**Jimma University**

**College of Natural Sciences**

**Department of Mathematics**

**Worksheet (Math2072)**

1. Find real numbers *a* and *b* if:  and *z* = *a* + *bi* .
2. Calculate and put answer in Cartesian form:
3. 
4. 

3. Compute the indicated powers

1. 
2. 

4. Write in polar form: 

5. Find all solution of the equation 

6. Sketch the graph of the given equations on the z-plane

1.



1. 

6. Find the principal part of 

*7. Solve:*

1. 
2. 

7. Determine whether the following functions are analytic or not

1. 
2. *f*(*z*) = 2*x*2 + *y* + *i*(*y*2 *− x*)
3. 
4. 
5. 
6. 

8. Find real constants *a*, *b*, *c*, and *d* so that the given function is analytic.

1. *f*(*z*) = 3*x − y* + 5 + *i*(*ax* + *by −* 3)
2. *f*(*z*) = *x*2 + *axy* + *by*2 + *i*(*cx*2 + *dxy* + *y*2)

9. Let

1. 
2. 
3. 
4. 

a. Verify that u(x; y) is a harmonic function.

b. Find a harmonic conjugate v(x; y) of u(x; y).

c. Find the analytic function f(z) = u(x; y) + iv(x; y).

11. Evaluate:

1. 
2. 
3. 
4. Use L'H^opital's rule to determine the following limits. 

12. Evaluate  where C is the straight line joining 0 to 1 + i.

13. Evaluate  where C is the straight line joining 0 to i first and then i to 1 + i.

13. Obtain the integralalong the straight-line paths

(a) from z = 2+2i to z = 5+2i

(b) from z = 5+2i to z = 5+5i

14. Cauchy’s integral formula to evaluate

15. Compute  is the of radius 2 center at 0 oriented counterclockwise.

16. Evaluate  counterclockwise for any contour enclosing 

17. Evaluate  counterclockwise for any contour enclosing i.

18. Evaluate 



19. Evaluate  counterclockwise for any contour enclosing the point 2 but having  outside

20., where *C* is the circle |*z –* 2*i* | = 4.

21. Evaluate by Cauchy’s integral formula  where C is the circle 

22. Evaluate  where C is the circle 

23. Evaluate by Cauchy’s integral formula  where C is the circle 

24. Evaluate by Cauchy’s integral formula , where C is

1. the circle 
2. the circle 

25. Evaluate, where C is the circle 

26. Use Cauchy’s integral formula to evaluate, where C is the boundary of a square with vertices 1 + i, -1 + i , -1 - i and 1 – i traversed counter clock wise.

27. Find poles and residues at these poles of  also find the sum of these residues

28. Find the sum of residues at poles of 

29. Show that, where C is the circle*.*

30. Evaluate by Cauchy Residue Theorem:  , where C is the Circle *.*

31. Evaluate by Cauchy Residue Theorem:  , where C is the Circle *.*

32. Evaluate:  by Cauchy’s Residue Theorem, where C is

1. the circle  ,
2. the circle 

 33. Evaluate the integral when the contour C is the square whose edges lie along the lines and with positive orientation.

34. Let C be the positively oriented circle. Evaluate the contour integral 

36. Find the integral, where and C is

(a) The circle |z − 2i| = 1;

(b) The circle |z + 2i| = 1;

(c) Any closed path enclosing both z = 2i and z = −2i.

37. Evaluate:

1.  Where C is an ellipse 
2.  , where.
3.  , where.
4. Use Cauchy’s integral formulae to evaluate ( any one )
5.  , Where C is the circle 
6. 
7.  , Where C is the circle 
8. 

38. Use Cauchy integral formula to evaluate  where C is the circle.

39. Use Cauchy integral formula to evaluate  where C is the circle.

40. Use Cauchy integral formula to evaluate  where C is the circle.

41. Use Cauchy integral formula to evaluate  where C is the circle.

42. Expand the following function in Laurent‘s series  , when

1. 
2. 

43. Expand the following function in Laurent‘s series about z=1

44. Expand the following function in Laurent‘s series  , when

1. 
2. 

45. Calculate the residues of at its pole.

46. Locate the poles and determine the order for the function 

**47. Give precise of the following expressions**

1. State Cauchy- Riemann equations.
2. State the necessary condition for the function f(x) to be analytic.
3. State the sufficient condition for the function f(x) to be analytic
4. Define an analytic function.
5. Define singular points of an analytic function f(z).
6. What is harmonic function?
7. State Cauchy’s integral formula for f(a).
8. State Cauchy’s integral formula for f′ (a).