Jimma University, College of Natural Sciences

Department of Physics

Mathematical Methods of Physics II (Phys 2032), Assignment I

1. The orbital angular momentum $\mathbf{L}$ of a particle is given by $\mathbf{L} = \mathbf{r} \times \mathbf{p} = m\mathbf{r} \times \mathbf{v}$, where $\mathbf{p}$ is the linear momentum. With linear and angular velocities related by $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$, show that

$$\mathbf{L} = \mathrm{mr}^2[\boldsymbol{\omega} - \hat{\mathbf{r}}(\hat{\mathbf{r}}.\boldsymbol{\omega})]$$

Here $\hat{\mathbf{r}}$ is a unit vector in the $\mathbf{r}$-direction.

2. The Pauli spin matrices are

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Show that

a. $(\sigma_i)^2 = I_2$, I_2 is a 2×2 unit matrix

b. $\sigma_j \sigma_k + i\sigma_l, (j, k, l) = (1, 2, 3), (2, 3, 1), (3, 1, 2)$ (cyclic permutations)

c. $\sigma_i \sigma_j + \sigma_j \sigma_i = 2\delta_{ij} I_2$; I_2 is the 2×2 unit matrix

d. Show that $(\boldsymbol{\sigma}.\mathbf{a})(\boldsymbol{\sigma}.\mathbf{b}) = \mathbf{a}.\mathbf{b} I_2 + i\boldsymbol{\sigma}.(\mathbf{a} \times \mathbf{b})$

Here, $\boldsymbol{\sigma} = \sigma_1 \hat{\mathbf{x}} + \sigma_2 \hat{\mathbf{y}} + \sigma_3 \hat{\mathbf{z}}$, $\mathbf{a}$ and $\mathbf{b}$ are ordinary vectors, and I_2 is the2×2 unit matrix

3. Find the eigenvalues and the normalized eigenvectors of the following matrices.

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 5 & 0 & \sqrt{3} \\ 0 & 3 & 0 \\ \sqrt{3} & 0 & 3 \end{pmatrix}$$

4. Solving the following system of linear equations

$$2a + 5b + 6c = 5$$

$$a + 2b + 3c = 9$$

$$a + b + c = 4$$

5. Show that charge density (ρ) and current density $(\mathbf{J})$ go together to make a 4-vector.

6. Solving the following system of linear equations and give the results to three decimal places

$$1.0a + 0.9b + 0.8c + 0.4d + 0.1e = 1.0$$

$$0.9a + 1.0b + 0.8c + 0.5d + 0.2e + 0.1f = 0.9$$

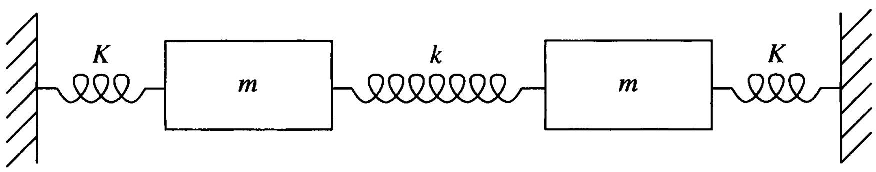
$$0.8a + 0.8b + 1.0c + 0.7d + 0.4e + 0.2f = 0.8$$

$$0.4a + 0.5b + 0.7c + 1.0d + 0.6e + 0.3f = 0.7$$

$$0.1a + 0.2b + 0.4c + 0.6d + 1.0e + 0.5f = 0.6$$

$$0.1b + 0.2c + 0.3d + 0.5e + 1.0f = 0.5$$

7. Two equal masses are connected to each other and to walls by springs as shown below. The masses are constrained to stay on a horizontal line.



a. Set up the Newtonian acceleration equation for each mass.
b. Solve the secular equation and determine the eigenvalues (the normal modes of vibration).
c. Determine the eigenvectors.