

**UNIT THREE**  
**INTRODUCTION TO LINEAR PROGRAMMING**

## Unit Objectives

By the end of studying this unit, you will be able to:

- Know the technicalities of formulating linear programming problems in both cases of minimization and maximization.
- Understand the meaning and concepts of linear programming.
- Develop familiarity with the graphical method for solving linear programming problems.

## Unit Introduction

A large number of decision problems faced by a business manager involve allocation of resources to various activities with the objective of increasing profits or decreasing costs, or both. When resources are in excess, no difficulty is experienced. Nevertheless, such cases are very rare. Practically in all situations, the managements are confronted with the problem of scarce resources. Normally, there are several activities to perform but limitations of either of the resources or their use prevent each activity from being performed to the best level. Thus, the manager has to take a decision as to how best the resources be allocated among the various activities.

The decision problem becomes complicated when a number of resources are required to be allocated and there are several activities to perform. Rule of thumb, even of an experienced manager, in all likelihood, may not produce the right answer in such cases. The decision problems can be formulated and solved as mathematical programming problems.

## **Section One: Basic Concepts**

### **Section Objectives:**

After covering this section, you will be expected to:

- Know the meaning of linear programming.
- Identify the conditions that should be fulfilled to apply linear programming.

## Section Overview:

### 3.1 Defining Linear Programming Model

### 3.2 Requirements to Apply Linear Programming

#### 3.1 Defining Linear Programming Model

*Linear Programming* (LP) is a mathematical process that has been developed to help management in decision-making involving the efficient allocation of scarce resources to achieve a certain objective. *Diagrammatically,*

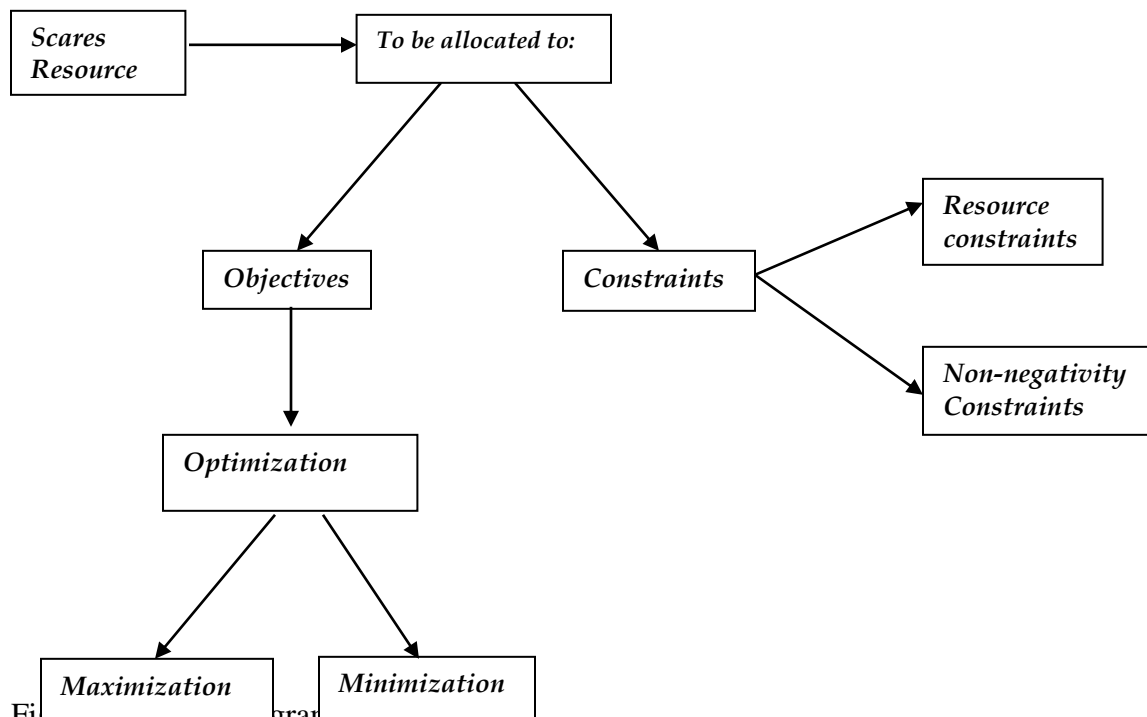


Fig 3.1 Linear Programming Model

LP is a method for choosing the best alternative from a set of feasible alternatives.

## 3.2 Requirements to Apply Linear Programming

To apply LP, the following conditions must be satisfied. These will be discussed next:

- a. There should be an objective that should be clearly identified and measurable in quantitative terms. Example, maximization of sales, profit, and minimization of costs etc.

- b. The activities to be included should be distinctly identifiable and measurable in quantitative terms.
- c. The resources of the system which are to be allocated for the attainment of the goal should also be identifiable and measurable quantitatively. They must be in limited supply. These resources should be allocated in a manner that would trade off returns on investment of the resources for the attainment of the objective.
- d. The relationship representing the objective and the resource limitation considerations represented by the objective function and the constraint equations or inequalities, respectively, must be linear in nature.
- e. There should be a series of feasible alternative courses of actions available to the decision-maker that is determined by the resource constraints.

When these stated conditions are satisfied in a given solution, the problem can be expressed in algebraic form called *linear programming problem (LPP)*, and then solved for optimal decision. We first illustrate the formulation of linear programming problems and then consider the method of their solution.

## **Section Two: Formulation of Linear Programming Model**

### **Section Objectives:**

After covering this topic, you will be in a position to:

- Identify the goal of the problem in terms of the objective function.
- Identify the limited resources problem in terms of inequality.
- Translate the problem from verbal form to standard mathematical statements showing all the objective and available scarce in supply of resource.

### **Section Overview:**

Problem Modeling: the Maximization Case

Problem Modeling: the Minimization Case

Assumptions Underlying Linear Programming

General Statement of Linear Programming Problem

### 3.3 Problem Modeling: The Maximization Case

*Problem Formulation or Modeling* is the process of translating the verbal statement of a problem into mathematical statements. Formulating a model is an art that can only be measured with practice and experience. Even though every problem has certain unique features, most problems have common features. Therefore, some general guidelines for model formulation are helpful. Next, you will have the illustration of some general guidelines by developing a mathematical model for some given problems. Consider the following example.

#### **Example 3.1**

A firm is engaged in producing two products, A and B. Each unit of product A requires two Kgs of raw material and four labor hours for processing whereas each unit of product B requires three kg of raw material and three hours of labor of the same type. Every week the firm has an availability of 60 kgs of raw material and 96 labor hours. One unit of product A sold yields Birr 40 and one unit of product B sold yields Birr 35 as profit.

Formulate this problem as a linear programming problem to determine as to how many units of each of the products should be produced per week so that the firm can maximize the profit. Assume that there is no marketing constraint so that all that is produced can be sold.

**The objective function:** the first major requirement of a linear programming problem (LPP) is that we should be able to identify the goal in terms of the objective function. This function relates mathematically the variables with which we are dealing in the problem.

For our problem, the goal is the maximization of profit, which would be obtained by producing (and selling) the products A and B. If we let  $x_1$  and  $x_2$  represent the number of units produced per week of the products A and B respectively, the total profit,  $Z$ , would be equal to  $40x_1 + 35x_2$ . Then, the objective function, relating the profit and the output level of each of the two items. Notice that the function is a linear one. Further, since the problem calls for a decision about the optimal values of  $x_1$  and  $x_2$ , these are known as the decision variables.

**The constraints:** As has been laid, another requirement of linear programming is that the resources must be in limited supply. The mathematical relationship which is used to explain this limitation is inequality. The limitation itself is known as a constraint.

Each unit of product A requires 2 kg of raw material while each unit of product B needs 3 kg. The total consumption would be  $2x_1$  and  $3x_2$ , which cannot be the total availability of 60 kg

every week. We can express this constraint as  $2x_1$  and  $3x_2 \leq 60$ . Similarly, it is given that a unit of A requires 4 labor hours for its production and one unit of B requires 3 hours. With an availability of 96 hours a week, we have  $4x_1$  and  $3x_2 \leq 96$  as the labor hour's constraint. It is important to note that for each of the constraint, inequality rather than equation has been used. This is because the profit maximizing output might not use all the resources to the full leaving some unused, hence the  $\leq$  sign. However, it may be noticed that all the constraints are also linear in nature.

**Non-negativity Condition:** Quite obviously,  $x_1$  and  $x_2$ , being the number of units produced, cannot have negative values. Thus both of them can assume values only greater than or equal to zero. This is the non-negativity condition, expressed symbolically as  $x_1 \geq 0$  and  $x_2 \geq 0$ .

Now we can write the problem in complete form as follows.

Maximize	$Z = 40 X_1 + 35 X_2,$	Profit
Subject to		
	$2X_1 + 3X_2 \leq 60$	Raw material constraint
	$4X_1 + 3X_2 \leq 96$	Labor hours constraint
	$X_1, X_2, \geq 0$	Non-negativity restriction

### 3.4 Problem Modeling: the Minimization Case

Dear student, can you guess the differences between maximization and minimization problems?

Consider the following example

#### Example 3.2

The agricultural Research institute has suggested to a farmer to spread out at least 4800 kg of a special phosphate fertilizer and no less than 7200 kg of a special nitrogen fertilizer to raise productivity of crops in his fields. There are two resources for obtaining these mixtures A and B. Both of these are available in bags weighing 100 kg each and they cost Birr 40 and Birr 24 respectively. Mixture A contains phosphate and nitrogen equivalent of 20 kg and 80 respectively, while mixture B contains these ingredients equivalent of 50 kg each.

Write this as a linear programming problem and determine how many bags of each type the farmer should buy in order to obtain the required fertilizer at minimum cost.

**The Objective Function:** In the given problem, such a combination of mixtures A and B is sought to be purchased as would minimize the total cost. If  $x_1$  and  $x_2$  are taken to represent the

number of bags of mixtures A and B respectively, the objective function can be expressed as follows:

$$\text{Minimize } Z = 40x_1 + 24x_2 \quad \text{Cost}$$

**The constraints:** In this problem, there are two constraints, namely, a minimum of 4800 kg of phosphate and 7200 kg of nitrogen ingredients are required. It is known that each bag of mixture A contains 20 kg and each bag of mixture B contains 50 kg of phosphate. The phosphate requirement can be expressed as  $20x_1 + 50x_2 \geq 4800$ . Similarly, with the given information on the contents, the nitrogen requirement would be written as  $80x_1 + 50x_2 \geq 7200$ .

**Non-negativity condition:** As before, it lays that the decision variables, representing the number of bags of mixtures A and B, would be non-negative. Thus  $x_1 \geq 0$  and  $x_2 \geq 0$ .

The linear programming problem can now be expressed as follows:

$$\text{Minimize } Z = 40x_1 + 24x_2 \quad \text{Cost}$$

Subject to

$$20x_1 + 50x_2 \geq 4800 \quad \text{Phosphate requirement}$$

$$80x_1 + 50x_2 \geq 7200 \quad \text{Nitrogen requirement}$$

$$x_1, x_2 \geq 0 \quad \text{Non negativity restriction}$$

### 3.5 General Statement of Linear Programming Problem

In general, linear programming problem can be written as

$$\text{Maximize } Z = c_1x_1 + c_2x_2 + \dots + c_n x_n \quad \text{Objective Function}$$

Subject to

$$a_{11}x_1 + a_{22}x_2 + \dots + a_{in} x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{in} x_n \leq b_2$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn} x_n \leq b_m$$

$$x_1, x_2, \dots, x_n \geq 0$$

Where  $c_j, a_{ij}, b_i$  ( $i = 1, 2, \dots, m; j = 1, 2, \dots, n$ ) are known as constants and  $x$ 's are decision variables  
 $c$ 's are termed as the profit coefficients  
 $a_{ij}$ 's the technological coefficients

b's the resource values

In shorter form, the problem can be written as:

$$\text{Maximize } Z = \sum_{j=1}^n c_j x_j$$

$$\text{Subject to } Z = \sum_{j=1}^n c_{ij} a_{ij} x_j \leq b_i$$

$$\text{for } \begin{matrix} i = 1, 2, \dots, m \\ x_i \geq 0 \\ j = 1, 2, \dots, n \end{matrix}$$

Where the objective is to minimize a function, the problem is,

$$\text{Minimize } Z = \sum_{j=1}^n c_j x_j$$

$$\text{Subject to } Z = \sum_{j=1}^n a_{ij} x_j \geq b_i$$

$$\text{for } j = 1, 2, \dots, n$$

In matrix notation, a LPP can be expressed as follows:

**Maximization Problem**

$$\text{Maximize } Z = cx$$

Subject to

$$ax \leq b$$

$$x' \geq 0$$

**Minimization Problem**

$$\text{Minimize } Z = cx$$

Subject to

$$ax \geq b$$

$$x' \geq 0$$

Where, c = row matrix containing the coefficients in the objective function,

x = Column matrix containing decision variables,

a = Matrix containing the coefficients in the constraints,

b = Column matrix containing the RHS values of the constraints

Although, generally, the constraints in the maximization problems are of the  $\leq$  type, and the constraints in the minimization problems are of  $\geq$  type, a given problem might contain a mix of the constraints, involving the signs  $\leq$ ,  $\geq$ , and /or =.

### 3.6 Assumptions Underlying Linear Programming

A linear programming model is based on the assumptions of proportionality, additivity, continuity, certainty, and finite choices. These are explained here next.

1. **Proportionality:** A basic assumption of linear programming is that proportionality exists in the objective function and the constraint inequalities. For example, if one unit of a product is assumed to contribute Birr 10 toward profit, then the total contribution would be equal to  $10x_1$  where  $x_1$  is the number of units of the product. For 4 units, it would equal Birr 40 and for 8 units it would be Birr 80, thus if the output (and sales) is doubled, the profit would also be doubled. Similarly, if one unit takes 2 hours of labor of a certain type, 10 units would require 20 hours, 20 units would require 40 hours...and so on. In effect, then, proportionality means that there are constant returns to scale and there are no economies of scale.
2. **Additivity:** Another assumption underlying the linear programming model is that in the objective function and constraint inequalities both, the total of all the activities is given by the sum total of each activity conducted separately. Thus, the total profit in the objective function is determined by the sum of the profit contributed by each of the products separately. Similarly, the total amount of a resource used is equal to the sum of the resource values used by various activities.
3. **Continuity:** It is also an assumption of a linear programming model that the decision variables are continuous. Therefore, combinations of output with fractional values, in the context of production problems, are possible and obtained frequently. For example, the best solution to a problem might be to produce  $5\frac{3}{4}$  units of product B per week. Although in many situations we can have only integer values, but we can deal with the fractional values, when they appear.
4. **Certainty:** A further assumption underlying a linear programming model is that the various parameters, namely, the objective function coefficients, the coefficients of the inequality/equality constraints and the constraint (resource) values are known with certainty. Thus, the profit per unit of the product, requirements of materials and labor per unit, availability of materials, labor etc. are given and known in a problem involving these. The linear programming is obviously deterministic in nature.
5. **Finite Choices:** A linear programming model also assumes that a limited number of choices are available to a decision maker and the decision variables do not assume negative values.



Thus, only non-negative levels of activity are considered feasible. This assumption is indeed a realistic one. For instance, in the production problems, the output cannot obviously be negative, because a negative production implies that we should be able to reverse the production process and convert the finished output back in to the raw materials!

## **Section Three: Solution Approaches to LPPs**

### **Section Objectives:**

Upon completion of this section, you will be able to:

- Solve Linear Programming Problems with two variables using the graphic approach.
- Solve Linear Programming Problems using the simplex algebra approach.

### **Section Overview:**

3.7 Approaches of Solving LPPs

3.8 Graphical Solution to Linear Programming Problems

### **3.7 Approaches of Solving LPPs**

Now we shall consider the solution to the linear programming problems. They can be solved by using graphic method or by applying algebraic method, called the Simplex Method. The graphic method is restricted in application – it can only be used when two variables are involved. Nevertheless, it provides an intuitive grasp of the concepts that are used in the simplex technique. The simplex method, whereas is the mathematical technique of solving linear programming problems with two or more variables.

### **3.8 Graphical Solution to Linear Programming Problems**

#### **Topic Objectives:**

**After studying this topic, you will be capable of:**

- Identifying the problem including the decision variables.
- Convert the inequalities in to equations and draw a graph that include all the constraints.
- Identify the feasible area of the solution that satisfies all constraints.
- Identify the corner points (coordinates) in the feasible region.

- Determine the value of the objective function by using the corner points (coordinates).
- Identify the optimal point and interpret the result to be obtained.

### 3.8.1 Steps in Graphic Method of Linear Programming Problems

To use the graphic method, the following steps are needed:

- i. *Identify the problem* – determine the decision variables, the objective function, and the constraints.
- ii. *Draw a graph* including all the constraints and identify the feasible region.
- iii. *Obtain a point on the feasible region* that optimizes the objective function – optimal solution.
- iv. *Interpret the results.*

**Note:** Graphical LP is a two-dimensional model.

### 3.8.2 The Maximization Problem

This is the case of *Maximize Z* with inequalities of constraints in  $\leq$  form.

#### Example 3.3

Consider two models of color TV sets; **Model A** and **B**, are produced by a company to maximize profit. The profit realized is \$300 from A and \$250 from set B. The limitations are

- a. Availability of only 40 hrs of labor each day in the production department,
- b. A daily availability of only 45 hrs on machine time, and
- c. Ability to sale 12 set of model A.

How many sets of each model will be produced each day so that the total profit will be as large as possible?

<i>Constraints</i>	<i>Resources used per unit</i>		<i>Maximum Available Hours</i>
	<i>Model A</i> $(x_1)$	<i>Model B</i> $(x_2)$	
<i>Labor Hours</i>	2	1	40
<i>Machine Hours</i>	1	3	45

<i>Marketing Hours</i>	1	0	12
<i>Profit</i>	<b>\$300</b>	<b>\$250</b>	

### Solution

1. Formulation of mathematical model of LPP

$$\text{Max } Z = 300X_1 + 250X_2$$

St:

$$\left. \begin{array}{l} 2X_1 + X_2 \leq 40 \\ X_1 + 3X_2 \leq 45 \\ X_1 \leq 12 \\ X_1, X_2 \geq 0 \end{array} \right\} \text{LP Model}$$

2. Convert constraints inequalities into equalities

$$2X_1 + X_2 = 40$$

$$X_1 + 3X_2 = 45$$

$$X_1 = 12$$

3. Draw the graph by finding out the x- and y-intercepts

$$2X_1 + X_2 = 40 \implies (0, 40) \text{ and } (20, 0)$$

$$X_1 + 3X_2 = 45 \implies (0, 15) \text{ and } (45, 0)$$

$$X_1 = 12 \implies (12, 0)$$

$$X_1, X_2 = 0$$

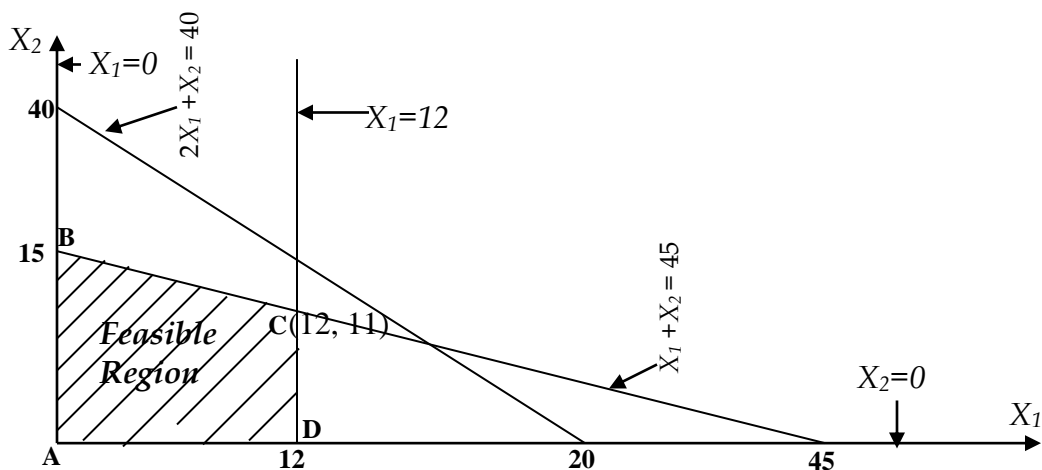


Fig. 3.3.1 Graphical Solution of LPP. (Maximization Problem)

4. Identify the feasible area of the solution which satisfies all constraints. The shaded region in the above graph satisfies all the constraints and it is called *Feasible Region*.
5. Identify the corner points in the feasible region. Referring to the above graph, the corner points are in this case are:

A (0, 0), B (0, 15), C (12, 11) and D (12, 0)

6. Identify the optimal point.

<i>Corners</i>	<i>Coordinates</i>	<i>Max Z = 300 X<sub>1</sub> + 250X<sub>2</sub></i>
<b>A</b>	(0, 0)	\$0
<b>B</b>	(0, 15)	\$3750
<b>C</b>	<b>(12, 11)</b>	<b>\$ 6350 (Optimal)</b>
<b>D</b>	(12, 0)	\$3600

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7. Interpret the result. Accordingly, the highlighted result in the table above implies that 12 units of Model A and 11 units of Model B TV sets should be produced so that the total profit will be \$6350.

**Example 3.4**

A manufacturer of Light Weight mountain tents makes two types of tents: *REGULAR* tent and *SUPER* tent. Each *REGULAR* tent requires one labor-hour from the cutting department and 3 labor-hours from the assembly department. Each *SUPER* tent requires 2 labor-hours from the cutting department and 4 labor-hours from the assembly department. The maximum labor hours available per week in the cutting department and the assembly department are 32 and 84 respectively. Moreover, the distributor, because of demand, *will not take more than 12 SUPER* tents per week. The manufacturer sales each *REGULAR* tents for \$160 and costs \$110 per tent to make. Where as *SUPER* tent ales for \$210 per tent and costs \$130 per tent to make.

**Required:**

- Formulate the mathematical model of the problem
- Using the graphic method, determine how many of each tent the company should manufacture each week so as to maximize its profit?
- What is this maximum profit assuming that all the tents manufactured in each week are sold in that week

### Solution

#### 1. The LP Model:

<i>Department</i>	<i>Labor Hours per Tent</i>		<i>Maximum Labor-hours Available per Week</i>
	<i>REGULAR (X<sub>1</sub>)</i>	<i>SUPER(X<sub>2</sub>)</i>	
Cutting department	1	2	32
Assembly department	3	4	84
<i>Selling price per tent</i>	\$160	\$210	
<i>Cost per tent</i>	\$110	\$130	
<i>Profit per tent</i>	<b>\$50</b>	<b>\$80</b>	

The distributor will not take more than 12 SUPER tents per week. Thus, the manufacturer should not produce more than 12 SUPER tents per week.

Dear student, please formulate the mathematical model based on the information in the above table before going to the solution part.

Let  $X_1 =$  The  $N^{\circ}$  of REGULAR tents produced per week.

$X_2 =$  The  $N^{\circ}$  of SUPER tents produced per week.

$X_1$  and  $X_2$  are called the decision variables.

$$\text{LP Model} \left\{ \begin{array}{l}
 \text{Max. } Z = 50X_1 + 80X_2 \\
 \text{St :} \\
 X_1 + 2X_2 \leq 32 \quad \dots\dots \text{Cutting department constraint} \\
 3X_1 + 4X_2 \leq 84 \quad \dots\dots \text{Assembly department constraint} \\
 X_2 \leq 12 \quad \dots\dots \text{Demand constraint} \\
 X_1, X_2 \geq 0 \quad \dots\dots \text{Non-negativity constraints}
 \end{array} \right.$$

2. *The Corners and Feasible Solution:*

<i>Corners</i>	<i>Coordinates</i>	<i>Max Z=50 X<sub>1</sub> +800X<sub>2</sub></i>
<b>A</b>	(0, 0)	\$0
<b>B</b>	(0, 12)	\$960
<b>C</b>	(8, 12)	\$1360
<b>D</b>	<b>(20, 6)</b>	<b>\$1480</b>
<b>E</b>	(28, 0)	\$1400

3. *The Interpretation:*

The manufacturer should produce and sale **20 REGULAR** tents and **6 SUPERS** tents to get a maximum weekly profit of \$1480.

Dear student, try to solve the above example by adding 5 to each coefficient and 10 to the right hand side values of the constraints.

### 3.8.3 The Minimization Problem

In this case, we deal with *Minimize Z* with inequalities of constraints in  $\geq$  form

#### Example 3.4

Suppose that a machine shop has two different types of machines; Machine 1 and Machine 2, which can be used to make a single product. These machines vary in the amount of product produced per hr., in the amount of labor used and in the cost of operation. Assume that at least a certain amount of product must be produced and that we would like to utilize at least the regular labor force. How much should we utilize on each machine in order to utilize total costs and still meets the requirement?

<i>Items</i>	<i>Resource Used</i>		<i>Minimum Required Hours</i>
	<i>Machine 1 (X<sub>1</sub>)</i>	<i>Machine 2 (X<sub>2</sub>)</i>	
Product produced/hr	20	15	100

Labor/hr	2	3	15
<b>Operation Cost</b>	<b>\$25</b>	<b>\$30</b>	

**Solution**

1. *The LP Model:*

$$\begin{array}{l}
 \text{Min. } Z = 25X_1 + 30X_2 \\
 \text{St:} \\
 20X_1 + 15X_2 \geq 100 \\
 2X_1 + 3X_2 \geq 15 \\
 X_1, X_2 \geq 0
 \end{array}
 \quad \left. \vphantom{\begin{array}{l} \\ \\ \\ \\ \end{array}} \right\} \text{LP Model}$$

Dear student, can you graph the above constraints? Please try to do so before going to the solution part.

2. *The Graph of Constraint Equations:*

$$20X_1 + 15X_2 = 100 \implies (0, 20/3) \text{ and } (5, 0)$$

$$2X_1 + 3X_2 = 15 \implies (0, 5) \text{ and } (7.5, 0)$$

$$X_1, X_2 = 0$$

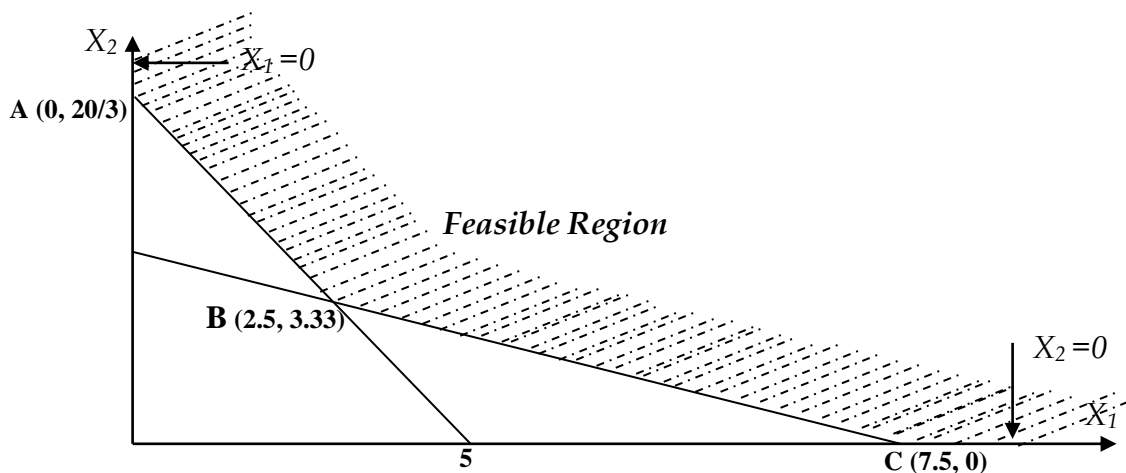


Fig 3.3.2 Graphical Solution of LPP. (Minimization Problem)

3. *The Corners and Feasible Solution:*

<b>Corners</b>	<b>Coordinates</b>	<b>Min <math>Z = 25X_1 + 30X_2</math></b>
<b>A</b>	(0, 20/3)	200
<b>B</b>	(2.5, 3.33)	<b>162.5 (Optimal)</b>

C	(7.5, 0)	187.5
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Since our objective is to minimize cost, the minimum amount (162.5) will be selected.

$$X_1 = 2.5$$

$$X_2 = 3.33 \text{ and}$$

$$\text{Min } Z = 162.5$$

**Note:**

- In maximization problems, our point of interest is looking the furthest point from the origin (Maximum value of Z).
- In minimization problems, our point of interest is looking the point nearest to the origin (Minimum value of Z).

**Exercise 3.1**

A company owns two flourmills (A and B) which have different production capacities for *HIGH*, *MEDIUM* and *LOW* grade flour. This company has entered contract supply flour to a firm every week with 12, 8, and 24 quintals of *HIGH*, *MEDIUM* and *LOW* grade respectively. It costs the Co. \$1000 and \$800 per day to run mill A and mill B respectively. On a day, mill A produces 6, 2, and 4 quintals of *HIGH*, *MEDIUM* and *LOW* grade flour respectively. Mill B produces 2, 2 and 12 quintals of *HIGH*, *MEDIUM* and *LOW* grade flour respectively. How many days per week should each mill be operated in order to meet the contract order most economically. Solve the problem graphically.

**Exercise 3.2**

Use graphical method to solve the following LPP.

1.  $\text{Max. } Z = 7/4X_1 + 3/2X_2$

St:

$$8X_1 + 5X_2 \leq 320$$

$$4X_1 + 5X_2 \leq 20$$

$$X_1 \geq 15$$

$$X_2 \geq 10$$

$$X_1, X_2 \geq 0$$

3.  $\text{Max. } Z = 3X_1 + 2X_2$

2.  $\text{Max. } Z = 3X_1 + 2X_2$

St:

$$-2X_1 + 3X_2 \leq 9$$

$$X_1 - 5X_2 \geq -20$$

$$X_1, X_2 \geq 0$$

4.  $\text{Max. } Z = X_1 + X_2$



St:

$$X_1 - X_2 \leq 1$$

$$X_1 + X_2 \geq 3$$

$$X_1, X_2 \geq 0$$

5. Max.  $Z = 6X_1 - 4X_2$

St:

$$2X_1 + 4X_2 \leq 4$$

$$4X_1 + 8X_2 \geq 16$$

$$X_1, X_2 \geq 0$$

St:

$$X_1 + X_2 \leq 1$$

$$-3X_1 + X_2 \geq 3$$

$$X_1, X_2 \geq 0$$

6. Max.  $Z = X_1 + 1/2X_2$

St:

$$3X_1 + 3X_2 \leq 12$$

$$5X_1 \leq 10$$

$$X_1 + X_2 \geq 8$$

$$-X_1 + X_2 \geq 4$$

$$X_1, X_2 \geq 0$$