The transportation Problem

Chapter 4: Special LP Models

Transportation Problem (TP)

- **TP** : aims to find the best way to fulfill the demand of n demand points using the capacities of m supply points.
- Minimizing total distribution cost meeting capacity and demand constraints.
- Can be used to locate new facilities (planning)
- Solved by Simplex, very large **simplex** tableaux and numerous iterations

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Special-purpose algorithms

- *Special-purpose algorithms (more efficient* than LP) exist for solving the TP. They involve
  - finding an initial solution
  - testing if it is optimal, and
  - developing an improved solution
  - repeating these steps until an optimal solution is reached

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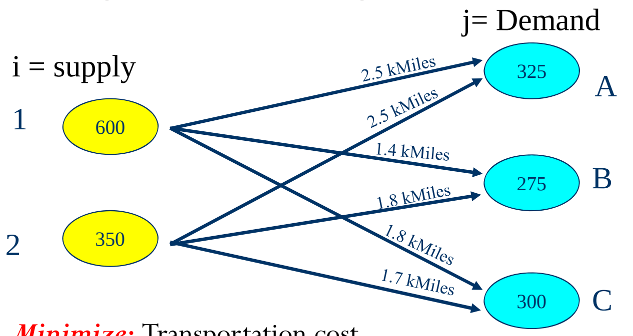
Common techniques

- Common techniques/methods for developing initial solutions are:
  - North West Corner
  - Minimum Cost
  - Vogel's approximation
- Optimality test Methods:
  - stepping-stone
  - modified distribution (MODI)

Transportation Example

- A toy problem!…
- 2 supply plants, 3 markets, and 1 commodity.
- Given: unit costs of shipping, c.
- How much to ship to minimize total transportation cost

Transportation Example



Minimize: Transportation cost

Subject to: Demand satisfaction and supply constraints

Transportation tableau

- A transportation problem is specified by the :
  - supply,
  - demand, and
  - shipping costs.
- Relevant data can be summarized in a transportation tableau.
- The transportation tableau implicitly expresses the supply and demand **constraints** and the shipping cost between each demand and supply point.

Transportation Example

Distances				
	Markets			
Plants	**A**	**B**	**C**	**Supply**
1	2.5	1.7	1.8	350
2	2.5	1.8	1.4	600
Demand	325	300	275	

Shipping costs are assumed to be constant = $90 per case per kMile.

Algebraic Representation

- ***Indices:***

  i = plants

  j = markets

- ***Given Data (or parameters):***

  a_i = supply of commodity of plant i (in cases)

  b_j = demand for commodity at market j (cases)

  c_{ij} = cost per unit shipment between plant i and market j ($/case)

Algebraic Representation

- ***Decision Variables:***

  x_{ij} = amount to ship from plant i to market j (cases),
  where $x_{ij} \geq 0$, for all i, j

- ***Constraints:***

  supply limit at plant i: $\sum_j x_{ij} \leq a_i$, *for all i* (cases)
  Satisfy demand at market j: $\sum_i x_{ij} \geq b_j$, *for all j* (cases)

- ***Objective Function:***

  Minimize $\sum_i \sum_j c_{ij}\, x_{ij}$

Solution

- Optimal shipment output
- Total transportation cost = 153.675 K$

	A	B	C
1	50.0	300.0	0
2	275.0	0	275.0

General Description of a Transportation Problem

1. A set of *m supply points* from which a good is shipped. Supply point i can supply at most s_i units.
2. A set of *n demand points* to which the good is shipped. Demand point j must receive at least d_i units of the shipped good.
3. Each unit produced at supply point i and shipped to demand point j incurs a *variable cost* of c_{ij}.

Mathematical model

- X_{ij} = number of units shipped from *supply point i* to *demand point j*

$$\min \sum_{i=1}^{i=m} \sum_{j=1}^{j=n} c_{ij} X_{ij}$$

$$s.t. \sum_{j=1}^{j=n} X_{ij} \le s_i (i = 1,2,\ldots,m)$$

$$\sum_{i=1}^{i=m} X_{ij} \ge d_j (j = 1,2,\ldots,n)$$

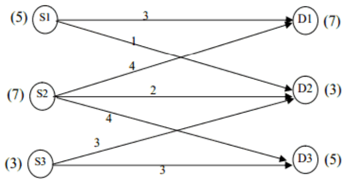
$$X_{ij} \ge 0 (i = 1,2,\ldots,m; \ j = 1,2,\ldots,n)$$

Transportation problem representation

		Cost per distributed unit				**Supply**
		Destination				
		1	**2**	**…**	**n**	
Source	**1**	c_{11}	c_{12}	…	c_{1n}	s_1
	2	c_{21}	c_{22}	…	c_{2n}	s_2
	:	:	:		:	:
	m	c_{m1}	c_{m2}	…	c_{mn}	s_m
Demand		d_1	d_2	…	d_n	

If shipment is impossible b/n a given source and destination, a large cost of M is entered (big M method)

Example



		Destinations			Supply
		1	2	3	
	1	3	1	M	5
Sources	2	4	2	4	7
	3	M	3	3	3
Demand		7	3	5	

Balanced Transportation Problem

- If total supply equals to total demand, the problem is said to be a **balanced transportation problem**: We restricted **focus** to balanced transportation problems $\sum_{i=1}^{i=m} s_i = \sum_{j=1}^{j=n} d_j$
- There are no slacks and so all constraints are equalities rather than inequalities

$$\min Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} X_{ij}$$

$$s.t. \quad \sum_{j=1}^{j=n} X_{ij} = s_i \quad (i = 1,2,\ldots,m)$$

$$\sum_{i=1}^{i=m} X_{ij} = d_j \quad (j = 1,2,\ldots,n)$$

Balancing a TP –Case 1

- If total supply exceeds total demand, we can balance the problem by adding dummy demand point (an extra market, *a dump*). Since shipments to the dummy demand point are not real, they are assigned a cost of zero.
- The demand of the extra market will be equal to

$$\sum_{i=1}^{m} s_i - \sum_{j=1}^{n} d_j$$

Balancing a TP- Case-2

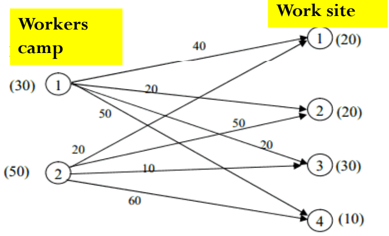
- If a transportation problem has a total supply that is less than total demand the problem has no feasible solution.
- We may introduce an extra source to make up the shortfall, and assign costs which reflect penalties for undersupplying markets.

$$\sum_{j=1}^{n} d_j - \sum_{i=1}^{m} s_i$$

- As one can guess the total penalty cost is desired to be minimum.

Example : Transporting workers

- A contractor transports workers to four work sites each day. The travel time in minutes between each camp and work site is shown in the figure. In order to maximize the number of productive hours per day of each worker, the contractor wishes to minimize the total travel time. Formulate a mathematical model to minimize total travel time.



Example : Transporting workers

- Tabular format for the TP problem

		Destination				Supply
		1	2	3	4	
Source	1	40	20	20	50	30
	2	20	50	10	60	50
Demand		20	20	30	10	

Example : Transporting workers

- Assume the variable x_{ij} number of workers to be dispatched from camp i to work site j, where i=1,2 and j= 1,2,3,4
- Balanced problem (total supply equals total demand)

Minimize Z = $40X_{11} + 20X_{12} + 20X_{13} + 50X_{14} + 20X_{21} + 50X_{22} + 10X_{23} + 60X_{24}$

S.T.:

$X_{11} + X_{12} + X_{13} + X_{14} + X_{15} = 30$ (Supply)

$X_{21} + X_{22} + X_{23} + X_{24} + X_{25} = 50$ (Supply)

$X_{11} + X_{21} = 15$ (Demand)

$X_{12} + X_{22} = 20$ (Demand)

$X_{13} + X_{23} = 30$ (Demand)

$X_{14} + X_{24} = 10$ (Demand)

$X_{ij} >= 0$ for all I and j (i= 1,2; j= 1,2,3,4,5)

Example : Transporting workers

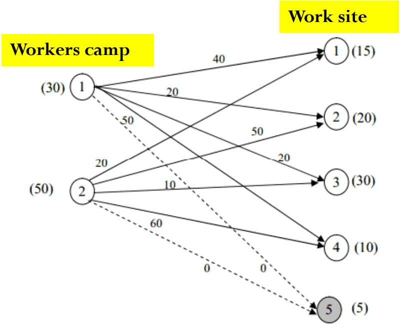
- Suppose that there are no workers dispatched from camp 1 to work site 2

		Destination				Supply
		1	2	3	4	
Source	1	M	20	20	50	30
	2	20	50	10	60	50
Demand		20	20	30	10	

- A very large time will be assigned b/n camp 1 and worksite 1 (this discourages the solution from using such cell)

Example : Transporting workers

- Suppose the demand at work site 1 is 15 instead of 20 workers
- In this case, no balance (supply =80, demand =75)
- A fictitious (dummy) destination will be added to balance



Example : Transporting workers

- The mathematical model in this case becomes

Minimize Z = $40X_{11} + 20X_{12} + 20X_{13} + 50X_{14} + 0X_{15} + 20X_{21} + 50X_{22} + 10X_{23} + 60X_{24} + 0X_{25}$

S.T.:

$X_{11} + X_{12} + X_{13} + X_{14} + X_{15} = 30$ (Supply Constraints)

$X_{21} + X_{22} + X_{23} + X_{24} + X_{25} = 50$ (Supply Constraints)

$X_{11} + X_{21} = 15$ (Demand Constraints)

$X_{12} + X_{22} = 20$ (Demand Constraints)

$X_{13} + X_{23} = 30$ (Demand Constraints)

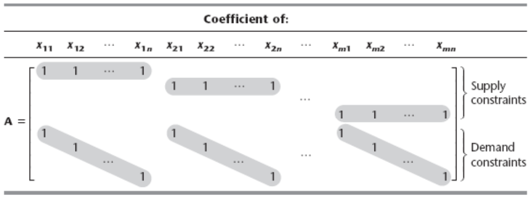
$X_{14} + X_{24} = 10$ (Demand Constraints)

$X_{15} + X_{25} = 5$ (Demand Constraints)

$X_{ij} >= 0$ (i= 1,2,; j= 1,2,3,4,5)

Simplex method for TP

- Specific structure of TP
  - *all coef. Of the variable in the constraint eqns are 1.*
  - *every variable appears in exactly two constraints,*
  - *all constraint eqns are formed using = signs*
- Coefficient matrix



- Simplified simplex methods (to achieve computational efficiency)

Basic variables for balanced TP

- Given m sources and n destinations, number of functional constraints is $m+n$.
- However, number of basic variables = $m + n - 1$, because of equality constraints. (any one of the constraints is automatically satisfied whenever $m+n-1$ constraints are satisfied)
- We have m+n-1 basic variables (non-zero variables)
- The sum of all allocations for each row and each column equals its supply or demand.
- How can we determine these m+n-1 basic variables in an easy way?

Methods to find the bfs for a balanced TP

- There are three basic methods:

1. Northwest Corner Method
2. Minimum Cost Method
3. Vogel's Method

1. Northwest Corner Method

- NWC method: is a procedure for constructing an initial basic feasible solution one at a time.
- The North West corner rule begins with an allocation at the top left-hand corner of the tableau
- It proceeds systematically along either a row or a column and make allocations to subsequent cells until the bottom right-hand corner is reached, by which time enough allocations will have been made to constitute an initial solution.

North West Corner Rule

1. Start by selecting the cell in the most "North-West" corner of the table.
2. Assign the maximum amount to this cell that is allowable based on the requirements and the capacity constraints.
3. Exhaust the capacity from each row before moving down to another row.
4. Exhaust the requirement from each column before moving right to another column.
5. Check to make sure that the capacity and requirements are met.

Example-1 NWC method

- A concrete company transports concrete from three plants, 1, 2 and 3 to three construction sites, A, B, C.
- The cost of transporting 1 m^3 of concrete in \$ to each site is shown below. (Transportation tableau)

	A	**B**	**C**	**Supply (m^3/week)**
1	4	3	8	300
2	7	5	9	300
3	4	5	5	100
Demand (m^3/week)	200	200	300	

Solution NWC contd..

- We start in the upper left hand cell and allocate units to shipping routes as follows:

1. Exhaust the supply (plant capacity) of each row before moving down to the next row.
2. Exhaust the demand (construction sites) requirements of each column before moving to the next column to the right.
3. Check that all supply and demand requirements are met.

Solution NWC contd..

- The initial shipping assignments are given in Table below.

	A	B	C	**Capacity**
1	200	100		300
2		100	200	300
3			100	100
Demand	200	200	300	**700**

- We can compute, the cost of shipping assignments: = 200*4+100*3+100*5+200*9+100*5 = \$ 3900
- Note that this is not necessarily equal to the optimal solution

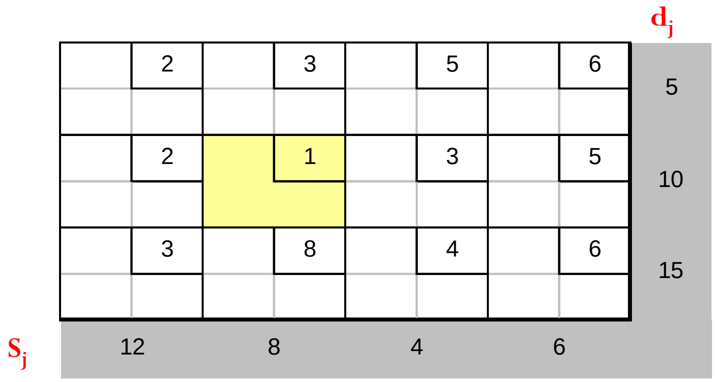
2. Minimum cost method

- The minimum cost method uses shipping costs in order come up with a bfs that has a lower cost.
- To begin the minimum cost method, first we find the decision variable with the smallest shipping cost (X_{ij}).
- Then assign X_{ij} its largest possible value, which is the minimum of s_i and d_j
- cross out row i and column j and reduce the supply or demand of the noncrossed-out row or column by the value of Xij.
- Then we will choose the cell with the minimum cost of shipping from the cells that do not lie in a crossed-out row or column and we will repeat the procedure.

Example 2 : Minimum Cost Method

- Step 1: Select the cell with minimum cost

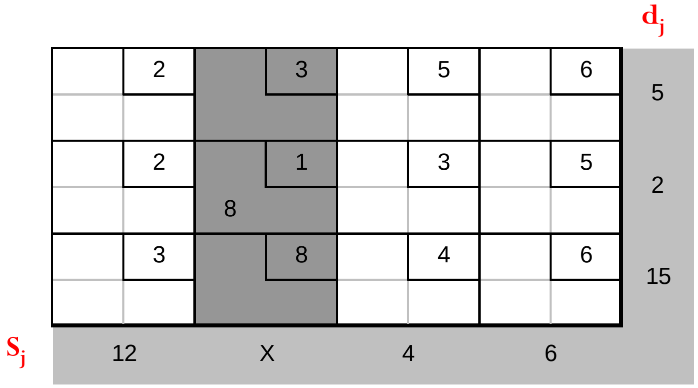
Cell : 22



				d_j
2	3	5	6	5
2	1	3	5	10
3	8	4	6	15
12	8	4	6	

S_j

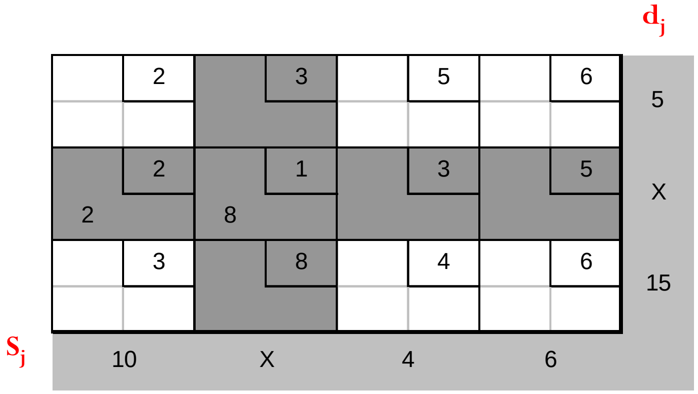
Step 2: Cross-out column 2



				d_j
2	3	5	6	5
2	1 8	3	5	2
3	8	4	6	15
12	X	4	6	

S_j

Step 3: Find the new cell with minimum shipping cost and cross-out row 2

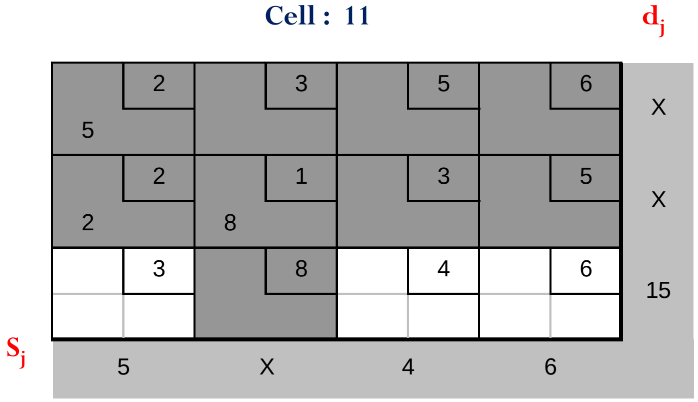


				d_j
2	3	5	6	5
2 2	1 8	3	5	X
3	8	4	6	15
10	X	4	6	

S_j

Step 4: Find the new cell with minimum shipping cost and cross-out row 1

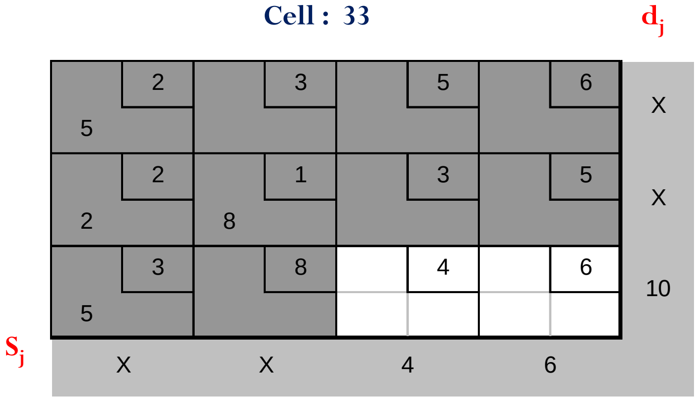
Cell : 11



					d_j
	2 5	3	5	6	X
	2 2	1 8	3	5	X
	3	8	4	6	15
S_j	5	X	4	6	

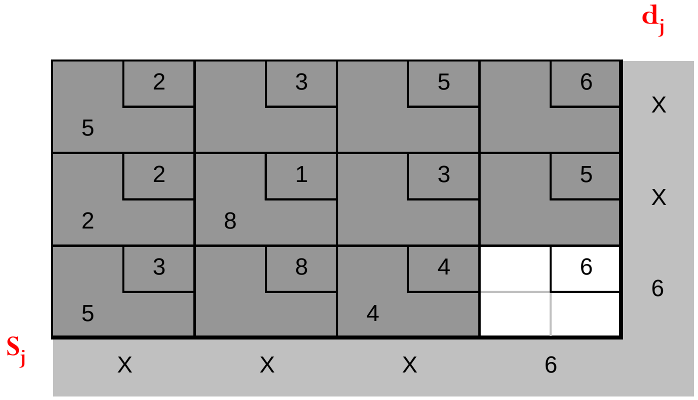
Step 5: Find the new cell with minimum shipping cost and cross-out column 1

Cell : 33



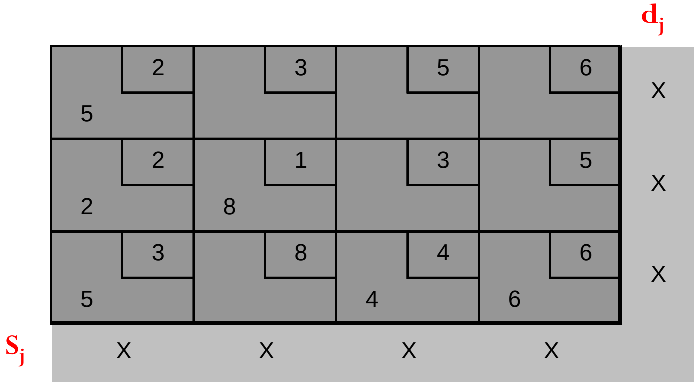
					d_j
	2 5	3	5	6	X
	2 2	1 8	3	5	X
	3 5	8	4	6	10
S_j	X	X	4	6	

Step 6: Find the new cell with minimum shipping cost and cross-out column 3



					d_j
	2 5	3	5	6	X
	2 2	1 8	3	5	X
	3 5	8	4 4	6	6
S_j	X	X	X	6	

Step 7: Finally assign 6 to last cell. The bfs is found as: $X_{11}=5$, $X_{21}=2$, $X_{22}=8$, $X_{31}=5$, $X_{33}=4$ and $X_{34}=6$



					d_j
	2 5	3	5	6	X
	2 2	1 8	3	5	X
	3 5	8	4 4	6 6	X
S_j	X	X	X	X	

3. Vogel's Method

- Begin with computing each row and column a penalty.
- The penalty will be equal to the difference between the two smallest shipping costs in the row or column.
- Identify the row or column with the largest penalty.
- Find the first basic variable which has the smallest shipping cost in that row or column.
- Then assign the highest possible value to that variable, and cross-out the row or column as in the previous methods.
- Compute new penalties and use the same procedure

Example 3 : Vogel's Method

Step 1: Compute the penalties

				Supply	Row Penalty
	6	7	8	10	7-6=1
	15	80	78	15	78-15=63
Demand	15	5	5		
Column Penalty	15-6=9	80-7=73	78-8=70		

Step 2: Identify the largest penalty and assign the highest possible value to the variable.

				Supply	Row Penalty
	6	7 5	8	5	8-6=2
	15	80	78	15	78-15=63
Demand	15	X	5		
Column Penalty	15-6=9	_	78-8=70		

Step 3: Identify the largest penalty and assign the highest possible value to the variable.

				Supply	Row Penalty
	6	7 5	8 5	0	_
	15	80	78	15	_
Demand	15	X	X		
Column Penalty	15-6=9	_	_		

Step 4: Identify the largest penalty and assign the highest possible value to the variable.

				Supply	Row Penalty
	6 0	7 5	8 5	X	_
	15	80	78	15	_
Demand	15	X	X		
Column Penalty	_	_	_		

Step 5: Finally the bfs is found as $X_{11}=0$, $X_{12}=5$, $X_{13}=5$, $X_{21}=15$, $X_{22}=0$, $X_{23}=0$

				Supply	Row Penalty
	6 0	7 5	8 5	X	_
	15 15	80	78	X	_
Demand	X	X	X		
Column Penalty	_	_	_		

Optimality test

- The next step is to determine whether the allocations at any stage of the solution process is optimal.
- one of the methods used to determine optimality of and improve a current solution is the Stepping Stone method

Stepping-stone Method

1. Select an unused square to be evaluated.
2. Beginning at this square, trace a closed path back to the original square via squares that are currently being used (only horizontal or vertical moves allowed). You can only change directions at occupied cells!.
3. Beginning with a plus (+) sign at the unused square, place alternative minus (-) signs and plus signs on each corner square of the closed path just traced.

Stepping-stone Method

4. Calculate an improvement index, I_{ij} by adding together the unit cost figures found in each square containing a plus sign and then subtracting the unit costs in each square containing a minus sign.
5. Repeat steps 1 to 4 until an improvement index has been calculated for all unused squares.
   - If all indices computed are greater than or equal to zero, an optimal solution has been reached.
   - If not, it is possible to improve the current solution and decrease total shipping costs.

Optimality criterion

- If all the cost index values obtained for all the currently unoccupied cells are nonnegative, then the current solution is optimal.
- If there are negative values the solution has to be improved.
- This means that an allocation is made to one of the empty cells (unused routes) and the necessary adjustments in the supply and demand effected accordingly.

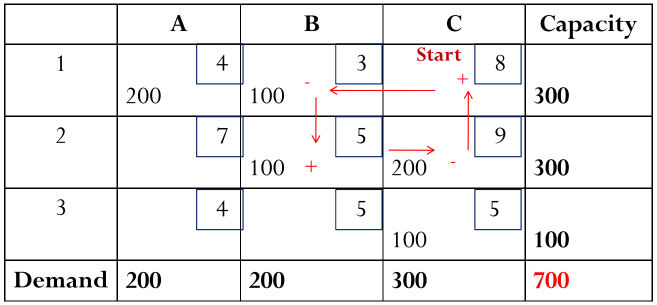
Optimality test for Example-1

- Apply these steps to the Contractor's problems

	A	B	C	Capacity
1	200	100		300
2		100	200	300
3			100	100
Demand	200	200	300	700

Solution

- Beginning at cell 1C route



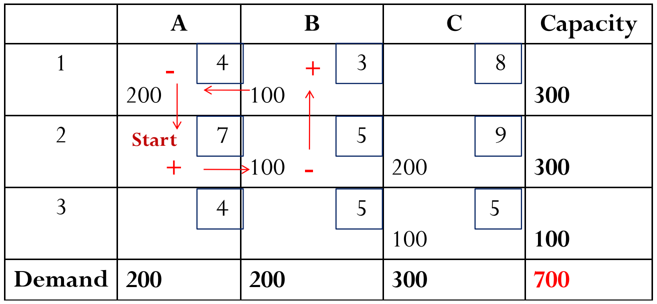
	A	B	C	Capacity
1	4 200	3 100 -	Start + 8	300
2	7	5 100 +	9 200 -	300
3	4	5	5 100	100
Demand	200	200	300	700

Solution

- Calculate the improvement index for 1C route
- I_{1c} = +8 – 3 + 5 -9 = +1
- This means that for every m3 concrete shipped via the 1-C route, total transportation costs will increase by $1 over their current level.
- Next we consider 2A unused route. The closed path we use is shown in the figure next slide

Solution

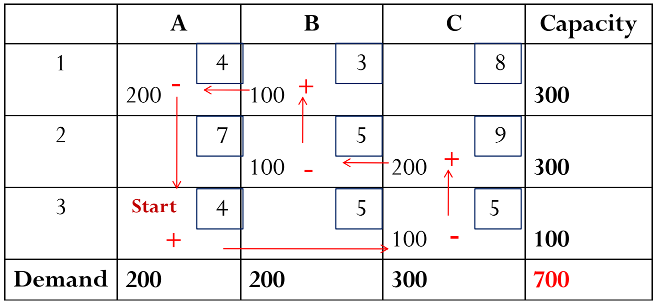
- 2-A route improvement index is calculated
- I_{2A} = +7 – 5 + 3 - 4 = +1



	A	B	C	Capacity
1	- 4 200	+ 3 100	8	**300**
2	Start 7 +	5 100 -	9 200	**300**
3	4	5	5 100	**100**
Demand	**200**	**200**	**300**	**700**

Solution

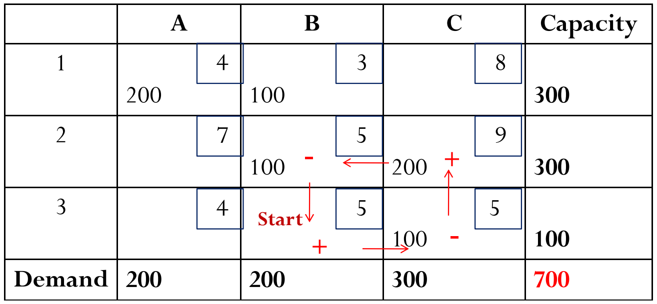
- Next 3-A route and the improvement index is calculated as:
- I_{2A} = +4 – 5 + 9 – 5 + 3 - 4 = +2



	A	B	C	Capacity
1	4 200 -	3 100 +	8	**300**
2	7	5 100 -	9 200 +	**300**
3	Start 4 +	5	5 100 -	**100**
Demand	**200**	**200**	**300**	**700**

Solution

- Last 3-B route and the improvement index is calculated as:
- I_{2A} = +5 – 5 + 9 – 5 = +4



	A	B	C	Capacity
1	4 200	3 100	8	**300**
2	7	5 100 -	9 200 +	**300**
3	4	Start 5 +	5 100 -	**100**
Demand	**200**	**200**	**300**	**700**

Solution

- The solution is the optimal because each improvement index for the Table is greater than or equal to zero.
- Therefore, the minimum cost is = 3900 $
- If we had one negative index, a cost saving may be attained by making use of that route i.e the cell can be improved.
- The quantity is found by referring to the closed path of plus signs and minus signs drawn for the route and selecting the smallest number found in those squares containing minus signs.
- To obtain a new solution, that number is added to all squares on the closed path with plus signs and subtracted from all squares on the path assigned minus signs.
- To determine whether further improvement is possible, we return to the first five steps to test each square that is now unused.

Software

- As with other linear programming problems, the usual software options (Excel, LINGO/LINDO, etc..)are available for setting up and solving transportation problems (and assignment problems),

Exercise

- Use NWC rule and Stepping Stone method to find the optimal solution for the following Contractor's TP Problem.

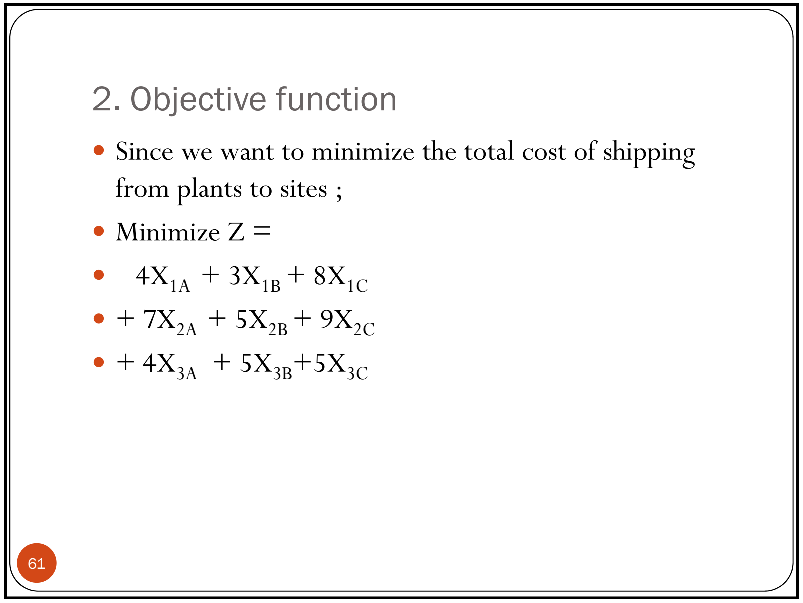
	A	B	C	Supply (m^3/week)
1	5	4	3	100
2	8	4	3	300
3	9	7	5	300
Demand (m^3/week)	300	200	200	

Example : using software

1. Decision Variable:

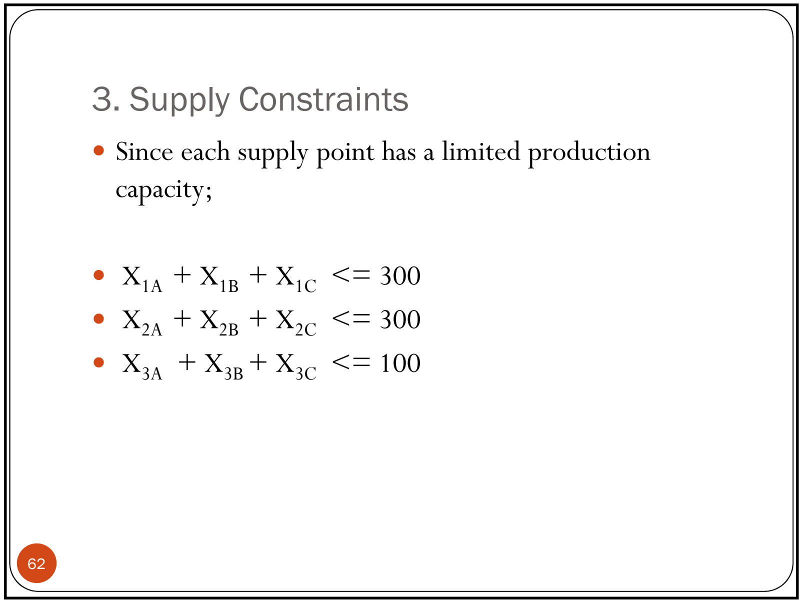
- X_{ij} = Amount of concrete (m^3) produced at plant i and sent to Site j

	A	B	C	Supply (m^3/week)
1	4	3	8	300
2	7	5	9	300
3	4	5	5	100
Demand (m^3/week)	200	200	300	



2. Objective function

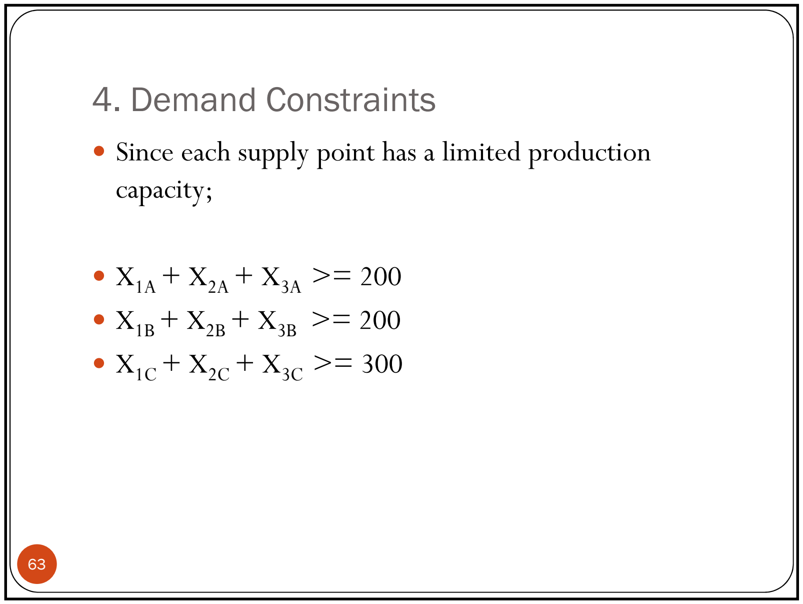
- Since we want to minimize the total cost of shipping from plants to sites ;
- Minimize Z =
- $4X_{1A} + 3X_{1B} + 8X_{1C}$
- $+ 7X_{2A} + 5X_{2B} + 9X_{2C}$
- $+ 4X_{3A} + 5X_{3B} + 5X_{3C}$



3. Supply Constraints

- Since each supply point has a limited production capacity;

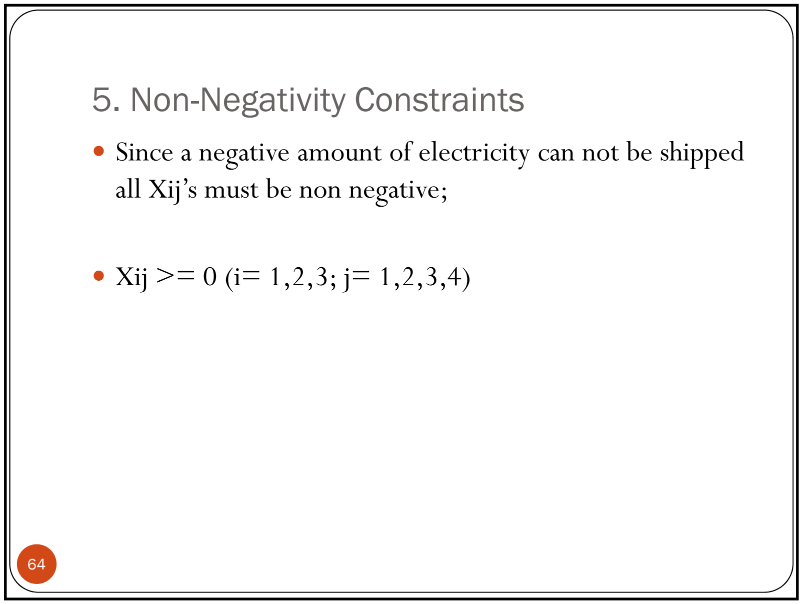
- $X_{1A} + X_{1B} + X_{1C} <= 300$
- $X_{2A} + X_{2B} + X_{2C} <= 300$
- $X_{3A} + X_{3B} + X_{3C} <= 100$



4. Demand Constraints

- Since each supply point has a limited production capacity;

- $X_{1A} + X_{2A} + X_{3A} >= 200$
- $X_{1B} + X_{2B} + X_{3B} >= 200$
- $X_{1C} + X_{2C} + X_{3C} >= 300$



5. Non-Negativity Constraints

- Since a negative amount of electricity can not be shipped all Xij's must be non negative;

- Xij >= 0 (i= 1,2,3; j= 1,2,3,4)

LP Formulation of Concrete Company Problem

Minimize $Z = 4X_{1A} + 3X_{1B} + 8X_{1C} + 7X_{2A} + 5X_{2B} + 9X_{2C} + 4X_{3A} + 5X_{3B} + 5X_{3C}$

S.T.: $X_{1A} + X_{1B} + X_{1C} <= 300$ (Supply Constraints)

$X_{2A} + X_{2B} + X_{2C} <= 300$

$X_{3A} + X_{3B} + X_{3C} <= 100$

$X_{1A} + X_{2A} + X_{3A} >= 200$ (Demand Constraints)

$X_{1B} + X_{2B} + X_{3B} >= 200$

$X_{1C} + X_{2C} + X_{3C} >= 300$

$X_{ij} >= 0$ (i= 1,2,3; j= 1,2,3)

Solution using Lindo

- MIN 4X1A + 3X1B + 8X1C + 7X2A + 5X2B + 9X2C + 4X3A + 5X3B + 5X3C
- ST
- X1A + X1B + X1C <= 300
- X2A + X2B + X2C <= 300
- X3A + X3B + X3C <= 100
- X1A + X2A + X3A >= 200
- X1B + X2B + X3B >= 200
- X1C + X2C + X3C >= 300
- END

Solution using Lindo

```
LP OPTIMUM FOUND AT STEP      6

   OBJECTIVE FUNCTION VALUE

   1)      3900.000

VARIABLE        VALUE          REDUCED COST
     X1A       200.000000          0.000000
     X1B       100.000000          0.000000
     X1C         0.000000          1.000000
     X2A         0.000000          1.000000
     X2B       100.000000          0.000000
     X2C       200.000000          0.000000
     X3A         0.000000          2.000000
     X3B         0.000000          4.000000
     X3C       100.000000          0.000000


     ROW   SLACK OR SURPLUS     DUAL PRICES
      2)         0.000000          2.000000
      3)         0.000000          0.000000
      4)         0.000000          4.000000
      5)         0.000000         -6.000000
      6)         0.000000         -5.000000
      7)         0.000000         -9.000000

NO. ITERATIONS=       6
```

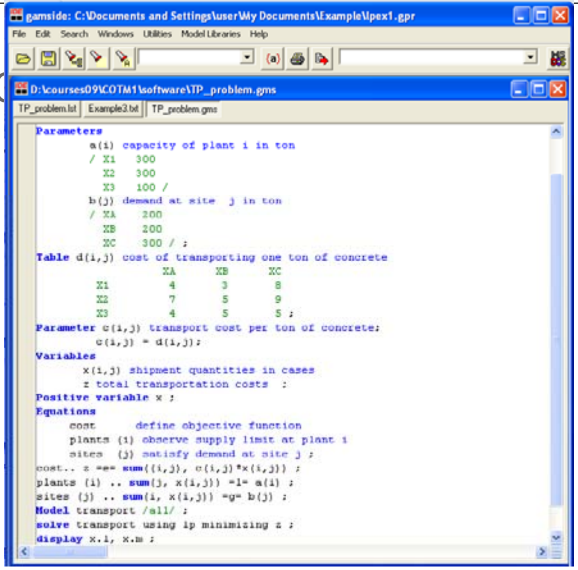
GAMS

```
GAMS Rev 236  WIN-VS8 23.6.0 x86/MS Windows          10/12/12 11:05:09 Page 6
General Algebraic Modeling System
Execution


----     33 VARIABLE x.L  shipment quantities in cases

             XA            XB             XC

X1     200.000       100.000
X2                   100.000        200.000
X3                                   100.000
```



```
Parameters
         a(i) capacity of plant i in ton
         / X1   300
           X2   300
           X3   100 /
         b(j) demand at site  j in ton
         / XA    200
           XB    200
           XC    300 / ;
Table d(i,j) cost of transporting one ton of concrete
                    XA        XB        XC
          X1         4         3         8
          X2         7         5         9
          X3         4         5         5 ;
Parameter c(i,j) transport cost per ton of concrete;
          c(i,j) = d(i,j);
Variables
        x(i,j) shipment quantities in cases
        z total transportation costs  ;
Positive variable x ;
Equations
     cost      define objective function
     plants (i) observe supply limit at plant i
     sites  (j) satisfy demand at site j ;
cost.. z =e= sum((i,j), c(i,j)*x(i,j)) ;
plants (i) .. sum(j, x(i,j)) =l= a(i) ;
sites (j) .. sum(i, x(i,j)) =g= b(j) ;
Model transport /all/ ;
solve transport using lp minimizing z ;
display x.l, x.m ;
```

Transshipment Problem

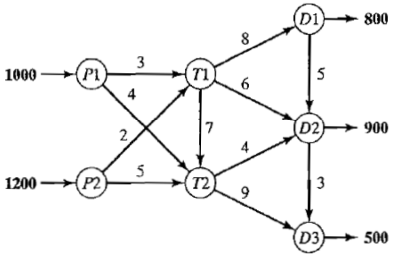
- More general than the transportation problem
- An extension of a transportation problem
- In this problem there are intermediate "transshipment points", through which goods can be transshipped on their journey from a supply point to a demand point.
- The transshipment model recognizes that it may be cheaper to ship through intermediate or transient nodes before reaching the final destination.

Transshipment Problem

- **Supply point**: it can send goods to another point but cannot receive goods from any other point
- **Demand point** : it can receive goods from other points but cannot send goods to any other point
- **Transshipment point**: it can both receive goods from other points send goods to other points

Example

- Two automobile plants, P_1 and P_2, are linked to three dealers, D_1, D_2, and D_3, by way of two transit centers, T_1 and T_2, according to the network shown in Figure. The shipping costs per car (in hundreds of dollars) are shown on the connecting links of the network.



Example

- Each node of the network with both input and output arcs (T1,T2, Dl, and D2) acts as both a source and a destination and is referred to as a **transshipment node**.
- The remaining nodes are either pure supply nodes (P1 and P2) or pure demand nodes (D3).
- The transshipment model can be converted into a regular transportation model with six sources (P_1, P_2, T_1, T_2, D_1, D_2) and five destinations (T_1, T_2, D_1, D_2, D_3).
- Solution using a regular transportation model using the idea of a buffer

Example

- The amounts of supply and demand at the different nodes are computed as:
  - Supply at a pure supply node = Original supply
  - Demand at a pure demand node = Original demand
  - Supply at a transshipment node = Original supply + Buffer amount
  - Demand at a transshipment node = Original demand + Buffer amount

Example

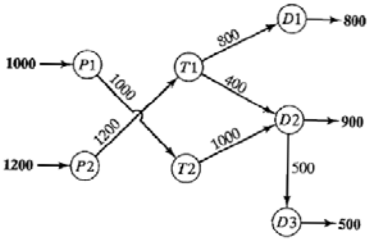
- The buffer amount should be sufficiently large to allow all of the original supply (or demand) units to pass through any of the Transshipment nodes.
- Let B be the desired buffer amount;
  - B = Total supply (or demand)
  - = 1000 + 1200 (or 800 + 900 + 500)
  - = 2200 cars
- Using the buffer B and the unit shipping costs given in the network; we construct the equivalent regular transportation model

Example

	T1	T2	D1	D2	D3	Sources
P1	3	4	**M**	**M**	**M**	1000
P2	2	5	**M**	**M**	**M**	1200
T1	0	7	8	6	**M**	B
T2	**M**	0	**M**	4	9	B
D1	**M**	**M**	0	5	**M**	B
D2	**M**	**M**	**M**	0	5	B
destinations	B	B	800+B	900+B	500	B

Example

- The solution of the resulting transportation model (determined by TORA) is shown in the Figure.
- Note the effect of transshipment: Dealer D2 receives 1400 cars, keeps 900 cars to satisfy its demand, and sends the remaining 500 cars to dealer D3.



Steps

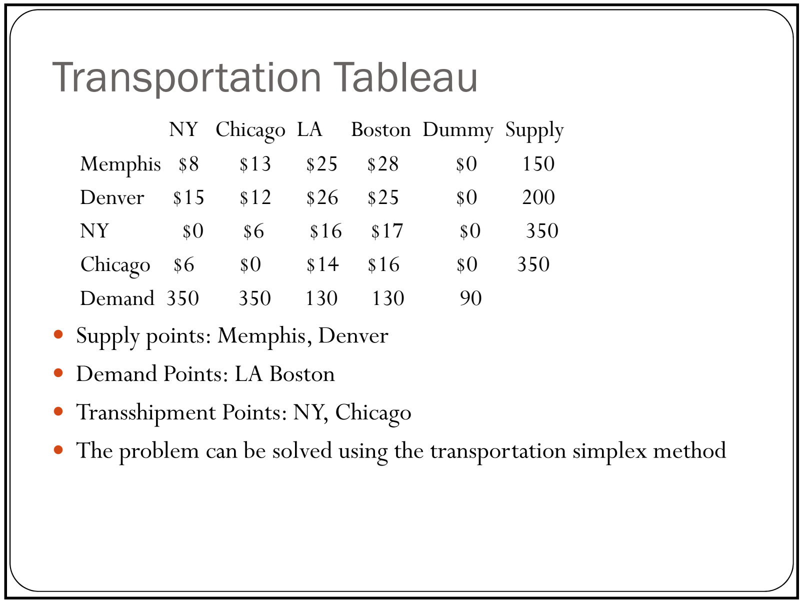
- The optimal solution to a transshipment problem can be found by solving a transportation problem.
- Step1. If necessary, add a dummy demand point (with a supply of 0 and a demand equal to the problem's excess supply) to balance the problem. Shipments to the dummy and from a point to itself will be zero. Let s= total available supply.
- Step2. Construct a transportation tableau as follows: A row in the tableau will be needed for each supply point and transshipment point, and a column will be needed for each demand point and transshipment point.

Steps

- Each supply point will have a supply equal to it's original supply, and each demand point will have a demand to its original demand.
- Let s= total available supply. Then each transshipment point will have a supply equal to (point's original supply)+s and a demand equal to (point's original demand)+s.
- This ensures that any transshipment point that is a net supplier will have a net outflow equal to point's original supply and a net demander will have a net inflow equal to point's original demand.
- Although we don't know how much will be shipped through each transshipment point, we can be sure that the total amount will not exceed s.

Transshipment Example

- Widgetco manufactures widgets at two factories, one in Memphis and one in Denver. The Memphis factory can produce as 150 widgets, and the Denver factory can produce as many as 200 widgets per day. Widgets are shipped by air to customers in LA and Boston. The customers in each city require 130 widgets per day. Because of the deregulation of airfares, Widgetco believes that it may be cheaper first fly some widgets to NY or Chicago and then fly them to their final destinations. The cost of flying a widget are shown next. Widgetco wants to minimize the total cost of shipping the required widgets to customers.



Transportation Tableau

	NY	Chicago	LA	Boston	Dummy	Supply
Memphis	$8	$13	$25	$28	$0	150
Denver	$15	$12	$26	$25	$0	200
NY	$0	$6	$16	$17	$0	350
Chicago	$6	$0	$14	$16	$0	350
Demand	350	350	130	130	90	

- Supply points: Memphis, Denver
- Demand Points: LA Boston
- Transshipment Points: NY, Chicago
- The problem can be solved using the transportation simplex method