# Lindo Program

Introduction

## LINGO and LINDO

- LINGO and LINDO are computer software packages developed by LINDO systems.
- Designed for formulating and solving a wide variety of optimization problems include :
  - Linear programming
  - Integer programming
  - Nonlinear programming

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## LINGO and LINDO

- Download page
  - http://www.lindo.com


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## LINDO

- LINDO (Linear Interactive and Discrete Optimizer) used to solve :
  - Linear programming
  - Integer programming
  - quadratic programming
- It can be applied in areas like manufacturing, scheduling, budgeting, and other industrial applications.

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# LINGO

- LINGO is an interactive computer-software package
  - Linear
  - Nonlinear (convex & nonconvex/Global), Quadratic, Quadratically Constrained, Second Order Cone,
  - Stochastic, and Integer optimization models
- LINGO provides a completely integrated package that includes a powerful language for expressing optimization models,
- LINGO provide a vast library of mathematical, statistical, and probability functions.
- The recently released LINGO 14.0
- Trial version can be downloaded

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# Lindo/Lingo

- **Trial Version Capacities:**

| | Constraints | Variables | Integer Variables | Nonlinear Formulas | Global Variables |
|---|---|---|---|---|---|
| Classic LINDO | 150 | 300 | 30 | N/A | N/A |
| LINDO API | 150 | 300 | 30 | 30 | 5 |
| LINGO | 150 | 300 | 30 | 30 | 5 |
| What's*Best* | 150 | 300 | 30 | 30 | 5 |

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# Example using Lindo

$$
\begin{aligned}
\max \quad & 12x_1 + 9x_2 \\
\text{s.t.} \quad & x_1 + x_3 = 1000 \\
& x_2 + x_4 = 1500 \\
& x_1 + x_2 + x_5 = 1750 \\
& 4x_1 + 2x_2 + x_6 = 4800 \\
& x_1, x_2, x_3, x_4, x_5, x_6 \geq 0
\end{aligned}
$$

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# LINDO Program

```
MAX 12 x1 + 9 x2
ST
x1 + x2 = 1000
x2 + x4 = 1500
x1 + x2 + x5 = 1750
4x1+ 2x2 + x6 = 4800
x1>=0
x2>=0
x3>=0
x4>=0
x5>=0
x6>=0
END
```

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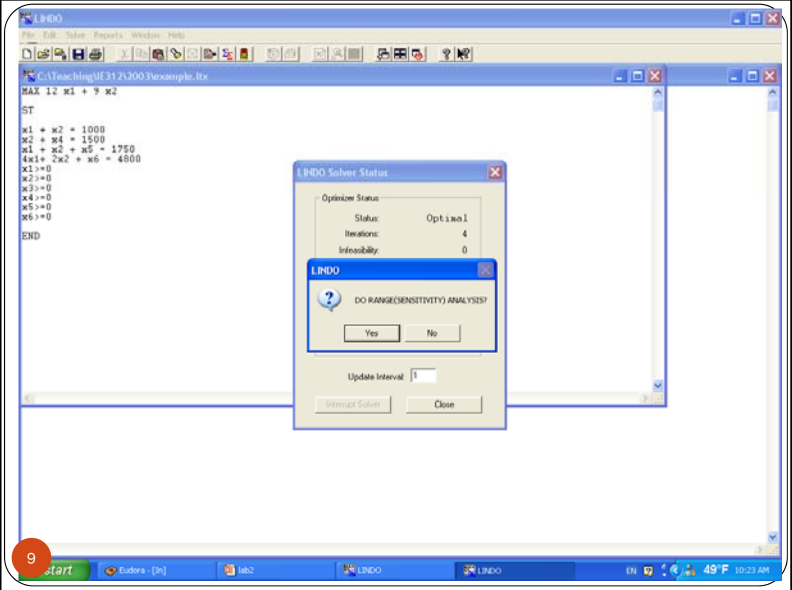


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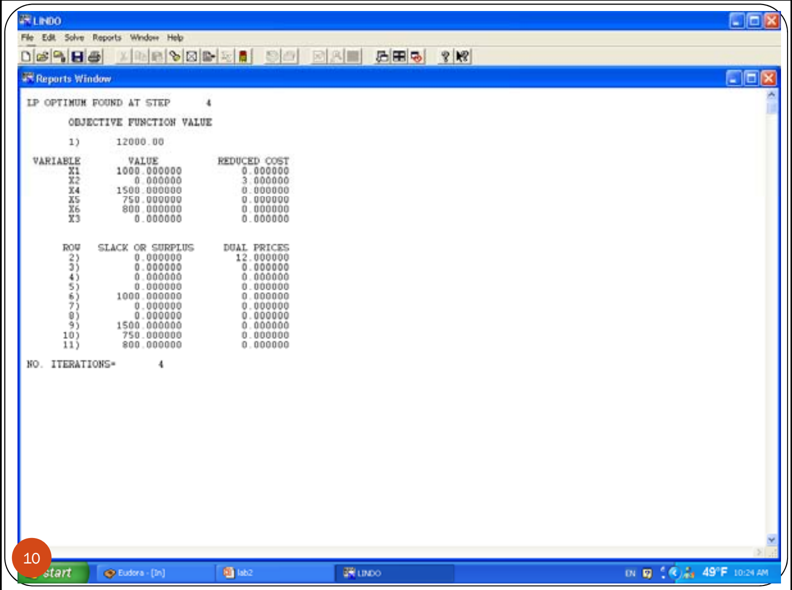


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## Output

OF coeff. for $x_2$ (9 ) must be improved by 3 in order for the optimal value of $x_2$ to become nonzero.

```
LP OPTIMUM FOUND AT STEP      4

        OBJECTIVE FUNCTION VALUE

        1)      12000.00

  VARIABLE        VALUE          REDUCED COST
        X1     1000.000000          0.000000
        X2        0.000000          3.000000
        X4     1500.000000          0.000000
        X5      750.000000          0.000000
        X6      800.000000          0.000000
        X3        0.000000          0.000000
```

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## Output (cont.)

```
     ROW   SLACK OR SURPLUS     DUAL PRICES
      2)         0.000000        12.000000
      3)         0.000000         0.000000
      4)         0.000000         0.000000
      5)         0.000000         0.000000
      6)      1000.000000         0.000000
      7)         0.000000         0.000000
      8)         0.000000         0.000000
      9)      1500.000000         0.000000
     10)       750.000000         0.000000
     11)       800.000000         0.000000

NO. ITERATIONS=       4
```

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## Reduced cost

- A variable's reduced cost is the amount by which the objective coefficient of the variable would have to improve (increase for maximization problems, decrease for minimization problems) before it would become profitable to bring that variable into the solution at a nonzero value.
- The reduced cost for a decision variable with a positive value is 0.
- A reduced cost may be interpreted also as the amount of penalty you would have to pay to introduce a variable into the solution.

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## Example

Consider the following objective function:

Min $2x_1 + 5x_2 + 4x_3$

Suppose the optimal value of $x_1$ is zero, with a reduced cost of 1.2

Since this is a minimization problem, this tells us that the current coefficient of $x_1$, which is 2, must be decreased by 1.2 in order for the optimal value of $x_1$ to be nonzero.

Thus if the objective function coefficient of $x_1$ was 0.8 (or less), resolving the LP would yield a nonzero value of $x_1$.

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## SLACK/SURLUS

- The slack for "<=" constraints is the difference between the right hand side of an equation and the value of the left hand side after substituting the optimal values of the decision variables.
- The slack represents the amount of unused units of the right hand side resources.
- The surplus for ">=" constraints is the difference between the right hand side of an equation and the value of the left hand side after substituting the optimal values of the decision variables.
- The surplus represents the number of units in which the optimal solution causes the constraint to exceed the right hand side lower limit.

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## Dual Prices

- The LINDO solution report also gives a DUAL PRICE figure for each constraint.
- You can interpret the dual price as the amount by which the objective would improve given a unit of increase in the right-hand side of the constraint
- Report dual prices
  - Gives us sensitivities to RHS parameter
  - Know how much objective function will change
- Dual prices are sometimes called *shadow prices, because they tell you how much you should be* willing to pay for additional units of a resource.

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## LINDO: Basic Syntax

- Objective Function Syntax: Start all models with MAX or MIN
- Variable Names: Limited to 8 characters
- Constraint Name: Terminated with a parenthesis
  - Land) X1+X2 < =200
- Recognized Operators (+, -, >, <, =)
- Order of Precedence: Parentheses not recognized

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## Syntax (cont.)

- Adding Comment: Start with an exclamation mark
- Splitting lines in a model: Permitted in LINDO
- Case Sensitivity: LINDO has none
- Right-hand Side Syntax: Only constant values
- Left-hand Side Syntax: Only variables and their coefficients

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## Why Modeling Language?

- More 'complicated' to use than LINDO (at least at first glance)
- Advantages
  - Natural representations
  - Similar to mathematical notation
  - Can enter many terms simultaneously
  - Much faster and easier to read

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## Why Solvers?

- Best commercial software has modeling language and solvers separated
- Advantages:
  - Select solver that is best for your application
  - Learn one modeling language use any solver
  - Buy 3rd party solvers or write your own!

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## Example Problem

Bisco's new sugar-free, fat-free chocolate squares are so popular that the company cnnot keep up with demand. Regional demands shown in the following table total 2000 cases per week, but Bisco can produce only 60% of that number.

| | NE | SE | MW | W |
|---|---|---|---|---|
| Demand | 620 | 490 | 510 | 380 |
| Profit | 1.6 | 1.4 | 1.9 | 1.2 |

The table also shows the different profit levels per case experienced in the regions due to competition and consumer tastes. Bisco wants to find a maximum profit plan that fulfills between 50% and 70% of each region's demand.

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## Problem Formulation

$$\max \sum_{i=1}^{4} p_i x_i$$

$$\sum_{i=1}^{4} x_i \le 1200$$

$$l_i \le x_i \le u_i, i = 1,2,3,4$$

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## LINDO Solution

```
max   1.60 x1 + 1.40 x2 + 1.90 x3 + 1.20 x4

st
    x1 + x2 + x3 + x4 <=1200
    x1 >= 310
    x1 <= 434
    x2 >= 245
    x2 <= 343
    x3 >= 255
    x3 <= 357
    x4 >= 190
    x5 <= 266
end
```

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## LINGO Solution

- Capacity constraint

```
@SUM(REGIONS(I): CASES(I))
    <=1200;
```

- Minimum/maximum cases

```
@FOR(REGIONS(I):
    CASES(I) <= UBOUND;
    CASES(I) >= LBOUND);
```

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## LINGO Solution

- Objective function

```
MAX = @SUM(REGIONS(I):
          PROFIT*CASES(I));
```

- We also need to define `REGIONS`, `CASES`, etc, and type in the data.

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## LINGO Solution

- Defining sets

```
SETS:
   REGIONS / NE SE MW W/: LBOUND,
       UBOUND, PROFIT, CASES;
ENDSETS
```

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## LINGO Solution

- Enter the data

```
DATA:
   LBOUND = 310 245 255 190;
   UBOUND = 434 343 357 266;
   PROFIT = 1.6 1.4 1.9 1.2;
ENDDATA
```

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## Sensitivity Analysis

- Basic Question: How does our solution change as the input parameters change?
  - The objective function?
    - More/less profit or cost
  - The optimal values of decision variables?
    - Make different decisions!
- Why?
  - Only have estimates of input parameters
  - May want to change input parameters

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## LINDO Sensitivity Analysis

RANGES IN WHICH THE BASIS IS UNCHANGED:

| | OBJ COEFFICIENT RANGES | | |
|---|---|---|---|
| VARIABLE | CURRENT COEF | ALLOWABLE INCREASE | ALLOWABLE DECREASE |
| X1 | 1.600000 | 0.300000 | 0.200000 |
| X2 | 1.400000 | 0.200000 | INFINITY |
| X3 | 1.900000 | INFINITY | 0.300000 |
| X4 | 1.200000 | 0.400000 | INFINITY |
| X5 | 0.000000 | 0.000000 | INFINITY |

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## Interpretation

- As long as prices for the NE region are between $1.4 and $1.9, we want to sell the same quantity to each region, etc.

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