Understanding Complex Systems

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Dimitri Volchenkov

# Survival under Uncertainty

## An Introduction to Probability Models of Social Structure and Evolution

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## An Introduction to Probability Models of Social Structure and Evolution

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*To all migrants from the past to the future*

# Preface

I want to live, and so do you.

Existential anxiety engenders in me an awareness of myself and others, as well as the contrast between the present and the uncertain future. All I can do is trust that, whatever happens, I will make the best of it for me and my family.

I have heard that only the fittest will survive the life test. But have you ever seen this hilarious guy? Would the state of being the fittest help him to avoiding taking a plane that is about to be bombed? And what about those among the fittest citizens unfortunate enough to be hijacked by gangs? In 1921, my great-great-grandfather and his five small daughters were mercilessly executed among other civilian hostages by the Soviet commissars during the uprising in Kronstadt, the naval guard post for the approaches to Saint Petersburg in the Gulf of Finland. The victims of terror were adapted enough to live, weren't they? Clearly, survival and death may depend upon an occasional confluence of favorable and unfavorable, yet profoundly accidental, circumstances, irrelevant to the individual level of fitness.

The principle of least action suggests that nature finds the most efficient course from one point to another. The principle of evolution by natural selection is consistent with this, stating that the traits enhancing performance in stable environments are inherited, so that teeth, horns, and claws are destined to dominate the landscape. However, if the rate of environmental changes exceeds the adaptation rate of species, the previous adaptations may rapidly turn into an evolutionary trap, and then it would rather be enduring runners would survive, being capable of evading evolutionary traps better than others. Although humans are poor sprinters compared with other running species, their capability for endurance running is unique among other animals. Volatile environments may change the rules of evolutionary selection, creating a situation of adaptive uncertainty, and this is what we discuss in this book.

In the face of uncertainty, nature promotes configurations allowing for the maximum number of "microscopic" states. A shattered cup is more likely to be found under uncertainty than an intact one, as smithereens allow for overwhelmingly more disordered "microscopic" states than a single state of an intact cup. A crowd is also more likely to occur under uncertainty than a well-coordinated team of individuals since, once individual identity disappears, crowd members are unable to resist any passing idea or emotion. While the survival of the fittest rewards the talented under certainty, community members experiencing uncertainty may cut

“tall poppies” down, since their talents and achievements distinguish them from their peers.

This book aims to bring about a more interdisciplinary approach across diverse fields of research, including survival analysis, strategic management and planning, demography, evolutionary biology, ecology, migration studies, economics, anthropology, communication science, business administration and management, psychology, sociology, political science, history, and philosophy. But first of all, we hope that our book will empower readers to develop their own life strategies based on the understanding that the uncertainty of life is not always an enemy, but rather a wonderful opportunity that can help us to face the challenge of temporal irreversibility of the world.

With typical frankness, my children have explained the major drawback of my book to me: there are too many mathematical formulas, so nobody is likely to read it. I have honestly tried to improve the situation by enhancing the readability of the text.

☞ The important conclusions are displayed in large print to make reading easier.

Probability models are introduced in gray boxes throughout the text.

The book was conceived on the basis of lectures delivered for the Interdisciplinary College *Wicked Problems, Complexity and Wisdom*, held in Günne (Germany) in March 2013 and improved later during the short lecture course *Survival Under Uncertainty* given at the opening of the Center for Nonlinear Physical Science in the Sichuan University of Science and Engineering in Zigong, Sichuan, China, in December 2015.

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My deepest thanks must go to my wife, Lyudmila, and our children. Without their love, support, and encouragement, this book would never have been written. Special thanks go to Dr. Bettina Bläsing for inviting me to take part in the work of the Interdisciplinary College *Wicked Problems, Complexity and Wisdom* in Günne, where the idea of the book first surfaced in 2013. I would also like to express profound thanks to Prof. Dr. Philippe Blanchard whose moral support and advice over many years were absolutely essential. I am deeply grateful to Prof. Dr. Helge Ritter, who has provided essential encouragement and support to me over many years. I am further indebted to Dr. Christian Caron for his competent advice, unfailing interest in my work, and invaluable assistance in the completion of the final manuscript. His help is gratefully acknowledged.

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Bielefeld, Germany Dimitri Volchenkov
January 2016

# Contents

# 1 Subsistence Under Uncertainty

*We demand rigidly defined areas of doubt and uncertainty!*[1]

**Abstract**

The characterization of uncertainty has a dual nature, because the factors responsible for the objective type of uncertainty (arising due to volatile environments) and for the subjective type of uncertainty (arising from subjective imperfections) may evolve on different time scales. The most favorable survival statistics obey the Zipf power law, but in precarious environments survival is always transitory.

## 1.1 Introduction

Any randomness revealing our incapacity to control events, or a lack of intelligible patterns and our inability to predict events in the future, brings fear into our hearts. In the attempt to avoid this at all costs, we adopt an attitude of always trying to find rational explanations whenever apparent order emerges, and then to underestimate the role of chance in all aspects of daily life [1].

On the one hand, by honing our skills of selective attention, we persistently tend to pick out and focus on the 'most interesting' episodes, those for which we were occasionally able to provide a rational explanation, while ignoring whole series of 'fairly uninteresting' cases, full of uncorrelated and thus inexplicable events. On the other hand, when faced with unpredictable or irrational behavior, we experience varying degrees of fear and anxiety with regard to incidents that happen despite our plans and best efforts. Since such an anxious feeling is frequently brought about by association with a past traumatic experience of uncertainty, it victimizes us again

[1]Douglas Adams, *The Hitchhiker's Guide to the Galaxy*.



D. Volchenkov, *Survival under Uncertainty*, Understanding Complex Systems,
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and again, and we may lose sight of regular predictable patterns that could otherwise be observed over a large number of trials for the singular unpredictable events.

Nevertheless, by gathering enough information over a sufficiently long period of time, informed estimates about the unfolding of the future can often be made [2].

## 1.2 Structure of Uncertainty

A first important question concerns the *structure of uncertainty*. According to Helton [3], uncertainties may be classified at a conceptual level into two major groups:

- *Objective uncertainty*, which results from environmental volatility and is a property of the habitat.
- *Subjective uncertainty*, which results either from uncertainty in measurements, or from a lack of knowledge about the environment (ignorance), or simply through indolence, i.e., a lack of ability of the individual to meet the challenges of the external world.

By distinguishing between the objective and subjective types of uncertainty, we can make rational assessments of whether risks will arise due to mingling of possible outcomes and their consequences which take place with different probabilities. In the present chapter, we emphasize the complex nature of uncertainty in the process of subsistence, clearly involving both its objective and its subjective modes and the dynamic interplay between them.

☞ A sudden volcanic eruption or failure to pass a test manifest the different types of uncertainty that we may face.

## 1.3 Principle of Subsistence

A species will subsist as long as the means available (food) suffice to maintain it. The problem of uncertainty in subsistence can be studied with the help of a simple model, in which both the exact amount of resources required to support life at a minimum level during a certain period of time and the amount of resources available through hunting, gathering, and subsistence agriculture for the same period of time are treated as random variables that can change inconsistently.

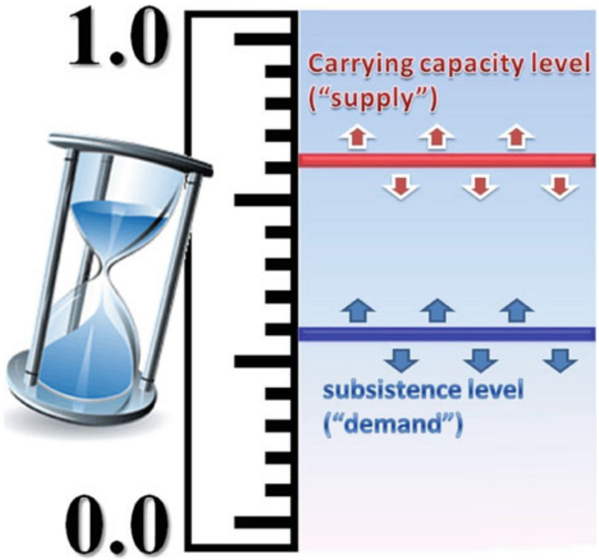


**Fig. 1.1** A species subsists as long as the means available suffice to maintain it

☞ A species subsists as long as the means available suffice to maintain it.

In such a stochastic process (see Fig. 1.1):

- randomness in the minimal level of needs required for subsistence (random demand) constitutes the subjective type of uncertainty, this being an inherent property of the species, and
- randomness in the available amount of resources (random supply) exemplifies the objective type of uncertainty.

In a subsistence economy which is not based on money but is wholly reliant on the self-provisioning of the community and available natural resources (such as food, water, or fuel), as well as on renewal and reproduction rates within the community, levels of supply and demand are essentially precarious. Then the proposed model can help to infer the statistics of subsistence durations between catastrophic events occurring in series. The mathematical models describing the behavior of systems close to a threshold of instability have been studied by ourselves in detail [7]. The statistics of survival durations in the proposed model are determined by the degree of inconsistency between the random revaluations of supply and demand.

## 1.4 Model of Subsistence Under Uncertainty

We suppose that the minimal level of subsistence needs (demand) for a given species during a certain period of time can be quantified by a real number $d \in [0, 1]$. Another real number, $s \in [0, 1]$, appraises the amount of nourishment (supply) available during the same period of time.

We assume that the species survives as long as $d \leq s$, but dies out immediately after the carrying capacity of the habitat, the maximum population size that the environment can sustain indefinitely [8], is exceeded, i.e., $d > s$.

In the setting of population biology, the level of demand ($d$) can be considered as the population density, and the level of supply ($s$) plays the role of the carrying capacity of the region, setting a ceiling on population growth. As soon as a population has increased to the point where all available territories are occupied, surplus animals become non-territorial floaters with poor survival rates and zero reproductive prospects [9]. Therefore, as soon as population numbers ($d$) reach a stability threshold ($s$), the carrying capacity determined by the total area of available territories, the population growth rate is reduced to zero without any time lag [10]. Since the carrying capacity of habitat is significantly affected by year-to-year fluctuations in the temperature and the amount of rainfall, by gradual changes in the climate, by the existing level of agricultural technology, and by the way this technology is employed, it may change accidentally and suddenly [9].

☞ Uncertainty arises due to limited information and innumerous volatile factors.

As information regarding the variability of random supply and demand is best conveyed using probability distribution functions, we assume that $d$ is a random variable distributed with respect to some probability distribution function $P\{d < x\} = F(x)$, and the random level of supply $s$ is drawn from another probability distribution function $P\{s < x\} = G(x)$.

Furthermore, it is natural to assume that living species and humans, in particular, may seek to reduce the level of objective uncertainty by applying special control measures in order to conserve and protect the carrying capacity of their habitat.

A society that approaches the current limits of population growth can invest in clearing forests, draining swamps, irrigation, and flood control [9]. Laws and policies that would effectively reduce the world's carbon emissions are enforced nowadays in an attempt to stabilize the global average temperature. Due to these efforts, the amount of supply may stay put at the same acceptable level or even increase, at least for a while, so that the level of objective uncertainty can be temporarily attenuated.

For instance, reduced metabolic rates are characteristic of organisms exposed to environmental stress. An organism can modify its cell structures to maximally protect them against degradation. Once this is done, it can switch off virtually all metabolic processes until the adverse environmental conditions end. However, the rate of spontaneous degradation of cell structures is never equal to zero [11].

☞ The degree of uncertainty can be reduced by implementing special control measures, but uncertainty can never be completely eliminated.

Throughout history, the survival of all living species including humans has always been precarious. Hence, it is a basic ecological principle that the population ($d$) tends to rise to meet all available food supply ($s$) [12]. Since the rate of spontaneous degradation of cell structures in an organism is never reduced to zero, its death will follow as soon as the amount of damage accumulated in its tissues goes over a critical threshold [11]. Thus, survival always occurs at a threshold of instability, where there is a delicate balance between volatile levels of supply and demand that can be violated at any time.

☞ Survival occurs at a threshold of instability, where the precarious levels of supply and demand hang in a delicate balance.

The concerted measures taken routinely by humans in order to preserve their habitat, property, health, etc., can lead to a substantial difference in the rate of variability of factors responsible for the objective and subjective types of uncertainty. Efforts aimed at attenuating objective uncertainty lead to a situation where the level of available supply ($s$) varies *more slowly* than the level of required demand ($d$).

There are also many examples of quite the opposite situation, where the rate of environmental variability appears to be significantly higher than the rate of variation in demand. An inability to adapt to profound and sudden changes in environmental conditions is considered to be the main cause of *mass extinctions* in Earth's history, in which abnormally large numbers of species died out simultaneously or within a limited time frame [5].

Since the development of any adequate adaptive trait—whether structural, behavioral, or physiological—in response to particular environmental stresses always takes a relatively long time, the factors contributing to the subjective type of uncertainty obviously evolve more slowly in such a situation than those contributing to the objective type of uncertainty.

Within evolutionary biology, a situation in which rapid environmental changes trigger presumably well adapted organisms to make maladaptive behavioral decisions leading to the extinction of the species is known as an *evolutionary trap* [6].

## ☞ Factors of objective and subjective uncertainty may challenge our survival across different time scales.

While individual failures are ubiquitous in life, supervolcanic eruptions and other natural calamities are relatively rare events, so the challenge to survive may take place over incomparable time scales.

In the proposed discrete time stochastic process, modeling subsistence under uncertainty, we describe the degree of inconsistency between the rates of random variations in supply and demand by the *degree of environmental stability* $\eta \in [0, 1]$.

In particular, we shall assume that the rate of variation at the demand level is greater than or equal to that of the supply level. However, it is important to mention that, under an elementary change of variables, the proposed probability model is equally applicable to the opposite situation describing the survival of an organism facing random environmental challenges occurring in series. In fact, it is the relative rate of random updates of supply and demand (the rate of environmental stability), described in our model by the degree of environmental stability $\eta \geq 0$, that actually determines a species' chances of survival.

The stochastic process of subsistence under uncertainty is defined in the following way:

At time $t = 0$, the level of demand $d$ is chosen with respect to the probability distribution function $F$, and the level of supply $s$ is chosen with respect to the probability distribution function $G$. If $s \geq d$, the species subsists, and the process keeps going to time $t = 1$. At time $t \geq 1$, either,

- with probability $\eta \geq 0$, the level of demand $d$ is drawn anew from the probability distribution function $F$, while the level of supply $s$ keeps the value it had at time $t - 1$, or
- with probability $1 - \eta$, the level of demand $d$ is updated anew from the probability distribution function $F$, and the level of supply $s$ is updated with respect to the probability distribution function $G$.

As long as the carrying capacity of the habitat is not exceeded ($d \leq s$), the species survives, but the process ends at some moment of time $\tau$, as soon as $d > s$. The flowchart for the probability model of subsistence under uncertainty is shown in Fig. 1.2.

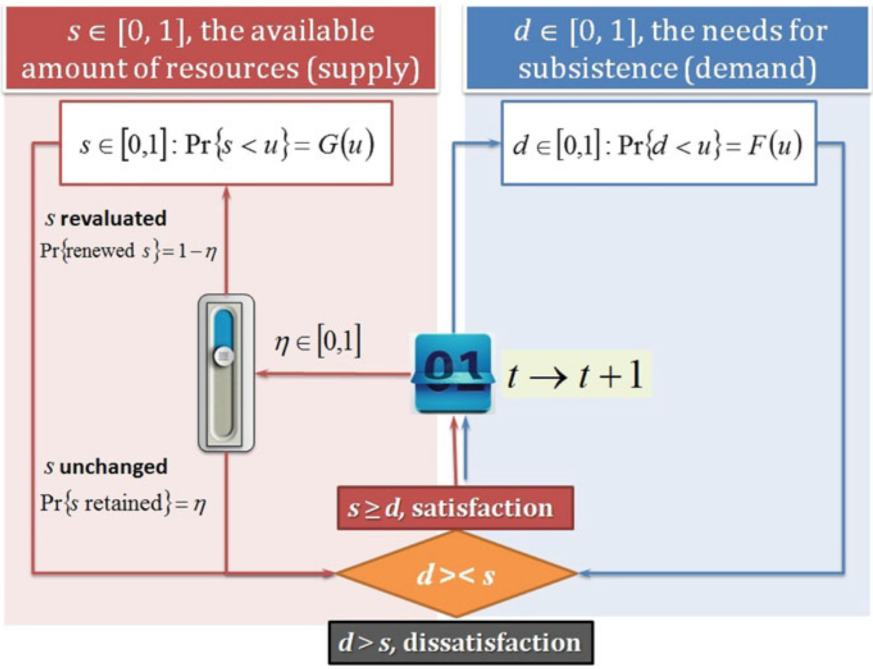


**Fig. 1.2** The flowchart for the model of subsistence under uncertainty

The degree of environmental stability $\eta \geq 0$ describing the expected relative rate of random variations in supply and demand determines the structure of uncertainty in our subsistence model. In particular, we often discuss the following opposite cases:

- *Dual uncertainty*, in which the objective and subjective types of uncertainty contribute simultaneously through consistent (and, perhaps, coherent) random updates of supply and demand ($\eta = 0$).
- *Singular uncertainty*, in which the amount of available supply remains unchanged during the entire process, so that objective uncertainty is efficiently excluded from the problem ($\eta = 1$).

By tuning the degree of environmental stability to the intermediate values $0 < \eta < 1$, we can adjust the degree of disparity between the two types of uncertainty.

It is important to mention that the value of the probability $\eta > 0$ can also be interpreted as the reciprocal characteristic time interval, during which the level of available supply $s$ remains unchanged due to successfully applied control measures.

Despite the inherent simplicity of the proposed probability model, it can help generate the main hypotheses concerning fundamental biological and evolutionary principles, and by adopting a new point of view, bring a variety of social mechanisms under a common perspective.

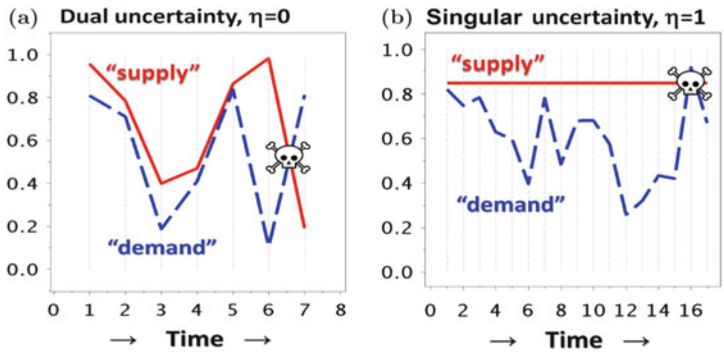


**Fig. 1.3** The expected sequence of survival events is longer in the case of a stable supply level (under singular uncertainty (**b**)) than in the case of consistent random updates of supply and demand (under dual uncertainty (**a**)). Time is measured by the number of random updates of the demand level

The main observation relating to the model of subsistence under uncertainty is that, in the case of a stable supply level (under singular uncertainty), the expected survival duration is always longer than under dual uncertainty, when the levels of supply and demand are updated consistently (see Fig. 1.3).

## 1.5 Probability of Subsistence Under Uncertainty

We study the probability distribution $P_\eta(\tau)$ of the survival durations $\tau$ for some probability distribution functions $F$ and $G$ and a given value of the probability $\eta \geq 0$. It is important to mention that the degree of environmental stability can be related to the characteristic duration of control $(1-\eta)^{-1}$ applied to the environment.

Although the levels of demand and supply have been introduced in the probability model as random variables, they can also be considered as deterministic dynamical variables—say, generated by iterated images of certain maps defined on the interval $[0, 1]$. If the demand and supply levels are considered to be the deterministic dynamical variables, we have to assume the existence of the invariant ergodic (Bernoulli) measures $\mathrm{d}F$ and $\mathrm{d}G$, for which the sequence of values of $d$ and $s$ constitute the generic orbits.

We are interested in the probability $P_\eta(\tau)$ that the stochastic subsistence process introduced in the previous section ends precisely at time $\tau$, at the moment it exhausts the carrying capacity of the habitat. When the subsistence process keeps going to $\tau \geq 1$, the survival and extinction events can take place either in a consistent way (with probability $1-\eta$), or in an inconsistent way (with probability $\eta$).

A straightforward computation shows that, independently of the value of $\eta$, the initial probability of choosing the level of demand below the supply level (to start the subsistence process) is $\int_0^1 \mathrm{d}G(x)F(x)$. The probability of extinction precisely at time $\tau = 1$ is

$$\begin{aligned} P_\eta(1) &= \eta \int_0^1 \mathrm{d}G(x)F(x)\big[1 - F(x)\big] \\ &\quad + (1-\eta)\int_0^1 \mathrm{d}G(x)F(x)\int_0^1 \mathrm{d}G(y)\big[1 - F(y)\big] , \\ &= \eta B(1) + (1-\eta)A(1)B(0) , \end{aligned} \tag{1.1}$$

and similarly,

$$\begin{aligned} P_\eta(2) = \eta^2 B(2) + \eta(1-\eta)\big[A(1)B(1) + A(2)B(0)\big] \\ +(1-\eta)^2 A(1)^2 B(0) , \end{aligned} \tag{1.2}$$

where we have defined, for $n = 0, 1, 2, \ldots$,

$$A(n) \equiv \int_0^1 \mathrm{d}G(y)F(y)^n ,$$

$$B(n) \equiv \int_0^1 \mathrm{d}G(y)F(y)^n\big[1 - F(y)\big] = A(n) - A(n+1) .$$

The general formula for $P_\eta(\tau)$ for all $\tau \geq 3$ can be found in [7].

The formulas describing the probability of subsistence duration in the case when the rate of environmental variability appears to be significantly higher than the rate of changes in demand can be readily derived from (1.1) and (1.2) by a simple change of variables $x \to 1 - x$, $y \to 1 - y$.

For further calculations, it is useful to introduce the generating function $\hat{P}_\eta(z) \equiv \sum_{\tau=0}^{\infty} z^\tau P_\eta(\tau)$ with the following generating property:

$$P_\eta(\tau) = \frac{1}{\tau!}\frac{\mathrm{d}^\tau \hat{P}_\eta(z)}{\mathrm{d}z^\tau}\bigg|_{z=0} . \tag{1.3}$$

Defining the auxiliary functions

$$\begin{aligned} &p(l) = \eta^l A(l+1) , \quad \text{for } l \geq 1 ,\ p(0) = 0 , \\ &q(l) = (1-\eta)^l A^{l-1}(1) , \quad \text{for } l \geq 1 ,\ q(0) = 0 , \\ &r(l) = \eta^l\Big[\eta B(l+1) + (1-\eta)A(l+1)B(0)\Big] , \quad \text{for } l \geq 1 ,\ r(0) = 0 , \\ &\rho = \eta B(1) + (1-\eta)A(1)B(0) , \end{aligned} \tag{1.4}$$

we obtain

$$\hat{P}_\eta(z) = B(0) + \rho z + \frac{z\Big[\hat{r}(z) + \rho\hat{p}(z)\hat{q}(z) + \rho A(1)\hat{q}(z) + A(1)\hat{q}(z)\hat{r}(z)\Big]}{1 - \hat{p}(z)\hat{q}(z)}, \tag{1.5}$$

where $\hat{p}(z)$, $\hat{q}(z)$, and $\hat{r}(z)$ are the generating functions of $p(l)$, $q(l)$, and $r(l)$, respectively.

The qualitative conclusions about the statistics of survival durations given in the forthcoming sections also retain their validity for the case when the factors contributing to the subjective type of uncertainty evolve more slowly than those contributing to the objective type of uncertainty.

## 1.6 Transitory Subsistence Under Dual Uncertainty

When both objective and subjective factors contribute equally to the chances of survival through consistent updates of the supply and demand levels (under dual uncertainty $\eta = 0$), the resulting probability function $P_{\eta=0}(\tau)$ decays exponentially fast with $\tau$, for any choice of the probability distribution functions $F$ and $G$.

☞ Subsistence under dual uncertainty is always transitory.

For $\eta = 0$, (1.4) and (1.5) give

$$\hat{P}_{\eta=0}(z) = \frac{B(0)}{1 - zA(1)} . \tag{1.6}$$

Applying the inverse formula (1.3)–(1.6), we get

$$P_{\eta=0}(\tau) = A^\tau(1)B(0) = \left[\int_0^1 \mathrm{d}G(y)F(y)\right]^\tau \int_0^1 \mathrm{d}G(y)\big[1 - F(y)\big] .$$

Therefore, for any choice of $F(x)$ and $G(x)$, the probability $P_{\eta=0}(\tau)$ decays exponentially fast with time, because $A(1) < 1$.

In particular, if the level of supply and demand are drawn uniformly at random, the subsistence process under dual uncertainty is equivalent to simply flipping a fair coin, for which heads and tails come up equiprobably. For the special case of uniform probability densities, $\mathrm{d}F(x) = \mathrm{d}G(x) = \mathrm{d}x$, for all $x \in [0, 1]$,

$$P_{\eta=0}(\tau) = \frac{1}{2^{(\tau+1)}} . \tag{1.7}$$

The vanishing probability of winning in a long enough sequence of coin flips features in the opening scene of the Tom Stoppard's play *Rosencrantz and Guildenstern Are Dead*, where the protagonists are betting on coin flips. Rosencrantz, who bets on heads each time, has won ninety-two flips in a row, leading Guildenstern to suggest that they are within the range of supernatural forces. And he was actually right, as the king had already sent for them.

☞ A species becomes extinct within a limited time frame in the face of dual uncertainty. There are no 'centenarians' in the population.

The expected duration of subsistence amid dual uncertainty is always finite:

$$\tau_* = \sum_{\tau=0}^{\infty} \tau\, P_{\eta=0}(\tau + 1) = \frac{A^2(1)B(0)}{\left[A(1) - 1\right]^2}\,. \tag{1.8}$$

The key feature of survival duration statistics described by simple coin tossing is that the species tends to have a fairly regular rate of extinction, in line with the observations of Van Valen to the effect that all groups of species go extinct (in a million years) at a rate that is constant for a given group [13].

## 1.7 Extraordinary Longevity Under Singular Uncertainty

When the level of supply is kept unchanged, i.e., under singular uncertainty $\eta = 1$, the factors contributing to the objective type of uncertainty are completely excluded from the proposed model. Then there are many different types of possible behavior for the probability function of survival duration, depending upon the particular choice of the distributions $F$ and $G$.

Under singular uncertainty ($\eta = 1$), (1.4) and (1.5) yield $\hat{P}_{\eta=1}(z) = \hat{B}(z)$, so that

$$P_{\eta=1}(\tau) = B(\tau) = \int_0^1 \mathrm{d}G(y)F(y)^{\tau}\left[1 - F(y)\right]\,. \tag{1.9}$$

For example, in the special case of uniformly random updates of the supply and demand levels, the probability function (1.9) decays algebraically, tending asymptotically to the quadratic hyperbola for $\tau \gg 1$ :

$$P_{\eta=1}(\tau) = \frac{1}{(\tau + 1)(\tau + 2)} \simeq \frac{1}{\tau^2}\,. \tag{1.10}$$

In particular, the following rule of thumb can be used in order to anticipate the expected duration of subsistence under a uniformly random demand: *a lifespan that lasts twice as long, occurs a quarter as often*. This rule of thumb follows from the quadratic hyperbolic tail of the probability function of survival duration (1.10).

☞ No mass extinction is observed in the population when the factors responsible for the objective and subjective types of uncertainty evolve on inconsistent time scales.

The quadratic hyperbolic tail of the probability function (1.10) suggests that the stochastic process of subsistence under subjective uncertainty alone leads to a situation with no characteristic time scale:

$$\tau_* = \sum_{\tau=0}^{\infty} \tau \, P_{\eta=1}(\tau) = \infty. \tag{1.11}$$

However, the actual lifespan of the species does indeed remain finite. The absence of a characteristic time scale in (1.11) indicates that virtually all ages may be present in a population subsisting under singular uncertainty, including individuals of extraordinarily long lifespans ('centenarians'). It is this inconsistency in the time scales for the factors of objective and subjective uncertainty that can lead to extraordinary longevity.

☞ Extraordinary longevity can occur in a population subsisting under singular uncertainty.

For a general family of invariant measures of a map of the interval [0, 1] with a fixed neutral point defined by the probability distributions $F$ and $G$, absolutely continuous with respect to the Lebesgue measure [14], i.e.,

$$\begin{aligned} \mathrm{d}F(x) &= (1+\alpha)x^{\alpha}\mathrm{d}x\,, \qquad \alpha > -1\,, \\ \mathrm{d}G(x) &= (1+\beta)(1-x)^{\beta}\mathrm{d}x\,, \qquad \beta > -1\,, \end{aligned} \tag{1.12}$$

Eq. (1.9) gives

$$P_{\eta=1}(\tau) = \frac{\Gamma(2+\beta)\;\Gamma\big(1+\tau(1+\alpha)\big)}{\Gamma\big(2+\beta+\tau(1+\alpha)\big)} - \frac{\Gamma(2+\beta)\;\Gamma\big(1+(\tau+1)(1+\alpha)\big)}{\Gamma\big(2+\beta+(\tau+1)(1+\alpha)\big)}\,,$$

where $\Gamma(x)$ is the Gamma function. Using the Stirling approximation, we obtain the power law asymptotic decay for $\tau \gg 1$:

$$P_{\eta=1}(\tau) \simeq \frac{(1+\beta)\Gamma(2+\beta)(1+\alpha)^{-1-\beta}}{\tau^{2+\beta}}\big[1+O(1/\tau)\big]\,. \tag{1.13}$$

Therefore, it is mainly the character of the probability function $G$ for the supply level that determines the rate of decay of the survival probability $P_{\eta=1}(\tau)$ with time. The asymptotic decay seems to be algebraic for any choice of the distributions $F$ and $G$. At least, we have not found any counterexample contradicting this conjecture. For instance, in the case of uniform $F$, $P_{\eta=1}(\tau)$ is nothing but a particular case of a Riemann–Liouville integral, and we have not found any case of non-algebraic decay for large $\tau$ in the tables [15].

Moreover, we can obtain all possible power law asymptotic decays for the tail of the probability function for different values of $\beta > -1$:

$$P_{\eta=1}(\tau) \simeq \frac{1}{\tau^{2+\beta}}\ , \tag{1.14}$$

for $\beta > -1$. It is worth mentioning that the exponent $2+\beta$ characterizing the decay of $P_{\eta=1}(\tau)$ is independent of the exponent $\alpha$ characterizing the distribution $F$ of the demand level in (1.12).

## 1.8 Zipfian Longevity in a Land of Plenty

We have seen that subsistence can be long-lasting, provided the factors responsible for one type of uncertainty are wholly excluded. *What are the best possible chances for survival under uncertainty?* It is obvious that very long-lasting subsistence is more probable when living resources are plentiful. In *Cockaigne*, an imaginary land of plenty in medieval myths, where physical comforts and pleasures are always immediately at hand and where harshness of life does not exist, the level of supply and carrying capacity of the habitat ($s$) is always close to the maximal value 1.

When the random level of supply is drawn from the probability density $dG(x)$ (1.12), the process of subsistence is characterized by an infinite characteristic lifetime (1.11), provided that the exponent $-1 < \beta \leq 0$, independently of the distribution $F$ characterized by the value of $\alpha$ (1.13).

In particular, the case of $\beta = 0$ in (1.12) corresponds to a situation where the supply level is chosen uniformly at random over the interval $[0, 1]$. And the supply level probability function $G(x)$ is a convex function on the interval $[0, 1]$ with maximal value at $x = 1$, for $-1 < \beta \leq 0$.

In the limiting case when the support of the probability distribution $G(x)$ determining the choice of the supply level is concentrated close to $x = 1$, i.e., is zero everywhere in the interval $[0, 1]$, except for a small interval of length $\varepsilon$ up to 1, the *Zipf power law* asymptote $\propto t^{-1-\varepsilon}$, $\varepsilon > 0$, follows directly from (1.13).

A possible modeling function for such a bountiful probability distribution, forming a thin spike as $x \to 1$, can be chosen in the form

$$G_\varepsilon(x) = 1 - (1-x)^\varepsilon \;, \quad \varepsilon > 0 \;, \tag{1.15}$$

with probability density in the interval $[0, 1[$ ,

$$\mathrm{d}G_\varepsilon(x) = \frac{\varepsilon \mathrm{d}x}{(1-x)^{1-\varepsilon}} \;. \tag{1.16}$$

The exponent $\varepsilon$ in the modeling probability density (1.16) can be viewed as the *degree of precariousness* in supply. If the degree of precariousness $\varepsilon > 0$ is small enough, the resources are plentiful and inexhaustible. However, the distribution of the supply threshold gets broader as $\varepsilon$ grows. Finally, when $\varepsilon = 1$, the degree of precariousness reaches the maximum, and the supply level density distribution over the interval $[0, 1]$ becomes uniform, i.e., any value of the supply is equally possible.

For any choice of the probability distribution $F(x)$ in (1.12), e.g., the demand level can be chosen to be uniformly distributed over $[0, 1]$, so that $\alpha = 0$ and $\mathrm{d}F(x) = \mathrm{d}x$, the asymptotic probability function of survival by time $\tau$ (1.13) is given by

$$P_{\eta=1}(\tau) \simeq \frac{\varepsilon}{\tau^{1+\varepsilon}} \;, \quad \tau \gg 1 \;, \;\; \varepsilon > 0 \;. \tag{1.17}$$

In particular, it follows from (1.13) and (1.14) that, for a small enough degree of precariousness in supply $\varepsilon > 0$, the most favorable longevity statistics obey the Zipf law (1.17), observed in many types of data studied in the physical and social sciences [16, 17].

The most favorable longevity statistics then abide by the following rule of thumb: *a lifespan that lasts twice as long, occurs half as often*, and these are the best survival statistics that we can hope for!

☞ The best possible chances for survival under uncertainty satisfy the Zipf law: a lifespan twice as long occurs half as often.

However, when $\varepsilon > 0$ increases, the chance for essentially long survival worsens. Eventually when the degree of precariousness reaches the maximum ($\varepsilon = 1$), the probability of survival by time $\tau$ asymptotically turns into the quadratic hyperbola (1.10). Nevertheless, even in the case of precarious resources, there are no mass extinctions possible, as long as the factors responsible for the objective and subjective types of uncertainty evolve on the inconsistent time scales.

## 1.9 A General Rule of Thumb for Subsistence Under Uncertainty

We now present the bounds for the survival probability until time $\tau$, valid for any value of the probability $\eta$ and for any $\tau \geq 3$. It can be shown that, as long as the structure of uncertainty is not strictly inconsistent (for any $0 \leq \eta < 1$), the decay of the distribution function $P(\tau)$ is always bounded by exponentials.

☞ Survival remains transitory and mass extinction is possible as long as the time scales of subjective and objective uncertainty are not radically different.

We use the fact that

$$A(1)^n \leq A(n) \leq A(1), \text{ and } 0 \leq B(n) \leq A(1), \text{ for } n = 1, 2, \ldots .$$

The upper bound for $A(n)$ is then trivial, since $0 \leq F(y) \leq 1$ for any $y \in [0, 1]$. The lower bound is a consequence of Jensen's inequality, and of the fact that the function $x \to x^n$ is convex on the interval $]0, 1[$ for any integer $n$. Following [7], we obtain

$$\begin{aligned} P_\eta(\tau) \leq & \Big[\eta^\tau B(\tau) + \eta^{\tau-1}(1-\eta)A(\tau)B(0)\Big] \\ & + \Big[\eta A(1) + (1-\eta)A(1)B(0)\Big] \sum_{k=1}^{\tau-1} \binom{\tau-1}{k} \big[(1-\eta)A(1)\big]^k \eta^{\tau-1-k} \end{aligned} \tag{1.18}$$

and

$$\begin{aligned} P_\eta(\tau) \geq & \Big[\eta^\tau B(\tau) + \eta^{\tau-1}(1-\eta)A(\tau)B(0)\Big] \\ & + (1-\eta)A(1)B(0) \sum_{k=1}^{\tau-1} \binom{\tau-1}{k} \big[(1-\eta)A(1)\big]^k \eta^{\tau-1-k} \ . \end{aligned} \tag{1.19}$$

This implies the upper bound

$$\begin{aligned} P_\eta(\tau) \leq & \ \eta^\tau B(\tau) + (1-\eta)A(1)B(0)\big[\eta + (1-\eta)A(1)\big]^{\tau-1} \\ & + \eta A(1)\Big\{\big[\eta + (1-\eta)A(1)\big]^{\tau-1} - \eta^{\tau-1}\Big\} \end{aligned} \tag{1.20}$$

and the lower bound

$$\begin{aligned} P_\eta(\tau) & \geq \eta^\tau B(\tau) + (1-\eta)A(1)^\tau B(0) \\ & = \eta^\tau P_{\eta=1}(\tau) + (1-\eta)P_{\eta=0}(\tau) \ . \end{aligned} \tag{1.21}$$

For any $0 \leq \eta < 1$, the decay of the distribution function $P(\tau)$ is bounded by exponentials. Furthermore, the bounds (1.20) and (1.21) are exact in the marginal cases $\eta = 0$ and $\eta = 1$.

In particular, for the special case of a uniformly random choice of the supply and demand levels, for any $\eta \in [0, 1]$ and for any $\tau \geq 3$, the bounds (1.20) and (1.21) become

$$\frac{\eta^\tau}{(\tau+1)(\tau+2)} + \frac{1-\eta}{2^{\tau+1}} \leq P_\eta(\tau) \leq \frac{1}{2}\left(\frac{1+\eta}{2}\right)^\tau . \tag{1.22}$$

From the bounds (1.22), it is obvious that asymptotically algebraic decay in the right tail of the probability function $P_\eta(\tau)$ is possible only for $\eta = 1$, since for any $0 \leq \eta < 1$, it is bounded by exponentials. Similar results can also be obtained for the case when the level of demand stays put while supply varies.

☞ A stable and secure habitat is crucially important for long-term survival, but plentiful resources increase the chances of extraordinary longevity.

## 1.10 On the Optimal Strategy of Subsistence Under Uncertainty

In the case of uniform probability densities for the random updates of the supply and demand levels, it is possible to get an exact expression for the probability of subsistence $P_\eta(\tau)$ for all times and for any value of the probability $\eta$ [7]:

$$P_\eta(\tau) = \frac{\eta^\tau}{(\tau+1)(\tau+2)} + \sum_{k=1}^{\tau} \frac{\eta^\tau}{k(\tau-k+1)(\tau-k+2)} \sum_{m=1}^{k} c_{m,k} \left(\frac{1-\eta}{\eta}\right)^m , \tag{1.23}$$

where $c_{m,k}$ is defined by

$$c_{m,k} = m! \sum_{\substack{l_1+\dots+l_m=k \\ l_i \geq 1}} \frac{\prod_{s=1}^{m} l_s}{(l_m+1)\prod_{s=1}^{m-1}\left[\,(l_s+1)\left(k-\sum_r^s l_r\right)\right]} . \tag{1.24}$$

In Fig. 1.4, we have plotted the empirical distributions $P_\eta(\tau)$ obtained in the numerical experiments modeling the subsistence under uncertainty for levels of supply and demand updated uniformly at random, for the three values of the degree of environmental stability $\eta = 0$, $\eta = 0.88$, and $\eta = 1$, along with the trend lines

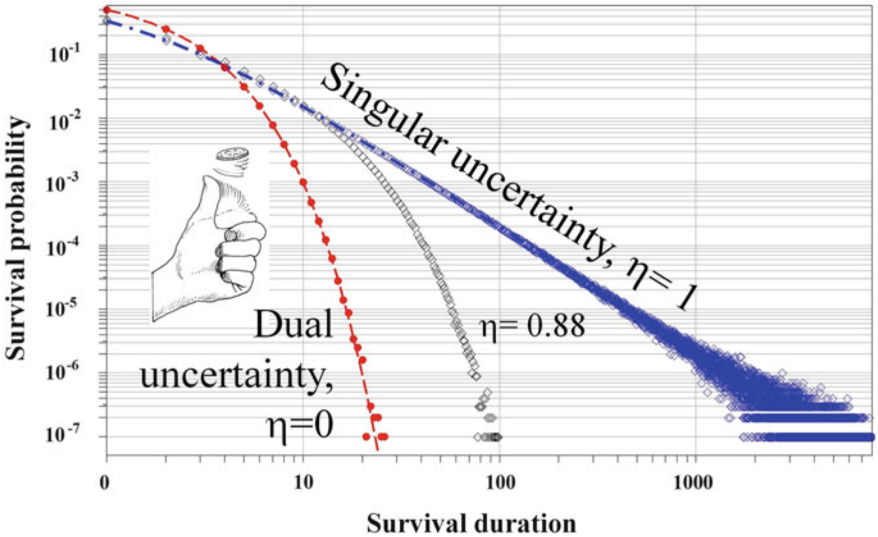


**Fig. 1.4** Distribution of survival times obtained in the numerical experiment, for random levels of supply and demand uniformly distributed over the interval [0, 1], shown on a log–log scale. The duration is measured by the number of random updates to the level of demand. Consistently with the analytical result $P_{\eta=0}(\tau) = 2^{-(\tau+1)}$ (shown by the *dashed trend line*), the empirical probability $P_{\eta=0}(\tau)$ decays exponentially fast with time. For $P_{\eta=1}(\tau)$, the *trend line* is $1/(\tau+1)(\tau+2)$. The experimental distribution of survival durations obtained for the intermediate value $\eta = 0.88$ is indicated by *diamonds*

representing the analytical results (1.23). It is remarkable that the experimentally observed duration of subsistence for a fixed level of supply would correspond to $\sim 7 \times 10^5$ successful random updates of demand in the numerical experiment with $10^7$ trials.

Interestingly, the chances of initial survival during the first few sequential updates of demand are more opportune in precarious environments ($\eta = 0$) than in stable environments ($\eta = 1$). Although the probability of initial survival under dual uncertainty is only a matter of flipping a fair coin, $P_{\eta=0}(0) = 1/2$, the probability of initial subsistence in more stable environments is even less favorable, i.e., $P_{\eta>0}(0) < 1/2$.

☞ Initial subsistence under uncertainty can never be more opportune than flipping a fair coin.

Precarious environments provide the most favorable chances for survival during the initial stage of the process, yet the decay of survival probability is essentially slower in more stable environments, which are therefore more opportune for long-term survival. Thus, individuals striving to boost their chances for subsistence under uncertainty can be motivated to change or destabilize the environment at each step (for example, by moving from one place to another) during the initial stage of

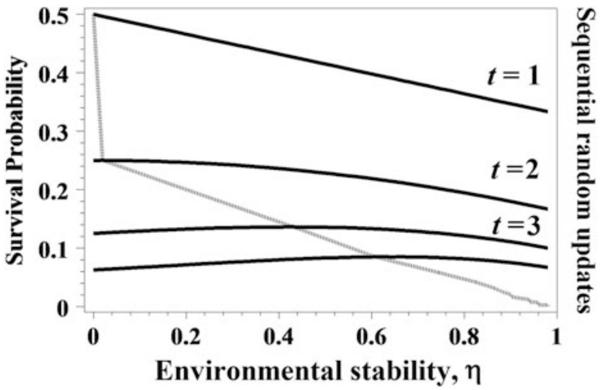


**Fig. 1.5** The survival probabilities (shown by *solid lines*) during the first few random updates of demand are represented as functions of the probability $\eta$ characterizing the degree of environmental stability, for uniformly random updates of the demand and supply levels. The *grey dotted line* connects the points of maximum probability for the sequence of time steps

survival, but keep the environment stable and safe later on, in order to maximize the chances for longer survival. The stability of environments is more important for enhancing the chances for survival during the intermediate times (ranging from 5 to 20 updates of demand) than affluence of available resources. However, in a land of plenty, the chances for survival clearly follow the Zipf law for very long survival series (Fig. 1.5).

We can summarize the results obtained for the optimal survival strategy under uniformly random updates of supply and demand (Fig. 1.6) as developing through three consecutive stages:

- *Initial destabilization* of the environment is required at each time step, with consistent updates of demand and supply, in order to boost the chances for survival during the initial stage. The initial destabilization stage can be viewed as a migration phase, with the intention of settling temporarily in a new location.
- *Intermediate stabilization* of the environment, by keeping the level of supply unchanged during the sequence of random updates of demand, boosts the chances for survival during intermediate times $5 \leq \tau \leq 20$.
- *A safe haven in a land of plenty* is required in order to enjoy extraordinary longevity.

We conclude the section by noting that the concept of *being in control* of the environment can have different meanings depending on the duration of the relevant time interval. In accordance with the optimal survival strategy, the concept of control involves the capacity of destabilization of the environment in the short-term, the capacity to preserve and protect the environment in the intermediate future, and, finally, the capacity to use the environment efficiently in order to maximize the amount of supply that will eventually be available in the long run.

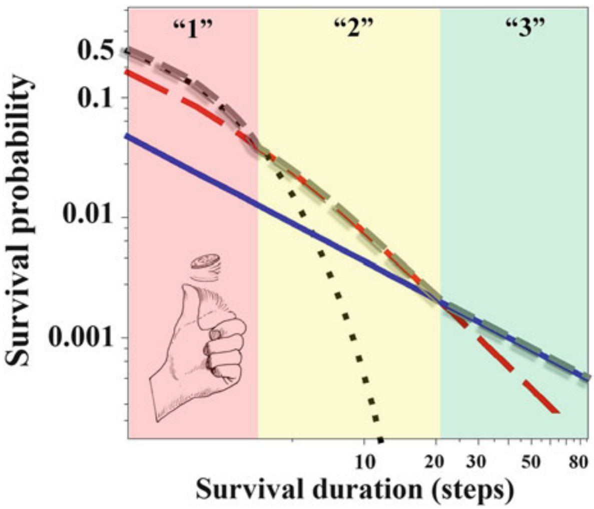


**Fig. 1.6** Trends of maximum survival probability in the model of subsistence under uncertainty, for uniformly random updates of the supply and demand levels (log–log scale): (1) $P_{\eta=0}(\tau) = 2^{-\tau-1}$, for short times $\tau = O(1)$. (2) $P_{\eta=1}(\tau) = 1/(\tau+1)(\tau+2)$, for intermediate times $5 \leq \tau \leq 20$. (3) The Zipf power law distribution $P(\tau) \simeq t^{-1-\varepsilon}$, $\varepsilon > 0$, for long enough survival sequences $\tau > 20$

## 1.11 Conclusion

In the present chapter, we have developed the mathematical theory of subsistence under uncertainty. We have pointed out that the characterization of uncertainty has a dual nature, because the factors responsible for the objective type of uncertainty (arising due to volatile environments) and for the subjective type of uncertainty (arising from subjective imperfections) may evolve on different time scales. By tuning the degree of environmental stability in the proposed model of subsistence, we were able to study the chances for subsistence under differently structured uncertainty and design the optimal survival strategy.

We have shown that the most favorable survival statistics obey the Zipf power law, but that survival is transitory in precarious environments.

# Love of Life and the Maximum Age Diversity Principle

# 2

*He, as a man, no longer strove. It was the life in him, unwilling to die, that drove him on*[1]

**Abstract**

The age composition of a population can be characterized by entropy as a way of assessing the uncertainty of survival. The population is likely to increase its survival entropy with time. Such striving (or entropic force) in the surviving population may be seen as the gradual demographic shift to older ages. We present an exact expression for the entropic force provided that age-specific survivorship depends on the current level of well-being, assessing the entire range of complex social, ecological, and economic factors. The official demographic statistics on the age structure of the global population provided by the UN Statistics Division suggests that age diversity in the world population has grown steadily during the whole period of observation.

## 2.1 Introduction

While trudging wearily across the frozen Canadian tundra to find food, the protagonist of the story *Love of Life*, perhaps the most famous of the Northern short stories written by Jack London in 1905, badly sprained his ankle, wore out his clothes, threw away the bag filled with gold for which he had come to those distant wild lands, lost his gun and hat, traveled by night as much as during the day, and eventually engaged in a fatal struggle with a wolf in which everything was put at stake. The love of life is an apt metaphor for a force that drive us on, even if personally we can strive no longer.

[1] Jack London, *Love of Life & Other Stories*, Published by Macmillan (1907).



D. Volchenkov, *Survival under Uncertainty*, Understanding Complex Systems,
DOI 10.1007/978-3-319-39421-3_2

The love of life might reveal itself through a phenomenon of collective intelligence, emerging in a group of many communicating individuals, that arises in consensus decision making on migrations. Millions of migrants set out to find a better life in other countries every year, even though their lives at home were not immediately threatened, but the living conditions in crudely assembled refugee villages are desperate, and they are unable to send home any of the little they earn. Many thousands of migrants risk their lives every day, fleeing upheavals and instability in their homelands, crossing the Mediterranean in small decrepit vessels in an attempt to reach European territories. However, aggregate models and economic approaches to migrations are hard-pressed to explain why migrants choose to travel to one location, when wages and opportunities are clearly superior elsewhere [18].

People might aspire to greater uncertainty, if it offers more opportunities for longer survival. Such striving can be seen in the gradual demographic shift to older ages in the population.

## 2.2 Age-Specific Survivorship

Population dynamics describing changes in the size and structure of population in a particular country due to fertility, mortality, and mobility is studied by demography. Demographic data are based on censuses and sample surveys set up to continuously record demographic events such as births, deaths, and changes in the usual place of living.

Demographic information is important because it can be used to analyze different life history tactics. This analysis is usually based on a convention about the equivalence of provisory stages in individual life histories, i.e., stages contingent upon a particular culture/tradition, called *age classes*, including young, adult, baby-boomer, or 969 lunar months old, useful for the purpose of interpretation within a particular culture.

Sampling a population with respect to age frequency is a stochastic process by nature, as it always involves a probability of age detection.[2] The higher the probability of a demographic event (survival to a given age), the higher the relative frequency of its occurrence over the population, and the more certain we are that an individual of that age will be found in future censuses.

The two most basic parameters in population demography are an individual's likelihood of surviving and an individual's likelihood of breeding. These basic parameters are usually combined in the life chronicle as *age-specific survivorships*,

$$p_a = \frac{N_a}{N} \approx \Pr\{\text{age} = a\} , \qquad (2.1)$$

[2]For instance, all 500-plus claims of unrivaled longevity over age 120 reported in the official Soviet censuses failed birth-record validation and other tests [19].

the fraction of individuals of age $a = 0, 1, \ldots$ in a total population of size $N$, which thus approximates the likelihood (probability) of living to a given age. Clearly, the survivorship (2.1) satisfies the natural normalization condition for probabilities, $\sum_{a \geq 0} p_a = 1$.

By interpreting age-specific survivorship as the approximate probability of survival, we view survival as a random process characterized by some probability function quantifying the chances of every age group to reach the older ages.

## 2.3 Population Pyramids and Survival Probability

A *population pyramid*, also called an age pyramid, is a traditional way of visualizing and explaining the age structure by portraying the distribution of various age groups in the population. When we draw this, each age group is represented by a horizontal bar, and each bar is placed one above the other with the youngest at the bottom and the oldest at the top. The shape of the pyramid is determined by a series of horizontal bars which eventually narrow to a pointed top, where lifespans are short and mortality rates high.

In an expansive population pyramid, children are the most numerous group, and old people the least, thereby shortening the lengths of bars for successive age groups (see Fig. 2.1a–c). In the figure, we have shown three example models of expansive population pyramids, which do not correspond to any particular country or region of the world, but are important for the forthcoming discussion. Population pyramids are wide at the base, indicating high birth and death rates. They are symmetric with respect to the central axis, since we have disregarded information about gender-specific population differences, and the sizes of horizontal bars are normalized in such a way that all pyramids have unit total area.

Figure 2.1a shows the pyramid for a population in which survival is a matter of flipping a fair coin, i.e., 'heads' and 'tails' are equally likely to occur ($p = 1/2$). The survival rate in such a case decreases uniformly:

$$s_a = \frac{N_{a+1}}{N_a} = \frac{1}{2} \,,$$

continually through all age classes, so that the probability of survival decays exponentially with the age $a$, i.e.,

$$\Pr\{\text{age} = a\} = 2^{-(a+1)} \,,$$

satisfying the natural normalization condition

$$\sum_{a=0}^{\infty} \Pr\{\text{age} = a\} = \sum_{a=0}^{\infty} 2^{-(a+1)} = 1 \,.$$

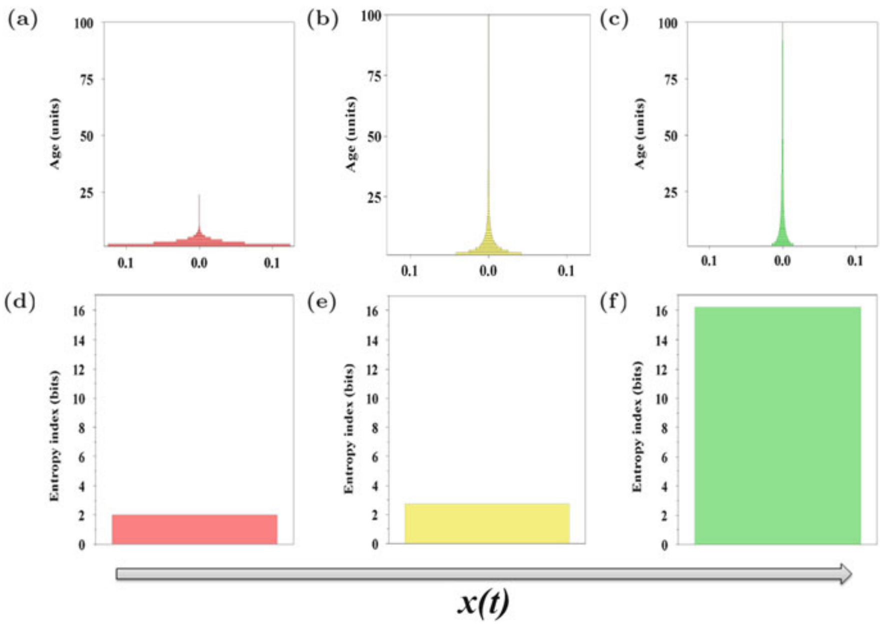


**Fig. 2.1** Models of expansive population pyramids, normalized to unit total area, for different survival probability distributions: (**a**) exponential $\Pr\{\text{age} = a\} = 2^{-(a+1)}$, (**b**) algebraic $\Pr\{\text{age} = a\} = 1/(a+1)(a+2)$, (**c**) Zipfian $\Pr\{\text{age} = a\} = Z_\varepsilon/a^{1+\varepsilon}$, with $\varepsilon = 0.1$. (**d**)–(**f**) Entropy values for the corresponding probability distributions

The probability of survival in a population modeled by the second pyramid (Fig. 2.1b) exhibits the algebraic decay

$$\Pr\{\text{age} = a\} = \frac{1}{(a+1)(a+2)}\,, \quad \sum_{a=0}^{\infty} \Pr\{\text{age} = a\} = 1\,,$$

with the survival rate

$$s_a = \frac{N_{a+1}}{N_a} = \frac{a+1}{a+3}\,.$$

Finally, the age structure represented by the third population pyramid (Fig. 2.1c) is characterized by an essential longevity of group members modeled by the famous discrete Zipf distribution [16],

$$\Pr\{\text{age} = a\} = \frac{a^{-1-\varepsilon}}{\zeta(1+\varepsilon)}\,, \quad \varepsilon = 0.1\,,$$

in which the normalization constant $\zeta(z) = \sum_{m=1}^{\infty} m^{-z}$ is the Riemann zeta function [20]. The Zipf distribution approximates many types of data studied in the physical and social sciences [17].

Despite the fact that the total areas of the population pyramids shown in Fig. 2.1a–c are equal, their shapes differ strikingly, revealing the alternative group survival tactics. While the first group (Fig. 2.1a) is characterized by the maximal birth rate and the highest mortality rate among the three model populations, the third group (Fig. 2.1c) has the minimal birth rate, but also the maximal likelihood of centenarians in the population.

In the next section, we show that every group survival tactic encoded by the shape of the population pyramid can be assessed by a certain degree of survival uncertainty/opportunity quantified by the entropy index.

## 2.4 Spring, Autumn, and Entropy

The ancient Chinese chronicle known as the *Spring and Autumn Annals* (*Chūnqiū*), which covers all the main events occurring each year of the State of Lu, from 722 to 481 BC, is tersely written, with each entry limited to only ten characters on average. It is not clear whether these annals were ever originally intended as a chronicle for human readers or as "ritual messages directed primarily to the ancestral spirits" [21], for the text contains virtually no elaboration on events, and a number of the annual entries are only a single character long, such as *zhōng* "a plague of insects", or *míng* "a bollworm". According to tradition, it was Confucius who edited the chronicle, deliberately giving it a terse style in order to convey lofty principles in just a few subtle words [21].

*But just how concise can a chronicle be and yet remain meaningful?* To find out, let us consider a terse life history chronicle in which only the sequence of survival events are recorded for every group member. Since there are only two possible outputs in the process of survival ('survived' or 'died'), each survival event in such a life history requires precisely one bit of data to be recorded:

$$\log_2 2 = 1 \text{ bit}.$$

In information theory, one bit is defined as the amount of information gained by an observer when the output of the binary random variable (0 or 1) becomes known. Therefore, in order to record a block of $\tau$ consequent survival events (the entire life duration) in the chronicle, formally speaking, a storage capacity of precisely $\tau$ bits would be required.

However, it is obvious that the storage capacity actually required can be substantially less than $\tau$ bits, since for every individual history we need only record the *age at death* for every individual in the population. The age at death has the most important meaning when assessing survivorship over the population. Two different information quantities are relevant to such a concise chronicle.

First, the age at death $\tau$ calls for $\log_2 \tau$ bits of storage capacity, so that

$$J = \lim_{\tau \to \infty} \sum_{t=0}^{\tau} \frac{N_\tau}{N} \log_2 \tau = \sum_{\tau \geq 0} p_\tau \log_2 \tau \equiv \langle \log_2 \tau \rangle \tag{2.2}$$

bits are required on average in order to record an entry in the chronicle, provided that the age-specific survivorship $p_\tau$ is known.

Second, we can assess the reported age of death $\tau$ in terms of how *often/rare* survivals to this age are in the population. The degree of ubiquity of the demographic event can be quantified by the reciprocal age-specific survivorship

$$\frac{1}{p_\tau} \approx \frac{N}{N_\tau} \,,$$

which calls for

$$I(\tau) \equiv \log_2 \frac{1}{p_\tau} \tag{2.3}$$

bits of stored data.

The quantity defined by (2.3) is often called the *information content* [24]. It quantifies the degree of ubiquity of the age at death $\tau$ over the population, provided that the age-specific survivorship $p_\tau$ is known. Taking the average of the information content (2.3) over all ages at death occurring in the population, we estimate the average required storage capacity to be

$$H = \sum_{t=0}^{\infty} p_\tau \log_2 \frac{1}{p_\tau} \equiv \langle I(t) \rangle \text{ bits} \,, \tag{2.4}$$

where we use the convention $0 \times \log_2 0 = 0$. The amount of information (2.4) arises from the classification of the reported demographic event with respect to its relative rarity of occurrence over the entire population. The quantity defined in (2.4) is nothing else but *entropy*, a key concept of information theory introduced by C. Shannon.[3]

It is clear that the amount of storage capacity required on average to record an entry of the chronicle does not coincide with the mean information content of the entry, i.e.,

$$J \neq H \,.$$

[3] "I thought of calling it 'information', but the word was overly used, so I decided to call it 'uncertainty'. Von Neumann told me, "You should call it entropy..." ". From the conversation between Claude Shannon and John von Neumann regarding what name to give to the attenuation in phone-line signals [22].

Furthermore, in a terse chronicle expected to be written under the concept of economy, the mean information content of an entry $H$ should be *maximal* for the available amount of storage capacity per entry $J$. This *principle of terseness* can be viewed as the version of the famous *principle of least effort* suggested by Zipf [16], which describes the tendency towards the minimum amount of effort that is necessary to achieve the maximum result.

Investigating speech as a natural phenomenon, Zipf had discovered that an inclination to economy can be a criterion regulating any aspect of human behavior, including linguistic changes such as word shortening, word borrowing, and changes of meaning. However, thanks to the principle of least effort, an equilibrium with a maximum of economy is always preserved. While only a few words are used very often, many or most are used quite rarely, so that the frequency of a word ranked $n$ th in the word frequency list satisfies

$$P_n \propto \frac{1}{n^{\alpha}} , \tag{2.5}$$

where the exponent $\alpha$ is almost 1.

Indeed, the maximum entropy (2.4) is attained for the power law distributed age-specific survivorship $p_\tau$, for the given average amount of information per entry of the chronicle $J$ [37].

We maximize the value of the entropy $H$ (2.4) subject to two constraints: the first constraint is the natural normalization condition for probabilities, viz., $\sum_{\tau\geq 1} p_\tau = 1$, and the second constraint is nothing but (2.2), i.e., $J = \langle \log_2 \tau \rangle$. Introducing a Lagrange multiplier $(1+\varepsilon)$ corresponding to the constraint (2.2) and another Lagrange multiplier $(\log_2 Z - 1)$ for the natural normalization condition for probabilities, we obtain the variation functional

$$\begin{aligned}\mathcal{H} = &-(1+\varepsilon)\Big(\sum_{\tau\geq 0} p_\tau \log_2 \tau - J\Big) \\ &-(\log_2 Z - 1)\Big(\sum_{\tau\geq 0} p_\tau - 1\Big) - \sum_{\tau\geq 0} p_\tau \log_2 p_\tau \ .\end{aligned} \tag{2.6}$$

Varying it with respect to $p_\tau$ yields the extremum condition

$$-(1+\varepsilon)\log_2 \tau - \log_2 Z - \log_2 p_\tau = 0 ,$$

with the explicit solution

$$p_\tau = \frac{\tau^{-1-\varepsilon}}{\zeta(1+\varepsilon)} , \tag{2.7}$$

in which $\zeta(1+\varepsilon)$ is the Riemann zeta function converging for $\varepsilon > 0$. The value of $1+\varepsilon$ is determined self-consistently by $J$, as

$$J = \frac{1}{\zeta(1+\varepsilon)} \sum_{\tau \geq 0} \frac{\log_2 \tau}{\tau^{1+\varepsilon}} \approx \frac{1}{\varepsilon} - \gamma + O(\varepsilon) \; , \tag{2.8}$$

where $\gamma = 0.577\,21\ldots$ is the Euler constant, so that

$$\varepsilon \approx \frac{1}{J+\gamma} \; . \tag{2.9}$$

Therefore, the maximum entropy (2.4) is equal to

$$\begin{aligned} H_{\max} &= \textstyle\sum_{\tau \geq 1} p_\tau \log_2 p_\tau \\ &= \log_2 \zeta(1+\varepsilon) + (1+\varepsilon) J \\ &\approx \frac{1}{\varepsilon} - \gamma + 1 - \log_2 \varepsilon + O(\varepsilon) \; , \end{aligned} \tag{2.10}$$

and, finally,

$$H_{\max} \approx J + 1 - \log_2 \varepsilon + O(\varepsilon) > J \; . \tag{2.11}$$

☞ In a terse chronicle, the mean information content of an entry exceeds the amount of storage capacity required for the entry.

Entropy (2.4) can also be viewed as a measure of uncertainty in the group age-structure resulted from the survival process and can be calculated for any given age-specific survival probability function $P(t)$ that decays fast enough with time.

Let us consider a demographic census consisting of $n \gg 1$ demographic events, in which every survival age, $t = 0, \ldots, \tau$, is observed precisely $n_t$ times, being sampled according to the survival probability $P(t)$. The total number of possible samples of size $n = \sum_{t=0}^{\tau} n_t$ with $n_t$ counts is then given by the multinomial coefficient

$$M_\tau = \frac{n!}{n_0! \cdots n_\tau!} \; , \tag{2.12}$$

quantifying the multiplicity of the population age structure. For large enough age groups, the use of Stirling's approximation, viz.,

$$\ln n! \approx -n + n \ln n \; , \tag{2.13}$$

yields the well known estimate for the total number of age-structure samples that can be found over a long enough chronicle:

$$M_\tau \approx 2^{nH_n(\tau)}\ , \quad H_n(\tau) = -\sum_{t=0}^{\tau} \frac{n_t}{n} \log_2 \frac{n_t}{n}\ , \tag{2.14}$$

where the fraction $n_t/n$ is nothing but the age-specific survivorship (2.1), approximating the probability $P(t)$ of survival up to the age $t$ in the population, for a large enough sample size $n$.

Taking into account all possible lifespans that might be observed in the chronicle, we then obtain the limiting functional $H = \lim_{\tau\to\infty} H_n(\tau)$, well defined for all survival probability functions $P(t)$ that decay fast enough with time. Therefore, in order to assess the uncertainty in the group age-structure (2.12) of the population, we may use the quantity

$$H = -\sum_{t\geq 0} P(t) \log_2 P(t)\ , \tag{2.15}$$

which essentially coincides with the entropy defined in (2.4). In thermodynamics, entropy is commonly understood as a measure of disorder, quantifying the number of specific microscopic ways in which a macroscopic system may be arranged [23]. With the use of the entropy (2.4), we can assign a score to every population age-structure according to the number of ways it can be achieved given the age-specific survival probability function $P(t)$.

The survival entropy introduced in (2.4) also quantifies the additional amount of information required to detail the exact age structure of the population that remains ambiguous under such a terse statistical description: the lower the entropy, the less information we lack about the actual age structure of the population. Conversely, if the entropy increases, we actually lose much of the information about the age-structure available initially.

The survival entropy (2.4) can also be interpreted as a measure of uncertainty of the survival process in a population characterized by the age-specific survivorship $p_\tau$. The maximum level of uncertainty is attained when all $n$ ages are present equiprobably in the population, so that $H_{\max} = \log_2 n$. The maximum score thus increases with the number of age classes $n$. However, no additional information is required ($H_{\min} = 0$) if all group members are of the same age, so that a single age class suffices to describe the age structure precisely.

## 2.5 Survival Entropy is Finite

The value of the entropy (2.4) calculated for a survival probability function that decays fast enough with time is finite.

Most survival durations are quite short in the model examples of expansive populations shown in Fig. 2.1a–c, and therefore the storage capacity required for recording the survival history over a large enough group of individuals can be further reduced. For an exponentially decaying survival probability function $\simeq r^{-t}$, with $r > 1$, it follows from (2.4) that

$$H \simeq r \log_2 \frac{r}{(r-1)^2} ,$$

and

$$H \simeq r \frac{|\zeta(1,r)|}{\ln 2} ,$$

for the algebraically decaying survival probability function $\propto t^{-r}$, with $r > 1$, where $|\zeta(1,z)|$ is the absolute value of the first derivative of the Riemann Zeta function, which converges when the real part of $z$ is greater than 1. The function $\zeta(z)$ and its first derivative $\zeta(1,z)$ diverge as $z \to 1$, so the value of the entropy function may be particularly big for survival distributions obeying Zipf's law, i.e., $\propto t^{-1-\varepsilon}$, for small enough $\varepsilon > 0$.

In all the cases considered, the value of the entropy remains finite. Figure 2.1d–f visualizes the relative values of the entropic index (2.4) by means of vertical bars for all three modeling populations. The population characterized by the maximal birth and death rates (Fig. 2.1a) has the minimal entropy value Fig. 2.1d, while the value of the entropy index is maximum for the population with the maximum likelihood of centenarians (Fig. 2.1f).

☞ Living longer offers more opportunities, yet increases the uncertainty of future survival.

## 2.6 The Maximum Age Diversity Principle

The entropy provides a unified quantitative way to analyze systems with broadly dissimilar physical properties, as it requires no assumptions about the types of correlations between variables [25].

The concept of entropy was originally introduced by Clausius in the context of the second law of thermodynamics, as a quantity that always increases in a spontaneous process of structural changes in an isolated system [26]. The second law of thermodynamics provides us with a variation principle for determining the equilibrium state of an isolated system as a state of maximum entropy: if at any instant a thermodynamic system is not in equilibrium, then its structure will evolve toward the equilibrium configuration characterized by maximum entropy.

The rationale behind maximizing entropy is that the distribution that can be achieved in the largest number of ways is the most likely distribution to be observed. Thus, the evolving thermodynamic system tends to increase its entropy and appears to be driven by an apparently macroscopic phenomenological *entropic force* (resulting from particular underlying microscopic forces [27]) towards the equilibrium state of maximum entropy [28]. A natural correspondence between statistical mechanics and information theory is well established [29,30]: the entropy of statistical mechanics and the information entropy of information theory are basically the same thing.

In our model example of three populations, with age structures governed by the simple survival probability functions (Fig. 2.1), the principle of maximum entropy can be interpreted in the following way:

☞ If there exists an opportunity to improve the quality of well-being (by improving access to food, water, and medical care, or simply by migrating to a safer and more prosperous place), then the population age structure will evolve toward the configuration of maximum entropy.

In our example, it is the third population pyramid (Fig. 2.1c) enjoying the more opportune Zipf probability distribution of ages, which is characterized by the maximum likelihood of longevity and the maximal value of entropy (Fig. 2.1f). This age composition will attract the evolution of the age structures in the other two populations if a continuous life-history path between them is possible.

There seems to be no mystery in the tendency of the entropy of a surviving population to increase, as the individuals belonging to the third group characterized by the maximal likelihood of longevity (Fig. 2.1f) will have greater chances for reproduction and will therefore occur more frequently than others as time goes on.

☞ A population striving towards the maximum age diversity (entropy) will undergo a demographic shift to older ages.

Nowadays, we are used to reading about people living much longer in various parts of the world, as compared with the situation two decades ago, since virtually all countries have taken great strides in reducing mortality [31]. The average age of death increased from 46.7 in 1990 to 59.3 in 2013, and global life expectancy for both sexes increased from 65.3 years in 1990 to 71.5 years in 2013 as a result of declining fertility and the demographic shift in the world population to older ages. The dramatic increase in average life expectancy is visible essentially within the older population as an increase in the number and proportion of people at very old ages.

In many countries, the 'oldest old' are now the fastest growing part of the total population, and the global number of centenarians is projected to increase tenfold between 2010 and 2050, or even more, as previous population projections often underestimated decreases in mortality rates among the oldest old [32]. Such a steady increase in survival in all age groups is viewed as part of a recent major transition in human health that is spreading around the globe at different rates and along different pathways [33], including a decline from high to low fertility, a steady increase in life expectancy at birth and at older ages, and a shift in the leading causes of death and illness from infectious and parasitic diseases to non-communicable diseases and chronic conditions.

Although it was not until the twentieth century that mortality rates began to decline substantially within the older age groups, the rise in life expectancy is by no means a recent phenomenon. The data on life expectancies between 1840 and 2007 show a steady increase, averaging about 3 months of life per year [33].

Interestingly, major changes in age structure can occur with time, not only in humans, but also among wild populations in other species. In particular, within- and between-seasonal trends, revealing extraordinary lifespans of wild-caught individuals of the Mediterranean fruit fly which survived in the laboratory for 170 days and more, far beyond the average lifespan of 30 days for laboratory-born reference cohorts, has been reported recently [34]. The causal mechanisms for this extended longevity in the field rather than in the laboratory remain uncertain. Indeed, the origin and evolution of aging remains one of the most important unsolved problems in biology [35].

## 2.7 Persistent Striving for More Opportunities

Let us assume for simplicity that variations in the age structure of a population (depending on a complex nexus of social, economic, and historical factors) can be assessed in a long run by using a single measurable parameter, referred to vaguely as ‘well-being’, or the quality-of-life index, which we denote by $x$. Such a synthetic measure is intended to link the degree of subjective life satisfaction to objective determinants of the quality of life underlying survivorship in the population. Then the formal expression for the entropic force driving a population to form a more opportune age structure can be obtained.

There have been numerous attempts to construct a measure of social and economic well-being in a single statistic by combining a variety of different factors that are thought to influence the quality of life. It has long been accepted that material well-being alone (measured by GDP per capita) cannot explain the broader quality of life in a country. The specific problems of measures suggested so far are the selection bias in the factors that are chosen to assess quality of life and the arbitrariness of weights assigned to different indicators. There is also a problem measuring the highly volatile factors and intangibles that may make up, for instance, a personal emotional experience.

We show that the variability of precarious factors makes the results of these measurements essentially dependent on the entire previous history of well-being in the population, as well as on the speed and scale of its improvements.

Let us assume that we have a comprehensive measure of life quality $x$, assessing the entire range of complex factors determining opportunities in the society for a healthy, safe, and prosperous life in the years ahead. The higher the value of $x$, the better the reported quality of life in the given society.

We suppose that the population develops over a very long time $T$, and that the population age structure can be specified by the current value of $x$, in such a way that growing $x$ corresponds to increasing longevity. We focus on possible life history paths, the trajectories of the quality-of-life index in time $x = \chi(t)$ that the society can follow during some time $0 \leq t \leq T$ into the prosperous future, which are the microscopic states of the population. It is clear that the higher values of $x$ (the macroscopic variable) correspond to many more different microscopic states, i.e., different trajectories $\chi(t)$.

Therefore, as time goes on, the evolution of the developing society toward the future will seem as though a force is pushing the population towards the more opportune age structure corresponding to the higher values of the macroscopic variable $x$ with greater microscopic support. This force is called entropic because it is proportional to the entropy gradient, as is shown below.

Let us follow the particular path $\chi(t)$ of a society striving into the prosperous future. Below, we follow the recently developed investigations in statistical mechanics [27, 36]. Unlike socio-economic parameters which can be measured in financial terms, it is harder to make objective or long-term measurements of the quality of life experienced by groups of people.

We assume that any quantitative measurement, in which respondents are basically asked about the quality of their everyday emotional experiences, is probabilistic by nature. The conditional probability of finding the value of quality-of-life index equal precisely to $x$ provided the measurement takes place at time $t$ is

$$P(x|t) = \delta\big(x - \chi(t)\big) \ , \tag{2.16}$$

where $\delta(\cdot)$ is the Dirac delta function, a distribution that is zero everywhere except at zero.

According to Bayes theorem, the unconditional probability density for finding a pair $(x, t)$ in a measurement performed at random is

$$P(x,t) = P(x|t)\pi(t) = P(t|x)\pi(x)\ , \tag{2.17}$$

where $\pi(t)$ is the probability density for the measurement of the quality-of-life index to be performed at time $t$, $\pi(x)$ is the probability density that the outcome of the measurement that took place at arbitrary time is precisely $x$, and $P(t|x)$ is the reciprocal conditional probability that we obtain the value $x$ as a result of measuring the quality-of-life exactly at time $t$.

Let us suppose that measurements are performed at uniformly random moments $t$, so that

$$P(x,t) = \frac{1}{T}\delta\big(x - \chi(t)\big)\ .$$

The probability density $\pi(x)$ which determines the outcomes of these measurements is nothing else but the stationary probability density of the macroscopic state $x$, since

$$\pi(x) = \int_0^T P(x,t)\mathrm{d}t = \frac{1}{T}\int_0^T \delta\big(\chi(t) - x\big)\mathrm{d}t = \big\langle P(x|t)\big\rangle\ . \tag{2.18}$$

Then, the time derivative of $P(x, t)$ is

$$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial x}\left(\dot{\chi}P\right)\ , \tag{2.19}$$

because $\chi(t)$ does not depend on $x$. Let us now multiply both sides of (2.19) by $\dot{\chi}(t)$, average the result over the time of measurements on record to obtain

$$\frac{1}{T}\int_0^T \dot{\chi}\frac{\partial P(x,t)}{\partial t}\mathrm{d}t = -\frac{1}{T}\int_0^T \frac{\partial}{\partial x}\big[\dot{\chi}^2 P(x,t)\big]\mathrm{d}t\ , \tag{2.20}$$

perform the partial integration on the left-hand side,

$$\begin{aligned}\frac{\dot{\chi}(T)P(x,t) - \dot{\chi}(0)P(x,0)}{T} - \frac{1}{T}\int_0^T \ddot{\chi}(t)P(x,t)\mathrm{d}t& \\ = -\frac{1}{T}\int_0^T \frac{\partial}{\partial x}\big[\dot{\chi}^2 P(x,t)\big]\mathrm{d}t\ ,&\end{aligned} \tag{2.21}$$

and note that, in all processes for which the function $\dot{\chi}(t)$ is well defined, the first term in the latter equation vanishes for long enough time $T \to \infty$ at every typical point $\chi(t)$ where $P(x, t)$ does not diverge.

Introducing the expression for the unconditional probability (2.17) into (2.21), we then obtain

$$\underbrace{\pi(x)\,\frac{1}{T}\int_0^T \ddot{\chi}P(t|x)\mathrm{d}t}_{\mathsf{F}(x)\ \text{'force'}} = \frac{\mathrm{d}}{\mathrm{d}x}\left[\underbrace{\pi(x)\,\frac{1}{T}\int_0^T \dot{\chi}^2P(t|x)\mathrm{d}t}_{\mathsf{T}(x)\ \text{'temperature'}}\right]. \quad (2.22)$$

With the use of the special notations for the conditional time averages, viz., $\mathsf{F}(x)$ for the time-averaged 'force', the mean conditional acceleration on record, and $\mathsf{T}(x)$ for the time-averaged 'temperature', the mean conditional squared rate on record, we get the relation for the conservative quantities depending only on the result of an instantaneous measurement of well-being $x$, but neither on the time $t$, nor on the particular path $\chi(t)$ the society followed to reach its current state $x$:

$$\pi(x)\mathsf{F}(x) = \frac{\mathrm{d}}{\mathrm{d}x}\big(\pi(x)\mathsf{T}(x)\big)\ . \quad (2.23)$$

In particular, it follows from the latter equation that if the mean conditional squared rate $\mathsf{T}(x) \approx 0$, then the time-averaged force is also $\mathsf{F}(x) \approx 0$ for any probability distribution $\pi(x)$.

☞ If the quality of well-being has not changed substantially for a long enough time, people would not strive for future improvements.

Equation (2.23) has a unique solution for the stationary probability density of the possible outcomes of quality-of-life measurements:

$$\frac{\pi(x)}{\pi_{\max}} = \frac{1}{\mathsf{T}(x)}\exp\left[\int_0^x \frac{\mathsf{F}(x')}{\mathsf{T}(x')}\mathrm{d}x'\right], \quad (2.24)$$

where $\pi_{\max}$ is the maximum probability of the outcome observed for the most probable value of the index $\tilde{x}$.

☞ The rating of life quality over the population is determined by the entire history of well-being, as well as by the speed and scale of its variations.

## 2.8 Potential Force Enhancing Age Diversity

It follows from (2.24) that the most probable value of the index $\tilde{x}$ is reached when

$$\mathsf{F}(\tilde{x}) = \frac{d}{dx}\mathsf{T}(\tilde{x}) \ . \tag{2.25}$$

In the vicinity of the most probable configuration (2.25), we can approximate the mean conditional squared velocity $\mathsf{T}(x)$ as

$$\mathsf{T}(x) \approx \mathsf{T}(\tilde{x}) + (x - \tilde{x})\mathsf{F}(\tilde{x}) \ . \tag{2.26}$$

As for any outcome of measurements $x$, the stationary probability distribution $\pi(x) \neq 0$, we can rewrite (2.23) as

$$\mathsf{F}(x) = \frac{1}{\pi(x)}\frac{d}{dx}\left[\pi(x)\mathsf{T}(x)\right] , \tag{2.27}$$

and then use the approximation (2.26) in order to obtain the expression for the potential force in the vicinity of the most probable outcome of measurements $\tilde{x}$:

$$\frac{\mathsf{F}(\tilde{x}) - \mathsf{F}(x)}{\mathsf{T}(\tilde{x})} = \frac{d}{dx}I(x) \ , \quad I(x) \equiv \ln\frac{1}{\pi(x)} \ . \tag{2.28}$$

Here $I(x)$ is the information content (actually, with ln instead of $\log_2$) of all well-being conditions ever experienced by the population.

> We assume that in equilibrium the value of the most probable rating of life quality $\tilde{x}$ has to be stable (at least for a while), and therefore the time-averaged 'force' $\mathsf{F}(\tilde{x}) = 0$, and the time-averaged 'temperature' is a constant independent of $x$, i.e., $\mathsf{T}(\tilde{x}) \equiv \tilde{\mathsf{T}}$.

Finally, we can transform (2.28) to the following expression

$$f(x) \equiv \frac{\mathsf{F}(x)}{\tilde{\mathsf{T}}} = \frac{d}{dx}I(x) \ , \tag{2.29}$$

with the gradient of the information content $I(x)$ playing the role of the potential. The conservative force (2.29) drives the possible outcomes of measurements towards the most probable configuration $\tilde{x}$.

In practice, age-specific approximations over the large, albeit finite groups of respondents are available from surveys rather than the probability density distribution $\pi(x)$. The probability that the quality of well-being reported at an arbitrary moment of time by an individual of age $a$ is precisely equal to $x$ is

$$p_a(x) \approx \frac{n_a(x)}{n(x)} ,$$

where $n_a(x)$ is the number of individuals of age $a$ who rated their life quality as $x$, and $n(x)$ is the total number of individuals reporting the value $x$. The age-specific information content (measured in bits) experienced by the cohort of age $a$ is then equal to

$$I_a(x) = \log_2 \frac{1}{p_a(x)} ,$$

and the information content $I(x)$ in (2.29) is given by the average of $I_a(x)$ taken over all age cohorts, i.e.,

$$I(x) \approx \sum_{a \geq 0} p_a(x) I_a(x) = H(x) , \tag{2.30}$$

that is, it is nothing but the entropy of survival by the age $a$ corresponding to the particular level of well-being $x$.

We conclude that, if the age structure of the population depends on the level of its well-being $x$, for which the age composition uncertainty is characterized by entropy $H(x)$, then a potential force

$$f(x) = \frac{\mathrm{d}}{\mathrm{d}x} H(x) \tag{2.31}$$

should be exerted on the population, driving its age structure toward the maximum age diversity possible for the given level of well-being $x$. When the entropy $H$ attains its maximum value, the entropic force (2.31) tends to zero, whence the age composition of the population reaches equilibrium.

☞ Persistent striving for more opportunities drives the population age structure toward the maximum age diversity.

## 2.9 Age Diversity in the Global Population Is Growing Steadily, Data of the UN Statistics Division Suggests

We have used the official demographic data on the population age structure collected over 231 countries and areas during the period of 66 years, from 1948 till 2014, as provided by the UN Statistics Division at http://data.un.org. The age diversity in the

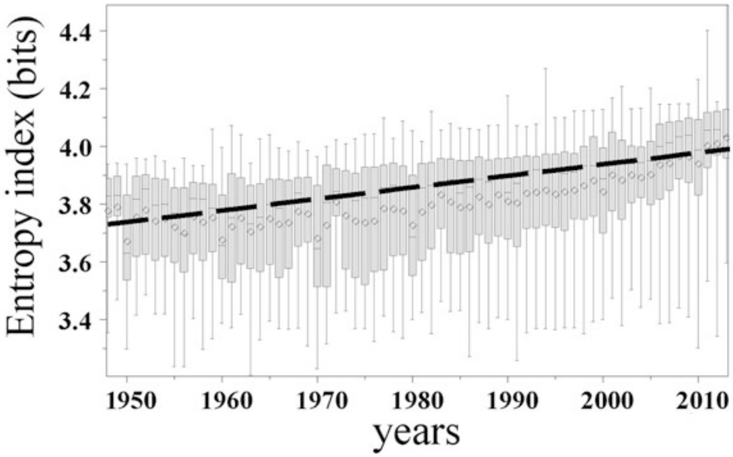


**Fig. 2.2** The box plots represent the distributions of the survival entropy indices (2.32) calculated over 231 countries and areas of the world for 66 consecutive years. Combined statistics (for men and women) on the survival entropy indices. The *trend line* is $3.73 + 4 \times 10^{-3}(\text{year} - 1948)$

global population has been assessed through the value of the survival entropy index:

$$H(\text{area, year}) = -\sum_{i=1}^{20} p_i(\text{area, year}) \log_2 p_i(\text{area, year}), \tag{2.32}$$

calculated over 20 consecutive 5-year periods $i = 0$–4, 5–9, 10–14, 15–19, 20–24, 25–29, 30–34, 35–39, 40–44, 45–49, 50–54, 55–59, 60–64, 65–69, 70–74, 75–79, 80–84, 85–89, 90–94, and 95–99, where $p_i(\text{area, year})$ is the survivorship during the age interval $i$, in the given area at the given year.

The combined and gender specific statistics on the survival entropy index (2.32) are summarized in the form of box plots shown in Figs. 2.2 and 2.3. Each box plot comprises a central line showing the median of the data, a lower line showing the first quartile, and an upper line showing the third quartile. Two lines extend from the central box of maximal length 3/2 times the interquartile range (if it does not extend past the range of the data).

☞ Age diversity in the global population has grown steadily during the whole period of observation covered by the official demographic statistics.

It is worth noting that the increase in life expectancy, the reduced fertility and mortality rates, as well as massive migrations leveling the age composition of the local populations are among the main factors contributing to the steady growth of the average survival entropy index in the global population.

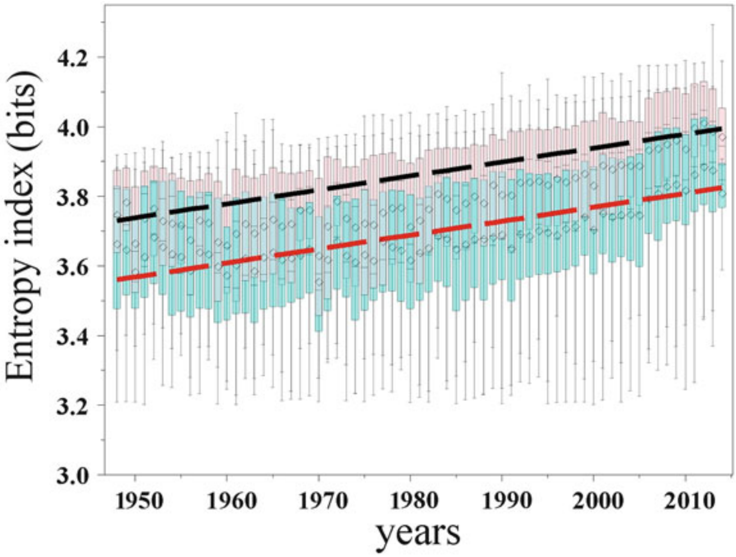


**Fig. 2.3** The box plots represent the distributions of the survival entropy indices (2.32) calculated over 231 countries and areas of the world for 66 consecutive years. The gender specific statistics on the survival entropy indices (*cyan colored boxes* are for statistics on the male survivorship, and *pink boxes* are for female statistics). The *upper trend line* is $3.73 + 4 \times 10^{-3}(\text{year} - 1948)$ and the *lower trend line* is $3.56 + 4 \times 10^{-3}(\text{year} - 1948)$

Late age migrations may play an important role in adjusting life duration statistics, as every immigrant to a new country raises that country's average longevity. If a 50-year-old man immigrates to a country, he can no longer die at an age lower than 50, and therefore ultimately contributes to the upper end of his adoptive country's age-at-death distribution. An aged individual that immigrates to some country may be attracted by the better healthcare facilities and then tend to live longer, further raising the longevity statistics, because she is already older than most people in that country who have already died.

Clearly, those developed countries that provide good healthcare facilities and massively accept migrants have the possibility of inflating their life expectancy figures.

Interestingly, age diversity in the female population is systematically higher than that in the male population (see Fig. 2.3). This is due to the fact that women live on average longer than men. In particular, for individuals born in 13 developed nations after 1880, female death rates in adults aged over 40 decreased 70 % faster than those of males [38].

## 2.10 Conclusion

In the present chapter, we have discussed the idea that the shape of a population pyramid portraying the age composition of the population can be characterized by an entropy that assesses the uncertainty of survival.

If there exists an opportunity to continuously improve the quality of well-being in finite time, then the population age structure will evolve toward the configuration of maximum entropy. Such a striving in the population would be seen as a gradual demographic shift in the population toward older ages.

We have derived the exact expression for the entropic force, provided that the age-specific survivorship depends on the current level of well-being, assessing the entire range of complex social, ecological, and economic factors.

The official demographic statistics on the age structure of the global population suggest that age diversity has grown steadily during the whole period of observation.

# 3 Life Tactics Amidst Uncertainty: Austerity Versus Progress

*Without continual growth and progress, such words as improvement, achievement, and success have no meaning*[1]

**Abstract**

There are two alternative strategies for promoting longevity in the face of uncertainty: either applying the strict austerity measures aimed at locking demand preferably at the minimal level, or choosing 'faster' behavioral strategies. A prominent solution to the dilemma of survival amidst uncertainty that has been found in Western societies is based on the development of a concept of abstract time that allows for a perpetual acceleration of the pace of life in a process of permanent modernization.

## 3.1 Introduction

Extraordinary longevity could occur when the time scales of factors contributing to the objective and subjective types of uncertainty are very different. Therefore, a passive strategy for attaining extraordinary longevity in the face of uncertainty might be to apply strict austerity measures aimed at locking the level of one's own demand at a minimum by dramatically reducing spending and increasing frugality.

In such a case, it is the level of demand that plays the role of a steady threshold (when subjective uncertainty is excluded), in the face of rapid environmental changes that aggravate the objective type of uncertainty. For instance, the survival of higher organisms during freezing and desiccation is likely to occur in the regime of abandoned metabolic control [11], when all metabolic processes are virtually switched off until the adverse environmental conditions come to an end.

[1] Benjamin Franklin.



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Special protection mechanisms may include, for example, degradation of mitochondria in frozen animals [39], since mitochondria could serve as undesirable hotspots of spontaneous chaotic biochemical reactions, so their elimination should contribute to biochemical stabilization of the cells [11].

By minimizing the level of demand, we can increase the chances for longer survival. However, a statistically efficient application of the ideal austerity strategy designed to block any metabolizing (or economic) activity, if committed once cannot be safely halted later, since it would result in an uncontrolled process of degradation. In economics, austerity just undermines any economic activity, causing a long, drawn-out period of decline in living standards, eroding society. In the same way, in a regime of 'slow death', where the rate of biochemical reactions in freezing animals abandoning metabolic control is similar to that of dead organisms, the benefits of energy savings can be nullified by the detrimental effects of chaotic biochemical processes occurring in the organism [11].

## 3.2 Where Are We Going? Entropy of Survival Under Uncertainty

Let us recall that, in the stochastic model of subsistence under uncertainty, the process of subsistence starts at time $t = 0$, when the random level of demand $d$ chosen with respect to the probability distribution function $F$ does not exceed the random level of supply $s$ chosen with respect to the probability distribution function $G$. If the carrying capacity of the habitat (quantified by $s$) is not exhausted immediately, the process keeps going up to time $t = 1$ when the level of demand $d$ is drawn anew, while the level of supply $s$ keeps the previous value with probability $\eta$, but is updated anew otherwise.

Thus, the random levels of demand and supply in the model are updated synchronously if $\eta = 0$, but when $\eta = 1$, the time scales of these updates are considered to be incomparable to each other. Therefore, the probability $\eta \geq 0$ can be viewed as the *degree of environmental stability* under systematic updates of demand. The environment is precarious when $\eta = 0$, but is perfectly stable if $\eta = 1$.

In Chap. 1, we calculated the probability $P_\eta(t)$ of survival under uncertainty by time $t$. In particular, we showed that the probability of survival decays exponentially fast with time in precarious environments ($\eta = 0$), but improves gradually as the degree of environmental stability grows. In perfectly stable environments ($\eta = 1$), the probability of survival decays algebraically, and the characteristic survival time can be infinite.

We can introduce an entropy of survival into the process of subsistence under uncertainty:

$$H(\eta) = -\sum_{t\geq 0} P_\eta(t) \log_2 P_\eta(t) \ . \tag{3.1}$$

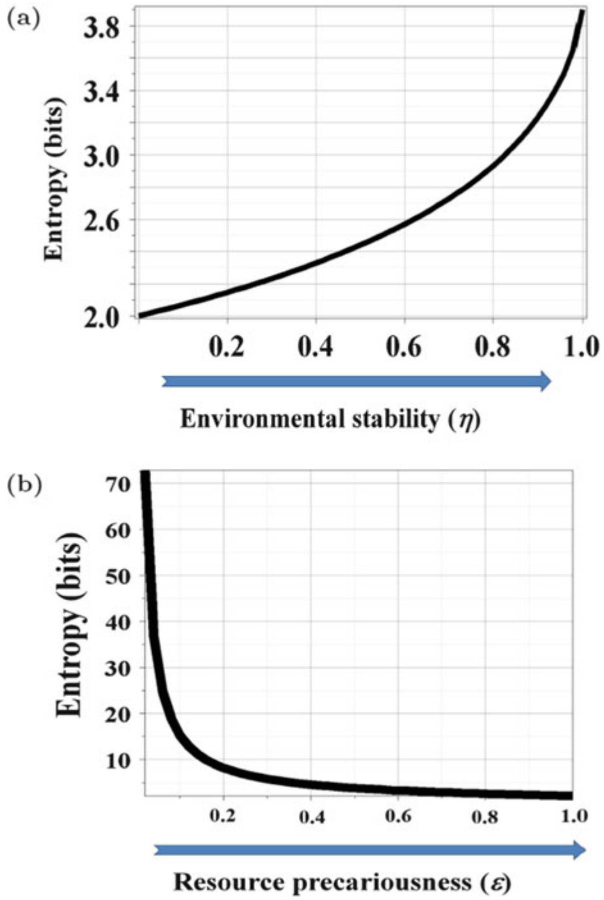


**Fig. 3.1** (**a**) The entropy of survival (3.1) grows steadily with increasing environmental stability in the stochastic model of subsistence under uncertainty, for uniformly random levels of supply and demand. (**b**) The entropy of survival (3.2) is maximal for Zipf longevity statistics attained for $\varepsilon > 0$, when available resources are plentiful, and decreases steadily as the available resources become precarious

If the probability of survival decays fast enough with time, the entropy function (3.1) remains finite for any $\eta \in [0, 1]$, except for the most favorable Zipf distribution of survival times. It is clear that the higher values of entropy correspond to enhanced survival chances in stable environments where longer survival is possible, and vice versa.

In Fig. 3.1a, we have plotted the values of the entropy (3.1) as a function of the degree of environmental stability $\eta \geq 0$, for levels of supply and demand chosen uniformly at random. The value of the entropy grows steadily with increasing environmental stability.

We have already discussed the fact that very long subsistence is more probable when living resources are always plentiful. In particular, we introduced the modeling function (1.15), viz.,

$$G_\varepsilon(x) = 1 - (1 - x)^\varepsilon \ ,$$

to provide a bountiful probability distribution for the supply threshold, in a land of plenty, forming a thin spike of probability as $x \to 1$, for small enough $\varepsilon > 0$.

The exponent $\varepsilon > 0$ in the probability distribution $G(x)$ can be viewed as the degree of precariousness in the available supply. When $\varepsilon = 1$, the supply is precarious: the density distribution of the supply threshold over the interval $[0, 1]$ is uniform, and any value of supply is possible, worsening the chances for essentially long survival. However, when the abundance of resources is guaranteed (for a small enough degree $\varepsilon > 0$), the probability of survival by time $t$ follows the Zipf law $P(t) \propto t^{-1-\varepsilon}$.

Similarly, we can introduce another entropy, with respect to the degree of precariousness of the available resources $\varepsilon > 0$ in perfectly stable environments $(\eta = 1)$:

$$H(\varepsilon) = -\sum_{t \geq 0} P_\varepsilon(t) \log_2 P_\varepsilon(t) \ , \tag{3.2}$$

in which

$$P_\varepsilon(t) = \frac{t^{-1-\varepsilon}}{\zeta(1+\varepsilon)}$$

is the power law degree distribution of survival up to time $t$ in a land of plenty, in perfectly stable environments $(\eta = 1)$. The normalization constant $\zeta(z)$ is the Riemann zeta function [20]. The entropy function (3.2) also remains finite for $\varepsilon > 0$.

The higher entropy values correspond to enhanced survival chances under plentiful resources (Fig. 3.1b). The tendency of a living system to maximize entropy should emerge over a large enough group of surviving individuals, resulting in two phenomenological entropic forces:

$$f(\eta) = \frac{\partial H(\eta)}{\partial \eta} \ , \quad f(\varepsilon) = \frac{\partial H(\varepsilon)}{\partial \varepsilon} \ . \tag{3.3}$$

Note in passing that the state of maximum entropy is attained in stable resource-rich environments, in a safe haven located in a land of plenty.

☞ Love of life drives us towards a safe haven in a land of plenty.

## 3.3 Will I Ever See My Great-Grandchildren?

My grandfather never saw his grandfather. My mother saw her grandfather once and talked with her grandmother on the phone. And I practically grew up in the care of my grandparents. As we live longer, communication between distant generations is becoming ubiquitous. *So will I ever see my great-grandchildren?*

The second law of thermodynamics deals solely with systems that do not change macroscopically with time. It determines whether or not a thermodynamic system is in equilibrium, quantifies the distance from equilibrium, and indicates the direction toward equilibrium [26]. However, it cannot help us to determine rates of changes in a non-equilibrium system, when the macrostate changes with time.

Since time is missing from the second law of thermodynamics, the only practical way of introducing it is via the rate of change of entropy, using the formal mathematical definition of the time derivative:

$$\dot{H}_\eta = \frac{\partial \eta}{\partial t} f(\eta) > 0 \,. \tag{3.4}$$

The *entropy production rate* characterizes the rate of increase in the number of age groups emerging in the population, in the given time frame, and this is always positive according to the second law of thermodynamics.

Suppose the instantaneous change to a macrostate, characterized by $\eta$, is infinitesimal during a single time step. Then, we can define the entropy production rate as the average rate of information content described by the probability function $P_\eta(t)$:

$$\dot{H}_\eta \equiv \langle \delta_t I_\eta(t) \rangle = -\sum_{t \geq 0} P_\eta(t) \log_2 \frac{P_\eta(t+1)}{P_\eta(t)} \,. \tag{3.5}$$

The entropy production rate $\dot{H}_\eta$ attains a maximum in precarious environments, i.e., in the state of dual uncertainty when the random levels of supply and demand are updated coherently, and the survival probability decays exponentially fast with time. Under singular uncertainty, the entropy production rate $\dot{H}_\eta$ is minimal within stable environments when the decay of survival probability is algebraic. The minimum entropy production rate corresponds to the situation where age diversity is maximum and therefore any specific age can play the role of a social marker, as individuals of virtually all ages are ubiquitous throughout the population.

☞ Love of life drives survivors to a state where their age is not a social marker.

We have already suggested that the entropic forces of survival drive us towards a safe haven in a land of plenty. The same state of maximal entropy is characterized by the minimum entropy production rate. Therefore, I shall hardly ever see my great-grandchildren.

☞ Although life expectancy has grown considerably over the past decades, delayed marriage and child-bearing negate the effect of improving longevity.

It is worth mentioning that, although our conclusion resembles the *minimum entropy production principle* formulated within the framework of linear irreversible thermodynamics by Prigogine [40, 41], it actually has nothing to do with the approximate variation characterization of steady states for thermodynamically open systems maintained out of equilibrium. We have considered a clearly defined stochastic model and do not need any assumption about the linearity of response processes and reciprocity of the thermodynamic system of the kind considered within linear irreversible thermodynamics.

## 3.4 Economies of Scale in the Age Production Process

The entropy production rate (3.5) provides a way to obtain a relation between the increment in the probability $\delta\eta > 0$ and the resulting age increment $\delta t > 0$. Figure 3.2a presents the relation between the entropy production rate given by (3.5) and the entropic force $f_\eta$ defined by (3.3), calculated numerically in the stochastic model of subsistence under uncertainty for uniformly random updates of the supply and demand levels.

As neither the entropy production rate $\dot{H}_\eta$, nor the entropic force $f_\eta$ is equal zero for any value of the degree of environmental stability $\eta$, the formal relation (3.4) can also be interpreted as an age production function

$$\delta t|_\eta = \left(f_\eta / \dot{H}_\eta\right)\Big|_\eta \delta\eta \,, \tag{3.6}$$

in which $f_\eta / \dot{H}_\eta$ plays the role of the *age production coefficient*.

The increment of probability $\delta\eta > 0$ characterizes the improvement in environmental stability, and can therefore be viewed as an estimate for the efficiency of applied environmental control measures. The age increment $\delta t > 0$ is then considered as an expected gain in the number of age classes emerging in the population due to the successful implementation of environmental control measures (or disease prevention policies).

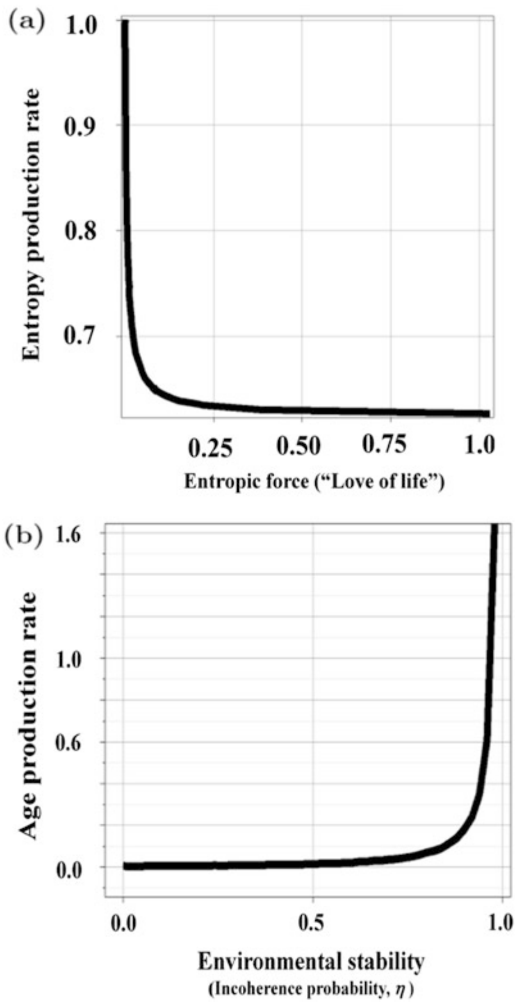


**Fig. 3.2** (**a**) The entropy production rate (3.5) versus the entropic force $f_\eta$, calculated numerically in the stochastic model of subsistence under uncertainty for the uniformly random levels of supply and demand. (**b**) The age production rate (3.6) is presented as a function of the environmental stability calculated through the entropy changes, for uniformly random updates of the supply and demand levels

Figure 3.2b presents the age production rate as a function of the degree of environmental stability assessed in the stochastic model of subsistence under uncertainty by the value of $\eta$ for the uniformly random updates of the supply and demand levels. As can be seen in Fig. 3.2b, the age production rate $\delta t/\delta\eta$ remains very small as long as the environment is not perfectly stable (when the probability $\eta$ is not close enough to 1).

Let us suppose that an individual has the flexibility to choose a particular scale for improving the control effectiveness $\delta\eta > 0$. Then, in the long run, returns to scale are important for understanding whether the resulting gain in the expected age $\delta t > 0$ also grows to scale. It is known from microeconomics [42] that increasing returns to scale in the production function, like the one defined by (3.6), typically yield economies of scale that have cost advantages for producers, i.e., the cost per unit of output generally decreases with increasing scale of production, as fixed costs are spread out over more units of output.

☞ There are economies of scale in age production under singular uncertainty, as any further improvement in the efficiency of control measures or health prevention policy will lead to increasing returns to scale.

We can conclude that there might also be *economies of scale in age production* in stable enough environments. It is obvious from Fig. 3.2b that the increasing returns to scale in age production occur mainly under singular uncertainty, when the time scales of subjective and objective uncertainty are already largely disconnected ($\eta \simeq 1$). Then the production process output, i.e., the expected increment in age $\delta t > 0$ gained by input efforts which aim to reduce the degree of coherence between objective and subjective uncertainty, more than scales in comparison to its inputs $\delta\eta > 0$. Economies of scale are known to be a key advantage for a business that is able to grow [42].

## 3.5 Why AIDS- and Other Disease-Prevention Strategies Would Not Work in Precarious Environments

Incremental control measures may be of no avail under dual uncertainty, as it appears that age production less than scales. The production rate of age (3.6) exhibits decreasing returns to scale, as $\delta t/\delta\eta < 1$, for values of $\eta$ substantially less than 1, and therefore the gain in age due to the implementation of a disease prevention policy less than scales, in comparison to a possible reduction in the degree of coherence between objective and subjective uncertainty. Moreover, any incremental control measure improving $\delta\eta > 0$ may be to no avail amidst dual uncertainty when $\eta \leq 1/2$, as it may produce virtually no output.When the objective and subjective types of uncertainty are interdependent, it is increasingly expensive (with respect to efforts made and costs incurred) to gain any significant increase in the ages observed over the population.

Speaking in terms of microeconomics, this undesirable phenomenon can be referred to as *diseconomies of scale in age production*. The costs incurred for the incremental control measures in the long run cannot be minimized if there are diseconomies of scale.

☞ Gradually improving the control efficiency brings no change to the population age structure in precarious environments.

The very idea that future diseases can be systematically avoided by present actions and by targeted modes of behavior has been developed mainly in Western societies, even though many other societies have instilled various rules of conduct designed to maintain good health [43]. However, it is noteworthy that, even in Europe, preventive measures in public health constitute a fairly recent phenomenon that has only developed over the past 200 years [44]. It has already been suggested that the very logic of inducing preventive measures by convincing people to behave today in a way that will eventually be rewarding for them in the future implies specific concepts of time and future in individuals [45].

For instance, in AIDS prevention discussed in [45], the individual is called upon to weigh up the present interest of having sex without a condom (i.e., to be in control of their current behavior) against the future interest of being healthy in 10 years—for some people an unimaginably long period of time that may have no meaning in certain societies. To begin with, it involves the prevalence of a concept of the linear flow of time independent of human interpretation.

☞ In precarious environments, people may have no reason to engage in any prevention program.

Furthermore, in a situation where there are diseconomies of scale in expected age production, people may have no reason to be deliberately engaged in any prevention program involving a change in their habits, since the long incubation period of HIV/AIDS, and of many other chronic diseases, makes the participation in such a program in precarious environments meaningless.

## 3.6 Austerity Measures Do Not Assume a Safe Exit Strategy

While under environmental stress (such as prolonged food deprivation), the organism can attempt to survive by continually sustaining its order and maintaining a low, albeit non-zero, metabolic rate, continually repairing the breakdown of cellular structures at the expense of internal energy resources. As long as the internal energy reserves of the animal persist, no detrimental changes need accumulate in the animal body [11].

In the present section, we study the speed of transition to an uncontrolled, exponentially rapid degradation (extinction), after removal of external control and inadvertent loosening of austerity measures. The crossover time between the austere life and exponentially fast decay can be calculated in the stochastic model of subsistence under uncertainty as a function of the probability $\eta$, for the case of a uniformly random choice of supply and demand levels.

On the one hand, we have seen that the survival probability function $P_\eta(\tau)$ decays according to a power law when $\eta = 1$. On the other hand, the decay is always exponential for any $\eta < 1$. Since $P_\eta(\tau)$ is a continuous function of $\eta$ for any fixed $\tau$, we have to study the behavior of $P_\eta(\tau)$ for $\eta \to 1$, where the continuity property cannot be uniform in $\tau$.

This means that, for any fixed interval of times $[\tau_-, \tau_+]$, with $\tau_-$ located in the range of validity of the power-law asymptotes for $P_{\eta=1}(\tau)$, the survival probability distribution function $P_\eta(\tau)$ may be arbitrarily close to the same power law if the value of $\eta$ is sufficiently close to 1. The asymptotic behavior of $P_\eta(\tau)$ will follow a power law until $\tau \simeq \tau_+$, but then for times $\tau \gg \tau_+$, the decay becomes exponential.

Recall that the generating function for the survival probabilities discussed in Chap. 1 is

$$\hat{P}_\eta(z) = \frac{1}{1 + (1-\eta)\gamma(z)} \left[ \frac{1+\gamma(z)}{z} - \eta\gamma(z) \right] , \tag{3.7}$$

where

$$\gamma(z) \equiv \frac{\ln(1-\eta z)}{\eta z} .$$

It follows from (3.7) that the asymptotic behavior of $P_\eta(\tau)$ is determined by the singularity of the generating function $\hat{P}_\eta(z)$ that is closest to the origin. In particular, the generating function has a simple pole for $\eta = 0$, viz.,

$$\hat{P}_{\eta=0}(z) = \frac{1}{2-z} , \tag{3.8}$$

and therefore $P_{\eta=0}(\tau)$ always decays exponentially fast with time according to the result in Chap. 1.

However, the generating function $\hat{P}_\eta(z)$ has two singularities, for the intermediate values $1 > \eta > 0$. The first pole corresponds to the vanishing denominator $1 + (1 - \eta)\gamma(z)$ in (3.7). This occurs when $z = z_0$, where $z_0$ is the unique nontrivial (positive) solution of the equation

$$-\ln(1-\eta z_0) = \frac{z_0\eta}{1-\eta} . \tag{3.9}$$

Another singularity, viz.,

$$z_1 = \frac{1}{\eta} , \tag{3.10}$$

corresponds to the vanishing argument of the logarithm in (3.7).

It is easy to see that, while $1 < z_0 < z_1$, for times much longer than the crossover time

$$\tau_c(\eta) \simeq \frac{1}{\ln z_0(\eta)} , \tag{3.11}$$

the dominant singularity of $\hat{P}_\eta(z)$ is of the polar type, and the corresponding decay of the survival probability function until time $\tau$ is exponential, i.e.,

$$P_\eta(\tau) \simeq \exp\left[-\frac{\tau}{\ln z_0(\eta)}\right] , \tag{3.12}$$

with decay rate $\ln z_0(\eta)$.

Eventually, when $\eta \to 1$, the two singularities $z_0$, defined by (3.9), and $z_1 = 1/\eta$ merge. More precisely, we have

$$\hat{P}_{\eta=1}(z) = \frac{z + (1-z)\ln(1-z)}{z^2} , \tag{3.13}$$

which obviously agrees with the exact result obtained in Chap. 1 for $\mathrm{d}F(u) = \mathrm{d}G(u) = \mathrm{d}u$:

$$P_{\eta=1}(\tau) = \frac{1}{(\tau+1)(\tau+2)} .$$

Equation (3.9) can be solved numerically and then the crossover time to the exponential decay (3.11) can be calculated for any intermediate value of the probability $\eta \in ]0,1[$. Since the value of $\eta$ can be interpreted in terms of the *characteristic time of control* applied to the level of supply

$$\tilde{t} = \frac{1}{1-\eta} , \tag{3.14}$$

we can find (numerically) the relation between the characteristic time of control and the characteristic time of crossover to the exponential decay of the probability function $P_\eta(\tau)$ (see Fig. 3.3).

As usual, the characteristic time of control as well as the characteristic crossover time to the exponential decay of the survival probability are assessed in terms of the number of random updates of demand remaining below the current level of supply. The dashed line in Fig. 3.3 shows the linear trend.

From the resulting relation it is clear that the decay in the probability of survival until time $\tau$ remains algebraic, as long as strict control is applied to the level of supply. However, once this control is removed, the species undergoes exponentially fast extinction.

In the alternative strategy, where the demand level is kept unchanged, the decay of the survival probability remains algebraic, at least as long as the strict austerity measures are intact. Therefore, maintaining the lowest possible level of consumption unchanged even if the environment is occasionally replete with resources constitutes an integral part of the subsistence strategy.

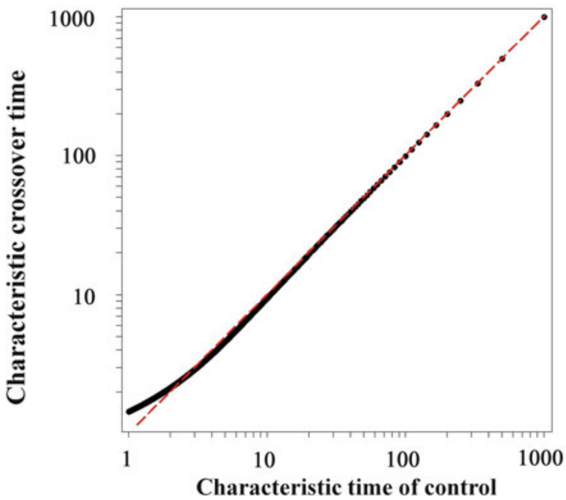


**Fig. 3.3** The characteristic time of crossover to the exponential decay of the survival probability distribution is shown versus the characteristic time of control applied to the level of supply. Both characteristic times are assessed in terms of the number of random updates to the level of demand remaining below the current level of supply. The *dashed line* shows the linear trend

☞ Survival could last as long as control measures are kept intact, although it is never guaranteed. Austerity measures do not assume a safe exit strategy.

Clearly, the passive strategy of longevity promotion by practising austerity is more vulnerable essentially in threatening environments than the active strategy based on the systematic use of control measures aimed at maintaining and increasing the carrying capacity of the habitat.

## 3.7 The Scarcity Mindset Amidst Uncertainty

People's minds work differently when they feel they may lack something [48], and it does not matter greatly what that something is, since having less always elicits greater focus [49]. As people look around the environment and decide whether to act, their perceptual experiences always take into account the available bio-energetic and psycho-energetic resources, and when these resources and energy are scarce, the environment appears to them more extreme, reflecting people's disinterest or unwillingness to act [51]. Since physical states of the body influence the way people see the environment, hills may appear steeper and distances longer, regarding the high energy costs of traversing certain environments, when people lack the energy to do so easily [69, 70].

Scarcity, essentially when it concerns subsistence, will capture a human mind by concentrating it on pressing needs and creating a tunnel vision, narrowing perspectives, and suppressing initiative and activity. On the one hand, the *scarcity mindset* helps people to focus on the survival threats and gives them a keener sense of the value of every minute or calorie. On the other hand, it shortens individual horizons, making people less forward-thinking, slower witted, and weaker willed [48].

One limited resource is also the central notion of the willpower metaphor. From the dominant social psychological perspective regarding human volition, the problem is not one task that depletes the human will, but rather the carrying out of a number of tasks that strongly rely on willpower, since this leads to a depletion of this resource (so-called *ego depletion*), and results in impaired performance in subsequent tasks also relying on willpower [46, 47].

☞ A bird in the hand is better than two in the bush: while subsisting, an invariable and troubleproof yet scarce supply is preferable to a volatile and uncertain supply that sometimes arrives in large quantities.

Since a lack of available resources makes each expense more obtrusive and more pressing, scarcity changes the way people allocate attention, eliciting a greater engagement in some problems, while neglecting others [49], and this might exacerbate the conditions of scarcity, further perpetuating survival threats. Therefore, subsistence in scarcity rapidly traps people in a certain mindset, since the stress and anxiety of not having enough resources to meet urgent needs can impair their ability to take decisions that would help alleviate the situation. It may even lead them to take irrational decisions instead [49, 71, 72]. The phenomenon of the scarcity mindset is much more widespread than it may appear at first sight.

Carrying out a self-control task required to maintain the lowest possible level of consumption under austerity tends to lead to reduced persistence when faced with difficult problem-solving tasks—willed behavior is very effortful and therefore requires more energy than behavior that does not rely on willpower [50]. In particular, processes involving willpower have been shown to be very sensitive to one's neural glucose level [56].

Since scarcity engages the mind, dominating the cognitive load of the absolute poor in developing countries, it causes neglect in other aspects of life, including the vaccination of children, washing hands, weeding crops, etc. Many ingenious schemes to better the lot of the poor fail just because the poor themselves often fail to stick to them [71].

With insufficient attention to whether benefits outweigh costs, scarcity of any kind creates a tendency to borrow [49]. In developed countries, low-income individuals borrow at great cost. This is because scarcity creates a focus on pressing expenses today, so attention will go to a loan's benefits, but not its costs. People who are less well-off really do appear to give in more readily to the temptation of making the very purchases they can't afford [73], or taking short-term, high-interest loans that make it easier to meet today's needs, while the deferred costs of such loans can make it difficult to meet future expenses [49, 72].

Busy people also tend to borrow. Facing tight deadlines, they borrow time by taking extensions, but postpone everything to the last day because they focus on urgent tasks, neglecting important tasks that seem less pressing [49]. Indeed, stressing out over a looming work deadline has the knock-on effect of poor planning for routine tasks, preventing people from figuring out the optimal use of their already scarce resources, and thereby diminishing performance [71].

An increase in the number of appealing alternatives that constitutes the nature of freedom results in an increase in the number of decisions, choices, and trade-offs that people have to make. However, chronic scarcity exhausts precious mental resources, rendering quality decision-making difficult [74]. The possibility of deliberate choice which is associated with the idea of freedom and autonomy would usually only multiply the chances of unfavorable outcomes in subsisting communities. Under such circumstances, defaults and guidelines that would reduce freedom but lessen the number of decisions that people must make on their own would dramatically improve their well-being.

☞ An individual subsisting in chronic scarcity should try to avoid freedom of choice at all costs, as it only generates more uncertainty.

It is known that if people grow up with severely limited resources, their level of existential security is causally prior to subjective emphasis on choice. In particular, although it is relatively easy to hold elections, democratic institutions are not efficient under conditions of existential insecurity [12].

Introducing ration cards that allow holders to obtain a limited amount of food which is in short supply is one of the most effective means to undermine democratic institutions, as Russia's historical experience has shown us. Under conditions of insecurity, people have a powerful need to see authority as both strong and benevolent, even in the face of evidence to the contrary [53].

## 3.8 Confronting Uncertainty by Stress Avoidance and Stress Resistance

Researchers studying centenarians agree that there is no evident specific pattern in the habits of everyday life, nor any particular level of nutrition that would guarantee longevity. Scientific explanations for longevity remain elusive, since the usual recommendations for average people to lead a healthy life would not appear to be entirely relevant to centenarians. Indeed, according to Brazilian statistics, 37 % of them were reported to be overweight at age 70, 8 % were obese, 37 % were smokers (for an average of 31 years), 44 % reported only moderate exercise, and 20 % never exercised at all [76].

As we have seen from the analysis of subsistence, it is the invariability of living conditions, the continuous support of a loving family, and the persistence of daily habits that would help to prolong life, rather than improving the degree of individual physical fitness, or a certain level of income (that might in fact be arbitrary). Interestingly, from interviews and surveys with centenarians, a behavioral pattern aimed at maintaining a stable habitat is clearly evident. In particular, the following themes come up in their reports time and time again [77]: a loving family providing for their basic needs, a faith sustaining their spirit and helping them to take no notice of their chronological age, clean living keeping them out of trouble by doing what they know is right and following their conscience.

☞ A behavioral pattern aiming at maintaining a stable habitat is clearly evident in interviews and surveys with centenarians.

Centenarians overwhelmingly cite stress as the most important thing to be avoided, by just "letting it go rather than dwelling on it" [76]. Indeed, stress occurs when changes in the external and internal environment are interpreted by an organism as a threat to its homeostasis [78, 79].

We have seen that the expected survival duration of any species is very limited when environmental changes are beyond their control. Therefore, stress plays the role of a natural indicator, revealing a lack of control over the environment. It is well-known that repeated exposure to a particular stressor favors the evolution of mechanisms that suppress an organism-wide stress reaction and, instead, activate stress-specific responses [79, 80]. Hormones associated with stress detection and avoidance also play a major role in modifications of neural circuits [81], and the repeated experience of successfully overcoming social stresses during ontogeny is known to be a prerequisite for the acquisition of a normal repertoire of behavioral strategies [82].

Finally, the ability to actively remove a stressor by either relocation or avoidance constitutes an important mechanism of adaptive evolution, which allows stress-induced effects and stress-resistance strategies to persist for many generations through maternal, ecological, and cultural inheritance [79]. Efficient stress avoidance and stress resistance strategies set the traits of longevity, providing successful acquirers with a sufficient lifespan for maturation and reproduction.

## 3.9 Human Progress as a Survival Strategy

The concept of *human progress* was originally formulated by medics in the second half of the nineteenth century, in connection with the fatal disease of *nostalgia* [52]. Much of the medical literature about nostalgia was written by military physicians, as the disease was so frequently found in the army, and young men between 20 and 30 seemed to be at greatest risk.

When the possibility of returning to the place of longing became easier, due to increased centralization and improvements in transportation and communication, the disease diminished day by day. By the 1870s the medical community had developed a narrative of progress to show how modernity had provided a cure for diseases by changing the human habitat and nature.

However, as long as humans were unable to exert significant control over their natural environment, almost no perceptible change took place from one generation to the next. Throughout most of human history, survival was insecure and the population experienced sequential cycles of growth and decline marked by catastrophic events, such as wars, famine, and epidemics [10].

The sustained economic growth that began to outpace population growth in the nineteenth century and brought unprecedentedly high levels of economic and physical security to Western societies can be understood in the framework of the proposed model of subsistence under uncertainty.

The optimal strategy for survival in the face of uncertainty should maximize the chances for longevity. There are two alternative strategies which seek to promote longevity by disconnecting the time scales on which the factors responsible for objective and subjective types of uncertainty evolve:

- either *strict control measures* must be permanently applied, in order to keep the level of supply unchanged forever, e.g., by preserving the present carrying capacity of the environment while achieving a zero rate of population growth,
- or the *rate of adaptability of one's own behavioral strategies* within the environment must be increased dramatically, to a much higher level than the maximal rate of possible environmental variations.

Since any type of control that might be applied to the environment and to the self within that environment, no matter how intelligent and efficient it may appear at present, is only temporary, the first extensive strategy of longevity promotion is always futile by nature.

☞ There is no alternative for a species pursuing survival but to maximize its own tempo of life.

An individual has often no alternative but to periodically increase the rate of its own behavior in order to promote adaptation to environmental challenges and to exert control over them, especially when in desperate ecological circumstances characterized by limited and unstable resources. For example, an inherent pattern in mammalian hibernation is periodic arousal accompanied by elevation of the metabolic rate [54].

The feeling of being in control is often referred to as the *sense of agency* [55]. Being in control seems to be a biological need and is therefore strongly linked to basic motivational processes, since it is adaptive for survival [56].

It is known that *desperation ecologies* (harsh and unpredictable ones) elicit a tendency to the 'faster' behavioral strategies, rather than the 'slower' ones implemented in more hopeful ecologies. In environments with high mortality rates, for instance, it tends to be beneficial to reproduce early to avoid the possibility of dying without reproducing [57]. Girls whose fathers are absent from home exhibit a fast suite of traits, including earlier age of menarche, first sex, and first child [58], as father absence signals high male mortality rates and unstable pair bonds, indicating more desperate local ecologies.

Individuals from desperation ecologies are interested primarily in short-term gains and are often perceived as having ‘fast’ behavioral characteristics, such as impulsivity, risk-taking, and promiscuity. They are less likely to plan ahead and less invested in their children. Alternatively, perceivers stereotype individuals from *hopeful ecologies* as having ‘slow’ life characteristics such as inclinations toward delay of gratification, cautious decision-making, and sexual restrictedness [57].

Experiments suggest that desperation and hopeful ecologies engage different psychological and behavioral strategies: when high mortality is made salient, individuals become more risk-taking and present-oriented, being interested primarily in short-term gains—particularly those who grew up with lower socioeconomic status [59].

Although more impulsive and automatic actions undertaken in emergency situations could help to avoid immediate danger to life by adrenalizing the subject, the time compression strategy nearly always exacerbates any difficult tactical situation by significantly reducing the disposable repertoire of actions and frittering away the time available for decisions. This in turn causes the involved subject to experience visual, auditory, and temporal distortions [61], and is therefore fraught with potentially maladaptive actions. Although the only reliable survival strategy is to increase the rate of one’s own behavior, time compression should be avoided if at all possible.

The logical answer to the dilemma of survival amidst uncertainty had been found empirically in Western societies through the long process of trial and error. Briefly, the solution was based on the development of a *concept of abstract time*, which is anchored to neither a natural cycle inherent in the human body, nor other environmental rhythms.

The main advantage of the new conceptualization of time is the possibility of almost infinitely dividing any temporal interval into smaller segments. On the one hand, this permanently refined the temporal structure by both thickening and thinning time, while on the other hand, it allowed us to continuously make use of previous experiences in order to deliberately maintain the time perception framework in an appropriate way. Once natural rhythms had been detached from experiences of time, we had the opportunity to identify with phenomena that had never been a part of lived and localized experiences [62].

☞ The dilemma of survival amidst uncertainty was solved by the development of a concept of abstract time, allowing for a permanent refinement of the temporal structure.

The earliest human societies were founded on agriculture and thus governed by natural time [63], i.e., by the rising and setting of the sun and by cycles of the lunar phases. Although the rise of cities and monasteries led to the establishment of temporal reference points for the coordination of various activities through the day, it was not until well into the nineteenth century that clock time was separated from the rhythms of daily life and ultimately synchronized with bodily time [64].

The shift to clock hours created a standard hour that never changed; it was the same, day or night, at any season of the year. Moreover, it made time the same across different latitudes, no matter how far north or south one traveled. Fixing the length of the hours detached the time scale from sunrise and sunset, the former cognitive reference points for time measurement, and this then made it possible to divide the hour further, into 60 min, each of which was divided further still, into 60 s.

In this manner, a mechanized device not only replaced the human and natural time scales, but also quickly reshaped our way of thinking, transforming also our way of seeing and way of perceiving the world [65]. Further indoctrination with notions of punctuality, regular attendance, and diligence was critical for the development of large-scale industry, as the coordination of large numbers of workers involved in different stages and processes of production was essential [75]. Mechanical clock time thus became what we now perceive as time and experience as time.

Meanwhile, the time fragmentation process which is such a distinctive feature of contemporary civilization is still in flux, shaped by the pressures of human activity. Although the idea of using atomic transitions to measure time was first suggested by Lord Kelvin as early as 1879 [66], the first accurate atomic clock, in which the reckoning of time is attached to the quantum fibrillations of electrons in an atom, was only built in 1955, in the UK. By 1967, the lapse of time known as the second had been further divided into 9,192,631,770 parts corresponding to the cycles of radiation in the transition between two energy levels of the caesium-133 atom.

The invention of the computer then brought meaning to these infinitesimal time intervals by using them for information processing, communication, and transactions. There then followed a process of habituation to the new technology, where information and events can be combined and recombined in endless nonlinear sequences, furnishing the infrastructure of globalization [67].

As digital networks extend ever further into the social environment, their ultrafast time-scale is affecting aspects of everyday life more and more [64]. Following the basic trends towards acceleration and nonstop activity, we have succeeded in developing and maintaining the principles of a perpetual fast-forward pace of life, according to which the less we need to set aside time specifically for certain activities, the more things we can squeeze into a single unit of time, and the more activities we subjectively believe we are having to forgo [68].

The concept of abstract time, anchored to neither a natural cycle inherent in the human body, nor other environmental rhythms, has allowed for a ceaseless acceleration of the pace of life in an incessant process of modernization. By overtaking the rate of critical changes in the environment (at least those observed in the past few centuries), we have so far succeeded in survival.

However, an elusive yet inescapable feeling of anxiety has arisen as things have been changing so rapidly across all the dimensions of life. Positing the self in the stream of human progress has turned time into a field of anxiety, because of the growing fear of not managing to get a job done on time, and in a broader context, due to the problem of coordinating one's own life stages with the pace of the metaphoric modernization train, for not only must it not stop anywhere, but it cannot help but increase its speed at every instant of time, while heading nowhere in particular.

## 3.10 Conclusion

In the present chapter, we have discussed the way the entropic forces of survival ('love of life') drive us to the state of maximum entropy, i.e., a safe haven in a land of plenty. We have also discussed the alternative strategies for promoting longevity in the face of uncertainty by applying strict austerity measures aimed at locking demand, preferably at the minimal level. Although by dramatically reducing spending and increasing frugality we could indeed increase the chances for longer survival, strict austerity measures do not assume any exit strategy.

Since the passive strategy of longevity promotion by practising austerity is vulnerable, an individual has often no alternative but to increase the rate of his/her own behavior when facing external challenges. Desperation ecologies (characterized by scarce and fluctuating amounts of resources) elicit a tendency to adopt 'faster' behavioral strategies, compared with the 'slower' ones favoured in more hopeful ecologies.

For those who are barely able to subsist, a scarce but invariable and thus reliable supply is preferable to a volatile, and thus uncertain, but abundant supply. Subsistence in scarcity forms a culturally specific way of life, as it provokes choices that exacerbate the conditions of scarcity, further perpetuating survival threats.

We have also studied the efficiency of control measures that might be used in order to confront uncertainty, and found that once such controls are removed, the survival probability in the model enters a regime of exponentially fast decay. Since stress generally indicates a lack of control over the environment, efficient stress avoidance and stress resistance strategies set longevity traits, providing successful acquirers with a sufficient lifespan for maturation and reproduction. Although the only reliable survival strategy is to increase the rate of one's own behavior, simple time compression should be avoided if at all possible, as it is fraught with potentially maladaptive actions. The most prominent solution to the problem of survival amidst uncertainty that has been found in Western societies is based on the development of a concept of abstract time, which has allowed for a ceaseless acceleration of the pace of life in a process of permanent modernization.

# Time and Institutions

# 4

*The personal reason why the discovery of that which is eternal and permanent behind all changes has a high value for people is, I suggest, their fear of their own transience—the fear of death*[1]

**Abstract**

The relationship between the past, present, and future can be characterized by the amounts of information shared between them. At least three distinct models of time are possible under the different types of uncertainty. In an indeterministic world, planning the future on the basis of knowledge of the past is impossible. In the context of the linear model of time, the future is seen as a space of potentials, where virtually anything can be shaped, exploited, and improved. Eventually, the future in the cyclic model of time is ensured by the past, being as much present as the actual present is. To act as though the future were predictable and certain, we ultimately need others. Social institutions constitute a mechanism reducing the uncertainty of the present by directly linking the past and the future. Tradition and archaic social institutions eliminate the need to decide one's own destiny. In modern societal institutions, attention is focused primarily on the search for universal laws that would allow every individual to explore the future alone, at their own risk. Organizations shape our communications into a framework of social memory, reconstructing our distant past from a common perspective and transforming it into a perspective on the common future. Probably the only way to remain oneself and avoid the fate of a group nowadays is to remain foreign to it, not understanding the language spoken by the local media.

[1]Elias, N., *Time: An Essay*, Oxford: Blackwell (1994).



D. Volchenkov, *Survival under Uncertainty*, Understanding Complex Systems,
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## 4.1 Introduction

If you are reading these words, it means that with probability one you were born some years ago and you are alive at present:

$$\Pr[\text{present}] = 1\ . \tag{4.1}$$

This seemingly trivial fact nevertheless contains a wealth of information, such as that you have had access to enough food, water, and medical care to see you through, and that adverse health conditions which could have affected you anytime and anywhere were never that critical. In short, the balance of the countless, mainly random factors responsible for objective and subjective uncertainty has been remained in your favor throughout your lifetime.

If the sequence of survival events, in the stochastic model of subsistence under uncertainty, introduced and studied in Chap. 1, was simply a matter of fortunate flips of a fair coin, in which heads and tails occur equiprobably ($p = 1/2$), then the single existential observation (4.1) provides us with a single bit of information about the present, quantified by the Shannon entropy, viz.,

$$H(p) = \Big[ -p\log_2 p - (1-p)\log_2(1-p)\Big]\Big|_{p=1/2} = 1\ \text{bit}\,, \tag{4.2}$$

telling us nothing about survival in the past, or in the future.

On the other hand, if we were sure that the observed sequential survival events were a part of a regular reincarnation process, in which a spirit is reborn to begin a new life in a new body after biological death, then the single existential observation (4.1) would tell us virtually everything, since the phase of reincarnation readily calculated from the actual age of the survivor would represent the single bit of information that such a regular process contains. However, the actual survival process is in general much more informative than simple coin flips and reincarnations.

First, by its very nature, survival is never a stationary stochastic process, as its joint probability distribution obviously *does* change when shifted in time—it is clear that, for newborns, the chances of living to their 100th birthday are much higher than the chances of longevity for the next 100 years for individuals who are already 50 years old.

Second, survival is obviously not an ergodic stochastic process, because death—at least for those not believing in reincarnation—constitutes a terminating absorbing state.

In the present chapter, we show that a single existential observation made in the process of subsistence under uncertainty can contain many kinds of information about interactions between the past, the present, and the future, in the context of the entire survival history. We show that, since human existence is finite, it is precisely the 'anticipation of one's own finiteness', the co-presence of death, that reveals the future to us as a temporal perspective full of possibilities and plans [83].

## 4.2 Concepts of Time Under Uncertainty

From a metaphysical point of view, death awareness is a cornerstone of our awareness of time, and it also generates behaviors aiming at supporting or abolishing its finiteness [84].

Psychologists know [85] that judgments about the amount of time passed made in retrospect are based on how much information was processed during the actual experience: not much happened in the waiting room, so time passed slowly, but time may pass quickly while we are watching an action-packed film. We can say that time is a metaphor by which we describe and look back on our survival experience.

In Western societies, we are used to thinking of time as a measure in which events can be ordered from the past through the present into the future [86]. Such an ordering is obviously derived from the linear fashion in which an observer originating from the European cultural region experiences time as a continuous movement from the known (past) to the unknown (future).

The *linear metaphor of time* is based on the continuity and uniqueness of social collective events and experiences constructed by our technological environment, which follow one another irreversibly in a single direction [87]. As a result, we can speak of a before and an after, naming the totality of events accessed through memory and recollection (perhaps perceived more than once) as the *past*, and a speculation anticipating the unknown, the *future*. Sometimes, we can even establish a strict relation of cause and effect between the before and after: previous events are seen to give rise to, or produce, the events that follow [87].

Thus, the European future is, in fact, a projection of what is actually already a part of memory and what arises in the form of desires, dreams, and hopes regarding what lies ahead of us, as if we are continuously moving towards these ultimate goals. The *present* is contrasted with the past and future, being located somewhere 'between' them.

The perceived relation between the past, present, and future is culturally determined, as can be seen in a variety of categories expressing time references in the grammar and semantics of natural languages. For instance, the words 'yesterday' and 'tomorrow' both translate to the same word in Hindi meaning 'the day remote from today' [88]. In many societies viewed nowadays as archaic, irreversibility of events is not clear, and time is perceived in cycles. In the course of such a cyclic pattern, time is interpreted as a permanently established script of sacral events that are reproducible with inevitable consequences [89].

These relations are often manifested by specific ways of speaking and thinking about the past, present, and future. *How can we know our future if we do not know what will be our past*, says a popular Russian proverb, reflecting the long-lasting national tradition of freely, creatively, and recurrently recomposing the narrative of official history in order to legitimise the present political regime and justify any of its actions intended to affect the course of events in the future.

Bearing in mind the model of survival under uncertainty, we can quantify information relations between the past, present, and future. It is worth mentioning that the concepts of entropy and information can also be extended to composite random variables, allowing for the analysis of interactions among arbitrary sets of them. For example, the mutual information [90]

$$I(X, Y) = \sum_{\{X,Y\}} \Pr(X, Y) \log_2 \frac{\Pr(X, Y)}{\Pr(X) \Pr(Y)} , \qquad (4.3)$$

with summation over all possible pairs of random variables $\{X, Y\}$, is used as a standard measure of the extent to which knowing the value of the random variable $X$ would reduce uncertainty about the value of $Y$, given the joint probability $\Pr(X, Y)$ that both $X$ and $Y$ occur simultaneously. If $X$ and $Y$ participate in an event in a statistically independent manner, so that $P_t(X, Y) = P_t(X)P_t(Y)$, then the amount of mutual information (4.3) associated which such a pair is zero.

☞ The relations between the past, present, and future can be characterized by the amounts of information shared between them.

Similar measures can be used to characterize interactions between semi-infinite blocks of consecutive observations in a series of random events [25], by quantifying the reduction in the degree of uncertainty about events occurring in one block using knowledge of those in the other. Following [25], we will use the word *information* to name measures which involve only subsets of entropy which are shared by multiple variables, and the word *entropy* for those measures which do not.

Perhaps the amount of information shared between the past and the present that has no repercussions in the future can be vaguely associated with *reminiscence*, taken as the sharing of past experiences with those living at present. In a similar way, the amount of information that is not explained by the past, but is present now and has repercussions in the future can be considered as *anticipation*, taken as characterizing the ability to predict the future. Finally, we call a *tradition* the amount of information shared directly between the past and the present, not being captured by the present but passed down (always within a group) to the future, with symbolic meaning and with origins in the past (see Fig. 4.1).

The simplest model in which such information relations between the past, present, and future can be readily estimated uses infinite discrete sequences,

$$\ldots, X_{t-2}, X_{t-1}, X_t, X_{t+1}, X_{t+2}, \ldots$$

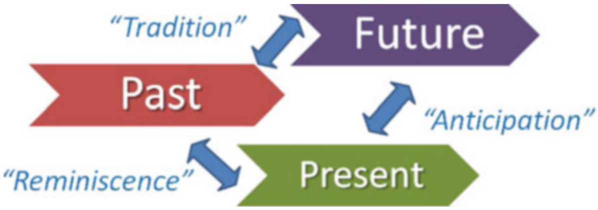


**Fig. 4.1** Information relations between the past, present, and future

of binary random variables $X_t \in \{0, 1\}$, indicating the state of being alive (1) and the state of being dead (0).

In the process of survival, the past and the future differ with respect to the present: if the present state is $X_t = 0$, the future literally does not exist, containing only zeros. However, the past can contain 1s even if the present state is 0. If the state of being alive was terminated once in the past, both the present state and the future states would be zero.

The statistics of survival processes under uncertainty can be characterized by a set of joint probabilities, such as the probability of the future $\Pr(X_{t+1:}) = \sum_{l\geq 1} P_\eta(t + l)$ (quantifying the cumulative chances of dying sometime after the present moment), the probability of the past $\Pr(X_{:t}) = P_\eta(t)$ (quantifying the chances of survival until the present moment), as well as the joint probabilities $\Pr(X_{:t}, X_t)$, $\Pr(X_t, X_{t+1:})$, $\Pr(X_{:t}, X_{t+1:})$, and $\Pr(X_{:t}, X_t, X_{t+1:})$, quantifying the chances of being alive over two or all three temporal segments together. Then, the joint probability functions characterizing the appearance of blocks of consecutive survival events can be used to define an algebra of information measures [91] for the survival process, although it is not a stationary stochastic process.

In the rest of the chapter, we shall denote the amount of information shared between the past and the present and which has no repercussions in the future (reminiscence) by $I\left[X_{t-1}; X_t | X_{t+1:}\right]$, the amount of information that is not explained by the past, but is present now and has repercussions in the future (anticipation) by $I\left[X_t; X_{t+1:} | X_{:t}\right]$, and the amount of information not captured by the present, but shared directly between the past and the present, by $I\left[X_{t-1}; X_{t+1:} | X_t\right]$.

Since an individual who lived in the past and is known to live in the future is certainly also alive at the present moment, the joint probability of the past and future equals the joint probability of the past, present, and future, so

$$\Pr(X_{:t}, X_{t+1:}) = \Pr(X_{:t}, X_t, X_{t+1:}) \ .$$

Therefore, the amount of uncertainty that remains at the present moment, after accounting for the uncertainty associated with every other moment (so-called *ephemeral* information component, to use the term coined in [25]) is obviously zero:

$$\begin{aligned} u &= H\left[X_t | X_{:t}, X_{t+1:}\right] \\ &= \sum_{t \geq 0} \Pr(X_{:t}, X_t, X_{t+1:}) \log_2 \frac{\Pr(X_{:t}, X_{t+1:})}{\Pr(X_{:t}, X_t, X_{t+1:})} \\ &= 0 \, . \end{aligned} \tag{4.4}$$

The amount of predictable information that is present now, is not explained by the past, but has repercussions in the future (so-called *bound* information [91]) is given by

$$\begin{aligned} & I\left[X_t; X_{t+1:} | X_{:t}\right] \\ &\quad = \sum_{t \geq 0} \Pr(X_{:t}, X_t, X_{t+1:}) \log_2 \frac{\Pr(X_{:t}, X_t, X_{t+1:}) \Pr(X_{:t})}{\Pr(X_{:t}, X_{t+1:}) \Pr(X_{:t}, X_t)} \\ &\quad = -u + \sum_{t \geq 0} \Pr(X_{:t}, X_t, X_{t+1:}) \log_2 \frac{\Pr(X_{:t})}{\Pr(X_{:t}, X_t)} \\ &\quad > 0 \, . \end{aligned} \tag{4.5}$$

This is always positive, because the probability of the past always exceeds the joint probability of the past and present, that is, $\Pr(X_{:t}) > \Pr(X_{:t}, X_t)$. The amount of predictable information shared between the past and present, but not captured by the future, is given by

$$\begin{aligned} & I\left[X_{:t}; X_t | X_{t+1:}\right] \\ &\quad = \sum_{t \geq 0} \Pr(X_{:t}, X_t, X_{t+1:}) \log_2 \frac{\Pr(X_{t+1:}) \Pr(X_{:t}, X_t, X_{t+1:})}{\Pr(X_t, X_{t+1:}) \Pr(X_{:t}, X_{t+1:})} \, . \end{aligned} \tag{4.6}$$

It is remarkable that, in contrast to the information measures defined for stationary stochastic processes previously discussed in the literature [25, 91], the information shared between the past and present (4.6) and the information shared between the future and present (4.5) are by no means equal, as the survival process is not stationary, being characterized by the essential asymmetry between the past and future with respect to the actual state at the present.

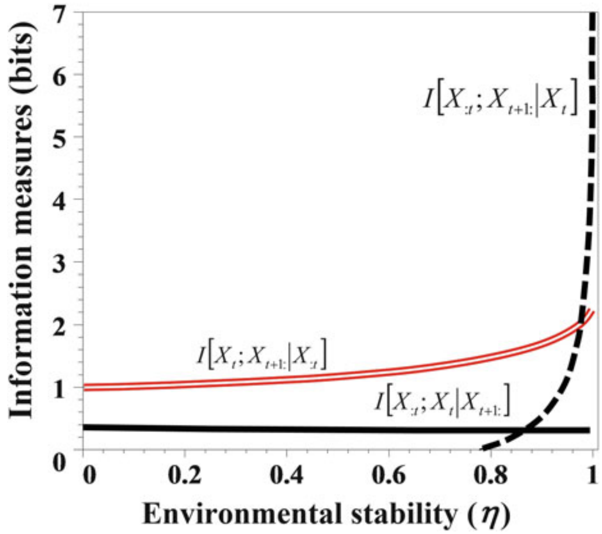


**Fig. 4.2** Multivariate information measures as functions of the degree of environmental stability $\eta$, in the stochastic model of subsistence under uncertainty, for uniformly random updates of supply and demand levels: $I\left[X_t; X_{t+1:}|X_{:t}\right]$ is the amount of predictable information that is present now and has repercussions in the future, but is not explained by the past (*double line*), $I\left[X_{:t}; X_t|X_{t+1:}\right]$ is the amount of predictable information shared between the past and present, but not captured by the future (*bold line*), and $I\left[X_{:t}; X_{t+1:}|X_t\right]$ is the amount of information shared between the past and future, but not captured by the present (*dashed line*)

Finally, the amount of predictable information shared between the past and future, but not captured by the present (so-called *elusive* information [91]) is given by

$$I\left[X_{:t}; X_{t+1:}|X_t\right] = \sum_{t\geq 0} \Pr(X_{:t}, X_t, X_{t+1:}) \log_2 \frac{\Pr(X_{:t}, X_t, X_{t+1:}) \Pr(X_t)}{\Pr(X_t, X_{t+1:}) \Pr(X_{:t}, X_t)} . \tag{4.7}$$

It is remarkable that similarly to the stationary time series analyzed in [25, 91], the value of elusive information $I\left[X_{:t}; X_{t+1:}|X_t\right]$ for the survival process can also be negative, since such a component of information may just not exist in the present observation (4.1) for the rapidly decaying survival probabilities under dual uncertainty. However, the past and future can indeed correlate for the slowly decaying survival probabilities under singular uncertainty, as $\eta \to 1$.

We have presented the behavior of these information components as functions of the environmental stability (assessed by the probability $\eta$), in the stochastic model of subsistence under uncertainty, for uniform densities $dF(x) = dG(x) = dx$ of the probability functions $F$ and $G$, describing the random updates of the supply and demand levels (see Fig. 4.2). The amount of predictable information shared between the present and future, but not explained by the past is shown in Fig. 4.2 by the double line. It grows monotonically with $\eta$.

☞ At least three distinct types of relations between the past, present, and future are possible under the different types of uncertainty.

## 4.3 Time Model of Russian Roulette

When survival is a matter of a fair coin flip ($\eta = 0$), the information shared between the present and future attains the minimal value, i.e.,

$$I\big[X_t; X_{t+1:}|X_{:t}\big]\big|_{\eta=0} = 1 \text{ bit},$$

encompassing the whole amount (a single bit) of information contained in the fact of survival at the present moment under dual uncertainty (4.2).

It is remarkable that in such a case the amount information shared between the present and future (anticipation) attains its minimal value of one bit as well, i.e., $I\big[X_t; X_{t+1:}|X_{:t}\big]\big|_{\eta=0} = 1$ bit. In such a case, the future literally does not exist, fully collapsing onto the present. If one plays the potentially lethal game of chance known as Russian roulette, in which a player places a single round in a revolver, spins the cylinder, places the muzzle against their head, and pulls the trigger, one potentially has no future, as it completely depends upon whether the loaded revolver chamber comes to rest in the 'correct' position at the present moment.

From the point of view of information theory, the present and future share the same information (on present survival) and virtually coincide. The corresponding model of time is presented diagrammatically in Fig. 4.3. The only difference between the present and the future under dual uncertainty is in their relation with the past. While the amount of predictable information shared between the past and present (reminiscence), shown by the bold line in Fig. 4.2, exists for $\eta = 0$, a tradition does not exist—the amount of information shared between the past and

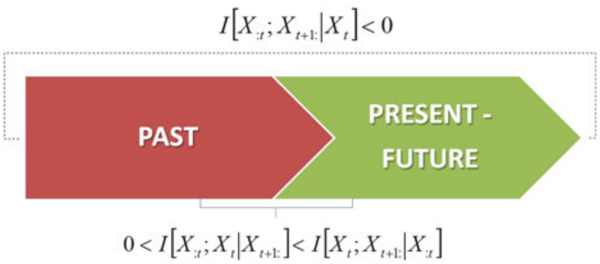


**Fig. 4.3** When survival is a matter of a flip of a fair coin (under dual uncertainty), the present and future share the same information

future, not captured by the present $I\,[X_{:t}; X_{t+1:}|X_t] < 0$, indicating that the past is irrelevant to the future under dual uncertainty (Fig. 4.3).

☞ In the time model of Russian roulette, planning the future based on knowledge of the past is impossible.

Research on uncertainty often portrays dramatic social and political upheaval, in which lives are made nonsensical as a result of profound changes over short time periods [95]. Suddenly befalling natural calamities, rapidly progressing human conflicts, as well as forceful implementations of new traditions—all these events are known to change the temporal structure of a culture [105]. When the assumption that the future is relevant to the past is undermined, the very notion of the future itself becomes increasingly tenuous, collapsing into an extended present [2]—the basis for planning, expectation, and forward movement of the self becomes difficult to sustain.

The feeling that the future has collapsed into the present has been reported in deportable individuals and failed asylum seekers, who are often transferred between various detention centers at short notice and without explanation [95]. The lack of temporal predictability prevents them not only from being able to plan for the future, but also from having the ‘stability’ of knowing that the present will remain uncertain. Instead of rational choice and future planning, survival under dual uncertainty requires a willingness to take what comes and make the best of it [96]. Radical uncertainty about the nature of the future undermines the very nature of being, and gives rise to a state of heightened ontological insecurity in those who experience it. When the sense of forward movement from the past through the present into the future breaks down, distortions of time perception can be experienced [95] and certain ‘pathologies’ of the self can arise in individuals, engendering in them a feeling of being transitory and unimportant [2].

However, when Russian roulette is played with a properly maintained weapon, with a single round inside the cylinder, the full chamber, which weighs more than the empty chambers, will usually end up near the bottom of the cylinder due to gravity, altering the odds in favor of the player.

## 4.4 Linear Time Model

Since the entropic force of survival (‘love of life’) continuously drives survivors to states of higher entropy of survival, we also expect the value of $\eta$ over a group of survivors to grow with time.

The amount of information shared between the present and the future (shown by the double line in Fig. 4.2) also increases with $\eta$, always exceeding the information shared between the past and the present, i.e., $I\,[X_t; X_{t+1:}|X_{:t}] > I\,[X_{:t}; X_t|X_{t+1:}] > 0$. Moreover, the information excess between anticipation and reminiscence, viz.,

$$\Delta I \equiv I\big[X_t; X_{t+1:}|X_{:t}\big] - I\big[X_{:t}; X_t|X_{t+1:}\big] > 0\ ,$$

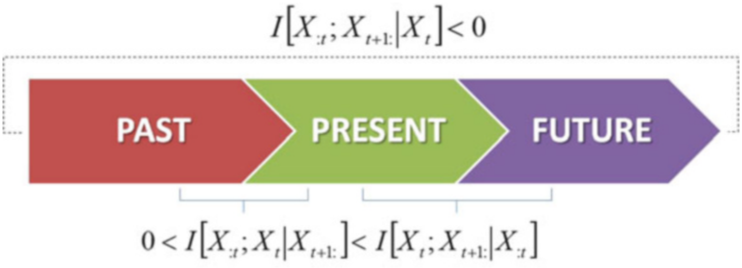


**Fig. 4.4** The linear model of time flow for $\eta > 0$

also grows steadily with the degree of environmental stability ($\eta$). Therefore, we may argue that the entropic force of survival induces an increasing flow of information from the past, through the present, to the future that might engender a feeling of time flow in survivors.

☞ 'Love of life' induces an increasing flow of information from the past, through the present, to the future that might engender a feeling of time flow in survivors.

However, as long as the degree of environmental stability $\eta$ is substantially less than 1, any interaction (information transmission) between the past and the future is possible only through the present, as the amount of information shared between the past and future, but not captured by the present (shown by the dashed line in Fig. 4.2), is still negative, i.e., $I[X_{:t}; X_{t+1:}|X_t] < 0$.

The resulting linear conceptualization of time is well known in Western societies (Fig. 4.4). Nowadays, the linear model of time flow is considered as a fundamental characteristic of nature. As time flows in a linear fashion, every present moment in this flow matters only once, being transient between the past and the future. The concept of an unending flow of time, independent of the events occurring within it, lies at the foundation of the Western notion of the future [45].

Since a part of the past information is shared with the present, present events can be considered as the consequence of past actions. Similarly, since a part of the present information is shared with the future, the present can be perceived by us as the past of an imagined future, urging us to plan ahead and make our present decisions in light of their possible later outcomes [45].

Since $\eta \leq 1$, the environment is relatively open to human control, and the future, although uncertain, can nevertheless be conceptualized as contingent and relatively reliable, as it might be influenced by deliberate present actions. Today we are more prone to think of our future lives as spans to be exploited like a resource [98], to be taken advantage of, and filled with as much as possible [62].

In the model of linear time flow, images of the future are not as real as the present, but can be interpreted as narratives on how life could have been, or how one wishes it to be [99]. The present is therefore experienced and made meaningful in perspectives of potentials, and the future becomes an arena in which one is going to improve the present situation [98].

Although the individual can neither understand, nor predict what lies ahead within the linear model of time, there is a presumption that the future can be influenced by deliberate present actions, such as buying an insurance plan or saving money in order to achieve higher rewards in the future.

☞ In the context of the linear model of time, the future is seen as a space of potentials where virtually anything can be shaped, exploited, and improved.

Nevertheless, the clock face of my wristwatch is round, and yours is round too, I suspect.

The earliest time-telling artifact, the shadow stick, conveys to us the idea of a repeating, cyclic day as a conceptual blend, built on the analogy between 1 day and the next [63], while a calendar conveys to us the idea of repeating seasons—so here we go!

## 4.5 Cyclic Time Model

The entropic force of survival pushes us further toward $\eta \to 1$, where the amount of information shared between the past and future directly, but not through the present (shown by the dashed line in Fig. 4.4), becomes positive, i.e., $I\,[X_{:t}; X_{t+1:}|X_t] > 0$. When the value of the probability $\eta$ is relatively close to 1, $I\,[X_{:t}; X_{t+1:}|X_t]$ increases rapidly, exceeding all other information segments. In a safe haven, the future, present, and past are not recognized as divided entities.

In fact, under singular uncertainty ($\eta = 1$), the past and future appear to be immediately related to each other, and the amount of information shared between them exceeds the information shared between both of them and the present. Quite literally, the present becomes redundant, against the background of the tight information connection between the past and future.

☞ In a pretty stable environment, the past and the future appear to be immediately related to each other, and most of the information shared between them is not captured by the present.

The cyclic time model (Fig. 4.5) is characterized by some peculiar conceptual features regarding the way the future is perceived, differing from those to which we are so accustomed today. While in the linear model of time (Fig. 4.4) the future is relatively open to various outcomes that may happen, the future in the cyclic model of time, in the limiting case, is more like the ‘forthcoming’, as it is known

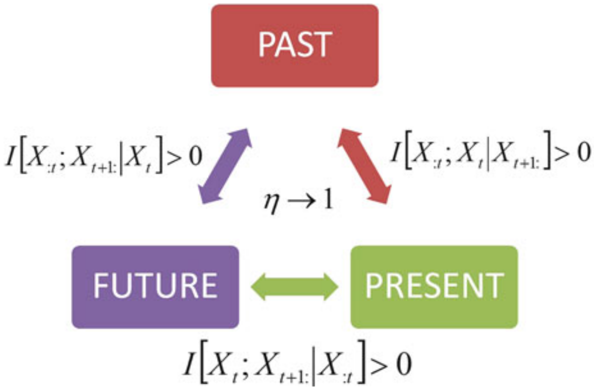


**Fig. 4.5** The cyclic model of time under singular uncertainty

in traditional societies [100], being formed by events that develop from the past in an expected way. Although the unfolding of events can be supported by deliberate present actions, they can neither be completely altered, nor substantially controlled at the present moment, being predetermined by the past. The future in such a cyclic model of time is not possible—at least, it cannot be apprehended as arising from an infinite number of possibilities—but it exists rather in potentiality, allowing for an immediate perception from the past (by a prophecy) which is as much present, as the actual present is, and thus virtually incapable of not coming about [45].

☞ The future in the cyclic model of time is ensured by the past, being as much present as the actual present is.

We are accustomed to think of the cyclic model of time as relevant to archaic, undeveloped societies, because of relatively recent changes—most notably, the weakening of the important social institutions of the past through processes of secularization and the undermining of absolutism—which have destroyed the basis of the providential determinism prevailing also in Europe during previous centuries [2].

The transformation of the concept of time was reflected most noticeably in physics, where *Newton's understanding of time* as a part of the fundamental structure of the universe, in which events occur in sequence, set the stage for the systematic study of natural phenomena that eventually, but inexorably, enriched the certainty and predictability of the causal laws of classical mechanics with the uncertainties and probabilities of the quantum realm [101].

However, there is no reason to leave the cyclic model of time in the care of anthropologists alone, as the entropic force (love of life) compels us to move toward the state of singular uncertainty $\eta = 1$ characterized by the minimal entropy production rate, but the maximal expected survival time production rate. Here, in the words of Eleanor Dark's novel *The Timeless Land* (1941), "the past and future might be interwoven with [our] own life by legend and unvarying tradition, where all time would be the frame for [our] mortality, and contentment [would be] our heritage."

## 4.6 Cultural Time of Rituals

A culture, taken as the totality of socially transmitted behavior patterns, beliefs, and institutions, provides reassurance that, even though an individual cannot anticipate the future, the universe follows a plan, which guarantees that either in this world or in the next, things will work out if she or he obeys the rules [53]. Time unfolding in popular culture, heavily influenced by the mass media, is thus modeled by the experience of crossing the border between the 'sacred' world, in which a higher power will ensure that everything will turn out well, and the 'profane' world, immersed in daily uncertainty.

By commissioning rituals and obeying laws, we put ourselves into a cultural time that comes in cycles—working days give way to weekends, and holidays follow weekdays, repeating periodically. Periodic repetitions and symbolic reiterations of events that happened 'once upon a time', written in codices and formulated in laws, then enacted in rituals and implemented by judicial authorities, are experienced as the actual, eternal present, since every ritual is done 'now', at this very minute, and happens for the first time, no matter how long ago the original event actually happened [89]. Rituals are the necessary actions for resuming and recreating the world according to strictly established deadlines, reducing stress and enabling people to shut out their anxiety, so that they can focus on coping with the immediate problems. Through rituals, immortality becomes the object of immediate consumption [92], as the way one lives through the moment makes it an 'immortal experience'.

*Cultural time*, being constructed by ritual actions, is continuous and uniform, albeit only within the framework of a particular culture. The actual discontinuity of cultural time is most clearly visible through the fact that the events follow each other because one of them is a direct consequence of another, resulting from it inevitably, like a punishment following a crime, rather than because one of them took place before another. Therefore cultural time exists and is meaningful only when certain significant events are happening, but it literally does not exist when nothing significant happens.

Since there is no clear separation between the cultural past, the cultural future, and the cultural present, all these modes of time are experienced as evenly real. The past and the future are always present here and now, either in the form of divinations, or in the decisions of popular referenda and plebiscites, or in scientific reports telling us the story about what happened 'at the beginning of everything' and foretelling the future of the Universe.

Since the story of the past is inextricably linked with daily life, the cultural past has a special status as being real, comprising the exactly known consequences of any action, and being constantly reproducible through the everyday activity of humans and the sacred ritual practices within their community. The historical reenactments, in which people follow a plan to recreate aspects of a historical or mythical event make everybody a participant in and contributor to that event, synchronizing the entire community in cultural time to be contemporary with the gods, the heroes, and the great scientists of the past, and thereby enhancing group solidarity. The important thing is to know the myths, laws, and theories and accurately follow their plot by reproducing, repeating, and demonstrating it in the appropriate rituals.

Cultural time, and the time of theoretical physics, in particular, is reversible,[2] and therefore lacks any certain direction either to the past, or to the future.[3] Since there is no specific development, cultural time may be essentially discontinuous and intermittent in true physical time and, of course, always finite, starting from a single miraculous Big Bang and ending with a Big Freeze, a Big Rip, or a Big Crunch—equally miraculous events, which are not clearly allocated in time.

## 4.7 Risk and the Open Future of Linear Time: A Desperate Need for Others

When the environment is relatively open to human control, the uncertainty of the future is understood through the concept of risk, and managed through the practice of insurance [2]. The term *risqué* was first used in the middle of seventeenth century and signaled a radical new way of thinking about temporality as an open future, signifying not something that will happen, but something that *might* happen [2].

*Risk* is a probable consequence of *individual actions* taken in spite of uncertainty over a stated period of time. As probability deals with aggregates over the long term, risk analysis can neither give a specific prediction about an exact event, nor make certain recommendations for individuals. Instead it estimates a potential danger that we construct in our head while attempting to protect an individual from the threat of possible events in the unknown future. The risk cannot be experienced directly, but is defined by gathering relevant information regarding the unfolding of future occurrences over a sufficiently long period of time, and it vanishes as soon as the anticipated event occurs [2]. Risk perceptions are socially constructed, and long-term risks that bring no immediate threat to the individual make it possible for much of the population to ignore these risks or view them as hypothetical [12].

In societies where control over the environment is limited, the concept of risk has no meaning, since hunger and economic scarcity presenting an immediate threat to individual survival provide direct first-hand experience [12]. When time comes

[2]The theoretical symmetry of physical laws under a time reversal transformation $t \mapsto -t$ is known as T-symmetry.

[3]For example, the time reversal of a *black hole* would be a hypothetical object known as a *white hole*, which appears similar from the outside. However, while a black hole has a beginning and is inescapable, a white hole has an ending and cannot be entered.

in cycles, the uncertainty of everyday life is rather viewed as fate, providence, or luck, i.e., symbolic representations of the will of gods [2], which does not require intellectual insight to perceive it. Interestingly, in many traditional religious communities, commercial insurance is still regarded as an avoidance of god's will, revealing a lack of faith, or at least a form of usury or speculation [93]. In contrast, the risks arising when the future is open are not immediately felt but have to be understood, being based not on first-hand experience, but rather requiring cognitive insights, high levels of information, and often a grasp of complex argumentation.

Although the calculation of risk itself neither guarantees the avoidance of catastrophes, nor provides assurances about the uncertain future, it renders a formally justifiable guide for behavior, based on the premise of present rational actions, independent of future success or failure. Armed with the appropriate information, individuals can have the comfort of knowing that they are taking active steps to protect their well-being and shape their future. In this respect, the ongoing exercise in risk calculation is nothing else but a metaphor of the flow of time, as it involves an expansion of the time frame and projection into the future, until the appearance of regularities and patterns creates a sense of predictability in us. Risk calculation and decision-making under uncertainty becomes a formal strategy for moving into the future. As N. Luhmann puts it, today "modern society experiences its future in the form of the risk of deciding" [102].

Although the management of risk through insurance is individualized, its treatment always involves sharing a possible loss among many others. The funds from many insured individuals are pooled to pay for the losses that some may incur [94], creating the means for acting as though the future were predictable and certain. In fact, insurers benefit from the law of large numbers, according to which predicted losses are similar to the actual losses.

☞ To act as though the future were predictable and certain, we ultimately need others.

By taking the lone individuals and placing them in groups of their own kind, risk acts as a stimulus for coalescence around "shared rituals of risk calculation and avoidance" [4]. Being insured at the price of membership within a large group of individuals exposed to the same types of risk, we are compelled to agree on a limited form of agency, not only because we commit ourselves to obey certain rules and to take part in common rituals enhancing group solidarity, but also because probabilistic assessments, although efficient in constructing future scenarios over large classes of similar exposure units, can never tell us about our individual case at any particular moment of time.

In order to be an insurable risk, the risk insured against must have certain characteristics, such as embracing a large number of similar exposure units; assuming a definite loss at a known time, in a known place, and from a known cause; being clearly outside the control of the beneficiary of the insurance; incurring a large loss, meaningful from the perspective of the insured; offering the affordable premium for a calculable loss; and, finally, limiting the risk of catastrophically large losses [97].

Thus, belonging to a large group of peers is worthwhile for low-probability, catastrophic losses, but not for high-probability, small losses which do not cause a disruption in our lives, even though they may be experienced almost every day.

## 4.8 In Search of Lost Time Cyclicity: Institutions

Evolutionary psychologists who study the cognitive structure of the human brain conclude that humans do not develop general analytical skills, but rather use a different approach to reasoning about what is forbidden, obligated, or permitted, in contrast to reasoning about what is true and false [103]. When reasoning about whether empirical relationships are true, they tend to use a confirmation strategy rather than logical arguments [104].

The ability of humans to obey rules, to reflect upon and rationalize past experiences and memories, and to match them with foreseeable future circumstances appears to be much more important for survival than highly developed skills in general logical problem-solving. At least, although still controversial, such a hypothesis seems plausible in view of the previous discussion. Indeed, the fear of time, being a metaphor for the fear of death experienced by virtually all people, would give rise to a natural instinct for eliminating time from everyday life, and the development of special behavior aiming at abolishing one's own finiteness. Reducing stress and anxiety that arise due to the fact of one's inevitable death in the future enables people to focus on coping with their immediate problems [53].

It is remarkable that, in contrast to proto-Sumerian pictographic texts created in the pre-state epoch circa 3100 BC, which contained a comprehensive classification of ages, later Sumerian texts of the state era, never specified a human age (Emelianov, 2015, private communication). We know neither the ages of kings, nor the ages of ordinary citizens, although the ages of cows, oxen, donkeys, mules, and slaves put on sale were always clearly indicated. It seems that the elimination of age from everyday life was one of the important functions of the state.

It has already been suggested that a person could delegate the establishment of the more permanent structures of his/her existence to collective regulations and *institutions*, in order to reduce the personal fear of time [105]. In particular, when confronted with the power of time to bring our human life to its final destination (death), several types of illusions are used in order to forget about one's mortality [84]: the 'religious illusion', opening up the hope of life after death; the 'moral illusion', awarding the hope of eternity on the basis of merit; the 'social illusion', working through prestige, fame, consideration, and glory; the 'ontological illusion', relying on possessions and the 'apparent consistency of reality' [83]; and the 'practical illusion', allowing for oblivion of personal mortality by focusing on practical tasks.

We can conclude that, given the state of personal finiteness, humans tend to create their own worlds—specific mental environments of signs and meanings—trying to reduce the level of objective uncertainty by shaping spatial and temporal order, routinizing and ritualizing activities, and stereotyping thoughts and actions, with the general aim of increasing trust and stability [105]. In the context of our work, the aim of societal institutions is to emphasize the continuity of human life by maximizing the amount of information shared between the past and the future, not captured by the present, i.e., $I\,[X_{:t}, X_{t+1:}|X_t] > 0$. Quite literally, social institutions constitute a mechanism for reducing the uncertainty of the present by directly linking the past and future.

☞ The role of social institutions—the fixed invariants of behavioral and mental patterns—is to maximize the amount of information shared between the past and the future, not captured by the present.

Indeed, the embedded patterns of behavior within a society—the values and objectives shared between many individuals forming a community—constitute the most natural mechanism for the direct sharing of information between the past and future without any reference to the present. By continually increasing the shared amount of information, we move from the model of linear time to the model of cyclical time, in which the past and the future are inextricably linked, regardless of the present.

Social institutions represent a minimal set of values and objectives shared between all individuals forming a group that is persistently maintained in the given community, as long as it exists. They arise from human behavior within a group and then shape this behavior, along with all interactions between the group members, through traditions, mores, customs, and markets [107]. In order to reduce the uncertainty of the present by making time cyclic, we ultimately need others—a community of people sharing our values.

Being driven by the entropic force of survival (love of life), we try to cope with uncertainty and to streamline the game of chance in our everyday lives by creating rigid behavioral and mental structures of rules, convictions, and beliefs that continue to operate automatically after the present moment, with the goal of arresting and controlling the flow of time by imposing the appropriate constraints on our present experience.

No society can exist unless its members have common feelings about what is the proper way of conducting its affairs, and these common feelings are expressed in ideology [108]. When survival is insecure, uncertainty dominates life strategies that are rapidly crystallized into certain cultural patterns. Tradition, customs, and other social institutions provide mechanisms through which individuals acquire common values and criteria for evaluation that can be used to make choices about their objectives and alternatives [107]. The younger birth cohorts that have grown up within already established institutions would take survival for granted [12], and thus inherit the cultural constants as a precondition for survival and, perhaps, as a natural characteristic of reality. The use of tradition and social institutions, rather than making personal choices, is a way of minimizing the use of analysis and reasoning, as there is already a set of ready-made choices [107].

☞ Tradition and archaic social institutions eliminate the need to decide one's own destiny.

In archaic societies, where time is already gauged by natural cycles that a person can hardly change at will, traditional institutions play a crucial role in societal life, which is in turn nothing other than the rhythm of a natural calendar.

Since insecurity raised by natural calamities could be evaded only by consolidating relationships within a group, it was not at all encouraged for individuals to take control of their own future. Duties to other members of the kinship or family and reciprocity play a major role in maintaining the activity of social institutions. Values and behavioral expectations justifying the legitimacy of a social institution in a traditional society are communicated in the form of unproven collective beliefs, i.e., *myths*, which are accepted uncritically by all members of the group and transmitted from generation to generation [107].

Personal future-orientation is disapproved as a presumptuous activity, contradicting the ethic of reciprocity [45]. Special cultural norms are instilled that strongly discourage people from exercising autonomous choices [12], establishing group membership as a central aspect of identity [109]. As the social context is prominent in people's perceptions and causal reasoning under conditions of collective identity, the future is seen as a predestined fate, common to the entire social unit, and every natural phenomenon is viewed as a sign of providential meaning, clearly understandable by prophets.

In modern societies, the concept of an abstract time which is not anchored to any environmental rhythm allows one to continuously use previous experiences in order to deliberately maintain the time perception framework. This new conception of temporality has laid the groundwork for a new way of reasoning about uncertainty that has been based on long-term predictions made over large sets of previously observed events, viz., the *probability theory* [2]. The focus of attention has been shifted away from awe in the face of unique miracles, consolidating the kinship group, to the observation and reproduction of almost infinite series of identical events, in search of regular and predictable patterns, or universal laws that would allow every individual to explore the future alone, at their own risk. The future can therefore be constructed by every individual member of a modern community, independently of the will of others, and thus looked upon as an individual potential [2].

☞ In modern societal institutions, attention is paid primarily to the search for universal laws that would allow every individual to explore the future alone, at their own risk.

However, no matter which society, traditional or modern, a social institution may serve, its main function always remains the same: to maximize the amount of information shared between the future and the past, not captured by the present, providing individuals with information and some degree of certainty in their social interactions. Therefore, for the smooth functioning of social institutions, we desperately need an inexhaustible and incessant flow of information from the past.

The transition from traditional to modern social institutions is always painful and even dangerous to people. A refusal to live in the present as that time which anticipates the future is known as nostalgia [52]. This disease first entered the medical lexicon in the seventeenth century and was considered worthy of medical attention, discussion, and intervention throughout the eighteenth century. During and after the wars of the Revolution and the Napoleonic period, in the early decades of the nineteenth century, there was a significant increase in medical writings about nostalgia which evince an intense longing to connect with the past and to stay alive through a connection with the past. The disease was considered primarily in spatial terms and stemmed from one's fidelity to one's origins.[4] However, improvements in transportation and the establishment of rapid communications made people more cosmopolitan, and the incidence of this disease diminished. Interest in nostalgia had dropped off considerably by the 1850s, and after the 1870s, it was no longer considered as an illness. Nevertheless, even Nietzsche diagnosed an excess of remembrance and a smothering of creativity under the burden of history as a disease of his day, emphasizing the need to forget in order to enjoy a healthy life [52].

## 4.9 A Desperate Need for the Past

Uncertainty regarding the past incurs a risk for a social institution as substantial as uncertainty of the present for a single individual. The mathematical approach developed in the previous sections shows that a group's aptitude for reconstructing its own past is a crucially important faculty, highly relevant to its survival prospects. A rich stream of information, flowing from the past and structuring the future, indicates the liveliness of the group's institutions and ensures its ability to reproduce itself in generations ahead. When a nation's newly discovered past is extremely ancient, its future also begins to seem endless [113].

☞ A group's ability to reconstruct its own past is a crucial faculty if it is to survive in the future.

[4] According to [52]: "J.B.F. Descuret (1841) described the case of a Parisian lodger whose cherished apartment was to be destroyed because of improvements to this street. For this man, there was simply no other place in which he could live. His life was tied to a past that was expressed in his home. No substitutes were possible, and we can say more generally that nostalgia as a disease was characterized by the refusal to accept substitute satisfactions. The lodger died in his room rather than accept the possibility that life could be lived elsewhere, apart from his past."

Consequently, members of the group share a collective memory [110], literally feeding their own future with information retrieved from the past, continuously reinterpreting it and legitimizing previous experiences [105]. It is remarkable that the new cultural content is created ceaselessly, not only in the present, but also in the past [112]. Previously unknown monuments and historical texts are constantly being found, uncovered, and dug out of the ground or from the dust in libraries.

Every social institution is correlated with the interests of a particular group of individuals that benefits from it and thus has a vested interest in preventing changes in the institution [107]. Therefore, it should be no surprise that every culture defines its own paradigm concerning what should be remembered and what has to be cast into oblivion [112]. Antique statues found in the pre-Renaissance era were neither restored, nor cherished, but were thrown away and destroyed.[5]

In this regard, the decision by Islamic militants to demolish cultural monuments of the past is noteworthy. The recent bulldozing by the Islamic State (IS) of the ancient cities of Nimrud, Hatra, Korsabad, Asshur, and Palmyra, the world's greatest archaeological and cultural sites, fits into an old and common pattern of destruction of historical artifacts [115, 116]. Perhaps, they consider the historical monuments and, more importantly, the interpretations of these monuments in the Western 'community of memory' [110], to be exclusive vestiges of the Western past, nurturing information to ensure the future of a Western culture so alien to them.

The permanent work of memory, which carries forth information from the past to the future through a present act of narration, is largely subordinate to the selection of matters that have to be remembered in order to reproduce power [117], or to be sacralized with the goal of producing a dominant narrative toolkit for the power-holders and later converted into novel social practice [105]. Collectivities that are capable of reconstructing their past at every moment [110] constitute the modern *units of survival*, replacing the kinships of archaic societies. As people master the future, they create, recreate, and discover the past [111]. It is also remarkable that all updates, revival, and progress in modern society always have the form of a reference to the past, as though they depend upon the resources of cultural memory, being synchronized with it in every detail [114].

☞ All updates, revival, and progress in modern society always have the form of a reference to the past.

It is important to note that Medieval European culture, in its early period, was aware of itself in the context of the life of Christ and his apostles, and subsequently experienced an attraction to Aristotelian philosophy and Hermeticism. The early Renaissance was fed by Late Hermeticism [118], and by the end, mastered the heritage of the classical era of Greek polis.

The epoch of European colonial empires, in the late eighteenth to early nineteenth centuries, was nourished by the spirit of ancient empires praised by Homer, so

[5] As noted in [114]: "Grandchildren are eager to learn the legacy of their grandfathers and great-grandfathers, while criticizing and rejecting the rules of life of their fathers."

that the leaders of conflicting European empires related themselves to the heroes of the Trojan War. Finally, the European totalitarian states of the twentieth century embodied many of the violent ideals of the Ancient Eastern empires [119, 120]. It appears that any further movement into the future, in one form or another, is constructed as a memory of the distant past, preceding the recent past [114]. In the process of decoding previously unknown and incomprehensible information from the past, the present not only acquires an understanding of the past, but also partially obeys it, by projecting it onto its own future.

## 4.10 Why Do We Think that the Majority Is Always Right?

Social institutions provide rules, conventions, and codes of conduct binding the behavior of individuals [106]. Although they determine virtually all aspects of human life, such as the size of one's family, forms of marriage, responsibility for children or parents, expectations about disposal of waste, use of resources, trust, theft, voting, creativity, etc. [107], they are useless in the absence of organizations providing a structure for everyday communications between the group members. Organizations shape our communications into a framework of social memory, reconstructing our distant past from a common perspective and transforming it into a perspective on the common future.[6]

As already discussed, the human tendency to obey the rules rather than to think logically [103] makes the media an excellent tool for shaping the social memory. Today, the media, which profoundly, intensively, and extensively mediate the frameworks of social memory, play a major role in the ongoing reconstruction of our past [121].

We like to be a part of a majority, because 'if many believe so, it is so', i.e., *argumentum ad populum*. The drive towards a majority is analogous to an insurance policy against the risks with which daily life is fraught [122]. There is a simple statistical reason, beyond the fact that a group is always at least as good at predicting the future outcome $x \in [0, \infty]$ of a random process as the average opinion of the individuals making up the group [123], i.e., the whole is never less accurate than the average of its parts.

The reason is that, given a set of individuals attempting to predict the value $x$ which may arise in the future, the squared error of the collective prediction $(x - \bar{\chi}_i)^2$ calculated over all individual guesses $\bar{\chi}_i = \sum_{i=1}^{n} \chi_i / n$ obviously

(continued)

[6] "Our stories are singular, but our destiny is shared, and a new dawn of American leadership is at hand", said Senator Barack Obama at a rally in Grant Park in Chicago, Illinois, after winning the race for the White House, on 4 November 2008.

does not exceed the averaged squared error $\sum_{i=1}^{n}(x-\chi_i)^2/n$, i.e.,

$$(x-\bar{\chi}_i)^2 \le \frac{1}{n}\sum_{i=1}^{n}(x-\chi_i)^2 \ , \tag{4.8}$$

in which equality is achieved if and only if all individuals guess the same value.

The existence of a majority assumes a certain diversity of predictions over the group, so that (4.8) is an inequality in this case, and the collective prediction of the outcome $x$ is always more precise than the simple average value $\bar{\chi}_i$. It is indeed the entropic force of survival (love of life) that urges us so insistently to choose the side of a majority, as the majority certainly follows the correct rules, being guided by the right behavioral codes to guarantee success in achieving longevity, while minorities look so miserable and are expected to vanish eventually.

Nowadays, the mass media form a picture of the world, so they can claim to represent a majority position. By sharing it, we find ourselves on the side of the majority, which is so reassuring. To swim against the stream, to resist the force of destiny by challenging the majority position, is always a difficult thing.[7] The use of new interactive technologies in television makes modern propaganda particularly effective. For example, polls have shown that the mediation of public opinion with the help of new information technologies actively conducted by the state-controlled Russian media during the Russian–Ukrainian conflict in 2014–2015 was able to form and maintain the public consensus continuously in favor of war, and at an unprecedentedly high level of 84–87 % (According to a report of the independent Levada Center pollster published on 22–25 August 2014, 84 % of respondents supported the activities of the Russian president. Earlier in August 2014, the Russian president's approval rating had crept up to 87 %. Between November 2014 and January 2015 the approval rating stood at 85 %, according to the Levada Center. http://www.bbc.co.uk/russian/russia/2014/08/140827_levada_center_survey_putin, http://www.themoscowtimes.com/news/article/putin-s-approval-rating-creeps-up-again-poll-shows/516580.html, http://www.themoscowtimes.com/news/article/putins-approval-rating-soars-to-87-poll-says/504691.html).

Interestingly, the voluntary renunciation of television that was so popular among intellectuals in the last century, as a way of holding onto personal convictions and attitudes in the face of systematic brainwashing, did not help much in this case. It turned out, that having just a few watchers of modern interactive television in one's personal entourage is still enough to become fully indoctrinated. However,

[7]However, as Seneca remarked: "It is not because things are difficult that we do not dare, it is because we do not dare that they are difficult."

people with a relatively low level of language proficiency demonstrated the highest resistance to forcible indoctrination by limiting communications with locals and refusing participation in society. Probably, the only way to remain true to oneself and to avoid the fate of the group nowadays is to stay foreign to it, by not understanding the language spoken by the local media.

## 4.11 Conclusion

We have discussed the perceived relations between the past, present, and future under uncertainty. In particular, we observed that the entropic force of survival should induce an increasing flow of information from the past, through the present, to the future that might engender a feeling of time flow in survivors. When the environment is relatively open to human control, the future, although uncertain, can be conceptualized as contingent and reliable. Under singular uncertainty, the future, present, and past are not recognized as divided entities: the past and the future appear to be immediately related to each other, as most of the information shared between them is not captured by the present. Being ensured by the past, the future in such a cyclic model of time is as much present as the actual present is.

In the information theory setting, the main role of institutions—the fixed invariants of behavioral and mental patterns—is to maximize the amount of information shared between the past and the future, not captured by the present. We pointed out that a group's ability to reconstruct its own past is a crucial indicator of its ability to survive in the future. In archaic societies, special cultural norms are instilled, strongly discouraging people from exercising autonomous choices, and the future tends to be defined as a predestined fate, common to the entire group. In modern social institutions, attention is focused primarily on the search for universal laws that would allow every individual to explore the future alone, at his or her own risk.

# 5 We Speak Up for Time, and Time Speaks Up for Us

*We can only preserve our unity by being able to 'open and close', to participate in and withdraw from the flow of messages. It therefore becomes vital to find a rhythm of entry and exit that allows each of us to communicate meaningfully without nullifying our inner being. Yet in this alternation between noise and silence we need an inner wholeness that must survive through change*[1]

**Abstract**

Time is a fundamental dimension of social interaction. We present a first study, integrating the analysis of temporal patterns of interaction, interaction preferences, and the local vs. global structure of communication in two organizations over a period of 3 weeks. Our results suggest that simple principles reflecting interaction propensities, time budget, and institutional constraints underlie the distribution of interaction events. As a result, the duration of interactions (as well as the interval between interactions) reveal deep aspects of social systems. Not only does the interaction duration reveal a multiplicity of regimes affecting interaction parameters, but it also offers differentiated windows over different social network structures corresponding to such regimes. We show that institutions never die, as once interrupted communication can be resumed anytime.

[1] Melucci, A., "Inner Time and Social Time in a World of Uncertainty", *Time Society* **7**, 179 (1998).



D. Volchenkov, *Survival under Uncertainty*, Understanding Complex Systems,
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## 5.1 Introduction

Communication is essentially a social process, any change of which immediately alters the nature of groups and, perhaps, the form of government [124]. The regularly renewable process of communication between group members plays an essential role in the continuous functioning of social institutions, serving as a mechanism for strengthening social integrity and group functional stability, as well as constituting a moral sign of group solidarity.

Organizations implement the codes and conventions maintaining the group's institutions by coordinating communications between group members and shaping the durations of intervals between all three types of communication within them. The design of complex organizations implies a division of labor, grouping tasks based on similarity in function, and their subsequent coordination and integration. The division of labor enables the creation of specialized units that are relatively autonomous and enables the creation of economies of scale, for localized adaptation within problematic parts of the organization, while simultaneously buffering the unaffected parts. Therefore, it is critically important to study the principles of communication in organizations, in order to understand the nature of institutional longevity.

The possibilities for observing the unfolding of human behavior in time have dramatically expanded due to the diffusion of digitally networked activities and the availability of wearable sensors. This has opened new opportunities for accurately monitoring the way humans interact in time. However, most often the time dimension of interactions has been flattened: the cumulative duration of interactions has frequently been used to measure the strength of social ties in a static portrait of social networks. Times of interaction have been used as a proxy for the strength of relations. But the way the temporal distribution of interactions relates to the network structure of interacting agents is a basic question that has not yet been addressed.

We analyze face-to-face interactions in two organizations over a period of 3 weeks. Data on interactions among about 140 individuals have been collected through a wearable sensors study, carried out in two start-up organizations in the north-east of Italy. The data was collected by C. Cattuto, M. Warglien, A. Cabigiosu, and coworkers from the Ca'Foscari University of Venice. Two new phenomena emerge. First, the duration of interactions between pairs of agents displays a non-monotonic relation with the interaction preferences of each of them, as measured by mutual information. Second, a similar non-monotonic relation can be observed by relating interaction durations to the ratio of the first passage time to a node and the recurrence time to it in the biggest connected components of the communication graphs for the different communication durations. Both phenomena can be explained by the existence of the same different regimes of interaction that explain interaction duration and intervals.

## 5.2 Data Collection for the Study of Communication Patterns in Organizations

Data collection was carried out in June and July 2010 in two firms: H-farm and H-art. While legally distinguished, the two organizations have the same origin and are located in distinct buildings in the same area in the countryside outside Treviso in Italy.

H-farm is a venture incubator founded in 2005. H-farm's mission is to encourage the creation of projects aimed at simplifying the use of digital tools and services by people and companies, helping them transform their processes into digital workflows. In 2010 H-farm had 75 employees and hosted 9 start-ups that included 54 team members. H-farm's staff, which supports start-up development, had 21 employees. H-farm and the start-ups all have a functional structure and all start-ups have dedicated space.

Since 2009, H-art works in the media industry and provides creative and innovative marketing plans for many different brands. In 2010 H-art had 71 employees. H-art has a modified functional structure in which employees belong to functions and are assigned, at the same time, to different projects.

H-art and H-farm employees and start-up members were asked to wear the radio-frequency identification sensors to report on occasions of physical proximity. Twelve sensor readers were placed all over the workplace, allowing full coverage. We monitored face to face interactions for 24 h a day and 7 days a week over the 4-week observation period. Our analysis is based on 18 working days.

At the time of the study, the start-ups hosted by the H-farm had a functional structure and dedicated desks. In contrast, H-art works in the media industry and provides creative and innovative marketing plans for multiple brands. It has a modified functional structure in which employees belong to functions and are assigned simultaneously to different projects managed by members of staff known as the Alpha team.

The radio-frequency sensors worn by those taking part in the experiment reported an interaction if they came within 1 m of each other. This distance was chosen after a field observation of work activities and layout in the two firms. This distance excludes the recording of an interaction when employees are facing each other, sitting at their desks during individual work. It thus allowed us to selectively record only face to face interactions which occurred when at least one of the two interactants had moved from his or her individual work station.

We ended up with two adjacency matrices, one for H-art and one for H-farm, for each minute of the analyzed days. The first column and row of these matrices lists, for the H-art matrix, the radio-frequency identification (RFID) of H-art's employees, and for the H-farm matrix, the RFID of H-farm's employees and of H-farm's incubator members. The entries are 0 or 1 and represent a lack (0) of interaction between two RFIDs or the existence of an interaction or a tick (1). Interactions are bidirectional and matrices are symmetric. We further recorded data to measure the duration of each interaction. Field observation suggested that, when in a single interaction, there was a 1 min interruption between two ticks. This interruption represented noise in the data recording rather than a separation between different interaction episodes.

## 5.3 Statistics on Interactions in Organizations

Communication is a networking process, evolving in time through continuous individual and collective decision-making about participation in or withdrawal from interaction. The communication processes in the two working teams were remarkable for the absence of a characteristic size of communicating groups (see Fig. 5.1). The distributions of the numbers of joint communication events are strongly skewed, with long right tails decreasing with the size of the communication group, approximately following power laws (see the trend lines shown in Fig. 5.1).

In general, smaller groups of team members communicated more frequently than larger groups. Meetings involving a significant fraction of the full working team were rare events (especially in the H-farm). It is remarkable that there is a clear difference between the power exponents characterizing the steepness of slope in the distribution tails. This may arise due to the difference in organization structure and pursued goals between H-farm and H-art.

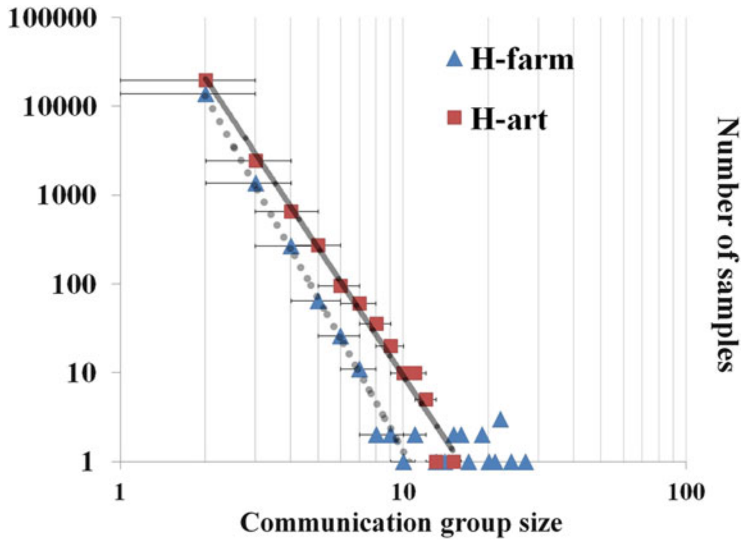


**Fig. 5.1** Statistics of communication group size. *Error bars* (along the horizontal axis) with the 5 % value are given for the selected chart series to show the statistically significant difference between the two empirical distributions. The *solid trend line* (fitting the data for H-art) is $N = 571\,032 \times s^{-4.793}$, with the goodness-of-fit linear regression $R^2 = 0.98$. The *dotted trend line* (fitting the data for H-farm) is $N = 681\,423 \times s^{-5.717}$, with the goodness-of-fit linear regression $R^2 = 0.98$

☞ Smaller groups communicate more often than larger groups, and brief communications are much more common than longer ones.

Basic temporal patterns of interaction, viz., duration and intervals between consecutive interactions, are identical in the two organizations, despite the structural differences between the firms. Our analysis of the data suggests further qualification of such patterns, leading us to simple models of the process generating them. Inherently different statistical properties reflected by the skewed distribution of intervals between consecutive communications suggest distinguishing between three different regimes of interaction: casual, spontaneous, and institutional.

### 5.3.1 Intervals Between Interactions

Every team member in the organization regularly comes into contact with others. The durations of intervals between consecutive communication acts is an important characteristic of organized interaction, allowing us to assess the degree of personal commitment to take part in business and social interactions, as well as on the ability of team members to dynamically schedule the emerging communication into the current working timetable. Some striking regularities appear and these are reflected in our observations. A typical pattern of communication activity demonstrated by individual team members contains bursts of communication activity separated by relatively long breaks, sometimes lasting longer than 2 h, as shown in Fig. 5.2.

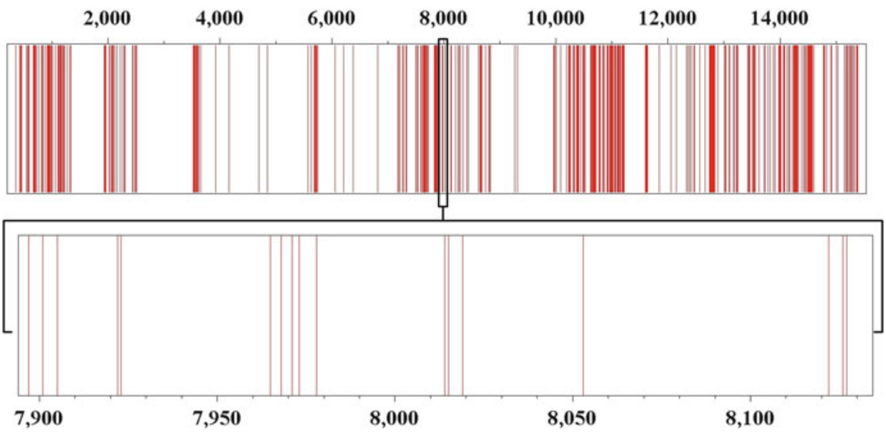


**Fig. 5.2** Typical pattern of communication activity for a group member. The *horizontal axis* denotes time in minutes and each *vertical line* corresponds to a communication event. The *upper diagram* shows an individual pattern of communication acts during the entire period of observation. The *lower diagram* is an enlargement of a short period of the recorded communication activity. The interval between two consecutive lines is the inter-event time

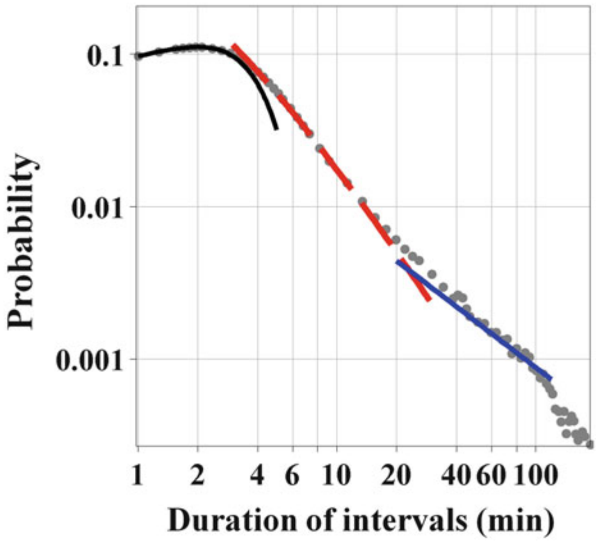


**Fig. 5.3** The distribution of intervals between consecutive communication events for all members of the two working teams (on a log–log scale). The first fitting curve (*black solid line*) holds for the normal distribution $\exp\left[-(t-\nu)^2/2\sigma^2\right]/\sqrt{2\pi\sigma^2}/2$, characterized by a mean interval of $\nu = 2.0$ min between consecutive communications, with standard deviation $\sigma = 1.78$. The crossover in the right tail of the distribution occurs between the algebraic decay shown by the *red dashed trend line* $\sim 1/(t+1)(t+2)$ and the Zipf asymptote shown by the *blue solid line* $\sim t^{-1-\varepsilon}$, with $\varepsilon = 10^{-4}$

The distribution of intervals between consecutive communication events for all members of the two organizations is summarized in the chart in Fig. 5.3, using a log–log scale. The distribution is remarkably skewed, indicating a significant proportion of abnormally long periods of inactivity. The data for the shortest and most probable intervals between consecutive communications is well fitted by the very common normal probability distribution, characterized by a mean interval between consecutive communications of 2.0 min, with standard deviation 1.78 (see the solid trend line in Fig. 5.1).

The normal distribution of interval durations lasting not longer than 4 min can be interpreted as an average outcome of many statistically independent processes that determine the majority of short interruptions in communication. While the value of the normal distribution should be practically zero when the duration of the interval lies more than a few standard deviations away from the mean, the distribution of intervals lasting longer than 4 min exhibits a long right tail, indicating the effect of management strategies for interaction resumption after longer interaction breaks. The right tail of the distribution displays a crossover between the algebraic decay (fitting the data well for intervals between 4 and 20 min) and the Zipf asymptote (fitting the data best for longer intervals of 25–120 min) observed in many types of data studied in the physical and social sciences [17]. Finally, an exponential cutoff of the distribution for intervals longer than 2 h is obviously due to ‘finite size effects’, the large fluctuations that occur in the tail representing large but rare events.

Below, we propose a simple model for the decision to interact after a break that is analogous to the probability model of subsistence under uncertainty which we discussed in Chap. 1. The model starts from the obvious remark that it takes at least two to speak.

In the model, interaction is the result of two parameters, one regulating the willingness to propose an interaction, the other the willingness to accept it. Let us assume that the propensity of an individual to engage in an act of interaction can be characterized by a certain threshold $x_c \in [0, 1]$. If the potential partner is able to motivate her at time $t$ to interact by providing a strong enough reason $x_t \geq x_c$, she accepts the invitation to interact, but evades it otherwise. We assume that at each moment of time the degree of motivation varies, and if it is considered over the working team, it is a random variable distributed in the interval [0,1], with respect to some given probability distribution function $\Pr\{x < u\} = F(u)$. We also think of the threshold $x_c$ as being chosen once, randomly from the interval [0,1], with respect to some given probability distribution function $\Pr\{x < u\} = G(u)$.

The proposed decision-making model is based on a number of essential simplifications. First, it is difficult, if it is even possible at all, to reliably estimate the instantaneous degree of motivation $x_t$ and the way such a degree of motivation can be expressed, as it might depend upon the permanently variable interaction context and can involve many personal factors that lie well beyond the scope of any reasonable modeling. Second, in contrast to the instantaneously varying degree of motivation $x_t$, the threshold value $x_c$ filtering out the unimportant interactions is very likely determined by the pressure exerted by competing activities and the opportunity cost they generate. We might expect the value of $x_c$ to be quite high if we are pressed by a heavy schedule, having a lot to do that day, or on the contrary, we might expect it to be relatively low during leisure time. No matter whether the threshold value $x_c$ is high or low, we assume that it is virtually invariable (at least during the time of observation) in comparison to the highly variable degree of motivation $x_t$.

From the analysis of the probability model of subsistence under uncertainty, we know that the probability of observing an interval of length $t$ between consecutive interactions is

$$\Pi(t) = \int_0^1 \mathrm{d}G(u) F^t(u) \big[1 - F(u)\big] . \tag{5.1}$$

In particular, if both probability distributions $F$ and $G$ are taken to be the invariant measures of maps of the interval [0, 1], viz.,

$$\mathrm{d}F(u) = (1+\alpha)u^{\alpha}, \ \ \mathrm{d}G(u) = (1+\beta)(1-u)^{\beta}, \ \ \alpha > -1 \, , \ \ \beta > -1 \, ,$$

the probability (5.1) reads as follows:

$$\Pi(t) = \frac{\Gamma(2+\beta)\Gamma\big(1+(1+\alpha)t\big)}{\Gamma\big(2+\beta+(1+\alpha)t\big)} - \frac{\Gamma(2+\beta)\Gamma\big(1+(1+\alpha)(t+1)\big)}{\Gamma\big(2+\beta+(1+\alpha)(t+1)\big)} \, , \tag{5.2}$$

where $\Gamma(x)$ is the Gamma function. For instance, when both probability distributions are taken to be uniform, so that

$$\mathrm{d}F(x) = \mathrm{d}G(x) = \mathrm{d}x \, ,$$

the probability $\Pi(t)$ exhibits an algebraic decay:

$$\Pi(t) = \frac{1}{(t+1)(t+2)} \approx \frac{1}{t^2} \, , \quad t \gg 1 \, . \tag{5.3}$$

The algebraically decaying function (5.3) describes the statistics of intervals between consecutive communications quite well for intermediate values of the intervals, between 4 and 20 min (see the dashed trend line in Fig. 5.3), but fails to explain the data for longer intervals, lasting from 30 to 130 min. It is remarkable that the slowly decaying far-right tail of the distribution for longer intervals, well fitted by the Zipf asymptote, can be explained as a limiting case of the same simple model for communication decision-making—for institutional (mandatory) communications.

This is the limiting case of an increasingly high threshold filtering out the unimportant interactions $x_c \to 1$, so that only mandatory (institutional) communications are attended. Mandatory communications may include urgent or exigent contacts made in an emergency, as well as some common rites and rituals that serve important functions for all team members. In most organizations, attending a mandatory meeting is an obligation that is difficult to avoid, even on days off. Referring to the discussion of the probability model of subsistence under uncertainty, we can choose the distribution of $x_c \to 1$ to be a spike-like probability distribution concentrated at 1:

$$G_{\varepsilon}(u) = 1 - (1-u)^{\varepsilon} \, , \quad \varepsilon > 0 \, .$$

Then the corresponding probability density over the interval $]0, 1]$ is $\mathrm{d}G_\varepsilon(u) = \varepsilon(1-u)^{1-\varepsilon}\mathrm{d}u$, and for any choice of the probability $F$, the probability of the interval between consecutive mandatory communication acts is dominated by the Zipf asymptote as $t \gg 1$:

$$\Pi_2(t) \approx \frac{t^{-1-\varepsilon}}{\zeta(1+\varepsilon)}\,, \quad \varepsilon > 0\,, \tag{5.4}$$

where $\zeta(x)$ is the Riemann zeta function. It is clear that, for long enough time intervals $t \gg 1$, the slowly decaying Zipf asymptote $\Pi_2(t) \propto t^{-1-0.01}$ effectuates the crossover between the trends $\Pi_1(t)$ and $\Pi_2(t)$ visible in Fig. 5.3 (solid trend line).

It is also worth mentioning that the normal distribution of interval durations lasting not longer than 4 min can be naturally interpreted in the framework of the proposed model as unmanaged short intervals characterized by the very low threshold $x_c \to 0$, so that any interaction can be resumed after an occasional short break not exceeding 4 min. Given that the probability of communication resumption is the same for all participants in all trials, the frequency distribution of the possible number of successful communication acts in a given number of trials is the binomial distribution, which is best approximated by the normal distribution, if the chances of undergoing a brief interruption are close to what would be given by fair coin tossing.

### 5.3.2 Interaction Durations

The distribution of communication durations is also strongly skewed. In general, brief communications are much more common than longer ones, and the shortest communication events (of 1 min) are the most frequent among all interactions (see Fig. 5.4). The statistics of the communication durations indicate a significant proportion of long interactions. Communications with the shortest and most probable durations (1–2 min) may also be fitted by a normal probability distribution with a standard deviation (dashed trend line in Fig. 5.4), although the two points are not enough for a reliable fitting. However, it is obvious that the distribution of interactions with durations exceeding 2 min has the right tail, indicating the effect of interaction management strategies. Similarly to the statistics of the intervals, the right tail of the distribution (Fig. 5.4) displays a crossover between the asymptotically algebraic decay (fitting the data well for intermediate durations of 2–20 min) and the Zipf asymptote shown by the solid trend line (fitting the particular data points for communication durations of 13–120 min). Finally, exceptional (unique) instances of very long interactions constitute outliers among the duration statistics.

The distribution of interaction durations can be interpreted with the help of a threshold model for decision-making that is similar to the one we used for the distribution of intervals between consecutive communication events. We assume that different pairs of individuals have different 'propensities' for interacting with

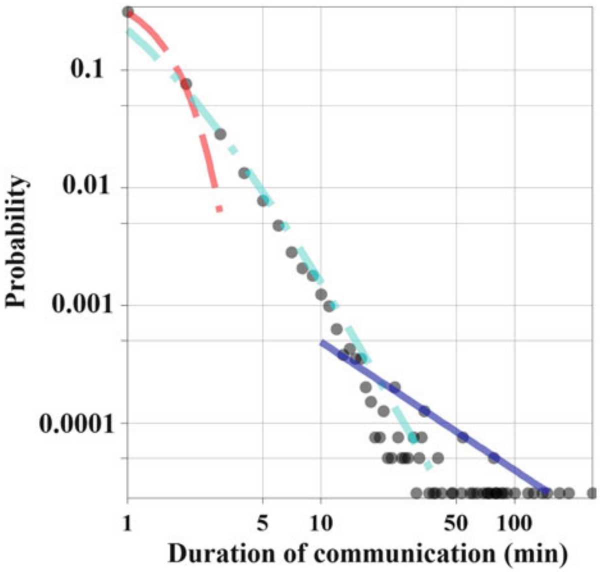


**Fig. 5.4** The distribution of communication durations for all members of the two organizations is given on a log–log scale (*dots*), along with three *trend lines*

each other. Such propensities should be taken as broader than simple 'liking': they may be due to homophily, task complementarities, organizational roles, spatial proximity, or many other factors. These propensities affect the relative likelihood that *A* interacts with *B* rather than *A* with *C*. At the same time, the duration of interactions with others can be limited by considerations of cost, by interrupting events, or other random causes of the interaction. The huge potential variety of factors limiting the continuation of interactions suggests treating those causes statistically in terms of an ensemble of random variables.

We assume that the propensity of an individual to keep the current interaction going can be characterized by a certain threshold $y_c \in [0, 1]$. If the communication partner challenges the already heavy schedule of the individual at time $t$ by $y_t \geq y_c$, the current interaction stops, but it keeps going otherwise. The proposed threshold model mimics the continuous decision-making process for unceasing interaction. We further assume that, at each moment of time, the parameter $y_t$ varies, being a random variable distributed over the interval $[0, 1]$ with some probability distribution $\Pr\{x < u\} = F(u)$, and the critical threshold value $y_c$ is chosen randomly once from the interval $[0, 1]$, according to another probability distribution $\Pr\{x < u\} = G(u)$, and kept unchanged during the interaction.

The statistics of communication events for the intermediate interaction durations between 2 and 20 min (Fig. 5.4) are best fitted by the probability function (5.2), with $\alpha = \beta = 1.0$ in the probability densities d$F$ and d$G$. The corresponding trend is shown in Fig. 5.4 by the dash-dotted line and can be approximated asymptotically for $t \gg 1$ by the cubic hyperbola $\Pi(t) \propto t^{-3}$. The choice $\alpha = \beta = 1.0$ in the model indicates that high values of the tolerance threshold $y_c$ are increasingly probable compared with lower values, but the high values of the motivation parameter $y_t$ are less and less probable compared with lower values: the communication process is statistically 'sticky' for intermediate times.

Eventually, in the case of the highest tolerance thresholds $y_c \to 1$, which that can be modeled by the spike-like probability distribution concentrated at 1, the probability of the interaction duration $t$ follows the Zipf law asymptote $\propto t^{-1-0.09}$, dominating the statistics for longer interaction durations as shown in Fig. 5.4 by the solid trend line. The Zipf asymptote may correspond to protracted institutional interactions, for which no characteristic time limits are imposed.

We conclude the analysis of temporal patterns of interaction with a remark about the three statistically different types of interaction reflecting the different valuation and management strategies applied to time intervals of different duration. These three interaction regimes can be parsimoniously represented by different distributions of communication durations and intervals between consecutive communication events. Short time intervals (interaction durations and intervals between consecutive communications alike) remain largely unmanaged and unregulated. Short occasional breaks in communication are tolerated. On the one hand, short interactions are unavoidable as soon as a person randomly bumps into someone else, but on the other hand they may be so undemanding that one can hardly refuse them—everybody can be engaged in a brief communication at any moment of time, so we call these *casual interactions*.

Time intervals of intermediate durations (lasting up to 20–25 min) are thoroughly managed by individuals demonstrating a high propensity to keep the current interaction going, while filtering out potentially unimportant forthcoming communications. We call such interactions *spontaneous*, as they are motivated by the propensity to interact with others.

Finally, where Zipf's law manifests itself, we suggest that a logic of *institutional interaction* prevails, where top-down, almost mandatory interaction occurs. The simple threshold models for the decision to interact and to keep the current interaction going support the proposed taxonomy.

## 5.4 Time and Social Structure of Interactions

Up to now we have analyzed time patterns of interactions, abstracting from the concrete relational structure within which they were taking place. However, interpreting the distribution of interaction durations and intervals has required us to introduce heterogeneous propensities of individuals to interact with each other. In this section, we take a closer look at the finer texture of the relational network of agents, and how it interlaces with the temporal unfolding of interactions. The non-monotonic relationships between interaction time and different network metrics suggest that the three regimes of interaction found in our former analysis may contribute to explaining how relational structures and temporal patterns of interaction affect each other.

We have already introduced a notion of interaction propensity in relation to spontaneous interactions occupying intermediate time intervals. Not all agents are equally likely to interact with each other spontaneously in structured contexts such as organizations, for a host of reasons including personal preferences, task requirements, and organizational roles. It is a reasonable to conjecture that such propensities may affect interaction durations occupying time intervals that last up to 20–25 min. Each individual should be expected to spend longer when interacting with other individuals with whom she or he has a higher propensity to interact. At the same time, there are competing demands on each individual time budget that may limit this effect—one cannot spend an infinite time with other people one likes. Beyond a reasonable time limit, one may expect other institutional factors to become the dominant driver of time allocation. For example, in the organizations we studied, periodic collective meetings can force face-to-face interactions over protracted times. In order to analyze how interaction propensities and the duration of interaction affect each other, we use mutual information [90] as a statistical measure of pairwise interaction propensities.

Given a random event $X_A$ that a subject $A$ is presently communicating (with anybody) during time $t$, described by the probability function $P_t(X_A)$, and a random event $X_B$ that another subject $B$ is communicating during the same time, characterized by the probability function $P_t(X_B)$, it is possible to analyze the pairwise interaction preferences of $A$ and $B$, as well as those of the entire working team, with the help of the mutual information [90]:

$$I(t) = \sum_{\{A,B\}} I_{AB}(t) = \sum_{\{A,B\}} P_t(X_A, X_B) \log_2 \frac{P_t(X_A, X_B)}{P_t(X_A)P_t(X_B)} \ , \tag{5.5}$$

where the summation is performed over all possible pairs of individuals $\{A, B\}$. If during the observation period $A$ and $B$ participated in meetings independently, in which case

$$P_t(X_A, X_B) = P_t(X_A)P_t(X_B) \ ,$$

then the amount of mutual information $I_{AB}(t)$ associated with such a pair is zero. As the amount of mutual information in a communicating pair obviously reaches its maximum if, whenever $X_A$ takes part in a communication event, $X_B$ always does too (since they may perhaps be speaking to each other), this value allows one to assess the degree of communication preferences in each pair and, when summed over all communicating pairs, the degree of communication preferences within the working team as a whole. The mutual information can be analyzed for every

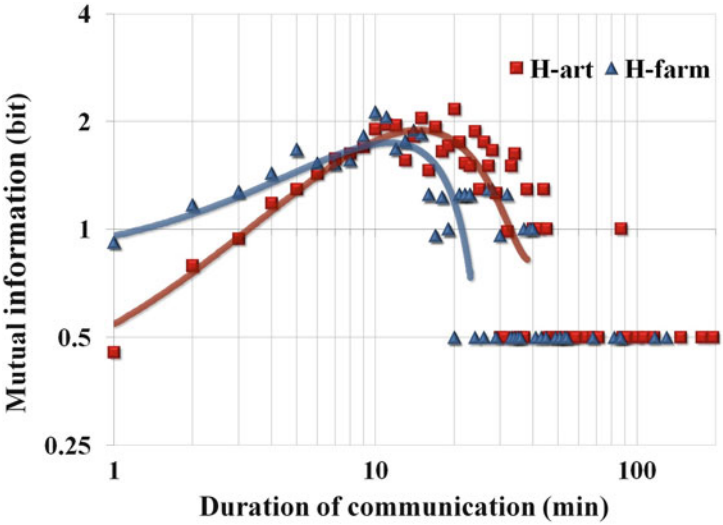


**Fig. 5.5** Mutual information vs. communication duration. *Trend lines* (given by cubic splines optimally fitting the collected data points) are shown to facilitate understanding of the dependencies

communication duration, serving as a measure of how much knowing that $A$ is communicating during time $t$ would reduce uncertainty about the possibility that $B$ is communicating, provided that the joint probability $P_t(X_A, X_B)$ for $A$ and $B$ is known.

We have used the mutual information to analyze interaction preferences in communications of every duration. Figure 5.5 shows the dependence of mutual information upon the durations of the observed communication acts. High values of mutual information show that team members demonstrate a high degree of selectivity when they choose an interaction partner, and conversely, interaction partners are selected at random if the level of mutual information is minimal.

☞ People are essentially selective in choosing partners for communications lasting between 10 and 20 min.

The analysis of mutual information carried out here shows that the degree of selectivity in both companies increases monotonically with the interaction duration, until their maximum values are attained, for durations ranging between 10 and 20 min, and then falls rapidly to the minimal values. For particularly long interactions, perhaps involving many group members at once, the values of mutual information are particularly small, as the statistical contribution from the rare long conversations occurring between pairs of individuals was insignificant. Thus, the structure of interactions in the two organizations reveals an essentially high degree of selectivity for the interactions whose duration is concentrated in the interval from a few minutes to several tens of minutes.

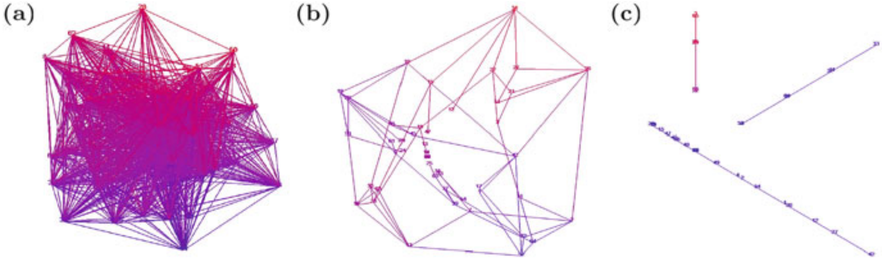


**Fig. 5.6** Interaction networks between the HA firm members, corresponding to three different sample durations: (**a**) 1 min, (**b**) 10 min, (**c**) 20 min

In the communication graphs, each vertex represents an individual, and edges connecting the vertices are characterized by weights assigned according to the intensity of communication between the two (for instance, the probability of communication). In our dynamic approach, instead of a single static communication graph, we have analyzed an ensemble of graphs, in which the probabilities of communications in all pairs of interlocutors are described by an individual graph, for each communication duration. The collected empirical data shows convincingly that the shortest communication events lasting 1 min are ubiquitous, encompassing all employees, and perhaps serving the basic communication needs within a working team (see Fig. 5.6a).

The communication graphs that describe the probabilities of pairwise communications of longer durations are more sparse, but also more rich in structure, accounting for a good deal of the personal and working communication preferences. For example, they can include micro-communities, consisting of just a few permanently communicating partners, loosely connected (in the sense of communication probability) with other members of the working teams. In particular, the communication graphs for the longer durations can contain a number of connected components of different sizes, sometimes including either a single pair of interlocutors, or just a disconnected vertex (if the corresponding subject never took part in a communication event of that duration during the whole observation period) (see Fig. 5.6b).

Interactions of different durations may possess very different structural properties and generate different graphs. Again, the data suggests the existence of three regimes. Shortest duration interactions correspond to low mutual information. The shortest interaction events lasting a few minutes are ubiquitous, and correspond to random, occasional, or aborted encounters—it also takes a short interaction just to say that you don't actually have time. Intermediate duration is where the graph of interactions strongly reflects pairwise interaction propensities. It is more sparse but has more structure. This corresponds to the regime that we have labeled as spontaneous interaction. Above approximately 20 min, there is a decline in mutual information that we interpret as the result of a substitution of motivations to interact, from spontaneous to institutional. The graph of interactions is structured for the main part by persistent chains of interactions reflecting collective work meetings where agents are sitting close to each other in a meeting room (as clearly suggested by the longest chain in Fig. 5.6c).

## 5.5 Time, Interaction Synchronization, and Information Transmission

We turn our attention now to the information transmission properties of face-to-face interactions. We look at interactions as communication episodes. The main objective of this analysis is to understand how the 'local' individual interaction propensities, described by the connectivity of subjects as nodes of a communication graph, determine the 'global' connectedness property of the whole communication process, described by the ensemble of communication graphs for all communication durations.

In order to address this problem in relation to all communication graphs, let us consider a model of simple random walks, a statistical metaphor of message transmission in a working team.

We suppose that a message (requiring $t$ time units to be transmitted) is passed on by each subject $X$ to another one $Y$, selected at random among all available companions according to the connection probability $T^{(t)}_{XY}$ determined by the communication graph of communication duration $t$. We can characterize a degree of variability in individual (local) communication preferences by the minimal amount of information required to record the choice of a partner $Y$ for communication made by $X$ in order to pass a message:

$$h^{(t)}_X = -\sum_{\{Y\}} \pi^{(t)}_X T^{(t)}_{XY} \log_2 T^{(t)}_{XY} , \quad (5.6)$$

where $\pi^{(t)}_X$ is a stationary distribution of the random walk, the left eigenvector of the matrix $T^{(t)}_{XY}$ belonging to the maximal eigenvalue 1, and, as usual, we suppose that $0 \times \log 0 = 0$. Then the minimal amount of information required to record a single random transition of a message in the communication graph as a whole, corresponding to the duration $t$, is defined by the entropy rate of random walks [125], viz.,

$$H^{(t)} = -\sum_{\{X,Y\}} \pi^{(t)}_X T^{(t)}_{XY} \log_2 T^{(t)}_{XY} , \quad (5.7)$$

summed over all pairs of interlocutors. The entropy rate reaches the maximal value if subjects have no communication preferences, transmitting the message equiprobably to any other member of the working team, but it takes the minimal values when a connected component of the communication graph constitutes a chain, in which only the subsequent communication partner is available. In homogeneous graphs, where all vertices and all transitions between them are supposed to be equiprobable, the transmission of a message can be viewed as a sequence of statistically independent transmission events, so that its entropy rate is the same as the entropy of any individual member in the communicating team.

The serial quantities,

$$H_2^{(t)} = -\sum_{\{X,Y\}} \overset{(t)}{\Pr}\left\{X \overset{2}{\rightarrow} Y\right\} \log_2 \overset{(t)}{\Pr}\left\{X \overset{2}{\rightarrow} Y\right\} \ , \tag{5.8}$$

$$H_3^{(t)} = -\sum_{\{X,Y\}} \overset{(t)}{\Pr}\left\{X \overset{3}{\rightarrow} Y\right\} \log_2 \overset{(t)}{\Pr}\left\{X \overset{3}{\rightarrow} Y\right\} \ , \tag{5.9}$$

and so on, where $\Pr^{(t)}\left\{X \overset{n}{\rightarrow} Y\right\}$ denotes the probability of observing a path of length $n$ connecting $X$ and $Y$ (which is readily calculated, as the transition probabilities in random walks are independent of $n$), sequentially define the Shannon entropy over the $n$ blocks [125], quantifying the amounts of information required to record a block of $2, 3, \ldots$ random transmissions of the message in the communication graph.

Complementary information on the global connectedness of communication graphs can be obtained by analyzing the level of correlations between infinitely long paths (along which a message would be transmitted) using the excess entropy [126], viz.,

$$\mathcal{E}^{(t)} = \lim_{N\to\infty}\left[H_N^{(t)} - NH^{(t)}\right] \ , \tag{5.10}$$

which expresses the amount of information required to describe the additional structural irregularities of message transmission that cannot be explained statistically by a simple superposition of individual communication propensities, while considering increasingly longer paths of message transmission. If the excess entropy is zero, the interaction process is perfectly synchronized within a single stream of sequential communication events.

However, large values of excess entropy indicate that the process of message transmission cannot be synchronized within a single communication stream in the same time slot. Since the group members have rather different individual interaction propensities during the different time intervals, several independent interaction streams are required in order to synchronize them simultaneously.

We can get an insight into the complexity of interaction schedules by juxtaposing the entropy rates expressing the connectivity property of the entire communication graph and the excess entropy of random walks describing the correlations of the very long message transmission paths (see Fig. 5.7).

We used the diagram showing the entropy rates vs. the excess entropy earlier, in order to study the graphs and their subgraphs at different scales [127]. The data on the values of excess entropy and entropy rates for the messages transmitted by random walkers on the communication graphs (Fig. 5.7) show that differences in

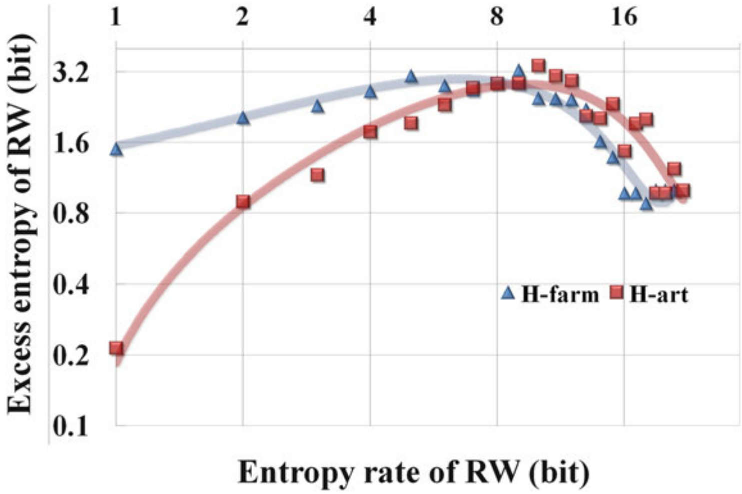


**Fig. 5.7** The excess entropy of random walks vs. the entropy rate of random walks for the ensembles of communication graphs corresponding to the different communication durations. The *trend lines* are shown to facilitate understanding of the dependencies

institutional structure are crucially important for the complexity and heterogeneity of interactions.

Interactions in H-art, with a modified functional structure, look more natural, as the low values of the excess entropy are associated with low values of the entropy rate, and the interaction complexity assessed by the excess entropy grows steadily with improvement of the local connectivity in the interaction graphs. However, the level of complexity of interactions in the H-farm, with a traditional functional structure, is already relatively high for low entropy rates, indicating the importance of institutional ties for structuring interactions. Nevertheless, independently of the difference in functional structure, the level of interaction complexity remains bounded by approximately 3.2 bits for the entropy rate of 10 bits, for both firms.

In a completely open communication environment where all 73 employees in the working team can talk to each other, the entropy rate of 10 bits means that the probability of interaction between any pair of members would amount to 0.78. Therefore, by promoting subjects to become more open and flexible when choosing a partner for transmitting messages, we can promote complexity of interactions in the group as a whole.

However, if the entropy rate exceeds 10 bits (i.e., subjects communicate all together on virtually every occasion), the level of interaction complexity within the teams decays rapidly, reducing the communication process to attendance at general meetings.

We conclude the discussion on structural properties of communication graphs by comparing the statistics of recurrence times and first-passage times of random walks over the biggest connected components of these graphs, observed for the different durations of interactions.

The recurrence time of random walks to a node in an undirected graph depends upon the connectivity of the node, i.e., the number of its nearest neighbors, a local structural property of the node in the graph. In contrast, the first-passage time to the node, the expected number of transmissions (steps) required for a message (a random walker) to reach the node from any other node chosen randomly over the communication graph, according to its propensity of interactions during time $t$, represents a global structural property of the node in the graph, as all possible paths for message transmission to it following a self-avoiding random walk (with no self-loops) are taken into account, even though some paths are rendered more probable than others. While the recurrence time can be viewed as a metaphor for circulating rumors, accounting for the average number of transmission acts required for a message to return to its source given that the random walk carrying the rumor can revisit any vertex many times, the first-passage time rather characterizes the directed message transmission through the uncertain communication environment.

While relatively low recurrence times might be typical for individuals with high communication propensity (connectivity), disregarding their role for the group structure as a whole, relatively low first-passage times indicate the importance of individuals making up message transmission paths of all lengths for the group structural integrity even if their communication propensity is relatively low. Indeed, a bridge connecting city districts situated on the opposite banks of a river is vital for the entire urban transportation system, despite its limited connectivity to the immediate city neighborhoods [129, 130]. Typical values of the recurrence time are close to those of first-passage times in structurally homogeneous graphs (such as the graphs of casual and institutional interactions involving virtually all team members). However, they can differ dramatically for certain nodes in graphs with more structure, such as the communication graphs of spontaneous interactions that strongly reflect pairwise interaction propensities and a high degree of communication selectivity. The application of first-passage times to the analysis of the structure of graphs and databases is discussed in detail in [128], along with calculation methods for first-passage times.

Furthermore, it is reasonable to suggest that a group of individuals constitutes a single communication entity whenever the global structural property of many individuals (characterizing connectedness of the corresponding communication graph) gets an edge over the local, individual connectivity property. Such a group would constitute a good transmission medium for messages addressed to every group member. The ability of the group to transmit messages directly, in short enough time, obviously reveals the level of its global connectedness with respect to interactions of a given duration, as the message can follow all paths available in the communication graph at once. On the other hand, a message can spread over the group, literally at random, as a rumor, due to the local, individual interaction propensities of group members. For the window of interaction durations typical for spontaneous interactions, we expect the quality of global connectedness of the relevant communication graphs to be superior to the net effect of local connectivity.

Any vertex $X$ in a finite weighted undirected connected graph can be characterized with respect to nearest neighbor random walks by the recurrence time to that vertex (how long one must wait to revisit the vertex on average):

$$R_x = \frac{1}{\pi_X}\,, \tag{5.11}$$

where $\pi_X$ is the stationary distribution of random walks on $X$. For finite weighted undirected connected graphs, the stationary distribution $\pi_X = \deg(X)/2E$ is nothing else but the connectivity $\deg(X)$ of the vertex $X$, normalized to twice the total weight of all edges in the graph $2E$ (as every edge can be traversed in both directions), and therefore characterizes a local property of the vertex, independent of the connectivity of other vertices [128, 131].

☞ Communication graphs rich in cycles can promote social interactions.

In the first-passage time to $X$, all possible transmission paths of any length ending at $X$ are taken into account (excluding the paths comprising loops), yet some of them are considered to be more probable then others, being weighted by the probability of being traversed by a random walker. Given the transition matrix $T_{XY}$ describing a nearest neighbor random walk on a finite connected undirected graph, the first-passage times can be calculated as the diagonal elements $\Phi_X = (L^{-1})_{XX}$ of the multiplicative inverse $L^{-1}$ of the Laplace operator

$$L_{XY} = \delta_{XY} - T_{XY} \tag{5.12}$$

describing the corresponding diffusion process on the graph. In general, in connected undirected graphs, the first-passage time to any vertex of the graph does not equal the recurrence time to it, although it is proportional to it, i.e.,

$$\Phi_X = c_X R_X. \tag{5.13}$$

In contrast to the recurrence time to $X$, the first-passage time to this vertex characterizes the role of $X$ with respect to the entire graph structure, as all non self-intersecting paths of all lengths ending at $X$ are taken into account. The value of the coefficient $c_X$ characterizes the role of the vertex $X$ for the entire graph structure, picking out structurally integrated and structurally isolated vertices [129]. In particular, a random walker would be trapped in the sites $X$, for which $c_X > 1$, as the time of recurrence is shorter than the first-passage time, i.e., $R_X < \Phi_X$, but would virtually fly by the sites where $R_X > \Phi_X$. In the context of the random walk model of the communication process, the ratio of the two characteristic times $c_X$ calculated for the nodes of a communication graph can spot key individuals playing an important role in communications of the given duration (Fig. 5.8).

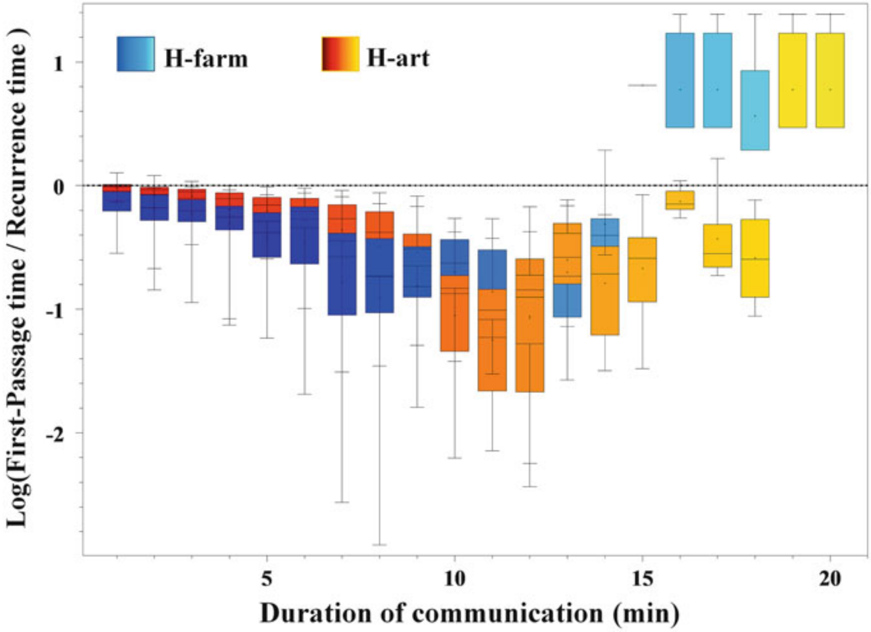


**Fig. 5.8** The box plots represent the distributions of the ratio of the first passage time to a node and the recurrence time to it in the biggest connected components of the communication graphs for different communication durations

In order to characterize the structural properties of the biggest connected components of communication graphs of every duration, we have summarized the data on the distributions of ratios $c_X$ for all $X$ in the form of box plots, which are useful for comparing similar data sets. Each box plot comprises a central line showing the median of the data, a lower line showing the first quartile, and an upper line showing the third quartile. Two lines extend from the central box of maximal length 3/2 of the interquartile range (if it does not extend past the range of the data). Finally, outliers indicate the data values that lie outside the extent of the previous elements.

Multiple cycles and structural heterogeneity are gradually effaced from the biggest connected components of communication graphs observed for durations longer than 12 min. These are structurally dominated by chain segments, due to the high level of selection in personal and working communication, promoting directed propagation of messages over the circulation of rumors. It is remarkable that, in the communication graphs for durations not exceeding 15 min, the recurrence times to most of the team members are typically longer than the corresponding first-passage times, i.e., $c_X < 1$, indicating that these graphs are very well integrated. The level of connectedness in graphs for short communication durations systematically surpasses the level of local connectivity based on the individual communication preferences of group members.

Once again, we find the non-monotonic shape characterizing the relationship between mutual information and interaction duration, suggesting that the same underlying principle unifies different observations.

## 5.6 Communication Pyramid and Longevity of Institutions

At the origin of the human race, in the first family or group, a single person (a father) spoke, and others imitated him, starting to talk to each other after numerous imitations [124]. A word that began as the word of a leader, ordering, warning, threatening, or condemning, belonged to a realm of mandatory communications that was not reciprocal, since it assumed neither objection, nor reply. Then, after being copied and repeated, it became the word for a deliberate communication between equals (peers), each with an equal chance of influencing the other. They approved the initial word until, finally, it turned into the word of a spoken language, a vehicle for casual communication.

To some extent, the words of mandatory institutional interactions prevail over those of both deliberate and casual communications by stimulating, enslaving, and providing a contextual framework for them that is crucially important for bringing about a change of opinion and a change of behavior, as well as for imposing the required public opinion. Equality of peers in deliberate communications requires active maintenance from every member of a group. The main feature of deliberate communication is that it occurs by mutual agreement between partners, which is the central point of the model for communication decision-making discussed in the last section.

Intentionally interacting peers have equal rights, either to accept communication or reject it. On the other hand, by requiring obedience to group discipline, mandatory communications maintain the superiority of authority and, once established, rule over the individual rights of group members, thus transforming a group of people into an organization that can be integrated by chains of commands.

☞ Institutional communications exploit the social solidarity in group members.

Our results reported in the previous section show that, while equality of deliberate communications requires active maintenance from every member of a group (by filtering out the unimportant interaction motives), mandatory communications would reward renunciation of dominance with a sense of full social acceptance, engendering a strong pull of social solidarity in group members that can be expressed in special rituals, rites, and other common social events.

It is therefore vitally important for an organization to find an optimal balance between these two types of communication. In view of that, we can summarize graphically our observations of the intervals between interactions with an 'onion dome', reflecting different types of interaction, i.e., in the form of a consecutive interval population pyramid, in which every axially centered horizontal bar indicates

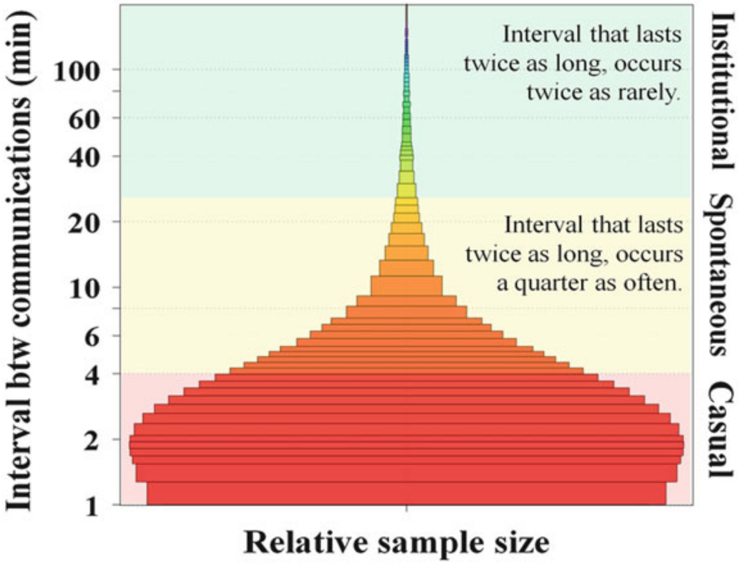


**Fig. 5.9** The interval dome. The majority of experimentally observed interactions were casual, being characterized with an average break of about 2 min between consecutive communication acts. There is a rule of thumb for spontaneous interactions: an interval that lasts twice as long, occurs a quarter as often. Finally, institutional interactions satisfy another rule of thumb: an interval between them that lasts twice as long, occurs twice as rarely

a fraction of consecutive interactions, sorted according to the intervals between them (see Fig. 5.9).

The pyramid takes the form of an 'onion dome' with a very wide bulge at the base, corresponding to the casual interactions which dominating the others in number, then rapidly contracts as one moves upwards from deliberate to mandatory communications.

☞ Institutions never die since, when interrupted, communication in them can be resumed anytime.

The contraction rates in the deliberate and mandatory levels of the pyramid are different. In deliberate communications maintained by every member of the working teams, an inter-event interval that lasts twice as long occurs on average a quarter as often. And in mandatory communications, exploiting the social solidarity in group members, an interval that lasts twice as long occurs twice as rarely. The slow decay rates of the distributions of intervals between consecutive intentional communications provide a statistical basis for the extraordinary longevity of organizations and institutions.

Since neither deliberate, nor institutional mandatory communications possess any characteristic time scale, i.e.,

$$\sum_{t\geq 0} t\Pi_2(t) \gg \sum_{t\geq 0} t\Pi_1(t) \to \infty\,, \quad t \to \infty\,. \tag{5.14}$$

Both deliberate and institutional mandatory communications in organizations, once interrupted, can be resumed at any time.

## 5.7 Conclusion

This study is the first to our knowledge to integrate the analysis of temporal patterns of interaction, interaction preferences, and the local vs. global structure of communication in networks of agents. Our results suggest that simple principles reflecting interaction propensities, time budget, and institutional constraints underlie the distribution of interaction events. As a result, the duration of interactions (as well as the intervals between interactions) reveal deep aspects of social systems. Not only does interaction duration reveal a multiplicity of regimes affecting interaction parameters, but it also offers differentiated time windows over different social network structures corresponding to such regimes.

Certainly, important aspects of the interaction process may have been affected by the setting in which we made our observations. Our focus on two business organizations may have led to a stronger emphasis on institutional factors of communication than one might find in less structured contexts. However, as no human interaction is completely devoid of any institutional constraints, we expect our results to hold in a variety of interaction contexts. This paper also extends the range of tools available for analyzing the dynamic properties of interaction. In particular, we have demonstrated that mutual information can be useful for assessing pairwise interaction propensities. Finally, we have shown that, by comparing the statistics of recurrence times and first-passage times of random walks over the biggest connected components of the graphs for different communication durations, one can appraise the quality of global connectedness of the working team for interactions of functionally important durations.

We have also demonstrated that equality of peers in deliberate communications requires active maintenance from every member of a group, and mandatory institutional communications exploit the social solidarity in group members. We conclude that institutions never die because, when interrupted, communication in them can be resumed at any time.

# Anatomy of Success, Hierarchy, and Inequality 6

*– Is this not the dwelling of the Chosen One and the Khan of Luck, Aladdin?*
*– Aladdin is my son.*
*– Here are presents for you from your uncle!*
*– I didn't know my husband was blessed with a brother.*
*– Is this not the house of Aladdin, son of Ali Al Marouf?*
*– From my uncle?*
*– Your uncle.*
*– They must have made a mistake.*
*– You'll see—They'll come and take it all back.*[1]

**Abstract**

Domains with a lot of uncertainty have the highest likelihood of skilled people failing. Those that succeed the most under uncertainty are often simply those that tried harder and whose early luck compounded. By fostering hierarchical organization in a group, uncertainty ultimately leads to inequality. Wealth inequality in a population arises from risky decisions being taken under uncertainty by the vital few: the more adventurous traders are, the greater their fortune, and the fewer lucky ones there are. Scarcity also promotes inequality by necessitating competition and fueling conspicuous consumption. Existing econometric data suggest that rising income inequality is a global phenomenon, occurring whenever the national economy is out of step with the world average. Rampant inequality may transform the uncertainty of national economic development into uncertainty of international relations.

[1]From the movie script *Aladdin's Magic Lamp*, directed by B. Rytsarev, M. Gorky Central Film Studio for Youth and Children, USSR (1966).



D. Volchenkov, *Survival under Uncertainty*, Understanding Complex Systems,
DOI 10.1007/978-3-319-39421-3_6

## 6.1 Introduction

Success comes with perseverance and improvements, opposing luck, over which we have no control.

The probability model of success can be viewed as correlated Bernoulli trials, in which the probability of winning in a random experiment with exactly two possible outcomes, 'success' and 'failure', would depend on the number of trials and previous (successful) outcomes. While the chances of getting lucky might be as small as $p_0 \ll 1$, perhaps in the same vein as the likelihood of being struck by lightning, success results from subsequent deliberate actions and skill acquisition, the aim being to work on the opportune occasion in such a way as to boost its chances of recurrence in the future:

$$p \to p + \delta p \, . \tag{6.1}$$

The positive probability increment $\delta p > 0$ in (6.1) can describe the effectiveness of learning, or a gain in advantageous skills that contribute toward a favorable outcome. It can also result from a preferential attachment mechanism, where the probability of the next outcome in a series is proportional to the number of previous successful outcomes.

Although a matter of pure chance $p_0 > 0$, the launch phase of success is often surprisingly controllable. For instance, since the initial phase of a new business is a search for customer needs that have not yet been addressed, identification of a critical point that would represent a major business opportunity in the future can be accomplished via a predictable process, which identifies a set of market hypotheses and seeks to validate them one after the other through controlled experimentation [132].

As soon as we have identified where to focus our efforts, it is time to put ourselves in a position to win. There are two factors contributing to the process of success: the number of times (the amount of time) we tried and the magnitude of the probability increment $\delta p$. However, the road to success is not that easy.

## 6.2 Enhancing the Chances of Success by Persistent Learning and Skill Acquisition

Individuals are capable of continually developing their abilities through persistence and effort. The degree to which early success (characterized by some probability $p_0 > 0$) causes subsequent success may be attributed to a learning process, in which existing knowledge, behavior, and skill are modified and reinforced. Progress over time does not happen all at once, but builds upon and is shaped by previous knowledge.

We suppose that the efficiency of a learning process can be described by the increase in the probability of achieving success in the future, after every successful trial, viz.,

$$\delta p = p_n - p_{n-1} = \omega = \text{Const.}, \tag{6.2}$$

which we assume to be a fixed constant $\omega > 0$ for simplicity. We suppose that $p_n = 1$ if $p_{n-1} + \omega > 1$, and $p_n = 0$ if $p_{n-1} + \omega < 0$.

Let us study the distributions of the numbers of successful outcomes in the model (6.2).

The positive probability increment $\omega > 0$ describes a positive feedback on the motivation to perform further trials after the previous success. The modification of a Bernoulli random process that includes a simple component of self-affirmation (6.2) has been introduced and studied in detail in [133].

We consider the trials as a series of $N$ Bernoulli random variables $u_i$, $i = 1, \dots, N$, with probabilities $1 - p_i$ and $p_i$ for the outcomes 0 (failure') and 1 (success), respectively. We are interested in the distribution $P_N\big(\sum_{i=1}^{N} u_i = n\big)$ of the $n$ successes over $N$ trials.

With no effect of learning, that is, $p_i = p$, all $u_i$ are independent identically distributed random variables, and $P_N$ is given by the binomial distribution

$$P_N\left(\sum_{i=1}^{N} U_i = n\right) = \binom{N}{n} p^n (1-p)^{N-n} . \tag{6.3}$$

The effect of positive feedback (6.2) for the bimodal model (6.3) is revealed by the geometric distribution of distances $D_i$ between consecutive successful events, viz.,

$$P\left(D_i = d_i\right) = p_i (1-p_i)^{d_i - 1} , \quad i = 0, \dots, n , \tag{6.4}$$

with respect to the probabilities $1 - p_n$. Therefore, the desired distribution of successes expressed by

$$P_N(n) = \sum_{\{\sum_i u_i = n\}} P(u_1, \dots, u_N) , \tag{6.5}$$

where $P(u_1, \ldots, u_N)$ is the joint probability distribution of the series of random variables $\{u_i\}$, can be calculated as

$$P_N(n) = \sum_{\{\sum_i d_i = N\}} p_0 \cdots p_{n-1} (1-p_0)^{d_0-1} \cdots (1-p_n)^{d_n-1} \,, \tag{6.6}$$

where $1 \le d_i \le N - n$.

This marginal distribution satisfies an intuitively plausible Pascal type recurrence relation for the probabilities $P_N(n)$, expressing the simple idea that $n$ successes in $N$ trials can be reached either from $n$ successes in $N-1$ trials plus a final failure, or from $n-1$ successes in $N-1$ trials and a final success:

$$P_N(n) = (1-p_n)\, P_{N-1}(n) + p_{n-1} P_{N-1}(n-1) \,, \tag{6.7}$$

supplemented by the two boundary conditions $P_0(0) = 1$ and $P_N(n) = 0$, for $n > N$.

Multiplying (6.7) by $x^n$ and summing over all $n = 0, \ldots, \infty$, one arrives at the equation

$$G_N(x) - G_{N-1}(x) = (x-1) H_{N-1}(x) \tag{6.8}$$

for the generating functions,

$$G_N(x) = \sum_{n=0}^{\infty} x^n P_N(n) \quad \text{and} \quad H_N(x) = \sum_{n=0}^{\infty} x^n p_n P_N(n) \,. \tag{6.9}$$

Another equation required in order to complete the system comes from the relation $p_n = p_0 + \omega n$, following for the probability gain (6.2):

$$H_N(x) = p_0 G_N(x) + \omega x \frac{\partial}{\partial x} G_N(x) \,. \tag{6.10}$$

Combining (6.8) and (6.10), we obtain the finite difference equation

$$G_N(x) - G_{N-1}(x) = (x-1) \left( p_0 + \omega x \frac{\partial}{\partial x} \right) G_N(x) \,. \tag{6.11}$$

For $N\omega < 1$, we can use the continuum approximation $N \to t$ and replace the finite difference in the left-hand side of (6.11) by the time derivative, whence

$$\frac{\partial G(x,t)}{\partial t} = (x-1) \left[ p_0 G(x,t) + \omega x \frac{\partial}{\partial x} G(x,t) \right] \,, \tag{6.12}$$

with the solution

$$G(x,t) = \left[ \mathrm{e}^{\omega t} - x(\mathrm{e}^{\omega t} - 1) \right]^{-p_0/\omega} \,. \tag{6.13}$$

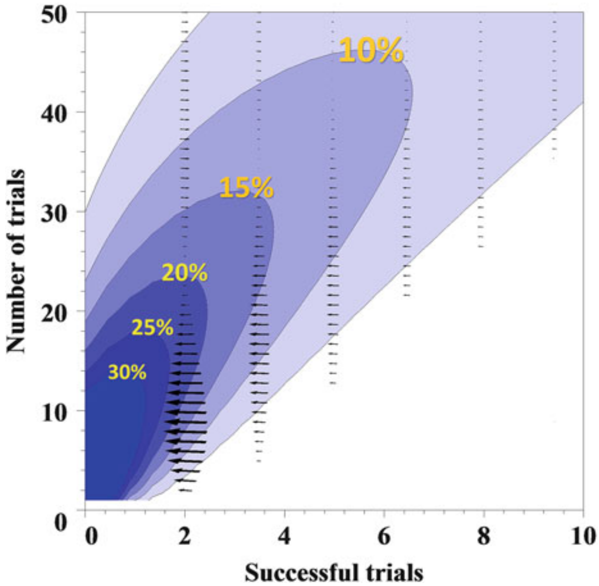


**Fig. 6.1** Probability density of successful trials in a process with persistent learning, in which the initial probability of success is $p_0 = 0.1$ and the probability increment $\omega = 0.02$. *Arrows* show the probability gradients

The time continuum analog of the probability function $P_N(n)$ that corresponds to the generating function (6.13) is nothing else but a negative binomial distribution, viz.,

$$P_t(n) = \mathrm{e}^{-p_0 t} \frac{\Gamma(p_0/\omega + n)}{n!\,\Gamma(p_0/\omega)} \left(1 - \mathrm{e}^{-\omega t}\right)^n = \binom{r+n-1}{n} p^n (1-p)^r \,, \qquad (6.14)$$

with time dependent probability function $p = 1 - \mathrm{e}^{-\omega t}$, where $r = p_0/\omega$. The derived continuum approximation (6.14) is valid for a total number of trials not exceeding $t_{\max} < \omega^{-1}$ and for a total number of successes less than $n_{\max} = (1 - p_0)/\omega$.

Figure 6.1 shows the probability density plot for the number of successful trials in the process with persistent learning, in which the initial probability of success is $p_0 = 0.1$ and the probability increment $\omega = 0.02$. When learning matters, the number of tries is attributed to skill. The probability gradients shown in Fig. 6.1 by arrows 'worsen' the chances for success if the number of trials is small, but 'enhance' these chances for longer trial sequences.

☞ Driving down cycle time in trials allows for more experiments, which can produce better results for those whose early luck is compounded.

## 6.3 Like a Squirrel in a Wheel: Freud's Repetition Compulsion

Factors over which we have no control may play an important role in determining performance amidst uncertainty. When cause and effect are not well understood, or the environment is permanently changing, even those who do everything 'right', will fail more likely than not, as acquired skill does not necessarily pay off over time.

☞ Areas involving high uncertainty are most likely to be the ones where skilled people will fail.

A natural mechanism that can help to improve the long-term performance under uncertainty is *diversification of activities*. By getting involved in many different projects, e.g., by making a portfolio of many investments, or by bearing and raising many children, one can dramatically increase the chance of that advantageous skill, and persistent efforts will redeem over time.

We consider a large group of individuals engaged in many different activities, each characterized by some probability of initial success $p_0 > 0$ and by some probability increment $\omega > 0$, after every successful trial. The state of getting precisely $n < n_{\max}$ successful outcomes after $t < t_{\max}$ trials in every activity is then characterized by the probability (6.14), where $n_{\max}$ and $t_{\max}$ are precisely determined by the given values of $p_0$ and $\omega$.

According to the second law of thermodynamics, the equilibrium state of such a group striving for success in a variety of activities can be determined as a state of maximum entropy, viz.,

$$H(p_0, \omega) = -\sum_{n=0}^{n_{\max}} \sum_{t=1}^{t_{\max}} \left\{ P_t(n) \log_2 P_t(n) + \left[1 - P_t(n)\right] \log_2 \left[1 - P_t(n)\right] \right\}, \tag{6.15}$$

which is attained for such a probability distribution $P_t(n)$ that can be achieved in the maximum number of ways—the most likely distribution to be observed over all individuals in the group. The phenomenological entropic force driving the group to increase its entropy of success can be expressed as the entropy gradient with respect to the parameters $p_0$ and $\omega$ determining the chances for success.

It is worth mentioning that the number of different $(n, t)$ states with precisely $n$ successful outcomes after $t$ trials grows unboundedly as the probability increment tends to zero, i.e., $\omega \to 0$, so the entropy of success (6.15) does too.

Figure 6.2 presents the density plot of entropy (6.15) as a function of learning efficiency (characterized by the probability increment $\omega$) and task simplicity

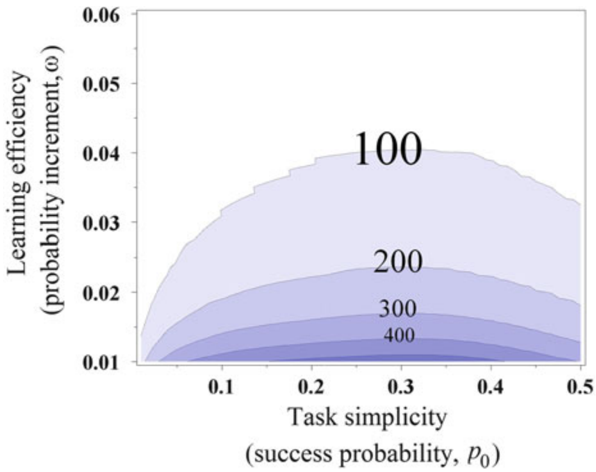


**Fig. 6.2** Density plot of entropy (6.15) as a function of learning efficiency ($\omega$) and probability of initial success ($p_0$). The contours correspond to entropy levels of 100, 200, 300, 400, and 500 bits

(quantified by the probability of initial success $p_0$). The contours shown in Fig. 6.2 correspond to rising entropy levels of 100, 200, 300, 400, and 500 bits, respectively.

As can be seen in Fig. 6.2, the entropy increases as the number of states grows, as $n_{\max}, t_{\max} \sim \omega^{-1}$, and tends to infinity as the probability increment decreases $\omega \to 0$. The maximum entropy gradient is observed for the initial probability of success $p_0 \approx 1/3$.

☞ When initial success is random, skill does not necessarily play a role over time for improving the future chances of success.

Therefore, when success breeds success, but initial success was random, the most likely behavior to be observed over a large enough group of individuals engaged in a variety of activities is to *make things into a matter of routine*, by continuously repeating actions over and over again, *without* searching for any improvement in the future chances of success.

Extreme uncertainty can be a very serious stress factor indeed. Interestingly, the endless repetition of behavior, or life patterns, which were difficult (or distressing) in earlier life was a key concept in Freud's understanding of mental life—this is known as *repetition compulsion* [134].

☞ Being in a group in the face of uncertainty, we are forced to repeat the same behavior pattern, without any improvement, as no lesson can been learnt from the previous experience.

The essential character traits which remain always the same and which are compelled to find expression in a repetition of the same experience appeared to Freud as ultimately contradicting the organism's search for pleasure, and this justified Freud's hypothesis [135]:

> [...] hypothesis of a compulsion to repeat—something that seems more primitive, more elementary, more instinctual than the pleasure principle which it overrides.

In later editions of his work, Freud expanded on this point: by stating that

> [...] such repetitions are of course the result of instincts intended to lead to satisfaction; but no lesson has been learnt from the old experience of these activities, which led only to unpleasure.

## 6.4 The Rich Get Richer: Pareto Principle

The sociologist R. Merton was the first to recognize the phenomenon of *accumulated advantage*, emerging when skill does not tell over time and diversification of activities is impossible. He dubbed it the *Matthew effect*, quoting a Bible passage in which the rich get richer and the poor get poorer.

The self-reinforcing behavior of certain probability distributions and stochastic processes has been known since the early works of Gibrat [136] and Yule [137]. The best known example is the stochastic urn process, in which discrete units of wealth, usually called 'balls' and denoted by $\circ$, are added continuously as an increasing function of the number of balls already present in a set of cells, usually called 'urns' and denoted by $|\ \ |$, arranged in linear order [138]:

$$|\circ\circ\circ\circ|\circ|\ |\circ\circ|\,.$$

Let the size $t_k$ of the $k$th cell be the number of balls in this cell plus one, i.e., the number of spaces existing in the cell: between two balls, or between two bars, or between a ball and a bar. Steady state distributions of cell sizes can be obtained if the number of cells $n$ is increased proportionally as the number of balls is increased.

> In each round of the urn process, either a bar or a ball is selected with probability $\alpha$ and $1-\alpha$, respectively. If a ball is selected, it is thrown in such a way that each space in all cells has an equal chance of receiving it. If a bar is selected, it is placed next to an existing bar, so that new cells of unit size emerge at a rate $\alpha$.

The average size of cells is therefore a random variable with mean $1/\alpha$. Note that the aggregate size

$$t = \sum_{k=1}^{n} t_k \tag{6.16}$$

of all cells is increased steadily by one at the end of the round, regardless of whether a bar or a ball is selected in any given round, either because the size of one of the cells is increased by one or because a new cell of size 1 is added. Thus, we can use $t$ not only as the aggregate size but also to count the number of rounds, i.e., time, in the urn process.

Let $p(x, t)$ be the expected value of the number of cells with size $x$ when the aggregate size of all cells is $t$. Then, for $x = 1$, we have

$$p(1, t + 1) - p(1, t) = \alpha - \frac{1 - \alpha}{t} p(1, t) \ , \tag{6.17}$$

where $\alpha$ is the probability that $p(1, t)$ is increased by one and $(1 - \alpha)p(1, t)/t$ is the probability that $p(1, t)$ is decreased by one as a result of a ball falling in one of the unit-sized cells.

It is clear that, in the steady state, for all $x = 1, 2, \ldots,$ we should have

$$P(x) = \frac{p(x, t + 1)}{\alpha(t + 1)} = \frac{p(x, t)}{\alpha t} \ , \tag{6.18}$$

where $\alpha t$ is the expected value of the total number of cells after $t$ rounds.

Setting $x = 1$, we can use the right-hand side of (6.18) to eliminate $p(x, t + 1)$ from (6.17):

$$p(1, t) = \frac{\alpha t}{2 - a} \ . \tag{6.19}$$

For $x > 1$, we have

$$p(xt + 1) - p(xt) = (1 - \alpha) \left[ \frac{(x - 1)p(x - 1, t)}{t} - \frac{xp(x, t)}{t} \right] . \tag{6.20}$$

The last equation assumes that the increase in the number of balls (accumulated wealth) in a cell is proportional to the current cell size; it is impossible to make any cell better off (by increasing its wealth), without making at least one cell worse off.

Using (6.18) in (6.20), we then obtain

$$\frac{p(x, t)}{p(x - 1, t)} = \frac{(1 - \alpha)(x - 1)}{(1 + (1 - \alpha)x)} \ . \tag{6.21}$$

If we define

$$\rho = \frac{1}{1-\alpha} \ , \tag{6.22}$$

it then follows from (6.21) that

$$\frac{p(x,t)}{p(x-1,t)} = \frac{x-1}{x+\rho} \ , \tag{6.23}$$

for any time $t$, and therefore, for the stationary distribution, it will also be true that

$$\frac{P(x)}{P(x-1)} = \frac{x-1}{x+\rho} \ . \tag{6.24}$$

Moreover, since the stationary probability of obtaining a single unit of wealth is

$$P(1) = \frac{p(1,x)}{\alpha t} = \frac{1}{2-\alpha} = \frac{\rho}{1+\rho} \ , \tag{6.25}$$

this stationary probability distribution can be calculated from the following product:

$$P(x) = \frac{x-1}{x+\rho} \prod_{r=1}^{x} \frac{r-1}{r+\rho} \ . \tag{6.26}$$

The product formula (6.26) immediately gives the expression for the Yule distribution:

$$P(x) = \rho \frac{\Gamma(x)\Gamma(\rho+1)}{\Gamma(x+\rho+1)} = \rho B(x,\rho+1) \ , \tag{6.27}$$

where $\Gamma(x)$ and $B(x)$ are the Gamma and Beta functions, respectively.

The cumulative distribution function for the Yule distribution (6.27), viz.,

$$F(x) = \sum_{i=x}^{\infty} \rho B(i,\rho+1) = \rho B(x,\rho) \ , \tag{6.28}$$

is characterized by the skewed heavy-tailed asymptote for $x \to \infty$, as

$$B(x,\rho) \ \to \ \Gamma(\rho)x^{-\rho} \ , \tag{6.29}$$

so that the limiting cumulative distribution follows a power law:

$$\lim_{x\to\infty} F(x) = \frac{\Gamma(\rho+1)}{x^{\rho}} \ . \tag{6.30}$$

According to (6.30), a relative change in the size of a cell (accumulated wealth) always results in a proportional change in the probability of its occurrence over all cells.

The exponent $\rho$ in (6.30) is the reciprocal probability of adding a ball (a unit of wealth) in a given round, and this is nothing else but the average wealth per cell in the urn model.

☞ Processes of accumulated advantage lead to skewed heavy-tailed (Pareto) wealth distributions.

Approximate power law distributions similar to (6.30) are observed over a wide range of magnitudes, in a wide variety of physical, biological, and man-made phenomena, where an equilibrium is found in the distribution of the 'small' to the 'large' [139].

In particular, a power law distribution—the well-known Pareto distribution [140]—was suggested in the context of the distribution of upper incomes and wealth among the population as early as 1896:

$$F(x) = 1 - \frac{1}{x^\rho}, \quad 1 \leq x \leq \infty, \tag{6.31}$$

where $\rho$ is a fixed parameter called the Pareto coefficient and $x$ is the variable size. It then follows that the probability density function for (6.31) can be described by

$$f(x) = \frac{\rho}{x^{\rho+1}}, \quad 1 \leq x \leq \infty, \tag{6.32}$$

whence Zipf's law may be thought of as a discrete counterpart of the Pareto distribution.

The Pareto distribution (6.31) has been used to describe the allocation of wealth over a population, since a larger portion of the wealth is usually owned by a smaller percentage of individuals in any society. Therefore, it is intuitive that as incomes increase, the number of cases of higher incomes would be expected to decline, following a law dictated by some constant parameter.

This idea is sometimes expressed more simply as the *Pareto principle*, or the *80–20 rule*, the law of the vital few, which says that 20 % of the population controls 80 % of the wealth. Although the 80–20 rule corresponds to a particular value of $\rho \approx 1.161$, it becomes a common rule of thumb in business, e.g., 80 % of sales come from 20 % of clients. Pareto suggested that the negative of the slope $\rho$ might be an indicator of inequality in the underlying population, implying that small values of $\rho$ relate to high inequality.

☞ Under uncertainty, only the vital few accumulate advantage.

## 6.5 Inequality Arising from Risk-Taking Under Uncertainty

An individual is risk averse if he is not willing to accept a fair gamble, with an expected return of zero. “Anyone who bet any part of his fortune, however small, on a mathematically fair game of chance acts irrationally”, wrote Daniel Bernoulli in 1738 [141]. It is the reluctance of a person to accept a bargain with an uncertain payoff rather than another bargain with a more certain, but possibly lower, expected payoff.

People’s preferences with regard to choices that have uncertain outcomes are described by the *expected utility hypothesis* [142]. This hypothesis states that, under the quite general conditions the subjective value associated with an uncertain outcome is the statistical expectation of the individual’s valuations over all outcomes. In particular, a decision-maker could use the expected value criterion as a rule of choice in the presence of risky outcomes. The individual’s *risk aversion* is accounted by a mathematical function called the utility function [141]. Utility refers to the perceived value of a good (or wealth), and the utility function (viewed as a continuous function of actual wealth) describes the attitude towards risky projects of a ‘rational trader’, whose objective is to maximize the growth of his or her wealth in the long term. Such a trader would attach greater weight to losses than to gains of equal magnitude. Thus, risk aversion implies that the utility functions of interest are concave (Fig. 6.3).

A plausible example of a utility function is given by

$$U_\lambda(w) = \frac{w^\lambda - 1}{\lambda}\,, \tag{6.33}$$

where $0 < \lambda < 1$ is the *risk tolerance* parameter—as $\lambda$ decreases, traders become more risk-averse and vice versa. In the limit of maximum risk avoidance, $\lambda \to 0$,

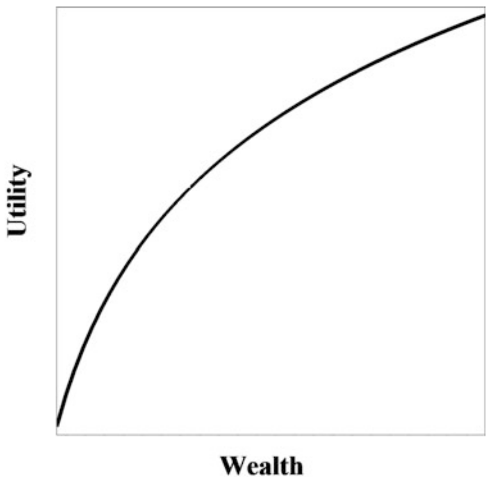


**Fig. 6.3** Risk aversion implies that utility functions are concave

and the function (6.33) turns into the Bernoulli logarithmic utility function [141],

$$\lim_{\lambda\to 0} U_\lambda(w) = \lim_{\lambda\to 0} \frac{\mathrm{d}\big(\mathrm{e}^{\lambda \ln w} - 1\big)/\mathrm{d}\lambda}{\mathrm{d}\lambda/\mathrm{d}\lambda} = \ln w\ . \tag{6.34}$$

Let us consider a population characterized by some distribution of wealth $p_w$. The expected utility over the population is

$$\upsilon = \langle \ln w \rangle = \sum_w p_w \ln w\ . \tag{6.35}$$

According to the maximum entropy principle [29, 30], the system would evolve toward the state of maximum entropy characterized by the probability distribution which can be achieved in the largest number of ways, this being the most likely distribution to be observed. We are interested in the probability distribution of wealth $p_w$ over the population with maximum entropy:

$$H_w = -\sum_w p_w \ln p_w\ , \tag{6.36}$$

under the condition of maximum risk avoidance. As soon as the expected logarithmic utility (6.35) is given, it is well known [37] that the maximum entropy (6.36) is attained for power law distributed wealth.

☞ The Pareto wealth distribution over a population arises under zero risk tolerance.

Given expected utility $\upsilon$, the Lagrangian functional for the entropy function $H_w$ subject to two constraints reads as

$$\mathcal{L} = -z\left(\sum_w p_w \ln w - \upsilon\right) - (\ln\zeta - 1)\left(\sum_w p_w - 1\right) - \sum_w p_w \ln p_w\ . \tag{6.37}$$

Then, the equation for the most likely wealth probability distribution to be observed is

$$-z \ln w - \ln\zeta - \ln p_w = 0\ . \tag{6.38}$$

The explicit solution of (6.38) is nothing but the Pareto distribution

$$p_w = \frac{w^{-z}}{\zeta(z)}\ , \quad w > 1\ , \tag{6.39}$$

where $\zeta(z) = \sum_w w^{-z}$ is the appropriate normalization constant. The value of the Lagrange multiplier $z$, which becomes the exponent in the power law (6.39), can be determined self-consistently as the solution of the equation $\upsilon(z) = -\mathrm{d}\ln\zeta(z)/\mathrm{d}z$.

In the case of a less risk averse population, when $\lambda > 0$, variation of the corresponding functional leads to the equation

$$-zU_\lambda(w) - \ln\zeta - \ln p_w = 0\,, \tag{6.40}$$

with a more uneven, exponential solution for the most likely wealth probability distribution:

$$p_w = \frac{1}{\zeta(z)} \mathrm{e}^{-z\left(w^\lambda+1\right)/\lambda}\,. \tag{6.41}$$

Therefore, we conclude that wealth inequality can be viewed as a direct statistical consequence of *making decisions under uncertainty under the condition of zero risk tolerance*. The more risk is taken by traders investing under uncertainty, the more unequal the distribution of assets that is likely to be observed among them in the long term.

☞ Wealth inequality among the population arises from the vital few taking risky decisions under uncertainty: the more adventurous traders are, the greater their fortune, and the fewer lucky ones there will be.

## 6.6 Generalized Mass-Action Principle: Emergence of Hierarchies Under Uncertainty

A graph is a representation of a set of objects (called *vertices*, or *nodes*), in which some pairs of objects may be connected by links (called *edges*). The *degree* of a vertex in a graph is the number of edges incident to it, called its *valence*.

Random graphs with scale-free probability distribution of node degrees are ubiquitous in many real world networks such as the World Wide Web and social, linguistic, citation, and biochemical networks. An excellent survey on scale-free properties of real world networks can be found in [145]. Many of them are formed according to the preferential attachment principle which, together with its various modifications, can be seen as a special case of the *degree-mass-action principle* since the degree of a node acts as a positive affinity parameter (a *mass*) quantifying the attractiveness of the node for new vertices [144]. ‘Preferential attachment’ is perhaps the most recent of many names that have been given to processes, in which a quantity is distributed over a number of units according to how much of it they already have.

Our first aim is to construct a family of static random graph models, in which vertex degrees are distributed according to a power law, while edges still have a high degree of independence. Our aim is to mimic the community formation process under uncertainty. As usual in random graph theory, we will deal entirely with asymptotic properties, in the sense that the graph size goes to infinity.

We consider graphs with vertex set $V = V_n = \{1, \ldots, n\}$, where an edge between the vertices $x$ and $y$ (denoted by $x \sim y$) is interpreted as a persistent contact between the two. Given $x \in V$, its degree will be denoted by $d(x)$. We view edges as being generated by a pair formation process, in which each individual vertex $x$ chooses a set of partners according to a specified $x$-dependent rule.

The set of individuals which has a contact with the vertex $x$ can be divided into two possibly not disjoint sets:

- the set of nodes which are chosen by $x$ herself/himself,
- the set of nodes which have chosen $x$ as one of their partners.

We call the size of the first set the out-degree $d_{\text{out}}(x)$ of $x$ and the size of the second one, the in-degree $d_{\text{in}}(x)$ of $x$. Obviously,

$$d(x) \leq d_{\text{out}}(x) + d_{\text{in}}(x) \ , \tag{6.42}$$

and if the choices of partners are sufficiently independent, one can expect the equality in (6.42) to hold almost surely for infinitely large graphs $n \to \infty$.

We partition the set of vertices $V_n$ into groups $\{C_i(n)\}_{i \geq 1}$, where all members of the group $C_i(n)$ choose exactly $i$ partners by themselves, i.e., $d_{\text{out}} = i$ on $C_i(n)$. Let $P^1_\alpha\,(n, j)$ the probability of $x$ choosing a fixed partner $y \in C_j\,(n)$ , among $n$ partners available for the choice, provided that just a single choice will be made, is

$$P^1_\alpha\,(n, j) = A_\alpha(n)\frac{j^\alpha}{n} \ . \tag{6.43}$$

Here $A_\alpha(n)$ is a normalization constant such that

$$A_\alpha(n)\left[\sum_{i \geq 1} |C_i(n)|\frac{i^\alpha}{n}\right] = 1 \ , \tag{6.44}$$

and $\alpha$ is a real parameter. Since we want $A_\alpha(n) \to A_\alpha$, as $n \to \infty$, we need the sum $\sum_i |C_i(n)| i^\alpha / n$ to be bounded as a function of $n$, and this will impose constraints on the constant $\alpha$.

The parameter $\alpha$ in (6.43) acts as the degree of affinity, tuning the tendency to choose a partner with a high out-degree, or with a low out-degree. So if $\alpha = 0$, the choice is made without any preference, and $A_\alpha(n) \equiv 1$. For $\alpha > 0$, the 'highly active' individuals are preferred, whereas individuals of 'low activity' are favored if $\alpha < 0$.

We then obtain the basic probability that $x$ and $y$ are connected in the graph:

$$\Pr[x \sim y] \simeq A_\alpha(n)\,\frac{i \cdot j^\alpha}{n}\;, \quad x \in C_i\;, \quad y \in C_j\;. \tag{6.45}$$

Concerning the size of the sets $C_i(n)$, we make the following assumption:

$$\frac{|C_i(n)|}{n} = p_i\,(n) \longrightarrow \frac{c_1}{i^\gamma}\;, \quad \text{as} \quad n \to \infty\;. \tag{6.46}$$

With this choice, we have to impose the restriction $\alpha < \gamma - 1$ to ensure convergence of $A_\alpha\,(n)$ as $n \to \infty$.

We also require $\gamma > 2$, since otherwise the expected in-degree for individuals from a fixed group would diverge. Note that the fixed out-degree distribution defines a probability distribution on each graph with vertex set $V_n$, and therefore a random graph space $\mathcal{G}_n\,(\alpha, \gamma)$.

In order to compute the important pairing probabilities, we start with the easier case $\alpha = 0$:

$$\Pr\left(x \sim y \mid x \in C_i\,; y \in C_k\right) = \frac{i+k}{n} - \frac{ik}{n^2} \underset{n\to\infty}{\sim} \frac{i+k}{n}\;. \tag{6.47}$$

Likewise, one can compute the corresponding probabilities for $\alpha \neq 0$. Dropping the details of the calculations, we just state the result:

$$\Pr\left(x \sim y \mid x \in C_i\,; y \in C_k\right) \simeq A_\alpha \frac{ki^\alpha + k^\alpha i}{n}\;, \quad n \to \infty\;. \tag{6.48}$$

It turns out that for $\alpha < 2$ the typical community graphs in the model of community formation still have a *power law* distribution for the node degree, although they are characterized by the different exponents for the in-degree and out-degree components. For $\alpha > 2$, we obtain a degree distribution which follows a power law on average.

In order to compare the two domains, we use the integrated tail distribution

$$F_k = \Pr\left(d(x) > k\right)\;. \tag{6.49}$$

We show that in both cases, $\alpha < 2$ and $\alpha > 2$, we nevertheless get the same tail for the degree distribution.

Since partner choice under uncertainty is sufficiently random and is not strongly biased toward high degree individuals (that is the meaning of the condition $\alpha \leq$

$\gamma - 1$), it is easy to see that the in-degree distribution of a vertex from the group $C_i$ converges to a Poisson distribution with mean const. $\times\, i^\alpha$ for $n \to \infty$:

$$\Pr(X{=}k) = \frac{\lambda^k \mathrm{e}^{-\lambda}}{k!}\,, \quad \lambda = \text{const.} \times i^\alpha\,. \tag{6.50}$$

Furthermore, there are essentially two regimes of community formation in the parameter space:

- one for which the expected in-degree is of smaller order than the out-degree over all groups, and
- one for which the in-degree is asymptotically of higher order.

In the first case, it is clear that the in-degree is too small to have an effect on the degree distribution exponent. In other words, the set of individuals with degree $k$ consists mainly of individuals whose out-degree is of order $k$. Further estimation shows that the expected in-degree of individuals from the group $i$ tends asymptotically to the power law

$$\left\langle \left(d_{\text{in}}(x) \middle| x \in C_i\right) \right\rangle \simeq \text{const.} \times i^\alpha\,. \tag{6.51}$$

Therefore the in-degree is of smaller order than the out-degree if $\alpha < 1$. In the case $\gamma - 1 \geq \alpha \geq 1$, the set of individuals with degree $k$ consists mainly of those whose index is of order $k^{1/\alpha}$.

We conclude that under uncertainty, when the cumulative advantage is at work, there may be no parity of chances to choose a partner and to be chosen as a partner by other individuals.

☞ In uncertain romantic relationships, love triangles are inevitable.

While aiming at exclusive relationships with a partner, we always get an arrangement unsuitable to one or more of the people that might be involved: we either have to compete with rivals for the love of the beloved, or we have to split our attention between many objects of desire, so that love triangles become inevitable.

Hierarchical organization within large human groups integrated by chains of command would emerge under uncertainty in pretty much the same way. As the group size grows, face-to-face interactions among all group members are not required, as the accumulated advantage featuring selection of partners for interactions makes probable only a few interactions between a superior and several subordinates, and this is enough to maintain integrity within the entire group. In the face of uncertainty, the group grows by adding hierarchical levels, and such a process obviously has no physical limit. However, the downside of hierarchical organization is that it inevitably leads to inequality [146].

☞ By fostering hierarchical organization in a group, uncertainty ultimately leads to inequality.

The egalitarian social ethos typical for most hunter-gatherers is possible only for small enough groups based on kinship and tribe membership, as equality requires active maintenance by means of continuous face-to-face interactions. While hierarchical organization emerges spontaneously amidst uncertainty (as seen among chimpanzees, forming themselves into hierarchies dominated by an alpha male), people living in small-scale societies possess numerous norms and institutions designed to control those individuals who attempt to dominate others [147]. However, such norms cannot be efficiently implemented within groups that are too large or among individuals that live far apart. Thus, the side-effect of selection for greater societal size was the appearance of permanent leaders and hierarchical organization [148].

## 6.7 Inverse Mass-Action Principle: Inequality Due to Uniqueness and Scarcity

In the *economics of location theory* introduced in [149] and developed by Henderson [150], a city or a particular city district may specialize in the production of special goods connected with a unique natural resource, the special education or capabilities of inhabitants, efficient policy, or unusually low expenditures. Cities and city districts can compete among themselves in a market of unique products that is not necessarily connected with the number of their inhabitants. The demand for these unique and precious products comes into the city district from everywhere and can be considered as exogenous. The degree of attractiveness of such a place can be specified by a real positive random variable $\omega > 0$ [143]. In the present section, we describe an edge formation principle in a random graph related to some degree of attractiveness imposed *a priori* on the vertex set in a large graph [144, 151].

We thus assume that attractiveness can be described by a real positive random variable $\omega$ that quantifies some important attractive property of an 'individual', i.e., an available place (such as its wealth or popularity), or the beauty and importance of the place. The degree of attractiveness is distributed over the population (or all available places) with a given probability distribution $\varphi(\omega)$. We shall demonstrate below that the particular form of the function $\varphi$ is not important for a qualitative understanding of the advantage accumulation process.

Furthermore, we assume that a link between two individuals (or places) $x$ and $y$ arises as a result of a directed choice made by either $x$ or $y$ (symbolized by $x \to y$ or $y \to x$, respectively). Although the edge creation is certainly a directed process, in the present section, we consider the resulting graph to be undirected since, for the majority of relevant transmission processes defined on the network, the original orientation of an edge is irrelevant.

We suppose that the pairing probability follows an *inverse mass-action principle*: the probability that $x$ decides to connect to $y$ characterized by its affinity value $\omega(y)$ reads as

$$\Pr\left\{x \to y\middle|\omega(y)\right\} \sim \frac{1}{N}\frac{1}{\varphi\big(\omega(y)\big)^{\alpha}}\,, \quad \alpha \in (0;1)\,, \tag{6.52}$$

where $N$ is the total number of vertices. Let us note that it is not the actual value $\omega(y)$ which plays a decisive role while pairing, but rather its relative frequency of appearance over the population.

The proposed principle captures the essence of antiquity markets:

☞ The rarer a property, the higher its value, and the more attractive it becomes for others.

The pairing probability model described above is called the *cameo principle*, thinking of the attractiveness, rareness, and beauty of a small medallion with the profile of a head in relief, called a cameo. And it is exactly their rareness and beauty which gives them their high value [144, 151].

In practice, it is indeed difficult if not impossible to estimate the exact value of $\omega$ for any individual (or place), since such an assessment is obviously referred to both economic and cultural factors, and these may vary greatly over different historical periods and across certain populations.

In the framework of a probabilistic approach, it seems therefore natural to consider $\omega$ as a real positive independent random variable distributed uniformly over the vertex set of the graph, according to a smooth monotonically decreasing probability density function $f(\omega)$.

When we introduce cameo graph model, we make the following assumptions:

- The parameter $\omega$ is an independent identically distributed (i.i.d.) random variable over the vertex set, with a smooth monotonically decreasing density function $\varphi(\omega)$.
- Edges are formed by a sequence of *choices*. By a choice we mean that a vertex $x$ chooses another vertex, say $y$ , to form an edge between $y$ and $x$. A vertex can make several choices, but all choices are assumed to be made independently of each other.
- If $x$ makes a choice, the probability of choosing $y$ as a partner depends only on the relative density of $\omega(y)$, which is of the form (6.52).
- A predefined out-degree distribution determines the number of choices made by the vertices. The total number of choices (and therefore the number of edges) is assumed to be $\propto$ const. $\times\, N$.

We focus on the striking observation that, under the above assumptions, a scale-free degree distribution emerges independently of the particular choice of the $\omega$ distribution. Furthermore, it can be shown that the exponent in the degree distribution becomes independent of $\varphi(\omega)$, if the tail of $\varphi$ decays faster than any power law.

Let $V_N = \{1, \ldots, N\}$ be the vertex set of a random graph space. We are mainly interested in the asymptotic properties for $N \gg 1$. We assign to each element $x$ from the set $V_N$ a continuous positive real random variable $\omega(x)$ drawn from a distribution with density function $\varphi(\omega)$. The variable $\omega$ can be interpreted as a parameterization of $V_N$. For a set

$$C_{\omega_0,\omega_1} = \left\{x : \omega\,(x) \in [\omega_0, \omega_1]\right\} , \tag{6.53}$$

we obtain

$$\left\langle \sharp C_{\omega_0,\omega_1} \right\rangle = N \int_{\omega_0}^{\omega_1} \varphi(\omega) \mathrm{d}\omega , \tag{6.54}$$

where $\sharp C_{\omega_0,\omega_1}$ denotes the cardinality of the set (6.53). Without loss of generality, we assume that $\varphi > 0$ on $[0, \infty)$ , and that the the tail of the distribution for $\varphi$ is a monotonic function of $\omega > \omega_0$.

Edges are created by a directed process in which the basic events are choices made by the vertices. All choices are assumed to be independently identically distributed. The number of times a vertex $x$ makes a choice is itself a random variable which may depend on $x$. We denote this random variable by $d_{\mathrm{out}}(x)$. The number of times a vertex $x$ is chosen in the edge formation process is called the in-degree $d_{\mathrm{in}}(x)$. Although each choice generates a directed edge, we are interested in the corresponding undirected graph. In what follows, we refer just to the original direction in the edge formation process.

The probability that a vertex $y$, with a fixed value of $\omega$, is chosen by $x$ is

$$p_\omega = \Pr\left\{x \to y \middle| \omega = \omega(y)\right\} . \tag{6.55}$$

For a given realization $\xi$ of the random variable $\omega$, we assume that

$$p_\omega(\xi, N) = \frac{1}{N} \frac{A(\xi, N)}{\left[\varphi(\omega)\right]^\alpha} , \tag{6.56}$$

where $\alpha \in (0, 1)$ and $A(\xi, N)$ is a normalization constant. It is easy to see that the condition

$$\int_0^\infty \left[\varphi(\omega)\right]^{1-\alpha} \mathrm{d}\omega < \infty \tag{6.57}$$

is necessary and sufficient in order to get

$$A\,(\xi,N) \;\to\; A > 0\;, \quad N \to \infty\;, \tag{6.58}$$

so that we should have $\alpha < 1$.

One might argue that the choice of probabilities should depend more explicitly on the actual realization $\xi$ of the random variable $\omega$ over $V_N$, and not only via the normalization constant. The reason for not doing so is twofold. First, it is mathematically unpleasant to work with the empirical distribution of $\omega$ induced by the realization $\xi$, since one has to use a somewhat artificial $N$-dependent coarse-graining. Second, the empirical distribution is not really 'observed' by the vertices (having in mind for instance individuals in a social network). What seems to be more relevant is the *common belief* about the distribution of $\omega$. In this sense our setting is a natural one.

The emergence of a power law distribution in the above setting is no surprise. The situation is best explained by the following example. Let us take

$$\varphi(\omega) = C\mathrm{e}^{-\omega}$$

and define a new variable

$$\omega^* = \frac{1}{\big[\varphi(\omega)\big]^\alpha} = \frac{\mathrm{e}^{\omega\alpha}}{C^\alpha}\;.$$

The new variable $\omega^*$ can be seen as the effective parameter to which the vertex choice process applies. Then, the induced distribution of $\omega^*$ is

$$F(z) = \Pr\{\omega^* < z\} = \int_0^{\frac{1}{\alpha} z \ln C^\alpha} \varphi(\omega)\mathrm{d}\omega = -\frac{1}{z^{1/\alpha}} - C\;, \tag{6.59}$$

and therefore the $\omega^*$ distribution is nothing but a power law, viz.,

$$\phi\left(\omega^*\right) = \frac{1}{\alpha}\frac{1}{\left(\omega^*\right)^{1+1/\alpha}}\;,$$

with an exponent depending only on $\alpha$.

We conclude that a common belief about rarity (or scarcity) of a resource or an item automatically fosters inequality between individuals (or places) (that is revealed by highly skewed distributions of accumulated advantage) with respect to accessibility to (or availability of) the scarce resource, independently of the particular choice of the $\omega$ distribution.

☞ Scarcity always promotes inequality.

It is remarkable that by no means does an item have to be important for survival for it to be scarce. However, people have to be ready to make a sacrifice by giving something up, or by making a trade-off in order to obtain more of the resource that is wanted so much (and viewed as scarce).

Thus, scarcity ultimately involves a sacrifice for the sake of *keeping up with the Joneses*, indicating benchmarks for a particular social class, and demonstrating the desire of an individual for upward social mobility.

☞ Scarcity necessitates competition and fuels conspicuous consumption.

Scarce resources necessitate competition, as "people strive to meet the criteria that are being used to determine who gets what" [152], and fuel conspicuous consumption, as people always care about their standard of living in relation to their peers [153].

## 6.8 Cross-Database Analysis Suggests the Worldwide Growth–Inequality Relation (U-Curve)

The American economist S. Kuznets suggested that, as an economy develops, market forces first increase and then decrease economic inequality (see Fig. 6.4) [155]. Kuznets demonstrated this relationship using cross-sectional data about income inequality [154] collected in the United States in the middle of the twentieth century.

According to Kuznets, as a country develops, more capital is accumulated by the owners of industry, introducing inequality. However, more developed countries move then back to lower levels of inequality through various redistribution mechanisms, such as social welfare programs. Kuznets himself expressed concern about the 'fragility of the data' which underpinned the hypothesis and added [156]:

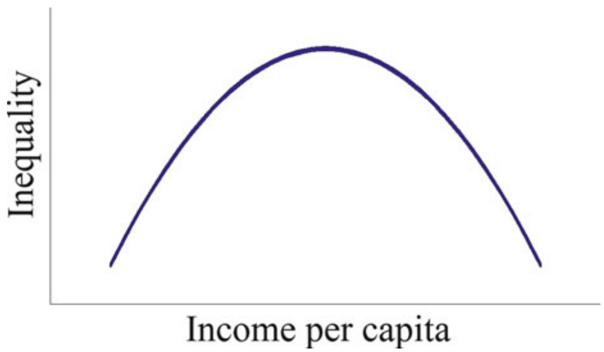


**Fig. 6.4** As an economy develops, market forces first increase and then decrease economic inequality, as S. Kuznets suggested

> [. . . ] even if the data turned out to be valid, they pertained to an extremely limited period of time and to exceptional historical experiences.

Recent data shows that, after the 1970s, the level of income inequality began to rise in the United States again [157], although there had been a worldwide pattern of declining inequality up until 1980 [158]. Then, a long and sharp period of increasing inequality was observed worldwide, starting from 1981 through to the end of the century. This portrayed inequality as a global macroeconomic phenomenon in the globalized world rather than a net result of the disparate effects of technology, trade in national labor markets, and particular national policy choices [158].

The new databases that have become available recently can help us to understand how economic growth and inequality are related: whether growth produces inequality, and whether inequality is a necessity for overall growth [159]. In the present section, we report on the unprecedented cross-database analysis of inequality and economic performance for different regions of the world and different periods of time.

First, we have used all existing data series (1870–2014) in the *World Top Incomes Database* [160] as the source of currently available inequality statistics in 30 countries,[2] over a long period of time, including scattered and isolated data patches, such as the top income data records for Denmark attributed to 1870. In contrast to existing international databases, generally restricted to the post-1970 or post-1980 period, the top income database covers (although only partially) nearly the whole of the twentieth century—a much longer period, which is important for the analysis, because structural changes in income and wealth distributions often span several decades [160]. It is also important that, by using data from the income tax records the studies underlying the database [157,161–164] used similar sources and methods to the pioneering work by Kuznets [154, 155] for the United States.

Second, we have used the GDP historical database of the Maddison Project[3] [165] on the gross domestic product (GDP) per capita (per person) as the main source of data on economic development and evolving living standards. GDP per capita is a measure of production within a country's borders (the sum of the gross values for all residents and all institutional units engaged in production) divided by the resident population on a given date, the primary indicator used to gauge the health and size of a country's economy. The major advantage of GDP per capita as an indicator of the standard of living is that the data is available for almost every country in the world and the technical definition of GDP per capita is consistent among different countries. The estimates of GDP per capita in the data of the Maddison Project are given in the US dollars of 1990. The underlying methodology

[2]The data series for the following countries have been used: Argentina, Australia, Canada, China, Colombia, Denmark, Finland, France, Germany, India, Indonesia, Ireland, Italy, Japan, Korea, Malaysia, Mauritius, Netherlands, New Zealand, Norway, Portugal, Singapore, South Africa, Spain, Sweden, Switzerland, Taiwan, United Kingdom, United States, and Uruguay.

[3]The Maddison Project was initiated in March 2010 by a group of close colleagues of Angus Maddison.

and main results concerning economic growth in the world between AD 1 and 2010 were reported by the Maddison Project in [166].

We have used the *inverted Pareto–Lorenz coefficient* (IPLC) proposed in [157] as the measure of inequality. The IPLC is related to the standard Pareto coefficient for top income shares, viz.,

$$\rho = \frac{1}{1 - \log_{10}(S_{10}/S_1)} , \tag{6.60}$$

by

$$\text{IPLC} = \frac{\rho}{\rho - 1} = \frac{1}{\log_{10}(S_{10}/S_1)} , \tag{6.61}$$

where $S_{10}$ and $S_1$ represent the income shares of the top 10 and 1 % of the population, respectively.

Higher values of the IPLC correspond to higher top income shares, while the opposite is true for the standard Pareto coefficient (6.60), so that the IPLC provides a direct snapshot of top incomes. Recent studies on top incomes [157] have shown that the IPLC is effectively stable for any income distribution, in any given year and country, although it can vary over time due to a combination of economic and political factors.

We have presented all existing data series in the *World Top Incomes Database* [160] against the relevant GDP per capita data of the Maddison Project [165] in Fig. 6.5. Since the analyzed historical data on the relation between economic growth and inequality is attributed to different countries and covers almost one and a half centuries of economic development (1870–2014), we think that it may reveal a global trend, independently of short-term national economic miracles and misadventures.

☞ Inequality is closely correlated with low growth, and also with high growth.

The entire data set (including marginal outliers) is best fitted by the parabolic trend line

$$y = 2.5 - 0.114x + 0.004x^2 ,$$

or equally well by the hyperbolic cosine

$$y = 0.72 + \cosh(0.084x - 1.18) .$$

As can be seen in Fig. 6.5, the level of inequality is maximum for low as well as for high values of the GDP per capita, but it is minimum (IPLC $\approx$ 1.6) for the intermediate GDP levels of approximately US$ 14 250. Interestingly, the observed

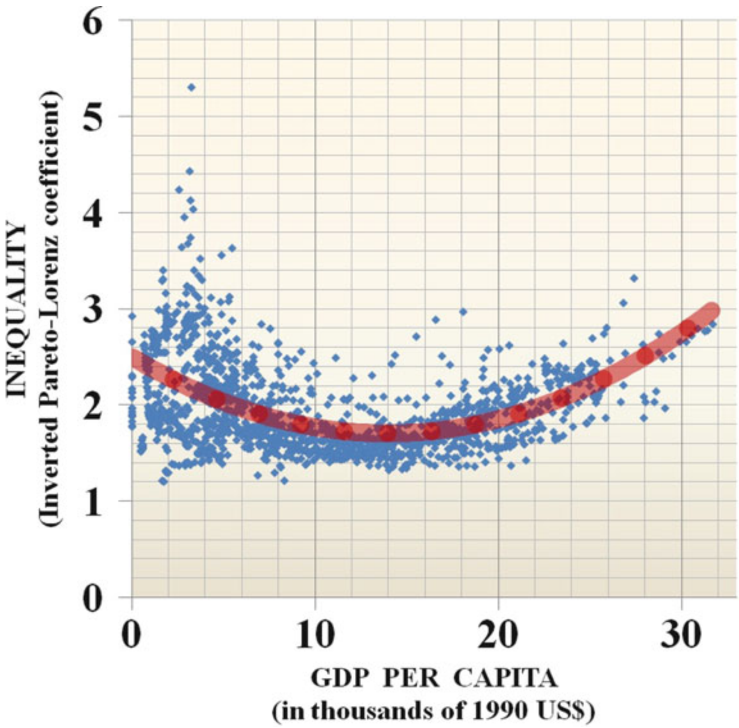


**Fig. 6.5** All presently available data series (1870–2014) in the *World Top Incomes Database* [160] are given against the GDP per capita data of the Maddison Project [165]. The parabolic *trend line* ($y = 2.5 - 0.114x + 0.004x^2$) fits the data best. The goodness-of-fit of linear regression over the entire data set, including marginal outliers, is $R^2 = 0.26$, and the goodness-of-fit of the parabolic central part of the U-curve is $R^2 = 0.84$

GDP level of minimum inequality is very close to the mean GDP level of the world (US$ 14 402 for 2013) as estimated by the World Bank [167]. The observed trend may be due partially to the fact that unprecedented economic growth observed in most countries in the last few decades and expressed by the very high GDP figures goes along with growing inequality. However, the trend suggests that, as economic growth is a global phenomenon, the rising inequality accompanying it must also be a global phenomenon, occurring whenever a national economy is out of step with the world average.

It is also remarkable that the observed trend in the relation between economic performance and inequality is precisely the opposite to Kuznets' hypothesis [155], as sketched in Fig. 6.4. As the economy developed, differences in inequality across rich countries could mostly be put down to the generosity of wealth redistribution. Economic and social policy regimes, providing important social protection for millions of industrial workers in developed countries, had proven to be adequate for a globally dominant industrial economy, underlying three decades of widely shared economic growth [168].

However, the ever increasing need to propel growth through risky entrepreneurship and innovation, into an era of globalization, deindustrialization, and economic dislocation fostered large financial rewards to the very top and pushed incentives toward the short-term maximization of share prices, rather than planning for long-term growth. The two factors have apparently promoted the unprecedented rise of inequality during the last 30 years [72, 169]. No surprise then that the present level of income inequality in the United States is comparable to that in Ghana and Turkmenistan. And, as reported in the *Global Wealth Report 2015* [174], just 0.7 % of the world's adult population own 45 % of all household wealth in 2015.

The empirically observed U-curve in the relationship between economic performance and income inequality (shown in Fig. 6.5) can be understood in the context of probability models discussed in the present chapter. A stagnant planned/command/palace economy, for example, aiming at the complete elimination of risk (by rationing food, for example) engenders scarcity of virtually everything in the society. In Sect. 6.7, we discussed the idea that scarcity might fuel ultimate inequality, as just a few redistributing people get unrestricted access to the scarce resources, while the shares of others dwindle steadily.

At the opposite end, the increasingly risky entrepreneurship and adventurous innovations necessary to propel economic growth foster inequality as well, as we demonstrated in Sect. 6.5. Therefore, both extremes of economic performance—the richest and the poorest alike—are prone to live under conditions of rising inequality.

☞ Equality calls for active maintenance, as usual.

We might suggest that on average the dynamics of the level of inequality is likely to obey the *pendulum law* (Fig. 6.6), since it rises at both extremes of economic performance, while it is minimum in the middle of the span, very close to the estimated mean GDP per capita of the world. The economy is growing there at a sufficient pace to meet fundamental human needs and to avoid shortages, although a considerable share of earned income is redistributed among the entire society by means of various social mechanisms such as taxation, monetary policies, and welfare, at the cost of a relative economic slowdown. The establishment and efficient functioning of such redistribution mechanisms naturally require a broad public consensus to be reached about the necessity, harmlessness, and fairness of changes that may take place within social institutions. However, such a social consensus based on a reciprocal willingness to sacrifice particular interests, including the quest for high domestic production and growth, for the common interest of raising social trust, is not so easy to achieve, unless people face an existential threat to their way of life and their ultimate security. War perhaps provides the most vital and compelling motivation to gladly embrace the "economy and equality of sacrifice-satisfied",[4] which may open the door to the efficient application of inequality-reducing policies.

[4] Franklin D. Roosevelt: 'A Call for Sacrifice', 28 April 1942.

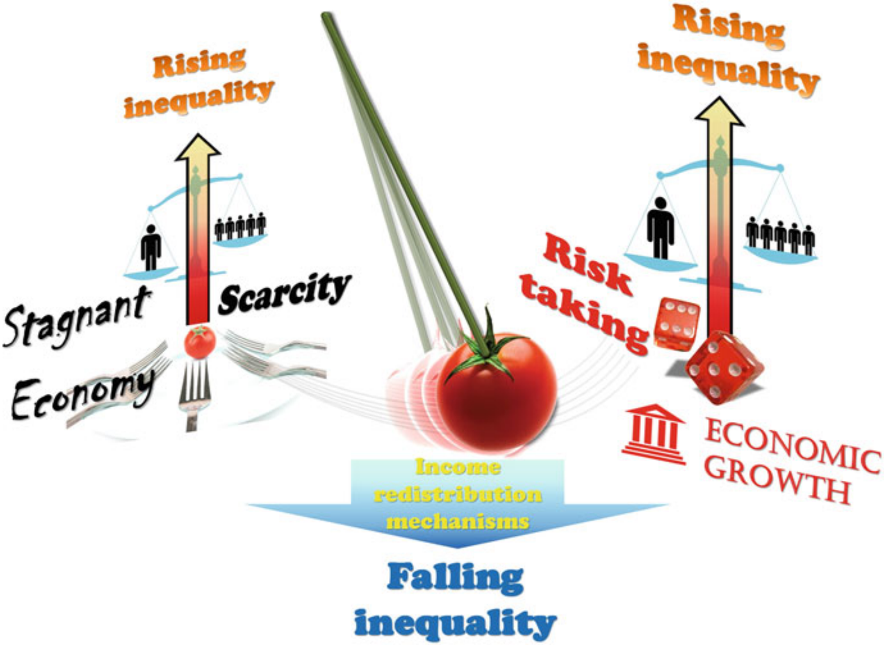


**Fig. 6.6** The pendulum law in the inequality-growth relation. The level of inequality experiences rises in both low and high performing economies

While running its course, the pendulum of rising inequality in one country inevitably pushes those in other countries. It is worth mentioning that rampant inequality in a 'winner-take-all economy' [169] breeds political polarization and boosts social mistrust, with a tendency to distort the functioning of democratic institutions and further erode social capital in the society [72, 168]. Since the service economy primarily rewards a highly educated and highly skilled elite, while leaving displaced workers and the unprotected middle class behind, the latter social groups find that they can no longer rely on the state for economic and social protection. As the state fails to carry out its basic social functions, it is no longer served and admired by the majority of the population, but rather endured and tolerated [170].

Rational country leaders facing common dissatisfaction with domestic policies and worsening economic conditions that prompt their removal from office are likely to gamble on a risky *diversionary war*, according to the diversionary war theory (DWT) [171]. The data on long-term inequality dynamics suggests that, during the last century, the sharp increases in inequality level occurred synchronously in many countries (see Fig. 6.7). Therefore, many governments may simultaneously get the idea that a diversionary foreign policy can only bring gain, especially when

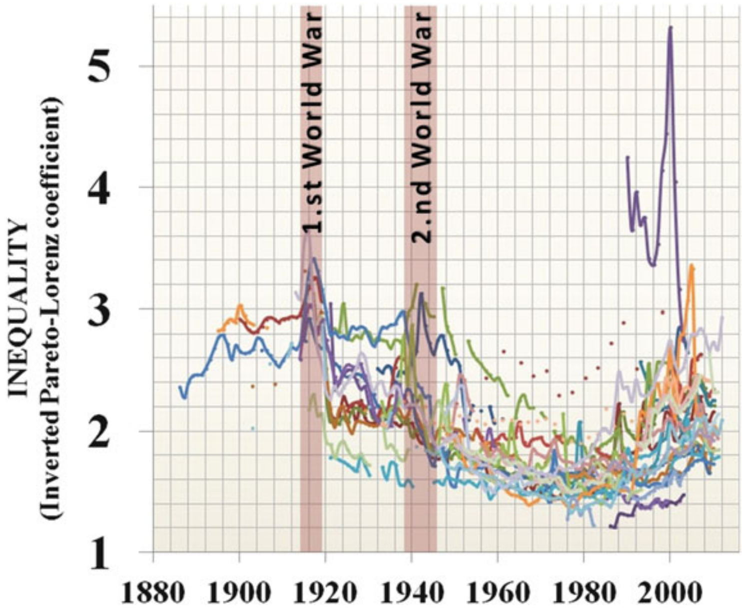


**Fig. 6.7** The long-term dynamics of inequality level measured by the inverted Pareto–Lorenz coefficient for 30 countries. The data is taken from the *World Top Incomes Database* [160]

preying on weaker states.[5] The use of modern hybrid warfare tactics which blend conventional warfare, irregular warfare, and subversive efforts, with the intention of avoiding attribution and retribution, makes such a temptation almost irresistible.

The primary purpose of diversionary conflicts is to divert the attention of the public away from domestic issues, increasing the time available for the government to address the internal troubles [172]. Furthermore, an external threat, either real or illusory, will certainly unify the country through the 'rally round the flag effect', by creating a new out-group, other than the government, to which the population can

[5] Consider Plato's perceptive commentary on diversionary wars in *The Republic*:

> But when he [the tyrant] has disposed of foreign enemies by conquest or treaty, and there is nothing to fear from them, then he is always stirring up some war or other, in order that the people may require a leader. To be sure. Has he not also another object, which is that they may be impoverished by payment of taxes, and thus compelled to devote themselves to their daily wants and therefore less likely to conspire against him? Clearly. And if any of them are suspected by him of having notions of freedom, and of resistance to his authority, he will have a good pretext for destroying them by placing them at the mercy of the enemy; and for all these reasons the tyrant must be always getting up a war. He must.

Translated by B. Jowett. Downloaded from *The Internet Classics Archive*, D.C. Stevenson, Web Atomics (C) 1994–2000.

direct its disapproval and by increasing short-term popular support for the country's leader. Moreover, when "freedom hath fled from the world", in the absence of any prospect of improving well-being and quality of life, guaranteeing personal freedom and civil rights, and promoting upward social mobility, it is well known that "the soldier, alone, is the freeman".[6] Taking part in a war as a mercenary or as a volunteer would become a cherished desire and a welcome breath of freedom for most young people vegetating in godforsaken provinces under conditions of total social despair.

Of course, we may disagree with the general message of DWT, which is hard to prove quantitatively [173]. Nevertheless, the data on the long-term dynamics of inequality over 30 countries presented in Fig. 6.7 shows convincingly that the sharp peaks of inequality level visible synchronously for many countries at once *do* announce periods of global conflict and uncertainty in international relations, and well in advance. The growing number and scale of international conflicts we are witnessing today apparently confirm this observation.

☞ Rampant inequality may transform the uncertainty of national economic development into uncertainty of international relations.

In the face of common challenges, moderate economic growth, although it might be considered as a relative step backwards, need not be such a big disgrace, just as rapid economic progress may not be a reason for such strong pride.

## 6.9 Conclusion

We have introduced and studied the probability model of success. The probability increments of getting success in the future can be maximized over time when consistent efforts are made in the direction of highest positive impact and when the personal role of an actor within a team becomes increasingly important. This can be achieved, in particular, with the help of followers and supporters by commitment to the ethic of reciprocity, focused on the simultaneous success of those around us.

We have shown that the ones that succeed the most under uncertainty, are simply the ones that tried harder. The gradients of probability for achieving success 'worsen' the chances for luck if the number of trials is small, but 'enhance' these

[6]From *The Camp of Wallenstein* by Friedrich Schiller, *Musen-Almanach* für das Jahr 1798, 137–140, Scene **XI**; translated by J. Churchill.

Now freedom hath fled from the world, we find
But lords and their bondsmen vile
And nothing holds sway in the breast of mankind
Save falsehood and cowardly guile.
Who looks in death's face with a fearless brow,
The soldier, alone, is the freeman now.

chances for longer trial sequences. Driving down cycle time in trials would allow for more experiments, which could produce better results for those with early luck compounded.

However, areas involving a lot of uncertainty have the highest likelihood of skilled people failing. When initial success is random and not highly probable, skill does not necessarily play a role over time for improving the future chances for success. We have also demonstrated that, being in a group facing uncertainty, we may be trapped within the repetition compulsion and forced always to repeat the same behavior pattern, without any improvement, as no lesson can been learnt from previous experience.

We have discussed the idea that the processes of accumulated advantage lead to highly skewed, heavy-tailed wealth distributions. In particular, under uncertainty, only the vital few would accumulate advantage. When a trader attaches greater weight to losses than she/he would do to gains of equal magnitude, we say that she/he is risk averse. We have shown that the Pareto distribution of wealth corresponds to the logarithmic wealth utility. Wealth inequality among the population rises from the vital few taking risky decisions under uncertainty: the more adventurous traders are, the greater their fortune, and the fewer lucky ones there will be.

When the cumulative advantage is at work, there may be no parity of chances to choose a partner and to be chosen as a partner by other individuals. By fostering hierarchical organization in a group, uncertainty ultimately leads to inequality. A common belief about rarity (or scarcity) of a resource or an item automatically fosters inequality between individuals regarding accessibility to the scarce resource. Scarcity also promotes inequality by necessitating competition and fueling conspicuous consumption.

We have analyzed all existing data series in the *World Top Incomes Database* [160] against the relevant GDP per capita data of the Maddison Project [165], and demonstrated that income inequality is closely correlated with low growth, but also with high growth. The discovered trend suggests that rising income inequality is a global phenomenon, occurring whenever a national economy is out of step with the world average. Finally, we have discussed the way rampant inequality may transform the uncertainty of national economic development into uncertainty of international relations.

# Evolution and Advantage Under Uncertainty 7

*A high man will not reach the sky,*
*A wide man will not cover the Earth,*
*A mighty man does not take all his bed,*
*(But) you, roaring as a storm, like a lion,*
*establish yourself.*[1]

**Abstract**

Since desperation ecologies would mark their inhabitants via cues to the 'faster', time compression behavioral strategies, the most likely planning strategy for a long enough period of time $T \gg 1$ would consist of approximately $T/\log T$ short-term decisions, which corresponds to the logarithmic utility function of time and hyperbolic discounting in time for risk aversion and prudence behavior. The optimal survival strategy in any foreseeable time period would consist of a regular change of scenery by innovation and migration to other environments. We have proposed and studied the model of stochastic competitive advantage, from which it follows in particular that, if competition is allowed in the group, any individual advantage is transitory. But if competition is avoided, then a progressively ineffective hierarchy arises in the group. In hierarchical societies under uncertainty, the 'fittest' traits are punished, and not rewarded. A community may seek to get rid of its prominent members, considering their traits of excellence maladaptive when environmental conditions change faster than the rate of possible adaptations to these changes. The high degree of behavioral adaptations in the society may not be that important for survival success under uncertainty. We also discuss the structural evolution of hierarchical societies and show that the structure of a hierarchical society would relax to a more democratic rank composition with time.

[1] Sumerian–Akkadian proverb.



D. Volchenkov, *Survival under Uncertainty*, Understanding Complex Systems,
DOI 10.1007/978-3-319-39421-3_7

## 7.1 Introduction

There are rarely 'right' answers under uncertain circumstances of life and business: *Is that the right decision or the wrong one?*—a perplexing question, and it may be useless to look for an answer to it, especially when we navigate uncharted waters. The information that could reduce ambiguity may be hard to find, or it just may not exist.

Any practical advice drawn up by a professional under certainty might become meaningless in the face of uncertainty. With regard to future success, it often turns out that it is not so important *what* one can do under uncertainty, but rather *when* one takes action and sets up the new course.

In the present chapter, we discuss an incremental approach that can help us to manage strategic uncertainty, but avoid taking on too many of the kind of risks that may come with sweeping large scale decisions. The *divide and conquer strategy* is useful for uncertainty management, since even though an immediate short-term decision may be deemed imperfect later, it has the benefit of reducing uncertainty in the near future. In particular, we show that the most likely planning strategy under uncertainty over a long enough period of time $T \gg 1$ would consist of a series of $T/\log T$ short-term segments, corresponding to the logarithmic utility function of time $\log(T)$.

In the stochastic model of subsistence under uncertainty (see Chap. 1), a species subsists as long as the random level of demand $d$ (accounting for the factors of subjective uncertainty) remains below the random level of supply $s$ (corresponding to objective uncertainty). The supply and demand levels were updated with a probability $0 \geq \eta \geq 1$, which could be viewed as the degree of environmental stability.

In the precarious environment when $\eta = 0$, both levels are updated synchronously, and the factors responsible for the subjective and objective types of uncertainty are manifested consistently. In particular, we have found that the probability of subsistence always decays exponentially fast with time in precarious environments.

However, the chances for longer subsistence under uncertainty improve gradually as the degree of environmental stability grows. When the environment is perfectly stable, i.e., $\eta = 1$, the random updates of demand and supply occur on incomparable time scales, and the probability of subsistence then decays algebraically with time. Moreover, we have shown that, when resources are plentiful, extraordinary longevity can occur as the probability of subsistence then follows the Zipf law $\propto t^{-1-\varepsilon}$, for some $\varepsilon > 0$.

However, as the actual lifetime of any individual is always finite, the survival of a species is not possible without new organisms, i.e., offspring produced by the parents. The species has no other choice but to respond to individual mortality by a combinatorial explosion of offspring (Fig. 7.1a).

Different species employ a wide range of reproductive strategies [175]. Some species, essentially those with most offspring, do not survive to adulthood, and they

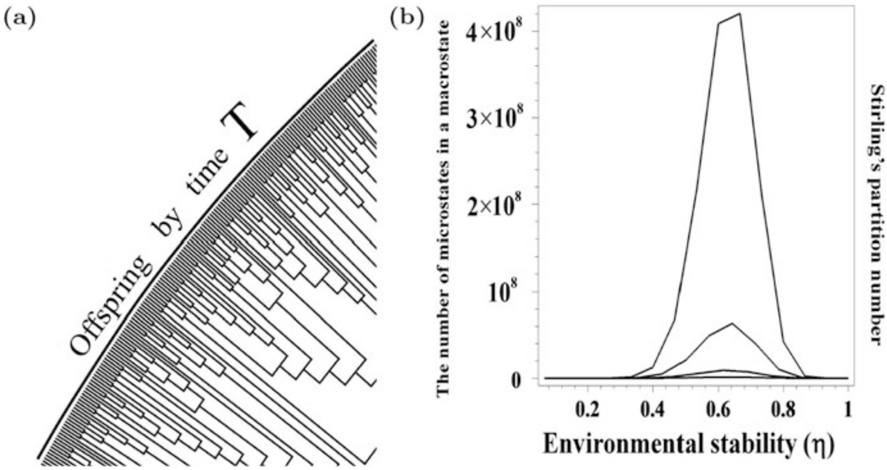


**Fig. 7.1** (**a**) The survival process with reproduction is represented by a probability tree rooted in a single ancestor, which includes all combinations of branches of all possible lifespan durations that fit up to time $T \gg 1$. (**b**) The number of different 'microstates' in a single 'macrostate', describing the quality of home ecology by the degree of environmental stability $\eta(m)$, are given by the Stirling partition numbers for $T = 12, \ldots, 15$

might follow the *r-selection strategy* of reproducing quickly and devoting scarce resources to their progeny. Others following the *K-selection strategy* produce few offspring but devote more resources to nurturing and protecting each individual offspring.

In the present chapter, we discuss the most likely strategies for surviving and taking advantage under uncertainty.

## 7.2 Survival with Reproduction

In a discrete time model of survival with reproduction, every single act of survival up to time $T \gg 1$ can be represented by a particular integer partition, e.g.,

$$T = t_1 + \cdots + t_m\,, \quad m = 1, \ldots T\,, \tag{7.1}$$

where $t_k$, $k = 1, \ldots, m$, are the consecutive times of reproduction, and $m$ is the number of generations required for progeny to reach time $T$. The entire survival process can be represented by a random tree like the one shown in Fig. 7.1a, which is rooted in a single ancestor and includes all combinations of branches of all possible lengths, representing all possible lifespan durations that fit up to time $T \gg 1$. The different reproductive strategies in the probability tree range from the *r-selection strategy*, which involves reproducing as quickly as possible, to the *K-selection strategy*, which produces just a few generations up to time $T$, to the marginal child-free strategy of promoting personal longevity until time $T$ and neglecting reproduction completely (a single generation). Survival success is assessed by the *expected number* of offspring that might reach the time horizon $T \gg 1$, originating from the single ancestor.

If reproduction neither affects, nor is affected by, the carrying capacity of the habitat, the optimal survival strategy maximizing the number of progeny at any given time is obvious: the individuals should reproduce as quick as possible in order to give combinatorics a full opportunity to work on their side against personal mortality, no matter whether the environment is stable or not.

However, it is natural to assume that reproduction *does* affect the local ecology and the current carrying capacity of the habitat of a biological species.

On the one hand, the unbounded increase in the number of descendants due to a combinatorial explosion of progeny can lead to depletion of local resources, while on the other hand, surplus offspring can either migrate to other regions with enough resources, or make improvements to existing living technologies, increasing the carrying capacity of the species in the habitat and reducing the environmental stress—in all cases, the living conditions of the population change.

It is known that ecologies could mark their inhabitants via cues to the ‘faster’ versus ‘slower’ behavioral strategies [57]. Ecologies toward the desperate end, which exert physical strain on the individual and are characterized by a high degree of random fluctuation in environmental events, are associated with having more children, risk-taking, and impulsivity. Individuals from desperation ecologies tend to reproduce earlier and faster, clearly emphasizing offspring quantity over quality.

Girls whose fathers are absent from home exhibit an earlier age of menarche, first sex, and first child, as father absence might signal high male mortality rates and unstable pair bonds, indicating more desperate local ecologies [58]. The ‘faster’ behavioral strategies associated with desperation ecologies would imply greater promiscuity and less stable partner bonds than the ‘slower’ behavioral strategies associated with more hopeful ecologies [57]. It has been demonstrated experimentally that, when high mortality is made salient, cueing a desperation ecology, individuals become more risk-taking and present-oriented [59]. In particular, those who grew up with lower socioeconomic status exhibit greater desires to have children in the near future [60].

We assume that the number of generations required for progeny to achieve the time horizon $T \gg 1$ in the future is determined by their home ecology. Many terms in the integer partition (7.1) would cue a desperation ecology, while a few terms would suggest more hopeful ecologies.

In the context of the stochastic model of subsistence under uncertainty introduced and studied in Chap. 1, we assume that the lifespan of every descendent organism (the length of every limb in the probability tree in Fig. 7.1a) is a random number drawn from the survival probability in the stochastic process of subsistence. The degree of environmental stability in the model of subsistence is characterized by the probability of inconsistency $\eta$ between the random updates of supply and demand. Desperation ecologies, viewed as harsh and unpredictable, obviously correspond to small values $\eta \to 0$, in which levels of supply and demand are updated consistently and subsistence is always transitory. On the contrary, the amount of supply is kept fixed, i.e., $\eta \to 1$, in a perfectly stable environment.

We assume the following relation between the number of terms $m$ (generations) in the integer partition (7.1) and the value of the probability $\eta$:

$$\eta(m) = 1 - \frac{m-1}{T} \,. \tag{7.2}$$

According to (7.2), the smaller values of $\eta$ elicit the 'faster' behavioral strategies associated with harsh and unpredictable living ecologies, with $m \to T$ as $\eta \to 0$ (assuming reproduction at every time). In contrast, the most stable environment, characterized by $\eta = 1$, corresponds to a single term $m = 1$ in the partition (7.1), and this might be viewed as the ultimately 'slow' behavioral strategy, promoting personal longevity (a single generation) by time $T$ at the cost of producing no offspring.

☞ A rapidly growing number of partitions for $T \gg 1$ is the main engine propelling the discrete time survival process to success.

The number of ways to partition $T$ time steps into $m$ non-empty subsets (generations) is given by a *Stirling partition number* (or *Stirling number of the second kind*), calculated using the following explicit formula:

$$S(T, m) = \frac{1}{m!} \sum_{j=0}^{m} (-1)^{m-j} \binom{m}{j} j^T \,, \quad m = 1, \ldots, T \,. \tag{7.3}$$

We can consider every possible partition of the time horizon $T$ into $m$ generations as a particular 'microstate' in the survival process with reproduction, and the value $m$ characterizing the 'rate' of the behavioral strategy (ranging from 'fast' to 'slow'), attuned to cues signaling the quality of the home ecology, as the 'macrostate'. Then, the Stirling partition number $S(T, m)$ shows how many 'microstates' correspond to the particular 'macrostate' $m$ in the model of survival with reproduction. It is clear that, if every partition of the time horizon $T$ into $m$ generations, a 'microstate' representing a particular reproductive strategy within the given local ecology, is taken as *equiprobable*, the reproductive strategy characterized by the maximum

Stirling partition number is the *most successful* survival strategy of all, providing the most numerous progeny.

It is known that the Stirling partition number (7.3) has a single maximum $S_{\max}(T, m)$ for large enough $T \gg 1$ [176], such that

$$\log S_{\max}(T, m) = T \log T - T \log\log T - T + O\left(\frac{T \log\log T}{\log T}\right) , \tag{7.4}$$

which is attained for at most two consecutive values of $m$ close to

$$m_T = \frac{T}{\log T} + O\left(\frac{T\sqrt{\log\log T}}{\sqrt{(\log T)^3}}\right) . \tag{7.5}$$

We conclude that, although the number of possible individual reproductive strategies grows enormously for long enough time $T \gg 1$, the most successful reproductive strategy (providing the maximum expected number of offspring) on average, consists of

$$m_T \simeq \frac{T}{\log T}$$

generations.

Similarly, while planning under uncertainty for the time horizon $T \gg 1$, we may apply the *divide and conquer strategy*, which helps us to avoid taking on too much risk when we have to make sweeping large-scale decisions. Let us suppose that uncertainty management involves $m$ immediate short-term decisions, reducing uncertainty in the near future. While the 'fast' behavioral strategies imply that many short-term decisions $m \to T$ have to be taken in unstable environments, the 'slow' behavioral strategy comprising just a few short-term decisions might be required under normal operating conditions, and a single strategy $m \to 1$ is enough in situations with perfect foresight.

☞ The most likely planning strategy for a long enough period of time $T \gg 1$ in the face of uncertainty, characterized by a degree of environmental stability $\eta$ defined as in (7.2), consists of a series of approximately $m_T \simeq T/\log T$ short-term decisions.

Substituting the most likely value of $m$ given by (7.5) into the expression for the degree of environmental stability (7.2), we obtain the most likely 'rate' of behavioral strategy as a function of the time horizon $T \gg 1$:

$$\eta_{\max}(T) \simeq 1 - \frac{1}{\log T} + \frac{1}{T} \underset{T\to\infty}{\longrightarrow} 1 . \tag{7.6}$$

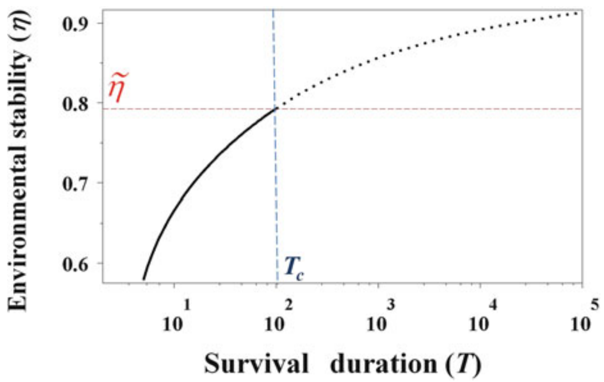


**Fig. 7.2** The most likely 'rate' of behavioral strategy $\eta_{\max}(T)$ is shown as a function of the survival time $T$. The characteristic survival time corresponding to a given degree of environmental stability $\tilde{\eta}$ is $T_c \simeq \exp(1-\tilde{\eta})^{-1}$

The most likely behavioral strategy $\eta_{\max}(T)$ (a 'macrostate'), providing the maximum number of offspring ('microstates') by time $T$, grows slowly with time (see Fig. 7.2), approaching unity as $T \to \infty$, e.g., $\eta_{\max}(10^{10}) \simeq 0.96$.

The combinatorial explosion in the number of progeny with time, viz.,

$$S_{\max}(T) \simeq \left(\frac{T}{\mathrm{e}\log T}\right)^{T}, \tag{7.7}$$

achieved by time $T$ when $\eta \to \eta_{\max}(T)$, more than compensates for loss due to mortality. Metaphorically speaking, however, in harsh environments, whenever $\eta \ll \eta_{\max}(T)$, combinatorics simply does not have enough time to work on survival. The expected number of offspring surviving in desperation ecologies does not grow with time, so long-term survival is a matter of chance.

☞ Harsh and perfectly stable ecologies threaten survival success equally.

It is remarkable that a species would also experience a population bottleneck if it belonged to an 'overly' stable environment, viz., $\eta \gg \eta_{\max}(T)$. Neglecting reproduction under conditions of environmental stability does not allow the species to respond to individual mortality by a combinatorial explosion of offspring. Even though the survival probability under singular uncertainty decays according to a power law, and extraordinary longevity could occur among the population in perfectly stable environments, the long-term survival of the species is not possible.

The highest expected number of progeny and the most likely survival strategies would be observed in the partially stable environments where the combinatorial explosion of offspring is balanced by an expected lifespan sufficient to reproduce new organisms. The reproductive strategies most likely to provide success within the time horizon $T$ correspond to the intermediate values of $\eta$, close to the optimal value $\eta_{\max}(T)$.

☞ The optimal survival strategy in any foreseeable time period consists of a regular change of scenery by innovation and migration to other environments.

Given the degree of environmental stability $\tilde{\eta}$ characterizing the ecology of the current habitat of a species, the most likely 'rate' (7.6) of a successful behavioral strategy determines the *characteristic time of adaptation* to the environment, i.e.,

$$T_{\mathrm{c}} \simeq \exp \frac{1}{1-\tilde{\eta}}\,, \tag{7.8}$$

in the model of survival with reproduction. So for $T \ll T_{\mathrm{c}}$, relatively stable environmental conditions in the habitat promote a combinatorial explosion in the number of progeny, rewarding those species capable of enhancing their reproductive performance in the environment. However, for $T \gg T_{\mathrm{c}}$, the most likely behavioral strategy of the species involves a steady decay of the reproduction rate in our model. Since the species (presumably well adapted to more stable environments) does not produce enough offspring to meet requirements in more precarious environments, its survival success is threatened by an *evolutionary trap*, in which previously successful adaptations may appear maladaptive. The species may experience a population bottleneck for $T \gg T_{\mathrm{c}}$, due to unstable environmental conditions, which may also reduce the variation in the gene pool of the population.

## 7.3 Survival by Endurance Running

From the discussion in the previous section it becomes clear that, in the face of uncertainty, there is no a single survival strategy that is optimal once and for all. The most likely strategies coping with uncertainty are similar in that they commence within the least stable, highly unpredictable environments ($\eta \ll 1$) and then continue to the more stable environments as time goes on, to arrive eventually at the perfectly stable environments ($\eta \to 1$) in very long times $T \gg 1$. In the framework of environmental determinism, the evolution of coping strategies towards progressive stabilization of the environment may be interpreted as an *adaptation* of the species to the local conditions of their habitat via natural selection for traits enhancing individual performance [177]. However, if the environmental conditions change suddenly, the previously successful adaptations which accumulated because they enhanced the fitness of the species in previous generations may suddenly be rendered maladaptive, and even lead to the extinction of the species, in the next generation [6, 178, 179]. It is the rate of environmental changes relative to the rate of adaptation to them that determines the chances for survival of the species under uncertainty. If the rate of environmental changes is higher than the rate of adaptation to the variable environment, the adaptations that once enhanced the fitness of the species become rather its liabilities, and the species can fall into the evolutionary trap.

It seems therefore reasonable to expect the evolution of a surviving species (especially the one dominating the planet) to be imprinted first of all on specific adaptations for evading evolutionary traps due to rapid and permanent environmental changes, either by running through them, in the early stages of the survival process, or by radically transforming (destroying) them, in the later stages of the process. Furthermore, it is remarkable that the high speed of running through uncertain and precarious environments is not as important for survival success from the statistical point of view as the exceptional capabilities for running endurance. It was suggested in [180] that the evolution of certain human characteristics can be viewed as evidence of selection for *endurance running*. Although humans are poor sprinters compared with other running animals, human endurance running capabilities, unique among primates, either match or exceed those of mammals adapted for running, including dogs and equids.

Running involves a 'mass spring' mechanism to exchange potential and kinetic energy, with the use of tendons and ligaments that is less energetically costly at faster speeds, but does not provide the benefits in walking. In response to the destabilization of the running gait, the human body appears to have evolved adaptations to increased stabilization, as well as for the mass-spring mechanism in general [180].

Furthermore, human sweat glands allow for the excretion of more sweat per unit surface area than any other species. By ridding themselves of an insulating fur coat, running humans are better able to dissipate the heat generated by exercise [181]. The upright orientation of bipedal hominids allows them to vary their breathing patterns with gait. This flexibility in respiration rate and running gait contributes to hominids having a broader range of energetically favorable running speeds [181].

Endurance running obviously played an important role for early hominids in obtaining food. Due to their superior thermoregulation capabilities, human hunters can continue the pursuit of a target prey over a period of hours. Extant human foragers use endurance running in persistence hunting during the midday heat to drive animals into hyperthermia and exhaustion so that they can no longer retaliate violently and can easily be killed [182].

Even though endurance running is no longer necessary for human survival, we are still witnessing the amazing achievements of ultra-runners, who can run for a few days and nights without stopping, surpassing all the achievements known in other animals. Surprisingly, the age of runners does not appear to matter for their degree of endurance, as Kim Allan, a mother of four, ran for 86 h 11 min, reaching 500 km, at the age of 47, and Dean Karnazes, reached 563 km in 80 h 44 min, at the age of 43, and made 4800 km in 75 days, at the age of 50. Apparently, human capabilities for endurance running are practically unlimited.

☞ Horns, claws, and teeth would appear to be less important for dominating the planet than exceptional endurance running capabilities.

Our statistical argument should help paleoanthropologists to solve the puzzle of identifying those past behaviors that favoured the evolution of endurance running capabilities. Although this survival strategy is not common, even among extant foragers, we have seen that endurance runners would gain a statistical advantage in the early stages of survival, because they could change their environment on a regular basis.

## 7.4 Logarithmic Usefulness of Time and Hyperbolic Discounting of the Future Under Uncertainty

The probability of inconsistency $\eta$ between the random updates of supply and demand is viewed as the degree of environmental stability in the model of subsistence under uncertainty. In particular, the characteristic time interval of environmental stability can be estimated by

$$\tilde{t} = \frac{1}{1-\eta} \, . \tag{7.9}$$

In the previous sections, we have introduced and studied a simple model for the survival process with reproduction. We have demonstrated that the maximum expected number of survivors by time $T$ corresponds to the ecology characterized by the probability $\eta_{\max}(T)$ ('macrostate') allowing for the maximum number of different behavioral strategies ('microstates'). We have suggested that reproduction affects the local ecology and environmental stability, so that the characteristic time interval of environmental stability in the process with reproduction can be estimated according to (7.9) as

$$u(T) = \frac{1}{1-\eta_{\max}} \simeq \log T + \frac{(\log T)^2}{T} + O\left(\frac{1}{T^2}\right) \, . \tag{7.10}$$

Similarly, in a situation that involves planning actions over the time horizon $T$ in the face of uncertainty, we may assume that any change in the operational situation will give rise to a need to decide and act immediately, or improve performance of the current action. Then, the expected duration of stable operation (during which operating procedures continue to work reliably) between the decisions in a sequence of intermediary decisions required by situational changes would be

$$u(T) \approx \log T \, ,$$

so that approximately $T/\log T$ short-term decisions would most likely be required by the time horizon $T$, in an incremental approach to uncertainty management.

☞ Operating under uncertainty is not all plain sailing, since within the time horizon $T$ things go overwhelmingly as planned only for a time $\log T$.

It is well known that people have a tendency to discount rewards and the importance of events as they approach a temporal horizon in the future or in the past. For humans, rewards that are expected 'now' usually have greater value than those close to their temporal horizons [204]. In particular, it follows from (7.10) that, for long enough times $T$, the *logarithmic utility of time*,

$$u(T) \simeq \log T \tag{7.11}$$

is a good enough approximation to usefulness in time.

The notion of utility goes back to Daniel Bernoulli (1738), who proposed the logarithmic 'moral value' of money, viz.,

$$u(x) = \log x \,,$$

as a standard of judgment on the available capital [141], where $u(x)$ can be viewed as the wealth or the gain of a decision-maker under uncertainty (utility of money).

Different utility functions of time $u(T)$ would indicate different attitudes and preferences of individuals towards risk, depending on their home environments, in order to secure the survival of progeny up to different time horizons. Moreover, it seems reasonable that different individuals would exhibit different patterns of risk aversion, and there is no a priori reason to believe that any particular utility function $u(T)$, such as the one obtained in (7.10), would describe the behavioral strategy of a particular species. Instead, the modeling function (7.10) corresponds to the most likely behavioral strategy that would secure survival success in the given time.

Following the standard account of utility theory, we shall consider the utility function $u(T)$ as a three times continuously differentiable function of $T > 0$. We suppose that a utility function of time in the survival process under uncertainty can be characterized by the following two self-evident features: *the longer, the better*, and *a bird in the hand is better than two in the bush*.

- **The longer, the better.** $u(T)$ is an *increasing* function of time, and the first derivative $u'(T) > 0$.

We believe that a human is never satiated with his/her actual life duration, since it will never be so long that living a bit longer would not be at least a little bit desirable.

- **A bird in the hand is better than two in the bush.** $u(T)$ is a *concave* function of time, and the second derivative $u''(T) < 0$.

Individuals can trade off time and resource allocations among various life tasks, prioritizing short-term but certain projects over long-term projects whenever they are more uncertain. Given $n$ different time horizons $T_1, \ldots, T_n$, for projects that might be accomplished with probabilities $p_i$, $i = 1, \ldots, n$, respectively, the utility of time that might be spent on them is

$$u(\langle T \rangle) \equiv u\left(\sum_{i=1}^{n} T_i p_i\right) , \tag{7.12}$$

and the expected utility of lifespan for future survival is

$$\langle u(T) \rangle \equiv \sum_{i=1}^{n} p_i u(T_i) . \tag{7.13}$$

It is then a consequence of the Jensen inequality that the expected utility is always less than the utility of the expected value provided that the utility function is concave:

$$u(\langle T \rangle) > \langle u(T) \rangle . \tag{7.14}$$

The latter principle is known as *risk aversion* in utility theory. The two modeling utility functions (7.10) and (7.11) are obviously concave.

The standard notions of utility theory that describe different aspects of individual preferences can be introduced and applied to the model of survival under uncertainty. First, the measure of *risk aversion* describes how much time the individual is willing to spend in the best case scenario (for example, playing sports in order to stay in shape) for securing a longer lifespan in the worst case scenario. The degree of risk aversion depends on the curvature of the utility function, and this leads naturally to the definition of the *Arrow–Pratt measure of risk aversion* [197, 198]:

$$R(T) = -\frac{u''(T)}{u'(T)} > 0 . \tag{7.15}$$

This is always positive due to the two properties of the utility function stated above.

It is reasonable to assume that the utility of time exhibits decreasing risk aversion with time. In particular, for the logarithmic utility function (7.11) obtained in the model of survival under uncertainty, the Arrow–Pratt measure of risk aversion, viz.,

$$R(T) = \frac{1}{T} , \tag{7.16}$$

is consistent with the hyperbolic time discount model of human and animal intertemporal choice [199].

☞ The degrees of risk aversion and prudence alike manifest themselves under uncertainty by hyperbolic discounting in time.

Second, the characteristic of *prudence* quantifies the extent to which an increase in uncertainty about future survival (say, under distress following a chronic disease diagnosis) will affect current time spending. Prudence is associated with the ability to govern and discipline oneself by the use of reason, and this requires convex marginal utility in addition to risk aversion, i.e., a positive third derivative of the utility function, $u'''(T) > 0$.

Following [200] and [201], we can define a measure of prudence as

$$\Pi(T) = -\frac{u'''(T)}{u''(T)} > 0\,, \tag{7.17}$$

which is also always positive. In particular, for the logarithmic utility function (7.11), the measure of prudence

$$\Pi(T) = \frac{2}{T} \tag{7.18}$$

is also consistent with the hyperbolic time discount model of human and animal intertemporal choice [199].

## 7.5 Hyperbolic Discounting of Time: Would You Prefer a Dollar Today or Three Dollars Tomorrow?

From the traditional point of view, a subject experiences a single 'self' that persists through time and plays an integral part in human motivation, cognition, and social identity [203]. If such a 'self' represents a decision-maker, his/her preferences are thought to be consistently aligned over time: a *Homo economicus* knows what she/he wants, and goes for it.

Traditional models of economics assume that humans show a preference for the rewards that arrive *sooner rather than later*, discounting the value of the later reward by a constant factor. Such a monotonic decrease in preference with increased time delay corresponds to a discounting function that is exponential in time, ensuring the consistency of human preferences over time [204].

However, a large number of studies have demonstrated that the constant discount rate is systematically violated [199]. In particular, spontaneous preferences exhibited by humans and animals alike convincingly follow a hyperbolic curve in time, rather than the exponential curve representing the preferences of *Homo economicus*

[205, 206]. The preferences of a real decision-maker—human or animal—change over time and can become inconsistent at another point in time.

☞ Any attempt to survive under uncertainty inevitably involves making choices today that our future 'self' would prefer not to have made.

It is remarkable that the logarithmic utility function of time (7.11) which arises naturally in the process of survival under uncertainty as the most likely behavioral strategy corresponds exactly to *hyperbolic discounting* in risk aversion behavior and prudence in handling one's own affairs.

In particular, under hyperbolic time discounting, valuations fall very rapidly for small delays, but then fall off slowly for longer periods, and this is inconsistent with a time constant discounting factor, as modelled by exponential discounting. Real individuals, at least those striving to survive, are most likely to be 'present-biased', making a much lower evaluation of the future than *Homo economicus* [207].

However, by valuing time differently, with regard to delay periods, we are condemned to make choices that are inconsistent over time, despite using the very same reasoning, as though there were many different 'selves' with mutually inconsistent preferences, with each 'self' representing us at a different point in time.

☞ If you wish you'd done things differently, it's a good sign that you are on the right trail to secure the future. You are really in danger if you have no regrets about anything.

This statement is by no means intended to be moralizing. Rather it follows from the statistical law for evaluating time under uncertainty (7.11).

Evaluating time according to delays is also consistent with the *matching law*, which states that subjects are likely to allocate their time and effort between non-exclusive, ongoing sources of reward in direct proportion to the rate and size of rewards, and in *inverse proportion* to their delays [208]. Furthermore, it may also explain why people display a consistent bias in their belief that they will have more time in the future than they have now [209].

## 7.6 Stochastic Model of Competitive Advantage Under Uncertainty

In order to produce offspring and contribute to a genetic pool of surviving individuals, we have to marry, since marriage is tied to important outcomes for the stability of partnerships and the well-being of children.

Although “poverty is not a vice, that’s a true saying”,[2] low-income men are less likely to marry [196]. Not only is the absolute level of income a strong predictor of marital status, but also the income relative to a ‘middle class ideal’ is determined by a local *reference group*. Hence, low-income men are less likely to be married and less likely to form long-term partnerships when they are farther from the median income in their reference group [196].

In particular, a standard partner search model suggests [196] that if income inequality is high, women will search longer for a mate with the highest reservation income. Women want men who can provide materially for their families, and a low-income man living in a high-income area may be less attractive than a man who earns the same real income in another, less expensive city.

We say that an individual (or a firm) has a *competitive advantage* over rivals if he is able to acquire (or develop) a certain attribute that allows him to become an enviable groom (or to outperform rivals on the economic market) at least for a while. The competitive advantage gained over the reference group then leads to offspring (or a profitable growth).

A sustainable competitive advantage rewards an individual with a reproductive edge over rivals and a firm with the ability to generate greater value for its shareholders.

We suppose that the fitness level of a reference group during a certain period of time can be quantified by a real number $\rho \in [0, 1]$. Another real number $\sigma \in [0, 1]$ appraises the competitive ability of an individual during the same period of time. We assume that the individual has an advantage (and is able to beget offspring) in comparison with his rivals as long as $\sigma > \rho$, but falls into disfavor whenever $\sigma \leq \rho$.

The proposed stochastic model of competitive advantage is almost identical with the stochastic model of subsistence under uncertainty which we introduced and studied in the first chapter. The only difference between them is that we are now interested in time durations of advantage when the abilities of the individual exceed the presumably low group fitness level (a threshold) (Fig. 7.3).

In this stochastic model:

- randomness in the level of instantaneous individual ability to get an edge over rivals attests to the subjective type of uncertainty, and
- randomness in the group fitness level manifests the objective type of uncertainty.

[2] Fyodor Dostoyevsky, *Crime and Punishment*.

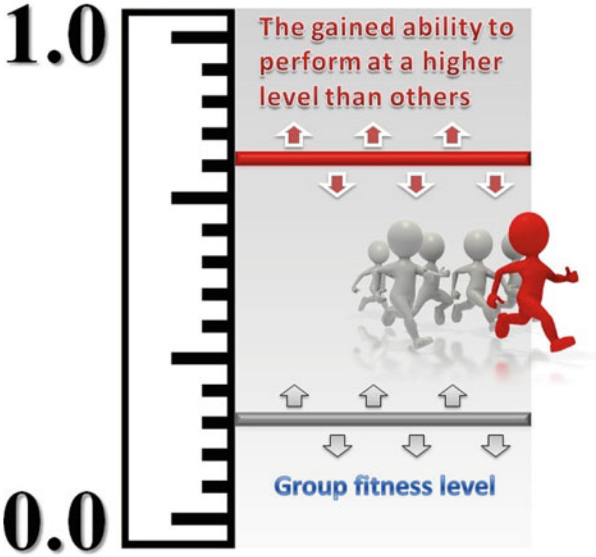


**Fig. 7.3** Stochastic competitive advantage over the stochastic group fitness level

The proposed stochastic model is oversimplified in the sense that it does not take into account the different types of competitive advantage. For example, if a firm is able to produce a good or service at a lower cost than its competitors, the *comparative advantage* obtained will give it the ability to sell its goods or services at a lower price than its competition. The *differential advantage* is created when a firm's products or services differ from and are seen as better than a competitor's products. In our model, however, we do not distinguish between these two (and possibly other) types of competitive advantage.

As before, in the stochastic model of subsistence under uncertainty, we assume that both levels of fitness are random variables, such that the performance level of the reference group $\rho \in [0, 1]$ is drawn from some probability distribution function $\Pr\{\rho < x\} = \Gamma(x)$, and the performance level of the pretender $\sigma \in [0, 1]$ is distributed over the unit interval with respect to another probability distribution, $\Pr\{\sigma < x\} = \Phi(x)$. We also describe the degree of inconsistency between random variations of the two random levels by the probability $\nu \in [0, 1]$.

The stochastic process of competitive advantage is then defined in the following way. At time $t = 0$, the performance level of the pretender $\sigma \in [0, 1]$ is chosen with respect to the probability distribution function $\Phi$, and the performance level of his reference group $\rho \in [0, 1]$ is chosen with respect to the probability distribution function $\Gamma$.

If $\sigma > \rho$, the pretender has an edge over rivals and the process keeps going to time $t = 1$. At time $t \geq 1$, there are two possibilities:

- with probability $\nu$, the performance level of the reference group $\rho$ is drawn anew from the probability distribution function $\Gamma$, while the performance level of the pretender $\sigma$ keeps the value it had at time $t - 1$, or
- with probability $1 - \nu$, the performance level of the reference group $\rho$ is updated anew from the probability distribution function $\Gamma$, and the level of the pretender $\sigma$ is also updated, with respect to the probability distribution function $\Phi$.

As long as the performance level of the pretender remains above the performance level of the group, i.e., $\sigma > \rho$, he has an advantage with respect to the chosen reference group until, at some time step $t = \tau$, as soon as $\sigma \leq \rho$, he falls into disfavor, and the process eventually ends. The flowchart of the model of competitive advantage under uncertainty is shown in Fig. 7.4.

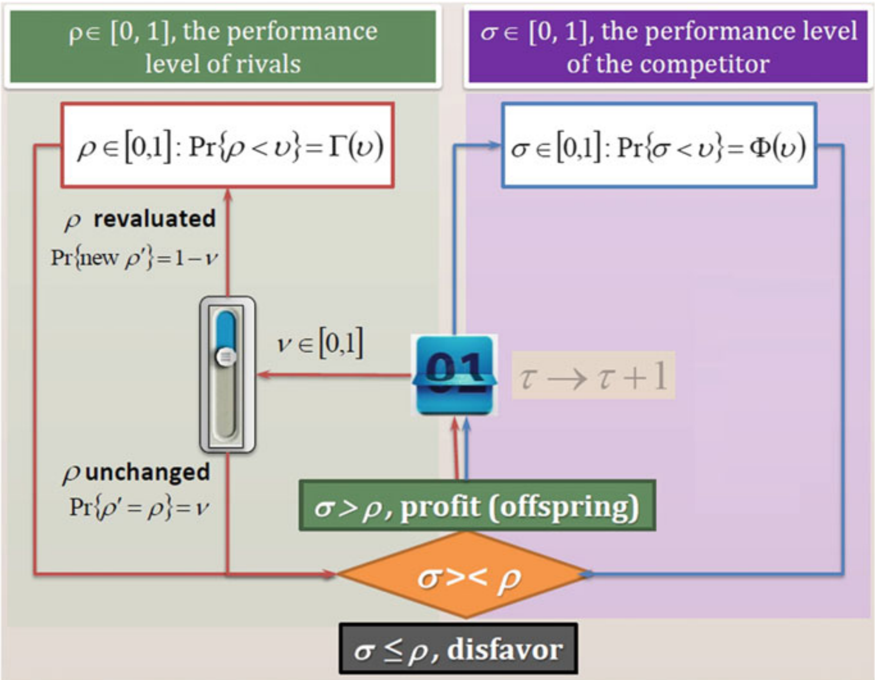


**Fig. 7.4** The flowchart of the model of competitive advantage under uncertainty

The probability $\nu$ determines the structure of uncertainty in the stochastic process of competitive advantage. In particular, under dual uncertainty when $\nu = 0$, both the objective and the subjective types of uncertainty contribute simultaneously through consistent random updates of the two performance levels. Under singular uncertainty when $\nu = 1$, the performance level of the group remains unchanged during the entire process, so that factors of objective uncertainty are completely excluded from the process. As in the stochastic model of subsistence under uncertainty, we can adjust the structure of uncertainty by tuning the probability $0 < \nu < 1$.

Given the probability distribution functions $\Phi$ and $\Gamma$ and the value of the probability $\nu$, we are interested in the probability distributions:

1. $Q^{>}_{\nu}(\tau)$ of the time intervals $\tau$ when the pretender has an advantage over his reference group, and
2. $Q^{<}_{\nu}(\tau)$ of the time intervals $\tau$ when the pretender is at a disadvantage with respect to his reference group.

## 7.7 The Fastest Way to Excel over Rivals Under Uncertainty

From the mathematical point of view, the proposed stochastic model of advantage is equivalent to the model of subsistence under uncertainty.

Following the discussion given in the first chapter, we can immediately conclude that the expected time required for two random numbers distributed over the interval [0, 1] to meet each other is minimum under dual uncertainty when the two numbers are updated synchronously. However, the expected time is maximum under singular uncertainty when random updates are inconsistent. Below, we briefly reinterpret the main mathematical results derived earlier for the model of subsistence under uncertainty in the context of attempts to gain advantage.

The mathematical analysis of the stochastic process given in the first chapter shows convincingly that, under dual uncertainty when both the objective and the subjective factors contribute equally to the process of competitive advantage ($\nu = 0$), the probability that the pretender is still performing at a level lower than the level of his reference group decays exponentially fast with time.

☞ A state of disfavor is always transitory under dual uncertainty.

In particular, if both performance levels are drawn from the uniform distribution over the interval $[0, 1]$, the probability that the pretender stays in disfavor will dwindle:

$$Q^{<}_{\nu=0}(\tau) = \frac{1}{2^{\tau+1}} \, . \tag{7.19}$$

Therefore, the rational strategy for quickly obtaining an edge over the reference group of rivals is to become actively engaged in solving problems and simultaneously to disturb the group fitness level at every instant of time—for better or for worse.

This can be accomplished, for instance, by stirring up conflicts within the group, by offering innovations, and by conceiving seductive and exciting projects that provoke competition within the group. Finally, the same task can be accomplished simply and peacefully, by randomly choosing another reference group.

☞ The fastest way to get an edge over rivals under uncertainty is to disturb the group fitness level at every instant of time—for better or for worse.

Under singular uncertainty ($\nu = 1$), so that the performance level of the group is unchangeable while the level of the pretender varies randomly, or conversely, the performance level of the group varies randomly while the level of the pretender is unchangeable, it may take too long to gain an advantage, even though many different types of behavior are possible, depending upon the particular choice of the distributions $\Phi$ and $\Gamma$. For example, for performance levels chosen uniformly at random, the decay of probability tends asymptotically to the quadratic hyperbola, i.e.,

$$Q^{<}_{\nu=1}(\tau) \propto \frac{1}{\tau^2}\,, \tag{7.20}$$

so that the characteristic time for remaining a loser is infinite.

It follows from the quadratic hyperbola tail (7.20) that, if an individual is unable to substantially change (improve) his own performance (or income) level, but simply searches for a prominent position over a variety of groups with performance levels distributed uniformly at random, the following rule of thumb can be used to anticipate the duration of search: *a search that lasts twice as long, occurs a quarter as often*.

The absence of a characteristic time scale in the process (7.20) indicates that an individual path to success may be of virtually any duration, so that the state of singular uncertainty could be a real pitfall on the way to fortune.

☞ Under singular uncertainty, a pretender might be cast almost indefinitely into lingering disfavor.

However, for many thousands of years, the idea of standing out and excelling over the group was absent from the sense of time dominant in traditional farming communities [75]. It is only recently that a social ideology of individual success enjoining personally striving rather than a cooperative community spirit has become ubiquitous throughout life and across virtually all societies.

## 7.8 Maintaining Control by Evading Competition

The reproductive (and financial) benefits of gaining advantage can be enormous, so a prominent position at the top is highly competitive and calls for continuous maintenance, since rivals from the reference group are not going to sit back and relax. The leader dominating the group has to continually detect, prevent, and contain threats to his hold on power [210].

In this case our model would work the other way round, since exceptionally long periods of dominance could occur in particular under singular uncertainty, when factors responsible for the objective and subjective types of uncertainty evolve on incomparable time scales. Two different, but not mutually exclusive, strategies are possible for maintaining prolonged control over a group in such a situation:

- **Control by instigating chaos.** Leaders that are unwilling or unable to change their own relatively high ability level (or the quality of management) may seek to disturb the group performance level at every possible opportunity, for instance, by stirring up a rumpus that would trigger a sense of insecurity among group members. Thus the leader himself is then the main source of chaos, disorder, and confusion, and, in the public eye, he also becomes the chosen one who can save the group from the chaos instigated by himself.
- **Tyranny.** If the leader is an absolute ruler, restrained neither by law, nor constitution, he could excel over the group by oppression, eliminating any suspected opponents.

In both cases, the mechanism for maintaining the monopoly of power consists in avoiding competition with any occasional rival at any price, since if both performance levels are allowed to vary, the probability of maintaining the advantage decays exponentially fast with time. Hence, dictators are notoriously insecure, as the power they wield today may slip out of their hands tomorrow [210].

☞ If competition is allowed, any advantage is always transitory.

The above strategies for maintaining the monopoly of power have been known since ancient times.[3]

However, as we have shown in the analysis of the subsistence problem in Chap. 1, no matter how horrible and oppressive the ruling regime—vigilantly seeking out real and imagined enemies and remorselessly weeding out people of genuine merit from a terrified population—it will inevitably come to an end, since the most favorable longevity statistics under uncertainty could not be better than one, following Zipf's

[3] As accounted in Livy's *History of Rome* [211], the Roman tyrant Tarquin the Proud was asked by his son Sextus Tarquinius about what he should do next in Gabii, since he had become all-powerful there. Tarquin went into his garden, took a stick, and swept it across his garden, thus cutting off the heads of the *tallest poppies* that were growing there. Sextus realized that his father wished him to put to death all of the most eminent people of Gabii, which he then did.

law, viz.,

$$Q(t) \propto t^{-1-\varepsilon} ,$$

for some $\varepsilon > 0$.

As discussed in the previous chapters, we can introduce an entropy of survival for authoritarian regimes:

$$H(\nu) = -\sum_{t \geq 0} Q_\nu(t) \log_2 Q_\nu(t) . \tag{7.21}$$

The entropy function (7.21) remains finite for all $\nu \in [0, 1]$, excepting for $\nu \to 1$, when Zipf's probability of survival can be attained as $\varepsilon \to 1$. Here, higher values of entropy correspond to enhanced chances for the regime to survive. Clearly, the value of the survival entropy grows steadily with increasing political stability $\nu \to 1$. Therefore, as discussed in Chap. 2, the entropic force ('love of life') should emerge in a closed system, arising from the tendency toward maximum entropy. The resulting state of maximum survival entropy—the *oppressive authoritarian regime*—is most likely to be observed with time, and thus may be viewed as a (quasi-)equilibrium state in the model of stochastic competitive advantage.

In democratic regimes, free and fair elections are held according to stable procedures, where rules are laid out beforehand and results are respected afterwards. Democratic elections are based on open electoral competition and therefore provide procedural certainty [210], although the outcomes of elections are indeterminate. Despite the fact that authoritarian regimes often try to use the same instruments, in an attempt at institutional mimesis, their 'elections' follow the opposite logic [210]: they are fostering procedural uncertainty (by meddling with procedures) in order to secure the outcome in advance. The equilibrium state in the resulting system of stochastic competitive advantage is the authoritarian regime.

☞ An authoritarian regime arises whenever the initial competitive advantage was gained under procedural uncertainty.

The authoritarian ruler can never know for sure how good he is at preventing, detecting, and containing threats to his survival in power [210]. Even though all opponents of the tyrant may have been eliminated, the resulting organisational hierarchy gradually becomes less and less effective and eventually loses stability through the steadily increasing incompetence of its members.

As a matter of fact, the person at the top of the hierarchy who wishes to remain in control over the group is not sovereign, since he depends for the exercise of power on actors who control economic wealth, arms, legitimacy, and organized violence, above all [210]. As these actors pose perennial threats to the capacity of leaders to govern and to survive in government, the dictator has to choose his subordinates with the prime criterion of incompetence, to ensure that they are less capable than the leader himself, and mimic him. And these negatively selected associates must do the same with those below them in the ruling hierarchy. As a result, the growing hierarchy is progressively filled with less and less competent people.

☞ If competition is avoided, a progressively less effective hierarchy will arise.

Since the most competent and potentially dangerous members of the hierarchy are concentrated at the top, the dictator will purge them regularly, filling the emptied positions from below with even less competent people. These occupational hazards inevitably translate into personal traits, making the dictator chase real as well as imaginary enemies [210]. Negotiations with opponents concerning mutual restraint on the use of organized violence seem ultimately inconclusive for the dictator himself, in his attempts to escape death. Although a repressive and corrupt authoritarian regime that enriches its elites and impoverishes its citizens may well be able to maintain social peace over long stretches of time [210], its ruling hierarchy will rapidly become grossly ineffective and collapse, on the first critical occasion, for instance, once the dictator dies.

## 7.9 In the Face of Uncertainty, the 'Fittest' is Punished, Rather than Rewarded

The evolutionary theory put forward by Jean-Baptiste Lamarck [212] consisted of two factors: the endogenous tendency of a species to become more advanced and an exogenous constraint that drove organisms into conformity with their environment.

In [213], Charles Darwin proposed to eliminate the endogenous tendency from the evolutionary scheme by suggesting that the conditions of life experienced by organisms are approximately uniform and that some organisms might be fitter than others, in the sense of being better able to compete and reproduce. The principle of natural selection rewards success by making the traits that are better adapted to local circumstances heritable.

In the modern theory of evolution, natural selection rather becomes a filter that punishes failure, and fitness rather becomes an attribute of a self-replicating unit of transmission, a gene [215]. Finally, the modern evolutionary synthesis constitutes a form of environmental determinism that explains human evolution in terms of 'heroic struggles' and selective winnowing, in which genes either have what it takes to survive or are eliminated [214].

However, provided the conditions of life are anything but uniform, the concept of competitive superiority would reveal serious doubts, at least in the societal context, since natural selection does not assume, in general, any selective multiplier to reward cooperative strategies. Moreover, when the primary purpose of cooperation within a group is the elimination of its most prominent members (because they may generate more uncertainty) the concept of competitive superiority of traits that enhance an individual's performance in the environment *is simply not true*.

The egalitarian ethos is known to be striking in societies of human hunters and gatherers, where no one is supposed to stand out from the rest of the group. For the sake of egalitarian group spirit, individuals who persistently seek to get an edge over their peers may be expelled from a group or even killed, so that their high-functioning, 'excessively' successful traits are not inherited, but mostly eliminated from the gene pool of the group. It is widely argued by paleoanthropologists that resistance to being dominated was a key factor driving the evolutionary emergence of human consciousness, language, and social organization [216]. The broad historical evidence suggests that people facing uncertainty can cooperate in order to expel their best performing peers. Expulsion of the best individual violating group integrity is a ubiquitous phenomenon, by no means limited to archaic societies. It is therefore surprising that, although this social phenomenon—the *tall poppy syndrome* [217]—is common to virtually all people, known in all societies, and recognisable at all times, it has never been considered an independent factor of societal evolution. The tall poppy syndrome constitutes a systematic persecution of people of genuine merit because their talents or achievements elevate them above their peers.

☞ In hierarchical societies in the face of uncertainty, the 'fittest' is discriminated against, not rewarded.

It was suggested by Max Weber [218] that the acquisition of prestige and power in a hierarchical society might be viewed as a zero-sum game, in which each participant's gain is exactly balanced by the losses of others. Since status is a relative value in such a disadvantaged society, for someone to rise in status, another person must fall, so that every 'excessively successful' community member undermines the standing of all other peers and is considered by them as an outlaw to be punished or, better, expelled.

From the point of view of evolutionary biology, the tall poppy syndrome can be explained in the following way:

☞ When the rate of social changes within a group is faster than the rate of possible social adaptations to these changes, the behavioral traits selected previously as enhancing individual performance can suddenly be treated as ineffective, outdated, and maladaptive. In the face of uncertainty, the community may seek to get rid of excessively successful peers to evade the evolutionary trap.

In essence, the tall poppy syndrome expressed by a permanent willingness to give up the best—including morality, social norms and cultural achievements, along with their most outstanding representatives—is nothing but a continuation of the 'endurance running strategy' for avoiding evolutionary traps, a strategy that has allowed us to survive and dominate this planet. And so our endurance racing continues!

Perhaps, the best known historical evidence for the validity of the 'expulsion of the best' principle is the continuous persecution and recurrent expulsion of Jewish communities playing a key role in the economy. Much less well known is the deportation and extrajudicial punishment of successful farmers (*kulaks*, the strong and sober) in Russia during the policy of 'eliminating kulaks as a class' proclaimed by Stalin in 1929–1933 and actively supported by the less successful remainder of the population. Approximately 20 % of a rural population of 120 million[4] were deported, while many of them were murdered in local violence and formally executed after conviction.

Throughout the history of mankind, it is easy to find a plethora of other examples in which the best and most prominent members and minorities within hierarchical communities were mercilessly eliminated in the course of genocidal cleansing. In all cases, the eliminated traits were not the worst of all, as one might expect according to the logic of competitive superiority that follows from the principles of natural selection. On the contrary, they were among the best, perfectly well adapted to their social environment.

☞ It may be that the fittest genes of all are silent genes.

We conclude this section by remarking that evolution in human populations may be an equilibrium-seeking process, in which the fittest individuals are those that evade selective winnowing, rather than those that acquire competitive superiority over their peers, thereby exposing themselves to the risk of subsequent elimination from their communities by a joint action of less successful group members.

In the case of biological evolution, the 'expulsion of the best' principle would mean that genes can maintain their fitness by hiding from natural selection rather than "go head-to-head with other alleles in a competition that only allows the strongest to win through" [214]. The fittest genes may be those that are incorporated into stable polymorphisms and hidden from any superiority/inferiority relation—perhaps, they are just silent.

[4]Our assessment of the number of people repressed during Stalin's campaign is based on the percentage of Russian peasants who participated in the agrarian reform instituted in 1906–1911, during the tenure of the Russian PM Peter Stolypin. He referred to his own programs as a "wager on the strong and sober", and believed that tying the peasants to their own private land holdings would produce profit-minded and politically conservative farmers. So these people, who had once dared to stand out from the serfdom community as being supported by the imperial government, were then 'eliminated as a class' in the course of Soviet reforms, reestablishing social norms and practices typical of mesolithic societies.

## 7.10 Emigration Can Give Us Yet Another Chance

My old friend and colleague professor Philippe Blanchard once said:

> Neither coming into power, nor getting a fabulous wealth, nor acquiring a splendid education, can save a family from decay and eventual extinction, but emigration can give it yet another chance.

This proposition might sound odd, since emigration is often associated with insecurity and uncertainty, both of which are deemed to be generally negative states. Migrants may not speak the local language sufficiently fluently, they may lack links with the local community, and their overseas qualifications may be either not recognized by local employers or treated unfairly. As a result, migrants are discriminated from suitable job vacancies far more often than natives, and are frequently overqualified for the jobs they do [183]. They may appear to be in a unenviable position, at the bottom of the hierarchy in the labor market.

For natives, establishing a stable, secure, and successful career is a normative expectation and a necessary precondition for family life and parenthood with an adequate standard of living. They may postpone the founding of their own family because they aspire to achieve career recognition first. Part-time work and non-permanent contracts significantly reduce their propensity to start a family, and indeed their chances of doing so. Upwardly mobile men usually delay the founding of a family due to long educations and investment in their career. But putting off the decision to have a first child has a crucial effect on fertility rates, since men who are older than 35 have a significantly lower probability of a transition to fatherhood than young men between the ages of 25 and 30, and becoming a father past middle age is the exception to the rule [184].

On the other hand, waiting for better economic conditions by accumulating resources and getting a secure workplace to cope later on with the liabilities of parenthood may not be a realistic option for migrants [184]. Moreover, discontent with their job and career situation may even intensify their desire to look for a steady and fulfilling personal relationship. The experience of insecurity in the labor market may only strengthen the need for trust and reliability in personal relationships, and the parent–child relationship will be the most indissoluble relationship of all [184]. Furthermore, the probability of becoming a father is significantly higher for those men who improve their situation than for those on a steady track, and migrants who succeeded in overcoming their social destiny by progressively adapting to a new host society regularly come out on top!

Therefore, although both natives and migrants face uncertainty in the labor market, they are likely to try to make their future more predictable by following quite opposite strategies: while natives would first increase investments, perhaps in the working sphere, migrants, instead, might prefer parenthood as a compensatory relationship, because they have nothing to lose, but something to win by investing in having children [185]. Interestingly, for many years after the German reunification, East Germans continue to become parents more often, even faced with the higher risks of having children when they are unemployed [186].

Not surprisingly, immigrants typically have more children than natives. First, they are usually poorer and live in a less predictable ecology than natives, so more children are probable. Constrained career opportunities make migrants less likely to find social and economic incentives to invest more in fewer children, rather than to invest less in many.

Second, most cultural and behavioral ideas and symbols generated by high-status individuals in developed societies and transmitted by their mass culture act to decrease the number of children. Popular TV shows, musical video clips, and commercials show successful people surrounded by beautiful things, but almost never burdened with children. On the one hand, being alien to the local culture due to the lack of restrictive social ties and perhaps insufficient knowledge of the new language, migrants are free from any need to share local ideas and to follow the social practices that presumably underlie the low fertility rates prevailing in the host society. On the other hand, for many migrants, the bicultural experience creates an opportunity to loosen the connection with their country of origin and liberate themselves from obedience to some of the traditions and odd laws of their home society, including those denunciating marriages between persons of different racially, religiously, or ethnically defined groups.[5] Migrants simply do not shy away from having more children than 'normal' and from miscegenation, even if it is viewed as somehow unrighteous in both the country of origin and the host country.

☞ A commitment to excellence in behavioral adaptations enhancing social fitness cannot save a family from decay and eventual extinction. However, emigration can give it another chance.

First and second generation immigrant youths are more likely to live in dual-biological parent households than their native peers, as the need for trust and reliability in personal relationships usually secures emigrant families [187]. Moreover, immigrant parents tend to be more authoritarian and authoritative than native-born parents [187]. Authoritarian parents are very strict, often exercising complete control over their children. This authoritative parenting style is highly demanding but supportive, setting firm limits and enforcing household rules through the use of parental authority [188]. First and second generation immigrant youths have parents who are in general stricter [189] and much less frequently permissive or uninvolved than native-born parents [187].

Parental involvement plays a key role in improving the chances of academic attainment for first and second generation immigrant youths, because parents are important socializing agents with regard to educational trajectories [189], providing the key scaffolding for children to navigate new or challenging tasks and remain

[5]For instance, anti-miscegenation laws enforcing racial segregation at the level of marriage and intimate relationships were upheld in certain US states until 1967 (and formally were still on the books in some states till 2000).

committed and steadfast in their behavior [190]. With clear and informative feedback from parents, children are able to navigate tasks more effectively, attesting to greater intrinsic motivation, while failure to provide parental support results in poorer school achievement [191]. Conversely, permissive and uninvolved parenting styles are negatively correlated with educational attainment.

Authoritarian and authoritative parenting styles seem to be most helpful in keeping immigrant youths on track toward higher grade completion, because they put the emphasis on their children doing the best they can, rather than the individualism customary in many native-born families [187]. Immigrant parents very likely index their own sense of self on their child's achievements, feeling proud when the child is performing successfully in school tasks, but embarrassed if the child is performing poorly there. Since academic achievements emphasize ego involvement [191], recent immigrants may have greater faith in the use of education to achieve upward mobility than their more established native-born peers [189]. Therefore, immigrant youths might have higher educational aspirations as well as stronger beliefs in the importance and usefulness of education than their more pessimistic native-born peers [187].

Much literature provides evidence of the importance of education in underpinning improved well-being [192]. Higher levels of education help to improve people's lives and facilitate skills which are needed to find a good job, earn sufficient income, and establish a stable family. Conversely, men with low educational levels also have low fertility [193]. As a manifestation of adopting the parent's family orientation, and also as an expression of the important social experiences gained within a large family, second generation immigrants who have grown up with siblings have a higher propensity to establish their own family [194]. In contrast, growing up as a single child significantly lowers the rate of transition to parenthood, while non-stability in the family of origin also reduces men's transition to fatherhood [184].

### 7.10.1 When Is the Right Time to Migrate?

However, the greater the length of residence in the new country, the lower the academic motivation and grade point averages of immigrant descendants [189]. For instance, while first-generation Hispanic and Asian immigrant youth in the US make tremendous strides in educational attainment relative to their parents, this trend in upward mobility reverses by the third generation [189]. Along with their native-born peers, third generation immigrants apparently fall into a fatalistic perspective, where academic success is no longer an option. Their discouragement when they see the many obstacles to upward mobility in their lives disillusions them about taking promising opportunities [189].

Perhaps, it is then the right time to start planning a new migration route, since when individuals emigrate into entirely new surroundings, they do not remain decadent, pessimistic, or immoral among the inhabitants of their new homeland [195]. Once they have broken away from their previous channels of thought, and after a short period of readjustment, they become normal members of their new communities. And then the virtuous cycle of survival may start again. After all, migrating at the right time can buffer tall poppies from the risk of being shorn in their home country.

## 7.11 An Even Equilibrium in a Hierarchical Society

The Russian 'Table of Ranks' was a formal list of positions and ranks in the military, government, and court of Imperial Russia, introduced in 1722 in order to reorganize the foundations of the feudal Russian nobility by recognizing service in the military, in the civil service, and at the imperial court as the new basis of an aristocrat's standing in society. The former feudal hierarchical system in Russia was based on the individual's seniority within an extended Russian aristocratic family, as well as on an order of precedence of the families that was calculated on the basis of historical records of past senior appointments.

The introduction of the new system, however, did not alter the nature of traditional feudal relations in society based on fief, granting of lands, governmental offices, hunting and fishing rights, monopolies in trade, and tax farms in return for a form of feudal allegiance and services. Although the 'Table of Ranks' was formally abolished by the Bolshevik government in 1917, they eventually restored the original system of fiefs in its most archaic form [219], ultimately encouraging rent-seeking behavior in the population and making corporate bribery and corruption a way of life.[6]

We are interested to learn about the possible equilibrium compositions in a hierarchically organized society, in which everyone is bestowed privileges on a scale determined by his own rank.

Let us suppose for simplicity that the socio-economic status, the grade of an individual in the societal 'table of ranks', takes values in the set of positive integers $n = 1, 2, 3, \ldots$, without any a priori upper bound. We estimate the 'usefulness' of the rank $n$ for the individual as $u(n)$, where $u$ is the utility function representing the ultimate opportunity to generate revenue from the office (by using legal fief or bribery).[7] An unqualified worker is of rank 1, being deprived of any privilege and in fact of any property rights $u(1) = 0$.

[6] As N.M. Karamzin, the author of the 12 volumes of *History of the Russian State*, once remarked (1826): "If you wanted to express what is happening in Russia in a single word, the answer would be: stealing."

[7] The grades in the actual 'Table of Ranks' were reversed: the Chancellor (or General field marshal) had rank 1, the Active privy councilor (or General) had rank 2, the Privy councilor (or Lieutenant general) had rank 3, etc.

We use the Bernoulli logarithmic utility function [141]

$$u(n) = \ln n \ , \tag{7.22}$$

since it is the sole isoelastic utility model satisfying the boundary condition $u(1) = 0$.

We also assume that a randomly chosen individual belongs to rank $n$ with probability $p_n$. The expected rank of an average individual in the society is $\langle n \rangle = \sum_n np_n$, with the corresponding scale of bestowed privileges

$$\ln(\langle n \rangle) \equiv \mu \ . \tag{7.23}$$

It is worth mentioning that, in an even enough society, the expected rank $\langle n \rangle$ might be infinite, provided that the probability distribution $p_n$ decays with rank $n$ not faster than the quadratic hyperbola $p_n \propto n^{-2}$, and the corresponding scale of privileges conferred on an arbitrary individual $\mu$ can thus also be infinite.

Therefore, the value $\mu$ can be viewed as the degree of social equality in the society: the higher $\mu$, the closer the actual rank composition $p_n$ to the Pareto-like distribution, and the less unequal the system of social ranking.

Another important characteristic of the model is the expected utility, that is, the expected scale of bestowed privileges, viz.,

$$\langle \ln n \rangle \equiv \chi \ , \tag{7.24}$$

which is concerned rather with individual rank in the society, being infinite only for a hypothetical person of infinitely high rank $n \to \infty$.

Let us assume that the rank composition in the society can be determined by analyzing the relation between the 'global' characteristic $\mu$, i.e., the scale of privileges conferred on an arbitrary individual in the society, and the 'local' characteristic $\chi$, i.e., the expected scale of conferred privileges over the entire society.

The meaning of our assumption is that the two parameters $\mu$ and $\chi$ may be considered for a certain time almost as life history invariants of the society, varying very slowly, perhaps over many generations. We suppose that both parameters can be viewed as constant during the time for an equilibrium structure to be achieved in the society. Our assumption is equivalent to saying that the society has a stable structure of *social classes*. Models of social stratification in which people are grouped into a set of relatively stable hierarchical social categories are common in sociology, economics, and politics.

It follows from Jensen's inequality [202] that $u(\langle n \rangle) \geq \langle u(n) \rangle$ for the concave utility function $\log n$. The condition (7.25) is analogous to the principle of risk aversion of utility theory. It is remarkable that

$$\mu \geq \chi \tag{7.25}$$

is nothing but the logarithm of the arithmetic mean–geometric mean inequality. Equality in (7.25) holds if and only if all individuals in the society belong to the same rank. Therefore the ratio $\chi/\mu$ can be viewed as a heuristic measure of *social uniformity*. Hence, all individuals in the society have the same rank (a state of complete uniformity) when $\chi/\mu = 1$. Otherwise, when $\chi/\mu \ll 1$, the diversity of grades in the society is extreme.

According to the maximum entropy principle [29, 30], we are interested in an equilibrium distribution $p_n$ that corresponds to a maximum of the entropy

$$H = -\sum_{n\geq 1} p_n \ln p_n \ , \tag{7.26}$$

under the two constraints (7.23) and (7.24), along with the usual normalization constraint for the probability, i.e., $\sum_{n\geq 1} p_n = 1$.

For every pair of parameters $(\mu, \chi)$, we can derive the most likely observable equilibrium distribution $p_n$ which characterizes the rank composition in the society. The variation problem for the entropy maximum under the constraints involving the logarithmic function is considered in [37].

The Lagrangian functional for the variation problem is

$$\begin{aligned} \mathcal{L} = -z\left(\sum_{n\geq 1} p_n \log n - \chi\right) + \ln w\left(\sum_{n\geq 1} p_n n - \mathrm{e}^{\mu}\right) \\ -(\ln Z - 1)\left(\sum_{n\geq 1} p_n - 1\right) - \sum_{n\geq 1} p_n \ln p_n \ , \end{aligned} \tag{7.27}$$

where $w$, $z$, and $Z$ are the Lagrangian multipliers. Varying the Lagrangian functional with respect to $p_n$ yields the following equation:

$$-z \ln n + n \ln w - \ln Z - \ln p_n = 0 \ . \tag{7.28}$$

The solution depends on two parameters:

$$p_n(w, z) = \frac{w^n n^{-z}}{\mathrm{Li}_z(w)} \ , \quad w < 1 \ , \tag{7.29}$$

where $\mathrm{Li}_z(w) = \sum_{n\geq 1} w^n n^{-z}$ is the poly-logarithm function.

The probability distribution (7.29) comprises two different factors, which we may refer to somewhat vaguely as the 'aristocratic' and 'industrial' factors. The aristocratic factor $\propto w^n$, decaying exponentially fast with the rank $n > 1$ as $w < 1$, characterises the probability distribution $p_n$ as $z \to 0$ and $w > 0$. The number of ranks in a society dominated by this factor is always finite,[8] with the majority of the

[8]The actual Russian 'Table of Ranks' was divided into just 14 grades.

population belonging to the minimal rank and just a few forming a privileged group of highly ranked individuals constituting the ruling class. Such a social structure was typical of agrarian societies of the pre-industrial age. We can call $0 < w < 1$ the *commonality parameter*, since the rank composition of the society is as unequal as possible when $w \to 0$, while the effect of aristocracy is leveled out completely when $w \to 1$.

The other, 'industrial' factor is the Pareto-like term $\propto n^{-z}$, which dominates the distribution (7.29) as $w \to 1$ and $z > 0$. Power law distributions are typically used to describe economic inequality in societies supporting a high capacity for division of labor and a wide range of different social positions. The parameter $z > 0$ is nothing other than a Pareto index, similar to the one quantifying the level of income inequality over the society. The larger the $z$ index, the smaller the proportion of high-status people. In general, the composition of society according to (7.29) is determined by both factors. An even society, in which all ranks $n$ are equally ubiquitous, is characterized by the minimal values $z \to 0$ and the maximal values $w \to 1$.

The expected scale of bestowed privileges $\chi$ and the scale of privileges conferred on an arbitrary individual $\mu > \chi$ can be calculated for $0 < w < 1$ and $z > 0$ as

$$\begin{cases} \chi(w,z) = \dfrac{1}{\mathrm{Li}_z(w)} \displaystyle\sum_{n\geq 1} \dfrac{w^n \ln n}{n^z}, \\ \mu(w,z) = \ln \left[ \dfrac{1}{\mathrm{Li}_z(w)} \displaystyle\sum_{n\geq 1} \dfrac{w^n n}{n^z} \right], \end{cases} \tag{7.30}$$

and both grow as $z \to 0$ and $w \to 1$, although at different rates.

Recall that the ratio $\chi/\mu$ can be viewed as a heuristic measure of social uniformity: the closer it comes to unity, the higher the proportion of individuals belonging to the same social rank. The degree of social uniformity (estimated by $\chi/\mu$) is presented in Fig. 7.5 as a function of the commonality parameter $w$ and the Pareto index $z$ in the distribution (7.29).

As expected, the maximal degree of social uniformity is achieved for the even society characterized by the minimal values $z \to 0$ and the maximal values $w \to 1$. Interestingly, the minimal degree of social uniformity ($\simeq 0.5$) is also relevant to the mass society—not to the aristocratic societal structure—when the commonality parameter $w \to 1$, but the Pareto index lies between 2 and 3.

☞ The rank composition in mass society can demonstrate patterns of highest uniformity as well as highest diversity.

The probability distribution (7.29) is the one most likely to be observed for the given values of $w$ and $z$, because the corresponding entropy of rank composition (7.26) is maximum.

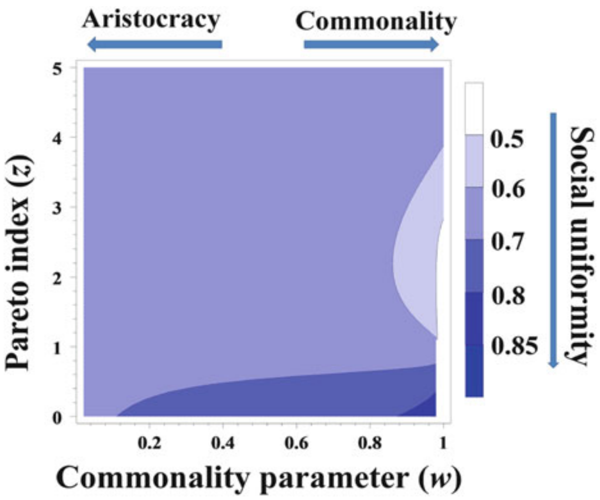


**Fig. 7.5** The degree of social uniformity (estimated by $\chi/\mu$) as a function of the commonality parameter $w$ and the Pareto index $z$ in the distribution (7.29)

However, as the entropy values are different for each pair $(w, z)$, we may suggest that according to the maximum entropy principle, the rank composition of the society would evolve with time towards the most likely, equilibrium state of maximum entropy, which is the one that can be achieved in the maximum number of ways.

### 7.11.1 What Would Such an Equilibrium State Be?

☞ The structure of a hierarchical society inevitably relaxes to a more democratic rank composition.

In Fig. 7.6, we have shown the contour plot for the entropy (in bits) of rank composition of the society. The entropy function is undetermined for the values $z < 1$ and $w \to 1$, indicating that our simple model of hierarchical society is not applicable for this region.

Nevertheless, it is clear that the maximum entropy of social structure is achieved when the society is close to an even state, for small enough Pareto index $z$ and large enough values of the commonality index $w$—in the state of democracy.

In the proposed model of a hierarchical society, we have suggested that its equilibrium structure can be determined under the assumption of constancy for the way privileges are conferred on the society. In particular, we propose that the social structure would evolve toward the equilibrium state of maximum entropy faster than the parameters $\mu$ and $\chi$ might change, so that relatively stable social classes exist in the society and we could use them as constraints in the variational problem.

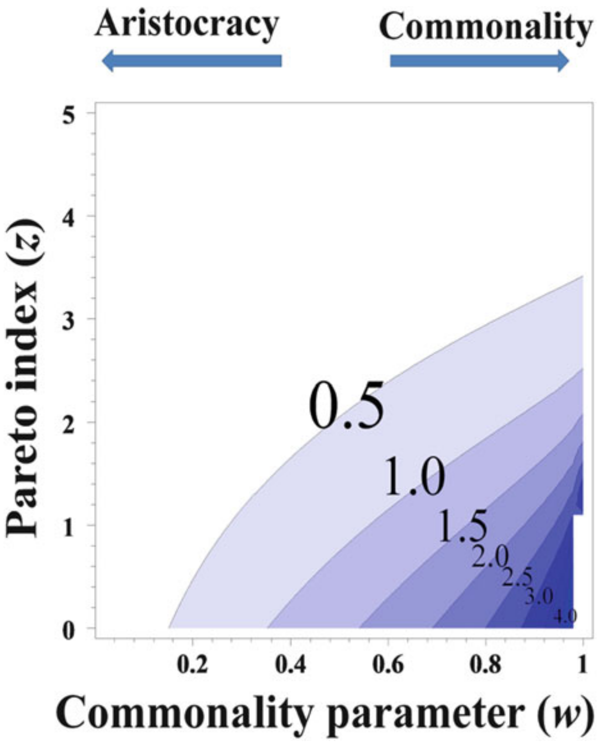


**Fig. 7.6** Entropy of rank composition of the society (levels are marked in bits) (7.26) as a function of the commonality parameter $w$ and the Pareto index $z$ in the distribution (7.29)

## 7.12 Conclusion

In the present chapter, we have discussed the most probable survival and planning strategies under uncertainty. Harsh and perfectly stable ecologies are equally threatening to survival success. While desperation ecologies threaten survival directly, stable ecologies would foster evolutionary traps, as the adaptations enhancing individual performance in stable environments can suddenly become maladaptive when living conditions change abruptly. The optimal survival strategy in any foreseeable time period would consist of a regular change of scenery by innovation and migration to other environments. Since desperation ecologies mark their inhabitants via cues to 'faster' behavioral strategies, the most likely strategy for planning under uncertainty for the time horizon $T$ would consist of $T/\log T$ short-term decisions, which corresponds to hyperbolic discounting of time in risk aversion and prudence behavior. Since also the preferences of a real-life decision-maker change over time and can become inconsistent at another point in time, any activity under uncertainty inevitably involves making choices today that his future 'self' would prefer not to have made.

We have proposed and studied the model of stochastic competitive advantage, from which it follows in particular that, under dual uncertainty, a state of disfavor is always transitory. The fastest way to get an edge over rivals is to disturb the group fitness level at every instant of time, for better or for worse. However, a pretender may be cast into lingering disfavor almost forever under singular uncertainty. If competition is allowed in the group, any individual advantage is transitory. But if competition is avoided, then a progressively ineffective hierarchy arises in the group. We have also discussed the fact that, in hierarchical societies, the 'fittest' is punished in the face of uncertainty, and not rewarded. A community may seek to get rid of its most prominent peers, considering their behavioral adaptations outdated and maladaptive, provided that the rate of social change is higher than the rate of possible social adaptations to these changes. The high degree of behavioral adaptations in the society may be not that important for survival success under uncertainty. We have supported the conclusion of [214], according to which genes can maintain their fitness by hiding from natural selection rather than go head-to-head with other alleles in a competition that only allows the strongest to win through—so it may be that the fittest genes of all are silent ones.

We have also discussed the structure of hierarchical societies and its evolution. We have shown that the structure of a hierarchical society inevitably relaxes to more democratic rank compositions with time.

# Impersonal Methods of Decision Making Under Uncertainty: Social Conformity, Market Economy, and Authoritarianism

# 8

*To be, or not to be—that is the question:*
*Whether 'tis nobler in the mind to suffer*
*The slings and arrows of outrageous fortune*
*Or to take arms against a sea of troubles,*
*And by opposing end them. To die- to sleep-*
*No more; and by a sleep to say we end*
*The heartache, and the thousand natural shocks*
*That flesh is heir to. 'Tis a consummation*
*Devoutly to be wish'd. To die—to sleep.*[1]

**Abstract**

Under conditions of uncertainty, people try to evade personal decision making by using impersonal methods of thought. We have introduced and analyzed a simple probability model for social conformity and a simple deterministic dynamical model of markets. We have discussed the idea that, when societal institutions dissolve but people's social identity remains intact, they tend to think that the end justifies the means, and would deliberately reject democracy in favor of authoritarianism as a way of getting an opportunity for social revenge.

## 8.1 Introduction

On the one hand rational choice under uncertainty is impossible, but on the other, it is also impossible not to choose in the face of uncertainty, as the very non-choice then also becomes a way of choosing [87] that may potentially threaten the survival of an individual or open a window of opportunity for routine-breaking. The deliberate choice—*To do, or not to do? To be, or not to be? To go, or not to go?*—

[1] W. Shakespeare, *Hamlet*, Act III, Scene 1.



D. Volchenkov, *Survival under Uncertainty*, Understanding Complex Systems,
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which is usually associated with the idea of freedom and autonomy, becomes an unbearable burden of destiny when it is to be made under uncertainty [220]. Not surprisingly, people then try to evade personal decision making at all costs, often with the use of a variety of exopsychic, impersonal methods of decision making, which we discuss in the present chapter.

## 8.2 Impersonal Methods of Decision Making

In a traditional society, under perfectly working societal institutions, the goals which people aspire to are mostly predefined by their social identity and status. The means to achieve these goals are clearly determined by the established societal institutions and traditions shared by all community members. A member of a traditional society is literally free *of* choice, as well as of any responsibility for its consequences. The future is revealed to him either through commandments and unspoken rules, or through divination and omens conveying god's will.

Since there is no room for randomness in the future, the concepts of chance and risk have no meaning in such a society, a result of any game of chance, such as throwing dice or spinning roulette wheels, is caused by none other than the deity, whose intentions are divined. *Exopsychic methods* of thought and decision making [225] are used to solicit divine direction. Steps taken to ensure randomness in a game of chance are intended merely to eliminate the possibility of human interference, so that the will of the gods can be discerned at first-hand [221]. There is an astonishing diversity among these methods, including various chance mechanisms, such as the movements of smoke rising from a censer, the form of hot wax dropped into water, the shapes of ashes, or simply heads and tails with a flipped coin.

Since any given game of chance is equated with the actual issue of uncertainty through a *metaphor*, the observed random patterns are identified as being symbolically the same as the gods' purpose for men in the given situation. We may add that the arbitrariness of the metaphor in the game of chance epitomizes the *uncertainty of means*, and the degree of indeterminacy of god's will represents the *uncertainty of ends* associated with the real situation. Thus, the outcome of such an exopsychic decision-making mechanism is always the result of an interplay between two types of uncertainty (Sect. 8.3).

When societal institutions disintegrate, people directly experience *uncertainty of means* (Sect. 8.3), as it becomes unclear which course of actions will best achieve their life goals. Lack of access to the functional systems of society means that they cannot continue to indulge in the usual illusions granting oblivion to personal mortality [84]. The stress and anxiety associated with the need to make decisions on their own would prevent people from coping with their immediate troubles, and these can then progressively turn into social and public problems, resulting in weakened bonds of mutual trust and increased social fragmentation. When the expected aims and apprised role in society become uncertain, people experience *uncertainty of ends* (Sect. 8.3). If their social identity in the community weakens, people join other social groups, where they can gain social support and rely on the exopsychic methods of decision making within the new community.

People tend to seek safety in closely knit groups when they experience existential threats [12]. By abiding by common rules and mimicking the behavior of peers, the benefits from evading personal decision making in the face of uncertainty far outweigh the cost of subordination to others. In Maslow's hierarchy of human needs [222], belonging to a community comes just above health and safety. Together with most of our peers, we often prefer to be in a low social position within the kin group, even tolerating the highest degree of inequality and injustice, rather than leave a safe haven of identity and try to find another one. We shall introduce and study a simple probability model of social conformity in Sect. 8.4.

When we are among our kin group or within the family, there is no problem of resource allocation, as resources are divided evenly between all family members of equal status. However, it is obvious that there are limits to how many people can share the unanimity of kinship (Sect. 8.7). Adequate prices have to be established for goods and services in order to enable the equitable allocation of resources over different groups and communities in the society.

A *market* constitutes a spontaneously emerging mechanism that allows us to evade personal decision making with regard to resource distribution by assigning prices to each tradable item. In Sect. 8.9, we introduce a deterministic dynamical model of price–quantity interactions representing the basic market processes and show that, although a stationary point of balance between supply and demand *does* exist in such a dynamical system, it is never stable in the long run (Sect. 8.10). In the best case, the emerging instability results in economic growth, i.e., a concerted increase in supply, demand, and price. However, if supply systematically lags behind demand, a market failure will occur (Sect. 8.11). The market force of supply may then engender a compulsory expropriation of goods and resources, and the market force of demand may lead to a situation in which food and other commodities in short supply are distributed according to the eligibility of members of society for such benefits.

Finally, in Sect. 8.12, we discuss the idea that, if the economy consistently generates a shortage of consumer goods and the market mechanism fails, relations between different groups and communities in the society will weaken. Sensing the powerlessness and irrelevance of societal institutions that have proven themselves unable to provide adequate goods and service, and feeling a subsequent disgust for such institutions, citizens will disassociate from the civil society, even though their social identity may remain intact. When societal institutions disintegrate, people experience *uncertainty of means* when it comes to the problem of achieving their life goals. Living under conditions of prolonged scarcity and worsening poverty makes them more likely to evade freedom of thought and action, as it may potentially generate even more uncertainty, by provoking political upheaval, social unrest, and conflict. Blind submission to an authority promising the uninterrupted distribution of food and goods and an opportunity to achieve desirable life goals strengthens patron–client ties between a small elite who concentrate supreme power in their hands and the body of the people. The decisions of an autocratic leadership are released from regularized mechanisms of popular control and, eventually, from any external legal restraint.

## 8.3 Uncertainty of Ends and Uncertainty of Means

Social identity forms through identification with the most significant and closest members of a kin group (primarily parents and the groups and social categories to which they belong). Others, as we perceive them, contribute to the formation of our life goals through the processes of idealistic identification and defensive contra-identification. We aspire to the characteristics, values, and beliefs of those we consider benign and seek to dissociate from the characteristics of malign individuals. We strive to achieve our life goals with the help of *social institutions*, the stable, valued, recurring patterns of behavior that govern the actions of individuals within our community [223].

However, during periods of uncertainty and confusion (such as adolescence), when a person's sense of identity becomes insecure, expected aims and the aspired role in society also become uncertain. We shall refer to this type of indeterminacy as *uncertainty of ends*, following the discussion given in [96]. In the course of substantial and rapid cultural changes in the society, or during their formative years, people may experience uncertainty about which goals are most worth pursuing, since the ends to which they previously aspired may become irreconcilable. When multiple social goals that are recognized as desirable contradict each other and the aim itself is under contention, the social actor may not be able to determine what future result or outcome he should try for [96] (Fig. 8.1).

In the particular example describing uncertainty of ends as experienced by female students in Cameroon trying to manage their reproductive lives [96], the widely sanctioned social goal of bearing many children on a regular basis appears to be in conflict with other aspirations and intentions of educated women. The goals of beginning childbearing while still young, staying in school, and marrying

**Fig. 8.1** Uncertainty of ends. When a person's sense of identity becomes insecure, expected aims and aspired role in society also become uncertain


**Fig. 8.2** Uncertainty of means. If social institutions dissolve, people experience uncertainty about how to achieve their goals, when the means become only erratically and unpredictably available

before having children are incompatible: achieving any two of these would preclude attainment of the third. By postponing marriage and childbearing, a woman may remain in school and bear her children within marriage. If a women does stay on in school, the only way to bear a child before getting socially old is to give birth while still a student. But it is almost impossible to combine school and marriage, as there are substantially greater limitations on being a married student than on being a student mother. Women negotiate through various acceptable, rather than preferable, alternatives, and the possibilities they eventually pursue depend on chance and circumstance [96] (Fig. 8.2).

When social institutions dissolve, people find themselves in an environment of uncertainty of another kind—*uncertainty of means* [96]. Although the aims and the aspired role in society may remain clear, people experience uncertainty about how to achieve their goals in a situation when the means to those ends become only erratically and unpredictably available. We experience uncertainty of means whenever it is unclear which course of action will best achieve the result which we desire.

People quite often experience both types of uncertainty simultaneously. With the breakdown of a state, as exemplified by the former Soviet Union in [224], its citizens may find themselves facing the question of their current identity: who are now the citizens of the former state? As the stakes are high and the goals uncertain (i.e., there is uncertainty of ends), the first step would be to retain or claim control over valuable resources, territories, and other legacies of the former state. At the same time, a series of basic issues arise, which involve deciding how possible conflicts between and within the new states are to be resolved and order ensured, and there are no ready-made answers here (i.e., uncertainty of means).

## 8.4 Social Conformity as an Exopsychic Method of Decision Making

When experiencing double uncertainty—of ends and of means simultaneously—people would rather get by than optimize their behavior [96]. In order to take troubles in their stride, people need reassurance from somebody strong and benevolent who tells them that things will work out, even though the individual alone cannot understand or predict what lies ahead [53]—and the exopsychic methods of decision making are then used to alleviate the burden of responsibility for the choice to be made.

Perhaps, the most common and natural exopsychic method of decision making consists in listening to what others tell us. On the one hand, listening to others can be viewed as deliberate and temporary suspension of our own identity, after which one either accepts or rejects what was said [225]. On the other hand, listening to others, as we demonstrate below, is nothing but the archetypal metaphor of an interplay between two types of uncertainty, as in every game of chance conveying god's will as the outcome (Sect. 8.2).

The probability model of social conformity that we consider in the present chapter was proposed in [226].

We assume that an individual $i$, $i = 1, \ldots, N-1$, takes a decision on a particular issue, being guided by both his personal opinion and some expected attitude towards the issue expressed by another $N-1$ individuals in his entourage. It is important to mention that most decisions we make in the face of uncertainty are of a binary nature, requiring a simple aye/nay answer: To do, or not to do? To be, or not to be? To go, or not to go? The individual has to make a choice, whether to move to a particular state or not.

Generally speaking, there can be any number of alternatives, either compatible or incompatible, and each of them can be viewed separately or all together as a single option, since we shall rather discuss the intentions of the individual than the actual stay in a particular state. Furthermore, the individual may be not aware of whether the alternatives she aspires to are compatible or mutually exclusive, and therefore she may express an intention to choose all options (or a part of them) at once, even though she will not succeed in realizing any contradictory intentions.

To build a mathematical model describing the behavior of an individual under uncertainty, we suggest that she has to make a binary decision (aye/nay) on a particular issue. We introduce two simple probability assessments:

- Let $\alpha_j \in [0, 1]$—the a priori attitude of the individual toward the state—denote the probability that an individual is deliberately ready to move into a state (by answering 'aye') before communicating with her kin group.
- The final, a posteriori attitude, adopted after communication with the kin group, is expressed by the probability $P_j \in [0, 1]$ that the final decision is made to move into the state (by answering 'aye').

Of course, the person may not succumb to the influence of others, if he is independent or stubborn enough. Therefore, we also introduce another characteristic of the individual:

- The *degree of obstinacy* $\mu_j \in [0, 1]$ toward the particular issue determines the probability that in this case the individual will behave as a fully independent person.

If $\mu_j = 1$, then the individual is absolutely obstinate, refusing to change his opinion or chosen course of action, despite attempts to persuade him to do so. On the contrary, if $\mu_j = 0$, we have an absolute dependent individual, who may not have any particular a priori opinion on that issue. It is clear that the degree of obstinacy $\mu_j$ does *not* characterize the entire psychological makeup of the individual $j$, but is rather determined by the particular circumstances. One and the same individual can manifest himself as a purely obstinate individualist in one situation, but as a flabby conformist in another situation.

For an obstinate individual ($\mu_j = 1$), the a posteriori probability that the final decision is made to move into the state obviously coincides with the a priori probability:

$$P_j^{(\mu=1)} = \alpha_j. \tag{8.1}$$

We now define the a posteriori probability $P_j^{(\mu=0)}$ for the absolutely dependent individual.

Let us assume that the possible influence of the $i$th kin on the opinion of the $j$th individual is independent of any possible influence from any other community member and is determined by the number $\lambda_{ij} > 0$, for $i \neq j$, assessing the probability that the $j$th individual mimics the decision of the $i$th individual, whatever it may be—answering 'aye' to the conundrum question with the same probability $P_i$ as the individual $i$.

Then the total transition probability of the $j$th fully dependent individual to the new state will be

$$P_j^{(\mu=0)} = \sum_{i=1}^{N} \lambda_{ji} P_i \,, \tag{8.2}$$

where $N$ is the number of members in the team (including the individual $j$), and $\lambda_{ii} = 0$ as an absolutely dependent individual does not decide for himself. We assume that the conformity parameters $\lambda_{ji}$ introduced above are the elements of a stochastic matrix $\sum_{i=1}^{N} \lambda_{ji} = 1$, since we suppose that every other member of the team can influence the fully dependent individual. We shall call the matrix $\Lambda$ with elements $\lambda_{ij} \geq 0$ the *conformity matrix* of a community.

We now find the a posteriori probability for the selected $j$th individual in the face of uncertainty.

Applying the formula for the total probability, we obtain

$$P_j = \mu_j \alpha_j + (1 - \mu_j) \sum_{i=1}^{N} \lambda_{ji} P_i \,, \quad j = 1, \ldots, N \,. \tag{8.3}$$

The latter equation represents the model of behavior under the simplifications that we have adopted. Given the parameters $(\alpha, \mu, \lambda)$, the model should determine the a posteriori probability $P_j$ that the individual $j$ would answer 'aye' in the face of uncertainty.

Note that the model (8.3) does indeed represent an archetypal metaphor of a state of double uncertainty. In particular, the degree of indeterminacy in opinions of the kin group and in the a priori attitude of the individual (assessed in the model by the probabilities $P_i$ and $\alpha_j$, respectively) correspond to uncertainty of ends. Uncertainty of means is then represented by the elements of the stochastic matrix ($\lambda_{ji} > 0$) and the degree of obstinacy $\mu$ assessing the probability that the opinion of the $i$th individual would determine the a posteriori probability $P_j$ for the individual

$j$. By calculating $P_j$ as a product of the probabilities corresponding to the factors responsible for the two types of uncertainty, we assume that these factors are statistically independent.

Despite the inherent simplicity of the model (8.3), it reveals many important features of social conformity. In particular, under conditions of existential insecurity, the survival values emphasize collective discipline and group conformity over human diversity [12]. In the setting of the model (8.3), such a case corresponds to a decreasing degree of individual obstinacy $\mu_j \to 0$, since the a posteriori probability $P_j$ is determined primarily by the opinions of the kin group, but not by the a priori attitude of the individual himself. It is well known that conformity among insiders has deindividualizing effects, making people readier to support limitations on individual liberties for the sake of group discipline [227].

The parameter complementary to the degree of obstinacy, viz., $1 - \mu_j$, can be viewed as a personal *strength of identification with the group*: when people have a strong sense of group identification, i.e., $1 - \mu_j \to 1$, their self is defined at the level of the group, rather than at the level of their personal identity, and pursuing the group's interests becomes a direct and natural expression of self-interest [109]. Conversely, when existential threats recede, self-expression values tend to become more widespread, and people adopt increasingly independent conceptions of themselves [12] that corresponds to the increasing degree of obstinacy in our model $\mu_j \to 1$.

In a vector setting, (8.3) can be written in the form

$$\mathbf{P} = \boldsymbol{\alpha}\mathbf{M} + (\mathbf{1} - \mathbf{M})\Lambda\mathbf{P} \,, \tag{8.4}$$

where $\Lambda$ is the stochastic conformity matrix, $\mathbf{M}$ is the diagonal matrix of obstinacy, $\mathbf{1}$ is the unit matrix, and the vectors $\boldsymbol{\alpha}$ and $\mathbf{P}$ are the a priori and a posteriori probabilities, respectively.

It can be readily shown that the system of equations (8.4) possesses a unique and positive solution $\mathbf{P}$, for all $0 \le P_j \le 1$, viz.,

$$\mathbf{P} = \boldsymbol{\alpha}\frac{\mathbf{M}}{\mathbf{1} - (\mathbf{1} - \mathbf{M})\Lambda} \,, \tag{8.5}$$

provided that all $\mu_j \neq 1$, so that the inverse matrix

$$[\mathbf{1} - (\mathbf{1} - \mathbf{M})\Lambda]^{-1}$$

exists. It is also clear that the norm of the matrix on the right-hand side of (8.5) defined by its largest eigenvalue does not exceed 1, as the largest eigenvalue of a stochastic matrix equals 1 and the matrix $\mathbf{M}$ is diagonal. Otherwise, if there are some obstinate individuals, $\mu_s = 1$, the corresponding matrix $(\mathbf{1} - \mathbf{M})\Lambda$ becomes decomposable, and a subgroup of fully independent individuals arises in the team, such that their a priori probabilities coincide with their a posteriori probabilities, $P_s = \alpha_s$.

## 8.5 Crowd Behavior and Circular Reaction

Let us consider the case where all individuals in the model of social conformity are completely dependent, so that $\mu_i = 0$. In this case, the matrix $\mathbf{M} = 0$, and (8.4) takes the form

$$(\mathbf{1} - \Lambda)\mathbf{P} = 0 \,. \tag{8.6}$$

The latter equation has many solutions, since $\det(\mathbf{1} - \Lambda) = 0$ and therefore the matrix $(\mathbf{1} - \Lambda)$ is not invertible. The multiplicity of possible solutions of (8.6) is intuitively understandable: there is no single individual with more or less certain aspirations, within a group of absolutely dependent individuals. Literally speaking, the whole group forms a 'majority', and it is far from clear what this 'majority' wants.

However, the behavior of individuals in such a crowd is not chaotic at all. Quite the opposite, all individuals mimic each other, as $P_j = \sum_{i=1}^{N} \lambda_{ij} P_i$, so that all the a posteriori probabilities $P_j = P$ are equal and all individuals behave in the same way, rather like a flock of birds or a street crowd without a leader.

☞ As soon as people come together in a group or crowd, they may spontaneously begin to obey just one member of the group.

In essence, the behavior of such a crowd is unpredictable, although they act as a single unit, always moving to a new 'collective' state uniformly, with a probability $P$, whose value is arbitrary. However, the crowd may be driven out of such a state of collective indifference by any provocative action—formally, a small perturbation of any of the parameters $\mu_i$ (the degree of obstinacy) or $1 - \mu_i$ (the strength of identification with the group).

Once a single individual somehow finds his way, it is enough for the whole crowd to follow him. If a $k$ th individual with $\mu_k > 0$ suddenly appears in the crowd, the situation changes dramatically, as a single solution for all individuals $P_j = \alpha_k$ arises at once, where $\alpha_k$ is an a priori probability of the $k$ th individual. Thus, the behavior of the $k$ th individual is immediately mimicked by everybody in the crowd.

Since the stochastic matrix of conformity $\Lambda$ is not generally symmetric, its left and right eigenvectors are not equal. Moreover, the left eigenvector, satisfying

$$\boldsymbol{\pi} = \boldsymbol{\pi}\Lambda \,, \tag{8.7}$$

and thus belonging to the same eigenvalue 1, is not uniform, i.e., $\pi_i \neq P$. What we have here is a strictly positive probability vector $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_N)$, $\sum_{i=1}^{N} \pi_i = 1$, with $\pi_i > 0$, and it satisfies the condition of detailed balance:

$$\pi_i \lambda_{ij} = \pi_j \lambda_{ji} \,. \tag{8.8}$$

A time reversible *random walk* on an undirected weighted connectivity graph of individuals joining the crowd is described by the stochastic conformity matrix $\Lambda$. It is also a random walk if considered backwards, and given the walker at a number of states, it is not possible to determine, after executing the walk, which state came first and which came later [128].

☞ The behavior of a crowd without a leader is irrational and intransitive, being characterized by the loss of individual responsibility for the outcome.

Random walks can be considered as an archetypal model for diffusion and autoinfection processes in various communities [128]. It is known that, for a stationary discrete-valued stochastic process, the expected recurrence time to return to a state is the reciprocal of the probability of this state. The expected *recurrence time* of random walks defined on $\Lambda$, which indicates how long a random walker must wait to revisit the individual *i*, is inversely proportional to $\pi_i$, i.e.,

$$R_i = \frac{1}{\pi_i}\,. \tag{8.9}$$

The characteristic duration of recurrence times depends upon the connectivity structure encoded by the conformity matrix $\Lambda$, but not on its size. In general, short recurrence times correspond to well connected individuals, strongly influenced by the group identity. Conversely, individuals weakly connected to the group are characterized by longer recurrence times.

In particular, it can help us to understand the special psychological mechanism of *circular reaction*, one of the natural forms of collective behavior in a crowd [229]. It is known that certain emotional states experienced by people in a crowd can circulate, recurrently, triggering emotional flooding in people and maintaining the emotional unity of the crowd. As autoinfection plays the central role in such a recurrence mechanism, a random walk described by the conformity matrix $\Lambda$ may be considered as an appropriate model for such a psychological phenomenon.

It is important to mention that a random crowd is usually formed under the influence of a certain emotion, such as curiosity or anger [229]. As curiosity continuously pulls new members into the crowd, the circular reaction could start and maintain itself at full speed. The information of interest or a particular emotional story is continuously retold to newcomers, creating a self-sustaining whirl of emotion that catches more and more people, and attracts them to the crowd. The random crowd could then be transformed into an expressive crowd, in which everyone expresses joy or sorrow, anger or protest, simultaneously and unanimously.

## 8.6 Public Choice and the Role of Mass Media: When Does Propaganda Work?

The electorate often appears to be inconsistent, as their choice can cycle indecisively among alternatives—for instance, when it votes for minimum wages that create unemployment, and then for government programs meant to create jobs, as remarked in [230]. This cycling in choice occurs when the preferences of voters are smeared across the entire spectrum of possibilities, precisely as suggested by our model of social conformity under uncertainty.

It thus seems natural to start examining the model (8.4) by assuming a certain type of structure for the conformity matrix $\Lambda$.

We start with the case where individuals do not give any personal preference to each other. In such an even community,

$$\lambda_{ji} = \frac{1-\delta_{ji}}{N-1}\ , \tag{8.10}$$

where $\delta_{ji}$ is the Kronecker delta symbol such that $\delta_{ii} = 1$, but $\delta_{ij} = 0$, for $i \neq j$.

Then Eq. (8.4) take the form

$$P_j = \alpha_j\mu_j + \frac{1-\mu_j}{N-1}\sum_{i=1}^{N}(1-\delta_{ji})P_i\ , \quad j = 1, 2, \ldots, N\ , \tag{8.11}$$

and can be easily interpreted. The sum on the right-hand side of (8.11) is nothing else but the expectation of the number of individuals (apart from $j$) who have already answered 'aye' (moved to the state of interest). Thus, the individual is guided by his own a priori attitude toward the state (determined by the a priori probability $\alpha_i$) and the fraction of other members of his community that have already passed to the state of interest.

The system (8.11) has the following analytic solution:

$$P_j = \frac{N-1}{N-\mu_j}\alpha_j\mu_j + N\frac{1-\mu_j}{N-\mu_j}f\ , \tag{8.12}$$

where we define

$$f \equiv N\sum_{i=1}^{N}P_i = \frac{\sum_{i=1}^{N}\alpha_i\mu_i/(N-\mu_i)}{\sum_{i=1}^{N}\mu_i/(N-\mu_i)}\ ,$$

the expected fraction of individuals who have already decided to move to the state (by answering the question positively).

It is important to mention that, for large enough $N \gg 1$, (8.12) can be simplified further to obtain

$$P_j = \alpha_j \mu_j + (1 - \mu_j) f \ , \quad f = \frac{\sum_{i=1}^{N} \alpha_i \mu_i}{\sum_{i=1}^{N} \mu_i} \ . \tag{8.13}$$

Let us suppose that the electoral programs of two presidential candidates disagree about some important issue, for example, *whether the government should raise taxes by n % or keep them unchanged, at the same level as before*.

Suppose that one candidate wants to raise taxes (so that his a priori attitude toward this issue is definitive $\alpha_1 = 1$) and his degree of obstinacy $\mu = \mu_1$, and suppose that the other candidate promises to keep the taxes unchanged, so that $\alpha_2 = 0$ and his degree of obstinacy is $\mu = \mu_2$. We also suppose that the electorate is characterized by $\mu = 0$, since people are expected to take both proposals seriously and keep an open mind. However, the range of a priori opinions in the electorate about the need to raise taxes may be quite wide. Indeed, all the $\alpha_j$ are different.

☞ Submissive individuals would easily give up their old beliefs in favor of the position of the more determined candidate.

From the simplified equation (8.13), it then follows that, for $\mu_j = 0$, the actual range of opinions does not play any role, since submissive individuals would easily renounce their old beliefs, provided that one of the candidates is more decisive than the other.

It is obvious that the electorate will then divide into two parts with respect to the given issue, in the ratio $\mu_1/\mu_2$: a fraction $\mu_1/(\mu_1 + \mu_2)$ will follow the first candidate, and the rest, i.e., a fraction $\mu_2/(\mu_1 + \mu_2)$, will follow the other candidate.

In the face of uncertainty, most of the electorate will join the more self-confident and decisive leader, with a higher degree of obstinacy, disregarding his actual political platform.

☞ In the face of uncertainty, the actual political platform of a leader does not play an important role in determining the outcome of elections, since people would rather vote for the more self-confident and decisive leader.

Let us now examine the role and influence of the mass media on campaigns and elections. We consider the process of taking decisions under uncertainty in an elementary social unit like a family or a kin group.

Let us consider a family consisting of $N_\mathrm{f}$ people and a 'television set' representing the mass media in our model. The television set will be characterized by an absolute degree of obstinacy ($\mu = 1$) and a definite political position, so that $P_0 = \alpha_0 = 1$. We do not distinguish between the other community members, assuming that they are characterized by some a priori attitude $\alpha_j = \alpha$ and a certain degree of obstinacy $\mu_j = \mu < 1$.

After the elementary calculations, the formula (8.12) yields $f$, the fraction of voters in the family that are ready to vote following the suggestion of mass media,

$$f = \frac{1 + N_\mathrm{f}\alpha\mu - \mu}{1 - \mu + N_\mathrm{f}\mu}. \tag{8.14}$$

We assume that the propaganda campaign is organized in the form of $n \gg 1$ sequential stages, during which the fraction of persuaded individuals $f^{(n-1)}$ achieved at the end of each previous stage, becomes the a priori probability $\alpha$ for input in the next round of the campaign:

$$f^{(n-1)} = \alpha^{(n)} . \tag{8.15}$$

The recurrence relation describing the *efficiency of a propaganda campaign* (8.15) lasting for $k$ steps has the following solution:

$$f^{(k)} = 1 - \left[1 - \alpha^{(1)}\right]\left[\frac{N_\mathrm{f}}{(1-\mu)/\mu + N_\mathrm{f}}\right]^k . \tag{8.16}$$

It is obvious that, as the degree of obstinacy of the community members $\mu < 1$ and the number of members of the kin group is not very large, i.e., $N_\mathrm{f} = O(1)$, the fraction in brackets in (8.16) will always be less than unity, so that $f^{(k)} \to 1$ inevitably, as $k \to \infty$.

☞ The mass media always win in a small enough community of docile and cooperative individuals.

Nevertheless, in the case of absolutely obstinate individuals, where $\mu = 1$, it is obvious that the fraction in the last brackets of (8.16) equals 1 for any number $N_f$, and therefore the propaganda campaign will fail to achieve its goals:

$$f^{(\infty)} = \alpha \; . \tag{8.17}$$

However, individuals that refuse cooperation with others because they are stubborn loners are unlikely to join any community.

☞ Propaganda is inefficient among stubborn loners.

Another strategy to reduce the influence of propaganda consists in increasing the number of community members to $N_f \gg 1$. It then follows from (8.16) that the fraction in brackets tends to unity for any value $\mu > 0$, and then the propaganda campaign once again fails to achieve its goals (8.17).

☞ Propaganda would not work on large enough groups of cooperating individuals.

For instance, all large-scale orchestrated trolling campaigns to promote pro-Russian propaganda in social networks launched during the last decade [231] largely failed, despite the best efforts of state-supported troll armies, the Internet groups linked to the Federal Security Service of the Russian Federation, with the practically unlimited resources and surge capacity that had been made available.

The proposed model of social choice based on social conformity in the face of uncertainty implies a serious questioning of the 'we' as political collectives. Apparently, there is no non-propagandistic, non-dictatorial way to aggregate different individual preferences and fuse them into one set of super-preferences, unless the individuals have identical preferences or they are unanimous [230]. But, as we have seen, unanimity of choice is possible only within small and closely knit groups like a couple, a family, or a few friends—and only because one individual in such a close group is the 'dictator' or the leader with unquestioned authority.

We may make one more remark about a popular belief, namely that *propaganda does not work on well-informed people*.[2] In our opinion, the reason for this confusion lies in a misunderstanding of the profound difference between propaganda and deceit (disinformation). As a matter of fact, propaganda would work just as well on well-informed and highly educated people as on uninformed, ignorant individuals with philistine tastes, simply because it has nothing to do with information or knowledge.

Propaganda does not start with distortion of any kind of information, but rather emerges in response to a common atmosphere of uncertainty and insecurity, such as the one arising under a threat of terror attacks or following explosions with multiple

[2]For example, this point of view is often attributed to Jacques Fresco, an American futurist and self-appointed social engineer.

civilian casualties. Submission to a decisive opinion by a self-confident leader in the face of uncertainty will always be the deliberate choice of the socialized majority.

## 8.7 What is the Maximum Size of a Working Team?

In the previous section, we showed that the size of a group whose members have similar preferences is naturally limited. It is impossible to maintain efficient social control over large groups of individuals, regardless of the degree of openness of the communication channels existing between them.

A related phenomenon known as *indivisibility of responsibility* was reported in [228]. When a group of individuals are involved in making a decision together or performing a joint task, the responsibility is expected to be evenly divided between them. However, if the number of actors involved increases beyond two, the individual responsibilities of the first two actors are not reduced as might be expected, but rather remain constant for each individual in the group, regardless of its size.

Consider a kin group or a working team headed by a chief for whom $\alpha_1 = 1$ and $\mu_1 = 1$, while for the other $N$ team members, we assume a *degree of diligence* $\alpha_j = \alpha < 1$ and a *degree of independence* $\mu_j = \mu < 1$, $j = 1, \dots, N$.

Using the exact solution (8.12), we may obtain the expected fraction of individuals expressing solidarity with their leader as

$$f = \frac{N - \mu + \alpha\mu(N-1)^2}{N - \mu + \mu(N-1)^2} \, . \tag{8.18}$$

This formula shows that, for large enough groups ($N \gg 1$), the team becomes practically unmanageable, since then $f \to \alpha$, so that the mutual influence of individuals on each other suppresses any opportunity for their chief to control and manage the group by imposing his preferences on others, except for the case where all team members are unanimous ($\alpha = 1$).

For the given degrees of diligence $\alpha$ and independence $\mu$, what is the maximum size of a team in which the fraction of members ready to follow the instructions of their leader is no less than $Q$?

To answer this question, one must solve the following inequality for the given values of $\alpha$, $\mu$, and $Q$:

$$(Q - \alpha)\mu(N-1)^2 - (1-Q)(N-1) - (1-Q)(1-\mu) \leq 0 \, . \tag{8.19}$$

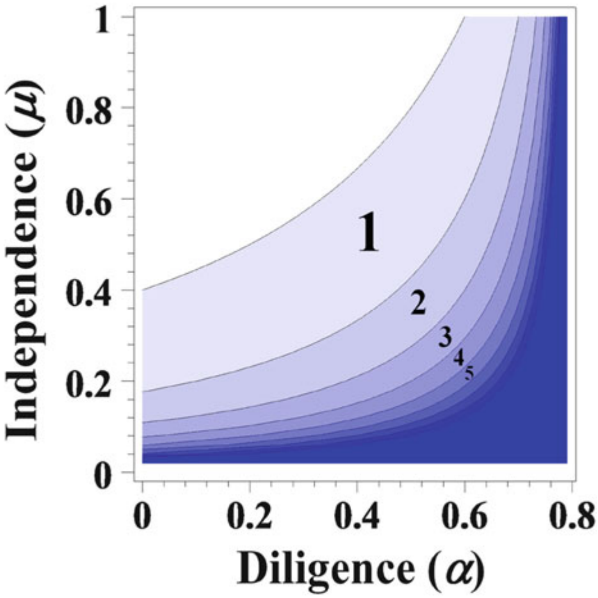


**Fig. 8.3** The maximum size of a team $N$, for the given degrees of diligence $\alpha$ and independence $\mu$, in which the fraction of its members that follows instructions from their leader is no less than $Q = 80\,\%$

This inequality holds for any $N$, if $\alpha \geq Q$, when the team is ready to do more than the minimum required of them. However, the most typical situation is when the degree of diligence in team members is less than required, i.e., $\alpha < Q$.

Figure 8.3 plots the maximum size of the team which can remain under control with 80 % efficiency as a function of the degree of independence ($\mu$) and the degree of diligence ($\alpha$) of its members. For instance, in the case of moderately independent ($\mu = 1/2$) and moderately diligent ($\alpha = 1/2$) individuals, the solution of (8.19) shows that the team with the chief should not exceed 2 people, i.e., the chief himself and a single subordinate.

However, the maximum size of the team for the given level of control efficiency would grow as the degree of independence of its members decreases, but the degree of diligence increases. It is remarkable that the maximum size of the group is unbounded when all group members are unanimous and diligent ($\alpha \to Q$), that is, when all of them are flabby conformists, i.e., absolutely dependent individuals.

Although the actual group could be of virtually any size, some team members may accept no responsibility for the decisions taken and the joint tasks performed. If the number of individuals in the team is increased arbitrarily, the influence of the leader, as we have already seen, will fall, and, for sufficiently large $N$, it may be that most of the staff will not participate in the joint work. The proper organization of a working team requires splitting it into a nested hierarchy of organization units.

## 8.8 Mastering Social Conformity in the Face of Uncertainty: *Divide et Impera*

The dictum *Divide et impera*[3] is common in politics. In order to gain and maintain power over a group of subjects, populations, or factions of different interests that might be able to oppose rule if united, the sovereign should endeavor with every art to divide them and to prevent smaller power groups from linking up. The use of the 'divide and rule' strategy has been ubiquitous throughout the history of mankind.

It is remarkable, however, that the commonly heard complementary wording, in the subjunctive mood, *Eritis insuperabiles, si fueritis inseparabiles*[4]—never works in history! Quite the contrary, groups that remain vigilant about maintaining their unanimity and guarding against any schism between their members are unable to take decisions efficiently, remain undecided on practically every occasion, and inevitably fall apart. Hence the subjunctive mood. It may be that another expression would be more appropriate: *Eritis dubius, si fueritis inseparabiles.*[5] In the present section, we try to understand the statistical basis that may underlie these principles.

Under uncertainty, the principle of maximum entropy favours distributions that allow for more microscopic states; these are the ones that are most likely to be observed in the long run. It is therefore interesting to determine the most likely configurations of the parameters $\alpha_j$ and $\mu_j$, $j = 1, \dots, N$, for which the entropy

$$H = -\sum_{j=1}^{N} \left[ P_j \log_2 P_j + (1 - P_j) \log_2 (1 - P_j) \right] \tag{8.20}$$

attains its maximum over a group of $N \gg 1$ individuals whose interactions are described by the simplified model

$$P_j = \alpha_j \mu_j + (1 - \mu_j) f \ , \quad f = \frac{\sum_{i=1}^{N} \alpha_i \mu_i}{\sum_{i=1}^{N} \mu_i} \ .$$

The problem of iterative maximization of the objective function (8.20), subjected to the obvious constraints $\alpha_j, \mu_j \in [0, 1]$, is a convex optimization problem that can be solved numerically, for any finite $N \gg 1$.

☞ In the face of uncertainty, a group attempting to maintain its unanimity might be unable to take any decision.

---

[3] *Lat.* Divide and rule.

[4] *Lat.* You would be insuperable if you were inseparable.

[5] *Lat.* You would be dubious if you were inseparable.

Numerical calculations show that the entropy attains the maximum value $H = N$ bits for the configuration $\mu_j = 1$ and $\alpha_j = 0.5$, uniformly for all individuals $j$ in the group. Therefore, in the face of uncertainty, the entire group would likely be in the same state, characterized by the highest degree of hesitation and indecision ($\alpha = 0.5$) with regard to every issue and, at the same time, with the maximum degree of obstinacy $\mu = 1$, so that no group member will ever change his mind.

Let us now consider a situation where someone is able to get people divided in opinion over some particular problem, so that all kinds of opinions about the problem are present in the society—each individual has her own view, different from the others. We can then place individuals in ascending order with respect to their a priori value

$$\alpha_i = \frac{i-1}{N-1} ,$$

and solve the problem of maximizing the entropy (8.20) once again, subject to the constraints $\mu_j \in [0, 1]$. The maximum entropy of $H = N$ bits is attained for another uniform configuration, where all $\mu_i = 0$, i.e., where all group members are absolute conformists.

☞ Dividing a society into factions of different interests will most likely result in the emergence of absolute social conformity in society members.

In the face of uncertainty, the highest degree of social conformity is established as soon as community members are divided, for instance, between two political parties—a ruling party and an opposition party. For simplicity, we assume that all community members conform equally, so that $\mu_j = \mu \geq 0$, and the community is divided into two equal parts: the supporters of the ruling party (for whom all $\alpha_j = 1$) and their opponents (for whom all $\alpha_j = 0$), so that $f = 0.5$ by default. The approximate equation then yields

$$P_j = \frac{1 \pm \mu}{2} , \tag{8.21}$$

where the plus sign holds for those who support the ruling party and the minus sign for those who support their opponents. The entropy function is then

$$H = -N\left[\frac{1+\mu}{2}\log_2\left(\frac{1+\mu}{2}\right) + \frac{1-\mu}{2}\log_2\left(\frac{1-\mu}{2}\right)\right] , \tag{8.22}$$

and its maximum value is attained for $\mu \to 0$ (see Fig. 8.4).

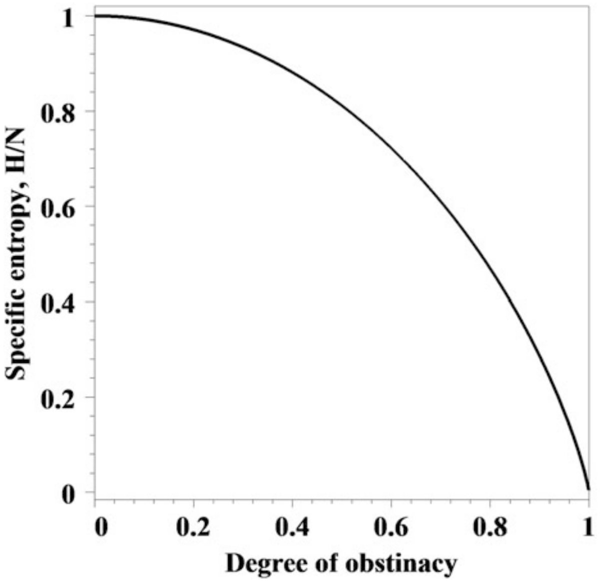


**Fig. 8.4** The specific entropy $H/N$ (bits per capita) vs. the degree of obstinacy ($\mu$) in a society evenly divided between two political parties

☞ Unanimity in one faction of a group is possible only in the face of another faction with opposing interests.

When all $\mu_j = 0$, the group turns into a crowd of the kind already discussed. In particular, we showed that the behavior of such a crowd might be quite unpredictable, despite the fact that all individuals mimic each other and the crowd might thus act as a single unit ($f = 1$). The case $f = 1$ corresponds to the situation of absolute unanimity in the group, which is observed in the model (8.13), provided that

$$\sum_{i=1}^{N} \alpha_i \mu_i = \sum_{i=1}^{N} \mu_i . \tag{8.23}$$

Since $\alpha_i \leq 1$, this is equivalent to the system of equalities

$$(1 - \alpha_i)\mu_i = 0 , \quad i = 1, \ldots, N . \tag{8.24}$$

The trivial solution of these equations ($\mu_i = 0$) is compatible with the most likely composition of a divided society under uncertainty. However, another possible solution of (8.24) is $\alpha_r = 1$ for members of the ruling class, $r$.

Thus, society is divided into two subgroups, both consisting of absolutely unanimous, like-minded people. The first subgroup (with $\alpha_r = 1$) is quite satisfied with the present state of society, and the other subgroup obediently follows them because of their absolute conformity ($\mu = 0$). Under uncertainty, such a configuration is a statistical equilibrium state of the society which can exist stably for quite a long time.

## 8.9 Market as an Impersonal Method of Decision Making

We have already discussed the fact that the preferences of a real life decision-maker change continuously over time. Real individuals striving to survive under uncertainty are more present-biased, attributing lower values to the future than the fully rational *Homo economicus*.

Under hyperbolic discounting of time, which is natural under uncertainty, valuations fall very rapidly for small delay periods, but then fall off slowly for longer delay periods. As a result, an individual's choices and decisions inevitably seem unjust and inconsistent over the differently valuated periods of time. Hence, the problem of equitable allocation of resources and determination of adequate prices for goods and services has no stable long-term solution in principle. As nobody wants to be blamed for causing negative and painful experiences to happen to others, an impersonal market mechanism is used for assigning prices to each tradable item or service.

In the present section, we introduce and study a simple deterministic dynamical model of interactions between the actual market price $\mathcal{P}(t)$, the supply $\mathcal{S}(t)$, and the demand $\mathcal{D}(t)$, all treated as functions of time. In particular, we demonstrate that very complex and volatile behavior is already possible in a single commodity market, in which both types of uncertainty (of means and of ends) can be identified: uncertainty about the current economic trend (uncertainty of ends) is exacerbated by uncertainty about the ways to achieve economic growth and gain profit (uncertainty of means).

Surprisingly, most economic studies have traditionally been preoccupied with equilibrium analysis, including considerations that can be described as static (an *equilibrium* strategy), statistical (fluctuations about an *equilibrium* state), game theoretic (Nash *equilibrium*, in which each player is assumed to know the *equilibrium* strategies of the other players), or psychological (expectations about *equilibrium*), while a full dynamical analysis of the principal market interactions that could be useful for a correct understanding of the specific conditions in which an equilibrium state is possible in the dynamical system of price–quantity interactions has attracted considerably less attention.

However, it is important to mention that the idea of market forces leading an economy to a point of equilibrium would be meaningless if there were no certitude about the existence of such a point. We think that a dynamical analysis can help to get an insight into the nature of price–quantity interactions and their occasional failures, causing collapse in the markets.

The problem of the stability of a supply-and-demand *equilibrium* state in single and multiple commodity markets was already discussed in terms of differential equations long ago [232, 233]. In particular, the cobweb models [234] explaining why prices might be subject to periodic fluctuations in certain types of markets have since become the prevailing theoretical tools for studying price–quantity dynamics.

In the present section, we study the possible types of dynamical behavior that might occur in a single commodity market dynamical system. We introduce the model by writing down linear differential equations expressing the universally accepted wisdom about the relations between supply, demand, and price.

> If at any price, demand exceeds supply, the price will rise. If supply exceeds demand, the price will fall [233].

It follows that the time derivative $\mathrm{d}\mathcal{P}(t)/\mathrm{d}t$ of the price should satisfy the following linear differential equation:

$$\frac{\mathrm{d}\mathcal{P}(t)}{\mathrm{d}t} = \gamma_{\mathrm{P}}\big[\mathcal{D}(t) - \mathcal{S}(t)\big] \,. \tag{8.25}$$

Here, the parameter $\gamma_{\mathrm{P}} > 0$ defines the rate at which the price reacts to current variations in supply and demand. In (8.25), we have assumed that all quantities and times are expressed in the proper units.

It is natural to suggest that the actual amount of supply $\mathcal{S}(t)$ would increase if the good or service is in demand. Let us combine the cost of producing the good and the desired rate of profit into a single *characteristic seller's price* denoted by $s$. It is clear that in a sustainable market the actual market price $\mathcal{P}(t)$ should exceed the characteristic seller's price $s$, in order that the actual amount of supply $\mathcal{S}(t)$ should increase.

> Supply increases for goods and services in demand. If the market price exceeds the characteristic seller's price, supply will rise. If the characteristic seller's price exceeds the market price, supply will fall.

Furthermore, let us suggest that the characteristic seller's price $s$ changes much more slowly than the actual values of the price, supply, and demand. Effectively, we assume it to be constant over time. Then, the corresponding linear differential equation for the rate of supply reads as follows:

$$\frac{\mathrm{d}\mathcal{S}(t)}{\mathrm{d}t} = \gamma_{\mathrm{S}}\big[\mathcal{P}(t) - s\big] + \gamma_{\mathrm{S}}\frac{\nu}{2}\mathcal{D}(t) \,. \tag{8.26}$$

Here, the parameter $\gamma_S > 0$ defines the characteristic time scale for the reaction of the supply to current variations in price and demand. Another parameter $(\nu/2 > 0)$ characterizes the extent to which supply is reinforced by expectations concerning the demand. The important role played by such reinforcement for the appearance of a stationary point in the dynamical system of a single-commodity market will become clear in the forthcoming discussion. For simplicity, we suggest that $\nu/2$ is constant over time and use it as a control parameter, representing the long-term behavior of dynamical solutions in the model.

Finally, the actual demand for a commodity certainly depends on whether the actual market price $\mathcal{P}(t)$ is affordable to buyers and the extent to which the supplied commodity fits their needs and tastes.

In order to describe those needs, tastes, and financial opportunities, which together determine the buyer's willingness to purchase the commodity, we introduce a *characteristic buyer's price*, denoted by $d$. It is then clear that the actual demand $\mathcal{D}(t)$ will rise if the characteristic buyer's price $d$ exceeds the actual market price $\mathcal{P}(t)$, but fall otherwise. We suggest that the value of $d$ is constant over time.

Furthermore, in its turn, a plentiful supply should also reinforce demand. Indeed, people may buy a good simply because it is offered to them; the good may be widely and intrusively advertised in the media, already purchased by friends and neighbors, or simply be affordable and available, while the commodity that might better fit the buyer's needs may be rather expensive or inaccessible.

If the characteristic buyer's price exceeds the market price, demand will rise. If the market price exceeds the characteristic buyer's price, demand will fall. Supply can reinforce demand.

The last differential equation in the proposed dynamical model therefore reads as follows:

$$\frac{\mathrm{d}\mathcal{D}(t)}{\mathrm{d}t} = \gamma_D\big[d - \mathcal{P}(t)\big] + \gamma_D \frac{\nu}{2}\mathcal{S}(t) \ , \tag{8.27}$$

in which $\gamma_D > 0$ defines the characteristic time scale on which the demand changes in response to current variations in price and supply.

For the sake of clarity and simplicity in our model, we have suggested in (8.27) that the strength $\nu/2 > 0$ with which supply would reinforce demand is the same as the reciprocal strength with which demand would reinforce supply. The parameter $\nu/2$ introduced in this way is nothing else but the feedback coefficient in a symmetric positive feedback loop describing the mutual reinforcement of supply and demand. Indeed, assuming a certain degree of asymmetry in the model, we can introduce a special parameter (say, $\nu'/2$), but this does not affect the qualitative conclusions we shall reach concerning the behavior of possible solutions.

It is important to mention that the characteristic time scales of the modeled processes may all be different, i.e., $\gamma_{\mathrm{P}} \neq \gamma_{\mathrm{S}} \neq \gamma_{\mathrm{D}}$, although they should be defined in relation to each other. For example, one can define the unit of time in our model to be associated with the characteristic time scale of price variations, assuming that $\gamma_{\mathrm{P}} \equiv 1$, and then determine the characteristic time scales of supply and demand variations ($\gamma_{\mathrm{D}}$ and $\gamma_{\mathrm{S}}$) in relation to $\gamma_{\mathrm{P}}$.

Without loss of generality, but for the sake of clarity, we may suppose that the characteristic time scales of price and supply variations are essentially the same, i.e.,

$$\gamma_{\mathrm{P}} = \gamma_{\mathrm{S}} = 1 , \tag{8.28}$$

but the (now relative) rate of variation $\gamma_{\mathrm{D}}$ in the demand may differ from them. This choice is equivalent to the main proposition of the cobweb model [234], which is based on a time lag between supply and demand decisions and predicts periodic fluctuations in certain types of markets. In particular, variations in the level of demand lag behind variations in price and supply, provided that $\gamma_{\mathrm{D}} < 1$, but run ahead of them when $\gamma_{\mathrm{D}} > 1$. In the following, we consider $\gamma_{\mathrm{D}}$ as another control parameter that defines the nature of possible solutions of the model.

## 8.10 The Law of Supply and Demand is a Saddle Point

In the present section, we study the possible solutions of the system of three dynamical equations (8.25)–(8.27), for positive values of the parameters $\nu/2 > 0$ and $\gamma_{\mathrm{D}} > 0$, in the non-negative quadrant $\mathcal{P}(t) \geq 0$, $\mathcal{S}(t) \geq 0$, $\mathcal{D}(t) \geq 0$.

First of all, note that, for $d > s$ and $\nu > 0$, the unique stationary state is attained in the model, precisely as suggested by the *law of supply and demand* [235], i.e., when the quantity demanded by consumers is correctly balanced by the quantity that sellers supply:

$$\mathcal{D}_* = \frac{d-s}{\nu} = \mathcal{S}_* . \tag{8.29}$$

The market clears at the equilibrium price

$$\mathcal{P}_* = \frac{d+s}{2} . \tag{8.30}$$

However, it is obvious from (8.29) that the stationary point is not determined if supply and demand do not reinforce each other, i.e., if $\nu \to 0$, despite the fact that the equilibrium price may still be defined (8.30).

☞ The quantity demanded by consumers is correctly balanced by the quantity that sellers supply if and only if supply and demand reinforce each other.

Let us now suppose that the equilibrium state (8.29)–(8.30) exists in the model, i.e., $d > s$ and $\nu > 0$, and we can analyze the behavior of the dynamical system (8.25)–(8.27) in the vicinity of this stationary point.

It is then convenient to introduce the deflections of price, demand, and supply from their equilibrium values, viz.,

$$\tilde{\mathcal{P}}(t) = \mathcal{P}(t) - \mathcal{P}_* , \quad \tilde{\mathcal{D}}(t) = \mathcal{D}(t) - \mathcal{D}_* , \quad \tilde{\mathcal{S}}(t) = \mathcal{S}(t) - \mathcal{S}_* , \tag{8.31}$$

and consider them as the components of a vector

$$\Phi(t) = \left\{\tilde{\mathcal{P}}(t), \tilde{\mathcal{D}}(t), \tilde{\mathcal{S}}(t)\right\} . \tag{8.32}$$

The system of dynamical equations (8.25)–(8.27) can be written in matrix form:

$$\dot{\Phi}(t) = \mathbf{W}\Phi(t) , \tag{8.33}$$

where

$$\mathbf{W} = \begin{pmatrix} 0 & -1 & 1 \\ \gamma_S & 0 & \gamma_S \nu/2 \\ -1 & \nu/2 & 0 \end{pmatrix} . \tag{8.34}$$

The trace $J_1 = \mathrm{Tr}(\mathbf{W})$ of the matrix (8.34) is zero, so its eigenvalues sum to zero, i.e., $\sum_{i=1}^{3} \lambda_i = 0$. The eigenvalues of (8.34) are the roots of the cubic polynomial

$$\det(\lambda \cdot 1 - \mathbf{W}) = \lambda^3 + J_2\lambda - J_3 = 0 , \tag{8.35}$$

in which

$$J_2 = 1 + \gamma_S - \frac{\gamma_S \nu^2}{4} , \quad J_3 = \gamma_S \nu . \tag{8.36}$$

The signature of the discriminant

$$\Delta = -4J_2^3 - 27J_3^2 \tag{8.37}$$

of the polynomial (8.35) gives information about the nature of the eigenvalues of the matrix (8.34). The following three situations are possible:

1. If the value of the discriminant is positive, i.e., $\Delta > 0$, the matrix (8.34) has three distinct real eigenvalues such that $\lambda_1 = -(\lambda_2 + \lambda_3)$. There are either two positive eigenvalues and one negative, or one positive eigenvalue and two negative, so that the corresponding solution is a *saddle point*, being unstable either in two directions or in one direction.
2. If $\Delta = 0$, then the matrix (8.34) has a multiple eigenvalue $\lambda_2 = \lambda_3$, $\lambda_1 = -2\lambda_2$, and all eigenvalues are real. Such a solution is also a *saddle point*.

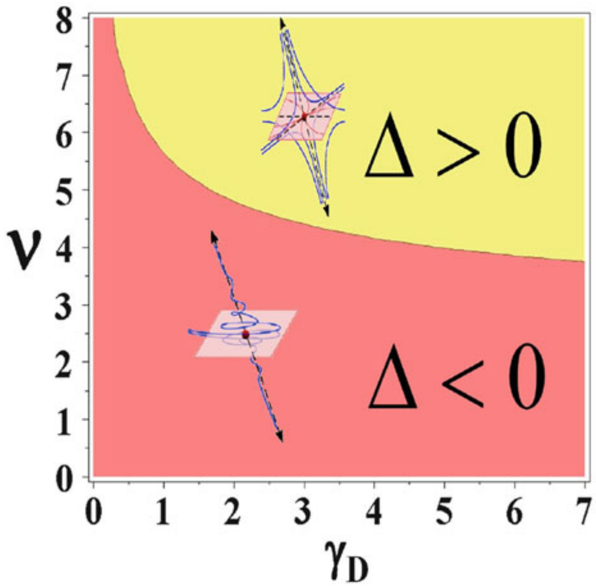


**Fig. 8.5** The sign of the discriminant (8.37) as a function of two parameters $\nu > 0$ and $\gamma_D > 0$. The stationary solution of the dynamical model is a saddle point when $\Delta \geq 0$, and a spiral saddle point when $\Delta < 0$

3. Finally, if $\Delta < 0$, the matrix (8.34) has one real eigenvalue and two non-real complex conjugate eigenvalues ($\lambda_2^* = \lambda_3$) responsible for periodic fluctuations in price and quantity as time goes by. It follows that the real parts of the eigenvalues are related by $\text{Re}(\lambda_{2,3}) = -\text{Re}(\lambda_1)/2$. Then, the corresponding solution is a spiral *saddle point*.

In any case, the unique stationary point (8.29) corresponding to the law of supply and demand in the dynamical market model is not a local extremum (a stable focus), but always has stable and unstable directions, and hence a saddle point. The signature of the discriminant (8.37) and the nature of the corresponding solutions is shown on the phase diagram in Fig. 8.5.

☞ The law of supply and demand is a saddle point of the dynamical market system and that is why economic growth is eventually possible.

The instability of the unique stationary solution (8.29) corresponding to the law of supply and demand obviously arises from the mechanism of mutual reinforcement of supply and demand taken into account by our model. Due to this mutual reinforcement, any quantity of goods that may be demanded by consumers will sooner or later be met by sellers, and any quantity of goods that sellers supply will eventually be consumed by customers. The emerging instability would result in economic growth, i.e., the consentaneous increase of supply, demand, and price.

☞ Insufficient mutual reinforcement between supply and demand would cause shifts over time between periods of boom and periods of recession.

It is also remarkable that fluctuations in price around the saddle point corresponding to the law of supply and demand may already emerge in the fully deterministic dynamical model of a single commodity market (when $\Delta < 0$), provided that the degree of mutual reinforcement between supply and demand is insufficient.

## 8.11 In Search of Persistent Economic Growth

Economic growth is the increase in the inflation-adjusted market value of the goods and services produced by an economy over time [235]. For economic growth to take place, the stationary point of the law of supply and demand should have at least one unstable direction that would foster a consentaneous increase of supply, demand, and price, i.e.,

$$\left\{\tilde{\mathcal{D}}(t), \tilde{\mathcal{S}}(t), \tilde{\mathcal{P}}(t)\right\} > 0 ,$$

from the stationary point, which is a saddle point of the single commodity dynamical market system.

If the matrix $\mathbf{W}$ of the dynamical system (8.33) is small enough and essentially depends upon just two parameters, its eigenvalues and corresponding eigenvectors can be readily calculated.

The eigenvalues with a negative real part are not important for our considerations, since their contributions rapidly decay with time and are not therefore related to long-term economic growth. Let us define the eigenvalue with a positive real part as $\lambda_1$. This value can be viewed as the growth rate of the economy in our dynamical model. In particular, it can be shown that

$$\lambda_1 = Z - \frac{1}{3}\frac{J_2}{Z} , \quad Z \equiv \frac{1}{2}\sqrt[3]{36J_3 + \sqrt{1296J_3^2 + J_2^3}} , \tag{8.38}$$

$$\lambda_{2,3} = -\frac{1}{2}\lambda_1 \pm \frac{\mathrm{i}\sqrt{3}}{2}\left(Z + \frac{1}{3}\frac{J_2}{Z}\right) . \tag{8.39}$$

If economic growth occurs, the trajectory of a dynamical solution should depart from the stationary point in a positive direction and stay indefinitely within the non-negative quadrant $\left\{\tilde{\mathcal{D}}(t), \tilde{\mathcal{S}}(t), \tilde{\mathcal{P}}(t)\right\} > 0$ of price–supply–demand space, so that the first eigenvector $\mathbf{v}_1$ belonging to the eigenvalue $\lambda_1$ has to be positive. The corresponding values of the control parameters can be found easily.

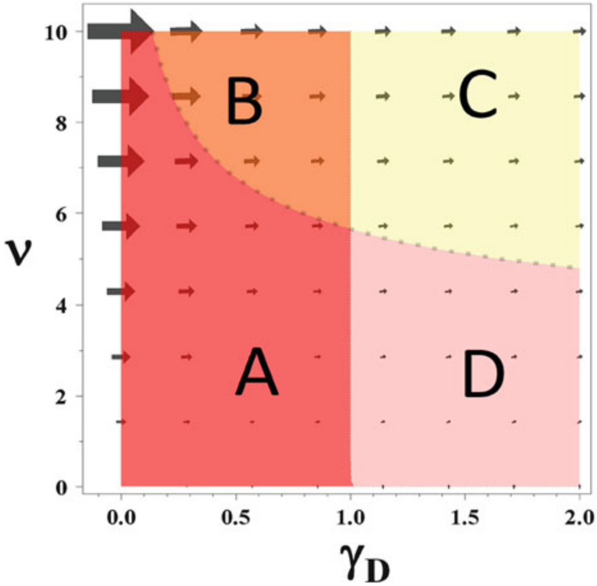


**Fig. 8.6** Dynamical regimes in the single-commodity market model: **A** ($\gamma_D < 1$, $\Delta < 0$), **B** ($\gamma_D < 1$, $\Delta > 0$), **C** ($\gamma_D > 1$, $\Delta > 0$), **D** ($\gamma_D > 1$, $\Delta < 0$). *Arrows* indicate the gradient and magnitude of the positive eigenvalue $\lambda_1$ with respect to the control parameters $\gamma_D$ and $\nu$

We conclude the analysis of the market model with the phase diagram shown in Fig. 8.6, which explains comprehensively the behavior of possible solutions of the dynamical model for different values of the control parameters ($\gamma_D$ and $\nu/2$).

The arrows in Fig. 8.6 indicate the gradient and magnitude of the positive eigenvalue $\lambda_1$ for the different values of $\gamma_D$ and $\nu$. The phase diagram Fig. 8.6 comprises four zones (the dynamical regimes in the single-commodity market model) marked by capital letters:

**Case A.** For $\gamma_D < 1$ and $\Delta < 0$, a dynamical solution will remain indefinitely within the non-negative quadrant of price–supply–demand space, and economic growth will therefore be persistent. The rate of variation in demand $\gamma_D < \gamma_{P,S} = 1$ lags behind the rates of variation in the price and supply, so that *supply always runs ahead of demand*.

Customers are permanently offered new products, improved services, and lucrative options, which appear so often that consumers have almost no time to make an informed decision regarding demand. They always have to choose between many different goods and options available to them on the market, all at the same time. In this dynamical regime, price and quantity would fluctuate periodically, because of the negative value of the discriminant $\Delta < 0$. The natural fluctuations of the economy between periods of growth and recession, i.e., *economic cycles*, are well known in economics. A typology of economic cycles according to their periodicity was proposed by Schumpeter [236] and other economists. We conclude that the dynamical regime A can be viewed as an *innovation economy*, in which sustainable economic growth is based on entrepreneurship and spurring higher productivity through greater innovation.

**Case B.** When supply goes ahead of demand ($\gamma_D < 1$) and the degree of mutual reinforcement between supply and demand increases considerably up to the level where $\Delta > 0$, the virtuous cycle accelerating economic growth would make the economy take off like a rocket, since price and quantity grow steadily and cohesively in this dynamical regime. Innovations constantly boost the level of consumption. People get familiar with the new technology and contribute to economies of scale that lead to reduced costs and improved production efficiencies. Lower average prices would foster consumption and aggregate output that would lead to more scale effects.

☞ Persistent economic growth is possible when supply always goes ahead of demand.

**Case C.** For the values of the control parameters belonging to zone C ($\gamma_D > 1$ and $\Delta > 0$), the price component of the first eigenvector $\mathbf{v}_1$ is negative, albeit small—money runs out, but only gradually. Depending on the initial conditions, the dynamical solution may stay within the non-negative quadrant of price–supply–demand space, and the corresponding economy may even exhibit a tendency to grow for some time. Moreover, as the value of the discriminant $\Delta >$ 0, prices and quantities do not fluctuate, so that quantities in the corresponding dynamical solutions may grow steadily.

However, zone C is special for $\gamma_D > \gamma_{S,P} = 1$, i.e., a high relative rate of variation in demand that exceeds the corresponding rates of variations in price and supply. Although, prices may seem stable for a while, demand always goes ahead of supply, and supply is always delayed with respect to demand. The actual needs of customers are never met in time, and thus a *permanent deficit* is the inherent feature of the dynamical regime C.

At the same time, regime C is special for the highest degree of mutual reinforcement between supply and demand. Living under conditions of scarcity, customers buy whatever is offered to them.[6] The manufacturer in turn makes every effort to produce a good in immense quantities once it is in demand, at the expense of a dramatic reduction in the variety of products and a progressive deterioration in their quality, since any item produced will definitely be sold anyway. People are

[6]My mother worked in one of the scientific research institutes in the Soviet Union. Employees of the institute drew lots for the right to buy essential goods, and once it fell to my mother—she could buy a pair of men's shoes! Although the available shoe size (41) was rather small for any member of our family, my mother was happy to buy the shoes in the hope of reselling them later.

exempt from having to make their own choices under scarcity, as they often have a single default option for any type of goods.

So that prices do not fluctuate, production is subject to *centralized administrative planning* and often based on large-scale industrial manufacturing. State control of investment and ownership of industrial assets ends up in a situation where the state itself becomes the main buyer and, at the same time, the main vendor on such a market, selling to and buying by itself the increasingly vast quantities of unsold and obsolete goods.

☞ When demand is ahead of supply, people will take whatever is on offer.[7] Scarcity liberates them from the burden of choice under uncertainty.

The frail trend of economic growth (as indicated by the eigenvalue gradients in Fig. 8.6) is achieved by the gradual deceleration of the supply response to an ever trembling demand, i.e., $\gamma_D \gg \gamma_S = 1$.

In contrast to the previously discussed dynamical regimes, innovations constitute a major threat to such a transient trend of growth in the dynamical regime C, as they may drain important resources and reduce the time available to increase the manufacture of goods that have already been mastered.

The dynamical regime C obviously describes the *economy of socialist countries* in their best years. Dynamical trajectories in regime C leave the non-negative quadrant of the price–quantity space.

**Case D.** Finally, when $\gamma_D > 1 = \gamma_S$ and $\Delta < 0$, the conditions of scarcity are aggravated by coherently fluctuating prices and quantities. The dynamical trajectories in our market model will periodically leave the non-negative quadrant of price–supply–demand space, so the market forces may alternate in nature: supply would turn into the compulsory expropriation of goods and resources ($\mathcal{S} < 0$), and demand would be replaced by centralized distribution of commodities (and food) in short supply ($\mathcal{D} < 0$).

It was suggested by Malthus [237] as early as in 1798 that the conflict between the population's natural tendency to increase and the limitations imposed by the availability of food results in a tendency for population numbers to oscillate, and this oscillation in population numbers should be accompanied by systematic changes in certain economic variables, most notably food prices [9]. The contemporary theory of *secular cycles* in agrarian societies has been developed in [9].

When the social system of the state cannot provide adequate goods and services, people became adept at using connections and backdoor channels [238]. They gradually disassociate from social institutions, meanings, norms, and values instilled by them, everything that once determined the social environment, in which people used to live. Uncertainty of means (about how to achieve desired life goals) and

[7] According to the Russian proverb: If it is given, take it; if beaten, run away.

poverty make people constantly preoccupied with predicting and foreseeing the unfolding of the political and economic environment [239]. However, innovations and any kind of prospective economic and political reform are perceived by them as a threat to the existing distribution system for food and goods, and are definitely rejected.

☞ Decay of societal institutions incites uncertainty of means in the population.

When supply systematically lags behind demand, the market mechanism for impersonal decision making about impartial resource allocation largely fails and connections weaken between the different groups making up the society. People start experiencing doubt that their behavior can affect the distribution of personal and social rewards, and the forces that shape their personal and social affairs become unclear to them [238]. Then, the societal institutions disintegrate, and once the social norms lose their regulative force, people start experiencing *uncertainty of means*, as commonly held values are rejected and socially unapproved means are believed to be necessary to achieve one's goals.

## 8.12 Living Under Disintegrated Societal Institutions: Authoritarianism

Living under conditions of prolonged scarcity and worsening poverty makes people shy away from freedom of thought and action, as it may potentially exacerbate the already high degree of uncertainty in their life. The overall trend in people's mentality in such a situation undermines the notion of self-help. They either look to great crises of the past, which have come to an end and argue that current troubles will likewise come to an end, or argue that spiritual realities are more important than the physical world. Then the fact of waiting for a better future develops in people, producing a pattern of behavior that is much harder to transform than the actual political or economic situation that originally created it.

☞ When societal institutions dissolve but people's social identity remains intact, they will deliberately reject democracy in favor of authoritarianism, since it can give them an opportunity for social revenge.

When a person's sense of identity remains intact but societal institutions dissolve in the course of rapid ongoing cultural changes or political transitions in the society, an individual will still be certain about his expected aims and his aspired role in society, but he would experience uncertainty about how to achieve his life goals in a situation where the means to the desired ends are only erratically available. People can experience uncertainty of means while being quite certain about their desired ends, so they may continue to pursue their life aims and take on their aspired role in society (because their identity remains intact), even though societal institutions have either dissolved or lost their purpose. Under such circumstances, bad or immoral methods can be used by people, as long as they may be able to accomplish something good by using them—and the end then always justifies the means!

☞ Whenever a good outcome excuses any wrongs committed to attain it, it shows that societal institutions are not functioning correctly.

It is remarkable, however, that in spite of the fact that there may be virtually no autonomous social organizations between family and government, such a society will not necessary fall into a state of chaos. Instead, it will look for another form of functioning under the lack of societal institutions and outside the space of institutionalized procedures. And this is precisely the point where authoritarianism emerges, because in the absence of intermediate social organizations any decision taken by an autocratic leadership is freed in these societies from any external legal restraint.

In the face of paralyzing uncertainty of means, any step outside the space of regular actions and procedures assumes a social revolution. It should be recalled that many national leaders in the Europe of the 1930s, and later in other parts of the world, were playing out a "permanent revolution without a plan" and only reached some kind of *Endlösung*[8] at a very late stage in a rather long political gamble aiming at first creating, then progressively aggravating the situation of uncertainty, in which the authoritarian leader was always a high-stakes player [240]. Civilian casualties and even the ultimate catastrophe did not convince those who initiated such politics of their absurdity. Quite the contrary, *permanent revolutionaries* used all the means at their disposal, and invested all available resources, to jeopardize the actual global order.

As the situation was deteriorating, the previously accepted social norms and social adaptations were rendered abnormal or maladaptive. Under aggravating uncertainty, relatively successful peers and simply people of genuine merit would be considered as 'tall poppies' to be resented, attacked, and eventually cut down. Indeed, there is no way to quench the thirst for social revenge within the regular, institutional political discourse.

☞ Authoritarian leaders are high-stakes players, creating and aggravating the situation of political uncertainty.

The relatively small elites of authoritarian regimes may rule by emulating some form of institutionalism, yet giving it no real importance, since actual government

[8] *Ger.* The Final Solution.

is always carried out beyond institutions. The authoritarian form of government is similar to a theological model of control over a millenarian sect [241]. In particular, the religious community is controlled by:

- expectations of a forthcoming catastrophe (the Apocalypse),
- a threat of 'violence' (a permanent threat of exclusion from the community),
- a demonstrative inequality (establishing and maintaining superiority of the leading group within the sect).

A typical *political myth* which underlies and legitimates the authoritarian regime is an archetypal millenarian myth, according to which:

- the regime arose miraculously after a disaster, as a reward for survivors, in recognition of their spiritual loyalty;
- it is exiled from the world where all spiritual values are lost;
- its fate is in constant flight from the sinful world and its salvation is connected with the personality of the leader and is virtually impossible without him.

The political tenure of such a millenarian regime might last almost indefinitely, since the politically weak population lacks social institutions and will protect the authoritarian system in every way, exchanging political power for the right to complain to their leader about their everyday problems. Despite initiating threats to people all around, an authoritarian leader usually meets no resistance due to the fear of the population, and yet there is always a secret thirst for a magical redemption [241].

## 8.13 Conclusion

Under uncertainty, people try to evade personal decision making at all costs, with the use of a variety of exopsychic methods. In the present chapter, we have discussed some methods of impersonal decision making.

We have introduced and analyzed a simple model for social conformity. Our conclusions suggest that the behavior of a crowd without a leader is irrational and intransitive, being characterized by a loss of responsibility for the outcome on the part of individuals. However, as soon as people come together in a group or crowd, they may spontaneously begin to obey one member of the group. Submissive individuals would easily renounce their old beliefs in favor of the position of the more determined leader. It is remarkable that, in the face of uncertainty, the actual political platform of a leader does not play an important role in determining the outcome of elections, as people would rather vote for the more self-confident and decisive candidate.

We have also discussed the problem of public choice and the role played therein by the mass media. We have shown that the mass media will always win in a small enough community of docile and cooperative individuals. Nevertheless, propaganda among stubborn loners is almost totally inefficient. And neither will propaganda work over large enough groups of cooperating individuals. We have found that, in the face of uncertainty, a group attempting to maintain its unanimity might be unable to take any decision. Dividing a society into factions of different interests most likely results in the emergence of absolute social conformity in society members following the suggestions of a self-confident leader. We have found that unanimity in one faction of a group is possible only in the face of another faction expressing alternative interests.

We have introduced and studied a simple deterministic dynamical market model as an impersonal mechanism for making decisions about equitable resource allocation. We have found that the quantity demanded by consumers is correctly balanced by the quantity that sellers supply if and only if supply and demand reinforce each other on the market.

The law of supply and demand is a saddle point of this dynamical market system and that is why economic growth is eventually possible. Insufficient mutual reinforcement between supply and demand would cause shifts between periods of economic boom and periods of recession as time goes by. In particular, persistent economic growth is possible provided that supply always runs ahead of demand.

On the other hand, if demand is ahead of supply, people will take whatever is given to them, as scarcity liberates them from the burden of choice under uncertainty. We have also discussed the way the decay of societal institutions incites uncertainty of means in the population. When societal institutions dissolve, but people's social identity remains intact, they will deliberately reject democracy and choose authoritarianism, as it can give them a chance for social revenge. Whenever social institutions dissolve, the end always justifies the means, and the door is left open for authoritarianism.

# 9 On the Edge of Uncertainty: Evolution in the Global Political System

*Seen politically, systems follow one another, each consuming the previous one. They live on ever-bequeathed and ever-disappointed hope, which never entirely fades. Its spark is all that survives, as it eats its way along the blasting fuse. For this spark, history is merely an occasion, never a goal.*[1]

**Abstract**

Historical trends regarding state disintegration and political transitions suggest that half of extant states will break up by the end of the century. Although the immediate consequences of political experiments disappear on average during the political lifetime of a single generation, they can generate a lasting situation of political uncertainty. Large-scale conflicts may open a window of opportunity for global political transformations.

## 9.1 Introduction

The whole of the world's inhabited land has been parceled up into areas claimed by various states since the early nineteenth century. Now, wherever we are, we live in organized political communities, under particular systems of government.

Patterns of authority in the world political system are extremely diverse, ranging from institutionalized democracies to institutionalized autocracies. The former are characterized by the presence of institutions and procedures through which citizens can express effective preferences about alternative policies and leaders. In these political systems, the exercise of power by the executive is often subjected to institutionalized constraints, and civil liberties are guaranteed to all citizens in their

[1] Ernst Jünger, *Eumeswil*.

 
D. Volchenkov, *Survival under Uncertainty*, Understanding Complex Systems,
DOI 10.1007/978-3-319-39421-3_9

daily lives [242]. However, the majority of other political forms of government lack any regularized political competition and have little concern for political freedom. Competitive political participation be suppressed in these systems, and their chief executives are chosen through a regularized process of selection within a small political elite, which has a tendency to exercise power with few institutional constraints.

Perhaps, the most important features of the state for the scope of our work on survival under uncertainty are the state's propensity to sequential political transitions and its tendency to disintegrate occasionally. Both these processes foster an increasing uncertainty about the future that is often accompanied by political violence. In its turn, by greatly enhancing the uncertainty of life, violence is above all a phenomenon viewed as a necessary condition for mobilization at the margins of the political spectrum, aiming for certain political objectives. Governments may also use violence to intimidate their populace into acquiescence.

In the present chapter, we study world historical data on state disintegration and political transitions that have occurred in the global political system since 1800.

## 9.2 The Process of State Secession in 1800–2014

States are not as stable as one could hope—old states fall apart and new ones come into being. The number of independent states has increased steadily due to the process of state disintegration over the last two centuries (see Fig. 9.1).

The surge of new states is best approximated by the exponential function

$$N(\text{year}) = 28.1 \exp \frac{\text{year} - 1799}{121.95} . \tag{9.1}$$

It then follows from (9.1) that the *mean lifetime of a state* observed during the last two centuries was $\tau = 121.95$ years, while the number of states in the world doubled every $\tau \ln 2 = 84.53$ years on average (*half-life* of a state). Although in many cases the collapse of the world's colonial systems did not lead to the disappearance of the parent state, its political landscape underwent dramatic and profound transformations anyway. According to the exponential trend (9.1), the number of previously sovereign states fell by half every 84.53 years on average, while the actual number of states doubled every 84.53 years.

☞ The observed trend suggests that half of extant states will break up by the end of the century.

Half-life is often used to describe something undergoing exponential decay, such as radioactive atoms. It makes it possible to give the probability that a single sovereign state will 'survive' during a given time interval, with 50 % probability for one half-life period of 84.5 years, 25 % probability for an interval twice as long (169 years), and so on. However, it is generally impossible to predict the time at which a given

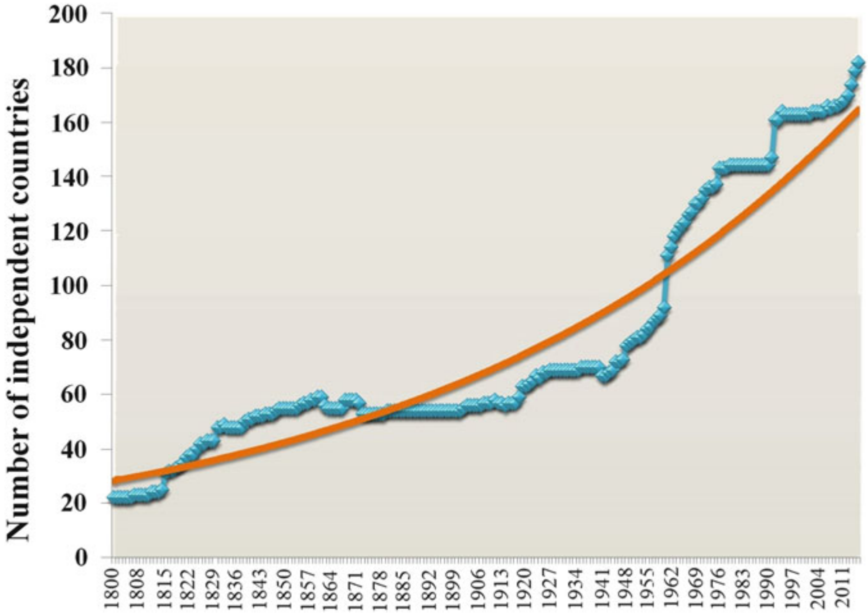


**Fig. 9.1** The increase in the number of independent states in the period 1800–2014 is best fitted by the exponential function $N(t) = 28.1\mathrm{e}^{(\text{year}-1799)/121.95}$, with the goodness-of-fit linear regression coefficient $R^2 = 0.9$

state will decay. Even if the probability of a decay within the next year is 99 %, it is nevertheless possible (though improbable) that the state will survive for many more decades.

The function (9.1) satisfies the following equation between the number of presently existing sovereign states and the current rate of their emergence:

$$\frac{\mathrm{d}N}{\mathrm{d}t}(\text{year}) = \frac{N(\text{year})}{121.95} \ . \tag{9.2}$$

It follows from (9.2) that the more independent states exist in the world today, the faster new states will appear during the next year.

☞ Modern states decay in a similar way to radioactive atoms: the more states there are in the world political system, the faster they split.

It seems that an avalanche of sovereignties covering the world amplifies the aggregate uncertainty of international relations and internal affairs, enhancing the trend toward disintegration of existing polities giving rise to new polities of smaller size than the initial state. The progressively increasing rate of state secession has become the main factor of political development in the contemporary global political system.

Our findings suggest that the main mechanism in the structuring of social and political organization in the near future may be the transition from the polities of the past to metropolises, that is, significant hubs for regional and international connections, commerce, and communications. The process of transfer of power from the state to the metropolis may already be gaining momentum in the European Union, under the guise of regionalism.

It is important to mention a certain relation existing between the political stability of states and the frequency and intensity of military conflicts in which they have participated. Historically, wars have impeded the process of political secession of polities [244] and given the state a reason and means for survival. Wars are usually accompanied by the destruction of urban infrastructure. And the struggle against separatism in autocracies is generally limited to the destruction of major urban infrastructure within some geographical region which expresses a common sense of identity and is able to shape collective actions. It is also remarkable that the growth in the number of states almost completely stopped during the epoch of intense military conflicts in Europe, in the late nineteenth century and in the first half of the twentieth century, as can be seen in Fig. 9.1.

However, the situation changed dramatically after WWII. On the one hand, once the spread of strategic nuclear weapons had made the risks of large-scale conflicts unacceptable, a major war between nuclear countries fraught with incalculable casualties and upheavals became highly improbable. On the other hand, the spread of institutionalized democracy, imposing certain constraints on the exercise of power by its executives and providing a guarantee of civil liberties to all citizens in their everyday lives and in acts of political participation, has made large-scale war within and between developed countries much less probable. Under these circumstances, over the past few decades, the process of political secession of states has rapidly caught up with a rate proportional to the number of currently extant states, as can be seen in Fig. 9.1.

## 9.3 Data Source and Concepts for the Study of Political Evolution in 1800–2013

In our study, we have used the data collected in the framework of the *Polity IV project*,[2] which characterizes the authority of polities around the world for the purposes of comparative and quantitative analysis [242]. The conceptual scheme for classification of polities as subsets of the class of ‘authority patterns’ was originally proposed in [243]. The data we have analyzed does not include information on the

[2] *Polity IV Project: Political Regime Characteristics and Transitions, 1800–2013*, M.G. Marshall, Director, T.R. Gurr, Founder, University of Maryland (Emeritus).

territorial coverage of central state authority or the existence of non-state polities within its borders.

Following the main concepts put forward in [243], polities are classified according to the three main traits of authority:

- *executive recruitment*, i.e., the ways in which superordinates come to occupy their positions,
- *independence of executive authority*, characterizing the extent to which the chief executive ruler must take into account the preferences of others when making decisions, and
- *participation*, the extent to which the political system enables non-elites to influence political elites in regular ways.

The classification of polities according to authority traits is expressed using the following six component variables [242]:

1. *Regulation of chief executive recruitment*: the extent of institutionalization of executive transfers indicating whether or not there are any established modes by which chief executives are selected.
2. *Competitiveness of executive recruitment*: the extent to which prevailing modes of advancement give subordinates equal opportunities to become superordinates.
3. *Openness of executive recruitment*: the extent to which the whole politically active population has an opportunity, in principle, to attain a position through a regularized process.
4. *Executive constraints (or decision rules)*: the extent of institutionalized constraints on the decision making powers of chief executives, whether individuals or collectivities.
5. *Regulation of participation*: the extent to which there are binding rules on when, whether, and how political preferences are expressed.
6. *Competitiveness of participation*: the extent to which alternative preferences for policy and leadership can be pursued in the political arena.

The values of these six component variables for each polity are further specified according to the gradations given in Table 9.1. Hence, the political regime of each polity in any given year can be formally encoded according to the classification given in Table 9.1 by a string of six symbols (or digits in parentheses given in Table 9.1), indicating the degrees of *regulation of chief executive recruitment*, *openness of executive recruitment*, *competitiveness of executive recruitment*, *executive constraints*, *regulation of participation*, and *competitiveness of participation*, respectively. The lexicon describing a variety of possible political situations is further supplemented by three auxiliary strings [242] indicating the following:

1. *Interruption periods*: when a country is occupied by foreign powers during war, thus terminating the old polity, and then reestablishes an independent polity after foreign occupation ends.
2. *Interregnum periods*: during which there is a complete collapse of central political authority that is most likely to occur during periods of internal war.

**Table 9.1** Classification of polities according to the component variables of their authority traits [242]

| Regulation of chief executive recruitment | Openness of executive recruitment | Competitiveness of executive recruitment | Executive constraints | Regulation of participation | Competitiveness of participation |
|---|---|---|---|---|---|
| (1) Unregulated | (1) Closed | (1) Unregulated | (1) Unlimited authority | (1) Unregulated | (1) Repressed |
| | | | (2) Intermediate | | |
| (2) Transitional | (2) Dual designation | (2) Selection | (3) Slight to moderate limitations | (3) Sectarian | (2) Suppressed |
| | | | | (2) Multiple identity | |
| | (3) Dual election | | (4) Intermediate | | (3) Factional |
| | | | (5) Substantial limitations | | (4) Transitional |
| (3) Regulated | (4) Open elections | (3) Dual hereditary/competitive | (6) Intermediate | (4) Restricted | |
| | | (4) Competitive | (7) Executive parity of subordination | (5) Regulated | (5) Competitive |

3. *Transition periods*: during which new institutions are planned, legally constituted in a procedure involving constitutional conventions and referenda, and put into effect.

For example, the political system of the USA in 2013 was characterized by the regulated recruitment of a chief executive, a dual election procedure for executive recruitment, a competitive procedure for executive recruitment, the executive parity of subordination, the regulated participation of the population in elections, and a competitive election participation scheme. It can thus be encoded according to the corresponding values of the component variables given in Table 9.1 by the string

$$[3\,,\;3\,,\;4\,,\;7\,,\;5\,,\;5]\,.$$

## 9.4 Political Distance for Comparative Analysis of Political Regimes

In information theory, the *Hamming distance* between two strings of equal length is the number of positions at which the corresponding symbols are different. We use the Hamming distance between two strings of six digits to encode the different political regimes as the *political distance* between them. The proposed political distance between two polities assesses the minimum number of political changes to the structures of political participation and legitimation of political power required formally in order to transform the current political system of one state into that of another state.

If we use the Hamming distance, the space of political regimes and possible transitions between them turns into a metric space of strings of length 6. For example, as already discussed, the political system of the USA in 2013 could be formally encoded by the word $[3, 3, 4, 7, 5, 5]$. According to the *Polity IV* dataset, using the same system, the political system of Russia in 2013 could be encoded by $[2, 2, 4, 4, 3, 4]$. Therefore, the political distance Pd between these two strings (matching in only one symbol 4 at the third position) is equal to

$$\mathrm{Pd}\,(\mathrm{USA}_{2013}, \mathrm{RUS}_{2013}) = 5\,. \tag{9.3}$$

Note, however, that this number does not mean that there exist just five 'magic' political reforms which could transform the autocratic political system of Russia into a liberal democracy. And neither does the political distance introduced above grasp the actual complexity of political transitions, or assess the probability that the relevant political reforms may one day be implemented and the political transitions actually brought about. Nevertheless, this metric assessing the formal political difference (or rather mismatch in the political structures) between any pair of polities can be used to provide a convenient representation of the global political landscape in the form of branching diagrams—*political phylogenetic*

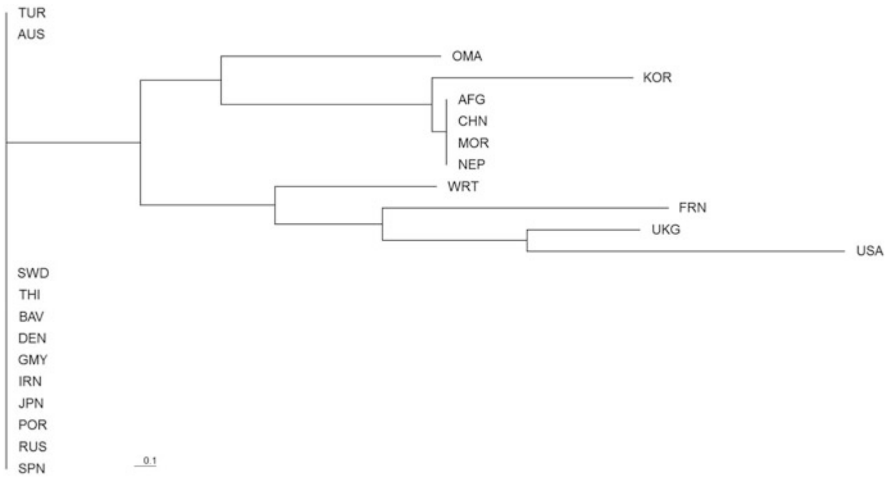


**Fig. 9.2** The tree representing the political landscape of 22 states in 1800: Afghanistan, Austria, Bavaria, China, Denmark, France, Prussia, Iran, Japan, Korea, Morocco, Nepal, Oman, Portugal, Russia, Spain, Sweden, Thailand, Turkey, United Kingdom, United States, Württemberg

*trees*—showing the inferred similarity and differences between authority patterns among various political regimes in any given year (or the historical epoch).

Tree diagrams are interpreted by summing up the length of the branches separating the strings encoding the political regimes in the symmetric matrix of political distances. If the lengths of these branches are short enough, the block in the tree diagram describes a group of polities having similar political regimes in a given year. If the branches separating two polities are long, the tree describes them as being committed to substantially different authority traits (see Figs. 9.2 and 9.3).

The political landscape in 1800 comprised just 22 states (see Fig. 9.2), the majority of which (12) were absolute monarchies (aligned uniformly along the vertical line on the left side of the tree). The new state created by the American revolution (USA) (with the political system most different from others, as shown on the right side of the political tree for 1800) marked the threshold between a political world dominated by monarchies and a political world, in which state power was based on widening control and mobilization of human and material resources in exchange for broadened rights of popular participation [242].

The global political landscape has been broadly reshaped during the last two centuries. The political tree for 2000 comprised 163 independent states, with the the biggest group of polities among them committed to liberal democracy (32). These polities are aligned along the vertical line, in the bottom left corner of the tree shown in Fig. 9.3. Those states manifesting relatively strong autocratic traits (in the given year) are presented in the upper right corner of the tree. The polities that appear furthest from the others were in a situation of fragmentation (such as Cyprus,

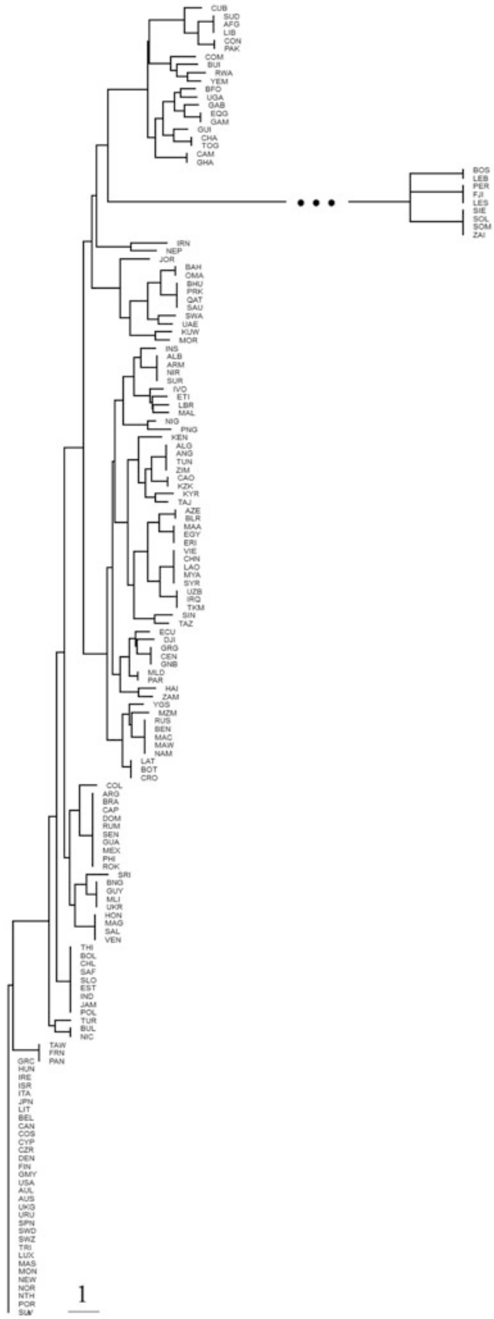


**Fig. 9.3** The tree representing the political landscape of 163 polities in 2000

Bosnia, and Colombia), or experiencing interruption or interregnum periods in their political history in 2000.

The impressive diversity and complexity of the global political landscape resulted from two centuries of differentiation in the structures of political participation and the legitimation of power. The political development had basically followed one of two paths, toward plural democracy or mass-party autocracy [242, 243]. In most Western European countries, the process of political democratization went through sequences of political crisis followed by successive reforms strengthening the societal institutions and procedures through which their citizens could express their political preferences.

In contrast, the Soviet state in Russia provided a new authoritarian model of government combining the deliberate and systematic destruction of the major societal institutions, in whole or in part, with near-absolute state control of social, economic, and political life. The deliberate destruction of supposedly *all* social institutions by a 'forcible overthrow' was by no means a casual affair in socialist autocracies. Indeed, it had been announced in the *Communist Manifesto*[3] as early as 1848 and planned for many decades prior to the actual establishment of these political regimes. Under the Soviet political system, the abolition of all social institutions involving communication between peers was accompanied by the establishment of *socialist institutions* based on highly asymmetric communication between each individual and the state. These fictitious institutions did away with the life of the individual by bringing it under government control. If the demolition of social institutions and the social safety net of peers under the regime of socialist autocracy had been carried out quite as thoroughly as the Communist Manifesto demanded, the subsequent processes of social recovery and political democratization would have become very difficult, if not impossible.

Both political models have been widely imitated, beginning with the establishment of derivative democracies in Latin America (in the nineteenth century) and in Eastern European countries after the decay of the Soviet Union, and ending with the different types of autocratic regimes established in many states of Africa and Asia after the collapse of the colonial system in the 1960s and 1970s, and in some post-Soviet states after the collapse of the Soviet Union.

## 9.5 Political Diversity in Authority Traits in 1800–2013

The classification scheme for political regimes given in Table 9.1 formally allows for 8400 different patterns of authority. However, not all of them can actually exist, as some values of the different component variables in Table 9.1 are obviously

[3]"In short, the Communists everywhere support every revolutionary movement against the existing social and political order of things [. . . ]. The Communists disdain to conceal their views and aims. They openly declare that their ends can be attained only by the forcible overthrow of all existing social conditions." Part IV of *The Communist Manifesto* by K. Marx, F. Engels (1848).

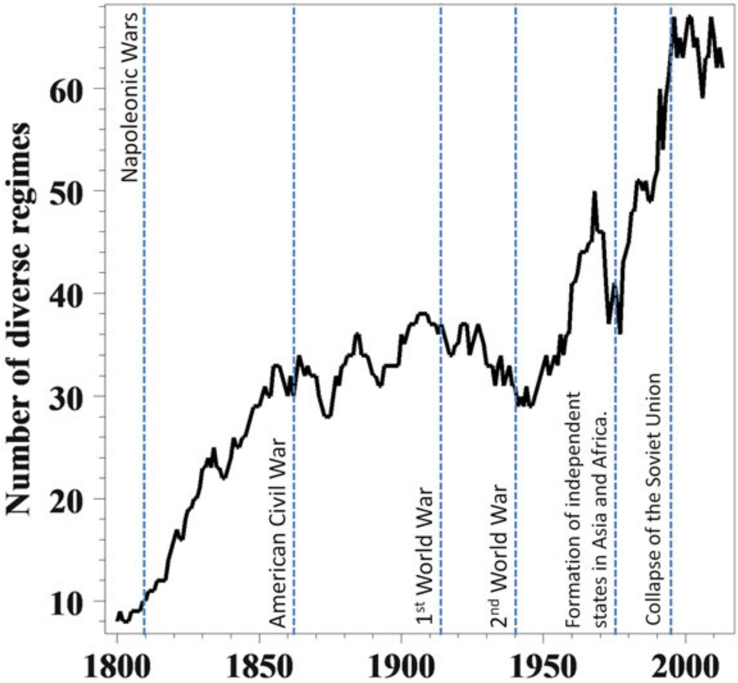


**Fig. 9.4** The number of different political regimes in the political system (1800–2013)

incompatible with each other. In other words, a polity cannot be committed to highly autocratic and highly democratic authority traits at the same time. Since 1800, only 232 different political regimes have been observed in the world political system.

The political diversity displayed in patterns of authority in the global political system increased persistently between the Napoleonic Wars (1803–1815) and the American Civil War (1861–1865), reaching a local maximum just before WWI (see Fig. 9.4). Despite the fact that a considerable reduction in political diversity occurred in the course of these intense military conflicts, the growth in political diversity resumed immediately after WWII, even though it had been temporarily held back by the almost simultaneous formation of a large group of independent states in Africa and Asia which established regimes of socialist autocracy. A further increase in the degree of political diversity occurred during the decay of the Soviet political block and the subsequent collapse of the Soviet state in Russia.

The degree of political diversity can also be characterized by an *entropy of political regimes*. Let us denote the fraction of polities committed to the political regime $X$ in a given year as $p_X(\text{year})$. Then, the entropy function is defined by

$$H(\text{year}) = -\sum_{\{X\}} p_X(\text{year}) \log_2 p_X(\text{year}) \; , \tag{9.4}$$

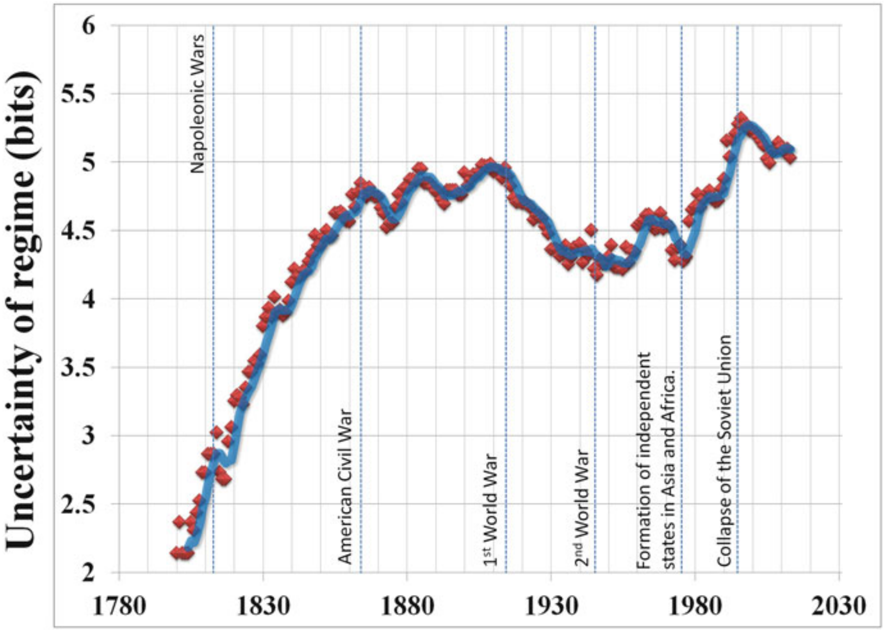


**Fig. 9.5** The degree of diversity in the global political system (1800–2013) as assessed by the entropy function (9.4)

where summation is carried out over all different regimes existing in the global political system in the given year (see Fig. 9.5). If all extant states are committed to the same pattern of authority, then $H(\text{year}) = 0$, but this quantity is positive otherwise. Therefore, the entropy value (9.4) can be viewed as a measure of uncertainty over the political regimes extant in a given year.

The dynamics of the entropy after the First World War shows (see Fig. 9.5) the polarization of the global political system with respect to divergent attitudes toward democratic and autocratic authority patterns—the degree of uncertainty fell by a quarter with respect to the present value. The degree of political polarization peaked during the Cold War, essentially at the end of the 1970s, when the Cold War intensified, with trends encouraging ideological polarization throughout the Third World. However, after the collapse of the Soviet block the global political system became essentially unipolar, and the uncertainty of political regimes resumed its growth.

The entropy value of the global political system (1800–2013) grows approximately logarithmically with the number of different political regimes $n(\text{year})$ extant in a given year (see Fig. 9.6). It is remarkable that the increase in diversity of political regimes did not result in a proportional increase in uncertainty—in many cases, the newly emerging states prefer to establish already familiar political regimes and implement already proven societal institutions, rather than move into uncharted waters at their own risk.

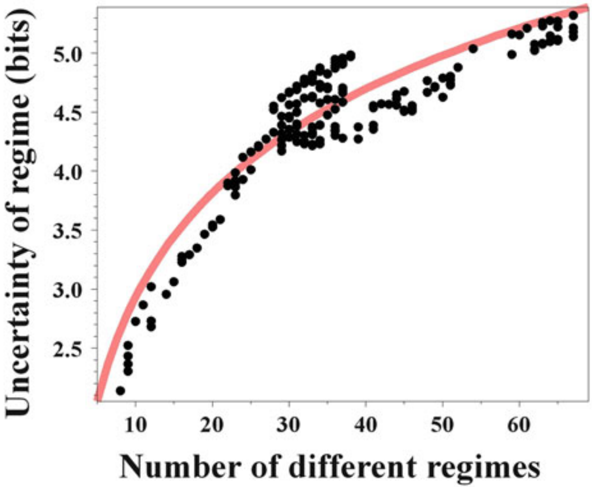


**Fig. 9.6** The approximately logarithmic growth of uncertainty among political regimes with the number of different regimes extant in the global political system in a given year. The *trend line* that best fits the data is $H(\text{year}) = 1.27 \ln n(\text{year})$, with the goodness-of-fit linear regression $R^2 = 0.9$

☞ Newly emerging states tend to adopt well-accepted patterns of authority.

## 9.6 The Statistical Account of Determinism in Political History

Are the current events of political history predetermined by the previous political developments?

The *Polity IV* database contains information about 7904 political transitions between different patterns of authority that have been observed in the world political system since 1800. Below, we present the results of the statistical analysis of these historical records, providing a rigorous statistical account of the strength of determinism in political history.

For any two authority patterns $X$ and $Y$, we can calculate the probability of political evolution of a single polity from $X$ to $Y$ in exactly $t$ years, viz.,

$$P_t(X, Y) = \Pr\left\{\underbrace{X \to Z \to \ldots \to S \to Y}_{t}\right\}, \tag{9.5}$$

provided that such a political development has actually been recorded in history. It is important to mention that the political evolution from $X$ to $Y$ during the lifetime of a single state may comprise many intermediate steps, with different time durations.

The 'diagonal' term $P_t(X, X)$ accounts for the probability that the state retains the political system $X$ during $t$ consecutive years.

We can assess the strength of determinism in global political history with the help of the mutual information

$$I(t) = \sum_{\{X,Y\}} P_t(X,Y) \log_2 \frac{P_t(X,Y)}{P_t(X,X)P_t(Y,Y)} , \tag{9.6}$$

where the summation is performed over all pairs of authority patterns $\{X, Y\}$, between which at least a single political transition has been recorded in the given time $t$. The value of the mutual information (9.6) can be calculated for every duration $t$ less then the total duration of the records (213 years), providing us with a statistically robust measure of the length of time a political regime must exist in order to determine a given political situation in the future (after $t$ years).

In Fig. 9.7, we have presented the mutual information (9.6) as a function of the elapsed time, calculated for all political transitions recorded in the period 1800–2013. The trend line

$$I(t) = 17.43 \exp\left(-\frac{t}{8.9\,\text{years}}\right) \tag{9.7}$$

fits the data best ($R^2 = 0.7$). The data presented in Fig. 9.7 show convincingly that global political history has a short memory, as the mean lifetime of any occasional political 'deviation' is limited on average to the *political lifetime of a single generation*, namely, $\tau = 8.9$ years.

☞ Global political history has a rather short memory. Social utopias fade on average during the political lifetime of a single generation.

Although the immediate consequences of any reckless political experiment may dissolve in a single generation, or even in just a few years, it may generate a situation of severe political uncertainty that can last much longer than the events that created it.

We can grasp the degree of uncertainty of political development in the global political system (1800–2013) by estimating the *entropy of political transitions* during time $t$:

$$\mathcal{H}(t) = -\sum_{\{X,Y\}} P_t(X,Y) \log_2 P_t(X,Y) , \tag{9.8}$$

where the summation is performed over all pairs of authority patterns $\{X, Y\}$. The entropy function defined by (9.8) is nothing else but a version of the block entropy

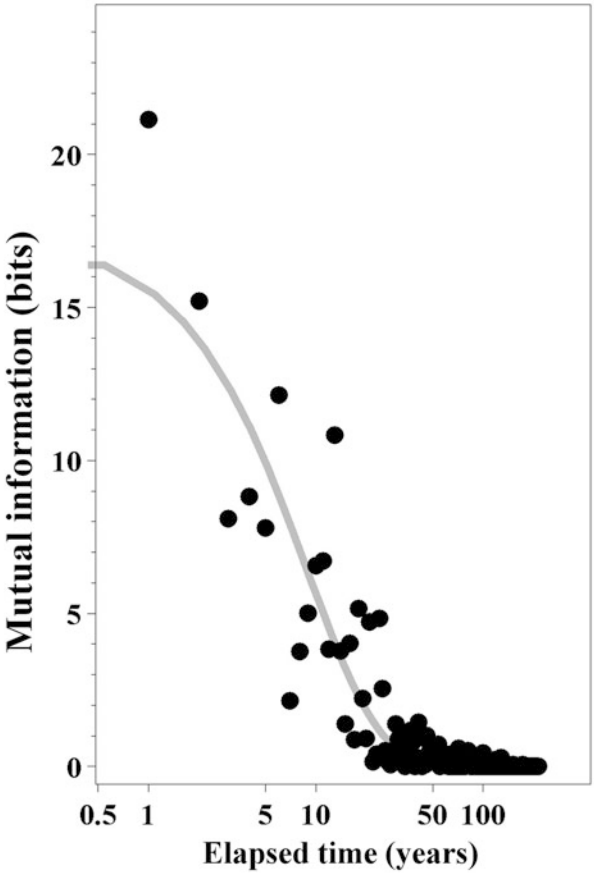


**Fig. 9.7** Log-linear plot of mutual information calculated over all recorded political transitions. The *trend line* is $I(t) = 17.43\exp(-t/8.9)$, with the goodness-of-fit linear regression $R^2 = 0.7$

calculated over all political developments occurring during the last two centuries (Fig. 9.8).

The data show that the highest degree of uncertainty is associated with immediate transitions within a single year, for which $\mathcal{H}(1) = 91.39$ bits. In the short term, a transition to virtually any political situation can happen. For example, a *coup d'état* may suddenly occur. However, the degree of uncertainty of political transitions decreases steadily for long durations, undergoing an approximately exponential decay:

$$H(t) = 82.25\exp\left(-\frac{t}{45.25\,\text{years}}\right) . \tag{9.9}$$

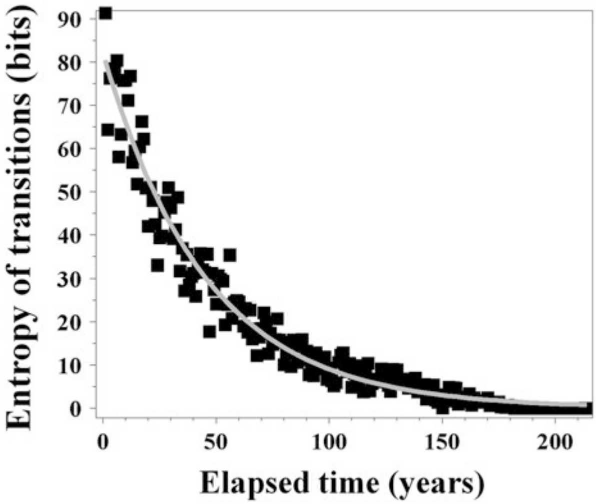


**Fig. 9.8** Uncertainty of political transitions recorded in the global political system (1800–2013). The *trend line* is $\mathcal{H}(t) = 82.25\exp(-t/45.25)$, with the goodness-of-fit linear regression $R^2 = 0.85$

In particular, the *mean duration of political uncertainty* arising in the past due to any particular political transition lasted on average for $\tau = 45.25$ years.

☞ The most fleeting political experiments can create a lasting situation of political uncertainty.

For time durations exceeding the mean lifetime of a state ($\approx$122 years), the number of political transitions recorded in the database falls dramatically, and so does the entropy of transitions (9.8).

## 9.7 Uncertainty of Political Evolution and Pivotal Moments in Global Political History

In 1800, it would have been hard to imagine a newly emerging state deliberately making a political choice in favor of liberal democracy, just as today it is difficult to envisage a new polity deliberately establishing the political regime of absolute monarchy. We now use our analysis to investigate political transitions specific to a given year. The main objective of our analysis is to estimate the degree of uncertainty of political evolution and identify the pivotal moments in global political history.

Let us recall that for any pair of political regimes, $X$ and $Y$, we are able to calculate the probability that the current political system of a state evolves from $X$ to $Y$ in exactly $t$ years, provided that such a political development has actually

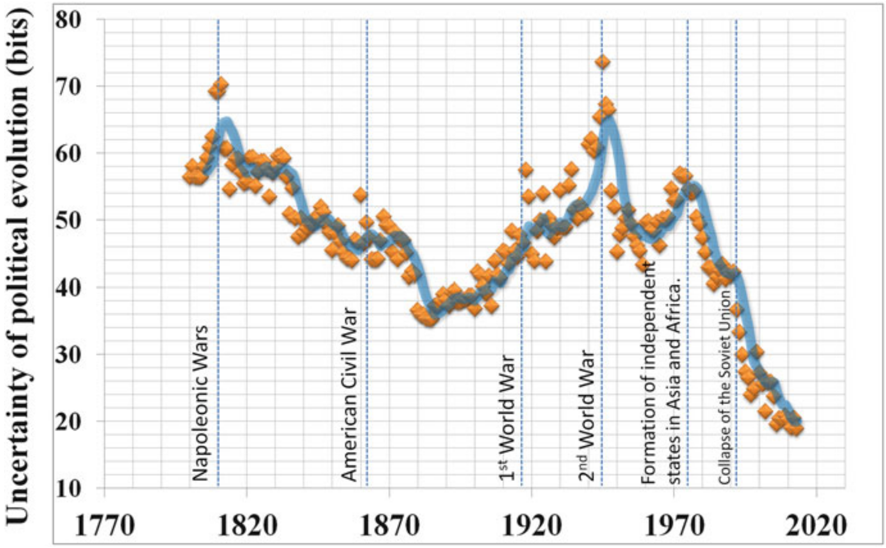


**Fig. 9.9** Uncertainty of political evolution in recorded political history (1800–2013). The *trend line* representing the moving average over the nearest data points is shown to facilitate understanding of the observed dependence

been recorded since 1800. Given the probability $\pi_X(\text{year})$ of the political regime $X$ in a certain year, we can estimate the degree of uncertainty of political evolution using the entropy rate of the year-specific random walks defined by the transition probabilities $P_t(X, Y)$:

$$h(\text{year}) = -\sum_{\{X\}} \pi_X(\text{year}) \sum_{\{Y\}} \sum_{t \geq 1} P_t(X, Y) \log_2 P_t(X, Y) \ . \qquad (9.10)$$

The uncertainty of political evolution in recorded political history (1800–2013) is shown in Fig. 9.9.

The peaks of maximum uncertainty in the political evolution indicate pivotal moments of global political history, providing windows of opportunities for global political transformation.

☞ Windows of opportunity for global political transformations may arise during the final stages of large-scale conflicts.

The first such period clearly visible in Fig. 9.9 relates to the time of the Napoleonic Wars (1803–1815), when a series of major conflicts led by Emperor Napoleon I of the French Empire shook the foundations of many European monarchies and reshaped the global political landscape.

The second pivotal period—marked by a local maximum of uncertainty—may be identified with the American Civil War, from which emerged a political system involving 34 united states. The uncertainty of political evolution increased progressively from the beginning of the twentieth century, through the upheavals of WWI, to culminate at the the end of WWII. The last important period in global political history occurred during the Cold War, when a number of new states in Africa and Asia acquired independence and made their political decisions about the patterns of authority they would adopt in the future.

It may seem that the degree of uncertainty of political evolution will decrease with time, as more and more countries seek to establish democratic social institutions. However, the falling entropy profile in Fig. 9.9 should be interpreted with caution, as the impact from historically recent traits in global political development is still statistically insufficient. History never ends!

## 9.8 Conclusion

We have studied world historical data on state disintegration and political transitions occurring in the global political system since 1800. We have shown that modern states decay in a similar way to radioactive atoms: the more independent states there are in the political system of the world, the faster they split. In particular, the observed trends suggest that half of extant states will break up within the next 84.5 years, i.e., by the end of the century.

In our study of political data collected in the framework of the *Polity IV project*, we have analyzed the global trends of political diversity in recorded history. We have found that newly emerging states tend to imitate well-accepted authority patterns rather than move through uncharted waters of the political ocean at their own risk.

We have also presented the results of a statistical analysis providing a rigorous statistical account for the strength of determinism in political history. Although the immediate consequences of any reckless political experiment would dissolve on average during the political lifetime of a single generation, they can generate a lasting situation of political uncertainty.

Finally, we have studied the degree of uncertainty of political evolution and the pivotal moments in global political history. Our analysis suggests that windows of opportunity for global political transformation may arise during the final stages of large-scale conflict.

# Epilogue: Keep Jogging Along! 10

A major survival strategy adopted recently by humans is a perpetual acceleration in the pace of life, in a process of permanent modernization based on the concept of abstract time. We ultimately need others if we are to act as though the future is certain. The social institutions we establish constitute a mechanism for reducing the uncertainty of the present by directly linking our social past with our social future. They are maintained by organizations providing a structure for communications, shaping them into a framework of social memory, reconstructing our distant past from a common perspective, and transforming it into a perspective on the common future.

Domains involving the most uncertainty have the highest likelihood of skilled people failing. Those that succeed best are often simply those that have tried harder and whose early luck has been compounded. Wealth inequality rises among the population when the vital few take risky decisions under uncertainty: the more adventurous traders there are and the greater their fortune, the fewer lucky ones there will be. Scarcity also promotes inequality by necessitating competition and fueling conspicuous consumption.

The most likely behavioral strategy under uncertainty over a long enough period of time $T$ would consist of approximately $T/\log T$ short-term decisions, and this corresponds to a logarithmic utility function of time and hyperbolic discounting in time for risk aversion and prudence behavior. The optimal survival strategy in any foreseeable period of time would consist of a regular change of scenery by innovation and migration to other environments.

Any individual advantage over rivals is always transitory, if competition is allowed in the group. If competition is avoided, a progressively ineffective hierarchy will arise in the group.

A high degree of behavioral adaptation in society may be not that important for survival success under uncertainty. In hierarchical societies facing uncertainty, the 'fittest' members are punished, rather than rewarded. A community may seek to get rid of 'excessively' successful peers, considering them maladaptive whenever



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the rate of environmental and social change is higher than than the rate of possible adaptations to these changes.

The observed historical trends in state disintegration and political transitions suggest that half of extant states will break up by the end of the century. Dramatic upcoming political change will potentially involve the destruction of important social institutions and a massive surge in the number of migrants coming to other countries to live and work, regardless of national borders. When the social identity of the displaced people remains intact, they will continue to pursue their life goals, but experience uncertainty about how to achieve these goals in the new society, where the means to the desired ends may be unavailable or unknown. For them, the end will always justify the means.

In the face of upcoming uncertainty, we are as much responsible for everything that happens to us in this life as we were once upon a time in the savannah, at the very dawn of our history. We managed to survive then, being perhaps the most enduring race among all other living species.

So let's keep jogging along!

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