Statistical physics II (Phys3092) Assignment II.

I. Answer all the questions.

1. Consider an ideal quantum gas of Fermi particles at a temperature T.

   (a) Write the probability P(n) that there are n particles in a given single particle state as a function of the mean occupation number, <n>.

   (b) Find the root-mean- square fluctuation $<(n-<n>)^2>^{1/2}$ in the occupation <n>. Sketch the result.

2. Consider a system of two atoms, each having only 3 quantum states of energies 0, E and 2E. The system is in contact with a heat reservoir at temperature T. Write down the partition function z for the system if the particles obey

   (a) Classical statistical and are distinguishable.

   (b) Classical statistical and are indistinguishable.

   (c) Fermi-Dirac statistics.

   (d) Bose-Einstein Statistics.

3. N weakly coupled particles obeying Maxwell-Boltzmann statistics may each exist in one of the 3 non-degenerate energy levels of energies -E, 0, +E. The system is in contact with thermal reservoir at temperature T.

   (a) What is the entropy of the system at T=0k?

   (b) What is the maximum possible entropy if the system?

   (c) What is the minimum possible energy of the system?

   (d) What is the partition function of the system?

   (e) What is the most probable energy of the system? (f) If C (T) is the heat capacity of the system, what is the value of $\int_0^\infty \frac{C(T)}{T} dT$?

4. Find the pressure, entropy and specific heat at constant volume of an ideal Boltzmann gas of indistinguishable particles in the extreme relativistic limit, in which the energy of particle is related to its momentum by E=CP. Express your answer as function of the volume V, temperature T and number of particles N.

5. A simple harmonic one-Dimensional oscillator has energy levels $E_n = (n + 1/2)\hbar\omega$ where ω is the characteristics oscillator (angular frequency) n=0, 1, 2...

   (a) Suppose the oscillator is in thermal contact with a heat reservoir at temperature T, with $KT/\hbar\omega << 1$. Find the mean energy of the oscillator as a function of the temperature T,

   (b) For a two dimensional oscillator, $n = (n_x + n_y)$ where $E_{nx} = (n_x + 1/2)\hbar\omega$, $E_{ny} = (n_y + 1/2)\hbar\omega$, n_x=0,1,2.. And n_y=0, 1, 2... What is the partition function for this case for any value of temperature? Reduce it in to degenerate case $\omega_x = \omega_y$.

   (c) If one-Dimensional classical an harmonic oscillator has potential energy $V(x) = CX^2 - gx^3$, where $gx^3 << cx^2$, at equilibrium temperature T, carry out the calculations as far as you can and give expressions as functions of temperature for

   1. The heat capacity per oscillator and

   2. The mean value of the position x of the oscillator.

6. What is the root-mean-square fluctuation in the number of photons of mode frequency w in a conducting rectangular cavity? Is it always smaller than the average number of photons in the mode?

7. Estimate

   (a) The number of molecules in the air in room.

   (b) Their energy, in joules or in ergs , per mole.

   (c) What quantity of heat in joules or in ergs must be added to warm one mole of air at 1 atm from 0oc to 20 ^{0}c?

   (d) What is the minimum energy that must be supplied to a refrigerator18^{0}c to cool 1 mole of air at 1 atm from 20^{0}c to 18^{0}c? The refrigerator acts in a cyclic process and gives out heat at 40^{0}c.

Statistical Physics II (3092) Assignment III. (Chapter 5)

1. Define the following terms: a) Collision time b) Scattering cross-section c) Viscosity d) Self-Diffusion
2. Using the mean molecular speed given in kinetic theory of gases that is $\bar{v} = \sqrt{\frac{8kt}{\pi m}}$,

   show that the viscosity coefficient η is given by $\eta = \frac{2}{3\sqrt{\pi}\,\delta_0}\sqrt{KTm}$
3. The coefficient of thermal conductivity is $\overline{K} = \frac{2C}{3\sqrt{\pi}\,\delta_0}\sqrt{\frac{KT}{m}},$ for monoatomic gases derive it.
4. Making use of diffusion equation show that the explicit form of d if given by $D = \frac{2}{3\sqrt{\pi}\,\overline{P}\delta_0}\sqrt{\frac{(KT)^3}{m}}.$
5. The electrical conductivity of materials are temperature dependent and it is also given by $\delta_{el} \approx \sqrt{\frac{\pi}{8}}\,\frac{ne^2}{n_1\delta_{im}\sqrt{mKT}},$ prove it.