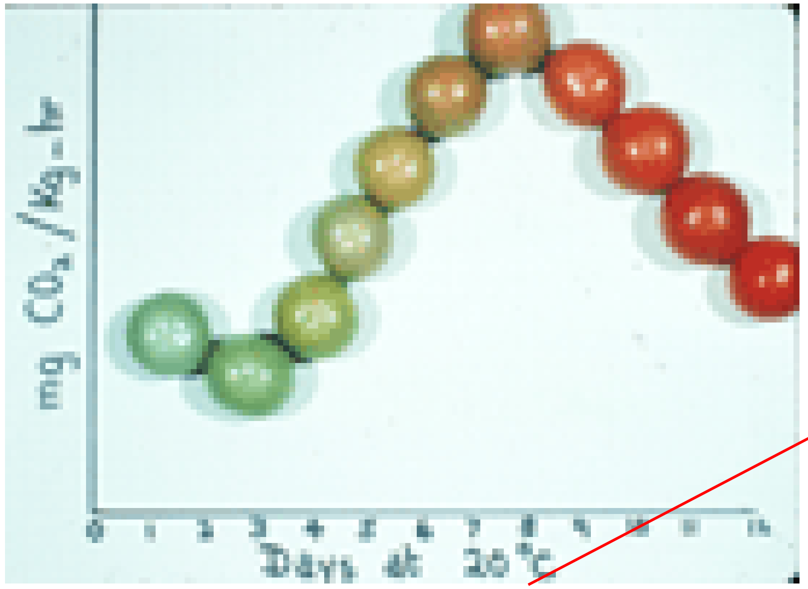
ADDIS ABABA UNIVERSITY
ADDIS ABABA INSTITUTE OF TECHNOLOGY SCHOOL OF CHEMICAL AND BIOENGINEERING

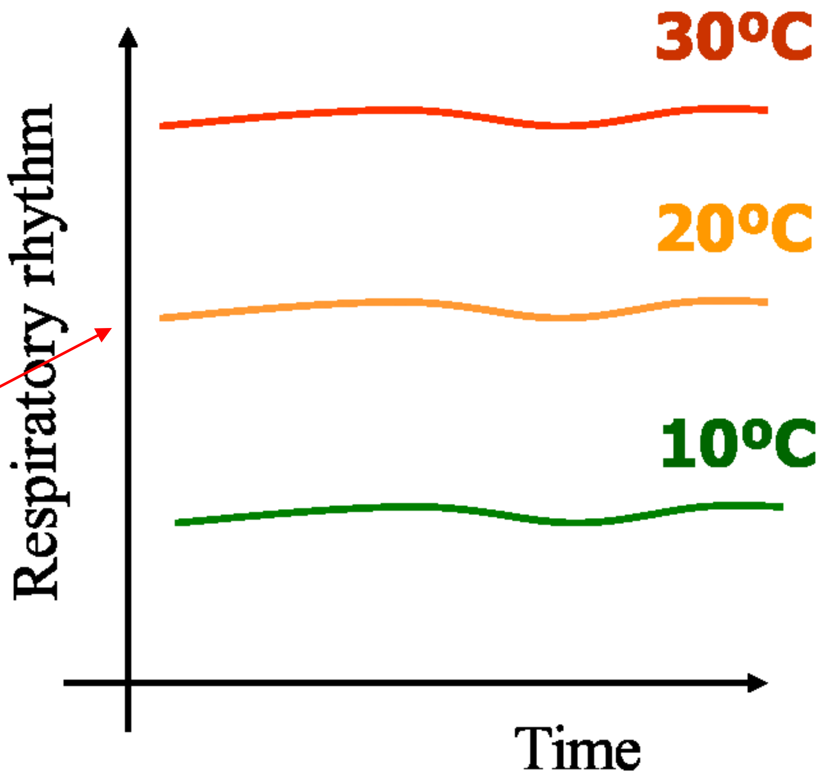
Course: Design and Development of Food products and Equipment

PROCESS DESIGN AND OPTIMIZATION

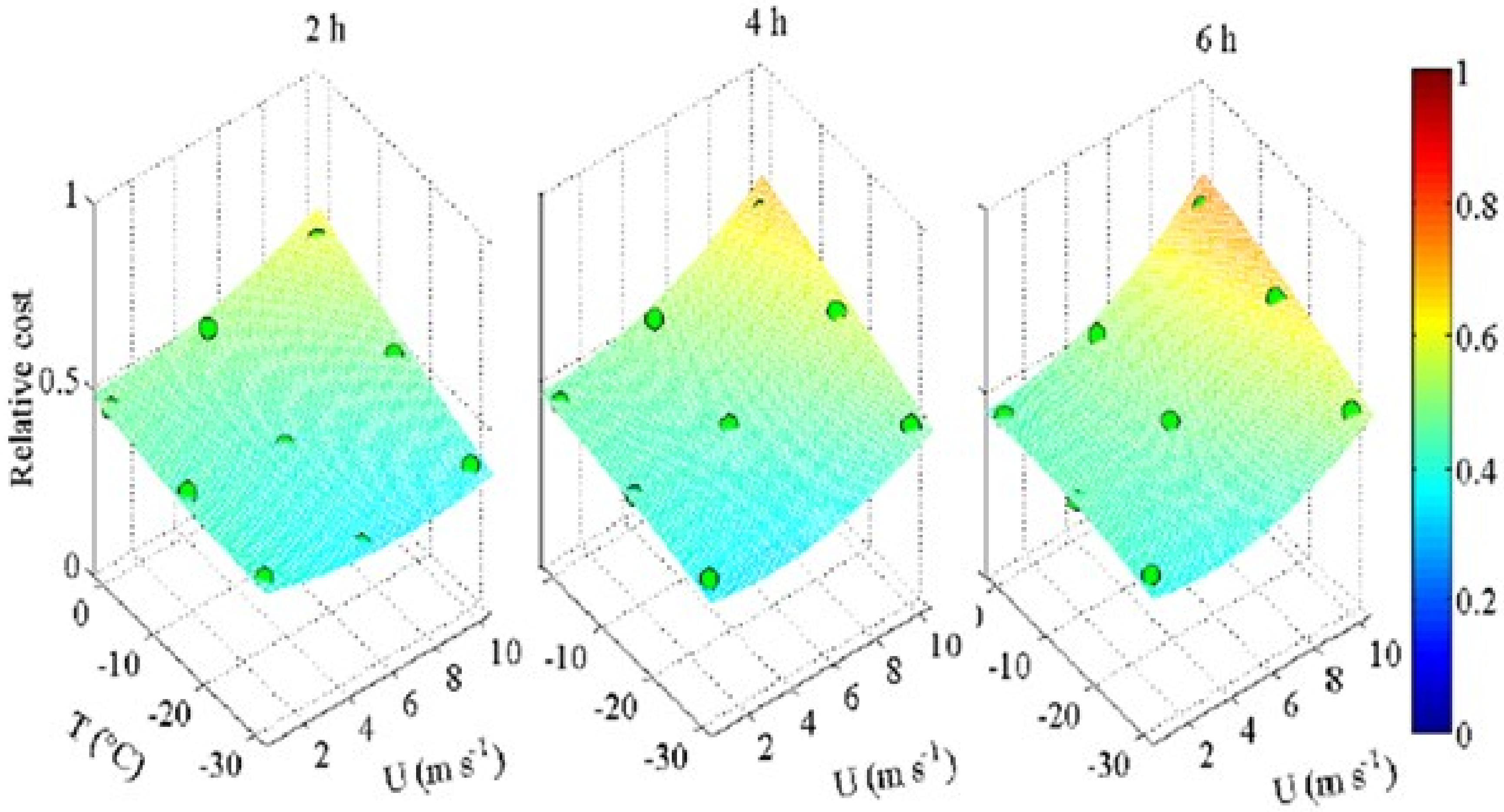
Background

- Natural process are filled ms



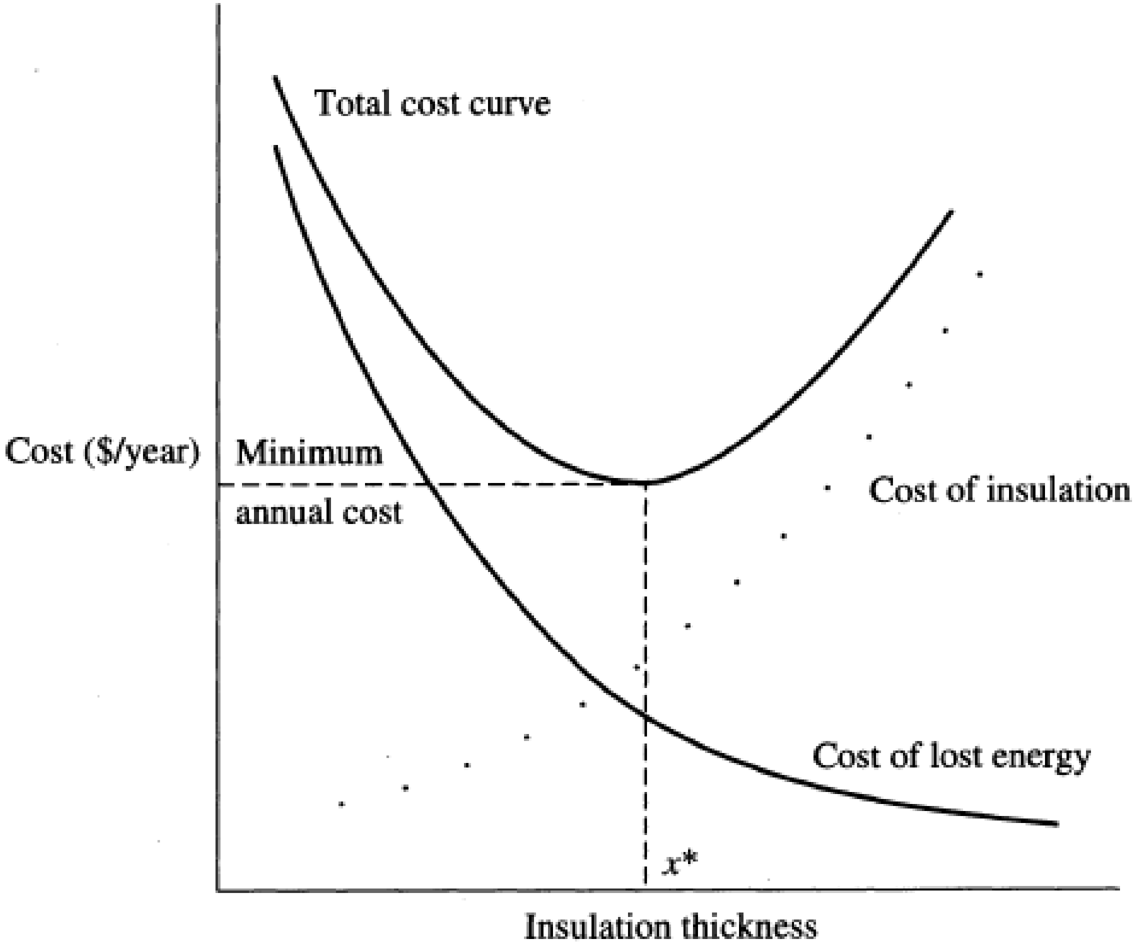


We can manipulate the ripening rate of fruits



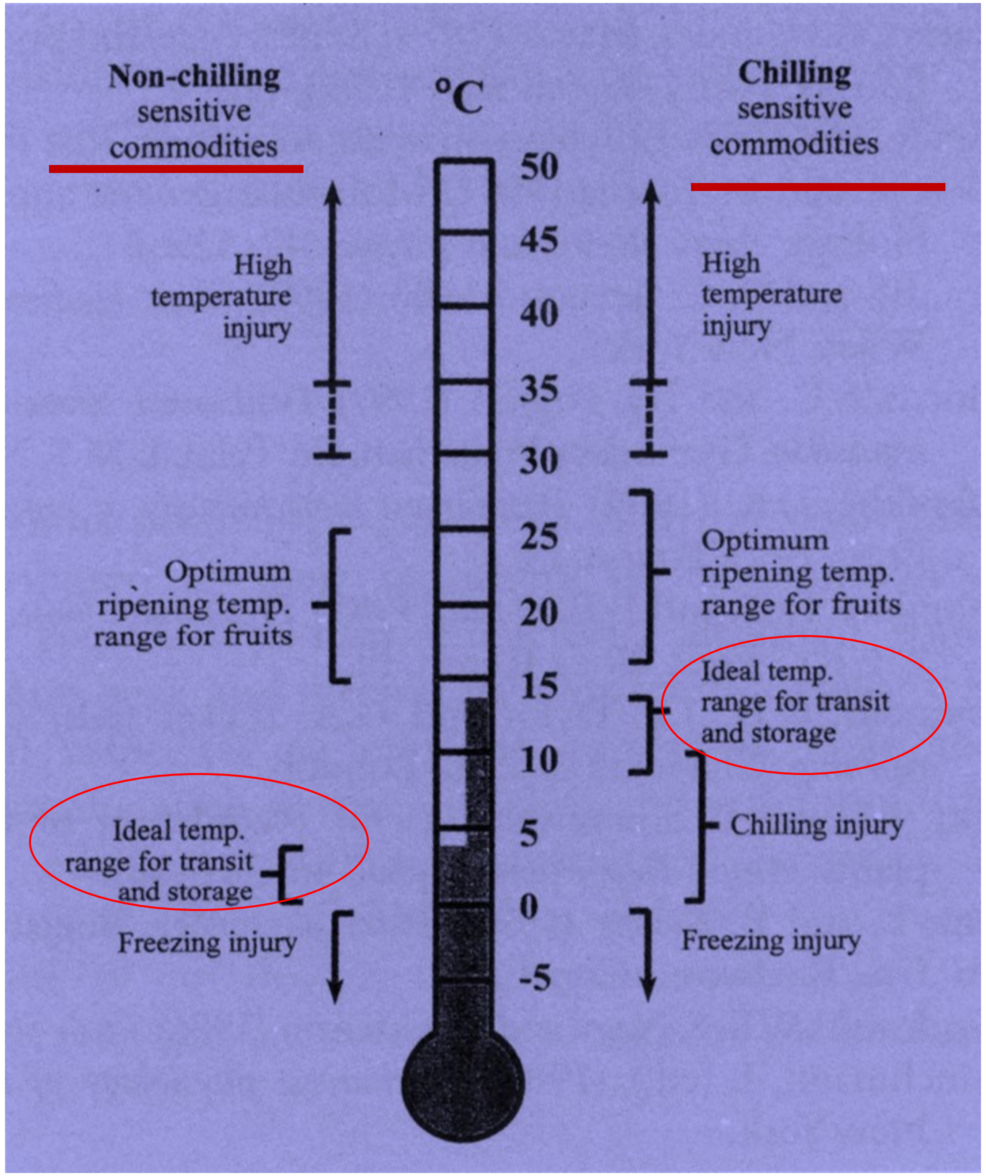
✓ Chilling process depends on cooling air speed, temperature and chilling duration

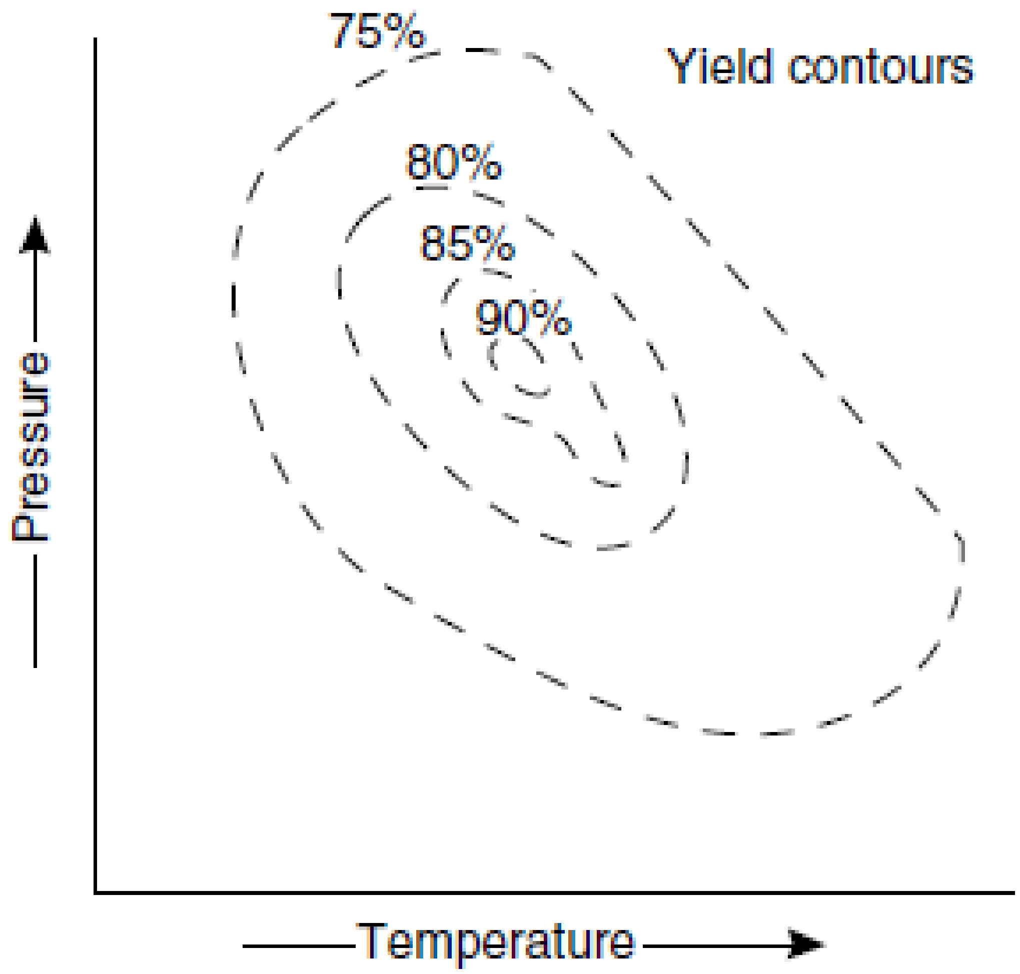
- Business owners make cost decisions to maximize profits



Source (Edgar et.al., 2001)

- Optimum storage temperature varies depending on commodities



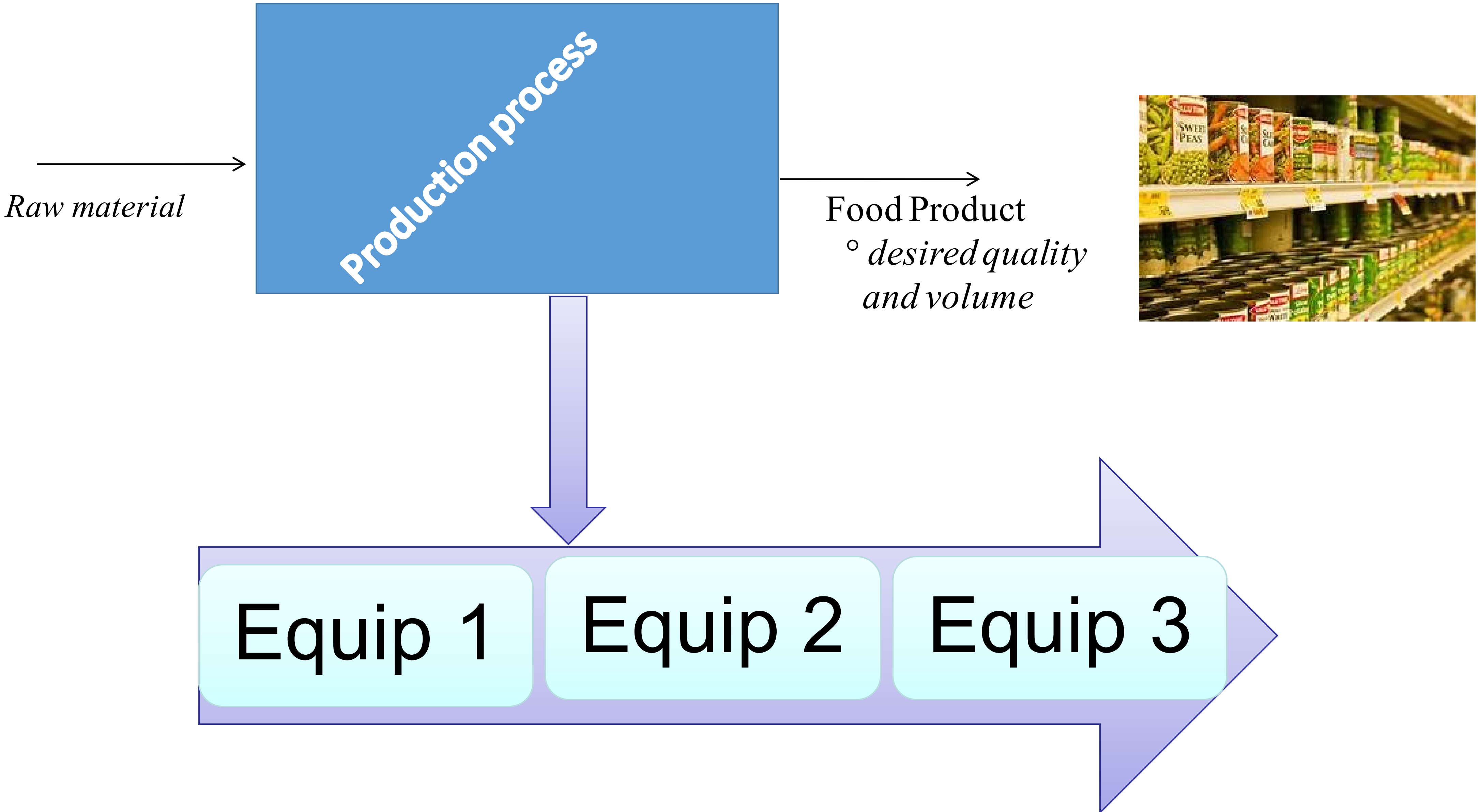


Yield as a function of reactor temperature and pressure (Coulson & Richardson, Vol 6)

Optimization

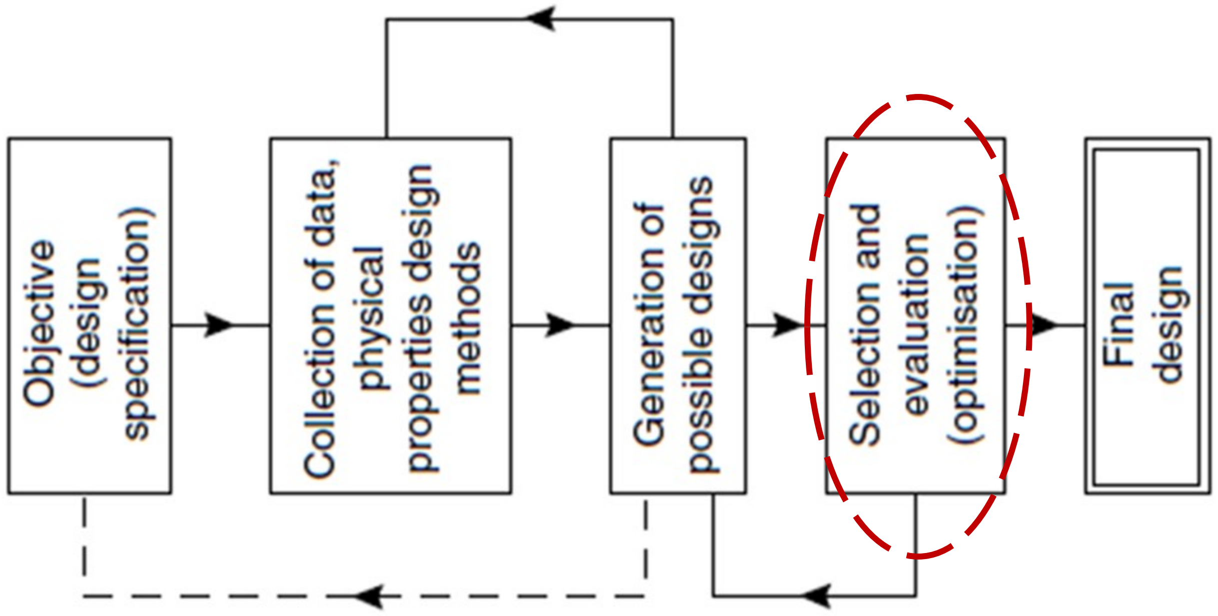
- Optimization: mathematically, the process of finding conditions that give the maximum or minimum values of a function
- It is the use of specific methods to determine the most cost-effective and efficient solution to a problem or design for a process
- It is one of the major quantitative tools in industrial decision making

At which production step do we need optimization?



Optimization (cont.)

- Typical problems in process design or plant operation have many solutions. Optimization is concerned with selecting the best among the entire set by efficient quantitative methods



Cont.

- In almost every case, these optimum conditions can ultimately be reduced to a consideration of costs or profits
- Thus, an optimum economic design could be based on conditions that give the least cost per unit of time or the maximum profit per unit of production
- A wide variety of optimization problems have amazingly similar structures

Cont.

- We can make use of this structural similarity to develop a framework or methodology within which any problem can be studied

➢ Any process problem, complex or simple, for which one desires the optimal solution should be organized. To do so, you must:

  ✓ Consider the model representing the process and

  ✓ Choose a suitable objective criterion to guide the decision making

The Essential Features of Optimization Problems

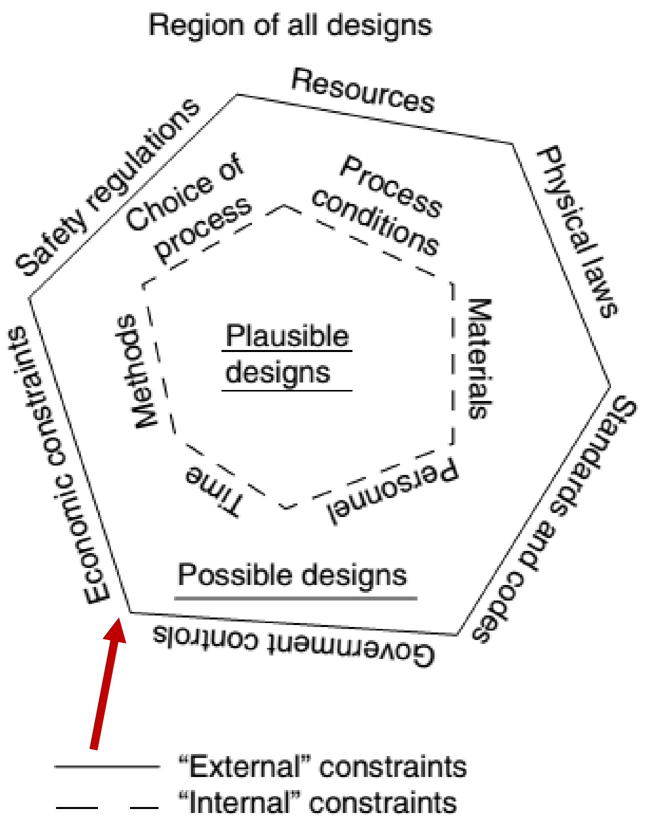
- Every optimization problem contains three essential categories
  - ✓ At least one objective function to be optimized (profit function, cost function, etc.)
  - ✓ Equality constraints (equations)
  - ✓ Inequality constraints (inequalities)

$$y = ax_1 + bx_2$$

$$x_1 + x_2 \leq c$$

Cont.

- Constraints in optimization arise because a process must describe the physical bounds on the variables, empirical relations, and physical laws that apply to a specific problem



Steps used to solve optimization problems

1) Analyze the process itself so that the process variables and specific characteristics of interest are defined; that is, make a list of all of the variables
2) Determine the criterion for optimization, and specify the objective function in terms of the variables defined in step 1 together with coefficients. This step provides the performance model

Cont.

3) Using mathematical expressions, develop a valid process or equipment model that relates the input-output variables of the process and associated coefficients. Include both equality and inequality constraints. Use well-known physical principles (mass balances, energy balances), empirical relations, implicit concepts, and external restrictions

Cont.

4) If the problem formulation is too large in scope:
    - break it up into manageable parts or
    - simplify the objective function and model
5) Apply a suitable optimization technique to the mathematical statement of the problem
6) Check the answers, and examine the sensitivity of the result to changes in the coefficients in the problem and the assumption

Mathematical Models

- The heart of the problem solving exercise
- Models provide insight into long term (steady state) and short term (dynamic) behavior of process systems
- They can obtained from Mechanistic/ Phenomenological/ First Principle models
  - ✓ Conservation of mass
  - ✓ Conservation of momentum – Newton's second law of motion
  - ✓ Conservation of energy: First law of thermodynamics
    - Equation of state: Fluid properties e.g., density as a function of pressure and temperature
    - Relationship between the stresses and the deformation of the material

- Model classification

  - ✓ Distributed parameter model: These models capture the relationship between the variables involved as a function of time and space
  - ✓ Lumped parameter models: These models lump all spatial variation and all the variables involved are treated as functions time alone

✓The above two classes of models together with the various scenarios under consideration give rise to different types of equation forms:

- Linear algebraic equations
- Nonlinear algebraic equations
- Ordinary differential equations: initial value problem/Bondary value problem
- Partial differential equation

Solution techniques

- Calculus method

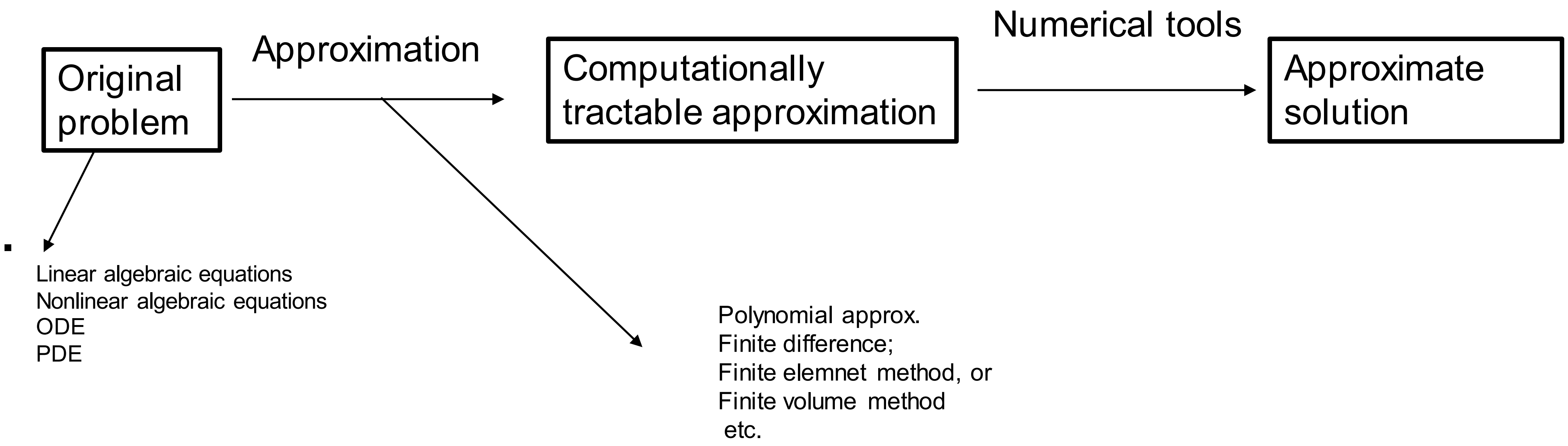
  ✓ If the objective function can be expressed as a mathematical function, the classical methods of calculus can be used to find the maximum or minimum (setting the partial derivative to zero)

- Search methods (Numerical method)

  ✓ The objective function is computed from arbitrary values of the independent variables

- Due to the complexity of most of the real problem, analytical solution is difficult/ impractical to obtain

➤ Steps for the numerical solution



Exaple 1

- A sugar processing industry receives 20000 kg/h of cane containing 10% sucrose and crystalizes it into two types of output (1) and (2). Type (1) has a purity of 99 % while type (2) has a purity of 96%. The selling price of type (1) 20 birr/kg while that of type (2) is 15 birr/ kg. Packaging facilities allow a maximum 2000kg/hr of type (1) and 2500 kg/h of type (2). The transportation cost for type (1) is 0.5 birr/ kg while that of type (2) 1 birr/kg. The total transportation cost should not exceed 3000 birr/h.

Cont.

- Set up the optimization problem that maximize profit; state all the constraints and indicate the feasible region using graphical method

Solution

Assign

type (1) = x_1

type (2) = x_2

- $\text{Profit} = (20x_1 + 15x_2) - 0.5x_1 - x_2$

Objective function

- Constraints

  a) mass balance

  $$0.99x_1 + 0.96x_2 \leq 0.1*20000$$

  b) Transportation cost

  $$0.5x_1 + x_2 \leq 3000$$

Inequality constraint

Cont.

c) Bounds for variables

$x_1 \leq 2000$

$x_2 \leq 2500$

d) Non- negativity constraints

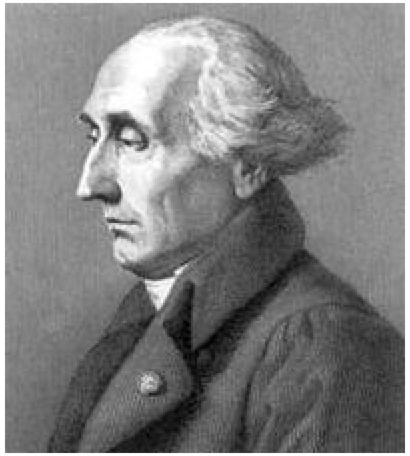
$x_1 \geq 0$

$x_2 \geq 0$

Use coordinates x_1 & x_2, to find the feasible region (the region that satisfy all the above conditions)

Lagrange multiplier method

- The method of Lagrange’s undetermined multipliers is a useful analytical technique for dealing with problems that have equality constraints



Joseph-Louis Lagrange
1736-1813

The Lagrange method writes the constrained optimization problem in the following form

Objective

Constraint(s)

$$\max_{x,y} f(x,y)$$

Choice variables

$$\text{subject to} \quad g(x,y) \geq 0$$

The problem is then rewritten as follows

$$\ell = f(x,y) + \lambda g(x,y)$$

Multiplier (assumed greater or equal to zero)

Cont.

- We need the derivatives with respect to both x and y to be zero

$$\ell_x = f_x(x,y) + \lambda g_x(x,y) = 0$$
$$\ell_y = f_y(x,y) + \lambda g_y(x,y) = 0$$

Eample 2

- Suppose you sell two products (X and Y). Your profits as a function of sales of X and Y are as follows:

$$\text{Profit} = 10x + 20y - .1(x^2 + y^2) \quad \Bigg\} \quad \begin{array}{l}\text{Objective} \\ f(x,y)\end{array}$$

- Your maximum production capacity is equal to 100 units

Cont.

- Choose X and Y to maximize profits subject to your capacity constraints

$$x + y \leq 100 \quad \Big\} \quad \text{Constraint } g(x, y)$$

$$g(x,y) \geq 0 \quad ; \quad x + y \leq 100 \Longrightarrow 100 - x - y \geq 0$$

Cont.

- Now, we can write the lagrangian

$$\ell = 10x + 20y - .1(x^2 + y^2) + \lambda(100 - x - y)$$

Take derivatives with respect to ‘x’ and ‘y’

$$\ell_x = 10 - .2x - \lambda = 0$$
$$\ell_y = 20 - .2y - \lambda = 0$$

Cont.

- And the multiplier conditions

$$\lambda \geq 0 \qquad 100 - x - y \geq 0 \qquad \lambda(100 - x - y) = 0$$

- First, let's suppose that lambda equals zero

$$\left.\begin{aligned} 10 - .2x = 0 \\ 20 - .2y = 0 \end{aligned}\right\} \Rightarrow \begin{aligned} x^* &= 50 \\ y^* &= 100 \end{aligned}$$

$$100 - x - y \geq 0 \Rightarrow \boxed{100 - 50 - 100 = -50 \geq 0}$$

wrong

Cont.

- Thus, lamda is greater than zero

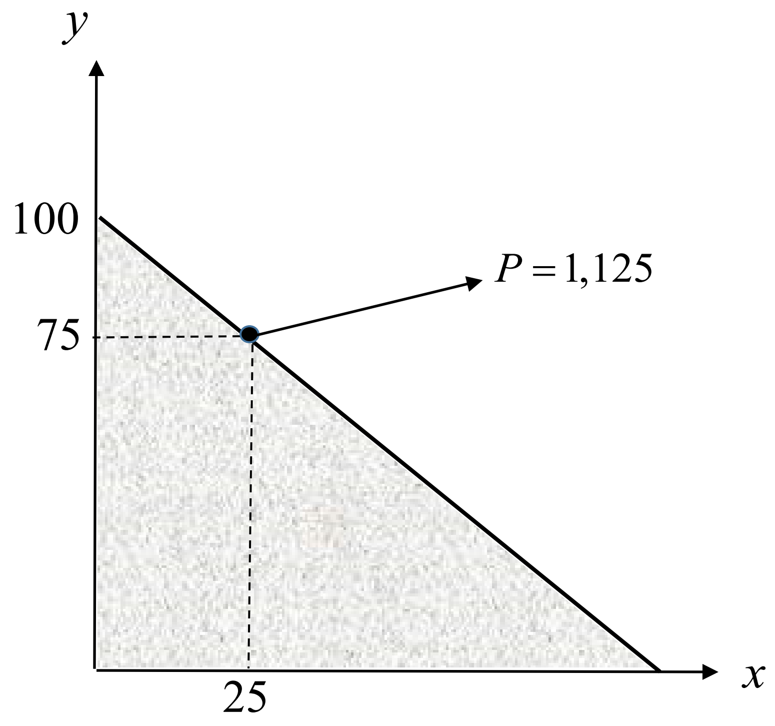
$$\left.\begin{aligned} 10 - .2x - \lambda = 0 \\ 20 - .2y - \lambda = 0 \end{aligned}\right\} \longrightarrow \begin{aligned} \lambda = 10 - .2x \\ \lambda = 20 - .2y \end{aligned}$$

Solve the first two expressions for lambda

$$10 - .2x = 20 - .2y \longrightarrow y = x + 50$$

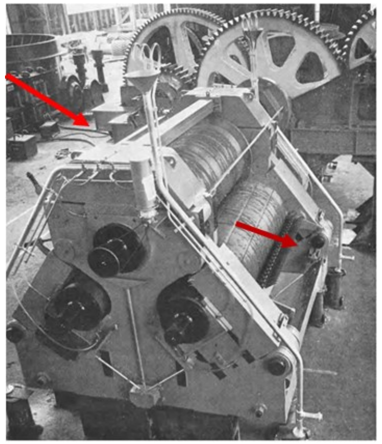
Cont.

$$100 - x - (x + 50) = 0 \quad \Rightarrow \quad x^* = 25,\; y^* = 75,\; \lambda = 5,\; P = 1{,}125$$

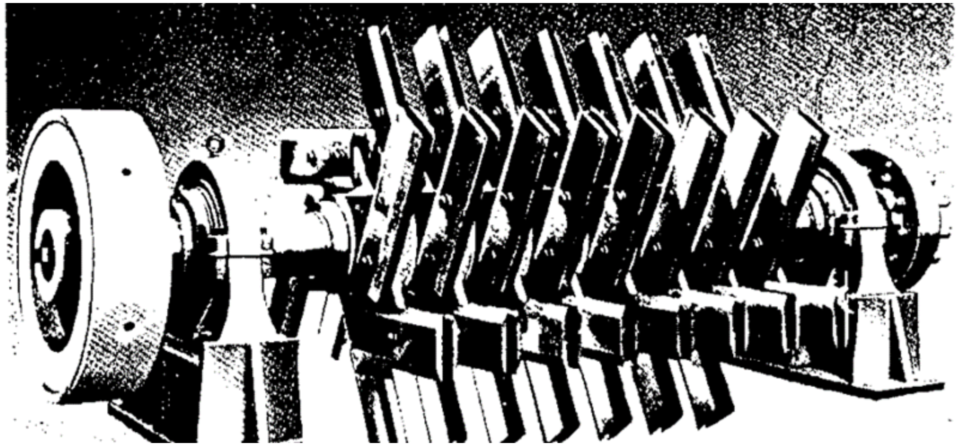


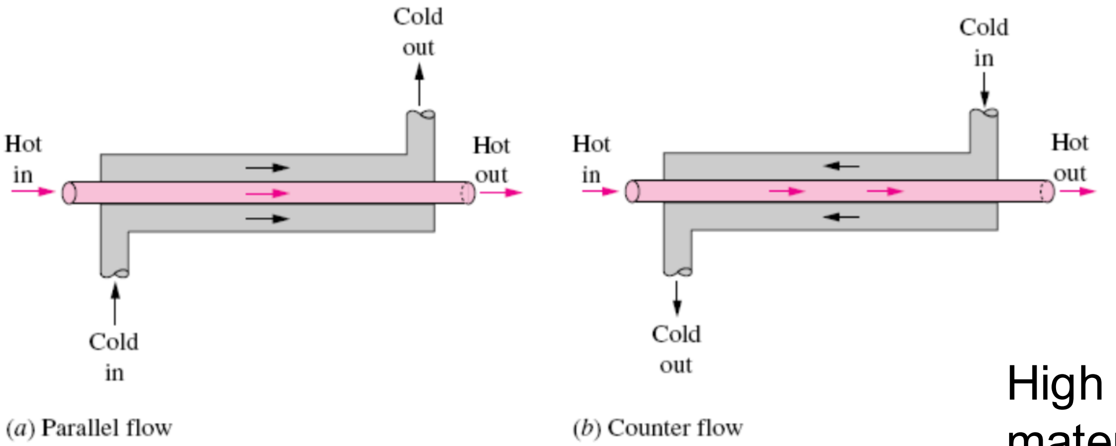
MATERIAL SELECTION FOR PROCESS EQUIPEMENT CONSTRUCTION

Background



Need material that has high mechanical





High thermal conductivity material

Construction materials

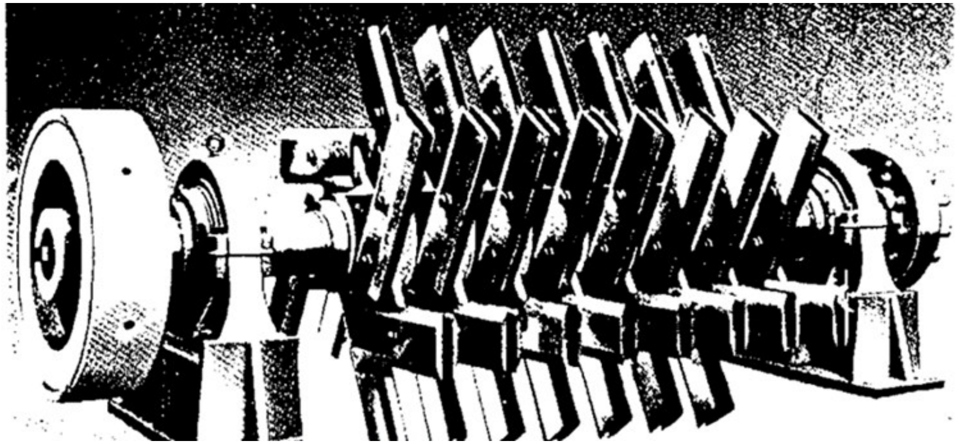
- The properties and cost of materials are very important factors in the design, construction, operation, and maintenance of general processing equipment
- The main materials being used for food processing equipment are metals, plastics, and glass/ceramics
- Wood and some natural fibers are used in some special applications

Cont.

- The materials used in the construction of food equipment must have the following properties:
  - ✓ Mechanical strength
  - ✓ Easy to fabricate
  - ✓ Easy to repair
  - ✓ Resistence to corrosion
  - ✓ Desirable thermal properties
  - ✓ Hygenic properties

Cont.

- The mechanical strength is especially important in the construction of equipment used in processing of large quantities in a relatively short time



Cont.

- In fabrication and repair of food processing equipment, the hardness and welding ability of the materials are also important
- The resistance against corrosion is especially important in
  - ✓ Wet processing (e.g., canning industry)
  - ✓ Processing of foods or food ingredients of relatively low pH, which may attack the equipment materials, and when corrosive chemicals are used, e.g. $S0_2$ for preservation, cleaning chemicals, etc

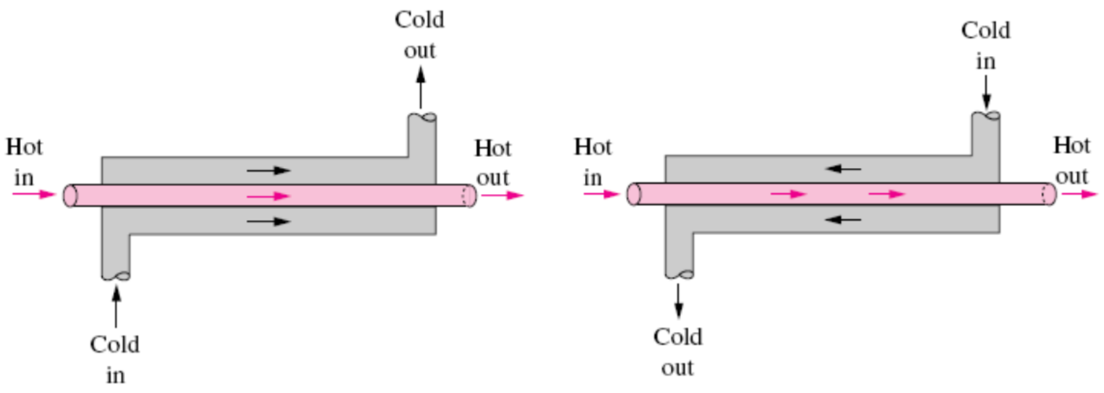
Cont.

- The hygienic (sanitary) materials do not exchange components with the food, i.e., they do not contaminate the food, or absorb components of the processed product

- They should have smooth or polished surfaces, not react with detergents, and they should be cleaned easily

Cont.

- Thermal properties are important when heat transfer to and from the processed products must take place (e.g., pasteurization of liquids in tubes, cooling down food in plate freezers)



(a) Parallel flow (b) Counter flow

$$\dot{Q} = \frac{\Delta T}{R} = UA\Delta T = U_iA_i\Delta T = U_oA_o\Delta T$$

$$R = R_{\text{total}} = R_i + R_{\text{wall}} + R_o = \frac{1}{h_iA_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{1}{h_oA_o}$$

Product	Tensile strength (MPa)	Thermal conductivity (W/m K)	Acetic acid	Nitric acid	HCl	H_2SO_4	NaOH	H_2O_2
Metals								
C-steel	350–490	35–46	−	−	−	−	++	−
304/316 SS	565	18.8	++	+++	−	+−	++	++
Cu	20–40		−	−	++	−	+−	−
Cu alloys	400–450	375.8						
Al	50–60	208.8	+++	+−	−	−	−	+++
Al alloys	150–470							
Monel	480–600		+	−	−	+−	++	++
Plastics								
Polyester	55–72	0.170	++	+	+++	+++	−	++
PVC (soft)	20–60	0.160	+++	+++	+++	+++	+++	+++
Polyethylene	6–37	0.334	+++	+++	+++	++	+++	++
Rubber			−	−	++	+++	+++	++

Data from Loncin (1969), Schimpke (1959), and Perry and Green (1997).

+++: very good resistance

++: sufficient resistance, if used under normal conditions

+: sufficient resistance for no permanent contact

−: insufficient resistance

Material types

- Metals
  - ✓ Metals are the most important materials used in the construction of food processing equipment. They can be classified into two main categories: ferrous metals and their alloys, and nonferrous metals
- Plastics-Rubber
  - ✓ Plastics are usually resistant to corrosion, but their mechanical strength is limited. Furthermore, their strength depends strongly on the temperature of the material. The upper temperature application limit of most temperature-resistant plastics lies at 250°C

Cont.

- ✓Therefore, in food processing equipment, plastics are mainly used for coating, and parts that are not under high and continuous stress (e.g., parts of ventilators and pumps, pipes, fittings, small tanks, covers of vessels, filters, gaskets). In all cases, plastics must fulfill the requirements concerning the interaction of materials with food. This is especially important for plasticizers, which are added to influence the properties of the plastics, and which are generally undesirable in the food system

Cont.

- Glass-Ceramics
  - ✓ Glass and ceramics are very resistant to acids and sufficiently resistant against lye. They are very hard and can withstand pressures of 100-400 MPa. However, they are very sensitive to bending (fragile). Their thermal conductivity is 0.62-1.45 W/m K. They are used in coating of other stable materials (e.g., in bins, vats) and in the construction of pipes and processing equipment for very sensitive products

Thank you