BA In Management

Course Material for Managerial Economics (MGMT331)

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CHAPTER ONE

1. SCOPE AND NATURE OF MANAGERIAL ECONOMICS

Objective

- *To introduce and define managerial economics*
- *To explain the nature and scope of managerial economics*
- *To outline the types of issue which are addressed by managerial economics*

1.1. What is Managerial Economics?

Managerial economics is the application of economic theory and quantitative methods (mathematics and statistics) to the managerial decision-making process. Simply stated, managerial economics is applied microeconomics with special emphasis on those topics of greatest interest and importance to managers. Managerial economics is the synthesis of microeconomic theory and quantitative methods to find optimal solutions to managerial decision-making problems.

Managerial economics applies economic theory and methods to business and administrative decision making. Managerial economics prescribes rules for improving managerial decisions. Managerial economics also helps managers to recognize how economic forces affect organizations and describes the economic consequences of managerial behaviour.

Managerial economics refers to the application of economic theory and the tools of analysis of decision science to examine how an organization can achieve its aims or objectives most efficiently. It links traditional economics with the decision sciences to develop vital tools for managerial decision-making. Managerial economics identifies ways to efficiently achieve goals. For example, suppose a small business seeks rapid growth to reach a size that permits efficient use of national media advertising. Managerial economics can be used to identify pricing and production strategies to help meet this short-run objective quickly and effectively.

Many different definitions have been given but most of them involve the application of economic theory and methods to business decision-making. As such it can be seen as a means to an end by managers, in terms of finding the most efficient way of allocating their scarce resources and reaching their objectives. As an approach to decision-making, managerial economics is related to economic theory, decision sciences and business functions. These relationships can be discussed.

- **Relationship with economic theory**

The main branch of economic theory with which managerial economics is related is microeconomics, which deals essentially with how markets work and interactions between the various components of the economy. In particular, the following aspects of microeconomic theory are relevant:

- theory of the firm
- theory of consumer behaviour (demand)
- production and cost theory (supply)
- price theory
- market structure and competition theory

- **Relationship with decision sciences**

The decision sciences provide the tools and techniques of analysis used in managerial economics. The most important aspects are as follows:

- numerical and algebraic analysis
- optimization
- statistical estimation and forecasting
- analysis of risk and uncertainty
- discounting and time-value-of-money techniques

These tools and techniques are introduced in the appropriate context, so that they can be immediately applied in order to understand their relevance, rather than being discussed in block or in isolation.

- **Relationship with business functions**

All firms consist of organizations that are divided structurally into different departments or units, even if this is not necessarily performed on a formal basis. Typically the units involved are:

- production and operations
- marketing
- finance and accounting
- human resources

All of these functional areas can apply the theories and methods mentioned earlier, in the context of the particular situation and tasks that they have to perform. Thus a production department may want to plan and schedule the level of output for the next quarter, the marketing department may want to know what price to charge and how much to spend on advertising, the finance department may want to determine whether to build a new factory to expand capacity, and the human resources department may want to know how many people to hire in the coming period and what it should be offering to pay them. It might be noted that all the above decisions involve some kind of quantitative analysis; not all managerial decisions involve this kind of analysis. There are some areas of decision-making where the tools and techniques of managerial economics are not applicable. The meaning of this relationship can be seen in the figure below.

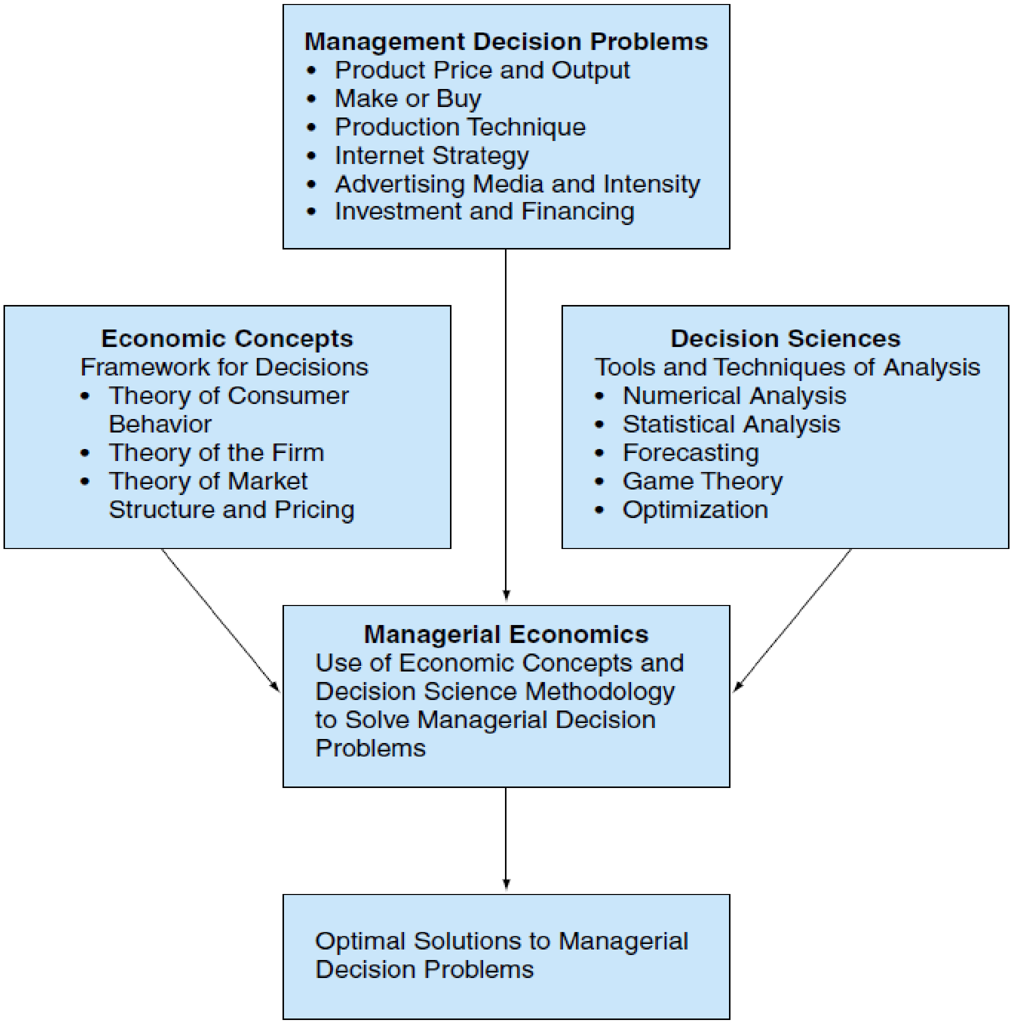


Figure 1.1. Nature of Managerial Economics

1.2. Scope of Managerial Economics

Two major managerial functions served by subject matter under managerial economics are;

i) Decision making

a. *Decision related to demand* – a manager has to take decisions regarding the quality and quantity of his product. For taking such decisions, a number of economic tools can be applied. Such as demand, elasticity of demand, demand forecasting etc.

b. *Decisions related to cost and production* **–** a manager has to analyse the cost and the types of cost he has to incur in the production process. He has to know the different laws governing production.

c. *Decisions related to price and market* **–** market analysis is part of managerial economics, as a manager should have the knowledge of various market structures and how price output is determined under the different types of market structure.

d. *Decision related to profit management* – some policies and theories related to profit management should be included.

e. *Macroeconomic factor* **–** a firm never works in isolation, so understanding of macro level factors by any business man is important. E.g. business cycle and inflation

ii) Forward planning – the other managerial function served by managerial economics is planning in which different economic theories like demand forecasting help the manager to appropriately plan for the firm.

Self Assessment Exercises

1. What is the relationship between the field of managerial economics and

   (a) Economic theories like microeconomics and macroeconomics?

   (b) Decision sciences?

   (c) The fields of accounting, finance, marketing, human resource, and production?

2. Define Managerial Economics using your own terms.
3. Explain the scope of Managerial Economics.
4. Explain how managerial economics is similar to and different from microeconomics.

5. Why is the subject of managerial economics relevant to the problem of decision making?
6. Discuss the nature and scope of managerial economics

CHAPTER TWO

FUNDAMENTAL ECONOMIC CONCEPTS

Objective

- *To introduce Equilibrium analysis*
- *To explain the effect of change in demand and supply on equilibrium price and quantity*
- *To explicate marginal analysis*
- *To portray time value of Money*

Introduction

Economic theory offers a variety of concepts and analytical tools which can be of considerable assistance to the managers in his decision making practice. These tools are helpful for managers in solving their business related problems. These tools are taken as guide in making decision. *Economic problems* involve tradeoffs.

An economic problem can be illustrated in the pricing decisions by Ethiopian Airlines whether or not to reallocate unsold first class seats to the discount status, if it is unsure whether the first class seats will ever be sold. This unit provides some of the basic tools of managerial economics.

2.1. Equilibrium Analysis: Demand and Supply Relationships

Successful managers understand how market forces create both opportunities and constraints for profitable decision making. Such managers understand the way markets work, and they are able to predict the prices and production levels of the goods, resources, and services that are relevant to their businesses.

Even though demand and supply analysis is deceptively simple to learn and apply, it is widely used by highly experienced-and well-paid-market analysts and forecasters.

Demand and supply analysis applies principally to markets characterized by many buyers and sellers and marketers in which a homogeneous or relatively not-differentiated good/service is sold. Such marketers are called competitive market.

2.1.1. Meaning of Demand

Demand in economics means effective demand that is one which meets with all its three crucial characteristics; *desire to have a good*, *willingness to pay for that good*, and *the ability to pay for that good*. In the absence of any of these three characteristics, there is no demand.

Factors that affect the Demand Function

Though economists emphasize the importance of price in purchasing decisions, they also recognize that a multitude of factors other than price affect the amount of a good or service people will purchase.

This section develops two types of demand relations:

(i). ***Generalized demand function***: which show how quantity demanded is related to product price and five other factors that affect demand, and

(ii). ***Ordinary demand function***: which show the relation between quantity demanded and the price of the product when all other variables affecting demand are held constant at specific values, it is derived from generalized demand functions.

The Generalized Demand Function

The six principal variables that influence the quantity demanded of a good or services are:

(1). the price of the good or service, (P),

(2). the incomes of consumers, (M),

(3). the prices of related goods and services, (P_R),

(4). the tastes or preference patterns of consumers, (T),

(5). the expected price of the product in the future periods, (P_e), and

(6). The number of consumers in the market (N).

The relation between quantity demanded and these six factors is referred to as the generalized demand function and is expressed as follows:

$$Q_d = f\ (P, M, P_R, T, P_e, \text{and } N)$$

Specifically;

$$Q_d = a + bP + cM + dP_R + eT + fP_e + gN$$

Where 'a' is the intercept parameter; it shows the value of Q_d when the variables P, M, P_R, T, P_e, and N are simultaneously equal to zero. The other parameters '*b, c, d, e, f,* and *g*' are slope parameter. i.e., parameters in a linear function that measure the effect on the dependent variable (Q_d) of changing one of the independent variables (P, M, P_R, T, P_e, and N) while holding the rest of these variables constant.

Summary of the Generalized (Linear) Demand Function:

$$Q_d = a + bP + cM + dP_R + eT + fP_e + gN$$

Variable	Relation to Quantity demanded	Sign of Slope Parameter
P	Inverse	$b = \Delta Q_d/\Delta P$ is negative
M	Direct for normal goods	$c = \Delta Q_d/\Delta M$ is positive
	Inverse for inferior goods	$c = \Delta Q_d/\Delta M$ is negative
P_R	Direct for substitute goods	$d = \Delta Q_d/\Delta P_R$ is positive
	Inverse for complement goods	$d = \Delta Q_d/\Delta P_R$ is negative
T	Direct	$e = \Delta Q_d/\Delta T$ is positive
P_e	Direct	$f = \Delta Q_d/\Delta P_e$ is positive
N	Direct	$g = \Delta Q_d/\Delta N$ is positive

2.1.2. Meaning of Supply

In the economic sense supply refers to the quantities that people are or would be willing to sell at different prices during a given time period, assuming that other factors affecting these quantities remain the same. When talking about the supply of products it is often the costs of production that are most important in determining the supply relationship, and generally there is a direct relationship between the quantity supplied and the price offered, with more being supplied the higher the price. However, in factor markets, in particular the labour market, supply is more complex. The availability of people with the relevant skills, the pleasantness of the work and the opportunity cost involved are all important factors.

As in the case of demand, economists ignore all the relatively unimportant variables in order to concentrate on those variables that have the greatest effect on quantity supplied. In general, economists assume that the quantity of a good offered for sale depends upon six major variables:

(1). the price of the good itself, (P),

(2). the price of the inputs used to produce the good , (P_I),

(3). the prices of goods related in production, (P_r),

(4). the level of available technology, (*T*),

(5). the expectation of the producers concerning the future price of the good, (P_e), and

(6). the number of firms or the amount of productive capacity in the industry (F)

The Generalized Supply Function

The generalized supply function shows how all six of these variables jointly determine the quantity supplied. It expressed as:

$$Q_s = g\ (P, P_I, P_r, T, P_e, \text{and } F)$$

Specifically;

$$Q_s = h + kP + lP_I + mP_r + nT + rP_e + sF$$

Summary of the Generalized (Linear) Supply Function:

$$Q_s = h + kP + lP_I + mP_r + nT + rP_e + sF$$

Variable	Relation to Quantity supplied	Sign of Slope Parameter

P	Direct	$k = \Delta Q_s/\Delta P$ is positive
P_I	Inverse	$l = \Delta Q_s/\Delta P_I$ is negative
P_r	Inverse for substitutes in production	$m = \Delta Q_s/\Delta P_r$ is negative
	Direct for complement in production	$m = \Delta Q_s/\Delta P_r$ is positive
T	Direct	$n = \Delta Q_s/\Delta T$ is positive
P_e	Inverse	$r = \Delta Q_s/\Delta P_e$ is negative
F	Direct	$s = \Delta Q_s/\Delta F$ is positive

2.1.3. Equilibrium Conditions

The intersection of the supply (SS) and demand (DD) curves at point E reflects an equilibrium price (P*) and quantity (Q*). This is the price and quantity that should prevail in the market place, given these supply and demand curves. The reason why this is true can be demonstrated with the following examples.

Suppose that the market prices were temporarily at price below P*. This price is represented by P_1 in Figure 2.1. In this situation, consumers would demand Q_1D but producers would only be willing to supply Q_1S. Since the quantity demanded exceeds the quantity supplied, some consumers will attempt to bid the price up. This results in upward price pressure in the direction of the equilibrium level

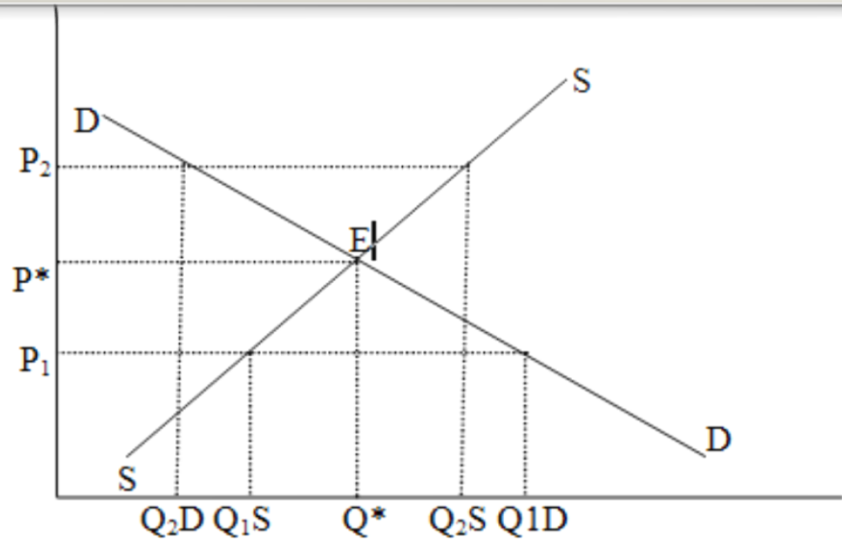


Figure 2.1 Supply and Demand Equilibrium Conditions

For example, during much of the decade of the 1980s, limits were set on the number of Japanese autos that could be imported into the United States. The list prices of these vehicles were often set below market equilibrium levels, which resulted in many dealers charging prices of $500 to $2000 above the list price.

Thus, the willingness of consumers to purchase more than was being supplied at the sticker prices caused upward price pressure toward an equilibrium price level, where the quantity demanded equals the quantity supplied.

Similarly, If the price is temporarily set above the equilibrium price P*, price pressure would be downward toward the equilibrium. For example, at a price P_2, the quantity demanded Q_2D would be less than the quantity producers are willing to supply Q_2S. Because of this surplus of supply, buyers will put pressure on sellers to lower prices. Thus, there is pressure for the price to return to an equilibrium level.

As an example, during much of the mid-to-late 1980s, the prices American car manufacturers and dealers placed on American-made cars were often perceived as being too high, relative to the demand for these vehicles. As a result, inventories of unsold cars built up at dealers. In order to move those inventories, manufacturers resorted to a seemingly endless stream of cash rebates and special financing offers;

Manufactures had misread the market demand and produced (supplied) more cars than consumers were willing to purchase at the listed prices. The resulting rebates brought the quantity demanded in line with the quantity supplied. At the same time, many manufacturers reduced output in order to avoid future occurrences of this problem.

In summary, the intersection of the supply and demand curves represents an equilibrium level of price and quantity. Once this equilibrium has been established, there is no economic incentive for consumers or producers to move from this position unless the other conditions affecting supply and demand causes a shift in the supply and demand curves.

Effects of an Increase or Decrease in Demand

The equilibrium level of price and quantity was determined in Figure 2.2 by the intersection of the current supply and demand curves. The equilibrium point is of some interest in its own right. The primary use of supply and demand analysis, however, is to examine the effects of changes in the factors affecting the supply and demand curves on the equilibrium price and quantity. The method of comparative statics in economies compares the beginning and final equilibrium points resulting from a change in supply and/or demand. The process of how the final equilibrium point is achieved is the concern of economic dynamics. The study of economic dynamics is beyond the scope of this chapter.

Consider, for example, the situation in Figure 2.2. The initial demand curve is DD and the supply curve is SS. At equilibrium (point E) the price is P* and the quantity demanded is Q*. Now suppose that there is suddenly an increase in demand caused by a change in consumer tastes or preferences, an increase in consumer income level, an increase in the price of a substituted good, or some similar factor. The demand curve DD will shift upward to the right and become D^1D^1, and new equilibrium will be established at point E^1, with a price of P_1* and a quantity Q_1*.

How is this new equilibrium established?

When demand shifts from DD to D^1D^1; consumers will demand Q_1 units of the commodity, but suppliers only will want to supply Q* units at the current price P*.

Thus, there will be pressure for a price increase due to the excess demand relative to supply. The price will ultimately be bid up to the new equilibrium level P*, and a new equilibrium quantity will be supplied and consumed at Q_1*

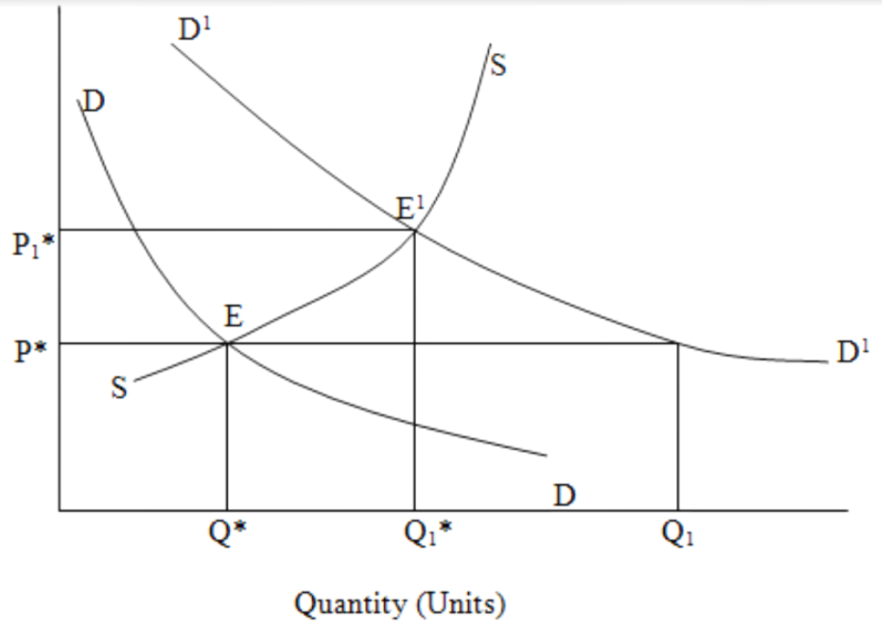


Figure 2.2 Effects of an Increase or Decrease in Demand

Thus, an increase in demand leads to an increase in both the equilibrium price and quantity. Similarly, a decrease in demand can be expected to lead to a decrease in the equilibrium price and quantity. For example, if the Soviet Union experiences an especially poor grain harvest, it will be forced to buy grain on the world market.

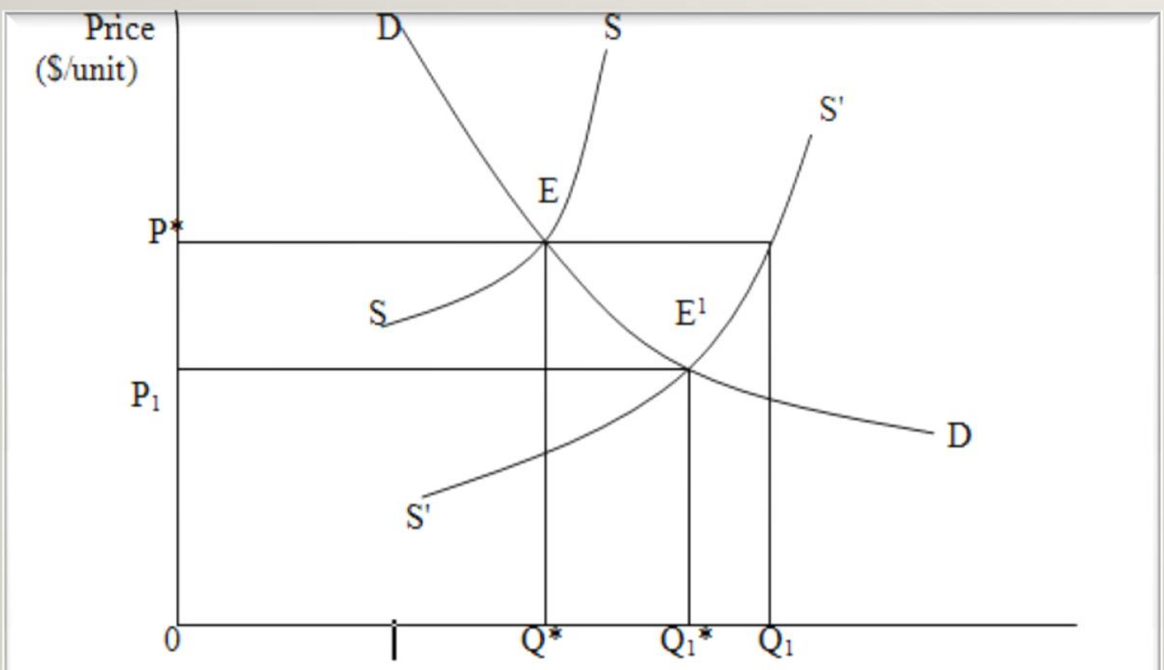
From the perspective of an American farmer, this is an increase in the demand for U.S. grain and it causes a shift in the demand curve upward and to the right. The effect of this increase in demand can be expected to be a higher market price for American grain and a greater level of output.

In summary, while holding constant the effects of all other factors, an increase in demand leads to a new equilibrium point where both the equilibrium price and quantity are higher than they were at the original equilibrium point. In contrast, a decrease in demand results in a new equilibrium point where price and quantity are lower than they were at the original equilibrium point.

Effects of an Increase or Decrease in Supply

An increase in supply can be represented by a shift in the supply curve to the right. For example, in Figure 2.3 the initial equilibrium point E occurs at the intersection of the initial supply curve SS and the initial demand curve DD.

Supply may increase because of many factors including a change in technology, such as the development of the computer chip that greatly increases the supply of micro computers, a change in the amount of an available resource, such as a major oil discovery, or a change in the price or quantity of a related good, such as an increase in the price of cotton, which would surely increase the supply of cotton seed oil.



igure 2.3 Effects of an Increase or Decrease in Supply

The rules of supply and demand analysis affect all markets, including the market for drugs. When cocaine prices increased dramatically in the early 1980s, huge crops of coca leaf, the raw material used in the production of cocaine, were planted in the producing countries.

As these plants reached maturity, there were record harvests. As a result, the price of one raw Corp declined by as much as so percent from its historically high levels. Although U.S. demand for cocaine also increased substantially, demand did not keep pace with supply.

In 1983-84, a gram of 35 percent pure cocaine cost between $100 and $125. In 1985, the price declined to $100, but purity rose to about 55 percent. In 1987, the price declined to a low $50 with purity as high as 75 percent. Thus, by 1987 the marketplace hade dramatically squeezed the profits of cocaine producers.

In summary, while holding constant the effects of all other factors, an increase in supply results in a new equilibrium point where the price is lower and the quantity is greater than they were at the original equilibrium point. In contrast, a decrease in supply results in a new equilibrium point with a higher price and lower quantity than those at the original equilibrium point.

2.2. Marginal Analysis

Marginal analysis is one of the most useful concepts of economic decision making. Resource-allocation decisions typically are expressed in terms of the marginal conditions that must be satisfied to attain an optimal solution. The familiar profit-maximization rule for the firm of setting output at the point where "marginal cost equals marginal revenue" is one such example. Long-term investment decisions (capital expenditures) also are made using marginal analysis decision rules. If the expected return from an investment project (that is, the *marginal return* to the firm) exceeds the cost of funds that must be acquired to finance the project (the *marginal cost* of capital), then the project should be undertaken. Following this important marginal decision rule leads to the maximization of shareholder wealth.

In the marginal analysis framework, resource-allocation decisions are made by comparing the marginal (or incremental) benefits of a change in the level of an activity with the marginal (or incremental) costs of the change. *Marginal benefit* is defined as the change in total benefits that are derived from undertaking some economic activity. Similarly, *marginal cost* is defined as the change in total costs that occurs from undertaking some economic activity.

In summary, marginal analysis instructs decision makers to determine the additional (marginal) costs and additional (marginal) benefits associated with a proposed action. *Only if the marginal benefits exceed the marginal costs* (that is, if net marginal benefits are positive) should the action be taken.

Total, Marginal, and Average Relationships

Economic relationships can be presented using tabular, graphic, and algebraic frameworks. Let us first use a tabular presentation. Suppose that the total profit

ΠT of a firm is a function of the number of units of output produced *Q*, as shown in columns 1 and 2 of Table 2.1. Marginal profit, which represents the change in total profit resulting from a one-unit increase in output, is shown in column 3 of the table. (a Δ is used to represent a "change" in some variable.) The marginal profit ΔΠ (*Q*) of any level of output *Q* is calculated by taking the difference between the total profit at this level ΠT (*Q*) and at one unit below this level ΠT (*Q*-1).

In comparing the marginal and total profit functions, we note that for increasing output levels, the marginal profit values remain positive as long as the total profit function is increasing. Only when the total profit function begins decreasing—that is, at *Q* = 10 units—does the marginal profit become negative. The average profit function values ΠA (*Q*), shown in column 4 of Table 2.1, are obtained by dividing the total profit figure ΠT (*Q*) by the output level *Q*.

In comparing the marginal and the average profit function values, we see that the average profit function ΠA (*Q*) is increasing as long as the marginal profit is greater than the average profit; that is, up to *Q* = 7 units. Beyond an output level of *Q* = 7 units, the marginal profit is less than the average profit and the average profit function values are decreasing. By examining the total profit function ΠT (*Q*) in Table 2.1, we see that profit is maximized at an output level of *Q* = 9 units.

Given that the objective is to maximize total profit, then the optimal output decision would be to produce and sell 9 units. If the marginal analysis decision rule discussed earlier in this section is used, the same (optimal) decision is obtained. Applying the rule to this problem, the firm would expand production as long as the *net* marginal return—that is, marginal revenue minus marginal cost (marginal profit)—is positive. From column 3 of Table 2.1, we can see that the marginal profit is positive for output levels up to *Q* = 9. Therefore, the marginal profit decision rule would indicate that 9 units should be produced— the same decision that was obtained from the total profit function.

Table 2.1 Total, Marginal, and Average Profit Relationships

(1) Number of Units of Output Per Unit of Time Q	(2) Total Profit $\Pi_T(Q)$ (\$)	(3) Marginal Profit $\Delta\Pi(Q) = \Pi_T(Q) - \Pi_T(Q-1)$ (\$/Unit)	(4) Average Profit $\Pi_A(Q) = \Pi T(Q)/Q$ (\$/Unit)
0	- 200	0	—
1	- 150	50	- 150.00
2	- 25	125	- 12.50
3	200	225	66.67
4	475	275	118.75
5	775	300	155.00
6	1,075	300	179.17
7	1,325	250	189.29
8	1,475	150	184.38
9	1,500	25	166.67
10	1,350	- 150	135.00

The relationships among the total, marginal, and average profit functions and the optimal output decision also can be represented graphically. A set of *continuous* profit functions, analogous to those presented above in Table 2.1 for discrete integer values of output (Q), is shown in Figure 2.1. At the break-even output level $Q1$, both total profits and average profits are zero. The marginal profit function, which equals the *slope* of the total profit function, takes on its maximum value at an output of $Q2$ units. This point corresponds to the *inflection point.*

Below the inflection point, total profits are increasing at an increasing rate, and hence marginal profits are increasing. Above the inflection point, up to an output level $Q4$, total profits are increasing at a decreasing rate and consequently marginal profits are decreasing. The average profit function, which represents the slope of a straight line drawn from the origin 0 to each point on the total profit function, takes on its maximum value at an output of $Q3$ units.

The average profit necessarily equals the marginal profit at this point. This follows because the slope of the 0*A* line, which defines the average profit, is also equal to the slope of the total profit function at point *A,* which defines the marginal profit. Finally, total profit is maximized at an output of 4 units where marginal profit equals 0. Beyond $Q4$ the total profit function is decreasing, and consequently the marginal profit function takes on negative values.

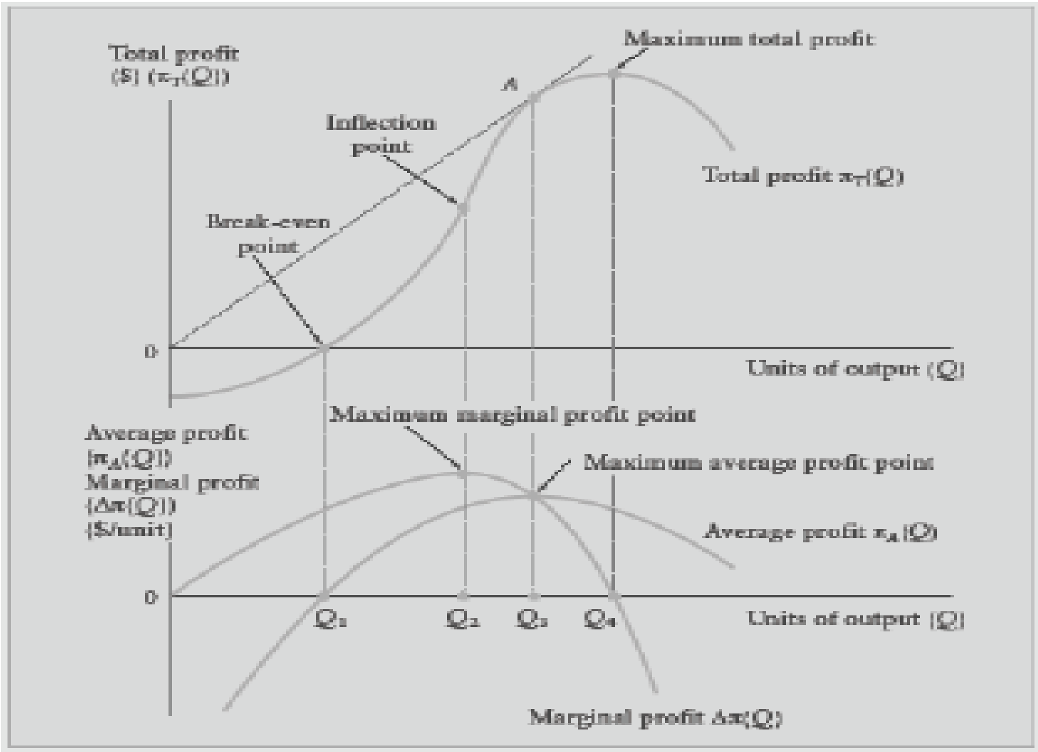


Figure 2.1 Total, Average, and Marginal Profit Functions

Another example of the application of marginal analysis is the capital budgeting expenditure decision problem facing a firm.

For example, consider the Deacon Corporation that has the following schedule of potential investment projects (all assumed to be of equal risk) available to it:

Project	Investment Required ($ millions)	Expected Rate of Return	Cumulative Investment ($ millions)
A	$ 25.0	27.0%	$ 25.0
F	15.0	24.0%	40.0
E	40.0	21.0%	80.0
B	35.0	18.0%	115.0
G	12.0	15.0%	127.0
H	20.0	14.0%	147.0
C	18.0	13.0%	165.0
I	13.0	11.0%	178.0
D	7.0	8.0%	185.0

In addition, the Deacon Corporation has estimated the cost of acquiring the funds needed to finance these investment projects as follows:

Block of funds ($ millions)	Cost of Capital	Cumulative Funds Raised ($ millions)
First $50.00	10.0%	$ 50.0
Next 25.00	10.5%	75.0
Next 40.00	11.0%	115.0
Next 50.00	12.2%	165.0
Next 20.00	14.5%	185.0

The expected rate of return on the projects listed above can be thought of as the marginal (or incremental) return available to Deacon as it undertakes each additional investment project.

Similarly, the cost of capital schedule may be thought of as the marginal cost of acquiring the needed funds. Following the marginal analysis rules means that Deacon should invest in additional projects as long as the expected rate of return on the project exceeds the marginal cost of capital funds needed to finance the project.

It is clear that project A, which offers an expected return of 27 percent and requires an outlay of $25 million, is acceptable because the marginal return exceeds the marginal cost of capital (10.0 percent for the first $50 million of funds raised by Deacon).

In fact, an examination of the tables indicates that project A, F, E, B, G, H, and C all meet the marginal analysis test because the marginal return from each of these projects exceeds the marginal cost of capital funds needed to finance these projects.

In contrast, projects I and D should not be undertaken because they offer returns of 11 and 8 percent respectively, compared with a marginal cost of capital of 14.5 percent for the $20 million in funds needed to finance these projects.

In summary, the marginal analysis concept instructs the decision maker to determine the additional (marginal) costs and additional (marginal) benefits associated with a proposed action. If the marginal benefits exceed the marginal costs (that is, if the net marginal benefits are positive), the action should be taken.

2.3. Time Value of Money

To achieve the objective of shareholder wealth maximization, a set of appropriate decision rules must be specified. We just saw that the decision rule of setting *marginal revenue* (*benefit*) *equal to marginal cost* (MR = MC) provides a framework for making many important resource-allocation decisions. The MR = MC rule is best suited for situations when the costs and benefits occur at approximately the same time. Many economic decisions require that costs be incurred immediately but result in a stream of benefits over several future time periods.

Consider the following situation. You have just inherited $1 million. Your financial advisor has suggested that you use these funds to purchase a piece of land near a proposed new highway interchange. Your advisor, who is also a state road commissioner, is certain that the interchange will be built and that in one year the value of this land will increase to $1.2 million. Hence, you believe initially that this is a risk less investment. At the end of one year you plan to sell the land. You are being asked to invest $1 million today in the anticipation of receiving $1.2 million a year from today, or a profit of $200,000. You wonder whether this profit represents a sufficient return on your investment.

You feel it is important to recognize that there is a one-year difference between the time you make your outlay of $1 million and the time you receive $1.2 million from the sale of the land. A return of $1.2 million received one year from today must be worth less than $1.2 million today because you could invest your $1 million today to earn interest over the coming year. *A dollar received in the future is worth less than a dollar in hand today because a dollar today can be invested to earn a return immediately.* Therefore, to compare a dollar received in the future with a dollar in hand today, it is necessary to multiply the future dollar by a *discount factor* that reflects the alternative investment opportunities that are available.

Instead of investing your $1 million in the land venture, you are aware that you could also invest in a one-year U.S. government bond that currently offers a return of 5 percent. The 5 percent return represents the return (the opportunity cost) forgone by investing in the land project. The 5 percent rate also can be thought of as the compensation to an investor who agrees to postpone receiving a cash return for one year.

In summary, Present value recognizes that a dollar received in the future is worth less than a dollar in hand today, because a dollar today could be invested to earn a return. To compare monies in the future with today, the future dollars must be discounted by a *present value interest factor*, PVIF = 1/(1+i), where i is the interest compensation for postponing receiving cash one period. For dollars received in *n* periods, the discount factor is PVIFn = [1/(1+i)]n.

We summarize below the types of time value analysis, using the data shown in Figure 2-2 to illustrate the various points. Refer to the figure constantly, and try to find in it an example of the points covered as you go through this summary.

• **Compounding** is the process of determining the **future value (FV)** of a cash flow or a series of cash flows. The compounded amount, or future value, is equal to the beginning amount plus the interest earned.

• **Future value:** (single payment)

$$FV_n = PV(1 + i)^n = PV(FVIF_{i,n}).$$

Example: $1,000 compounded for 1 year at 4 percent:

$$FV_1 = \$1{,}000(1.04)^1 = \$1{,}040.$$

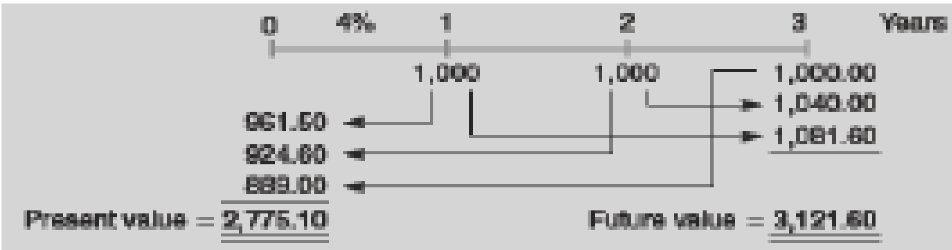


Figure 2.2

• **Present value:** (single payment)

$$PV = \frac{FV_n}{(1+i)^n} = FV_n\left(\frac{1}{1+i}\right)^n = FV_n(PVIF_{i,n}).$$

Example: $1,000 discounted back for 2 years at 4 percent:

$$PV = \frac{\$1{,}000}{(1.04)^2} = \$1{,}000\left(\frac{1}{1.04}\right)^2 = \$1{,}000(0.9246) = \$924.60.$$

• An **annuity** is defined as a series of equal periodic payments (PMT) for a specified number of periods

• **Future value:** (annuity)

$$\begin{aligned} FVA_n &= PMT(1+i)^{n-1} + PMT(1+i)^{n-2} + PMT(1+i)^{n-3} + \cdots + PMT(1+i)^0 \\ &= PMT\sum_{t=1}^{n}(1+i)^{n-t} \\ &= PMT\left(\frac{(1+i)^n - 1}{i}\right) \\ &= PMT(FVIFA_{i,n}). \end{aligned}$$

• **Present value:** (annuity)

$$\begin{aligned} PVA_n &= \frac{PMT}{(1+i)^1} + \frac{PMT}{(1+i)^2} + \cdots + \frac{PMT}{(1+i)^n} \\ &= PMT\sum_{t=1}^{n}\left[\frac{1}{1+i}\right]^t \\ &= PMT\left(\frac{1 - \frac{1}{(1+i)^n}}{i}\right) \\ &= PMT(PVIFA_{i,n}). \end{aligned}$$

Example: PVA of 3 payments of $ 1,000 when I = 4% per period

PVA3 = $1,000(2.7751)

= $2,775.10

• An annuity whose payments occur at the *end* of each period is called **ordinary annuities.** The Formulas above are for ordinary annuities.

• If each payment occurs at the beginning of the period rather than at the end, then we have an **annuity due.** In Figure 2-2, the payments would be shown at due would also be larger because each payment would be compounded for an ordinary annuity to an annuity due:

PVA (annuity due) = PVA of an ordinary annuity x (1 + i).

FVA (annuity due) = FVA of an ordinary annuity x (1 + i).

Example: PVA of 3 beginning-of-year payments of $1,000 when I = 4%:

PVA (annuity due) = $1,000(2.7751) (1.04)

= $2,886.10.

Example: FVA of 3 beginning-of-year payments of $1,000 when I = 4%:

FVA (annuity due) = $1,000(3.1216) (1.04)

= $3,246.46.

Self-Assessment Exercises

1. Why all sales and purchases are take place at the same price?
2. Why market price falls when the price of the product is above the equilibrium price?
3. Why market price raises when the price of the product is below the equilibrium price?
4. From the following data, determine the number of units that maximizes average profit and maximizes marginal profit.

Output (Q)	0	1	2	3	4	5	6	7
Total Profit (π)	-100	50	116	180	220	250	270	280

5. Why money has time value?
6. Why marginal analysis is important for function units in a given industry or for the industry in general?

CHAPTER THREE

ECONOMIC OPTIMIZATION

Objective

- *To introduce and define optimization*
- *To explain Optimization Techniques*

Introduction

Effective managerial decision making is the process of arriving at the best solution to a problem. If only one solution is possible, then no decision problem exists. When alternative courses of action are available, the best decision is the one that produces a result most consistent with managerial objectives. The process of arriving at the best managerial decision is the goal of economic optimization and the focus of managerial economics.

Normative economic decision analysis involves determining the action that best achieves a desired goal or objective. This means finding the action that optimizes (i.e., maximizes or minimizes) the value of an objective function. For example, in a price-output decision-making problem, we may be interested in determining the output level that maximizes profits. In a production problem, the goal may be to find the combination of inputs (resources) that minimizes the cost of producing a desired level of output. In a capital budgeting problem, the objective may be to select those projects that maximize the net present value of the investments chosen. There are many techniques for solving optimization problems such as these. Optimization techniques are a powerful set of tools that are important in efficiently managing a firm's resources and thereby maximizing shareholder wealth.

I. Types of Optimization Techniques

Recall that in chapter 1 we defined the general form of a problem that managerial economics attempts to analyze. The basic form of the problem is to identify the alternative means of achieving a given objective and then to select the alternative that accomplishes the objective in the most efficient manner, subject to constraints on the means. In programming terminology, the problem is optimizing the value of some objective function, subject to any resource and/or other constraints such as legal, input, environmental, and behavioural restrictions.

Mathematically, we can represent the problem as

$$\text{Optimize } y = f(x_1, x_2 \ldots x_n) \qquad [3.1]$$

$$\text{subject to } g_j(x_1, x_2, \ldots, x_n) \begin{vmatrix} \le \\ = \\ \ge \end{vmatrix} b_j \qquad j = 1, 2, \ldots, m \qquad [3.2]$$

where Equation 3.1 is the objective function and Equation 3.2 constitutes the set of constraints imposed on the solution. The xi variables, $x1$, $x2 \ldots xn$, represent the set of decision variables, and y $f(x1, x2, \ldots, xn)$ is the objective function expressed in terms of these decision variables. Depending on the nature of the problem, the term *optimize* means either *maximize* or *minimize* the value of the objective function. As indicated in Equation 3.2, each constraint can take the form of an equality (=) or an inequality (≤ or ≥) relationship.

Complicating Factors in Optimization

The following are several factors that can make optimization problems fairly complex and difficult to solve.

❖ ***The existence of multiple decision variables in a problem.***

Relatively simple procedures exist for determining the profit-maximizing output level for the single-product firm. However, the typical medium- or large-size firm often produces a large number of different products, and as a result, the profit-maximization problem for such a firm requires a series of output decisions—one for each product.

❖ ***The complex nature of the relationships between the decision variables and the associated outcome.***

For example, in public policy decisions on government spending for such items as education, it is extremely difficult to determine the relationship between a given expenditure and the benefits of increased income, employment, and productivity it provides. No simple relationship exists among the variables. Many of the optimization techniques discussed here are only applicable to situations in which a relatively simple function or relationship can be postulated between the decision variables and the outcome variable.

❖ ***The possible existence of one or more complex constraints on the decision variables.***

For example, virtually every organization has constraints imposed on its decision variables by the limited resources—such as capital, personnel, and facilities— over which it have control. These constraints must be incorporated into the decision problem. Otherwise, the optimization techniques that are applied to the problem may yield a solution that is unacceptable from a practical standpoint.

- ***The presence of uncertainty or risk.***

The presences of uncertainty or risk illustrate the difficulties that may be encountered and may render a problem unsolvable by formal optimization procedures.

The mathematical techniques used to solve an optimization problem represented by Equations 3.1 and 3.2 depend on the form of the criterion and constraint functions. The simplest situation to be considered is the *unconstrained* optimization problem. In such a problem no constraints are imposed on the decision variables, and *differential calculus* can be used to analyze them. Another relatively simple form of the general optimization problem is the case in which all the constraints of the problem can be expressed as *equality* (=) relationships. The technique of *Lagrangian multipliers* can be used to find the optimal solution to many of these problems.

Often, however, the constraints in an economic decision-making problem take the form of *inequality* relationships (≤ or ≥) rather than equalities. For example, limitations on the resources—such as personnel and capital—of an organization place an *upper bound* or budget ceiling on the quantity of these resources that can be employed in maximizing (or minimizing) the objective function. With this type of constraint, all of a given resource need not be used in an optimal solution to the problem. An example of a *lower bound* would be a loan agreement that requires a firm to maintain a *current ratio* (that is, ratio of current assets to current liabilities) of at least 2.00. Any combination of current assets and current liabilities having a ratio greater than or equal to 2.00 would meet the provisions of the loan agreement. Such optimization procedures as the Lagrangian multiplier method are not suited to solving problems of this type efficiently; however, modern mathematical programming techniques have been developed that can efficiently solve several classes of problems with these inequality restrictions.

Linear-programming problems constitute the most important class for which efficient solution techniques have been developed. In a linear-programming problem, both the objective and the constraint relationships are expressed as linear functions of decision variables. Other classes of problems include *integer-programming* problems, in which some (or all) of the decision variables are required to take on integer values, and *quadratic-programming* problems, in which the objective relationship is a quadratic function of the decision variables. Generalized computing algorithms exist for solving optimization problems that meet these requirements.

The various types of optimization problems and techniques can, thus, be represented as follows:

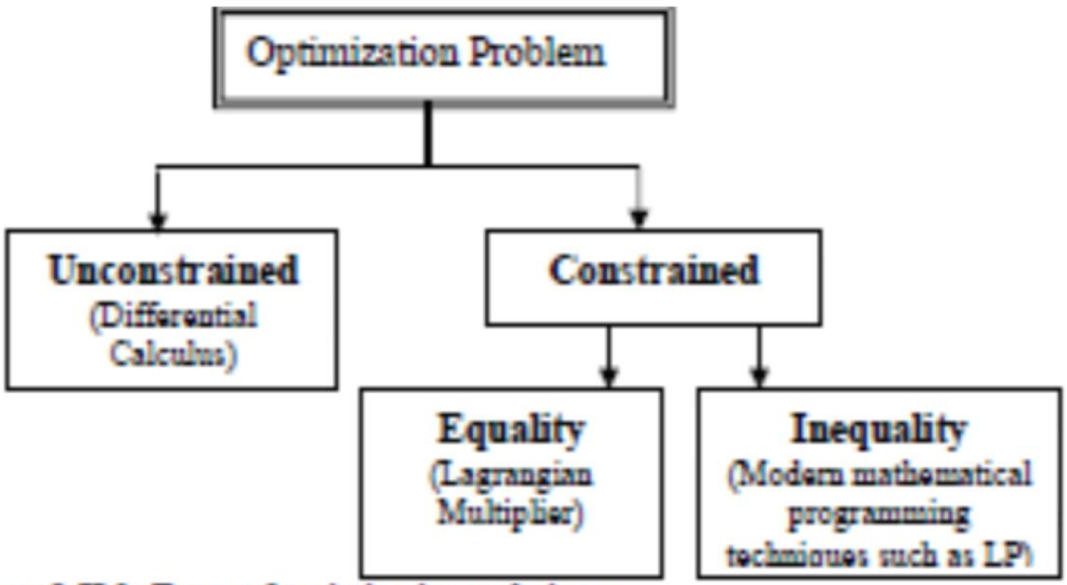


Figure 3-II-1: Types of optimization techniques

II. Differential Calculus and Bi-variate Optimization

Recall that in chapter 2, marginal analysis was introduced as one of the fundamental concepts of economic decision making. In the marginal analysis framework, resource-allocation decisions are made by comparing the marginal benefits of a change in the level of an activity with the marginal costs of the change. A change should be made as long as the marginal benefits exceed the marginal costs. By following this basic rule, resources can be allocated efficiently and profits or shareholder wealth can be maximized.

In the profit-maximization example developed in chapter 2, the application of the marginal analysis principles required that the relationship between the objective (profit) and the decision variable (output level) be expressed in either tabular or graphic form. This framework, however, can become cumbersome when dealing with several decision variables or with complex relationships between the decision variables and the objective. When the relationship between the decision variables and criterion can be expressed in *algebraic* form, the more powerful concepts of differential calculus can be used to find optimal solutions to these problems

Relationship between Marginal Analysis and Differential Calculus

Initially, let us assume that the objective we are seeking to optimize, Y, can be expressed algebraically as a function of *one* decision variable, X,

$$Y = f(X) \qquad \textbf{[3.3]}$$

Recall that marginal profit is defined as the change in profit resulting from a one-unit change in output. In general, the marginal value of any variable Y, which is a function of another variable X, is defined as the change in the value of Y resulting from a one-unit change in X. The marginal value of Y, M_y, can be calculated from the change in Y, ΔY *that* occurs as the result of a given change in X, ΔX:

$$M_y = \Delta Y / \Delta X \qquad \textbf{[3.4]}$$

When calculated with this expression, different estimates for the marginal value of Y may be obtained, depending on the size of the change in X that we use in the computation. The true marginal value of a function (e.g., an economic relationship) is obtained from Equation 3.4 when X is made as small as possible. If X can be thought of as a continuous (rather than a discrete) variable that can take on fractional values, then in calculating M_y by Equation 3.4, we can let X approach zero. In concept, this is the approach taken in differential calculus. The derivative or, more precisely, *first derivative*, dY/dX, of a function is defined as the *limit* of the ratio $\Delta Y/\Delta X$ as ΔX approaches zero; that is,

$$\frac{dY}{dX} = \text{limit}_{\Delta X \to 0} \frac{\Delta Y}{\Delta X} \qquad \textbf{[3.5]}$$

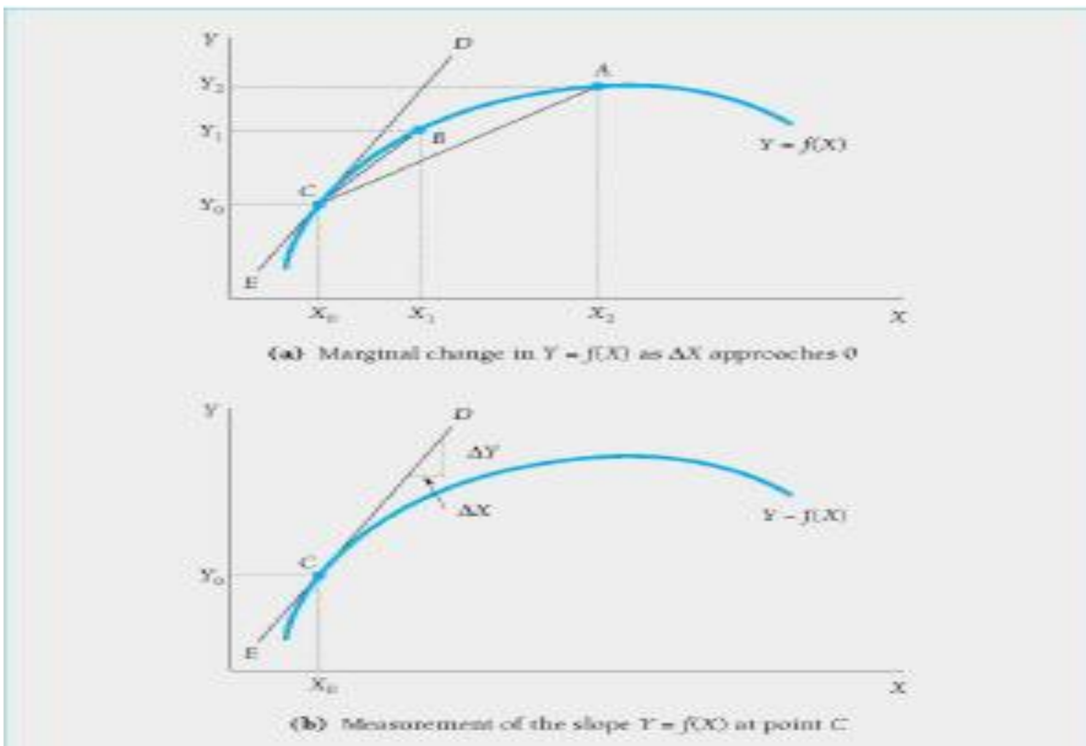
Graphically, the first derivative of a function represents the *slope* of the curve at a given point on the curve. The definition of a derivative as the limit of the change in Y (that is, ΔY) as ΔX approaches zero is illustrated in Figure 3.2 (a). Suppose we are interested in the derivative of the $Y = f(X)$ function at the point X_0. The derivative dY/dX measures the slope of the tangent line *ECD*. An estimate of this slope, albeit a poor estimate, can be obtained by calculating the marginal value of Y over the interval X_0 to X_2. Using Equation 3.4, a value of

$$M_y = \frac{\Delta Y}{\Delta X} = \frac{Y_2 - Y_0}{X_2 - X_0}$$

is obtained for the slope of the *CA* line. Now let us calculate the marginal value of *Y* using a smaller interval, for example, X_0 to X_1. The slope of the *CB* line, which is equal to

$$M_y = \frac{\Delta Y}{\Delta X} = \frac{Y_1 - Y_0}{X_1 - X_0}$$

gives a much better estimate of the true marginal value as represented by the slope of the *ECD* tangent line. Thus we see that the smaller the *ΔX* value, the better the estimate of the slope of the curve. Letting *ΔX approach* zero allows us to find the slope of the *Y* = *f*(*X*) curve at point *C*. As shown in Figure 3.2 (b), the slope of the *ECD* tangent line (and the *Y* = *f*(*X*) function at point *C*) is measured by the change in *Y*, or rise, *ΔY*, divided by the change in *X*, or run, *ΔX*.



(a) Marginal change in Y = f(X) as ΔX approaches 0

(b) Measurement of the slope Y = f(X) at point C

Figure 3.2

Process of Differentiation

The process of differentiation—that is, finding the derivative of a function— involves determining the limiting value of the ratio $\Delta Y/\Delta X$ as ΔX approaches zero. Before offering some general rules for finding the derivative of a function, we illustrate with an example the algebraic process used to obtain the derivative without the aid of these general rules.

Example:

Process of Differentiation: Profit Maximization at Mekelle Power Co.

Suppose the profit, π, of Mekelle Power Co. can be represented as a function of the output level Q using the expression:

$$\pi = -40 + 140Q - 10Q^2 \quad \textbf{[3.6]}$$

We wish to determine $d\pi/dQ$ by first finding the marginal-profit expression $\Delta\pi/\Delta Q$ and then taking the limit of this expression as ΔQ approaches zero.

Let us begin by expressing the new level of profit ($\pi = \Delta\pi$) that will result from an increase in output to ($Q + \Delta Q$). From **Equation 3.6**, we know that

$$(\pi + \Delta\pi) = -40 + 140(Q + \Delta Q) - 10(Q + \Delta Q)^2 \quad \textbf{[3.7]}$$

Expanding this expression and then doing some algebraic simplifying, we obtain

$$(\pi + \Delta\pi) = -40 + 140Q + 140\Delta Q - 10\,[Q^2 + 2Q\,\Delta Q + (\Delta Q\,)^2]$$

$$= -40 + 140Q + 140\Delta Q - 10Q^2 - 20Q\,\Delta Q - 10(\Delta Q\,)^2 \quad \textbf{[3.8]}$$

Subtracting **Equation 3.6** from **Equation 3.8** yields

$$\Delta\pi = 140\,\Delta Q - 20Q\,\Delta Q - 10(\Delta Q)^2 \quad \textbf{[3.9]}$$

Forming the marginal-profit ratio $\Delta\pi/\Delta Q$, and doing some canceling, we get

$$\frac{\Delta\pi}{\Delta Q} = \frac{140\Delta Q - 20Q\Delta Q - 10(\Delta Q)^2}{\Delta Q} = 140 - 20Q - 10\Delta Q \quad \textbf{[3.10]}$$

Taking the limit of **Equation 3.10** as ΔQ approaches zero yields the expression for the derivative of Mekelle Power Co.'s profit function (**Equation 3.6**)

$$\frac{d\pi}{dQ} = \lim_{\Delta Q \to 0} [140 - 20Q - 10\Delta Q]$$
$$= 140 - 20Q \qquad [3.11]$$

If we are interested in the derivative of the profit function at a particular value of *Q*, **Equation 3.11** can be evaluated for this value. For example, suppose we want to know the marginal profit, or slope of the profit function, at *Q* = 3 units. Substituting *Q* = 3 in Equation 3.11 yields

$$\text{Marginal profit} = \frac{d\pi}{dQ} = 140 - 20(3) = \$80 \text{ per unit}$$

Rules of Differentiation

Fortunately, we do not need to go through a lengthy process every time we want the derivative of a function. A series of general rules exists for differentiating various types of functions as shown in figure 3.3

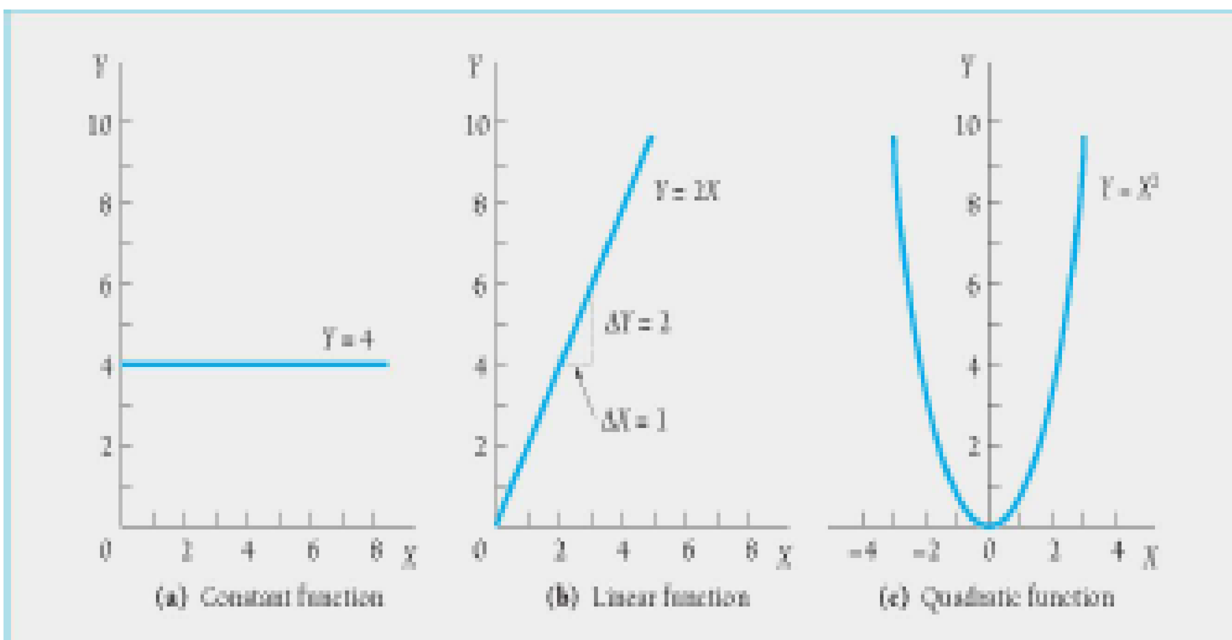


Figure 3.3 Constant, Linear, and Quadratic Functions

These rules for differentiating functions are summarized in Table 3.1 below:

Table 3.1Summary of Rules for Differentiating Functions

Function	Derivative
1. Constant Function	
$Y = a$	$\frac{dY}{dX} = 0$
2. Power Function	
$Y = aX^b$	$\frac{dY}{dX} = b \cdot a \cdot X^{b-1}$
3. Sums of Functions	
$Y = f_1(X) + f_2(X)$	$\frac{dY}{dX} = \frac{df_1(X)}{dX} + \frac{df_2(X)}{dX}$
4. Product of Two Functions	
$Y = f_1(X) \cdot f_2(X)$	$\frac{dY}{dX} = f_1(X) \cdot \frac{df_2(X)}{dX} + f_2(X) \cdot \frac{df_1(X)}{dX}$
5. Quotient of Two Functions	
$Y = \frac{f_1(X)}{f_2(X)}$	$\frac{dY}{dX} = \frac{f_2(X) \cdot \frac{df_1(X)}{dX} - f_1(X) \cdot \frac{df_2(X)}{dX}}{[f_2(X)]^2}$
6. Functions of a Function	
$Y = f_1(Z)$, where $Z = f_2(X)$	$\frac{dY}{dX} = \frac{dY}{dZ} \cdot \frac{dZ}{dX}$

Example:

Rules of differentiation: profit maximization at *Mekelle Power Co. (continued)*

As an example of the application of these rules, consider again the profit function for Mekelle Power Co., given by **Equation 3.6**, which was discussed earlier:

$$\pi = -40 + 140Q - 10Q^2$$

In this example Q represents the X variable and π represents the Y variable; that is, $\pi = f(Q)$. The function $f(Q)$ is the sum of *three* separate functions—a constant function, $f_1(Q) = -40$, and two power functions, $f_2(Q) = 140Q$ and $f_3(Q) - 10Q^2$

.

Therefore, applying the differentiation rules yields

$$\frac{d\pi}{dQ} = \frac{df_1(Q)}{dQ} + \frac{df_2(Q)}{dQ} + \frac{df_3(Q)}{dQ}$$

This is the same result that was obtained earlier in Equation 3.11 by the differentiation process.

Applications of Differential Calculus to Optimization Problems

The reason for studying the process of differentiation and the rules for differentiating functions is that these methods can be used to find optimal solutions to many kinds of maximization and minimization problems in managerial economics.

Maximization Problem

As you recall from the discussion of marginal analysis, a necessary (but not sufficient) condition for finding the maximum point on a curve (for example, maximum profits) is that the marginal value or slope of the curve at this point must be equal to zero. We can now express this condition within the framework of differential calculus. Because the derivative of a function measures the slope or marginal value at any given point, an equivalent necessary condition for finding the maximum value of a function $Y = f(X)$ is that the derivative dY/dX at this point must be equal to zero. This is known as the *first-order condition* for locating one or more maximum or minimum points of an algebraic function.

Example: First-order condition: Profit maximization at Mekelle Power Co. (contd.)

Using the profit function (Equation 3.6)

$$\pi = -40 + 140Q - 10Q^2$$

discussed earlier, we can illustrate how to find the profit-maximizing output level Q by means of this condition. Setting the first derivative of this function (which was computed previously) to zero, we obtain

$$\frac{d\pi}{dQ} = 140 - 20Q$$

$$0 = 140 - 20Q$$

Solving this equation for Q yields $Q^* = 7$ units as the profit-maximizing output level. The profit and first derivative functions and optimal solution are shown in Figure 3.4. As we can see, profits are maximized at the point where the function is neither increasing nor decreasing; in other words, where the slope (or first derivative) is equal to zero.

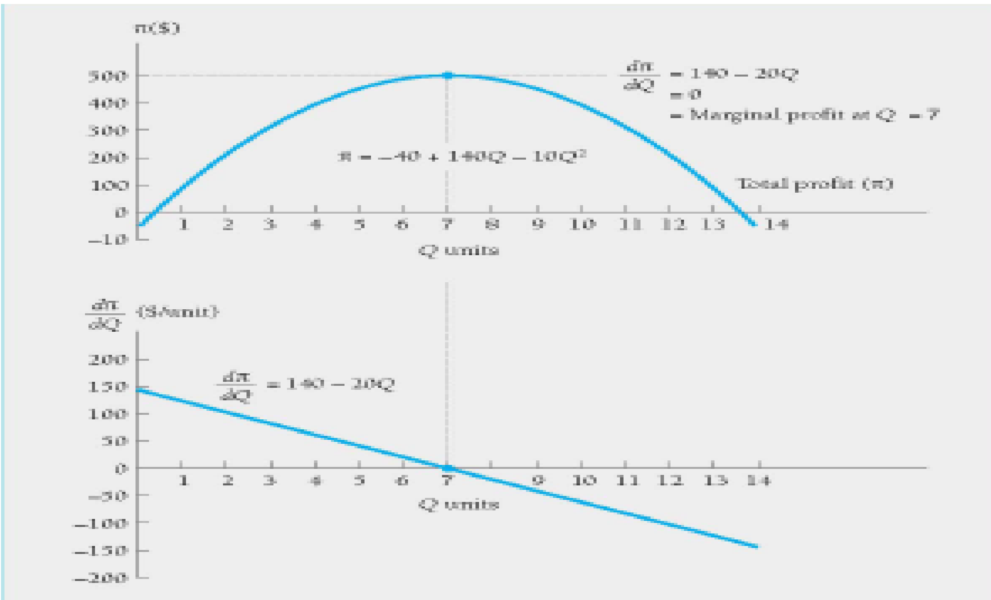


Figure 3-4 Profit and First derivative functions

Second Derivatives and the Second-Order Condition

Setting the derivative of a function equal to zero and solving the resulting equation for the value of the decision variable does not guarantee that the point will be obtained at which the function takes on its maximum value. The slope of a U-shaped function will also be equal to zero at its low point and the function will take on its *minimum* value at the given point. In other words, setting the derivative to zero is only a *necessary* condition for finding the maximum value of a function; it is not a *sufficient* condition.

Another condition, known as the *second-order condition,* is required to determine whether a point that has been determined from the first-order condition is either a maximum point or minimum point of the algebraic function.

This situation is illustrated in Figure 3.5. At both points *A* and *B* the slope of first-order condition is either a maximum point or the function (first derivative, dY/dX) is zero; however, only at point *B* does the function take on its maximum value. We note in Figure 3.5 that the marginal value algebraic function. (Slope) is continually *decreasing* in the neighborhood of the maximum value (point *B*) of the *Y f*(*X*) function. First the slope is positive up to the point where *dY*/*dX* = 0, and thereafter the slope becomes negative. Thus we must determine whether the slope's marginal value

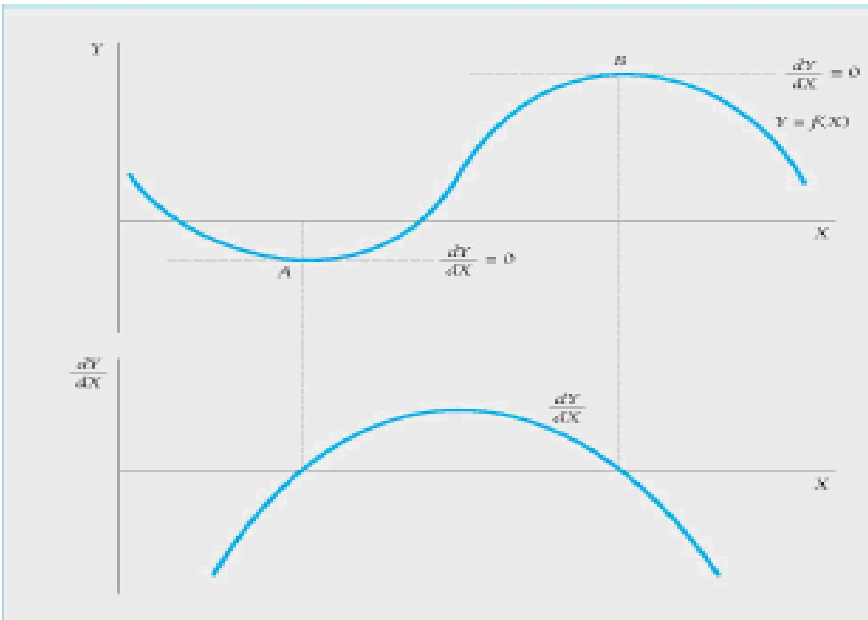


Figure 3-5 Maximum and Minimum Values of a Function

(slope of the slope) is declining. A test to see whether the marginal value is decreasing is to take the derivative of the marginal value and check to see if it is negative at the given point on the function. In effect, we need to find the derivative of the derivative—that is, the *second derivative* of the function—and then test to see if it is less than zero. Formally, the second derivative of the function *Y f*(*X*) is written as d^2Y/dX^2 and is found by applying the previously described differentiation rules to the first derivative. A *maximum point is obtained if the second derivative is negative; that is,* $d^2Y/dX^2 < 0$.

Example: Second-order condition: Profit maximization at Mekelle Power Co. (continued)

Returning to the profit-maximization example, the second derivative is obtained from the first derivative as follows:

$$\frac{d\pi}{dQ} = 140 - 20Q$$

$$\frac{d^2\pi}{dQ^2} = 0 + 1 * (-20)\, Q^{1-1} = -20$$

Because $d^2\pi/dQ^2<0$, we know that a maximum-profit point has been obtained. An opposite condition holds for obtaining the point at which the function takes on a minimum value. Note again in Figure 3.5 that the marginal value (slope) is continually *increasing* in the neighborhood of the minimum value (point *A*) of the *Y* (*X*) function. First the slope is negative up to the point where $dY/dX = 0$, and thereafter the slope becomes positive. Therefore, we test to see if $d^2Y/dX^2 > 0$ at the given point. *A minimum point is obtained if the second derivative is positive; that is,* $d^2Y/dX^2 > 0$.

Minimization Problem

In some decision-making situations, cost minimization may be the objective. As in profit-maximization problems, differential calculus can be used to locate the optimal points.

Example: *Cost Minimization: KeySpan Energy*

Suppose we are interested in determining the output level that minimizes average total costs for KeySpan Energy, where the average total cost function might be approximated by the following relationship (*Q* represents output):

$$C = 15 - .040Q + .000080Q^2$$

Differentiating *C* with respect to *Q* gives

$$\frac{dC}{dQ} = -.040 + .000160Q$$

Setting this derivative equal to zero and solving for *Q* yields

$$0 = .040 + .000160Q$$

$$Q^* = 250$$

Taking the second derivative, we obtain

$$\frac{d^2C}{dQ^2} = +.000160$$

Because the second derivative is positive, the output level of $Q = 250$ is indeed the value that minimizes average total costs.

Activity

Differentiate the following functions:

(a) $TC = 50 + 100Q - 6Q^2 + .5Q^3$ (b) $ATC = 50/Q + 100 - 6Q + .5Q^2$

III. Partial Differentiation and Multi-variate Optimization

Until now we have examined the relationship between two variables only. For example, variable *Y* (say, total revenue, total cost, or total profit) was assumed to be a function of or to depend on only the value of variable *X* (total output or quantity). Most economic relationships, however, involve more than two variables. For example, total revenue may be a function of or depend on both output and advertising, total costs may depend on expenditures on both labor and capital, and total profit on sales of commodities *X* and *Y*. Thus, it becomes important to determine the marginal effect on the dependent variable, say, total profit, resulting from changes in the quantities of each individual variable, say, the quantity sold of commodity *X* and commodity *Y*, *separately*. These marginal effects are measured by the *partial derivative,* which is indicated by the symbol ∂ (as compared to *d* for the derivative). The partial derivative of the dependent or left-hand variable with respect to each of the independent or right-hand variables is found by the same rules of differentiation presented earlier, except that all independent variables other than the one with respect to which we are finding the partial derivative are held constant.

For example, suppose that the total profit (π) function of a firm depends on sales of commodities *X* and *Y* as follows:

$$\pi = f(X, Y) = 80X - 2X^2 - XY - 3Y^2 + 100Y \qquad [3\text{-}12]$$

To find the partial derivative of π with respect to *X*, $\partial\pi/\partial X$, we hold *Y* constant and obtain

$$\pi = 80 - 4XY - Y$$

This isolates the marginal effect on π from changes in the quantity sold of commodity *X* only (i.e., while holding the quantity of commodity *Y* constant). Note that the derivative of the third term of the π function is –*Y* (since the implicit exponent of *X* is 1) and that *Y* is treated as a constant. The fourth and the fifth terms of the π function drop out in the partial differentiation because they contain no *X* term. Similarly, to isolate the marginal effect of a change of *Y* on π, we hold *X* constant and obtain

$\pi = -X - 6Y + 100$

We can visualize geometrically the concept of the partial derivative with a three dimensional figure, with π on the vertical axis and with the *X* axis and the *Y* axis forming the (plane surface, rather than the line) base of the figure. Then, $\partial\pi/\partial X$ measures the marginal effect of *X* on π, in the cross section of the three dimensional figure along the *X* axis. Similarly, $\partial\pi/\partial Y$ examines the marginal effect of *Y* on π in the cross section of the three-dimensional figure along the *Y* axis. Note also that the value of $\partial\pi/\partial X$ depends also on the level at which *Y* is held constant. Similarly, the value of $\partial\pi/\partial Y$ depends also on the level at which *X* is held constant. This is the reason that the expression for the $\partial\pi/\partial X$ *found* above also contains a *Y* term, while $\partial\pi/\partial Y$ also has an *X* term.

$\pi = 80X - 2X^2 - XY - 3Y^2 + 100Y$ **[3-12]**

We set $\partial\pi/\partial X$ and $\partial\pi/\partial Y$ (found earlier) equal to zero and solve for *X* and *Y*.

Specifically,

$\partial\pi/\partial X = 80 - 4X - Y = 0$

$\partial\pi/\partial Y = - X - 6Y + 100 = 0$

Multiplying the first of the above expressions by –6, rearranging the second, and adding, we get

$$\begin{aligned} -480 + 24X + 6Y &= 0 \\ 100 - X - 6Y &= 0 \\ \hline -380 + 23X \quad &= 0 \end{aligned}$$

Therefore, $X = 380/23 = 16.52$. Substituting $X = 16.52$ into the first expression of the partial derivative set equal to zero, and solving for *Y*, we get

$80 - 4(16.52) - Y = 0$

Therefore, $Y = 80 - 66.08 = 13.92$.

Thus, the firm maximizes π when it sells 16.52 units of commodity *X* and 13.92 units of commodity *Y*. Substituting these values into the π function; we get the maximum total profit of the firm of

$\pi = 80(16.52) - 2(16.52)^2 - (16.52)(13.92) - 3(13.92)^2 + 100(13.92)$

=$1,356.52

Activity

(a) What is meant by the "partial derivative"? How is it determined?

(b) Why is the concept of the partial derivative important in managerial economics?

(c) How can we use partial derivatives to optimize a multivariate function?

Activity

Determine the partial derivatives with respect to all of the variables in the following functions:

(a) $TC = 50 + 5Q_1 + 10Q_2 + .5Q_1Q_2$ (b) $Q = 1.5L^{.60} K^{.50}$ (c) $QA = 2.5PA^{-1.30}Y^{.20}PB^{.40}$

IV. Constrained Optimization and Lagrangian Multiplier Techniques

Most organizations have constraints on their decision variables. The most obvious constraints, and the easiest to quantify and incorporate into the analysis, are the limitations imposed by the quantities of resources (such as capital, personnel, facilities, and raw materials) available to the organization. Other more subjective constraints include legal, environmental, and behavioural limitations on the decisions of the organization.

Substitution Technique

When the constraints take the form of equality relationships, classical optimization procedures can be used to solve the problem. Substitution method can be employed when the objective function is subject to only *one* constraint equation of a relatively simple form, is to solve the constraint equation for one of the decision variables and then substitute this expression into the objective function. This procedure converts the original problem into an unconstrained optimization problem, which can be solved using the calculus procedures developed.

This procedure can be clarified by examining its use in a constrained minimization problem. Suppose a firm produces its product on two assembly lines and operates with the following total cost function:

$$TC = \$3X^2 + \$6Y^2 - \$1XY$$

Where X represents the output produced on one assembly line and Y the production from the second.

Management seeks to determine the least-cost combination of X and Y, subject to the constraint that total output of the product is 20 units. The constrained optimization problem is

$$\text{Minimize } TC = \$3X^2 + \$6Y^2 - \$1XY$$

Subject to

$$X + Y = 20$$

Solving the constraint for X and substituting this value into the objective function results in

$X = 20 - Y$ and

$$TC = \$3(20 - Y)^2 + \$6Y^2 - \$1(20 - Y)Y$$

$$= \$3(400 - 40Y + Y^2) + \$6Y^2 - \$1(20Y - Y2) \qquad \textbf{[3.13]}$$

$$= \$1,200 - \$120Y + \$3Y^2 + \$6Y^2 - \$20Y + Y^2$$

$$= \$1,200 - \$140Y + \$10Y^2$$

Now it is possible to treat Equation 3.13 as an unconstrained minimization problem. Solving it requires taking the derivative of the total cost function, setting that derivative equal to zero, and solving for the value of Y:

$$\frac{dTC}{dY} = -\$140 + \$20Y = 0$$

$$20Y = 140$$

$$Y = 7$$

A check of the sign of the second derivative evaluated at that point ensures that a minimum has been located:

$$\frac{dTC}{dY} = -\$140 + \$20Y$$

$$\frac{d^2 TC}{dY^2} = 20$$

Because the second derivative is positive, $Y = 7$ is indeed a minimum.

Substituting 7 for Y in the constraint equation allows one to determine the optimal quantity to be produced on assembly line X:

$$X + 7 = 20$$

$$X = 13$$

Thus, production of 13 units of output on assembly line X and seven units on line Y is the least-cost combination for manufacturing a total of 20 units of the firm's product. The total cost of producing that combination is

$$TC = \$3(132) + \$6(72) – \$1(13 \times 7)$$

$$= \$507 + \$294 – \$91$$

$$= \$\underline{\underline{710}}$$

Lagrangian technique

Unfortunately, the substitution technique used in the preceding section is not always feasible. Constraint conditions are sometimes too numerous or complex for substitution to be used. In these cases, the technique of *Lagrangian multipliers* can be used.

The Lagrangian technique for solving constrained optimization problems is a method that calls for optimizing a function that incorporates the original objective function and the constraint conditions. This combined equation, called the Lagrangian function, is created in such a way that when it is maximized or minimized the original objective function is also maximized or minimized, and all constraints are satisfied.

A reexamination of the constrained minimization problem shown previously illustrates this technique. Recall that the firm sought to minimize the function $TC = \$3X^2 + \$6Y^2 – \$1XY$, subject to the constraint that $X + Y = 20$. Rearrange the constraint to bring all terms to the right of the equal sign:

$$0 = 20 – X – Y$$

This is always the first step in forming a Lagrangian expression. Multiplying this form of the constraint by the unknown factor λ and adding the result to the original objective function creates the Lagrangian expression:

$$LTC = \$3X^2 + \$6Y^2 – \$1XY + \lambda (20 – X – Y) \qquad \textbf{[3.14]}$$

LTC is defined as the Lagrangian function for the constrained optimization problem under consideration. Because it incorporates the constraint into the objective function, the Lagrangian function can be treated as an unconstrained optimization problem. The solution to the unconstrained Lagrangian problem is *always* identical to the solution of the original constrained optimization problem.

To illustrate, consider the problem of minimizing the Lagrangian function constructed in Equation 3.14. At a minimum point on a multivariate function, all partial derivatives must equal zero. The partials of Equation 3.14 can be taken with respect to the three unknown variables, *X*, *Y*, and λ, as follows:

$$\frac{\partial LTC}{\partial X} = 6X - Y - \lambda$$

$$\frac{\partial LTC}{\partial Y} = 12Y - X - \lambda \quad \text{and}$$

$$\frac{\partial LTC}{\partial \lambda} = 20 - X - Y$$

Setting these three partials equal to zero results in a system of three equations and three unknowns:

$$6X - Y - \lambda = 0 \qquad \textbf{[3.15]}$$

$$-X + 12Y - \lambda = 0 \qquad \textbf{[3.16]}$$

$$20 - X - Y = 0 \qquad \textbf{[3.17]}$$

Notice that Equation 3.17, the partial of the Lagrangian function with respect to λ, is the constraint condition imposed on the original optimization problem. The Lagrangian function is constructed so that the derivative of the function taken with respect to the Lagrangian multiplier, λ, always gives the original constraint. So long as this derivative is zero, as it must be at a local extreme (maximum or minimum), the constraint conditions imposed on the original problem are met.

Further, because the last term in the Lagrangian expression must equal zero (0 = 20 – *X*– *Y*), the Lagrangian function reduces to the original objective function, and the solution to the unconstrained Lagrangian problem is always the solution to the original constrained optimization problem.

Completing the analysis for the example illuminates these relations. To begin, it is necessary to solve the system of equations to obtain optimal values of *X* and *Y*.

Subtracting Equation 3.16 from Equation 3.15 gives

$$7X - 13Y = 0 \quad \textbf{[3.18]}$$

Multiplying Equation 2A.7 by 7 and adding Equation 3.18 to this product gives the solution for *Y*:

$$140 - 7X - 7Y = 0$$

$$7X - 13Y = 0$$

$$140 - 20Y = 0$$

$$140 = 20Y$$

$$Y = 7$$

Substituting 7 for *Y* in Equation 3.17 yields *X* = 13, the value of *X* at the point where the Lagrangian function is minimized. Because the solution of the Lagrangian function is also the solution to the firm's constrained optimization problem, 13 units from assembly line *X* and seven units from line *Y* is the least-cost combination of output that can be produced subject to the constraint that total output must be 20 units. This is the same answer obtained previously, using the substitution method. The Lagrangian technique is a more powerful technique for solving constrained optimization problems than the substitution method; it is easier to apply with multiple constraints, and it provides valuable supplementary information. This is because the Lagrangian multiplier itself has an important economic interpretation. Substituting the values of *X* and *Y* into Equation 3.15 gives the value of λ:

$$6 \times 13 - 7 - \lambda = 0$$

$$\lambda = \$71$$

Here, λ is interpreted as the marginal cost of production at 20 units of output. It means that if the firm were allowed to produce only 19 instead of 20 units of output, total costs would fall by approximately $71. If the output requirement were 21 instead of 20 units, costs would increase by roughly that amount. Because λ = $71 can be interpreted as the marginal cost of production, an offer to purchase another unit of output for $100 is acceptable because it results in a $29 marginal profit.

Conversely, an offer to purchase an additional unit for \$50 would be rejected because a marginal loss of \$21 would be incurred. λ can be thought of as a planning variable, because it provides valuable information concerning the effects of altering current activity levels.

Another example provides additional perspective on the Lagrangian method. Recall from the discussion of Equation 3.16 and Figure 3.15 that the profit function,

$$\pi = -\$10{,}000 + \$400Q - \$2Q^2$$

where π is total profit and Q is output in units, is maximized at $Q = 100$ with $\pi = \$10{,}000$. The impact of constraints in the production process, and the value of the Lagrangian method, can be portrayed by considering the situation in which each unit of output requires 4 hours of skilled labor, and a total of only 300 hours of skilled labor is currently available to the firm. In this instance, the firm seeks to maximize the function $\pi = -\$10{,}000 + \$400Q - \$2Q^2$, subject to the constraint $4Q = 300$ (because $L = 4Q$). Rearrange the constraint to bring all terms to the right of the equal sign:

$$0 = 300 - 4Q$$

Multiplying this form of the constraint by λ and adding the result to the original objective function creates the Lagrangian expression:

$$L\pi = -\$10{,}000 + \$400Q - \$2Q2 + \lambda\,(300 - 4Q) \qquad \textbf{[3.19]}$$

with the following partials:

$$\frac{\partial L\pi}{\partial Q} = 400 - 4Q - 4\lambda$$

and

$$\frac{\partial L\pi}{\partial \lambda} = 300 - 4Q$$

Setting these two partials equal to zero results in a system of two equations and two unknowns. Solving provides the values $Q = 75$, $\lambda = \$25$, and, from the objective function, $\pi = \$8{,}750$. The constraint on skilled labor has reduced output from 100 to 75 units and has reduced total profits from \$10,000 to \$8,750. The value $\lambda = \$25$ indicates that should a one-unit expansion in output become possible, total profits would rise by \$25. This information indicates that the maximum value of additional skilled labor is \$6.25 per hour, because each unit of output requires 4 hours of labor. Assuming there are no other costs involved, \$6.25 per hour is the most the firm would pay to expand employment.

The effects of relaxing the constraint as progressively more skilled labor becomes available are illustrated in Figure 3.12. If an additional 100 hours of skilled labor, or 400 hours in total, is available, the output constraint would become $0 = 400 - 4Q$, and solved values $Q = 100$, $\lambda = \$0$, and $\pi = \$10,000$ would result. The value $\lambda = \$0$ indicates that skilled labor no longer constrains profits when 400 hours are available. Profits are maximized at $Q = 100$, which is the same result obtained in the earlier unconstrained solution to this profit maximization problem.

In this instance, the output constraint becomes nonbinding because it does not limit the profit-making ability of the firm. Indeed, the firm is not willing to employ more than 400 hours of skilled labor. To illustrate this point, consider the use of 500 hours of skilled labor and the resulting constraint $0 = 500 - 4Q$. Solved values are $Q = 125$, $\lambda = -\$25$, and $\pi = \$8,750$.

The value $\lambda = -\$25$ indicates that one additional unit of output, and the expansion in employment that results, would reduce profits by \$25. Conversely, a one-unit reduction in the level of output would increase profits by \$25. Clearly, the situation in which $\lambda < 0$ gives the firm an incentive to reduce input usage and output, just as $\lambda > 0$ provides an incentive for growth.

To generalize, a Lagrangian multiplier, λ, indicates the marginal effect on the objective function of decreasing or increasing the constraint requirement by one unit. Often, as in the previous examples, the marginal relation described by the Lagrangian multiplier provides economic data that help managers evaluate the potential benefits or costs of relaxing constraints.

Thus, the Lagrangian multiplier technique creates an additional artificial variable for each constraint. Using these artificial variables, the constraints are incorporated into the objective function in such a way as to leave the value of the function unchanged. If a problem has two or more constraints, then a separate variable is defined for each constraint and incorporated into the Lagrangian function. In general, λ measures the marginal change in the value of the objective function resulting from a one-unit change in the value on the right hand side of the equality sign in the constraint relationship

Activity

a) What is meant b constrained optimization? (b) How important is this to managerial economics? (c) How can a constrained optimization problem be solved?

b) Suppose that a firm's profit function is given by the following relationship:

$$\pi = -25 + 100Q_1 + 95Q_2 - 10Q_1^2 - 5Q_2^2 - 5Q_1Q_2$$

where Q_1 and Q_2 are the respective quantities of the two products that the firm manufactures and sells.

Each unit of the two products requires 10 and 5 units respectively of a certain raw material. During the forthcoming period, the firm only has 50 unit of this raw material available to produce the two products. The firm desires to maximize profits subject to the raw materials constraint.

(a) Formulate the problem in a programming framework.

(b) Solve the problem using Lagrangian Multiplier techniques. What are the optimal quantities (Q_1 and Q_2) of the two products?

CHAPTER FOUR

THEORY OF DEMAND

Objective

- *To introduce Theory of demand*
- *To explain Types of Demand*
- *To explicate demand function*
- *To portray elasticity and revenue analysis*

4.1.Introduction

Demand theory plays a crucial role in the business decision-making. Business executives have to take decision on such matters as what to produce, and how much to produce. Naturally production manager would seek information regarding the commodity, and its respective quantities demanded by the consumers, at different prices.

Theory of demand provides an insight into the above problems. Through the analysis of demand, business executives can know

1. The factors which determine the size of demand,
2. Elasticities of demand i.e. how responsive or sensitive is the demand to the changes in its determinants,
3. Possibility of sales promotion through manipulation of prices,
4. Responsiveness of demand to the advertisement, and
5. Optimum levels of sales, inventories and advertisement cost, etc.

4.1.1. Meaning of Demand:

The term demand implies a desire backed by ability and willingness to pay.

The term demand for a commodity has always a reference to price, a period of time and a place.

Any statement regarding the demand for a commodity without reference to its price, time of purchase and place is meaningless and is of no practical use.

The amount of a commodity or service which will be bought at any given price per unit of time is called demand.

4.1.1.1. Types of Demand:

The demand for the various kinds of goods is generally classified on basis of the number of consumers of a product, suppliers of the product, nature of goods, duration of consumption of a commodity, interdependence of demand, period of demand, and nature of use of the goods.

1. **Individual Demand and Market Demand:**

***Individual Demand*:**

The quantity of a commodity which an individual is willing to buy at particular price of the commodity during a specific time period, given his money income, his taste, and prices of other commodities is known as individual demand for a commodity.

Individual's demand for a commodity thus depends on the price of the commodity, his money income, taste and prices of the substitutes and complements.

Market Demand:

The total quantity which all the consumers of a commodity are willing to buy at a given price per time unit, given their money income, taste, and prices of other commodities is known as market demand for the commodity.

The market demand for a commodity is the total of individual demands by all the consumers of the commodity, over a time period, and at a given price, other factors remaining the same.

Table 1: Price and Quantity Demanded

Price of commodity X (Birr per unit)	Quantity of X demand by			Market Demand
	C_1	C_2	C_3	
10	5	1	0	6
8	7	2	0	9
6	10	4	1	15
4	13	7	2	22
2	17	10	3	30
0	24	14	8	46

In the above table, there are three consumers viz, C_1, C_2, C_3 of a commodity X, and their individual demand at its different prices is as given. The last column presents the market demand i.e. the aggregate of individual demand by three consumers at different prices.

2. **Firm's Demand and Industry Demand:**

***Firm's Demand*:**

The quantity of a firm's produce that can be disposed of at a given price over a time period connotes the demand for the firm's product.

***Industry Demand*:**

Aggregate of demand for the product of the entire firm's of an industry is known as demand for industry's product.

3. **Autonomous Demand and Derived Demand:**

***Autonomous Demand*:**

Autonomous demand for a commodity is one that arises independent of the demand for any other commodity.

Example: Biological and physical needs of the human beings, demand for food, clothes, shelter etc. due to demonstration effect and advertisement of new products leads to autonomous demand.

***Derived Demand*:**

Derived demand is one i.e. tied to the demand for some parent product. In other words derived demand is one that, demand for a commodity that arises because of the demand for some other commodity may be considered as derived demand.

Example: 1. Demand for land, fertilizers and agricultural tools.

2. Demand for cotton, bricks, cement etc.

3. Demand for complementary commodities, which complement the use of other commodities or for supplementary commodities, which supplement or provides additional utility from the use of other goods is a derived demand; *For example*: (i). Power -regulator is a complementary goods of refrigerators and TV sets. (ii). Chair is a complement and table gloss is supplement to the use of table. Therefore, demand for power regulator, chair and table glass would be considered as derived demand.

4. Durable Goods Demand and Non - Durable Goods Demand:

***Durable Goods Demand*:**

Durable goods are those whose total utility is not exhaustible by single use. Such goods can be used repeatedly or continuously over a period. Durable goods may be consumers as well as producers goods.

Example:

1. Durable consumer goods: clothes, shoes, owner occupied residential houses, furniture, utensils, refrigerators, scooters, cars etc.
2. The producers durable goods:- Fixed assets, building, plant, machinery, office furniture and fixture etc.

***Non - Durable Goods Demand*:**

Non - durable goods are those which can be used or consumed only once and their total utility is exhausted in single use.

Example: Food items.

1. Non - durable consumer goods:- All food items, drinks, soaps, cooking fuel, gas, coal, mobile lighting, cosmetics etc.
2. Non - durable producer goods: - raw materials, fuel and power, finishing materials and packing items etc.

5. **Short - Term Demand and Long - Term Demand:**

***Short - Term Demand*:**

Short - term demand refers to the demand for such goods as are demanded over a short period. In this category fall mostly the variety consumer goods, goods of seasonal use, inferior substitutes during the scarcity period of superior goods etc.

Example:-

1. Demand for cowboy trousers, slacks, high-heel plat form shoes, broad ties, etc.
2. (A). Demand for umbrella, rain - coats, gum - boots, cold-drinks etc is of seasonal use. The demand for such goods lasts till the season lasts.

   (B). Although some goods are used only seasonally but are of durable nature e.g. electric fans, woolen garments etc.

- The short -term demand depends by and large, on the price of commodities, price of their substitutes, current disposable income of the consumer, consumption pattern etc.

***Long - Term Demand*:**

Long - term demand refers to the demand, which exists over a long period. The change is long term demand is perceptible only after a long period. Most generic goods have long - term demand.

Example:- Demand for consumer's and producer's goods.

Demand for durable and non - durable goods.

- The long - term demand depends on the long - term income trends, availability of better substitutes, sales promotion etc.

4.1.2. Demand Function:

Demand function shows the relationship between the quantity demanded by consumers and the price of the product.

A demand function is a list of prices and the corresponding quantities that consumers are willing and able to purchase at each price in the list, all other things remaining constant. Consumers are willing and able to purchase more of an item the lower its price, i.e. quantity demanded varies inversely with price.

In mathematical language, a function is a symbolic statement of relationship between the dependent and independent variables. Demand function states the relationship between demand for a product and its determinants.

$D_x = f(P_x)$

If we assume that all determinants of demand, except price remain constant, then the quantity of a commodity demanded will depend on its price. In the above function demand for a commodity (D_x) depends on its price (P_x). In which D_x is a dependent and P_x is an independent variable. There are two common forms of demand - price relationship. They are linear and non - linear.

Linear Demand Function:

A demand function is said to be linear when slope of the demand curve i.e., $\Delta D/\Delta P$ remains constant throughout its length.

$$Dx = a - bPx$$

Where: a = is an intercept or a constant, denoting total demand at zero price

b = $\Delta D/\Delta P$ is also a constant.

if values of 'a' and 'b' are known the total demand (D_x) for any given price (P_x) can easily be obtained and a demand schedule can be prepared. Let us assume:

a = 100

b = 5

D = 100-5P

Now the value of D_x can be easily obtained for any value of P_x.

Example: If $P_x = 5$ $D_x = 100-5(5) = 75$

If $P_x = 10$ $D_x = 100- 5(10) = 50$

If $P_x = 15$ $D_x = 100- 5(15) = 25$

If $P_x = 20$ $D_x = 100- 5(20) = 0$

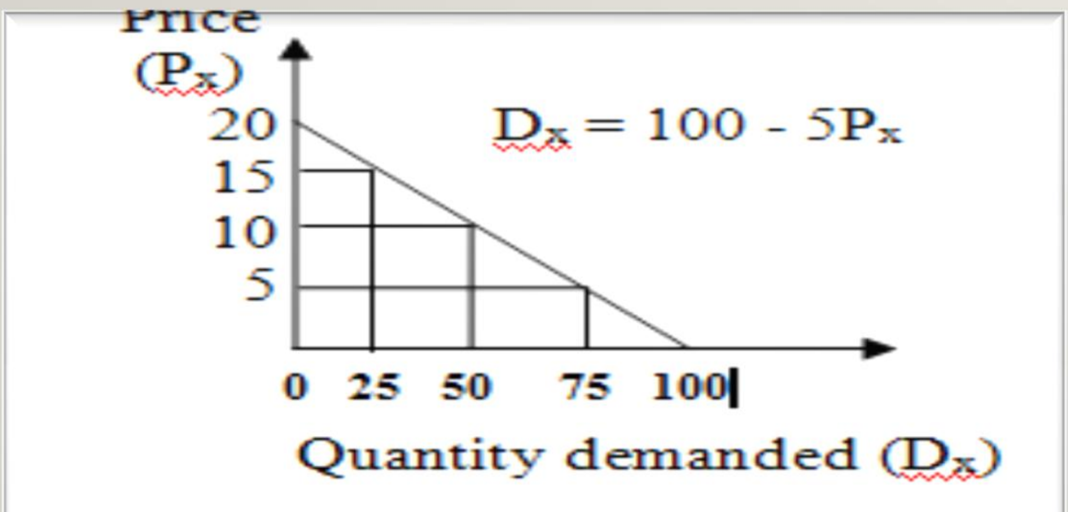


Figure 1: Linear demand function

Non-Linear Demand Function:

A demand function is said to be non-liner or curvilinear when the slope of a demand curve i.e. $\Delta D/\Delta P$ Changes all along the demand curve. Nonlinear demand function yields a demand curve instead of a demand line.

$D_x = aP_x^{-b}$

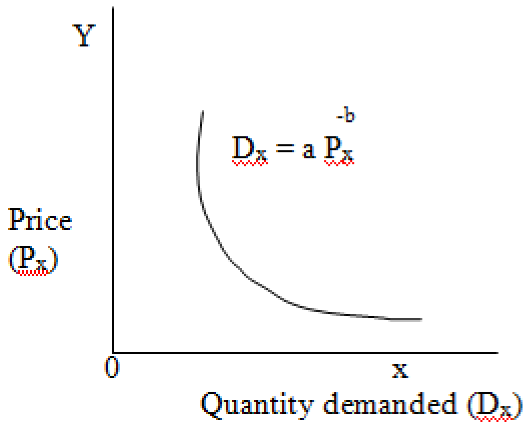


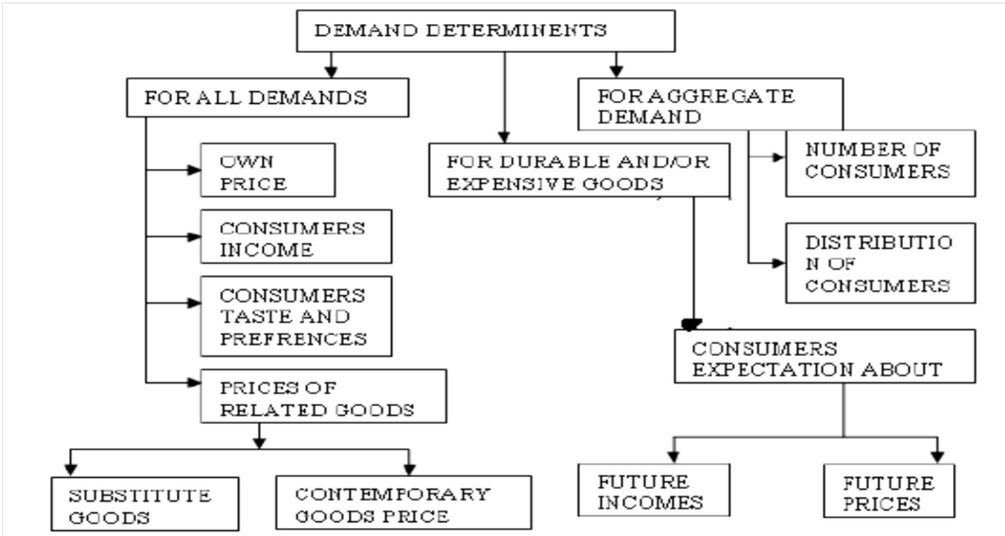
Figure 2: Non - Linear demand function

4.1.3. Demand Determinants:

The information regarding the prospective magnitude of demand for a product is indispensable in many business decisions, particularly those pertaining to production planning, sales management, inventories, advertisement etc.

Demand determinant factors for a product are price of the product, price and availability of the substitutes, consumer's income, his own preference for a commodity, utility derived from the good, demonstration effect, advertisement, credit facilities, off season discounts, population of the country etc. But all the above factors are not equally important. Some of them are not quantifiable e.g. consumer preferences, utility, demonstration effect, expectations etc.

To achieve the above objectives, demand analysis must include the factors which have bearing on the demand. No list of demand determinants could be exhaustive. The major ones would include as shown in the chart on below, arranged in a convenient way.



4.2. ELASTICITIES OF DEMAND

A measure of responsiveness used in demand analysis is elasticity, defined as the percentage change in a dependent variable, Y, resulting from a 1 percent change in the value of an independent variable, X. The equation for calculating elasticity is

$$\text{Elasticity} = \frac{\text{Percentage Change in Y}}{\text{Percentage Change in X}}$$

The concept of elasticity simply involves the percentage change in one variable associated with a given percentage change in another variable. The elasticity concept is also used in finance, where the impact of changes in sales on earnings under different production levels (operating leverage) and different financial structures (financial leverage) are measured by an elasticity factor. Elasticities are also used in production and cost analysis to evaluate the effect of changes in input on output, and the effect of output changes on costs.

Point Elasticity and Arc Elasticity

Elasticity is measured in two ways, ***point elasticity*** and ***arc elasticity***. Point elasticity measures elasticity at a given point on a function. The point elasticity concept is used to measure the effect on a dependent variable Y of a very small or marginal change in an independent variable X. Although the point elasticity concept can often give accurate estimates of the effect on Y of very small (less than 5 percent) changes in X, it is not used to measure the effect on Y of large scale changes. Elasticity typically varies at different points along a function. To assess the effects of large-scale changes in X, the arc elasticity concept is used.

Arc elasticity measures average elasticity over a given range of a function. Using the lowercase epsilon as the symbol for point elasticity, the point elasticity formula is written

$$\text{Point Elasticity} = \epsilon_X = \frac{\text{Percentage Change in Y}}{\text{Percentage Change in X}} = \frac{\partial Y / Y}{\partial X / X} = \frac{\partial Y}{\partial X} \times \frac{X}{Y}$$

The ∂Y/∂X term in the point elasticity formula is the marginal relation between Y and X, and it shows the effect on Y of a one-unit change in X. Point elasticity is determined by multiplying this marginal relation by the relative size of X to Y, or the X/Y ratio at the point being analyzed.

Point elasticity measures the percentage effect on Y of a percentage change in X at a given point on a function. If $\varepsilon_X = 5$, a 1 percent increase in X will lead to a 5 percent increase in Y, and a 1 percent decrease in X will lead to a 5 percent decrease in Y. Thus, when $\varepsilon_X > 0$, Y changes in the same positive or negative direction as X. Conversely, when $\varepsilon_X < 0$, Y changes in the opposite direction of changes in X. For example, if $\varepsilon_X = -3$, a 1 percent increase in X will lead to a 3 percent decrease in Y, and a 1 percent decrease in X will lead to a 3 percent increase in Y.

Advertising Elasticity Example

An example can be used to illustrate the calculation and use of a point elasticity estimate. Assume that management of a movie theater is interested in analyzing the responsiveness of movie ticket demand to changes in advertising. Also assume that analysis of monthly data for the past year suggests the following demand function:

$$Q = 8{,}500 - 5{,}000P + 3{,}500PV + 150I + 1{,}000A \quad \ldots\ldots\ldots\ldots \quad (4.2)$$

Where Q is the quantity of movie tickets, P is average ticket price (in dollars), P_V is the 1-day movie rental price at video outlets in the area (in dollars), I is average disposable income per household (in thousands of dollars), and A is monthly advertising expenditures (in thousands of dollars). (Note that I and A are expressed in thousands of dollars in this demand function.) For a typical theater, P= $7, P_V = $3, and income and advertising are $40,000 and $20,000, respectively. The demand for movie tickets at a typical theater can be estimated as

$$Q = 8{,}500 - 5{,}000(7) + 3{,}500(3) + 150(40) + 1{,}000(20) = 10{,}000$$

Numbers that appear before each variable in Equation 4.2 are called coefficients or parameter estimates. They indicate the expected change in movie ticket sales associated with a one-unit change in each relevant variable. For example, the number -5,000 indicates that the quantity of movie tickets demanded falls by 5,000 units with every $1 increase in the price of movie tickets, or $\partial Q/\partial P = -5{,}000$. Similarly, a $1 increase in the price of videocassette rentals causes a 3,500-unit increase in movie ticket demand, or $\partial Q/\partial P_V = 3{,}500$; a $1,000 (one-unit) increase in disposable income per household leads to a 150-unit increase in demand. In terms of advertising, the expected change in demand following a one-unit ($1,000) change in advertising, or $\partial Q/\partial A$, is 1,000. With advertising expenditures of $20,000, the point advertising elasticity at the 10,000-unit demand level is,

$$\begin{aligned}
\epsilon_A &= \text{Point Advertising Elasticity} \\
&= \frac{\text{Percentage Change in Quantity}}{\text{Percent Change in Advertising}} \\
&= \frac{\partial Q/Q}{\partial A/A} \\
&= \frac{\partial Q}{\partial A} \times \frac{A}{Q} \\
&= 1{,}000 \times \frac{\$20}{10{,}000} \\
&= 2
\end{aligned}$$

Thus, a 1 percent change in advertising expenditures results in a 2 percent change in movie ticket demand. This elasticity is positive, indicating a direct relation between advertising outlays and movie ticket demand. An increase in advertising expenditures leads to higher demand; a decrease in advertising leads to lower demand.

For many business decisions, managers are concerned with the impact of substantial changes in a demand-determining factor, such as advertising, rather than with the impact of very small (marginal) changes. In these instances, the point elasticity concept suffers a conceptual shortcoming.

To see the nature of the problem, consider the calculation of the advertising elasticity of demand for movie tickets as advertising increases from $20,000 to $50,000. Assume that all other demand-influencing variables retain their previous values. With advertising at $20,000, demand is 10,000 units. Changing advertising to $50,000 ($\partial A = 30$) results in a 30,000-unit increase in movie ticket demand, so total demand at that level is 40,000 tickets. Using the above Equation to calculate the advertising point elasticity for the change in advertising from $20,000 to $50,000 indicates that

$$\text{Advertising Elasticity} = \frac{\partial Q}{\partial A} \times \frac{A}{Q}$$

$$= \frac{30{,}000}{\$30} \times \frac{\$20}{40{,}000} = 2$$

The advertising point elasticity is $\varepsilon_A = 2$, just as that found previously. Consider, however, the indicated elasticity if one moves in the opposite direction—that is, if advertising is decreased from $50,000 to $20,000. The indicated point elasticity is

$$\text{Advertising Elasticity} = \frac{\partial Q}{\partial A} \times \frac{A}{Q}$$

$$= \frac{-30{,}000}{-\$30} \times \frac{\$50}{40{,}000} = 1.25$$

The indicated elasticity $\varepsilon_A = 1.25$ is now different. This problem occurs because elasticities are not typically constant but vary at different points along a given demand function. The advertising elasticity of 1.25 is the advertising point elasticity when advertising expenditures are $50,000 and the quantity demanded is 40,000 tickets

To overcome the problem of changing elasticities along a demand function, the arc elasticity formula was developed to calculate an average elasticity for incremental as opposed to marginal changes. The arc elasticity formula is

$$E = \text{Arc Elasticity} = \frac{\dfrac{\text{Change in } Q}{\text{Average } Q}}{\dfrac{\text{Change in } X}{\text{Average } X}} = \frac{\dfrac{Q_2 - Q_1}{(Q_2 + Q_1)/2}}{\dfrac{X_2 - X_1}{(X_2 + X_1)/2}}$$

$$= \frac{\dfrac{\Delta Q}{(Q_2 + Q_1)}}{\dfrac{\Delta X}{(X_2 + X_1)}} = \frac{\Delta Q}{\Delta X} \times \frac{X_2 + X_1}{Q_2 + Q_1}$$

The percentage change in quantity demanded is divided by the percentage change in a demand-determining variable, but the bases used to calculate percentage changes are averages of the two data endpoints rather than the initially observed value. The arc elasticity equation eliminates the problem of the elasticity measure depending on which end of the range is viewed as the initial point. This yields a more accurate measure of the relative relation between the two variables over the range indicated by the data. The advertising arc elasticity over the \$20,000 to \$50,000 range of advertising expenditures can be calculated as

$$\text{Advertising Arc Elasticity} = \frac{\text{Percentage Change in Quantity } (Q)}{\text{Percent Change in Advertising } (A)}$$

$$= \frac{(Q_2 - Q_1)/(Q_2 + Q_1)}{(A_2 - A_1)/(A_2 + A_1)}$$

$$= \frac{\Delta Q}{\Delta A} \times \frac{A_2 + A_1}{Q_2 + Q_1}$$

$$= \frac{30{,}000}{\$30} \times \frac{\$50 + \$20}{40{,}000 + 10{,}000}$$

$$= 1.4$$

Thus, a 1 percent change in the level of advertising expenditures in the range of \$20,000 to \$50,000 results, on average, in a 1.4 percent change in movie ticket demand. To summarize, it is important to remember that point elasticity is a marginal concept. It measures the elasticity at a specific point on a function. Proper use of point elasticity is limited to analysis of very small changes, say 0 percent to 5 percent, in the relevant independent variable. Arc elasticity is a better concept for measuring the average elasticity over an extended range when the change in a relevant independent variable is 5 percent or more. It is the appropriate tool for incremental analysis.

4.2.1. Revenue Analysis

- A major motive of any firm is to maximize profit. So it should have a price policy, keeping the long run prospects in mind, to attract maximum revenue.
- Both revenue and the cost concepts are of immense importance in Business economics.
- The relationship between revenue and price elasticity of demand has practical applications in real business life. A producer can use the knowledge of these two concepts to decide the price of his/her products.

Different types of revenues

- Revenue is the income received by the firm.
- The total revenue concept is related to
  - Total revenue – the total income of a firm by selling a commodity at a price. TR=PxQ
  - Average revenue – is the total revenue divided by the number of units sold. AR=TR/Q
- We can say that, AR of a firm is in fact the price of the commodity at each level of output.
  - Marginal revenue – is the addition to the total revenue as a result of increase in the sale of an addition unit by the firm. MR=ΔTR/ ΔQ

Relationship between TR and Price Elasticity of Demand

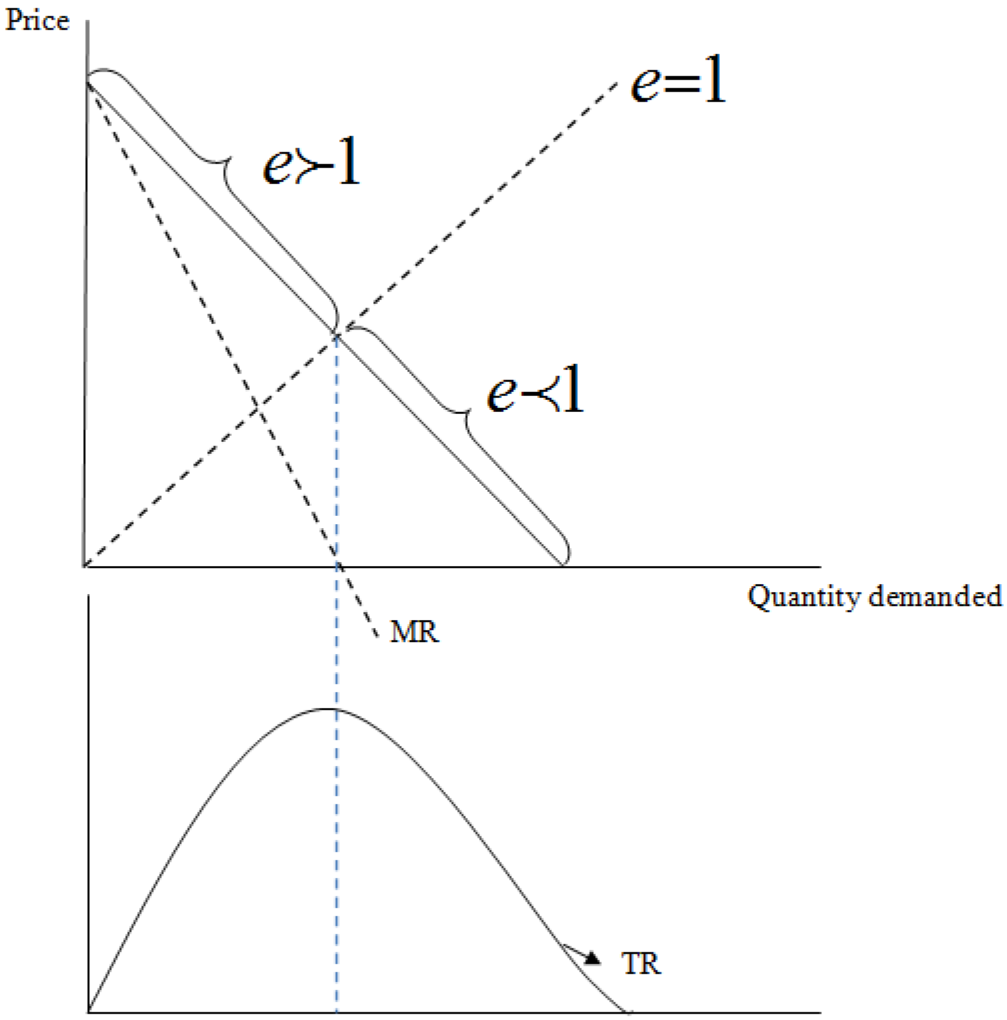
- The relationship is important because every firm has to decide whether to increase or decrease the price depending on the price elasticity of demand of the product.
- If the price elasticity of demand for his/her product is relatively inelastic, then he/she can increase the price to increase TR (in case of normal good).
- If the price elasticity of demand for the product is relatively elastic, increase In price will decrease his/her total revenue (in case of normal goods).
  - **Note**: Total Revenue always moves in the same direction as the variable (Q or P) having the dominant effect.
  -

Price change	Relatively Elastic ($\varepsilon_p > 1$) $\vert\%\Delta Q\vert > \vert\%\Delta P\vert$ **Q-effect dominate**	Relatively unit elastic ($\varepsilon_p = 1$) $\vert\%\Delta Q\vert = \vert\%\Delta P\vert$ **No dominant effect**	Relatively inelastic ($\varepsilon_p < 1$) $\vert\%\Delta Q\vert <> \vert\%\Delta P\vert$ **P-effect dominates**
When price increases	Total revenue will decrease	No change on Total revenue	Total revenue will increase
When price decreases	Total revenue will increase	No change on Total revenue	Total revenue will decrease

4.2.2. Price Elasticity and Marginal Revenue:

The relationship between price elasticity and the total revenue (TR) can be more precisely known though the relationship between price elasticity and marginal revenue (MR).

Price elasticity affects the MR which affects the TR. The responsiveness of consumers to changes in the price of a good must be considered in the profit maximizing decisions of a manager.


Price
e≻1
e=1
e≺1
Quantity demanded
MR
TR

Marginal revenue is the addition to total revenue attribute to selling one additional unit of output.

$$MR = \frac{\Delta TR}{\Delta Q}$$

Example: Suppose given output Q and P

TR = PQ

Now whether a change in P will increase or decrease or leave the TR unaffected. Whether addition to the TR i.e. MR > 0 or MR < 0 or MR = 0. The MR can be obtained by differentiating TR = PQ with respect to P. Thus

$$MR = \frac{\partial TR}{\partial P} = P + Q\frac{\partial P}{\partial Q}$$

$$= P\left(1 + \frac{Q}{P} \cdot \frac{\partial P}{\partial Q}\right) \text{...} Equation - 1$$

Note that $\frac{Q}{P} \cdot \frac{\partial P}{\partial Q}$ is the reciprocal of the elasticity. Thus,

$$\frac{Q}{P} \cdot \frac{\partial P}{\partial Q} = \frac{-1}{\varepsilon}$$

By substituting $\frac{-1}{\varepsilon}$ for $\frac{Q}{P} \cdot \frac{\partial P}{\partial Q}$ in *equation-1* we get:

$$MR = P\left(1 - \frac{1}{\varepsilon}\right) \text{..} Equation - 2$$

Given this relationship between MR and price elasticity of demand, the decision - makers can easily know whether is worthwhile to change price.

If e = 1, MR = 0.

Therefore, the change in price will not cause any change in TR.

If e < 1, MR < 0: when P↓ → TR↓ and vice versa.

If e > 1, MR > 0: when P↑ → TR↓ and vice versa.

Sample Applications:

1. Suppose a marginal revenue (MR) from a product is $10 and the price elasticity of demand (ε_p) is -1.5. What is the price of the product?
2. Assume Sheba tannery, which had monthly sneaker sales of 10,000 pairs (at $10 per pair) before a price cut by its major competitor. After this competitor's price reduction, Sheba's sales declined to 8,000 pairs a month. From the past experience Sheba has estimated the price elasticity of demand (ε_p) to be about -2.0 in this price-quantity range. If Shaba wishes to restore its sales to 10,000 pairs a month, determine the price that must be charged.
3. The demand function for truck assembled in MIE has been estimated to be:

   $$\mathbf{Q_d = 1{,}200 - 600P + 20Y}$$

   Where Y is in thousands of dollar; when P=$20 and Y=$50,000, determine

   a). price elasticity of demand

   b). Income elasticity of demand

CHAPTER FIVE

DECISION THEORY

- *To explain decision theory*
- *To explain the characteristics of problem formulation*
- *To portray types of decision environment*

5.0.Definition of Decision Theory

The success or failure of an individual or organization experiences, depends upon the ability to make appropriate decision. For making appropriate decision it requires certain course of action or strategies which should be feasible (possible) and viable (exists) in nature. Decision theory provides an analytical and systematic approach to depict the expected result of a situation when alternative managerial actions and outcomes are compared.

Decision theory is the combination of descriptive and prescriptive business modelling approach i.e., it is concerned with identifying the best decision to take, assuming an ideal decision maker who is fully informed, able to compute with perfect accuracy, and fully rational. The practical application of this prescriptive approach (how people *actually* make decisions) is called decision analysis, and aimed at finding tools, methodologies and software to help people make better decisions which can be classified as a degree of knowledge. The knowledge of degree is divided into four categories which are given below:-

5.1.Characteristics of Problem Formulation

A. **Decision alternatives:** In this case, N numbers of alternatives are available with the decision maker whenever the decision is made. These alternatives may depend on the previous decisions made. These alternatives are also called courses of action which are under control and known to decision maker.

B. **States of nature:** These are the future conditions (also known as consequences, events, or scenarios) which are not under the control of decision maker. A state of nature can be inflation, a weather condition, a political development etc. it usually is not determined by an action of an individual or an organization. But it may identify through some technique such as scenario analysis. Ex- stakeholders, long-time managers.

C. **Pay off:** A numerical outcome resulting from each possible combination of alternatives and states of nature is called payoff. The payoff values are always conditional values because of unknown states of nature. The payoff is measured within a specified period (e.g. after one year). This period is sometimes called decision horizon. Payoff can be measured in terms of money market share, or other measures.

D. **Pay off table**: A tabular arrangement of these conditional outcomes (profit or loss values) is known as payoff matrix. To construct a payoff matrix, the decision alternatives (courses of action or strategies) and states of nature are represented in the tabular form as below:

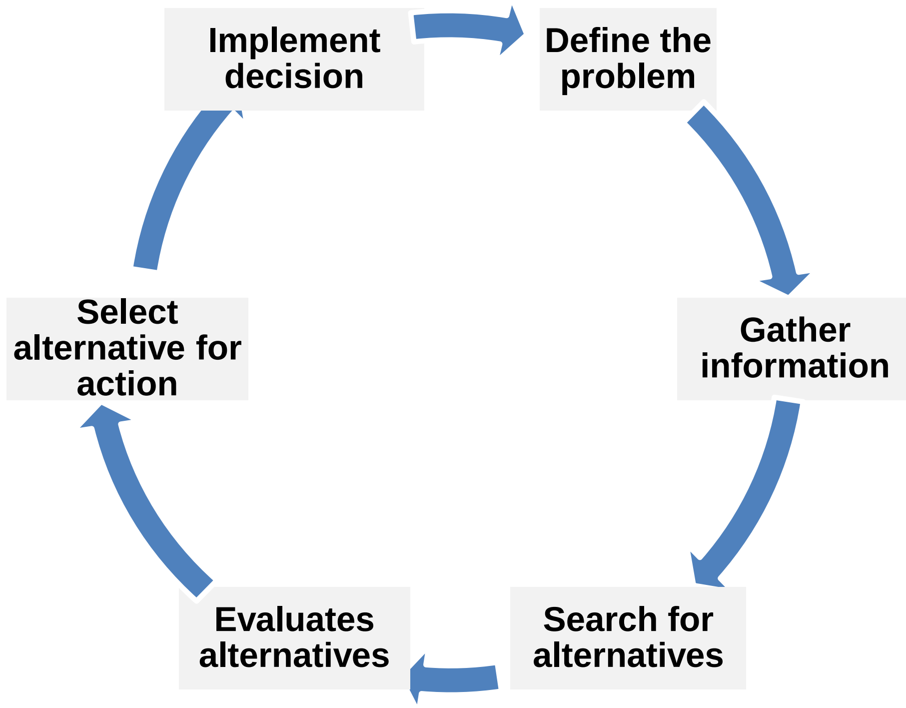
States of nature(events)	**Decision alternative (courses of action)**				
	A_1	A_2	A_3	……	A_M
E_1	A_{11}	A_{12}	A_{13}	……	$A1_m$
E_2	A_{21}	A_{22}	A_{23}	……	$A2_m$
E_3	A_{31}	A_{32}	A_{33}	……	$A3_m$
…..	…..	……	…..	……	……
E_N	A_n1	A_n2	A_n3	……	A_{mn}

5.1.1. Steps in Decision Theory Approach

- Identify and define the problem
- Listing of all possible future events, called states of nature. Such events are not under control of decision maker.
- Identification of all the courses of action which are available to the decision-maker.
- Evaluating the alternatives such as, cost effectiveness, performance, quality, output, profit.
- Expressing the pay-offs resulting from each pair of course of action and state of nature.

○ Choosing an appropriate course of action from the given list on the basis of some criterion that result in the optimal pay-off.

○ The next step is to implement the decision.



5.2.Types of Decision Making Environments

5.2.1. Decision Making Under Certainty:

In this type the decision maker has the perfect information about the consequences of every course of action or alternatives with certainty. Definitely he selects an alternative that gives the maximum return (pay-off) for the given state of nature. For ex- one has choices either to purchase national saving certificate, Indira Vikas Patra or deposit in national saving scheme. Obviously he will invest in one the scheme which will give him the assured return. In these decision models only one possible state of nature exists.

5.2.2. Decision-Making Under Risk

In this type, the decision maker has less information about the certainty of the consequence of every course of action because he is not sure about the return. In these decision model more than one state of nature exists for which he makes an assumption of the probability with each state of nature which will occur. For ex- probability of getting head in the toss of a coin is 50%.

5.2.3. Decision-Making Under Uncertainty

In this type, the decision-maker is unable to predict the probabilities of the various states of nature which will occur. Here the possible states of nature are known but still there is a less information than the decision under risk. For ex- The probability that Mr. Y will be the captain of the Indian cricket team for coming 10 years from now is not known.

5.2.4. **Decision-Making Under Conflict**

In this type, the consequences of each act of the decision maker are influenced by the acts of opponent. An example of this is the situation of conflict involving two or more competitors marketing the same product. The technique used to solve this category is the game theory.

5.3.2. Decision under Risk

As it already explained above, it is a probabilistic decision situation, in which more than one state of nature exists and the decision maker has sufficient information to assign probability values to the likely occurrence of each of these states. Knowing the probability distribution of the states of nature, the best decision is to select the course of action which has the largest expected payoff value. The expected (average) payoff of an alternative is the sum of all possible payoffs of that alternative weighted by the probabilities of those payoffs occurring.

The most widely used criterion for evaluating various courses of action under risk:

I. Expected Monetary Value (EMV) or Expected utility.

II. Expected opportunity Loss (EOL).

III. Expected value of Perfect Information (EVPI)

5.3.2.1. Expected Monetary Value (EMV)

The expected value (EMV) for a given course is the weighted sum of possible payoffs for each alternative. It is obtained by summing the payoffs for each course of action multiplied by the probabilities associated with each state of nature. The expected (or mean) value is the long-run average value that result if the decision were repeated a large number of times.

Steps for calculating EMV: The various steps involved in the calculation of EMV are as follow:

1. Construct a payoff matrix listing all possible courses of action and states of nature. Enter the conditional payoff values associated with each possible combination of course of action and state of nature along with probabilities of the occurrence of each state of nature.
2. Calculate the EMV for each course of action by multiplying the conditional payoffs by the associated probabilities and add these weighted values for each course of action.
3. Select the course of action that yields the optimal EMV.

5.3.2.2. Expected Opportunity Loss (EOL)

An alternative approach to maximizing expected monetary value (EMV) is to minimize the expected opportunity loss (EOL) also called expected value of regret. The EOL is defined as the difference between the highest profit (highest payoffs) for a state of nature and the actual profit obtained for the particular course of action taken. In other words, EOL is the amount of payoff that is lost by not selecting the course of action that has greatest payoff for the state of nature that actually occurs. The course of action due to which EOL is minimum is recommended.

Since EOL is an alternative decision criterion for decision making under risk, therefore the results will always be the same as those obtained by EMV criterion. Thus only one of the two methods should be applied to reach to a decision. It is stated as follows:

Steps for calculating EOL: The steps which are involved in the calculation of EOL are as follows:

1. Prepare a conditional profit table for each course of action and state of nature combination along with the associated probabilities.

2. For each state of nature calculate the conditional opportunity loss (COL) values by subtracting each payoff from the maximum payoff for that outcome.

3. Calculate EOL for each course of action by multiplying the probability of each state of nature with the COL value and then adding the values.

4. Select a course of action for which the EOL value is minimum

5.3.2.3. Expected Value of Perfect Information (EVPI)

In these decisions making under risk each state of nature as associated with the probability of its occurrence. Perfect information about the future demand would remove uncertainty for the problem. With these perfect information the decision maker would know in advance exactly about the future demand and he will be able to select a course of action that yields the desired payoff for whatever state of nature that actually occurs.

EVPI represents the maximum amount the decision maker has to pay to get to this additional information about the occurrence of various events.

EVPI = (expected profit with perfect information)- (expected profit without perfect information).

Example1: A shopkeeper buys apple for Rs 20/kg and sells them for Rs 30/kg. The past records of the sales are as follows:

Number of customers:	50	80	100	120	150
Number of Days:	20	30	20	10	20

Answer: Profit = Selling price – Cost price.

= 30-20, = **10**

Probability	Demand	Supply				
		50	80	100	120	150
20/100= 0.20	50	500	-100	-500	-900	-1500
30/100= 0.30	80	500	800	400	0	-600
20/100= 0.20	100	500	800	1000	600	0
10/100= 0.10	120	500	800	1000	1200	600
20/100= 0.20	150	500	800	1000	1200	1500
Total :		2500	3100	2900	2100	0

Maximum value :	500	800	1000	1200	1500
Minimum value :	500	-100	-500	-900	-1500

Maxi max: 1500 out of all maximum values.

Maxi min: 500 out of all minimum values.

Laplace:

2500/5 3100/5 2900/5 2100/5 0/5

= 500 = 620 = 580 = 420 = 0

= 620 in 80 units

Step 1: Write the demand & probability in column and supply in row.

Step 2: Calculate the Expected Pay off table (probability × pay off)

		Supply				
Probability	Demand	50	80	100	120	150
20/100= 0.20	50	100	-20	-100	-180	-300
30/100= 0.30	80	150	240	120	0	-180
20/100= 0.20	100	100	160	200	120	0
10/100= 0.10	120	50	80	100	120	60
20/100= 0.20	150	100	160	200	240	300
Total		500	620	520	300	-120

Expected monetary value (EMV): 620 (the highest one among the total).

Expected profit on perfect information (EPPI): (100+240+200+120+300) = 960.

Expected value of perfect competition (EVPI): 960 – 620 = 340.

Expected opportunity loss table: Deduct the highest number from expected payoff table from each row.

Probability	Demand	Supply				
		50	80	100	120	150
20/100= 0.20	50	0	120	200	280	400
30/100= 0.30	80	90	0	120	240	420
20/100= 0.20	100	100	40	0	80	200
10/100= 0.10	120	70	40	20	0	60
20/100= 0.20	150	200	140	100	60	0
Total		460	340	440	660	1080

Expected opportunity loss table will be 340, the minimum among all total.

Expected value of perfect information (EVPI) = Expected opportunity of loss table (EOL).

5.3.3. Decision Making Under Uncertainty

In the absence of information about the probability of any state of nature occurring, the decision-maker must arrive at a decision only on the actual conditional pay-offs values, together with a policy. There are several different criteria of decision making in these situation. The criteria are as follows:-

I. Optimism (Maximax or Minimin) criterion.

II. Pessimism (Maximin or Minimax) criterion.

III. Equal probabilities (Laplace) criterion.

IV. Coefficient of optimism (hurweiz) criterion.

V. Regret (salvage) criterion.

I. <u>Optimism criterion:</u> In this criterion the decision-maker always looks for the maximum possible profit (Maximax) or lowest possible (Minimin).Therefore he selects the alternatives that maximum of the maxima (or minimum of the minima) pay-offs. The methods are as follows:

A. Find the maximum (or minimum) payoff values corresponding to each alternative courses of action.

B. Select the alternative with the best anticipated payoff value i.e., maximum profit and minimum profit.

Examples 2: Based on the following information, which strategy should the company adopt?

Strategies →

States of nature	S1	S2	S3
P_1	2,00,000	5,00,000	3,00,000
P_2	4,00,000	1,50,000	9,00,000
P_3	0	4,50,000	7,00,000

Strategies →

States of nature	S1	S2	S3
P_1	2,00,000	5,00,000	3,00,000
P_2	4,00,000	1,50,000	9,00,000
P_3	0	4,50,000	7,00,000
Column maximum	4,00,000	5,00,000	9,00,000

Maximax

Answer: The maximum of column maxima is 9, 00,000

Hence the company should adopt strategy S3.

I. **Pessimism criterion:** in this criterion the decision-maker ensures that he should not

earn no less (or pay no more) than some specified amount. Thus, he selects the alternative that represents the maximum of the minima payoff in case of profits. The methods are as follows :

A. Find the minimum (or maximum in case of profits) payoff values in case of loss (or cost) data corresponding to each alternative.

B. Select an alternative with the best anticipated payoff value (maximum for profit and minimum for loss or cost).

Examples 3:

Strategies ⟶

States of nature	S1	S2	S3
P_1	2,00,000	5,00,000	3,00,000
P_2	4,00,000	1,50,000	9,00,000
P_3	0	4,50,000	7,00,000

Solution:-

Strategies →

States of nature	S1	S2	S3
P_1	2,00,000	5,00,000	3,00,000
P_2	4,00,000	1,50,000	9,00,000
P_3	0	4,50,000	7,00,000
Column minimum	0	1,50,000	3,00,000

Maximin

The row with the maximum value is the answer

i.e., **3, 00,000**

I. <u>Equal probabilities (Laplace) criterion:</u> The probabilities of states of nature are not known, so it is assumed that all states of nature will occur with equal probability. As states of nature are mutually exclusive and collectively exhaustive, so the probability of each of these must be 1/ (number of states of nature). The methods are as follows:-

1. Assign equal probability value to each state of nature by using the formula:

= 1/ (number of states of nature).

2. Calculate the expected (or average) payoff for each alternative (course of action) by adding all the payoffs and dividing by the number of possible states of nature or by applying the formula:

= (probability of state of nature) *(payoff value for combination of alternative, and state of nature)

3. Selected the best expected payoff value (maximum profit and minimum cost).

Strategy	**Expected return**
S1	2,00,000 + 4,00,000 + 0 = 6,00,000/3= 2,00,000

S2	5,00,000 + 1,50,000 + 4,50,000 = 11,00,000/3 = 3,66,666
S3	3,00,000 + 9,00,000 + 7,00,000 = 19,00,000/3 =6,33,333

Since the largest expected return is from strategy $_{S3}$. The executive must select strategy $_{S3.}$

I. **<u>Coefficient of optimism (Hurwitz) criterion</u>:** In this criterion a decision maker should

neither be completely optimistic nor of pessimistic. It should be a mixture of both. Hurwitz, who suggested this criterion, introduced the idea of coefficient of optimism (denoted by α) to measure the degree of optimism. This coefficient lies between 0 and 1 represents a complete pessimistic attitude about future and 1 a complete optimistic attitude about future. Thus if α is the coefficient of optimistic, then (1-α) will represent the coefficient of pessimism.

Hurwicz approach suggests that the decision maker must select an alternative that maximizes

II. (criterion of realism) =α (maximum in column)+ (1-α) minimum in column.

The methods are as follows:

A. Decide the coefficient of optimism α and then coefficient of pessimism(1 – α)

B. For each alternative select the largest and the lowest payoff value and multiply these with α and (1-α) values, respectively. Then calculate the weighted average, H by using above formula.

C. Select an alternative with best anticipated weighted average payoff value.

Example 4: Let the degree of optimism being 0.7.

Strategies →

States of nature	S1	S2	S3
P_1	2,00,000	5,00,000	3,00,000
P_2	4,00,000	1,50,000	9,00,000
P_3	0	4,50,000	7,00,000

Strategies →

States of nature	Maximum pay off (i)	Minimum pay off (ii)	H =α(i) +(1-α) (ii)
P_1	5,00,000	2,00,000	0.7×5,00,000 +0.3×2,00,000=4,10,000
P_2	9,00,000	1,50,000	0.7×9,00,000 +0.3×1,50,000=6,75,000
P_3	7,00,000	0	0.7×7,00,000+0.3×0=4,90,000

The maximum value of **H = 6, 75,000**

2. **Regret (savage) criterion: In** this, criterion is also known as opportunity loss decision criterion or minimax regret decision criterion because decision maker feels regret after adopting a wrong course of action resulting in an opportunity loss of payoff. Thus he always intends minimize this regret. The method is as follows:

A. Find the best payoff corresponding to each state of nature.

B. Subtract all other entries (payoff values) in that row from this value.

C. For each course of action identify the worst or maximum regret table . Record this number in a new row.

D. Select the course of action with the smallest anticipated opportunity- loss value.

Examples 5:

Strategies

States of nature	**S1**	**S2**	**S3**
P_1	**2,00,000**	**5,00,000**	**3,00,000**
P_2	**4,00,000**	**1,50,000**	**9,00,000**
P_3	**0**	**4,50,000**	**7,00,000**

Solution:-

Strategies →

States of nature	S1	S2	S3
P_1	5,00,000-2,00,000=3,00,000	5,00,000-5,00,000=0	5,00,000-3,00,000=2,00,000
P_2	9,00,000-4,00,000=5,00,000	9,00,000-1,50,000=7,50,000	9,00,000-9,00,000=0
P_3	7,00,000-0=7,00,000	7,00,000-4,50,000=2,50,000	7,00,000-7,00,000=0
Column Maximum	7,00,000	7,50,000	2,00,000

MiniMax Regret

Self-Assessment Exercises

1. A small building contractor has recently experienced two successive years in which work opportunities exceeded the firm's capacity. The contractor must now make a decision on capacity for next year. Estimated profits under each of the three possible states of nature are as shown in the table below

Alternatives	Demand		
	Low	Middle	High
Do nothing	$ 50	$40	$60
Expand	$20	$50	$80
Subcontract	$40	$30	$70

Which alternative should be selected if the decision criterion is:

a) Maximax?

b) Maximin?

c) Equal probability

d) Minimax regret?

CHAPTER SIX

PRODUCTION & COST DECISIONS

- *To describe production and cost in the short run*
- *To explain production and cost in the long run*
- *To describe economic of scope*

Introduction

No doubt almost all managers know that profit is determined not only by the revenue a firm generates but also by the costs associated with production of the firm's good or service. Many managers, however, find managing the revenue portion of the profit equation more interesting and exciting than dealing with issues concerning the costs of production. After all, revenue-oriented decisions may involve such tasks as choosing the optimal level and mix of advertising media, determining the price of the product, and making decisions to expand into new geographic markets or new product lines. Even the decision to buy or merge with other firms may be largely motivated by the desire to increase revenues.

When revenue-oriented tasks are compared with those involved in production issues – spending time with production engineers discussing productivity levels of workers or the need for more and better capital equipment, searching for lower cost

Suppliers of production inputs, adopting new technologies to reduce production costs, and perhaps even engaging in a downsizing plan - it is not surprising that managers may enjoy time spent on revenue decisions more than time spent on production and cost decisions.

This unit builds on the knowledge gained in the course on Microeconomics-I, specifically the theory of production decisions and the theory of cost analysis.

Here we will show you some statistical techniques that can be used to estimate production and cost functions. The focus will be on estimating short-run production functions and short-run cost functions. These are the functions that managers need to make a firm's pricing, output and hiring decisions. Although long-run production and cost functions can help managers make long-run decisions about investments in plant and equipment, most of the analysis in this unit concerns short-run operational decisions. Application of regression analysis to the estimation of short-run production and cost functions is rather a straightforward task. However, because of difficult problems with the data that are required to estimate long-run production and cost functions – as well as the more complex regression equations required – managers typically restrict their use of regression analysis to estimation of short-run production and cost functions.

Before we begin any further the following two sections will first present the lightning review of the fundamentals of the theory of production and the theory of cost in the short-run and the long-run as discussed in the course on Microeconomics-I. The section following begin by showing how to use regression analysis to estimate short-run production functions. Next, we explain how to estimate parameters of the short-run production function and test for statistical significance. After developing the techniques of empirical production analysis, we turn to estimation of short-run cost equations.

We must stress at the outset that the purpose here is not so much to teach you how to do the actual estimations of the functions but, rather, to show how to use and interpret the estimates of production and cost equations.

I. Production and Cost in the Short Run

The production function gives the maximum amount of output that can be produced from any given combination of inputs, given the state of technology.

The production function assumes technological efficiency in production, because technological efficiency occurs when the firm is producing the maximum possible output with a given combination of inputs. Economic efficiency occurs when a given output is being produced at the lowest possible total cost.

In the short run, at least one input is fixed. In the long run, all inputs are variable.

The short-run situation occur when only one input is variable, labor (*L*), and one fixed, capital (*K*). In the short run, the total product curve, which is a graph of the short-run production relation

$$Q = f(L, \overline{K})$$

with *Q* on the vertical axis and *L* on the horizontal axis, gives the economically efficient amount of labor for any output level when capital is fixed at *K* units. The average product of labor is the total product divided by the number of workers:

$AP = Q/L$.

The marginal product of labor is the additional output attributable to using one additional worker with the use of capital fixed:

$MP = \Delta Q/\Delta L$.

The law of diminishing marginal product states that as the number of units of the variable input increases, other inputs held constant, there exists a point beyond which the marginal product of the variable input declines. When marginal product is greater (less) than average product, average product is increasing (decreasing).

When average product is at its maximum—that is, neither rising nor falling and marginal product equals average product.

In the short run when some inputs are fixed, short-run total cost (*TC*) is the sum of total variable cost (*TVC*) and total fixed cost (*TFC*):

$TC = TVC + TFC$

Average fixed cost is total fixed cost divided by output:

$AFC = TFC/Q$

Average variable cost is total variable cost divided by output:

$AVC = TVC/Q$

Average total cost is total cost divided by output:

$ATC = TC/Q = AVC + AFC$

Short-run marginal cost (*SMC*) is the change in either total variable cost or total cost per unit change in output:

$SMC = \Delta TVC/\Delta Q = \Delta TC/\Delta Q$

A typical set of short-run cost curves is characterized by the following features:

(1) *AFC* decreases continuously as output increases,

(2) *AVC* is U-shaped,

(3) *ATC* is U-shaped,

(4) *SMC* is U-shaped and crosses both *AVC* and *ATC* at their minimum points, and

(5) *SMC* lies below (above) both *AVC* and *ATC* over the output range for which these curves fall (rise).

The link between product curves and cost curves in the short run when one input is variable is reflected in the following relations:

$$SMC = w/MP \text{ and } AVC = w/AP$$

Figure 6-1 shows the link between Short-Run Cost and Production Functions.

When *MP* (*AP*) is increasing, *SMC* (*AVC*) is decreasing. When *MP* (*AP*) is decreasing, *SMC* (*AVC*) is increasing. When *MP* equals *AP* at *AP*'s maximum value, *SMC* equals *AVC* at *AVC*'s minimum value. Similar but not identical relations hold when more than one input is variable.

II. Production and Cost in the Long Run

In the long run all inputs are variable. Isoquants show all possible combinations of labor and capital capable of producing a given level of output. Isoquants are downward-sloping to reflect the fact that if larger amounts of labor are used, less capital is required to produce the same output level.

The marginal rate of technical substitution (*MRTS*) is the absolute value of the slope of an isoquant and measures the rate at which the two inputs can be substituted for one another while maintaining a constant level of output:

$$MRTS = -\Delta K/\Delta L.$$

The marginal rate of technical substitution can be expressed as the ratio of the two marginal products:

$$-\frac{\Delta K}{\Delta L} = MRTS = \frac{MP_L}{MP_K}$$

As labor is substituted for capital, *MPL* declines and *MPK* rises, causing *MRTS* to diminish along the isoquant.

Producers must consider relative input prices in order to find the least cost combination of inputs to produce the give level of output. An extremely useful tool for analyzing the cost of purchasing inputs is an isocost curve. The isocost curves show the various combinations of inputs that may be purchased for a given dollar outlay. The equation of an isocost curve is given by

$$K = \frac{\bar{C}}{r} - \frac{w}{r}L$$

where C is the cost of any of the input combinations on this isocost curve and *w* and *r* are the prices of labor and capital, respectively. The slope of an isocost curve is the negative of the input price ratio (−*w/r*).

A manager minimizes the total cost of producing a given level of output or maximizes output for a given level of cost (expenditure on inputs) by choosing an input combination at the point of tangency between the relevant isoquant and isocost curves. The point of tangency indicates the *lowest* isocost curve that includes an input combination that is capable of producing the desired output level. Alternatively, the point of tangency indicates the largest output (the highest isoquant) that is attainable from any combination on the given isocost curve.

Since the cost-minimizing or output-maximizing input combination occurs at the point of tangency between the isoquant and the isocost curve, the slopes of the two curves are equal at the optimal input combination. The optimization condition may be expressed as

$$MRTS = \frac{MP_L}{MP_K} = \frac{w}{r}$$

or

$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

Thus the marginal product per dollar spent on the last unit of each input is the same. Equating marginal product per dollar spent on all variable inputs is the rule managers should follow both in the long run when all inputs are variable and in the short run when two or more inputs are variable.

The expansion path shows the equilibrium (or optimal) input combination for every level of output. An expansion path shows how input usage changes when output changes, input prices remaining constant. Along the expansion path the marginal rate of technical substitution is constant, because the ratio of input prices (*w/r*) is constant. All points on the expansion path are both cost-minimizing and output-maximizing combinations of labor and capital.

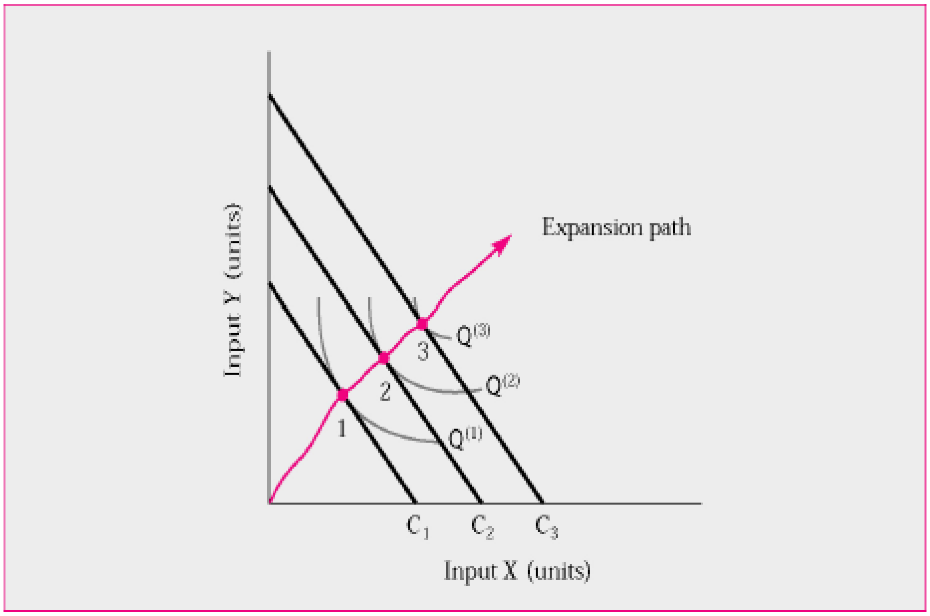


Figure 6-2: Expansion Path

Returns to scale, a long-run concept, involve the effect on output of changing all inputs by equi-proportionate amounts. If all inputs are increased by a factor of c and output goes up by a factor of z, then a firm experiences increasing returns to scale if $z > c$, constant returns to scale if $z = c$, and decreasing returns to scale if $z < c$.

The long-run cost curves are derived from the expansion path. Since the expansion path gives the efficient combination of labor and capital used to produce any particular level of output, the long-run total cost of producing that output level is the sum of the optimal amounts of labor and capital times their prices. Long-run average cost (*LAC*) is defined as

$$LAC = LTC/Q$$

and is U-shaped. Long-run marginal cost (*LMC*) is defined as

$$LMC = \Delta LTC/\Delta Q$$

and is also U-shaped. *LMC* lies below (above) *LAC* over the output range for which *LAC* is decreasing (increasing). *LMC* crosses *LAC* at the minimum point on *LAC*. When *LAC* is decreasing, economies of scale are present. When *LAC* is increasing, diseconomies are present.

Multi-Product Cost Function

Up until now, our analysis of the production process has focused on situations where the firm produces a single output. There are also numerous examples of firms that produce multiple outputs.

Generalmotors producers both cars and trucks (and many varieties of each); Hewlett Packard produces many different types of computers and printers. While our analysis for the case of a firm that produces a single output also applies to a multi-product firm, the latter raises some additional issues. This section will highlight these concepts.

In this section, we will assume that the cost function for a multi product firm is given by C(Q1, Q2), where Q1 is the number of units produced of product 1 and Q2 is the number of units produced of product 2. The multi product cost function thus defines the cost of producing Q1 units of good 1 and Q2 units of good 2 assuming all inputs are used efficiently.

Notice that the multi product cost function has the same basic interpretation as a single output cost function. Unlike with a single product cost function, however, the costs of production depend on how much of each type of output is produced.

This gives rise to what economists call *economies of scope* and *cost complementarities,* discussed next.

Economies of scope

When a firm produces more than one good or service, economies of scope may be present. Economies of scope exist when the joint cost of producing two or more goods is less than the sum of the separate costs of producing the goods. In the case of two goods *X* and *Y*, economies of scope are measured by

$$SC = \frac{C(X) + C(Y) - C(X,Y)}{C(X,Y)}$$

Self-Assessment Exercises

1. Does production in short run and long run depicts time horizon?
2. Explain the difference between short run production and long run production?
3. What does it mean economic of scope, support your explanation with practical local example?

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