Advanced Process Control

CBEg 6142

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Chapter 1:part 2

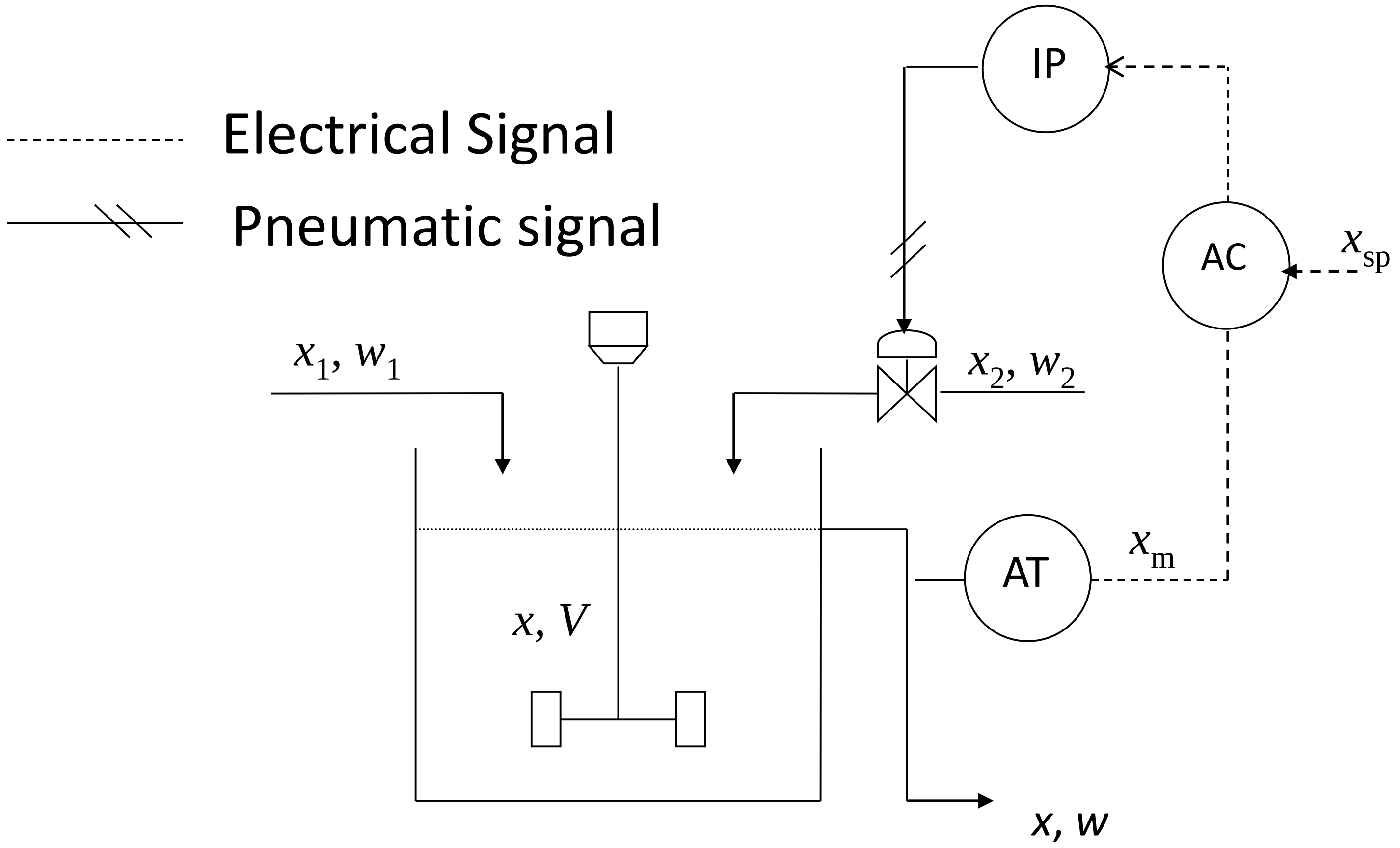
Feed Back Control Systems

Chapter Objectives

End of this chapter, you should be able to:

1. Explain the concept of feedback controllers
2. Explain P, I and D controllers

Consider the continuous blending process, shown below



Control system

Control objective:

To keep the tank exit composition x at the desired set-point by adjusting w_2.

Measurement : *Composition Analyzer-Transmitter (AT)*

Feedback controller: AC *Composition Controller*

Final control element: *Pneumatic control valve*

Current-to-pneumatic transducer: I/P

Basic Control Modes

- Proportional, Integral and Derivative

Proportional Control

In feedback control, the objective is to reduce the error signal to zero.

Define an error signal, *e*, by

$$e(t) = y_{SP}(t) - y_m(t) \qquad (6.1)$$

where y_{sp} = set point

y_m = measured value of the controlled variable (or equivalent signal from transmitter)

Basic Control Modes

- For proportional control, the controller output is proportional to the error signal

$$p(t) = \bar{p} + K_c e(t) \qquad (6.2)$$

Where

$p(t)$ = controller output

$\bar{p}(t)$ = bias (steady-state) value

K_c = controller gain (usually dimensionless)

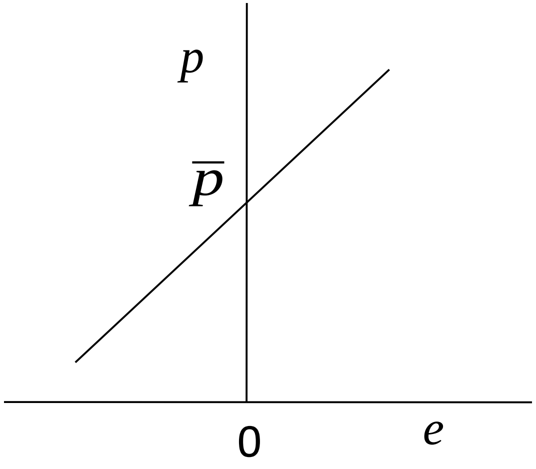
Proportional Band, PB:

- Some controllers have a proportional band setting instead of a controller gain. The *proportional band PB* (*in %*) is defined as

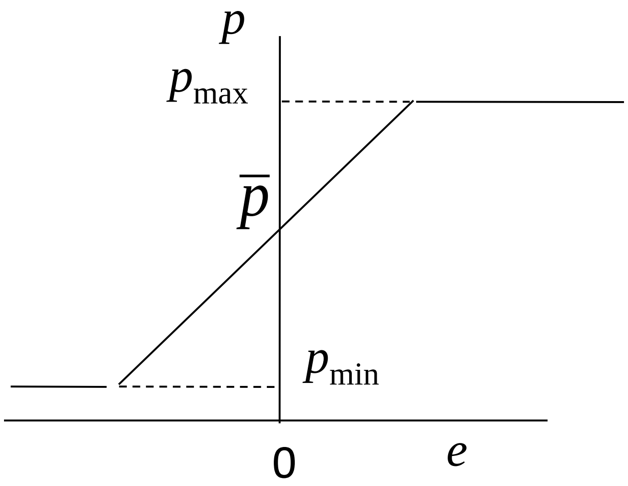
$$PB = \frac{100\%}{K_c} \qquad (6.3)$$

- Applies when K_c is dimensionless
- Small (narrow) PB corresponds to large K_c
- Large (wide) PB corresponds to small K_c

Ideal vs. actual



Proportional controller: ideal behavior



Proportional controller: actual behavior

Ideal controller does not include physical limits

A controller *saturates* when its output reaches a physical limit, either p_{max} or p_{min}.

Proportional controller transfer function

In order to derive the transfer function for an ideal proportional controller, define a deviation variable as

$$p'(t) = p(t) - \bar{p} \qquad (6.4)$$

Then (6.2) can be written as

$$p'(t) = K_c e(t) \qquad (6.5)$$

Taking Laplace transform of (6.5) and rearranging we get

$$\frac{P'(s)}{E(s)} = K_c \qquad (6.6)$$

Proportional controller limitation

- An inherent limitation of proportional controller is that a steady-state error (*offset)* occurs after a set-point change or a sustained disturbance.

Integral Control

Integral control (reset control, floating control)

For integral control action, the controller output depends on the integral of the error signal over time,

$$p(t) = \bar{p} + \frac{1}{\tau_I}\int_0^t e(t')dt' \qquad (6.7)$$

where τ_I is an adjustable parameter and referred to as the *integral time constant* or *reset time,* has units of time.

The transfer function:

$$\frac{P'(s)}{E(s)} = \frac{1}{\tau_I s} \qquad (6.8)$$

Integral Control

- An important practical advantage: *Eliminates offset.*
- Eq. (6.7) implies that p changes with time unless $e(t) = 0$.
- This desirable situation occurs unless the controller output or the final control element saturates.
- The control action by the integral controller is very little until the error signal has persisted for sometime.
- On the other hand, proportional controller takes immediate corrective action as soon as an error is detected.

PI Controller

Integral control is used in conjunction with proportional control as the *proportional-integral* (PI) controller:

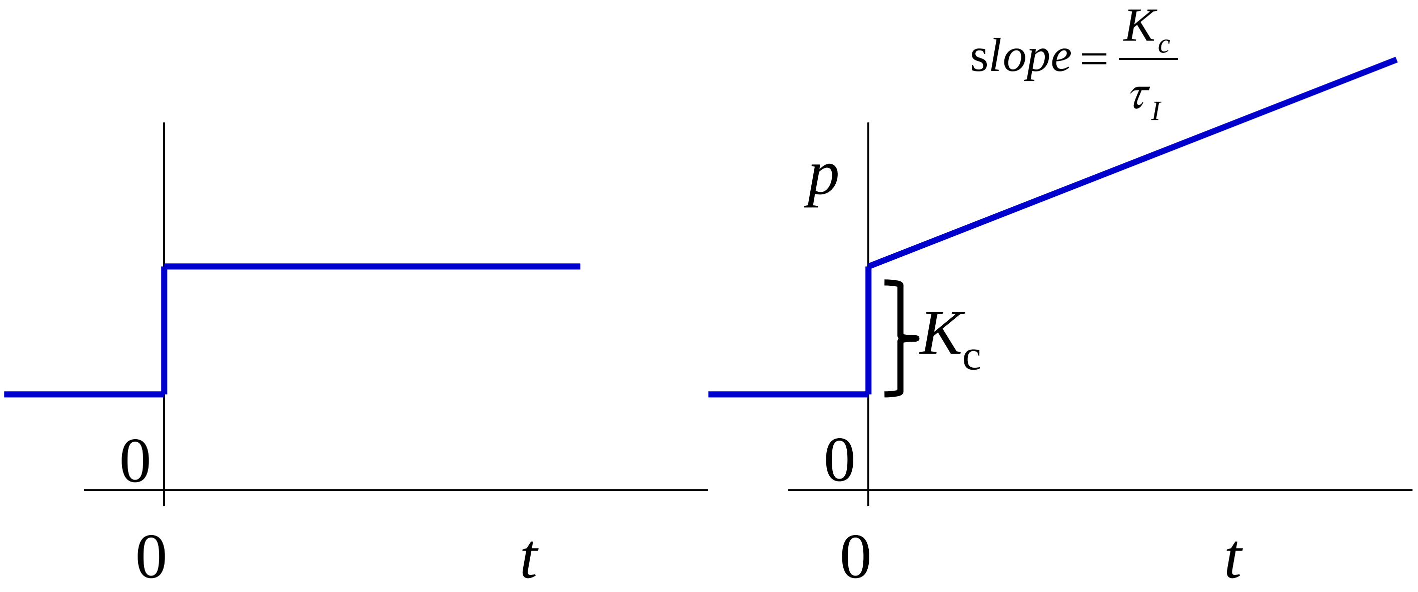
$$p(t) = \bar{p} + K_c \left[e(t) + \frac{1}{\tau_I} \int_0^t e(t')dt' \right] \qquad (6.9)$$

The corresponding transfer function is:

$$\frac{\mathrm{P}'(\mathrm{s})}{\mathrm{E}(\mathrm{s})} = K_c \left(1 + \frac{1}{\tau_I s} \right) \qquad (6.10)$$

PI Controller

The response of the PI controller to a unit step change in $e(t)$ is shown in Fig.



$1/\tau_I$ - repeats per minute

PI Controller

- Disadvantages:
  - Produces oscillatory response
  - Reset windup

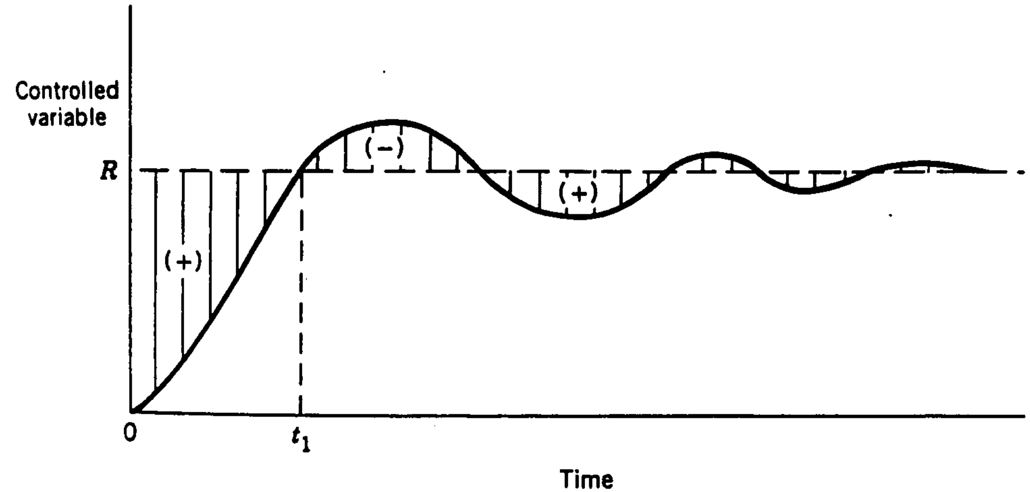


Figure 8.7. Reset windup during a set-point change.

When a sustained error occurs, the integral term becomes quite large and the controller output eventually saturates – *reset windup* or *integral windup.*

Anti reset windup: Temporarily halting the integral action whenever the control output saturates.

Reset windup explained

When the output of the controller becomes limited because process conditions cause it to be fully open or fully closed, and the PV is still not at the setpoint value, the reset remainder term continues to increase by the remaining error. When the process conditions change to allow the control valve to once again do its work, the reset remainder term is so large that even when the sign of the error changes, the output may not respond until all of the reset remainder term is "used up." The normal solution (anti reset windup) is to stop accumulating reset remainder when the output is limit-stopped. Other solutions cause the controller to go into Manual then reinitialize when the limit-stop conditions change.

Derivative control

Rate action, pre-act, anticipatory control

- Anticipate the future error by considering its rate of change.
- For ideal derivative action,

$$p(t) = \bar{p} + \tau_D \frac{de(t)}{dt} \qquad (6.11)$$

where τ_D is the derivative time, and has units of time.

As long as the error is constant *de/dt = 0,* the controller output is equal to $\bar{p}$.

Derivative control

- Derivative action is never used alone.
- Always used in conjunction with P or PI control.

PD controller has the transfer function

$$\frac{P'(s)}{E(s)} = K_c\left(1+\tau_D s\right) \tag{6.12}$$

The derivative control action tends to stabilize the controlled process.

PID Controller

PID control algorithm is given by

$$p(t) = \bar{p} + K_c \left[e(t) + \frac{1}{\tau_I} \int_0^t e(t')dt' + \tau_D \frac{de}{dt} \right] \quad (6.13)$$

Transfer function of an ideal controller (*parallel form*)

$$\frac{P'(s)}{E(s)} = K_c \left(1 + \frac{1}{\tau_I s} + \tau_D s \right) \quad (6.14)$$

Transfer function – actual (*Series form*)

$$\frac{P'(s)}{E(s)} = K_c \left(\frac{\tau_I s + 1}{\tau_I s} \right) \underbrace{\left(\frac{\tau_D s + 1}{\alpha \tau_D s + 1} \right)}_{\text{lead / lag units}} \quad (6.15)$$

lead / lag units

Typical responses of Feedback control systems

Consider response of a controlled system after a sustained disturbance occurs (e.g. step change in load variable)

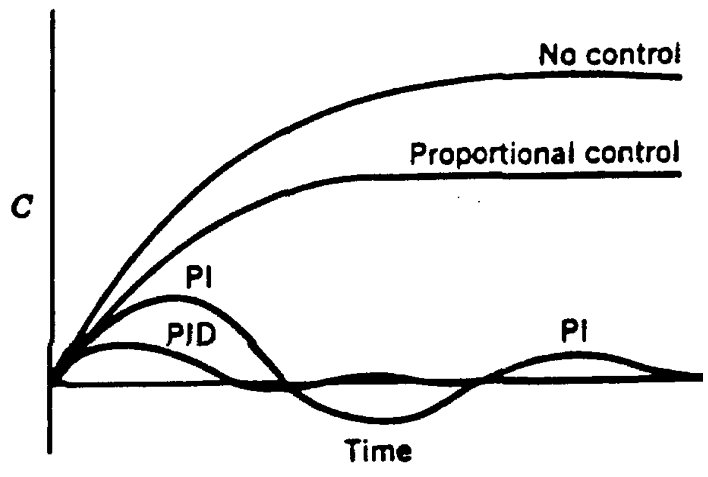


Figure 8.9. Typical process responses with feedback control.

No control

New steady state is reached

P control

Offset reduced

PI control

Offset eliminated

Oscillatory response

PID control

Oscillations reduced

No offset

Effect of controller parameters

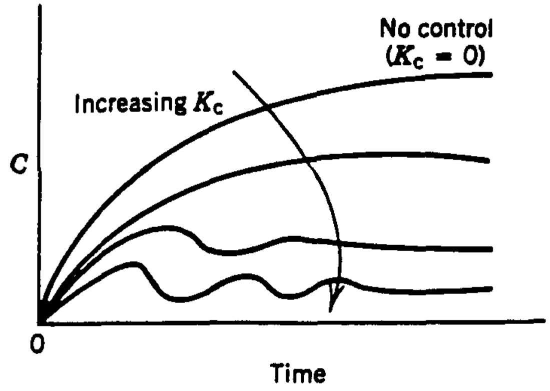


Figure 8.10. Proportional control: effect of controller gain.

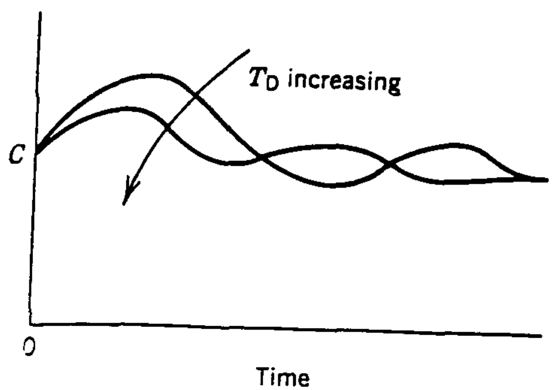


Figure 8.12. PID control: effect of derivative time.

-Too small a value of K_c

Sluggish response

Larger deviation

-Too large a value of K_c

Exhibit oscillatory or unstable behavior

-Intermediate values of K_c is desirable

-Increasing τ_D tends to improve the response by reducing the maximum deviation, response time, and degree of oscillation

-If τ_D is too large, measurement noise is amplified and the response may become oscillatory.

Effect of controller parameters

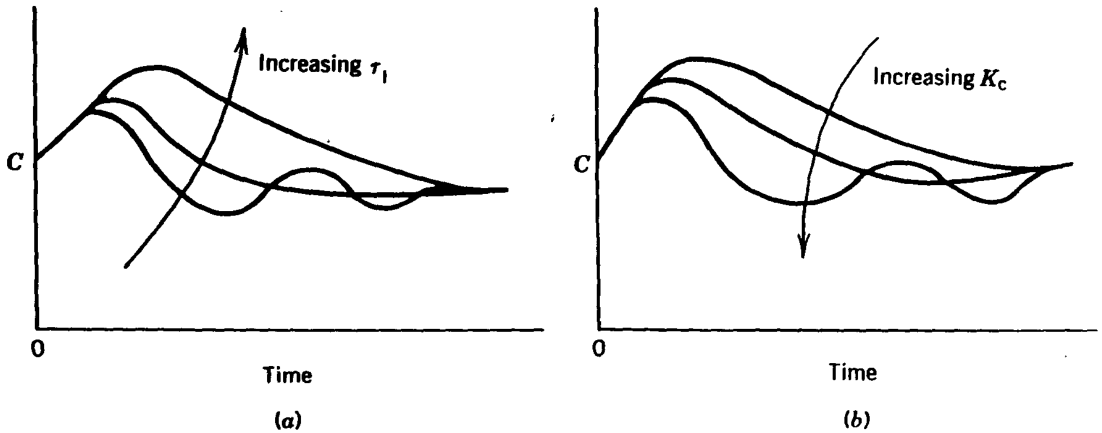


Figure 8.11. PI control: (a) effect of reset time (b) effect of controller gain.

- Increasing the integral time makes the controller more sluggish.
- Offset will be eliminated for all values τ_I
- For large values of τ_I, it takes very long time to return to the set-point.

Summary of the Characteristics of the Most Commonly Used Controller Modes

1. Two Position (ON-OFF)*:*

- Inexpensive
- Extremely simple
- Cause continual cycling of the CV
- Produces excessive wear on the control valve

2. Proportional:

- Simple
- Inherently stable when properly tuned
- Easy to tune
- Experiences offset at steady state

3. Proportional plus reset (PI)*:*

- No offset
- Better dynamic response than reset alone
- Possibilities exist for instability due to lag introduced

4. Proportional plus rate(PD)*:*

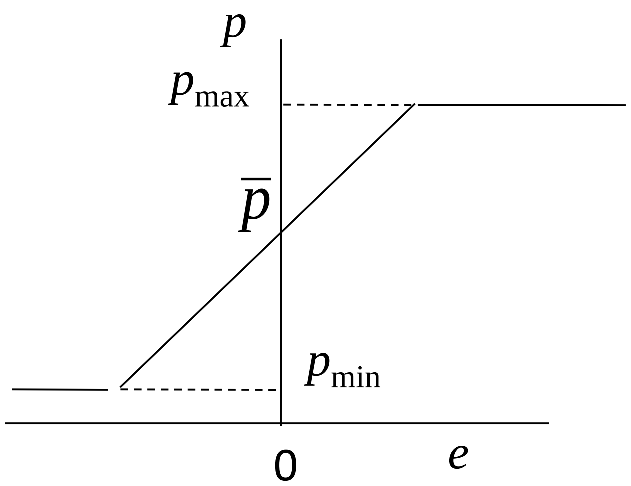
- Stable
- Less offset than proportional alone (use of higher gain possible).
- Reduces lags, i.e., more rapid response.

5. Proportional plus reset plus rate (PID)*:*

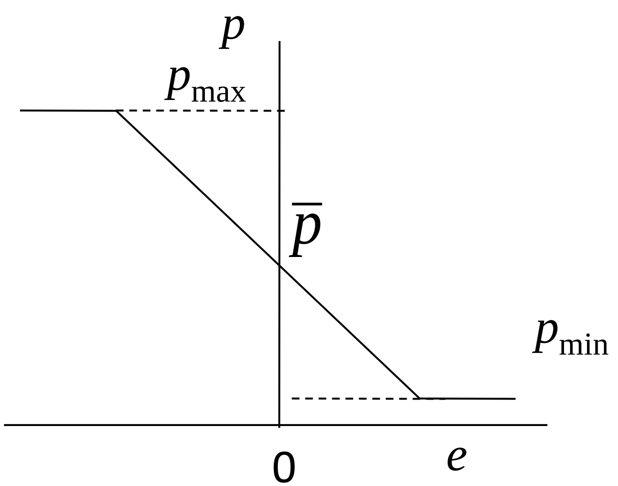
- Most complex
- Rapid response
- No offset
- Difficult to tune
- Best control if properly tuned.

Reverse or Direct Acting Controller

K_c can be made positive or negative



- **Reverse-Acting** ($K_c > 0$)
- "output increases as input decreases (measured value)"



- **Direct-Acting** ($K_c < 0$)
- "controller output increases as input increases (measured value)"

Conclusion!

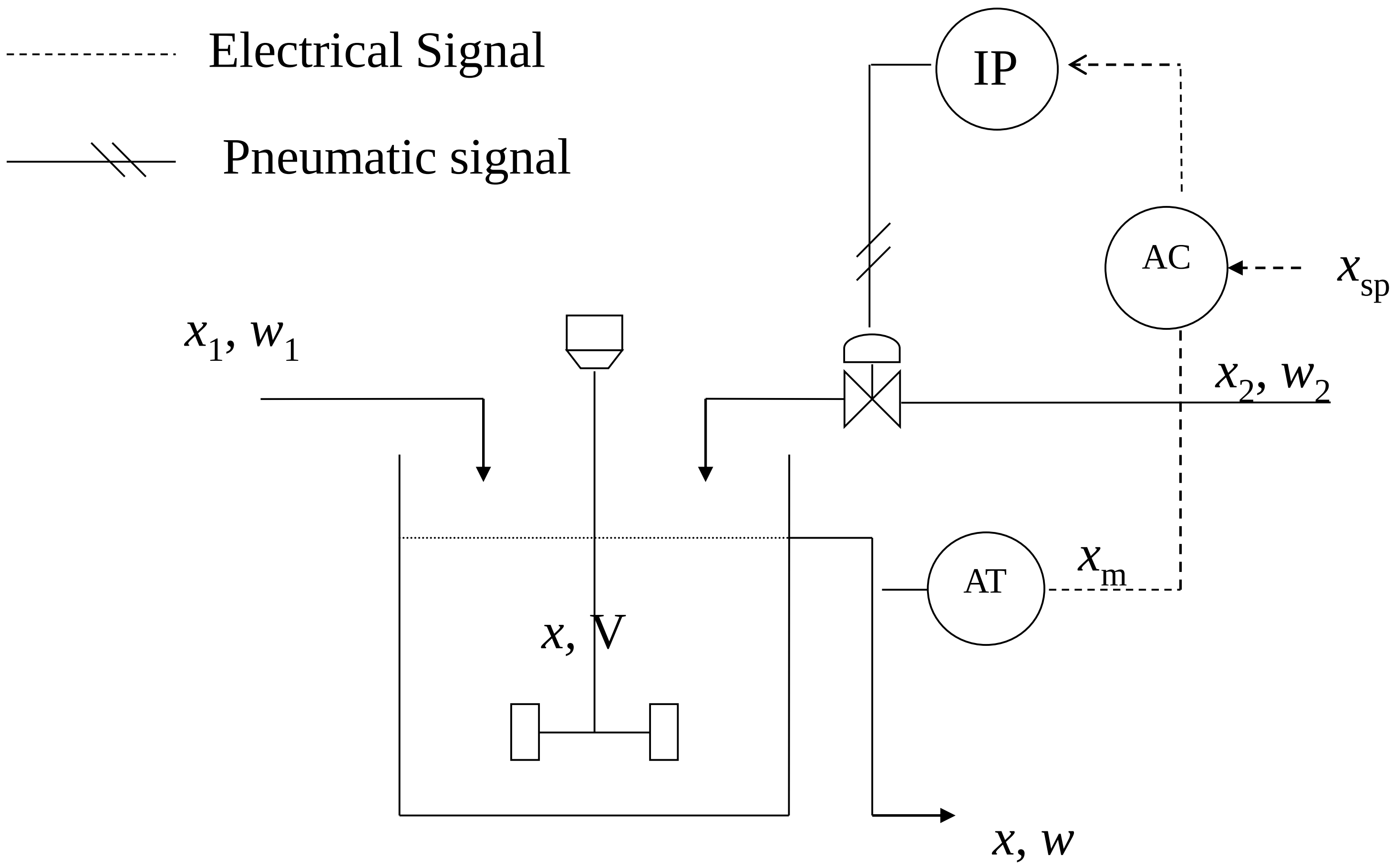
- Concept of feedback control
- P, I, D controller modes
- Advantages and disadvantages
- Motivation for additional modes

Chapter 8

Introduction to Feedback Control Systems:

Block Diagram, Closed-Loop Transfer Function, Closed-Loop Response

Schematic diagram of blending system



Consider the blending process

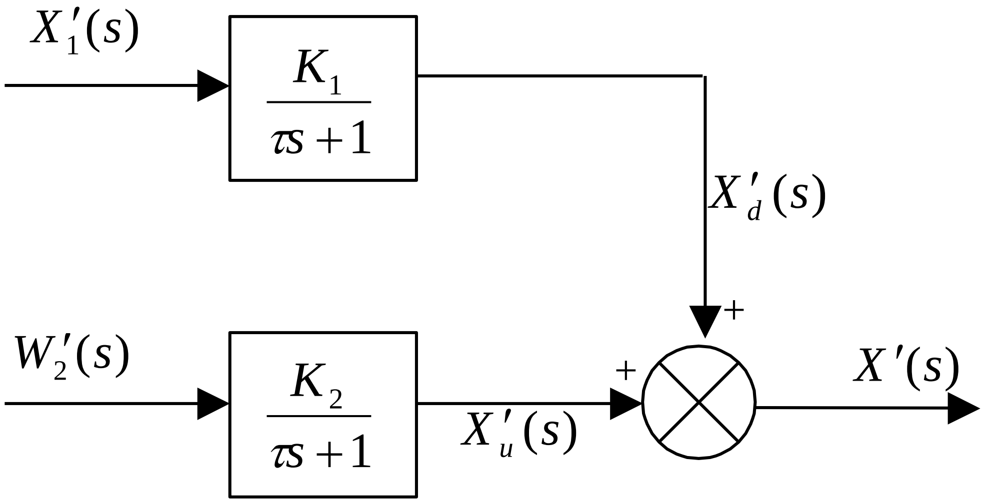
- The dynamic model of a stirred-tank blending process was developed as

$$X'(s) = \left(\frac{K_1}{\tau s + 1}\right) X_1'(s) + \left(\frac{K_2}{\tau s + 1}\right) W_2'(s) \qquad (8.1)$$

where $\tau = \dfrac{\bar{V}}{\bar{q}}$, $K_1 = \dfrac{\bar{w}_1}{\bar{w}}$, and $K_2 = \dfrac{1-\bar{x}}{\bar{w}}$ (8.2)

Block diagram development

- Figure below provides a block diagram representation of information in (8.1) and (8.2).

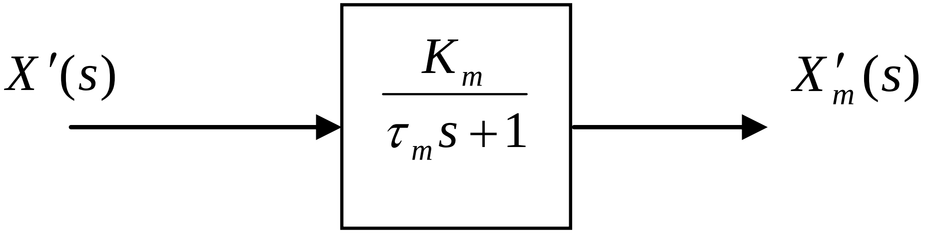


Sensor dynamics

- Composition Sensor-Transmitter (Analyzer)

The dynamic behavior of the analyzer can be approximated by

$$\frac{X'_m(s)}{X'(s)} = \left(\frac{K_m}{\tau_m s + 1}\right) \qquad (8.3)$$

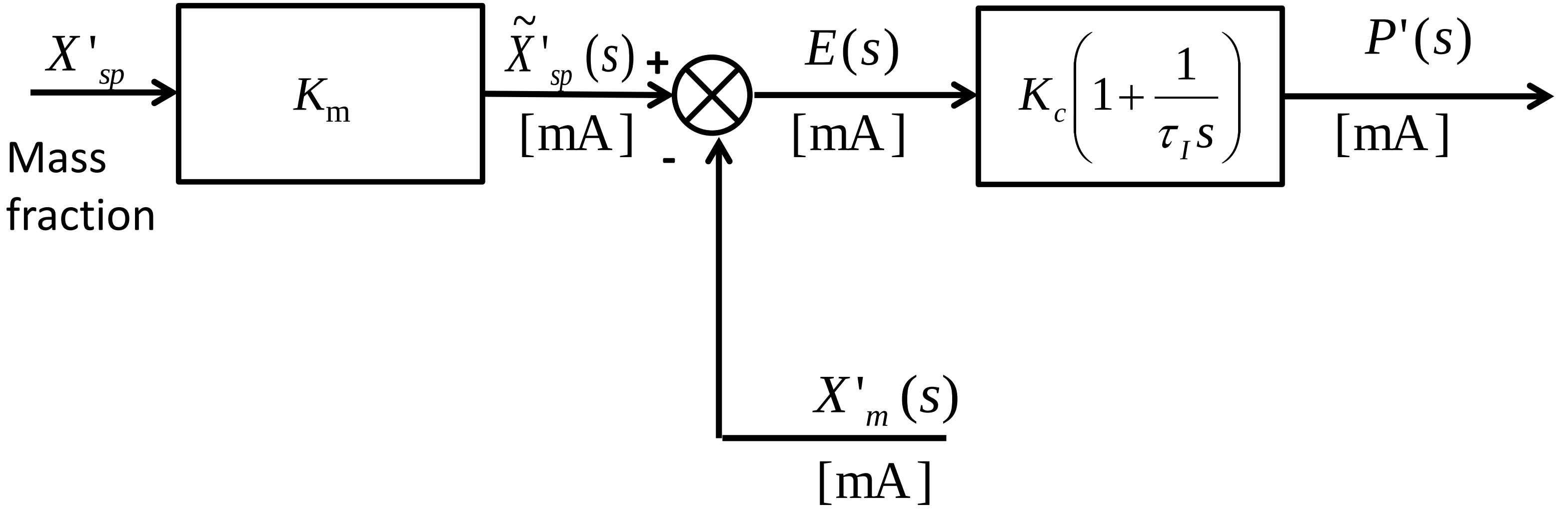


A useful approximation: $\tau_m = 0$ since $\tau_m << \tau$.

Controller

Suppose an electronic proportional plus integral controller is used. The controller transfer function is

$$\frac{P'(s)}{E(s)} = K_c\left(1+\frac{1}{\tau_I s}\right) \qquad (8.4)$$



The error signal is expressed as

$$e(t) = \tilde{x}'_{sp}(t) - x'_m(t) \qquad (8.5)$$

or after taking the Laplace transforms,

$$E(s) = \tilde{X}'_{sp}(s) - X'_m(s) \qquad (8.6)$$

$\tilde{x}'_{sp}(t)$ denotes the internal set-point expressed as an equivalent electrical current signal.

It is related to the actual composition set-point by the composition-transmitter gain K_m :

$$\tilde{x}'_{sp}(t) = K_m x'_{sp}(t) \qquad (8.7)$$

I/P Converter (Transducer)

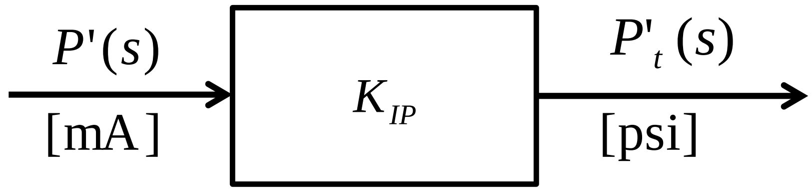
Thus

$$\frac{\tilde{X}'_{sp}(s)}{X'_{sp}(s)} = K_m \qquad (8.8)$$

The symbol that represents the subtraction operation is called a *comparator.*

Transducer transfer function consists of a steady-state gain K_{IP}:

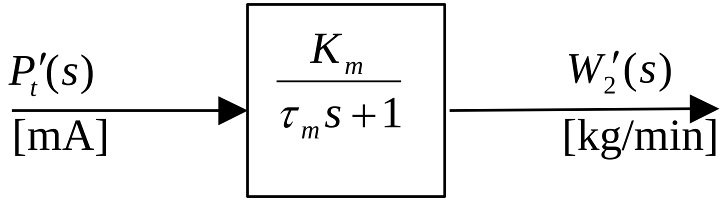
$$\frac{P_t'(s)}{P'(s)} = K_{IP} \qquad (8.9)$$



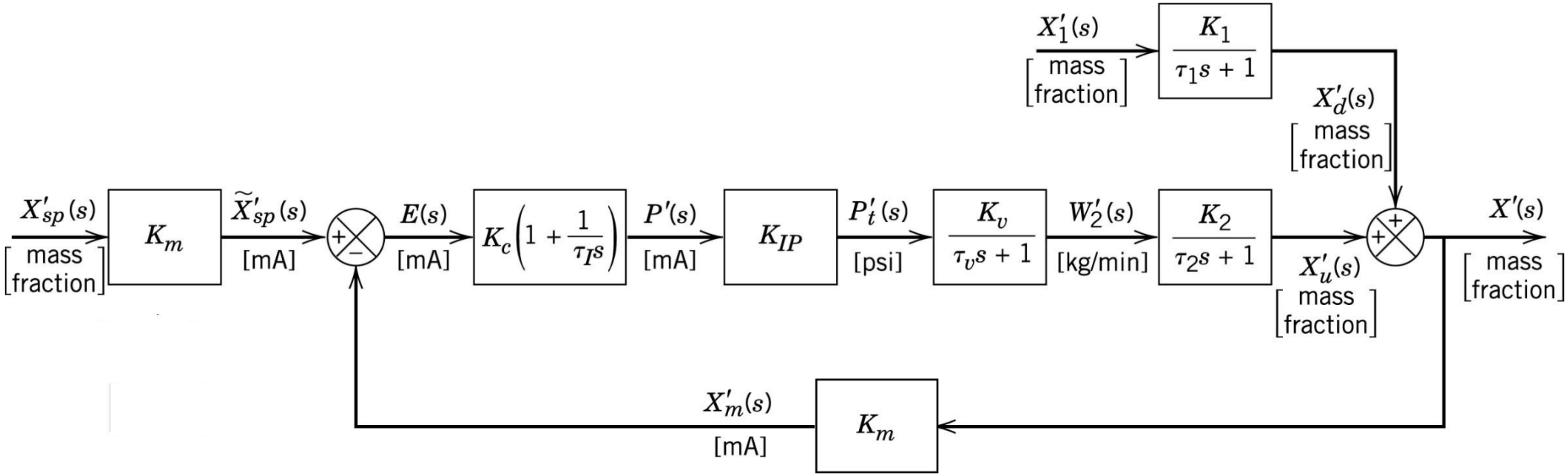
Control Valve

A first-order transfer function provides an adequate model for control valves. Thus

$$\frac{W_2'(s)}{P_t'(s)} = \left(\frac{K_v}{\tau_v s + 1}\right) \qquad (8.10)$$



Block diagram for the control system

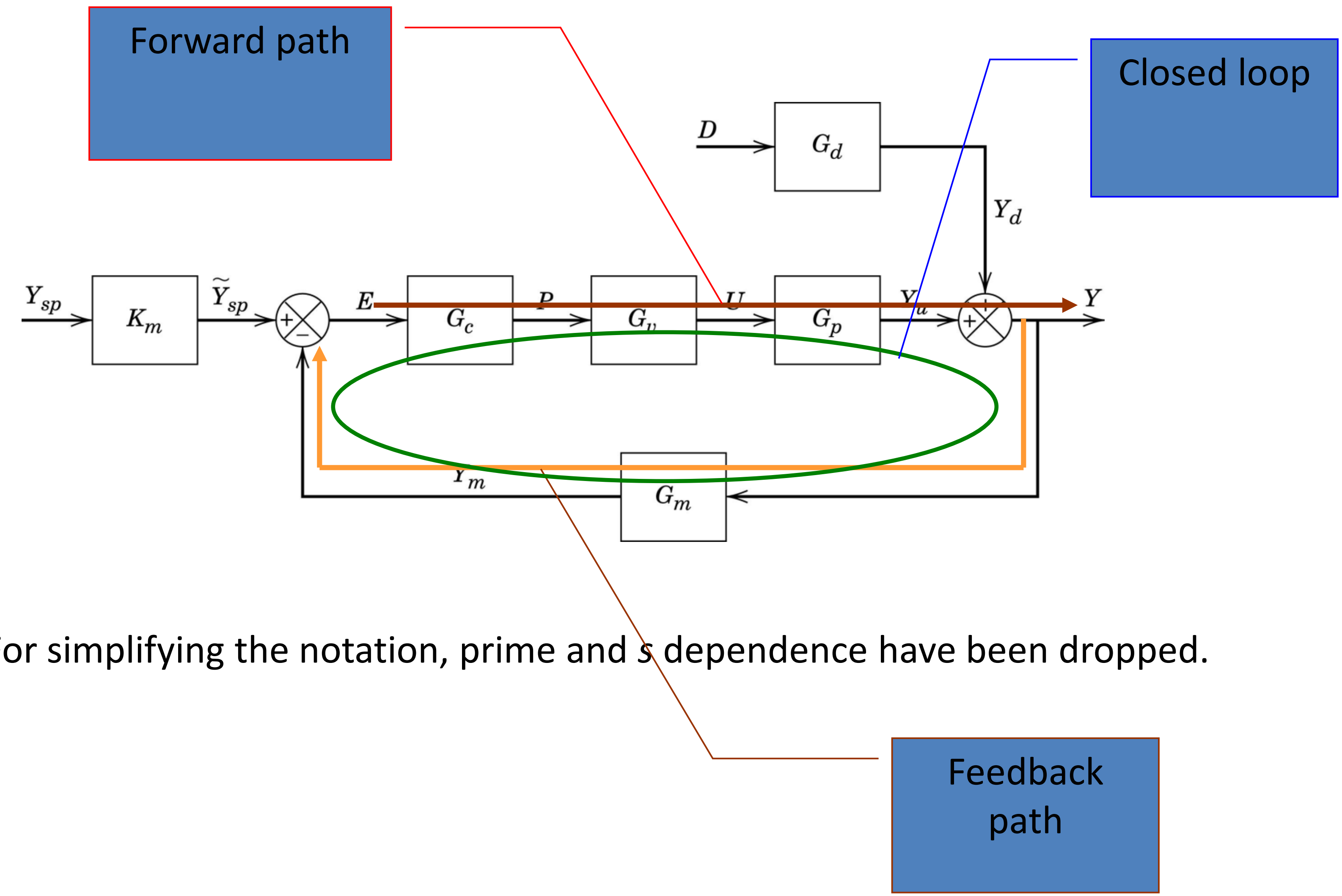


Closed-loop Transfer Functions

The standard notations are:

Y = controlled variable
U = manipulated variable
D = disturbance variable
P = controller output
E = error signal
Y_m = measured value of Y
Y_{sp} = set-point
$\tilde{Y}_{sp}$ = internal set-point (used by the controller)
G_c = controller transfer function
G_v = transfer function for final control element
G_p = process transfer function
G_d = disturbance transfer function
G_m = transfer function for measuring element and transmitter
K_m = steady-state gain for G_m

Standard block diagram



For simplifying the notation, prime and *s* dependence have been dropped.

Standard block diagram

Y_{sp} and D are the independent input signals for the controlled process because they are not affected by the operation of the control loop.

To evaluate the performance of the control system, we need to know how the controlled process responds to changes in Y_{sp} and D.

We derive expressions for the *closed-loop transfer functions* $Y(s)/Y_{sp}(s)$ and $Y(s)/D(s)$.

Closed-loop Transfer functions

Derivation of Closed loop Equation

$$Y = G_c G_v G_p E(s) + G_d D$$

$$E = K_m Y_{sp} - G_m Y$$

$$Y = G_c G_v G_p (K_m Y_{sp} - G_m Y) + G_d D$$

$$Y = \frac{K_m G_c G_v G_p}{1 + G_c G_v G_p G_m} Y_{sp} + \frac{G_d}{1 + G_c G_v G_p G_m} D$$ **Closed loop equation** (8.11)

Servo problem T.F.

$$G_{sp} = \frac{Y}{Y_{sp}} = \frac{K_m G_c G_v G_p}{1 + G_c G_v G_p G_m} \quad (8.12)$$

Regulator problem T.F.

$$G_{load}(s) = \frac{Y(s)}{D(s)} = \frac{G_d}{1 + G_c G_v G_p G_m} \quad (8.13)$$

Closed-loop Transfer functions

- Comparison of (8.12) and (8.13) indicates that both closed-loop transfer functions have the same denominator,

  $1+ G_p G_v G_c G_m$.

- The roots the denominator determines the nature of the closed loop response ,

  $1+ G_p G_v G_c G_m = 0$. **Characteristic Equation**

- The denominator is often written as $1+G_{OL}$ where G_{OL} is the *open-loop transfer function,*

  - $G_{OL} = G_p G_v G_c G_m$.

General Expression for Feedback Control Systems

- Closed-loop transfer functions for more complicated block diagrams can be written in the general form: (For negative feedback only)

$$\frac{Z}{Z_i} = \frac{\prod_f}{1 + \prod_l} \qquad (8.22)$$

where Z = the output variable or any internal variable within the control loop.

Z_i = an input variable

$\prod_f$ = product of transfer functions in the forward path from Z_i to Z.

$\prod_l$ = product of every transfer function in the feedback loop

Example

- Find the closed-loop transfer function *C/R* for the complex control system shown in fig.

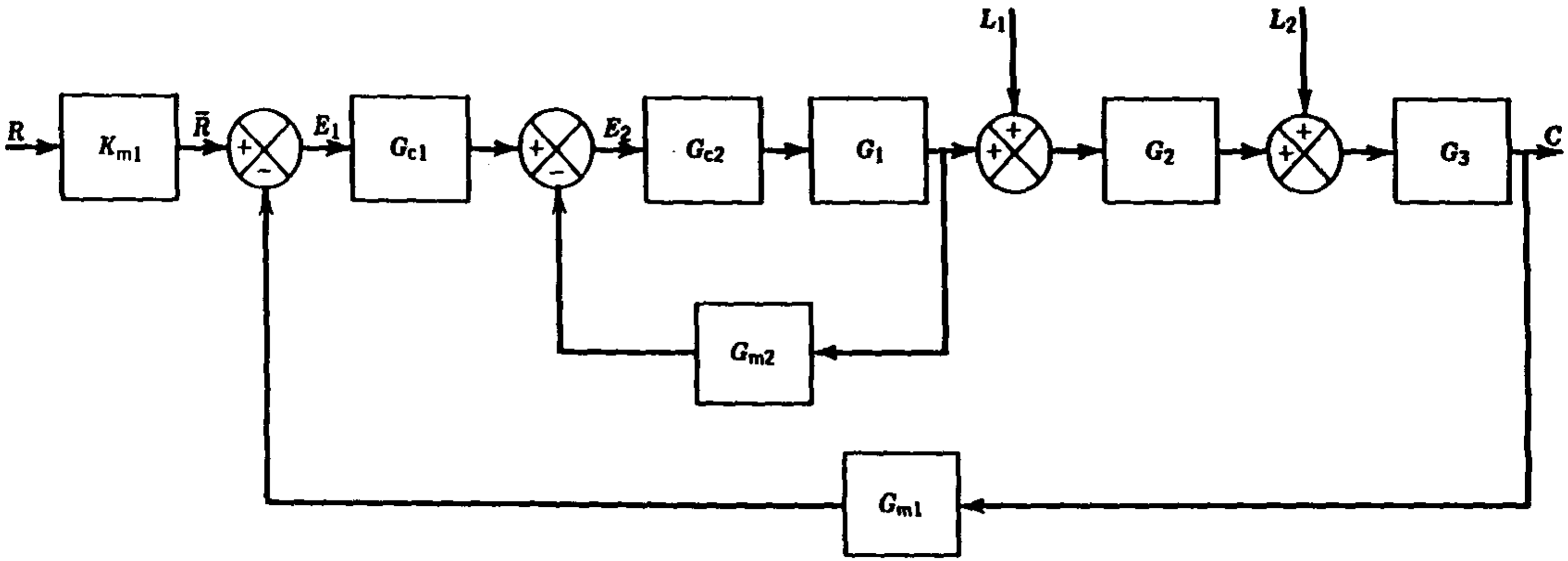


Figure 10.12. Complex control system.

Closed-loop Response

Effect of P- controller on closed-loop response

1. Offset

Offset = Desired steady state response − Attained steady state response

Servo problem

$$Y(s)=\frac{K_m G_c G_v G_p}{1+G_c G_v G_p G_m} Y_{sp}(s)$$

Closed-loop Response

Consider a step change in Y_{sp} of magnitude A

Desired steady state response = A

Closed-loop response

$$Y(s) = \frac{K_m G_c G_v G_p}{1 + G_c G_v G_p G_m} \frac{A}{s}$$

After the closed-loop response reaches steady state

$$y(t \to \infty) = \frac{K_m K_c G_v G_p}{1 + K_c G_v G_p G_m} \frac{A}{s} \cdot s \bigg|_{s=0} = \frac{A K_m K_c G_v(0) G_p(0)}{1 + K_c G_v(0) G_p(0) G_m(0)}$$

Closed-loop Response

$$y(t \to \infty) = \frac{AK_m K_c K_v K_p}{1 + K_m K_c K_v K_p}$$

$$Attained\ steady\ state\ response = \frac{AK_m K_c K_v K_p}{1 + K_m K_c K_v K_p}$$

$$Offset = A - \frac{AK_m K_c K_v K_p}{1 + K_m K_c K_v K_p}$$

Offset for servo Problem with P-Controller

$$Offset = \frac{A}{1 + K_m K_c K_v K_p}$$

$$Offset = \frac{100}{1 + K_m K_c K_v K_p}\%$$

Closed-loop Response

Regulator problem

The effect of disturbance should be removed therefore, the desired steady state response = 0 for regulator problem

Closed-loop response for regulator problem

$$Y(s) = \frac{G_d}{1 + G_c G_v G_p G_m} D_{sp}(s)$$

$$Y(s) = \frac{G_d}{1 + G_c G_v G_p G_m} \frac{A}{s}$$

$$y(t \to \infty) = \frac{G_d}{1 + K_c G_v G_p G_m} \frac{A}{s} \cdot s \Big|_{s=0} = \frac{AG_d(0)}{1 + K_c G_v(0) G_p(0) G_m(0)}$$

$$y(t \to \infty) = \frac{AK_d}{1 + K_m K_c K_v K_p}$$

$$offset = 0 - \frac{AK_d}{1 + K_m K_c K_v K_p}$$

Offset for Regulator Problem for with P-Controller

$$offset = -\frac{AK_d}{1 + K_m K_c K_v K_p}$$

$$offset = -\frac{100K_d}{1 + K_m K_c K_v K_p}\%$$

Effect of Proportional Controller

2. Effect of P- controller on the order and speed of closed-loop response

The nature of the closed-loop response depends on characteristics equation:

$$1 + G_c G_v G_p G_m = 0$$

For proportional controller

$$1 + K_c G_v G_p G_m = 0$$

The order of the closed loop response is not affected by Proportional controller.

Effect on the speed of the response

Consider a second order $G_vG_pG_m$ with proportional controller

$$1 + K_c \frac{K}{\tau^2 s^2 + 2\zeta s + 1} = 0$$

$$\tau^2 s^2 + 2\zeta s + (1 + KK_c) = 0$$

$$\tau'^2 s^2 + 2\zeta' s + 1 = 0$$

$$\tau' = \frac{\tau}{\sqrt{1 + KK_c}} \qquad \zeta' = \frac{\zeta}{\sqrt{1 + KK_c}}$$

Increasing proportional controller gain, K_c, can cause much oscillation by reducing the damping coefficient.

Effect of integral action on closed-loop response

1. Offset

Servo problem

- Generalize the I,PI and PID controllers

I: $$G_c = \frac{K_c}{\tau_I s}$$

PI: $$G_c = K_c\left(1 + \frac{1}{\tau_I s}\right) = K_c\left(\frac{\tau_I s + 1}{\tau_I s}\right)$$

PID: $$G_c = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right) = K_c\left(\frac{\tau_I \tau_D s^2 + \tau_I s + 1}{\tau_I s}\right)$$

$$G_c = \frac{N(s)}{\tau_I s}$$

Closed-loop Response

Consider a step change in Y_{sp} of magnitude A

Desired steady state response = *A*

Closed-loop response with $G_c = \dfrac{N(s)}{\tau_I s}$

$$Y(s) = \frac{K_m \dfrac{N(s)}{\tau_I s} G_v G_p}{1 + \dfrac{N(s)}{\tau_I s} G_v G_p G_m} \frac{A}{s}$$

Rearranging

$$Y(s) = \frac{K_m N(s) G_v G_p}{\tau_I s + N(s) G_v G_p G_m} \frac{A}{s}$$

Closed-loop Response

Attained steady state

$$y(t \to \infty) = \frac{K_m N(s) G_v G_p}{\tau_I s + N(s) G_v G_p G_m} \frac{A}{s} \cdot s \Bigg|_{s=0} = \frac{A K_m N(0) K_v K_p}{0 + N(0) K_v K_p K_m} = A$$

$$Offset = A - A = 0$$

Integral action eliminates offset for servo problem

Regulator problem

desired steady state response = 0 for regulator problem

Closed-loop response for regulator problem

$$Y(s) = \frac{G_d}{1 + G_c G_v G_p G_m} D_{sp}(s)$$

$$Y(s) = \frac{G_d}{1 + \dfrac{N(s)}{\tau_I s} G_v G_p G_m} \frac{A}{s}$$

$$Y(s) = \frac{\tau_I s G_d}{\tau_I s + N(s) G_v G_p G_m} \frac{A}{s}$$

Closed-loop Response

Attained steady state

$$y(t \to \infty) = \frac{(\tau_I s)G_d}{\tau_I s + N(s)G_v G_p G_m} \frac{A}{s} \cdot s \bigg|_{s=0} = \frac{0}{\tau_I 0 + N(s)G_v G_p G_m} = 0$$

$$Offset = 0 - 0 = 0$$

Integral action eliminates offset for regulator problem

Conclusion

Integral action eliminates offset for both for regulator and servo problem

Closed-loop Response

2. Effect of integral action on the order of the closed loop response

$$G_v(s)=\frac{K_v}{\tau_v s+1} \quad G_m(s)=\frac{K_m}{\tau_m s+1} \quad G_p(s)=\frac{N_p(s)}{D_p(s)} \quad G_c(s)=\frac{K_c}{\tau_I s}$$

Where $D_p(s)$ is the denominator of the process transfer function

The characteristic equation

$$1+G_c G_v G_p G_m=0$$

Introducing the transfer functions in the characteristic equation

$$1+\frac{K_c}{\tau_I s}\frac{K_v}{(\tau_v s+1)}\frac{N_p}{D_p}\frac{N_m}{(\tau_p s+1)}=0$$

Closed-loop Response

Rearranging

Order increases by one

$$\tau_I s(\tau_v s+1)(\tau_m s+1)D_p + K_c K_v K_m N_p = 0$$

The order increases by one due to integral controller. Therefore integral controller can make the closed-loop response sluggish.

Effect of PD controller

Effect of PD controller on closed-loop response

1. Offset

Servo problem

$$Y(s) = \frac{G_c G_v G_p K_m}{1 + G_c G_v G_p G_m} Y_{sp} \qquad (1)$$

TF of PD controller

$$G_c = K_c(1 + \tau_D s) \qquad (2)$$

Using (2) in (1) and introducing a step change of magnitude A in set point

$$Y(s) = \frac{K_c(1 + \tau_D s) G_v G_p K_m}{1 + K_c(1 + \tau_D s) G_v G_p G_m} \frac{A}{s} \qquad (3)$$

Effect of PD controller

Applying the final value theorem to find the steady state value

$$y(t \to \infty) = \frac{K_c(1+\tau_D s)G_v G_p K_m}{1+K_c(1+\tau_D s)G_v G_p G_m}\frac{A}{s}\cdot s\Bigg|_{s=0} = \frac{AK_c K_v K_p K_m}{1+K_c K_v K_p K_m}$$

$$Attained\ steady\ state\ response = \frac{AK_m K_c K_v K_p}{1+K_m K_c K_v K_p}$$

$$Offset = A - \frac{AK_m K_c K_v K_p}{1+K_m K_c K_v K_p}$$

Effect of PD controller

Offset for Servo Problem with Derivative Action

$$Offset = \frac{A}{1 + K_m K_c K_v K_p}$$

$$Offset = \frac{100}{1 + K_m K_c K_v K_p} \%$$

The same as P controller

A similar analysis for regulator problem leads to

Offset for Regulator Problem with Derivative Action

$$offset = -\frac{AK_d}{1 + K_m K_c K_v K_p}$$

$$offset = -\frac{100K_d}{1 + K_m K_c K_v K_p} \%$$

The same as P controller

Effect of PD controller

Effect of derivative action on the order of closed-loop response

TF of derivative action

$$G_c = K_c \tau_D s$$

Using the derivative action in the Characteristic Equation

$$1 + K_c \tau_D s G_v G_p G_m = 0$$

The order of closed-loop response is not affected by derivative action

The effect of derivative action: damping

Consider the characteristics equation when $G_m G_v G_p$ is second order with derivative action

$$1 + K_c \tau_D s \frac{K}{\tau^2 s^2 + 2\zeta s + 1} = 0$$

Rearranging

$$\left(\tau^2 s^2 + 2\zeta s + 1\right) + K K_c \tau_D s = 0$$

$$\tau^2 s^2 + (2\zeta + K K_c \tau_D) s + 1 = 0$$

The damping coefficient increases with K_c. Therefore, derivative action enables to increase the controller gain K_c without increasing the oscillations.

Stability Analysis of Feedback Control Systems

Introduction

- An important consequence of feedback control is that it can cause oscillatory responses.

- Under certain circumstances, the oscillations may be undamped or even have amplitude that increases with time until a physical limit is reached.

- In these situations, the closed-loop system is said to be *unstable.*

Control system:

Consider the feedback control system with the following transfer functions:

$$G_c = K_c \qquad G_v = \frac{1}{2s+1} \qquad G_p = G_d = \frac{1}{5s+1} \qquad G_m = \frac{1}{s+1} \qquad (9.1)$$

The transfer function for set-point changes is:

$$\frac{Y}{Y_{sp}} = \frac{K_m G_c G_v G_p}{1+G_c G_v G_p G_m} \qquad (9.2)$$

Consider a step change in set-point . $Y_{sp}(s) = 1/s$

Substituting (9.1) in (9.2), and rearranging gives us

$$Y(s) = \frac{K_c(s+1)}{10s^3+17s^2+8s+1+K_c}\frac{1}{s}$$

After K_c is specified, $y(t)$ can be obtained.

Fig. below demonstrates that as K_c increases, the response become more oscillatory and is unstable for $K_c = 15$.

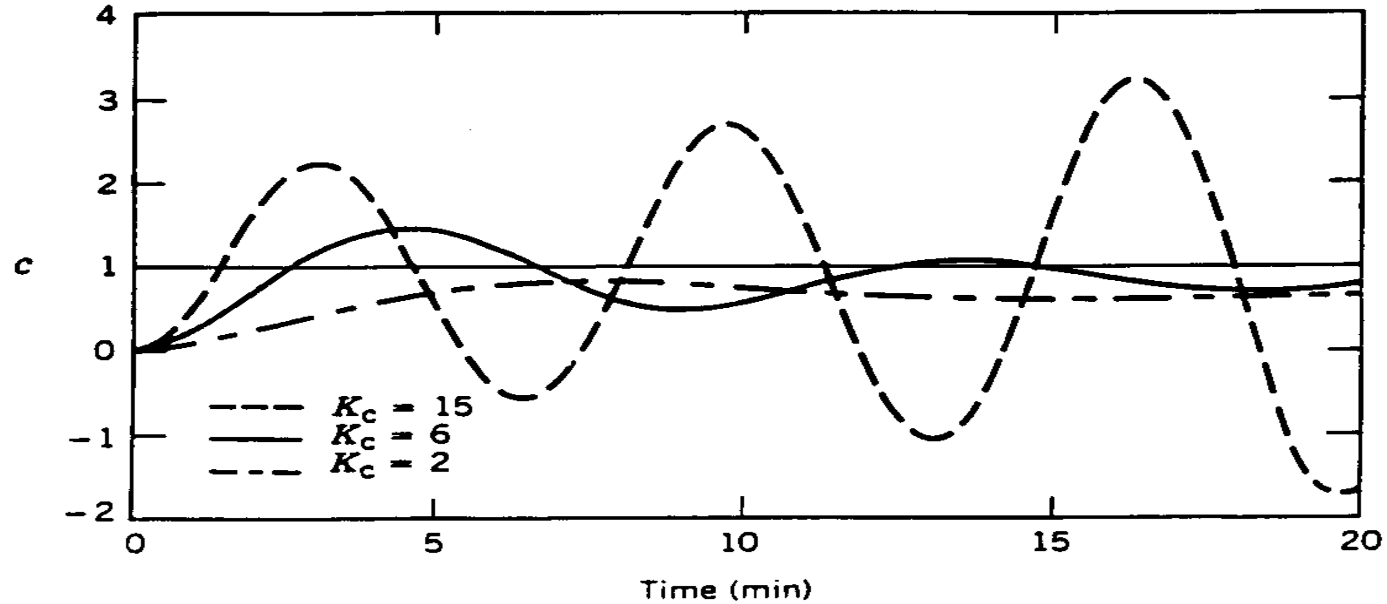


Figure 11.2. Effect of controller gains on closed-loop response to a unit step change in set point (example 11.1).

General Stability Criterion:

- Most industrial processes are stable without feedback controllers. They are said to be *open-loop stable* or *self-regulating.*

- *Definition of stability:* An unconstrained linear system is said to be stable if the output response is bounded for all bounded inputs. Otherwise, it is said to be unstable.

- By a *bounded input*, we mean an input variable that stays within upper and lower limits for all values of time.

- The term unconstrained refer to the ideal situation where there is no physical limits on the input and output variables.

Characteristic Equation

Consider the closed-loop equation. It is already developed for feedback control system :

$$Y = \frac{K_m G_c G_v G_p}{1+G_{OL}} Y_{sp} + \frac{G_d}{1+G_{OL}} D \qquad (9.1)$$

Where, $G_{OL} = G_c G_v G_p G_m$

The stability of the closed-loop system is determined by the poles of the closed-loop transfer function. The poles of the transfer function are the roots of the **Characteristic Equation:**

$$1 + G_{OL} = 0 \qquad (9.2)$$

Characteristic equation

Characteristic equation

If G_{OL} is a ratio of polynomials in *s*, then the closed-loop transfer function also a rational function. Then, it can be factored into *poles* (p_i) and zeroes (z_i) as

$$\frac{Y}{Y_{sp}} = K' \frac{(s - z_1)(s - z_2)....(s - z_m)}{(s - p_1)(s - p_2)....(s - p_n)} \qquad (9.3)$$

where *K*' is a multiplicative constant selected to give the correct steady-state gain. To have a physically realizable system, the number of poles must be greater than or equal to the number of zeroes.

The poles are also the roots of the *characteristic equation* of the closed-loop system:

For a unit step change in set-point, (9.3) becomes

$$Y = \frac{K'}{s} \frac{(s-z_1)(s-z_2)....(s-z_m)}{(s-p_1)(s-p_2)....(s-p_n)} \quad (9.4)$$

If there are no repeat roots (*all distinct poles*), then the partial fraction expansion of (9.8) has the form

$$Y = \frac{A_0}{s} + \frac{A_1}{(s-p_1)} + \frac{A_2}{(s-p_2)} + ... + \frac{A_n}{(s-p_n)} \quad (9.5)$$

Taking the inverse Laplace transform of (9.5) gives

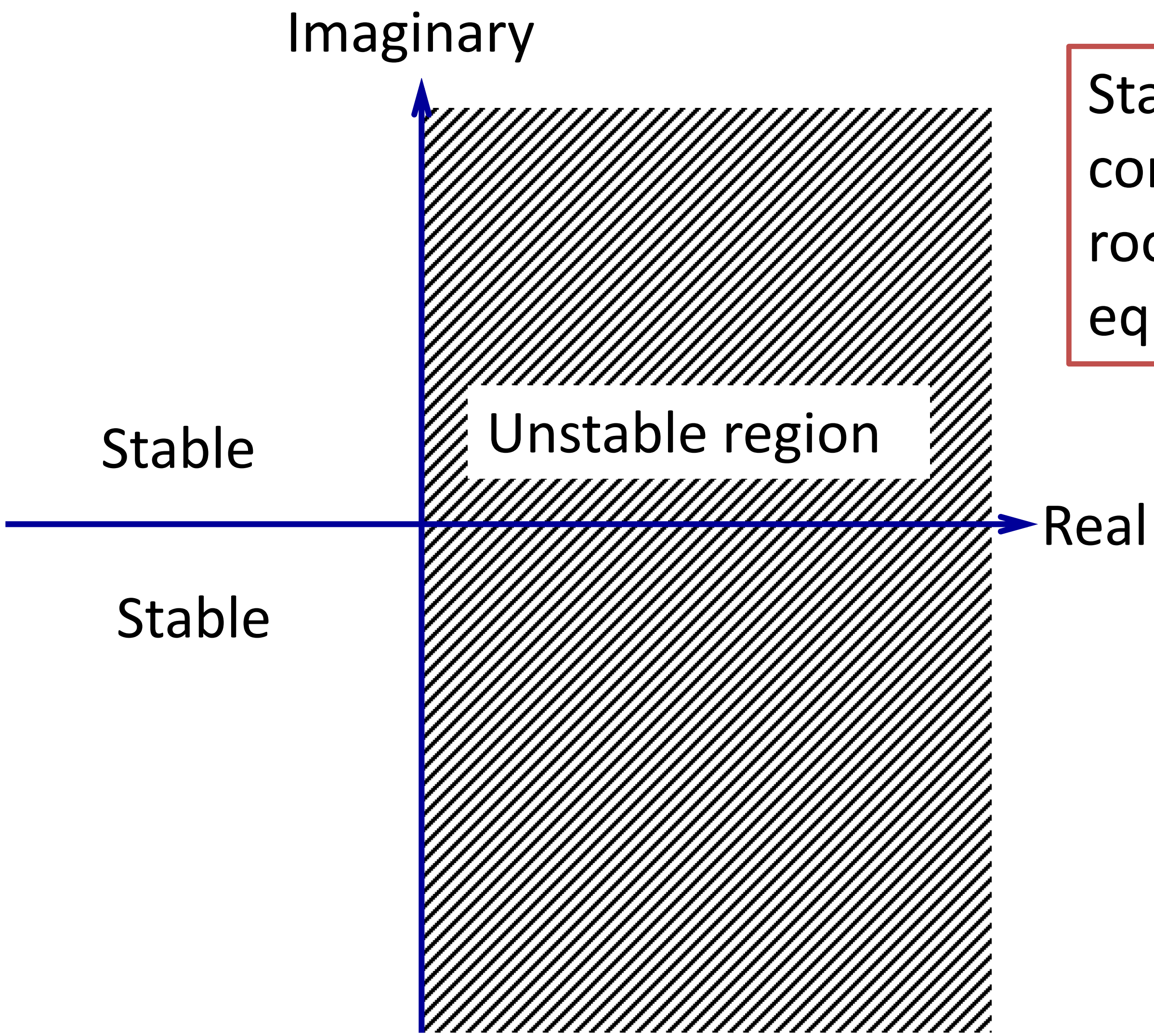
$$y(t) = A_o + A_1 e^{p_1 t} + A_2 e^{p_2 t} + ... + A_n e^{p_n t} \quad (9.6)$$

Suppose that one of the poles is a positive real number; i.e., $p_k > 0$.

Then it is clear from (9.6) that $y(t)$ is unbounded and thus the closed-loop system is unstable.

If p_k is a complex number, with a positive real part, then the system is also unstable.

If all the poles are negative (or have negative real parts) then the system is stable.



Stability region in the complex plane for the roots of the characteristic equation.

General stability criterion:

A feedback control system is stable if and only if all roots of the characteristic equation are negative or have negative real parts. Otherwise, the system is unstable.

Graphical interpretation of stability criterion:

Roots of
$1 + G_c G_v G_p G_m = 0$

(Each test is for different value of K_c)

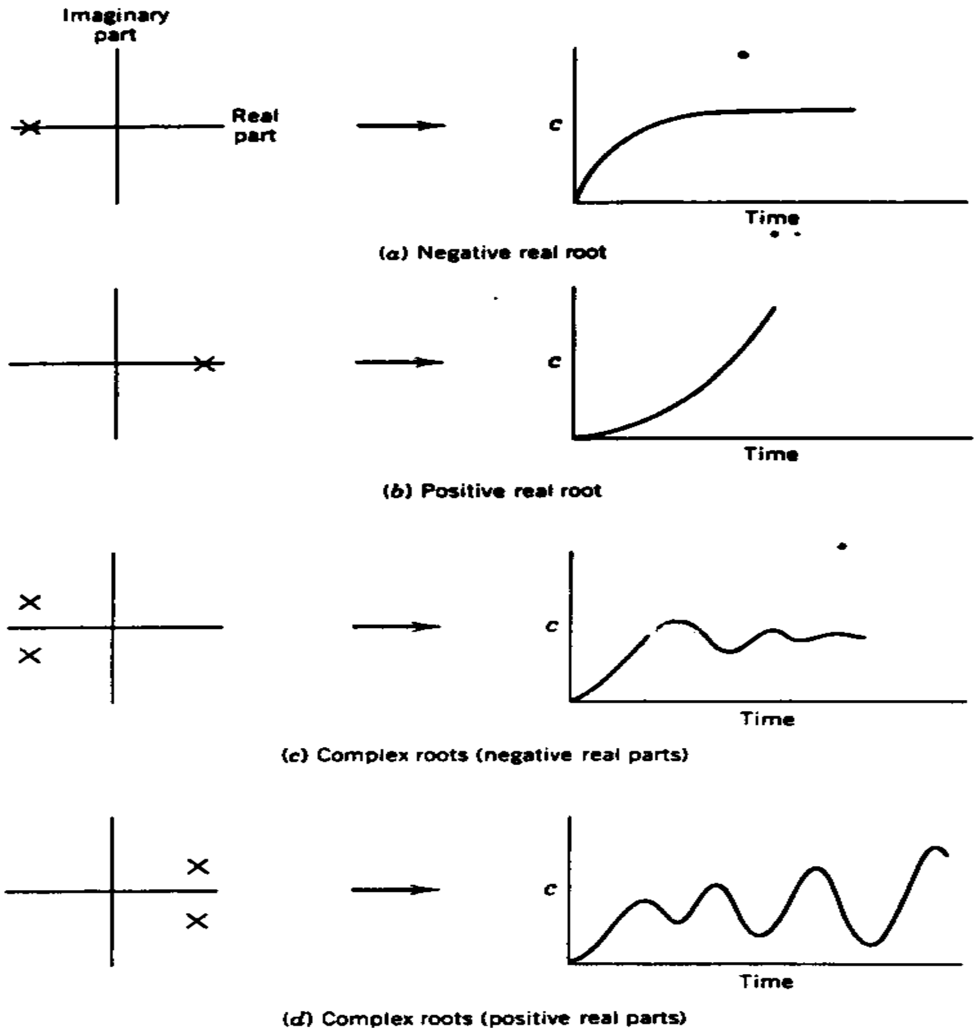


Figure 11.5. Contributions of characteristic equation roots to closed-loop response.

- If the characteristic equation is either first-order or second-order, we can find the roots analytically.
- For higher-order polynomials, we have to use 0ther techniques.

Routh Stability Criterion

Uses an analytical technique for determining whether any roots of a polynomial have positive real parts.

Characteristic equation

$$a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 = 0 \qquad (9.7)$$

where $a_n > 0$. According to the Routh criterion, if any of the coefficients a_0, a_1, a_K, a_{n-1} are negative or zero, then at least one root of the characteristic equation lies in the RHP, and thus, the system is unstable. On the other hand, if all of the coefficients are positive, then one must construct the Routh Array.

Routh Array

ROW				
1	a_n	a_{n-2}	a_{n-4}	$\cdots$
2	a_{n-1}	a_{n-3}	a_{n-5}	$\cdots$
3	b_1	b_2	b_3	$\cdots$
4	c_1	c_2	$\cdots$	
$\vdots$	$\vdots$			
$n+1$	d_1			

For stability, all elements in the first column **must** be positive.

Routh Array

The first two rows of the Routh Array are comprised of the coefficients in the characteristics equation. The elements in the remaining rows are calculated from coefficients from the using the formulas:

$$b_1 = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}} \qquad b_2 = \frac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}}$$

$$c_1 = \frac{b_1 a_{n-3} - a_{n-1} b_2}{b_1} \qquad c_2 = \frac{b_1 a_{n-5} - a_{n-1} b_3}{b_1}$$

(n+1 rows must be constructed
n = order of the characteristic eqn.)

Routh Stability Criterion:

A necessary and sufficient condition for all roots of the characteristic equation to have negative real parts is that all of the elements in the left column of the Routh array are positive.

Example 9.1

Determine the stability of a system that has the characteristic equation

$$s^4 + 5s^3 + 3s^2 + 1 = 0$$

Solution: Because the *s* term is missing, its coefficient is zero. Thus the system is unstable.

Example 9.2

Determine the stability of a system that has the characteristic equation

$$6s^4 + 12s^3 + 8s^2 + 9s + 7 = 0$$

Routh Array

Row			
1	6	8	7
2	12	9	0
3	c_1=3.5	c_2 =7	0
4	d_1=-15	0	0
5	e_1= 7	0	0

$$c_1 = \frac{(12\times 8) - (6\times 9)}{12} = 3.5$$

$$c_2 = \frac{(12\times 7) - (6\times 0)}{12} = 7$$

$$d_1 = \frac{(3.5\times 9) - (12\times 7)}{3.5} = -15$$

$$e_1 = \frac{(-15\times 7) - (3.5\times 0)}{-15} = 7$$

Example

Example 9.3

The transfer functions of a process, the control valve and the measurement are given below. Determine the values of the controller gain for which a simple feedback control system with proportional controller will be stable.

$$G_p = \frac{0.2}{5s+1} \qquad G_v = \frac{5}{s+1} \qquad G_m = \frac{1}{2s+1}$$

Solution

Characteristic equation

$$1 + G_c G_v G_p G_m = 0$$

Example

Inserting, the transfer functions

$$1 + K_c \frac{5}{(s+1)} \frac{0.2}{(5s+1)} \frac{1}{(2s+1)} = 0$$

Rearranging we get

$$10s^3 + 17s^2 + 8s + 1 + K_C = 0$$

All coefficients are positive provided that 1+K_c > 0 or K_c > -1. Therefore, we have to construct the Routh Array to determine the satbility.

Example

The Routh array is:

1	10	8
2	17	$1+K_c$
3	c_1	0
4	d_1	0

$$c_1 = \frac{17(8) - (10)(1+K_C)}{17}$$

$$d_1 = 1 + K_C$$

To have a stable system, each element in the left column must be positive, $c_1 > 0$ *and* $d_1 > 0$

Example

$$c_1 = \frac{17(8) - (10)(1 + K_C)}{17} > 0$$

$$K_C < \frac{17(8)}{10} - 1 = 12.6$$

From d_1

$$K_C > -1$$

Therefore, for the closed-loop system to be stable $-1 < K_c < 12.6$

TUNING OF PID CONTROLLER

Objectives

End of this unit, you should be able to :

1. Explain tuning criteria
2. Tune P,PI, and PID controllers

Introduction

- The stability and performance of a feedback control system highly depends on the controller settings, i.e., the values of K_c, τ_I, τ_D .
- PID controller settings can be determined by a number of alternatives techniques:
  - Direct synthesis (DS) method
  - Internal model control method
  - Controller tuning relations
  - Frequency response techniques
  - Computer simulation
  - **Online tuning (Ziegler –Nichols, Tyreus-Luyben)**

Controller tuning criteria

1. Integral Error Criteria

Integral of the absolute value of the error

$$IAE = \int_0^\infty |e(t)|dt$$

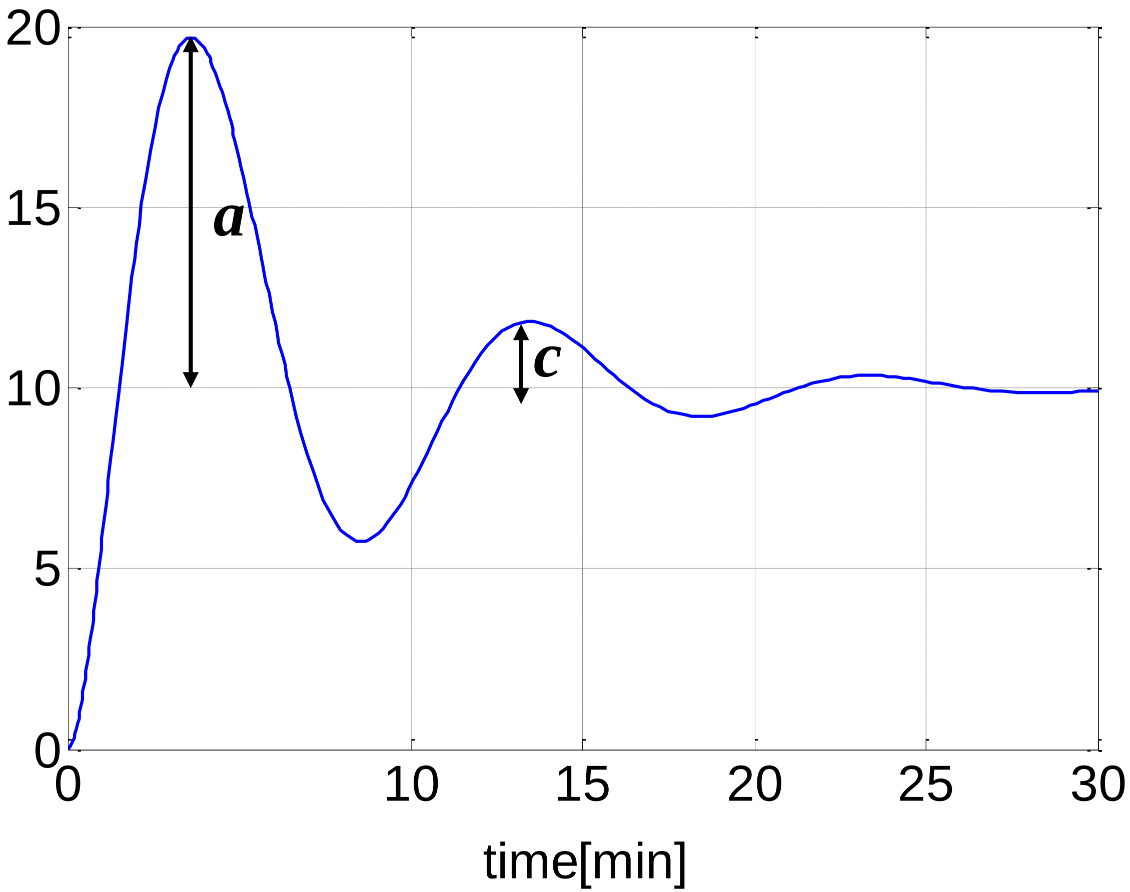
Integral of the squared error

$$ISE = \int_0^\infty e(t)^2 dt$$

Integral of the time-weighted absolute error

$$ITAE = \int_0^\infty t|e(t)|dt$$

2. Quarter decay ratio



Quarter decay ratio
$= c/a \leq 0.25$

Ziegler-Nichols Method

Online Procedure (Ziegler-Nichols)

Step 1. After the process has reached steady state, eliminate the integral and derivative action by setting τ_D to zero and τ_I to the highest possible value.

Step 2. Set K_c equal to a small value and place the controller in the automatic mode.

Step 3. Introduce a small, momentary set point change so that the controlled variable moves away from the set-point. Gradually increase, K_c, until a continuous cycling occurs.

Step 4. Calculate the PID controller settings using the Ziegler- Nichols or Tyreus-Luben settings

Step 5. Evaluate the Z-N or T-L settings by introducing a small set- point change and observing the closed-loop response. Fine tune the settings if necessary.

Ziegler- Nichols and Tyreus - Luyben method

Table 10.1 : Ziegler-Nichols and Tyreus-Luyben settings

Ziegler-Nichols method

	K_c	τ_I	τ_D
P	$\frac{K_{cu}}{2}$		
PI	$\frac{K_{cu}}{2.2}$	$\frac{T_u}{1.2}$	
PID	$\frac{K_{cu}}{1.7}$	$\frac{T_u}{2}$	$\frac{T_u}{8}$

Tyreus-Luyben method

PI	$0.31K_{cu}$	$2.2T_u$	
PID	$0.45K_{cu}$	$2.2T_u$	$T_u/6.3$

- Offline procedure

Step 1. With proportional controller determine the characteristic equation.

$$1 + K_c G_v G_p G_m = 0$$

Step 2. Replace $s = \omega j$ in the characteristic equation to get a complex equation with unknowns ω and K_c

Step 3. Find the value of ω and K_c by equating the imaginary part to zero and the real part to zero.

- Step 4. Determine T_u and K_{cu} as follows

$$K_{cu} = K_c$$

$$T_{cu} = \frac{2\pi}{\omega}$$

- Step 5. Determine the Ziegler-Nichols or Tyreus-Luyben settings using Table 10.1

Example

Example 10.1

The transfer functions of a process, the control valve and the measurement are given below. Determine settings of a PID controller using the Ziegler-Nichols method..

$$G_p = \frac{0.2}{5s+1} \qquad G_v = \frac{5}{s+1} \qquad G_m = \frac{1}{2s+1}$$

Solution

Characteristic equation

$$1 + G_c G_v G_p G_m = 0$$

Inserting, the transfer functions

$$1 + K_c \frac{5}{(s+1)} \frac{0.2}{(5s+1)} \frac{1}{(2s+1)} = 0$$

Rearranging we get

$$10s^3 + 17s^2 + 8s + 1 + K_C = 0$$

Replacing $s = \omega j$

$$-10\omega^3 j - 17\omega^2 + 8\omega j + 1 + K_C = 0$$

Rearranging

$$(-10\omega^3 + 8\omega) j + (-17\omega^2 + 1 + K_C) = 0$$

For a complex number to be zero, both the imaginary and real part should be zero

Equating the imaginary part to zero

$$-10\omega^3 + 8\omega = 0$$

$$\omega = \sqrt{8/10} = 0.8944$$

Equating the real part to zero

$$-17\omega^2 + 1 + K_{cu} = 0$$

$$K_{cu} = 17\omega^2 - 1 = 17(0.8944)^2 - 1$$

$$K_{cu} = 12.6$$

$$T_u = \frac{2\pi}{0.8944} = 7.0248$$

Using, PID-controller setting from Ziegler-Nichols setting (Table 10.1)

$$K_c = \frac{12.6}{1.7}$$

$$\tau_I = \frac{7.0248}{2} = 3.5124$$

$$\tau_D = \frac{7.0248}{8} = 0.8781$$

Example

Example 10.2

The dynamic model of a process is given by Equation (1), where M(s) is the manipulated variable and D(s) is the disturbance variable.

$$Y = \frac{6.5}{s(4s+1)} M(s) + \frac{0.21}{4s+1} D(s) \qquad (1)$$

The transfer functions for the transmitter, Gm, and the valve are below:

$$G_m = \frac{1}{0.5s+1} \qquad G_v = \frac{0.82}{0.3s+1}$$

Determine

(1) The stability of the open-loop system

(2) The range for which a closed-loop system with proportional controller will be stable

(3) Determine the Zeigler-Nichols setting for

  (a) proportional controller

  (b) PID controller

(4) Determine the offset for **part (3) (a)** for a unit step change in set-point and load.