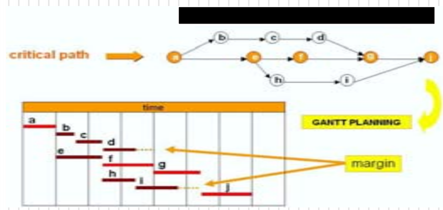
Critical Path Method (CPM)



Critical Path

- Critical activity : any delay on the start or finish of a critical activity will result in a delay in the entire project
- Critical path :
  - It represents a series of activities for which each activity has a zero float time.
  - longest time for the project from start to its completion and
  - decides the time of completion of the project

Major Steps in CPM

- CPM includes four major steps:
- 1- Determine the work activities: Project breakdown
  - Project must be divided into smaller activities or tasks (WBS)
  - Long durations should be avoided (14-20 days)
  - Prepare a list of all activities

Steps in CPM

- 2- Determine activity duration
  - Durations are calculated in workdays, (calendar) (“a 5 or 6-day workweek”)
  - Duration = Total quantity/Crew productivity
  - Sources
    - From company’s record
    - From standard estimating guide
    - Interviewing field personnel
  - Consider non-workdays, such as holidays, rain (weather) days

Steps in CPM

- 3- Determine the logical relationships :
  - Determine which activity must precede, succeed or may be done concurrently
  - Consider resource (labor, equipment) limitation
- 4- Draw the logic network and perform the CPM calculations:
  - finish date of the project, the critical path, and the available float for all noncritical activities.
  - CPM network using one of the commercially available **computer software programs**, such as Primavera Project Manager or Microsoft (MS) Project

Supplemental Steps in CPM

- 5- Review and analyze the schedule:
  a. Review the logic
  b. Make sure the activity has the correct predecessor
  c. Make sure there is no redundant activity
- 6- Implement the schedule:
  - take the schedule from paper to the execution.
- 7-Monitor and control the schedule:
  - comparing what is planned with what actually done.
- 8-Revise the database and record feedback.
  - cost and time estimates for activities are based on **past experience**.
- 9-Resource allocation and leveling
  (will be discussed later)

end-of-day convention

- Any date for an activity means the end of that day.
- Projects usually start at the beginning of day 1, which becomes the end of day 0. Example:

- Activity A starts at the beginning of day 1 (end of day 0). It takes 5 days to finish; it finishes on day 5 (end of the day).
- Activity B takes 8 days; it can start on day 5 (directly after activity A finishes), so it can finish as early as day 13.

Terminologies

- **Earliest start (ES) :** is earliest time at which an activity can be started
- **Earliest finish (EF) :** is earliest time by which an activity can be finished
  - **EF= ES + D (D is Activity duration)**
- **Latest start (LS) :** is latest time at which an activity can be started without delaying the completion of project
  - **LS = LF – D**
- **Latest finish (LF) :** is latest time by which an activity can be finished without delaying the completion of project.
  - **LF= LS + D**
- **Float (F):** is difference between the time available to do a job and the time required to do a job.

CPM calculations with AON (Example 1)

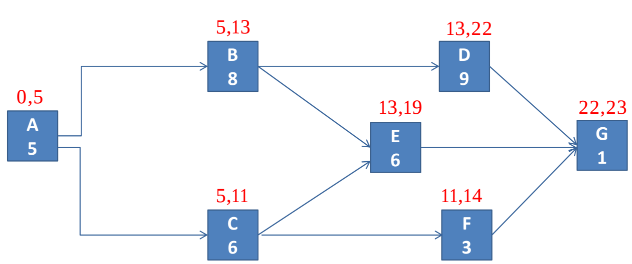
- Draw the logic network and perform the CPM calculations:

Activity	IPA	Duration
A	-	5
B	A	8
C	A	6
D	B	9
E	B,C	6
F	C	3
G	D,E,F	1

Forward pass calculations

- In mathematical terms, the ES for activity j is as follows :
  - ES_i= max (EF_i)
- Where (EF_i) represents the EF for all preceding activities. Likewise, the EF time for activity j is as follows :
  - EF_j= ES_j + D_j
  - where D_j is the duration of activity j
- Forward pass: The process of navigating through a network from start to end and calculating the completion date for the project and the early dates for each activity.

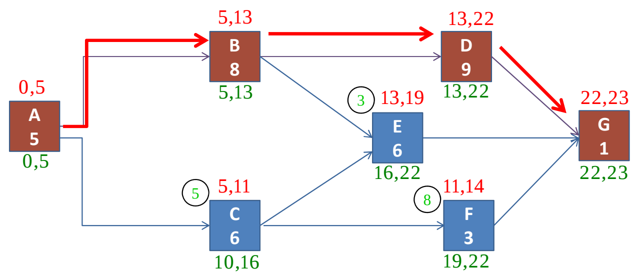
Solution



Backward pass calculations

- In mathematical terms, the late finish LF for activity j is as follows :
  - LF_j =min (LS_k)
- where (LS_k) represents the late start date for all succeeding activities. Likewise, the LS time for activity j is as follows :
  - LS_j= LF_j - D_j
  - where D_j is the duration of activity
- **Backward pass:** The process of navigating through a network from end to start and calculating the late dates for each activity.
- The late dates (along with the early dates) determine the critical activities, the critical path, and the amount of float each activity has.

Graphical Solution



Project completion time = 23 days

CPM (ES = LS , EF = LF , and TF = FF = 0)

Float calculations (Four types of floats)

- Total float (TF): The maximum amount of time an activity can be delayed from its early start without delaying the entire project

  TF = LS – ES
  or
  TF = LF - EF **or**
  TF = LF - D - ES

- Free Float (FF): the maximum amount of time an activity can be delayed without delaying the early start of the succeeding activities

  $$FF_i = \min(ES_{i+1}) - EF_i$$

Float calculations

- **Interfering float (Int. F):** the maximum amount of time an activity can be delayed without delaying the entire project but causing delay to the succeeding activities.

  Int. F = TF - FF

- **Independent float (Ind. F):** the maximum amount of time an activity can be delayed without delaying the early start of the succeeding activities and without being affected by the allowable delay of the preceding activities.

  $$Ind.\ F_i = \min(ES_{i+1}) - \max(LF_{i-1}) - D_i$$

Tabular solution for Example 1

In the previous example, for example :
Activity C's free float, FF = 11 - 11 = 0 days
Activity C's total float, TF =16 - 11= 5 days
Activity C's Ind. float, Ind. F =11 – 5 – 6 = 0 days

Activity	Duration	ES	EF	LS	LF	TF	FF
A	5	0	5	0	5	0	0
B	8	5	13	5	13	0	0
C	6	5	11	10	16	5	0
D	9	13	22	13	22	0	0
E	6	13	19	16	22	3	3
F	3	11	14	19	22	8	8
G	1	22	23	22	23	0	0

- **Critical activity**
- **Note : We must always realize that FF ≤ TF**

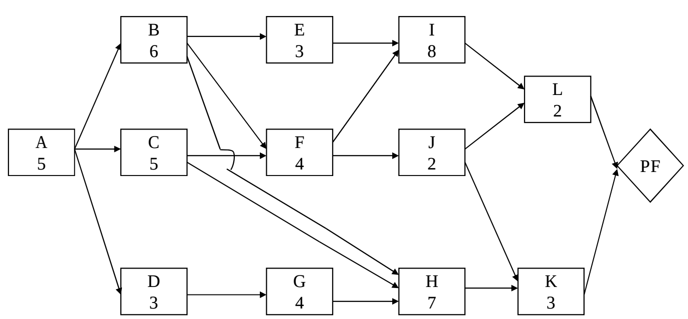
Floats

- In construction project scheduling,
  - Total float, TF, is the most frequently used type of float.
  - Free float, FF, comes in a distant second
  - Interfering float and independent float are almost unheard of, except in some delay-claim resolution cases.
  - Total float may be used for resource leveling

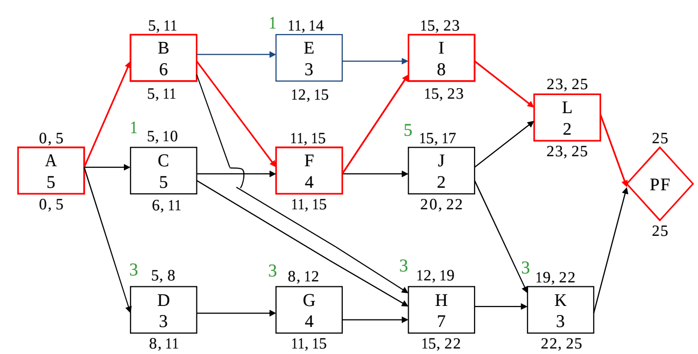
Example 2 (solution using computers)

Activity	Duration	IPA
A	5	-
B	6	A
C	5	A
D	3	A
E	3	B
F	4	B, C
G	4	D
H	7	B,C,G
I	8	E, F
J	2	F
K	3	H, J
L	2	I, J

AON diagram for Example 2



Graphical Solution for Example 2



Tabular Solution for Example 2

Activity	ES	EF	LS	LF	TF	FF
A	0	5	0	5	0	0
B	5	11	5	11	0	0
C	5	10	6	11	1	1
D	5	8	8	11	3	0
E	11	14	12	15	1	1
F	11	15	11	15	0	0
G	8	12	11	15	3	0
H	12	19	15	22	3	0
I	15	23	15	23	0	0
J	15	17	20	22	5	2
K	19	22	22	25	3	3
L	23	25	23	25	0	0

Lags in Node Networks

- A **lag** is a minimum compulsory waiting period between the start/finish of an activity and the start/finish of the successor
  - Actual waiting period may be greater, but never less than the lag
  - Lags are very common with SS and FF relationships
  - A lead is a negative lag
- The lag is **added** in the CPM's forward pass calculations and **subtracted** in the backward pass

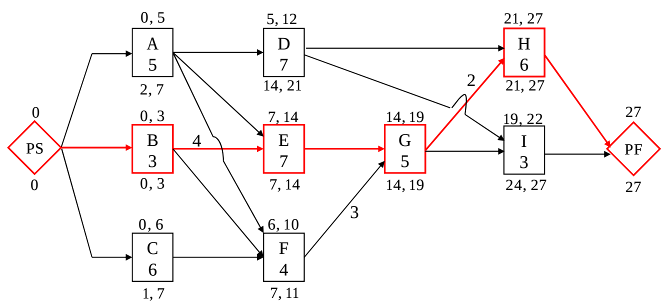
Examples of Lag

- Concrete curing (before formwork stripping)
- Asphalt curing (before striping)
- Waiting for a permit to be issued
- Waiting for the delivery of a custom material or equipment

Example 3 (with lag)

Activity	Duration	IPA	Lag
A	5	-	
B	3	-	
C	6	-	
D	7	A	
E	7	A	
		B	4
F	4	A,B,C	
G	5	E	
		F	3
H	6	D	
		G	2
I	3	D,G	

Graphic Solution for example 3



Tabular Solution for example 3

Activity	ES	EF	LS	LF	TF	FF
A	0	5	2	7	2	0
B	0	3	0	3	0	0
C	0	6	1	7	1	0
D	5	12	14	21	9	7
E	7	14	7	14	0	0
F	6	10	7	11	1	1
G	14	19	14	19	0	0
H	21	27	21	27	0	0
I	19	22	24	27	5	5

Calculated finish date

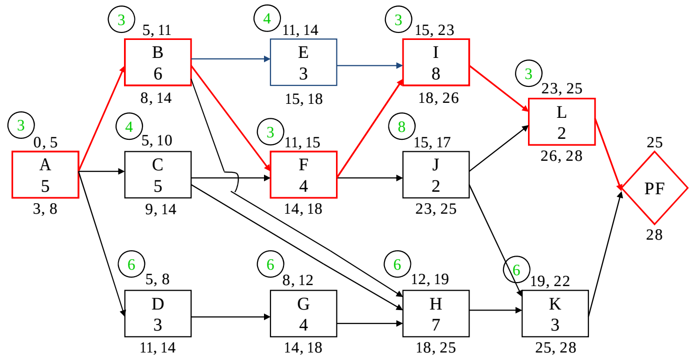
- In CPM ,
  - calculations based on the duration estimate for each activity, along with the logic between these activities.
  - The result is **a calculated finish date**.
  - The calculated finish date is different from that set by the owner
  - The contractor must adjust the schedule to meet the owner's imposed finish date (**Accelerating the schedule Project)**

Effect of Imposed Finish Date

- Imposed Finish Date is the project's completion date, as specified in the contract or stipulated by the owner
- When compared to the calculated finish date:
  - Calculated finish date < imposed finish date
    - You are in good shape
    - What happens if you enter the imposed date?
  - Calculated finish date > imposed finish date
    - Negative float appears when you enter the imposed date
    - You need to accelerate / crash the schedule

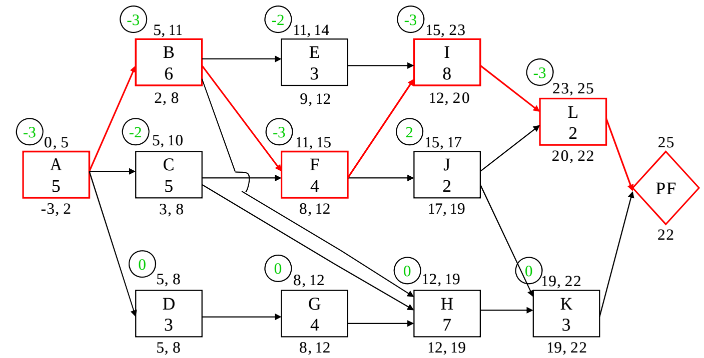
Example : Imposed Finish Date > Calculated Finish Date

- **Repeat Example 2 with imposed finish date of 28 days**



Example : **Imposed Finish Date < Calculated Finish Date**

- **Repeat Example 2 with imposed finish date of 22 days**

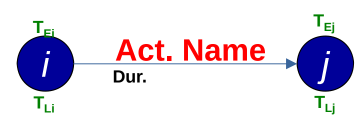


Negative Float

- Negative float is a situation that occurs when performing an activity even on its early dates, fails to meet the project's imposed finish date or other constraint
- It may occur in one of two cases:
  - Before construction starts
  - During construction (after normal start)

CPM with AOA Networks

- The preceding logic is similar to that of the forward and backward passes:
  - The early event time, T_E, is the largest (latest) date obtained to reach an event (going from start to finish).
  - The late event time, T_L, is the smallest (earliest) date obtained to reach an event (going from finish to start).



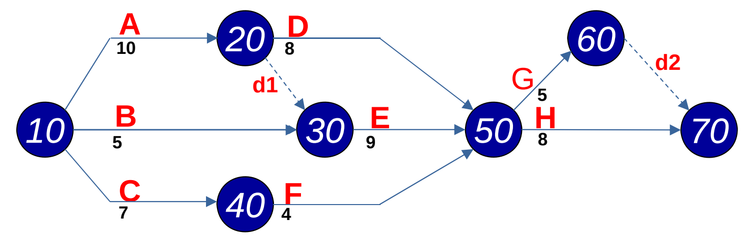
Arrow network Example

Perform the CPM calculations, using the arrow network diagram

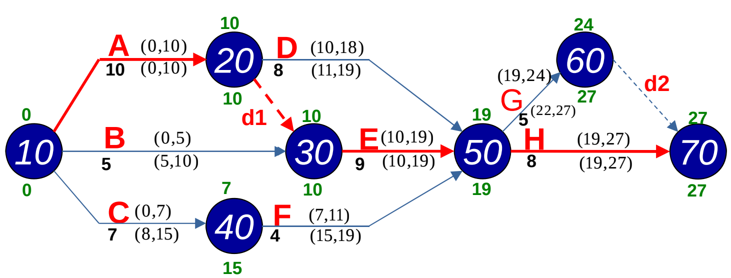
Activity	IPA	Duration
A	-	10
B	-	5
C	-	7
D	A	8
E	A, B	9
F	C	4
G	D,E,F	5
H	D,E,F	8

Arrow network Example

The arrow network shown below:



Arrow network solution



Node formats

- Each activity has several data items, depicting these data items inside the activity node is necessary.
- When drawing the CPM network manually or by computer, we may follow different node formats or we may devise our own format
- There is no correct or incorrect format
- Primavera Project Manager and MS Project provide ways to customize nodes.

Node Format

- A typical format may look like

ES	Activity ID	**EF**
	Activity Name	
LS	Duration	**LF**
TF		**FF**