**Mekelle University**

**College of Business and Economics**

**Department of Management**

**Worksheet for Operations Research**

**MGMT3132**

1. A firm manufactures two types of products A and B and sells them at a profit of $2 on type A and $3 on type B. Each product is processed on two machines G and H. Type A requires 1 minute of processing time on G and 2 minutes on H; type B requires 1 minute on G and 1 minute on H. The machine G is available for not more than 6 hours 40 minutes while machine H is available for 10 hours during any working day.
2. The numbers of units of vitamin A, B, and C in a kilogram of foods x and y are as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| Food | Vitamin A | Vitamin B | Vitamin C |
| X | 5 | 2 | 6 |
| Y | 4 | 6 | 2 |

A mixture of the two foods is made which has to contain at least 20 units of vitamin A, at least 24 units of vitamin of B, and at least 12 units of vitamin C. The unit cost of each food ( x and y) is 5 and 10 dollar respectively.

1. A company produces two products A and B which possess raw materials 400 quintals and 450 labour hours. It is known that 1 unit of product A requires 5 quintals of raw materials and 10 man hours and yields a profit of Rs 45. Product B requires 20 quintals of raw materials, 15 man hours and yields a profit of Rs 80.

**Required:** For each of the above questions:-

1. Formulate the LPP.
2. Solve the problem using graphical solution method.
3. Are there unused resources at the optimal solutions? What about surplus variables?
4. Do the problems have multiple optimal solutions? Why?

Min Z = 45x1 + 12x2

St: x1 +x2 ≥ 300

x1 ≥ 250 and

x1, x2 ≥ 0

Given the model above, answer the questions follow (use simplex solution method)

Standardize the model.

Generate an initial solution and the initial simplex tableau.

Determine the value of x1 and x2 at the optimal simplex tableau

What is the z-value at the optimal solution?

Does the problem have a multiple solution? Why?

1. Assume you are given the following primal model.

Max Z = 8x1 + 5x2

St:

2x1 + 4x2 ≤ 16 hours of labour where

6x1 + 3x2 ≤ 18 board feet of wood x1 = number of tables produced

x1, x2 ≥ 0 x2 = number of chairs produced

1. Solve the problem using the graphical method.
2. Solve the problem using the simplex method.
3. Develop the dual model.
4. Solve the dual model graphically.
5. Solve the dual model using the simplex method.
6. Does the problem have a unique solution? Why?
7. Based on question #5 above,
8. Develop a dual model from the LP model you developed (primal).
9. Solve the dual model using graphical and simplex method.
10. What are the marginal values of the constrained resources?
11. Compute the optimality range of the OFC and the RHS without violating the solution of the decision variables and the shadow prices.
12. Given the optimal simplex tableau below, answer the subsequent questions.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Cj B.V Qty** | | **3**  **X1** | **5**  **X2** | **4**  **X3** | **0**  **S1** | **0**  **S2** | **0**  **S3** |
| **5 x2** | 50/41 | 0 | 1 | 0 | 15/41 | 8/41 | -10/41 |
| **4 x3** | 62/41 | 0 | 0 | 1 | -6/41 | 5/41 | 4/41 |
| **3x1** | 89/41 | 1 | 0 | 0 | -2/41 | -12/41 | 15/41 |
| **Zj** | **$765/41** | 3 | 5 | 4 | 45/41 | 24/41 | 11/41 |
| **(Cj-Zj)** | | **0** | **0** | **0** | **-45/41** | **-24/41** | **-11/41** |

* 1. What are the ranges over which the OFC can vary for each of the decision variables?
  2. What are the shadow prices and which resource has the highest marginal value as per the optimal simplex tableau given?
  3. Over what range in each of the RHS values are these shadow prices valid?
  4. What are the solution values of the basic variables at the optimal tableau?
  5. Does the solution have multiple optimal solutions? Why?
  6. What will be the solution values of the basic variables at the optimal simplex tableau if you change the given model (primal) into dual?

1. A firm manufactures two types of products A and B and sells them at a profit of $2 on type A and $3 on type B. Each product is processed on two machines G and H. Type A requires 1 minute of processing time on G and 2 minutes on H; type B requires 1 minute on G and 1 minute on H. The machine G is available for not more than 6 hours 40 minutes while machine H is available for 10 hours during any working day. (Note that, when you answer, try to show all steps). Formulate the LP Model.
2. Solve the problem using graphical solution method.
3. Solve the problem using simplex solution method.
4. Does the problem have multiple optimal solutions? Why?
5. Develop the dual model from the given primal.
6. Solve the dual model using graphical and simplex method.
7. What are the marginal values of the constrained resources?
8. Compute the optimality range of the OFC and the RHS without violating the solution of the decision variables and the shadow prices.
9. Apply the simplex method to compute the optimality range of OFC and RHS without violating the conditions.
10. Given the optimal simplex tableau below, answer the subsequent questions.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Cj B.V Qty** | | **3**  **X1** | **5**  **X2** | **4**  **X3** | **0**  **S1** | **0**  **S2** | **0**  **S3** |
| **5 x2** | 50/41 | 0 | 1 | 0 | 15/41 | 8/41 | -10/41 |
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| **(Cj-Zj)** | | **0** | **0** | **0** | **-45/41** | **-24/41** | **-11/41** |

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  6. What will be the solution values of the basic variables at the optimal simplex tableau if you change the given model (primal) into dual?

1. Assume that Oranges are grown, picked and stored in three warehouses; A, B and C. It is required to deliver the orange from these warehouses to three markets; M1, M2 and M3. Each warehouse is able to supply the following number of tons of orange to the markets on a monthly basis.

|  |
| --- |
| Warehouse: A B C |
| No of Units in ton 150 175 275 |

And each market requirements are given below.

|  |
| --- |
| Market: M1 M2 M3 |
| No of Units in ton 200 100 300 |

The table below shows the costs (in birr) of transporting one unit from warehouse to market.

|  |  |  |  |
| --- | --- | --- | --- |
| Warehouse | M1 | M2 | M3 |
| A | Br. 6 | 8 | 10 |
| B | 7 | 11 | 11 |
| C | 4 | 5 | 12 |

Required:

1. Set up a transportation tableau for this problem.
2. Formulate this problem as a general linear programming model.
3. Is this a balanced or unbalanced transportation problem? Explain.
4. Determine the initial solution using:
5. NWCM II. LCM and III. VAM.
6. Does the solutions are degenerate? Explain. If it is degenerate, show how it would be put into proper form.
7. Solve the optimal solution using (take your answer in question ‘***D***’ as starting solution):
8. Stepping-stone solution method
9. Modified distribution method
10. Are there multiple solutions? Explain. If so, identify them.
11. Now, assume that capacity of warehouse **‘*B’*** is increase from 175 tons of orange to 200.
12. What happens to the issue of balanced and unbalanced TP?
13. If it is unbalanced, do we need to add dummy row or column and how many tons of orange should be assigned to it? Explain.
14. What change do you think will happen if we revise question ‘***a’*** to ‘***g’***?
15. Now, again assume that the demand of market 1, ***M1***, is reduced into 150 tons of orange while other things remain constant.
16. Apply the NWCM to compute the IBFS.
17. Using the NWCM as tentative solution, find the optimal solutions. Apply either the stepping-stone solution method or MODI method. (Hint: make sure that the IBFS is non-degenerate solution).
18. Taking the unit cost shipment of the table above as an assignment cost (assume: rows represent people whereas columns represent tasks), answer the following questions accordingly:
19. Compute the optimal assignment (route) that minimizes the total cost.
20. Compute the optimal cost (z-value).
21. Does the problem have a multiple optimal solutions? Explain.
22. A television repairman finds that the time spent on his jobs has an exponential distribution with a mean of 30 minutes. If he repairs sets of TV in the order in which they came in, and if the arrivals of TV sets follow a passion distribution approximately with an average rate of 10 per 8 hours day, compute the;
23. Expected number of TV set in the system.
24. Average time the TV sets spend waiting in line.
25. Expected idle time of repairmen each day.
26. Expected time the TV sets spend in the system.
27. Expected number of TV waiting to be served in the queue.
28. Four different Airplanes are to be assigned to handle three cargo consignments with a view to maximize profit. The profit matrix, in thousands of Birr, is given as follows:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Airplane | Cargo Consignment | | | |  |  |
|  | **I** | **II** | **III** |
| W |  | 8 | 11 | 12 |  |  |
| X |  | 9 | 10 | 10 |  |  |
| X |  | 10 | 10 | 10 |  |  |
| Z |  | 12 | 8 | 9 |  |  |

1. Compute the optimal assignment.
2. Compute the optimal assignment profit.
3. Does the problem have multiple optimal solutions? If your answer is yes, put the optimal assignment with its corresponding optimal profit.
4. Assume that Helen is a graduating class of 2013 in Mekelle University in the department of management. She received new word processing software for her birthday from her parents. She also received a check with which she intends to purchase a new computer. Helen’s Operations Research course instructor has given her an assignment with a due date of two days after her final exam. She has decided that she will prepare the paper on the new computer. She has contacted three different instructors who have well experienced about project management scheduling. Based on the advice of the instructors, She has made a list of the activities she will need to do and their estimated times as follow;

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Activity | Precedence Relationship | | Estimated time in days  Optimistic(TO) Most likely(Tm) | | |  | Pessimistic (Tp) |
| A | | None | 1 | | 1 | 7 | |
| B | | A | 3 | | 3 | 3 | |
| C | | A | 1 | | 3 | 5 | |
| D | | B | 1 | | 4 | 7 | |
| E | | D,C | 2 | | 2 | 8 | |
| F | | D,C | 3 | | 4 | 11 | |
| G | | E | 2 | | 2 | 2 | |
| H | | E | 1 | | 4 | 7 | |
| I | | F | | 3 | 4 | 5 | |
| J | | I,H | | 4 | 5 | 6 | |
| K | | G | | 1 | 3 | 5 | |
| L | | J.K | | 1 | 2 | 3 | |

Required:

1. Develop the Project Network.
2. Compute the expected time of each activity of the project**.**
3. Identify all possible project Paths with their corresponding lengths.
4. Determine the critical path and critical path duration.
5. Calculate the ES, EF, LS, LF, and slack value for each project activity
6. Compute the variance and standard deviation of the project length.
7. Suppose Helen wants to submit the paper 5 days earlier than the normal project duration, what is the probability that she will complete her paper by that time?
8. What project duration does a project with a 95% chance of completion have?
9. The following table provides the crash data for the network project described in question number 5 above. The normal activity times are considered to be deterministic, not probabilistic.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Activity | A | B | C | D | E | F | G | H | I | J | K | L |
| Crash Time | 1 | 2 | 2 | 3 | 2 | 4 | 1 | 3 | 3 | 3 | 2 | 2 |
| Normal Cost | 150 | 200 | 100 | 300 | 500 | 600 | 120 | 250 | 500 | 800 | 180 | 200 |
| Crash Cost | 200 | 300 | 150 | 320 | 620 | 800 | 150 | 280 | 700 | 110 | 260 | 200 |

**N.B:** The indirect cost she spends per day is Birr 80.

1. Calculate the crashing cost per day for each project activity.
2. If Helen wants to complete the project 5 days earlier than the normal project duration, by how much will the total direct cost (TDC) increase?
3. Taking total project cost as the criterion for crashing, compute;
4. The optimal project duration?
5. The total crashing cost?
6. The total direct cost?
7. The total indirect cost?
8. The total cost?
9. Taking time as a criterion for crashing, compute**;**
10. The optimal project duration?
11. The total crashing cost?
12. The total direct cost?
13. The total indirect cost?
14. The total project cost?
15. A construction company wants cement at three of its project sites: P1, P2 and P3. It procures cement from four plants: C1, C2, C3 and C4. Transportation costs per ton, capacities and requirements are as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | P1 | P2 | P3 | Capacity(tons) |
| C1 | 5 | 8 | 12 | **300** |
| C2 | 7 | 6 | 10 | **600** |
| C3 | 13 | 4 | 9 | **700** |
| Requirement | **400** | **800** | **400** |  |

Required:

1. Develop the initial feasible solution using VAM. (Indicate: basic variables, non basic variables and corresponding total transportation cost).
2. Based on the initial feasible solution of VAM, find the optimal solution using stepping stone method and calculate the optimum transportation cost.
3. Justify whether the final transportation schedule has multiple solution or not.
4. Assume that the Photo Dere Company is going to introduce a new instant camera into its product line and hopes to capture as large an increase in its market share as possible. In contrast, Photo Desta Company hopes to minimize Dere’s market share increase. Photo Dere and photo Desta Companies dominate the camera market and any gain in a market share for Photo Dere will result in a subsequent identical loss in market share for Photo Desta. The strategies for each company are based on their proportional campaigns, packaging and cosmetic differences between the products.

The payoff table which includes the strategies and outcomes for each company is as follows. The values in the table are the percentage increase in market share for Photo Dere Company.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Strategies of Photo Desta Company | | | |  |  |
|  | **B1** | **B2** | **B3** |
| Strategies of | **A1** | 9 | 7 | 8 |  |  |
| Photo Dere | **A2** | 7 | 10 | 12 |  |  |
| Company | **A3** | 5 | 6 | 11 |  |  |

1. Reduce the size of the game using the principle of dominance (if possible).
2. Compute the expected value of Photo Dere Company.
3. Compute the expected value of Photo Desta Company.
4. Compute the game value.
5. Does the game deterministic? Why?
6. Mr. Joseph, a corporate raider, has acquired a textile company and is contemplating the future of one of its major plants located in South Carolina. Three alternative decisions are being considered : (1) expand the plant and produce lightweight , durable materials for possible sales to the military , a market with little foreign competition ; (2) maintain the status quo at the plant , continuing production of textile goods that are subject to heavy foreign competition; or (3) sell the plant now. If one of the first two alternatives is chosen, the plant will still be sold at the end of a year. The amount of profit that could be earned by selling the plant in a year depends on foreign market conditions, including the status of the trade embargo bill in Congress. The following payoff table describes this decision situation.

|  |  |  |
| --- | --- | --- |
| Decision | States of Nature | |
| Good Foreign Competitive Conditions | Poor Foreign Competitive Conditions |
| Expand | $800,000 | $500,000 |
| Maintain Status Quo | 1,300,000 | -150,000 |
| Sell now | 320,000 | 320,000 |

Required:

1. Determine the best decision using the following decision criteria
2. Maximax
3. Maximin
4. Minimax regret
5. Hurwicz ( α = 0.3)
6. Equal likelihood
7. Assume it is now possible to estimate a probability of 0.70 that good foreign competitive conditions will exist and a probability of 0.3 that poor conditions will.

Determine the best decision using expected value and expected opportunity loss.

1. Compute the expected value of perfect information.

