

Program Evaluation and Review Technique (PERT)

Scheduling with Uncertain Durations

- The uncertainty in activity duration estimates considered by using the probabilistic distribution of activity durations.
- Duration of a particular activity is assumed to be a random variable that is distributed in a particular fashion.

NETWORK TECHNIQUES

PERT

- Program Evaluation and Review Technique
- developed by the US Navy with Booz Hamilton Lockheed
- on the Polaris Missile/Submarine program 1958

CPM

- Critical Path Method
- Developed by El Dupont for Chemical Plant Shutdown Project- about same time as PERT

- ✓ Both use same calculations, almost similar
- ✓ Main difference is probabilistic and deterministic in time estimation

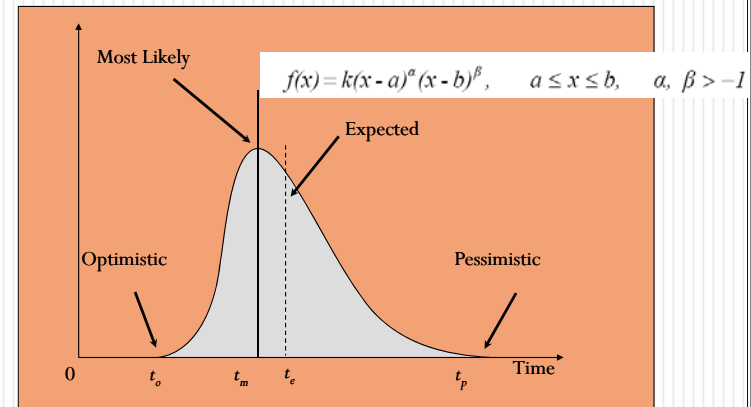
Program Evaluation and Review Technique (PERT)

- PERT is a network analysis technique used to estimate project duration when there is a high degree of **uncertainty** about the individual activity duration estimates.
- PERT uses probabilistic time estimates based on using optimistic, most likely, and pessimistic estimates of activity durations.

PERT: time estimates

- Three time estimates :
 - **Pessimistic time** (t_p) - the time the activity would take if things did not go well or the max. possible time that would be required to complete the activity.
 - **Most likely time** (t_m) (t_L) - the consensus best estimate of the activity's duration or most realistic estimate of the time an activity might consume.
 - **Optimistic time** (t_o) - the time the activity would take if things did go well or the shortest possible time in which an activity can be completed under ideal conditions.

Activity time β -distribution



Expected time

- Expected activity duration is calculated as *average of the 3 time estimates*.
- An assumption is made that the optimistic and pessimistic activity time t_o and t_p respectively are equally likely to occur. It is also assumed that the most likely activity time t_m is 4 times more likely to occur than either of t_o and t_p .

$$\text{Mean (expected time): } t_e = \frac{t_p + 4t_m + t_o}{6}$$

$$\text{Variance: } V_t = \sigma^2 = \left(\frac{t_p - t_o}{6} \right)^2$$

PERT analysis

- Using expected activity durations and critical path scheduling, a critical path of activities can be identified.
- This critical path is then used to analyze the duration of the project incorporating the uncertainty of the activity durations along the critical path.
- The expected project duration is equal to the sum of the expected durations of the activities along the critical path.

PERT analysis

- Assuming that activity durations are independent random variables, the variance or variation in the duration of this critical path is calculated as the sum of the variances along the critical path.
- With the mean and variance of the identified critical path known, the distribution of activity durations can also be computed.

Probability computation

Determine probability that project is completed within specified time

$$Z = \frac{x - \mu}{\sigma}$$

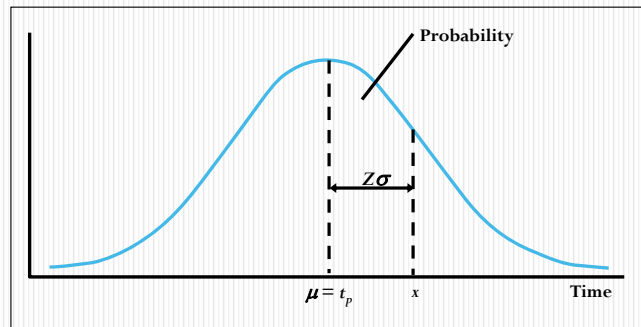
where $\mu = t_p$ = project mean time

σ = project standard mean time

x = (proposed) specified time

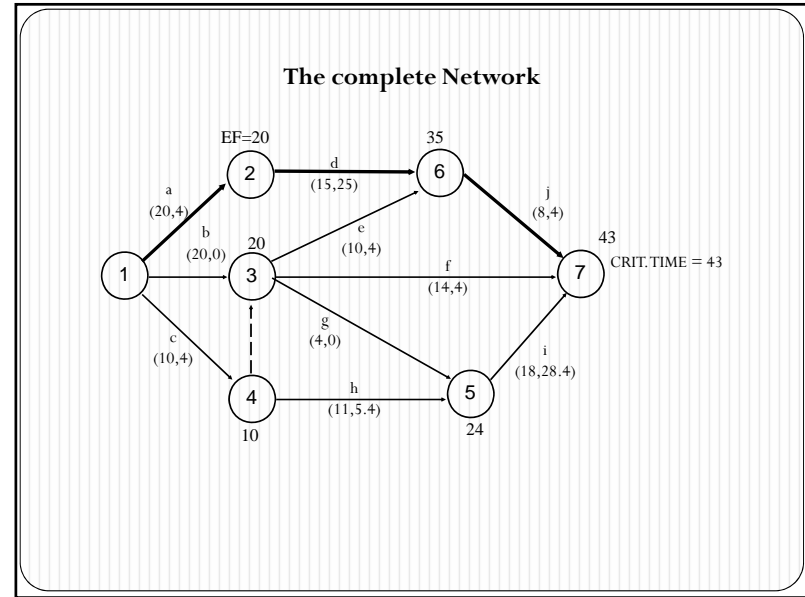
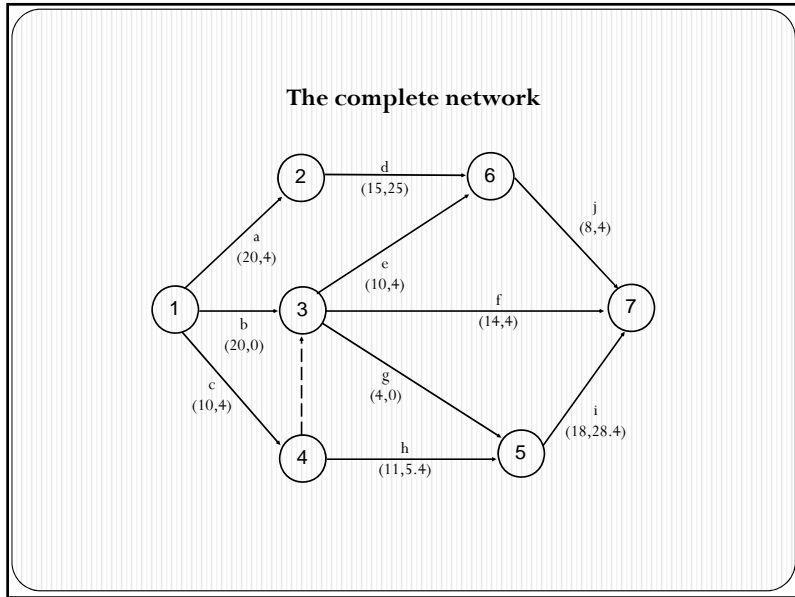
Normal Distribution of Project Time

The normal distribution is a good approximation to the beta distribution in the center of the distribution and is easy to work with, so it is often used as an approximation.



PERT Example

Activity	Immediate Predecessor	Optimistic time, to	Most Likely time, tm	Pessimistic time, tp	EXP te	Var V	S.Dev σ
a	-	10	22	22	20	4	2
b	-	20	20	20	20	0	0
c	-	4	10	16	10	4	2
d	a	2	14	32	15	25	5
e	b,c	8	8	20	10	4	2
f	b,c	8	14	20	14	4	2
g	b,c	4	4	4	4	0	0
h	c	2	12	16	11	5.4	2.32
I	g,h	6	16	38	18	28.4	5.33
j	d,e	2	8	14	8	4	2



Comparison Between CPM and PERT

Activity	LS	ES	TF	Critical ?
a	0	0	0	Yes
b	1	0	1	
c	4	0	4	
d	20	20	0	Yes
e	25	20	5	
f	29	20	9	
g	21	20	1	
h	14	10	4	
i	25	24	1	
j	35	35	0	Yes

Calculations PERT

Assume, PM promised to complete the project in the fifty days.
 What are the chances of meeting that deadline?
 Calculate Z, where

$$Z = \frac{x - \mu}{\sigma}$$

Example,
 $X = 50; \mu$ (project mean time) $= 20+15+8 = 43; V = (4+25+4) = 33$
 $Z = (50 - 43) / 5.745$
 $= 1.22$

The probability value of $Z = 1.22$, is 0.888

Calculations PERT

What deadline are you 95% sure of meeting

Z value associated with 0.95 is 1.645 (Table)

$$\begin{aligned} X &= \mu + 5.745 (1.645) \\ &= 43 + 9.45 \\ &= 52.45 \text{ days} \end{aligned}$$

Thus, there is a 95 percent chance of finishing the project by 52.45 days.

Exercise

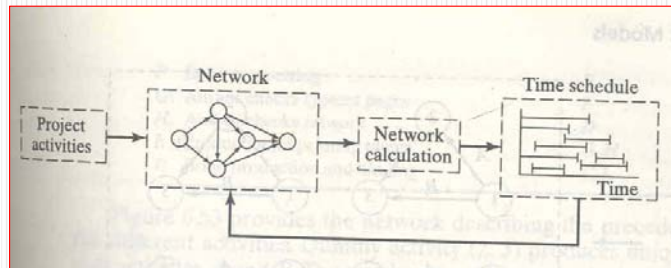
- Consider the activities given below with three time estimates



- Calculate the expected time for each activity and for the network-ending (**27.65**)
- Calculate the st. dev. of the network-ending (**2.39**)
- What is the chance (probability) of finishing on time if the latest allowable time is 25 weeks? (**14%**)

Network –based methods (CPM/ PERT)

- Assist in planning, scheduling and control of projects
- Provide analytical means for scheduling the activities



Comparison Between CPM and PERT

	CPM	PERT
1	Uses network, calculate float or slack, identify critical path and activities, guides to monitor and controlling project	Same as CPM
2	Uses one value of activity time	Requires 3 estimates of activity time Calculates mean and variance of time
3	Used where times can be estimated with confidence, familiar activities	Used where times cannot be estimated with confidence. Unfamiliar or new activities
4	Minimizing cost is more important	Meeting time target or estimating percent completion is more important
5	Example: construction projects, building machines, ships, etc	Example: Involving new activities or products, research and development, etc

BENEFITS OF CPM / PERT NETWORK

Consistent framework for planning, scheduling, monitoring, and controlling project.

- Shows interdependence of all tasks, work packages, and work units.
- Helps proper communications between departments and functions.
- Determines expected project completion date.
- Identifies critical activities, which can delay the project completion time.

BENEFITS OF CPM / PERT NETWORK (cont.)

- Identified activities with slacks that can be delayed for specified periods without penalty, or from which resources may be temporarily borrowed
- Determines the dates on which tasks may be started or must be started if the project is to stay in schedule.
- Shows which tasks must be coordinated to avoid resource or timing conflicts.
- Shows which tasks may run in parallel to meet project completion date

Benefits of CPM/PERT

CPM/PERT can answer the following important questions:

- How long will the entire project take to be completed? What are the risks involved?
- Which are the critical activities or tasks in the project which could delay the entire project if they were not completed on time?
- Is the project on schedule, behind schedule or ahead of schedule?
- If the project has to be finished earlier than planned, what is the best way to do this at the least cost?

Limitations to CPM/PERT

- Clearly defined, independent and stable activities
- Specified precedence relationships
- Over emphasis on critical paths
- Deterministic CPM model
- Activity time estimates are subjective and depend on judgment
- PERT assumes a beta distribution for these time estimates, but the actual distribution may be different
- PERT consistently underestimates the expected project completion time due to alternate paths becoming critical

Computer Software for Project Management

- *Microsoft Project* (Microsoft Corp.)
- *Primavera Project Planner* (Primavera)
- *MacProject* (Claris Corp.)
- *PowerProject* (ASTA Development Inc.)
- *Project Scheduler* (Scitor Corp.)
- *Project Workbench* (ABT Corp.)