Course Title:	Linear optimization	**Code**:	Math 2062
ECTS:	5	Cr. Hrs.	3
Course category:	Core	**Pre-requisites:**	Math 2041

Course Objectives:
On completion of the course, successful students will be able to:

- understand the interplay between geometry and linear algebra,
- define a linear programming,
- understand matrix algebra,
- solve systems of linear equations,
- understand the fundamental principles of linear programming,
- formulate optimization problems,
- understand decision process with respect to an optimization problem,
- solve linear programs graphically,
- test convexity of sets,
- understand theorems and algorithms of the simplex method,
- solve linear programming problems by the simplex method,
- understand duality theorems,
- apply theorems and algorithms in duality theory,
- do sensitivity analysis,
- comprehend the transportation problem,
- solve transportation problems,
- solve pure strategy games.

Course Contents

Chapter 1: Introduction to matrices
- 1.1 Introduction (definition of LP, Motivation, …)
- 1.2 Matrices (Rank, Elementary row operations)
- 1.3 System of linear equations
  - 1.3.1 $n \times n$ system: Gaussian Elimination Method
  - 1.3.2 $m \times n$ system: Basic solutions
- 1.4 System of linear inequalities

Chapter 2: Linear programming models of practical problems
- 2.1 Introduction
- 2.2 Decision process and relevance of optimization
- 2.3 Model and model building

Chapter 3: Geometric methods
- 3.1 Graphical solution methods
- 3.2 Convex sets
- 3.3 Polyhedral sets and extreme points
- 3.4 The Corner Point theorem

Chapter 4: The Simplex method
- 4.1 Linear programs in standard form
- 4.2 Basic feasible solutions
- 4.3 Fundamental theorem of linear programming
- 4.4 Algebra of the simplex method
- 4.5 Optimality test and basic exchange
- 4.6 The simplex algorithm
- 4.7 Degeneracy and finiteness of simplex algorithm
- 4.8 Finding a starting basic feasible solution
  - 4.8.1 The two-Phase method
  - 4.8.2 The Big-M method
- 4.9 Using solver (MS EXCEL) in solving linear programming

Chapter 5: Duality theory and further variations of the simplex method
- 5.1 Dual linear programs
- 5.2 Duality theorems
- 5.3 The dual simplex method

5.4 The Primal-Dual simplex method

Chapter 6: Sensitivity analysis

6.1 Introduction
6.2 Variation of coefficients of objective function
6.3 Variation of vector requirement
6.4 Variation of constraints
6.5 Addition of new constraints or variables
6.6 Solver outputs and interpretations

Chapter 7: Interior point methods

7.1 Basic ideas
7.2 One iteration of Karmarkar's projective algorithm
7.2.1 Projective transformation
7.2.2 Moving in the direction of steepest descent
7.2.3 Inverse transformation
7.3 The algorithm and its polynomiality
7.4 A purification scheme
7.5 Converting a given LP into the required format

Chapter 8: Transportation problem & Theory of games

8.1 Introduction
8.2 Transportation table
8.3 Determination of an initial basic feasible solution
8.3.1 North-West corner rule
8.3.2 Row minima rule
8.3.3 Cost minima rule
8.4 Optimality conditions
8.5 Unbalanced transportation problems and their solutions
8.6 Degenerate transportation problems and their solutions
9.1 Introduction to Theory of games
9.2 Formulation of two-person zero-sum games
9.3 Pure and mixed strategies
9.4 Solving pure strategy games
9.4.1 Reduction by dominance
9.4.2 The minimax (or maxmin) criterion
9.5 Some basic probabilistic considerations
9.6 Solving games with the simplex methods
9.7 $2 \times n$ and $m \times 2$ games

Teaching & learning Methods Questioning and answering, Lecture, Pair discussion, Class Work, Gapped Lecture, etc

Assessment/Evaluation & Grading System

1. Quiz (1) …….5%
2. Assignment (1) ……..……..15%
3. Test (1&2)…….30%
4. Final Exam ……...50%

Total……...100%

Attendance Requirements 85%

References:

- Bertsimas and J. Tsitsiklis, **Introduction to linear optimization**, Athena Scientific, 1997
- Brian D. Bunday, **Basic linear programming**, Edward Arnold, 1984
- H. A. Taha, **Operations research, an introduction**, Macmillan publishing company, 2002
- F. S. Hillier and G. J. Lieberman, **Introduction to Operation research**, Holde-day, 2001
- Robert Fourer, David M. Gay, and Brian W. Kernighan, **A modeling language for mathematical programming**, Boyd & Fraser publishing company, 1997
- R. J. Vanderbei, **Linear programming: Foundations and extensions**, 2001