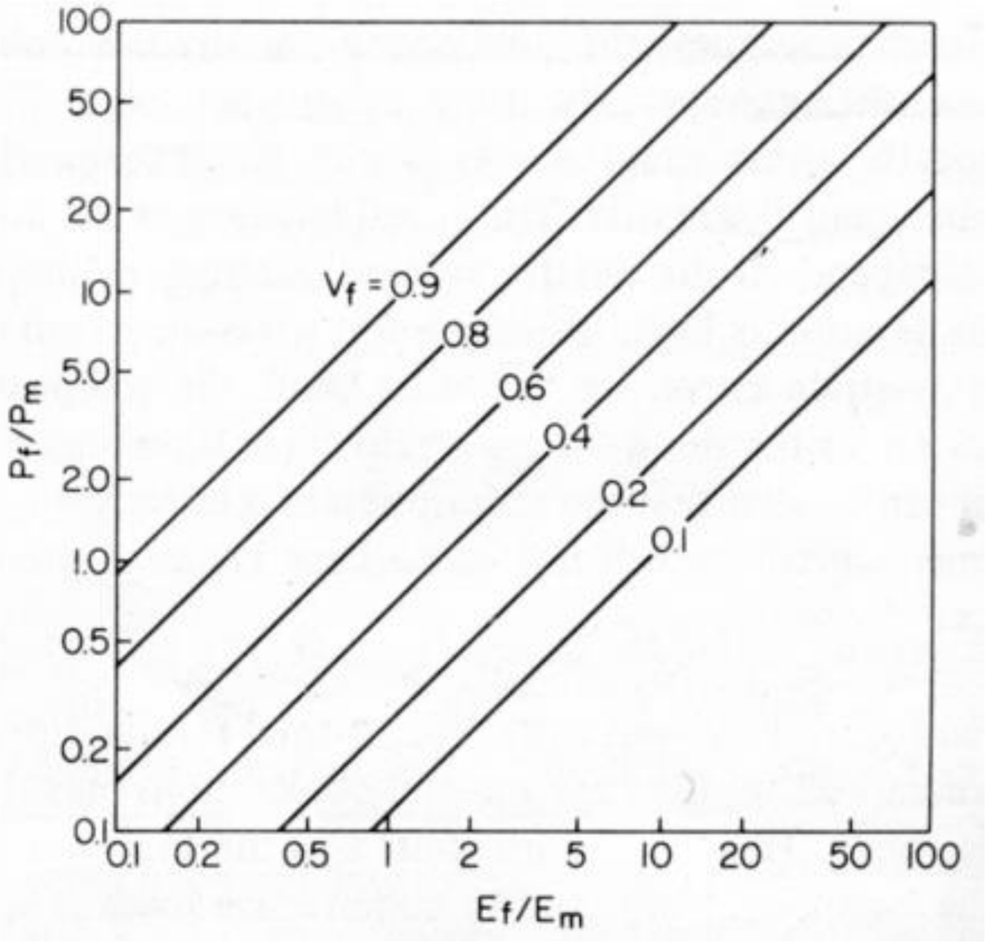


Composite micromechanics and mechanics theory

Micromechanics

Mechanical Properties

To attain high stresses in the fibers and thereby use high strength fibers more efficiently it is necessary for the fiber modulus to be much greater than the matrix modulus. The ratio of loads carried by the fibers and the matrix depends also on the fiber volume content.



Composite micromechanics and mechanics theory

Micromechanics　　　Mechanical Properties

Maximum volume fraction

The previous results indicate that the volume fraction of fibers must be maximized if we want the fibers to carry a higher portion of the composite load. Although the maximum theoretical volume fraction is around 90%, when it reaches about 80% properties of a composite tend to decrease because of the inability of the matrix to wet and infiltrate the fibers.

Exercise 2:

Calculate the fraction of load carried by the fibers in two composites of glass fibers and epoxy matrix, one of them containing 10 % of fibers by volume and the other one 50 %. Elastic moduli for glass fibers and epoxy resin are 72 Gpa and 3.6 Gpa, respectively

Micromechanics **Mechanical Properties**

Notes on the rule of mixture

In developing the rule of mixture for the elastic modulus, we have made the assumption that both matrix and fibers behave elastically till the failure.

In general the deformation of a composite may proceed in four stages:

1. Both fibers and matrix deform elastically
2. The fibers continue to deform elastically, but the matrix deforms plastically
3. Both fibers and matrix deform plastically
4. The fibers fracture is followed by the composite fracture

In the case 2, for instance, it holds: $E_c = E_f \phi_f + \left(\frac{d\sigma_m}{d\varepsilon_m} \right)_{\varepsilon_c} \phi_m$

Composite micromechanics and mechanics theory

Micromechanics Mechanical Properties

Major Poisson's Ratio

What is Poisson's ratio?

When the fiber is strained along the direction 1, due to the Poisson's effect:

$$\varepsilon_{f2} = -\nu_f \, \varepsilon_{f1} = -\nu_f \varepsilon_{c1}$$

$$\varepsilon_{m2} = -\nu_m \varepsilon_{m1} = -\nu_m \varepsilon_{c1}$$

$$\Delta_{c2} = \Delta_{f2} + \Delta_{m2} = \varepsilon_{f2} t_2 + \varepsilon_{m2} t_m = -\left(\nu_f t_f + \nu_m t_m\right)\varepsilon_{c1}$$

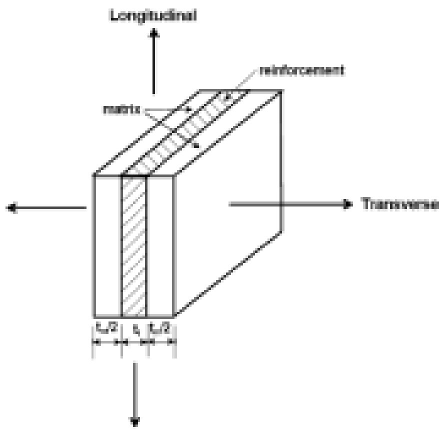
$$\varepsilon_{c2} = \frac{\Delta_{c2}}{t_c} = -\left(\nu_f \phi_f + \nu_m \phi_m\right)\varepsilon_{c1}$$

and finally $\nu_{12} = -\dfrac{\varepsilon_{c2}}{\varepsilon_{c1}} = \nu_f \phi_f + \nu_m \phi_m$

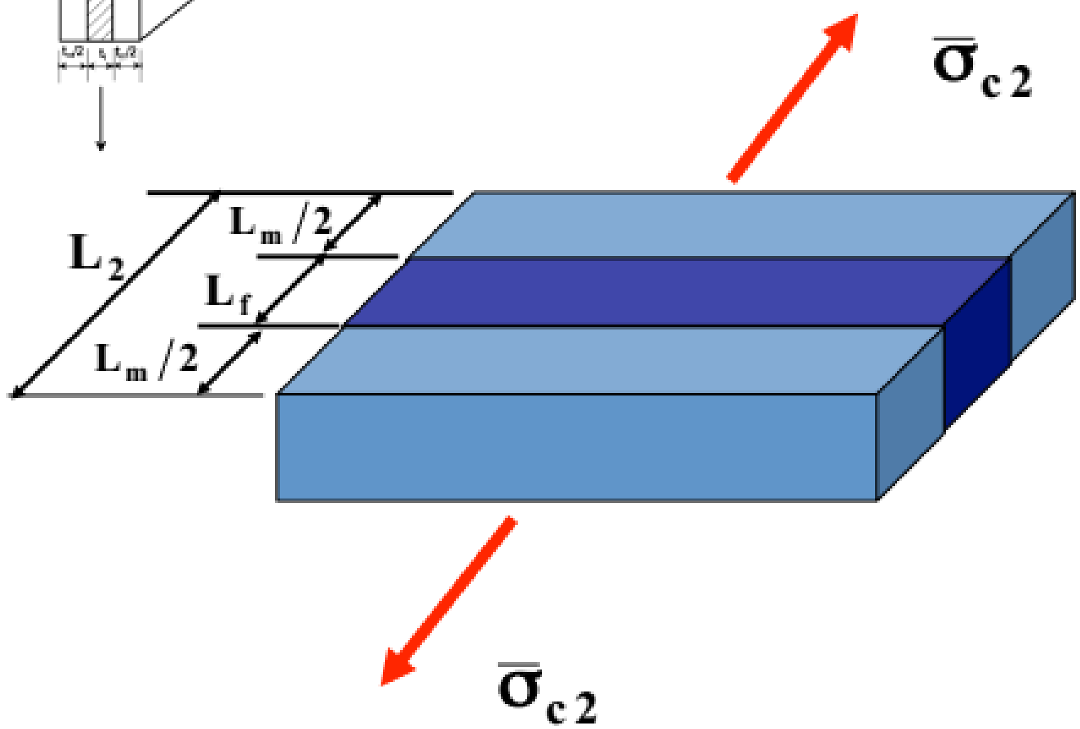
Composite micromechanics and mechanics theory

Micromechanics   Mechanical Properties

Transverse Modulus



RVE (representative volume element)

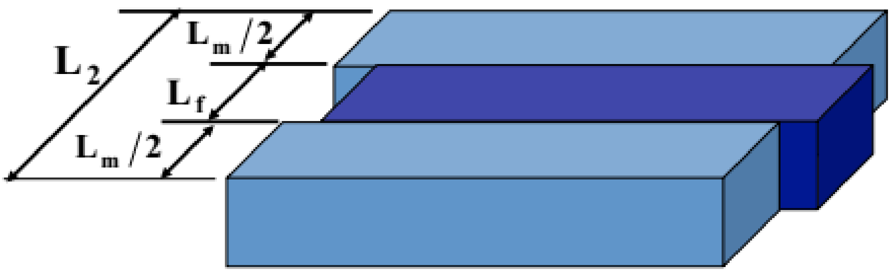


Composite micromechanics and mechanics theory

Micromechanics **Mechanical Properties**

Transverse Modulus

Volume Fractions



$$\frac{L_m}{L_2} = \varphi_m$$

and

$$\frac{L_f}{L_2} = \varphi_f$$

$$\bar{\varepsilon}_2 = \bar{\varepsilon}_{m2}\phi_m + \bar{\varepsilon}_{f2}\phi_f$$

Rule of Mixture

Hooke's Law

$$\bar{\sigma}_{c2} = E_2 \bar{\varepsilon}_{c2}$$

$$\bar{\sigma}_{f2} = E_{f2} \bar{\varepsilon}_{f2}$$

$$\bar{\sigma}_{m2} = E_m \bar{\varepsilon}_{m2}$$

$$\frac{\bar{\sigma}_{c2}}{E_2} = \frac{\bar{\sigma}_{f2}\phi_f}{E_{f2}} + \frac{\bar{\sigma}_{m2}\phi_m}{E_m}$$

Composite micromechanics and mechanics theory

Micromechanics　　　Mechanical Properties

Transverse Modulus

Stresses in composite, fiber, and matrix in transverse direction are all equal

$$\bar{\sigma}_{c1} = \bar{\sigma}_{m1} = \bar{\sigma}_{f1}$$

$$\frac{1}{E_2} = \frac{\phi_f}{E_{f2}} + \frac{\phi_m}{E_m}$$

$$\frac{1}{E_2} = \frac{\phi_f}{E_{f2}} + \frac{\phi_m}{E_m}$$

$$\frac{1}{E_2} = \frac{E_m \phi_f + E_{f2} \phi_m}{E_{f2} E_m}$$

$$E_2 = \frac{E_{f2} E_m}{E_m \phi_f + E_{f2} \phi_m}$$

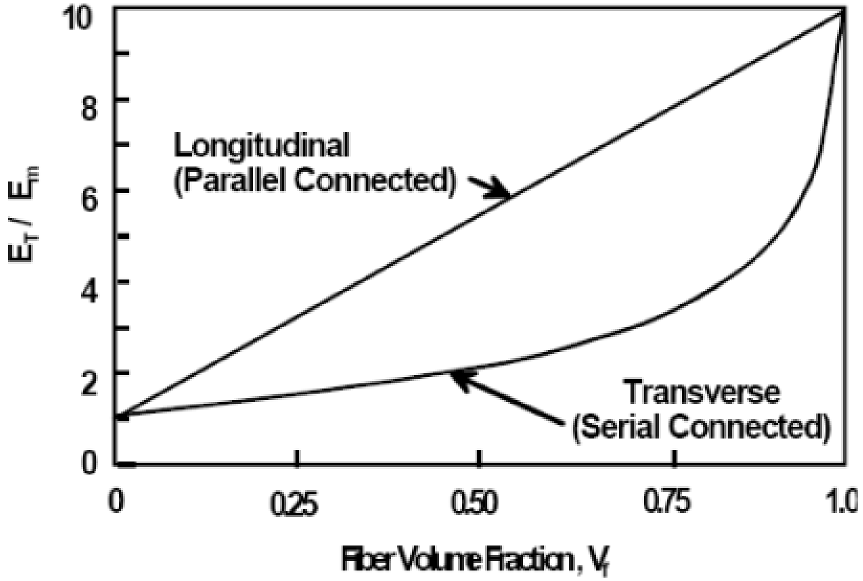
$$\frac{E_2}{E_m} = \frac{E_{f2}}{E_m \phi_f + E_{f2} \phi_m} = \frac{1}{\frac{E_m}{E_{f2}} \phi_f + \phi_m}$$

Composite micromechanics and mechanics theory

Micromechanics Mechanical Properties

Transverse Modulus

E_L and E_T as predicted by the mixture rules



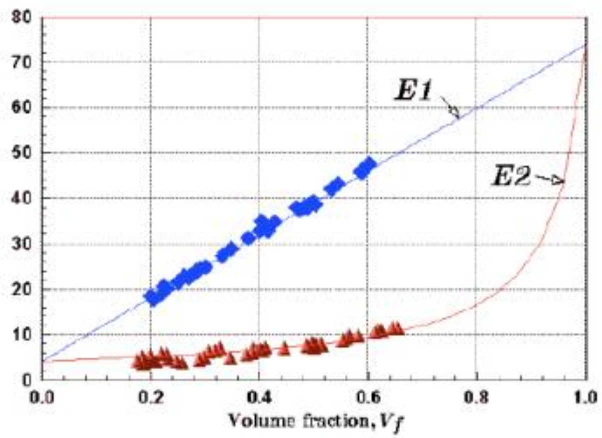
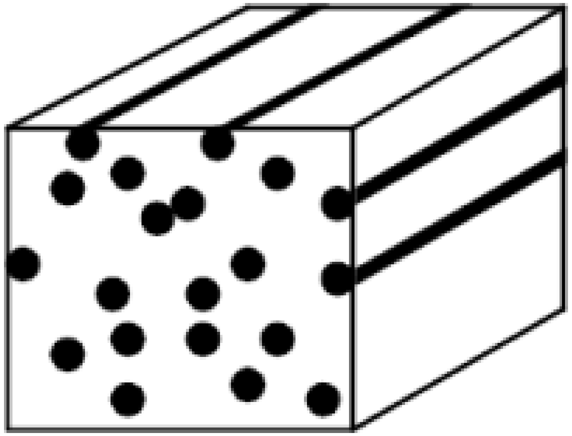


Figure 4: Rule-of-mixtures predictions for longitudinal (E_1) and transverse (E_2) modulus, for glass-polyester composite (E_f = 73.7 MPa, E_m = 4 GPa). Experimental data taken from Hull (1996).

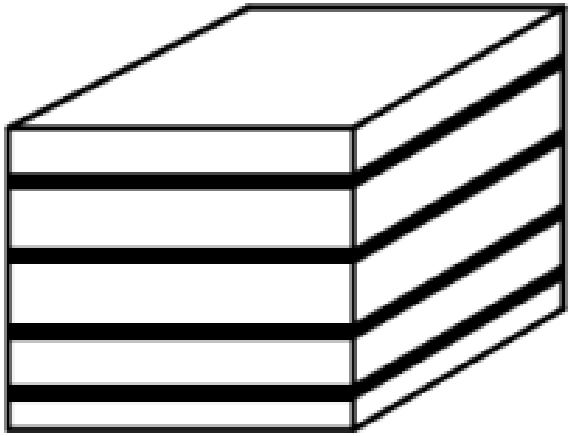
Limitations of slab models

The slab model has a number of very serious limitations, not the least of which is that it does not represent many common composites

- In realistic model, both reinforcement and matrix are present in any cross section.
- In the slab model either matrix or reinforcement is present



a) realistic model



b) slab model

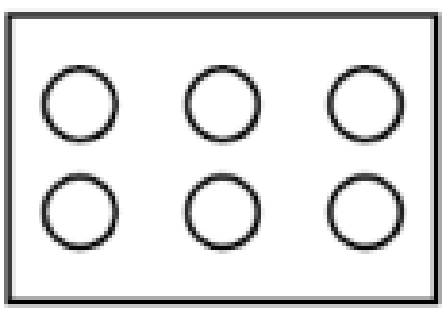
Micromechanics

Mechanical Properties

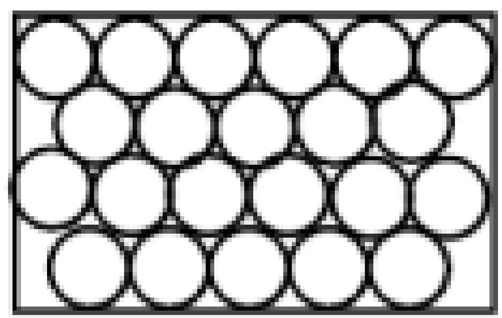
Elasticity model

In a realistic model the arrangement of the reinforcement will influence the transverse stiffness as well as other properties. Since the actual arrangement for the reinforcement cannot be known precisely some method must be devised to approximately described it. For this purpose one can use models of fiber **contiguity** shown below.

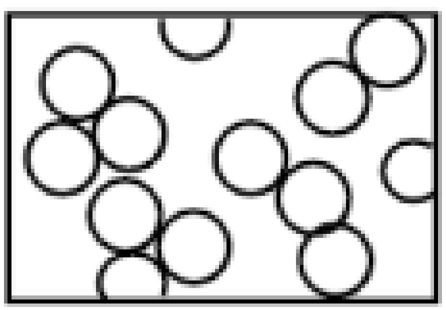
- ✓ If round fibers have the maximum number of neighbors touching them the contiguity value, C is 1
- ✓ If no fibers are touching the contiguity value C = 0
- ✓ Most composites have partial contiguity, therefore, C < 0 < 1



C = 0
Isolated fibers
Matrix Contiguous



C = 1
Isolated matrix
Fibers Contiguous



0 < C < 1
Partial Contiguity

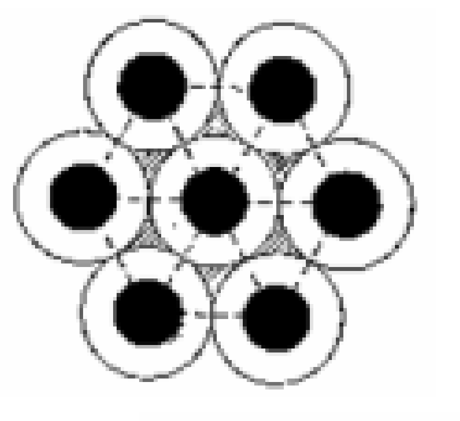
Composite micromechanics and mechanics theory

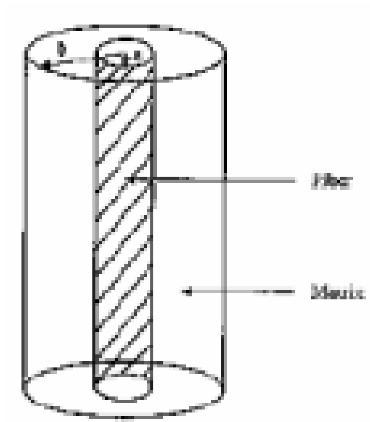
Micromechanics Mechanical Properties

Composite cylinder assemblage model (CCA)

Basic assumptions:

1. All fibers have same radii
2. Perfect bond between fibers and matrix
3. No voids
4. Fibers are uniformly arranged in a hexagonal packing pattern
5. Both fiber and matrix are linear elastic isotropic materials
6. Neglect the matrix between the cylinders and assuming axisymmetric loading





Composite micromechanics and mechanics theory

Micromechanics Mechanical Properties

Composite cylinder assemblage model (CCA)

The solutions of the model are:

$$E_1 = E_f V_f + E_m V_m + \frac{4(\nu_f - \nu_m)^2 V_f V_m}{\frac{V_m}{K_f} + \frac{V_f}{K_m} + \frac{1}{G_m}}$$

$$\nu_{12} = \nu_f V_f + \nu_m V_m + \frac{(\nu_f - \nu_m)(\frac{1}{K_m} - \frac{1}{K_f}) V_f V_m}{\frac{V_m}{K_f} + \frac{V_f}{K_m} + \frac{1}{G_m}}$$

where

$$K_m = \frac{E_m}{3(1-2\nu_m)}, K_f = \frac{E_f}{3(1-2\nu_f)}$$

$$G_{12} = G_m \frac{G_m V_m + G_f(1+V_f)}{G_f V_m + G_m(1+V_f)}$$

Composite micromechanics and mechanics theory

Micromechanics Mechanical Properties

Composite cylinder assemblage model (CCA)

The solutions of the model are:

The transverse Young's modulus base on an elasticity solution using the contiguity models is given as

$$E_T = 2\left[1-\nu_f+(\nu_f-\nu_m)V_m\right]\left[\begin{array}{l}(1-C)\dfrac{K_f(2K_m+G_m)-G_m(K_f-K_m)V_m}{(2K_m+G_m)+2(K_f-K_m)V_m}+\\ C\dfrac{K_f(2K_m+G_f)+G_f(K_m-K_f)V_m}{(2K_m+G_f)-2(K_m-K_f)V_m}\end{array}\right] \quad (5.25)$$

where $K_f=\dfrac{E_f}{2(1-\nu_f)}$, $K_m=\dfrac{E_m}{2(1-\nu_m)}$, $G_f=\dfrac{E_f}{2(1+\nu_f)}$, and $G_m=\dfrac{E_m}{2(1+\nu_m)}$

The shear modulus is given as

$$G_L=(1-C)G_m\left[\frac{2G_f-(G_f-G_m)V_m}{2G_m+(G_f-G_m)V_m}\right]+CG_f\left[\frac{(G_f+G_m)-(G_f-G_m)V_m}{(G_f+G_m)+(G_f-G_m)V_m}\right]$$

The Poisson's ratio is

$$\nu_{LT}=(1-C)\frac{K_f\nu_f(2K_m+G_m)V_m+K_m\nu_m(2K_f+G_m)V_m}{K_f(2K_m+G_m)-G_m(K_f-K_m)V_m}+C\frac{K_m\nu_m(2K_f+G_f)V_m+K_f\nu_f(2K_m+G_f)V_f}{K_f(2K_m+G_m)+G_f(K_m-K_f)V_m}$$

The CCA model does not yield an exact result for E_2 but only a pair of bounds (upper and lower bound). An alternative is obtaining an approximate solution of E_2 by using a method called Generalized-Self Consistent-Scheme (GSCS).

Composite micromechanics and mechanics theory

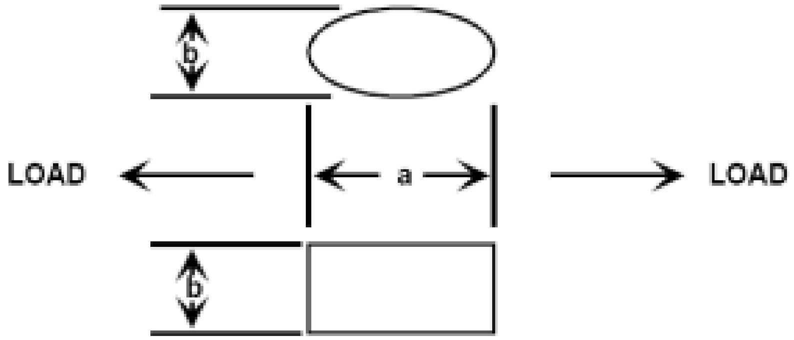
Micromechanics

Halpin-Tsai equation

The slab model for transverse elastic modulus uses unrealistic assumptions while the elasticity solution utilizing contiguity model require the evaluation of C that is not easy.

Halpin and Tsai (1969) proposed a semi-empirical approach that led to the following equation:

$$\frac{M_c}{M_m} = \frac{1+\xi\eta V_f}{1-\eta V_f}$$



Where

$$\eta = \frac{\dfrac{M_f}{M_m} - 1}{\dfrac{M_f}{M_m} + \xi}$$

Composite micromechanics and mechanics theory

Micromechanics   Halpin-Tsai equation

Semi-Empirical Models

$$\frac{E_2}{E_m} = \frac{1+\xi\eta v_f}{1-v_f\eta}$$

$$\eta = \frac{\left(\frac{E_{f2}}{E_m} - 1\right)}{\left(\frac{E_{f2}}{E_m} + \xi\right)}$$

$\xi \Rightarrow$ *curve - fitting parameter*

$\xi = 2$ *matches some previous work*

$$\frac{G_{12}}{G_m} = \frac{1+\xi\eta v_f}{1-v_f\eta}$$

$$\eta = \frac{\left(\frac{G_{f12}}{G_m} - 1\right)}{\left(\frac{G_{f12}}{G_m} + \xi\right)}$$

$\xi = 1$ *matches some previous results*

These equations are quite accurate at low volume fractions, a bit less so at higher volume fractions. They have been modified to include the maximum packing fraction. Specific values include the following:

Modulus	ξ
E_{11}	2(l/d)
E_{22}	0.5
G_{12}	1.0
G_{21}	0.5
K	0

Note finally that when $\xi = 0$, the 'inverse' rule of mixtures is obtained, and when $\xi = \infty$, the direct rule of mixtures is obtained.

Composite micromechanics and mechanics theory

Micromechanics **Tsai- Hahn equation**

$$\frac{1}{E_2} = \frac{1}{v_f + \eta_2 v_m}\left(\frac{v_f}{E_f} + \frac{\eta_2 v_m}{E_m}\right)$$

$$\frac{1}{G_{12}} = \frac{1}{v_f + \eta_{12} v_m}\left(\frac{v_f}{G_{f12}} + \frac{\eta_{12} v_m}{G_m}\right)$$

$$\eta_2, \eta_{12} \Rightarrow \textit{stress partioning parameters}$$

$$\bar{\sigma}_{m2} = \eta_2 \bar{\sigma}_{f2} \quad 0 < \eta_2 \le 1$$

$$\bar{\sigma}_{m12} = \eta_{12} \bar{\sigma}_{f12} \quad 0 < \eta_{12} \le 1$$

Composite micromechanics and mechanics theory

Micromechanics

Shear Modulus: Inverse Rule of Mixtures

$$\frac{1}{G_{12}} = \frac{v_f}{G_{f12}} + \frac{v_m}{G_m}$$

$$\Downarrow$$

$$G_{12} = \frac{G_m G_{f12}}{v_m G_{f12} + v_f G_m}$$

$$\Downarrow$$

$$\frac{G_{12}}{G_m} = \frac{1}{v_m + v_f\left(\frac{G_m}{G_{f12}}\right)}$$

Shear Modulus:

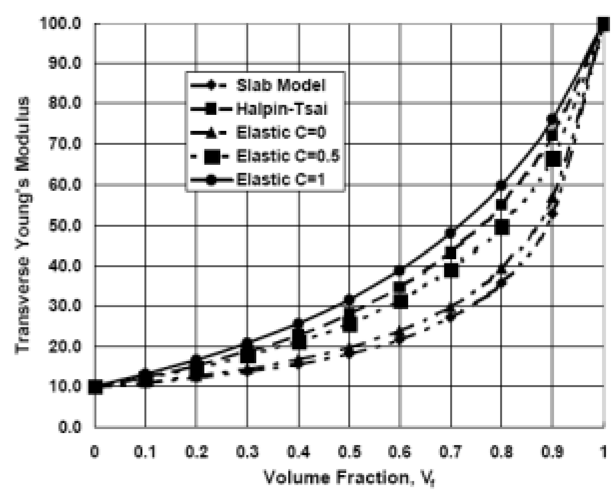
Inverse Rule of Mixtures

Assume: $G_m << G_f$

$$\frac{G_{12}}{G_m} = \frac{1}{v_m + v_f\left(\frac{G_m}{G_{f12}}\right)}$$

$$\Downarrow$$

$$\frac{G_{12}}{G_m} = \frac{1}{v_m} \Rightarrow G_{12} = \frac{G_m}{v_m} \Rightarrow G_{12} = \frac{G_m}{1 - v_f}$$



Composite micromechanics and mechanics theory

Micromechanics

Summary

1. Many models exists
2. Complexity of model depends on needed accuracy
3. Rule of Mixtures works well for fiber dominated properties (E_1 and ν_{12})
4. For E_1 and ν_{12} the mixture rule is accurate enough;
5. Better models needed for matrix dominated properties (E_2 and G_{12})
6. For other elastic constant use Halpin-Tsai equation.

Composite micromechanics and mechanics theory

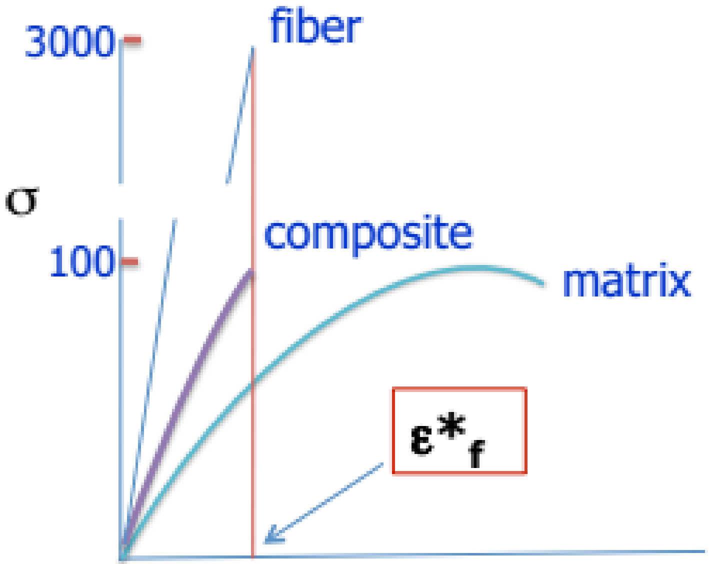
Micromechanics

Strength prediction

Ultimate tensile strength in the fiber direction

The ultimate tensile strength can be predicted by the rule of mixtures as follows:

$$\sigma_{cu} = \sigma_{fu}\phi_f + \left(\sigma_m\right)_{\varepsilon*_f}\left(1-\phi_f\right)$$



Important: The strength of the fiber is usually much higher than that one of the matrix, i.e., in the above equation the contribution of the second term is very small.
Therefore, the above equation can be modified as follows:

$$\sigma_{cu} = \sigma_{fu}\phi_f + \sigma_{mu}\phi_m$$

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Micromechanics

Strength prediction

But: do the fibers reinforce the matrix, always?

This happens if:

$$\sigma_{cu} = \sigma_{fu}\phi_f + (\sigma_m)_{\varepsilon *_f}(1-\phi_f) \geq \sigma_{mu}$$

Where σ_{mu} is the ultimate strength of the matrix

A critical fiber volume fraction must be exceeded to produce strengthening

$$\phi_{crit} = \frac{\sigma_{mu} - (\sigma_m)_{\varepsilon *_f}}{\sigma_{fu} - (\sigma_m)\varepsilon *_f}$$

… moreover, …….

Composite micromechanics and mechanics theory

Micromechanics

Strength prediction

… Moreover, …….

The composite will fail at a stress higher than the matrix strength only if:

$$\sigma_{cu} = \sigma_{mu}(1-\phi_f)$$

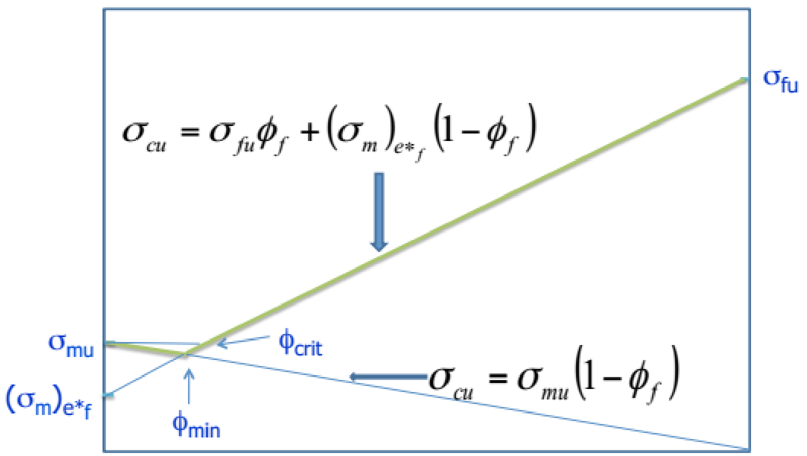
⇓

$$\sigma_{cu} = \sigma_{fu}\phi_f + (\sigma_m)_{\varepsilon*_f}(1-\phi_f) \geq \sigma_{mu}(1-\phi_f)$$

⇓

$$\phi_{min} = \frac{\sigma_{mu} - (\sigma_m)_{\varepsilon*_f}}{\sigma_{fu} + \sigma_{mu} - (\sigma_m)_{\varepsilon*_f}}$$

ϕ_{min} and ϕ_{crit}



Critical volume fraction for metal matrix composites

Matrix	σ_{fu}=0.7 GPa	σ_{fu}=1,75 GPa	σ_{fu}=3,5 GPa	σ_{fu}=7,0 GPa
Aluminum	8,33	3,25	1,61	0,80
Copper	25,53	9,84	4,86	2,41
Nickel	39,56	14,94	7,33	3.63
Stainless Steel	53,33	17,78	8,42	4,10