College of Natural Sciences

Department of Statistics

Course Title: Statistical Inference
Course code: **Stat 3052**

Credit: 5 EtCTS

Credit hours: 3 (3Lecture hrs+2 hrs tutorial)

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Course description

Definition, basic concepts and properties of a point estimator; Sufficiency, Completeness, exponential family of distributions; Parametric point estimation, methods of finding point estimates: unbiased estimation, method of moments, maximum likelihood and Bayesian; Parametric interval estimation, Confidence interval, Method of finding interval estimates: pivotal quantity method and large sample asymptotic distributions, some applications; Tests of hypotheses: basic concepts, Most powerful test, Neyman-Pearson lemma, Generalized likelihood ratio test, uniformly most powerful test, monotone likelihood ratio, unbiased test, applications: tests on the mean, the variance, several means, several variances; chi-square tests; relationship between tests and confidence intervals; Sequential test: sequential probability ratio test and some of its properties; Introduction to non-parametric methods: inference concerning the empirical distribution, quantiles and equality of two distributions.

Objectives

- to provide students with the Theory of Statistical Inference: Point and Interval Estimation, Tests of Hypotheses, and Applications.
- to introduce estimation and hypothesis testing methods based on likelihood and other methods
- to introduce students to the principles of efficient estimation and hypothesis testing and acquaint them with best methods of estimation and construction of test procedures

Learning outcomes

On successful completion of this course students are expected to:

- understand the underlying statistical theory of estimation and hypothesis testing;
- determine how good an estimator or test procedure is based on a number of criteria;
- construct estimators and test procedures based both on the maximum likelihood and Bayesian principles.

Course Outline

1. Parametric Point Estimation (15 lecture hours)

1.1 Introduction: notion of parameter, parameter space, vector of parameters, problems of estimation

- 1.2 Definition of point estimator
- 1.3 General properties of estimators
    - 1.3.1 Unbiasedness
    - 1.3.2 Mean squared error
    - 1.3.3 Efficiency
    - 1.3.4 Consistency
    - 1.3.5Asymptoticaly normal estimators
- 1.4 Uniformly minimum variance unbiased estimators
    - 1.4.1 Introduction
    - 1.4.2 Sufficient statistics
    - 1.4.3 Factorization theorem
    - 1.4.4 Minimal sufficient statistics
    - 1.4.5 Exponential family of distributions

1.4.6 Theorem of Rao and Blackwell
1.4.7 Completeness
1.4.8 Theorem of Lehmann and Scheffe
1.4.9 Cramer-Rao lower bound and UMVUE

1.5 Methods of finding point estimators

1.5.1 Maximum likelihood estimation
1.5.1.1 Likelihood function
1.5.1.2 Regularity conditions
1.5.1.3 Finding maximum likelihood estimator
1.5.1.4 Score function, Fisher information and variance
1.5.1.5 Optimum properties of maximum likelihood estimator
1.5.1.6 UMVUE and maximum likelihood estimator

1.5.2 The method of moments
1.5.3 Minimax and Bayes estimation
1.5.4 The method of least squares

2. **Interval Estimation (10 lecture hours)**

2.1 Introduction: notion of interval estimation
2.2 Confidence intervals: definition, length of confidence interval
2.3 Review of sampling distributions
2.4 A method for finding interval estimates

2.4.1 Pivotal quantity method
2.4.1.1 Confidence interval for the mean (when population variance is known and unknown)
2.4.1.2 Confidence interval for the variance (when population mean is known and unknown)
2.4.1.3 Simultaneous confidence interval for the mean and variance
2.4.1.4 Confidence interval for the difference between two means for samples from two independent normal populations
2.4.1.5 Confidence interval for ratio of variances for samples from two independent normal populations

2.4.2 Large sample confidence intervals
2.4.3 Bayesian interval estimates: credible interval

3. **Parametric Tests of Hypotheses (15 lecture hours)**

3.1 Introduction

3.1.1 Statistical hypothesis, null and alternative hypothesis, simple and composite hypotheses
3.1.2 Test of hypothesis, critical region, type I and type II errors, probabilities of type I and type II errors, problem of controlling these probabilities
3.1.3. Power function and size of a test

3.2 Simple null hypothesis versus simple alternative

3.2.1 Simple likelihood ratio test
3.2.2 Most powerful test
3.2.3 Neyman-Pearson lemma

3.3 Composite Hypotheses

        3.3.1 Generalized likelihood ratio test
        3.3.2 Uniformly most powerful test
        3.3.3 Monotone likelihood ratio
    3.4 Unbiased tests: definition
    3.5 Examples using sampling from the normal population
        3.5.1 Tests on the mean
        3.5.2 Tests on the variance
        3.5.3 Tests on equality of two and several means
        3.5.4 Tests on two variances
    3.6 Relationship between test of hypotheses and confidence intervals
    3.7 Chi - square tests
        3.6.1 Asymptotic distribution of generalized likelihood ratio test
        3.6.2 Goodness-of-fit test
        3.6.3 Test of independence in contingency tables
    3.8 Sequential tests of hypotheses
        3.8.1 Definition of sequential analysis and comparison with fixed sample test
        3.8.2 Sequential probability ratio test
    3.9 Bayesian Hypothesis testing

4. **Non parametric Methods (8 lecture hours)**
    4.1 Introduction
    4.2 Inference concerning a cumulative distribution function
        4.2.1 Sample cumulative distribution function: definition and convergence
        4.2.2 Kolmogorov-Smirnov goodness-of-fit test
    4.3 Inference concerning quantiles
        4.3.1 Definition of quantiles
        4.3.2 Point estimate of a quantile
        4.3.3 Interval estimate of a quantile
    4.4 Equality of two distributions
        4.4.1 Two-sample sign test
        4.4.2 Run test
        4.4.3 Median test
        4.4.4 Rank-sum test

Textbook

Mood, Graybill, and Boes: Introduction to the theory of Statistics, 3rd Edition,

References

1. Casella, G. and Berger, R.L. (2001). Statistical Inference (2nd Edition). Duxbury press.
2. Cox, D.R. (2006). Principles of Statistical Inference. Cambridge University Press.
3. Hoel, P.G. Port, S.C. and Stone, C.J. (1972). Introduction to Statistical Theory. Houghton-Mifflin Series in Statistics.
4. Hogg, R.V. and Tanis, E. (2009). Probability and Statistical Inference (8th edition). Prentice-Hall.
5. Lindgren, B. W. (1993). Statistical Theory (4th Edition). Chapman - Hall.

6. Migon, H.S. and Gamerman, D. (1999). Statistical Inference: An integrated Approach. A Hodder Arnold Publication.
7. Mukhopadhyay, N. (2000). Probability and Statistical Inference. Statistics: a Series of Textbooks and Monographs. CRC.
8. Rao, C.R. (1973). Linear statistical Inference and its Applications. Wiley.
9. Rohatgi, V.K. (2003). Statistical Inference. Dover Publications.
10. Wasserman, L. (2004). All of Statistics: A Concise Course in Statistical Inference. Springer Texts in Statistics, Springer.
11. Roussas, G. G. (2006). Introduction to Probability. Academic Press.

Teaching and Learning Methods

Lectures, tutorials and assignments

Modes of Assessment

Continuous Assessment 50%

Final Exam 50%

Method	Assignment	Test	Quiz	Final
Percent	20	20	10	50
Frequency	2	2	At least 2	1