

# SOIL PHYSICS

2017-2018

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## Course Outline

# Soil Physics

Course Code: **WRIE - 3151**

Pre-requisites: **None**

ECTS Credits: **3**

Contact Hours (per week): **2**

Instructor: **Biruk B.**

Semester: **II**

Academic Year: **2012 E.C**

# Course Objectives

At the end of the course, students will be able to understand:

- ➡ Different physical properties of soils, different relationships of soil properties,
- ➡ Process of water flow in soil,
- ➡ Process of gas transport in soil,
- ➡ Heat transfer in soil,
- ➡ Solute transport in soil,
- ➡ Different energy potentials in soil water atmosphere,
- ➡ Water and energy equilibrium in soil-water-plant atmosphere





# Course Content

## **CH 1: Composition and Physical Properties of Soils**

- » Composition of soils
- » Phases/States of Soils
- » Soil Texture and structure

## **CH 2: Soil Water Content and Water Potential Relationships**

- » Soil water content
- » Soil water energy
- » Soil water content–energy relationships



## **CH 3: Soil Water Movement**

- » Introduction
- » Flow in Saturated Soil
- » Flow in Unsaturated Soil

## **CH 4: Solute Transport In Soils**

- » Mass Flow
- » Diffusion and Hydrodynamic Dispersion
- » Solute-Soil interaction
- » Sources and Sinks of Solutes
- » Effect of salt on water flow



## CH 5: Gas Transport In Soils

- » Amount and composition of soil air
- » Equilibrium conditions for soil air
- » Bulk flow of gas
- » Gas diffusion
- » CO<sub>2</sub> diffusion in soil profile

## CH 6: Heat Transport In Soils

- » Introduction
- » Thermal soil properties
- » Heat conduction in dry soil



- » The heat balance of the soil surface
- » Combined heat and water vapour transport

## **CH 7: Water And Energy Balances at Soil–Plant– Atmosphere Interfaces**

- » Introduction
- » Energy Balance Equation
- » Water Balance Equation





# REFERENCE



Arthur W. Warrick. (2001), **Soil physics companion**



Gaylon s. Campbell, First edition (1985), **Soil physics with basic transport models for soil – plant systems**



P. Koorevaar, G. MENELIK And C. Dirksen First edition (1983), **Elements of soil physics**



Any other related books and journals

# CHAPTER ONE

## COMPOSITION AND PHYSICAL PROPERTIES OF SOILS

# Introduction

## What is Soil?

- Soils is the **weathered** and **fragmented** outer/upper layer of earth crust, and **supports all terrestrial life**.
- It is formed initially through disintegration, decomposition, and re-composition of mineral material contained in exposed rocks by physical, chemical, and biological processes.
- They are the **storehouse** of **water**, **nutrients** and **air** which are **necessary** for **plant growth**. Therefore, plants grow on soils that provide them water and nutrients.

Both excess and deficit of soil water affects the plant growth and results in yield reduction.

- The **plants need water**, the **soil stores the water needed by the plants**, and the **atmosphere** provides the **energy** needed by the plant to withdraw water from the soil.

- Therefore **knowledge** of the *process of water transport in soil, into plants and from soil and plants to the atmosphere* are the **basics** of irrigation practice.
- Often the important questions that rise in irrigation practices like “**when to irrigate** and **how much to irrigate**”, which the answers depend on *soil-water-plant-atmosphere relationships*.
- To understand these relationships, requires the study of **physical characteristics of soils** relevant to the rate of entry of water into the soil, moisture retention, storage, and transport, **availability to plants as well as mechanisms of water absorption, conduction and transpiration by plants**



## What is Soil Physics?

- Generally deals with the **state** and **movement of matter** and with the **fluxes** and **transformations** of energy in the soil.
- Specifically the areas of study include *structure and texture of the solid matrix*, *retention and flow of water*, *the movement of solutes*, *soil temperature and the flow of heat*, and *the diffusion of gases*, including measurement and prediction under natural and managed ecosystems.
- It also consists of the study of **soil components** and **phases**, their interaction with one another and the environment, and their temporal and spatial variations in relation to natural and managed ecosystems.

# *Composition of soils*

- Soil is the natural material that covers the land surface of the earth. It is the product of physical, chemical and biological weathering of parent material.
- Soils consists four constituent fractions or components:
  - ✓ **Mineral matter**
  - ✓ **Organic matter**
  - ✓ **Water solution**
  - ✓ **Soil air**
- **Inorganic components** are **primary** and **secondary** minerals derived from the parent material.
- **Organic components** are derived from **plants** and **animals**.
- The **liquid component** consists of a **dilute aqueous solution of inorganic and organic compounds**.

# Soil Phases

- The components of soil, like all other matter in nature, exist in the three states of matter, i.e. **solid**, **liquid** and **gas**. With some exceptions, all three states occur in soils side by side. Hence, these states are called '**phases**' of the soil. These phases are;

👍 **Solid Phase**

👍 **Liquid Phase**

👍 **Gas Phase**

- The **solid phase** is composed of **mineral** and/or **organic matter**
- The **liquid phase**, also called **soil solution**,
- The **gas phase**, also known as **soil air** or **soil atmosphere**, is composed mainly of **N<sub>2</sub>**, **O<sub>2</sub>**, **water vapour** and **CO<sub>2</sub>**, with traces of other gases.

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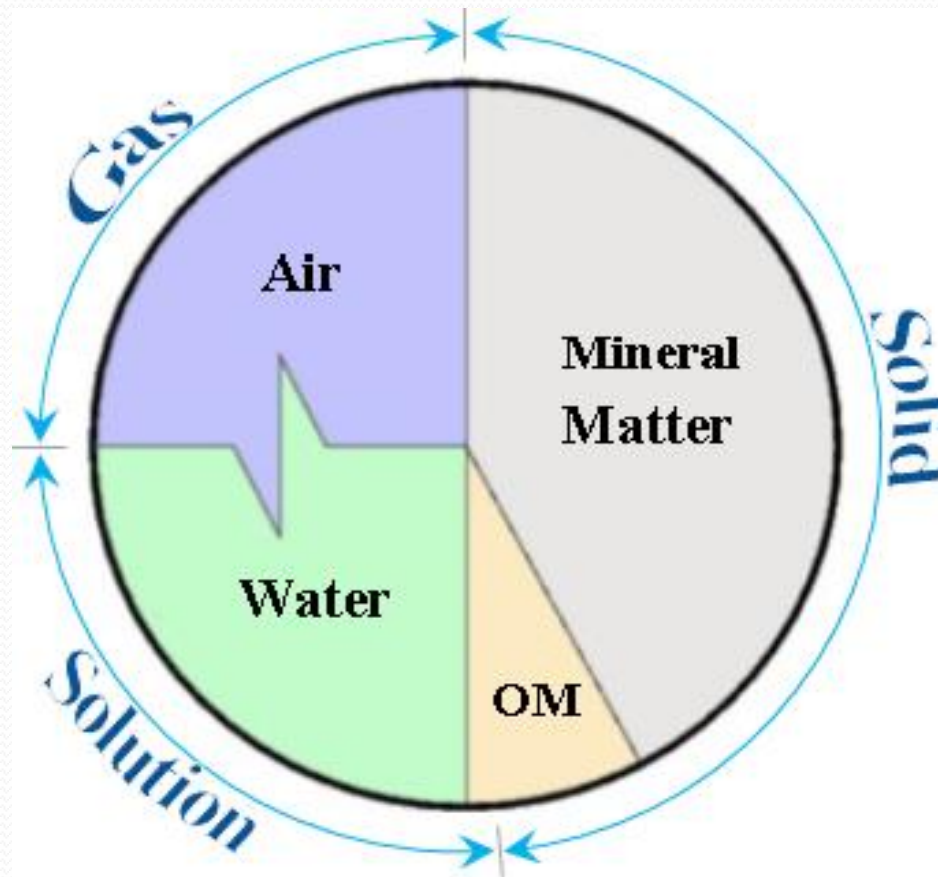


Figure: Components and Phases of soil

# *Texture and Structure of soil*

**Soil texture** : The term texture refers to the size range of particles in the soil. i.e., whether the particles of which a particular soil is composed are mainly **large, small**, or of some **intermediate size or range of sizes**.

- The term carries both **qualitative** and **quantitative** connotations.
- **Qualitatively**: it represents the “**feel**” of the soil material, whether coarse and gritty or fine and smooth.
- **Quantitative**: it represents the “**classes**” of the particular soil might belong. (or denotes measured distribution of particle sizes or the proportions of various size ranges of particles which occur in a given soil). ( Classes: sand, silt, and clay).

## Cont...

- Generally the soils occurring in nature are a combination of **sand**, **silt** and **clay**. And their relative proportion in a soil mass determines the soil texture.
- According to textural classification, soils may be broadly classified as **light**, **medium** and **heavy textured** soils.
  - ➔ The **light textured** soils contain **very low content of silt and clay** and hence these soils are coarse or sandy.
  - ➔ The **medium textured** soils contain **sand, silt and clay in sizable proportions**. In general loam is a soil which has all the three major size fractions in sizable proportions, and hence the loams are medium textured soils.
  - ➔ The **heavy textured** soils contain **high content of clay**. Thus clayey soils are tight or heavy textured soils.



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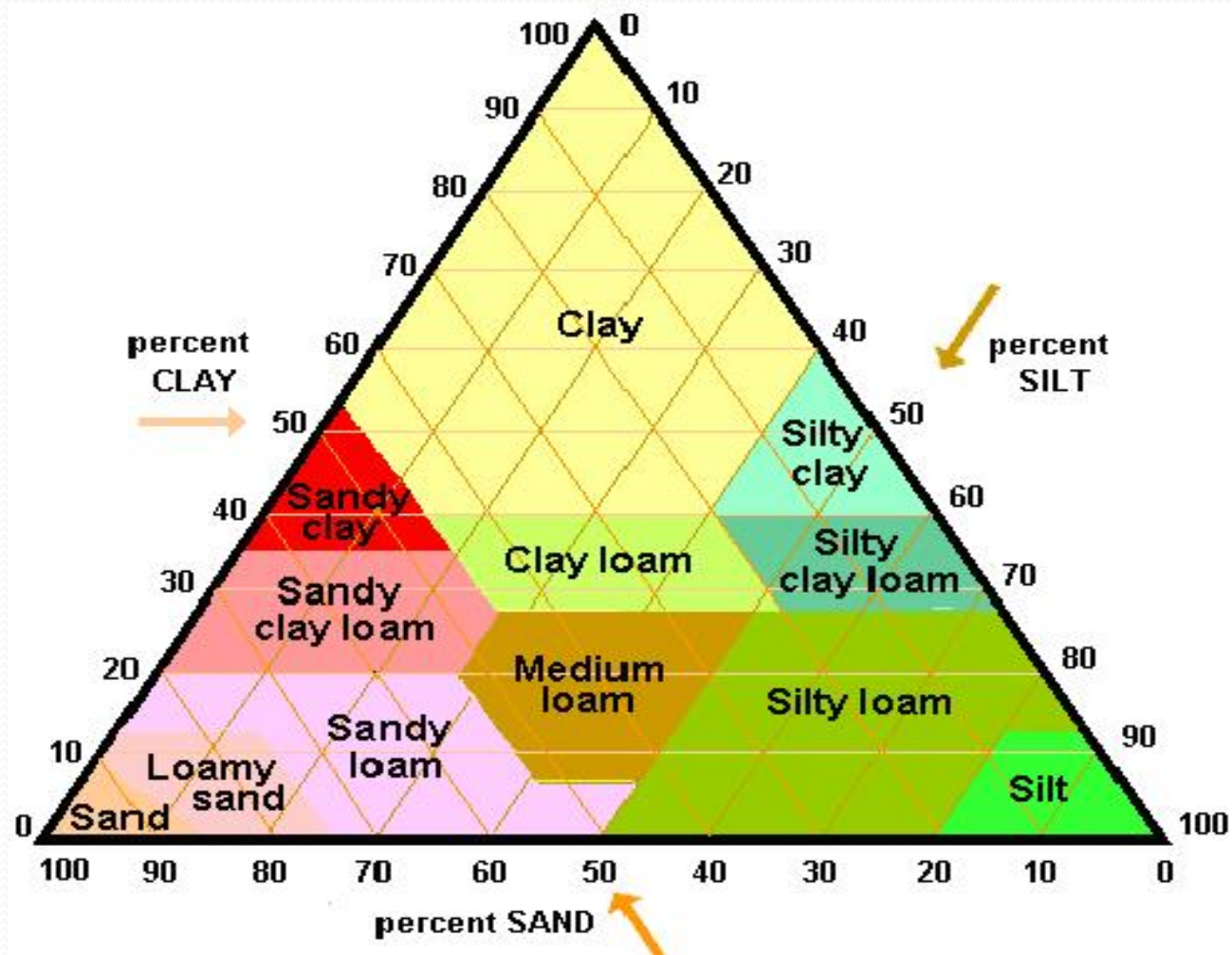


Figure: The soil textural triangle

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**Soil Structure**: describes the arrangement of individual soil particles and the aggregates with respect to each other into a pattern. The structure depends on what the soil developed from.

- The arrangement and placement of soil particles determines the response of soil to exogenous stresses such as tillage, traffic, and raindrop impact,
- The pattern of pores and particles defined by soil structure greatly influences water movement, heat transfer, aeration, and porosity in soils
- Types (shapes) of soil structures include: Granular, Blocky (angular and subangular), Platy, Columnar and prismatic, Single grain (non-structure) and Massive (non-structure).




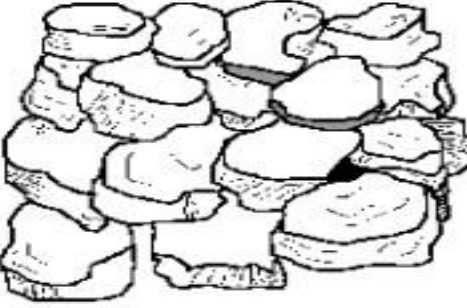




|  |  |   |
|--|--|---|
|   |                      |    |
| <p><b>Granular:</b> Resembles cookie crumbs and is usually less than 0.5 cm in diameter. Commonly found in surface horizons where roots have been growing.</p> | <p><b>Blocky:</b> Irregular blocks that are usually 1.5 - 5.0 cm in diameter.</p>                      | <p><b>Prismatic:</b> Vertical columns of soil that might be a number of cm long. Usually found in lower horizons.</p>   |
|    |                     |   |
| <p><b>Columnar:</b> Vertical columns of soil that have a salt "cap" at the top. Found in soils of arid climates.</p>   | <p><b>Platy:</b> Thin, flat plates of soil that lie horizontally. Usually found in compacted soil.</p> | <p><b>Single Grained:</b> Soil is broken into individual particles that do not stick together. Always accompanies a loose consistence. Commonly found in sandy soils.</p> |

Figure: Soil structure

# CHAPTER TWO

## *SOIL WATER CONTENT AND WATER POTENTIAL RELATIONSHIPS*

# INTRODUCTION

## Water . . . . ?

- Water is the most common of all liquids, and it is *indispensable for life*.
- It accounts for 60 - 95% of the material in all organisms and it is *a means of transporting nutrients dissolved or suspended in it to all parts of plants and other biological and porous bodies like soil*.
- Water is thus a solvent to make solutions or suspension.
- Hidden from casual view, highly substantial volumes of water are commonly **stored** in soils in their pore spaces.
- This water **enable consistent plant growth** in areas having scattered or sporadic (irregular) precipitation.
- As that of plant soil organisms also rely heavily on the **water holding characteristics of soils** for their existence.

## Cont...

- Soil water is a highly dynamic entity, *exhibiting substantial variation in both time and space*. This is particularly true near the soil surface, and in the presence of active plant roots.
- Changes in soil water content and its energy status **affect many soil mechanical properties** including *strength*, *compact ability* and *penetrability*, and may cause *changes in the bulk density of swelling soils*.
- The liquid phase characteristics of the soil **affect the soil gaseous phase and the rates of exchange between these phases**, as well as other important soil properties such as the **hydraulic conductivity** which governs the rate of water and soluble chemical flow.

The purpose of this chapter: is to introduce basic concepts related to the amount and energy state of water in soil.

- These concepts are **prerequisite** to *quantify and manage soil water storage, to obtain predictions concerning rates and directions of water flow and solute transport, to utilize soils as building or foundation materials*, and for many other purposes.
- The term soil water is used here to represent the soil liquid phase which is typically a water solution containing dissolved salts, organic substances and gases.

Cont...

The water status in soils is defined by:

- 1) The amount of water in the soil, or soil water content ( $\theta$ ), and
- 2) The force by which water is held in the soil matrix, soil energy content or soil water potential ( $\psi$ ).

These soil water attributes(x-ics) are related to each other through a function known as the **soil water characteristic (SWC)**.



# SOIL WATER CONTENT

- Many agronomic, hydrologic and geotechnical practices require knowledge of the **amount of water contained** in a particular soil volume.

Some of the most **common methods used to characterize and determine soil water content** are:-

## **1. Soil Water Content on Mass Basis (Gravimetric), $\theta_m$**

Mass or gravimetric soil water content is expressed relative to the mass of oven dry soil according to:

$$\theta_m = \frac{\text{mass of water}}{\text{mass of dry soil}} = \frac{(\text{mass of wet soil}) - (\text{mass of oven dry soil})}{\text{mass of oven dry soil}}$$

Cont...

## 2. Soil Water on Volume Basis, $\theta_v$

- It is often desirable to express water content on a volume basis. The volumetric water content is defined as the volume of water per bulk volume of soil:

$$\theta_v = \frac{\text{volume of water}}{\text{bulk volume of soil}} = \frac{(\text{mass of water} / \text{density of water})}{\text{bulk soil volume}}$$

- It also represents the depth ratio of soil water (i.e., the depth of water per unit depth of soil).
- The conversion between gravimetric and volumetric water contents requires knowledge of the soil dry bulk density ( $\rho_b$ ) which is the ratio of oven dry soil mass to its original volume, and the density of water. The conversion formula is given by:

$$\theta_v = \theta_m \frac{\rho_b}{\rho_w}$$



### 3. Water Content on Relative Saturation Basis, $\Theta$

- An additional means of characterizing the soil water content is in terms of the degree of saturation:

$$\Theta = \frac{\text{volume of water filled pore space}}{\text{total volume of soil pore space}} = \frac{\theta_v}{\theta_{vs}}$$

- where  $\theta_{vs}$  is volumetric soil water content under completely water-saturated conditions. This index ranges from **zero** in completely dry soil to **unity** in a saturated soil.
- The degree of saturation is also commonly termed effective saturation or relative water content.

Cont...

#### **4. Soil Water Storage:**

- It is often convenient to express the quantity of soil water in a specific soil depth increment in terms of soil water storage or equivalent depth of soil water (units of length).

**Equivalent depth of soil water,  $D_e$  (m),** is calculated as:

$$D_e = \theta_v D$$

where  $D$  is the soil depth increment (m)  
having volume water content  $\theta_v$ .

- This quantity is useful to relate aboveground water dimensions of rainfall, irrigation or evaporation ( $L$ ) to belowground dimensions ( $\theta_v$ ,  $\text{m}^3 \text{ water } \text{m}^{-3} \text{ soil}$ ).

Cont...

- For suitable accuracy it is often necessary to sum the equivalent depth relationship over discrete soil depth layers having distinct water contents:

$$D_e = \sum_{i=1}^n \theta_{vi} D_i$$

where  $i$  denotes depth increments

## 5. Water holding capacity of soils:

- Water holding capacity of soil is one of the dominant factors influencing irrigation.
- Is mainly depends on its **porosity**.
- In general there are **two types of soil pores**. Those are
  - ➞ Capillary or small pores and
  - ➞ Non-capillary or large pores.
- ▮ Capillary pores hold tightly by *capillarity* a large amount of water at saturation and prevent it from being drained off under gravity. It *induce greater water holding capacity*.
- ▮ Non-capillary pores do not hold water tightly and hence a large amount of water held by the soil at saturation is drained off under gravity. It *induce drainage and aeration*.

## Cont...

- The **relative magnitude of these types of pores** in a soil **depends on its *texture* and *structure***.
  - **Sandy soil** has more **non-capillary pores** which result in better drainage and aeration but **lower water holding capacity**.
  - **Clayey soil** has more **capillary pores** which result in **better water holding capacity** but poor drainage and aeration.
- . . .
- The water held by the soil is extracted by the roots of the plants for being used by the plants.
- The extraction of water from the soil by roots of the plants is resisted by some forces, but **the resisting forces are more in clayey soils than in sandy soil**.

## Cont...

- Water cannot be easily extracted by the roots of the plants in clayey soils although large amount of water is held by these soils.
- On the other hand relatively less amount of water is held by sandy soils, but water can be easily extracted from these soils by the roots of the plants.
- Thus an **ideal soil for irrigation** is that **which has its pore space almost *equally divided between capillary and non-capillary pores***.
  - └ Such a soil has enough small pores to provide adequate water holding capacity and also enough large pores to permit adequate drainage and aeration, and easy extraction of water by the roots of the plants.
  - └ The **loam soils** are therefore ideal soils as they possess *good water holding capacity, have good drainage and aeration, and allow extraction of water by roots of the plants without much resistance*.

# Application of Soil Water Content Information: The Water Balance

- A primary use of **soil water content information** is for evaluation of the **hydrologic water balance** described by:

$$\mathbf{P + I = ET + D + R - \Delta W}$$

where **P** is precipitation, **I** is irrigation, **ET** is evapotranspiration, **D** is drainage or deep percolation, **R** is surface runoff, &  $\Delta W$  is change in water storage within the profile (soil water depletion).

The convention used here is that **inputs to the soil profile** are taken as **positive**, and **outputs negative**.

- **W** is defined as the equivalent depth of water ( $\mathbf{D_e = \theta_v * D}$ ) stored in the soil profile under consideration, and

$$\Delta W = (\mathbf{W_{initial} - W_{final}}) \text{ (figure below)}$$

- $\Delta W$  is fairly significant over the short term (weeks to months), but generally evens out to about zero over one to several years.



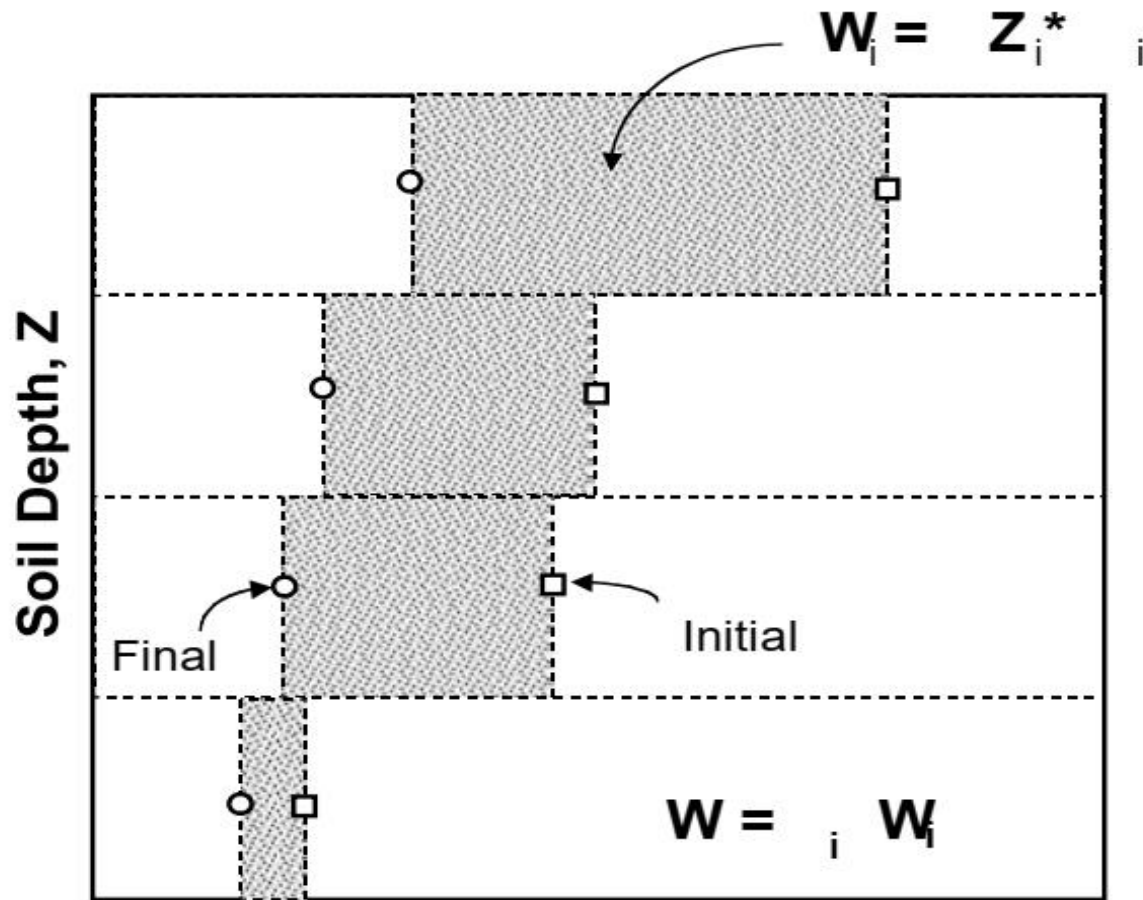


Figure: Schematic of soil water depletion calculations for a soil profile divided into four depth increments. Total change in water storage is the sum of depletion in each layer.



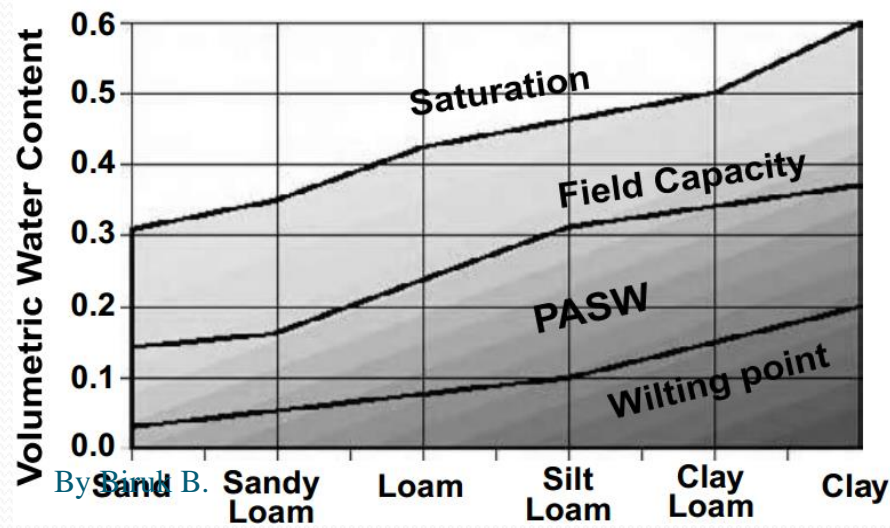
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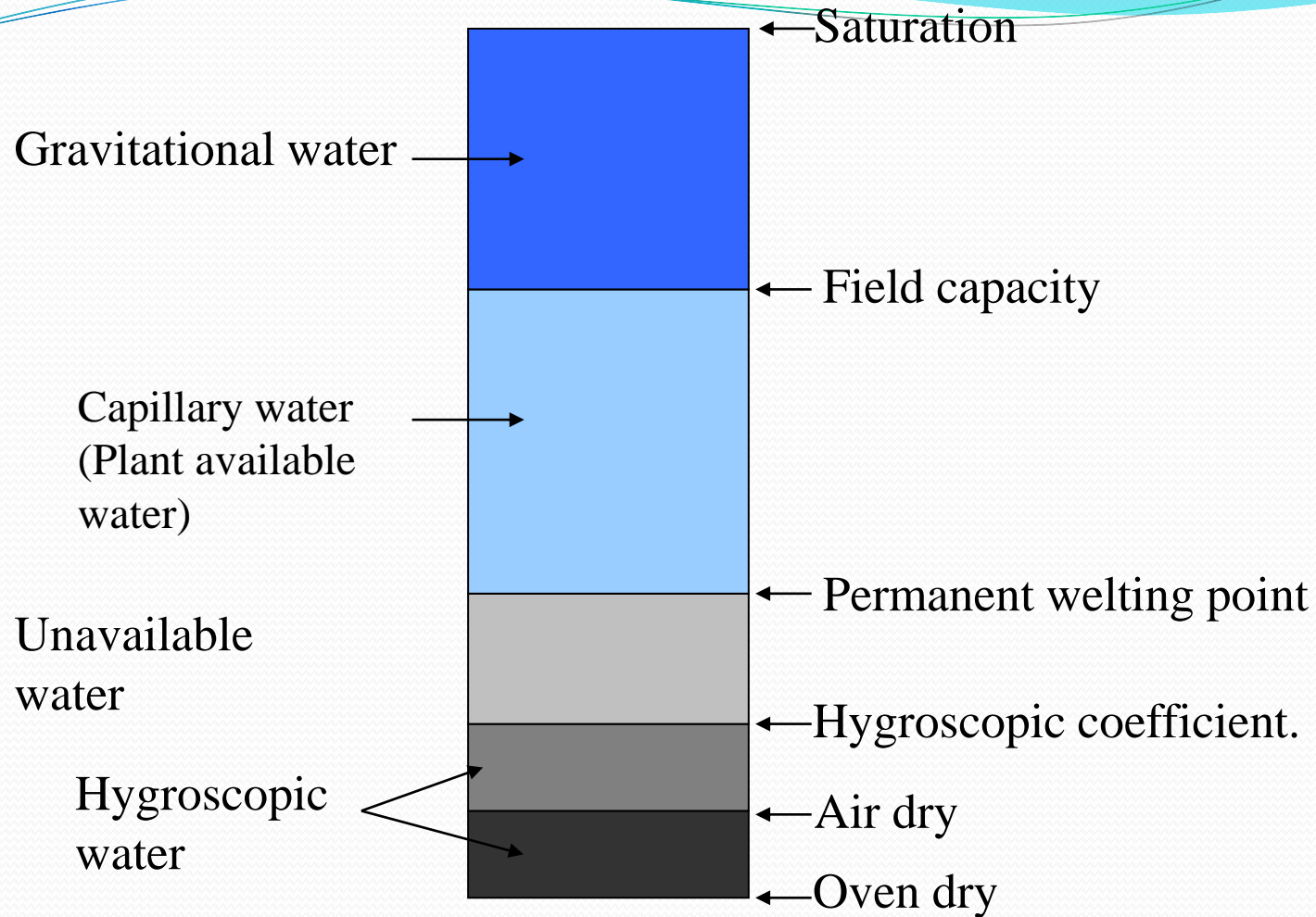
Soil water content also gives information on **Field Capacity**, **Wilting Point** & **Plant-Available Soil Water**.

- **Field Capacity**: is the amount of water remaining in the soil after the large pores have drained. Medium and small pores are still filled with water held against the force of gravity. Soil water at field capacity is readily available to plants and sufficient air is available in the soil for root and microbial respiration. .
- **Wilting Point**: is defined as the water content at which plants can no longer extract soil water at a rate sufficient to meet physiological demands imposed by loss of water to the atmosphere, and thus irreversibly wilt and die.
  - The water content ( $\theta_{vWP}$ ) at this point is primarily dependent on the soil's ability to transmit water, but also to some degree on the plant's ability to withstand or mitigate drought.

## Cont...

- A primary practical use of field capacity and wilting point concepts is the determination of a **plant available soil water** range (**PASW**) .
- Soil water storage available for plant use is generally calculated as being between field capacity and wilting point ( $\theta_{vFC} - \theta_{vWP}$ ).
- Plant-available soil water storage is an important factor in the determination of irrigation amounts for a cropped field or other soil-plant system.





**Figure:** Illustration of soil water constants

# SOIL WATER ENERGY

- **Water status** in soils is characterized by both the amount of **water present** and by its **energy state**.
- Soil water is subjected to forces of variable **origin** and **intensity**, thereby acquiring different *quantities* and *forms of energy*.
- The **difference in energy level** of water from *one site* or *one condition* (e.g., **wet soil**) to another (e.g., **dry soil**) **determines the direction and rate of water movement** in soils and plants.

In a **wet soil**:

- Most of the **water** is retained in large pores or thick water films around particles.
- Therefore, most of the **water molecules** are not very close to a particle surface and are not held very tightly by the soil solids (soil matrix).
- So, the **water molecules** have considerable freedom of **movement**, so their energy level is near that of water molecules in a pool of pure water outside the soil.

In a **drier soil**:

- ┐ the water that remains is located in small pores and thin water films, and is therefore held tightly by the soil solids.
- ┐ Thus the water molecules have little freedom of movement, and the energy level of the water is much lower than that of water in wet soil.
- If wet and dry soils are brought in touch with each other, water will move from the wet soil (higher energy state) to the drier soil (lower energy).
- The energy status of soil water in a particular location in the soil profile is compared to that of the reference state (*pure and free water*) at a standard pressure and temperature, unaffected by the soil and located at some reference elevation.
- The difference in energy levels between this pure water in the reference state and that of soil water is known as **Soil Water Potential**. Thus the term potential implies a difference in energy status.

- The two primary forms of energy of interest here are **kinetic** and **potential**.
  1. The **Kinetic Energy** :- is acquired due to virtue of motion of water through soils. =  $\frac{1}{2} m V^2$ 
    - negligible due to the slow mov't of water (usually  $< 0.1$  m/h)
  2. The **Potential Energy**:- due to position of soil water within soil body & by internal conditions. (e.g = **mgh**)
    - largely responsible for determining the state & movement of water in the soil.
- Knowing the potential energy in soil can help us **estimate how much work the plant must expend to extract a unit amount of water**.
- Potential energy rather than kinetic energy is used to describe soil water flow because kinetic energy is considered negligible by virtue of slow movement of soil water.



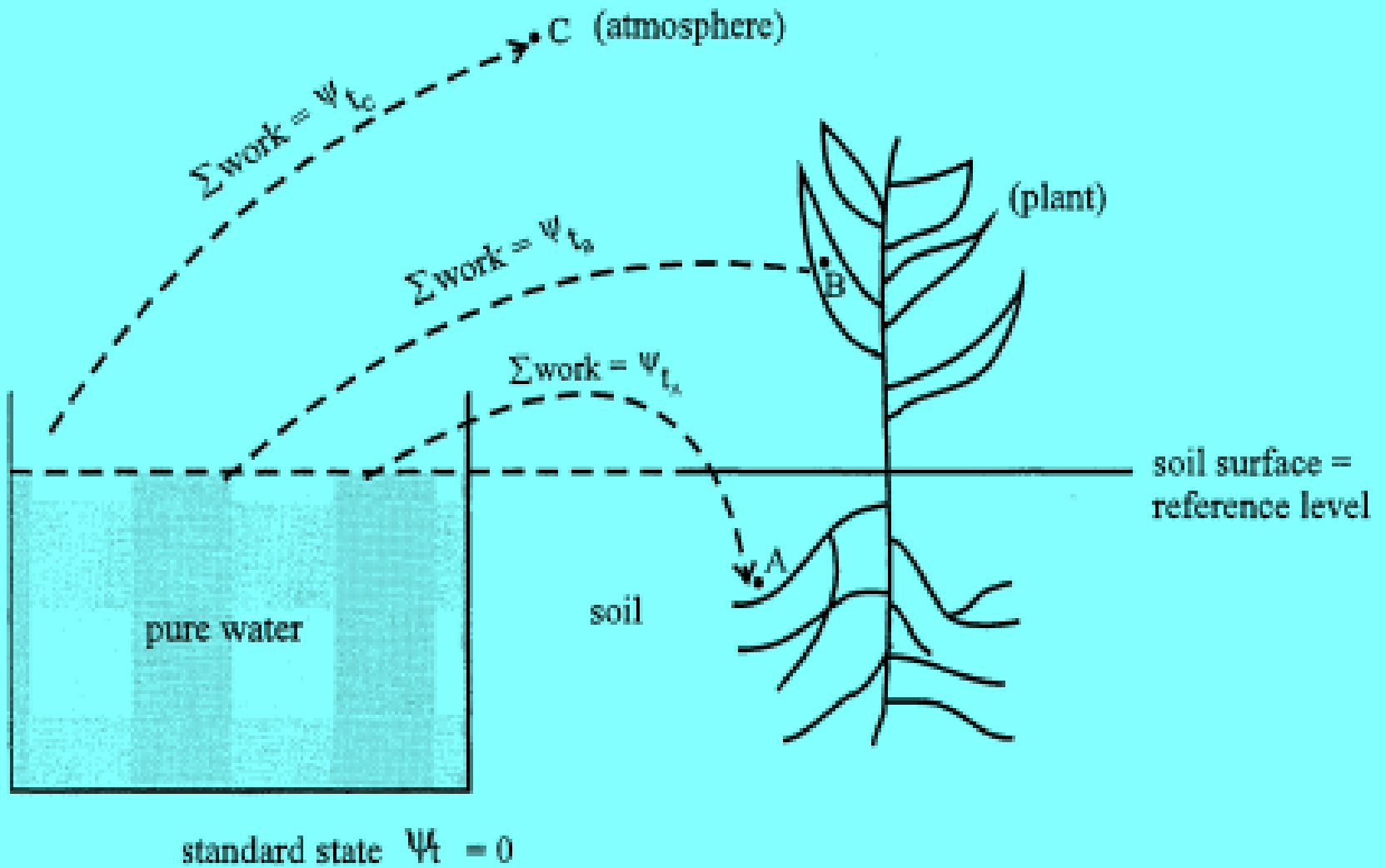
- The **magnitude of the driving force** for water movement in soil is a difference in potential energy across a distance b/n two points of interest. Or *water flows in response to gradients in water potential (water potential gradients)*
  - E.g From Darcy's law:  $f_w = -k \frac{d\psi}{dx}$
- *When water potential is uniform* across a boundary, *no water will flow*, even though water content may be different on the two sides of the boundary.
  - The water potential is the potential energy per unit mass (or volume) of water in a system, compared to that of pure, free water.
  - If potential energy per unit mass is used, the units of potential are **Joules/kg**
  - Potential energy per unit volume has units of **Joules m<sup>-3</sup>**, but this dimensionally equivalent to **Newtons m<sup>-2</sup>**, or **Pascal**, w/c is a pressure unit.
  - Since there are about **1000 kg of water in a m<sup>3</sup>**, **1 J/kg = 1 kPa**.
  - The pressure unit, Bar, has been used for water potential. A bar is equal to 10<sup>5</sup> Pa, so 1 bar = 100 J/kg or 100 kPa



## Components of Soil Water Potential

- Soil water is subject to a number of force fields, which cause its potential to differ from that of pure, free water.
- The combined effects these forces result in a deviation in potential energy relative to the reference state called the **Total Soil Water Potential** ( $\psi_T$ ) and is defined as:

“The amount of work that an infinitesimal quantity of soil water is capable of doing when it moves (isothermally and reversibly) to a pool of pure water at similar standard (reference) state (similar pressure, elevation, temperature and chemical composition)”
- If work is required the potential is positive, but if water in the reference state can accomplish work in moving into the soil the potential is negative.



- The components of the total soil water potential are result from:
  - ┐ the attraction of the solid matrix for water
  - ┐ the presences of solutes and
  - ┐ the action of external gas pressure and gravitation.
- The total potential can be expressed as is the summation of the component potentials corresponding to the different fields acting on soil water:

$$\Psi_T = \Psi_m + \Psi_o + \Psi_p + \Psi_g + \Psi_a + \Psi_\Omega$$

where:-  $\Psi_m$  : matric potential

$\Psi_z$  : gravitational potential

$\Psi_s$  : solute/osmotic potential

$\Psi_a$  : pneumatic potential

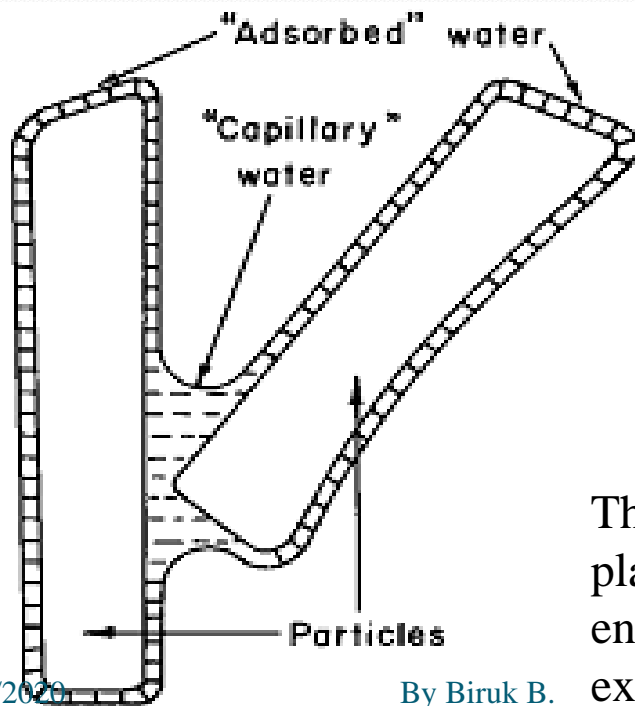
$\Psi_p$  : pressure potential

$\Psi_\Omega$  : overburden potential

- Usually, only one or two of the component potentials needs to be considered in any given flow problem, but gradients in any of these potentials can result in water flow when conditions are right.

❑ Matric Potential,  $\psi_m$  : is resulting from interactions of the solid phase with the liquid and gaseous phases. /or it is water potential due to attraction between water and soils./

- Or resulting from combined effects of **capillarity** and **adsorptive** forces within the soil matrix.
- These **interactions reduce the potential of water**, giving it a *negative sign*.



- The matric component of water potential is **important** as a driving force for **flow in unsaturated soil** and in the **cell walls of root cortex** and **leaf mesophyll tissue**.
- Matric potential is always *negative* or *zero*.

The movement of soil water, the availability of water to plants and the solutions to many civil engineering problems are determined to a considerable extent by matric potential.

- Dominating mechanisms for these effects include:
  - 1) Adhesion of water molecules to solid surfaces
  - 2) Capillarity caused by liquid-gas interfaces and liquid-solid-gas interfaces interacting within the irregular geometry of soil pores, and
  - 3) Ion hydration and water participating in diffuse double layers (particularly near clay surfaces)
- The water potential under a curved air-water interface, such as might exist in a capillary tube or a soil pore, is given by the capillary rise equation:

$$\psi_m = -2\sigma/r\rho_w$$

where  $r$  is the radius of curvature of the interface,  $\sigma$  is the surface tension ( $7.27 \times 10^{-2} \text{ J m}^{-2}$  at  $20^\circ\text{C}$ ) and  $\rho_w$  is the density of water.

This equation is often used to find the equivalent radius of the largest water-filled pore in a soil at a given matric potential.

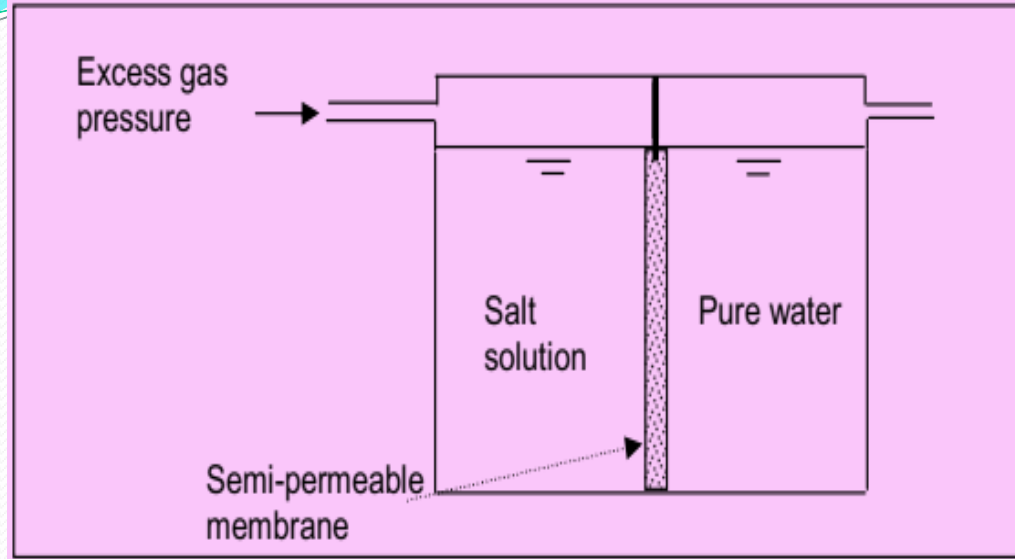
For example, in a soil at  $\psi_m = -100 \text{ J/kg}$  we would expect pores with radii larger than  $r = (-2 \times 7.27 \times 10^{-2}) / (10^3 \times (-100)) = 1.45 \text{ } \mu\text{m}$  to be air filled.

- ❑ **Solute/Osmotic Potential,  $\psi_o$**  : determined by the presence of **solutes** in soil water. The solute may be inorganic salts or organic compounds and etc.
- Presence of **solutes in soil water** *decreases* the **potential energy of water in the soil**, b/c of the reduced freedom of movement of the water molecules that cluster around each solute ion or molecule.
- The **potential energy of water in the solution** is *lower than* that of **pure water**.
- The greater the concentration of solutes, the more the osmotic potential is lowered. Water will tend to move to where the energy level will be lower, in this case where place where the solute concentration is high.

- The effects of  $\psi_s$  are important in the presence of:
  - 1) Appreciable amounts of solutes, and
  - 2) A semi permeable membrane or a diffusion barrier, which transmits water more readily than salts.
    - The effects of  $\psi_s$  are negligible when only liquid water flow is considered and no diffusion barrier exists.
- Liquid water will tend to move in response to differences in osmotic potential (the process termed osmosis) only if semi permeable membrane exists b/n the zones of high and low osmotic potential allowing water through but preventing the movement of the solute.
- If no membrane is present, the solute rather than the water generally moves to equalize concentrations.



- The osmotic pressure can be illustrated by Fig. below.



A container, divided into two parts by semi-permeable membrane, holds salt solution at one side of the membrane and pure water at the other side. Both sides are at atmospheric pressure.

### Notes:

The concentration of water molecules is higher in the pure water than in the solution, more water molecules will move in the direction of the solution than in the other direction. As the result, the surface of the salt solution will rise slowly and that of the pure water will drop. One can prevent this flow of water across the membrane and maintain static equilibrium by exerting on the salt solution a gas pressure in excess of atmospheric pressure.

The excess gas pressure needed to maintain equilibrium is equal to the osmotic pressure.

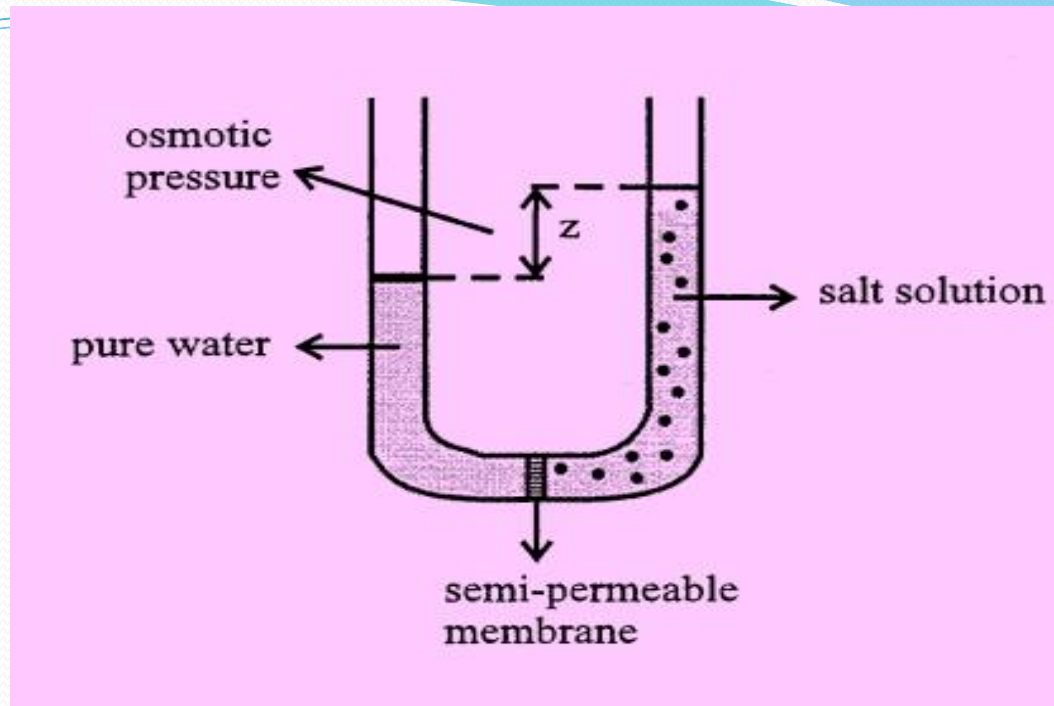


Figure: Schematic presentation of osmosis

Note: But Soil zones are not generally separated by membranes, so the osmotic potential has little effect on the mass movement of water in soils.

Its major effect is on the uptake of water by plant root cells that are isolated from the soil solution by their semi permeable cell membranes.

**Notes:** In soils high in soluble salts,  $\Psi_o$ , may be lower (have a greater negative value) in soil solution than in plant root cells. This leads to constraints in the uptake of water by the plants. In very salty soil, the soil water osmotic potential may be low enough to cause cells in young seedlings to collapse (plasmolyze) as water moves from the cells to the lower osmotic potential zone in the soil.

**Notes:** The random movement of water molecules causes a few of them to escape a body of liquid water, enter the atmosphere, and become water vapor.

Since the presence of solutes restricts the movement of water molecules, fewer water molecules escape into the air as the solute concentration of liquid water is increased. Therefore water vapor pressure is lower in the air over salty water than in the air over pure water. By affecting water vapor pressure, osmotic potential affects the movement of water vapor in soils.

- **Pressure Potential,  $\psi_p$**  : defined as the hydrostatic pressure exerted by unsupported water (i.e., saturating the soil) overlying a point of interest.
- Pressure potential and matric potential are mutually exclusive because *pressure potential occurs in saturated soil* while *matric potential occurs in unsaturated soil*.
  - └ For saturated conditions:  $\psi_p$  is positive and  $\psi_m$  is zero
  - └ For unsaturated conditions:  $\psi_m$  is negative and  $\psi_p$  is zero
  - └ At the free watertable elevation:  $\psi_p = \psi_m = 0$
- Pressure potential is positive and greater than atmospheric pressure while matric potential is a negative pressure lower than the atmospheric.
- The hydrostatic pressure **P** of water with reference to the atmosphere is:

$$\mathbf{P} = \rho gh \quad (\text{energy/volume} = \text{pressure})$$

- **Gravitational Potential,  $\psi_z$**  : The gravitational potential of soil water at each point is determined by the **elevation** of the point relative to some arbitrary reference level.
- If the point in question is **above the reference**,  $\psi_z$  is **positive**. If the point in question is **below the reference**,  $\psi_z$  is **negative**.
- Is **independent** of **soil properties**, **chemical** and **pressure conditions** of soil water. It depends only on the **vertical distance** between the reference and the point in question.
- Gravitational potential energy **E<sub>g</sub>** is expressed:

$$E_g = - mgz = \rho_w Vgz$$

- Gravitational potential can be expressed
  - in terms of the potential energy per unit mass is:  $\psi_z = - gz$
  - in terms of potential energy per unit volume is:  $\psi_{z,v} = - \rho_w gz$
  - in terms of potential energy per unit weight is:  $\psi_{z,w} = - z$

- ❑ **Envelope/Overburden Potential,  $\psi_\Omega$ :** is potential due to an external mechanical pressure such as the overburden pressure of the topsoil layers acts upon the soil.
- The **overburden potential is similar to the pressure potential**, in that it is the increase in potential of water in a porous system w/c results from the appl<sup>n</sup> of pressure to the water in the system. Their d/nce is that:
  - with the **pressure potential, the pressure is applied directly to the water**. This can only occur if the system is saturated.
  - With the **overburden potential, the pressure is applied to the water by the matrix**.
- The overburden potential can be present in an unsaturated porous medium.
- It is mainly important in soil at depth, where the pressure of the overburden can be substantial.
- Which is usually **negligible for sandy soils** and becomes more important for soils having greater clay contents.

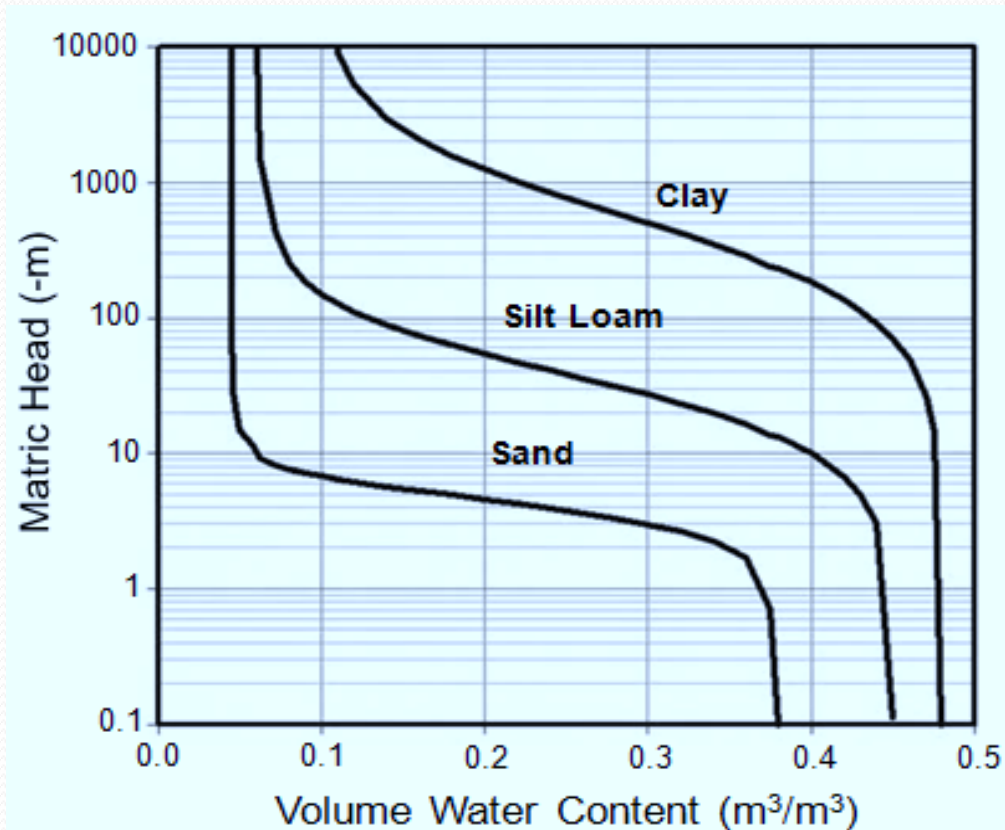


- ❑ **Pneumatic Potential,  $\psi_a$**  : which accounts for air pressure inside the soil pores being different from the outside atmospheric air pressure acting upon the reference water.

## Soil Water Content–Energy Relationships

**Soil Water Characteristic (SWC) curve:** is also known as *soil water retention curve* (**SWRC**)

It describes the functional relationship between **soil water content** ( $\theta_m$  or  $\theta_v$ ) and **matric potential** under equilibrium conditions.

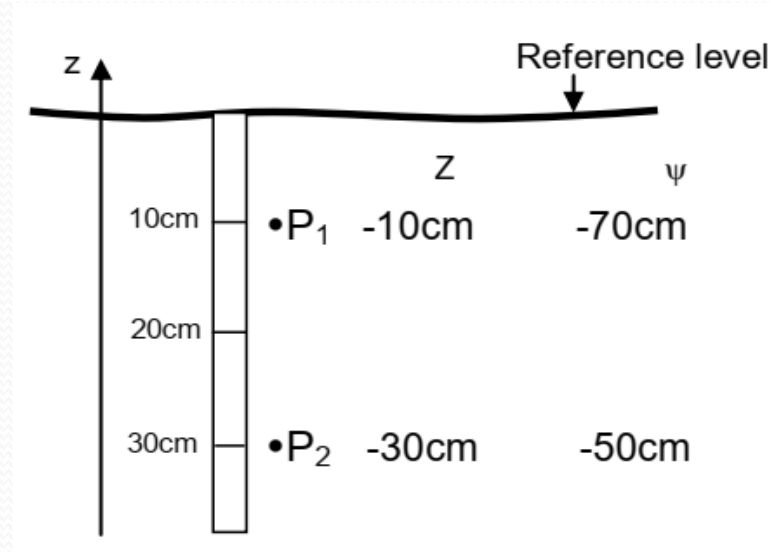


5 Example soil water retention relationships for three soil textures.



# Examples

1) Does the water flow takes place between  $P_1$  and  $P_2$ ?



## Answer

To answer this question we have to calculate the total hydraulic potential (H) at both points. Consider the gravitational head to be positive in upward direction and negative in downward direction.

$$H_1 = z_1 + \psi_1 = -10 + (-70) = -80 \text{ cm}$$

$$H_2 = z_2 + \psi_2 = -30 + (-50) = -80 \text{ cm}$$

**$\therefore$  There is no flow of water between the two points**, b/c the total potential at both points are equal. The hydraulic gradient is also equals zero.

## Examples

- 2) For question number 1, If the matric potential at  $P_2$  is measured to be equal to  $-70\text{cm}$ , calculate the hydraulic potential and evaluate whether there exist flow of water between the points.

### Answer

The total potential at point  $P_1$  and  $P_2$  are:

$$H_1 = z_1 + \psi_1 = -10 + (-70) = -80 \text{ cm}$$

$$H_2 = z_2 + \psi_2 = -30 + (-70) = -100 \text{ cm}$$

**$\Rightarrow$  Even though the matric potential at  $P_1$  and  $P_2$  are equal there is a flow of water from  $P_1$  and  $P_2$  as the result of differences in gravitational head only.**

The hydraulic gradient is :

$$\frac{\Delta H}{\Delta z} = \frac{(\psi_1 + z_1) - (\psi_2 + z_2)}{z_1 - z_2} = \frac{(-70 + (-10)) - (-70 + (-30))}{-10 - (-30)} = 1 \text{ cm/cm}$$

- 3) From a double ring infiltrometer the following infiltration rate were observed.

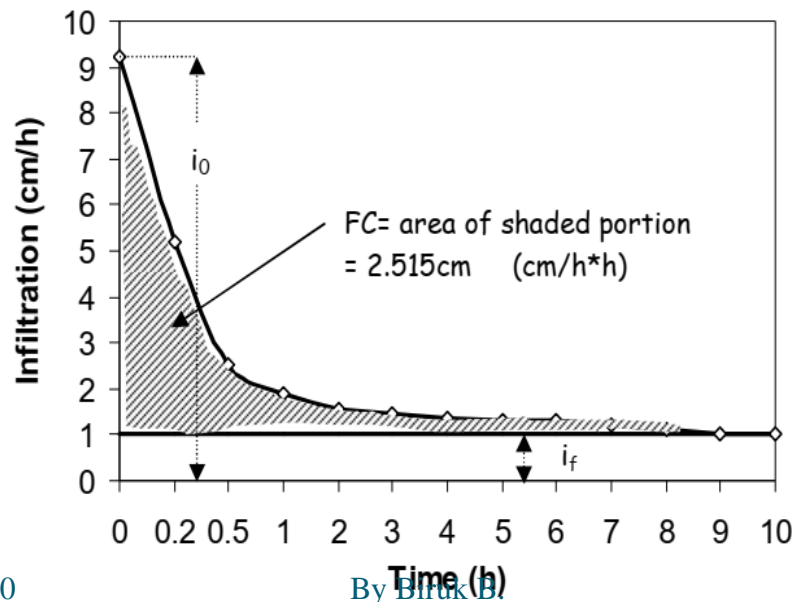
| Time (hr)   | 0.0167 | 0.0583 | 0.125 | 0.25 | 0.50 | 0.75 | 1.25 |
|-------------|--------|--------|-------|------|------|------|------|
| $i$ (cm/hr) | 8.76   | 7.90   | 6.45  | 4.68 | 2.75 | 1.76 | 1.10 |

Determine the constants  $i_0$ ,  $i_c$  and B of Horton's equation to describe the above data

### Solution:

The Horton's equation is given by:  $i = i_c + (i_0 - i_c)e^{-Bt}$

The given data are plotted on an ordinary graph as shown in fig. below:



From the graph

$$i_0 = 9.2 \text{ cm/hr}$$

$$i_c = 1.0 \text{ cm/hr}$$

The value of B may then be obtained as follows:

The Horton's equation is rewritten as

$$(i - i_c) = (i_0 - i_c) e^{-Bt}$$

Integrating the above equation on both sides in terms of t, we get

$$\text{LHS} = \int_0^{\infty} (i - i_c) dt = \text{the area of shaded portion in fig} = F_c$$

$$\text{RHS} = \int_0^{\infty} (i_0 - i_c) e^{-Bt} dt = \frac{(i_0 - i_c)}{B}$$

From these two results we can determine B from the equation

$$B = \frac{i_0 - i_c}{F_c} = \frac{9.2 - 1.0}{2.515} = 3.26$$

∴ The Horton's equation that can describe the given data is therefore given by

$$i = 1.0 + 8.2 e^{-3.26t}$$

# Reading Assignment

- 1) Measurement of Soil Water Potential Components
- 2) Measurement of SWC Relationships

# CHAPTER THREE

## *SOIL WATER MOVEMENT*

# INTRODUCTION

- **Water movement** in soils occurs under both **saturated** and **unsaturated** conditions.
  - ✓ **Saturated conditions:** occur below the WT where water mov't is predominately horizontal, with lesser components of flow in the vertical direction.
    - Saturated soils occur when soil pores are entirely filled with water ( $\theta = \eta_T$  and air filled porosity  $\theta_a = 0$ )
  - ✓ **Unsaturated conditions:** generally predominate above the water table (the vadose zone),
    - As a general rule, water mov't in the unsaturated zone is vertical, but can also have large lateral components.
- Water flow can be either steady or transient.



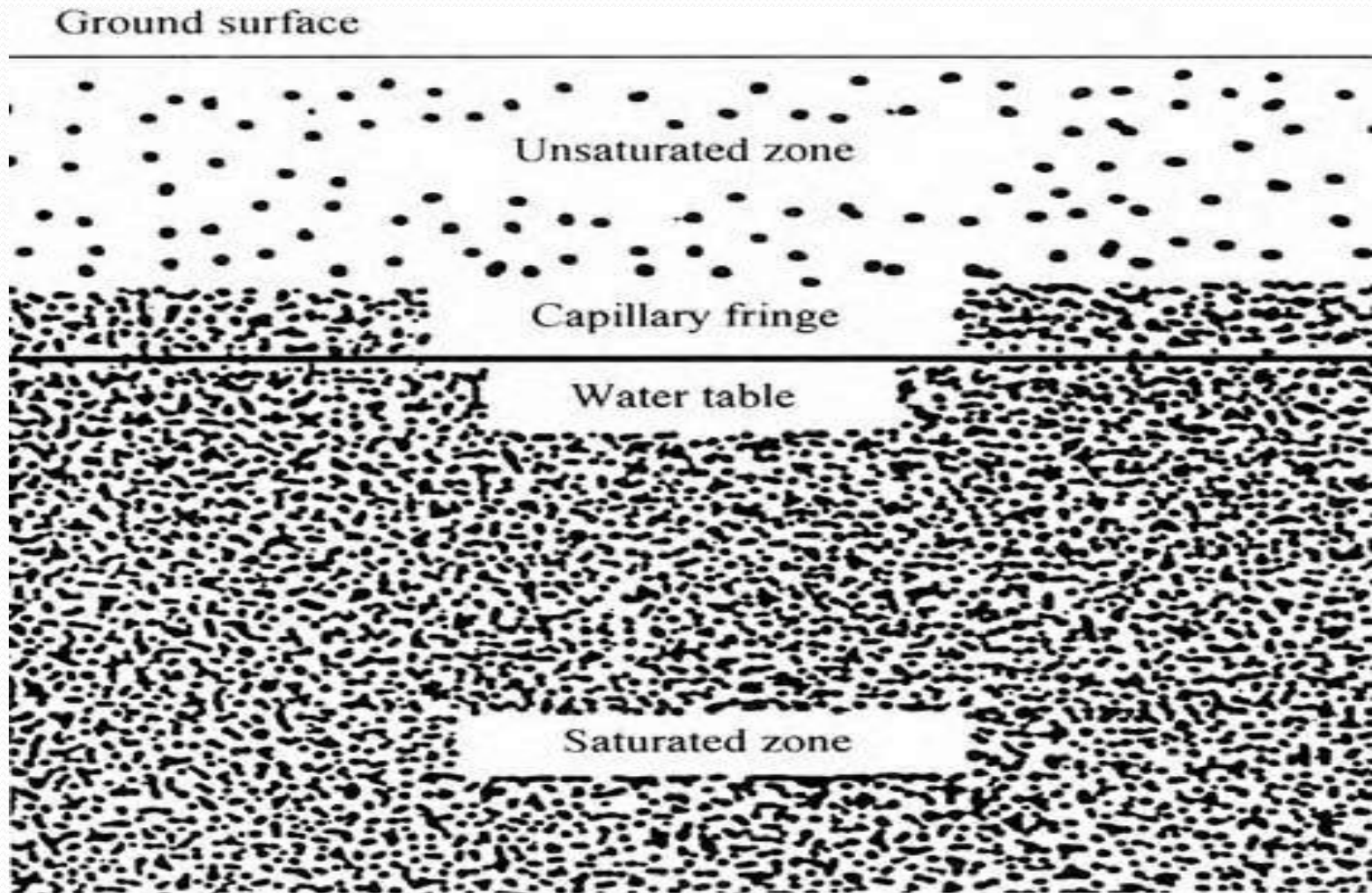


Figure 3.1: Saturated and unsaturated zones

## Three different types of flow will be considered:

1. Steady flow in saturated soil:  $\frac{\partial \theta}{\partial t} = 0$  and  $\theta = \eta$   
 $q$ ,  $K$  (or  $k$ ) and  $\theta$  are constant in time and position.
  2. Steady flow in unsaturated soil:  $\frac{\partial \theta}{\partial t} = 0$  and  $\theta < \eta$   
 $q$  is constant in time and position, but  $K$  (or  $k$ ) and  $\theta$  only in time
  3. Non-steady flow in unsaturated soil:  $\frac{\partial \theta}{\partial t} \neq 0$  and  $\theta < \eta$   
 $q$ ,  $K$  (or  $k$ ) and  $\theta$  vary with time and position.
- Non-steady flow in saturated soil, in which  $\theta$  is constant but  $q$  varies in time, occurs in drainage processes. This type of flow is usually treated separately and will not be considered here.

## Some Basic Terms

**Hydraulic Gradient**: is the driving force behind water flow.

- Is a vector that describes the slope of the energy distribution within the soil.

**Hydraulic Conductivity,  $K$** : is the rate of flow of water.

- **Saturated Hydraulic Conductivity ( $K_s$ )**: is the principal parameter required to predict saturated flow.
- Predicting unsaturated flow requires the **Unsaturated Hydraulic Conductivity [ $K(h)$ ]** and **Water Retention [ $\theta(h)$ ] Functions**.
- $K_s$ ,  $\theta(h)$  and  $K(h)$  are all affected by soil texture and structure.

$$K = \frac{1}{8\eta\tau} \Sigma(\Delta\theta)_i r_i^2$$

$$K_i = \eta K = \frac{1}{8\tau} \Sigma(\Delta\theta)_i r_i^2$$

## ***FLOW IN SATURATED SOIL***

### **1) Saturated Hydraulic Conductivity ( $K_s$ ):**

- For saturated flow, the most important soil parameter is saturated hydraulic conductivity, which is a function of the fluid and soil properties:

$$K_s = \frac{K_i \rho g}{\nu}$$

where  $K_i$  is the intrinsic permeability of the soil

- Soils with low porosity, few large pores and poor interconnectivity between pores have low values of  $K_s$
- Highest in coarse-textured soils and declined in fine-textured soil, due to larger pores in the coarse-textured soil.

Cont...

| Texture Class   | $K_s$<br>cm h <sup>-1</sup> | $\eta$<br>cm <sup>3</sup> cm <sup>-3</sup> | $h_a$<br>cm | $\lambda_c$<br>cm | $\lambda_m$<br>cm     |
|-----------------|-----------------------------|--|-------------|-------------------|-----------------------|
| Sand            | 21.00                       | 0.437                                      | -16.0       | 2.62              | $2.83 \times 10^{-2}$ |
| Loamy sand      | 6.11                        | 0.437                                      | -20.6       | 3.61              | $2.06 \times 10^{-2}$ |
| Sandy loam      | 2.59                        | 0.453                                      | -30.2       | 7.48              | $9.92 \times 10^{-3}$ |
| Sandy clay loam | 0.43                        | 0.398                                      | -59.4       | 16.05             | $4.63 \times 10^{-3}$ |
| Loam            | 1.32                        | 0.463                                      | -40.1       | 6.72              | $1.11 \times 10^{-2}$ |
| Silt loam       | 0.68                        | 0.501                                      | -50.9       | 12.74             | $5.83 \times 10^{-3}$ |
| Clay loam       | 0.23                        | 0.464                                      | -56.4       | 16.49             | $4.50 \times 10^{-3}$ |
| Sandy clay      | 0.12                        | 0.430                                      | -79.5       | 19.32             | $3.84 \times 10^{-3}$ |
| Silty clay loam | 0.15                        | 0.471                                      | -70.3       | 22.46             | $3.31 \times 10^{-3}$ |
| Silty clay      | 0.09                        | 0.479                                      | -76.5       | 24.60             | $3.02 \times 10^{-3}$ |
| Clay            | 0.06                        | 0.475                                      | -85.6       | 26.83             | $2.77 \times 10^{-3}$ |

TABLE 3.1 Saturated hydraulic conductivity ( $K_s$ ), total porosity ( $\eta$ ), air-entry matric potential ( $h_a$ ), macroscopic capillary length ( $\lambda_c$ ), and microscopic capillary length ( $\lambda_m$ ) of soils of various textures.



## 2) Steady Downward Flow In Homogeneous Soil

- The rate of movement (flux density) of water in saturated soil,  $q$  expressed as:

$$q = -K \frac{\partial H}{\partial s} = -K \left( \frac{\partial h}{\partial s} + \frac{\partial z}{\partial s} \right)$$

This relationship is called Darcy's law

hydraulic potential gradient      pressure potential gradient      gravitational potential gradient

- In a homogeneous, water-saturated soil **K** is constant with position and in time.
- ▮ For steady one-dimensional flow, as in the soil column in Figure 3.2,  $q$  is also constant in time and with height.
- ▮ From above Equation  $\partial H / \partial s$  is also constant with height, or that  $H$  varies linearly with height.

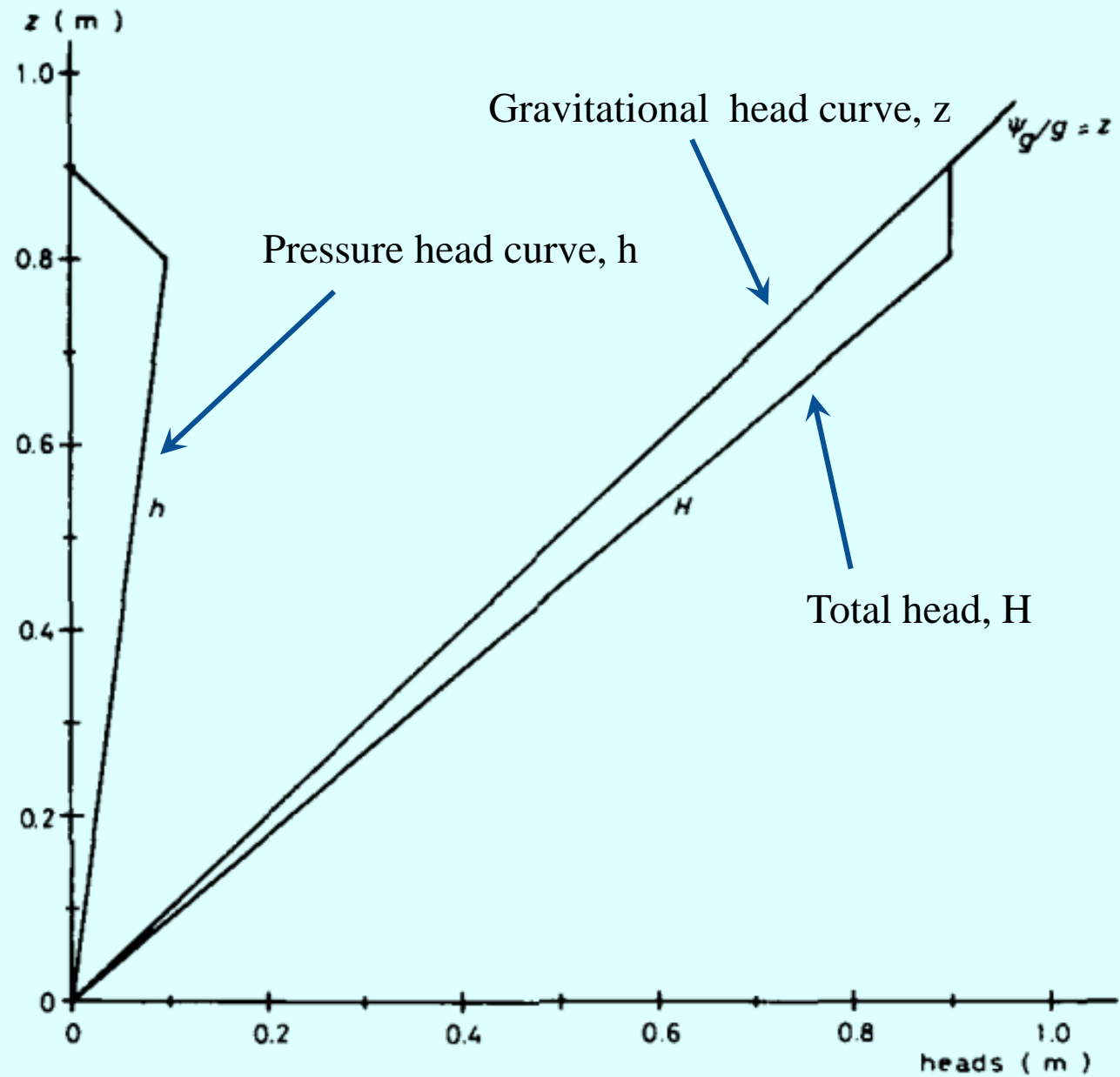
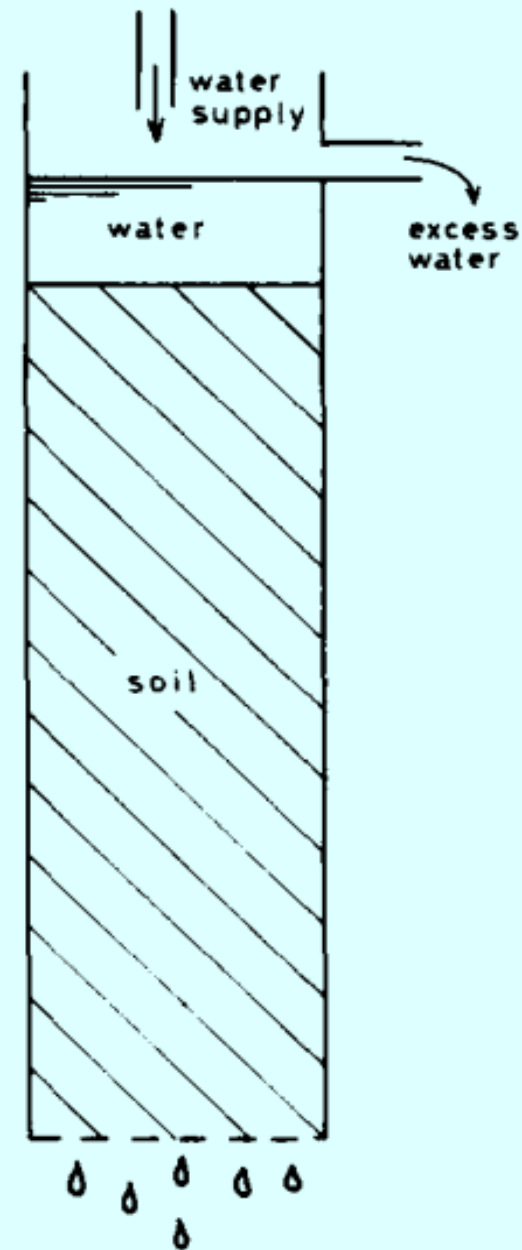


Figure 3.2 Steady saturated flow



## Cont...

- The lower end of the soil column in Figure 3.2 is supported by a screen, such that water can flow out freely under atmospheric pressure. This means that at that height,  $\mathbf{h} = \mathbf{0}$ .
- If the reference level for the gravitational potential is chosen at the same height, then there also  $\mathbf{z} = \mathbf{0}$  and thus,  $\mathbf{H} = \mathbf{h} + \mathbf{z} = \mathbf{0}$
- At the free water surface above the soil column,  $\mathbf{z} = \mathbf{0.9m}$  and  $\mathbf{h} = \mathbf{0}$ , thus  $\mathbf{H} = \mathbf{h} + \mathbf{z} = \mathbf{0.9m}$ .
- The water in the layer from  $\mathbf{z} = \mathbf{0.8m}$  to  $\mathbf{z} = \mathbf{0.9m}$  can be assumed hydrostatic. This means that in this layer  $\mathbf{H}$  is constant. Thus at the soil surface,  $\mathbf{H} = \mathbf{0.9m}$ ,  $\mathbf{z} = \mathbf{0.8m}$  and thus  $\mathbf{h} = \mathbf{0.1m}$ .
- Since within the soil  $\mathbf{H}$ , and also  $\mathbf{h}$ , vary linearly with height, their values are now determined as indicated in Figure 3.2

### 3) Steady Downward Flow In Layered Soil

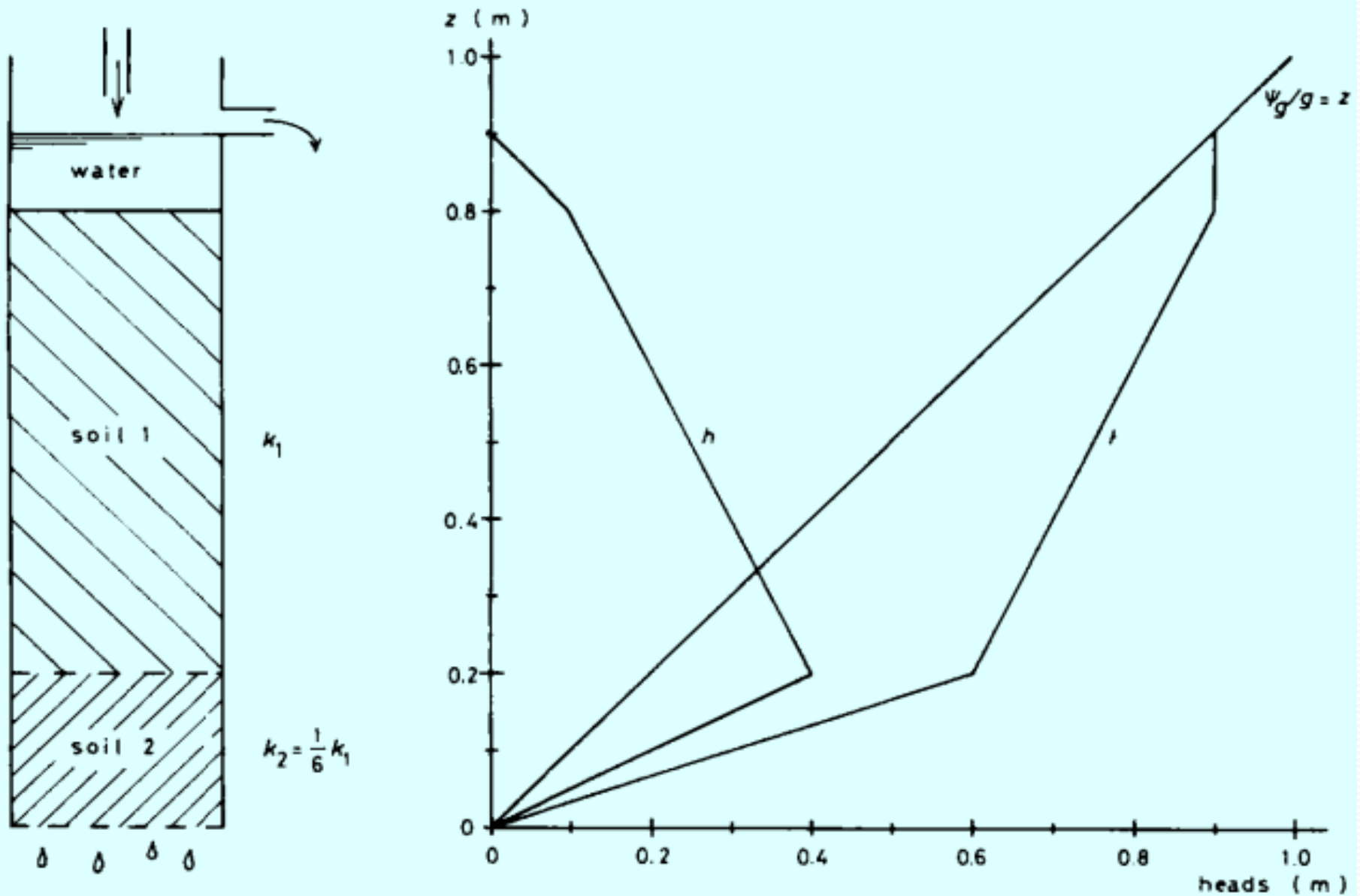
- For saturated flow of water in a soil profile or soil column with layers of different hydraulic conductivity, the flux density is the same in all the layers.

#### Why is this so?

- Because the flux density is everywhere the same, the gradient of **H** is inversely proportional to the values of **K** in the different layers. From this **H** and **h** can be calculated.
- This will be illustrated for the soil column in Figure 3.3
- In figure 3.2 the bottom 0.2 m is replaced by a layer of which **K** is **one sixth** of that in the overlying soil (Figure 3.3)

$$K_2 = \frac{1}{6} K_1$$

Cont...



$$q_1 = -k_1 (dH/dz), = -k_2(dH/dz), = q_2$$

$$k_1 \frac{\Delta H_1}{0.6m} = k_2 \frac{\Delta H_2}{0.2m}$$

- ›  $\Delta H$  is the difference of  $H$  between the **upper** and **lower** boundaries of a layer.
- › With  $k_1 = 6k_2$ , one then finds  $\Delta H_1 = \frac{1}{2} \Delta H_2$ .
- › Because of  $H = 0.9m$  at the upper surface of the soil column and  $H = 0$  at the lower surface,  $\Delta H_1 + \Delta H_2 = 0.9m$ . It easily follows that  $\Delta H_1 = 0.3m$  and  $\Delta H_2 = 0.6m$ .
- › The hydraulic gradients of  $H$  in the two layers can now be calculated and the values of  $h$  can be found from the equation  $h = H - z$ .

## 4) Steady Upward Flow

- Upward flow of water may occur in a soil profile due to **transpiration** by vegetation and/or **evaporation** from the soil surface.
- If the water is supplied by groundwater, this upward flow will lower the groundwater table, unless the groundwater is replenished by water from elsewhere.
- Replenishment of GW may occur by seepage from higher surroundings, seepage from irrigation canals or irrigated fields to the groundwater in neighboring fields.
- If the seepage balances the evapotranspiration, the groundwater level remains stable. In the laboratory such a situation can be simulated by a system as depicted in Figure 3.4.

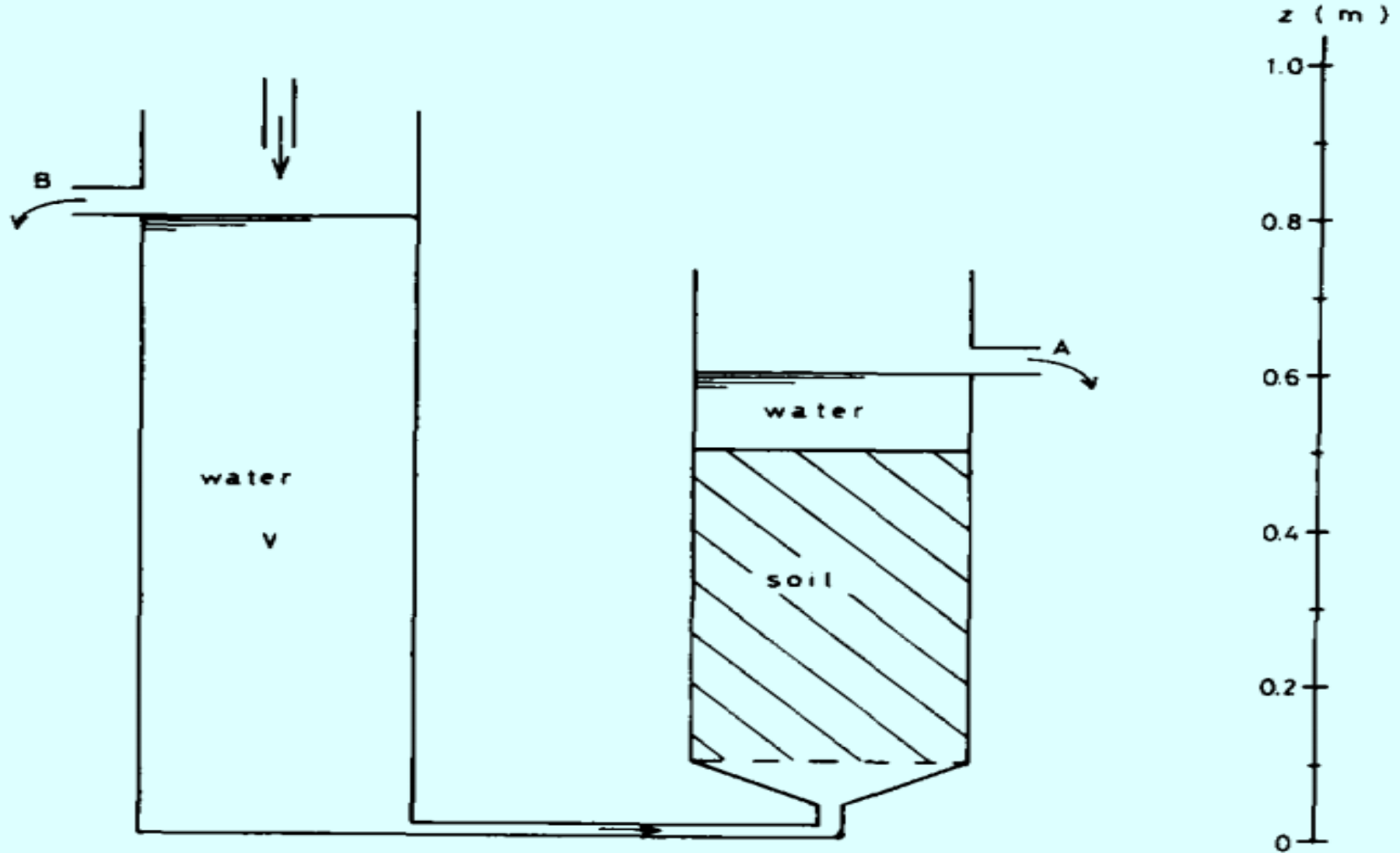
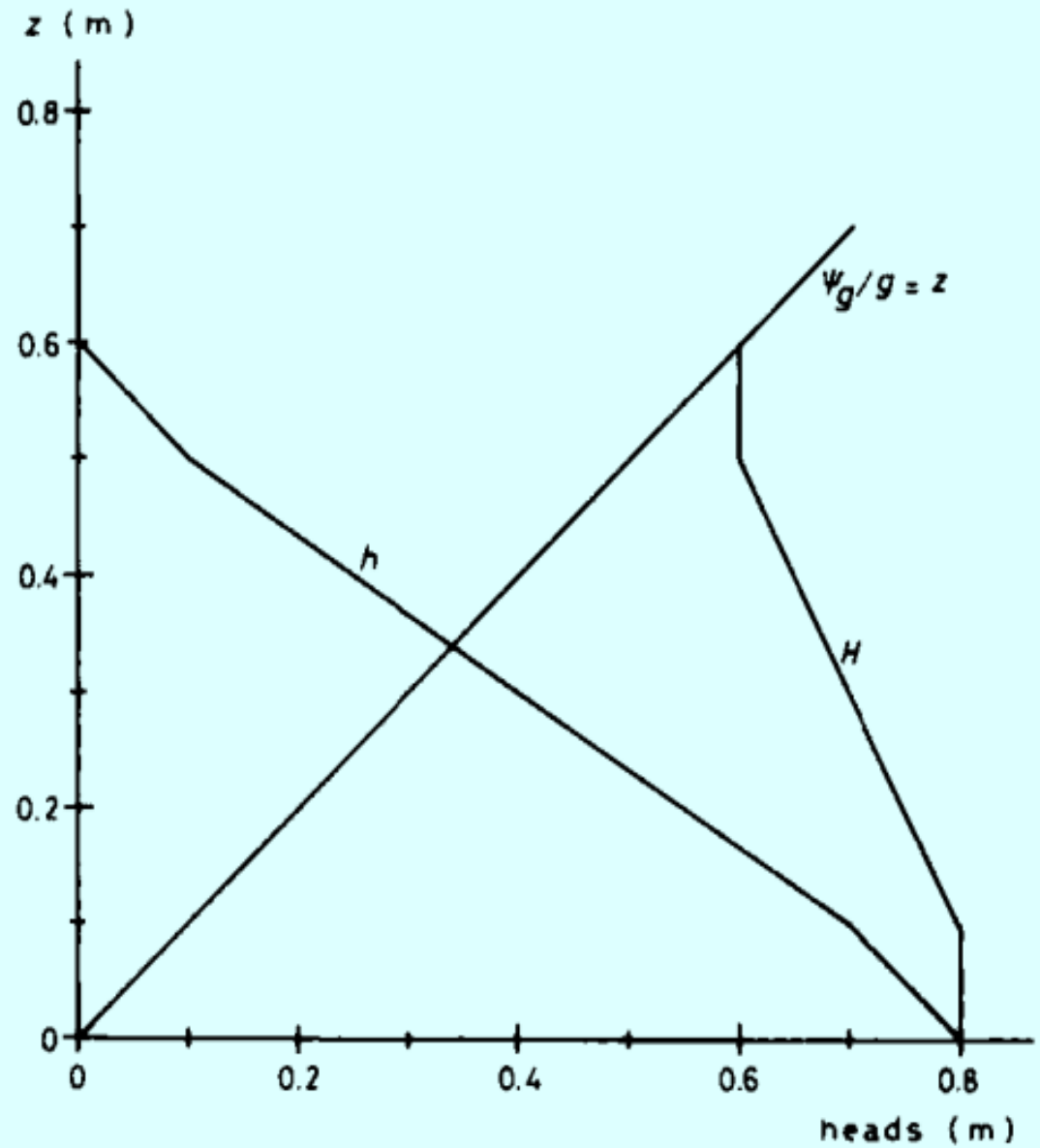
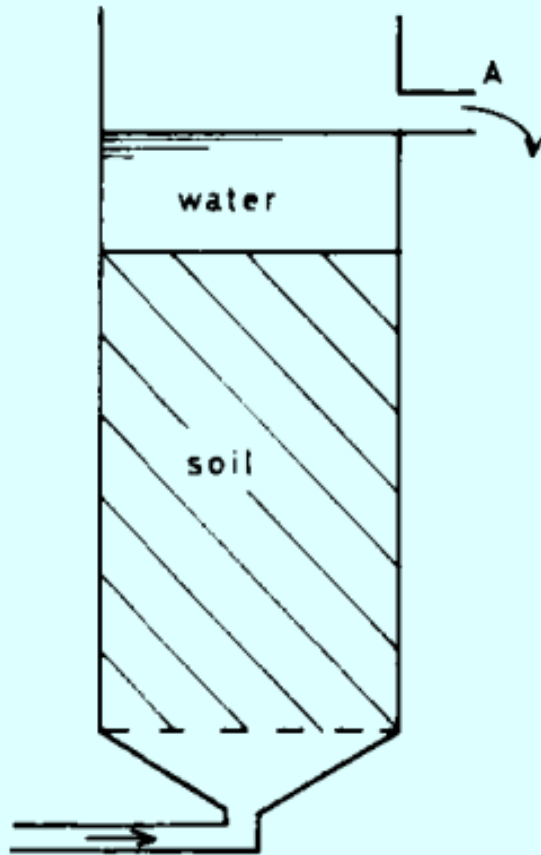


Figure 3.4: Simulation of steady saturated upward flow of water

## Cont...

- At the bottom of the vessel **V**,  **$h = 0.8\text{m}$** . If the reference level for **z** is chosen at the same place,  **$H = 0.80\text{ m}$  at  $z = 0$** .
- The resistance of the connection pipe between the vessel and the bottom of the soil column can be assumed so small, that the hydraulic head loss there is negligible and thus  **$H = 0.8\text{m}$  at  $z = 0.1\text{m}$** , at the bottom of the soil column.
- At  **$z = 0.6\text{m}$** ,  **$h = 0$**  and thus  **$H = 0.6\text{m}$** .
- As before, **H** is constant in the water layer above the soil column. Thus, at  **$z = 0.5\text{m}$** ,  **$H = 0.6\text{m}$**  and  **$h = 0.1\text{m}$** .
- With this the potential diagram is determined.





## 5) Intergranular Pressure, $\sigma_g$

- Is defined as the total contact force between the grains divided by area.
- In a **dry soil**, the vertical intergranular pressure ,  $\sigma_g$  **equals** the soil pressure,  $\sigma_s$  because at any depth the weight of the overlying soil material is borne solely by the contact forces between the grains at that depth. Hence:

$$\sigma_s = \sigma_g$$

- Is **causes** friction b/n the grains, which in turn **determines the** resistance against motion of grains relative to each other.
- For instance, the friction determines the resistance of the soil against ploughing, its 'bearing capacity' for buildings and its resistance against compaction by heavy farm machinery.

- In a **water-saturated soil**, the weight of the overlying soil is borne not only by the contact force between the grains, but also by the contact force in the water:

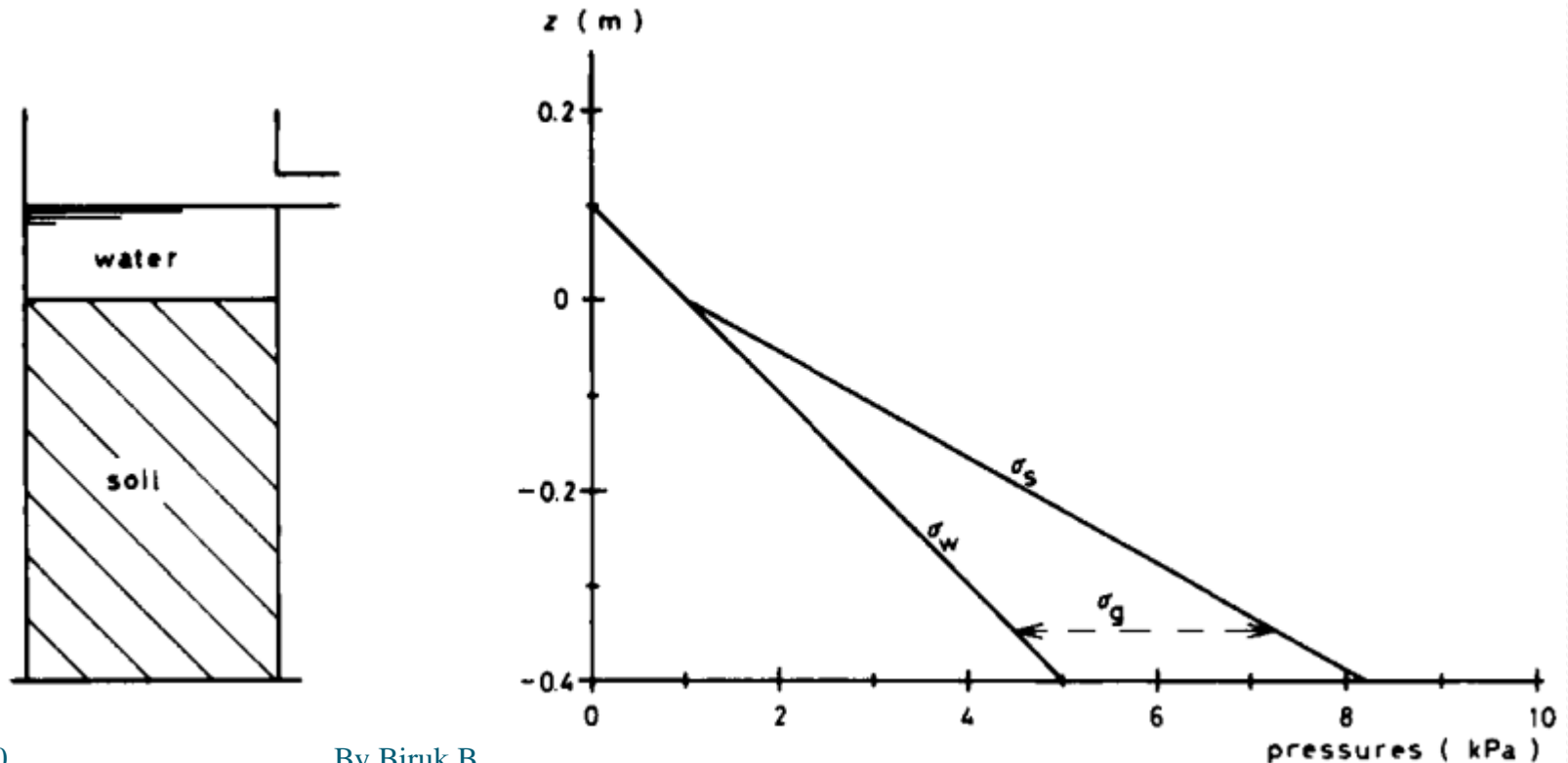
$$\sigma_s = \sigma_g + \sigma_w$$

where  $\sigma_w$  - is called water pressure  $= \rho\psi_p$

- The water pressure is active only in the water-filled pore space, while  $\sigma_s$ ,  $\sigma_g$  and  $\sigma_w$  are defined as if they act over the total cross-sectional area.
- For an unsaturated, soil:  $\sigma_s = \sigma_g + \frac{\theta}{\eta} \sigma_w$

## Cont...

- The relationships between the intergranular pressure,  $\sigma_g$ , the soil pressure,  $\sigma_s$ , and the water pressure,  $\sigma_w$  are illustrated in figure below, which shows the pressure diagram of a sand column with a layer of water on top, in a cylinder closed at the bottom



Question 1)

- a) Verify the pressure diagram in Figure above.
- b) What is the value of  $\sigma_g$  at the bottom?

Question 2) If the bottom of the cylinder in Figure is replaced by a screen ,

- a) Draw a new pressure diagram for this situation.
- b) Determine the intergranular pressure at the bottom of the soil column.

Question 3 Suppose the flow conditions in the soil column of question 2 are changed to those in Figure 3.4.

- a) Draw a diagram for  $u_s$  and  $u_w$  and determine the intergranular pressure at the bottom of the column.
- b) Calculate the intergranular pressure in the soil column if the overflow  $B$  is  $= 0.92$  m.

# ***FLOW IN UNSATURATED SOIL***

- **The unsaturated zone, also called the vadose zone**, lies between the water table and the soil surface.
- In this region the water content of the soil is less than saturation ( $\theta < \eta$ ) many pores are air-filled ( $\eta_a > 0$ ), and pressure heads are generally negative ( $h < 0$ ).
- The **water table**, which separates the saturated zone from the unsaturated zone, is the surface **where the water potential equals the mean atmospheric pressure**.
- When the soil desaturates, the **driving force** for flow becomes the **gradient in matric** and **gravitational potential**.
- As the largest pores in the soil drain, the hydraulic conductivity is rapidly reduced

## 1) Unsaturated Hydraulic Conductivity

- In the unsaturated zone, larger pores drain more readily than smaller ones. Therefore, the **hydraulic conductivity is much smaller under unsaturated than saturated** conditions due to water moving through small pores or as films along the walls of larger pores (or b/c only the pores which contain water can contribute to the flow of water)
- At very low water contents, continuous fluid paths may not exist and water may move in the vapor phase.
- At low pressure head (water contents), the unsaturated **hydraulic conductivity is greater in the clay than in the sand due to** more water-filled pores and continuous water films in the clay.



- Conductivity drops **rapidly** in the sand as pores empty since there is a more narrow distribution of pore sizes compared to the clay.
- Conductivity declines more **gradually** in the clay because each decrease in potential empties only a few pores due to the wide distribution in pore size.
- The **unsaturated hydraulic conductivity** is therefore represented as a function of **pressure head** [ $K(h)$ ] or as a function of **water content** [ $K(\theta)$ ].

Cont...

- The **Gardner** equation (1958):

$$K(h) = K_s \exp(\alpha h)$$

Where  $\alpha$  is a constant

- Other commonly used equations are the **Brooks** and **Corey**:

$$K(h) = K_s \left( \frac{h}{h_a} \right)^{-2-3\lambda}, \quad h < h_a$$

$$K(h) = K_s, \quad h \geq h_a$$

Where  $h_a$  is the air-entry pressure and  $\lambda$  is a constant

Cont...

- The **Campbell** (1974) equation: 
$$K(\theta) = K_s \left( \frac{\theta}{\theta_s} \right)^m$$

Where  $m$  is a constant

- The **Haverkamp** et al., 1977: 
$$K(h) = \frac{K_s}{1 + \left( \frac{h}{a} \right)^N}$$

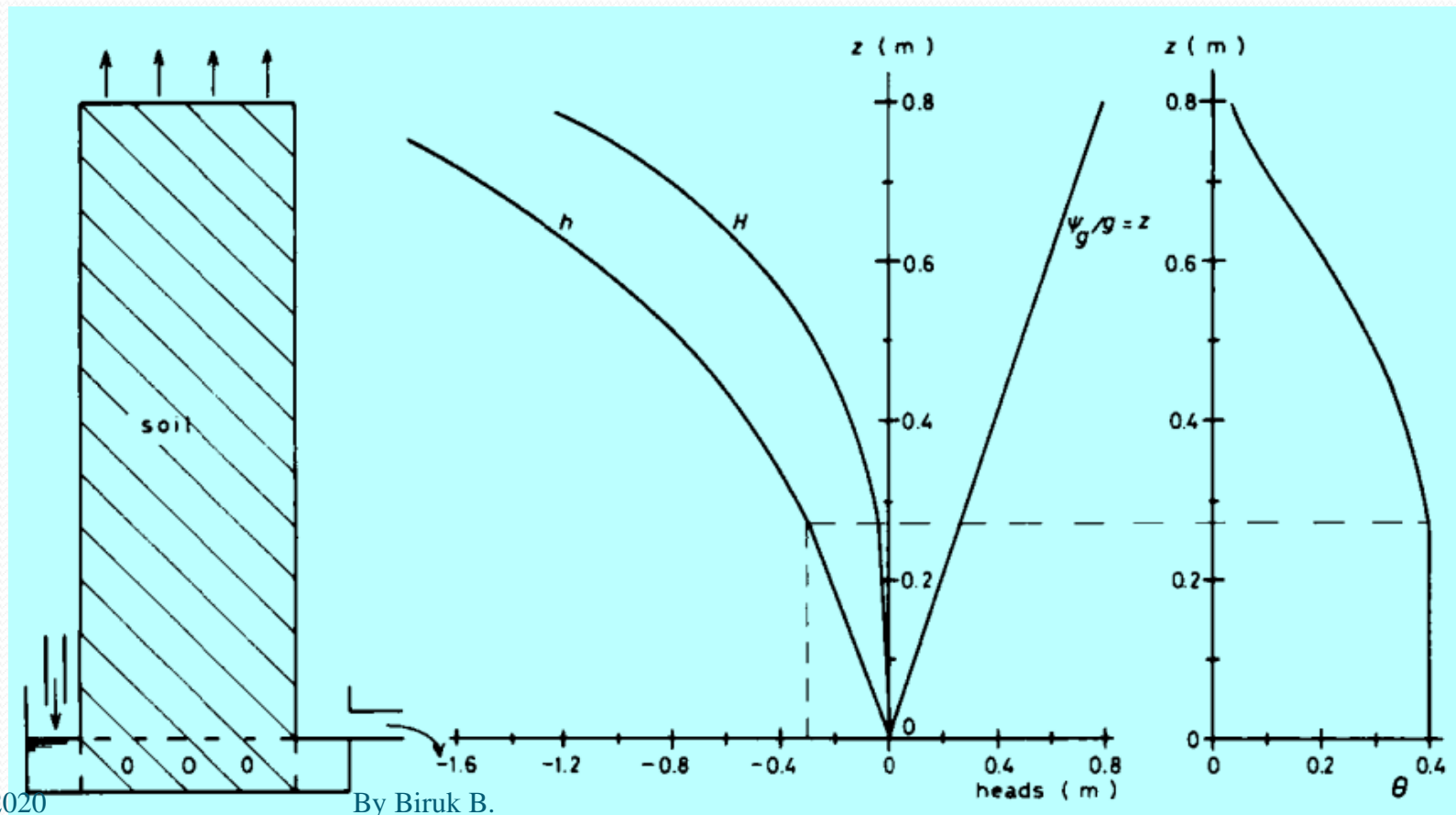
Where  $a$  and  $N$  are constants

Cont...

# 1) Steady Flow of Water In Unsaturated Soil

## a) Steady upward flow

- To study steady, upward flow/evaporation/ of water in unsaturated soil, let we consider a soil column as shown in figure 3.6.



- According to the continuity equation

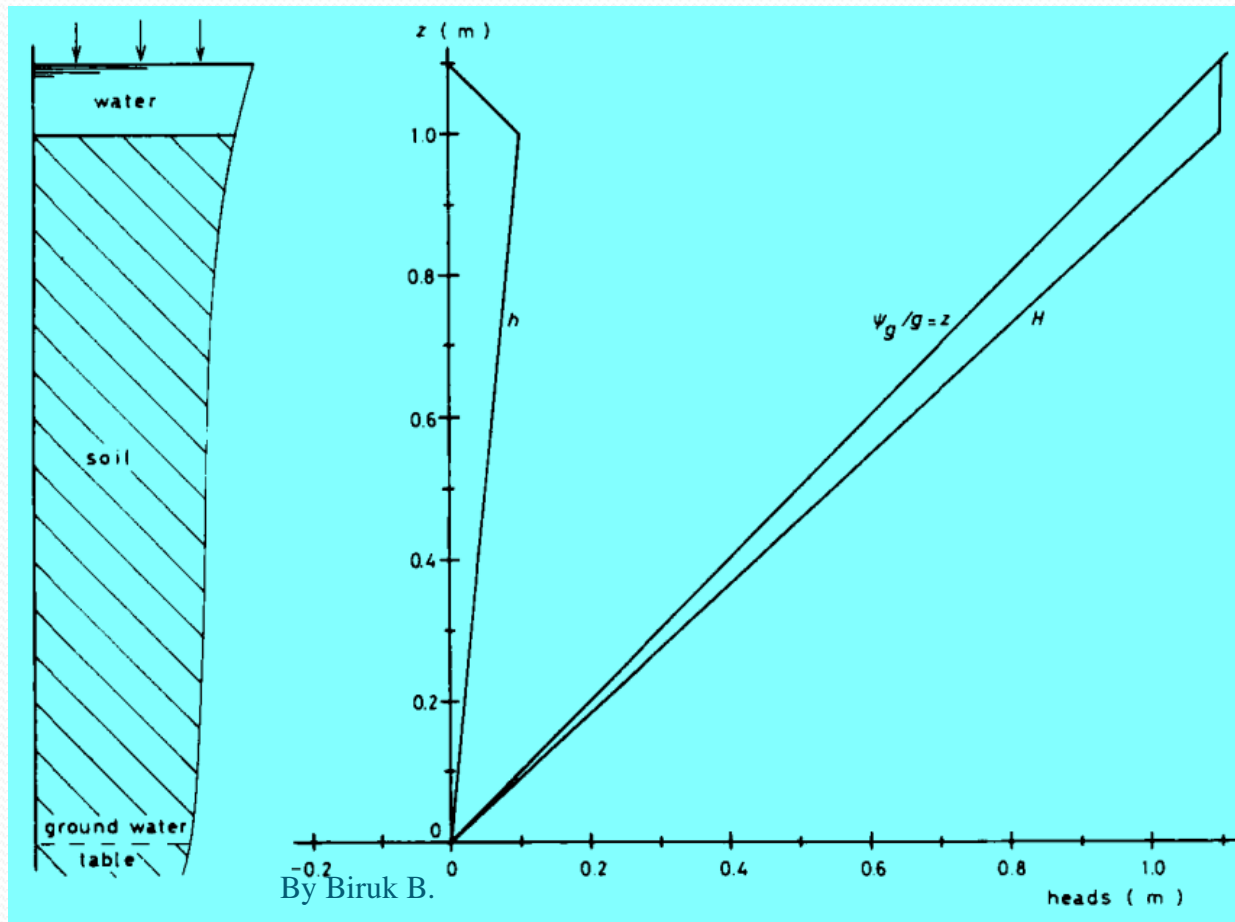
$$q = -k \frac{dH}{dz} = \text{constant and uniform}$$

- For the flow to be directed upwards,  $H$  must decrease with height, making  $dH/dz$  negative and  $q$  positive. Since  $K$  decreases fast with  $\theta$ , it decreases fast with height.
- To maintain the same flux density at all heights, a certain rate of decrease of  $K$  with height must be compensated by the same rate of increase with height of the absolute value of  $dH/dz$ . Thus the  $H$  curve in the potential diagram must be increasingly flatter towards the soil surface, as indicated in Figure 3.6.
- The above situation is often found in the field during periods of drought, when a rather sharp transition from a dry upper layer to a moist lower zone is developed due to the sharp decrease of  $k$  with decreasing water content.

Cont...

## b) Steady downward flow

- Figure 3.7 shows the potential diagram for a field situation.
- The water at the surface is applied by sprinklers and the groundwater table is stabilized by tile drains. The sprinkling rate is such that, without runoff, the layer of water on the soil surface remains constant.



*Figure 3.7*  
Steady  
downward  
flow of water  
at a high  
sprinkling rate  
(flooded soil).

### 3) Infiltration

- It is a typical example of non-steady unsaturated flow of water into soils.
- **Infiltration** is a process in which water enters the surface strata of the soil mass.
- The infiltrated water first meets the soil moisture deficiency, if any, and thereafter the excess water moves vertically downwards to reach the groundwater table. This vertical movement is called **percolation**.
- Infiltration is a **key process** because it determines how much water from **rainfall** and *irrigation* enters the soil and how much becomes **runoff**.



## Infiltr... Cont...

- It is also a key process in erosion in that there can be no erosion without runoff to transport and scour sediment.
- **Infiltration capacity of the soil** is defined as **the maximum rate at which it is capable of absorbing water.**
- Infiltration rate decreases during rain or irrigation. The rate of decrease is rapid initially and the infiltration rate tends to approach a constant value.
- In general infiltration rate is high in the early stages of infiltration, particularly where the soil is initially dry, but tends to decrease monotonically and eventually to approach asymptotically a constant rate, which is often termed the final infiltration capacity but which is preferably called steady-state infiltration.

## Factors Affecting Infiltration

Major factors governing the rate of infiltration of water are:

- i. Conditions and characteristics of surface soil :
- ii. Duration and intensity of irrigation or rainfall,
- iii. Vegetation cover,
- iv. Tillage and crop management practices

Conditions and characteristics of surface soil : primarily involved in the process are the soil water content, soil texture, soil structure, soil compaction, soil organic matter content, soil surface sealing, presence of cracks in surface soil, soil depth, depth of water table and the soil hydraulic conductivity.

**Duration and intensity of irrigation or rainfall** : The duration of sustained heavy rainfall results in a steady reduction in the infiltration capacity until the rate of infiltration attains a nearly constant value. When the rainfall rate is smaller than the infiltration capacity, there is still a steady reduction of the infiltration capacity, but the rate of reduction is lower. Under this condition, all rainfall will be infiltrated as long as rain falls.

**Vegetation cover** : A soil surface with vegetative cover favors a greater infiltration than a bare soil as the vegetative cover encourages slow movement of water over the surface.

**Tillage and crop management practices** : is increase the looseness of the soil and by that increase the infiltration rate.

## Infiltration Equations

- Numerous formulations (empirical and theoretically based) have been proposed over the years in repeated attempts to express **infiltration rate** as a function of **time** or **the total quantity of water infiltrated** into the soil.
- We use the symbol ***I*** to represent the ***cumulative volume of water infiltrated*** in **time *t*** per **unit area of soil surface** and the symbol ***i*** for the ***infiltration rate*** as a volume flux (***i.e., the volume of water entering a unit soil surface area per unit time***).

$$i = \frac{dI}{dt} \quad \text{and} \quad I = \int_0^t i \, dt$$

## Infilt... Cont...

$$i = \frac{dI}{dt} \quad \text{and} \quad I = \int_0^t i \, dt$$

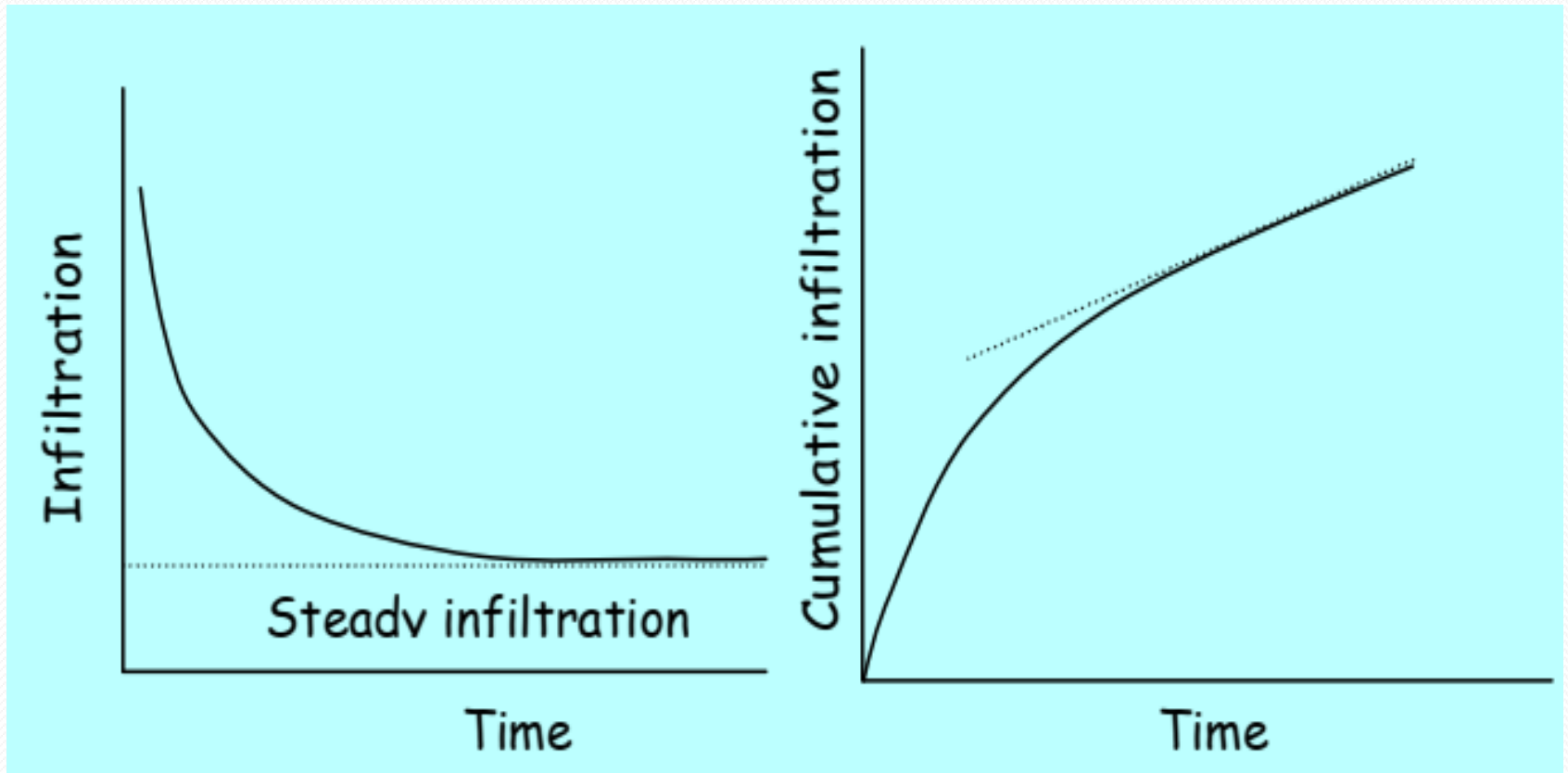


Figure: Time dependence of infiltration and cumulative infiltration

### Green and Ampt (1911) equation:

- The earliest eq<sup>n</sup> can be expressed by Green & Ampt as:

$$i = i_c + b/I$$

Here  $i_c$  and  $b$  are the characterizing constants, with  $i_c$  being the asymptotic steady infiltration flux reached when  $t$  (and hence  $I$ ) become large. Note that at  $t = 0$ ,  $I$  is also zero, so that the above eq<sup>n</sup> predicts  $i$  to be infinite initially and then to decrease gradually to its eventual value  $i_c$ . The constant  $b$  influences the form of initial infiltration.

- Example, Suppose  $b_1 = 5$  and  $b_2 = 50$   $i_c = 1 \text{ cm/min}$  and the values of cumulative infiltration obtained from experiment are as follows

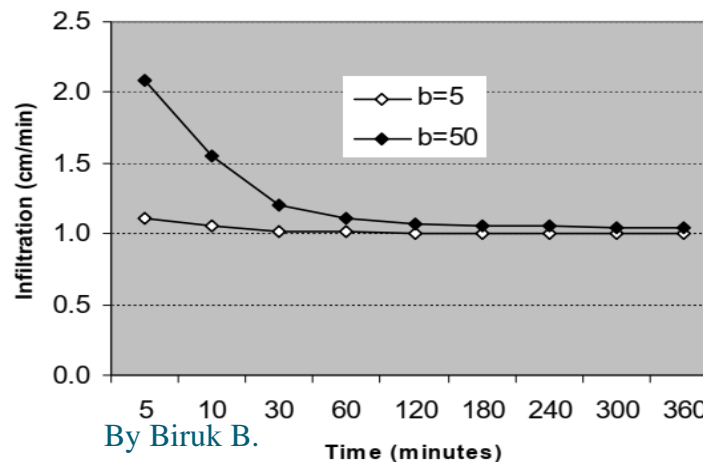


Fig. calculated infiltration rate

## Kostiakov equation (1932):

- The eq<sup>n</sup> shows the r/nship b/n cumulative infiltration depth and elapsed time since the start of infiltration.

$$i = B n t^{(1-n)} + i_c$$

$$I = B t^n + i_c t$$

where  $B$  = empirical coefficient (L/T) and  $n$  = is empirical exponent (-)  
 $i_c$  is called the basic infiltration rate

- Example, Suppose value of  $B = 5$  and variable value for  $n$  are given to be (-0.2, -0.5 and -1.0)

| Time (min) | Cumm vol. of water added (cm <sup>3</sup> ) | Time (min) | Cumm vol. of water added (cm <sup>3</sup> ) |
|------------|---|------------|---|
| 0          | 0   | 180        | 842   |
| 5          | 46  | 240        | 1000  |
| 10         | 90  | 300        | 1154  |
| 30         | 246   | 360        | 1300  |
| 60         | 435   |            |   |
| 120        | 662   |            |   |

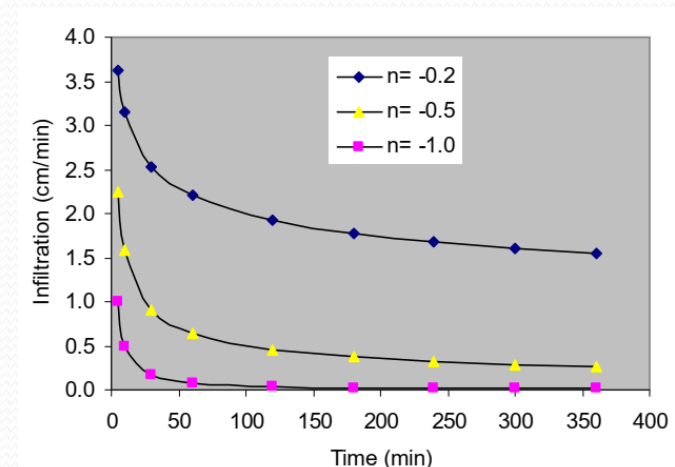


Fig. : Infiltration curve of Kostiakov formula



### Horton's Equation (1939, 1940):

- Is one of the best known models in hydrology..

$$i = i_c + (i_0 + i_c)e^{-\beta t}$$

$$I = i_c t + \frac{i_0 - i_c}{\beta} [1 - e^{-\beta t}]$$

where  $i$  = infiltration capacity or potential infiltration rate ( $\text{LT}^{-1}$ )

$i_c$  = final constant infiltration rate ( $\text{LT}^{-1}$ )

$i_0$  = infiltration capacity at  $t = 0$  ( $\text{LT}^{-1}$ )

$\beta$  = soil parameter ( $\text{T}^{-1}$ ) that controls the rate of decrease of infiltration and depends on initial water content and application rate

$t$  = time after start of infiltration

- The parameters  $i_c$ ,  $\beta$ ,  $i_0$  must be determined from measured infiltration data.

## Philip Equation (1957, 1969):

- Is one of the best known models in hydrology..

$$i = \frac{1}{2} S t^{-1/2} + K t$$

$$I = S t^{1/2} + K t$$

where  $S$  is a parameter called sorptivity, which is a function of the soil suction potential, and

$K$  is hydraulic conductivity

- As  $t \rightarrow \infty$ ,  $i$  tends to  $K$ . The two terms in Philip's equation represent the effects of soil suction head and gravity head, respectively.
- For a horizontal column of soil, soil suction is the only force drawing water into the column, and Philip's equation reduces to:

$$I = S t^{1/2}$$



## Examples

- 4) A small tube with a cross-sectional area of  $40 \text{ cm}^2$  is filled with soil and laid horizontally. The open end of the tube is saturated, and after 15 minutes,  $100 \text{ cm}^3$  of water have infiltrated into the tube. If the saturated hydraulic conductivity of the soil is  $0.4 \text{ cm/h}$ , determine how much infiltration would have taken place in 30 minutes if the soil column had initially been placed upright with its upper surface saturated. (Philip Equation )

### Solution

The cumulative infiltration depth in the horizontal column is:

$$I = V/A = 100 \text{ cm}^3 / 40 \text{ cm}^2 = 2.5 \text{ cm}$$

For horizontal infiltration, cumulative infiltration is a function of **soil suction** alone so that after  $t = 15 \text{ min} = 0.25 \text{ h}$ ,

$$I = S t^{1/2}$$

$$2.5 = S (0.25)^{1/2} \Rightarrow S = 5 \text{ cm/h}^{1/2}$$

For infiltration down a vertical column, Philip Equation applies with  $K = 0.4 \text{ cm/h}$ . Hence, with  $t = 30 \text{ min} = 0.5 \text{ h}$

$$I = S t^{1/2} + K t$$

$$I = (5) (0.5)^{1/2} + (0.4) (0.5) = 3.74 \text{ cm}$$

## Examples 5

If the effective saturated hydraulic conductivity and the suction of the soil are  $2 \times 10^{-4}$  cm/sec and 0.1 cm/sec respectively, estimate the cumulative infiltration and infiltration rate values for a vertical column at 1, 4, and 16 hr. If the initial wetness ( $\theta_i$ ) is  $0.05 \text{ cm}^3$  and the saturated wetness ( $\theta_s$ ) is  $0.45 \text{ cm}^3$ , estimate the depth of the wetting front at the same times.)

### Solution

To calculate cumulative infiltration into a vertical column, we use Philip Equation:

$$I = S t^{1/2} + K t$$

$$\text{At 1 hr (3600 sec), } I = (0.1) (3600)^{1/2} + (2 \times 10^{-4}) (3600) = 6.72 \text{ cm}$$

$$\text{At 4 hr (14400 sec), } I = (0.1) (14400)^{1/2} + (2 \times 10^{-4}) (14400) = 14.88 \text{ cm}$$

$$\text{At 16 hr (57600 sec), } I = (0.1) (57600)^{1/2} + (2 \times 10^{-4}) (57600) = 35.52 \text{ cm}$$

To calculate the infiltration rate into a vertical column, we differentiate the equation:  $i = \frac{1}{2} S t^{-1/2} + K$

$$\text{At 1 hr (3600 sec), } i = (0.5) (0.1) (3600)^{-1/2} + (2 \times 10^{-4}) = 10.33 \times 10^{-4} \text{ cm/sec}$$

$$\text{At 4 hr (14400 sec), } i = (0.5) (0.1) (14400)^{-1/2} + (2 \times 10^{-4}) = 6.166 \times 10^{-4} \text{ cm/sec}$$

$$\text{At 16 hr (57600 sec), } i = (0.5) (0.1) (57600)^{-1/2} + (2 \times 10^{-4}) = 4.083 \times 10^{-4} \text{ cm/sec}$$

## Solution cont...

To calculate the depth of the wetting front  $s_f$  we use a variant of equation:

$$I = (\theta - \theta_i) s_f = (0.45 - 0.05) s_f = 0.4 s_f$$

At 1 hr (3600 sec),

$$s_f = 6.72/0.4 = 16.8 \text{ cm}$$

At 4 hr (14400 sec),

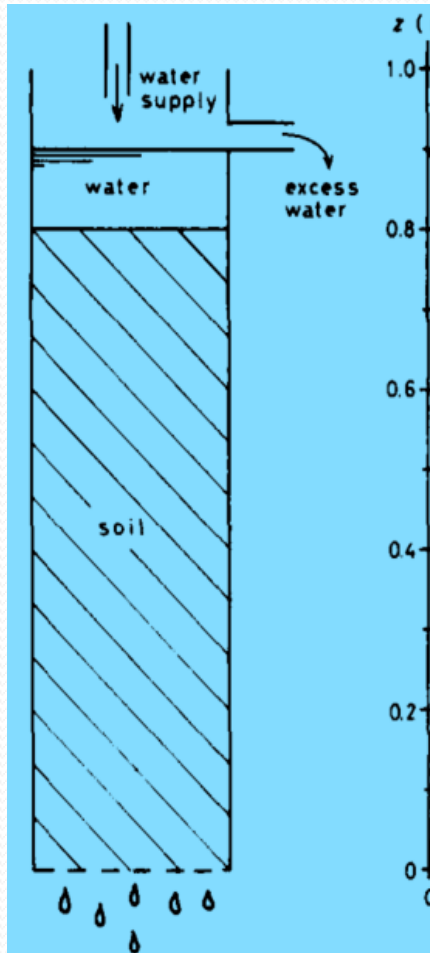
$$s_f = 14.88/0.4 = 37.2 \text{ cm}$$

At 16 hr (57600 sec),

$$s_f = 35.52/0.4 = 88.8 \text{ cm}$$

## Example 6

Calculate the saturated hydraulic conductivity,  $k$ , of the soil in Figure, if the rate of discharge of water at the bottom of the column is  $1.62 \text{ cm}^3$  per minute and the internal cross-sectional area of the cylinder is  $20 \text{ cm}^2$ .



### Solution

The flux density is downwards, so it is negative. The rate of discharge is:

$$Q = 1.62 \text{ cm}^3 / 1 \text{ min} = 1.62 \times 10^{-6} \text{ m}^3 / 60 \text{ s} = 2.7 \times 10^{-8} \text{ m}^3 / \text{s}$$

The cross-sectional area,  $A$  is  $20 \text{ cm}^2 = 20 \times 10^{-4} \text{ m}^2$

$$\text{Hence, } q = -\frac{Q}{A} = -\frac{2.70 \times 10^{-8}}{20 \times 10^{-4}} = -1.35 \times 10^{-5} \text{ m}^3 / \text{s}$$

$$\text{The hydraulic gradient, } i = \frac{dH}{dz} = \frac{\Delta H}{\Delta z} = \frac{0.9 \text{ m}}{0.8 \text{ m}} = 1.125$$

Then, according to Darcy's law,

$$k = \frac{-q}{i} = \frac{1.35 \times 10^{-5} \text{ m/s}}{1.125} = 1.2 \times 10^{-5} \text{ m/s}$$

## Example 7

The soil profile of a rice field consists of an upper layer of 0.60 m and a lower layer of 0.20 m. Below these layers is coarse gravel. The saturated hydraulic conductivity of the upper layer is 6 times that of the lower layer. The water level in the field is maintained at 0.10 m above the soil surface.

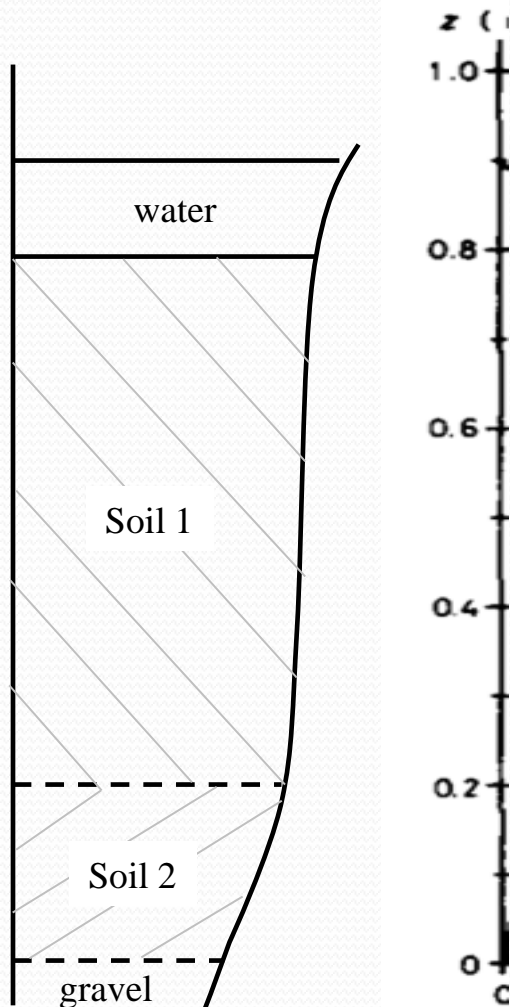
- a) Make a schematic drawing of the system and a potential diagram.
- b) Calculate the flux density to the gravel layer if the hydraulic conductivity of the lower layer is  $1.2 \times 10^{-5} \text{ m s}^{-1}$ .



Cont...

## Solution

Let first develop the figure



a) From  $q_1 = q_2$ , we can get the following r/n

$$\frac{\Delta H_1}{\Delta H_2} = \frac{k_2 \Delta z_1}{k_1 \Delta z_2} = \frac{1}{6} \times \frac{0.60}{0.20} = \frac{1}{2} \Rightarrow \Delta H_2 = 2\Delta H_1$$

At  $z = 0.80$  m,  $h = 0.10$  m,  $H = 0.90$  m

At  $z = 0$ ,  $h = 0$ ,  $H = 0$

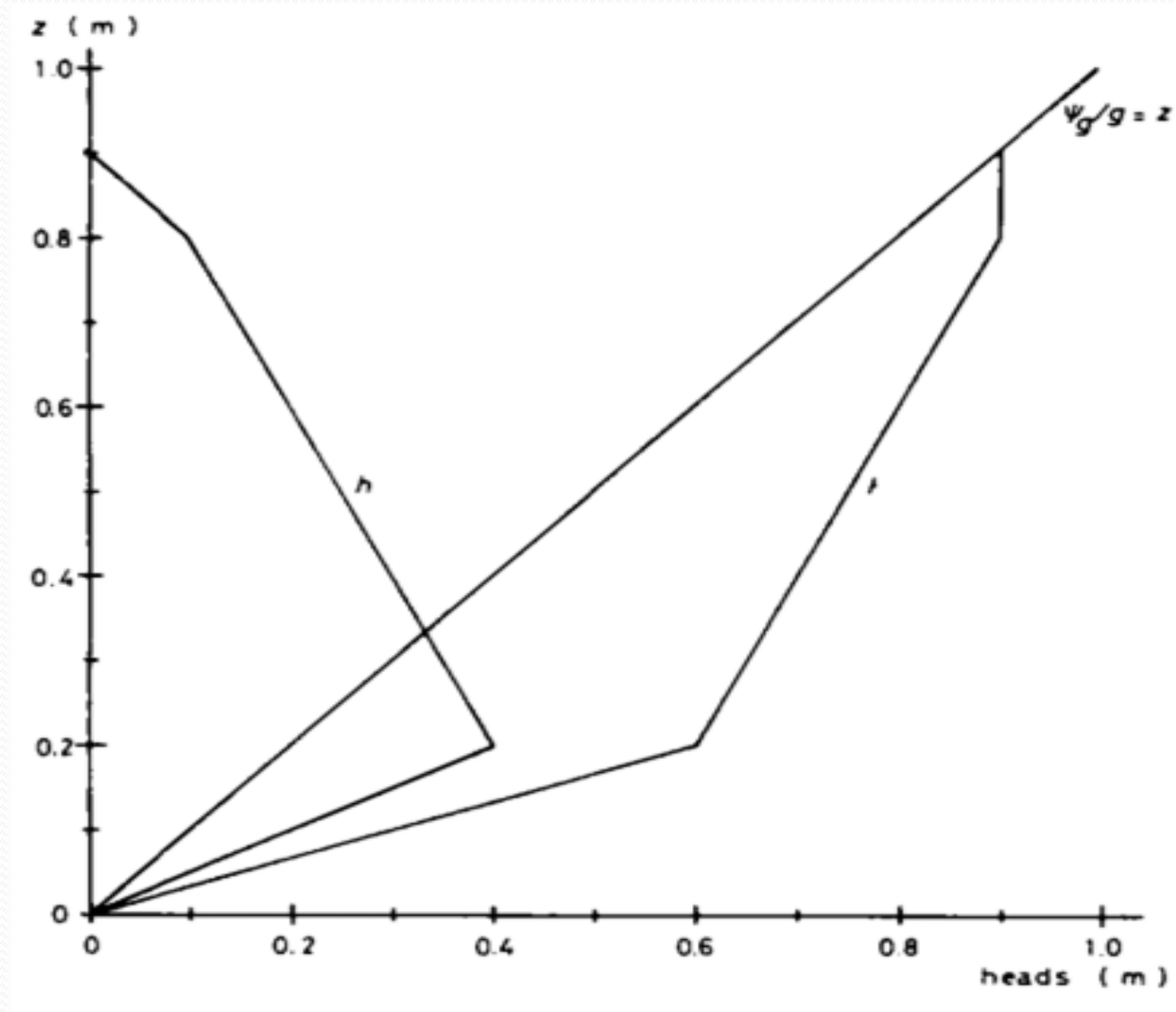
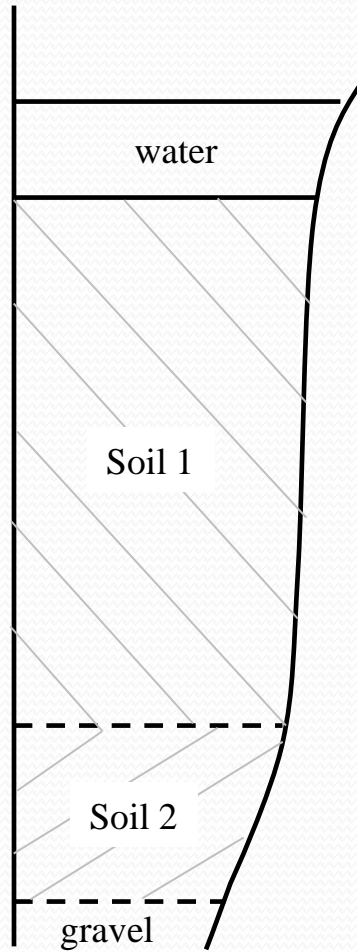
Thus  $\Delta H_1 + \Delta H_2 = \Delta H_1 + 2\Delta H_1 = 0.90$  m

Then  $\Delta H_1 = 0.30$  m and  $\Delta H_2 = 0.60$  m

With this the potential diagram is determined.

b) The flux density in layer 2 is:

$$q = k_2 \frac{\Delta H_2}{\Delta z_2} = 1.2 \times 10^{-5} \times \frac{0.60}{0.20} \text{ m s}^{-1} = -3.60 \times 10^{-5} \text{ m s}^{-1}$$



# Exercises 1

Assume that the soil of question 5 as an irrigated farm. The farmer wants to saturate it down to 25 cm depth.

- a) Calculate the volume of water he has to apply per  $1 \text{ cm}^2$  of soil surface.
- b) If this were to be applied as a single application, what would be the depth of flooding?

## Exercises 2

A soil that has porosity of 0.4 is used as an irrigated farm. The farmer wants to saturate it down to 25 cm depth.

- a) Calculate the volume of water he has to apply per 1 cm<sup>2</sup> of soil surface.
- b) If this were to be applied as a single application, what would be the depth of flooding?

## Exercises 3

The cross-sectional area of the soil column in Figure 3.4 (refer ppt) is  $100 \text{ cm}^2$  and the rate of discharge measured at A is  $180 \text{ cm}^3$  per hour. Calculate the saturated hydraulic conductivity of the soil column.

# *CHAPTER FOUR*

## ***SOLUTE TRANSPORT IN SOILS***

# INTRODUCTION

- Soil Scientists and agricultural engineers have traditionally been interested in the **behavior** and **effectiveness** of **agricultural chemicals** (fertilizers, pesticides) applied to soils for enhancing crop growth, as well as in the **effect** of **salts** and other **dissolved substances** in the soil profile on plant growth.
- More recently, concern for the quality of the vadose zone and possible contamination of groundwater has provided a major impetus for studying **solute transport in soils**.
- The **movement** and **fate** of solutes in the subsurface is affected by a large number of **physical**, **chemical** and **microbiological** processes requiring a broad array of mathematical and physical sciences to study and describe solute transport.



## Cont...

- **Transport** of a dissolved substance (solute) depends on the **magnitude** and **direction** of the solvent (water) flux.
- The ability to predict the **rate of movement** of solutes in soils and the **effect of the soil** on this rate would be useful for many purposes.
- **Predicting/Knowing** of the **amount** and the **rate of mov't** of solutes:
  - would assist in forming more accurate nutrient budgets and increase the understanding of nutrient cycling.
  - helps to know the **amount** and **concentration** of fertilizer nutrient below the root zone in order to **design management schemes that minimize fertilizer losses and keep concentration of certain solutes**, such as nitrate, within acceptable water quality standards.

*Cont...*

- essential to **predicting their impact on the environment**
- helps to reclaim salt-affected soils and to devise(plan) management practices for irrigated crop land to prevent excessive salt accumulation in soil.
- allow better prediction of the effect of soil on plant nutrition.

# MASS FLOW

- Chemical species dissolved in water are transported with the **water**. This type of transport is termed the *mass flow* of **solutes**.
- The change in concentration with **time**, at a given point in the soil, of a solute moving by steady mass flow, can be described by the equation:

$$\rho_b \partial S / \partial t = f_w \partial c / \partial z \quad \dots\dots\dots 4.1$$

where c - solute concentration in the soil solution (kg/kg),

S - solute present per unit mass of soil,

$f_w$  - water flux density ( $\text{kg m}^{-2}\text{s}^{-1}$ ), z - soil depth (m),

$\rho_b$  - bulk density ( $\text{kg m}^{-3}$ ), and t - time (s).

- If we assume that flow through all the soil pores has uniform velocity, the flux of a solute **into** and **out** of a soil can be described by above equation, if the initial solute concentration and water flux density in the soil are known.

- A typical method used to **compare calculated solute flow to actual/initial solute flow** is to establish a saturated column of soil in the laboratory where water flow is initiated in the column at a rate,  $f_w$ .
- A **slug** of solute then is intruded at the top of the column and the **amount of water moving through the column**, as well as the **solute concentration** in the water at the bottom of the column, is **measured**.
- If the solute moved **only** through *mass flow*, and the *flow* of all the water in all pores were *uniform*, then the *breakthrough* curve of the solute appear as in Figure (a) below. This is termed *piston flow*.

Cont...

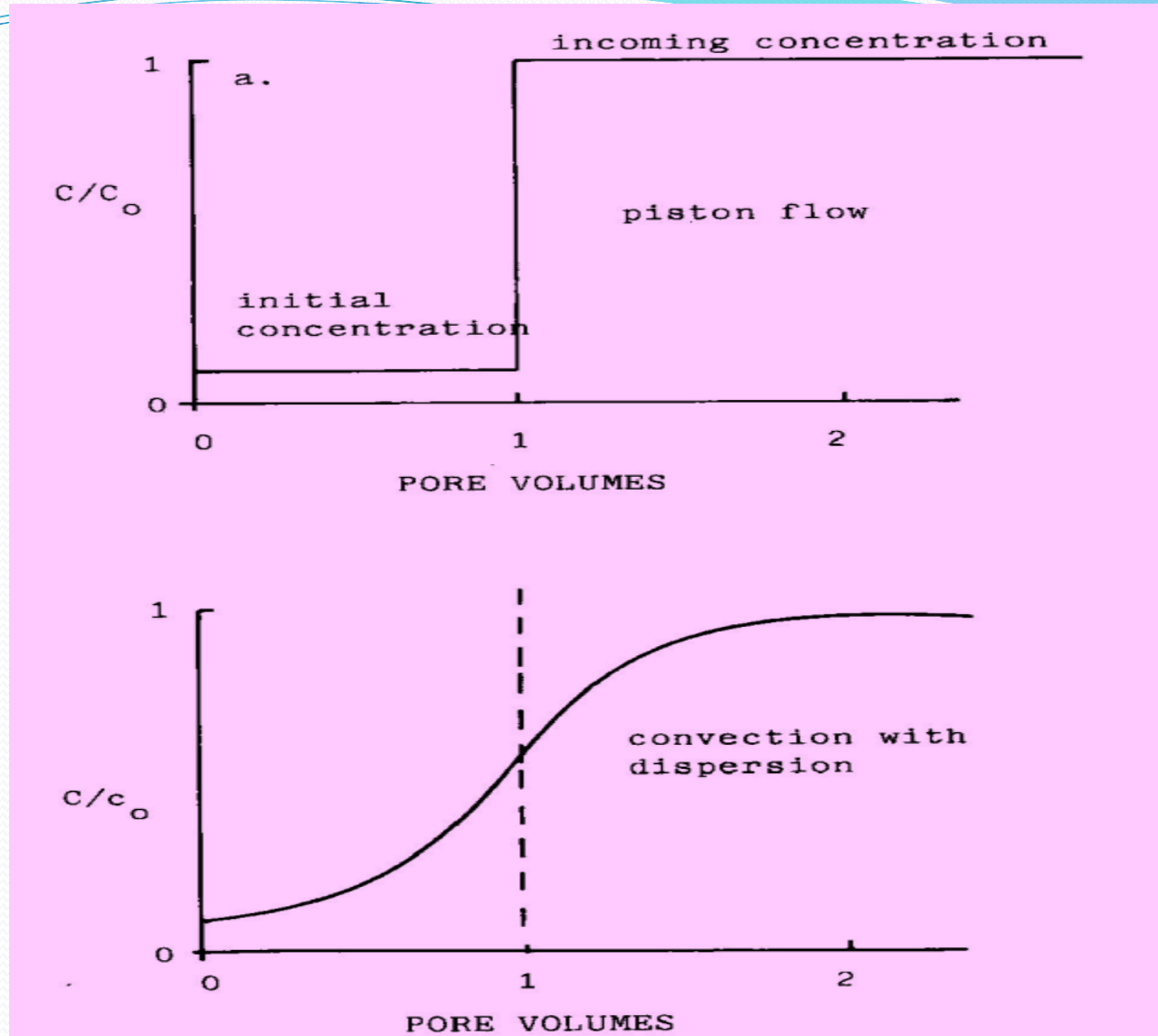


Figure: Convection (a) without and (b) with dispersion

## Cont...

- Initial concentration of the solute is low. At time 0, solute with concentration  $c_0$  is introduced at the top of the column.  $c/c_0$  is the ratio of outflow to inflow concentration.
- The amount of time required for the solute to appear in the outflow is equal to the amount of time taken to replace all the water in the soil (one pore volume) with the introduced water. This depends on  $f_w$ , the water flux density in the column.
- Increasing or decreasing the hydraulic conductivity would increase or decrease the time taken for the solute to appear. but would change the shape of breakthrough curve.

## Dispersion

- Local variations in water flow in a porous medium will lead to mechanical dispersion.
- Dispersion may occur because of
  - 1) The development of a velocity profile within an individual pore such that the highest velocity occurs in the center of the pore, and presumably little or no flow at the pore walls;
  - 2) Different mean flow velocities in pores of different sizes;
  - 3) The mean water flow direction in the porous medium being different from the actual streamlines within individual pores, which differ in shape, size and orientation; and
  - 4) Solute particles converging to or diverging from the same pore.
- All of these processes contribute to increased spreading, in which initially steep concentration fronts become smoother during movement along the main flow direction.



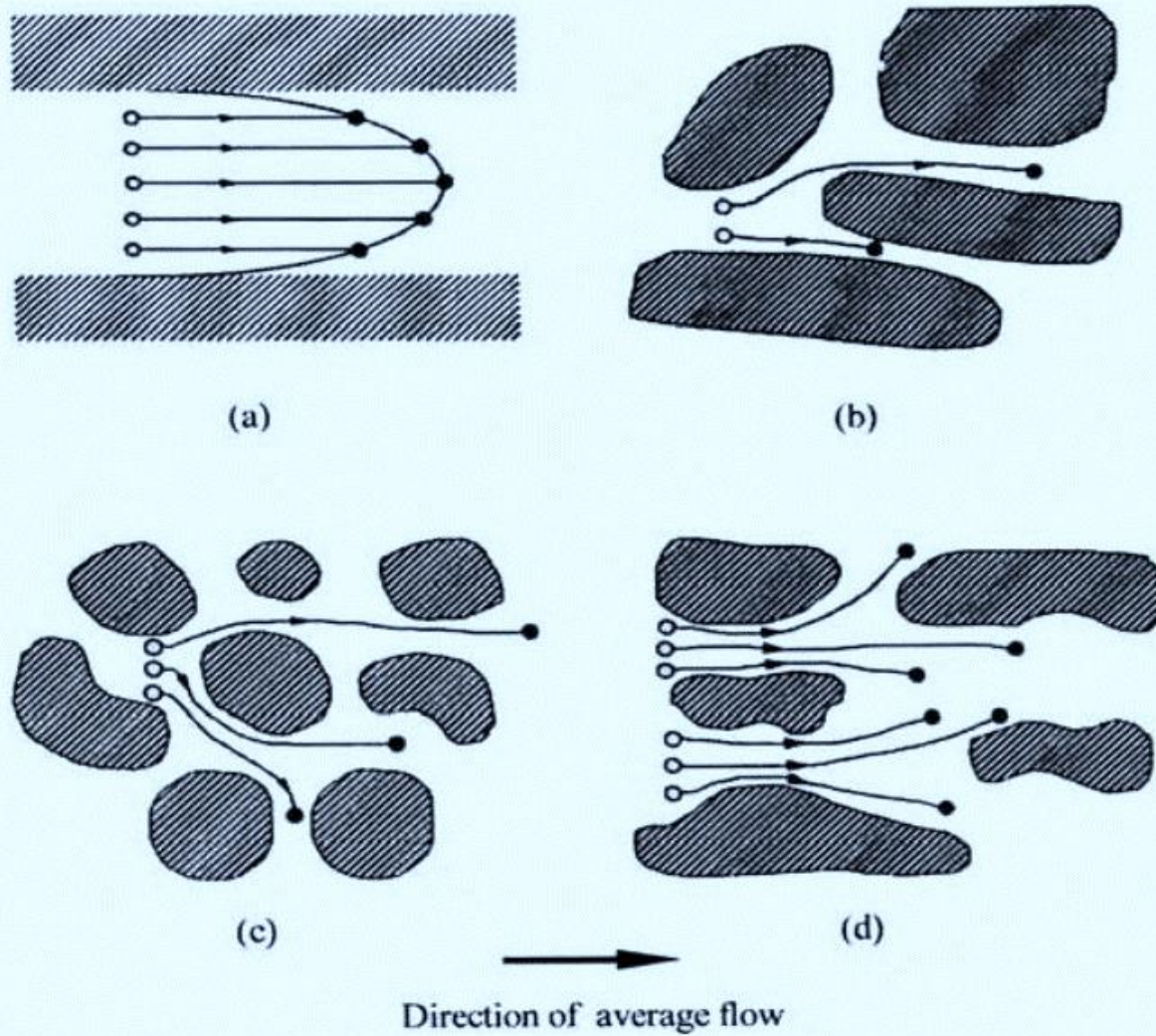


Figure: Schematic concepts contributing to mechanical dispersion

Cont...

- But it is **not realistic** to assume that all the water in the pores is **moving at the same speed/uniform**.
- Water along with the solutes dissolved in it, will **move fastest through the largest pores**.
- Solute in **larger channels** would **flow sooner(faster) than the average**, and water in **small pores** would come through **well behind the average**. The result would be a breakthrough curves like the above Figure (b).
- In this case, Solute transport due to mass flow, therefore, might be more precisely described by the equation

$$\rho_b \partial S / \partial t = \partial c / \partial z \sum f_{wi}$$

where  $f_{wi}$  - the water flux density per pore.

$$f_{wi} = -k_i \partial \psi / \partial z.$$

$k_i$  – is a function of both pore size & the number of pores in a pore size class.

## Cont...

- Laminar flow through a porous medium further complicates the mass flow of solutes because flow within a single pore is not uniform (i.e. there is a gradient in velocity within the pore).
- According to Newton's law of viscosity, this gradient in velocity will be proportional to the shear force in the flowing liquid, divided by the viscosity.
- The water flux density at any one point in the pore is therefore a function of distance from the center of the pore

$$f_w = \rho_w(a^2 - r^2)/4\eta (\Delta P/\Delta x)$$

where  $a$  - radius of the pore,

$r$  - radial distance from the center of the pore

$\eta$  – viscosity

- In other words, **solute dissolved in water near the side of the pore will flow more slowly than those in the middle of the pore.**

Cont...

- In summary, **Two** major problems involved with predicting the mass flow of solutes in soil are:
  - 1) Determining the effect of differing water velocities in different sized pores, and
  - 2) Determining the effect of differing water velocities within a single pore.
- One solution to these problems has been to place the dispersing effect that different pore sizes has on solute flow in the diffusion rather than the mass flow portion of the solute transport equation.

# ***DIFFUSION AND HYDRODYNAMIC DISPERSION***

- Solutes move in response to concentration gradients, as well as moving with water in response to hydraulic potential gradients.
- The equation for such movement is called the **Fick diffusion equation**, and can be written as:

$$\rho_b \frac{\partial S}{\partial t} = \frac{\partial [\rho_w D(\theta, f_w) \partial c / \partial z]}{\partial z}$$

..... 4.4

where  $D(\theta, f_w)$  – is diffusion coefficient ( $\text{m}^2 \text{s}^{-1}$ ) which can include both **molecular diffusion,  $D_m$**  and **hydrodynamic dispersion**.

- **Molecular or ionic diffusion** is an important mechanism for **solute transport** in soils in directions where there is little or no water flow.
- From this eq<sup>n</sup>, in order to predict the change in concentration of a solute with time due to diffusion, **both** the concentration gradient of the solute and the diffusion coefficient must be known.



- When there are no sources or sinks for the solute in the soil, then in order to predict concentration,  $c$  as a function of depth and time, we need only to know the initial concentration of the solute in the soil and the amount of solute being introduced at the surface of the soil as a function of time.
- The cases involving active sources and sinks are more complex and will be dealt with later in the chapter.
- Diffusion coefficients for various solutes in free water are available in handbooks.
- However, to predict the diffusion of **an ion** in soil we must use a diffusion coefficient *that accounts for both the tortuosity in a porous medium such as soil and the cross-sectional area of water available for diffusion* (which is a function of the water content of the soil).

Cont...

- ❖ The diffusion coefficient,  $D(\theta, f_w)$  can be calculated from **Bresler (1973)** equation as:

$$D = D_m + kf_w/\theta$$

where  $k$  is a soil dependent constant.

$(kf_w/\theta)$  is the dispersion effect, is a function of  $f_w$ ,

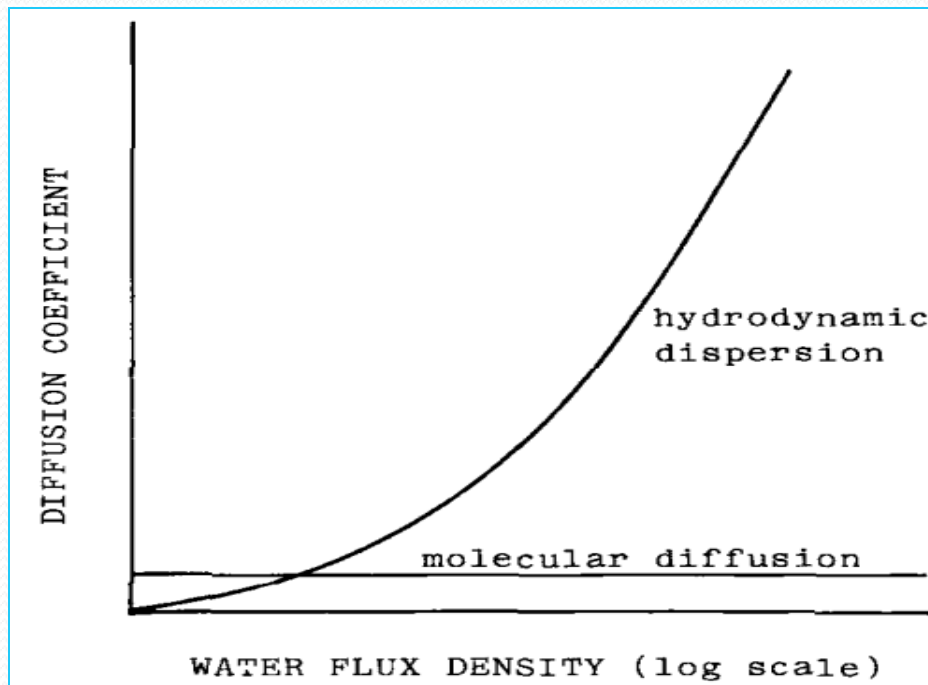


Fig. Comparison of **molecular diffusion** and **hydrodynamic dispersion** as a function of flow velocity



- Several equations have been proposed for calculating the molecular diffusion of an ion in soil.

- **Bresler (1973)**: One commonly used equation

$$D_m = D_o a e^{b\theta}$$

where  $D_o$  is the diffusion coefficient of the ion in water ( $\text{m}^2 \text{s}^{-1}$ ) and  $a$  &  $b$  are soil-dependent constants.

- **Papendick and Campbell (1980)**: later proposed

$$D_m = D_o a \theta^3$$

where  $a$  is a constant (around 2.8) to account for **tortuosity**.

- The above two equations give similar results except in dry soil where the second appears to give more realistic values.
- Diffusivity is predicted to decrease rapidly with decreasing water content by both equations.

# *SOLUTE-SOIL INTERACTION*

- In eqs 4.1 and 4.4, the flux of solute is determined by the **concentration in solution**, **c**, while the storage at any point in the soil is shown by changes in the variable, **S**.
- Before the transport equation can be solved, the relationship between **c** and **S** must be specified. In addition to the **solute in solution**, that present in a **solid phase**, or in complexed and exchangeable forms must be included.
- The **total amount of solute** in the soil is the sum of the **adsorbed phase** and the **solution phase**, and is given by:

$$S = N + w c \quad \dots\dots\dots 4.9$$

where **N** - mass of adsorbed-phase solute in soil  
(kg/kg)

**w** - is the mass-basis water content (kg/kg)

Cont...

- The adsorbed,  $N$  and solution-phase,  $c$  solute concentrations can be related by a **Langmuir** equation:

$$N = kQc / (1+kc)$$

where  $k$  and  $Q$  are constants for a given soil and solute

- Solute concentrations in soil, often are sufficiently low that the denominator of the above eq<sup>n</sup> is near unity. For these conditions, **total amount of solute** in the soil is:

$$S = (kQ + w)c \quad \dots\dots\dots 4.10$$

- Equation 4.10 allows the comparison of quantities of adsorbed and solution-phase solutes.

Cont...

Table: Values of the constants  $k$  and  $Q$

|            | $k$                | $Q$                | $kQ$               |
|------------|--------------------|--------------------|--------------------|
| Solute     | (kg water/kg salt) | (kg salt/kg soil)  | (kg water/kg soil) |
| Phosphorus | $3 \times 10^5$    | $2 \times 10^{-4}$ | 60                 |
| Potassium  | $8 \times 10^2$    | $8 \times 10^{-3}$ | 6.4                |
| Ammonium   | $2 \times 10^3$    | $3 \times 10^{-3}$ | 5.6                |

# *SOURCES AND SINKS OF SOLUTES*

- **Sources of solutes:** that produce solute
- **Sinks of solutes:** that consume of solute

Eg: Sugar moves through phloem in plants. It moves from a source cell, where sugar is made through photosynthesis, to a sink cell, where the sugar is used.

- Nitrification consumes ammonium and produces nitrite and nitrate. Ammonium is a product of decomposition.
- Microbes (Microorganisms) and roots take up nutrients from the soil solution.

# *EFFECT OF SALT ON WATER FLOW*

- A gradient in osmotic potential can provide a **driving force for water flow** under some conditions.
- **Osmotic potential gradients** are usually present whenever there are solutes in the soil solution.
- It is therefore important to know whether these gradients significantly influence water flow.
- Solutes which are completely free to move within the soil cannot provide a driving force for water flow.
- On the other hand, solutes which cannot move can give rise to osmotic potential gradients which are as effective as matric or gravitational potential gradients in causing water to move.

Cont...

- The steady water flow equation is usually written as:

$$f_w = -K(d\psi_m/dz + d\psi_g/dz + \sigma_s d\psi_o/dz)$$

where  $k$  is the hydraulic conductivity and

$\sigma_s$  is a reflection coefficient.

- The reflection coefficient is a measure of the effectiveness of osmotic potential gradients in driving water flow.

- A vapor gas gives  $\sigma_s = 1$ , for low vapor pressure solutes
- Plant membranes generally have  $\sigma_s = 1$ .
- Soil, however, generally has as values close to zero.





# *CHAPTER FIVE*

## *GAS TRANSPORT IN SOIL*

*GAS TRANSPORT IN SOIL*

# AMOUNT & COMPOSITION OF SOIL AIR

- The gas phase of soil is characterized by its **amount/volume** and **composition**
- The **volume fraction of gas** generally decreases with **depth** in a soil profile **due to decreasing porosity** as a result of compaction, biological activity, etc. and due to increasing water content.
- The **volume fractions of gas and water** are each others **complement**; together they fill the available pore space.
- The critical volume fraction of gas for plant roots depends on the kind of plant and many other factors. A volume fraction of gas of 5 -15 % is generally considered adequate.

## Cont...

- The **composition of the soil air also varies with depth and time.**
- Due to free diffusion of the atmospheric components through the soil surface, the composition of the soil air near the surface is almost the same as in the atmosphere, **i.e. 80%  $N_2$ , with  $O_2$  and  $CO_2$  together forming the remaining 20%.**
- With depth, the concentration of  $CO_2$  increases and that of  $O_2$  decreases because of biological activity in the soil (respiration of plant roots, micro-organisms and soil fauna).

# EQUILIBRIUM CONDITIONS FOR SOIL AIR

- The water pressure increases proportionally with depth, b/c  $\rho_w$ , was uniform (incompressible medium).
- If density is dependent on pressure, the r/nship b/n pressure & height is more complex. Such is the case, for instance, in the atmosphere, b/c air is compressible.
- If the air in the atmosphere behaves as an **ideal gas** and temperature is constant with height, the r/nship b/n **air pressure** & **height** can be calculated for the equilibrium state.
- Then the total potential,  $\psi_t = \psi_g + \psi_p$ , is constant
- Hence:

$$\psi_t = gz + \int \frac{1}{\rho_a} dp$$

where  $\rho_a$  is the density of the air.

Cont...

- The r/nship b/n  $\rho_a$  and  $p$  can be derived from the **ideal gas law**:

$$pV = nRT$$

where  $p$  – pressure of air,  $V$  - volume,  $n$  - amount of gas (mol),  
 $R$  - molar gas constant and  $T$  - temperature (K).

$$\text{Also } \rho_a = \frac{nM}{V} = \frac{pM}{RT} \quad \text{where } M \text{ is molar mass of air}$$

$$\psi_t = gz + \int \frac{RT}{pM} dp = C_1 \quad \text{or} \quad gz + \frac{RT}{pM} \ln p = C_2 \quad \text{or}$$

$$\ln p = -\frac{M}{RT} gz + C_3 \quad \text{or} \quad p = C \exp\left(-\frac{M}{RT} gz\right) \quad \text{where } C_1, C_2, C_3 \text{ and } C \text{ are constants.}$$

Taking  $p = p_o$  at  $z = 0$  (sea level) leads to

$$p = p_o \exp\left(-\frac{M}{RT} gz\right) \rightarrow \text{pressure of air}$$

$p_o$  - pressure at sea level

Cont...

- If  $z = 1\text{m}$  &  $T = 290\text{K}$ , the above Eq<sup>n</sup> gives  **$p = 0.99988 p_o$** .
- Thus, air pressure changes by 0.012% over a height of 1m.
- If this small gradient ( $0.12 \text{ mbar m}^{-1}$ ) is ignored, air pressure can be regarded as independent of height (i.e.  $p$  is uniform).
- This assumption applies not only to the total gas pressure, but also to the partial pressures of the different gaseous components.
- Therefore, soil air and the atmosphere at the soil surface are in equilibrium with each other only if the total gas pressure and the composition of the soil air are the same as in the atmosphere.

# BULK FLOW OF GAS

- Bulk flow of gas in soil results from a gradient of **total gas pressure**. This can be caused, for instance, by
  - ✓ variations in barometric pressure of the atmosphere,
  - ✓ displacement of soil air by infiltrating rain or irrigation water,
  - ✓ rise of the groundwater table or temperature changes.
- We can ignore compression of soil air, because of the very small gas-pressure differences encountered in soils.
- The bulk flow of soil air can then be expressed analogously to the transport of soil water:

$$f_a = -K_a \frac{\partial p}{\partial s}$$

where  $f_a$  - is the air flux density in  $\text{m}^3 \text{m}^{-2} \text{s}^{-1}$ , or  $\text{m s}^{-1}$ ,

$\partial p / \partial s$  - is the air pressure gradient in  $\text{Pa m}^{-1}$

$K_a$  – is the air conductivity (has the same unit as the water  $\text{m}^2 \text{Pa}^{-1} \text{s}^{-1}$ )



## Cont...

- The air conductivity depends on the volume fraction, size, continuity and geometry of the gas-filled pores.
- The air conductivity of a completely dry soil is about 50 times the water conductivity of the same soil when completely saturated, provided the porosity and pore geometry of the soil are the same in both conditions. Of course, for most soils this is not true.
- The factor 50 is the ratio of the viscosities of water and air ( $\nu_w \approx 1.0 \text{ mPa s}$ ,  $\nu_a \approx 20 \text{ }\mu\text{Pa s}$ ).
- The intrinsic permeability of a soil for air,  $k_{ia}$ , can be calculated by an equation similar to that used for water:

$$k_{ia} = \nu_a K_a$$

The value of  $k_{ia}$ , depends only on the **geometry** of the soil pores.

# GAS DIFFUSION

- The composition of soil air changes due to consumption of  $O_2$ , and production of  $CO_2$ , as a result of respiration of soil organisms and plant roots.
- The volume fraction of  $CO_2$ , in soil air is more than the 0.03 % in the atmosphere. In general, the volume fractions of  $CO_2$ , and  $O_2$ , together are nearly constant at about 20%.
- Root growth and other activities of most plants are hindered if the volume fraction of  $CO_2$ , is more than about 5% ( $O_2$ , fraction < 15%), while the upper limit for plants is about 10%  $CO_2$ ,.
- Supply of  $CO_2$ , and removal of  $CO_2$ , by exchange with the atmosphere is called **Soil Aeration**.
- Since bulk flow of soil air contributes very little to the aeration of the root zone, exchange of  $O_2$ , and  $CO_2$ , with the atmosphere takes place mainly by **diffusion**.

- So, **Gas Diffusion** is the transport/movement of gas components (e.g.  $O_2$ ,  $CO_2$ ) only due to **gradients of their partial pressures** ( $\partial p/\partial s$ ).
- The driving force for gas diffusion of a component  $i$  can be represented by the **partial pressure gradient**  $\partial p_i/\partial s$ ,  
where  $p_i$  is the pressure due to gaseous component  $i$ .
- One can also express the driving force as a gradient of the **mass concentration,  $x_i$** , which is the mass of component  $i$  divided by volume of gas ( $kg\ m^{-3}$ ).

$$x_i = \frac{m_i}{V} = \frac{nM_i}{V} = \frac{M_i}{RT} p_i$$

$x_i$  is proportional to the partial pressure at low total gas pressures.

Cont...

- Using mass concentration gradient as driving force, the flux density equation for gas diffusion is:

$$f_i = -D_g \frac{\partial x_i}{\partial s}$$

This Equation is called **Fick's law**

where  $D_g$  is the diffusion coefficient for the particular gaseous component in the particular soil.

$D_g$  is expressed by:

$$D_g = \frac{1}{2} \eta_g D_o$$

where  $\eta_g$  - is the air-filled porosity

$D_o$  – is diffusion coefficient of gas in free air

- The flux density equation then becomes:

$$f_i = -\frac{1}{2} \eta_g D_o \frac{\partial x_i}{\partial s}$$

Cont...

- Gas diffusion in soil due to a **partial pressure gradient** of one component is always accompanied by **counter-current diffusion** of other components.

**Note:** Explain why **counter-current diffusion** takes place during diffusion of gases.

**Answer:** *If the pressure in a gas mixture is to remain constant, the increase in partial pressure of one component must be compensated by an equal decrease in partial pressure of other components. Thus, the partial pressure gradient of the one component is compensated by reverse gradients of other components. These gradients cause the other gas components to move (diffuse) in an opposite direction. That is **counter-current diffusion**.*

# *CO<sub>2</sub> DIFFUSION IN SOIL PROFILES*

- The CO<sub>2</sub> concentration in the gas phase of a soil generally increases with depth. This is one of the reasons why root development and activity is limited to the topsoil.
- Under average conditions in the root zone of natural soils, the production rate of CO<sub>2</sub>,  $\alpha_{CO_2}$ , varies between 0.2 and 0.6 mg m<sup>-3</sup>s<sup>-1</sup>.
- If the production rate and the diffusion coefficient of CO<sub>2</sub> for the soil are known, the CO<sub>2</sub> concentration as a function of depth can be calculated for relatively simple conditions,

Note: when the CO<sub>2</sub> concentration does not change with time (steady state) and the production rate is constant throughout the root zone and zero below the root zone.

## Cont...

- At steady state, the gradient of the CO<sub>2</sub> concentration at a certain depth must be such that all CO<sub>2</sub> produced below that depth is removed by upward diffusion.
- Thus, the CO<sub>2</sub> flux density is the highest at the soil surface.
- It is somewhat easier to solve the problem if a height scale is used, rather than a depth scale, and
- If  $z = 0$  is taken at the bottom of the root zone. The diffusion and continuity equations are then, respectively

$$f_{CO_2} = -D_g \frac{\partial x_{CO_2}}{\partial z} \text{ and } \frac{\partial G_{CO_2}}{\partial t} = -\frac{\partial f_{CO_2}}{\partial z} + \alpha_{CO_2}$$

Where  $G_{CO_2}$  - mass of CO<sub>2</sub> per volume of soil

- At steady state,  $\partial G / \partial t = 0$ , the above eq<sup>n</sup> can be simplified as:

$$\frac{df_{CO_2}}{dz} = \alpha_{CO_2} \text{ or } df_{CO_2} = \alpha_{CO_2} dz$$

Note that normal differentials can be used because, at steady state,  $f$  and  $x$  are no longer time-dependent, but only a function of height.



Cont...

- If a  $\text{CO}_2$  is assumed constant throughout the root zone, integration of above Equation yields:

$$f_{\text{CO}_2} = \alpha_{\text{CO}_2} z + C_1$$



$$f_{\text{CO}_2} = \alpha_{\text{CO}_2} z$$

where the integration constant,  $C_1$ , is zero b/c  $f_{\text{CO}_2} = 0$  @  $z = 0$ .

“ **Question:** Why is  $f_{\text{CO}_2} = 0$  at  $z = 0$  ?

**Answer:** There is no  $\text{CO}_2$ , production below  $z = 0$ . Hence, no  $\text{CO}_2$ , has to be removed from below  $z = 0$ . A downward flux also is unlikely, because there usually are layers in the subsoil which block gas diffusion, for instance a groundwater table or rocks. So, below the root zone,  $f_{\text{CO}_2} = 0$ . “

- Substitution of  $\alpha_{\text{CO}_2}$  for  $f_{\text{CO}_2}$  in diffusion yields:

$$\alpha_{\text{CO}_2} z = -D_g \frac{\partial x_{\text{CO}_2}}{\partial z} \quad \text{or} \quad \alpha_{\text{CO}_2} z dz = -D_g dx_{\text{CO}_2}$$

- If  $D_g$  is uniform, integration yields:

$$\alpha_{CO_2} z^2 = -2D_g x_{CO_2} + C_2$$

- The integration constant,  $C_2$ , can be found from the condition that  $x_{CO_2} = 0$  at the soil surface where  $z = z_s$  (because  $CO_2$  in the atmosphere is negligible):  $\Rightarrow C_2 = z_s^2 \alpha_{CO_2}$
- So ,

$$x_{CO_2} = \frac{\alpha_{CO_2}}{2D_g} (z_s^2 - z^2)$$

- Thus, the concentration of  $CO_2$  is a quadratic function of height

## Example

- For a soil with a production rate of  $\text{CO}_2$  of  $0.6 \text{ mg m}^{-3} \text{ s}^{-1}$ , a  $0.6 \text{ m}$  root zone and  $\eta_g = 0.3$ ,  $D_o = 2 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$

$$x_{\text{CO}_2} = 100 \text{ g m}^{-5} [ (0.6 \text{ m})^2 - z^2 ]$$

- At the lower end of the root zone (where  $z = 0$ ) and below the root zone, the concentration of  $\text{CO}_2$  is uniform at  **$36 \text{ g m}^{-3}$** .

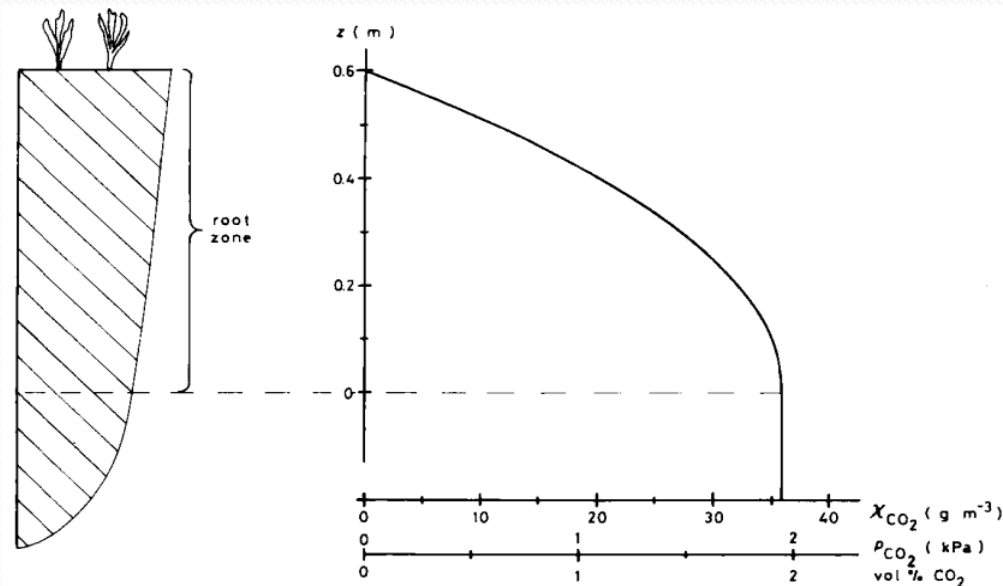


Figure: Concentration of  $\text{CO}_2$  in soil with constant and uniform production of  $\text{CO}_2$  and diffusion coefficient.

*Cont...*

- Mass concentrations of CO<sub>2</sub> can be converted to partial pressures by the ideal gas law

$$p_{CO_2} = \frac{RT}{M_{CO_2}} x_{CO_2}$$

- Using  $RT = 2.4 \text{ kJ mol}^{-1}$  and  $M_{CO_2} = 44 \text{ g mol}^{-1}$ ,  
⇒ So the partial pressure of CO<sub>2</sub> at the bottom of the root zone is about 1963.6 Pa.



## Assignments on Chapter Five

- 1) Explain why the mass concentration of  $\text{CO}_2$ , is uniform below the root.
- 2) Express equation  $x_{\text{CO}_2} = \frac{\alpha_{\text{CO}_2}}{2D_g} (z_s^2 - z^2)$  in terms of depth,  $s$ , if  $s = 0$  at the soil surface.
- 3) A soil profile has a root zone of 0.7 m. At steady state, the oxygen consumption is  $0.4 \text{ mg m}^{-3} \text{ s}^{-1}$  uniform throughout the root zone. Consumption of  $\text{O}_2$  below the root zone is negligible. Calculate the partial pressure of  $\text{O}_2$ , as a function of height with the following information:  $\eta_g = 0.32$ ,  $D_o = 2 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ ,  $RT = 2.4 \text{ kJ mol}^{-1}$ ,  $M_{\text{O}_2} = 32 \text{ g mol}^{-1}$  and volume fraction of  $\text{O}_2$  in the atmosphere = 20% (partial pressure = 20 kPa). .

# *CHAPTER SIX*

## HEAT TRANSPORT IN SOIL

## 6.1 INTRODUCTION

- An important aspect (characteristic) of the **behavior** and **use** of soils is their **thermal regime**.
- This regime is characterized to a large extent by the **temperature**.
- Many processes occurring in soils are strongly influenced by temperature. This is true especially for:
  - » **Biological processes**, such as germination of seeds, plant growth, root development and activity, microbial activity, etc.
  - » **Physical and chemical processes**, such as frost heaving, weathering, decomposition of organic matter, etc.
- **Soil temperatures are determined by:**
  - ⇒ The transport processes of heat within the soil and
  - ⇒ Exchange of heat between the soil and the atmosphere.



## Cont...

- There are basically **three** different processes whereby heat can be transported:
  - i. **Conduction**: conduction of heat occurs by the transmission of thermal energy of motion from one microscopic particle to another.
  - ii. **Convection**: is the transport of heat by a fluid in motion.
  - iii. **Radiation**: is transfer of thermal energy from a body to its surroundings by electromagnetic waves.
- Thus, in contrast to conduction and convection, radiation can occur through a vacuum.

## Cont...

- Transport of heat within soils can occur by conduction and by convection, with or w/t latent heat transport.
- Heat conduction is governed by the:
  - ✓ **Thermal soil properties:**
    - ⇒ Volumic heat capacity and
    - ⇒ Heat conductivity.
- The thermal exchange processes at the soil surface are dominated by the **meteorological conditions** and occur by **radiation**, **conduction** and **convection**, with or without phase changes/latent heat transport/.
- The thermal soil properties are strongly dependent on water content. /heat capacity of a soil is a linear function of the water content/

## 6.2 THERMAL SOIL PROPERTIES

### 6.2.1 Volumic Heat Capacity, $C_h$

- Is defined as the change of heat content of soil divided by volume and change of temperature. ( $\text{J m}^{-3} \text{K}^{-1}$ )
- The volumic heat capacity of a soil can be obtained by summing the contributions of the different soil components:

$$C_h = \sum_i C_{h,i} \eta_i = \sum_i \rho_i c_i \eta_i$$

Where:  $C_{h,i}$  - volumic heat capacities of component i  
 $c_i$  - specific heat capacities of component i,  
 $\eta_i$  - volume fraction of component i &  
 $\rho_i$  - density of component i.

Cont...

**Table:** Thermal properties of soil components at 10° C

|                                   | $c$<br>$\text{kJ kg}^{-1} \text{K}^{-1}$ | $C_h$<br>$\text{MJ m}^{-3} \text{K}^{-1}$ | $\lambda$<br>$\text{J m}^{-1} \text{s}^{-1} \text{K}^{-1}$<br>( $\text{W m}^{-1} \text{K}^{-1}$ ) |
|-----------------------------------|--|---|---|
| quartz                            | 0.76                                     | 2.0                                       | 8.8   |
| clay minerals                     | 0.73                                     | 2.0                                       | 2.9   |
| organic matter                    | 1.8                                      | 2.5                                       | 0.25  |
| water                             | 4.2                                      | 4.2                                       | 0.57  |
| ice (0°C)                         | 2.1                                      | 1.9                                       | 2.18  |
| air (saturated with water vapour) | 1.0                                      | 0.0013                                    | 0.025   |

### 6.2.2 Heat Conductivity, $\lambda$

- Is defined as **the heat flux density by conduction through the soil divided by the temperature gradient**. ( $\text{J m}^{-1} \text{s}^{-1} \text{K}^{-1}$ )
- The flux density equation for heat conduction in soils is:

$$f_h = -\lambda \frac{\partial T}{\partial s} \quad \leftarrow \text{Fourier's law}$$

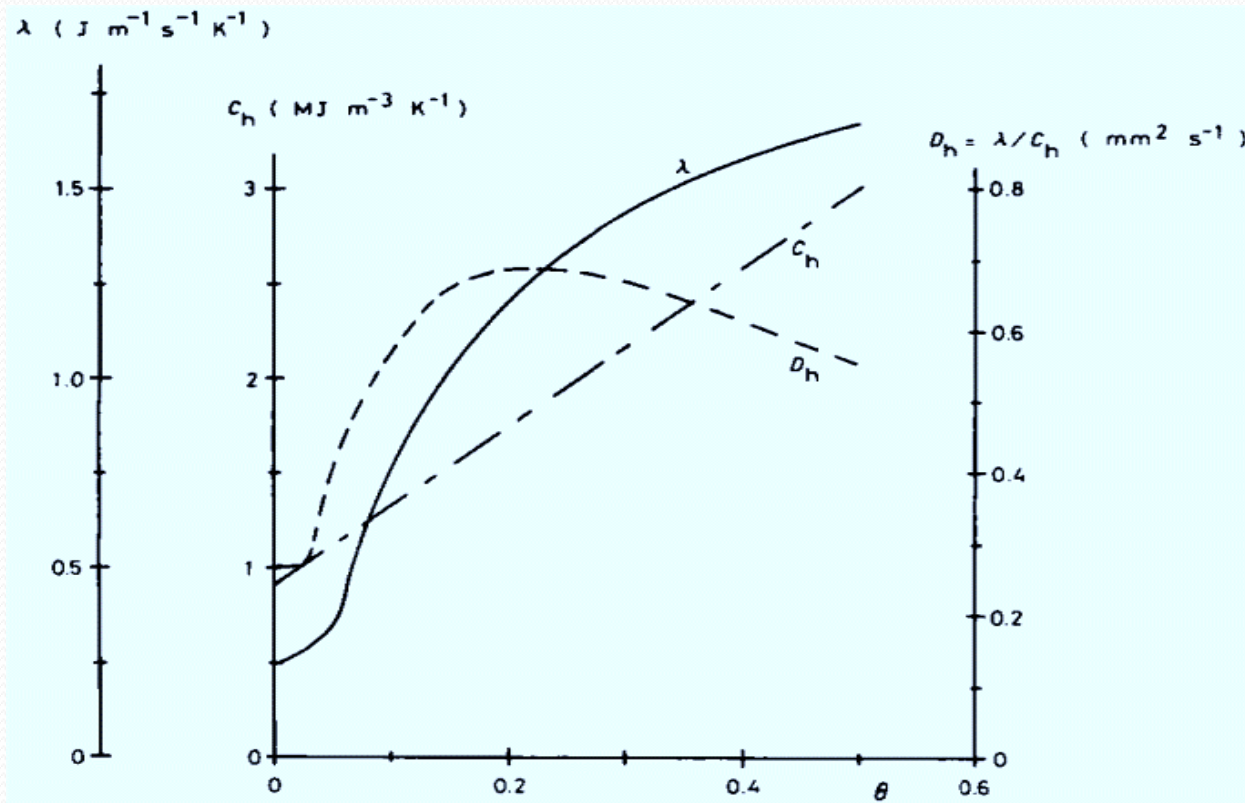
Where  $\lambda$  – transport coefficient/heat conductivity

$\partial T / \partial s$  - temperature gradient ( $\text{K m}^{-1}$ )

- The value of  $\lambda$  depends highly on the way in which the best conducting mineral particles are:
  - ✓ interconnected by the less conducting water phase and
  - ✓ separated by the poorly conducting gas phase.

Cont...

Figure below shows the **heat conductivity of a soil /Thermal soil properties/** as a function of volume fraction of liquid.



- At very low water contents,  $\lambda$  is generally smaller than  $0.5 \text{ J m}^{-1} \text{s}^{-1} \text{K}^{-1}$ . The heat transport then takes place mainly through the narrow points of contact b/n the soil particles. The contribution of the soil air is very small due to the very low heat conductivity of air.

- A small increase in water content of a dry soil causes only a modest increase of  $\lambda$ , b/c this water forms thin films around the soil particles.
- Further increases in water content cause a sharp increase of the heat conductivity, b/c water has a much higher heat conductivity than air and this water collects around the contact points b/n the soil particles.
- Water around contact points forms very effective '**bridges**' for conduction of heat.
- Upon further increase of the water content, the value of  $\lambda$  increases ever more gradually, b/c the conducting cross-sectional area of the water '**bridges**' increases ever more slowly.
- The **maximum** value of  $\lambda$  is reached at **water saturation**.



## 6.3 HEAT CONDUCTION IN DRY SOIL

- The general **one-dimensional heat conduction equation** is given as:

$$\frac{\partial T}{\partial t} = \frac{1}{C_h} \frac{\partial}{\partial s} \left( \lambda \frac{\partial T}{\partial s} \right)$$

This equation is valid if  $\lambda$  is independent of position, which is usually not true.

- Temperatures can be measured easily, as well as  $\lambda$  and  $C_h$ , since they are nearly independent of temperature. This makes heat conduction problems much easier to solve.
- If  **$\lambda$  is independent of  $s$ , and thus also of  $T$**  (homogeneous soils with uniform water content), the above equation can be simplified to:

$$\frac{\partial T}{\partial t} = \frac{\lambda}{C_h} \frac{\partial^2 T}{\partial s^2} = D_h \frac{\partial^2 T}{\partial s^2}$$

Where:  **$D_h$  is the heat diffusivity** = is a measure for the rate at which an imposed temperature gradient in the soil is dissipated by conduction.

### 6.3.1 Steady Heat Conduction In Dry Soil

- With steady heat conduction the temperature, per definition, does not change in time, i.e.

$$\frac{\partial T}{\partial t} = 0$$

- Since  $\lambda$  and  $C_h$  are constant and non-zero for dry soil. The above general eq<sup>n</sup> reduced to:

$$\frac{d}{ds} \left( \frac{dT}{ds} \right) = 0$$

- Integrating twice yields:  **$T = C_1 s + C_2$**
- The integration constants,  $C_1$  and  $C_2$ , can be determined if the temperature is known for two values of  $s$ .

## Example 1

**Question:** In a steady situation the temperature in a homogeneous dry soil profile is 30°C at the soil surface ( $z = 0$ ) and 15°C at a depth of 1 m ( $z = -1$  m).

- Express the temperature as a function of height.
- Calculate the temperature gradient.

## Solution

- To express the temperature as a function of height. Substitute  $z$  for  $s$  in the equation.  $T = C_1 z + C_2$

Since  $T = 30^\circ\text{C} = 303$  K at  $z = 0$ ,  $C_2 = 303$  K and  $T = C_1 z + 303$  K.

Substitution of  $T = 15^\circ\text{C} = 288$  K and  $z = -1$  m gives:

$$288\text{K} = -1 \text{ m} \times C_1 + 303 \text{ K} \quad \text{or} \quad C_1 = 15 \text{ K m}^{-1}.$$

Thus  **$T = 15 \text{ K m}^{-1} \times z + 303 \text{ K}$ .**

- The temperature gradient is:

$$\frac{dT}{dz} = C_1 = 15 \text{ K m}^{-1}$$

### 6.3.2 Cyclic Variation of Surface Temperature

- In nature, soil temperatures fluctuate due to more or less **cyclic variations** of the surface temperature,  $T_{O,t}$ .
- These variations are the diurnal (day-night) and the annual (summer-winter) cycles.
- Both can be approximated by a sinus function (Figure below).
- The temperature at depth  $s$  and time  $t$ ,  $T_{S,t}$  expressed as:

$$T_{S,t} = T_{avg} + A_o \exp(-s/d) \sin(\omega t - s/d)$$

Where:  $A_o$  - amplitude of the sinusoidal temperature variation at the soil surface,

$\omega$  – angular frequency of the temperature variation (wave),  
 $t$  - time and  $d$  - the damping depth.

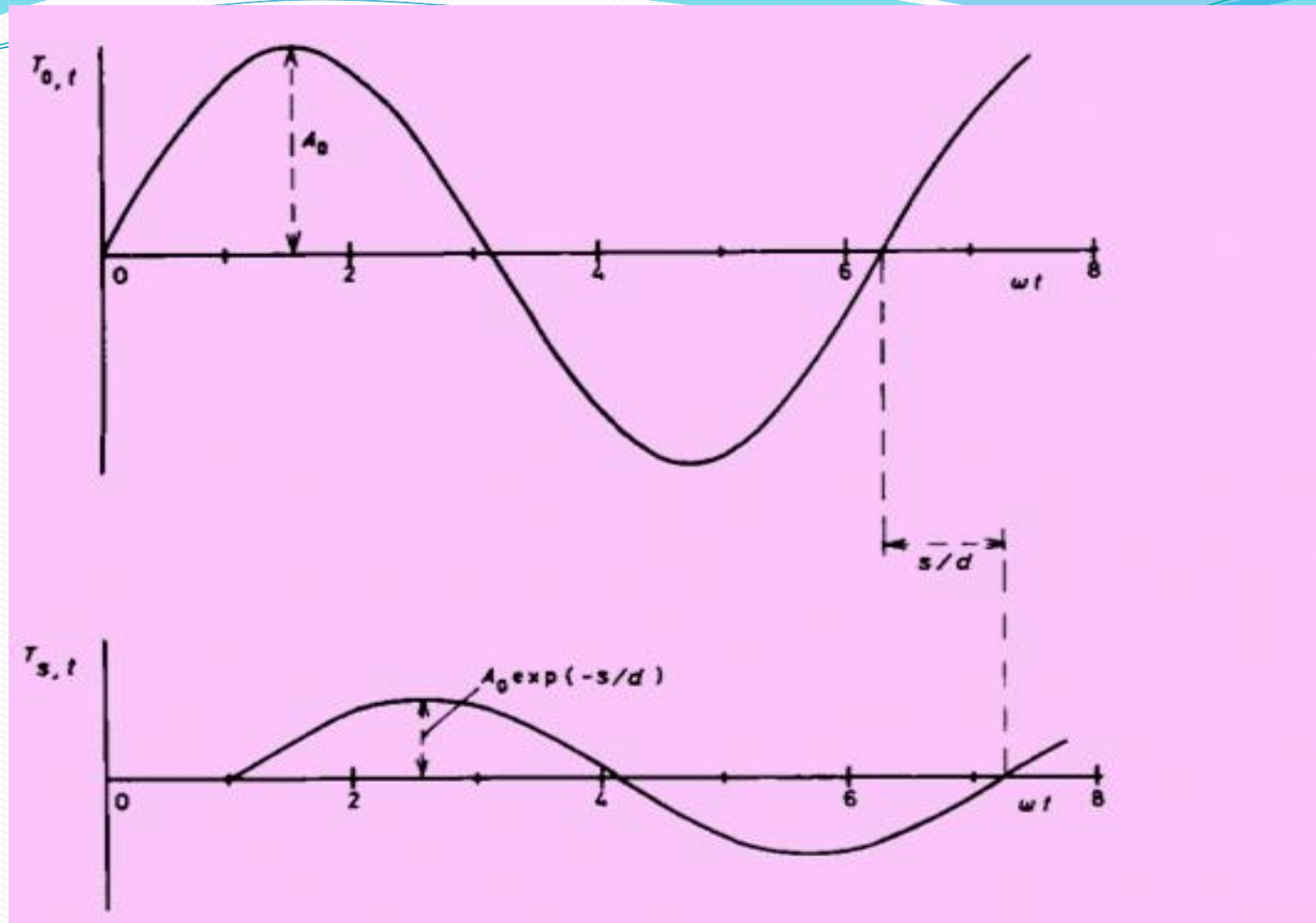


Figure: Cyclic variation of temperature at the soil surface and at depth  $s = d$  .

- The **angular frequency** is:  $\omega = \frac{2\pi}{t_c}$

where  $t_c$  is the time needed to complete one cycle of the wave

- The **damping depth** depends on the thermal properties of the soil and on the angular frequency of the temperature variation according to:

$$d = \sqrt{\frac{2\lambda}{\omega C_h}} = \sqrt{\frac{2D_h}{\omega}}$$

- The **finite heat diffusivity** of the soil also causes a **phase shift** b/n the heat wave at the soil surface and at a **certain depth  $s$** . /due to the finite heat diffusivity of the soil, the amplitude of the heat wave decreases with soil depth, (Figure above). In the  $T_{s,t}$  equation, this phenomenon is accounted for by the introduction of the so-called **damping depth**, which is the depth where the amplitude of the temperature fluctuation has decreased (at  $s = d$  is  $A_0/e \approx 0.37 \times A_0$ ).
- The phase difference is represented in above eq<sup>n</sup> by the variable -  **$s/d$**  in the sinus function ( **$-s/d$  is the phase angle**).

Cont...

## Example 2

**Question:** Make a graph of the temperature at the soil surface,  $T_{0,t}$ , as function of time, if  $T_{avg} = 20^\circ\text{C}$ ,  $A_0 = 10^\circ\text{C}$  and  $\omega = 2\pi/24$  radians per hour ( $t_c = 24\text{h}$ ).

### Solution

- For the soil surface, where  $s = 0$ , the  $T_{s,t}$  equation reduces to:

$$T_{S,t} = T_{avg} + A_o \sin(\omega t) = 20^\circ\text{C} + 10^\circ\text{C} \sin\left(\frac{\pi t}{12 h}\right)$$

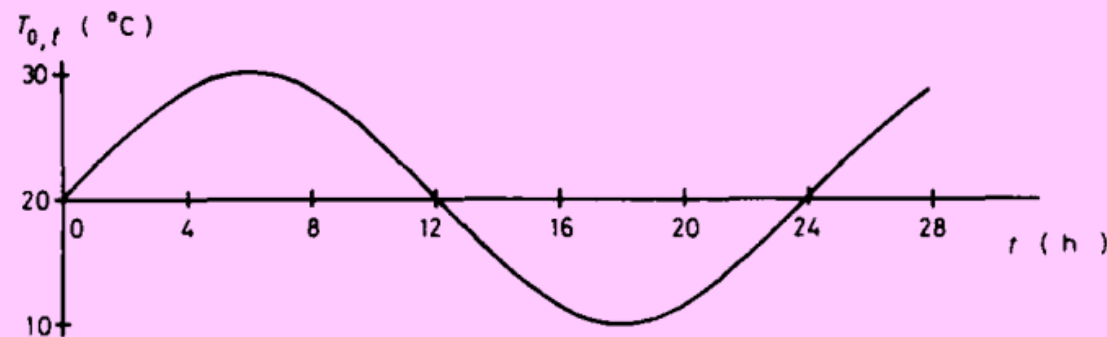
$$t = 0 \rightarrow T_{0,0} = (20 + 10 \sin 0)^\circ\text{C} = 20^\circ\text{C}$$

$$t = 6 \text{ h} \rightarrow T_{0,6} = (20 + 10 \sin \frac{\pi}{2})^\circ\text{C} = 30^\circ\text{C}$$

$$t = 12 \text{ h} \rightarrow T_{0,12} = (20 + 10 \sin \pi)^\circ\text{C} = 20^\circ\text{C}$$

$$t = 18 \text{ h} \rightarrow T_{0,18} = (20 + 10 \sin \frac{3\pi}{2})^\circ\text{C} = 10^\circ\text{C}$$

$$t = 24 \text{ h} \rightarrow T_{0,24} = (20 + 10 \sin 2\pi)^\circ\text{C} = 20^\circ\text{C}$$





### Example 3

**Question:** For a soil that have the following thermal properties  $\lambda = 0.6 \text{ J m}^{-1} \text{ s}^{-1} \text{ K}^{-1}$  and  $C_h = 1.6 \text{ MJ m}^{-3} \text{ K}^{-1}$ , Calculate the damping depth for a diurnal and for an annual temperature fluctuation

### Solution

a) For the daily fluctuation,  $\omega = \frac{2\pi}{t_c} = \frac{2\pi}{24 \times 3600} \text{ s}^{-1}$

$$\text{Thus, } d = \sqrt{\frac{2\lambda}{\omega C_h}} = \sqrt{\frac{2 \times 0.6 \times 24 \times 3600}{2\pi \times 1.6 \times 10^6}} \text{ m}^2 = 0.10 \text{ m}$$

b) For the annual wave,  $\omega = \frac{2\pi}{t_c} = \frac{2\pi}{365 \times 24 \times 3600} \text{ s}^{-1}$

$$\text{Thus, } d = \sqrt{\frac{2\lambda}{\omega C_h}} = \sqrt{\frac{2 \times 0.6 \times 365 \times 24 \times 3600}{2\pi \times 1.6 \times 10^6}} \text{ m}^2 = 1.94 \text{ m}$$

## Example 4

### Question:

- a) Calculate the amplitude of the daily temperature fluctuation at 0.3 m depth in the soil above (example 3a), if the amplitude at the soil surface is,  $A_o = 10$  K.
- b) Make a graph of the temperature wave and compare it with the graph in example 2 (take  $T_{avg} = 20^\circ\text{C}$ ).

### Solution

- a) At 0.3 m depth the amplitude equals:

$$A_o \exp(-s/d) = 10 \text{ K} \exp(-0.3/0.1) \approx 0.5 \text{ K}$$

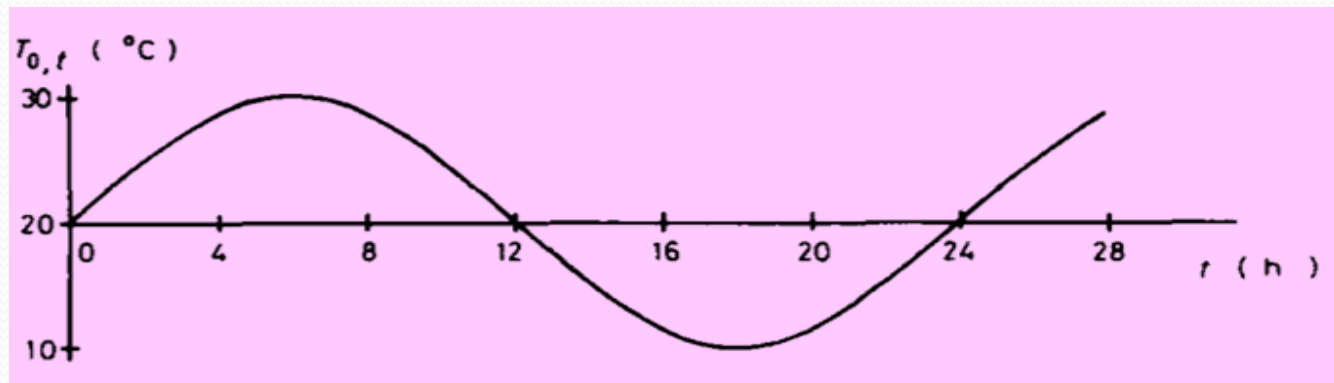
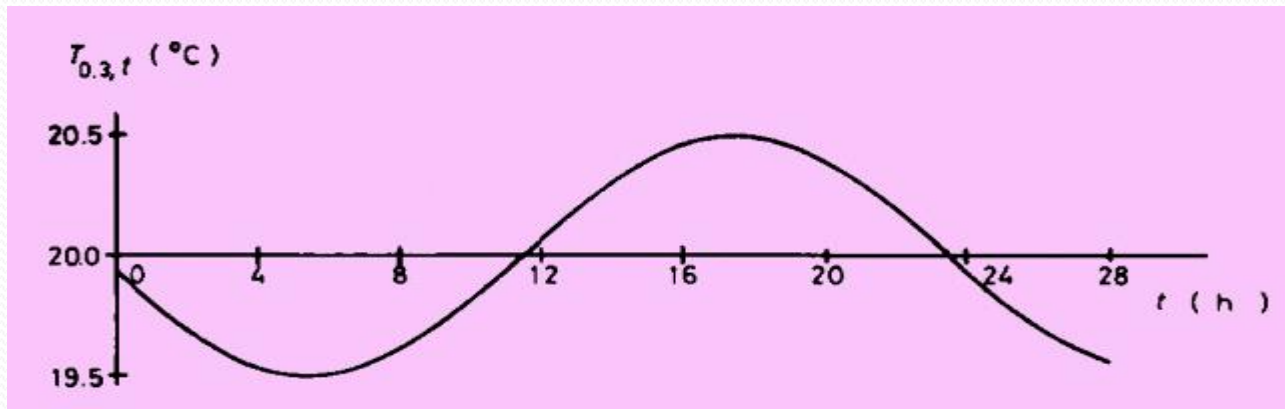
- b) The phase shift with the soil surface equals:

$$-s/d = -0.3/0.1 = -3 \text{ rad} \approx -172^\circ$$

Cont...

### Solution example 4 cont...

A comparison with example 2 shows that the temperature at 0.3 m depth attains its minimum value when the surface temperature is near its maximum value.



## 6.4 THE HEAT BALANCE OF THE SOIL SURFACE

- As mentioned above the temperature at the soil surface varies **periodically**.
- During daylight, most of the incoming direct and indirect short-wave radiation is absorbed by the soil surface, the remainder is reflected back into the atmosphere.
- During darkness as well as daylight, the soil surface loses energy by emitting long-wave heat radiation.
- The **net radiation flux density**,  $f_n$  ( $\text{J m}^{-2} \text{s}^{-1}$ ): is the difference between the unreflected incoming short-wave radiation and the outgoing long-wave heat radiation.
- Normally, during the day  $f_n > 0$ , and at night  $f_n < 0$ .

- The net radiation flux density is used for:
  - ✓ evaporation of water at the soil surface ( $f_e$ )
  - ✓ transpiration of water by the vegetative cover ( $f_t$ )
  - ✓ heating of the soil ( $f_s$ )
  - ✓ heating of the atmospheric air by conduction at the soil surface ( $f_a$ ).
- ❖ The heat balance of the soil can thus be described by:

$$f_n = f_e + f_t + f_s + f_a$$

- The distribution of  $f_n$  over  $f_e$ ,  $f_t$ ,  $f_s$ , and  $f_a$  is important, because  $f_e$  and  $f_t$  determine the amount of water that can be evaporated, where as  $f_s$  determines the soil temperature and, thus, the growth of crops.

## Example 5

**Question:** The net radiation flux density at a bare, dry soil surface ( $C_h = 0.8 \text{ MJ m}^{-3} \text{ K}^{-1}$ ) is  $20 \text{ W m}^{-2}$ .

Assuming  $f_s/f_a = 1$ , calculate:

- a) The thickness of the surface layer in which the temperature is raised, on average, 1 K per hour;
- b) The thickness of the layer of air ( $C_h = 1.3 \text{ kJ m}^{-3} \text{ K}^{-1}$ ) above the soil surface that can be heated, on average, 1 K per hour.

## Solution

At a bare, dry soil surface,  $f_e = f_t = 0$ . Therefore,  $f_n = f_s + f_a = 20 \text{ W m}^{-2}$ . Assuming  $f_s/f_a = 1$ ,  $f_s = f_a = 10 \text{ W m}^{-2} = 10 \text{ J m}^{-2} \text{ s}^{-1}$ .

- a) The flux density for heating the soil in one hour is ,  $f_s = 36000 \text{ J m}^{-2} \text{ h}^{-1}$

From the definition of differential heat capacity, this radiation flux density heats the soil according to:

$$\frac{f_s}{C_h} = \frac{36000 \text{ J m}^{-2} \text{ h}^{-1}}{0.8 \times 10^6 \text{ J m}^{-3} \text{ K}^{-1}} = 4.5 \times 10^{-2} \text{ m K h}^{-1}$$

Thus, the soil layer heated 1 K in one hour is  **$4.5 \times 10^{-2} \text{ m}$**  or  **$4.5 \text{ cm}$** .

- b) Similarly, for air,  $f_a = 36000 \text{ J m}^{-2} \text{ h}^{-1}$  :

$$\frac{f_a}{C_h} = \frac{36000 \text{ J m}^{-2} \text{ h}^{-1}}{1.3 \times 10^3 \text{ J m}^{-3} \text{ K}^{-1}} = 27.7 \text{ m K h}^{-1}$$

Thus, the air layer heated 1 K in one hour is  **$27.7 \text{ m}$**  thick.



## 6.5 COMBINED HEAT AND WATER VAPOUR TRANSPORT

### 6.5.1 Isothermal Water Vapour Diffusion

- Water vapour is one of the components of the gas phase of the soil.
- The principal transport mechanism of water vapour in soil is diffusion.

- So the water vapour flux density by diffusion,  $f_v$ , according to Fick's law is:

$$f_v = -D_g \frac{\partial x_v}{\partial s}$$

- The diffusion coefficient,  $D_g$  is the corrected form of the molecular diffusion coefficient of water vapour in free air,  $D_o$ :

$$D_g \approx \frac{1}{2} \eta_g D_o$$

- The concentration gradient,  $(\partial x_v / \partial s)$  can be replaced by the vapour pressure gradient according to the ideal gas law

## Exercise

- 1) Explain why much incoming radiation is required to heat up the surface layer of a wet soil.
- 2) Does water vapour diffusion take place in the direction of higher or lower temperatures?
- 3) What is the vapour pressure gradient induced by a temperature gradient of  $1 \text{ K cm}^{-1}$  at  $15^\circ\text{C}$ ?
- 4) What is the concentration gradient of the water vapour for the pressure gradient of Answer 3? ( $RT \approx 2.4 \text{ kJ mol}^{-1}$ ).
- 5) Calculate the water vapour flux density resulting from the concentration gradient of Answer 4, if  $D_o \approx 2 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$  and  $\eta_g = 0.30$ .
- 6) Convert the water vapour flux density in Answer 5 to an equivalent liquid water flux density.

# Reading Assignment

- 1) Evaporation
- 2) Evapotranspiration

# *CHAPTER SEVEN*

## **WATER AND ENERGY BALANCES AT SOIL – PLANT – ATMOSPHERE INTERFACES**

## 7.1 INTRODUCTION

- Energy fluxes at soil-atmosphere and plant-atmosphere interfaces can be summed to **zero** because the surfaces have no capacity for energy storage.
- The resulting energy balance equations may be written in terms of *physical descriptions of these fluxes*; and
- This equations have been the basis for *problem casting* and *solving* in diverse fields of environmental and agricultural science such as
  - ✓ Estimation of evapotranspiration (ET) from plant canopies,
  - ✓ Estimation of evaporation from bare soil,
  - ✓ Rate of soil heating in spring (important for timing of seed germination),
  - ✓ Rate of residue decomposition (dependent on temperature and water content at the soil surface) and many others.

## 7.2 ENERGY BALANCE EQUATION

- The surface energy balance is:

$$0 = R_n + G + LE + H$$

where  $R_n$  - net radiation;  $G$  - soil heat flux;

$LE$  - latent heat flux (evaporation to the atmosphere) and is the product of the *evaporative flux*,  $E$ , and the *latent heat of vaporization*,  $L$ ; and

$H$  - sensible heat flux (all terms taken as positive when flux is toward the surface)

- Each term may be expressed more completely as the sum of sub terms that describe specific physical processes, some of which are shown in Fig. 7.1.

- **Net radiation,  $R_n$** : which includes :-
  - » *Absorption* and *Reflection* of shortwave radiation (sunlight,  $R_{si}$  & the reflected portion  $\alpha R_{si}(R_{sr})$ ), as well as
  - » *Emission* and *Reception* of long-wave radiation ( $L\uparrow$  and  $L\downarrow$ , respectively, Figure 7.1)
- **Soil heat flux**: which involves
  - » *Diffusion\_of\_heat,  $G$* , as expressed by Fourier's law,
  - » *Convective heat flux,  $G_{J_w}$* , which is the flow rate of heat when water at temp.  $T$  flows at rate  $J_w$  into soil at another temp.  $T'$ .
- Both evaporation from the soil and from plants are examples of **latent heat flux,  $LE$**  but so also is *dew formation*, whether it wets the soil surface or plant canopy.
- **Sensible heat flux,  $H$**  may occur b/n soil & atmosphere or b/n plant & atmosphere, & may be short circuited b/n soil & plant,
- Values of these energy fluxes change **diurnally** and **seasonally**.



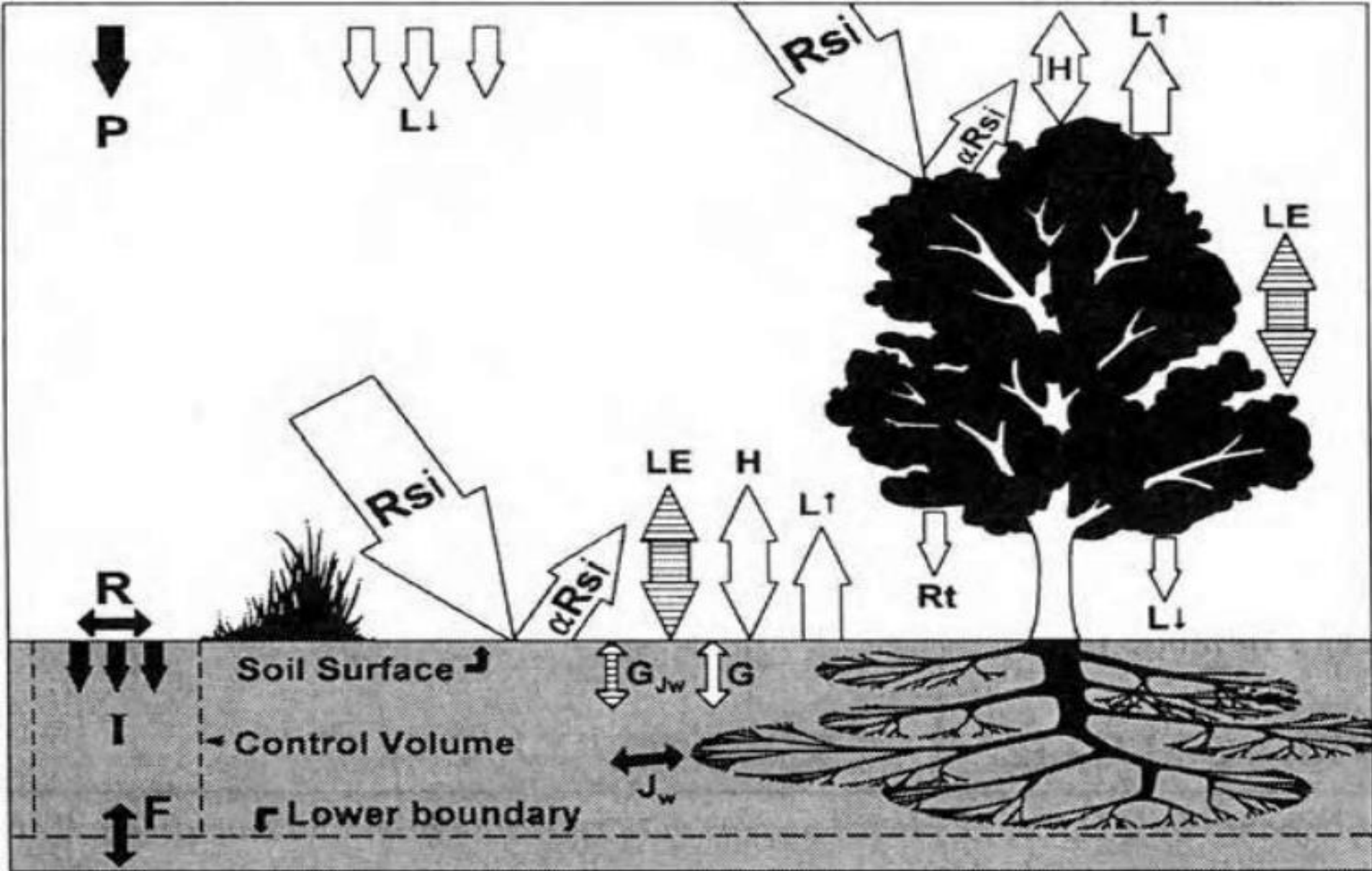


Figure 7.1 Water and energy balance components. Water balance components are in black, energy balance in white. The shared term  $LE$  is shaded.

## 7.2.1 Net Radiation

- Net radiation is the sum of incoming and outgoing radiation:

$$R_n = R_{si}(1 - \alpha) + (L_{\downarrow} - L_{\uparrow}) = R_{si}(1 - \alpha) + (L_{\downarrow} - \epsilon\sigma T^4)$$

where  $R_{si}$  - solar irradiance at the surface,

$\alpha$  - the albedo or surface reflectance (0 to 1),

$\epsilon$  - the surface emissivity (0 to 1),

$\sigma$  - the Stefan-Boltzmann constant ( $5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ ),

$T$  - surface temperature (K) and

$L_{\downarrow}$  - long-wave irradiance from the sky

- The sun radiates energy like a black body at about 6000K while the earth radiates at about 285K.

**Note that:** the radiance of the earth is about 4 million times lower than that of the sun.

- Net radiation may be measured by a net radiometer

## Cont...

- Its components may be measured separately using *pyranometers* to measure *incoming* and *reflected* short wave radiation, and *pyrgeometers* to measure *incoming* and *outgoing* long wave radiation.

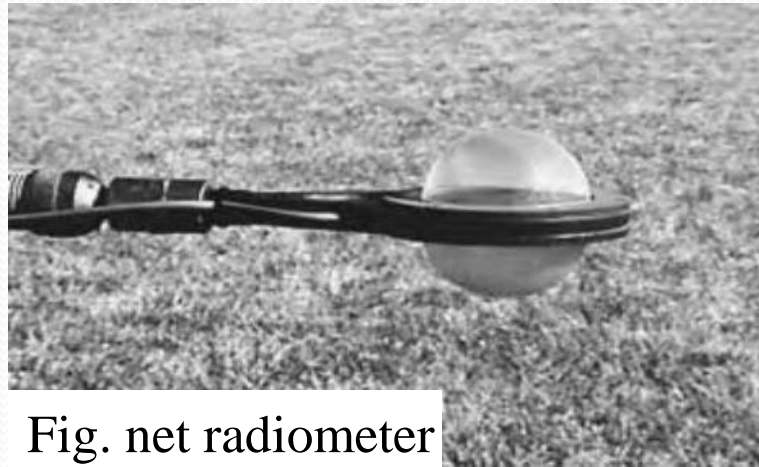


Fig. net radiometer

- The **emitted/outgoing longwave radiance** of the earth's surface,  $L_{\uparrow}$  is given by the *Stefan-Boltzmann law* for radiance from a surface at *temperature*  $T$  & with *emissivity*  $\epsilon$ :

$$L_{\uparrow} = \epsilon \sigma T^4$$

- Surface temperature,  $T$  is often measured by suitably placed & shielded *thermocouples*, or by *infrared-thermometer* (IRT).

**Incoming Long-Wave Radiation** from the sky,  $L_{\downarrow}$ : can be estimated from:

$$L_{\downarrow} = \varepsilon \sigma (T_a + 273.16)^4$$

where  $T_a$  ( $^{\circ}\text{C}$ ) is air temperature at the reference measurement level (often 2 m), and

$\varepsilon$  is the emissivity - may be estimated from the vapor pressure of water in air at reference level ( $e_a$ ) (kPa), or using both  $e_a$  and  $T_a$ .

- The vapor pressure is:  $e_a = \text{RH}(e_s)$

where RH is the relative humidity of the air and

$e_s$  is the saturation vapor pressure (kPa) at  $T_a$  ( $^{\circ}\text{C}$ ) given by (Murray, 1967):

$$e_s = 0.61078 \exp\left(\frac{17.269T_a}{237.3 + T_a}\right)$$

- If the dew point temperature,  $T_{\text{dew}}$  rather than the RH, is known, then:

$$e_a = 0.61078 \exp\left(\frac{17.269T_{\text{dew}}}{237.3 + T_{\text{dew}}}\right)$$

- Emissivity,  $\varepsilon$  is estimated from the following equation:

- » Idso's (1981) equation:

$$\varepsilon = 0.70 + 5.95 \times 10^{-4} e_a \exp\left(\frac{1500}{273.1 + T_a}\right)$$

- » Brunt's (1932) equation:

$$\varepsilon = 0.52 + 0.206 e_a^{0.5}$$

- » Brutsaert's (1982) equation:

$$\varepsilon = 0.767 e_a^{1/7}$$

**Note:** All three equations gave good predictions for *clear sky conditions*, but probably underestimated  $\varepsilon$  for *cloudy* and *nighttime* conditions.

- ✓ For regressions of predicted versus measured  $\varepsilon$ , the Idso's equation gave a slightly higher correlation coefficient and a slope closer to unity.
- ✓ Under *heavy clouds*, sky emissivity approaches *unity*

**Solar irradiance ( $R_{si}$ ):** is defined as the radiant energy reaching a horizontal plane at the earth's surface, includes both direct beam and diffuse shortwave.

- It may be easily measured by **pyranometer**

Note that: net radiation was negative at night.



## 7.2.2 Latent Heat Flux, $\mathcal{LE}$

- Latent heat flux is the product of the *evaporative flux*,  $E$  ( $\text{kg s}^{-1} \text{m}^{-2}$ ), & the *latent heat of evaporation*,  $L$  ( $2.44 \times 10^6 \text{ J kg}^{-1}$  at  $25^\circ\text{C}$ ).
- The value of  $L$  is temperature dependent, but is well described (in  $\text{J kg}^{-1} \times 10^6$ ) by:

$$L = 2.501 - 2.370 \times 10^{-3}T \quad \text{where } T \text{ is in } ^\circ\text{C}$$

- The evaporative flux according to *Rosenberg et al.*, 1983:

$$E = \left( \frac{M_w}{M_a P} \right) \rho_a \overline{w' e_a'}$$

where the overscores indicate time averages of vertical wind speed ( $w'$ ) and vapor pressure ( $e_a'$ ), the primes indicate instantaneous deviations from the mean,

$P$  is atmospheric pressure ( $P_{\text{atm}}$ ),  $\rho_a$  is air density, and  $M_w$  and  $M_a$  are the molecular weights of water and air.



## 7.2.2.1 Penman-Monteith Estimates of Latent Heat Flux, LE

- The Penman-Monteith equation for the estimation of latent heat flux is given by:

$$LE = \frac{\Delta(R_n + G) + \rho_a c_p (e_s - e_a) / r_a}{\Delta + \gamma(1 + r_s / r_a)}$$

where LE - latent heat flux,  $R_n$  - net radiation, and G - soil heat flux (all in  $\text{MJ m}^{-2} \text{s}^{-1}$ );

$\Delta$  - the slope of the saturation vapor pressure-temperature curve ( $\text{kPa } ^\circ\text{C}^{-1}$ ),  $\rho_a$  - air density ( $\text{kg m}^{-3}$ ),

$c_p$  - the specific heat of air ( $\text{kJ kg}^{-1} ^\circ\text{C}^{-1}$ ),

$e_a$  - vapor pressure of the air at reference measurement height z,

$e_s$  - the saturated vapor pressure at a dew point temperature equal to the air temperature at z ( $\text{kPa}$ ),

$(e_s - e_a)$  - the vapor pressure deficit,

$r_a$  - the aerodynamic resistance ( $\text{s/m}$ ),

$r_s$  - the surface (canopy) resistance ( $\text{s/m}$ ), and

$\gamma$  - the psychrometric constant ( $\text{kPa } ^\circ\text{C}^{-1}$ ).

- The values of  $r_a$  and  $r_s$  may be difficult to obtain.
- The surface(canopy) resistance is known for only a few crops and is dependent on *plant height, leaf area, irradiance and water status of the plants.*
- Aerodynamic resistance was estimated for neutral atmospheric conditions from:

$$r_a = \frac{\ln\left(\frac{z_m - d}{z_{0m}}\right) \ln\left(\frac{z_H - d}{z_{0H}}\right)}{k^2 u_z}$$

where  $z_m$  (m) is the measurement height for wind speed,  $u_z$ , (m/s),  
 $z_H$  (m) is measurement height for air temp. & relative humidity,  
 $z_{0m}$  and  $z_{0H}$  are the roughness length parameters for momentum  
 (wind) and sensible heat transport,  
 $d$  is the zero plane displacement height and  
 $k$  is von Karman constant = 0.41,

- Surface resistance,  $r_s$  was calculated from:

$$r_s = \frac{r_1}{0.5LAI}$$

where  $r_1$  - the stomatal resistance taken as 100 s/m, and  
LAI - the leaf area index, is given by:

$$LAI = 5.5 + 1.5 \ln(h_c)$$

where  $h_c$  - the crop height, was taken as 0.12cm  
for grass, and 0.5m for alfalfa.

- The zero plane displacement height,  $d$  is calculated as:

$$d = \frac{2}{3} h_c$$

- The roughness length for momentum,  $z_{0m}$ , is calculated as:

$$z_{0m} = 0.123 h_c$$

- The roughness length for sensible heat transport is:

$$z_{0H} = 0.1 z_{0m}$$

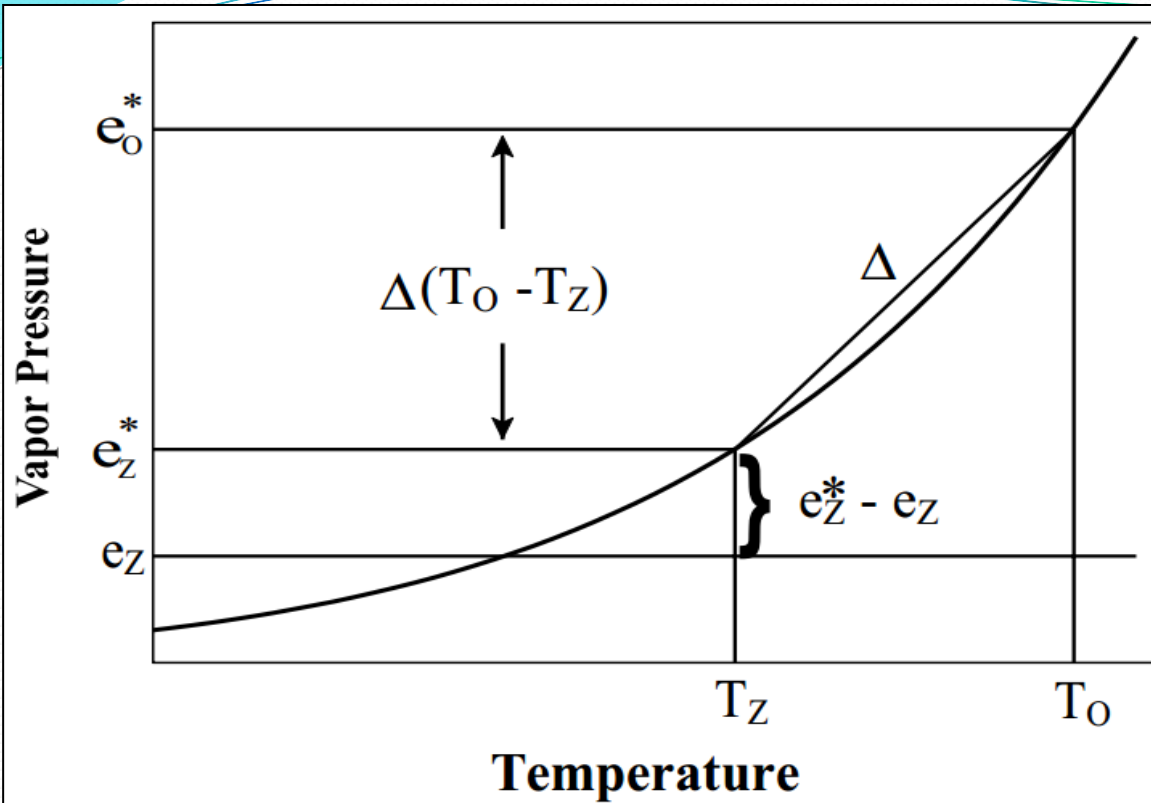
- Penman (1948) restated the above developed Eq<sup>n</sup> for a *wet surface*, and substituted transport mechanisms for LE and H to give a combination equation:

$$LE = f(u)(e_z - e_o^*) = -[R_n + G + \gamma f(u)(T_z - T_o)]$$

where  $f(u)$  is a wind speed dependent conductance or transport coefficient,  $\gamma$  is the psychrometric constant ( $c_p P / (0.622 L)$ ),  $e_o^*$  is the saturation vapor pressure at the surface temperature,  $e_z$  is the air vapor pressure at measurement height  $z$ ,  $T_z$  is the air temperature at measurement height,  $T_o$  is the surface temperature,

- Because of surface temperature is difficult to measure, Penman (1948) introduced an approximation for  $(T_o - T_z)$  that is derived from the *slope of the saturation vapor pressure vs. temperature curve*.

Cont...



$$\Delta = (e_o^* - e_z^*) / (T_o - T_z)$$

$$e_o^* - e_z = e_z^* - e_z + \Delta(T_o - T_z)$$

Rearranging to find  $(T_z - T_o)$  and substituting into the above developed LE Eq<sup>n</sup> gives:

$$f(u)(e_z - e_o^*) = -[R_n + G + (\gamma/\Delta)f(u)(e_z - e_o^*) + (\gamma/\Delta)f(u)(e_z^* - e_z)]$$

Finally by rearranging, LE can be written as:

$$LE = -[\Delta(R_n + G + \gamma)f(u)(e_z^* - e_z) / (\Delta + \gamma)]$$

w/c is the Penman eq<sup>n</sup> & is analogous to the previous Eq<sup>n</sup>

### 7.2.3 Soil Heat Flux, G

- Heat conduction in one dimension is described by a diffusion equation:

$$C_h \frac{\partial T}{\partial t} = \lambda \frac{\partial}{\partial z} \left( \frac{\partial T}{\partial z} \right)$$

where the volumetric heat capacity,  $C_h$  ( $\text{J m}^{-3} \text{K}^{-1}$ ), and the thermal conductivity,  $\lambda$  ( $\text{J s}^{-1} \text{m}^{-1} \text{K}^{-1}$ ), are assumed constant in space, and vertical distance is denoted by  $z$ , time by  $t$ , and temperature by  $T$ .

- The one-dimensional soil heat flux ( $G$ ) for a homogeneous medium is described by:

$$G = -\lambda \frac{\partial T}{\partial z}$$

- The thermal conductivity is a single-valued function of water content and is related to the thermal diffusivity,  $D_T$  ( $\text{m}^2 \text{s}^{-1}$ ), by:

$$\lambda = D_T C_h$$

where

$$C_h = \frac{2.0 \times 10^6 \rho_b}{2.65} + 4.2 \times 10^6 \theta_v + 2.5 \times 10^6 f_o$$

$f_o$  - a volume fraction of organic matter

### 7.2.3 Sensible Heat Flux, H

- Sensible heat flux is the *transfer of heat away from or to the surface by conduction or convection*.
- Because air is not a very good conductor of heat, most sensible heat flux is by convection(movement) of air.
- Perhaps the most common method of evaluating *sensible heat flux* is to measure the other terms in surface energy balance eq<sup>n</sup> as accurately as possible and then set H equal to the residual:

$$H = -R_n - G - LE$$

- Or directly determined from *eddy correlation* as:

$$H = \overline{\rho_a} c_p \overline{w' T'}$$

where the overbars denote short time averages of air density ( $\rho_a$ ), vertical wind speed ( $w'$ ), and air temperature ( $T'$ ), measured at some height within the constant flux layer.



## Cont...

- The *Bowen ratio method* can be applied to *sensible heat flux* as well as to *latent heat flux*. Which is the
- The Bowen ratio,  $\beta$  is the ratio of sensible to latent heat flux:

$$\beta = H/LE$$

- For sensible heat flux, the Bowen ratio is (Rosenberg et al., 1983):

$$H = -\frac{(R_n + G)}{(1 + 1/\beta)}$$

- Sensible heat flux sometimes estimated using a straightforward resistance equation:

$$H = \frac{\rho_a c_p (T_z + T_0)}{r_{aH}}$$

where  $\rho_a$  is the density of air ( $\rho_a = 1.291 - 0.00418T_a$ , the  $-5$  to  $40^\circ\text{C}$  range,  $T_a$  in  $^\circ\text{C}$ ),  $c_p$  is the heat capacity of air ( $1.013 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$ ),  $T_z$  is the air temp. at measurement height,  $r_{aH}$  is the aerodynamic resistance to sensible heat flux ( $\text{s m}^{-1}$ ), and  $T_0$  is the temperature of the surface.

## Cont...

- A general form for  $r_a$  is: 
$$r_a = \frac{\ln((z - d)/z_0)^2}{k^2 u_z}$$

where  $k$  is the von Karman constant = 0.41,  $z_0$  is the roughness length (m),  $z$  is the reference measurement height (m),  $u_z$  is the wind speed (m/s) at that height and  $d$  is the zero plane displacement height (m).

- For *bare soil* **Kreith** and **Sellers** (1975) simplified above Eq<sup>n</sup> to:

$$r_a = \frac{\ln(z/z_0)^2}{k^2 u_z}$$

They found a value of  $z_0 = 0.003\text{m}$  worked well for smooth bare soil.

- **Campbell** (1977) estimates **d** & **z** from plant height,  $h$  as:

$$d = 0.64 h$$

$$z_{0m} = 0.31 h$$

$$z_{0H} = 0.2 z_{0m}$$

## 7.3 WATER BALANCE EQUATION

- The water balance is written for a control volume of unit surface area, and with a vertical dimension that extends from the soil surface to a lower boundary that is usually assigned a depth at or below the bottom of the root zone:

$$0 = \Delta S - P + R - F - E$$

where  $\Delta S$  is the change in soil water storage in the profile,

$P$  is precipitation or irrigation,

$R$  is the sum of runoff and run-on,

$F$  is flux across the lower boundary of the profile, &

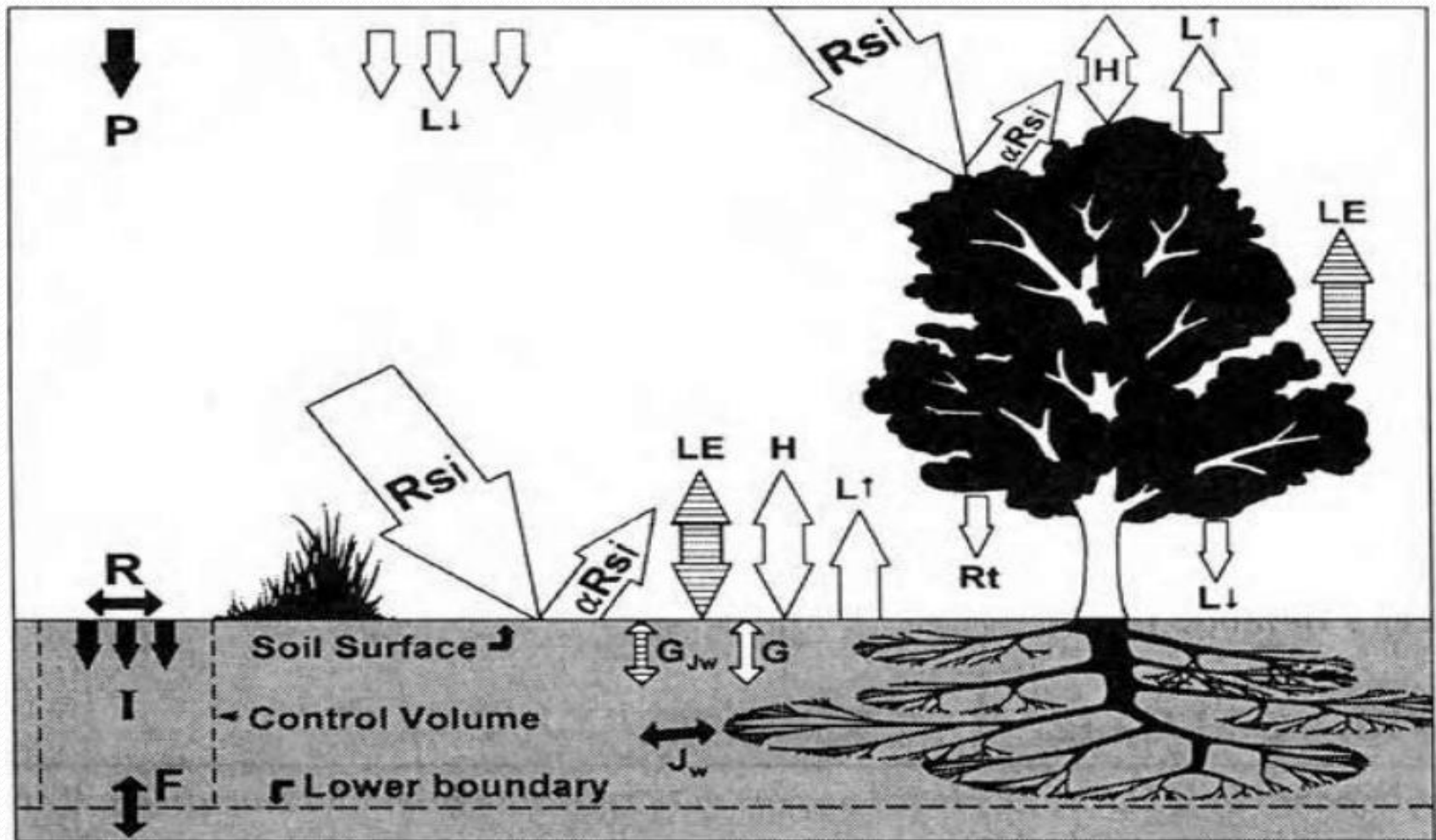
$E$  is water lost to the atmosphere through evaporation from the soil or plant or gained by dew formation.

## Cont...

- The value of **P** is always *positive* or *zero*, but values of  $\Delta S$ , **R**, **F**, and **E** may have either sign.
- By convention, **R** is taken to be *positive* when there is more runoff than run-on, and
- **E** is often taken to be *positive* when flux is out of the control volume.
- Here, in order to be compatible with the energy balance equation, **E** is positive toward the surface of the soil.
- The equation is often rearranged to provide values of **E** when suitable measurements or estimates of the other terms are available, but
  - it can and has been used to estimate runoff, soil water available for plants, and deep percolation losses (flux downward out of the profile).

## Cont...

- Here,  $F$  is *positive* when flux is upward across the lower boundary into the control volume.



- The change in storage ( $\Delta S$ ) is often determined by measuring soil water content changes by methods that give volumetric water content,  $\theta_v$  ( $\text{m}^3 \text{m}^{-3}$ ).
  - Multiplying the water content by the depth of the layer gives the ***depth of water stored***.
- In the United States, the term evapotranspiration (ET) is used to represent the sum of evaporative fluxes from the soil and plant.
- The water balance may be rearranged as:

$$\Delta S = P - R + F - ET$$

- By convention, **ET** is *positive* for fluxes from plant or soil surface to the atmosphere.

- We can examine from the eq<sup>n</sup> that soil water storage:
  - ✓ *increases* with *precipitation*,
  - ✓ *decreases* if runoff from precipitation occurs,
  - ✓ *decreases* with increasing ET, and
  - ✓ increases with flux upward into the control volume.



# *Reading Assignment*

## ➤ **Measuring $\Delta S$ And ET**