JIMMA UNIVERSITY NATURAL SCIENCES COLLEGE

DEPARTMENT OF MATHEMATICS: *OPTIMIZATION THEORY I (MATH.356).*

WORKSHEETS: 1 , 2 , 3 , 4 , 5 & 6

WORKSHEET – 1 (On Convex Set).

Attempt Each Of The Following Problems:

1. **Define a convex set.**

2. **Prove that a vertex is boundary point but all boundary points are not vertices. Give example. Identify the vertices, if any, of the following sets.**

   a) $\{x : |x| \le 1, x\varepsilon R^n\}$ **b)** $\{x : x = (1-\lambda)x_1 + \lambda x_2 \ , \lambda \ge 0, x_1, x_2 \varepsilon R^n\}$

3. **Which of the following sets are convex; if so, why?**

   $X = \{(x_1, x_2) : x_1 x_2 \le 1, x_1 \ge 0, x_2 \ge 0\}$, $X = \{(x_1, x_2) : x_1^2 + x_2^2 \le 9\}$

   $X = \{(x_1, x_2) : x_2^2 \le 4x_1 ; x_1, x_2 \ge 0\}$

4. **Discuss whether the following sets are convex or not. And find the convex hull of the set in each case: (i) Set of point on the line** $y = mx + c$ **(ii) Set of point of the union of the half lines x = 0, y >0 ,y = 0 x> 0 on the xy plane. (iii) Points (0, 0), (0, 1), (1, 0), (1, 1) on the (x,y) plane.**

5. **Prove that the set B =** $\{(x_{1,} x_2) : x_1^2 + x_2^2 \le 4\}$ **is a convex set.**

6. **Show that the set** $S = \{(x_1, x_2) : 3x_1^2 + 2x_2^2 \le 6\} \textit{ is convex.}$

7. **Determine the convex hull of the following sets:** $X = \{(x_1, x_2) : x_1^2 + x_2^2 = 1\}$

8. **Graph the convex hull of the following points: (0, 0), (0, 1), (1, 2), (1, 1) , (4,0) . Which of these points are in an interior point of the convex hull?**

9. **Find the extreme points of the convex polygon given by the inequalities:**

$$2x_1 + x_2 + 9 \ge 0 \ , \ -x_1 + 3x_2 + 6 \ge 0 \ , \ x_1 + 2x_2 - 3 \le 0 \ , \ x_1 + x_2 \le 0$$

10. **What do we mean by convex combination of a finite number of points** $x_1 , x_2 , \ldots x_m$ **?**

11. **Draw a convex polygon & a non – convex polygon.**

12. **Show the intersection of two convex sets is a convex set.**

WORKSHEET –2 (*On Formulation an LP model for LPP*). OPTIMIZATION THEORY I

Formulate an LP model for the following problems.

1. **A manufacturer has 3 machines *A, B, C* with which he produces three different articles P, Q, R. The different machine times required per article, the amount of time available in any week on each machine and the estimated profits per article are furnished in following table:**

Article	Machine time (in hrs.)			Profit per article (in rupees)
	A	B	C	
P	8	4	2	20
Q	2	3	0	6
R	3	0	1	8
Available Machine hrs.	250	150	50	

Formulate the problem as a linear programming problem.

2. **Two alloys A and B are made from four different metals I, II, III and IV according to the following specifications: A: at most 80% of I, at most 30% of II, at least 50% of III, B:between 40% & 60% of II, at least 30% of III, at most 70% of IV. The four metals are extracted from three different ores whose constituent's percentage of these metals, maximum available quantity and cost per ton are as follows:**

Constituent %

Ore	Max. Quantity (tons)	I	II	III	IV	Others	Price (Rs.per ton)
1	1000	20	10	30	30	10	30
2	2000	10	20	30	30	10	40
3	3000	5	5	70	20	0	50

Assuming the selling prices of alloys A and B are Rs. 200 and Rs. 300 per ton respectively.

Formulate the above as a linear programming problem selecting appropriate objective and constraint functions.

3. **Formulate the following linear programming problem.**

A used-car dealer wishes to stock-up his lot to maximize his profit. He can select cars A, B and C which are valued wholesale at Rs.5000, Rs.7000 and Rs.8000 respectively. These can be sold at Rs.6000, 8500 and 10500 respectively.

For each car type, the probabilities of sale are:

Type of car	:	A	B	C
Prob. of sale in 90 days	:	*0.7*	*0.8*	*0.6*

For every two cars of B, he should buy one car of type A or C. If he has Rs, 1, 00,000 to invest,

What should he buy to maximize his expected gain ?

[Hint. Let x_1 , x_2 , x_3 number of cars be purchased of type A,B,C respectively. Gain per car for A,B,C will be Rs. (6000-50000),Rs (8500-7000),Rs.(10500-8000) respectively.

Therefore total expected will be $z = 1000x_1\times 0.7 + 1500x_2\ 0.8 + 2500x_3\times 0.6$] Investment constraints will be given by: $5000x_1 + 7000(2x_2) \leq 1,00{,}000$ and $7000(2x_2) + 8000x_3 \leq 1,00{,}000$].

4. A feed mixing company purchases and mixes one or more of the three types of grain, each

Containing different amount of three nutritional elements, the data is given below:

Item Nutritional ingredient	*One unit weight of*			*Minimum total Requirement, over Planning horizon*
	Grain 1	*Grain 2*	*Grain 3*	
A	*2*	*4*	*6*	*≥125*
B	*0*	*2*	*5*	*≥24*
C	*5*	*1*	*3*	*≥80*
Cost per unit wt.(Rs.)	*25*	*15*	*18*	*Minimize*

The production manager specifies that any feed mix for his livestock must meet at least minimal nutritional requirement; and seeks the least costly among all such mixes. Suppose his planning horizon is a two week period that is he purchases enough to fill his needs for two weeks. Formulate this as an L.P.P.

[Hint Let x_1 , x_2 , x_3 denote the weight levels of three different grains. Then by considering the nutritional ingredient in the three grains, linear programming problem is:

To minimize $C = 25x_1 + 15x_2 + 18x_3$, subject to constraints:

$2x_1 + 4x_2 + 6x_3 \geq 125$, $2x_2 + 5x_3 \geq 24$, $5x_1 + x_2 + 3x_3 \geq 80$, and $x_1 , x_2 , x_3 \geq 0$]

5. A person require 10, 12 and 12 unit of chemicals A,B and C respectively for his garden. A liquid product contains 5, 2 and one unit of A,B and C respectively per jar. A dry product contains,1,2 and 4 unit of A,B and C per carton. If the liquid product sells for Rs. 3 per jar and the dry product sells for Rs. 2 per carton, how many of each should be purchased to minimize the cost and meet the requirements.

[Hint. Formulation: Min.$z = 3x_1 + 2x_2$, s.t.$5x_1 + x_2 \geq 10$, $2x_1 + 2x_2 \geq 12$, $x_1 + 4x_2 \geq 12$ and

$x_1, x_2 \geq 0$.The vertices of the feasible region are :(12, 0),(4, 2),(1,5),(0, 10)].

6. The manager of an oil refinery must decide on the optimal mix of two possible blanding

Processes of which the inputs and outputs per production run are as follows:

Process	*Input(unit)*		*Output(unit)*	
	Crude A	*Crude B*	*Gasoline X*	*Gasoline Y*
1	*5*	*3*	*5*	*8*
2	*4*	*5*	*4*	*4*

The maximum amounts available of crude A and B are 200 units and 150 units respectively.

Market requirements show that at least 100units of gasoline x and 80 units of gasoline Y must

Be produced. The profits per production run from process 1 and process 2 are Rs. 300 & Rs.400 respectively. Solve the LP problem by graphical method.

[Hint. Formulation is: Max. $z = 300x_1 + 400x_2$, s.t.

$5x_1 + 4x_2 \leq 200$, $3x_1 + 5x_2 \leq 150$, $5x_1 + 4x_2 \geq 100$, $8x_1 + 4x_2 \geq 80$, $x_1, x_2 \geq 0$.

The vertices of the feasible region are: (20, 0),(40, 0),(400/13, 150/13),(0, 30), (0, 25)]

7. **A television company has three major department for manufacture of its models, A and B. monthly capacities are given as follows:**

Department	*Per unit time requirement(hours)*		*Hours available This month*
	Model A	*Model B*	
I	*4.0*	*2.0*	*1600*
II	*2.5*	*1.0*	*1020*
III	*4.5*	*1.5*	*1600*

The margin profit of model A is Rs. 400 each and that of model B is Rs. 100 each. Assuming that the company can sell any quantity of either product due to favorable market conditions, determine the optimum out-put for both the models, the highest possible profit for this month and the slack time in the three departments.

[Hint. Formulation of this problem is: Max, $z = 400x_1 + 100x_2$, subject to

$4x_1 + 2x_2 \leq 1600$, $2.5x_1 + x_2 \leq 1200$, $4.5x_1 + 1.5x_2 \leq 1600$; $x_1, x_2 \geq 0$]

8. **A manufacture produces two different models, X and Y of the same product. The raw material r_1, $r_2$are required for production. At least 18 kg of r_1 and 12 kg of r_2 must be used daily. Also, at most 34 hours of labor are to be utilized. 2 kg of r_1'are needed for model X and 1kg of r_1 for each model Y.**

For each model of X and Y, 1 kg of r_2 is required. It takes 3 hours manufacture a model X and 2 hours to manufacture a model Y. The profit is Rs. 50 for each model X and Rs. 30 for each model Y. How many units of each model should be produced to minimize the profit?

[Hint. The formulation of the problem is;

Max $z = 50x_1 + 30x_2$, st. $2x_1 + x_2 \geq 18$, $x_1 + x_2 \geq 12$, $3x_1 + 2x_2 \leq 34$, $x_1 \geq 0$, $x_2 \geq 0$.

WORKSHEET -3 (On Graphical method Of Solution & Types of Solutions). ***OPTIMIZATION THEORYI.***

I. Solve the following LP problems by Graphical method.

1) $Min.z = 5x_1 - 2x_2$, $Subjects\, to: 2x_1 + 3x_2 \geq 1$; $x_1, x_2 \geq 0.$

2) $Max.z = 5x_1 + 3x_2$, $Subjects\, to: 3x_1 + 5x_2 \leq 15$; $5x_1 + 2x_2 \leq 10$; $x_1, x_2 \geq 0$

3) $Max.z = 2x_1 + 3x_2$, $Subjects\, to: x_1 + x_2 \leq 1$; $3x_1 + 1x_2 \leq 4$; $x_1, x_2 \geq 0$

4) $Min.z = -x_1 + 2x_2$, $Subjects\, to: -x_1 + 3x_2 \leq 10$; $x_1 + x_2 \leq 6$; $x_1 - x_2 \leq 2$; $x_1, x_2 \geq 0$

5) $Max.z = 3x_1 + 4x_2$, $Subjects\, to: x_1 - x_2 \leq 1$; $-x_1 + x_2 \leq 0$; $x_1, x_2 \geq 0$

6) $Max.z = x_1 + 3x_2$, $Subjects\, to: 3x_1 + 6x_2 \leq 8$; $5x_1 + 2x_2 \leq 10$; $x_1, x_2 \geq 0$

7) $Max.z = 7x_1 + 3x_2$, $Subjects\, to: x_1 + 2x_2 \geq 3$; $x_1 + x_2 \leq 4$; $0 \leq x_1 \leq \frac{5}{2}$; $0 \leq x_2 \leq \frac{3}{2}.$

8) $Max.z = x_1 + 2x_2$, $Subjects\, to: 2x_1 - x_2 \geq -2$; $x_1 + 2x_2 \geq 8$; $x_1, x_2 \geq 0$

9) $Find\ the\ Maximum\ \&\ Minimum\ value\ of$, $z = 5x_1 + 3x_2$ $subject to: x_1 + 2x_2 \leq 6$;
$2x_1 + 3x_2 \geq 3$; $0 \leq x_1 \geq 0$; $0 \leq x_2 \geq 3.$

II. Answer The Following Question Clearly.

10. Establish the difference between : i) Feasible Solution , ii) Basic Feasible Solution, iii) Degenerate Basic Feasible Solution.

11. a) Define a basic solution to a given system of m simultaneous linear equations in n unknowns.

b) How many Basic Feasible Solution are thereto a given system of 3 simultaneous linear equations in 4 unknowns.

12. Define the following terms.

a) Basic Variable. b) Basic Solution. c) Basic Feasible Solution. d) Degenerate Solution.

13. What do you mean by two-phase method for solving a given LPP. Why is it used.

14. What are the various methods known to you for solving LP problems?

15. Name the 3 basic parts of the simplex technique?

16. In the course of simplex table calculations , describe how you will detect a degenerate , un unbounded and non existing feasible solutions.

17. What is degeneracy in simplex? Solve the following LPP using simplex:

$Max.z = 3x_1 + 9x_2$, $Subjects\, to: 4x_1 + 4x_2 \leq 8$; $x_1 + x_2 \leq 4, and\ x_1, x_2 \geq 0$

18. With reference to the solution of LPP by simple method/table when do you conclude as follows: a) LPP has multiple solutions, b) LPP has no limit for the improvement of the objective function, c) Lpp has no feasible solution.

WORKSHEET: 4 (On Simplex Algorithm). OPTIMIZATION THEORY I.

I. Solve the following Linear Programming Problems using Simplex Method

1. $Max.z = 2x + y$

   $subject \quad to: x + 4y \le 24, \quad x + y \le 9, \quad x - y \le 3, \quad x - 2y \le 2, \quad and \quad x \ge 0, y \ge 0$

2. $Max.z = -3x + 4y$

   $subject \quad to: 3x - 4y \le 24, \quad 5x + 4y \le 36, \quad -x + 3y \le 8, \quad -3x + y \le 0, \quad and \quad x \ge 0, y \ge 0$

3. $Max.z = x + 2y$

   $subject \quad to: -2x + y \le 1, \quad -x + y \le 2, \quad x + y \le 6, \quad 2x - 3y \le 2, \quad and \quad x \ge 0, y \ge 0$

4. $Max.z = -3x + 4y + 5z$

   $subject \quad to: 5x + -4y - 8z \le 40, 4x + y + 2z \le 40, x + 4y + z \le 50, 3x + 3y + 4z \le 60, and \quad x \ge 0, y, z \ge 0 \ge 0$

5. $Max.z = 5x_1 + 7x_2$

   $subject \quad to: x_1 + x_2 \le 4, 3x_1 - 8x_2 \le 24, 10x_1 + 7x_2 \le 35 \ and \quad x_1, x_2 \ge 0$

6. $Max.z = 3x_1 + 2x_2$

   $subject \quad to: 2x_1 + x_2 \le 40, x_1 + x_2 \le 24, 2x_1 + 3x_2 \le 60 \ and \quad x_1, x_2 \ge 0$

7. $Max.z = 3x_1 + 2x_2$

   $subject \quad to: 2x_1 + x_2 \le 5, x_1 + x_2 \le 3, and \quad x_1, x_2 \ge 0$

8. $Max.z = 2x_1 + 4x_2$

   $subject \quad to: 2x_1 + 3x_2 \le 48, x_1 + 3x_2 \le 42, x_1 + x_2 \le 21 \ and \quad x_1, x_2 \ge 0$

9. $Max.z = 3x_1 + 4x_2$

   $subject \quad to: x_1 - x_2 \le 1, -x_1 + x_2 \le 2, \ and \quad x_1, x_2 \ge 0$

10. $Max.z = 3x_1 + 2x_2$

    $subject \quad to: 2x_1 + x_2 \le 10, x_1 + 3x_2 \le 6 \ and \quad x_1, x_2 \ge 0$

11. $Max.z = 2x_1 + 5x_2$

$subject \quad to: x_1 + 3x_2 \le 3, 3x_1 + 2x_2 \le 6,\ and \quad x_1, x_2 \ge 0$

12. $Max.z = 3x_1 + 5x_2$

$subject \quad to: 3x_1 + 2x_2 \le 18, x_1 \le 4, x_2 \le 6\ and \quad x_1, x_2 \ge 0$

13. $Max.z = 2x_1 + x_2$

$subject \quad to: x_1 + 2x_2 \le 10, x_1 + x_2 \le 6, x_1 - x_2 \le 2, x_1 - 2x_2 \le 1 and \quad x_1, x_2 \ge 0$

14. $Max.z = 8x_1 + 19x_2 + 7x_3$

$subject \quad to: 3x_1 + 4x_2 + x_3 \le 25, x_1 + 3x_2 + 3x_3 \le 50\ and \quad x_1, x_2, x_3 \ge 0$

15. $Max.z = x_1 + x_2 + 3x_3$

$subject \quad to: 3x_1 + 2x_2 + x_3 \le 3, 2x_1 + x_2 + 2x_3 \le 2\ and \quad x_1, x_2, x_3 \ge 0$

16. $Max.z = 4x_1 + 3x_2 + 4x_3 + 6x_4$

$subject \quad to: x_1 + 2x_2 + 2x_3 + 4x_4 \le 80, 2x_1 + 2x_3 + x_4 \le 60, 3x_1 + 3x_2 + x_3 + x_4 \le 80 and \quad x_1, x_2, x_3, x_4 \ge 0$

17. $Max.z = 2x_1 + 4x_2 + x_3 + 4x_4$

$subject \quad to: x_1 + 3x_2 + x_4 \le 4, 2x_1 + x_2 \le 3, x_2 + 4x_3 + x_4 \le 3 and \quad x_1, x_2, x_3, x_4 \ge 0$

18. $Max.z = 8x_1 + 11x_2$

$subject \quad to: 3x_1 + x_2 \le 7, x_1 + 3x_2 \le 8,\ and \quad x_1, x_2 \ge 0$

II. Solve The following LP Problems by Two-Phase Method.

20. $Max.z = 3x_1 - x_2$

$subject \quad to: 2x_1 + x_2 \ge 2, x_1 + 3x_2 \le 2,\ x_2 \le 4 and \quad x_1, x_2 \ge 0$

21. $Max.z = 5x_1 + 8x_2$

$subject \quad to: 3x_1 + 2x_2 \ge 3, x_1 + 4x_2 \ge 4,\ x_1 + x_2 \le 5\ and \quad x_1, x_2 \ge 0$

22. $Max z = 3x_1 + 2x_2 + x_3 + 4x_4$

$subject\ \ to: 4x_1 + 5x_2 + x_3 - 3x_4 = 5, 2x_1 - 3x_2 - 4x_3 + 5x_4 = 7, x_1 + 4x_2 + 2.5x_3 - 4x_4 = 6\ and\ \ x_1, x_2, x_3, x_4 \geq 0$

23. $Max z = 2x_1 + 3x_2 + 5x_3$

$subject\ \ to: 3x_1 + 10x_2 + 5x_3 \leq 15, x_1 + 2x_2 + x_1 \geq 4, 33x_1 - 10x_2 + 9x_3 \leq 33\ and\ \ x_1, x_2, x_3 \geq 0$

III. Solve The following LP Problems (Use Artificial Variables if necessary).

24. $Max.z = 5x_1 + 3x_2$

$subject\ \ to: x_1 + x_2 \leq 2, 5x_1 + 2x_2 \leq 10\ , 3x_1 + 8x_2 \leq 12, and\ \ x_1, x_2 \geq 0$

25. $Max.z = 3x_1 + 5x_2$

$subject\ \ to: x_1 + x_3 = 4,\ x_2 + x_4 = 6\ , 3x_1 + 2x_2 + x_5 = 12\ and\ \ x_1, x_2, x_3, x_4, x_5 \geq 0$

26. $Max.z = 2x_1 + 3x_2 + 10x_3$

$subject\ \ to: x_1 + 2x_3 = 1,\ x_2 + x_3 = 1\ ,\ \ and\ \ x_1, x_2, x_3 \geq 0$

27. $Max.z = 2x_1 + 3x_2 + 10x_3$

$subject\ \ to: x_1 + 2x_3 = 0,\ x_2 + x_3 = 1\ ,\ \ and\ \ x_1, x_2, x_3 \geq 0$

28. a) Compute all the basic feasible solution of the LP problem : $Max.z = 2x_1 + 3x_2 + 4x_3 - 7x_4$, $Subjects\,to: 2x_1 + 3x_2 - x_3 + 4x_4 = 8\ ;\ x_1 - 2x_2 + 6x_3 - 7x_4 = -3$, **and choose that one which maximizes z.**

b) Find the maximum and minimum value of :

$z = 5x_1 + 3x_2$, $Subjects\,to: x_1 + x_2 \leq 6\ ;\ 2x_1 + 3x_2 \geq 3, 0 \leq x_1 \geq 3, 0 \leq x_2 \geq 3.$

WORKSHEET: 5 (*Dual Simplex Algorithm*). *OPTIMIZATION THEORY I.*

I. Obtain The Dual Of The Following LP Problems.

1. $Max.z = 3x_1 + 4x_2$; $subject\ \ to: 2x_1 + 6x_2 \le 10,\ 5x_1 + 2x_2 \ge 20$, $and\ \ x_1, x_2 \ge 0$
2. $Max.z = x_1 - x_2 + 3x_3$; $subject\ to: x_1 + x_2 + x_3 \le 10,\ 2x_1 - x_3 \le 2,\ 2x_1 - 2x_2 + 3x_3 \le 6\ and\ x_1, x_2, x_3 \ge 0$
3. $Min.z = 7x_1 + 3x_2 + 8x_3$

   $subject\ \ to: 8x_1 + x_2 + x_3 \ge 3,\ 3x_1 + 6x_2 + 4x_3 \ge 4,\ 4x_1 + x_2 + 5x_3 \ge 1,\ x_1 + 5x_2 + 2x_3 \ge 7\ and\ \ x_1, x_2, x_3 \ge 0$

II. Use Duality *In Obtaining An Optimal Solution , if any for the following* LP Problems.

4. $Max.z = 8x_1 + 6x_2$; $subject\ \ to: x_1 - x_2 \le \frac{3}{5},\ x_1 - x_2 \ge 2$, $and\ \ x_1, x_2 \ge 0$
5. $Max.z = 2x_1 + x_2$; $subject\ \ to: x_1 + x_2 \ge 2,\ x_1 + 3x_2 \le 3$, $and\ \ x_1, x_2 \ge 0$
6. $Min.z = -2x_1 + 3x_2 + 4x_3$;

   $subject\ \ to: -2x_1 + x_2 \ge 3,\ -x_1 + 3x_2 + x_3 \ge -1$, $and\ \ x_1, x_2, x_3 \ge 0$

III. Use Dual Simplex Method To Solve The Following LP Problems.

7. $Max.z = -2x_1 - 2x_2 - 4x_3$; $subject\ \ to: 2x_1 + 3x_2 + 5x_3 \ge 2,\ 3x_1 + x_2 + 7x_3 \le 3\ and\ \ x_1, x_2, x_3 \ge 0$
8. $Min.z = 80x_1 + 60x_2 + 80x_3$

   $subject\ \ to: x_1 + 2x_2 + 3x_3 \ge 4,\ 2x_1 + 3x_3 \ge 3,\ 2x_1 + 2x_2 + x_3 \ge 4,\ 4x_1 + x_2 + x_3 \ge 6\ and\ \ x_1, x_2, x_3 \ge 0$
9. $Max.z = -3x_1 - 2x_2$; $subject\ to: x_1 + x_2 \ge 1,\ x_1 + x_2 \le 7,\ x_1 + 2x_2 \le 10,\ x_2 \le 3\ and\ x_1, x_2 \ge 0$
10. $Min.z = 6x_1 + x_2$; $subject\ \ to: 2x_1 + x_2 \ge 3,\ x_1 - x_2 \ge 0\ \ and\ \ x_1, x_2 \ge 0$
11. $Min.z = x_1 + x_2$; $subject\ \ to: 2x_1 + x_2 \ge 2,\ -x_1 - x_2 \ge 1\ \ and\ \ x_1, x_2 \ge 0$
12. $Min.z = 3x_1 + 2x_2 + x_3 + x_4$

    $subject\ \ to: 2x_1 + 4x_2 + 5x_3 + x_4 \ge 10,\ 3x_1 - x_2 + 7x_3 - 2x_4 \ge 2,\ 5x_1 + 2x_2 + x_3 + 6x_4 \ge 15\ and\ \ x_1, x_2, x_3, x_4 \ge 0$
13. $Max.z = -4x_1 - 6x_2 - 18x_3$; $subject\ \ to: x_1 + 3x_3 \ge 3,\ x_2 + 2x_3 \ge 5\ and\ \ x_1, x_2, x_3 \ge 0$
14. $Min.z = 2x_1 + 2x_2$; $subject\ \ to: x_1 + 2x_2 \ge 1,\ 2x_1 + x_2 \ge 1,\ x_1 \ge 0\ and\ x_2 \ge 0$

WORKSHEET: 6 (Sensitivity Analysis). OPTIMIZATION THEORY I.

1. *In a LPP,* $Max.z = 3x_1 + 5x_2$, *Subject to:* $x_1 + x_2 \le 1, 2x_1 + 3x_2 \le 1$ *and* $x_1, x_2 \ge 0$
   obtain the variation in c_j *which are permitted without changing the optimal solution.*

2. *Given a LPP:* $Max.z = -x_2 + 3x_3 - 2x_5$
   Subject to: $3x_2 - x_3 + 2x_5 \le 7, -2x_2 + 4x_3 \le 12, -4x_2 + 3x_3 + 8x_5 \le 10$ *and* $x_2, x_3, x_5 \ge 0$
   The optimal table of this problem is:–

		$C_j \rightarrow$	0	-1	3	0	-2	0
Basic Var.	C_B	X_B	X_1	X_2	X_3	X_4	X_5	X_6
X_2	-1	4	$\frac{2}{5}$	1	0	$\frac{1}{10}$	$\frac{4}{5}$	0
X_3	-3	5	$\frac{1}{5}$	0	1	$\frac{3}{10}$	$\frac{2}{5}$	0
X_6	0	11	1	0	0	$\frac{-1}{2}$	10	1
	$z = 11$		$\frac{1}{5}$	0	0	$\frac{4}{5}$	$\frac{12}{5}$	0 $\leftarrow \Delta_j$

   a) *Formulate the dual problem for this primal problem*
   b) *What are optimal values of dual variables?*
   c) *How much* c_5 *be decreased before* x_2 *goes into basis?*
   d) *How much can the 7 in first constraint be increased before the basis would change?*

3. *Given a LPP:* $Max.z = 10x_1 + 3x_2 + 6x_3 + 5x_4$
   Subject to: $x_1 + 2x_2 + x_4 \le 6, 3x_1 + 2x_3 \le 5, x_2 + 4x_3 + 5x_4 \le 3$ *and* $x_1, x_2, x_3, x_4, x_5 \ge 0$
   a) *Determine an optimum solution to the problem*
   b) *If element* a_{11} *is changed to* $a_{11} + \Delta a_{11}$, *determine the limits for descrite change* Δa_{11} *so as to maintain the optimality of the current optimum solution.*