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Kinetic Theory of the Inner Magnetospheric Plasma

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To the loving memory of my parents
Vladimir and Polina Khazanov
To my wife, Tamara, and my sons
Igor and Max

Preface

The inner magnetosphere is an important region of space plasma because it is one of the “kitchens” for space weather effects. The scientific understanding of this region is important for predicting the interaction between space environmental conditions and human activities.

The inner magnetospheric plasma is a unique composition of different plasma particles and waves. It covers a huge plasma energy range with spatial and time variations of many orders of magnitude. In such a situation, the kinetic approach is the key element, and must be the starting point of a proper theoretical description of these inner magnetospheric plasma phenomena; a book dedicated to this particular area of research is required.

The kinetic theory of the inner magnetosphere has many different aspects and could not be described in one book. This particular book is based on my research and Space Plasma Physics and Advance Plasma Physics courses that I have delivered at different universities of the United States and Russia. The major topics in this book are: Kinetic Theory of Superthermal Electrons, Kinetic Foundation of the Hydrodynamic Description of Space Plasmas (including wave–particle interaction processes), and Kinetic Theory of the Terrestrial Ring Current.

The analytical solutions of simplified transport equations are the distinguishable features of this book. Approximate analytic solutions of transport phenomena are very useful because they help us gain physical insight into how the system responds to varying sources of mass, momentum, and energy and also to various external boundary conditions. They also provide us a convenient method to test the validity of complicated numerical models, a task that is usually tedious and time consuming.

This book provides a broad introduction to the theory of space plasma physics for students who intend to carry out independent research in upper atmospheric science and space physics. It is designed to provide a comprehensive description of the different kinds of transport equations for both plasma particles and waves with an emphasis on the applicability and limitations of each set of equations. It is hoped that this book will provide students and space researchers with an understanding of

how to determine the best approach to any upper atmospheric or space physics problem.

It is my pleasant duty to thank (in alphabetical order) A. A. Chernov, K. V. Gamayunov, G. D. Gefan, T. I. Gombosi, O. A. Gorbachev, V. B. Ivanov, D. V. Khazanov, M. A. Koen, Yu. V. Konikov, J. U. Kozyra, I. A. Krinberg, E. N. Krivorutsky, V. I. Kuzivanov, M. W. Liemohn, E. V. Mishin, T. E. Moore, A. F. Nagy, V. N. Oraevskii, V. M. Polyakov, G. V. Popov, and A. A. Trukhan for the numerous useful discussions, valuable help, and remarks during various stages of the work on the problems considered in this book.

Greenbelt, MD
September 2009

George V. Khazanov

Biographical Sketch

Dr. George V. Khazanov is a senior scientist in the Heliosphysics Science Division at the NASA Goddard Space Flight Center. Prior to joining NASA, he was a Tenured Professor of Physics at the University of Alaska Fairbanks. Dr. Khazanov has extensive experience in space plasma physics and simulation of geophysical plasmas. His specific research areas include: space plasma energization and transport, kinetic theory of superthermal electrons in the ionosphere and plasmasphere, hydrodynamic and kinetic theory of space plasma in the presence of wave activity, theoretical and numerical-modeling investigations of ionosphere-plasmasphere interactions, theoretical studies of the artificial injection of charged and neutral particles into the ionosphere, and wave and beam-induced plasma instabilities in the ionosphere and magnetosphere. Dr. Khazanov was the Dean of the College of Physics, and the Theoretical Physics Department Chair at Altai State University, Russia; Director of the Ionospheric Plasma Physics Laboratory at Irkutsk State University, Russia. He supervised and directed more than 30 M.S. and 15 Ph.D. graduates. He is the author or coauthor of 6 books and about 250 peer reviewed publications.

Book Summary

This book provides a broad introduction to the kinetic theory of space plasma physics with the major focus on the inner magnetospheric plasma. It is designed to provide a comprehensive description of the different kinds of transport equations for both plasma particles and waves with an emphasis on the applicability and limitations of each set of equations. The major topics are: Kinetic Theory of Superthermal Electrons, Kinetic Foundation of the Hydrodynamic Description of Space Plasmas (including wave-particle interaction processes), and Kinetic Theory of the Terrestrial Ring Current. Distinguishable features of this book are the analytical solutions of simplified transport equations. Approximate analytic solutions of transport phenomena are very useful because they help us gain physical insight into how the system responds to varying sources of mass, momentum and energy and also to various external boundary conditions. They also provide us a convenient method to test the validity of complicated numerical models, a task that is usually tedious and time consuming. It is hoped that this book will provide students and space researchers with an understanding of how to determine the best approach to any upper atmospheric or space physics problem.

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Abbreviations

AMIE	Assimilative Mapping of Ionospheric Electrodynamics
AMPTE	Active Magnetospheric Particle Tracer Explorers
AMPTE/CCE	Active Magnetospheric Particle Tracer Explorers/Charge Composition Explorer
BGK	Bhatnagar-Gross-Krook
CGL	Chew-Goldberger-Low
CPCP	Cross Polar Cap Potential
CRRES	Combined Release and Radiation Effects Satellite
DE	Dynamics Explorer
DMSP	Defense Meteorological Satellites Program
ELF	Extremely Low Frequency
EMIC-waves	ElectroMagnetic Ion Cyclotron waves
EUV	Extreme UltraViolet
FAC	Field-Aligned Current
FFT	Fast Fourier Transform
FLIP	Field Line Interhemispheric Plasma
FMS	Fast MagnetoSonic
FWHM	Full Width at Half Maximum
GPS	Global Positioning System
HENA	High-Energy Neutral Atom
ICME	Interplanetary Coronal Mass Ejection
IM/S	Inner Magnetosphere/Storm
IMAGE	Imager for Magnetopause-to-Aurora Global Exploration
IMF	Interplanetary Magnetic Field
IRI	International Reference Ionosphere
IRM	Ion Release Module
ISEE	International Sun–Earth Explorer
ISIS	International Satellites for Ionospheric Studies
LANL	Los Alamos National Lab
LCB	Loss Cone Boundary

LFW	Low-Frequency Waves
LHR	Lower-Hybrid Resonance
LHW	Lower-Hybrid Waves
LINAC	LINear Accelerator
MENA	Medium-Energy Neutral Atom
MHD equations	MagnetoHydroDynamic equations
MLT	Magnetic Local Time
MSIS	90-Atmosphere Model
PE	PhotoElectron
PIC	Particle-In-Cell
PSE	Plasma Sheet Electron
PT	Plasma Turbulence
PTE	Plasma Transfer Event
RB	Radiation Belt
RC	Ring Current
RIS	Radio wave Incoherent Scatter
RPI	Radio Plasma Imager
SAPS	SubAuroral Polarization Stream
SAR	Stable Auroral Red
UKS	United Kingdom Subsatellite
VHF	Very High Frequency
VLF	Very Low Frequency

Chapter 1

Introduction

Before discussing the kinetic theory of the inner magnetosphere, it is necessary to describe the physical characteristics of this region and its position in the near-Earth's space plasma environment. It is also useful to give a brief overview of the plasma parameters of this focused area, and to provide a qualitative discussion of the major processes that control the formation of this region of the space plasma.

1.1 General Overview

A region of outer space usually referred to as the near-Earth plasma extends from altitudes of approximately 50–60 km (the lower layer of the D region of the ionosphere) to the outer boundary of magnetosphere, i.e., the magnetopause, where the magnetospheric plasma interacts with the solar wind. There are a number of reviews and monographs (Lyons and Williams 1984; Rees 1989; Kozyra and Nagy 1991; Lemaire and Gringauz 1998; Schunk and Nagy 2000; Gombosi 2004; Walt 2005; Lemaire et al. 2007) summarizing numerous studies of the structural properties of the various zones in the near-Earth plasma (see Fig. 1.1) and the energy distributions of the charged particles in these zones.

In giving a general description of the near-Earth plasma, it is convenient to distinguish the following three regions: the ionosphere, the plasmasphere and inner magnetosphere, and the outer magnetosphere.

The ionosphere is the region with the highest density of charged particles, with characteristic regions D, E, and F. The upper boundary of the ionosphere is usually defined as the region where the concentration of H^+ ions exceeds the concentration of O^+ ions. Depending on the level of solar activity, the altitude of this boundary varies in a broad range, from 600 km up to 2,000 km (Schunk and Nagy 2000). In this book we consider the lower boundary of the ionosphere to be at the level of 90–100 km, thus, excluding from consideration the D region, which is characterized by an extremely complicated formation mechanism and constitutes a separate important topic.

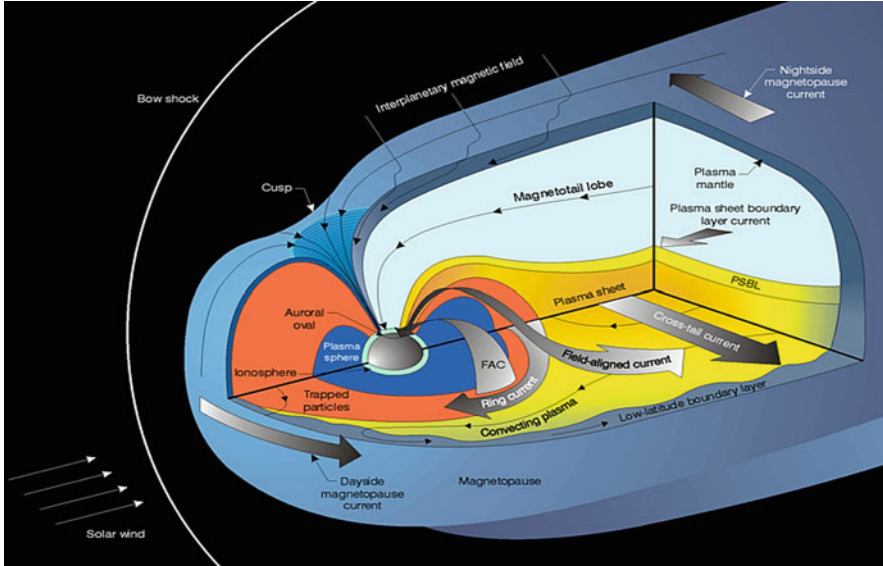


Fig. 1.1 Earth's magnetosphere

Molecular ions dominate at altitudes of up to approximately 150 km, whereas O^+ is the dominant ion beginning from 200 km and continuing up to the region populated by light H^+ and He^+ ions. During periods of maximum solar activity, a region with a predominance of He^+ ions can appear (Schunk and Nagy 2000).

A distinctive feature of the ionospheric plasma is the presence of superthermal electrons. These electrons result from the ionization of the neutral atmosphere by solar short-wave radiation with the wavelength $\lambda < 1,026 \text{ \AA}$ or by precipitating particles of magnetospheric origin. Using generally accepted terminology, the electrons in the Earth's ionosphere may be divided into three groups:

1. *High-energy electrons of magnetospheric origin with energy $E \sim 1\text{--}100 \text{ keV}$.* The density of these electrons is of the order of $10^{-5}\text{--}10^0 \text{ cm}^{-3}$; they lose their energy mainly due to the ionization of neutral particles.
2. *Superthermal electrons with $E \sim 1 - 10^3 \text{ eV}$.* They are produced as a result of the ionization of neutral particles by solar radiation (photoelectrons) and by high-energy electrons (secondary electrons). Their density is of the order of $n_e \sim 10^0\text{--}10^2 \text{ cm}^{-3}$. They transform to thermal electrons by losing their energy either by transferring it to thermal electrons or by the excitation or ionization of neutral particles. Superthermal electrons play an important role in the kinetics of microprocesses of ionospheric plasma. They are usually invoked to account for thermal plasma heating and the intense excitation of energy levels of atoms and molecules, which gives rise to various atmospheric emissions and to an increase in rates of some aeronomical reactions.
3. *Thermal electrons with $E < 1 \text{ eV}$.* They are produced as a consequence of the thermalization of superthermal electrons. They disappear due to dissociative

recombination with molecular ions. Depending on the height and geophysical conditions, their density is of the order of $n_e \sim 10^2 - 10^6 \text{ cm}^{-3}$. The velocity distribution of thermal electrons is close to Maxwellian.

The geomagnetic field topology causes some regional peculiarities in the dynamics of the ionospheric plasma at low and high latitudes. For example, there is a constant, supersonic outflow of thermal plasma at high altitudes in the regions of open geomagnetic field lines. This phenomenon, by analogy with the well-known outflow of solar plasma (Parker 1958), has been given the name of polar wind (Axford 1968).

In the regions with closed geomagnetic field lines, a considerable role in the dynamics of the ionospheric plasma belongs to plasmasphere, with which the ionosphere exchanges particles and energy fluxes (Lemaire and Gringauz 1998). The plasmasphere is the coldest part of the inner magnetosphere. It is an important region of space inside the geosynchronous orbit because it is one of the primary locations for space weather effects. In addition to the plasmasphere, there are two other main plasma populations in the inner magnetosphere: the ring current and the radiation belts. Along with the plasmasphere, each of these populations has distinct features that make it a vital component of the inner magnetosphere.

The plasmasphere may be considered as an extension of the ionosphere into the outer layers of the near-Earth environment because, as with the ionosphere, the plasmasphere is filled with dense and cold ($E \leq 1 \text{ eV}$) plasma. That is why the term “peripheral part of the ionosphere” is sometimes used instead of “plasmasphere.” A characteristic feature of the plasmasphere is its sharp upper boundary, the plasma-pause, where the electron density falls rapidly from hundreds of particles per cubic centimeter to just a few particles over a distance of several hundred kilometers (Lemaire and Gringauz 1998).

The plasmasphere, in turn, can be divided into two zones: the inner zone, with $L \leq 3$ (where L is the McIlwain parameter), and the outer zone, with $L \geq 3 - 5$ (Lemaire and Gringauz 1998). The first of these two zones is weakly affected by geomagnetic disturbances. There seem to be no energy sources in this zone (except photoelectrons) and the transport of heat and mass is caused by ionosphere–plasmasphere coupling. The second zone varies considerably during magnetospheric storms and substorms. There seem to be additional energy sources (Lemaire and Gringauz 1998) and, along with processes in the inner zone, convection also plays an important role leading to transport of energy and particles across the field lines and outside the plasmasphere. This convection is caused by the large-scale electric field produced by the solar wind streaming past the magnetosphere. Although the magnetic field lines in a large part of this region remain closed, plasma can stream toward the front boundary of the magnetosphere (the magnetopause), which considerably and abruptly reduces the plasmaspheric electron density forming the plasmopause.

Magnetic storms in space cause severe distortions of the electric and magnetic fields of near-Earth space, and the various inner magnetosphere plasma populations undergo dramatic dropouts, enhancements, and complicated dynamics. Table 1.1

Table 1.1 Characteristics of inner magnetospheric plasma populations (Liemohn 2006)

Population	Density	Temperature	Source	Composition	Driver	Importance
Plasmasphere	100s cm^{-3} to 1,000s	<1 eV, maybe up to 10s of eV	Subauroral ionosphere	H^+ , some He^+ and O^+	E-field	Dominates mass density
Ring current	$\sim$ few cm^{-3} up to 10s	1–400 keV	Plasma sheet (SW and ionosphere)	H^+ , O^+ in storms	E and B fields	Dominates energy density
Radiation belts	$<<1 \text{ cm}^{-3}$	100s of keV to MeV	Plasma sheet, SEPs, local acc.	Mostly e^- , some H^+	B-field	Dominates S/C damage

(Liemohn 2006) summarizes some of the major plasmasphere, ring current, and radiation belts characteristics. They have very different plasma densities and temperatures, as well as source populations and dominant driver terms in their transport and dynamics. The plasmasphere dominates the mass density and is therefore a catalyst for plasma-wave excitation and interaction with the charged particles in the region. The ring current dominates the energy density and therefore has a strong influence on the near-Earth electric and magnetic fields. Yet another influence is that the electron component of the ring current is responsible for spacecraft surface charging, which is a large source of adverse space weather effects on satellites. The radiation belts dominate the damaging effects to near-Earth spacecraft, including deep dielectric charging within satellites, causing single-event upsets in electronics and harm to biological cells.

So, the inner magnetosphere, which is the major focus of this book, is strategically important for space weather, as amply demonstrated by the renewed interest in transport and loss processes operating in the radiation belt. These electrodynamic processes are multifaceted, involving wave–particle interactions and coupling between cold and hot plasmas originating in distinctly different regions of the magnetosphere. During intense magnetic storms, and the periods immediately following them, the transport and mixing of hot and cold plasmas, the generation of waves, the acceleration of plasma particles, energy diffusion, and the pitch-angle diffusion of energized electrons and their loss constitute a set of mutually interacting complex processes determining space weather conditions (Liemohn 2006).

The inner magnetosphere plasma is a very unique composition of different plasma particles and waves. It covers a huge energy plasma range with the spatial and time variations of many orders of magnitude. In such situations, the kinetic approach is the key element and the starting point of the theoretical description of this inner magnetosphere plasma phenomena.

The outer part of the magnetosphere extends behind the inner magnetosphere and is filled with high-energy plasma. The following regions of the outer magnetosphere are of particular importance to the inner magnetosphere: the nightside cusp, which, when disturbed, is filled with hot ($E = 1\text{--}10$ keV) plasma characterized by the highly anisotropic and varying-with-time distribution of particles over their pitch angles; and the dayside cusp, formed by the injection of particles from the transition region that have energies of a few kiloelectron volts. The dayside cusp is also characterized by strong spatial plasma density gradients. It includes a plasma

layer filled with particles having energies of a few kiloelectron volts, which, in this region, also have anisotropic pitch-angle distributions with the predominance of particles with magnetic field-aligned velocities and the high-latitude part of the magnetotail – the polar wind region.

One of the most characteristic features of the outer magnetosphere is the distinct day–night asymmetry in the near-Earth plasma distribution as well as in the magnetic field configuration. The outer magnetosphere is characterized by a high plasma mobility, which makes it extremely difficult to obtain a general picture of the ionization distribution. In addition, there is a permanent electromagnetic noise of sufficient intensity to considerably influence particle behavior.

However, one thing can be said with certainty, the distribution of plasma density and the energy of the particles in the outer magnetosphere is dependent on magnetic activity and, above all, on magnetic substorms.

This book is intended to provide a broad introduction to the kinetic theory of space plasma physics for students who intend to carry out independent research in upper atmospheric science and space physics. It is designed to provide a comprehensive description of the different kinds of plasma transport equations for both plasma particles and waves, with an emphasis on the applicability and limitations of each set of equations. It is hoped that this book will leave students and space researchers with an understanding of how to approach any upper atmospheric or space physics problem and how to determine the best method to describe the problem.

1.2 Complexity of the Inner Magnetosphere

Figure 1.2 presents a schematic of the processes and interactions in the inner magnetosphere, as well as the linkages coupling all of these components that will be considered in this book. The plasma sheet is the primary source of particles for the hot electron and ion populations in the inner magnetosphere. The inflow of particles contains both electrons (e.g., Khazanov et al. 1998) and ions (e.g., Liemohn et al. 1999). The flow of these particles is governed by the topology of the near-Earth magnetic and electric fields (e.g., Alfvén and Fälthammar 1963; Khazanov et al. 2003a). Through a variety of processes, some of the electrons and ions are pitch-angle scattered into the loss cone where they precipitate into the thermosphere and ionosphere (e.g., Hultqvist et al. 1976; Khazanov et al. 2002). In their descent through the atmosphere, they ionize and heat the background particles, resulting in changes to the electrical conductance of the subauroral ionosphere (Khazanov et al. 2003a). In addition, longitudinal asymmetries in the magnetospheric currents must be closed through field-aligned currents (FACs) into and out of the ionosphere (e.g., Iijima and Potemra 1976; Liemohn et al. 2001b). The ionospheric conductance allows perpendicular currents to flow (to connect and balance the FACs), resulting in perpendicular (to the magnetic field lines) electric fields (Ridley and Liemohn 2002). Assuming the field-aligned conductance to be infinite, the electric fields can be mapped along the field lines to the inner magnetosphere, where they will govern

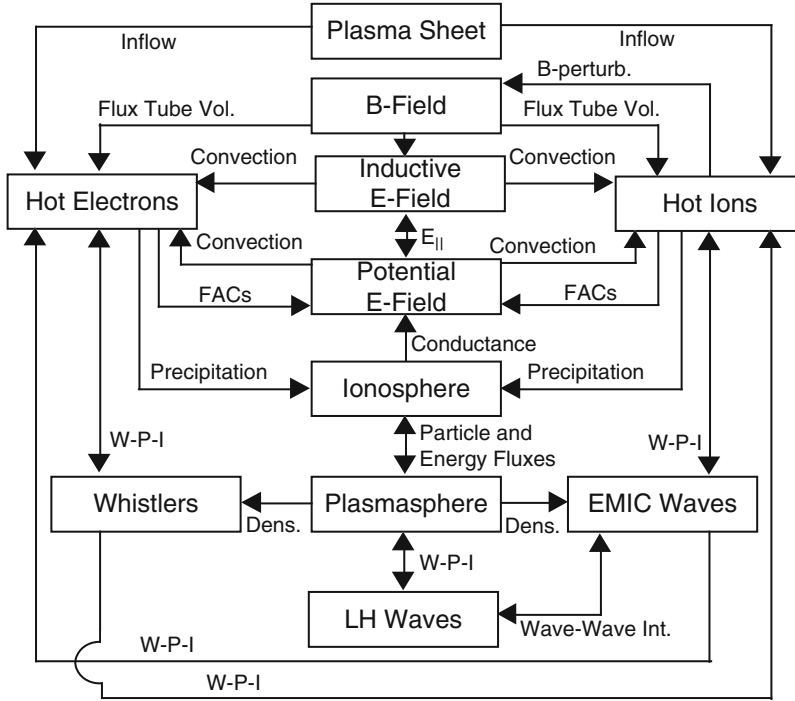


Fig. 1.2 Coupling between processes in the inner magnetosphere and the subauroral ionosphere

the subsequent flow of the hot electrons and ions. Including this feedback loop in a numerical simulation of the ring current is known as a self-consistent potential electric field calculation, because the location of the hot particles in the inner magnetosphere modifies the electric fields that determine the locations of the hot particles (e.g., Fok et al. 2001; Khazanov et al. 2003a; Liemohn et al. 2004). Of course, the high-latitude electric potential distribution also influences the mid-latitude electric potential pattern and any self-consistent solution of the subauroral potential must have a high-latitude boundary condition imposed on it.

Many methods have been used to impose a non-self-consistent potential electric field within the inner magnetosphere to convect the storm-time ring current particles (e.g., Chen et al. 1993; Liemohn et al. 2001a). The choice of these various fields can lead to large differences in the resulting plasma distributions (e.g., Jordanova et al. 2001; Khazanov et al. 2003a), including differences in adiabatic energization (Khazanov et al. 2004a, b).

This convective flow is also the major loss of hot particles and energy from the inner magnetosphere. Flow out through the dayside magnetopause has been shown to be the dominant loss mechanism during the main phase of storms, and usually well into the recovery phase (Liemohn et al. 1999, 2001a; Kozyra and Liemohn 2003). It is also responsible for the adiabatic de-energization of the particles while they are still close to the Earth (Liemohn et al. 2002).

This central feedback loop between the hot plasma and the potential electric field is influenced by a number of other processes. One such process is the magnetic field perturbation caused by the inner magnetospheric currents (e.g., Liemohn 2003). These perturbations can be large (Tsyganenko 2002), altering the magnetic drift paths of the particles (another feedback loop). In addition, the magnetic field can explosively reconfigure, e.g., substorms, resulting in transient induced electric fields that can influence the particle flow. The induction electric field of substorms, however, has been shown to be a less efficient injection mechanism than the large-scale convective flow (e.g., Fok et al. 1999; Liemohn and Kozyra 2002).

Another factor is the plasmasphere. The plasmasphere is the cold, dense particle population in the inner magnetosphere that is slowly filled from the subauroral ionosphere (Lemaire and Gringauz 1998). Because this filling process is slow, only those field lines that co-rotate with the Earth (i.e., only those within a few R_E of the Earth) for several days can become populated with plasmaspheric material (e.g., Carpenter 1963). During storms, increased convection strips away the outer plasmasphere, shrinking its size and creating a large drainage plume that extends out to the dayside magnetopause (e.g., Rasmussen et al. 1993). Several processes occur in the overlap between the hot and cold particles. The first is Coulomb collisions, which cause scattering and energy transfer between the populations (e.g., Liemohn et al. 2000; Khazanov et al. 2000a, b). This energy transfer, particularly that from the hot O^+ to the cold electrons, is the cause of stable auroral red arcs (Kozyra et al. 1997). Another is the excitation of plasma waves. With the hot electrons, whistler waves are excited, resulting in plasmaspheric hiss (Liemohn 1967; Church and Thorne 1983), which can scatter the hot electrons and ions. With the hot ions, electromagnetic ion cyclotron (EMIC) waves are excited (e.g., Kennel and Petschek 1966; Khazanov et al. 2002, 2003b), which can interact with (i.e., scatter) the hot ions (Jordanova et al. 1997; Khazanov et al. 2002, 2003b) and electrons (Summers and Thorne 2003). Given the proper plasmaspheric conditions, these low-frequency EMIC waves can excite lower hybrid waves, which are very efficient at energizing the plasmasphere (e.g., Khazanov et al. 1996b, 2000a). This is an additional energy transfer process from the hot particles to the cold plasma via wave-wave interactions. Finally, the plasmasphere can deposit this excess energy (as well as some of its particles) back into the ionosphere, influencing the subauroral conductance.

Another source of hot electrons in the inner magnetosphere is photoionization and impact ionization of the upper thermosphere. Above ~ 250 km, warm electrons from the ionosphere can escape along the magnetic field lines (e.g., Takahashi 1973; Khazanov and Liemohn 1995, 2002). Some of these electrons are scattered and trapped in the magnetosphere, where they will drift around and eventually redeposit their energy into the ionosphere (e.g., Khazanov et al. 1996a, 1998). Because these particles can be either long or short lived in the magnetosphere, they are quite efficient at altering the energy deposition pattern into the ionosphere.

Presently there is no single technique that can provide a uniformly solvable valid description of the space plasma and the inner magnetosphere in particular. Simplified MHD equations have been used to model large-scale structures of the solar wind and planetary magnetospheres. Generalized transport equations are

traditionally used to describe mesoscale transport of plasmas of thermal origin. Simplified kinetic equations are used to describe the global transport properties of plasmas of nonthermal origin. Plasma simulations are used to describe small-scale microphysical processes, while Monte Carlo methods can be used to study both large-scale and small-scale plasma properties. Each method has advantages and disadvantages; therefore they should not be regarded as mutually exclusive, or competing approaches, but rather as complementary techniques.

The units used in the book are a mixture of MKSA and Gaussian because of the corresponding usage by practitioners in the field.

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Chapter 2

Kinetic Equations and Particle Collisions

The inner magnetospheric plasma is a unique composition of different plasma particles and waves. It covers a huge plasma energy range, with spatial and time variations of many orders of magnitude. In such a situation, the kinetic approach is the key element, and must be the starting point of a proper theoretical description of these inner magnetospheric plasma phenomena. The highly nonequilibrium inner magnetosphere particle and energy flows are characterized by large temperature differences between the interacting components and the flow conditions changing from collision-dominated to collisionless regimes. That is why the questions of how to handle the different kinds of kinetic equations, and the kinetic description of the particle collisions are very important. Therefore, it is instructive to trace the derivations of the different kinetic collision terms in order to understand the strengths and limitations of different approaches.

2.1 Kinetic and Maxwell Equations

Practically, the most comprehensive description of plasma is given by the velocity distribution function in the framework of the standard approach based on the Boltzmann–Vlasov kinetic equation. The distribution function $f_a(\mathbf{v}_a, \mathbf{r}_a, t)$ is defined so that the value $f_a(\mathbf{v}_a, \mathbf{r}_a, t)d\mathbf{v}_a d\mathbf{r}_a$ represents the number of particles of the sort a , at time t in a small volume $d\mathbf{r}_a$ with coordinates from $\mathbf{r}_a$ to $\mathbf{r}_a + d\mathbf{r}_a$ and velocities from $\mathbf{v}_a$ to $\mathbf{v}_a + d\mathbf{v}_a$. In other words, the distribution function f represents the density of particles of a given kind in the six-dimensional phase space $(\mathbf{v}_a, \mathbf{r}_a)$.

The distribution function is defined by the following kinetic equation

$$\frac{\partial f_a}{\partial t} + \mathbf{v}_a \nabla f_a + \frac{1}{m_a} \left\{ \mathbf{F}_a + e_a \left(\mathbf{E} + \frac{1}{c} [\mathbf{vB}] \right) \right\} \nabla_{\mathbf{v}} f_a = S_a + q_a. \quad (2.1.1)$$

Here, $\mathbf{E}$ and $\mathbf{B}$ are the total electric and magnetic fields, being the sum of the external and self-consistent (internal) parts produced by plasma particles; e_a and m_a

are the charge and mass of particles of the sort a ; $\mathbf{F}_a$ are forces of nonelectromagnetic nature; c is the speed of light; and $\nabla_{\mathbf{v}}$ is the gradient in the velocity space. The distribution function variations caused by particle collisions are formally described by the terms in the right-hand side of the kinetic equation denoted by S_a . These terms are called *collision integrals* and, in turn, depend on the distribution function. The last term in the kinetic equation, q_a , is the production of particles of the sort a .

The kinetic equation (2.1.1) is only complete if the electric and magnetic fields that exist in the plasma are known. Therefore, the Maxwell equations for electromagnetic field must be solved along with the kinetic equation (2.1.1). In plasma, these equations are given by

$$\begin{aligned}\nabla \cdot \mathbf{E} &= 4\pi(\rho + \rho_0), \quad \nabla \cdot \mathbf{B} = 0, \\ \nabla \times \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{B} = \frac{4\pi}{c} (\mathbf{j} + \mathbf{j}_0) + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t},\end{aligned}\tag{2.1.2}$$

where ρ_0 and $\mathbf{j}_0$ are the external charge density and current; ρ and $\mathbf{j}$ are the charge density and current produced by plasma particles and defined through the distribution function

$$\rho(\mathbf{r}, t) = \sum_a e_a \int f_a(\mathbf{v}_a, \mathbf{r}_a, t) d\mathbf{v}_a.\tag{2.1.3}$$

$$\mathbf{j}(\mathbf{r}, t) = \sum_a e_a \int \mathbf{v}_a f_a(\mathbf{v}_a, \mathbf{r}_a, t) d\mathbf{v}_a.\tag{2.1.4}$$

The summation in (2.1.3) and (2.1.4) extends over all kinds of charged particles present in plasma.

The kinetic equation is simplified substantially when particle collisions can be neglected. In that case, setting $S_a = 0$ in (2.1.1), we arrive to a collisionless approximation. The collisionless kinetic equation together with (2.1.2)–(2.1.4) was first considered by Vlasov (1938), and is called the kinetic equation with self-consistent field or the Vlasov equation. It is often used in the studies of plasma oscillations and stability (see, e.g., Akhiezer et al. 1975; Alexandrov et al. 1988; Stix 1992).

The use of the kinetic equation (2.1.1) in the study of various processes in plasma encounters major difficulties because of its complexity. That is why the class of problems that could be solved using the kinetic approach is quite limited. Besides, for a large number of problems, the detailed kinetic description is even excessive and only macroscopic parameters such as density, bulk velocity, pressure, viscosity tensor, and heat flux are important. These quantities are defined as corresponding velocity moments of the distribution function. Let us first give the definitions and a brief description of the basic macroscopic parameters of the plasma expressed through the moments of the distribution function.

The zero-order moment is the number density

$$n(\mathbf{r}, t) = \int f(\mathbf{v}, \mathbf{r}, t) d\mathbf{v}. \quad (2.1.5)$$

Here and throughout the book, whenever possible, we will omit the species subscript for simplicity.

The first moment is the macroscopic (hydrodynamic or directional) velocity

$$\mathbf{V}(\mathbf{r}, t) = \frac{1}{n} \int \mathbf{v} f(\mathbf{v}, \mathbf{r}, t) d\mathbf{v}, \quad (2.1.6)$$

where $\mathbf{v} = \mathbf{V} + \mathbf{w}$ is the total velocity, and $\mathbf{w}$ is intrinsic (chaotic) part of it, which, according to (2.1.6) satisfies the condition

$$\int \mathbf{w} f d\mathbf{v} = \mathbf{0}.$$

The second moment is the stress tensor and is defined as

$$P_{\alpha\beta}(\mathbf{r}, t) = m \int w_\alpha w_\beta f(\mathbf{v}, \mathbf{r}, t) d\mathbf{w}. \quad (2.1.7)$$

Finally, let us define the third-order moment, which is the heat flux tensor

$$S_{\alpha\beta\gamma}(\mathbf{r}, t) = m \int w_\alpha w_\beta w_\gamma f(\mathbf{v}, \mathbf{r}, t) d\mathbf{w}. \quad (2.1.8)$$

The kinetic description of plasma thus could be substituted by the system of equations for the velocity moments. These moment equations are usually called hydrodynamic or transport equations and are presented in [Chap. 4](#) of this book.

2.2 Elastic Collisions

Collisions play a very important role in the kinetics of the inner magnetosphere/plasmasphere/ionosphere coupling processes. They are responsible for the plasma production in these regions as well as for the energy and momentum transfer between the different plasma populations. The collisional processes can be either elastic or inelastic. This section presents a short description of binary *elastic collisions*. In such collisions, the mass, momentum, and energy of the colliding particles are conserved in the collision process.

For binary elastic collisions between a and b particles, the appropriate collision operator is the *Boltzmann collision integral*, which can be presented as

$$S_{ab}^0 = \iiint [f_a(\mathbf{v}') f_b(\mathbf{v}_1') - f_a(\mathbf{v}) f_b(\mathbf{v}) f_b(\mathbf{v}_1)] u I_{ab}(u, \chi) \sin \chi d\chi d\epsilon d\mathbf{v}_1. \quad (2.2.1)$$

Here $\mathbf{v}'$ and $\mathbf{v}'_1$ are the velocity of the a particle and of the b particle prior to the collision (their velocities after the impact are, respectively, $\mathbf{v}$ and $\mathbf{v}_1$); $I_{ab}(u, \chi)$ is differential scattering cross-section; $u = |\mathbf{v}' - \mathbf{v}'_1| \equiv |\mathbf{v} - \mathbf{v}_1|$ is the relative speed of the colliding particles a and b . The integration in (2.2.1) is over the velocities of the particle b (we call it particle 1 inside the collision integral) and over the scattering angles $d\Omega = \sin \chi d\chi d\varepsilon$, where χ is the angle between $\mathbf{v} - \mathbf{v}_1$ and $\mathbf{v}' - \mathbf{v}'_1$, or the center-of-mass scattering angle. In (2.2.1) the first term in the brackets corresponds to the particles scattered into a given region of velocity space, $\mathbf{v}$ (production term), and the second term corresponds to the particle scattering out of the same region of the velocity space (loss term).

The Boltzmann collision integral (2.2.1) can be applied to both self-collisions ($a = b$) and collisions between unlike particles. It can be applied to elastic electron–neutral and ion–neutral collisions, and to collisions between different neutral species. In addition, to a resonant charge exchange interaction between an ion and its parent neutral because the charge exchange process is pseudo–elastic. The net energy loss is small.

Although the Boltzmann collision integral (2.2.1) was originally intended to describe a gas of particles interacting through binary collisions, Landau in 1936 applied (2.2.1) to plasma and found the collision integral for the interacting charged particles. This result, known as the Landau equation, was obtained by retaining only the small-angle scattering events in the Boltzmann collision integral (2.2.1) and introducing a cut-off at the Debye shielding distance in the integral over the impact parameters, which would otherwise be logarithmically divergent for the Coulomb scattering law (Landau 1936). The Landau equation can be presented in differential form as (Gurevich 1978)

$$S_{ab} = -\frac{\partial}{\partial \mathbf{v}} \cdot \mathbf{j}_{ab}. \quad (2.2.2)$$

Here, $\mathbf{j}_{ab}$ is the particle flux in the velocity space due to the collisions between a and b particles. It is convenient to express it as

$$\mathbf{j}_{ab} = \frac{A_{ab}}{m_a} \int \left(\frac{\mathbf{I}}{u} - \frac{\mathbf{u}\mathbf{u}}{u^3} \right) \left[\frac{1}{m_b} f_a(\mathbf{v}) \frac{\partial f_b}{\partial \mathbf{v}_1} - \frac{1}{m_b} f_b(\mathbf{v}_1) \frac{\partial f_a}{\partial \mathbf{v}} \right] d\mathbf{v}_1. \quad (2.2.3)$$

Here the dyadic notation has been used for the second-rank tensors, with $\mathbf{I}$ being the unit dyadic, equivalent to the Kronecker delta in subscript notation. The following parameters were also introduced

$$A_{ab} = 2\pi e_a^2 e_b^2 \ln \Lambda; \quad \Lambda = \frac{\lambda_D}{b_0}; \quad \lambda_D = \left(\frac{T}{4\pi n_e e^2} \right)^{1/2}; \quad b_0 \equiv \frac{e^2}{3T}, \quad (2.2.4)$$

where $\ln \Lambda$ is the Coulomb logarithm, λ_D is the Debye shielding distance, and b_0 is a typical “distance of closest approach” for a thermal particle.

In some cases it is convenient to express $\mathbf{j}_{ab}$ with the help of functions G_b and H_b , the so-called *Rosenbluth potentials* (Shkarofsky et al. 1966),

$$\mathbf{j}_{ab}(\mathbf{v}) = \frac{c_{ab}}{m_a^2} \left(\frac{m_a}{m_b} \frac{\partial H_b}{\partial \mathbf{v}} f_a - \frac{\partial}{\partial \mathbf{v}} \frac{\partial}{\partial \mathbf{v}} G_b \frac{\partial f_a}{\partial \mathbf{v}} \right), \quad (2.2.5)$$

$$G_b(\mathbf{v}) = \int u f_b(\mathbf{v}_1) d\mathbf{v}_1; \quad H_b(\mathbf{v}) = 2 \int \frac{1}{u} f_b(\mathbf{v}_1) d\mathbf{v}_1, \quad (2.2.6)$$

where, as above, $\mathbf{u} = \mathbf{v} - \mathbf{v}_1$, $u = |\mathbf{u}|$.

The collision integrals (2.2.1) and (2.2.2) can be significantly simplified to describe collisions between space plasma electrons ($a = e$) and heavy ionospheric compositions, neutral particles ($b = n$) and ions ($b = i$). In an elastic impact of an electron against the heavy atom or molecule, the velocity of the electron can readily change direction, but the absolute value of the velocity or energy of the electron can change only insignificantly. Recognizing that the electron velocity is much larger than that of the atom or molecule, and assuming the Maxwellian background of the neutral atmospheric particles

$$f_n = n_n \left(\frac{M_n}{2\pi T_n} \right)^{3/2} \exp \left(-\frac{M_n v^2}{2T_n} \right), \quad (2.2.7)$$

the Boltzmann collision integral that describes elastic electron–neutral particle collisions can be presented as (Khazanov 1979)

$$\begin{aligned} S_{en}^0(E, \theta, \varphi) = n_n v \left\{ \frac{2m}{M_n} \frac{1}{E} \left[E^2 \sigma_{en}^{(1)} \left(f + T_n \frac{\partial f}{\partial E} \right) \right] \right. \\ \left. + \int I_{en}(E, \chi) f(E, \theta', \varphi') d\Omega' - \sigma_{en}^{(0)} f(E, \theta, \varphi) \right\}. \end{aligned} \quad (2.2.8)$$

To present this collision term, the spherical coordinate system in the velocity space has been introduced, $\mathbf{v} = \{v, \theta, \varphi\}$, with a polar axis along the magnetic field direction, and the connection between the modulus of electron velocity and its energy, $E = mv^2/2$. In this coordinate system, the angle θ is the electron pitch angle ($\mu = \cos \theta = \mathbf{v} \cdot \mathbf{B}/vB$), and angle φ is the azimuthal angle of the rotation around the magnetic field line. In this coordinate system, the $\mu_0 \equiv \cos \chi = \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\phi - \phi')$ relation between the angles in (2.2.8) holds. In addition, the following notations have been used for the transport ($\sigma_{en}^{(1)}$) and total ($\sigma_{en}^{(0)}$) cross-sections of the scattering of the electron on neutral particle

$$\sigma_{en}^{(1)}(E) = \int I_{en}(E, \chi) (1 - \cos \chi) d\Omega; \quad \sigma_{en}^{(0)}(E) = \int I_{en}(E, \chi) d\Omega. \quad (2.2.9)$$

Now the Landau equation (2.2.2) and expression (2.2.5) are used to simplify the description of the electron ion collisions. Assuming the Maxwellian background of the thermal ions and considering that the relative electron ion velocity u is approximately equal to the electron velocity v , Rosenbluth potentials (2.2.6) could be easily calculated and presented as (Shkarofsky et al. 1966)

$$G_i(\mathbf{v}) = v_i n_i \left[\frac{1}{\sqrt{\pi}} \exp(-w^2) + \left(w + \frac{1}{2w} \right) \text{erf}(w) \right]; \quad H_i(\mathbf{v}) = \frac{2n_i}{v_i w} \text{erf}(w). \quad (2.2.10)$$

Here, $w = v/v_i$; $v_i = \sqrt{2T_i/M_i}$ is the thermal ion velocity, and $\text{erf}(w) = 2/\sqrt{\pi} \int_0^w e^{-t^2} dt$ is the probability integral. After substituting (2.2.10) into the Landau collision term, the electron ion collision integral can be written as (Khazanov 1979)

$$S_{ei}^0(E, \theta, \varphi) = An_i \frac{v}{E} \left\{ \frac{m}{M_i} \frac{\partial}{\partial E} \left(f + T_i \frac{\partial f}{\partial E} \right) + \frac{1}{4E \sin \theta} \left[\frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{\sin \theta} \frac{\partial^2 f}{\partial \varphi^2} \right] \right\}. \quad (2.2.11)$$

It is also assumed here that $A = A_{ee} = A_{ei}$.

In the case of collisions between electrons, the collision integral is described by (2.2.2) and (2.2.5). It is necessary only to take into account that $a = b = e$. If the electron distribution function has a symmetry with respect to the magnetic field direction, then $f_e \equiv f(\mathbf{v}) = f(v, \theta)$, and the electron/electron collision term can be presented as

$$S_{ee}(E, \theta) = An_e \frac{v}{E} \left\{ \frac{\partial}{\partial E} \left(A_1 f + A_2 E \frac{\partial f}{\partial E} + \frac{1}{2} A_3 \frac{\partial f}{\partial \theta} \right) + \frac{1}{4E \sin \theta} \frac{\partial}{\partial \theta} \left[\sin \theta \left(2A_4 \frac{\partial f}{\partial \theta} + 2A_3 E \frac{\partial f}{\partial E} + A_5 \frac{\partial f}{\partial \theta} \right) \right] \right\}. \quad (2.2.12)$$

Here, the dimensionless coefficients A_k , following from (2.2.5) and (2.2.6), are determined by the equalities

$$\begin{aligned} A_1 &= -\frac{v^2}{2n_e} \frac{\partial H_e}{\partial v}, \quad A_2 = \frac{v}{n_e} \frac{\partial^2 G_e}{\partial v^2}, \quad A_3 = \frac{v}{n_e} \frac{\partial}{\partial v} \left(\frac{1}{v} \frac{\partial G_e}{\partial \theta} \right), \\ A_4 &= -\frac{v}{2n_e} \frac{\partial H_e}{\partial \theta}, \quad A_5 = \frac{1}{n_e} \left(\frac{1}{v} \frac{\partial^2 G_e}{\partial \theta^2} + \frac{\partial G_e}{\partial v} \right). \end{aligned} \quad (2.2.13)$$

2.3 Inelastic Collisions

The exact nature of the collision process depends both on the relative kinetic energy of the colliding particles and on the type of the particles. In general, however, for low energies, elastic collisions in the ionosphere dominate, but, as relative kinetic energy increases, inelastic collisions become progressively more important. The order of importance is from elastic to rotational, vibrational, and electronic excitation, and then to ionization as the relative kinetic energy increases. However, the different collision process may affect the continuity, momentum, and energy equations in different ways (see [Chap. 4](#) for details).

Inelastic collisions of electrons with molecules, atoms, and ions resulting in excitation of rotational, vibrational, and optical levels, ionization and recombination, are a very important mechanism of thermalization of the electron gas in the near-Earth space plasma. This section is mostly devoted to the inelastic description of electron–neutral collision processes, leaving some of the details of inelastic ion–neutral interactions for further chapters of this book.

As in a previous section, it is convenient to introduce the spherical coordinate system in the velocity space, $\mathbf{v} = \{v, \theta, \varphi\}$, with a polar axis along the magnetic field direction, and connect the modulus of electron velocity with its energy $E = mv^2/2$. To simplify the inelastic collision integrals structure, it is also convenient to transfer from the electron distribution function to the flux

$$\Phi(E, \theta, \varphi) = \frac{2Ef(E, \theta, \varphi)}{m^2}, \quad (2.3.1)$$

determined in such way that $\Phi(t, \mathbf{r}, E, \theta, \varphi)dEd\Omega$ is the quantity of electrons with energy from E to $E + dE$ and the direction of velocity $\mathbf{v}$ within the solid angle $d\Omega = \sin \theta d\theta d\varphi$, which intersect at point $(t, \mathbf{r})$ in a unit of time the unit of area.

When constructing a nonelastic integral of collision, it is convenient to start from an analogy with elastic collisions of electrons with neutral particles, which is described by an integral of collision in the Boltzmann form ([2.2.1](#)). The first term of this integral characterizes the entering of electrons in the current phase volume, and the second term characterizes electrons leaving from it. If one considers that the distribution of neutral particles in the velocity space is fixed, then the expression for the integral of collisions that describes the process of excitation by electron of certain level of molecules or atoms with energy E_k from the state that characterized energy $E_j < E_k$, can be presented in the form ([Khazanov 1979](#))

$$S_{en}^{ijk} = n_n^j \left[\iint W_{jk}(E' \rightarrow E, \chi, \varepsilon) \Phi(E', \theta', \varphi') dE' d\Omega \right. \\ \left. - \iint W_{jk}(E \rightarrow E', \chi, \varepsilon) \Phi(E, \theta, \varphi) dE' d\Omega \right], \quad (2.3.2)$$

where n_n^j is the density of neutral particles of the n sort that are in the state characterized by energy E_j ; and $W_{jk}(E \leftrightarrow E', \chi, \varepsilon)dE'd\Omega$ is the probability that electrons found in the phase volume close to $E', \theta', \varphi'(E, \theta, \varphi)$ have undergone a nonelastic collision characterized by angles of scattering χ and ε , which scatter the electrons to a current volume (from a current volume) of phase space.

From the expressions of dimensionality and the law of conservation of energy, the probability that characterizes the process of scattering in (2.3.2) can be presented in a form (Khazanov 1979)

$$\begin{aligned} W_{jk}(E' \rightarrow E, \chi, \varepsilon)dE'd\Omega &= I_{jk}^{en}(E', \chi)\delta(E' - E - E_{jk})dE'd\Omega, \\ W_{jk}(E \rightarrow E', \chi, \varepsilon)dE'd\Omega &= I_{jk}^{en}(E, \chi)\delta(E - E' - E_{jk})dE'd\Omega, \end{aligned} \quad (2.3.3)$$

where $I_{jk}^{en}(E, \chi)d\Omega$ is the differential electron cross-section, depending on the energy of the bombarding electron and the polar scattering angle; $\delta(x)$ is the delta function; and $E_{jk} = E_k - E_j > 0$ is the energy of excitation.

Then, substituting (2.3.3) in (2.3.2) and integrating according to dE' , we find

$$S_{en}^{ljk} = n_n^j \left[\int \Phi(E + E_{jk}, \theta', \varphi') I_{en}^{jk}(E + E_{jk}, \chi) d\Omega - \sigma_{en}^{jk}(E) \Phi(E, \theta, \varphi) \right]. \quad (2.3.4)$$

Here, $\sigma_{en}^{jk}(E)$ is the total cross-section of nonelastic scattering of an electron on a neutral particle of the n type, during which excitation of the latter occurs from the E_j level to the E_k

$$\sigma_{en}^{jk}(E) = \iint I_{en}^{jk}(E, \chi) \sin \chi d\chi d\varepsilon. \quad (2.3.5)$$

In a case of a collision of a second type (de-excitation) of energy of an excited state of an atom or molecule, a bombarding electron can be transmitted ($E_{jk} < 0$). Then, presenting arguments similar to the preceding, it is not difficult to find (Khazanov 1979)

$$S_{en}^{lkj} = n_n^k \left[\int \Phi(E - E_{jk}, \theta', \varphi') I_{en}^{kj}(E - E_{jk}, \chi) d\Omega - \sigma_{en}^{kj}(E) \Phi(E, \theta, \varphi) \right], \quad (2.3.6)$$

where n_n^k is the density of neutral particles of the n sort that are in the state characterized by energy E_k ; and I_{en}^{kj} , σ_{en}^{kj} are corresponding cross-sections of inverse transitions involving values I_{en}^{jk} and σ_{en}^{jk} by a principle of detailed equilibrium.

In this way, the integral of collisions that describes the process of excitation and deactivation of energy levels of neutral particles can be presented in the form

$$S_{en}^* = \sum_{j,k>j} (S_{en}^{ljk} + S_{en}^{lkj}). \quad (2.3.7)$$

Here, a summation of all possible transitions of neutral particles of the n is presented. The integral of collisions with the ions, S_{ei}^* , has a similar structure that, in a view of the small concentration of ions, can be ignored.

Now, the ionization collision integral that accounts for the electron–neutral collisions resulting in the ionization of the neutral particles will be constructed. It will be assumed that all such collisions result in single ionization, i.e., that only one electron is removed from the neutral atom or molecule. In this case, both the ejected and the scattered electron must be considered. Presenting considerations similar to those for constructing (2.3.2), we find (Khazanov 1979)

$$S_{en}^+ = n_n \left[\iint W_1(E' \rightarrow E, E_2, \chi_1, \varepsilon_1) \Phi(E', \theta', \varphi') dE' d\Omega_1 \right. \\ \left. + \iint W_2(E', E_2, \chi_2, \varepsilon_2) \Phi(E', \theta', \varphi') dE' d\Omega_2 \right. \\ \left. - \iint W(E, E', \chi_1, \varepsilon_1, \chi_2, \varepsilon_2) \Phi(E, \theta, \varphi) dE' d\Omega_1 d\Omega_2 \right]. \quad (2.3.8)$$

Here, $W_1 dE' d\Omega_1$ is the probability of transition of the bombarded electron in the energy interval from E to $E + dE$ (with simultaneous formation close to E_2 of a secondary electron) and scattering of it in the element of the solid angle $d\Omega_1 = \sin \chi_1 d\chi_1 d\varepsilon_1$; $W_2 dE' d\Omega_2$ is the probability of the fact that the bombarding electron formed in the energy interval from E to $E + dE$ a secondary electron with angles of scattering χ_2, ε_2 ; and $W dE' d\Omega_1 d\Omega_2$ is the corresponding probability of output of the electron from the element of phase volume considered.

Ionizing collision of a neutral particle and electron with energy E is usually characterized by differential cross-section of ionization, as a result of which, the bombarded electron is scattered in the element of the solid angle, $d\Omega_1$, and the secondary electron is formed with the energy of E_2 in the element of $d\Omega_2$ (Khazanov 1979)

$$I^{(1,2)}(E, E_2, \chi_1, \chi_2) dE_2 d\Omega_1 d\Omega_2. \quad (2.3.9)$$

Here, as in the case of nonelastic scattering (2.3.3) resulting in excitation, let us assume an angular dependence only on the polar angle of scattering.

Considering dimensionality of S_{en}^+ and the laws of conservation of energy, the corresponding probabilities of transitions in (2.3.8) can be presented in the form

$$W_1 dE' d\Omega_1 = I^{(1)}(E', E' - E - E^+, \chi_1) dE' d\Omega_1, \\ W_2 dE' d\Omega_2 = I^{(2)}(E', E, \chi_2) dE' d\Omega_2, \\ W dE' d\Omega_1 d\Omega_2 = I^{(1,2)}(E, E', \chi_1, \chi_2) dE' d\Omega_1 d\Omega_2, \quad (2.3.10)$$

where E^+ is the threshold of ionization, and $I^{(1)}$ and $I^{(2)}$ correspondingly equal

$$I^{(1)}(E, E_2, \chi_1) dE_2 d\Omega_1 = \int I^{(1,2)} d\Omega_2, \\ I^{(2)}(E, E_2, \chi_1) dE_2 d\Omega_2 = \int I^{(1,2)} d\Omega_1; \quad (2.3.11)$$

where $I^{(1,2)}$ is determined by (2.3.9).

Substituting (2.3.10) in (2.3.8) and conventionally assuming that the energy of the primary electron E_1 is larger than the energy of the secondary electron (Khazanov 1979), we find

$$S_{en}^+ = n_n \left\{ \int_{E+E_n^+}^{2E+E_n^+} \left[\int_0^{2\pi} \int_0^\pi I_n^+(E', E' - E - E_n^+, \chi_1) \Phi(E', \theta', \varphi') \sin \chi_1 d\chi_1 d\varepsilon_1 \right] dE' \right. \\ \left. + \int_{2E+E_n^+}^\infty \left[\int_0^{2\pi} \int_0^\pi I_n^+(E', E, \chi_2) \Phi(E', \theta', \varphi') \sin \chi_2 d\chi_2 d\varepsilon_2 \right] dE' - \sigma_n^+(E) \Phi(E, \theta, \varphi) \right\}. \quad (2.3.12)$$

Here,

$$\sigma_n^+(E) = \int_0^{(E-E_n^+)/2} I_n^+(E, E_2) dE_2 \quad (2.3.13)$$

is the total cross-section of ionization, and $I_n^+(E, E_2)$ is the corresponding differential cross-section

$$I_n^+(E, E_2) = \int I^{(1,2)} d\Omega_1 d\Omega_2. \quad (2.3.14)$$

In the case where, during ionization, an excitation of the ion occurs, the integral of collisions S_{en}^+ can be rewritten similarly to (2.3.12), introducing partial values that characterize excitation of the l level of the ion $I_{nl}^+, \sigma_{nl}^+, E_{nl}^+ = E_n^+ + E_{nl}$ (E_{nl} is the energy of excitation of the l level of the ion from its basic state), and making a summation of all the l levels possible in its state.

Finally, for integrals of collision, which describe the process of recombination of electrons and ions, S_{ei}^r , which only removes electrons from the current phase volume, the expression (Khazanov 1979)

$$S_{ei}^r = -n_i \sigma_i^r(E) \Phi(E, \theta, \varphi), \quad (2.3.15)$$

is used, where $\sigma_i^r(E)$ is the total recombination cross-section for an ion of the i sort.

2.4 Plasma Source

Let us now establish a clear form of the last component of the kinetic equation (2.1.1), q , for the case of electrons. The basic processes leading to formation of the ionosphere plasma at altitudes $h > 100$ km are processes of ionization of the L- and

K-shells of neutral components of the upper atmosphere by solar radiation with wavelengths $\lambda < 1,026 \text{ \AA}$ (middle latitudes) and electrons of magnetospheric origin poured out with energies $E \geq 1 \text{ KeV}$ (high latitudes).

The number of electrons with energies from E to $E + dE$ at point s of the geomagnetic field line that has formed during ionization of the external L-shells of neutral atoms and molecules of the upper atmosphere n_n by a solar source with wavelength λ and intensity Φ_λ per unit of volume per second, with the direction of velocity within the solid angle $d\mu d\varphi$, can be defined by

$$\begin{aligned} dq(s, E, \mu, \varphi) &\equiv q(s, E, \mu, \varphi) dE d\mu d\varphi \\ &= dE d\mu d\varphi \sum_{n,l} n_n(s) \Phi_\lambda(s, E + E_{nl}^+, \mu', \varphi') I_{nl}^\lambda(E + E_{nl}^+) d\mu_0 d\varepsilon_0. \end{aligned} \quad (2.4.1)$$

Here, $I_{nl}^\lambda(E + E_{nl}^+)$ is the differential cross-section of photoionization of photons with energies $hc/\lambda = E + E_{nl}^+$, as a result of which an ion forms at the l energy level; μ_0, ε_0 are angles that characterize the directions of motion of a photon and knocked-on electron; $\Phi_\lambda(\mu', \varphi') = \bar{\Phi}_\lambda \delta(\mu' - \mu_v) \delta(\varphi' - \varphi_v)$ is the unidirectional flux of photons; and $\theta_v = \arccos \mu_v, \varphi_v$ are angles that characterize its motion in relation to the magnetic field.

If the process of ionization is isotropic ($I_{nl}^\lambda = \sigma_{nl}^\lambda/4\pi$ for O_2 and N_2), then, on the basis of (2.4.1), the source of photoelectrons can be presented in the form

$$q(s, E) = \frac{1}{4\pi} \sum_{\lambda} \sum_n \sum_l n_n(s) \sigma_{nl}^\lambda(E + E_{nl}^+) \bar{\Phi}(s, E + E_{nl}^+), \quad (2.4.2)$$

where σ_{nl}^λ is the partial cross-section of photoionization.

For the atomic oxygen, according to the data of Kennedy and Manson (1972), when $E > 5 \text{ eV}$, the differential cross-section of ionization has an angular dependence

$$I_{nl}^\lambda(E + E_{nl}^+) = \frac{\sigma_{nl}^\lambda(E + E_{nl}^+)}{4\pi} [1 - P_2(\mu_0)]. \quad (2.4.3)$$

From the connection of angular variables in (2.4.1) with consideration of (2.4.3), we find the following expression for the source of photoelectrons during ionization of the external shell of the atomic oxygen

$$q(s, E, \mu) = \frac{[1 - P_2(\mu_v)P_2(\mu)/2]}{4\pi} n(O) \sum_{\lambda} \sum_n \sum_l \sigma_{nl}^\lambda(E + E_{nl}^+) \bar{\Phi}(s, E + E_{nl}^+). \quad (2.4.4)$$

Here, $\mu_v \equiv \cos \theta_v = -\sin \delta \cos(J + \Lambda) + \cos \delta \sin(J + \Lambda) \cos \Phi$; and δ, J, Λ , and Φ are solar declination, magnetic inclination, latitude, and hour angle, respectively.

The second source of ionosphere electrons whose calculation can be made similarly to (2.4.1) involves ionization of the K-shells of the atom. The internal vacancy formed here has a very small lifetime ($\tau \approx 10^{-14}$ s) (Avakyan et al. 1977) and is filled by an electron with more external shells. Here an X-ray quantum can be emitted or radiationless transition can occur with emission of an electron from one of the external electron shells (Auger effect), which carries off energy stored by the atomic system during primary ionization of the K-shell of the atom (Avakyan et al. 1977).

During ionization of the K-shells of atmospheric gases of nitrogen and oxygen in the initial system, radiationless Auger transitions occur basically and the output of X-ray fluorescence is very insignificant (Avakyan et al. 1977). Specifically, N₂ Auger electrons form a series of peaks from 315 to 367 eV, atomic oxygen forms Auger peaks from 474 to 509 eV, and the O₂ Auger electron peaks range from 456 to 507 eV.

If one assumes that angular distribution of the Auger electrons has an isotropic character, and filling of the appropriate energy intervals by the electrons occurs uniformly, then the expression for the source of electrons as the result of the Auger transition can be presented in the form

$$q(s, E) = \frac{1}{4\pi} \sum_{\lambda} \sum_n n_n(s) \sigma_{nl}^{\lambda}(E + E_{nl}^+) \bar{\Phi}(s, E + E_{nl}^+) \frac{\theta(E - E_{1n}) \theta(E_{2n} - E)}{E_{2n} - E_{1n}}, \quad (2.4.5)$$

where $\theta(x)$ is the unit function

$$\theta(x) = \begin{cases} 0, & \text{if } x < 0, \\ 1, & \text{if } x \geq 0, \end{cases}$$

and E_{1n} and E_{2n} are the lower and upper boundaries of the energy intervals where formation of the Auger electrons takes place.

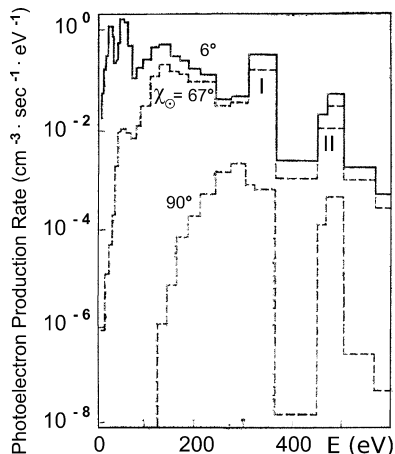
Altitude variations of photons in (2.4.2), (2.4.4), and (2.4.5) are determined by (Schunk and Nagy 2000)

$$\bar{\Phi}_{\lambda}(h) = \bar{\Phi}_{\lambda\infty} \exp \left[- \sum_n \sigma_n^{\lambda} \int_h^{\infty} n(h') ch(\chi_o, h') dh' \right], \quad (2.4.6)$$

where $\bar{\Phi}_{\lambda\infty}$ is the flux of photons with wavelength λ outside the atmosphere; σ_n^{λ} is the cross-section of absorption, and $ch(\chi_o, h)$ is the Chapman function. Note that, when calculating electrons along the geomagnetic field line, (2.4.6) usually is recalculated for the appropriate coordinates s of the magnetic force line.

If ionization occurs with ultraviolet and X-ray radiation of the sun, then the primary energy spectrum of the photoelectrons actually reflects the spectral distribution of solar radiation slightly distorted by the presence of numerous thresholds of ionization in atmospheric gases. Figure 2.1 illustrates the energy distribution of

Fig. 2.1 Energy distribution of the photoelectron source at the altitude 125 km with different zenith angles of the Sun



the electron source at an altitude of 125 km with different zenith angles of the sun. The character of the spectrum above 100 eV only slightly changes in the range of zenith angles from 0° to 70° . There is a broad maximum at 110–150 eV caused by the intense flux of solar in the region 75–100 Å, then, right up to 600 eV, one observes a constant drop on which there are two intensive peaks: I is the Auger electrons from a molecule of nitrogen and II is from an atom and molecule of oxygen. It is apparent from Fig. 2.1 that when $\chi_o = 90^\circ$, peak I blends at the base on the entire spectrum with peak 245–330 eV, and peak II increases greatly in relation to the remaining part of the spectrum. This is due to the strong absorption of the X-ray radiation, which is more intense in the range of wavelengths 31–20 Å; at the same time, radiation with wavelengths <20 Å (which the Auger transitions excite in oxygen) and above 31 Å (giving the base of the peak 245–300 eV) is even weaker because the corresponding cross-sections of photoabsorption of nitrogen and oxygen are fairly small.

If ionization of neutral component of the upper atmosphere occurs by flux of the precipitated electrons of magnetospheric origin, $\Phi_e(s, E, \mu)$, then the source of the secondary electron can be presented in the form (Khazanov 1979)

$$q(s, E, \mu) = \sum_n \sum_l n_n(s) \int_0^{2\pi} \int_{-1}^1 \int_{2E+E_{nl}^+}^{\infty} \Phi_e(s, E', \mu') I_{nl}^+(E', E, \mu_\chi) dE' d\mu_\chi d\varepsilon, \quad (2.4.7)$$

where I_{nl}^+ is the differential cross-section of ionization with electron impact of the L-shell of the atom or molecule.

For calculation of the spectrum of the secondary electrons, it is necessary to have the altitude distribution of precipitated electrons. This should be found from the solution of an appropriate electron kinetic equation. Some aspects of secondary

electron production and calculations of secondary electrons distribution functions are discussed in [Chaps. 5 and 7](#).

2.5 Superthermal Electron Kinetic Equation Simplification

Superthermal electrons are produced as a result of the ionization of neutral particles by solar radiation (photoelectrons) and by high-energy electrons of magnetospheric origin (secondary electrons). They transform to thermal electrons, thus losing their energy due to the excitation and ionization of neutral particles, or transferring it to thermal electrons. Superthermal electrons play an important role in the kinetics of microprocesses of ionospheric plasma. They are usually invoked to account for the thermal plasma heating and the intense excitation of energy levels of atoms and molecules, which gives rise to the various atmospheric emissions and to increased rates of some aeronomical reactions. For superthermal electrons, the most obvious simplification in the kinetic description is made by recognizing that the magnetic field dominates the motion of these particles. In a background magnetic field, charged particles gyrate about the magnetic lines of force, and the frequency of this gyration, $\Omega = qB/m$, is very fast for electrons (10^4 – 10^7 rad s $^{-1}$). Therefore, the foremost reduction of (2.1.1) is to average over this gyrational motion. In this case, the velocity space reduces to two variables, and instead of following the individual particles in configuration space, the equation solves for the motion of their gyrational guiding center. Yet another simplification that is made possible by the magnetic dominance is that most spatial transport is along the magnetic field lines, and therefore cross-field drifts can be omitted in a first-order calculation. Under these assumptions, (2.1.1) can be rewritten as the field-aligned, guiding-center kinetic equation (Khazanov 1979; Khazanov et al. 1994)

$$\frac{\beta}{\sqrt{E}} \frac{\partial \Phi}{\partial t} + \mu \frac{\partial \Phi}{\partial s} - \frac{1 - \mu^2}{2} \left(\frac{1}{B} \frac{\partial B}{\partial s} - \frac{F}{E} \right) \frac{\partial \Phi}{\partial \mu} + EF\mu \frac{\partial \Phi}{\partial E} = \langle S_e \rangle + \langle q_e \rangle, \quad (2.5.1)$$

which calculates the time-dependent distribution of the differential flux function, $\Phi = 2Ef/m^2$, as a function of time, distance along the field line, energy, and pitch angle. In (2.5.2), t is time; s is the distance along the field line; E is the particle energy; and μ is the cosine of the pitch angle. The inhomogeneity of the geomagnetic field, B , is included, as well as other forces, such as electric fields, in F . $\langle S_e \rangle$ and $\langle q_e \rangle$ are averaged over the azimuth angle; the superthermal electron collision terms, representing interactions with thermal electrons and ions, elastic and inelastic scattering with neutral particles that have been described in Sects. 2.2 and 2.3; and the superthermal electron source presented in the previous section. Combining all collision terms, the kinetic equation (2.5.2) can be written as (Khazanov 1979; Khazanov et al. 1994).

$$\begin{aligned}
& \frac{\beta}{\sqrt{E}} \frac{\partial \Phi}{\partial t} + \mu \frac{\partial \Phi}{\partial s} - \frac{1 - \mu^2}{2} \left(\frac{1}{B} \frac{\partial B}{\partial s} - \frac{F}{E} \right) \frac{\partial \Phi}{\partial \mu} + EF\mu \frac{\partial \Phi}{\partial E} \\
& = An_e \left\{ \frac{\partial}{\partial E} \left[\left(1 + \frac{m}{M_i} + 2 \frac{m}{M_n} \frac{\sigma_{en}^{tr} E^2 n_n}{An_e} \right) \frac{\Phi}{E} \right. \right. \\
& \quad \left. \left. + \left(T_e + T_i \frac{m}{M_i} + 2 \frac{m}{M_n} T_n \frac{\sigma_{en}^{tr} E^2 n_n}{An_e} \right) \frac{\partial}{\partial E} \left(\frac{\Phi}{E} \right) \right] \right. \\
& \quad \left. + \frac{1}{2E^2} \frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial \Phi}{\partial \mu} \right] \right\} + n_n \left\{ \int_0^{2\pi} \int_{-1}^1 I_{en}(E, \mu_\chi) \Phi(\mu') \sin \chi d\chi d\varepsilon \right. \\
& \quad \left. + \sum_{j,k>j} [\Delta_n^j \sigma_{en}^{jk}(E + E_{jk}) \Phi(E + E_{jk}) + \Delta_n^k \sigma_{en}^{kj}(E - E_{jk}) \Phi(E - E_{jk})] \right. \\
& \quad \left. + \int_{E+E_n^+}^{2E+E_n^+} I_n^+(E, E' - E - E_n^+) \Phi(E') dE' \right. \\
& \quad \left. + \frac{1}{2\pi} \int_{2E+E_n^+}^{\infty} I_n^+(E, E') \left[\int_0^{2\pi} \Phi(E', \sqrt{1 - \mu^2} \cos \varepsilon) d\varepsilon \right] dE' \right. \\
& \quad \left. - \left[\sigma_{en} + \sum_{j,k>j} \left(\Delta_n^j \sigma_{en}^{jk} + \Delta_n^k \sigma_{en}^{kj} + \sigma_n^+ + \frac{n_i}{n_n} \sigma_i^r \right) \right] \Phi \right\} + q. \tag{2.5.2}
\end{aligned}$$

Here, a summation is made over all types of ions and neutral particles; for shortening of the equation, only the current arguments for all functions have been used, and the following notations are introduced

$$\Delta_n^{j(k)} = \frac{n_n^{j(k)}}{n_n}, \quad \sigma_{en}(E) = \int_0^{2\pi} \int_{-1}^1 I_{en}(E, \mu_\chi) d\mu_\chi d\varepsilon.$$

When describing Coulomb interaction of superthermal electrons with charged particles, the Landau collision integrals have been presented in a linearized form, assuming that the thermal electrons and ions can be described by the Maxwellian distribution function (see Sect. 2.2). Expressions describing inelastic collisions are also derived using a number of well-known approximations (Khazanov 1979; Khazanov et al. 1994), assuming that the dissipating (initial) electron does not change direction (is forward scattered), and if ionization occurs, that the secondary electron is emitted perpendicular to the velocity of the initial electron before the collision.

Equation (2.5.2) can be further simplified depending on its application to the different space plasma phenomena. This is discussed in the proceeding chapters.

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Chapter 3

General Description of Wave–Particle Interaction Phenomena

Wave–particle interactions play fundamental roles in the dynamics and energetics of the inner magnetosphere. To understand these phenomena in detail, some initial knowledge of plasma-wave electrodynamics is required and is presented in Sects. 3.1–3.8. This short introductory material is based on excellent plasma physics books written by Ginzburg and Rukhadze (1975), Akhiezer et al. (1975), Alexandrov et al. (1988), and Stix (1992). The focus in this chapter is on some fundamental modes that can propagate in a magnetized plasma and, specifically, on those that will be considered in later chapters. All the derivations and results presented here, however, are given in a very general form, with a major focus on the kinetic plasma description, that will allow us to use this material for the different wave plasma phenomena in the near-Earth’s plasma. In the final two sections of this chapter, the quasilinear plasma theory and nonlinear drift-kinetic equation are discussed in order to prepare the reader for later material covered in this book.

3.1 Plasma Electrodynamics

The Maxwell equations describing the electromagnetic field in the plasma must contain induced (self-consistent) charges and currents. These equations are for the plasma as for any medium:

$$\begin{aligned}\nabla \times \mathbf{B} &= \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} (\mathbf{j} + \mathbf{j}_0), & \nabla \cdot \mathbf{B} &= 0, \\ \nabla \times \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, & \nabla \cdot \mathbf{E} &= 4\pi(\rho + \rho_0),\end{aligned}\tag{3.1.1}$$

where $\mathbf{B}$ is the magnetic induction and $\mathbf{E}$ is the electric field; $\mathbf{j}$ and ρ are the densities of the current and the space charge densities induced in the medium, respectively; and $\mathbf{j}_0$ and ρ_0 are current and charge densities of the external field sources.

The influence of the medium (or plasma in our case) is characterized by the presence of one more vector quantity compared with the vacuum case, $\mathbf{j}$. It is convenient to introduce the vector quantity $\mathbf{D}$, called the vector of electric displacement, through

$$\mathbf{D}(t, \mathbf{r}) = \mathbf{E}(t, \mathbf{r}) + 4\pi \int_{-\infty}^t dt' \mathbf{j}(t', \mathbf{r}). \quad (3.1.2)$$

Using this definition and the continuity equation:

$$\frac{\partial(\rho + \rho_0)}{\partial t} + \nabla \cdot (\mathbf{j} + \mathbf{j}_0) = 0, \quad (3.1.3)$$

one can eliminate the induced charge and current densities from the system of equations (3.1.1) for the electromagnetic field in the medium and write

$$\begin{aligned} \nabla \times \mathbf{B} &= \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c} \mathbf{j}_0, & \nabla \cdot \mathbf{B} &= 0, \\ \nabla \times \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, & \nabla \cdot \mathbf{D} &= 4\pi \rho_0. \end{aligned} \quad (3.1.4)$$

Exercise 3.1. Using (3.1.1) verify (3.1.3).

The system of equations (3.1.1) or (3.1.4) is not closed, unless the relation of the densities of the induced currents and charges to the electric field $\mathbf{E}$ (or the relation of $\mathbf{D}$ to $\mathbf{E}$) is given. The problem of any physical model is to establish such a relation. In particular, we can obtain it for the plasma from the equations of motion of the particles. However, by general reasoning (independent of a specific model for the medium), one can state that in linear electrodynamics this relation must have the form

$$j_i(t, \mathbf{r}) = \int_{-\infty}^t dt' \int d\mathbf{r}' \sigma_{ij}(t, t', \mathbf{r}, \mathbf{r}') E_j(t', \mathbf{r}') \quad (3.1.5)$$

and

$$D_i(t, \mathbf{r}) = \int_{-\infty}^t dt' \int d\mathbf{r}' \varepsilon_{ij}(t, t', \mathbf{r}, \mathbf{r}') E_j(t', \mathbf{r}'), \quad (3.1.6)$$

respectively. The indices i and j refer to the three space dimensions.

These relations, called the *material equations of linear electrodynamics*, account for the dependence of the currents and charges induced in the medium at a given

moment t and a space point $\mathbf{r}$ upon the field values at all previous times and at any point of the medium. This is how time (frequency) and space (wavelength) dispersions in the medium appear. Physically the frequency dispersion is related to the inertia of particles and relaxation processes in the medium. Spatial dispersion concerns the propagation of the field action from one point of the medium to another due to transport processes or thermal motion of particles. The functions $\sigma_{ij}(t, t', \mathbf{r}, \mathbf{r}')$ and $\varepsilon_{ij}(t, t', \mathbf{r}, \mathbf{r}')$, called influence functions, are the kernels of the integrals (3.1.5) and (3.1.6), characterizing the efficiency of field action transfer from one space-time point to another. The problem within physical models of the medium is to find explicit expressions for these functions.

Let us now analyze the properties of the functional relations (3.1.5) and (3.1.6) without referring to a specific plasma model. We consider the medium homogeneous in space and time. In this case, the kernels of (3.1.5) and (3.1.6) depend on $t - t'$ and $\mathbf{r} - \mathbf{r}'$ only. It follows

$$j_i(t, \mathbf{r}) = \int_{-\infty}^t dt' \int d\mathbf{r}' \sigma_{ij}(t - t', \mathbf{r} - \mathbf{r}') E_j(t', \mathbf{r}') \quad (3.1.7)$$

$$D_i(t, \mathbf{r}) = \int_{-\infty}^t dt' \int d\mathbf{r}' \varepsilon_{ij}(t - t', \mathbf{r} - \mathbf{r}') E_j(t', \mathbf{r}'). \quad (3.1.8)$$

By applying the Fourier expansion, the electromagnetic field in the medium may be presented as the sum of plane monochromatic waves of the type

$$\mathbf{E}(t, \mathbf{r}) = \mathbf{E}(\omega, \mathbf{k}) \exp(-i\omega t + i\mathbf{k} \cdot \mathbf{r}), \quad (3.1.9)$$

where ω is the frequency and $\mathbf{k}$ the wave vector. A corresponding dependency is valid for all other field quantities. Then (3.1.7) reads

$$j_i(\omega, \mathbf{k}) = \int_{-\infty}^t dt' \int d\mathbf{r}' \sigma_{ij}(t - t', \mathbf{r} - \mathbf{r}') \exp[-i\omega(t' - t) + i\mathbf{k} \cdot (\mathbf{r}' - \mathbf{r})] E_j(\omega, \mathbf{k}) \quad (3.1.10)$$

or

$$j_i(\omega, \mathbf{k}) = \int_0^{\infty} dt_1 \int d\mathbf{r}_1 \sigma_{ij}(t_1, \mathbf{r}_1) \exp[i\omega t_1 - i\mathbf{k} \cdot \mathbf{r}_1] E_j(\omega, \mathbf{k}), \quad (3.1.11)$$

with $t_1 = t - t'$, $\mathbf{r}_1 = \mathbf{r} - \mathbf{r}'$. Therefore, we immediately get the relation between the amplitudes $j_i(\omega, \mathbf{k})$ and $E_i(\omega, \mathbf{k})$:

$$j_i(\omega, \mathbf{k}) = \sigma_{ij}(\omega, \mathbf{k}) E_j(\omega, \mathbf{k}), \quad (3.1.12)$$

where

$$\sigma_{ij}(\omega, \mathbf{k}) = \int_0^{\infty} dt_1 \int d\mathbf{r}_1 \sigma_{ij}(t_1, \mathbf{r}_1) \exp[i\omega t_1 - i\mathbf{k} \cdot \mathbf{r}_1]. \quad (3.1.13)$$

The quantity $\sigma_{ij}(\omega, \mathbf{k})$ is called the *tensor of complex conductivity* of the medium.

The *tensor of complex dielectric permittivity* (*dielectric tensor*) can be introduced in the same way. From (3.1.8) it follows that

$$D_i(\omega, \mathbf{k}) = \varepsilon_{ij}(\omega, \mathbf{k}) E_j(\omega, \mathbf{k}), \quad (3.1.14)$$

where

$$\varepsilon_{ij}(\omega, \mathbf{k}) = \int_0^{\infty} dt_1 \int d\mathbf{r}_1 \varepsilon_{ij}(t_1, \mathbf{r}_1) \exp[i\omega t_1 - i\mathbf{k} \cdot \mathbf{r}_1] \quad (3.1.15)$$

holds.

The dependency of these tensors on ω defines the frequency (time) dispersion and on the wave vector $\mathbf{k}$ the spatial dispersion of the electromagnetic field in the medium. Taking into account (3.1.13) and (3.1.15), one can easily establish from the electric displacement (3.1.2) the following relation between the tensors $\sigma_{ij}(\omega, \mathbf{k})$ and $\varepsilon_{ij}(\omega, \mathbf{k})$:

$$\varepsilon_{ij}(\omega, \mathbf{k}) = \delta_{ij} + \frac{4\pi i}{\omega} \sigma_{ij}(\omega, \mathbf{k}). \quad (3.1.16)$$

δ_{ij} is the unit tensor. Naturally, we suppose $\omega \neq 0$.

It must be noted that the tensors $\sigma_{ij}(t, \mathbf{r})$ and $\varepsilon_{ij}(t, \mathbf{r})$ are real functions of their variables, because they connect the real quantities $\mathbf{E}(t, \mathbf{r})$, $\mathbf{j}(t, \mathbf{r})$, and $\mathbf{D}(t, \mathbf{r})$. The tensors $\sigma_{ij}(\omega, \mathbf{k})$ and $\varepsilon_{ij}(\omega, \mathbf{k})$, however, appear as complex functions even when their variables ω and $\mathbf{k}$ are real. The reality of $\sigma_{ij}(t, \mathbf{r})$ and $\varepsilon_{ij}(t, \mathbf{r})$ results in the following relations for $\sigma_{ij}(\omega, \mathbf{k})$ and $\varepsilon_{ij}(\omega, \mathbf{k})$:

$$\begin{aligned} \varepsilon_{ij}(\omega, \mathbf{k}) &= \varepsilon_{ij}^*(-\omega, -\mathbf{k}) \\ \operatorname{Re}\{\varepsilon_{ij}(\omega, \mathbf{k})\} &= \operatorname{Re}\{\varepsilon_{ij}(-\omega, -\mathbf{k})\} \\ \operatorname{Im}\{\varepsilon_{ij}(\omega, \mathbf{k})\} &= -\operatorname{Im}\{\varepsilon_{ij}(-\omega, -\mathbf{k})\}. \end{aligned} \quad (3.1.17)$$

Here, $\operatorname{Re}\{\varepsilon_{ij}(\omega, \mathbf{k})\}$ and $\operatorname{Im}\{\varepsilon_{ij}(\omega, \mathbf{k})\}$ are the real and imaginary parts of the tensor $\varepsilon_{ij}(\omega, \mathbf{k})$, respectively.

For isotropic media, the dielectric tensor simplifies further. In this case, rotational invariance requires that the second rank tensor is composed of δ_{ij} and $k_i k_j$. Consequently, for the isotropic medium, the tensor $\varepsilon_{ij}(\omega, \mathbf{k})$ can be written as

$$\varepsilon_{ij}(\omega, \mathbf{k}) = \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) \varepsilon^{tr}(\omega, k) + \frac{k_i k_j}{k^2} \varepsilon^{lo}(\omega, k), \quad (3.1.18)$$

meaning that among the nine tensor components $\varepsilon_{ij}(\omega, \mathbf{k})$ only two components $\varepsilon^{tr}(\omega, k)$ and $\varepsilon^{lo}(\omega, k)$ are independent. These are called the *transverse and longitudinal dielectric permittivity*, respectively, because the tensors $k_i k_j / k^2$ and $(\delta_{ij} - k_i k_j / k^2)$ are projections into the directions parallel and perpendicular to the wave vector $\mathbf{k}$. Therefore, the quantity $\varepsilon^{lo}(\omega, k)$ characterizes electromagnetic properties of the medium with respect to the longitudinal field and $\varepsilon^{tr}(\omega, k)$ characterizes those with respect to the transverse field.

For isotropic media, the tensor of complex conductivity takes the same form

$$\sigma_{ij}(\omega, \mathbf{k}) = \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) \sigma^{tr}(\omega, k) + \frac{k_i k_j}{k^2} \sigma^{lo}(\omega, k). \quad (3.1.19)$$

Relation (3.1.16) holds for both components separately

$$\varepsilon^{tr, lo}(\omega, \mathbf{k}) = 1 + \frac{4\pi i}{\omega} \sigma^{tr, lo}(\omega, \mathbf{k}). \quad (3.1.20)$$

The reality conditions (3.1.17) for the isotropic medium read

$$\begin{aligned} \operatorname{Re}\{\varepsilon^{tr, lo}(\omega, \mathbf{k})\} &= \operatorname{Re}\{\varepsilon^{tr, lo}(-\omega, -\mathbf{k})\}, \\ \operatorname{Im}\{\varepsilon^{tr, lo}(\omega, \mathbf{k})\} &= -\operatorname{Im}\{\varepsilon^{tr, lo}(-\omega, -\mathbf{k})\}. \end{aligned} \quad (3.1.21)$$

It is obvious that the tensor $\sigma_{ij}(\omega, \mathbf{k})$ satisfies reality conditions of the same form.

3.2 Energy of the Electromagnetic Field

Proceeding with the study of the general properties of the tensor of dielectric permittivity in a material medium, we consider the problem of electromagnetic field energy. External forces producing a field in a medium naturally change its energy due to the interaction of the electromagnetic field with the sources. To calculate this work, we form the scalar product of the first equation of the system (3.1.4) with $\mathbf{E}$, of the third one with $\mathbf{B}$, and take the difference. As a result, we obtain

$$\frac{1}{4\pi} \left(\mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} + \mathbf{B} \cdot \frac{\partial \mathbf{B}}{\partial t} \right) = -\frac{c}{4\pi} \nabla \cdot [\mathbf{E} \times \mathbf{B}] - \mathbf{E} \cdot \mathbf{j}_0. \quad (3.2.1)$$

Integrating this relation over a volume V , confined by the surface S , and using the Gauss theorem, we find

$$\frac{1}{4\pi} \int_V d\mathbf{r} \left(\mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} + \mathbf{B} \cdot \frac{\partial \mathbf{B}}{\partial t} \right) = -\frac{c}{4\pi} \oint d\mathbf{S} [\mathbf{E} \times \mathbf{B}] - \int_V d\mathbf{r} \mathbf{E} \cdot \mathbf{j}_0. \quad (3.2.2)$$

Exercise 3.2. Verify (3.2.1) and (3.2.2).

Passing to the limit of unbounded medium and taking into account that the fields $\mathbf{E}$ and $\mathbf{B}$ vanish at infinity, we may neglect the first term on the right-hand side of (3.2.2). The second term describes the work A of the field against the external sources per unit time

$$\frac{dA}{dt} = \int_V d\mathbf{r} \mathbf{E} \cdot \mathbf{j}_0. \quad (3.2.3)$$

This work is balanced by a change in the field energy W . It follows

$$\frac{dW}{dt} = -\frac{dA}{dt} = - \int_V d\mathbf{r} \mathbf{E} \cdot \mathbf{j}_0 = \frac{1}{4\pi} \int_V d\mathbf{r} \left(\mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} + \mathbf{B} \cdot \frac{\partial \mathbf{B}}{\partial t} \right). \quad (3.2.4)$$

Next we consider the case in which the fields $\mathbf{E}$, $\mathbf{B}$, and $\mathbf{D}$ are monochromatic plane waves $\exp(-i\omega t + i\mathbf{k} \cdot \mathbf{r})$. Taking into account the reality of the functions $\mathbf{E}(t, \mathbf{r})$, $\mathbf{B}(t, \mathbf{r})$, and $\mathbf{D}(t, \mathbf{r})$, we write

$$\mathbf{E}(t, \mathbf{r}) = \mathbf{E}(\omega, \mathbf{k}) e^{-i\omega t + i\mathbf{k} \cdot \mathbf{r}} + \mathbf{E}^*(\omega, \mathbf{k}) e^{i\omega t - i\mathbf{k} \cdot \mathbf{r}}. \quad (3.2.5)$$

The quantities $\mathbf{B}$ and $\mathbf{D}$ are written analogously. Substituting these expressions into (3.2.4) and averaging over the time, we obtain

$$\frac{d\overline{W}}{dt} = \frac{i\omega}{4\pi} \int_V d\mathbf{r} (\mathbf{E} \cdot \mathbf{D}^* - \mathbf{E}^* \cdot \mathbf{D}) = \frac{i\omega}{4\pi} V (\mathbf{E} \cdot \mathbf{D}^* - \mathbf{E}^* \cdot \mathbf{D}), \quad (3.2.6)$$

where V contains the medium.

Exercise 3.3. Verify (3.2.6).

Note that by using (3.2.4) for the monochromatic fields of type (3.2.5), which do not vanish at infinity, strictly speaking, we are acting inconsistently. One can show, however, that the surface integral on the right-hand side of (3.2.2) does not increase as fast as the volume integral in the limit $V \rightarrow \infty$, so that it can be neglected.

Substituting (3.1.14) into (3.2.6), we obtain an expression for the average energy, dissipated in the medium per unit time

$$Q = \frac{d\bar{W}}{dt} = \frac{i\omega}{4\pi} \int_V d\vec{r} [\varepsilon_{ij}^*(\omega, \mathbf{k}) - \varepsilon_{ji}(\omega, \mathbf{k})] E_i E_j^* = \frac{i\omega}{4\pi} V [\varepsilon_{ij}^*(\omega, \mathbf{k}) - \varepsilon_{ji}(\omega, \mathbf{k})] E_i E_j^*. \quad (3.2.7)$$

Exercise 3.4. Verify (3.2.7), using the properties of the dielectric tensor (3.1.17).

From (3.2.7) it follows directly that the quantity of heat delivered per unit volume and unit time is

$$\frac{Q}{V} = \frac{i\omega}{4\pi} [\varepsilon_{ij}^*(\omega, \mathbf{k}) - \varepsilon_{ji}(\omega, \mathbf{k})] E_i E_j^*. \quad (3.2.8)$$

This equation allows us to draw an important conclusion: in a medium with a Hermitian dielectric tensor for real ω and $\mathbf{k}$, i.e., when $\varepsilon_{ij}^*(\omega, \mathbf{k}) = \varepsilon_{ji}(\omega, \mathbf{k})$ holds, heat is not generated. This implies that a plane monochromatic wave is not absorbed in such a medium. Thus, it follows that the absorption of the electromagnetic field in the medium is related to the anti-Hermitian part of the dielectric tensor, i.e., when $\varepsilon_{ij}^*(\omega, \mathbf{k}) = -\varepsilon_{ji}(\omega, \mathbf{k})$.

Equation (3.2.8) takes a very simple form and allows further conclusions regarding the properties of the dielectric tensor for the isotropic medium when the tensor $\varepsilon_{ij}(\omega, \mathbf{k})$ is expressed by (3.1.18). In this case, we obtain from (3.2.8)

$$\frac{Q}{V} = \frac{\omega}{2\pi k^2} \left(\text{Im}\{\varepsilon^{lo}(\omega, \mathbf{k})\} |\mathbf{k} \cdot \mathbf{E}|^2 + \text{Im}\{\varepsilon^{tr}(\omega, \mathbf{k})\} |[\mathbf{k} \times \mathbf{E}]|^2 \right), \quad (3.2.9)$$

where $\text{Im}\{\varepsilon^{lo}(\omega, \mathbf{k})\}$ and $\text{Im}\{\varepsilon^{tr}(\omega, \mathbf{k})\}$ are the imaginary parts of the components $\varepsilon^{lo}(\omega, \mathbf{k})$ and $\varepsilon^{tr}(\omega, \mathbf{k})$ with real ω and $\mathbf{k}$. The first term of this expression defines the absorption of the longitudinal field in the medium ($\mathbf{E} \parallel \mathbf{k}$), and the second term that of the transverse field ($\mathbf{E} \perp \mathbf{k}$).

From (3.2.9), an important property of the tensor $\varepsilon_{ij}(\omega, \mathbf{k})$ for the isotropic medium in thermodynamic equilibrium follows. It is clear that in this case any electromagnetic wave must be absorbed. From $Q > 0$ it follows that in thermodynamic equilibrium

$$\text{Im}\{\varepsilon^{lo}(\omega, \mathbf{k})\} > 0, \quad \text{Im}\{\varepsilon^{tr}(\omega, \mathbf{k})\} > 0 \quad (3.2.10)$$

must be fulfilled for positive frequencies $\omega > 0$.

The violation of the inequalities (3.2.10) means a change in sign of Q , i.e., an energy transfer from the medium to the electromagnetic field. In such an active medium, some random field fluctuation can increase with time thus decreasing

the energy of the medium. Obviously, this is only possible if the medium is in a nonequilibrium state. We note here that the violation of any of the inequalities (3.2.10) is not sufficient for the development of an instability and for the increase of electromagnetic field energy in the medium. In the range where $\text{Im}\{\varepsilon^{lo}\} \leq 0$ and $\text{Im}\{\varepsilon^{tr}\} \leq 0$ holds, electromagnetic waves must be able to propagate in the medium.

Project 3.1. Obtain an expression for the average energy, dissipated in the medium per unit time, when the electromagnetic field in the medium consists of a superposition of monochromatic components, i.e., instead of (3.2.5), we have

$$\mathbf{E}(t, \mathbf{r}) = \mathbf{E}(\omega, \mathbf{k}, t)e^{-i\omega t + i\mathbf{k} \cdot \mathbf{r}} + \mathbf{E}^*(\omega, \mathbf{k}, t)e^{i\omega t - i\mathbf{k} \cdot \mathbf{r}}$$

where function $\mathbf{E}(\omega, \mathbf{k}, t)$ is a function slowly varying in time.

3.3 Electromagnetic Waves

From vacuum electrodynamics, it is well known that monochromatic electromagnetic waves $\exp(-i\omega t + i\mathbf{k} \cdot \mathbf{r})$ can exist in the absence of external sources. In a vacuum, ω and $\mathbf{k}$ have real values, connected by the relation

$$\omega = kc. \quad (3.3.1)$$

An equation connecting the frequency with the wave vector $\omega(\mathbf{k})$ is called a *dispersion equation*. Equation (3.3.1) is an example valid for electromagnetic waves in vacuum. Now we investigate the dispersion equation $\omega(\mathbf{k})$ for electromagnetic waves in media. If the medium is nonabsorbing, ω and $\mathbf{k}$ are real quantities. The tensor $\varepsilon_{ij}(\omega, \mathbf{k})$ is Hermitian for this case. For absorbing media, they become complex.

Considering nontrivial solutions of the field equations (3.1.1) when the external sources are absent and assuming the field to depend on time and space coordinates in the form $\exp(-i\omega t + i\mathbf{k} \cdot \mathbf{r})$, we can write these equations as follows

$$\begin{aligned} [\mathbf{k} \times \mathbf{B}]_i &= -\frac{\omega}{c} \varepsilon_{ij}(\omega, \mathbf{k}) E_j, \quad \mathbf{k} \cdot \mathbf{B} = 0, \\ [\mathbf{k} \times \mathbf{E}] &= \frac{\omega}{c} \mathbf{B}, \quad k_i \varepsilon_{ij}(\omega, \mathbf{k}) E_j = 0. \end{aligned} \quad (3.3.2)$$

By elimination of $\mathbf{B}$, a system of three homogeneous equations for the components of the field $\mathbf{E}$ is easily derived, i.e.,

$$\left[k^2 \delta_{ij} - k_i k_j - \frac{\omega^2}{c^2} \varepsilon_{ij}(\omega, \mathbf{k}) \right] E_j = 0. \quad (3.3.3)$$

Exercise 3.5. Verify (3.3.3).

A nontrivial solution of system (3.3.3) exists only if

$$\Lambda = \left| k^2 \delta_{ij} - k_i k_j - \frac{\omega^2}{c^2} \varepsilon_{ij}(\omega, \mathbf{k}) \right| = 0 \quad (3.3.4)$$

holds, where Λ is the determinant of the coefficient in (3.3.3). Equation (3.3.4) is the dispersion equation, relating the frequency ω to the wave vector $\mathbf{k}$. For an isotropic medium with a dielectric tensor of the form (3.1.18), the system (3.3.3) decomposes into two equations

$$\varepsilon^{lo}(\omega, \mathbf{k}) \mathbf{E}^{lo} = 0, \quad \left[k^2 - \frac{\omega^2}{c^2} \varepsilon^{tr}(\omega, \mathbf{k}) \right] \mathbf{E}^{tr} = 0, \quad (3.3.5)$$

where $\mathbf{E}^{lo} = \mathbf{k}(\mathbf{k} \cdot \mathbf{E})/k^2$ is the component of the electric field $\mathbf{E}$ parallel to the wave vector $\mathbf{k}$, i.e., the longitudinal field, and $\mathbf{E}^{tr} = \mathbf{E} - \mathbf{E}^{lo}$ is the electric field component perpendicular to the vector $\mathbf{k}$, i.e., the transverse field.

The dispersion equation for an isotropic medium consequently consists of two separate equations

$$\varepsilon^{lo}(\omega, \mathbf{k}) = 0, \quad k^2 - \frac{\omega^2}{c^2} \varepsilon^{tr}(\omega, \mathbf{k}) = 0 \quad (3.3.6)$$

giving the condition of existence for longitudinal and transverse waves, respectively.

Denoting the roots of the dispersion equation (3.3.4) by $\omega_n(\mathbf{k})$, we find the following dependence of the field on time:

$$\mathbf{E}(t, \mathbf{k}) \sim \sum_n C_n \exp[-i\omega_n(\mathbf{k})t], \quad (3.3.7)$$

i.e., the electric field is a superposition of plane waves with frequencies defined by (3.3.4).

The roots $\omega_n(\mathbf{k})$ of the dispersion equation generally are complex. The sign of the imaginary part shows whether the oscillations with the corresponding frequencies $\text{Re}\{\omega_n\}$ increase or decrease with time. When, as for the stable system, all the imaginary parts $\text{Im}\{\omega_n(\mathbf{k})\}$ are negative, then the terms of (3.3.7) decrease with time. The quantity $\delta_n = |\text{Im}\{\omega_n(\mathbf{k})\}|$ is called the *damping decrement* of the corresponding monochromatic perturbation. Under these conditions, after an initial interval of time, the summand with the minimal damping decrement contributes dominantly to the field.

If among the roots of the dispersion equation, there is a root with $\text{Im}\{\omega_n(\mathbf{k})\} = 0$, then the corresponding term of (3.3.7) describes a long-lived natural oscillation in the medium.

Finally, if there is a root with $\text{Im}\{\omega_n(\mathbf{k})\} > 0$, then the corresponding mode will increase with time. This is possible only when the medium is unstable. The quantity

$\delta_n = \text{Im}\{\omega_n(\mathbf{k})\}$ then is called the *increment of oscillations* or *growth rate*. Note that the appearance of roots with a positive imaginary part is a sufficient condition for the instability of the medium.

As already stated, the dispersion equation (3.3.4) establishes the relation between ω and $\mathbf{k}$ for the electromagnetic waves that can exist in the medium, the so-called eigenmodes. It determines the complex frequencies $\omega(\mathbf{k})$ for a given real value $\mathbf{k}$ or, in other words, the spectrum of natural oscillations of the medium. On the other hand, for any real value ω we can find a projection of the complex wave vector $\mathbf{k}(\omega)$ in the directions of space. These two approaches correspond to two different formulations of the problem of solving the integro-differential equations for the electromagnetic field. The first case corresponds to the initial value problem in which the time development is calculated from a known initial state of the electromagnetic field in the medium. The latter corresponds to the boundary value problem in which the spatial dependency of the field in the medium is determined from known values on some surface.

3.4 Dielectric Tensor of Collisionless Plasma

The presence of thermal motion of electrons and ions in the plasma leads to the appearance of qualitatively new features when electromagnetic waves propagate in a magneto-active plasma. First of all, there appear a number of new branches of oscillations in a magneto-active plasma due to the thermal motion of the particles in the plasma; these are not present in a cold plasma. When there is no magnetic field, taking into account the thermal motion of the electrons in a plasma with cold ions and hot electrons leads to the appearance of only one extra branch of oscillations, namely, the ion-sound waves. Secondly, in such plasmas the interaction of resonance particles with the electric field of the wave leads to Landau damping of the wave if we are dealing with a plasma whose initial state is an equilibrium state. Such an effect also occurs in a magneto-active plasma. However, besides this, yet another damping mechanism arises in a magneto-active plasma: cyclotron damping, which is connected with the emission and absorption of electromagnetic waves by the charged particles in the plasma, which move along spirals in the magnetic field at frequencies that are equal to the cyclotron frequency or multiples of it.

To study these effects, we start from the kinetic equations for the particles in the plasma with self-consistent interactions. First, we use the kinetic theory to obtain the dielectric permittivity tensor for a plasma in a magnetic field, and then we use that tensor to study the propagation of waves in a magneto-active plasma, taking the thermal motion of electrons and ions into account.

We recall that when there are no oscillations, the distribution function f_{x0} of the plasma particles of the a -th kind satisfies the equation

$$\frac{e_x}{m_x c} \left([\mathbf{v} \times \mathbf{B}_0] \cdot \frac{\partial f_{x0}}{\partial \mathbf{v}} \right) = S_x, \quad (3.4.1)$$

from which it follows that $f_{\alpha 0}$ is a Maxwell distribution function

$$f_{\alpha 0} = n_0 \left(\frac{m_\alpha}{2\pi T} \right)^{3/2} \exp \left(-\frac{m_\alpha v^2}{2T} \right)$$

where T is the plasma temperature and n_0 the equilibrium particle density.

In the case of a high-temperature plasma, the relaxation time is very large and we can consider a quasi-equilibrium state in which the distribution function varies slowly due to collisions and satisfies a zeroth approximation (3.4.1) without the collision integral

$$\frac{e_\alpha}{m_\alpha c} \left([\mathbf{v} \times \mathbf{B}_0] \cdot \frac{\partial f_{\alpha 0}}{\partial \mathbf{v}} \right) = -\Omega_{B\alpha} \frac{\partial f_{\alpha 0}}{\partial \varphi} = 0. \quad (3.4.2)$$

Exercise 3.6. Verify (3.4.2).

From this it follows that the initial distribution function $f_{\alpha 0}$ is an arbitrary function of two variables, $v_{\parallel}$ and $v_{\perp}$, which are the velocity components parallel and at right angles to the magnetic field, and is independent of the azimuthal angle, φ , in velocity space (see Fig. 3.1),

$$f_{\alpha 0} = f_{\alpha 0}(v_{\parallel}, v_{\perp}).$$

Let us now consider small oscillations of the plasma around an initial state described by the distribution function $f_{\alpha 0}$. If the oscillations are high-frequency oscillations, $\Omega_{B\alpha} \tau \gg 1$, then the kinetic equation for the distribution function F_α

$$\frac{\partial F_\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial F_\alpha}{\partial \mathbf{r}} - \frac{e}{m} \left(\mathbf{E} + \frac{1}{c} [\mathbf{v} \times (\mathbf{B} + \mathbf{B}_0)] \right) \cdot \frac{\partial F_\alpha}{\partial \mathbf{v}} = S_\alpha$$

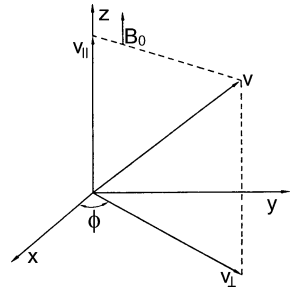


Fig. 3.1 Coordinate system in the velocity space

can be considered collisionless ($S_\alpha = 0$) and F_α can be written

$$F_\alpha(\mathbf{r}, \mathbf{v}, t) = f_{\alpha 0}(v_{\parallel}, v_{\perp}) + f_\alpha(\mathbf{r}, \mathbf{v}, t),$$

where f_α is a small oscillating correction to $f_{\alpha 0}$. Linearizing the kinetic equation, we get

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial f_\alpha}{\partial \mathbf{r}} - \frac{e_\alpha}{m_\alpha} \left(\mathbf{E} + \frac{1}{c} [\mathbf{v} \times \mathbf{B}] \right) \cdot \frac{\partial f_{\alpha 0}}{\partial \mathbf{v}} - \Omega_{B\alpha} \frac{\partial f_\alpha}{\partial \varphi} = 0. \quad (3.4.3)$$

This equation, together with the Maxwell equations for the variable fields $\mathbf{E}$ and $\mathbf{B}$, determines the linear oscillations of a magneto-active plasma. Because of the linearity of (3.4.3) and the equations of the perturbing fields $\mathbf{E}$ and $\mathbf{B}$, it is sufficient to restrict our considerations to the case of plane waves $\propto \exp[i(\mathbf{k}\mathbf{r}) - i\omega t]$ if we want to find the dispersion equation, and to find by means of the kinetic equation the dielectric permittivity tensor ε_{ij} .

In the case of plane monochromatic waves, (3.4.3) becomes

$$-i[\omega - (\mathbf{k}\mathbf{v})]f_\alpha - \Omega_{B\alpha} \frac{\partial f_\alpha}{\partial \varphi} = -\frac{e_\alpha}{m_\alpha} \left\{ \mathbf{E} \left(1 - \frac{(\mathbf{k}\mathbf{v})}{\omega} \right) + \frac{\mathbf{k}(\mathbf{k}\mathbf{E})}{\omega} \right\} \cdot \frac{\partial f_{\alpha 0}}{\partial \mathbf{v}}. \quad (3.4.4)$$

Exercise 3.7. Using $\mathbf{B} = (c/\omega)[\mathbf{k} \times \mathbf{E}]$, verify (3.4.4).

Integrating (3.4.4), we find

$$\begin{aligned} f_\alpha = & \frac{e_\alpha}{m_\alpha \Omega_{B\alpha}} \exp \left\{ \frac{i}{\Omega_{B\alpha}} \int_0^\varphi (\mathbf{k}\mathbf{v} - \omega) d\varphi' \right\} \\ & \times \left[\int_0^\varphi \exp \left\{ -\frac{i}{\Omega_{B\alpha}} \int_0^{\varphi'} (\mathbf{k}\mathbf{v} - \omega) d\varphi'' \right\} \left(\left\{ \mathbf{E} \left(1 - \frac{(\mathbf{k}\mathbf{v})}{\omega} \right) + \frac{\mathbf{k}(\mathbf{k}\mathbf{E})}{\omega} \right\} \cdot \frac{\partial f_{\alpha 0}}{\partial \mathbf{v}} \right) d\varphi' + C \right], \end{aligned} \quad (3.4.5)$$

where the constant $C = C(v_{\parallel}, v_{\perp})$, which is independent of φ and must be determined from the periodicity conditions $f_\alpha(\varphi + 2\pi) = f_\alpha(\varphi)$, is equal to

$$\begin{aligned} C = & \left\{ \frac{i}{\Omega_{B\alpha}} \int_0^{2\pi} (\mathbf{k}\mathbf{v} - \omega) d\varphi \right\} \\ & \times \int_0^{2\pi} d\varphi \left(\left\{ \mathbf{E} \left(1 - \frac{(\mathbf{k}\mathbf{v})}{\omega} \right) + \frac{\mathbf{k}(\mathbf{k}\mathbf{E})}{\omega} \right\} \cdot \frac{\partial f_{\alpha 0}}{\partial \mathbf{v}} \right) \left\{ -\frac{i}{\Omega_{B\alpha}} \int_0^\varphi (\mathbf{k}\mathbf{v} - \omega) d\varphi' \right\}. \end{aligned} \quad (3.4.6)$$

It is convenient to write (3.4.5) in a slightly different form. To do this, we introduce a coordinate system in which $k_y = 0$, so that $\mathbf{k} = (k_x, 0, k_z) = (k \sin \theta, 0, k \cos \theta)$; we then have

$$\int_0^\varphi (\mathbf{k}\mathbf{v} - \omega) d\varphi' = (k_z v_{\parallel} - \omega)\varphi + k_x v_{\perp} \sin \varphi. \quad (3.4.7)$$

Substituting this expression into (3.4.6) and putting the following series expansion under the integral sign:

$$e^{-i\lambda \sin \varphi} = \sum_{l=-\infty}^{+\infty} J_l(\lambda) e^{-il\varphi}, \quad (3.4.8)$$

where $J_l(\lambda)$ are Bessel functions and $\lambda = k_x v_{\perp} / \Omega_{Bz}$, we get the following expression for f_α

$$f_\alpha = \frac{ie_\alpha}{m_\alpha} e^{i\lambda \sin \varphi} \sum_{l=-\infty}^{+\infty} \frac{\mathbf{a} \cdot \mathbf{E}}{k_z v_{\parallel} + l\Omega_{Bz} - \omega} e^{-il\varphi}, \quad (3.4.9)$$

where

$$\begin{aligned} a_x &= \left[\frac{k_z v_{\perp}}{\omega} \frac{\partial f_{\alpha 0}}{\partial v_{\parallel}} + \left(1 - \frac{k_z v_{\parallel}}{\omega} \right) \frac{\partial f_{\alpha 0}}{\partial v_{\perp}} \right] \frac{l}{\lambda} J_l(\lambda) \\ a_y &= \left[\frac{k_z v_{\perp}}{\omega} \frac{\partial f_{\alpha 0}}{\partial v_{\parallel}} + \left(1 - \frac{k_z v_{\parallel}}{\omega} \right) \frac{\partial f_{\alpha 0}}{\partial v_{\perp}} \right] i J'_l(\lambda) \\ a_z &= \frac{\partial f_{\alpha 0}}{\partial v_{\parallel}} J_l(\lambda) + \left[\frac{k_x v_{\parallel}}{\omega} \frac{\partial f_{\alpha 0}}{\partial v_{\perp}} - \frac{k_x v_{\perp}}{\omega} \frac{\partial f_{\alpha 0}}{\partial v_{\parallel}} \right] \frac{l}{\lambda} J_l(\lambda). \end{aligned}$$

It can be seen from (3.4.9) that, in the case of weakly damped oscillations, the perturbation of the distribution function has a steep maximum when

$$v_{\parallel} \approx v_{\parallel res} \equiv \frac{\omega(k) - l|\Omega_{Bz}|}{k_z}, \quad l = 0, \pm 1, \pm 2, \dots \quad (3.4.10)$$

We will call particles with such a velocity *resonance* particles.

Putting $l = 0$ in (3.4.10), we get the relation

$$\omega(k) = k_z v_{\parallel}, \quad (3.4.11)$$

which is the condition for Cherenkov resonance between a wave moving along the magnetic field $\mathbf{B}_0$ with a phase velocity $\omega(k)/k_z$ and a resonance particle with a

velocity component along the magnetic field $v_{||}$. The absorption (excitation) of waves by the resonance particles is called Cherenkov absorption (excitation) of waves.

We can interpret condition (3.4.10) when $l \neq 0$ as follows. We can consider a charged particle moving in the plasma along a spiral with a velocity $v_{||}$ along the magnetic field $\mathbf{B}_0$ to be an oscillator that, in its eigenframe of reference, has the eigenfrequencies $|\Omega_{Bz}|, 2|\Omega_{Bz}|, 3|\Omega_{Bz}|, \dots$. The resonance condition (3.4.10) for $l = 0, 1, 2, \dots$,

$$\omega(k) = l|\Omega_{Bz}| + k_z v_{||}, \quad (3.4.12)$$

is nothing but the condition that the wave frequency and the frequency of the oscillator are equal, taking into account the Doppler shift $k_z v_{||}$. The phase velocity of the wave along $\mathbf{B}_0$, which is equal to $\omega(k)/k_z$, is larger than the particle velocity $v_{||}$ in this case and the Doppler effect is the normal one. We will call the absorption (excitation) of waves by these resonance particles cyclotron absorption (excitation) under normal Doppler effect conditions.

When $l = 0, -1, -2, \dots$, the condition (3.4.10)

$$\omega(k) = k_z v_{||} - l|\Omega_{Bz}| \quad (3.4.13)$$

also means that the wave frequency and the eigenfrequency of the oscillator in the laboratory frame of reference are equal, but when $l < 0$, the phase velocity of the wave along $\mathbf{B}_0$ is less than particle velocity $v_{||}$, that is, the Doppler effect is anomalous. Correspondingly, we will call the absorption (excitation) of waves by these resonance particles cyclotron absorption (excitation) under anomalous Doppler effect conditions.

Using (3.4.9) for f_α , we can find the current

$$\mathbf{j} = \sum_\alpha e_\alpha \int \mathbf{v} f_\alpha d\mathbf{v} = \sum_\alpha e_\alpha \int d\mathbf{v} \cdot \mathbf{v} \left\{ \frac{ie_\alpha}{m_\alpha} e^{i\lambda \sin \varphi} \sum_{l=-\infty}^{+\infty} \frac{\mathbf{a} \cdot \mathbf{E}}{k_z v_{||} + l\Omega_{B\alpha} - \omega} e^{il\varphi} \right\} \quad (3.4.14)$$

and then, using Ohm's law, $j_i = \sigma_{ij} E_j$, to calculate the conductivity tensor σ_{ij} of a magnetized plasma. In the frame of reference in which $k_y = 0$, this tensor has the form

$$\begin{aligned} \sigma_{ij} = -i \sum_\alpha \frac{e_\alpha^2}{m_\alpha \omega} \left\{ \sum_l \int d\mathbf{v} \left(\frac{\omega - k_{||} v_{||}}{v_\perp} \frac{\partial f_{\alpha 0}}{\partial v_\perp} + k_{||} \frac{\partial f_{\alpha 0}}{\partial v_{||}} \right) \frac{\Pi_{ij}^l(\mathbf{v})}{\omega - k_z v_{||} - l|\Omega_{B\alpha}|} \right. \\ \left. - b_i b_j \int d\mathbf{v} \left(f_{\alpha 0} + \frac{v_{||}^2}{v_\perp} \frac{\partial f_{\alpha 0}}{\partial v_\perp} \right) \right\}. \end{aligned} \quad (3.4.15)$$

Accordingly, the tensor of dielectric permittivity ε_{ij} can be written as

$$\varepsilon_{ij} = \delta_{ij} + \sum_{\alpha=e,i} \frac{4\pi e_\alpha^2}{m_\alpha \omega^2} \left\{ \sum_{l=-\infty}^{+\infty} \int d\mathbf{v} \left(\frac{\omega - k_{\parallel} v_{\parallel}}{v_{\perp}} \frac{\partial f_{\alpha 0}}{\partial v_{\perp}} + k_{\parallel} \frac{\partial f_{\alpha 0}}{\partial v_{\parallel}} \right) \frac{\Pi_{ij}^l(\mathbf{v})}{\omega - k_z v_{\parallel} - l |\Omega_{B\alpha}|} - b_i b_j \int d\mathbf{v} \left(f_{\alpha 0} + \frac{v_{\parallel}^2}{v_{\perp}} \frac{\partial f_{\alpha 0}}{\partial v_{\perp}} \right) \right\}, \quad (3.4.16)$$

where

$$\Pi_{ij}^l(\mathbf{v}) = \begin{pmatrix} \frac{l^2 \Omega_{B\alpha}^2}{k_x^2} J_l^2 & i v_{\perp} \frac{l \Omega_{B\alpha}}{k_x} J_l J_l' & v_{\parallel} \frac{l \Omega_{B\alpha}}{k_x} J_l^2 \\ -i v_{\perp} \frac{l \Omega_{B\alpha}}{k_x} J_l J_l' & v_{\perp} J_l'^2 & -i v_{\parallel} v_{\perp} J_l J_l' \\ v_{\parallel} \frac{l \Omega_{B\alpha}}{k_x} J_l^2 & i v_{\parallel} v_{\perp} J_l J_l' & v_{\parallel}^2 J_l^2 \end{pmatrix}, \quad (3.4.17)$$

$J_l = J_l(\lambda)$ and $J_l'(\lambda) = dJ_l/d\lambda$ are Bessel functions and their derivatives, $\lambda = k_x v_{\perp} / \Omega_{B\alpha}$, and $\vec{b} = \vec{B}_0 / B_0$ is a unit vector in the direction of the external magnetic field.

Exercise 3.8. Calculate σ_{zy} and σ_{zz} components of conductivity tensor.

Exercise 3.9. Using (3.4.16), derive the tensor of dielectric permittivity ε_{ij} for an isotropic distribution function.

3.5 Maxwellian Plasma

If the initial state of the plasma is an equilibrium state, that is, it is determined by a Maxwellian velocity distribution for the particles

$$f_{\alpha 0} = n_0 \left(\frac{m_\alpha}{2\pi T} \right)^{3/2} \exp \left(-\frac{m_\alpha v^2}{2T} \right),$$

one can greatly simplify (3.4.16) for the dielectric permittivity tensor. One can in that case perform the integration over $v_{\perp}$ using the well-known expression

$$\int_0^\infty e^{-p^2 t^2} J_l(at) J_l(bt) t dt = \frac{1}{2p^2} \exp \left(-\frac{a^2 + b^2}{4p^2} \right) I_l \left(\frac{ab}{2p^2} \right),$$

where $|\arg p| < \pi/4$ and where $I_l(x)$ is a modified Bessel function.

Secondly, the integrals over $v_{\parallel}$ are reducible to the probability integral

$$\int_C dv_{\parallel} \frac{k_z \exp(-v_{\parallel}^2/2v_{\alpha}^2)}{k_z v_{\parallel} - (\omega - l\Omega_{B\alpha})} = i\pi W(z_l), \quad (3.5.1)$$

where

$$W(z_l) = \exp(-z_l^2) \left[\frac{k_z}{|k_z|} + \frac{2i}{\sqrt{\pi}} \int_0^{z_l} e^{-t^2} dt \right] \quad (3.5.2)$$

$$z_l = \frac{\omega - l\Omega_{B\alpha}}{\sqrt{2}k_z v_{\alpha}}, \quad v_{\alpha} = \sqrt{\frac{T_{\alpha}}{m_{\alpha}}}.$$

Let us now examine some limiting cases of (3.5.2). If $|z| \ll 1$, we have

$$W(z) \approx 1 + \frac{2iz}{\sqrt{\pi}} + \dots \quad (3.5.3)$$

If $|z| \gg 1$, the following asymptotic expansion is valid

$$W(z) \approx \frac{i}{\sqrt{\pi}z} \left(1 + \frac{1}{2z^2} + \frac{3}{4z^4} + \dots \right) + e^{-z^2}. \quad (3.5.4)$$

As a result we obtain the expressions for the components of the dielectric permittivity tensor of a magneto-active plasma

$$\varepsilon_{ij}(\mathbf{k}, \omega) = \delta_{ij} + \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega^2} \left[\sum_{l=-\infty}^{+\infty} i\sqrt{\pi} z_0 W(z_l) \Pi_{ij}^{(\alpha)} + 2z_0^2 b_i b_j \right], \quad (3.5.5)$$

where

$$\Pi_{ij} = \begin{pmatrix} \frac{l^2}{a_{\alpha}} A_l(a_{\alpha}) & ilA'_l(a_{\alpha}) & \sqrt{2/a_{\alpha}} l z_l A_l(a_{\alpha}) \\ -ilA'_l & \frac{l^2}{a_{\alpha}} J_l^2 A_l(a_{\alpha}) - 2a_{\alpha} A'_l(a_{\alpha}) & -i\sqrt{2a_{\alpha}} z_l A'_l(a_{\alpha}) \\ \sqrt{2/a_{\alpha}} l z_l A_l(a_{\alpha}) & i\sqrt{2a_{\alpha}} z_l A'_l(a_{\alpha}) & 2z_l^2 A_l(a_{\alpha}) \end{pmatrix}$$

$$A_l(x) = e^{-x} I_l(x), \quad A'_l(x) = \frac{dA_l}{dx}, \quad \sqrt{a_{\alpha}} = \frac{k_x v_{\alpha}}{\Omega_{B\alpha}}, \quad \text{sgn} \sqrt{a_{\alpha}} = \frac{e_{\alpha}}{|e_{\alpha}|}.$$

Exercise 3.10. Calculate ε_{yy} and ε_{zz} components of the dielectric permittivity tensor.

We note that the tensor ε_{ij} is non-Hermitian. This is connected with the presence of absorption caused by the interaction with resonance particles. We note further that the following relations hold

$$\varepsilon_{21} = -\varepsilon_{12}, \quad \varepsilon_{31} = \varepsilon_{13}, \quad \varepsilon_{32} = -\varepsilon_{23}.$$

Knowing the components of the dielectric permittivity tensor, we can write down the dispersion equation for electromagnetic waves in a magneto-active plasma (see also (3.3.4)) in a form

$$\Lambda = \text{Det}|\Lambda_{ij}| = An^4 + Bn^2 + C = 0, \quad (3.5.6)$$

where $n = kc/\omega$ is the refractive index and

$$\begin{aligned} A &= \varepsilon_{11}\sin^2\theta + 2\varepsilon_{13}\sin\theta\cos\theta + \varepsilon_{33}\cos^2\theta \\ B &= -\varepsilon_{11}\varepsilon_{33} - (\varepsilon_{22}\varepsilon_{33} + \varepsilon_{23}^2)\cos^2\theta - (\varepsilon_{11}\varepsilon_{22} + \varepsilon_{12}^2)\sin^2\theta \\ &\quad + 2(\varepsilon_{12}\varepsilon_{23} - \varepsilon_{22}\varepsilon_{13})\sin\theta\cos\theta + \varepsilon_{13}^2 \\ C &= \text{Det}|\varepsilon_{ij}| = \varepsilon_{33}(\varepsilon_{11}\varepsilon_{22} + \varepsilon_{12}^2) + \varepsilon_{11}\varepsilon_{23}^2 + 2\varepsilon_{12}\varepsilon_{13}\varepsilon_{23} - \varepsilon_{22}\varepsilon_{13}^2. \end{aligned} \quad (3.5.7)$$

Exercise 3.11. Verify (3.5.6) and (3.5.7).

Here we will give only the expression for the coefficient A in the case of a plasma with a Maxwellian particle velocity distribution

$$A(\mathbf{k}, \omega) = 1 + \sum_{\alpha=e,i} \frac{\omega_{p\alpha}^2}{k^2 v_\alpha^2} \left[1 + i\sqrt{\pi} z_0 \sum_{l=-\infty}^{+\infty} e^{-a_\alpha} I_l(a_\alpha) W(z_l) \right]. \quad (3.5.8)$$

It is easy to see that $A(\mathbf{k}, \omega)$ coincides with the “longitudinal” dielectric permittivity of the magneto-active plasma

$$A(\mathbf{k}, \omega) = \frac{k_i k_j}{k^2} \varepsilon_{ij}(\mathbf{k}, \omega) = 0. \quad (3.5.9)$$

This equation corresponds to electrostatic perturbations with the magnetic field much smaller than the electric field.

3.6 Cold Plasma Approximation

The detailed analysis of the oscillations of a collisionless magneto-active plasma is very complex because the number of the solution branches is practically infinitely large compared with the number of modes existing in the unmagnetized plasma. Therefore, we will deal with the most interesting and thoroughly studied oscillation types only. To begin with, we entirely neglect the thermal motion of the particles. In this cold plasma approximation, the following inequalities holds:

$$\frac{k_\perp v_\alpha}{\Omega_{B\alpha}} \ll 1, \quad \frac{k_z v_\alpha}{\omega} \ll 1, \quad \frac{\omega \pm l\Omega_{B\alpha}}{k_z v_\alpha} \gg 1, \quad (3.6.1)$$

where $V_\alpha = \sqrt{T_\alpha/m_\alpha}$.

In this case, the dielectric tensor (3.5.5) has the relatively simple form

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_\perp & ig & 0 \\ -ig & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix}. \quad (3.6.2)$$

The elements of ε_{ij} are

$$\begin{aligned} \varepsilon_\perp &= \varepsilon_{xx} = \varepsilon_{yy} = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2 - \Omega_{B\alpha}^2}, & \varepsilon_\parallel &= \varepsilon_{zz} = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2}, \\ \varepsilon_{xy} &= -\varepsilon_{yx} = ig = i \sum_\alpha \frac{\omega_{p\alpha}^2 \Omega_{B\alpha}}{(\omega^2 - \Omega_{B\alpha}^2)}, & \varepsilon_{xz} &= \varepsilon_{zx} = \varepsilon_{yz} = \varepsilon_{zy} = 0. \end{aligned} \quad (3.6.3)$$

Exercise 3.12. Verify (3.6.2) and (3.6.3) based on (3.5.5).

Because dissipative processes are neglected in the cold plasma approximation, the tensor (3.6.2) is Hermitian, i.e., $\varepsilon_{ij} = \varepsilon_{ji}^*$.

Substituting (3.6.2) and (3.6.3) into the wave equation

$$\left[k^2 \delta_{ij} - k_i k_j - \frac{\omega^2}{c^2} \varepsilon_{ij}(\omega) \right] E_j = 0, \quad (3.6.4)$$

we obtain

$$\begin{aligned} \left(k_z^2 - \frac{\omega^2}{c^2} \varepsilon_\perp \right) E_x - i \frac{\omega^2}{c^2} g E_y - k_\perp k_z E_z &= 0 \\ i \frac{\omega^2}{c^2} g E_x + \left(k^2 - \frac{\omega^2}{c^2} \varepsilon_\perp \right) E_y &= 0 \\ -k_\perp k_z E_x + \left(k_\perp^2 - \frac{\omega^2}{c^2} \varepsilon_\parallel \right) E_z &= 0. \end{aligned} \quad (3.6.5)$$

The condition for the existence of solutions of this system yields the dispersion equation for electromagnetic waves in the cold magneto-active plasma:

$$\begin{aligned} \Lambda(\omega, \mathbf{k}) &= k^2 (\varepsilon_\perp k_\perp^2 + \varepsilon_\parallel k_z^2) - \frac{\omega^2}{c^2} [(\varepsilon_\perp^2 - g^2 - \varepsilon_\perp \varepsilon_\parallel) k_\perp^2 \\ &\quad + 2k^2 \varepsilon_\perp \varepsilon_\parallel] + \frac{\omega^4}{c^4} \varepsilon_\parallel (\varepsilon_\perp^2 - g^2) = 0. \end{aligned} \quad (3.6.6)$$

It follows from (3.6.5) that, in contrast to the magnetic-field-free case, the longitudinal and transverse waves are not independent in the magnetic-active plasma. Equation (3.6.6) does not separate into independent equations for the

transverse and longitudinal waves in general. The separation is only complete for pure longitudinal wave propagation when the wave vector is directed along the magnetic field $k_{\perp} = 0$. Then, the first two equations of the system (3.6.5) describe transverse waves obeying the dispersion equation

$$k^2 c^2 = \omega^2 (\varepsilon_{\perp} \pm g). \quad (3.6.7)$$

The two signs of the right-hand side correspond to two independent modes with different polarization. The left-handed mode with

$$\frac{E_{y1}}{E_{x1}} = -i \quad (3.6.8)$$

is called the *ordinary wave*, and the right-handed one with

$$\frac{E_{y2}}{E_{x2}} = i \quad (3.6.9)$$

is the *extraordinary wave*.

For pure longitudinal propagation ($k_{\perp} = 0$), a third solution described by the third equation of the system (3.6.5) exists. These longitudinal oscillations of the cold magneto-active plasma obey the dispersion equation

$$\varepsilon_{||} = 0 \quad (3.6.10)$$

because $\mathbf{E} || \mathbf{k} || \mathbf{B}_0$. The frequency of these longitudinal oscillations is given by

$$\omega^2 = \omega_{pe}^2. \quad (3.6.11)$$

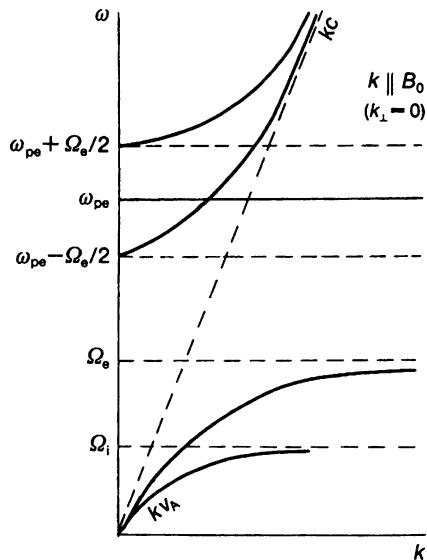
The solution of (3.6.7) for the transverse waves with $k_{\perp} = 0$ are rather complicated in general. A simple analytic expression for the frequency of these waves can be obtained by an asymptotic evaluation

$$\begin{aligned} \omega^2 &= k^2 c^2 + \omega_{pe}^2 \left(1 \mp \frac{\Omega_{Be}}{\sqrt{k^2 c^2 + \omega_{pe}^2}} \right) \quad \text{for } \omega \gg \Omega_{Be} \\ \omega &= \frac{k^2 c^2 \Omega_{Be}}{\omega_{pe}^2} \quad \text{for } \Omega_{Bi} \ll \omega \ll \Omega_{Be}, \omega_{pe}^2 / \Omega_{Be} \\ \omega^2 &= \frac{k^2 v_A^2}{1 + v_A^2 / c^2} \left[1 \mp \frac{v_A k}{\Omega_{Bi} (1 + v_A^2 / c^2)^{3/2}} \right] \quad \text{for } \omega \ll \Omega_{Bi}, \end{aligned} \quad (3.6.12)$$

where $v_A \equiv c \Omega_{Bi} / \omega_{pi} = B_0 / \sqrt{4\pi N_i M}$ is the Alfvén velocity.

Using these expressions and taking into account that, due to (3.6.3), solutions $\omega \rightarrow \Omega_{Bi}$ and $\omega \rightarrow \Omega_{Be}$ exist for $k^2 \rightarrow \infty$, we can sketch the general course $\omega(k)$ of the solution branches. For $\omega_{pe} > \Omega_{Be}$ the result is shown in Fig. 3.2. In the cold

Fig. 3.2 Spectra of the electromagnetic waves in cold magneto-active plasma: longitudinal propagation



magneto-active plasma, there exist five branches of oscillations propagating strictly along the external magnetic field. Four of them describe transverse waves; one branch represents longitudinal waves.

In the frequency range $\omega < \Omega_{Bi}$, the transverse waves are called *magnetohydrodynamic* (MHD) waves: the oscillation branch with the limit $\omega \rightarrow \Omega_{Bi}$ for $k \rightarrow \infty$ is called the *Alfvén wave*. The oscillation branch with the limit $\omega \rightarrow \Omega_{Be}$ for $k \rightarrow \infty$ is called the *fast magnetosonic wave* in the range $\omega < \Omega_{Bi}$ and the *spiral wave* or *helicon* in the range of intermediate frequencies $\Omega_{Bi} < \omega < \Omega_{Be}$. Note that these oscillations are undamped in the collisionless limit.

Exercise 3.13. Using (3.6.6), investigate the case of wave propagation across the magnetic field $k_z = 0$ and sketch the general course $\omega(k)$ of the solution branches.

Inner magnetosphere observations have shown that the simple, one-ion theory for MHD waves, $\omega < \Omega_{Bi}$, is not sufficient, because both cold and hot heavy ions are present in the magnetosphere, He^+ and O^+ are the most important. The heavy ions in the cold plasma can have profound effects on the generation and propagation of these waves and these phenomena are discussed in Chaps. 9 and 10 of this book.

3.7 Wave Damping and Growth

The account of the thermal particle motion leads to a damping of plasma waves. Using (3.4.16), let us consider pure longitudinal wave propagation when the wave vector is directed along the magnetic field. With $k_{\perp} = 0$, the dispersion relation is

greatly simplified, because all the J_n^2 terms vanish for $n > 0$ and only the J_0^2 term remains.

The elements of ε_{ij} in this case are

$$\begin{aligned}\varepsilon_{xx} &= 1 + \sum_{\alpha} \frac{4\pi e_{\alpha}^2}{m_{\alpha} \omega^2} \frac{1}{4} \sum_{l=\pm 1} \int d\mathbf{v} \left(\frac{\omega - k_{\parallel} v_{\parallel}}{v_{\perp}} \frac{\partial f_{\alpha 0}}{\partial v_{\perp}} + k_{\parallel} \frac{\partial f_{\alpha 0}}{\partial v_{\parallel}} \right) \frac{v_{\perp}^2}{\omega - k_z v_{\parallel} - l |\Omega_{B\alpha}|} \\ \varepsilon_{xy} &= \frac{i}{4} \sum_{\alpha} \frac{4\pi e_{\alpha}^2}{m_{\alpha} \omega^2} \sum_{l=\pm 1} l \int d\mathbf{v} \left(\frac{\omega - k_{\parallel} v_{\parallel}}{v_{\perp}} \frac{\partial f_{\alpha 0}}{\partial v_{\perp}} + k_{\parallel} \frac{\partial f_{\alpha 0}}{\partial v_{\parallel}} \right) \frac{v_{\perp}^2}{\omega - k_z v_{\parallel} - l |\Omega_{B\alpha}|} \\ \varepsilon_{zz} &= 1 + \sum_{\alpha} \frac{4\pi e_{\alpha}^2}{m_{\alpha} \omega^2} \left\{ \int d\mathbf{v} \left(\frac{\omega - k_{\parallel} v_{\parallel}}{v_{\perp}} \frac{\partial f_{\alpha 0}}{\partial v_{\perp}} + k_{\parallel} \frac{\partial f_{\alpha 0}}{\partial v_{\parallel}} \right) \frac{v_{\parallel}^2}{\omega - k_z v_{\parallel}} \right. \\ &\quad \left. - \int d\mathbf{v} \left(f_{\alpha 0} + \frac{v_{\parallel}^2}{v_{\perp}} \frac{\partial f_{\alpha 0}}{\partial v_{\perp}} \right) \right\}\end{aligned}\quad (3.7.1)$$

$$\varepsilon_{xx} = \varepsilon_{yy}, \quad \varepsilon_{xy} = -\varepsilon_{yx}, \quad \varepsilon_{xz} = \varepsilon_{zx} = 0, \quad \varepsilon_{yz} = -\varepsilon_{zy} = 0.$$

Exercise 3.14. Verify (3.7.1).

Knowing the components of the dielectric permittivity tensor, we can write down the dispersion equation for electromagnetic waves

$$\varepsilon_{zz} \left(n^4 - 2\varepsilon_{xx} n^2 + \varepsilon_{xx}^2 + \varepsilon_{xy}^2 \right) = 0. \quad (3.7.2)$$

As we see from (3.7.2), for pure longitudinal propagation ($k_{\perp} = 0$), this equation separates into three modes

$$\varepsilon_{zz} = 0 \quad (3.7.3)$$

$$n_{1,2}^2 = \varepsilon_{xx} \pm i\varepsilon_{xy}. \quad (3.7.4)$$

Equation (3.7.3) describes pure longitudinal oscillations. It exactly coincides with the dispersion equation for electrostatic waves in a field-free plasma and, for high-frequency plasma waves, it can be presented in the form

$$\varepsilon^{lo}(\omega, \mathbf{k}) \equiv \varepsilon_{zz} = 1 + \frac{4\pi e^2}{mk^2} \int \frac{\mathbf{k} \cdot \frac{\partial f_{e0}}{\partial \mathbf{v}} d\mathbf{v}}{\omega - \mathbf{k} \cdot \mathbf{v}}. \quad (3.7.5)$$

Now our concern is the treatment of the poles at $\omega = \vec{k} \cdot \vec{v}$ in this integral. At $\omega = \vec{k} \cdot \vec{v}$, this equation has no exact meaning because the integration result depends on the prescription of how to treat the singularity. To eliminate this ambiguity, one must take into account that the perturbation of the distribution function δf_{e0}

disappears at $t \rightarrow -\infty$. For the given time-dependence $\delta f_{e0} \sim \exp(-i\omega t)$, this means that we must have an infinitesimal positive imaginary part of ω when $\mathbf{k}$ is real. So, the integral (3.7.5) can be calculated by assuming that ω is real and replacing ω in the denominator by $\omega + i\Delta$ (Landau prescription), where Δ is an infinitesimally positive number. Then, using the well-known relation

$$\frac{1}{\omega - \mathbf{k}\mathbf{v} + i\Delta} = \frac{P}{\omega - \mathbf{k}\mathbf{v}} - i\pi\delta(\omega - \mathbf{k}\mathbf{v}),$$

we can represent the dielectric permittivity in the form

$$\varepsilon^{lo}(\omega, \mathbf{k}) = \text{Re } \varepsilon^{lo}(\omega, \mathbf{k}) + i \text{Im } \varepsilon^{lo}(\omega, \mathbf{k}), \quad (3.7.6)$$

where

$$\text{Re } \varepsilon^{lo} = 1 + \frac{4\pi e^2}{mk^2} P \int \frac{\mathbf{k} \frac{\partial f_{e0}}{\partial \mathbf{v}} d\mathbf{v}}{\omega - \mathbf{k}\mathbf{v}} \quad (3.7.7)$$

$$\text{Im } \varepsilon^{lo} = -\frac{4\pi e^2}{mk^2} \int \mathbf{k} \frac{\partial f_{e0}}{\partial \mathbf{v}} \delta(\omega - \mathbf{k}\mathbf{v}) d\mathbf{v}. \quad (3.7.8)$$

In (3.7.7) the symbol P denotes the principal value of the integral.

It follows from these relations that ε^{lo} is complex if the distribution function has a nonvanishing derivative near the phase velocity of the perturbation. If ε^{lo} calculated for real ω is complex, the frequency of the plasma oscillations is also complex. The imaginary part of the frequency can be readily found if it is small compared with the real part; this is possible if $\text{Im } \varepsilon^{lo}$ is small. Let us consider this case in more detail.

If $\text{Im } \omega \ll \text{Re } \omega$, it is sufficient to take into account the imaginary correction to the frequency in only $\text{Re } \varepsilon^{lo}(\omega, \mathbf{k})$. Expanding the latter in a series in $\text{Im } \omega$, substituting the result into the equation $\varepsilon^{lo}(\omega, \mathbf{k}) = 0$, and separating the real and imaginary parts in the latter, we obtain

$$\text{Re } \varepsilon^{lo}(\omega_k, \mathbf{k}) = 0 \quad (3.7.9)$$

$$\gamma_k = -\frac{\text{Im } \varepsilon^{lo}(\omega_k, \mathbf{k})}{\frac{\partial \text{Re } \varepsilon^{lo}(\omega_k, \mathbf{k})}{\partial \omega_k}}. \quad (3.7.10)$$

Here

$$\omega_k \equiv \text{Re } \omega(\mathbf{k}), \quad \gamma_k \equiv \text{Im } \omega(\mathbf{k}).$$

Let us consider the high-frequency plasma waves in a Maxwellian plasma in the fast wave range limit where the phase velocity exceeds the thermal velocities

$$\frac{\omega}{k} \gg v_{Te}, v_{Ti}. \quad (3.7.11)$$

Using (3.5.8), the dispersion equation for electrostatic waves in a field-free weakly damped plasma in the limit of (3.7.11) can be presented in the form

$$\varepsilon^{lo}(\omega_k, k) \approx 1 - \frac{\omega_{pe}^2}{\omega^2} \left(1 + 3 \frac{k^2 v_{Te}^2}{\omega^2} \right) + i \sqrt{\frac{\pi}{2}} \frac{\omega \omega_{pe}^2}{k^3 v_{Te}^3} \exp\left(-\frac{\omega^2}{2k^2 v_{Te}^2}\right) = 0. \quad (3.7.12)$$

The real part of ω is obtained from the real part of this equation

$$\text{Re } \varepsilon^{lo}(\omega_k, k) = 1 - \frac{\omega_{pe}^2}{\omega^2} \left(1 + 3 \frac{k^2 v_{Te}^2}{\omega^2} \right), \quad (3.7.13)$$

which yields

$$\omega_k^2 = \omega_{pe}^2 (1 + 3k^2 r_{De}^2). \quad (3.7.14)$$

Here $r_{De} = v_{Te}/\omega_{pe}$ is the electron Debye length. We have taken into account that $\omega \approx \omega_{pe}$ and substituted ω_{pe} for ω in small terms.

Due to the condition (3.7.11), i.e., $k^2 r_{De}^2 \ll 1$, the second term in (3.7.14) is a small correction. Thus, weakly damped longitudinal oscillations with high phase velocity have a wavelength $\lambda \sim 1/k \gg r_{De}$. The longitudinal waves with dispersion relation (3.7.14) are called electron plasma waves (electron Langmuir waves) or simply plasma waves. They form the high frequency ($\omega \gg \omega_{pe}$) branch of longitudinal oscillations in the isotropic plasma, which is also called the electron branch, because the ion contribution is negligible.

The damping decrement γ of the plasma waves can be obtained based on (3.7.10) and presented in the form

$$\gamma_k \approx -\sqrt{\frac{\pi}{8}} \frac{\omega_{pe}}{k^3 r_{De}^3} \exp\left(-\frac{1}{2k^2 r_{De}^2} - \frac{3}{2}\right). \quad (3.7.15)$$

Hence, if $k^2 r_{De}^2 \ll 1$, the damping of the plasma waves is exponentially small. The collisionless damping of oscillations was established by Landau and is usually called *Landau damping*.

Let us now consider the two solutions of (3.7.4). Using (3.7.1), they can be presented in the form

$$n_{1,2}^2 \equiv n_{L,R}^2 = 1 + \frac{1}{2} \sum_{\alpha} \frac{4\pi e_{\alpha}^2}{m_{\alpha} \omega^2} \int d\mathbf{v} \left(\frac{\omega - k_{\parallel} v_{\parallel}}{v_{\perp}} \frac{\partial f_{\alpha 0}}{\partial v_{\perp}} + k_{\parallel} \frac{\partial f_{\alpha 0}}{\partial v_{\parallel}} \right) \frac{v_{\perp}^2}{\omega - k_z v_{\parallel} \mp \Omega_{B\alpha}}. \quad (3.7.16)$$

These equations describe the transverse ordinary (left-hand polarized) and extraordinary (right-hand polarized) wave propagating along the field, respectively. It can be shown that, in the case of the Maxwellian distribution function, these waves will experience weak collisionless Landau damping.

Project 3.2. It is instructive to calculate the damping in some special cases, assuming in (3.7.16) f_{x0} to be a Maxwell distribution:

- (a) Alfvén waves: $\omega \ll \Omega_{Bi}$
- (b) Whistler waves: $\Omega_{Bi}^2 \ll \omega^2 \ll \Omega_{Be}^2$
- (c) Cyclotron waves: $\omega = \pm \Omega_{Be}$

Exercise 3.15. Derive (3.7.16) for the case of the cold plasma with the distribution function

$$f_{x0} = N_{x0} \frac{1}{2\pi} \delta(v_{\perp}) \delta(v_{\parallel}) \frac{1}{v_{\perp}}.$$

Let us now consider the case in which the distribution function of the particles in the plasma is not an equilibrium one. Such distributions may, naturally, be anisotropic and may be functions of the particle velocities, which are not monotonically decreasing. In that case, the oscillations of the plasma, caused by the self-consistent field, may not necessarily be damped – as in the case of an equilibrium distribution – because the energy that the waves give off to the particles is not necessarily larger than the energy obtained by the waves from the particles in the case of nonequilibrium distributions. If the particle distribution is such that the waves obtain more energy from the particle than the particles from the waves, the plasma oscillations will grow in time and the particle distribution will be unstable.

Such a situation occurs, for instance, in a plasma in which the particle velocity distribution has the form of a sum of a Maxwell distribution and a narrow peak around some velocity. Such a distribution corresponds to the passage of a beam of charged particles moving through an equilibrium plasma.

In this case, the fluctuations in the density and in the velocities of the particles in the beam propagate thanks to the collective interaction of the beam particles and the plasma in the form of waves with increasing amplitudes and the electric field that then arises is of the same nature. As a result of the growth of the waves, the initial particle distribution becomes unstable, that is, the beam–plasma system is unstable.

Plasmas are distinguished from other states of matter by the great variety of instabilities to which they are subject. Depending on the circumstances, instability may be desirable or undesirable. For example, instabilities make possible the turbulent heating of plasmas but they are also the main obstacle in the way of controlled thermonuclear fusion. However, these instabilities are not of the same nature in the two cases.

The final stage of the nonequilibrium plasma due to instabilities can be determined on the basis of the nonlinear theory only. The initial stage of development of instabilities, however, presents a problem of linear electrodynamics according to which the growth rate of small perturbations is determined by (3.3.4)

$$\left| k^2 \delta_{ij} - k_i k_j - \frac{\omega^2}{c^2} \varepsilon_{ij}(\omega, \mathbf{k}) \right| = 0. \quad (3.7.17)$$

Using this relation and the tensor of dielectric permittivity ε_{ij} from Sect. 3.4, we consider here the instabilities of the nonequilibrium plasma with anisotropic distribution functions of the particles.

Let us consider only pure longitudinal wave propagation when the wave vector is directed along the magnetic field. As we see from (3.7.2), for pure longitudinal propagation ($k_{\perp} = 0$) (3.7.17) separates into three modes

$$\varepsilon_{zz} = 0, \quad n_{1,2}^2 = \varepsilon_{xx} \pm i\varepsilon_{xy}. \quad (3.7.18)$$

The last equations describe the transverse ordinary (left-hand polarized) and extraordinary (right-hand polarized) wave propagating along the field, respectively. In this section, we derive the conditions of instability of these waves in the cold plasma with a small addition of hot electrons or ions.

The dispersion equation for such waves was derived in the previous section (see (3.7.16)) for the case of an arbitrary distribution function. Using this formula and substituting in it the distribution function

$$f_{\alpha 0} = f_{\alpha 0}^{(0)} + f_{\alpha}^h, \quad f_{\alpha 0}^{(0)} \gg f_{\alpha}^h, \quad (3.7.19)$$

where

$$f_{\alpha 0}^{(0)} = N_{\alpha 0} \frac{1}{2\pi} \delta(v_{\perp}) \delta(v_{\parallel}) \frac{1}{v_{\perp}},$$

we can present (3.7.16) in the form

$$\begin{aligned} \frac{k^2 c^2}{\omega^2} - n_{L,R}^2 = & -i \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega^2} \frac{\pi^2}{N_{\alpha 0}} \int v_{\perp}^2 dv_{\perp} dv_{\parallel} \delta(\omega - k_{\parallel} v_{\parallel} \mp \Omega_{B\alpha}) \\ & \times \left\{ \omega \frac{\partial f_{\alpha}^h}{\partial v_{\perp}} - k_{\parallel} v_{\parallel} \left[\frac{\partial f_{\alpha}^h}{\partial v_{\perp}} - \frac{v_{\perp}}{v_{\parallel}} \frac{\partial f_{\alpha}^h}{\partial v_{\parallel}} \right] \right\}. \end{aligned} \quad (3.7.20)$$

Here,

$$n_{L,R}^2 = 1 - \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega} \frac{1}{\omega \mp \Omega_{B\alpha}} \quad (3.7.21)$$

is the refractive index of the cold plasma.

Exercise 3.16. Derive (3.7.20) using the Landau prescription (3.7.5).

Keeping in mind that

$$\delta(\omega - k_{\parallel} v_{\parallel} \mp \Omega_{B\alpha}) = \delta \left\{ k_{\parallel} \left(\frac{\omega \mp \Omega_{B\alpha}}{k_{\parallel}} - v_{\parallel} \right) \right\} = \frac{1}{k_{\parallel}} \delta(u_{\alpha} - v_{\parallel}),$$

where

$$u_\alpha = \frac{\omega \mp \Omega_{B\alpha}}{k_{||}},$$

the dispersion equation (3.7.20) may be presented as

$$k^2 - \frac{\omega^2}{c^2} n_{L,R}^2(\omega) + i \sum_\alpha \frac{\omega_{p\alpha}^2}{c^2} \frac{\pi^2 \Omega_{B\alpha}}{N_{\alpha 0} k} \int_0^\infty v_\perp^2 dv_\perp \left\{ \frac{k v_\perp}{\Omega_{B\alpha}} \frac{\partial f_\alpha^h}{\partial v_{||}} \pm \frac{\partial f_\alpha^h}{\partial v_\perp} \right\}_{v_{||}=u_\alpha} = 0. \quad (3.7.22)$$

Using (3.7.10), we can obtain the condition of instability for transverse ordinary and extraordinary waves propagating along the magnetic field. Deriving the imaginary part of the frequency ω

$$\gamma = \frac{\pi^2 V_g}{2c^2 k^2} \sum_\alpha \frac{\omega_{p\alpha}^2 \Omega_{B\alpha}}{N_{\alpha 0}} \int_0^\infty v_\perp^2 dv_\perp \left\{ \frac{k v_\perp}{\Omega_{B\alpha}} \frac{\partial f_\alpha^h}{\partial v_{||}} \pm \frac{\partial f_\alpha^h}{\partial v_\perp} \right\}_{v_{||}=u_\alpha}, \quad (3.7.23)$$

where $V_g = c/\partial(\omega n_{L,R})/\partial\omega$, the instability requirement $\gamma > 0$ can be presented as

$$\left\{ \frac{k v_\perp}{\Omega_{B\alpha}} \frac{\partial f_\alpha^h}{\partial v_{||}} \pm \frac{\partial f_\alpha^h}{\partial v_\perp} \right\}_{v_{||}=u_\alpha} > 0. \quad (3.7.24)$$

Let us evaluate (3.7.23), using a bi-Maxwellian distribution function

$$f_\alpha^h = \frac{N_\alpha^h}{(2\pi)^{3/2}} \left(\frac{m_\alpha}{T_{\alpha||}} \right)^{1/2} \left(\frac{m_\alpha}{T_{\alpha\perp}} \right) \exp \left\{ -\frac{m_\alpha}{2} \left[\frac{v_{||}^2}{T_{\alpha||}} + \frac{v_\perp^2}{T_{\alpha\perp}} \right] \right\}. \quad (3.7.25)$$

The final result of integration (3.7.23) for the a component of the plasma can be written as

$$\frac{\gamma}{\Omega_{B\alpha}} = -\frac{\sqrt{\pi}}{2} \frac{\omega_{p\alpha}^2 N_\alpha^h}{N_{\alpha 0}} \frac{V_g}{c^2 k^2 w_{\alpha||}} \left\{ \frac{\omega}{\Omega_{B\alpha}} \frac{w_{\alpha\perp}^2}{w_{\alpha||}^2} \mp \left(\frac{w_{\alpha\perp}^2}{w_{\alpha||}^2} - 1 \right) \right\} \exp \left\{ -\left(\frac{\omega \mp \Omega_{B\alpha}}{k w_{\alpha||}} \right) \right\}. \quad (3.7.26)$$

Here,

$$w_{\alpha\perp}^2 = \frac{2T_{\alpha\perp}}{m_\alpha}, \quad w_{\alpha||}^2 = \frac{2T_{\alpha||}}{m_\alpha}.$$

Using the notation

$$\Delta_\alpha = 1 - \frac{w_{\alpha\perp}^2}{w_{\alpha||}^2},$$

the instability criteria of the α component of the plasma with bi-Maxwellian distribution function can be presented in the form

$$\frac{\omega}{\Omega_{B\alpha}}(1 - \Delta_\alpha) \pm \Delta_\alpha < 0. \quad (3.7.27)$$

The sign $(-)$ here corresponds to the left-hand polarized wave, $(+)$ corresponds to the right-hand polarized wave.

Exercise 3.17. Using (3.7.27), derive the condition of instability for electron and ions.

The anisotropic velocity distribution of the particles (3.7.25) is typical in the space plasma, and (3.7.27) plays a fundamental role in the analysis of the instability of the radiation belts in the Earth's magnetosphere. This cyclotron instability is due to the resonance interaction of the high-energy electrons and protons with electromagnetic waves at the cyclotron resonance. The development of the cyclotron instability in the region of the radiation belts leads to the generation of Alfvén waves and whistlers, which, in their turn, have a significant effect on the trapped particles.

3.8 The Quasilinear Approximation

When studying small (linear) oscillations in plasma, we split the distribution function into two terms – a nonoscillating part (the initial distribution function) and a small correction to it, which oscillates with the frequency of the plasma oscillations. We also assumed that the nonoscillating part was not connected at all with the oscillations occurring in the plasma. Plasma oscillations, however, affect the nonoscillating part of the distribution function and this effect increases with increasing amplitude of oscillations.

When the amplitude of the oscillations increases, one also violates the basic property of linear oscillations – the independence of propagation of oscillations with different wave vectors and frequencies (superposition principle) because processes involving the interactions between different oscillations begin to play an ever-larger part.

In this section, we will assume the amplitude of the oscillations to be so small that we can neglect wave–wave interactions, that is, that we can neglect the violation of the superposition principle. The theory of plasma oscillations that takes into account the effect of the oscillations on the nonoscillating part of the distribution function but still assume that superposition principle of the oscillations is valid is called the quasilinear theory or the quasilinear approximation.

Let us consider small oscillations of the plasma around an initial state described by the distribution function f_{x0} in the presence of the external magnetic field $\mathbf{B}_0$. The

kinetic equation for the distribution function F_α that describes such oscillations in the collisionless limit can be written as

$$\frac{\partial F_\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial F_\alpha}{\partial \mathbf{r}} - \frac{e}{m} \left(\mathbf{E} + \frac{1}{c} [\mathbf{v} \times (\mathbf{B} + \mathbf{B}_0)] \right) \cdot \frac{\partial F_\alpha}{\partial \mathbf{v}} = 0. \quad (3.8.1)$$

Here $\mathbf{E} = \mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B} = \mathbf{B}(\mathbf{r}, t)$ are self-consistent electric and magnetic fields of plasma oscillations. We also assumed that F_α in (3.8.1) is presented as

$$F_\alpha(\mathbf{r}, \mathbf{v}, t) = f_{\alpha 0}(\mathbf{v}_\parallel, \mathbf{v}_\perp) + f_\alpha(\mathbf{r}, \mathbf{v}, t), \quad (3.8.2)$$

where f_α is small oscillating correction to $f_{\alpha 0}$. We also assume that $f_{\alpha 0} = \langle F_\alpha \rangle$ is the average value of the distribution function, which is independent of the spatial coordinates, and f_α , the oscillating part of the distribution function, $\langle f_\alpha \rangle = 0$. Now we expand the function f_α and the electrical and magnetic field wave strengths in Fourier series in the spatial coordinates:

$$\begin{aligned} f(\mathbf{r}, \mathbf{v}, t) &= \sum_{k,j} f_{\mathbf{k}}(\mathbf{v}, t) e^{i(\mathbf{k} \cdot \mathbf{r}) - i\omega(\mathbf{k})t}, \\ \mathbf{E}(\mathbf{r}, \mathbf{v}, t) &= \sum_{k,j} \mathbf{E}_{\mathbf{k}}^j(\mathbf{v}, t) e^{i(\mathbf{k} \cdot \mathbf{r}) - i\omega^{(j)}(\mathbf{k})t}, \quad \mathbf{B}(\mathbf{r}, \mathbf{v}, t) = \sum_{k,j} \mathbf{B}_{\mathbf{k}}^j(\mathbf{v}, t) e^{i(\mathbf{k} \cdot \mathbf{r}) - i\omega^{(j)}(\mathbf{k})t}, \end{aligned} \quad (3.8.3)$$

where summations are over all values of the wave vector $\mathbf{k}$ and over all branches j of the oscillations, whereas $\omega^{(j)}(\mathbf{k})$ is the eigenfrequency of oscillations with wave vector $\mathbf{k}$.

Substituting (3.8.2) into the kinetic equation (3.8.1) after presenting it in a form (3.4.3), and averaging over the phases, we next obtain the equation for the background distribution function $f_{\alpha 0}$, taking into account the reaction of oscillations on the particles (Akhiezer et al. 1975)

$$\frac{\partial f_{\alpha 0}}{\partial t} - \Omega_{B\alpha} \frac{\partial f_{\alpha 0}}{\partial \varphi} = -\frac{e_\alpha}{m_\alpha} \left\langle \left(\left\{ \mathbf{E} + \frac{1}{c} [\mathbf{v} \times \mathbf{B}] \right\} \cdot \frac{\partial f_\alpha}{\partial \mathbf{v}} \right) \right\rangle. \quad (3.8.4)$$

The oscillating part f_α of the distribution function is determined from the equation

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial f_\alpha}{\partial \mathbf{r}} - \frac{e_\alpha}{m_\alpha} \left(\mathbf{E} + \frac{1}{c} [\mathbf{v} \times \mathbf{B}] \right) \cdot \frac{\partial f_\alpha}{\partial \mathbf{v}} - \Omega_{B\alpha} \frac{\partial f_\alpha}{\partial \varphi} = 0, \quad (3.8.5)$$

in which we have dropped the nonlinear terms

$$\frac{e_\alpha}{m_\alpha} \left\langle \left(\left\{ \mathbf{E} + \frac{1}{c} [\mathbf{v} \times \mathbf{B}] \right\} \cdot \frac{\partial f_\alpha}{\partial \mathbf{v}} \right) \right\rangle - \left\langle \left(\left\{ \mathbf{E} + \frac{1}{c} [\mathbf{v} \times \mathbf{B}] \right\} \cdot \frac{\partial f_\alpha}{\partial \mathbf{v}} \right) \right\rangle,$$

responsible for nonlinear wave–wave interactions.

Bearing in mind that oscillations with different values k and j are statistically independent, we get from (3.8.4) (see Akhiezer et al. 1975 for the details)

$$\frac{\partial f_{x0}}{\partial t} - \Omega_{Bx} \frac{\partial f_{x0}}{\partial \varphi} = Q(\mathbf{v}, t), \quad (3.8.6)$$

where

$$Q(\mathbf{v}, t) = -\frac{e_x}{m_x} \left(\frac{\partial}{\partial \mathbf{v}} \cdot \sum_{k,j} \left\{ \mathbf{E}_k^{(j)} + \frac{1}{c} [\mathbf{v} \times \mathbf{B}_k^{(j)}] \right\} \cdot f_k^{(j)} \right).$$

The Fourier component $f_k^{(j)}(\mathbf{v}, t)$ of the oscillating part of the distribution function is here, in accordance with (3.8.3) and (3.8.5) determined by the equation

$$\frac{\partial f_k^{(j)}}{\partial t} + i \left[(\mathbf{k} \cdot \mathbf{v}) - \omega^{(j)}(\mathbf{k}) \right] - \Omega_{Bx} \frac{\partial f_k^{(j)}}{\partial \psi} = R(\mathbf{k}, t), \quad (3.8.7)$$

where

$$R(\mathbf{v}, t) = -\frac{e_x}{m_x} \left(\left\{ \mathbf{E}_k^{(j)*} + \frac{1}{c} [\mathbf{v} \times \mathbf{B}_k^{(j)*}] \right\} \cdot \frac{\partial f_{x0}}{\partial \mathbf{v}} \right),$$

$\psi = \phi - \varphi$, with φ the azimuthal angle in wave vector space with the polar axis parallel to $\mathbf{B}_0$.

Our problem now consists in using (3.8.5) or (3.8.7) to express f in terms of f_{x0} ,

$$f = f\{f_0\},$$

and substituting this expression into (3.8.4) to obtain an equation containing only f_{x0} . Depending on the application of the quasilinear theory (see Chaps. 4, 8, and 9 of this book for the details), this can be done using different approaches based on the specific space plasma physics problems. One sees easily, however, that in this way we get a second-order differential equation (Akhiezer et al. 1975),

$$\frac{\partial f_{x0}}{\partial t} = \sum_{i,j} \frac{\partial}{\partial v_i} D_{ij} \frac{\partial f_{x0}}{\partial v_j}, \quad (3.8.8)$$

where $D_{ij} = D_{ij}(\mathbf{v}, t)$ is the function velocity and the time; (3.8.8) has the form of a diffusion equation in velocity space. This is the reason why the D_{ij} are called the *diffusion coefficients in velocity space*.

3.9 Nonlinear Drift-Kinetic Equation

This section presents equations of motion for a single particle and the nonlinear kinetic equation in the drift approximation for a plasma in the presence of a circularly polarized wave of arbitrary frequency with its wave vector directed along the ambient magnetic field. The nonlinear effect of electromagnetic waves on plasmas is a subject of interest in laboratory experiments as well as in space physics. Wave–particle interactions described by the ponderomotive force, which arises in such situations, are responsible for different nonlinear processes that can be important for plasma redistribution, particle energization, and acceleration in space plasmas. A detailed discussion of the history of this work is given by Khazanov et al. (2004).

In space plasmas, as well as in some laboratory devices, a circularly polarized wave propagating parallel to the external magnetic field is often a valid description of the real situation. An example of such waves in space are whistler mode waves in the vicinity of the magnetopause observed on board the AMPTE-UKS satellite, propagating parallel or almost parallel to the ambient magnetic field (Sazhin et al. 1991). Another example is the observation of strong ion cyclotron waves with a wave vector almost parallel to the magnetic field (less than 7°) observed during Galileo’s December encounter with Io (Crary and Bagenal 2000). The frequency of such waves can be comparable to the particles’ gyrofrequency and the wave amplitudes can vary over a wide spatiotemporal scale.

Let us consider a plasma immersed in an external magnetic field $\mathbf{B}(s, x)$ with the s -coordinate along this field and the x -axis along the principal normal to the line of force. The plasma is then subjected to a circularly polarized wave

$$E(t, s, x)(\mathbf{e}_x \pm i\mathbf{e}_y) \exp(-i \int \omega(t, s, x) dt + i \int k(t, s, x) ds) + c.c.,$$

where the wave vector k is directed along the external magnetic field. It is assumed below that the initial plasma distribution is independent of the y -coordinate. The magnetic field and the wave amplitude spatiotemporal variations are assumed to be weak on the scale of the wave frequency, wavelength, particle Larmor radius, and the radius of particle drift rotation in the wave field (Khazanov et al. 2000). In the collisionless limit, the kinetic equation can be written as (Hastie et al. 1967)

$$\frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{r}} + F_\varepsilon \frac{\partial f}{\partial \varepsilon} + F_\mu \frac{\partial f}{\partial \mu} + F_\varphi \frac{\partial f}{\partial \varphi} = 0. \quad (3.9.1)$$

Here, the kinetic energy $\varepsilon = \frac{m}{2} (v_s^2 + v_\perp^2)$, magnetic moment $\mu = \frac{mv_\perp^2}{2B}$, and φ , the azimuthal angle about the external magnetic field direction, are introduced in

velocity space. In these variables, the characteristic equations of (3.9.1) can be presented as

$$\begin{aligned}
 \frac{dx}{dt} &= v_{\perp} \cos \varphi, \quad \frac{dy}{dt} = v_{\perp} \sin \varphi, \quad \frac{ds}{dt} = v_s, \quad \frac{d\varepsilon}{dt} = ev_{\perp} E e^{i(\phi+\varphi)} \\
 \frac{d\mu}{dt} &= -v_{\perp} \rho_1 \cos \varphi \left(\mu + \frac{mv_s^2}{B} \right) - \rho_2 v_s \mu \cos 2\varphi + e \frac{v_{\perp} e^{i\phi}}{B} \\
 &\quad \times \left[e^{i\varphi} \frac{\tilde{\omega} + i v_s \hat{L}_E}{\omega} + \frac{\rho_2 v_s}{\omega} \cos \varphi \right] E + c.c. \\
 \frac{d\varphi}{dt} &= -\omega_B + \frac{\rho_1 v_s^2}{v_{\perp}} \sin \varphi + \frac{\rho_2 v_s}{2} \sin 2\varphi + i \frac{e}{m} \frac{e^{i\phi}}{v_{\perp}} \\
 &\quad \times \left[e^{i\varphi} \frac{\tilde{\omega} + i v_s \hat{L}_E}{\omega} - \frac{\rho_2 v_s}{\omega} \sin \varphi + i \frac{v_{\perp}}{\omega} \frac{\partial}{\partial x} \right] E + c.c., \tag{3.9.2}
 \end{aligned}$$

where $\omega_B = \frac{eB}{mc}$, $\phi = -\int \tilde{\omega} dt$, $\tilde{\omega} = \omega - kv_s$, $\hat{L}_E = \frac{\partial}{\partial t} \frac{k}{\omega} + \frac{\partial}{\partial s}$, and the magnetic field line curvature is determined by the coefficients $\rho_1 = \frac{1}{B} \frac{\partial B}{\partial x}$, $\rho_2 = -\frac{1}{B} \frac{\partial B}{\partial s}$. Equations (3.9.2) are written for the right-hand polarized mode, $E_y = iE_x$. For left-hand polarization, the corresponding equations can be found changing E, E^* to E^*, E and (ω, k) to $(-\omega, -k)$, respectively. In Maxwell's equation, the wave magnetic field in the Lorentz force is replaced with the electric one,

$$\mathbf{B} e^{i\phi} = \frac{ce^{i\phi}}{i\omega} \left[-\mathbf{e}_x(ik + \hat{L}_E)E_y + \mathbf{e}_y(ik + \hat{L}_E + \rho_2)E_x + \mathbf{e}_s \frac{\partial E_y}{\partial x} \right],$$

and only the first-order terms over large-scale spatiotemporal variations were taken into account. In this expression, the magnetic field perturbation due to the slow time variation of the electric field is described by the terms with the operator $\hat{L}_E$, which includes the time derivative.

It can be noted from the structure of (3.9.2) that the second-order terms, i.e., the terms with the second derivatives, will lead to a third-order correction after averaging these equations. The cause is that these terms appear in (3.9.2) as the real correction to the zeroth-order terms. This is because in the zeroth-order approximation, the wave field has no impact on the average particle energy, drift velocity, and magnetic moment. The same argument is valid for the second-order correction, which simply redefines the zeroth-order terms.

The drift equations for a particle can be found from (3.9.2) by averaging over the Larmor rotation and oscillation in the wave electromagnetic field in the usual way (Morozov and Solov'ev 1966), as it is presented in Appendix A. The left-hand side terms in these equations are periodic functions with respect to the arguments φ and ϕ . In fact, they are periodic with respect to the arguments φ and $\varphi + \phi$, where the main term in φ is proportional to the particle gyrofrequency ω_B . The exception is the term proportional to $\partial E / \partial x$ in the last equation of (3.9.2). This term affects only the drift velocity along the y -axis, and this velocity after averaging is still

proportional to the product of periods $1/\omega_B$, $1/(\omega_B + \tilde{\omega})$. As a result, even when the wave period is large (low frequency waves), this term is still proportional to the squared gyroperiod. The drift equations can be obtained by averaging (3.9.2) over these periods with second-order accuracy with respect to the small parameters, $1/\omega_B$, $1/(\omega_B + \tilde{\omega})$. These second-order terms should be taken into consideration not only for high-frequency waves but also if the mass dependence of the wave–particle interaction in the limit of low-frequency waves ($|\tilde{\omega}| \ll \omega_B$) is the subject of interest. The third-order terms over these small parameters can be neglected if the restrictions

$$\left| \frac{kv_{\perp}}{\Omega} \right|, \left| \frac{k^2 v_{\perp}^2}{\Omega \omega} \right| \ll 1 \quad (3.9.3)$$

are satisfied. Here, $\Omega = \omega_B + \tilde{\omega}$. The resulting drift equations for the particle are:

$$\begin{aligned} v_x = 0, \quad v_y = \frac{2\rho_1}{m\omega_B} \left(\varepsilon - \frac{\mu B}{2} \right) - \frac{e^2}{4m^2} \frac{\tilde{\omega}}{\Omega \omega_B \omega^2} \frac{\partial}{\partial x} |E_0|^2, \quad \mu - \frac{e^2}{2m} \frac{\tilde{\omega}^2}{\omega^2 \Omega^2 B} |E_0|^2 = \text{const} \\ \frac{d\varepsilon}{dt} = -\frac{e^2}{2m} \frac{v_s}{\omega} \frac{\partial}{\partial s} \frac{\omega_B}{\Omega^2} |E_0|^2 + \frac{e^2}{2m^2} \frac{\mu B}{\Omega^2} \frac{\partial}{\partial t} \frac{k^2}{\omega^2} |E_0|^2 + \frac{4e^2}{m} \frac{E}{\Omega} \left(\frac{\partial}{\partial t} \frac{\tilde{\omega} E}{\Omega \omega} - \frac{1}{\omega} \frac{\partial}{\partial t} \frac{kv_s}{\omega} E \right), \end{aligned} \quad (3.9.4)$$

where $|E_0|^2 = 4E^2$. Equations (3.9.4) describe the particle motion in the drift approximation for a nonstationary, inhomogeneous plasma in the range of velocities restricted by the inequalities (3.9.3). In the framework of these restrictions, the particle velocity can be larger than the wave phase velocity.

Because the magnetic field and wave amplitude change only in the x, s -plane, the drift velocity is directed along the y -axis. The first term in this velocity, v_y in (3.9.4), is the usual drift due to the magnetic field curvature and its gradient. A particle subjected to a circularly polarized wave with a wave vector along the magnetic field line rotates under the influence of this magnetic field and the electric field of the wave. Averaged over these rotations, the motion results in additional drift due to the wave electric field inhomogeneity and leads to the second term in the drift velocity v_y .

It can be noted that the result for the magnetic moment μ in (3.9.4) immediately follows from the character of the particle motion in the circularly polarized wave. Neglecting the slow drift along the y -axis, the guiding center rotates with a velocity that can be written as (Khazanov et al. 2000)

$$v_{\pm} \equiv v_x \pm iv_y = \pm i \frac{e}{m} \frac{\tilde{\omega}}{\omega \Omega} E \exp(\pm i\phi)$$

The particle is also involved in Larmor rotation. The total average energy of the perpendicular motion is the sum of the energy of the Larmor rotation and the guiding center rotation $mv_+v_-/2$. This last energy of perpendicular rotation in the

wave field divided by the external magnetic field leads to exactly the same expression as in the equation for the magnetic moment μ .

The second term in the energy equation, in the limit of a stationary medium and neglecting the velocity dependence in Ω , turns out to be the magnetization energy per particle as found by Similon et al. (1986). This term also can be found from a simple consideration for the stationary case. The guiding center trajectory is wound along a cylinder with a radius of $r_{\pm} = \mp i_{\pm} v_{\pm} / \tilde{\omega}$ (Khazanov et al. 2000). The rate of change of the magnetic field flux through the surface restricted by such a circle divided by the guiding center rotation period ($2\pi/\omega$) leads to the same magnetization energy. The μ -dependent term in the equation for the energy is the same as the corresponding term in the ponderomotive Hamiltonian found by Grebogi and Littlejohn (1984).

The force component acting on the particle along the external magnetic field, $m dv_s/dt$, which frequently appears in different applications for space plasma, can be calculated from (3.9.4). The part of this force that depends on the inhomogeneity is

$$F^{(s)} = -\mu \frac{\partial B}{\partial s} - \frac{e^2}{2m} \left(\frac{1}{\omega} \frac{\partial}{\partial s} \frac{\omega_B}{\Omega^2} |E_0|^2 + B \frac{\partial}{\partial s} \frac{\tilde{\omega}^2}{\Omega^2 B \omega^2} |E_0|^2 \right).$$

The μ -dependent term in this expression is the usual mirroring force. The ponderomotive term is the same as found by Aamodt and Vella (1977) in the quasilinear approximation, if ω is constant and the terms kv_s in $\tilde{\omega}, \Omega$ are omitted. That is, if

$$F^{(s)} = -\frac{e^2}{2m} \frac{B}{\omega(\omega_B + \omega)} \frac{\partial}{\partial s} \frac{|E_0|^2}{B}.$$

The part of the ponderomotive force in $m dv_s/dt$ that arises due to the nonstationarity is:

$$F^{(t)} = -\frac{4e^2}{m} \left(\frac{E}{\omega\Omega} \frac{\partial}{\partial t} \frac{kE}{\omega} - \frac{kE}{\omega\Omega} \frac{\partial}{\partial t} \frac{\tilde{\omega}E}{\omega\Omega} - \frac{v_{\perp}^2}{8\Omega^2 v_s} \frac{\partial}{\partial t} \frac{k^2 |E_0|^2}{\omega^2} \right). \quad (3.9.5)$$

Note that dv_s/dt for this case describes the total particle acceleration along the magnetic field line because the external magnetic field and the unit vectors are time independent in our consideration. The last term in this expression can be essential for particles with a velocity comparable to the wave phase velocity. The first and the second terms of this force, for a constant wave frequency, ω , and omitting the terms kv_s in $\tilde{\omega}, \Omega$, lead to the force

$$F^{(t)} = -\frac{e^2}{m} \left(\frac{|E_0|^2}{\omega^2(\omega + \omega_B)} \frac{\partial k}{\partial t} + \frac{k\omega_B}{2\omega^2(\omega + \omega_B)^2} \frac{\partial}{\partial t} |E_0|^2 \right). \quad (3.9.5a)$$

For this part of the ponderomotive force, the same result was found by Barash and Karpman (1983) using a macroscopic approach based on the energy relations in a moving media. To compare the results between their approach and the one presented here, the external magnetic field in their calculations should be taken as time independent. The part of the ponderomotive force, $F^{(t)}$, acting along the external magnetic field can be calculated directly starting with (3.9.2) (i.e., the nonaveraged equations) for the acceleration term dv_s/dt . The result for the time-dependent part is still the same as that presented by (3.9.5) and (3.9.5a). Note that this differs from the results of Lee and Parks (1983) and Lee (2000), who based their results on the equations of motion. Their result follows from the ponderomotive Hamiltonian calculated by Grebogi and Littlejohn (1984). The time-dependent part of the acceleration found as a derivative of the guiding center drift velocity is the same as that calculated from the results of Lee and Parks (1983) and Lee (2000).

Results for the interaction of an Alfvén wave (Khazanov et al. 2000) with a particle undergoing acceleration along the magnetic field due to some external potential nonelectric force follows from (3.9.4) in the limit of the stationary and homogeneous system when only the particle velocity v_s changes due to this force.

The averaged characteristic equations in (3.9.4) permit one to write the kinetic equation in the drift approximation. Note, that we started with the characteristic equations (3.9.2), with the right-hand side oscillating due to the dependence on the angle variables, φ and ϕ . The corresponding system of equations is the system (A1) in Appendix A. The averaged equations (3.9.4) or (A3) are independent of these oscillations. We did not include in (3.9.4) the averaged equation for the angle variable φ but, as it can be seen from (A3), the right-hand side of this equation is also angle independent. The kinetic equation (3.9.1), with the use of the chain rule, can be rewritten in the new averaged variables ξ_k, α_m (A6). In our case, ξ_k are the averaged variables $x, y, z, \mu, \varepsilon$ with the time derivatives determined by (3.9.4), and α_m are two new angle variables instead of ϕ and φ . Because of the time derivatives of the new variables, (3.9.4) and (A3) are independent of the angle variables, the characteristic equations of the transformed kinetic equation (A6) are also independent of them. As discussed in Appendix A, in this case the distribution function is independent of the angle variables ϕ and φ . This leads to a kinetic equation for the distribution function f_0 in the new variables,

$$\frac{\partial f_0}{\partial t} + v_s \frac{\partial f_0}{\partial s} + v_y \frac{\partial f_0}{\partial y} + \frac{d\varepsilon}{dt} \frac{\partial f_0}{\partial \varepsilon} = 0. \quad (3.9.6)$$

Here v_y , v_s , and $d\varepsilon/dt$ are determined by (3.9.4) and conservation of the magnetic moment is taken into account.

To calculate macroscopic quantities such as plasma density, current, and heat, the distribution function f_0 should be transformed to the old variables (Bernstein and Catto 1985). The dependence between these two sets of variables is presented by (A2). This can be easily transformed to (A7), i.e., the averaged variables can be

expressed as a function of the old, nonaveraged variables. Then the distribution function can be expanded in series and, with the help of (A8) and (A7), we have:

$$\begin{aligned}
 f = & f_0 + \frac{1}{\omega_B} v_{\perp} \left(\sin \varphi \frac{\partial f_0}{\partial x} - \cos \varphi \frac{\partial f_0}{\partial y} \right) + \frac{e^2}{m\omega\Omega} \left\{ -\frac{\partial f_0}{\partial \varepsilon} \frac{v_{\perp} \sin \varphi}{4\omega_B} \frac{\partial}{\partial x} |E_0|^2 \right. \\
 & - \frac{\partial f_0}{\partial \mu} \frac{1}{\omega} \left[\frac{\tilde{\omega}}{B} \frac{v_{\perp} \sin \varphi}{4\omega_B} \frac{\partial}{\partial x} + \rho_2 \frac{\mu k}{2m} \frac{\sin 2\varphi}{\omega_B - \tilde{\omega}} \right] |E_0|^2 \\
 & \left. + \frac{\mu}{2} |E_0|^2 \left[\frac{\omega B}{\Omega} \frac{\partial^2}{\partial \varepsilon^2} + \frac{\tilde{\omega}}{B\omega} \left(\frac{\tilde{\omega}}{\Omega} - \frac{\rho_2 v_s \sin 2\varphi}{\omega_B - \tilde{\omega}} \right) \frac{\partial^2}{\partial \mu^2} + \left(\frac{2\tilde{\omega}}{\Omega} - \frac{\rho_2 v_s \sin 2\varphi}{\omega_B - \tilde{\omega}} \right) \frac{\partial^2}{\partial \mu \partial \varepsilon} \right] f_0 \right\}.
 \end{aligned} \tag{3.9.7}$$

where v_s and $v_{\perp}$ are determined by the expressions $\varepsilon = \frac{m}{2} (v_s^2 + v_{\perp}^2)$, $\mu = \frac{mv_{\perp}^2}{2B}$.

Because the velocity components depend only on the angle φ ($v_x = v_{\perp} \sin \varphi$, $v_y = v_{\perp} \cos \varphi$) and we are interested in the slowly changing part of the macroscopic quantities (calculated with the help of these velocity components), the terms in the transformed distribution function dependent on the wave phase ϕ can be omitted.

The elementary volume in the five-dimensional space $x, y, z, \mu, \varepsilon$ can be defined as $\frac{B dx dy dz d\mu d\varepsilon}{m\sqrt{2m(\varepsilon - \mu B)}}$. This volume is invariant under the transformation presented by (A2) and (A3) assuming that the conditions in (3.9.3) are satisfied. Then Liouville's theorem, with an accuracy of second order in the small parameters $1/\omega_B$, $1/(\omega_B + \tilde{\omega})$ is satisfied and the moments of the distribution function (3.9.7) can be calculated.

Ponderomotive forces are essential for the understanding of a wide variety of phenomena in the solar wind and magnetospheres. Examples of their importance include the cold plasma redistribution toward the Earth's magnetospheric equator and the energization of ionospheric ions. Circularly polarized waves (ion cyclotron waves and whistlers) are widely prevalent throughout space. The wave vector of these waves is often directed along the external magnetic field (or close to it). Under these conditions, a kinetic description of the ponderomotive wave-particle interaction is significantly simplified and can be used for a number of applications.

The role of additional velocity-dependent terms in the time-dependent part of the ponderomotive force can be illustrated by the problem of upward acceleration of ions in the auroral region. As pointed out by Shukla et al. (1996), parallel ion acceleration is strongly dependent on the spatiotemporal variation of the wave electric field energy density. Assuming for simplicity a magnetic moment equal to zero, for an ion in the field of ion cyclotron wave from (3.9.4) the time-dependent term can be presented as

$$\frac{d\varepsilon}{dt} = \frac{4e^2}{m} \frac{E}{\omega_{Bi} - \tilde{\omega}} \left(\frac{\partial}{\partial t} \frac{\tilde{\omega} E}{(\omega_{Bi} - \tilde{\omega})\omega} + \frac{1}{\omega} \frac{\partial}{\partial t} \frac{kv_s}{\omega} E \right).$$

For an altitude of $\sim 2,000$ km, the ion velocity is in the range of approximately 5 km/s, the wave phase velocity is approximately 6,000 km/s, and the wave

frequency $\omega \sim 2$ Hz (Boehm et al. 2000). Therefore, the additional second term for hydrogen ions is at least of the same order of magnitude as the first, and can essentially change the rate of acceleration. Note the dependence of the last term on the velocity and wave vector orientation.

Another problem for which the same additional term can be important is related to the nonlinear ionospheric effects produced by lightning discharge. For example, Yukhimuk and Roussel-Dupre (1997) proposed the parametric decay of a whistler mode wave into a lower hybrid wave and an ultralow-frequency electromagnetic wave as an explanation of an observed electrostatic wave and magnetic pulse. A complete description of such phenomena includes the energy density evolution of the whistler wave. The whistler wave amplification is also of interest for some other problems related to lightning-produced whistlers. According to observations (Kelley et al. 1985), the whistler burst duration is 10–20 ms. There is a case that the whistler electric field changes 2.4 times the distance from the lightning changes from 300 to 650 km. The whistler, electron gyro- and plasma frequencies are $\omega = 3 \times 10^4 \text{ s}^{-1}$, $\omega_{Be} = 2 \times 10^5 \text{ s}^{-1}$, $\omega_{pe} = 4 \times 10^7 \text{ s}^{-1}$. Assuming the magnetic moment is equal to zero, it follows from (3.9.4) that the main term in the parallel ponderomotive force is the term with the time derivative. The whistler phase velocity is approximately 550 km/s and both components of this term can be of the same order.

Results obtained for a single particle (3.9.4) can be useful for a particle-in-cell simulation. The kinetic equation (3.9.6) along with the distribution in (3.9.7) permit the analysis of the interaction of a multicomponent plasma with a circularly polarized wave, taking into account the system nonstationarity and inhomogeneity, and allow for the calculation of the macroscopic plasma parameters. The results are found under the assumption that the wave field varies on the same scale as the background magnetic field.

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Chapter 4

Hydrodynamic Description of Space Plasma

Because of its complexity, major difficulties are encountered in the use of the kinetic equation in the study of various processes in plasma. Thus, the class of problems that can be solved using the kinetic approach is quite limited. Additionally, for a large number of problems, the detailed kinetic description is excessive and only macroscopic parameters like density, bulk velocity, pressure, viscosity tensor, and heat flux are important. These quantities are defined as the corresponding moments of the velocity distribution function. The plasma kinetic description could thus be replaced by a system of velocity moment equations. These moment equations are usually called hydrodynamic or transport equations. The transport equation approach has received a lot of attention because it can handle the core plasma particle and energy flow conditions in the solar-terrestrial environment. Many of the highly nonequilibrium flows found in space plasma are characterized by appreciable temperature anisotropies, i.e., unequal species temperatures parallel and perpendicular to the magnetic field direction. Also, wave-particle interaction processes can dramatically contribute to the formation of these nonequilibrium plasma flows. Therefore, it is necessary to trace the derivation of the different sets of the transport equations in order to understand the applicability of space plasma hydrodynamic theory.

4.1 Moments of the Kinetic Equation

Derivation of the system of hydrodynamic equations is based on averaging (integration over velocity space) of the kinetic equation multiplied by a corresponding weight function. This approach allows one to get strict rules of applicability of the obtained hydrodynamic equations. The obtained system of transport equations appears as an infinite chain of engaged equations because there is always a $(k + 1)$ -order moment in an equation for the moment of the order k .

In practical usage, a few first moments having clear physical interpretation are usually sufficient, which means that a finite system of hydrodynamic equations is

used to describe plasma. To break the chain of moment equations and calculate the moments, one should know the expression for the particle distribution function or make certain assumptions about its properties.

Existing methods in kinetic theory allow this problem to be solved (Chapman and Cowling 1958; Grad 1949; Braginskii 1965; Burgers 1969; Shkarofsky et al. 1966). We will consider the moment method because it will be used extensively below.

Because it has a clear physical sense and mathematical conveniences, the method of moments is particularly useful in the study of multicomponent plasmas (Tanenbaum 1967; Burgers 1969). This fact partly explains the advantages of this method over other approaches in the construction of hydrodynamic models of charged and neutral components in the upper atmosphere of the Earth (Schunk 1975).

The distribution function in the method of moments is assumed to possess the following structure (Kogan 1967)

$$f(\mathbf{r}, \mathbf{v}, t) = F(\mathbf{v}, M^{(0)}, M^{(1)}, \dots), \quad (4.1.1)$$

where $M^{(k)}$, its k -moment, is defined as follows

$$M^{(k)} \equiv M_{\alpha_1 \dots \alpha_k}^k(\mathbf{r}, t) = \int \prod_{i=1}^k v_{\alpha_i} f d\mathbf{v}, \quad \alpha_i = 1, 2, 3. \quad (4.1.2)$$

Substitution of this expression into the equations for moments shows that the Boltzmann equation (2.1.1) is equivalent to an infinite system of macroscopic equations for quantities $M^{(k)}$. The chain of equations reduces to a finite set and the distribution function for only for a small number of problems, and, hence, the state of the medium is described by the finite number of moments (Kogan 1967). Various assumptions could be used to represent the distribution function by the finite number of moments to get an approximate solution of the kinetic equation.

The moment approach was first used by Grad (1949), who used the fact that the distribution function can be expanded into series in orthogonal tensor polynomials of particle velocities. Assuming the fluid to be close to an equilibrium state, local Maxwell distribution and Hermite polynomials were chosen in this paper as a zero-order approximation. Expansion coefficients are effectively the macroscopic parameters of the medium, each of them being described by the separate transport equation. For practical purposes, a finite number of moments is used. The most popular seems to be the so-called 13-moment approximation (Grad 1949; Silin 1971; Tanenbaum 1967; Burgers 1969; Schunk 1975), which corresponds to the first three nonzero terms in the series of Hermite polynomials. In this case, the following macroscopic parameters are present in the equations: density, bulk velocity, temperature, viscosity tensor, and heat flux vector. These quantities comprise the set of 13 independent moments of the distribution function.

Formally speaking, this approach does not impose any restrictions on relations between the particle mean free paths λ and collision times τ and, correspondingly,

characteristic space L and time t_0 scales of the problem under consideration. It is only necessary that the distribution function be approximated with a required precision by the first few terms of the expansion. However, the macroscopicity condition

$$\lambda \ll L, \quad \tau \ll t_0 \quad (4.1.3)$$

guarantees sufficiently rapid convergence of the series of Hermite polynomials, which allows confining oneself to just a few terms in this expansion.

Relations (4.1.3) imply that the particles are localized in volumes of scales λ , where over the time τ , the local Maxwell velocity distribution is attained. In this sense, (4.1.3) in the Grad approach are equivalent to the requirement that the distribution function is close enough to the Maxwell distribution function.

The system of hydrodynamic equations simplifies considerably under the assumption that variations of the macroscopic plasma parameters over the distances comparable with the mean free path and over times comparable with the collision time are small enough. In this case, from equations for the nonequilibrium quantities, viscosity and heat flux, it is possible to obtain simple relations between these quantities as well as gradients of basic thermodynamic parameters and external forces using kinetic or transport coefficients. The results thus obtained using the Grad approach correspond to the first approximation in the Chapman–Enskog method (Chapman and Cowling 1958; Kogan 1967). This is explained by the linear relations between the Hermite polynomials and Sonin polynomials, which are used in the Chapman–Enskog expansion method.

It should be pointed out that (4.1.3) are sufficient but not necessary for the rapid convergence of the series in the Grad method. That is why it is possible that, in certain situations, having specific boundary and initial conditions (4.1.3) does not hold but the distribution function is still well approximated by just a few terms in the expansion used in the Grad method (Grad 1949). The macroscopicity conditions (4.1.3) as mentioned above, allow not only the truncation of the series of Hermite polynomials for the distribution function but also the obtaining of simple linear formulae relating the nonequilibrium macroscopic quantities to the gradients of basic thermodynamic parameters.

The closed system of transport equations obtained from the Grad method is applicable when the distribution function is close to Maxwell distribution function, which is used as a zero-order approximation. This causes a sufficiently rapid convergence of the series of Hermite polynomials and allows the restriction of the consideration to a few first terms in the expansion. Nevertheless, in many situations, the distribution function due to various reasons may have considerable deviations from the Maxwell function. In these cases, the Grad approach is inapplicable because taking Maxwell function as the zero-order approximation would lead to a drastic deterioration of the convergency of the series (or to its divergency) for the distribution function and would make it necessary to consider large (or infinite) number of terms and, hence, hydrodynamic equations.

In this connection, as a generalization of the Grad approach, a new method was developed (Mintzer 1965) that allows one to find the solution to the kinetic equation represented as a generalized series in orthogonal polynomials $P_\alpha^{(k)}(\mathbf{v}, \mathbf{r}, t)$:

$$f(\mathbf{v}, \mathbf{r}, t) = f_0(\mathbf{v}, \mathbf{r}, t) \sum_{k, \alpha} a_\alpha(\mathbf{r}, t) P_\alpha^{(k)}(\mathbf{v}, \mathbf{r}, t). \quad (4.1.4)$$

Here f_0 is the weight function of the expansion (zero-order approximation); a_α are expansion coefficients; α is a set of subscripts designating coordinates over which the summation should be performed, the summation should also be performed over k , the order of polynomials.

The zero-order approximation f_0 and the system of polynomials $P_\alpha^{(k)}$ depend on the specific features of a problem under consideration (Mintzer 1965; St.-Maurice and Schunk 1976; Schunk 1977). If a Maxwell function is selected as the zero-order approximation, then expansion into the generalized series is equivalent to Grad's approach, which is used for plasmas near thermal equilibrium.

Unfortunately, there are no universal rules for the selection of f_0 . The zero-order function could be obtained from the consideration of the simplified version of the initial problem or from its solution obtained under certain restrictions. General theoretical considerations can also help to find the form of f_0 . For example, it is natural to accept the Maxwell function for weakly nonequilibrium states. For weakly ionized plasma at high latitude ionosphere, the bi-Maxwellian or toroidal distribution can be accepted depending on the magnitude of the perpendicular electric field (St.-Maurice and Schunk 1976, 1977, 1979; Hubert 1982).

It is important to select an adequate function f_0 because a good choice of the zero-order approximation will secure rapid convergency of the series (4.1.4) and will allow the number of significant terms in the expansion to be restricted. To every choice of the zero-order approximation, there corresponds a specific system of transport equations for the expansion coefficients a_α . Although an adequate choice of the zero-order approximation is important, the complexity of f_0 can considerably hamper the solution of the system of macroscopic equations or even make the solution impossible. That is why the zero-order approximation should not only be consistent with the nature of the physical problem and provide for rapid convergence of the series (4.1.4) but should also allow for practical resolution of the resulting system of transport equations.

After the zero-order approximation is selected, the next step should be the construction of the system of orthogonal polynomials $P_\alpha^{(k)}$ (Mintzer 1965), with f_0 serving as the weight function in the orthogonality relations:

$$\int f_0(\mathbf{v}, \mathbf{r}, t) P_\alpha^{(k)} P_\beta^{(n)} d\mathbf{v} = N_\alpha(\mathbf{r}, t) \delta_{\alpha, \beta}^{k, n}, \quad (4.1.5)$$

where N_α is the normalizing factor and δ is the Kronecker symbol. The a_α coefficients in the expansion of distribution function can be expressed through their

moments (4.1.2). Multiplying (4.1.4) by the corresponding orthogonal polynomial and making use of the orthogonality equations (4.1.5), we obtain:

$$a_\alpha(\mathbf{r}, t) = \frac{1}{N_\alpha} \int f(\mathbf{v}, \mathbf{r}, t) P_\alpha^{(k)} d\mathbf{v}. \quad (4.1.6)$$

If the polynomials $P_\alpha^{(k)}$ constitute an ordered sequence of monomials $1, v_\alpha, v_\alpha v_\beta, v_\alpha v_\beta v_\gamma, \dots$, then the resulting set of polynomials form the complete system of orthogonal velocity polynomials. Selection of the zero-order distribution function f_0 , the initial system of polynomials, and the domain of integration completely specifies the system of orthogonal polynomials. For example, for the local Maxwell distribution chosen as the weight function and the rectangular system of coordinates in the phase space, the corresponding system of polynomials is the Hermite polynomials, which are used in the Grad method (Grad 1949; Kogan 1967; Silin 1971).

Significant difference in masses of electrons and ions and, as a result, in the values of mean free paths and collision times, results in a difference of relaxation times due to intercomponent collisions. This allows the introduction of different distribution functions for each component, with each component having its own set of macroscopic parameters and hydrodynamic equations, and considers the plasma as a mixture of two fluids: electron and ion fluids. An approximation of plasma as a mixture of ion and electron fluids (two-fluid or multifluid approximation) has a rather wide area of applicability. This approach is often used to describe various processes not only in collisional but also in collisionless plasma. Application of these equations to collisionless plasma does not mean that Coulomb collisions are neglected. Coulomb interaction of the charged particles produces a self-consistent electromagnetic field in a plasma, which serves as a mechanism of interconnection for the ion–electron fluid. Apart from the self-consistent field, these two fluids are also “tied” together by interparticle collisions transferring energy and momentum between the components, in other words, equalizing the two temperatures and producing a friction force between the two components.

In a partially ionized plasma, the neutral component has to be taken into account as well. The interaction between charged and neutral particles goes through collisions only, in contrast with the ion–electron interactions. Besides, in addition to the momentum and energy exchange, various transformations of the particles can take place: particle excitations, formation of new ions, charge exchanges, etc.

A hydrodynamic approximation works well when the characteristic space and timescales of the problem are much greater than the mean free paths and corresponding collision times (4.1.3). However, the hydrodynamic approach can also be used for collisionless plasmas. The possibility of using a hydrodynamic approximation for perpendicular motions of plasma was first demonstrated by Chew et al. (1956). The results obtained showed a striking similarity with the collisional case. Such a coincidence in seemingly totally different situations is explained by the fact that the magnetic field causing the charged particles to rotate around the lines of force leads to symmetrization of the velocity distribution in the

directions perpendicular to the magnetic field direction. The rotating particles are thus localized in the perpendicular direction over the Larmor radius, which, for motions across a magnetic field, plays the role of the mean free path. In this sense, the magnetic field acts similar to collisions. That is why, for perpendicular motions with characteristic space and timescales far in excess of the Larmor radius and period of the Larmor rotation, a collisionless plasma behaves much like a fluid.

A hydrodynamic approach can also be used to describe longitudinal motions of a collisionless plasma. In this case, particles are bound together by the self-consistent electric field rather than by collisions or a magnetic field. The applicability of such an approximation depends on the values of electric force and pressure gradient and requires that thermal velocity be small compared with the bulk velocity. We will not discuss in detail the question of applicability of the hydrodynamic approximation for the collisionless plasma because this question is considered in detail in the next chapter, along with the derivation of corresponding systems of hydrodynamic equations.

Frequent collisions “stir” the energy of intrinsic particle motion, changing velocity vectors and causing isotropization of their distribution functions. Pressure in this case remains isotropic. In the collisionless plasma, such stirring is absent and pressures in various directions may differ. Magnetic field produces the effect of stirring in two directions perpendicular to the magnetic field lines of force even if there are no collisions. For times much greater than the period of Larmor rotation, the averaging over fast cyclotron rotation produces uniform distribution of energy over the transversal degrees of freedom. The energies of parallel and perpendicular motions thus remain “unstirred” and, as a result, the possibility of two different pressures existing for the parallel and perpendicular directions arises. Because the underlying assumptions justifying the use of the hydrodynamic approximation for parallel and perpendicular motions are different, the laws of parallel and perpendicular pressure behavior during plasma motion also differ. Even if the two pressures are equal initially, the difference will evolve later as a result of plasma motion.

The geomagnetic field produces an anisotropy in energy and mass transport properties of the near-Earth plasma. In those regions where collisions are practically absent, the thermal energy distribution of the charged particles over the degrees of freedom along and perpendicular to the magnetic field direction may be different. The energy anisotropy of the thermal plasma can exist even in the presence of collisions. For this to happen, the average collision time has to be greater than the typical timescales of the processes, leading to the unbalanced supply of energy to various degrees of freedom. These could be various heating (cooling) processes with different energy fluxes for various degrees of freedom or heat transfer processes going with different velocities for longitudinal and transversal degrees of freedom. These situations are observed in the near-Earth plasma and hence it is reasonable to expect thermal energy anisotropy for charged particles over different degrees of freedom relative to the direction of the external magnetic field.

The kinetic theory approaches mentioned above (Chapman and Cowling 1958; Grad 1949; Braginskii 1965; Shkarofsky et al. 1966), and among them the Grad

method, do not take into account possible nonuniformity of the plasma energy distribution over the degrees of freedom. (It should be pointed out that the 13-moment approximation in the Grad method allows one to account for a small pressure anisotropy through small terms in the diagonal part of the viscous stress tensor. This approach is limited by the requirement that the viscous tensor elements, and, hence, the anisotropy, be small, and was used for example in a number of studies devoted to simulation of the near-Earth plasma with the temperature anisotropy; Schunk and Watkins 1979, 1981, 1982; Mitchell and Palmadesso 1983. The Grad method is inapplicable for the description of a plasma with an arbitrary pressure anisotropy.) That is why an adequate mathematical description of a plasma with anisotropic energy distribution is impossible for the hydrodynamic equations obtained in the framework of these approaches. For example, one cannot exploit such systems of transport equations to describe the ionosphere and plasmasphere with an anisotropic electron temperature that was discovered experimentally (Clark et al. 1973; Likhter et al. 1979). It is necessary therefore to develop a method that would allow one to obtain a system of hydrodynamic equations for a plasma with an anisotropic energy distribution.

The approach based on the generalized series representation of the distribution function (4.1.4) allows one to account for the above-mentioned property. Bearing in mind the difference in thermal energy distribution over the degrees of freedom, it looks feasible as the first step in the solution of the problem to accept a quasiequilibrium bi-Maxwellian distribution as the zero-order approximation

$$f = n \left(\frac{m}{2\pi T_{\perp}} \right) \left(\frac{m}{2\pi T_{\parallel}} \right)^{1/2} \exp \left(-\frac{mw_{\perp}^2}{2T_{\perp}} - \frac{mw_{\parallel}^2}{2T_{\parallel}} \right), \quad (4.1.7)$$

which means separate equilibrium Maxwell distributions for perpendicular and parallel degrees of freedom with corresponding temperatures $T_{\perp}$ and $T_{\parallel}$. The bi-Maxwellian distribution constitutes a natural first step from the isotropic Maxwell function toward accounting for anisotropy in energy distribution. It should be noted here that (4.1.7) belongs to a class of functions $F(v^2, v_{\parallel}, \mathbf{r}, t)$ (Oraevskii et al. 1985) that describe the zero-order approximation in the solution of the kinetic equation

$$[\mathbf{v}' \times \mathbf{B}] \frac{\partial F}{\partial \mathbf{v}'} = 0, \quad \mathbf{v}' = \mathbf{v} - c \frac{[\mathbf{E} \times \mathbf{B}]}{B^2} \quad (4.1.8)$$

for a plasma immersed in the magnetic field and when the Larmor radius for the plasma particles is much less than the characteristic scales of the problem under consideration and the mean free path of the particles. Choosing the distribution function (4.1.7) represented as the product of Maxwell functions with different temperatures allows one to obtain a system of macroscopic equations for the expansion coefficients (4.1.4) that has the same level of complexity as the analogous system for an anisotropic plasma obtained from the Grad method. It should be pointed out also that the knowledge of the distribution function (4.1.7) underlies the

analysis of the observation data (Clark et al. 1973; Likhter et al. 1979). The correct interpretation of experimental data using the models of ionosphere require that the theory be based on f taken in the form (4.1.7).

A method leading to the system of hydrodynamic equations with temperature anisotropy was first suggested in 1965 by Oraevskii and was published later (Oraevskii et al. 1968). The approach is a generalization of the well-known Grad method. The distribution function in Oraevskii et al. (1968) is also represented as the series of Hermite polynomials, but the zero-order approximation, in contrast to the Grad method, is taken in the form of bi-Maxwellian function (4.1.7). In addition, due to the temperature anisotropy the two heat flux vectors are introduced that describe separately transport of parallel and perpendicular thermal energies. In contrast with the 13-moment approximation of Grad, the distribution function in Oraevskii et al. (1968) at the same level of approximation by the Hermite polynomials is expressed through 16 independent moments (the 16-moment approximation). The truncation of the series representing the distribution function and thus the closure criterion is obtained in Oraevskii et al. (1968).

4.2 General Transport Equations

Let us first give the definitions and a brief description of the basic macroscopic parameters of the plasma expressed through the moments of the distribution function. See also Sect. 2.1.

The zero-order moment is the number density:

$$n(\mathbf{r}, t) = \int f(\mathbf{v}, \mathbf{r}, t) d\mathbf{v}. \quad (4.2.1)$$

The first moment is the macroscopic (hydrodynamic or directional) velocity:

$$\mathbf{V}(\mathbf{r}, t) = \frac{1}{n} \int \mathbf{v} f(\mathbf{v}, \mathbf{r}, t) d\mathbf{v}, \quad (4.2.2)$$

where $\mathbf{v} = \mathbf{V} + \mathbf{w}$ is the total velocity, and $\mathbf{w}$ is the intrinsic (chaotic) part of it that, according to (2.1.6), satisfies the condition

$$\int \mathbf{w} f d\mathbf{v} = \mathbf{0}.$$

The second moment is the stress tensor and is defined as follows

$$P_{\alpha\beta}(\mathbf{r}, t) = m \int w_{\alpha} w_{\beta} f(\mathbf{v}, \mathbf{r}, t) d\mathbf{w}. \quad (4.2.3)$$

In the absence of external fields (no selected directions), as is well known, there exists only one scalar quantity that could be constructed from the tensor $P_{\alpha\beta}$, namely

$$3p \equiv P_{\alpha\beta}\delta_{\alpha\beta} = \text{Sp}P_{\alpha\beta}. \quad (4.2.4)$$

Here $\delta_{\alpha\beta}$ is the Kronecker symbol ($\delta_{\alpha\beta} = 1$ when $\alpha = \beta$ and $\delta_{\alpha\beta} = 0$ when $\alpha \neq \beta$); the Sp notation indicates trace of a tensor; and p is the isotropic pressure of plasma.

In the presence of an external magnetic field, there is another scalar quantity

$$p_{||} = P_{\alpha\beta}\tau_{\alpha\beta} \equiv m \int w_{||}^2 f d\mathbf{w}, \quad (4.2.5)$$

where $w_{||} = (\mathbf{w}, \mathbf{h})$;

$$\tau_{\alpha\beta} = h_{\alpha}h_{\beta}; \quad (4.2.6)$$

here $\mathbf{h} = \mathbf{B}/B$ is a unit vector along the direction of external magnetic field that defines the longitudinal plasma direction. We can now rewrite (4.2.4) as $\text{Sp}P_{\alpha\beta} = 2p_{\perp} + p_{||}$. Using (4.2.5) and introducing the notation

$$n_{\alpha\beta} = \delta_{\alpha\beta} - \tau_{\alpha\beta}, \quad (4.2.7)$$

we get the definition for $p_{\perp}$:

$$p_{\perp} \equiv \frac{1}{2}P_{\alpha\beta}n_{\alpha\beta} \equiv \frac{m}{2} \int w_{\perp}^2 f d\mathbf{w}, \quad (4.2.8)$$

$$\mathbf{w}_{\perp} = \mathbf{w} - (\mathbf{w}, \mathbf{h})\mathbf{h},$$

which is called the “perpendicular pressure.” The relations (4.2.5)–(4.2.8) now allow us to represent the pressure tensor as

$$P_{\alpha\beta} = p_{\perp}n_{\alpha\beta} + p_{||}\tau_{\alpha\beta} + \pi_{\alpha\beta} = p_{\alpha\beta} + \pi_{\alpha\beta}. \quad (4.2.9)$$

Here

$$p_{\alpha\beta} = p_{\perp}n_{\alpha\beta} + p_{||}\tau_{\alpha\beta} \quad (4.2.10)$$

is the pressure tensor; and $\pi_{\alpha\beta}$ is the viscous stress tensor, which according to (4.2.5), (4.2.8), and (4.2.9) satisfies

$$\pi_{\alpha\beta}\tau_{\alpha\beta} = 0, \quad \pi_{\alpha\beta}n_{\alpha\beta} = 0. \quad (4.2.11)$$

Expression (4.2.10) for the pressure tensor can also be obtained using a different approach. For example, one can seek the zero-order approximation in the Larmor radius (4.1.8) of the kinetic equation, as it is done in Oraevskii et al. (1968), or one can look for a solution of the full equation for $P_{\alpha\beta}$ in the same approximation (Schunk 1977).

In the isotropic case ($p_{\parallel} = p_{\perp}$), $P_{\alpha\beta} = p\delta_{\alpha\beta} + \pi'_{\alpha\beta}$, and the viscous stress tensor should satisfy only one condition, $\pi'_{\alpha\beta}\delta_{\alpha\beta} = 0$ (see, e.g., Braginskii 1965). In this case, it only has five independent components. When the pressure is anisotropic, the number of independent components of the viscosity tensor $\pi_{\alpha\beta}$, as it follows from (4.2.11), equals four. This also means that the longitudinal viscosity (usually included in $\pi'_{\alpha\beta}$) is included in $p_{\parallel}$. The same could be said about $p_{\perp}$.

Let us define the third-order moment, the heat flux tensor, as follows

$$s_{\alpha\beta\gamma}(\mathbf{r}, t) = m \int w_{\alpha} w_{\beta} w_{\gamma} f(\mathbf{v}, \mathbf{r}, t) d\mathbf{w}. \quad (4.2.12)$$

Because we now have two pressures, $p_{\parallel}$ and $p_{\perp}$, we need two heat flux vectors, $\mathbf{S}^{\parallel}$ and $\mathbf{S}^{\perp}$, that describe the transport of parallel and perpendicular energies and are defined by the following relations

$$S_{\alpha}^{\parallel} \equiv s_{\alpha\beta\gamma} \tau_{\beta\gamma} \equiv m \int w_{\alpha} w_{\parallel}^2 f d\mathbf{w}, \quad (4.2.13)$$

$$S_{\alpha}^{\perp} \equiv s_{\alpha\beta\gamma} n_{\beta\gamma} \equiv \frac{m}{2} \int w_{\alpha} w_{\perp}^2 f d\mathbf{w}. \quad (4.2.14)$$

Then by analogy with (4.2.9), the full heat flux tensor (4.2.12) could be written as

$$s_{\alpha\beta\gamma} = S_{\alpha\beta\gamma} + \sigma_{\alpha\beta\gamma}, \quad (4.2.15)$$

where

$$\begin{aligned} S_{\alpha\beta\gamma} = & \frac{1}{2} \left[S_{\alpha}^{\perp} n_{\beta\gamma} + S_{\beta}^{\perp} n_{\alpha\gamma} + S_{\gamma}^{\perp} n_{\alpha\beta} + (S_e^{\perp} \tau_{e\alpha}) n_{\beta\gamma} + (S_e^{\perp} \tau_{e\beta}) n_{\alpha\gamma} + (S_e^{\perp} \tau_{e\gamma}) n_{\alpha\beta} \right] \\ & + S_{\alpha}^{\parallel} \tau_{\beta\gamma} + S_{\beta}^{\parallel} \tau_{\alpha\gamma} + S_{\gamma}^{\parallel} \tau_{\alpha\beta} - \frac{2}{3} \left[(S_e^{\parallel} \tau_{e\alpha}) \tau_{\beta\gamma} + (S_e^{\parallel} \tau_{e\beta}) \tau_{\alpha\gamma} + (S_e^{\parallel} \tau_{e\gamma}) \tau_{\alpha\beta} \right], \end{aligned} \quad (4.2.16)$$

and tensor $\sigma_{\alpha\beta\gamma}$ satisfies the conditions

$$\sigma_{\alpha\beta\gamma} \tau_{\beta\gamma} = 0, \quad \sigma_{\alpha\beta\gamma} n_{\beta\gamma} = 0. \quad (4.2.17)$$

The macroscopic parameters introduced above: number density, hydrodynamic velocity, pressure, and heat flux tensors, are described by the moment equations

obtained by averaging (using certain weight coefficients) of the kinetic equation over the phase space. To demonstrate that, multiply the kinetic equation (2.1.1) by m , mV_α , $mV_\alpha V_\beta$, $mV_\alpha V_\beta V_\gamma$ and integrate over $\mathbf{v}$. After performing some routine transformations, we get the system of equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = J^{(0)}, \quad (4.2.18)$$

$$\rho \frac{dV_\alpha}{dt} + \frac{\partial}{\partial x_\beta} P_{\alpha\beta} - \rho \left(\frac{1}{m} F_\alpha + \frac{e}{m} E_\alpha + \omega_B [\mathbf{V} \times \mathbf{h}]_\alpha \right) = J_\alpha^{(1)}, \quad (4.2.19)$$

$$\begin{aligned} \rho \frac{dP_{\alpha\beta}}{dt} + \frac{\partial}{\partial x_\gamma} S_{\alpha\beta\gamma} + \frac{\partial V_\gamma}{\partial x_\varepsilon} (\delta_{\gamma\varepsilon} P_{\alpha\beta} + \delta_{\alpha\gamma} P_{\beta\varepsilon} + \delta_{\beta\gamma} P_{\alpha\varepsilon}) \\ - \omega_B h_\varepsilon (\varepsilon_{\beta\gamma\varepsilon} P_{\alpha\gamma} + \varepsilon_{\alpha\gamma\varepsilon} P_{\beta\gamma}) = J_{\alpha\beta}^{(2)}, \end{aligned} \quad (4.2.20)$$

$$\begin{aligned} \rho \frac{ds_{\alpha\beta\gamma}}{dt} + \frac{\partial}{\partial x_\varepsilon} Q_{\alpha\beta\gamma\varepsilon} - \frac{1}{\rho} \frac{\partial P_{ve}}{\partial x_\varepsilon} (\delta_{\alpha\varepsilon} P_{\beta\gamma} + \delta_{\beta\varepsilon} P_{\alpha\gamma} + \delta_{\gamma\varepsilon} P_{\alpha\beta}) \\ + \frac{\partial V_v}{\partial x_\varepsilon} (\delta_{ve} S_{\alpha\beta\gamma} + \delta_{\alpha v} S_{\beta\gamma\varepsilon} + \delta_{\beta v} S_{\alpha\gamma\varepsilon} + \delta_{\gamma v} S_{\alpha\beta\varepsilon}) - \omega_B h_\varepsilon (\varepsilon_{\alpha v\varepsilon} S_{\beta\gamma v} + \varepsilon_{\beta v\varepsilon} S_{\alpha\gamma v} + \varepsilon_{\gamma v\varepsilon} S_{\alpha\beta v}) \\ = J_{\alpha\beta\gamma}^{(3)} - \frac{1}{\rho} (P_{\alpha\beta} J_\gamma^{(1)} + P_{\alpha\gamma} J_\beta^{(1)} + P_{\beta\gamma} J_\alpha^{(1)}), \end{aligned} \quad (4.2.21)$$

where $\rho = mn$ is the mass density; $d/dt = \partial/\partial t + (\mathbf{V} \cdot \nabla)$ is the material derivative; $\varepsilon_{\alpha\beta\gamma}$ is the totally antisymmetric unit tensor of the third order; and $Q_{\alpha\beta\gamma\varepsilon} = m \int w_\alpha w_\beta w_\gamma w_\varepsilon f d\mathbf{w}$ is the fourth order moment. Terms at the right hand side of (4.2.18)–(4.2.21)

$$J^{(0)} = m \int S d\mathbf{w}, \quad (4.2.22)$$

$$J_\alpha^{(1)} = m \int w_\alpha S d\mathbf{w}, \quad (4.2.23)$$

$$J_{\alpha\beta}^{(2)} = m \int w_\alpha w_\beta S d\mathbf{w}, \quad (4.2.24)$$

$$J_{\alpha\beta\gamma}^{(3)} = m \int w_\alpha w_\beta w_\gamma S d\mathbf{w} \quad (4.2.25)$$

are moments of the collision integrals S of the kinetic equation and describe variation of the corresponding macroscopic parameters due to collisions.

As has already been mentioned in Sect. 4.1 and is clearly seen from the above moment equations, the system (4.2.18)–(4.2.21) is not closed, but, rather, forms a

sequence of engaged equations. For example, (4.2.21) for the heat flux tensor $s_{\alpha\beta\gamma}$ contains the divergence term of the next-order moment, $Q_{\alpha\beta\gamma\epsilon}$. In order to obtain a closed system of equations in the approximation that would include the heat flux tensor, $Q_{\alpha\beta\gamma\epsilon}$ should be expressed through the lower-order moments of the distribution function. To this end, certain assumptions should be made about the relations between the moments of different order or the distribution function should be known. The second choice seems to be more consistent and preferable as the knowledge of the distribution function allows also the calculation of the collisional terms (4.2.22)–(4.2.25) in the equations.

Following Oraevskii et al. (1968), we will be looking for the distribution function in the form of the series of tensor's Hermite polynomials $H_{i_1 \dots i_k}^{(k)}(\xi)$:

$$f(\xi, \mathbf{r}, t) = f_0(\xi, \mathbf{r}, t) \sum_{k=0}^{\infty} \frac{1}{k!} a_{i_1 \dots i_k}^{(k)}(\mathbf{r}, t) H_{i_1 \dots i_k}^{(k)}(\xi), \quad (4.2.26)$$

where ξ is the weighted velocity related to $\mathbf{w}$ by the following transformation

$$w_i = R_{ik} \xi_k, \quad R_{ik} = \sqrt{1/\beta_{\perp}} n_{ik} + \sqrt{1/\beta_{\parallel}} \tau_{ik}, \quad \beta_{\perp} = m/T_{\perp}, \quad \beta_{\parallel} = m/T_{\parallel}. \quad (4.2.27)$$

In accordance with Sect. 4.1, we will choose the bi-Maxwellian distribution function (4.1.7) as the zeroth-order approximation f_0 in the expansion (4.2.26). When expressed through the weighted velocity ξ it can be written, according to (4.2.27), as follows

$$f_0 = (2\pi)^{-3/2} n \beta_{\perp} \beta_{\parallel}^{1/2} e^{-\xi^2/2}. \quad (4.2.28)$$

This function, accurate to the velocity-independent factor is the weight function in the orthogonality relations of the Hermite polynomials, which are defined by

$$H_{i_1 \dots i_k}^{(k)}(\xi) = (-1)^k e^{\xi^2/2} \frac{\partial^k e^{-\xi^2/2}}{\partial \xi_{i_1} \dots \partial \xi_{i_k}}, \quad (4.2.29)$$

where the subscripts $i_1, \dots, i_k$ may take values 1–3. Consider a first few polynomials

$$H^{(0)} = 1, \quad H_i^{(1)} = \xi_i, \quad H_{ij}^{(2)} = \xi_i \xi_j - \delta_{ij}, \quad H_{ijk}^{(3)} = \xi_i \xi_j \xi_k - \delta_{ik} \xi_j - \delta_{ij} \xi_k - \delta_{jk} \xi_i. \quad (4.2.30)$$

Polynomials (4.2.29) form the normalized orthogonal system. The orthogonality relations read

$$(2\pi)^{-3/2} \int e^{-\xi^2/2} H_{i_1 \dots i_n}^{(n)} H_{j_1 \dots j_m}^{(m)} d\xi = \delta_{nm} \delta_{i_1 \dots i_n j_1 \dots j_m}, \quad (4.2.31)$$

where $\delta_{i_1 \dots i_n, j_1 \dots j_m} = 1$, if the sequence of subscripts $i_1 \dots i_n$ can be obtained by permutation of a group of subscripts $j_1 \dots j_m$, otherwise $\delta_{i_1 \dots i_n, j_1 \dots j_m} = 0$.

The orthogonality relations (4.2.31) allow us to obtain the expansion coefficients (4.2.26)

$$a_{i_1 \dots i_k}^{(k)} = \frac{1}{n\beta_{\perp}\sqrt{\beta_{\parallel}}} \int f H_{i_1 \dots i_k}^{(n)} d\zeta = \frac{1}{n} \int f H_{i_1 \dots i_k}^{(n)} d\mathbf{w}. \quad (4.2.32)$$

Using (4.2.30), (4.2.32), as well as definitions of the moments of distribution function (4.2.1)–(4.2.3), (4.2.9), and (4.2.12), one can express the coefficients $a_{i_1 \dots i_k}^{(k)}$ through the macroscopic plasma parameters:

$$a^{(0)} = 1, \quad a_i^{(1)} = 0, \quad a_{ij}^{(2)} = \frac{1}{\rho} R_{ix}^{-1} R_{j\beta}^{-1} \pi_{\alpha\beta}, \quad a_{ijk}^{(3)} = \frac{1}{\rho} R_{ix}^{-1} R_{j\beta}^{-1} R_{k\gamma}^{-1} s_{\alpha\beta\gamma}. \quad (4.2.33)$$

To obtain a closed system of moment equations (4.2.18)–(4.2.21), it is necessary to approximate the distribution function by the finite number of terms in the series (4.2.26). If we restrict ourselves to the moments having clear physical sense, up to the heat flux tensor, for example, this will correspond to retaining the terms with $k \leq 3$ in the expansion (4.2.26). Thus, using the expressions for the coefficients (4.2.33), the distribution function finally takes the form

$$f = f_0 \left[1 + \frac{\pi_{\alpha\beta}}{2\rho} G_{\alpha i} G_{\beta j} w_i w_j + \frac{s_{\alpha\beta\gamma}}{2\rho} \left(G_{\alpha i} G_{\beta j} G_{\gamma k} w_i w_j w_k - G_{\beta\gamma} G_{\alpha i} w_i - G_{\alpha\gamma} G_{\beta j} w_j - G_{\alpha\beta} G_{\gamma k} w_k \right) \right],$$

$$G_{\alpha\beta} = R_{\alpha i}^{-1} R_{\beta i}^{-1} = \beta_{\perp} n_{\alpha\beta} + \beta_{\parallel} \tau_{\alpha\beta}. \quad (4.2.34)$$

Using the above results, it can be readily shown that in this approximation $Q_{\alpha\beta\gamma\epsilon}$ can be expressed through the second-order moments of the distribution function

$$Q_{\alpha\beta\gamma\epsilon} = \frac{1}{\rho} (p_{\alpha\beta} p_{\gamma\epsilon} + p_{\alpha\gamma} p_{\beta\epsilon} + p_{\beta\gamma} p_{\alpha\epsilon} + \chi_{\alpha\beta\gamma\epsilon}),$$

$$\chi_{\alpha\beta\gamma\epsilon} = p_{\alpha\beta} \pi_{\gamma\epsilon} + p_{\alpha\gamma} \pi_{\beta\epsilon} + p_{\beta\gamma} \pi_{\alpha\epsilon} + p_{\gamma\epsilon} \pi_{\alpha\beta} + p_{\beta\epsilon} \pi_{\alpha\gamma} + p_{\alpha\epsilon} \pi_{\beta\gamma}. \quad (4.2.35)$$

This expression for $Q_{\alpha\beta\gamma\epsilon}$ closes the system of equations (4.2.18)–(4.2.19). Retaining in (4.2.34) the total heat flux tensor $s_{\alpha\beta\gamma}$ having 10 independent components, we arrive at the description of distribution function by 20 moments, the 20-moment approximation. This approximation, with rare exceptions (see, e.g., Barakat and Schunk 1982b), is practically never used because of the huge complexity of the system of hydrodynamic equations. That is why a further simplification is usually made at this point by performing a summation of $s_{\alpha\beta\gamma}$ over the two subscripts and changing thus to the heat flux vectors.

In order to estimate the degree of approximation of the obtained result (2.34), let us, following Grad (1949), define a measure of deviation of the distribution function from the zero-order approximation using the Hilbert's "distance":

$$\left[\int \frac{(f - f_0)^2}{f_0} d\mathbf{w} \right]^{1/2}.$$

Substituting (4.2.34) and using the orthogonality of Hermit's polynomials, one can demonstrate that f is close to f_0 (in other words, the series (4.2.26) rapidly converges and approximation (4.2.34) can be used as an approximation of the distribution function), if

$$\alpha = \left[\frac{1}{2} a_{ij}^2 a_{ji}^2 + \frac{1}{6} a_{ijk}^3 a_{kji}^3 \right]^{1/2} \quad (4.2.36)$$

is small (Oraevskii et al. 1968). Explicit criteria for the characteristic spatiotemporal scales justifying the truncation of the series (4.2.26), and representations of the distribution function in the form (4.2.34) will be given in the next section where the system of hydrodynamic equations for the anisotropic plasma is presented. These criteria, as demonstrated in Oraevskii et al. (1968), imply that $\pi_{\alpha\beta\gamma}$ should be small compared with $S_{\alpha\beta\gamma}$ in (4.2.15). Assuming in (4.2.34), $\sigma_{\alpha\beta\gamma} = 0$, we obtain the distribution function that is described by 16 moments. This representation of the distribution function is called "the 16-moment approximation." The representation is defined by the function (2.34), where $s_{\alpha\beta\gamma}$ should be substituted by $S_{\alpha\beta\gamma}$, defined by (4.2.16). Finally, we can write the 16-moment representation of the distribution function in an orthogonal system of coordinates with the unit vector $\mathbf{e}_3$ aligned with the magnetic field:

$$\begin{aligned} f = f_0 & \left\{ 1 + \frac{\beta_{\perp}}{2\rho} \left[\beta_{\perp} \left(w_1^2 - w_2^2 \right) \pi_{11} + 2\beta_{\perp} w_1 w_2 \pi_{12} + 2\beta_{\parallel} w_3 \left(w_1 \pi_{13} + w_2 \pi_{23} \right) \right] \right. \\ & + \frac{1}{\rho} \left[\left(w_1^2 + w_2^2 \right) \frac{\beta_{\perp}^2}{2} \left[\frac{\beta_{\perp}}{2} \left(w_1 S_1^{\perp} + w_2 S_2^{\perp} \right) + \beta_{\parallel} w_3 S_3^{\perp} \right] \right. \\ & + \frac{\beta_{\parallel}^2}{2} w_3^2 \left[\beta_{\perp} \left(w_1 S_1^{\parallel} + w_2 S_2^{\parallel} \right) + \frac{\beta_{\parallel}}{3} w_3 S_3^{\parallel} \right] \\ & \left. \left. - \beta_{\perp}^2 \left(w_1 S_1^{\perp} + w_2 S_2^{\perp} \right) - \beta_{\perp} \beta_{\parallel} w_3 S_3^{\perp} - \frac{\beta_{\perp} \beta_{\parallel}}{2} \left(w_1 S_1^{\parallel} + w_2 S_2^{\parallel} \right) - \frac{\beta_{\parallel}^2}{2} w_3 S_3^{\parallel} \right] \right\}, \\ f_0 = & \left(2\pi \right)^{-3/2} n \beta_{\perp} \sqrt{\beta_{\parallel}} \exp \left[-\frac{\beta_{\perp}}{2} \left(w_1^2 + w_2^2 \right) - \frac{\beta_{\parallel}}{2} w_3^2 \right]. \end{aligned} \quad (4.2.37)$$

This expression will be used in the next chapter in calculations of the moments of the collision integral.

4.3 The 16-Moment Approximation

Let us now derive from the generalized transport equations (4.2.18)–(4.2.21), the system of hydrodynamic equations for an anisotropic plasma in the 16-moment approximation. The equation for the parallel pressure $p_{\parallel}$ can be obtained from (4.2.20) by multiplication on $\tau_{\alpha\beta}$. In a similar way, multiplying (4.2.20) by $\frac{1}{2}n_{\alpha\beta}$, the equation for $p_{\perp}$ is obtained. After the subtraction of the obtained equations from (4.2.20), the equation for the viscosity tensor $\pi_{\alpha\beta}$ is obtained. The heat flux vectors $\mathbf{S}^{\parallel}$ and $\mathbf{S}^{\perp}$ are described by the equations that can be obtained by multiplying (4.2.21) by $\tau_{\alpha\beta}$ and $\frac{1}{2}n_{\alpha\beta}$, where it is necessary to set $s_{\alpha\beta\gamma} = S_{\alpha\beta\gamma}$ and to make use of (4.2.16) and (4.2.35). Now, taking into account relations (4.2.9) and (4.2.10), we can finally write the system of hydrodynamic equations in the 16-moment approximation:

Continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = J^{(0)}, \quad (4.3.1)$$

Momentum

$$\begin{aligned} \rho \frac{d\mathbf{V}}{dt} + \nabla_{\parallel} p_{\parallel} + \nabla_{\perp} p_{\perp} + (p_{\parallel} - p_{\perp}) [\mathbf{h} \nabla \cdot \mathbf{h} + (\mathbf{h} \nabla) \mathbf{h}] \\ + \nabla \cdot \hat{\pi} - \rho \left(\frac{1}{m} \mathbf{F} + \frac{e}{m} \mathbf{E} + \omega_B [\mathbf{V} \times \mathbf{h}] \right) = \mathbf{J}^{(1)}, \end{aligned} \quad (4.3.2)$$

Parallel energy

$$\begin{aligned} \frac{dp_{\parallel}}{dt} + \frac{dV_{\alpha}}{dx_{\beta}} (p_{\parallel} n_{\alpha\beta} + 3p_{\parallel} \tau_{\alpha\beta} + 2\pi_{\beta\gamma} \tau_{\alpha\gamma}) + \nabla \cdot \mathbf{S}^{\parallel} - 2S_{\alpha}^{\parallel} (\mathbf{h} \nabla) h_{\alpha} \\ - 2(\mathbf{S}^{\perp}, \mathbf{h}) \nabla \cdot \mathbf{h} - \pi_{\alpha\beta} \frac{d\tau_{\alpha\beta}}{dt} = J_{\parallel}^{(2)}, \quad J_{\parallel}^{(2)} = J_{\alpha\beta}^{(2)} \tau_{\alpha\beta}, \end{aligned} \quad (4.3.3)$$

Perpendicular energy

$$\begin{aligned} \frac{dp_{\perp}}{dt} + \frac{dV_{\alpha}}{dx_{\beta}} (p_{\perp} \tau_{\alpha\beta} + 2p_{\perp} n_{\alpha\beta} + \pi_{\beta\gamma} n_{\alpha\gamma}) + \nabla \cdot \mathbf{S}^{\perp} + S_{\alpha}^{\parallel} (\mathbf{h} \nabla) h_{\alpha} \\ + (\mathbf{S}^{\perp}, \mathbf{h}) \nabla \cdot \mathbf{h} + \frac{1}{2} \pi_{\alpha\beta} \frac{d\tau_{\alpha\beta}}{dt} = J_{\perp}^{(2)}, \quad J_{\perp}^{(2)} = \frac{1}{2} J_{\alpha\beta}^{(2)} n_{\alpha\beta}, \end{aligned} \quad (4.3.4)$$

Stress tensor

$$\begin{aligned} -\omega_B h_{\epsilon} (\varepsilon_{\alpha\gamma\epsilon} \pi_{\beta\gamma} + \varepsilon_{\beta\gamma\epsilon} \pi_{\alpha\gamma}) + p_{\parallel} \left(\frac{dV_{\alpha}}{dx_{\gamma}} \tau_{\beta\gamma} + \frac{dV_{\beta}}{dx_{\gamma}} \tau_{\alpha\gamma} - 2 \frac{dV_{\gamma}}{dx_{\epsilon}} \tau_{\gamma\epsilon} \tau_{\alpha\beta} \right) \\ + p_{\perp} \left(\frac{dV_{\alpha}}{dx_{\gamma}} n_{\beta\gamma} + \frac{dV_{\beta}}{dx_{\gamma}} n_{\alpha\gamma} - \frac{dV_{\gamma}}{dx_{\epsilon}} n_{\gamma\epsilon} n_{\alpha\beta} \right) + (p_{\parallel} - p_{\perp}) \frac{d\tau_{\alpha\beta}}{dt} - O_1(\pi, S) \\ = J_{\alpha\beta}^{(2)} - \left(J_{\parallel}^{(2)} \tau_{\alpha\beta} + J_{\perp}^{(2)} n_{\alpha\beta} \right), \end{aligned} \quad (4.3.5)$$

Parallel heat flow

$$\begin{aligned}
& h_\alpha \left(\mathbf{h} \frac{d\mathbf{S}^\parallel}{dt} \right) + p_{\gamma\beta} \frac{\partial}{\partial x_\gamma} \left[\frac{p_\parallel}{\rho} (n_{\alpha\beta} + 3\tau_{\alpha\beta}) \right] - \omega_B [\mathbf{S}^\parallel \times \mathbf{h}]_\alpha \\
& - \frac{1}{\rho} (p_{\alpha\beta} p_{\gamma\varepsilon} + p_{\alpha\gamma} p_{\beta\varepsilon} + p_{\beta\gamma} p_{\alpha\varepsilon}) \frac{\partial \tau_{\beta\gamma}}{\partial x_\varepsilon} + O_2(\pi, S) \\
& = J_{\alpha\beta\gamma}^{(3)} \tau_{\beta\gamma} - \frac{1}{\rho} [p_\parallel (n_{\alpha\beta} + 3\tau_{\alpha\beta}) + 2\pi_{\alpha\gamma} \tau_{\gamma\beta}] J_\beta^{(1)}, \tag{4.3.6}
\end{aligned}$$

Perpendicular heat flow

$$\begin{aligned}
& h_\alpha \left(\mathbf{h} \frac{d\mathbf{S}^\perp}{dt} \right) + p_{\gamma\beta} \frac{\partial}{\partial x_\gamma} \left[\frac{p_\perp}{\rho} (2n_{\alpha\beta} + \tau_{\alpha\beta}) \right] - \omega_B [\mathbf{S}^\perp \times \mathbf{h}]_\alpha \\
& - \frac{1}{2\rho} (p_{\alpha\beta} p_{\gamma\varepsilon} + p_{\alpha\gamma} p_{\beta\varepsilon} + p_{\beta\gamma} p_{\alpha\varepsilon}) \frac{\partial n_{\beta\gamma}}{\partial x_\varepsilon} \\
& + O_3(\pi, S) = J_{\alpha\beta\gamma}^{(3)} n_{\beta\gamma} - \frac{1}{\rho} [p_\perp (2n_{\alpha\beta} + \tau_{\alpha\beta}) + \pi_{\alpha\gamma} n_{\gamma\beta}] J_\beta^{(1)}, \tag{4.3.7}
\end{aligned}$$

wherein terms O_1 , O_2 , and O_3 (4.3.5)–(4.3.7) include higher-order quantities compared with those retained in the equations.

Let us discuss the applicability of the 16-moment approximation for the distribution function and the corresponding system of hydrodynamic equations (4.3.1)–(4.3.7). First, let us consider the case of collisionless plasma assuming all terms in the right-hand side to be zero.

The sufficient condition of applicability of (4.2.34) or (4.2.37) for the distribution function ($\alpha \ll 1$ (4.2.36)) means the smallness of the second and third terms as compared with the first one, equal to unity. Taking into account the form of the expansion coefficients (4.2.33), we get the requirement of the smallness of the viscosity tensor components compared with $p_\parallel$ and $p_\perp$, and also of the heat fluxes $S^\perp$ and $S^\parallel$ compared with the quantity of the order ρv_T^3 , where $v_T = (T/m)^{1/2}$ is the thermal velocity. When considering the dynamics of plasma in the transversal directions, the above requirements, as they follow from (4.3.5)–(4.3.7), are satisfied if

$$r_B/L \ll 1, \tag{4.3.8}$$

$$\tau_B/t_0 \ll 1, \tag{4.3.9}$$

where L is the characteristic scale length of the problem in the direction perpendicular to the external magnetic field direction. These conditions coincide with the relations obtained in Chew et al. (1956) for the hydrodynamic description of the perpendicular motions of a collisionless plasma (see also Sect. 4.1).

Inequalities (4.3.8) and (4.3.9), in addition to the viscosity tensor, only impose restrictions on perpendicular heat flux components. The analysis of plasma motions along the magnetic field requires an additional relation for the longitudinal components of the vectors $\mathbf{S}^{\parallel}$ and $\mathbf{S}^{\perp}$, which would secure the smallness of the part of the term $S_{\alpha\beta\gamma}$ in (4.2.34) and (4.2.37) containing $S^{\perp}$ and $S^{\parallel}$. It is easy to show using (4.3.6) and (4.3.7) that this is true if

$$a_{\parallel} = v_{T\parallel} t_0 / L_{\parallel} \ll 1, \quad (4.3.10)$$

where $v_{T\parallel}$ is the parallel component of the plasma thermal velocity; $L_{\parallel}$ is the scale length in the direction of magnetic field. It should be noted that the obtained equations are applicable to the collisionless case as well when $a = v_{T\parallel} t_0 / L \ll 1$ (Oraevskii et al. 1985).

The smallness of the parameter $a_{\parallel}$ in most cases means the smallness of the pressure forces (thermal plasma motion) as compared with the electromagnetic forces. In the limit $a_{\parallel} \rightarrow 0$, thermal motion of particles is not important and the plasma is then called “cold.” Its distribution function reduces to the delta function and the stress tensor term in the equations vanishes. For the zero approximation in a , the equation for hydrodynamic velocity is nothing else but an equation for a single charged particle subject to the electromagnetic forces, multiplied by particle density. The closed system of equations in this case consists of the equation of continuity, equations of motion for each plasma component, and Maxwell equations for electromagnetic field.

When collisions are present and inequalities (4.1.3) are fulfilled, instead of the hydrodynamic approximation applicability criterion for parallel plasma motions (2.51), it is possible to use relations (4.1.3) with L substituted by $L_{\parallel}$. If, moreover, $r_B < \lambda$ and $\tau_B < \tau$, then inequalities (4.3.8) and (4.3.9) will remain the criterion for hydrodynamic description of perpendicular motions. Otherwise, (4.1.3) should be used as the criterion of applicability of the hydrodynamic description.

It should be stressed (see Sect. 4.1) that the above applicability criteria for the 16-moment approximation are only sufficient conditions. The necessary condition is a sufficiently good approximation of the distribution function by (4.2.34) and (4.2.37). In this case, in addition to the above “ordinary” hydrodynamic criteria for the zero-order approximation (4.1.7), the stability condition against the development of the instability driven by anisotropic energy distribution should be added.

Equations (4.3.5)–(4.3.7) for the viscosity tensor and heat flux vectors are written in the form with “ordering” introduced over the above-mentioned small parameters. It is evident that to the lowest order of approximation heat fluxes do not enter (4.3.5) for the viscosity tensor and, correspondingly, the viscosity tensor does not enter the (4.3.6) and (4.3.7) in the same approximation.

The terms O_1 , O_2 , and O_3 in (4.3.5)–(4.3.7) include higher-order quantities compared with those retained in the equations. This form of equations, as will be seen later, is more convenient for the analysis of plasma dynamics in different conditions. In the limit $T_{\perp} \rightarrow T_{\parallel}$ for f_0 from (4.1.7) follows the isotropic Maxwellian function and the well-known 13-moment approximation. Further

simplifications are still possible, reducing the number of moments through which the distribution function is expressed (Oraevskii et al. 1985).

4.4 Coulomb Collision Terms

Calculations of the moments of collision integrals (4.2.22)–(4.2.25) is one of the main difficulties in the derivation of the system of hydrodynamic equations and is necessary for its practical applicability. The difficulties lie both in the knowledge of the distribution functions of the interacting particles and in the knowledge of the effective sections of their interactions. In addition to these difficulties of the principal character, there are purely technical difficulties connected with the mathematical complexity and the fact that integration can be performed only in rare cases and under certain restricting assumptions.

Calculations of the moments of collision integrals in the 16-moment approximation were originally performed in Chodura and Pohl (1971) for the Coulomb interactions between the charged particles in the fully ionized plasma, where the corresponding transport coefficients were obtained. These results were later generalized and extended in Konikov and Khazanov (1981, 1982, 1983) to account for other types of particle collisions. In this and following sections, the moments of collision integrals will be obtained for the main types of the charged component interactions in a partially ionized ionospheric plasma.

4.4.1 General Expressions

Moments of the Coulomb collisions integral for the distribution function of charged particles in the 16-moment approximation (4.2.37) was considered in Chodura and Pohl (1971). First-order moments, $J_\alpha^{(1)}$, (4.2.23) and second-order moments, $J_{\alpha\beta}^{(2)}$, (4.2.24) that describe friction force, rates of collisional energy exchange, and viscosity changes in the fully ionized multispecies plasma were obtained. Third-order moments $J_{\alpha\beta\gamma}^{(3)}$ (4.2.25) for the two-component ion–electron plasma are calculated that allowed one to obtain expressions for the ion and electron heat fluxes. These results were generalized in Konikov et al. (1983), where moments $J_{\alpha\beta\gamma}^{(3)}$ for the multispecies fully ionized plasma were found as well as corresponding expressions for the heat fluxes.

Following Chodura and Pohl (1971), in order to obtain moments (4.2.23)–(4.2.25), we will use the Coulomb collision integral for the charged particles of the kinds s and r written in Landau form (2.2.2)

$$S_{sr} = -\frac{\partial}{\partial v_{sz}} j_z, \quad (4.4.1)$$

where

$$j_\alpha = A_{sr} \int u_{\alpha\beta} \left[\frac{f_s(v_s)}{m_r} \frac{\partial f_r(v_r)}{\partial v_{r\beta}} - \frac{f_r(v_r)}{m_s} \frac{\partial f_s(v_s)}{\partial v_{s\beta}} \right] d\mathbf{v},$$

$$u_{\alpha\beta} = \delta_{\alpha\beta}/g - g_\alpha g_\beta / g^3, \quad A_{sr} = \frac{1}{m_s} 2\pi e_s^2 e_r^2 \ln \Lambda, \quad (4.4.2)$$

where $\mathbf{g} = \mathbf{v}_s - \mathbf{v}_r$ is relative velocity of colliding particles; and $\ln \Lambda$ is Coulomb logarithm.

In the derivation, it was assumed that the difference between the directional velocities of the particles, $\mathbf{u} = \mathbf{V}_s - \mathbf{V}_r$, is small compared with the effective thermal velocity:

$$u_\perp \ll \sqrt{\beta_{s\perp}^{-1} + \beta_{r\perp}^{-1}}, \quad u_\parallel \ll \sqrt{\beta_{s\parallel}^{-1} + \beta_{r\parallel}^{-1}}. \quad (4.4.3)$$

Distribution functions of the colliding particles f_s and f_r were taken in the form (4.2.37), and, in the products $f_s f_r$, only terms linear in u , π , and S were retained (except for the special cases discussed below).

Let us introduce the notations:

$$\beta_{sr\perp} = \frac{\beta_{s\perp} \beta_{r\perp}}{\beta_{s\perp} + \beta_{r\perp}}, \quad \beta_{sr\parallel} = \frac{\beta_{s\parallel} \beta_{r\parallel}}{\beta_{s\parallel} + \beta_{r\parallel}}, \quad a_{sr} = \frac{\beta_{sr\parallel}}{\beta_{sr\perp}}, \quad b_{s\perp} = \frac{\beta_{sr\perp}}{\beta_{r\perp}}, \quad b_{s\parallel} = \frac{\beta_{sr\parallel}}{\beta_{r\parallel}}, \quad \mu_s = \frac{\mu_{sr}}{m_r}.$$

The following substitutions were used in the calculations of the collision integral moments:

$$\mathbf{x} = \mathbf{w}_s - \mathbf{w}_r, \quad \mathbf{z}_\perp = b_{s\perp} \mathbf{w}_{s\perp} + b_{r\perp} \mathbf{w}_{r\perp}, \quad \mathbf{z}_\parallel = b_{s\parallel} \mathbf{w}_{s\parallel} + b_{r\parallel} \mathbf{w}_{r\parallel}. \quad (4.4.4)$$

The transformation Jacobian is $\partial(\mathbf{w}_s, \mathbf{w}_r)/\partial(\mathbf{x}, \mathbf{z}) = 1$.

Below are the collisional moments for the Coulomb interaction, which enter (4.3.2)–(4.3.7), for the charged component of the kind s .

First-order moments – friction force:

$$J_\alpha^{(1)} = -4 \sum_r v_{sr} \left[K_{200} \rho_s u_\alpha + (2K_{220} - K_{200}) \beta_{sr\perp} \left(S_{s\alpha}^\perp - \frac{\rho_s}{\rho_r} S_{r\alpha}^\perp \right) \right. \\ \left. + \frac{1}{2} (2a_{sr} K_{202} - K_{200}) \beta_{sr\parallel} \left(S_{s\alpha}^\parallel - \frac{\rho_s}{\rho_r} S_{r\alpha}^\parallel \right) \right], \quad \alpha = 1, 2, \quad (4.4.5)$$

where

$$v_{sr} = \sqrt{\frac{\pi}{2}} \frac{n_r e_r^2 e_s^2 \ln \Lambda}{m_s \mu_{sr}} \beta_{sr\perp} \sqrt{\beta_{sr\parallel}} \quad (4.4.6)$$

is the Coulomb collision frequency in the presence of temperature anisotropy:

$$K_{LMN} \equiv K_{LMN}(a_{sr}) = \frac{1}{\pi} \int \frac{y_1^L y_2^M y_3^N}{y^3} \exp[-(y_1^2 + y_2^2 + a_{sr} y_3^2)] d^3 y \quad (4.4.7)$$

(values of these functions for quantities L, M, N found in this book are given in Appendix B);

$$J_{\alpha}^{(1)} = -4 \sum_r v_{sr} \left\{ K_{002} a_{sr} \rho_s u_3 + (2K_{202} - K_{002}) \beta_{sr||} \left(S_{s3}^{\perp} - \frac{\rho_s}{\rho_r} S_{r3}^{\perp} \right) + \frac{a_{sr}}{2} \left(\frac{2}{3} a_{sr} K_{004} - K_{002} \right) \beta_{sr||} \left(S_{s3}^{\parallel} - \frac{\rho_s}{\rho_r} S_{r3}^{\parallel} \right) \right\}. \quad (4.4.8)$$

Second-order moments – collisional energy exchange between parallel and perpendicular degrees of freedom in (4.3.3) and (4.3.4):

$$J_{s\perp}^{(2)} = 4n_s \sum_r v_{sr} \left\{ \mu_s \left[2K_{200}(T_{r\perp} - T_{s\perp}) + \frac{m_r}{\beta_{sr\perp}} (K_{002} - K_{200}) \right] + m_s b_{r\perp} K_{200} u_{\perp}^2 \right\}, \quad (4.4.9)$$

$$J_{s||}^{(2)} = 8n_s \sum_r a_{sr} v_{sr} \left\{ \mu_s \left[2K_{002}(T_{r||} - T_{s||}) + \frac{m_r}{\beta_{sr||}} (K_{200} - K_{002}) \right] + m_s b_{r||} K_{002} u_{||}^2 \right\}. \quad (4.4.10)$$

In the above expressions of the second-order terms, only the terms that contain squares of the relative velocity u components are retained. Other terms in this approximation are proportional to products (u, S) and are neglected here. Their expressions are given in Chodura and Pohl (1971).

The first terms in the square brackets in (4.4.9) and (4.4.10) containing temperature differences describe energy exchange between different particle species whereas the second terms describe the temperature anisotropy relaxation. In contrast with the case of isotropic energy distribution, where collisions between identical particles make no contribution to the corresponding second-order moment (Silin 1971; Tanenbaum 1967; Burgers 1969; Schunk 1975), here, these collisions take part in the temperature anisotropy relaxation process contributing to the moments (4.4.9) and (4.4.10). It is also necessary (in general, for the interaction potential $U \propto r^{-\alpha}$, see Oraevskii et al. 1985) to stress that, during the Coulomb interaction of particles with strongly differing masses (e.g., ions and electrons), the anisotropy relaxation process according to (4.4.9), (4.4.10) goes approximately m_i/m_e faster than the energy exchange between them. This statement is true for elastic electron–neutral collisions as well (see formulae below). Terms with

differences of mean velocities in (4.4.9) and (4.4.10) describe frictional heating and, in particular, Joule dissipation of energy. It is clear from the above expressions that, depending on the relation between parallel and perpendicular components of the bulk velocity difference, such heating may lead to uneven energy supply to different degrees of freedom being thus one of the possible sources of the temperature anisotropy formation. It should be noted that this effect is present for arbitrary difference in the bulk velocities as well (Barakat and Schunk 1981). However, in this case both parallel and perpendicular components of $\mathbf{u}$ will contribute to the heating of both degrees of freedom. For small values of the bulk velocity difference (4.4.3), this contribution in the first approximation separates (see (C15) and (C16) in Appendix C) – parallel and perpendicular components of $\mathbf{u}$ only control the heating of a corresponding degree of freedom.

The moments that define viscosity changes due to Coulomb collisions in (4.3.5) take the form

$$J_{s12}^{(2)} = -8 \sum_r v_{sr} \left\{ [(2b_{r\perp} + \mu_r)K_{220} + b_{s\perp}K_{200}] \pi_{s12} + \frac{\rho_s}{\rho_r} [(2b_{r\perp} + \mu_r)K_{220} - b_{s\perp}K_{200}] \pi_{r12} \right\}. \quad (4.4.11)$$

Changing π_{12} to π_{11} in (4.4.11), we get an equation for $1/2(J_{11}^{(2)} - J_{22}^{(2)})$:

$$J_{s\alpha 3}^{(2)} = -4 \sum_r v_{sr} \left\{ [2a_{sr}(b_{r\perp} + b_{r\parallel} + \mu_r)K_{202} + b_{s\parallel}K_{200} + a_{sr}b_{s\perp}K_{002}] \pi_{s\alpha 3} + \frac{\rho_s}{\rho_r} [a_{sr}(b_{r\perp} + b_{r\parallel} + \mu_r)K_{202} - b_{s\parallel}K_{200} - a_{sr}b_{r\perp}K_{002}] \pi_{r\alpha 3} \right\}, \quad \alpha = 1, 2. \quad (4.4.12)$$

The third-order moments are collisional variations of heat fluxes. Collisional transport of thermal energy is partially defined by third-order moments (4.2.25) (see (4.3.6) and (4.3.7)):

$$J_{\alpha 33}^{(3)} = J_{\alpha\beta\gamma}^{(3)} \tau_{\beta\gamma}, \quad \frac{1}{2} (J_{\alpha 11}^{(3)} + J_{\alpha 22}^{(3)}) = J_{\alpha\beta\gamma}^{(3)} n_{\beta\gamma}.$$

Following Konikov et al. (1983), these moments can be written as:

$$J_{\alpha 33}^{(3)} = \sum_r v_{sr} \left[R_{\alpha} u_{\alpha} + R_{\alpha}^{\perp} S_{s\alpha}^{\perp} + R_{\alpha}^{\parallel} S_{s\alpha}^{\parallel} + \frac{\rho_s}{\rho_r} (P_{\alpha}^{\perp} S_{r\alpha}^{\perp} + P_{\alpha}^{\parallel} S_{r\alpha}^{\parallel}) \right], \quad (4.4.13)$$

$$\frac{1}{2} (J_{\alpha 11}^{(3)} + J_{\alpha 22}^{(3)}) = \frac{1}{2} \sum_r v_{sr} \left[C_{\alpha} u_{\alpha} + C_{\alpha}^{\perp} S_{s\alpha}^{\perp} + C_{\alpha}^{\parallel} S_{s\alpha}^{\parallel} + \frac{\rho_s}{\rho_r} (D_{\alpha}^{\perp} S_{r\alpha}^{\perp} + D_{\alpha}^{\parallel} S_{r\alpha}^{\parallel}) \right], \quad \alpha = 1, 2, \quad (4.4.14)$$

$$J_{333}^{(3)} = \sum_r v_{sr} \left[R_3 u_3 + R_3^\perp S_{s3}^\perp + R_3^\parallel S_{s3}^\parallel + \frac{\rho_s}{\rho_r} \left(P_3^\perp S_{r3}^\perp + P_3^\parallel S_{r3}^\parallel \right) \right], \quad (4.4.15)$$

$$\frac{1}{2} \left(J_{113}^{(3)} + J_{223}^{(3)} \right) = \sum_r v_{sr} \left[C_3 u_3 + C_3^\perp S_{s3}^\perp + C_3^\parallel S_{s3}^\parallel + \frac{\rho_s}{\rho_r} \left(D_3^\perp S_{r3}^\perp + D_3^\parallel S_{r3}^\parallel \right) \right]. \quad (4.4.16)$$

Coefficients R , P , C , and D entering (4.4.13)–(4.4.16) are obtained for collisions of the charged particles of arbitrary kind in Konikov et al. (1983) and are also given in Appendix C. In the special case of ion–electron collisions, they coincide with the corresponding expressions found in Chodura and Pohl (1971).

4.4.2 Collisions with Suprathermal Electrons

Before proceeding to detailed calculations of the collision integral moments for electrons, let us discuss the question of the form of the distribution function of the electron component in the near-Earth plasma. Basic processes of the electron formation in the ionosphere are the ionization of neutral components of the upper atmosphere by solar radiation with the wavelength $\lambda < 1,026 \text{ \AA}$ and the precipitating electrons of the magnetospheric origin with energies $E > 1 \text{ keV}$ (see also Sect. 2.6 for details). The excess of energy from the ionization process goes to ion excitation and enters the ionospheric plasma in the form of kinetic energy of the produced electron reaching values of hundreds of electron volts. Later, the newly born suprathermal electrons experience a series of elastic as well as inelastic collisions with the ionospheric particles and finally transform to thermal electrons with energies of the order of tens of percent of electron volts (Khazanov 1979).

The presence of constant background of the sources of ion formation (especially in the daytime) results in the fact that, starting from certain small energies ($\sim 1\text{--}2 \text{ eV}$), the electron distribution function differs significantly from the Maxwell function, and a kinetic description of the electron component of the near-Earth plasma based on the kinetic equation (Khazanov 1979) is necessary

$$\begin{aligned} \frac{\partial f}{\partial t} + \mathbf{v} \nabla f + \left\{ \mathbf{g} - \frac{e}{m} \left(\mathbf{E} + \frac{1}{c} [\mathbf{v} \mathbf{B}] \right) \right\} \nabla_{\mathbf{v}} f \\ = S_{ee} + S_{ei} + S_{en} + S_{en}^* + S_{en}^+ + S_{ei}^- + q_v. \end{aligned} \quad (4.4.17)$$

Here S represents collision integrals, which describe elastic collisions of electrons with electrons S_{ee} , ions S_{ei} , and neutral particles S_{en} , as well as inelastic interactions leading to excitation and deactivation of various energy levels of

neutral particles S_{en}^* , ionization of neutral particles S_{en}^+ , and electron recombination S_{ei}^- ; q_v is a source term defined so that $q_v(\mathbf{v}, \mathbf{r}, t)d\mathbf{v}$ is the number of electrons with velocities in the range $\mathbf{v}$ to $\mathbf{v} + d\mathbf{v}$ produced in the unit volume per unit time. The explicit form of the collision integrals and the structure of the source is described in detail in Chap. 2.

The extremely complicated structure of the kinetic equation for electrons (4.4.17) precludes one from finding its solution not only by analytical methods, but even numerically. It is thus necessary for the analysis of (4.4.17) to make certain simplifying assumptions about the structure of this equation.

One of the basic simplifications in the solution of (4.4.17) is the assumption that electrons in the near-Earth space plasma could be divided into thermal, with the distribution function f_T close to Maxwellian (but not necessarily coinciding with it!), and suprathermal electrons with arbitrary distribution function f_e whose structure is defined by the nonequilibrium spectrum of the electron source.

This division is equivalent to looking for the solution to (4.4.17) in the form (Khazanov 1979)

$$f = f_T + f_e. \quad (4.4.18)$$

After the substitution of (4.4.18) into the kinetic equation for electrons (4.4.17), (4.4.17) can be transformed to a couple of kinetic equations, one of which could be referred to as the kinetic equation of thermal electrons

$$\begin{aligned} \frac{\partial f_T}{\partial t} + \mathbf{v} \nabla f_T + \left\{ \mathbf{g} - \frac{e}{m} \left(\mathbf{E} + \frac{1}{c} [\mathbf{v} \mathbf{B}] \right) \right\} \nabla_{\mathbf{v}} f_T \\ = S_{ee}(f_T f_e) + S_{ee}(f_T f_T) + S(f_T f), \end{aligned} \quad (4.4.19)$$

whereas the other could be referred to as the kinetic equation of suprathermal electrons

$$\begin{aligned} \frac{\partial f_e}{\partial t} + \mathbf{v} \nabla f_e + \left\{ \mathbf{g} - \frac{e}{m} \left(\mathbf{E} + \frac{1}{c} [\mathbf{v} \mathbf{B}] \right) \right\} \nabla_{\mathbf{v}} f_e \\ = S_{ee}(f_e f_T) + S_{ee}(f_e f_e) + S(f_e f) + q_v. \end{aligned} \quad (4.4.20)$$

In the right-hand sides of (4.4.19) and (4.4.20), only electron–electron collisions are written explicitly, whereas all electron collisions with heavy particles are included in the term S .

Separation of (4.4.17) into two equations, (4.4.19) and (4.4.20), where the electron source term is placed in the equation for suprathermal electrons, is purely heuristic in nature and is based on the analysis of the cause and effect relations of the considered system. Namely, as long as the electron source possesses a strongly nonequilibrium energy distribution (Khazanov 1979), it should be related to the

kinetic equation of the superthermal electrons. This assumption can be proved more rigorously using the solutions of the simplified kinetic equation for electrons (Khazanov 1979).

When dividing the kinetic equation (3.17) into two equations, (4.4.19) and (4.4.20), it seems to be not necessary to introduce the separation energy of thermal and suprathermal electrons (E_M) for which $f_T = f_e$, as was done in Khazanov (1979). This is connected with the fact that the nonequilibrium electron source spectrum referred to the kinetic equation for the suprathermal electrons is also defined for energies $E < E_M$. Introduction of the separation energy for breaking (4.4.17) into two equations, (4.4.19) and (4.4.20), would mean a zero nonequilibrium part of the electron distribution function connected with the source in the region $E < E_M$.

Assuming that f_T and f_e are defined in the whole energy region, we will not only account for the indirect action of the electron source on the parameters characterizing the thermal electron distribution function f_T , but will also find its distortion in the region $E < E_M$ caused by the nonequilibrium character of the source. As it was demonstrated in Khazanov (1979), this distortion is actually the continuation of the solution (4.4.20) into the energy region $E < E_M$.

For the thermal electrons in the ionosphere and plasmasphere of the Earth, the hydrodynamic description applicability conditions (4.1.3) are satisfied and it is possible to substitute the kinetic equation for the distribution function f_T by less complex macroscopic transport equations for the following thermal electron parameters: number density, bulk velocity, temperature, and so on (see Sect. 4.3). To find these parameters, it is necessary to know the distribution function of suprathermal electrons defined by the kinetic equation (4.4.20). The methods of solution of the latter are investigated in detail in Khazanov (1979), as well as in Chaps. 5 and 7 of this book.

The aforementioned division of the electron component of the near-Earth plasma into thermal and suprathermal allows one to consider their contribution to the hydrodynamic equations separately. For example, moments of collision integrals of the interaction of thermal electrons with each other and ions is described in the 16-moment approximation by (4.4.5)–(4.4.16), where corresponding subscript reassignments should be made. Here we will consider collisions between thermal and suprathermal electrons. It should be noted that, in the equations of motion and the equations for the viscosity and heat flux tensor components, the influence of suprathermal electrons can be neglected (Khazanov 1979) due to their low number density n_e and pressure p_e as compared with the number density n_e and pressure p_e of the thermal plasma and also because of rapid decay of the Coulomb interaction section with the growing energy ($\sigma \propto E^{-2}$). Suprathermal electrons will give contributions to the continuity equation and energy transport equation. The corresponding moments of the Coulomb collision integral accounting for temperature anisotropy were calculated in Konikov and Khazanov (1981).

Collisions cause energy losses of suprathermal electrons and their slow transition into the regions of lower energy. This process of energy losses or thermalization ends up transforming suprathermal electrons into thermal electrons thus representing a source of thermal electrons in the equation of continuity. To evaluate it

quantitatively, it is necessary to find the zero-order moment $J^{(0)}$ (4.2.22) of the Coulomb collision integral (4.4.1). Substituting (4.4.1) into (4.2.22) and, in accordance with Khazanov (1979), denoting the source in the equation of continuity due to interaction between suprathermal and thermal electrons by $q_e = J^{(0)}/m_e$, we obtain

$$q_e = \frac{A_e}{m_e} \iint d\mathbf{v}_e d\mathbf{v} \left[f_e \left(\frac{\partial^2 f_T}{\partial v_{ex} \partial v_{ey}} u_{x\beta} - \frac{\partial f_T}{\partial v_{ex}} \frac{2g_x}{g^3} \right) + \frac{\partial f_e}{\partial v_x} \left(\frac{2g_x}{g^3} f_T - \frac{\partial f_T}{\partial v_{ey}} u_{x\beta} \right) \right], \quad (4.4.21)$$

where v is the suprathermal electron velocity. Taking into account that $v \gg v_e$, let $g_x \approx v_x$ in (4.4.2), then

$$q_e = \frac{4\pi A_e}{m_e} n_e \int_{-1}^1 f_e(E \rightarrow 0, \mu) d\mu. \quad (4.4.22)$$

Here $\mu = \cos \theta$; θ is the pitch-angle. From (4.4.22), it is clear that finding q_e requires the knowledge of a rather subtle characteristics of f_e in the low-energy region. Calculation of these parameters is a very complex task. On the other hand, the q_e could be represented in a different form, expressed through rates of the photoionization of ions q_0 and secondary ionization Δq_0 (Khazanov 1979):

$$q_e = q_0 + \Delta q_0, \quad (4.4.23)$$

where

$$q_0 = 4\pi \int_0^\infty q(E) dE; \quad \Delta q_0 = 4\pi \int_0^\infty dE \sum_n n_n \int_{2E+E_n^+}^\infty I_n^+(E', E) \Phi(E') dE'. \quad (4.4.24)$$

Here $q(E)$ is the primary electron spectrum (the source in (4.4.17)); n_n is the number density of neutral particles of the kind n ; I_n^+ is the differential section of the ionization process; and E_n^+ is the ionization threshold by electron collision for the n -th species of neutral particles. See also Sect. 2.3 in Chap. 2 for details.

A portion of energy is transferred from suprathermal to thermal electrons during their collisions thus heating the latter. This source of plasma heating turns out to be the main source in the ionosphere and plasmasphere of the Earth at low and medium latitudes. The energy supply from suprathermal electrons to thermal electrons is defined by second-order collisional moments (4.2.24). The heating rates for the parallel and perpendicular degrees of freedom are given by (Oraevskii et al. 1985)

$$J_{e\perp}^{(2)} = \frac{2\pi A_e}{m_e} n_e \int_0^\infty dE \int_{-1}^1 (1 + \mu^2) f_e(E, \mu) d\mu + T_\perp q_e, \quad (4.4.25)$$

$$J_{e\parallel}^{(2)} = \frac{4\pi A_e n_e}{m_e} \int_0^\infty dE \int_{-1}^1 (1 - \mu^2) f_e(E, \mu) d\mu + T_{\parallel} q_e. \quad (4.4.26)$$

If we express pressures $p_{\parallel}$ and $p_{\perp}$ in the transport equations (4.3.3) and (4.3.4) through temperatures, then only the first terms in (4.4.25) and (4.4.26) will remain. Because the pitch-angle distribution of suprathermal electrons is anisotropic (Khazanov 1979), the energy supply from them to parallel and perpendicular degrees of freedom will be different, as is clear from (4.4.25) and (4.4.26). From this, it follows that suprathermal electrons may be considered as one of the sources of the thermal electrons energy distribution anisotropy formation process, which will be quantitatively considered in Chap. 6.

If we represent the distribution function of suprathermal electrons as a series of Legendre polynomials $P_n(\mu)$ (see also Chap. 5):

$$f_e(E, \mu) = \sum_n f_n(E) P_n(\mu), \quad (4.4.27)$$

then we get accurate to the second harmonics (terms with the source are omitted)

$$J_{e\perp}^{(0)} = \frac{16\pi}{3m_e} A_e n_e \left[\int f_0(E) dE + \frac{1}{10} \int f_2(E) dE \right], \quad (4.4.28)$$

$$J_{e\parallel}^{(0)} = \frac{16\pi}{3m_e} A_e n_e \left[\int f_0(E) dE - \frac{1}{5} \int f_2(E) dE \right]. \quad (4.4.29)$$

Here terms with f_0 describe isotropic heating whereas terms containing the f_2 harmonics in (4.4.28) and (4.4.29) are of the opposite sign and define electron temperature anisotropy formation. The total energy supply $J_0^{(2)} = J_{\perp}^{(2)} + \frac{1}{2} J_{\parallel}^{(2)}$, as follows from (4.4.25) and (4.4.26), or from (4.4.28) and (4.4.29), coincides with expressions obtained in Khazanov (1979).

4.5 Collisions with Neutral Particles

4.5.1 Electrons

4.5.1.1 Elastic Electron–Neutral Collisions

Neutral particles present in the ionospheric plasma facilitate energy and momentum exchange between charged components through collisions. This fact brings forth the necessity to account for interaction processes between neutral and charged

particles in the hydrodynamic equations. The additional terms in the equations are the moments of the corresponding collision integrals. Such moments for elastic collisions between thermal electrons and basic neutral components of the ionospheric plasma in the framework of the 16-moment approximation were considered in Konikov and Khazanov (1981, 1982, 1983).

Elastic collisions between electrons and neutral particles are usually described by the Boltzmann collision integral

$$S_{en}^0 = \iint [f_e(\mathbf{v}')f_n(\mathbf{v}_1') - f_e(\mathbf{v})f_n(\mathbf{v}_1)]\mathbf{g}I_{en}(\mathbf{g}, \chi)d\Omega d\mathbf{v}_n. \quad (4.5.1)$$

Here primes denote function values after the collision; $I_{en}(g, \chi)$ is the differential section of the elastic scattering of electrons by neutral particles of the kind n ; and $d\Omega = \sin \chi d\chi d\varepsilon$ (where χ and ε are the scattering angles).

It is known that, for low-energy electrons, scattering by neutral particle is close to isotropic. Thus it is possible not to distinguish between the total $\sigma_{en}^{(0)} = 2\pi \int_0^\pi I_{en} \sin \chi d\chi$ and transport $\sigma_{en}^{(1)} = 2\pi \int_0^\pi (1 - \cos \chi) I_{en} \sin \chi d\chi$ sections ($\sigma_{en}^{(0)} = \sigma_{en}^{(1)}$) of the elastic scattering of thermal electrons by neutral components in the ionosphere.

The review of theoretical and experimental results on the sections of elastic scattering of low-energy electrons by basic neutral components of the upper atmosphere of the Earth N_2 , O_2 , O , H , He is given in Banks (1966). For example, according to Banks (1966), the total elastic cross-sections are

$$\begin{aligned} \sigma_{eN_2} &= (18.3\sqrt{E} - 7.3E) \times 10^{-16} \text{ cm}^2, \\ \sigma_{eO_2} &= (2.2 + 5.1\sqrt{E}) \times 10^{-16} \text{ cm}^2, \\ \sigma_{eO} &= (3.4 \pm 1.0) \times 10^{-16} \text{ cm}^2, \\ \sigma_{eH} &= (54.7 - 28.7E) \times 10^{-16} \text{ cm}^2, \\ \sigma_{eHe} &= (5.6 \pm 0.6) \times 10^{-16} \text{ cm}^2, \end{aligned} \quad (4.5.2)$$

where $E = \mu_{en}g^2/2$ is the energy of relative motion in eV.

For compactness of the expressions and convenience of later usage and following Konikov and Khazanov (1981, 1982, 1983), we will represent the effective cross-sections (4.5.2) in the general form as follows:

$$\sigma_{en}(E) = A_n \left(1 + B_n \sqrt{E} - C_n E \right), \quad (4.5.3)$$

where A_n , B_n , and C_n are constants characterizing the specific neutral species. The collision section for each component is easy to obtain from (4.5.3) accounting for corresponding terms.

In the calculation of the moments of Boltzmann collision integral (4.5.1), the electron distribution function was taken in the 16-moment approximation (4.2.37), whereas neutral particles were assumed to be described by the Maxwell distribution

$$f_n = n_n(\beta_n/2\pi)^{3/2} \exp(-\beta_n w_n^2/2), \quad \beta_n = m_n/T_n. \quad (4.5.4)$$

The same assumptions as well as changes of variables were made as in Sect. 4.4.1. Taking into account the smallness of the electron mass compared with the mass of neutral particles ($m_e \ll m_n$),

$$\beta_{en\perp} \approx \beta_{e\perp} = m_e/T_{e\perp}, \quad \beta_{en\parallel} \approx \beta_{e\parallel} = m_e/T_{e\parallel}, \quad a_{en} \approx T_{e\perp}/T_{e\parallel}.$$

Below are the expressions for the moments of the integral of elastic electron–neutral collisions:

$$\begin{aligned} J_\alpha^{(1)} = & -\rho_e v_{en} \left[3K_{0,2} - K_{2,2} + \frac{3\sqrt{\pi}}{2} B_n \sqrt{T_{e\perp}} (2K_{0,5/2} - K_{2,5/2}) \right. \\ & \left. - 3C_n T_{e\perp} (5K_{0,3} - 3K_{2,3}) \right] u_x - v_{en} \beta_{e\perp} \left\{ 2[3K_{0,4} - 2(2+X_e)K_{2,4} + (3+2X_e)K_{4,4}] \right. \\ & + \frac{15}{16} B_n \sqrt{\pi T_{e\perp}} [3K_{0,9/2} - 2(5+2X_e)K_{2,9/2} + (7+4X_e)K_{4,9/2}] \\ & \left. - 12C_n T_{e\perp} [K_{0,5} - (3+X_e)K_{2,5} + (2+X_e)K_{4,5}] \right\} S_{e\alpha}^\perp \\ & - v_{en} \beta_{e\parallel} \left\{ 2[-K_{0,4} + (7+5X_e)K_{2,4} - (6+5X_e)K_{4,4}] \right. \\ & + \frac{15}{8} B_n \sqrt{\pi T_{e\perp}} [-K_{0,9/2} + 2(4+3X_e)K_{2,9/2} - (7+6X_e)K_{4,9/2}] \\ & \left. - 6C_n T_{e\perp} [-K_{0,5} + (9+7X_e)K_{2,5} - (8+7X_e)K_{4,5}] \right\} S_{e\alpha}^\parallel, \quad \alpha = 1, 2, \end{aligned} \quad (4.5.5)$$

where

$$v_{en} = 2n_e A_n T_{e\perp} / \sqrt{2\pi m_e T_{e\parallel}} \quad (4.5.6)$$

is the frequency of elastic electron–neutral collisions, and the quantity

$$X_e = a_{en} - 1 = (T_{e\perp} - T_{e\parallel})/T_{e\parallel} \quad (4.5.7)$$

is called the degree of the electron temperature anisotropy. The function

$$K_{a,b} \equiv K_{a,b}(X_e) = \int_0^1 t^a (1 + X_e t^2)^{-b} dt \quad (4.5.8)$$

is a special case of the function $Y_{e,m}^{(n)}$ introduced in Appendix D. The $K_{a,b}$ values for variables a and b are presented in Appendix B. The longitudinal moment of the first order is:

$$\begin{aligned}
 J_3^{(1)} = & -2\rho_e v_{en} \left[K_{0,2} + K_{2,2} + \frac{3}{4} B_n \sqrt{\pi T_{e\perp}} (K_{0,5/2} + 2K_{2,5/2}) - 2C_n T_{e\perp} (K_{0,3} + 3K_{2,3}) \right] u_3 \\
 & - 2v_{en} \beta_{e\parallel} \left\{ 4[2K_{2,4} - (3 + X_e)K_{4,4}] + \frac{15}{8} B_n \sqrt{\pi T_{e\perp}} [5K_{2,9/2} - (7 + 2X_e)K_{4,9/2}] \right. \\
 & \left. - 6C_n T_{e\perp} [3K_{2,5} - (4 + X_e)K_{4,5}] \right\} S_{e3}^\perp - 2v_{en} \beta_{e\parallel} \frac{\beta_{e\parallel}^2}{\beta_{e\perp}} \left\{ 2[-K_{2,4} + (2 + X_e)K_{4,4}] \right. \\
 & \left. + \frac{5}{8} B_n \sqrt{\pi T_{e\perp}} [-3K_{2,9/2} + (7 + 4X_e)K_{4,9/2}] - C_n T_{e\perp} [-3K_{2,5} + (8 + 5X_e)K_{4,5}] \right\} S_{e3}^\parallel,
 \end{aligned} \tag{4.5.9}$$

$$\begin{aligned}
 J_\perp^{(2)} = & 4n_e v_{en} T_{e\perp} \left\{ \mu_n \left(\frac{5}{3} K_{0,3} - K_{2,3} \right) - 2b_{n\perp} (K_{0,3} - K_{2,3}) + \frac{15}{16} B_n \sqrt{\pi T_{e\perp}} \right. \\
 & \times \left[\mu_n \left(\frac{5}{3} K_{0,7/2} - K_{2,7/2} \right) - 2b_{n\perp} (K_{0,7/2} - K_{2,7/2}) \right] \\
 & \left. - 3C_n T_{e\perp} \left[\mu_n \left(\frac{5}{3} K_{0,4} - K_{2,4} \right) - 2b_{n\perp} (K_{0,4} - K_{2,4}) \right] + F_\perp(\mathbf{u}) \right\},
 \end{aligned} \tag{4.5.10}$$

$$\begin{aligned}
 J_\parallel^{(2)} = & 8n_e v_{en} T_{e\perp} \left\{ \mu_n \left(\frac{1}{3} K_{0,3} + K_{2,3} \right) - 2b_{n\parallel} K_{2,3} + \frac{15}{16} B_n \sqrt{\pi T_{e\perp}} \left[\mu_n \left(\frac{1}{3} K_{0,7/2} + K_{2,7/2} \right) \right. \right. \\
 & \left. \left. - 2b_{n\parallel} K_{2,7/2} \right] - 3C_n T_{e\perp} \left[\mu_n \left(\frac{1}{3} K_{0,4} + K_{2,4} \right) - 2b_{n\parallel} K_{2,4} \right] + F_\parallel(\mathbf{u}) \right\}.
 \end{aligned} \tag{4.5.11}$$

The terms $F_\perp(\mathbf{u})$ and $F_\parallel(\mathbf{u})$ in (4.5.10) and (4.5.11) describe the frictional heating:

$$\begin{aligned}
 F_\perp(\mathbf{u}) = & \frac{\beta_{e\perp}}{4} \left\{ \left(\frac{5}{3} u_\perp^2 + \frac{1}{2} u^2 \right) K_{0,2} + \left(\frac{4}{3} u_\parallel^2 - \frac{2}{3} u_\perp^2 + \frac{1}{2} u^2 \right) K_{2,2} + \frac{3}{4} B_n \sqrt{\pi T_{e\perp}} \right. \\
 & \times \left[\left(3u_\perp^2 + \frac{2}{3} u_\parallel^2 - \frac{1}{3} u^2 \right) K_{0,5/2} + \left(\frac{8}{3} u_\parallel^2 - \frac{4}{3} u_\perp^2 + u^2 \right) K_{2,5/2} \right] \\
 & \left. - 2C_n T_{e\perp} \left[\left(4u_\parallel^2 - 2u_\perp^2 \right) K_{2,3} + \left(\frac{2}{3} u_\parallel^2 + \frac{11}{3} u_\perp^2 + u^2 \right) K_{0,3} \right] \right\},
 \end{aligned} \tag{4.5.12}$$

$$\begin{aligned}
 F_\parallel(\mathbf{u}) = & \frac{\beta_{e\perp}}{4} \left\{ \left(\frac{4}{3} u_\parallel^2 + \frac{2}{3} u_\perp^2 + \frac{1}{2} u^2 \right) K_{0,2} + \left(\frac{2}{3} u_\parallel^2 - \frac{1}{3} u_\perp^2 - 2u^2 \right) K_{2,2} + \frac{3}{4} B_n \sqrt{\pi T_{e\perp}} \right. \\
 & \times \left[\left(\frac{4}{3} u_\perp^2 + \frac{1}{3} u_\parallel^2 \right) K_{0,5/2} + \left(2u_\parallel^2 - \frac{5}{3} u^2 \right) K_{2,5/2} \right] - 2C_n T_{e\perp} \left[\frac{5}{6} u^2 K_{0,3} \right. \\
 & \left. \left. + \left(2u_\parallel^2 - u_\perp^2 - \frac{3}{2} u^2 \right) K_{2,3} \right] \right\}.
 \end{aligned} \tag{4.5.13}$$

In the approximation of weak anisotropy $|X_e| \ll 1$, (4.5.10) and (4.5.11) take a simpler form after making use of expansion of the functions $K_{a,b}$ into series in powers of X_e . Without frictional terms (4.5.12) and (4.5.13), and in the linear approximation in X_e , they take the form

$$J_{\perp}^2 = \frac{8}{3} n_e v_{en} \left\{ 2 \frac{m_e}{m_n} (T_n - T_e) \left[1 + \frac{15}{16} B_n \sqrt{\pi T_e} - 3 C_n T_e \right] - T_e \left[\frac{2}{5} + \frac{7}{16} B_n \sqrt{\pi T_e} - \frac{8}{5} C_n T_e \right] X_e \right\}, \quad (4.5.14)$$

$$J_{\parallel}^2 = \frac{8}{3} n_e v_{en} \left\{ 2 \frac{m_e}{m_n} (T_n - T_e) \left[1 + \frac{15}{16} B_n \sqrt{\pi T_e} - 3 C_n T_e \right] + T_e \left[\frac{4}{5} + \frac{7}{8} B_n \sqrt{\pi T_e} - \frac{16}{5} C_n T_e \right] X_e \right\}, \quad (4.5.15)$$

where $T_e = (2T_{e\perp} + T_{e\parallel})/3$.

In the linear approximation, the terms with X_e in (4.5.14) and (4.5.15) define electron temperature anisotropy relaxation caused by elastic electron–neutral collisions. These terms exceed the value of total energy variation $J_0^{(2)}$ approximately by the factor of m_n/m_e . From this, it follows that the elastic electron–neutral collisions determine primarily the process of temperature anisotropy relaxation rather than total energy losses.

As it follows from (4.5.12) and (4.5.13), energy supply to the parallel and perpendicular degrees of freedom caused by frictional heating is different (as has been mentioned in Sect. 4.4 for the Coulomb interaction), which can lead to the anisotropy in the electron energy distribution.

In the limit $X_e \rightarrow 0$, (4.5.10) and (4.5.11), and (4.5.14) and (4.5.15) coincide with the corresponding results obtained for the isotropic electron energy distribution (Banks 1966).

The collisional viscosity is described by the moments

$$J_{12}^{(2)} = -6v_{en} \left[K_{0,4} - 2K_{2,4} + K_{0,4} + \frac{35}{32} B_n \sqrt{\pi T_{e\perp}} (K_{0,9/2} - 2K_{2,9/2} + K_{4,9/2}) - 4C_n T_{e\perp} (K_{0,5} - 2K_{2,5} + K_{4,5}) \right] \pi_{e12}, \quad (4.5.16)$$

$$J_{\alpha 3}^{(2)} = -24v_{en} \frac{T_{e\perp}}{T_{e\parallel}} \left[K_{2,4} - K_{4,4} + \frac{35}{32} B_n \sqrt{\pi T_{e\perp}} (K_{2,9/2} - K_{4,9/2}) - 4C_n T_{e\perp} (K_{2,5} - K_{4,5}) \right] \pi_{e\alpha 3}, \quad \alpha = 1, 2. \quad (4.5.17)$$

Third-order moments of the elastic electron–neutral collisions integral have a much more complicated structure and can be found in Konikov et al. (1980).

4.5.1.2 Inelastic Electron–Neutral Collisions

To obtain the moments of the integral of inelastic collisions of thermal electrons with neutral particles, the following expression is usually used (Shkarofsky et al. 1966; Gurevich 1978; Khazanov 1979)

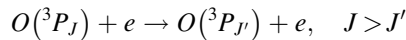
$$S_{en}^* = n_n \frac{v_e}{E} \frac{\partial}{\partial E} \left[E \sum_{\alpha} \sum_{\beta > \alpha} \delta_{\alpha} [1 - \exp(-E_{\alpha\beta}/T_n)] E_{\alpha\beta} \sigma_{\alpha\beta}^*(E) \left(f_e + T_n \frac{\partial f_e}{\partial E} \right) \right], \quad (4.5.18)$$

which has been obtain in the approximation of continuous electron energy losses and the assumption of the Boltzmann energy distribution of neutral particles. Here δ_{α} is the share of neutral particles in the state with energy E_{α} ; $E_{\alpha\beta}$ is the transition energy for level α and β ; $\sigma_{\alpha\beta}^*(E)$ is the total cross-section corresponding to that transition state.

Using (4.5.18) in Konikov and Khazanov (1981, 1982, 1983) in the 16-moment approximation for the electron distribution function, an inelastic interaction between electrons and neutral particles was considered, leading to excitation and deactivation of the atomic oxygen fine structure levels and molecular rotational degrees of freedom. These inelastic processes together with the Coulomb interaction are the main sources of the energy losses by thermal electrons in the low- and high-latitude ionosphere of the Earth (Schunk and Nagy 2000).

Fine Structure Levels of the Atomic Oxygen

The excitation process for the sublevels of the atomic oxygen in the state ^3P (fine structure levels) goes on according to the following scheme



where $J = 0, 1, 2$ – the total atomic momentum values in the state ^3P with energies, correspondingly, of $E_J = 0.0281, 0.0197$, and 0 eV (Takayanagi and Itikawa 1970).

According to Breig and Lin (1966), the excitation section can be written as follows (Oraevskii et al. 1985)

$$\sigma_{JJ'}^*(E) = \pi a_0^2 Ry E_{\Omega}^{-3/2} \frac{\sqrt{E}}{2J+1}. \quad (4.5.19)$$

Here $a_0 = 5.29 \times 10^{-9}$ cm is the Bohr radius and $Ry = 13.6$ eV. For the transitions $1 \rightarrow 2, 0 \rightarrow 2, 0 \rightarrow 1$, E_{Ω} equals 1.8, 5.4, and 3.4 eV, correspondingly.

Substituting (4.2.37) and (4.5.19) into (4.5.18), performing the summation over all transitions, and integrating, we obtain for the moments (4.2.23)–(4.2.25) the following expressions (Konikov and Khazanov 1981, 1982, 1983):

$$J_{\alpha}^{(1)} = v_{en}^* \frac{T_n}{4\sqrt{T_{e\perp}T_{e\parallel}}} \left\{ \frac{1}{2} \beta_{e\perp} [-K_{0,5/2} - 2(1 + 2X_e)K_{2,5/2} + (3 + 4X_e)K_{4,5/2}] S_{e\alpha}^{\perp} + \beta_{e\parallel} [-K_{0,5/2} + 2(2 + X_e)K_{2,5/2} - (3 + 2X_e)K_{4,5/2}] S_{e\alpha}^{\parallel} \right\}, \quad (4.5.20)$$

where

$$v_{en}^* = \frac{A^* n_n}{T_n} \sqrt{\frac{2}{m_e}} \quad (4.5.21)$$

is the collision frequency in the excited state $O(^3P_J)$,

$$A^* = \frac{\pi a_0^2 Ry}{Z} \sum_{J,J'} E_{JJ'}^2 E_{\Omega}^{-3/2}, \quad Z = \sum_{J=2}^0 (2J+1) \exp(-E_J/T_n),$$

$$J_3^{(1)} = v_{en}^* \frac{T_n}{2T_{e\parallel}} \sqrt{\frac{T_{e\perp}}{T_{e\parallel}}} \left\{ \beta_{e\perp} [K_{2,5/2} - (3 + 2X_e)K_{4,5/2}] S_{e3}^{\perp} + \beta_{e\parallel} [-K_{2,5/2} + K_{4,5/2}] S_{e3}^{\parallel} \right\}, \quad (4.5.22)$$

$$J_{\perp}^{(2)} = -n_e v_{en}^* \left[T_{e\perp} - \frac{3}{2} T_n \sqrt{T_{e\perp}/T_{e\parallel}} (K_{0,3/2} - K_{2,3/2}) \right], \quad (4.5.23)$$

$$J_{\parallel}^{(2)} = -n_e v_{en}^* \left[T_{e\parallel} - 3T_n \sqrt{T_{e\perp}/T_{e\parallel}} K_{2,3/2} \right]. \quad (4.5.24)$$

Under the assumption of small anisotropy ($|X_e| \ll 1$), (4.5.23) and (4.5.24) reduce to

$$J_{\perp}^{(2)} = n_e v_{en}^* [T_n - T_e - (T_e/3 - T_n/5)X_e], \quad (4.5.25)$$

$$J_{\parallel}^{(2)} = n_e v_{en}^* [T_n - T_e + (T_e/3 - T_n/5)X_e], \quad (4.5.26)$$

$$J_{12}^{(2)} = -v_{en}^* \left[1 - \frac{9T_n}{8(T_{e\perp}T_{e\parallel})^{1/2}} (K_{0,5/2} - 2K_{2,5/2} + K_{4,5/2}) \right] \pi_{e12}, \quad (4.5.27)$$

$$J_{\alpha 3}^{(2)} = -v_{en}^* \left[1 - \frac{9T_n}{2T_{e\parallel}} \frac{\sqrt{T_{e\perp}}}{T_{e\parallel}} (K_{2,5/2} - K_{4,5/2}) \right] \pi_{e\alpha 3}, \quad \alpha = 1, 2, \quad (4.5.28)$$

$$\frac{1}{2} \left(J_{\alpha 11}^{(3)} + J_{\alpha 22}^{(3)} \right) = -\frac{3}{2} v_{en}^* \left\{ S_{e\alpha}^{\perp} - \frac{3}{2} \frac{T_n}{\sqrt{T_{e\perp} T_{e\parallel}}} [K_{2,7/2} - 2(3 + 2X_e)K_{4,7/2} + (11 + 8X_e)K_{6,7/2} \right. \\ \left. - (5 + 4X_e)K_{6,7/2}] \left(\frac{1}{2} S_{e\alpha}^{\perp} - \frac{T_{e\perp}}{T_{e\parallel}} S_{e\alpha}^{\parallel} \right) \right\}, \quad (4.5.29)$$

$$J_{\alpha 33}^{(3)} = -\frac{3}{2} v_{en}^* \left\{ S_{e\alpha}^{\parallel} - 3 \frac{T_n}{\sqrt{T_{e\perp} T_{e\parallel}}} [K_{2,7/2} - 2(3 + 2X_e)K_{4,7/2} + (5 + 4X_e)K_{6,7/2}] \right. \\ \left. \times \left(\frac{1}{2} S_{e\alpha}^{\perp} - \frac{T_{e\perp}}{T_{e\parallel}} S_{e\alpha}^{\parallel} \right) \right\}, \quad \alpha = 1, 2, \quad (4.5.30)$$

$$\frac{1}{2} \left(J_{113}^{(3)} + J_{223}^{(3)} \right) = -\frac{3}{2} v_{en}^* \left\{ S_{e3}^{\perp} - 3 \frac{T_n}{T_e} \left(\frac{T_{e\perp}}{T_{e\parallel}} \right)^{3/2} [3K_{0,7/2} - 2(4 + X_e)K_{2,7/2} \right. \\ \left. + (5 + 2X_e)K_{6,7/2}] \times \left(S_{e3}^{\perp} - \frac{T_{e\perp}}{3T_{e\parallel}} S_{e3}^{\parallel} \right) \right\}, \quad (4.5.31)$$

$$J_{333}^{(3)} = -\frac{3}{2} v_{en}^* \left\{ S_{e3}^{\parallel} - 2 \frac{T_n}{T_{e\perp}} \left(\frac{T_{e\perp}}{T_{e\parallel}} \right)^{3/2} [3K_{4,7/2} - (5 + 2X_e)K_{6,7/2}] \right. \\ \left. \times \left(3S_{e3}^{\perp} - \frac{T_{e\perp}}{3T_{e\parallel}} S_{e3}^{\parallel} \right) \right\}. \quad (4.5.32)$$

Rotational Degrees of Freedom of Molecules

The rotational energy levels of nonpolar molecules are excited with the effective section (Gerjony and Stein 1955)

$$\sigma_{J,J+2}^*(E) = \frac{8\pi a_0^2 Q_n^2}{15} \frac{(J+1)(J+2)}{(2J+1)(2J+3)} \sqrt{1 - \frac{2(2J+3)B_n}{E}}, \quad (4.5.33)$$

where $J = 0, 1, 2, \dots$ is the rotational quantum number; and Q_n and B_n are the quadrupole momentum and molecule constant of rotation, correspondingly. According to Takayanagi and Itikawa (1970), for the molecules N_2 and O_2 : $Q_{N_2} = 1.1$ a.u., $B_{N_2} = 2.48 \times 10^{-4}$ eV, $Q_{O_2} = 1.8$ a.u., $B_{O_2} = 2.48 \times 10^{-4}$ eV.

The first-order moments are:

$$J_{\alpha}^{(1)} = \frac{1}{2} v_{er}^* \left\{ \beta_{e\perp} \left[(1 + X_e)(K_{2,3} - K_{4,3}) - \frac{T_n}{4T_{e\perp}} [K_{0,2} + 2X_e K_{2,2} - (1 + 2X_e)K_{4,2}] \right] S_{e\alpha}^{\perp} \right. \\ \left. + \frac{1}{2} \beta_{e\parallel} \left[K_{0,3} - (5 + 3X_e)K_{2,3} + (4 + 3X_e)K_{4,3} + \frac{T_n}{2T_{e\perp}} [-K_{0,2} + (3 + X_e)K_{2,2} \right. \right. \\ \left. \left. - (2 + X_e)K_{4,2}] \right] S_{e\alpha}^{\parallel} \right\}, \quad \alpha = 1, 2, \quad (4.5.34)$$

where

$$v_{er}^* = \frac{128}{15} \pi a_0^2 Q_n^2 B_n n_n (2\pi m_e T_{||})^{-1/2} \quad (4.5.35)$$

is the electron collision frequency for excited rotational molecular degrees of freedom; and

$$\begin{aligned} J_3^{(1)} = v_{er}^* \frac{T_{e\perp}}{T_{e||}} & \left\{ \beta_{e\perp} \left[-K_{2,3} + (2 + X_e)K_{4,3} - \frac{T_n}{2T_{e\perp}} (1 + X_e)K_{4,2} \right] S_{e3}^\perp \right. \\ & \left. + \frac{1}{6} \beta_{e||} [3K_{2,3} - (4 + X_e)K_{4,3} + \frac{T_n}{2T_{e\perp}} [-3K_{2,2} + (2 - X_e)K_{4,2}]] S_{e3}^\perp \right\}. \end{aligned} \quad (4.5.36)$$

The second-order moments are:

$$J_\perp^{(2)} = -n_e v_{er}^* [T_{e\perp} (K_{0,2} - K_{2,2}) - T_n (K_{0,1} - K_{2,1})], \quad (4.5.37)$$

$$J_{||}^{(2)} = -2n_e v_{er}^* [T_{e\perp} K_{2,2} - T_n K_{2,1}], \quad (4.5.38)$$

$$J_{12}^{(2)} = -v_{er}^* [K_{0,3} - 2K_{2,3} + K_{4,3} - T_n/2T_{e\perp} (K_{0,2} - 2K_{2,2} + K_{4,2})] \pi_{e12}, \quad (4.5.39)$$

$$J_{12}^{(2)} = -4v_{er}^* \frac{T_{e\perp}}{T_{e||}} [K_{2,3} - K_{4,3} - T_n/2T_{e\perp} (K_{2,2} - K_{4,2})] \pi_{e\alpha 3}, \quad \alpha = 1, 2. \quad (4.5.40)$$

Third-order moments of this type of electron–neutral collisions integral have a much more complicated structure and can be found in the paper (Konikov et al. 1980).

In conclusion, note that in the absence of temperature anisotropy (X_e), (4.5.23) and (4.5.24) and (4.5.37) and (4.5.38) reduce to known expressions for the inelastic energy dissipation for the Maxwellian distribution of thermal electrons (Schunk and Nagy 1978).

4.5.2 Ions

The process of ion scattering by neutral particles goes through the polarization interaction and resonance charge exchange. The expressions describing the contributions of these processes in the equations of continuity, motion, and ion energy conservation were obtained in Banks (1966), Banks and Holzer (1968), and Horwitz and Banks (1973) for the Maxwellian distribution functions (for the 13-moment

approximation, see, e.g., the reviews Schunk 1975, 1977 and references therein). The results obtained in these works were summarized in Demars and Schunk (1979), Barakat and Schunk (1981), and Oraevskii et al. (1985), where the moments of collision integrals were calculated for the polarization interaction and resonance charge exchange in the presence of ion energy distribution anisotropy in the 16-moment approximation.

4.5.2.1 Polarization Interaction

When an ion moves in the surrounding gas of neutral particles, its scattering is determined by the polarization interaction (gas of Maxwellian molecules) with the section (Dalgarno et al. 1958)

$$\sigma_{in} = 2.21\pi e_i \sqrt{\alpha_n / \mu_{in} g^2}, \quad (4.5.41)$$

where α_n is the polarizability of neutral particles, which for N_2 , O_2 , O , H , He equals 1.76, 1.60, 0.89, 0.67, and $0.21 \times 10^{-24} \text{ cm}^3$, respectively.

The moments were calculated using the Boltzmann collision integral (5.5.1). The distribution function for neutral particles was taken as Maxwellian (4.5.4), and the 16-moment approximation (4.2.37) was used for the ions. Because the effective section of this process is the reciprocal of the relative velocity, it was possible to obtain the expressions for the collisional moments, in contrast with the previous results, in the closed form and for the arbitrary values of difference between the bulk velocity $\mathbf{u}$ and thermal particle velocities.

Following Konikov and Khazanov (1982), the results of calculations are:

$$\mathbf{J}^{(1)} = -\rho_i v_{in} \mathbf{u}, \quad (4.5.42)$$

where

$$v_{in} = 2.21\pi e_i n_n \sqrt{\alpha_n \mu_n / m_i} \quad (4.5.43)$$

is the ion–neutral particle collision frequency for polarization scattering;

$$J_{\perp}^{(2)} = \rho_i v_{in} \left[\frac{\mu_n}{3} \left(\frac{5}{\beta_{in\perp}} + \frac{1}{\beta_{in\parallel}} + \frac{5}{2} u_{\perp}^2 + u_{\parallel}^2 \right) - \frac{2}{\beta_{i\perp}} \right], \quad (4.5.44)$$

$$J_{\parallel}^{(2)} = \rho_i v_{in} \left[\frac{\mu_n}{3} \left(\frac{2}{\beta_{in\perp}} + \frac{4}{\beta_{in\parallel}} + u_{\perp}^2 + 4u_{\parallel}^2 \right) - \frac{2}{\beta_{i\parallel}} \right], \quad (4.5.45)$$

$$J_{12}^{(2)} = \rho_i v_{in} \left[\mu_n u_1 u_2 + \frac{1}{\rho_i} (\mu_n - 2) \pi_{i12} \right], \quad (4.5.46)$$

$$J_{\alpha 3}^{(2)} = \rho_i v_{in} \left[\mu_n u_\alpha u_3 + \frac{1}{\rho_i} (\mu_n - 2) \pi_{i\alpha 3} \right], \quad \alpha = 1, 2, \quad (4.5.47)$$

$$\begin{aligned} \frac{1}{2} (J_{\alpha 11}^{(3)} + J_{\alpha 22}^{(3)}) = & \frac{1}{16} \rho_i v_{in} \left\{ \left[-\frac{12}{\beta_{i\perp}} + \frac{32\mu_n}{\beta_{i\perp}} - \mu_n^2 \left(\frac{10}{\beta_{in\perp}} + \frac{4}{\beta_{in\parallel}} + 7u_\alpha^2 + 7u_{3-\alpha}^2 + 4u_3^2 \right) \right] u_\alpha \right. \\ & + \frac{1}{\rho_i} [2(-3 + 10\mu_n - 7\mu_n^2)(\pi_{\alpha 1} u_\alpha + \pi_{\alpha 2} u_{3-\alpha}) + 8\mu_n \mu_i \pi_{\alpha 3} u_3 \\ & \left. + 2(-9 + 13\mu_n - 7\mu_n^2) S_{i\alpha}^\perp + 4\mu_n \mu_i S_{i\alpha}^\parallel] \right\}, \end{aligned} \quad (4.5.48)$$

$$\begin{aligned} J_{\alpha 33}^{(3)} = & \frac{1}{3} \rho_i v_{in} \left\{ \left[-\frac{3}{\beta_{i\parallel}} + 2\mu_n \left(\frac{1}{\beta_{i\perp}} + \frac{3}{\beta_{i\parallel}} \right) - \mu_n^2 \left[4 \left(\frac{1}{\beta_{in\perp}} + \frac{1}{\beta_{in\parallel}} \right) + u_1^2 + u_2^2 + 4u_3^2 \right] \right] u_\alpha \right. \\ & + \frac{1}{\rho_i} [2\mu_n \mu_i (\pi_{\alpha 1} u_1 + \pi_{\alpha 2} u_2) + 2(-3 + 7\mu_n - 4\mu_n^2) \pi_{\alpha 3} u_3 + 2\mu_n \mu_i S_{i\alpha}^\perp \\ & \left. + (-9 + 10\mu_n - 4\mu_n^2) S_{i\alpha}^\parallel] \right\}, \quad \alpha = 1, 2, \end{aligned} \quad (4.5.49)$$

$$\begin{aligned} \frac{1}{2} (J_{113}^{(3)} + J_{223}^{(3)}) = & \frac{1}{6} \rho_i v_{in} \left\{ \left[-\frac{6}{\beta_{i\perp}} + 4\mu_n \left(\frac{1}{\beta_{i\parallel}} + \frac{3}{\beta_{i\perp}} \right) \right. \right. \\ & \left. \left. - \mu_n^2 \left[\frac{10}{\beta_{in\perp}} + \frac{6}{\beta_{in\parallel}} + 5(u_1^2 + u_2^2) + 2u_3^2 \right] \right] u_3 \right. \\ & + \frac{1}{\rho_i} \left[(-6 + 16\mu_n - 10\mu_n^2) (\pi_{13} u_1 + \pi_{23} u_2) \right. \\ & \left. \left. + (-24 + 22\mu_n - 10\mu_n^2) S_{i3}^\perp + 2\mu_n \mu_i S_{i3}^\parallel \right] \right\}, \end{aligned} \quad (4.5.50)$$

$$\begin{aligned} J_{333}^{(3)} = & \frac{1}{3} \rho_i v_{in} \left\{ \left[\frac{1}{\beta_{i\parallel}} (-9 + 8\mu_n) - 3\mu_n^2 \left[\frac{2}{\beta_{in\perp}} + \frac{9}{\beta_{in\parallel}} + u^2 + u_3^2 \right] \right] u_3 \right. \\ & \left. + \frac{1}{\rho_i} [6\mu_n \mu_i (\pi_{13} u_1 + \pi_{23} u_2) + 6\mu_n \mu_i S_{i3}^\perp + (-9 + 4\mu_n - 6\mu_n^2) S_{i3}^\parallel] \right\}. \end{aligned} \quad (4.5.51)$$

4.5.2.2 Resonance Charge Exchange

The charge exchange plays a crucial role in the collisional processes between the ions and neutral particles of the same kind. Because the total internal energy of the interacting particles during this process does not change, the resonance charge

exchange is classified among the elastic processes. The interaction section of the process of resonance charge exchange is in logarithmic dependence on the particle interaction energy (Dalgarno et al. 1958) and, for the main components of the ionospheric plasma, is given in Banks and Kockarts (1973). For this case, the moments of collision integral in the 16-moment approximation were found in Demars and Schunk (1979), Barakat and Schunk (1981), and Konikov and Khazanov (1982). However, the obtained results are extremely awkward and are not very convenient for practical usage. Let us then use another approach (Oraevskii et al. 1985).

The charge exchange reaction can, in general, be written as follows:



This form accounts for the resonance charge exchange reaction $O^+ + H \leftrightarrow O + H^+$ (Banks and Kockarts 1973), which plays an important role in the upper ionosphere of the Earth.

With the charge exchange interaction section weakly depending on energy (Banks and Kockarts 1973), the collision integral describing the reaction (4.5.52) in the kinetic equation for the distribution function $f_A(\mathbf{v})$ of ions A^+ can be represented as follows (Banks and Holzer 1968)

$$S = \sigma^+ \int f_A(\mathbf{v}) f_{B^+}(\mathbf{v}') |\mathbf{v} - \mathbf{v}'| d^3 v' - \sigma^- \int f_{A^+}(\mathbf{v}) f_B(\mathbf{v}') |\mathbf{v} - \mathbf{v}'| d^3 v', \quad (4.5.53)$$

where σ^- and σ^+ are the averaged sections of the direct and reverse reactions in (4.5.52).

The moments of the collision integral (4.5.53) were found under the assumption that neutral particles are distributed with the Maxwellian function (4.5.4), and ions are bi-Maxwellian (4.1.7). In addition, it was assumed that the difference between the directional particle velocities is small compared with the thermal velocity.

The zero-order moment in this case describes the rate of the ions A^+ production and the disappearance as the result of the charge exchange reaction (4.5.52):

$$\frac{1}{m_A} J^{(0)} = n(A) n(B^+) \gamma^+ K_{0,2}^+ - n(A^+) n(B) \gamma^- K_{0,2}^-. \quad (4.5.54)$$

Here

$$\gamma^\pm = 2\sigma^\pm / \beta_\perp^\pm \sqrt{2\beta_\parallel^\pm / \pi}, \quad (4.5.55)$$

where the constants of the forward (−) and reverse (+) reactions are:

$$\beta_{\perp,\parallel}^- = \frac{\beta_{\perp,\parallel}(A^+) \beta(B)}{\beta_{\perp,\parallel}(A^+) + \beta(B)}, \quad \beta_{\perp,\parallel}^+ = \frac{\beta_{\perp,\parallel}(B^+) \beta(A)}{\beta_{\perp,\parallel}(B^+) + \beta(A)},$$

$$K_{a,b}^\pm \equiv K_{a,b}^\pm(X^\pm) X^\pm = \frac{1}{\beta_\perp^\pm} (\beta_\parallel^\pm - \beta_\perp^\pm).$$

The first-order moment (friction force) is

$$\begin{aligned}
 \mathbf{J}^{(1)} = & m_A n(A) n(B^+) \gamma^+ \left\{ \frac{\beta_{\perp}^+}{2\beta_{\perp}(B^+)} (K_{0,2}^+ - K_{2,2}^+) [\mathbf{V}_{\perp}(A) - \mathbf{V}_{\perp}(B^+)] \right. \\
 & \left. + \frac{\beta_{\parallel}^+}{\beta_{\parallel}(B^+)} K_{2,2}^+ [\mathbf{V}_{\parallel}(A) - \mathbf{V}_{\parallel}(B^+)] \right\} - m_A n(A^+) n(B) \gamma^- \\
 & \times \left\{ \frac{\beta_{\perp}^-}{2\beta_{\perp}(A^+)} (K_{0,2}^- - K_{2,2}^-) [\mathbf{V}_{\perp}(A^+) - \mathbf{V}_{\perp}(B)] + \frac{\beta_{\parallel}^-}{\beta_{\parallel}(A^+)} K_{2,2}^- [\mathbf{V}_{\parallel}(A^+) - \mathbf{V}_{\parallel}(B)] \right\}.
 \end{aligned} \tag{4.5.56}$$

The second-order moments (parallel and perpendicular energy variations caused by charge exchange) are

$$\begin{aligned}
 J_{\perp}^{(2)} = & m_A n(A) n(B^+) \frac{\gamma^+}{\beta_{\perp}(B^+) + \beta(A)} \left[\frac{2\beta(A)}{\beta_{\perp}(B^+)} (K_{0,3}^+ - K_{2,3}^+) + K_{0,2}^+ \right] \\
 & - m_A n(A^+) n(B) \frac{\gamma^-}{\beta_{\perp}(A^+) + \beta(B)} \times \left[\frac{2\beta(B)}{\beta_{\perp}(A^+)} (K_{0,3}^- - K_{2,3}^-) + K_{0,2}^- \right],
 \end{aligned} \tag{4.5.57}$$

$$\begin{aligned}
 J_{\parallel}^{(2)} = & m_A n(A) n(B^+) \frac{\gamma^+}{\beta_{\parallel}(B^+) + \beta(A)} \left[\frac{4\beta(A)}{\beta_{\perp}(B^+)} \frac{\beta_{\perp}(B^+) + \beta(A)}{\beta_{\parallel}(B^+) + \beta(A)} K_{2,3}^+ + K_{0,2}^+ \right] \\
 & - m_A n(A^+) n(B) \frac{\gamma^-}{\beta_{\parallel}(A^+) + \beta(B)} \times \left[\frac{4\beta(B)}{\beta_{\perp}(A^+)} \frac{\beta_{\perp}(A^+) + \beta(B)}{\beta_{\parallel}(A^+) + \beta(B)} K_{2,3}^- + K_{0,2}^- \right].
 \end{aligned} \tag{4.5.58}$$

Relations (4.5.54)–(4.5.58) constitute a generalization to the case of anisotropic ion energy distribution of the results obtained in (Banks and Holzer 1968), where the corresponding expressions were obtained for the Maxwell distribution functions.

4.6 Wave–Particle Interaction Terms

The wave–particle interaction may be included if we add the corresponding moments of the quasilinear collision integral to the hydrodynamic equations. Similar attempts were made in articles (Konikov et al. 1989; Gamayunov et al.

1991; Gorbachev et al. 1992). The authors of these papers examined the interaction of the ion-cyclotron waves generated by hot anisotropic protons of the ring current with electrons and protons of the background plasma. These results cannot be used to describe the interactions of other types of observed waves that interact with a multicomponent plasma.

In this section, the system of hydrodynamic equations governing the behavior of a multicomponent anisotropic plasma will be supplemented with terms that take into account the quasilinear interactions of particles with arbitrary waves. Resonant quasilinear interaction of waves with particles (Konikov et al. 1989; Gamayunov et al. 1991) may be insufficient to describe a system. For example, both resonant and adiabatic interactions must be used to describe the interaction of Langmuir waves with electrons. Therefore, below, when obtaining the moments of the quasilinear collision integral, we will consider both resonant and adiabatic interactions.

4.6.1 *Quasilinear Interaction in Hydrodynamics: General Relationships*

Let us consider the case of a weakly turbulent plasma. The expression for the quasilinear integral that considers both resonant and adiabatic interaction was obtained previously (Belikov et al. 1974). Because it will subsequently be used in a somewhat different form, we will outline its calculation. The following equations for the regular (f_a^R) and turbulent ($f_a^{(1)}$) components of the distribution function are obtainable using a standard procedure (for more details, see Tsytovich 1971):

$$\frac{\partial f_a^R}{\partial t} + \mathbf{v} \frac{\partial f_a^R}{\partial \mathbf{r}} + \frac{1}{m_a} \left(\mathbf{F}_a + \frac{e_a}{c} [\mathbf{v} \mathbf{B}_0] \right) \frac{\partial f_a^R}{\partial \mathbf{v}} = - \frac{e_a}{m_a} \left\langle \left(\mathbf{E} + \frac{1}{c} [\mathbf{v} \mathbf{B}] \right) \frac{\partial f_a^{(1)}}{\partial \mathbf{v}} \right\rangle, \quad (4.6.1)$$

$$\frac{\partial f_a^{(1)}}{\partial t} + \mathbf{v} \frac{\partial f_a^{(1)}}{\partial \mathbf{r}} + \frac{1}{m_a} \left(\mathbf{F}_a + \frac{e_a}{c} [\mathbf{v} \mathbf{B}_0] \right) \frac{\partial f_a^{(1)}}{\partial \mathbf{v}} = - \frac{e_a}{m_a} \left(\mathbf{E} + \frac{1}{c} [\mathbf{v} \mathbf{B}] \right) \frac{\partial f_a^R}{\partial \mathbf{v}}, \quad (4.6.2)$$

where $\mathbf{F}_a$ is the regular component of force, and, for brevity, the integral of interparticle collisions is omitted on the right-hand side of (4.6.1). The regular force may include the gravity and electrical terms. We write the magnetic term in the evident form. In the further description, the regular force is considered as constant on the time-space scale of the turbulent quantities.

Then if we use an accompanying coordinate system moving with the mean velocity of particles of the sort α in considering a point in time and space,

$(\mathbf{u}(\mathbf{r}, t) = \frac{1}{N} \int \mathbf{v} f(\mathbf{r}, \mathbf{v}, t) d\mathbf{v}, N = \int f(\mathbf{r}, \mathbf{v}, t) d\mathbf{v})$, (4.6.2) takes the form (we omit index α):

$$\frac{\partial f^{(1)}(t', \mathbf{r}', \mathbf{v}')}{\partial t'} + \vec{\mathbf{v}}' \frac{\partial f^{(1)}}{\partial \mathbf{r}'} + \frac{e}{m'c} [\mathbf{v}' \mathbf{B}_0] \frac{\partial f^{(1)}}{\partial \mathbf{v}'} = -\frac{e}{m'} \left(\mathbf{E}' + \frac{1}{c} [\mathbf{v}' \mathbf{B}'] \right) \frac{\partial f^R}{\partial \mathbf{v}'}. \quad (4.6.3)$$

The primes indicate that all quantities are taken in a coordinate system moving with the plasma. If one neglects the background plasma's inhomogeneity and nonstationarity, (4.6.3) has the following solution (axis $z \parallel \mathbf{B}_0$) (Akhiezer et al. 1975):

$$f_k^{(1)} = e^{i \frac{\mathbf{k}'_{\perp} \mathbf{v}'_{\perp}}{\omega'_B} \sin(\varphi' - \chi')} \sum_n \frac{e^{-in\varphi'}}{2\pi i (\omega' - k'_{\parallel} |\mathbf{v}'_{\parallel}| - n\omega'_B)} \int_0^{2\pi} d\varphi'' e^{-i \frac{\mathbf{k}'_{\perp} \mathbf{v}'_{\perp}}{\omega'_B} \sin(\varphi'' - \chi')} \\ \times e^{in\varphi''} \frac{e}{m'} \left\{ \mathbf{E}'_{k'} \left(1 - \frac{\mathbf{k}' \mathbf{v}'}{\omega'} \right) + \frac{\mathbf{k}' (\mathbf{v}' \mathbf{E}'_{k'})}{\omega'} \right\} \frac{\partial f^R}{\partial \mathbf{v}'}, \quad (4.6.4)$$

where $\mathbf{k}$ is the wave vector, $\cos \chi^{\parallel} = \frac{k_x^{\parallel}}{k_{\perp}^{\parallel}}$, $\cos \varphi^{\parallel} = \frac{\mathbf{v}_x^{\parallel}}{\mathbf{v}_{\perp}^{\parallel}}$, $\omega_B^{\parallel} = \frac{eB_0^{\parallel}}{m'c}$.

Assuming that the mean velocities $\mathbf{u}$ ($|\mathbf{u}| \ll c$) on small scales are coordinates and time independent, the transformations (see, e.g., Alexandrov et al. 1988):

$$\omega' = \omega - \mathbf{k} \mathbf{u}, \quad \mathbf{k}' = \mathbf{k}, \quad E'_{ki} = \beta_{ij}(\mathbf{u}) E_{kj}, \quad \beta_{ij}(\mathbf{u}) = \frac{\omega - \mathbf{k} \mathbf{u}}{\omega} \delta_{ij} + \frac{k_i u_j}{\omega} \quad (4.6.5)$$

(where δ_{ij} is the Kronecker delta), permit (4.6.4) to be rewritten in a laboratory coordinate system:

$$f_k^{(1)} = \hat{L} e^{i \frac{\mathbf{k}_{\perp} \mathbf{v}'_{\perp}}{\omega_B} \sin(\varphi' - \chi)} \sum_n \frac{e^{-in\varphi'}}{2\pi i (\omega - \mathbf{k} \mathbf{u} - k_{\parallel} |\mathbf{v}'_{\parallel}| - n\omega_B)} \int_0^{2\pi} d\varphi'' e^{-i \frac{\mathbf{k}_{\perp} \mathbf{v}'_{\perp}}{\omega_B} \sin(\varphi'' - \chi)} \\ \times e^{in\varphi''} \frac{e}{m} \left\{ -\frac{\mathbf{E}_k(\mathbf{k} \mathbf{u}) - \mathbf{k}(\mathbf{u} \mathbf{E}_k)}{\omega} + \mathbf{E}_k \left(1 - \frac{\mathbf{k} \mathbf{v}'}{\omega} \right) + \frac{\mathbf{k} (\mathbf{v}' \mathbf{E}_k)}{\omega} \right\} \frac{\partial f^R}{\partial \mathbf{v}'}. \quad (4.6.6)$$

Here $\hat{L} g(\mathbf{v}') \equiv g(\mathbf{v} - \mathbf{u})$.

On substituting (4.6.6) into the right-hand side of (4.6.1), multiplying by $mh(\mathbf{v} - \mathbf{u})$, and integrating over velocity, we obtain the following expression for the moments of the quasilinear integral:

$$\begin{aligned}
M &= \frac{e^2}{m} \int d\mathbf{k} I_{\mathbf{k}}^{\sigma}(\mathbf{r}, t) \int d\mathbf{v} \left\{ \mathbf{d}_{\mathbf{k}}^{\sigma*} - \frac{\mathbf{e}_{\mathbf{k}}^{\sigma*}(\mathbf{k}\mathbf{v}) - \mathbf{k}(\mathbf{v}\mathbf{e}_{\mathbf{k}}^{\sigma*})}{\omega_{\mathbf{k}}^{\sigma}} \right\} \frac{\partial h(\mathbf{v})}{\partial \mathbf{v}} \\
&\times \sum_n \frac{e^{-in\varphi}}{2\pi i(\omega_{\mathbf{k}}^{\sigma} - \mathbf{k}\mathbf{u} - k_{\parallel}v_{\parallel} - n\omega_B)} \int_0^{2\pi} d\varphi' e^{-i\frac{k_{\perp}v'_{\perp}}{\omega_B} \sin(\varphi' - \chi)} \\
&\times e^{in\varphi'} \left\{ \mathbf{d}_{\mathbf{k}}^{\sigma} - \frac{\mathbf{e}_{\mathbf{k}}^{\sigma}(\mathbf{k}\mathbf{v}) - \mathbf{k}(\mathbf{v}\mathbf{e}_{\mathbf{k}}^{\sigma})}{\omega_{\mathbf{k}}^{\sigma}} \right\} \frac{\partial f^R}{\partial \mathbf{v}}, \tag{4.6.7}
\end{aligned}$$

where the primes were omitted for $\mathbf{v}'$ and the asterisk refers to complex conjugation.

We make the following definitions:

$$\mathbf{d}_{\mathbf{k}}^{\sigma} = \frac{\mathbf{e}_{\mathbf{k}}^{\sigma}(\omega_{\mathbf{k}}^{\sigma} - \mathbf{k}\mathbf{u}) + \mathbf{k}(\mathbf{u}\mathbf{e}_{\mathbf{k}}^{\sigma})}{\omega_{\mathbf{k}}^{\sigma}}; \quad I_{\mathbf{k}}^{\sigma}(\mathbf{r}, t) = |E_{\mathbf{k}}^{\sigma}(\mathbf{r}, t)|^2;$$

where $\mathbf{e}_{\mathbf{k}}^{\sigma}$ is a polarization vector of the wave oscillations of type σ , and f^R refers to a distribution function in the reference frame moving with velocity $\mathbf{u}$.

By making the following definitions (cf. with (4.6.7)),

$$\begin{aligned}
\sigma \begin{bmatrix} ijk \\ ij \\ i \\ 0 \end{bmatrix} &= \frac{e^2}{m} \int d\mathbf{v} e^{v_{\perp t}} (a_x v_y - a_y v_x) \begin{bmatrix} v_i v_j v_k \\ v_i v_j \\ v_i \\ 1 \end{bmatrix} \sum_n \frac{e^{-in\varphi}}{2\pi i(\omega_{\mathbf{k}}^{\sigma} - \mathbf{k}\mathbf{u} - k_{\parallel}v_{\parallel} - n\omega_B)} \\
&\times \int_0^{2\pi} d\varphi' e^{-i\frac{k_{\perp}v_{\perp}}{\omega_B} \sin(\varphi' - \chi) + in\varphi'} \left\{ \mathbf{d}_{\mathbf{k}}^{\sigma} - \frac{\mathbf{e}_{\mathbf{k}}^{\sigma}(\mathbf{k}\mathbf{v}) - \mathbf{k}(\mathbf{v}\mathbf{e}_{\mathbf{k}}^{\sigma})}{\omega_{\mathbf{k}}^{\sigma}} \right\} \frac{\partial f^R}{\partial \mathbf{v}}; \\
v_{\perp t}^2 &= \frac{T_{\perp}}{m}; \quad (i, j, k) = x, y, z \equiv 1, 2, 3; \tag{4.6.8}
\end{aligned}$$

one may rewrite (4.6.7) in the form used in a hydrodynamic description (Oraevskii et al. 1985), i.e., $h = v_{\alpha}, v_{\alpha}v_{\beta}, v_{\alpha}v_{\beta}v_{\gamma}$, etc.:

$$J_{\alpha} = \int d\mathbf{k} I_{\mathbf{k}}^{\sigma} \left\{ d_{\mathbf{k}\alpha}^{\sigma*} \sigma_0 - \frac{e_{\mathbf{k}\alpha}^{\sigma*} k_i - k_{\alpha} e_{\mathbf{k}i}^{\sigma*}}{\omega_{\mathbf{k}}^{\sigma*}} \sigma_i \right\} \Big|_{a=\frac{k_{\perp}v_{\perp t}}{\omega_B}}, \tag{4.6.9}$$

$$J_{\alpha\beta} = \int d\mathbf{k} I_{\mathbf{k}}^{\sigma} \left\{ \left[d_{\mathbf{k}\alpha}^{\sigma*} \sigma_{\beta} - \frac{e_{\mathbf{k}\alpha}^{\sigma*} k_i - k_{\alpha} e_{\mathbf{k}i}^{\sigma*}}{\omega_{\mathbf{k}}^{\sigma*}} \sigma_{i\beta} \right] + [\beta, \alpha] \right\} \Big|_{a=\frac{k_{\perp}v_{\perp t}}{\omega_B}}, \tag{4.6.10}$$

$$J_{\alpha\beta\gamma} = \int d\mathbf{k} I_{\mathbf{k}}^{\sigma} \left\{ \left[d_{\mathbf{k}\alpha}^{\sigma*} \sigma_{\beta\gamma} - \frac{e_{\mathbf{k}\alpha}^{\sigma*} k_i - k_{\alpha} e_{\mathbf{k}i}^{\sigma*}}{\omega_{\mathbf{k}}^{\sigma*}} \sigma_{i\beta\gamma} \right] + [\gamma, \alpha, \beta] + [\beta, \gamma, \alpha] \right\} \Big|_{a=\frac{k_{\perp} \mathbf{v}_{\perp t}}{\omega_B}} \quad (4.6.11)$$

It is obvious that, during the quasilinear interaction, the number of particles is unchanged, i.e., for the $h = 1$ moment $J_0 = 0$. Here the meaning of α , β , and γ are any x , y , and z . The notations $[\beta, \alpha]$, $[\gamma, \alpha, \beta]$, etc. mean that the first term must be copied with the substitution $\alpha \rightarrow \beta$, $\beta \rightarrow \alpha$ in (4.6.10); $\alpha \rightarrow \gamma$, $\beta \rightarrow \alpha$, $\gamma \rightarrow \beta$ in the second term, and $\alpha \rightarrow \beta$, $\beta \rightarrow \gamma$, $\gamma \rightarrow \alpha$ in the third term in (4.6.11).

For receiving (4.6.9)–(4.6.11), different functions h ($h = v_{\alpha}$, $v_{\alpha} v_{\beta}$, $v_{\alpha} v_{\beta} v_{\gamma}$) are replaced in (4.6.7). Then the derivative $\frac{\partial h}{\partial v_i}$ is multiplied by the bracket $\left\{ \mathbf{d}_{\mathbf{k}}^{\sigma*} - \frac{\mathbf{e}_{\mathbf{k}}^{\sigma*}(\mathbf{k}\mathbf{v}) - \mathbf{k}(\mathbf{v}\mathbf{e}_{\mathbf{k}}^{\sigma*})}{\omega_{\mathbf{k}}^{\sigma*}} \right\}_i$, and the definitions (4.6.8) are used. We introduced parameter $\mathbf{a}$, which is equal to $\frac{k_x v_{\perp t}}{\omega_B}$, $\frac{k_y v_{\perp t}}{\omega_B}$, in the first exponential multiplier (4.6.8) in order to distinguish k_x, k_y in this term from the others. Parameter $\mathbf{a}$ will be used later for determining the connection between $\sigma_{\alpha\beta}$ and $\sigma_{\alpha}, \sigma_{\alpha\beta\gamma}$ and $\sigma_{\alpha\beta}$, etc. ((4.6.13)–(4.6.16)).

Equations (4.6.9)–(4.6.11) can be reduced to a simpler form if relationships of continuity for the moments following from (4.6.3) are used. By introducing

$$P \begin{bmatrix} ij \\ i \\ 0 \end{bmatrix} = -\frac{e^2}{mk_{\parallel} 2\pi i} \int d\mathbf{v} e^{\frac{i}{v_{\perp t}}(a_x v_y - a_y v_x)} \begin{bmatrix} v_i v_j \\ v_i \\ 1 \end{bmatrix} \sum_n e^{-in\varphi} \int_0^{2\pi} d\varphi' e^{-i\frac{k_{\perp} \mathbf{v}_{\perp}}{\omega_B} \sin(\varphi' - \chi)} \times e^{in\varphi'} \left\{ \mathbf{d}_{\mathbf{k}}^{\sigma} - \frac{\mathbf{e}_{\mathbf{k}}^{\sigma}(\mathbf{k}\mathbf{v}) - \mathbf{k}(\mathbf{v}\mathbf{e}_{\mathbf{k}}^{\sigma})}{\omega_{\mathbf{k}}^{\sigma}} \right\} \frac{\partial f^R}{\partial \mathbf{v}}, \quad (4.6.12)$$

we get

$$J_{\alpha} = \int d\mathbf{k} I_{\mathbf{k}}^{\sigma} \left\{ \frac{k_{\alpha}}{\omega_{\mathbf{k}}^{\sigma*}} [(\mathbf{u}\mathbf{e}_{\mathbf{k}}^{\sigma*}) \sigma_0 + \mathbf{e}_{\mathbf{k}i}^{\sigma*} \sigma_i] + \mathbf{e}_{\mathbf{k}\alpha}^{\sigma*} \frac{\omega_{\mathbf{k}}^{\sigma*} - \omega_{\mathbf{k}}^{\sigma}}{\omega_{\mathbf{k}}^{\sigma*}} \sigma_0 \right\} \Big|_{a=\frac{k_{\perp} \mathbf{v}_{\perp t}}{\omega_B}} \quad (4.6.9')$$

$$J_{\alpha\beta} = \int d\mathbf{k} I_{\mathbf{k}}^{\sigma} \left\{ \left[\frac{k_{\alpha}}{\omega_{\mathbf{k}}^{\sigma*}} [(\mathbf{u}\mathbf{e}_{\mathbf{k}}^{\sigma*}) \sigma_{\beta} + \mathbf{e}_{\mathbf{k}i}^{\sigma*} \sigma_{i\beta}] + e_{\mathbf{k}\alpha}^{\sigma*} \frac{\omega_{\mathbf{k}}^{\sigma*} - \omega_{\mathbf{k}}^{\sigma}}{\omega_{\mathbf{k}}^{\sigma*}} \sigma_{\beta} - \frac{k_{\parallel}}{\omega_{\mathbf{k}}^{\sigma*}} e_{\mathbf{k}\alpha}^{\sigma*} P_{\beta} - i \frac{\omega_B}{\omega_{\mathbf{k}}^{\sigma*}} e_{\mathbf{k}\alpha}^{\sigma*} (\delta_{\beta 2} \sigma_1 - \delta_{\beta 1} \sigma_2) \right] + [\beta, \alpha] \right\} \Big|_{a=\frac{k_{\perp} \mathbf{v}_{\perp t}}{\omega_B}} \quad (4.6.10')$$

$$J_{\alpha\beta\gamma} = \int d\mathbf{k} I_{\mathbf{k}}^{\sigma} \left\{ \left[\frac{k_{\alpha}}{\omega_{\mathbf{k}}^{\sigma*}} [(\mathbf{u}\mathbf{e}_{\mathbf{k}}^{\sigma*}) \sigma_{\beta\gamma} + \mathbf{e}_{\mathbf{k}i}^{\sigma*} \sigma_{i\beta\gamma}] + e_{\mathbf{k}\alpha}^{\sigma*} \frac{\omega_{\mathbf{k}}^{\sigma*} - \omega_{\mathbf{k}}^{\sigma}}{\omega_{\mathbf{k}}^{\sigma*}} \sigma_{\beta\gamma} - \frac{k_{\parallel}}{\omega_{\mathbf{k}}^{\sigma*}} e_{\mathbf{k}\alpha}^{\sigma*} P_{\beta\gamma} - i \frac{\omega_B}{\omega_{\mathbf{k}}^{\sigma*}} e_{\mathbf{k}\alpha}^{\sigma*} (\delta_{\beta 2} \sigma_{1\gamma} - \delta_{\beta 1} \sigma_{2\gamma} + \delta_{\gamma 2} \sigma_{1\beta} - \delta_{\gamma 1} \sigma_{2\beta}) \right] + [\gamma, \alpha, \beta] + [\beta, \gamma, \alpha] \right\} \Big|_{a=\frac{k_{\perp} \mathbf{v}_{\perp t}}{\omega_B}} \quad (4.6.11')$$

Using (4.6.8) and (4.6.12), we have the following:

$$\sigma_1 = i\mathbf{v}_{\perp t} \frac{\partial \sigma_0}{\partial a_y}; \quad \sigma_2 = -i\mathbf{v}_{\perp t} \frac{\partial \sigma_0}{\partial a_x}; \quad \sigma_3 = P_0 + \frac{\omega_{\mathbf{k}}'}{k_{\parallel}} \sigma_0 + i \frac{\omega_B}{k_{\parallel}} \left[a_y \frac{\partial}{\partial a_x} - a_x \frac{\partial}{\partial a_y} \right] \sigma_0; \quad (4.6.13)$$

$$\sigma_{1\alpha} = i\mathbf{v}_{\perp t} \frac{\partial \sigma_{\alpha}}{\partial a_y}; \quad \sigma_{2\alpha} = -i\mathbf{v}_{\perp t} \frac{\partial \sigma_{\alpha}}{\partial a_x}; \quad \sigma_{33} = P_3 + \frac{\omega_{\mathbf{k}}'}{k_{\parallel}} \sigma_3 + i \frac{\omega_B}{k_{\parallel}} \left[a_y \frac{\partial}{\partial a_x} - a_x \frac{\partial}{\partial a_y} \right] \sigma_3; \quad (4.6.14)$$

$$\sigma_{1\beta\gamma} = i\mathbf{v}_{\perp t} \frac{\partial \sigma_{\beta\gamma}}{\partial a_y}; \quad \sigma_{2\beta\gamma} = -i\mathbf{v}_{\perp t} \frac{\partial \sigma_{\beta\gamma}}{\partial a_x}; \quad \sigma_{333} = P_{33} + \frac{\omega_{\mathbf{k}}'}{k_{\parallel}} \sigma_{33} + i \frac{\omega_B}{k_{\parallel}} \left[a_y \frac{\partial}{\partial a_x} - a_x \frac{\partial}{\partial a_y} \right] \sigma_{33}. \quad (4.6.15)$$

Relations (4.6.13)–(4.6.15) may be verified by the direct substitution of (4.6.8) and (4.6.12) in these formulae.

If we use the following relationships

$$\begin{aligned} P_1 &= i\mathbf{v}_{\perp t} \frac{\partial P_0}{\partial a_y}; \quad P_2 = -i\mathbf{v}_{\perp t} \frac{\partial P_0}{\partial a_x}; \\ P_{1\alpha} &= i\mathbf{v}_{\perp t} \frac{\partial P_{\alpha}}{\partial a_y}; \quad P_{2\alpha} = -i\mathbf{v}_{\perp t} \frac{\partial P_{\alpha}}{\partial a_x}; \end{aligned} \quad (4.6.16)$$

then, when calculating the functions, P , one can put $\mathbf{a} = \frac{\mathbf{k}_{\perp} \mathbf{v}_{\perp t}}{\omega_B}$:

$$P_0|_{\mathbf{a}=\frac{\mathbf{k}_{\perp} \mathbf{v}_{\perp t}}{\omega_B}} = 0 \quad (4.6.17)$$

$$P_{\alpha}|_{\mathbf{a}=\frac{\mathbf{k}_{\perp} \mathbf{v}_{\perp t}}{\omega_B}} = \frac{e^2}{imk_{\parallel}} \int d\mathbf{v} f^R \left\{ d_{\mathbf{k}\alpha}^{\sigma} - \frac{e_{\mathbf{k}\alpha}^{\sigma}(\mathbf{k}\mathbf{v}) - k_{\alpha}(\mathbf{v}e_{\mathbf{k}}^{\sigma})}{\omega_{\mathbf{k}}^{\sigma}} \right\} \quad (4.6.18)$$

$$P_{\alpha\beta}|_{\mathbf{a}=\frac{\mathbf{k}_{\perp} \mathbf{v}_{\perp t}}{\omega_B}} = \frac{e^2}{imk_{\parallel}} \int d\mathbf{v} f^R \left\{ v_{\alpha} \left[d_{\mathbf{k}\beta}^{\sigma} - \frac{e_{\mathbf{k}\beta}^{\sigma}(\mathbf{k}\mathbf{v}) - k_{\beta}(\mathbf{v}e_{\mathbf{k}}^{\sigma})}{\omega_{\mathbf{k}}^{\sigma}} \right] + [\beta, \alpha] \right\} \quad (4.6.19)$$

Thus, we have written the expressions for the moments of the quasilinear integral of collisions in terms of the function σ_0 (its physical meaning is clear from its relationship to the charge density of α particles: $\rho'_{\alpha} = \sigma_0^{\alpha} E'_{\mathbf{k}}|_{\mathbf{a}=\frac{\mathbf{k}_{\perp} \mathbf{v}_{\perp t}}{\omega_B}}$) and P . The hydrodynamic equations should be supplemented with an expression governing the energy density evolution of the plasma oscillations. The resulting system takes into account both the resonant and adiabatic quasilinear interactions.

4.6.2 Moments in a Bi-Maxwellian Plasma

By virtue of the assumption made above (see (4.6.4)), the distribution function f^R can depend on coordinates and on time (in (4.6.8) and (4.6.12)) only as the parameters. If, moreover, interparticle collisions dominate and there is no need to consider plateau formation on f^R , the following expression (in the plasma frame) is often used:

$$f^R(\mathbf{v}) = \frac{N_\alpha}{(2\pi)^{3/2} v_{\perp t}^2 v_{\parallel t}} e^{-\frac{v_{\parallel}^2}{2v_{\parallel t}^2} - \frac{v_{\perp}^2}{2v_{\perp t}^2}}, \quad (4.6.20)$$

where N_α is the density of α particles; and $v_{\parallel t}, v_{\perp t}$ are the longitudinal (along the external magnetic field) and transverse thermal velocities. The density and temperatures in (4.6.20) depend parametrically on $\mathbf{r}$ and t . Note that calculation of moments of the quasilinear integral of collisions using (4.6.20) does not contradict the fact that, on scales in excess of the scale of averaging in (4.6.1), the distribution function can contain dissipative terms (e.g., the Grad approximation).

In the case of Coulomb collisions, the question of the consistency of the approximations, in terms of which cross-sections and moments of the integral of collisions are calculated, does not arise.

On substituting (4.6.20) into (4.6.8) and integrating over the velocity, we obtain:

$$\begin{aligned} \sigma_0 = & \frac{e^2 N}{imv_{\perp t} \omega_{\mathbf{k}}^\sigma} \sum_n e^{in(\chi - \chi')} \left\{ e^{-\frac{z^2 + \lambda^2}{2}} I_n(a\lambda) \left[\frac{\omega_{\mathbf{k}}^\sigma}{k_{\parallel} v_{\perp t}} d_{k_{\parallel}}^\sigma (A+1)(1 + i\sqrt{\pi} z_n w(z_n)) \right. \right. \\ & + \frac{n}{\lambda} \left(\frac{\mathbf{d}_{\mathbf{k}_{\perp}}^\sigma \mathbf{k}_{\perp}}{k_{\perp}} i\sqrt{\pi} z_0 w(z_n) + A \left[\frac{\mathbf{e}_{\mathbf{k}_{\perp}}^\sigma \mathbf{k}_{\perp}}{k_{\perp}} - e_{k_{\parallel}}^\sigma \frac{k_{\perp}}{k_{\parallel}} \right] (1 + i\sqrt{\pi} z_n w(z_n)) \right) \Big] \\ & + i\lambda e^{-\frac{a^2 + \lambda^2}{2}} \left[-I_n(a\lambda) + \frac{a}{\lambda} I'_n(a\lambda) \right] \\ & \times \left[\frac{[\mathbf{k} \mathbf{d}_{\mathbf{k}}^\sigma]_{\parallel}}{k_{\perp}} i\sqrt{\pi} z_0 w(z_n) + \frac{[\mathbf{k} \mathbf{e}_{\mathbf{k}}^\sigma]_{\parallel}}{k_{\perp}} A (1 + i\sqrt{\pi} z_n w(z_n)) \right] \Big\}. \end{aligned} \quad (4.6.21)$$

The designations used here are:

$$\tan \chi' = \frac{a_y}{a_x}; \quad \lambda = \frac{k_{\perp} v_{\perp t}}{\omega_B}; \quad A = \frac{T_{\perp}}{T_{\parallel}} - 1; \quad z_n = \frac{\omega_{\mathbf{k}}^{\sigma'} - n\omega_B}{\sqrt{2} k_{\parallel} v_{\parallel t}}; \quad z_0 = \frac{\omega_{\mathbf{k}}^\sigma}{\sqrt{2} k_{\parallel} v_{\parallel t}};$$

where I_n is the modified Bessel's function and $w(z_n)$ is the probability integral of a complex argument

$$w(z_n) = \exp(-z_n^2) \left[1 + \frac{2i}{\sqrt{\pi}} \int_0^{z_n} \exp(t^2) dt \right].$$

The expressions for the functions P (4.6.12) assume the form:

$$P_{\alpha}|_{\mathbf{a}=\frac{\mathbf{k}_{\perp}\mathbf{v}_{\perp t}}{\omega_B}} = \frac{\omega_p^2}{4\pi i k_{\parallel}} d_{\mathbf{k}\alpha}^{\sigma}; \quad P_{\alpha\beta}|_{\mathbf{a}=\frac{\mathbf{k}_{\perp}\mathbf{v}_{\perp t}}{\omega_B}} = \frac{\omega_p^2(e_{\mathbf{k}\alpha}^{\sigma}k_{\beta} - e_{\mathbf{k}\beta}^{\sigma}k_{\alpha})}{4\pi i k_{\parallel}\omega_{\mathbf{k}}^{\sigma}} \int d\mathbf{v} (v_{\alpha}^2 - v_{\beta}^2) f^R \quad (4.6.22)$$

where $\omega_p^2 = \frac{4\pi N_{\alpha}e^2}{m}$ is the plasma frequency of the considered particles.

It is most convenient to calculate the derivatives of σ_0 in parameters a_x and a_y in a cylindrical coordinate system. In this case, as follows from (4.6.21), $(\partial/\partial x')^l = (-in)^l$, and the relationships:

$$\lambda \frac{\partial}{\partial a} Q_n \Big|_{a=\frac{k_{\perp}\mathbf{v}_{\perp t}}{\omega_B}} = \lambda^2 \left[e^{-\lambda^2} I_n(\lambda^2) \right]' \equiv x A_n'(x), \quad (4.6.23)$$

$$\lambda^2 \frac{\partial^2}{\partial a^2} Q_n \Big|_{a=\frac{k_{\perp}\mathbf{v}_{\perp t}}{\omega_B}} = x^2 A_n''(x) - x A_n(x), \quad (4.6.24)$$

$$\lambda^3 \frac{\partial^3}{\partial a^3} Q_n \Big|_{a=\frac{k_{\perp}\mathbf{v}_{\perp t}}{\omega_B}} = x^3 A_n'''(x) - 3x^2 A_n'(x), \quad (4.6.25)$$

$$\lambda \frac{\partial}{\partial a} R_n \Big|_{a=\frac{k_{\perp}\mathbf{v}_{\perp t}}{\omega_B}} = x A_n''(x) + e^{-x} I_n'(x), \quad (4.6.26)$$

$$\lambda^2 \frac{\partial^2}{\partial a^2} R_n \Big|_{a=\frac{k_{\perp}\mathbf{v}_{\perp t}}{\omega_B}} = x^2 A_n'''(x) + x A_n''(x) + x [e^{-x} I_n'(x)]', \quad (4.6.27)$$

$$\lambda^3 \frac{\partial^3}{\partial a^3} R_n \Big|_{a=\frac{k_{\perp}\mathbf{v}_{\perp t}}{\omega_B}} = x^3 A_n^{IV}(x) + 3x^2 A_n'''(x) - 3x e^{-x} I_n'(x) \quad (4.6.28)$$

are valid, where

$$Q_n = e^{-\frac{a^2+\lambda^2}{2}} I_n(a\lambda)$$

and

$$R_n = e^{-\frac{a^2+\lambda^2}{2}} \left[-I_n(a\lambda) + \frac{a}{\lambda} I_n'(a\lambda) \right].$$

Using (4.6.21)–(4.6.28), one can rewrite the moments of the quasilinear integral (4.6.9)–(4.6.11) for the case under consideration. The resulting expressions

are rather unwieldy, however, for a number of particular situations (small Larmor radius, absence of transverse drift, resonant interaction, etc.), they simplify substantially.

4.6.3 Moments in the Absence of a Transverse Drift

If $\mathbf{u} = 0$, then $I_{\mathbf{k}}^\sigma$, σ_0 , and the derivatives on x' and a (after identifying $\mathbf{a} = \frac{\mathbf{k}_\perp \mathbf{v}_\perp t}{\omega_B}$), do not depend on x , and this allows us to simplify (4.6.9)–(4.6.11). Choose the polarization vector in the form (Akhiezer et al. 1975):

$$\mathbf{e}_{\mathbf{k}}^\sigma = \frac{1}{|g^\sigma|} \left\{ \cos \chi + iT^\sigma \sin \chi, \quad \sin \chi - iT^\sigma \cos \chi, \quad e_3^\sigma |g^\sigma| \right\}, \quad (4.6.29)$$

where T^σ is a combination of the components of the tensor of dielectric permeability $\varepsilon_{ij}(\omega_{\mathbf{k}}, \mathbf{k})$; $\varepsilon_3 \equiv \varepsilon_{zz}(\omega_{\mathbf{k}}, \mathbf{k})$ and $|g^\sigma|$ is the normalization. Now we can integrate (4.6.9')–(4.6.11') over χ . Then

$$J_\alpha = \delta_{\alpha 3} \int d\mathbf{k} I_{\mathbf{k}} \frac{1}{\omega_{\mathbf{k}}^*} \sum_n \left\{ e_{\mathbf{k}||}^* (\omega_{\mathbf{k}}^* - n\omega_B) + \frac{k_{||}\omega_B}{k_\perp |g|} (T^* \lambda \frac{\partial}{\partial a} + n) \right\} \sigma_0^n \Big|_{a=\frac{k_\perp \mathbf{v}_\perp t}{\omega_B}}, \quad (4.6.30)$$

$$\begin{aligned} J_{\alpha\beta} = & \int d\mathbf{k} I_{\mathbf{k}} \frac{1}{\omega_{\mathbf{k}}^* k_{||}} \left\{ (\delta_{\alpha 1} \delta_{\beta 1} + \delta_{\alpha 2} \delta_{\beta 2} - 2\delta_{\alpha 3} \delta_{\beta 3}) \frac{\omega_p^2}{4\pi i} [[\mathbf{k} \mathbf{e}_{\mathbf{k}}^*] \mathbf{d}_{\mathbf{k}}]_{||} \right. \\ & + (\delta_{\alpha 1} \delta_{\beta 1} + \delta_{\alpha 2} \delta_{\beta 2}) \sum_n [n\omega_B e_{\mathbf{k}||}^* (\omega_{\mathbf{k}} - n\omega_B) + (n\omega_B + \omega_{\mathbf{k}}^* - \omega_{\mathbf{k}}) \frac{\omega_B k_{||}}{k_\perp |g|} \\ & \times (T^* \lambda \frac{\partial}{\partial a} + n)] \sigma_0^n + 2\delta_{\alpha 3} \delta_{\beta 3} \sum_n (\omega'_{\mathbf{k}} - n\omega_B) [e_{\mathbf{k}||}^* (\omega_{\mathbf{k}}^* - n\omega_B) \\ & \left. + \frac{\omega_B k_{||}}{k_\perp |g|} (T^* \lambda \frac{\partial}{\partial a} + n)] \sigma_0^n \right\} \Big|_{a=\frac{k_\perp \mathbf{v}_\perp t}{\omega_B}}, \quad (4.6.31) \end{aligned}$$

$$\begin{aligned} J_{\alpha\beta\gamma} = & \int d\mathbf{k} I_{\mathbf{k}} \frac{1}{\omega_{\mathbf{k}}^* k_{||}^2} \times \left\{ -(\delta_{\alpha 3} (\delta_{\beta 1} \delta_{\gamma 1} + \delta_{\beta 2} \delta_{\gamma 2}) + \delta_{\beta 3} (\delta_{\gamma 1} \delta_{\alpha 1} + \delta_{\gamma 2} \delta_{\alpha 2}) \right. \\ & + \delta_{\gamma 3} (\delta_{\alpha 1} \delta_{\beta 1} + \delta_{\alpha 2} \delta_{\beta 2}) - 3\delta_{\alpha 3} \delta_{\beta 3} \delta_{\gamma 3}) \\ & \times \frac{\omega_p^2}{4\pi i} \left(\frac{k_{||} \mathbf{v}_{||t}^2}{\omega_{\mathbf{k}}} A [\mathbf{e}_{\mathbf{k}}^* \vec{k}]_\perp^2 + i\omega_B [\mathbf{e}_{\mathbf{k}}^* \mathbf{k}]_\perp \mathbf{d}_{\mathbf{k}\perp} \right) \\ & \left. - 3\delta_{\alpha 3} \delta_{\beta 3} \delta_{\gamma 3} \frac{\omega_p^2}{4\pi i} \left(\omega'_{\mathbf{k}} [[\mathbf{k} \mathbf{e}_{\mathbf{k}}^*] \vec{k}]_{||} - \omega_{\mathbf{k}}^* k_{||} |e_{\mathbf{k}||}|^2 + \omega_{\mathbf{k}}^* e_{\mathbf{k}||}^* \mathbf{k}_\perp \mathbf{d}_{\mathbf{k}\perp} \right) \right\} \end{aligned}$$

$$\begin{aligned}
& + 3\delta_{x3}\delta_{\beta3}\delta_{\gamma3} \sum_n (\omega'_{\mathbf{k}} - n\omega_B)^2 \times \left[(\omega_{\mathbf{k}}^* - n\omega_B) e_{\mathbf{k}\parallel}^* + \frac{\omega_B k_{\parallel}}{k_{\perp}|g|} (T^* \lambda \frac{\partial}{\partial a} + n) \right] \sigma_0^n \\
& + (\delta_{x3}(\delta_{\beta1}\delta_{\gamma1} + \delta_{\beta2}\delta_{\gamma2}) + \delta_{\beta3}(\delta_{\gamma1}\delta_{x1} + \delta_{\gamma2}\delta_{x2}) + \delta_{\gamma3}(\delta_{x1}\delta_{\beta1} + \delta_{x2}\delta_{\beta2})) \\
& \times \sum_n (\omega'_{\mathbf{k}} - n\omega_B) \left[n\omega_B e_{\mathbf{k}\parallel}^* (\omega_{\mathbf{k}} - n\omega_B) + (n\omega_B + \omega_{\mathbf{k}}^* - \omega_{\mathbf{k}}) \frac{\omega_B k_{\parallel}}{k_{\perp}|g|} \right. \\
& \times \left. (T^* \lambda \frac{\partial}{\partial a} + n) \right] \sigma_0^n - (\delta_{x3}(\delta_{\beta1}\delta_{\gamma1} + \delta_{\beta2}\delta_{\gamma2}) + \delta_{\beta3}(\delta_{\gamma1}\delta_{x1} + \delta_{\gamma2}\delta_{x2})) \\
& + \delta_{\gamma3}(\delta_{x1}\delta_{\beta1} + \delta_{x2}\delta_{\beta2})) \times \frac{\omega_B^2 k_{\parallel}^2}{2k_{\perp}^2} \sum_n \left[e_{\mathbf{k}\parallel}^* (\omega_{\mathbf{k}}^* - n\omega_B) \left(\lambda^2 \frac{\partial^2}{\partial x^2} + \lambda \frac{\partial}{\partial x} - n^2 \right) \right. \\
& + \frac{\omega_B k_{\parallel}}{k_{\perp}|g|} \left(T^* \left(\lambda^3 \frac{\partial^3}{\partial x^3} + \lambda^2 \frac{\partial^2}{\partial x^2} - (1 + n^2) \lambda \frac{\partial}{\partial x} + 2n^2 \right) \right. \\
& + \left. n \left(\lambda^2 \frac{\partial^2}{\partial x^2} + \lambda \frac{\partial}{\partial x} - n^2 \right) \right) \left. \right] \sigma_0^n + \frac{\omega_p^2}{4\pi i} (\delta_{x3}(\delta_{\beta1}\delta_{\gamma1} + \delta_{\beta2}\delta_{\gamma2}) \\
& + \delta_{\beta3}(\delta_{\gamma1}\delta_{x1} + \delta_{\gamma2}\delta_{x2}) + \delta_{\gamma3}(\delta_{x1}\delta_{\beta1} + \delta_{x2}\delta_{\beta2})) \\
& \times \left(k_{\parallel} \omega_{\mathbf{k}} e_{\mathbf{k}\perp}^* \vec{d}_{\perp} + \omega_{\mathbf{k}} [[\mathbf{k} e_{\mathbf{k}}^*] \mathbf{d}_{\mathbf{k}}]_{\parallel} \right) \Big|_{a=\frac{k_{\perp} v_{\perp t}}{\omega_B}}, \tag{4.6.32}
\end{aligned}$$

where σ_0 has the form:

$$\begin{aligned}
\sigma_0 &= \sum_n \sigma_0^n \\
&= \frac{\omega_p^2}{4\pi i v_{\perp t} \omega_{\mathbf{k}}} \sum_n \left\{ e^{-\frac{a^2 + \lambda^2}{2}} I_n(a\lambda) e_{\mathbf{k}\parallel} \left[\frac{\omega_{\mathbf{k}}}{k_{\parallel} v_{\perp t}} \left(1 + A \left(1 - \frac{n\omega_B}{\omega_{\mathbf{k}}^{\sigma}} \right) \right) (1 + i\sqrt{\pi} z_n w(z_n)) \right. \right. \\
&+ \left. \frac{n\omega_B}{\omega_{\mathbf{k}}} \frac{u_{\parallel}}{v_{\perp t}} i\sqrt{\pi} z_n w(z_n) \right] + \frac{\lambda}{|g|} [i\sqrt{\pi} z_0 w(z_n) + A(1 + i\sqrt{\pi} z_n w(z_n))] e^{-\frac{a^2 + \lambda^2}{2}} \\
&\times \left[I_n(a\lambda) \frac{n}{\lambda^2} + T(-I_n(a\lambda) + \frac{a}{\lambda} I'_n(a\lambda)) \right] \Big\}. \tag{4.6.33}
\end{aligned}$$

When substituting the derivatives $(\frac{\partial}{\partial a})^l$ into (4.6.30)–(4.6.32), it is more convenient to use not (4.6.23)–(4.6.28) but their combinations. Expressions for these combinations are given in Appendix E.

Thus, we have obtained the moments of the quasilinear collision integral that take into account the resonant and adiabatic interactions of arbitrary types of oscillations with arbitrary plasma particles. Expressions (4.6.9')–(4.6.11'), (4.6.13)–(4.6.17), and (4.6.21) and (4.6.22) are suitable for describing the plasma in the field of arbitrarily directed regular force $\mathbf{F}_x$ (4.6.30)–(4.6.33), in the absence of a transverse drift.

As mentioned above, the system of hydrodynamic equations, taking into account the contributions from the quasilinear interaction, should be solved together with the equation describing the evolution of the spectral energy of waves $I_{\mathbf{k}}^{\sigma}$. However,

as the first approximation, we can use the energy spectra of the given type of waves registered on satellites in the near terrestrial space.

In conclusion, we will derive the expressions for the moment of the quasilinear integral of collisions that can be used when modeling the interactions of ion–cyclotron waves with ions in the ring current region.

As we mentioned above, the anisotropic protons of the ring current generate intensive left-hand polarized ion–cyclotron waves in the region of the plasmasphere. Moreover, the generation takes place during the cyclotron interaction. Using (4.6.30)–(4.6.33), (E1)–(E6) for $n = 1$, assuming $\lambda \ll 1$ and neglecting $e_{k_{||}}$, we get fairly simple expressions taking into account the interaction of ring current protons and ion–cyclotron waves:

$$J_\alpha = \delta_{\alpha 3} \frac{\omega_p^2}{8\sqrt{\pi}} \int d\mathbf{k} I_{\mathbf{k}} \frac{k_{||}}{\omega_R^2 |g|^2} |1 + T|^2 (z_0 + Az_1) e^{-z_1^2} \text{sign } k_{||} \quad (4.6.34)$$

$$J_{\alpha\beta} = \frac{\omega_p^2 \omega_B}{8\sqrt{\pi}} \int d\mathbf{k} I_{\mathbf{k}} \frac{|1 + T|^2}{\omega_R^2 |g|^2} \left[\delta_{\alpha 1} \delta_{\beta 1} + \delta_{\alpha 2} \delta_{\beta 2} + 2\delta_{\alpha 3} \delta_{\beta 3} \left(\frac{\omega'_R}{\omega_B} - 1 \right) \right] \\ \times (z_0 + Az_1) e^{-z_1^2} \text{sign } k_{||}, \quad (4.6.35)$$

$$J_{\alpha\beta\gamma} = \frac{\omega_p^2 \omega_B^2}{8\sqrt{\pi}} \int d\mathbf{k} I_{\mathbf{k}} \frac{|1 + T|^2}{\omega_R^2 k_{||} |g|^2} \\ \times \left\{ \left[\delta_{\alpha 3} (\delta_{\beta 1} \delta_{\gamma 1} + \delta_{\beta 2} \delta_{\gamma 2}) + \delta_{\beta 3} (\delta_{\gamma 1} \delta_{\alpha 1} + \delta_{\gamma 2} \delta_{\alpha 2}) + \delta_{\gamma 3} (\delta_{\alpha 1} \delta_{\beta 1} + \delta_{\alpha 2} \delta_{\beta 2}) \right] \right. \\ \left. \times \left[\frac{\omega'_R}{\omega_B} - 1 + \frac{2k_{||}^2 v_{||t}^2}{\omega_B^2} \right] + 3\delta_{\alpha 3} \delta_{\beta 3} \delta_{\gamma 3} \left(\frac{\omega'_R}{\omega_B} - 1 \right)^2 \right\} (z_0 + Az_1) e^{-z_1^2} \text{sign } k_{||}, \quad (4.6.36)$$

where

$$\omega_R = \text{Re } \omega_{\mathbf{k}}; \quad \omega'_R = \omega_R - k_{||} u_{||}; \quad z_1 = \frac{\omega'_R - \omega_B}{\sqrt{2} k_{||} v_{||t}}; \\ T = \frac{i e_{xy}}{\varepsilon_{xx} - N^2}; \quad N^2 = \frac{k^2 c^2}{\omega_k^2}.$$

The tensor of dielectric permeability of the cold magnetized plasma in previous expressions may be used.

If there is a small addition of helium ions in the ring current region, then the dispersive dependence of ion–cyclotron waves is greatly modified (Young et al. 1981). The presence of helium leads to three branches in the dispersion relation for oblique propagation of waves. The first branch (class I) is the left-hand polarized

and lies below the gyrofrequency of helium ions (He^+) (Korth et al. 1984). The interaction of such waves with He^+ will also be described by (4.6.34)–(4.6.36), with substitution of the anisotropic protons parameters on the parameters of He^+ .

In conclusion, we stress that the moments of the quasilinear collision integrals obtained in this section take into account both resonant and adiabatic interactions of arbitrary waves and particles. This allows the use of the system of hydrodynamic equations for investigation of the multicomponent plasma in which the developed wave activity takes place.

The results obtained in previous sections have been used in papers by Khazanov et al. (1984, 1994, 1996), Konikov et al. (1989), and Gorbachev et al. (1992) in order to model the different aspects of ionosphere–magnetosphere coupling processes, and may be useful for a hydrodynamic description of the solar wind, plasmas of other planets, and so on. Note that in some of the papers cited above, we also derive and use (Gorbachev et al. 1992; Khazanov et al. 1994, 1996) the moment equations that describe the ion-induced scattering process. That approach was very similar to the approach presented in this section because the description of the ion-induced scattering process could be presented in the diffusive form (3.8.8) like the quasilinear relaxation of the thermal electrons (see Gamayunov et al. 1991 for details).

4.7 Heat Balance Equation in the Presence of Temperature Anisotropy

To conclude this chapter, let us consider the consequences of temperature anisotropy on the heat balance equations. In the anisotropic hydrodynamic description of the thermal plasma, the energy distribution is characterized by different temperatures for the longitudinal ($T_{\parallel}$) and transverse ($T_{\perp}$) degrees of freedom with respect to an external magnetic field, given by

$$T_{r\parallel} = \frac{m_r}{n_r} \int v_{r\parallel}^2 f_r d^3 v_r, \quad (4.7.1)$$

$$T_{r\perp} = \frac{m_r}{n_r} \int (v_{r\perp}^2/2) f_r d^3 v_r, \quad (4.7.2)$$

where m_r , n_r , v_r , and f_r are the mass, the number density, the characteristic velocity component, and the distribution function of species r particles, respectively.

Contrary to the isotropic approximation, two heat-flux vectors, $\mathbf{S}_r^{\parallel}$ and $\mathbf{S}_r^{\perp}$, are introduced to describe separately the transfer of longitudinal and transverse thermal energies. These are defined as

$$\mathbf{S}_r^{\parallel} = m_r \int \mathbf{v}_r v_{r\parallel}^2 f_r d^3 v_r \quad (4.7.3)$$

$$\mathbf{S}_r^\perp = m_r \int \mathbf{v}_r (v_{r\perp}^2/2) f_r d^3 \mathbf{v}_r \quad (4.7.4)$$

The magnetization conditions are satisfied over the altitude range considered in this study, so we can disregard the transverse components of the heat flux vectors (4.7.3) and (4.7.4) and the contribution of the viscosity tensor in the heat balance equations is neglected. Because the transfer processes, under the magnetization conditions, are controlled by the magnetic field, one of the coordinates (z) can be conveniently chosen along the magnetic field-line direction. The set of anisotropic hydrodynamics equations for the heat balance (Barakat and Schunk 1982a; Oraevskii et al. 1985) may then be presented in the following form (here and henceforth, the particle-specie subscript will be omitted wherever possible to simplify notation):

$$\frac{dp_{\parallel}}{dt} + p_{\parallel} \left(\text{div} \mathbf{u}_{\perp} + 3 \frac{\partial u_{\parallel}}{\partial z} - \frac{u_{\parallel}}{B} \frac{\partial B}{\partial z} \right) + \frac{\partial S^{\parallel}}{\partial z} + (2S^{\perp} - S^{\parallel}) \frac{1}{B} \frac{\partial B}{\partial z} = \frac{\delta p_{\parallel}}{\delta t} \quad (4.7.5)$$

$$\frac{dp_{\perp}}{dt} + p_{\perp} \left(2 \text{div} \mathbf{u}_{\perp} + \frac{\partial u_{\parallel}}{\partial z} - 2 \frac{u_{\parallel}}{B} \frac{\partial B}{\partial z} \right) + \frac{\partial S^{\perp}}{\partial z} - \frac{2S^{\perp}}{B} \frac{\partial B}{\partial z} = \frac{\delta p_{\perp}}{\delta t} \quad (4.7.6)$$

$$\frac{dS^{\parallel}}{dt} + S^{\parallel} \left(\text{div} \mathbf{u}_{\perp} + 4 \frac{\partial u_{\parallel}}{\partial z} - \frac{u_{\parallel}}{B} \frac{\partial B}{\partial z} \right) + 3p_{\parallel} \frac{\partial}{\partial z} \left(\frac{p_{\parallel}}{\rho} \right) = \frac{\delta S^{\parallel}}{\delta t} \quad (4.7.7)$$

$$\frac{dS^{\perp}}{dt} + 2S^{\perp} \left(\text{div} \mathbf{u}_{\perp} + \frac{\partial u_{\parallel}}{\partial z} - \frac{u_{\parallel}}{B} \frac{\partial B}{\partial z} \right) + p_{\parallel} \frac{\partial}{\partial z} \left(\frac{p_{\perp}}{\rho} \right) - \frac{p_{\perp}}{\rho B} (p_{\parallel} - p_{\perp}) \frac{\partial B}{\partial z} = \frac{\delta S^{\perp}}{\delta t}, \quad (4.7.8)$$

where $p_{\parallel,\perp} = nT_{\parallel,\perp}$ is pressure; $d/dt = \partial/\partial t + u_{\parallel}\partial/\partial z + (\mathbf{u}_{\perp}\nabla)$ is the total (Lagrangian) derivative; $\mathbf{u} = \mathbf{u}_{\parallel} + \mathbf{u}_{\perp}$ is the macroscopic velocity of particles of a given specie; $\mathbf{u}_{\parallel}$ and $\mathbf{u}_{\perp}$ are the longitudinal and transverse velocity components; B is the magnitude of magnetic flux density; and $S^{\parallel}$ and $S^{\perp}$ are the longitudinal and transverse components of the heat flux vectors.

The terms on the right-hand sides of (4.7.5)–(4.7.8) describe the variations of macroscopic parameters due to collisions of different types and are expressible through the respective velocity moments of the collision integrals. One of the principal difficulties in deriving the closed set of hydrodynamic equations arises from calculating the collision terms, because the colliding-particle distribution functions as well as the interaction cross-sections have to be prescribed. Besides these difficulties, pure methodological difficulties also arise when solving the sextuple collision integral. In most cases, the integration can be performed only after imposing restrictions on the plasma parameters.

Pioneering calculations of the collision terms in the 16-momentum approximation were carried out for the Coulomb interactions of charged particles (Chodura et al. 1969; Chodura and Pohl 1971). The potential $U \propto 1/r^{\alpha}$ was applied to

examining particle interactions in Demars and Schunk (1979) and Barakat and Schunk (1981). The collision terms for ion scattering from neutrals (polarization interaction and resonance charge exchange) were calculated in Demars and Schunk (1979), Barakat and Schunk (1981), Konikov and Khazanov (1982), and Oraevskii et al. (1985). Collisions of electrons with the basic components of neutral atmosphere were examined in Konikov et al. (1980), Konikov and Khazanov (1981, 1982, 1983), and Oraevskii et al. (1985). Konikov (1990) calculated the contribution to the 16-moment equations from the resonance interactions of thermal electrons with ion-cyclotron waves in the Earth's outer plasmasphere. Gombosi and Rasmussen (1991) calculated the collision terms of the anisotropic-hydrodynamics equations in 20-moment approximation for the BGK form of the collision integral.

Given the distributions of number density and of macroscopic velocity, (4.7.5)–(4.7.8) are used to simulate the thermal conditions for the various plasma components. Equations (4.7.5) and (4.7.6) were previously used to study the formation of the electron and ion temperature anisotropy in terrestrial plasma and to propose a sufficiently simple method for estimating the anisotropy (Konikov and Khazanov 1982; Oraevskii et al. 1985). In addition, some feasible anisotropization mechanisms of the thermal plasma energy distribution in the Earth's ionosphere and plasmasphere were explored. For example, the asymmetry of (4.7.5) and (4.7.6) with respect to transposition of $S^{\parallel}$ and $S^{\perp}$ therein corresponds to temperature anisotropy formation in case heat fluxes occur in the plasma (see Introduction). In turn, the difference between heat fluxes $S^{\perp}$ and $S^{\parallel}$ arises from the magnetic-mirror effect in an inhomogeneous magnetic field, which is described by the last term in the left-hand side of (4.7.8) and is the result of forces pushing the Larmor gyration of particles away to the region of a weaker magnetic field.

Equations (4.7.5)–(4.7.8) may also be presented in a somewhat different form (Barakat and Schunk 1982a; Oraevskii et al. 1985). Assuming that the transverse velocity component coincides with the electric drift velocity, the equations take the form

$$\frac{d}{dt} \left(\frac{p_{\parallel} B^2}{\rho^3} \right) = - \left(\frac{B}{\rho} \right)^3 \frac{\partial}{\partial z} \left(\frac{S^{\parallel}}{B} \right) - \frac{S^{\perp}}{\rho^3} \frac{\partial B^2}{\partial z} + \frac{B^2}{\rho^3} \frac{\delta p_{\parallel}}{\delta t} \quad (4.7.5a)$$

$$\frac{d}{dt} \left(\frac{p_{\perp}}{\rho B} \right) = - \frac{B}{\rho} \frac{\partial}{\partial z} \left(\frac{S^{\perp}}{B^2} \right) + \frac{1}{\rho B} \frac{\delta p_{\perp}}{\delta t} \quad (4.7.6a)$$

$$\frac{d}{dt} \left(\frac{S^{\parallel} B^3}{\rho^4} \right) = - \frac{3}{2} \left(\frac{B}{\rho} \right)^3 \frac{\partial}{\partial z} \left(\frac{p_{\parallel}}{\rho} \right)^2 + \frac{B^3}{\rho^4} \frac{\delta S^{\parallel}}{\delta t} \quad (4.7.7a)$$

$$\frac{d}{dt} \left(\frac{S^{\perp}}{\rho^2} \right) = - \frac{p_{\parallel}}{\rho^2} \frac{\partial}{\partial z} \frac{p_{\perp}}{\rho} - \frac{p_{\perp} (p_{\perp} - p_{\parallel})}{\rho^3 B} \frac{\partial B}{\partial z} + \frac{1}{\rho^2} \frac{\delta S^{\perp}}{\delta t}. \quad (4.7.8a)$$

If the contributions from heat fluxes and from collision processes in (4.7.5a) and (4.7.6a) are disregarded, we obtain the well-known CGL (Chew–Goldberger–Low) equations (Chew et al. 1956):

$$\frac{d}{dt} \left(\frac{p_{\parallel} B^2}{\rho^3} \right) = 0, \quad \frac{d}{dt} \left(\frac{p_{\perp}}{\rho B} \right) = 0,$$

which describe the laws governing the variations of the trajectories in terms of the adiabatic approximation.

The expressions for the collision terms in the anisotropic hydrodynamic equations are very complex; therefore, only the collision terms $\delta S^{\parallel}/\delta t$ and $\delta S^{\perp}/\delta t$, which are used in deriving the heat conductivities, are treated here.

At altitudes where Coulomb interactions are dominant, the collision terms are given by

$$\frac{\delta S_r^{\parallel}}{\delta t} = \sum_t v_{rt} \left(R_{rt}^{\parallel} S_r^{\parallel} + R_{rt}^{\perp} S_r^{\perp} \right) + \sum_{t \neq r} v_{rt} \left(P_{rt}^{\parallel} S_t^{\parallel} + P_{rt}^{\perp} S_t^{\perp} \right) \quad (4.7.9)$$

$$\frac{\delta S_r^{\perp}}{\delta t} = \sum_t v_{rt} \left(C_{rt}^{\parallel} S_r^{\parallel} + C_{rt}^{\perp} S_r^{\perp} \right) + \sum_{t \neq r} v_{rt} \left(D_{rt}^{\parallel} S_t^{\parallel} + D_{rt}^{\perp} S_t^{\perp} \right) \quad (4.7.10)$$

where v_{rt} is the Coulomb collision frequency. The factors R , P , C , and D are complicated functions of the masses, temperatures, and temperature anisotropies of colliding particles (see Chodura and Pohl 1971; Oraevskii et al. 1985 for more details). For simplicity, the terms containing the differences in the macroscopic velocities of various particle species and describing the thermoelectrical effect are omitted in (4.7.9) and (4.7.10).

The collision terms in (4.7.9) and (4.7.10) depend on the heat fluxes, which correspond to a given degree of freedom for the interacting particle species, as well as on the heat fluxes corresponding to another degree of freedom. This is because both energy exchange (energy transfer) between particles and energy redistribution among various degrees of freedom occur during collisions. The above-mentioned structure of the collision terms (4.7.9) and (4.7.10) arises from a mathematical point of view from the dependence of the integrand of the collision term on the distribution functions of interacting particle species, which, in turn, depend on heat fluxes. If the model collision integral of the BGK form is used, the collision terms will only contain the heat flux of a given particle specie and a given degree of freedom (Gombosi and Rasmussen 1991). This is because the BGK collision integral describes the collisions approximately and fails to reflect all of the features of the collision process. Because the collision term of the BGK form is proportional to $(f_0 - f)$, where f_0 is an equilibrium distribution function, the form of the distribution function f has yet to be specified. This makes the calculations much

easier, resulting in a simplification of the expressions for the collision terms in the hydrodynamic equations.

For simplicity, let the thermal plasma be treated as if composed of only one ion specie. Because of a great difference in the particle masses, the collision terms (4.7.9) and (4.7.10) for the electron and ion components become separated in the following manner:

$$\frac{\delta S_e^{\parallel}}{\delta t} = v_e \left(R_e^{\parallel} S_e^{\parallel} + R_e^{\perp} S_e^{\perp} \right) \quad (4.7.11)$$

$$\frac{\delta S_e^{\perp}}{\delta t} = v_e \left(C_e^{\parallel} S_e^{\parallel} + C_e^{\perp} S_e^{\perp} \right) \quad (4.7.12)$$

$$\frac{\delta S_i^{\parallel}}{\delta t} = v_i \left(R_i^{\parallel} S_i^{\parallel} + R_i^{\perp} S_i^{\perp} \right) \quad (4.7.13)$$

$$\frac{\delta S_i^{\perp}}{\delta t} = v_i \left(C_i^{\parallel} S_i^{\parallel} + C_i^{\perp} S_i^{\perp} \right) \quad (4.7.14)$$

The expressions for the factors R and C and the collision frequency are presented in Appendix C. Allowance for the many-component character of the ion composition gives rise to additional terms in relations (4.7.13) and (4.7.14), which address the influence on the energetics of a given ion specie produced by heat fluxes of other ion species.

Equations (4.7.7) and (4.7.8) can be greatly simplified if large spatial and temporal scales for variations in heat fluxes, macroscopic velocity, and magnetic field are assumed. In this case, the transition from (4.7.7) and (4.7.8) to simple linear relations of heat fluxes to temperature gradients through heat conductivities becomes feasible. Substituting (4.7.11)–(4.7.14) in (4.7.7) and (4.7.8) results in sets of algebraic equations that can be solved to obtain $S_e^{\parallel,\perp}$ and $S_i^{\parallel,\perp}$ (see also Chodura and Pohl 1971; Oraevskii et al. 1985), given by

$$S_r^{\parallel} = -\chi_{r\parallel}^{\parallel} \frac{\partial T_{r\parallel}}{\partial z} - \chi_{r\perp}^{\parallel} \frac{\partial T_{r\perp}}{\partial z}, \quad (4.7.15)$$

$$S_r^{\perp} = -\chi_{r\perp}^{\perp} \frac{\partial T_{r\perp}}{\partial z} - \chi_{r\parallel}^{\perp} \frac{\partial T_{r\parallel}}{\partial z}, \quad (4.7.16)$$

where $r = e, i$ and c is the heat (thermal) conductivity. It should be noted that, because the terms containing the difference in the macroscopic velocities of plasma components are omitted in (4.7.9) and (4.7.10), (4.7.15) and (4.7.16) do not include the respective terms describing the thermoelectric effect.

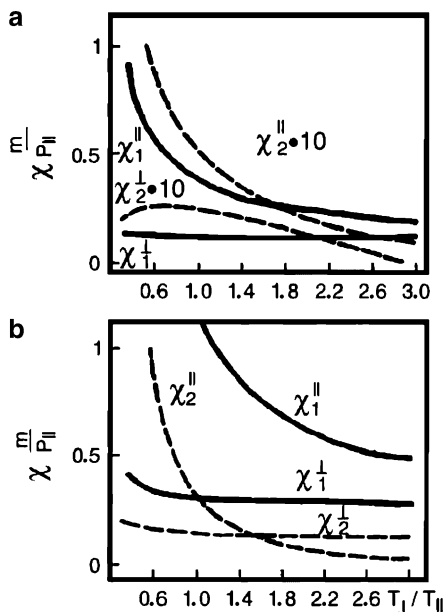
The heat conductances in (4.7.15) and (4.7.16) are of the form

$$\begin{aligned}
 \chi_{r\parallel}^{\parallel} &= -3p_{r\parallel}C_r^{\perp}/\Delta_r, \\
 \chi_{r\perp}^{\parallel} &= p_{r\parallel}R_r^{\perp}/\Delta_r, \\
 \chi_{r\perp}^{\perp} &= -p_{r\parallel}R_r^{\parallel}/\Delta_r, \\
 \chi_{r\parallel}^{\perp} &= 3p_{r\parallel}C_r^{\parallel}/\Delta_r.
 \end{aligned} \tag{4.7.17}$$

Equations (4.7.15) and (4.7.16) show that the heat fluxes are defined not only by the gradient in the temperature corresponding to a given degree of freedom (“intrinsic” temperature), but also by the spatial variations in the temperature characterizing the energy distribution for another degree of freedom (“extrinsic” temperature). In this respect, (4.7.15) and (4.7.16) are similar to the expressions for ion heat fluxes in a many-component plasma with isotropic energy distribution, where the heat flux of a given ion specie depends on the gradient of the intrinsic temperature and on the gradients of other ion components (Schunk 1977). Thus, a charged component, exhibiting a temperature anisotropy, may be treated formally as if composed of two different components whose temperatures are $T_{\parallel}$ and $T_{\perp}$, respectively. If the plasma contains two ion species of different temperatures, their heat fluxes will involve four terms each, rather than two terms each, as in (4.7.15) and (4.7.16), to include the additional temperature gradients of the other ion specie. As the number of ion components increases, the number of respective terms will rise accordingly.

Figure 4.1 shows the electron and ion heat conductivities, calculated by formula (4.7.17), versus the ratio $a = T_{\perp}/T_{\parallel}$. The general trend in the heat conductivities to rise with increasing longitudinal temperature (with decreasing value of a) should be noted. This behavior is the result of an increasing field-aligned extension of the distribution function as the $T_{\parallel}$ value rises and a resulting enhancement in the longitudinal transfer of thermal energy. In this case, as seen from (4.7.3) and (4.7.4), the $S^{\parallel}$ value rises more rapidly than the $S^{\perp}$ value and consequently the heat conductivities $\chi_{\parallel,\perp}^{\parallel}$ increase more rapidly than $\chi_{\parallel,\perp}^{\perp}$ with rising longitudinal temperature. The heat conductivities $\chi_{\parallel}^{\parallel}$ and $\chi_{\perp}^{\perp}$ enter into the expressions for the heat fluxes $S^{\parallel}$ and $S^{\perp}$ as multipliers of the gradients in $T_{\parallel}$ and $T_{\perp}$, respectively. These temperatures can be thought of as the “intrinsic” temperatures for their corresponding degrees of freedom. The “intrinsic” conductivities are in excess of the $\chi_{\perp}^{\parallel}$ and $\chi_{\parallel}^{\perp}$ values, which define the contributions from other degrees of freedom or the “extrinsic” temperatures. The differences prove to be substantial, and may reach an order of magnitude, in the case of electrons. Subject to the condition that $\partial T_{e\parallel}/\partial z \cong \partial T_{e\perp}/\partial z$, the terms with $\chi_{\perp}^{\parallel}$ and $\chi_{\parallel}^{\perp}$, containing the gradients of the “extrinsic” temperatures, may be disregarded in the expression for the electron heat flux. As seen from Fig. 4.1b, the above-mentioned differences are not significant in the case of ions, so all terms in (4.7.15) and (4.7.16) must be retained.

Fig. 4.1 Heat conductivities (in relative units) versus the anisotropy parameter $a = T_{\perp}/T_{\parallel}$ for electrons (a) and ions (b)



4.8 Equations of Anisotropic Hydrodynamics for Modeling the Ionosphere–Magnetosphere Plasma

The system of equations of anisotropic hydrodynamics in the 16-moment approximation (4.3.1)–(4.3.7) is also simplified under certain assumptions with respect to the coordinate dependence of macroscopic parameters. To illustrate this, let us consider its form in conformity with the problem of modeling the ionosphere–magnetospheric transport in the near terrestrial plasma.

First, let us consider the case of longitudinal macroscopic movement of the plasma: $\mathbf{u} = u\mathbf{b}$. In this case, plasma parameters will depend only on time t (or longitude) and the coordinate along the line of force of the geomagnetic field, s . Thus, the problem of modeling the real three-dimensional movement of the plasma is brought to two-dimensional one in the variables s and t by way of choosing one of the coordinates along the geomagnetic field. Such a formulation is used fairly often when modeling ionosphere–plasmasphere interactions (see Oraevskii et al. 1985 and references therein).

Omitting the index denoting the sort of particles, let us write the equation of continuity (4.3.1) in the form

$$\frac{\partial n}{\partial t} + \frac{1}{\sigma} \frac{\partial}{\partial s} (\sigma n u) = q - \beta n, \quad (4.8.1)$$

where $q - \beta n = (1/m)J^{(0)}$, q is the term describing the source of charged particles at the expense of the ionization process and chemical reactions, the term βn

describes the losses with the effective recombination coefficient β , and $\sigma \propto 1/B$ is the cross-section of the magnetic tube of force.

Introducing the force of gravity, $\mathbf{F} = m\mathbf{g}$, and neglecting the effects of viscosity, the momentum equation (4.3.2) after projecting on the direction of the magnetic field is

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial s} \right) + \frac{\partial p_{\parallel}}{\partial s} + (p_{\parallel} - p_{\perp}) \frac{1}{\sigma} \frac{\partial \sigma}{\partial s} - \rho \left(g_{\parallel} + \frac{e}{m} E_{\parallel} \right) = J_{\parallel}^{(1)}. \quad (4.8.2)$$

Using the condition of the plasma, one may neglect the lateral (transverse) components of thermal fluxes in (4.3.3) and (4.3.4). Additionally, if we pass from pressures to temperatures in these equations using the continuity equation (4.3.1), we get

$$n \left(\frac{\partial T_{\parallel}}{\partial t} + u \frac{\partial T_{\parallel}}{\partial s} \right) + T_{\parallel} \left[n \left(2 \frac{\partial u}{\partial s} - \beta \right) + q \right] + \frac{\partial S_{\parallel}^{\parallel}}{\partial s} + (S_{\parallel}^{\parallel} - 2S_{\parallel}^{\perp}) \frac{1}{\sigma} \frac{\partial \sigma}{\partial s} = J_{\parallel}^{(2)}, \quad (4.8.3)$$

$$n \left(\frac{\partial T_{\perp}}{\partial t} + u \frac{\partial T_{\perp}}{\partial s} \right) + T_{\perp} \left[n \left(\frac{u}{\sigma} \frac{\partial \sigma}{\partial s} - \beta \right) + q \right] + \frac{\partial S_{\parallel}^{\perp}}{\partial s} + 2S_{\parallel}^{\perp} \frac{1}{\sigma} \frac{\partial \sigma}{\partial s} = J_{\perp}^{(2)}. \quad (4.8.4)$$

For longitudinal components of thermal fluxes, (4.3.5), (4.3.6), or linear transport relationships (Chodura and Pohl 1971; Oraevskii et al. 1985) may be used.

In the presence of a transverse component, the velocity of the plasma movement is $\mathbf{u} = \mathbf{u}_{\parallel} + \mathbf{u}_{\perp}$. Let us assume now that the component $\mathbf{u}_{\perp}$ is conditioned by external factors (e.g., the drift caused by the electric field of the magnetospheric convection) and is a known coordinate function.

Then the continuity equation takes the form

$$\frac{\partial n}{\partial t^*} + \frac{1}{\sigma} \frac{\partial}{\partial s} (\sigma n u_{\parallel}) = q - \beta^* n, \quad (4.8.1a)$$

where $\partial/\partial t^* = \partial/\partial t + (\mathbf{u}_{\perp} \nabla)_{\perp}$, t^* is the time in the material coordinate system moving at a speed $\vec{u}_{\perp}$ along the trajectories of a transverse movement, and $\beta^* = \beta + \text{div} \mathbf{u}_{\perp}$ is the modified recombination coefficient. The transverse transport, depending on the sign of the term, $\text{div} \mathbf{u}_{\perp}$, will cause either increase or decrease of concentration in the moving element of the plasma volume, acting like a source or a flow of particles.

The equation of movement is

$$\rho \left(\frac{\partial u_{\parallel}}{\partial t^*} + u_{\parallel} \frac{\partial u_{\parallel}}{\partial s} \right) + \frac{\partial p_{\parallel}}{\partial s} + (p_{\parallel} - p_{\perp}) \frac{1}{\sigma} \frac{\partial \sigma}{\partial s} - \rho \left(g_{\parallel}^* + \frac{e}{m} E_{\parallel} \right) = J_{\parallel}^{(1)*} \quad (4.8.2a)$$

Here

$$g_{\parallel}^* = g_{\parallel} - \mathbf{b} \frac{\partial \mathbf{u}_{\perp}}{\partial t^*}, \quad J_{\parallel}^{(1)*} = J_{\parallel}^{(1)} - \rho \mathbf{u}_{\parallel} \frac{\partial \mathbf{u}_{\perp}}{\partial s}.$$

The additional term in $g_{\parallel}^*$ describes the additional acceleration to which plasma is subjected due to the change in the transverse velocity component. Modification of friction is conditioned by a breaking (or accelerating) effect, determined by the nature of change of $\mathbf{u}_{\perp}$ along $\mathbf{u}_{\parallel}$. By its structure, this term is similar (analogous) to the force of friction $J_{\parallel}^{(1)} \propto -\rho v u_{\parallel}$, where the value $\mathbf{b} \partial \mathbf{u}_{\perp} / \partial s$ plays the role of “the frequency of collisions.”

The thermal energy transport equations are:

$$n \left(\frac{\partial T_{\parallel}}{\partial t^*} + u_{\parallel} \frac{\partial T_{\parallel}}{\partial s} \right) + T_{\parallel} \left[n \left(2 \frac{\partial u_{\parallel}}{\partial s} - \beta \right) + q \right] + \frac{\partial S_{\parallel}^{\parallel}}{\partial s} + (S_{\parallel}^{\parallel} - 2S_{\parallel}^{\perp}) \frac{1}{\sigma} \frac{\partial \sigma}{\partial s} = J_{\parallel}^{(2)}, \quad (4.8.3a)$$

$$n \left(\frac{\partial T_{\perp}}{\partial t^*} + u_{\parallel} \frac{\partial T_{\perp}}{\partial s} \right) + T_{\perp} \left[n \left(\operatorname{div} \mathbf{u}_{\perp} + \frac{u_{\parallel}}{\sigma} \frac{\partial \sigma}{\partial s} - \beta \right) + q \right] + \frac{\partial S_{\parallel}^{\perp}}{\partial s} + 2S_{\parallel}^{\perp} \frac{1}{\sigma} \frac{\partial \sigma}{\partial s} = J_{\perp}^{(2)}. \quad (4.8.4a)$$

It is seen that adiabatic effects of the transverse plasma transport (the term with $\operatorname{div} \mathbf{u}_{\perp}$ in (4.8.4.a)) make a contribution only into the transverse degree of freedom, being, thus, one of the causes of the formation of anisotropy of the thermal plasma energetic distribution.

Equations (4.8.1a)–(4.8.4a) obtained by the performed changes have been reduced to two-dimensional equations (with the known distribution $\mathbf{u}_{\perp}$) and, in their structure, coincide with (4.8.1)–(4.8.4). In both cases, this allows one to use practically the same methods of numerical integration of equations when modeling the near-Earth plasma.

The results obtained in this chapter have been used in a previous papers by Khazanov et al. (1984, 1994, 1996), Konikov et al. (1989), and Gorbachev et al. (1992) in order to model the different aspects of ionosphere–magnetosphere coupling processes. The formalism developed here also maybe useful for the hydrodynamic description of the solar wind and other planetary ionospheres.

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Chapter 5

Transport of Superthermal Electrons: General Analysis

Superthermal electrons are one of the major energy players in the inner magnetosphere, and are responsible for the formation of self-consistent electric potentials in some space plasma regions. Numerous processes are involved in the determination of their distribution function, and only a kinetic approach provides the proper tool to treat this component of the inner magnetosphere. Because of the relative complexity of the kinetic equation solution, analytical investigations of some simplified kinetic problems are very useful because they help us gain physical insight into how the system responds to various physical processes and external boundary conditions. Solutions to these simplified problems also provide us a convenient method to test the validity of complicated numerical models where superthermal electrons are involved.

5.1 Exact Solution of the Collisionless Drift Kinetic Equation

5.1.1 Introduction

Modeling the behavior of a plasma through the use of the stationary collisionless drift kinetic equation (3.9.1) (the Vlasov equation) is a commonly used technique in plasma physics, especially when the primary factors influencing the changes in the distribution function are potential energy structures. It has broad applications across many disciplines: in near-Earth for describing the hot plasma along inner magnetospheric field lines, high-latitude ionospheric outflows, and other situations involving a field-aligned potential; for rotating planetary magnetospheres to determine the effects of the centrifugal force; in astrophysics for describing objects such as pulsars; for modeling the solar wind acceleration region; and in laboratory plasmas to describe collisionless expansion into a vacuum.

The collisionless kinetic description has been used for many scenarios in near-Earth space. Models of this type have been used for the description of the hot

plasma along an inner magnetospheric field line (Alfvén and Fälthammar 1963; Persson 1963; Eviatar et al. 1964; Whipple 1977; Chiu and Schulz 1978; Serizawa and Sato 1984; Stasiewicz 1985; Miller and Khazanov 1993; Olsen et al. 1994), for planetary magnetospheres (Lemaire 1976; Huang and Birmingham 1992), and for high-latitude outflow and precipitation (Lemaire and Scherer 1970; Knight 1973; Barakat and Schunk 1984; Washimi and Katanuma 1986; Pierrard and Lemaire 1996; Khazanov et al. 1997; Wilson et al. 1997). They all have the same basic approach of knowing the distribution function at some reference altitude and then mapping this distribution along the field line according to a given or calculated potential energy structure.

Alfvén and Fälthammar (1963) were the first to study the generation of an electrostatic potential structure due to trapped particles in a magnetic mirror (magnetic flux tube). They demonstrated that an anisotropic collisionless plasma in a magnetic field can be in quasineutral equilibrium without a parallel electric field only if the magnetic field is homogeneous or if the pitch-angle anisotropy is the same for both electrons and ions. They considered a single particle approach and determined that the electrostatic potential energy difference between the ionosphere and equator is of the order of the magnetic field mirror ratio multiplied by the mean perpendicular particle energy. For a typical mirror ratio ($\gamma \approx 40$ for an L shell of 4 and an altitude of 2,000 km), the potential energy determined by Alfvén and Fälthammar (1963) can be well in excess of the total kinetic energy of the trapped plasma that creates this potential difference.

In steady state, Persson (1963) evaluated the particle distribution function using Liouville's theorem and confirmed the results of Alfvén and Fälthammar (1963). However, Persson (1963) never computed the total electrostatic potential necessary for quasineutrality when the electrons and ions have different anisotropies. Whipple (1977) considered a generalization of the work by Alfvén and Fälthammar (1963). The integral formalism of Whipple (1977), using a delta function for the trapped plasma at the equator, reproduces the results of Alfvén and Fälthammar (1963); however, the electric potential, due to a variety of trapped distributions (in the absence of other plasma sources), was not investigated and compared with the Alfvén and Fälthammar (1963) model. Chiu and Schulz (1978) considered a self-consistent treatment for the electrostatic potential; however, their analysis included ionospheric plasma extracted upward by parallel electrostatic fields and backscattered electrons. Chiu and Schulz (1978) state that the Alfvén and Fälthammar (1963) model would produce unrealistically large potential drops (of the order of several hundred kiloelectron volts), but they never discuss the validity of the Alfvén and Fälthammar (1963) model. They do, however, mention that the large electric potentials would decrease in the presence of ionospheric plasma. Chiu and Schulz (1978) found potential drops of the order of the plasma kinetic energy. They derived accessibility criteria that quantify the limits of validity of their results, and the smoothness imposed by these constraints prohibits double layers or electrostatic shocks to form. Also, the magnitude of the electrostatic potential in the absence of ionospheric plasma and backscattered electrons is unclear from their analysis. This last issue has been addressed by Miller and Khazanov (1993).

For ion outflows, the first description was the polar exosphere model of Lemaire and Scherer (1970, 1971, 1972), which used a Maxwellian distribution function instead of a single particle trajectory, allowing for infinite mirror points for the incoming (precipitating) particles. They present a full description of the possible trajectories of particles (ballistic, escaping, trapped, etc.) and calculate the densities, particle and energy fluxes, and pressures for each of these classes along the field line. This model has continued to be used since this time and was recently extended for the solution of a Lorentzian (κ) distribution function instead of a Maxwellian distribution function (Pierrard and Lemaire 1996).

A similar approach was used by Knight (1973), who calculated the field-aligned potential from a zero current assumption. He also limits the solution to a monotonic potential distribution from the plasma sheet to the ionosphere, and the resulting equations only depend on the beginning reference point for that population and the local spatial point. He also gives a similar analysis of the various regions of existence in velocity space for different populations along the field line. An important development here and by Lemaire and Scherer (1973, 1974) was the use of different distribution functions for the various regions, allowing for the self-consistent calculation of widely disparate plasma populations, such as the precipitating plasma sheet particles and the ionospheric outflow. Results similar to Knight (1973), except for a partially filled loss cone distribution for the trapped population, were obtained by Stasiewicz (1985), and the result readily reduces to Knight's formula.

The model of Olsen et al. (1994), which calculated the generation of a self-consistent potential due to warm plasmaspheric plasma, took into account only the local spatial point in defining the filled integration region, thus filling in any low-energy holes in the distribution function due to drops in the potential energy. As mentioned in that study, this implies that some scattering mechanism is filling in these holes. While it might be reasonable to assume that such holes will be filled in for low-energy populations (up to a few electron volts temperature), the scattering is inconsistent with the use of the Vlasov equation. A similar assumption was probably made by Huang and Birmingham (1992), because the formulation is the same except for a more generalized potential energy. Similarly, the Heaviside step function introduced in the distribution function by Miller and Khazanov (1993) to keep the total energy greater than zero is also inconsistent with the Vlasov equation because it is the kinetic energy of the particle that has this constraint.

Two models (Khazanov et al. 1997; Wilson et al. 1997) combine a Vlasov equation solution for the ions and photoelectrons with a hydrodynamic description for the thermal electrons for calculating polar wind outflows. Although it is a mixture of collisionless and collision-dominated treatments, this is applicable here because the ionospheric thermal electrons are much more collisional than the outflowing ions or hot populations. While both models use a kinetic formulation similar to that of Lemaire and Scherer (1970), their hydrodynamic approaches are different. For the thermal electrons, Khazanov et al. (1997) use the momentum and energy equations for the density and temperature and a zero current assumption to find the velocity along the field line, while Wilson et al. (1997) use a Boltzmann

distribution for the density, which assumes no velocity and isothermal temperature, and then use a zero current condition at some altitude to generate a large potential barrier, reducing the large upward photoelectron flux.

A common trait of the models mentioned above is that they all depend only on the final position of the particles in space. The assumed accessibility of these particles to this point introduces a constraint on the functional dependence of the potential energy with respect to the magnetic field, which was recognized and discussed for the first time in detail by Chiu and Schulz (1978). By considering only the electrostatic potential φ , they found two constraints on its derivatives with respect to the magnetic field B ,

$$d\varphi/dB > 0 \quad (5.1.1)$$

$$d^2\varphi/dB^2 \leq 0. \quad (5.1.2)$$

These constraints must be satisfied when the calculation of the moments of the velocity distribution at a point along the field line only includes the parameters (such as potential energy and magnetic field strength) at the endpoints of the simulation region and those for the local spatial point. It is quite possible, however, to violate these constraints during any collisionless plasma scenario where this type of a solution is appropriate.

The purpose of this section is to develop a generalization of these previous studies to allow for the calculation of a collisionless plasma along a magnetic field line in the presence of an arbitrary potential energy structure (Khazanov et al. 1998; Liemohn and Khazanov 1998).

5.1.2 Moments of the Velocity Distribution Function

The collisionless nature of the plasma means that the distribution function depends only on the two constants of motion (Whipple 1977), namely the total energy E ,

$$E = \frac{mv_{\parallel}^2}{2} + \frac{mv_{\perp}^2}{2} + \Pi \quad (5.1.3)$$

and the first adiabatic invariant μ ,

$$\mu = \frac{mv_{\perp}^2}{2B}, \quad (5.1.4)$$

where μ is the plasma particle mass, $v_{\parallel}$ and $v_{\perp}$ are the particle velocities parallel and perpendicular to B , and Π is the total potential energy. This choice is quite beneficial because it removes the spatial dependence in the distribution function f

and only the region of integration in E - μ velocity space changes. The zeroth, first, second, and third moments of the velocity distribution function transformed to these variables from their standard definitions (Lemaire and Scherer 1971) and are written as

$$n = \frac{\sqrt{2}\pi B}{m^{3/2}} \iint \frac{f(E, \mu)}{\sqrt{E - \mu B - \Pi}} dE d\mu \quad (5.1.5a)$$

$$\phi = \frac{2\pi B}{m^2} \iint f(E, \mu) dE d\mu \quad (5.1.5b)$$

$$P_{\parallel} = \left(\frac{2}{m}\right)^{3/2} \pi B \iint \sqrt{E - \mu B - \Pi} f(E, \mu) dE d\mu \quad (5.1.5c)$$

$$P_{\perp} = \frac{\sqrt{2}\pi B^2}{m^{3/2}} \iint \frac{\mu f(E, \mu)}{\sqrt{E - \mu B - \Pi}} dE d\mu \quad (5.1.5d)$$

$$\varepsilon_{\parallel} = \frac{2\pi B}{m^2} \iint (E - \Pi) f(E, \mu) dE d\mu, \quad (5.1.5e)$$

where (5.1.5) has already been integrated over the velocity azimuthal angle.

It should be noted that Π is an arbitrary potential energy structure, and can include an electrostatic potential energy of the form $e\varphi$, a gravitational potential energy of the form $m\psi_g$, or other potential energies. Also, the squaring of $v_{\parallel}$ in (5.1.3) obscures the hemispherical direction of the distribution function. For the integration, then, f must be split into hemispherical descriptions f^+ and f^- . The sign of $v_{\parallel}$ must then be taken into account in (5.1.5) such that the functions are subtracted in the flux calculation and summed in the others.

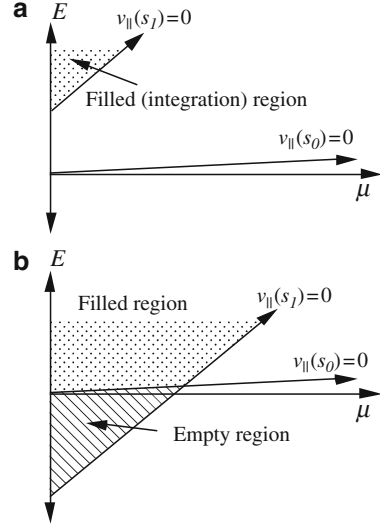
The function $f(E, \mu)$ is then defined at the reference altitude and can be mapped to any point along the field line. The problem that should be solved is the calculation of the region of the E - μ plane filled by particles for an arbitrary point along the field line. For a given plasma in a given magnetic field, this region is determined by the location of the source of the particles (reference point s_o on the magnetic field line), and by the conservation laws (5.1.3) and (5.1.4),

$$v_{\parallel} \geq 0; \quad E - \mu B - \Pi \geq 0. \quad (5.1.6)$$

We will restrict our analysis to the case of a one-to-one relation between distance and field strength, i.e., the magnetic field is a monotonic function of s .

Figure 5.1 shows the regions of integration for (a) decelerated and (b) accelerated particles when the reference point s_o is located at $B = B_{min}$. While the region of integration for the decelerated species is defined simply by the $v_{\parallel} = 0$ line for the local spatial point, the accelerated species requires two lines to define the region. The space below the $s_o v_{\parallel} = 0$ line and above the $s_1 v_{\parallel} = 0$ line is an empty region in

Fig. 5.1 Regions of integration in E - μ velocity space for (a) decelerated and (b) accelerated plasma populations at s_1 for a reference point at $B = B_{min}$. The dotted areas are the regions of integration, the striped areas are empty regions in velocity space at s_1 . The distribution function is defined above the $s_o v_{||} = 0$ line, with no particles below it



velocity space at s_1 , created by the acceleration from the potential drop the particles have experienced. The region above the s_o line yet below the s_1 line includes particles that are magnetically mirrored between s_o and s_1 . For anisotropic trapped plasma in the magnetosphere, this situation corresponds to s_o at the equatorial plane with the decelerated species being the ions and the accelerated species being the anisotropic electrons.

5.1.3 The Generalized Approach

Because the previous collisionless kinetic models include only the calculation of the local and reference point $v_{||} = 0$ lines in their definition of the filled region of velocity space, constraints must be imposed on the potential energy to ensure that the solution is valid. These were defined above for the electrostatic potential during magnetospheric precipitation, given in (5.1.1) and (5.1.2). Here, these criteria will be extended for an arbitrary potential energy Π . This will be analyzed using three neighboring points: s_{i-1} , s_i , and s_{i+1} , where i is the spatial grid index. The boundary lines for these points are found from (5.1.6),

$$E = \mu B_\alpha + \Pi_\alpha; \quad \alpha = i-1, i, i+1. \quad (5.1.7)$$

Let us begin by supposing that s_{i-1} is the reference point and that $\Pi_{i-1} = 0$, but the argument is easily generalized for any three points in the spatial domain. The region above the line $E = \mu B_{i-1} + \Pi_{i-1}$ is therefore filled at s_{i-1} .

In order for the region of integration to be defined by only the $v_{||} = 0$ lines from the reference point and the local point, there is just one necessary and sufficient condition that must be met: if the $s_{i+1}v_{||} = 0$ line is ever above the reference point (here, s_{i-1}) $v_{||} = 0$ line for $\mu > 0$, then it must also be above the $s_i v_{||} = 0$ line for these μ values. If the potential energy distribution is nonmonotonic, then this condition will not be met for the decelerated species. Because magnetic field and distance are one-to-one, this requirement can be written as follows: the first derivative of the potential energy with respect to the magnetic field must be monotonic throughout the spatial range.

Beginning with this constraint of monotonicity, a second criteria can be derived by considering the intersection of the $v_{||} = 0$ lines. The point of intersection for the lines of spatial points s and $s + ds$ (any two adjacent spatial points) is

$$\mu^* = -\frac{d\Pi}{dB}; \quad E^* = \Pi - B \frac{d\Pi}{dB}. \quad (5.1.8)$$

If $d\Pi/dB > 0$, μ^* is negative and the lines do not cross in the region of definition ($\mu > 0$). This is shown in Fig. 5.2a for the case when the minimum magnetic field is at the reference point (e.g., the magnetospheric trap). When this derivative is positive, the only criteria on the potential energy for validity of the previous models is monotonicity.

If $d\Pi/dB < 0$, then the lines intersect at some $\mu^* > 0$. If the points of intersection between lines $i - 1$ and i ($\mu_{i-1,i}^*$) and the intersection between i and $i + 1$ ($\mu_{i,i+1}^*$) are arranged so that $\mu_{i-1,i}^* < \mu_{i,i+1}^*$ (i.e., $\mu^*(s) < \mu^*(s + ds)$) and $dB/ds < 0$, then $d^2\Pi/dB^2 > 0$. Similarly, if the μ^* s are arranged so that $\mu_{i-1,i}^* > \mu_{i,i+1}^*$ (i.e.,

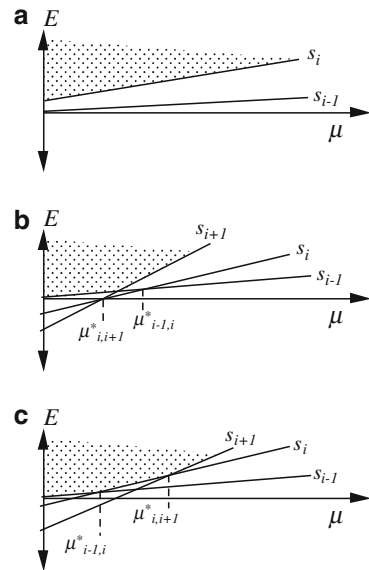


Fig. 5.2 Schematics of the definition of the filled region of velocity space (dotted area) at s_{i+1} based on three neighboring spatial points. This region is shown for (a) $d\Pi/dB > 0$, (b) $d\Pi/dB < 0$ with $d^2\Pi/dB^2 > 0$, and (c) $d\Pi/dB < 0$ with $d^2\Pi/dB^2 < 0$

$\mu^*(s) > \mu^*(s + ds)$) and $dB/ds > 0$, then again $d^2\Pi/dB^2 > 0$. This situation is shown in Fig. 5.2b, and the filled region is determined by only two lines, from the reference point ($i - 1$) and from the local point (i or $i + 1$, but not both). Therefore, when the first derivative is monotonically negative and the second derivative is positive, then the previous models will be valid.

Finally, if $d\Pi/dB < 0$ and either (a) $\mu^*(s) < \mu^*(s + ds)$ with $dB/ds > 0$, or (b) $\mu^*(s) > \mu^*(s + ds)$ with $dB/ds < 0$, then the second derivative of Π with respect to B will be negative. When this occurs, the boundary of the filled region for point $i + 1$ will consist of three line segments. For a situation analogous to that in Fig. 5.2c, the boundary is defined first by the s_{i-1} line $E = \mu B_{i-1} + \Pi_{i-1}$ up to $\mu_{i-1,i}^*$, then by the s_i line up to $\mu_{i,i+1}^*$, and finally by the ray $E = \mu B_{i+1} + \Pi_{i+1}$. In general, for these cases, the filled region of E - μ space depends not only on the reference and local point $v_{\parallel} = 0$ lines, but also on the potential energy and magnetic field of other (perhaps all) nonlocal points. Therefore, the functional dependence of Π with respect to B , then, that ensures the validity of the previous models can be written as two conditions:

$$\frac{d\Pi}{dB} \text{ must be monotonic} \quad (5.1.9)$$

$$\frac{d^2\Pi}{dB^2} > 0 \text{ if } \frac{d\Pi}{dB} < 0. \quad (5.1.10)$$

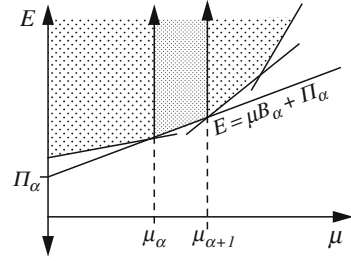
These constraints must be met for all populations in the calculation. Violation of these constraints leads to other nonlocal $v_{\parallel} = 0$ lines entering into the definition of the region of integration for the moments in (5.1.5). These extra lines define additional empty regions in velocity space where the particles that would have occupied that region have been reflected before reaching the local spatial point. Although some of the new empty regions in velocity space can be quite small, their proximity to the low-energy range (where f is typically larger) means that they could cause a significant change in the moment calculations.

It is clear that situations can arise that violate these constraints, and the region of integration in E - μ velocity space requires more than just the $v_{\parallel} = 0$ line defined in (5.1.6) for the reference point and the local spatial point. Deriving the moments for each possible scenario would be cumbersome, and a general formulation is needed that is applicable for any number of lines defining the integration region at any spatial point along the field line.

Consider the integration region in Fig. 5.3. The shaded area defines a region of E - μ space above an arbitrary $v_{\parallel} = 0$ line between two arbitrary μ endpoints. Integration over this area (at spatial point s) represents an arbitrary integral of the form

$$I_j(B_z, \Pi_z, \mu_1, \mu_2) = \int_{\mu_1}^{\mu_2} \left[\int_{\mu B_z + \Pi_z}^{\infty} F_j(E, \mu) dE \right] d\mu, \quad (5.1.11)$$

Fig. 5.3 Integration over a region of E - μ space (dotted area) above an arbitrary $v_{||} = 0$ line with arbitrary μ endpoints



where F_j is the appropriate kernel from the definition in (5.1.5), with $B = B_s$ and $\Pi = \Pi_s$. Note that B_α , Π_α , μ_1 , and μ_2 do not need to be specifically defined to perform this integral, but will eventually be defined by a length of some $v_{||} = 0$ line defining an integration region. Note that these integration boundaries could be defined by any point in the spatial range. The solution over this arbitrary region of E - μ space is what is needed for a generalization of the previous models. From this, a summation of these integrals can be constructed to obtain the needed moments of the distribution function. These moments can then be used in Maxwell's equations to couple the plasma parameters to the magnetic and electric fields.

5.1.4 Implementation

It is now necessary to define the distribution function at the reference altitude. Many functions have been used in the previous studies, particularly the Maxwellian and kappa (Lorentzian) distribution functions. Delta functions in $v_{||}$ and $v_{\perp}$ have also been used, as well as more complex distributions like bi-Maxwellians. The choice of this function primarily depends on the application of interest. A concise table of the common Lorentzian distribution functions and their associated Maxwellian limits for $\kappa \rightarrow \infty$ is available (Summers and Thorne 1991).

While the use of any function that satisfies Liouville's theorem is possible, the most general is the bi-Lorentzian loss cone distribution (Summers and Thorne 1991), written in E - μ velocity space as

$$f = \left(\frac{m}{2\pi}\right)^{3/2} \frac{n_o A_1}{\tau_{||}^{3/2}} \left(\frac{\mu B_o A_1}{\tau_{||}}\right)^\sigma \frac{\Gamma(\kappa + 1)}{\kappa^{\sigma+3/2} \Gamma(\kappa - 1/2) \Gamma(\sigma + 1)} \times \left(1 + \frac{E - \Pi_o + \mu B_o A}{\kappa \tau_{||}}\right)^{-(\kappa+\sigma+1)}, \quad (5.1.12)$$

where $\tau_{||} = T_{||}(\kappa - 3/2)/\kappa$, $A_1 = A + 1 = T_{||}/T_{\perp}$, the i subscript indicates the point of maximum B , and s is quantifies the level of depletion in the loss cone.

Various forms of loss cone distributions have been used for the past few decades. The bi-Maxwellian loss cone description of this form was originally used for analyzing plasma wave generation by geomagnetically trapped plasma (Dory et al. 1965).

A bi-Maxwellian loss cone distribution using a subtractive Maxwellian instead of a $v_{\perp}$ multiplier was used for a similar application (Ashour-Abdalla and Kennel 1978). This subtractive loss cone distribution,

$$f = \left(\frac{m}{2\pi}\right)^{3/2} \frac{n_o A_1 \exp\left(-\frac{E - \mu B_o - \Pi_o}{T_{\parallel}}\right)}{T_{\parallel}^{3/2}} \left\{ \delta \exp\left(-\frac{\mu B A_1}{T_{\parallel}}\right) + \frac{1 - \delta}{1 - \beta} \left[\exp\left(-\frac{\mu B A_1}{T_{\parallel}}\right) - \exp\left(-\frac{\mu B A_1}{\beta T_{\parallel}}\right) \right] \right\}, \quad (5.1.13)$$

was also used for field-aligned mapping of precipitating particles (Stasiewicz 1985). This distribution defines a partially filled loss cone, where d and b range from 0 to 1 and indicate the size and filling ratio of the loss cone at $v_{\parallel} = 0$. These two functions are complementary, and limit of $\beta \rightarrow 1$ in (5.1.13) reduces to the $\sigma = 1$ distribution of (5.1.12).

Because the original use of these functions was for dispersion relation calculations, a delta function at the resonant $v_{\parallel}$ was immediately applied, and thus the loss cone was a fixed size in $v_{\perp}$. For this application, however, there is no delta function specifying $v_{\parallel}$, and so the size of the loss cone must vary with $v_{\perp}$. In fact, the loss cone size depends on the field-aligned distribution of the total potential energy. In this case, then, a more rigorous approach to a loss cone definition is to define the loss cone as the region of integration at $B = B_{max}$ and define a distribution function to be subtracted from the trapped population in this “loss cone region” of velocity space.

Several other distributions are also commonly used. One is the isotropic Lorentzian distribution function,

$$f = \left(\frac{m}{2\pi}\right)^{3/2} \frac{n_o}{\tau^{3/2}} \frac{\Gamma(\kappa + 1)}{\kappa^{3/2} \Gamma(\kappa - 1/2)} \left(1 + \frac{E - \Pi_o}{\kappa \tau}\right)^{-(\kappa + 1)}, \quad (5.1.14)$$

which has been used for describing the solar wind acceleration region (Scudder 1992) as well as near-Earth plasma temperature profiles (Pierrard and Lemaire 1996). The bi-Maxwellian distribution is another function that is often used in collisionless plasma modeling, having the form

$$f = \left(\frac{m}{2\pi}\right)^{3/2} \frac{n_o A_1}{T_{\parallel}^{3/2}} \exp\left(-\frac{E - \Pi_o + \mu B_o A}{T_{\parallel}}\right). \quad (5.1.15)$$

This function is primarily used for modeling magnetic field-aligned potential energy distributions caused by mirroring anisotropic plasmas (Miller and Khazanov 1993), and also for rotating planetary magnetospheres (Huang and Birmingham 1992). A further simplification that is also used extensively is the isotropic Maxwellian distribution function,

$$f = \left(\frac{m}{2\pi}\right)^{3/2} \frac{n_o}{T^{3/2}} \exp\left(-\frac{E - \Pi_o}{T}\right), \quad (5.1.16)$$

and was used, for instance, in an ion-exosphere model (Lemaire and Scherer 1970, 1971) and in several polar wind outflow studies (Khazanov et al. 1997; Wilson et al. 1997).

In principle, each of these distribution functions can be used in the general model to obtain an analytical form of the velocity moments along a magnetic field line.

While there are many applications for mapping a collisionless plasma along a magnetic field in the presence of an arbitrarily complex field-aligned potential distribution, two applications will be discussed here: obtaining a relation between the field-aligned current and potential drop; and quantifying the plasma density and temperature changes along the field line.

5.1.4.1 Current Calculations

Finding the relation between the current density and the field-aligned potential has long been a topic of critical importance (Knight 1973; Lemaire and Scherer 1974; Pierrard et al. 2007). From the present model, the final result of the particle flux calculation along the field line will yield a general relation for the current density. The distributions presented above can be applied to (5.1.5b) using the general integration region limits of (5.1.11). As discussed above, the loss cone is best handled by defining the loss cone region of velocity space and subtracting an integration over that area, and so $s = 0$ will be used here. For comparison with the formulation, though, (5.1.13) has not been reduced. The resulting general flux integrals are

$$\begin{aligned} \phi_{gen}^{bi-L} = & \frac{n_o}{2} \sqrt{\frac{2\tau_{||}}{m\pi}} \frac{B}{B_z} \frac{\xi_\kappa^0}{\kappa^{1/2}(\kappa-1)} \frac{A_1}{1 + \frac{B_o A}{B_z}} \left[\left(1 + \frac{\mu_1(B_z + B_o A) - \Pi_o + \Pi_z}{\kappa\tau_{||}} \right)^{-(\kappa-1)} \right. \\ & \left. - \left(1 + \frac{\mu_2(B_z + B_o A) - \Pi_o + \Pi_z}{\kappa\tau_{||}} \right)^{-(\kappa-1)} \right] \end{aligned} \quad (5.1.17)$$

$$\begin{aligned}
\phi_{gen}^{LCM} = & \frac{n_o}{2} \sqrt{\frac{2T_{||} B_s}{\pi m B_x}} A_1 \exp\left(\frac{\Pi_o - \Pi_x}{T_{||}}\right) \left\{ \frac{\delta + \left(\frac{1-\delta}{1-\beta}\right)}{1 + \frac{B_o}{B_x} A} \left[\exp\left(-\frac{\mu_1(B_x + AB_o)}{T_{||}}\right) \right. \right. \\
& - \exp\left(-\frac{\mu_2(B_x + AB_o)}{T_{||}}\right) \left. \right] - \frac{\left(\frac{1-\delta}{1-\beta}\right)}{1 + \frac{B_o}{B_x} \left(\frac{A_1}{\beta} - 1\right)} \left[\exp\left(-\frac{\mu_1(B_x + B_o(A_1/\beta - 1))}{T_{||}}\right) \right. \\
& \left. \left. - \exp\left(-\frac{\mu_2(B_x + B_o(A_1/\beta - 1))}{T_{||}}\right) \right] \right\} \quad (5.1.1.8)
\end{aligned}$$

$$\phi_{gen}^L = \frac{n_o}{2} \sqrt{\frac{2\tau}{\pi m B_x}} \frac{B}{\kappa^{1/2}(\kappa - 1)} \frac{\xi_{\kappa}^0}{\kappa^{1/2}(\kappa - 1)} \left[(\zeta_1^{\alpha})^{-(\kappa-1)} - (\zeta_2^{\alpha})^{-(\kappa-1)} \right] \quad (5.1.19)$$

$$\begin{aligned}
\phi_{gen}^{bi-M} = & \frac{n_o}{2} \sqrt{\frac{2T_{||} B_s}{\pi m B_x}} \frac{A_1}{1 + \frac{B_o}{B_x} A} \exp\left(\frac{\Pi_o - \Pi_x}{T_{||}}\right) \\
& \times \left[\exp\left(-\frac{\mu_1(B_x + AB_o)}{T_{||}}\right) - \exp\left(-\frac{\mu_2(B_x + AB_o)}{T_{||}}\right) \right] \quad (5.1.20)
\end{aligned}$$

$$\phi_{gen}^M = \frac{n_o}{2} \sqrt{\frac{2T_{||} B_s}{\pi m B_x}} \exp\left(\frac{\Pi_o - \Pi_x}{T_{||}}\right) \left[e^{-\frac{\mu_1 B_x}{T}} - e^{-\frac{\mu_2 B_x}{T}} \right], \quad (5.1.21)$$

where $\xi_{\kappa}^0 = \Gamma(\kappa + 1)/\Gamma(\kappa - 1/2)$ and $\zeta_{1,2}^{\alpha} = 1 + (\mu_{1,2} B_x + \Pi_x - \Pi_o)/\kappa\tau$.

To obtain the appropriate line segments to integrate above, the potential energy structure must be known on a spatial grid from the reference point to the point of interest. If constraints (5.1.9) and (5.1.10) are met, then the summation of regions yields the previous relation for the current as a function of the field-aligned potential difference between the local point and the reference point. The potential energy distribution can be more complicated, though, and in this case a calculation along the field line is needed to obtain the $v_{||} = 0$ lines defining the region of integration for the flux.

5.1.4.2 Density, Temperature, and Heat Flux

It is often important to determine the local density, temperature, and heat flux of a plasma species, and these can also be obtained by using the functions in the moments in (5.1.5) with the limits of (5.1.11). This will yield general relations for the density, parallel pressure, perpendicular pressure, and heat flux that can be

piecewise summed over the appropriate $v_{||} = 0$ lines to obtain the total density or temperature component of the plasma at any given point along the field line. Here, the expressions are presented only for two of the most useful distribution functions: the Lorentzian and bi-Maxwellian. Note that both of these will reduce to the isotropic Maxwellian solution.

The general relations for n , $P_{||}$, $P_{\perp}$, and $\varepsilon_{||}$ for (5.1.14) are

$$n_{gen}^L = \frac{n_o}{2} \zeta_{\kappa}^1 \left\{ \frac{\left(1 - \frac{B}{B_{\alpha}}\right)^{\kappa}}{\zeta_0^{\kappa-\frac{1}{2}}} H_1(z_1^*, z_2^*) + \frac{H_1(0, z_1)}{\zeta_1^{\kappa-\frac{1}{2}}} - \frac{H_1(0, z_2)}{\zeta_2^{\kappa-\frac{1}{2}}} \right\} \quad (5.1.22)$$

$$P_{||,gen}^L = \frac{n_o \tau}{3} \kappa \zeta_{\kappa}^1 \left\{ \frac{\left(1 - \frac{B}{B_{\alpha}}\right)^{\kappa}}{\zeta_0^{\kappa-\frac{3}{2}}} H_2(z_1^*, z_2^*) + \frac{H_2(0, z_1)}{\zeta_1^{\kappa-\frac{3}{2}}} - \frac{H_2(0, z_2)}{\zeta_2^{\kappa-\frac{3}{2}}} \right\} \quad (5.1.23)$$

$$\begin{aligned} P_{\perp,gen}^L = P_{||,gen}^L + \frac{n_o \tau}{2} \kappa \zeta_{\kappa}^1 \left\{ \frac{\frac{B}{B_{\alpha}} \left(1 - \frac{B}{B_{\alpha}}\right)^{\kappa}}{\zeta_0^{\kappa-\frac{3}{2}}} \left[H_1(z_1^*, z_2^*) + H_2(z_1^*, z_2^*) \right] \right. \\ \left. + \frac{\mu_1 B}{\kappa \tau} \frac{H_1(0, z_1)}{\zeta_1^{\kappa-\frac{3}{2}}} - \frac{\mu_2 B}{\kappa \tau} \frac{H_1(0, z_2)}{\zeta_2^{\kappa-\frac{3}{2}}} \right\} \end{aligned} \quad (5.1.24)$$

$$\begin{aligned} \varepsilon_{||,gen}^L = (\Pi_{\alpha} - \Pi) \phi_{gen}^L + \frac{n_o}{2} \sqrt{\frac{2\tau}{\pi m B_{\alpha}}} \frac{\zeta_{\kappa}^0}{\kappa^{1/2}(\kappa-1)} \left\{ \mu_1 B_{\alpha} (\zeta_1^{\alpha})^{-(\kappa-1)} \right. \\ \left. + \mu_2 B_{\alpha} (\zeta_2^{\alpha})^{-(\kappa-1)} + \frac{2\kappa\tau}{\kappa-2} \left[(\zeta_1^{\alpha})^{-(\kappa-2)} - (\zeta_2^{\alpha})^{-(\kappa-2)} \right] \right\}, \end{aligned} \quad (5.1.25)$$

where $\zeta_{\kappa}^1 = \zeta_{\kappa}^0 / \Gamma(3/2)$, the new z terms are

$$\zeta_0 = 1 - \frac{B}{B_{\alpha}} + \frac{\Pi - \Pi_o + \frac{B}{B_{\alpha}}(\Pi_o - \Pi_{\alpha})}{\kappa \tau} \quad \zeta_{1,2} = 1 + \frac{\mu_{1,2} B + \Pi - \Pi_o}{\kappa \tau}, \quad (5.1.26)$$

the z limits are

$$z_{1,2} = \frac{\zeta_{1,2}}{1 + \frac{\mu_{1,2} B_{\alpha} + \Pi_{\alpha} - \Pi_o}{\kappa \tau}}, \quad z_{1,2}^* = \frac{\zeta_0}{\left(1 + \frac{B}{B_{\alpha}}\right) \left(1 + \frac{\mu_{1,2} B + \Pi - \Pi_o}{\kappa \tau}\right)}, \quad (5.1.27)$$

and the two H functions are the integrals

$$H_1(u_1, u_2) = \int_{u_1}^{u_2} t^{\kappa-\frac{3}{2}}(1-t)^{1/2} dt, \quad H_2(u_1, u_2) = \int_{u_1}^{u_2} t^{\kappa-\frac{5}{2}}(1-t)^{3/2} dt, \quad (5.1.28)$$

which can be readily written in series form.

The general relations for n , $P_{||}$, $P_{\perp}$, and $e_{||}$ for (5.1.15) are

$$\begin{aligned} n_{gen}^{bi-M} = & \frac{n_o}{2} \frac{A_1}{1 + A \frac{B_o}{B_s}} \exp\left(\frac{\Pi_o - \Pi_s}{T_{||}}\right) \left\{ \operatorname{erfc}(\sqrt{x_1}) \exp\left(-\frac{\mu_1(B_s + AB_o)}{T_{||}}\right) \right. \\ & \left. - \operatorname{erfc}(\sqrt{x_2}) \exp\left(-\frac{\mu_2(B_s + AB_o)}{T_{||}}\right) + G_1(y) \exp\left(\frac{B_s + AB_o \Pi_{\alpha} - \Pi_s}{B_{\alpha} - B_s} \frac{\Pi_{\alpha} - \Pi_s}{T_{||}}\right) \right\} \end{aligned} \quad (5.1.29)$$

$$P_{||,gen}^{bi-M} = n_{gen}^{bi-M} T_{||} + \frac{1}{2} n_o T_{||} \frac{A_1}{1 + A \frac{B_o}{B_s}} G_3(y) \times \exp\left(\frac{\Pi_o - \Pi_s}{T_{||}} + \frac{B_s + AB_o}{B_{\alpha} - B_s} \frac{\Pi_{\alpha} - \Pi_s}{T_{||}}\right) \quad (5.1.30)$$

$$\begin{aligned} P_{\perp,gen}^{bi-M} = & \frac{P_{||,gen}^{bi-M}}{1 + A \frac{B_o}{B_s}} + \frac{n_o T_{||}}{2} \exp\left(\frac{\Pi_o - \Pi_s}{T_{||}}\right) \frac{A_1}{\left(1 + A \frac{B_o}{B_s}\right)^2} \left\{ \frac{\mu_1(B_s + AB_o)}{T_{||}} \operatorname{erfc}(\sqrt{x_1}) \right. \\ & \times \exp\left(-\frac{\mu_1(B_s + AB_o)}{T_{||}}\right) - \frac{\mu_2(B_s + AB_o)}{T_{||}} \operatorname{erfc}(\sqrt{x_2}) \exp\left(-\frac{\mu_2(B_s + AB_o)}{T_{||}}\right) \\ & \left. + \frac{\Pi_{\alpha} - \Pi_s}{T_{||}} G_2(y) \exp\left(\frac{B_s + AB_o}{B_{\alpha} - B_s} \frac{\Pi_{\alpha} - \Pi_s}{T_{||}}\right) \right\} \end{aligned} \quad (5.1.31)$$

$$\begin{aligned} \varepsilon_{||,gen}^{bi-M} = & \left(\Pi_{\alpha} - \Pi + T_{||} \left[1 + \left(\frac{B_{\alpha}}{B_{\alpha} + B_o A} \right)^2 \right] \right) \phi_{gen}^{bi-M} + \frac{n_o}{2} \sqrt{\frac{2T_{||}}{\pi m}} e^{\frac{\Pi_o - \Pi_{\alpha}}{T_{||}}} \\ & \times \frac{BA_1 B_{\alpha}}{B_{\alpha} + B_o A} \left[\mu_1 \exp\left(-\frac{\mu_1(B_{\alpha} + B_o A)}{T_{||}}\right) - \mu_2 \exp\left(-\frac{\mu_2(B_{\alpha} + B_o A)}{T_{||}}\right) \right], \end{aligned} \quad (5.1.32)$$

where x and y are

$$x_{1,2} = \frac{\mu_{1,2}(B_{\alpha} - B_s) + \Pi_{\alpha} - \Pi_s}{T_{||}}, \quad y = \frac{B_{\alpha} + AB_o}{B_{\alpha} - B_s}, \quad (5.1.33)$$

and the three G functions are defined as

$$G_1(y) = \begin{cases} y = 0: & G_1 = \frac{2}{\sqrt{\pi}} [\sqrt{x_1} - \sqrt{x_2}] \\ y > 0: & G_1 = \frac{1}{\sqrt{y}} [\operatorname{erf}(\sqrt{yx_1}) - \operatorname{erf}(\sqrt{yx_2})] \\ y < 0: & G_1 = \frac{2}{\sqrt{-y\pi}} [e^{-yx_1} D(\sqrt{-yx_1}) - e^{-yx_2} D(\sqrt{-yx_2})] \\ y \rightarrow \infty: & G_1 = 0 \end{cases} \quad (5.1.34)$$

$$G_2(y) = \begin{cases} y = 0: & G_2 = \frac{2y}{\sqrt{\pi}} \left(\frac{B_s + AB_o}{B_x + AB_o} \right) [\sqrt{x_1} - \sqrt{x_2}] \\ y > 0: & G_2 = \sqrt{y} \left(\frac{B_s + AB_o}{B_x + AB_o} \right) [\operatorname{erf}(\sqrt{yx_1}) - \operatorname{erf}(\sqrt{yx_2})] \\ y < 0: & G_2 = -2\sqrt{\frac{-y}{\pi}} \left(\frac{B_s + AB_o}{B_x + AB_o} \right) [e^{-yx_1} D(\sqrt{-yx_1}) - e^{-yx_2} D(\sqrt{-yx_2})] \\ y \rightarrow \infty: & G_2 = 0 \end{cases} \quad (5.1.35)$$

$$G_3(y) = \begin{cases} y = 0: & G_3 = \frac{2y}{3\sqrt{\pi}} \left(\frac{B_s + AB_o}{B_x + AB_o} \right) [x_1^{3/2} - x_2^{3/2}] \\ y > 0: & G_3 = \left(\frac{B_s + AB_o}{B_x + AB_o} \right) \left[\frac{\operatorname{erf}(\sqrt{yx_1})}{\sqrt{y}} - \frac{\operatorname{erf}(\sqrt{yx_2})}{\sqrt{y}} - \sqrt{\frac{x_1}{\pi}} e^{-yx_1} + \sqrt{\frac{x_2}{\pi}} e^{-yx_2} \right] \\ y < 0: & G_3 = \frac{\left(\frac{B_s + AB_o}{B_x + AB_o} \right)}{\sqrt{\pi}} \left[e^{-yx_1} \left(\frac{D(\sqrt{-yx_1})}{\sqrt{-y}} - \sqrt{x_1} \right) \right. \\ & \quad \left. - e^{-yx_2} \left(\frac{D(\sqrt{-yx_2})}{\sqrt{-y}} - \sqrt{x_2} \right) \right] \\ y \rightarrow \infty: & G_3 = 0. \end{cases} \quad (5.1.36)$$

These equations can be used for many applications, such as polar wind outflows, magnetospheric precipitation, or trapped plasma anisotropies (Khazanov et al. 1998; Liemohn 2003; Pierrard et al. 2007).

Exercise. Obtain moments (5.1.5a)–(5.1.5e) using distribution (5.1.16) and assuming that the generalized constraints (5.1.9) and (5.1.10) are met, to get the same result from the more general calculations of the moments that were outlined above.

5.2 Legendre Polynomial Expansion

Considering the solution of the kinetic equation (2.5.2) for the superthermal electrons in the ionosphere for the energies $E \gg T_e$, some additional simplification could be made. First, the process of superthermal electron formation in the ionosphere is quasistationary one, and all forces in this equation can be omitted. Small terms proportional to the $m/M_{i,n} \ll 1$, term with the second energy derivative, the inelastic collisions of de-excitation, and recombination also should be neglected (see Appendix I for details). In this case, kinetic equation (2.5.2) can be written as

$$\begin{aligned}
 \mu \frac{\partial f}{\partial s} - \frac{1 - \mu^2}{2B} \frac{\partial B}{\partial s} \frac{\partial f}{\partial \mu} = \frac{An_e}{E} \left\{ \frac{\partial f}{\partial E} + \frac{1}{2E} \frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial f}{\partial \mu} \right] \right\} \\
 + n_n \left\{ \int_0^{2\pi} \int_{-1}^1 I_{en}(E, \mu_\chi) f(\mu') \sin \chi d\chi d\varepsilon + \sum_{j,k>j} \frac{\Delta_n^j \sigma_{en}^{jk} (E + E_{jk}) (E + E_{jk}) f(E + E_{jk})}{E} \right. \\
 + \frac{1}{E} \int_{E+E_n^+}^{2E+E_n^+} I_n^+(E, E' - E - E_n^+) E' f(E') dE' \\
 + \frac{1}{2\pi E} \int_{2E+E_n^+}^{\infty} I_n^+(E, E') \left[\int_0^{2\pi} E' f(E', \sqrt{1 - \mu^2} \cos \varepsilon) d\varepsilon \right] dE' \\
 \left. - \left[\sigma_{en} + \sum_{j,k>j} (\Delta_n^j \sigma_{en}^{jk} + \sigma_n^+) \right] f \right\} + \frac{q}{E}, \quad (5.2.1)
 \end{aligned}$$

where $f = \Phi/E$ is the distribution function for superthermal electrons, and Φ is electron flux.

Because of the form of the terms in the kinetic equation (5.2.1), it is convenient to expand the superthermal electron distribution function in a Legendre polynomial series,

$$f(\mu, E, s) = \sum_{m=0}^M f_m(E, s) P_m(\mu) \quad (5.2.2)$$

(Popov and Khazanov 1974). Substituting this expression into (5.2.1), we obtain a coupled set of differential equations for the expansion coefficients $f_m(E, s)$. Because the geomagnetic field does not change significantly with altitude in the ionosphere, the second term on the left-hand side of (5.2.1) can be neglected in comparison with the first term and the system of equations for the expansion coefficients can be written in the form

$$a_m \frac{\partial f_{m+1}}{\partial h} + c_m \frac{\partial f_{m-1}}{\partial h} + b_m f_m - d_m = 0 \quad (5.2.3)$$

(Khazanov 1979), where

$$\begin{aligned}
 a_m &= \frac{m+1}{2m+3} \sin I (1 - \delta_{mp}), \quad c_m = \frac{m}{2m-1} \sin I, \\
 b_m &= \frac{An_e}{E} \left(\frac{1}{\Delta E} + \frac{m^2 + m + 2}{2E} \right) + \sum_n n_n \left[(1 - \delta_{m0}) \sigma_{en}^i + \sum_{j,k>j} \Delta_n^j \sigma_{en}^{jk(i)} + \sigma_{en}^{+(i)} \right], \\
 d_m &= Q_m^i + \frac{An_e}{E\Delta E} f_m^{(i+1)} + \frac{1}{E} \sum_n n_n \left[\sum_{j,k>j} \Delta_n^j \sigma_{en}^{jk(i+n_{jk})} f_m^{i+n_{jk}} \right. \\
 &\quad \left. + \int_{E+E_n^+}^{2E+E_n^+} I_n^+(E', E' - E - E_n^+) f_m(E') dE' + P_m(0) \int_{2E+E_n^+}^{\infty} I_n^+(E', E) f_m(E') dE' \right].
 \end{aligned}$$

Here the energy derivative has been expressed as a finite difference with an energy step size of ΔE and derivatives along s have been projected along the radial direction, i.e., $ds = dh/\sin I$. In addition, the pitch-angle distribution of the super-thermal source is considered to be isotropic. Additionally, the following notations have been introduced: δ_{mp} is the Kronecker delta, $i = E/\Delta E$, $n_{jk} = E_{jk}/\Delta E$, and

$$Q_m(s, E) = \frac{2m+1}{8\pi} \int_{-1}^1 q(s, E, \mu) P_m(\mu) d\mu.$$

A distinctive feature of the system of equations represented by (5.2.3) is the existence of the spatial derivative with respect to the $m+1$ and $m-1$ expansion coefficients. To solve this system of equations numerically by iterative methods, the derivative $\partial f_{m+1}/\partial h$ is calculated from the previous iteration. Because derivatives of the grid functions are very sensitive to small variations in the functions, the method of successive approximations turns out to be divergent. Therefore, the initial system of equations is transformed to reduce the coupling between the equations.

Successively substituting the derivatives of the odd coefficients $\partial f_{2n+1}/\partial h$ in (5.2.3) from equation number $m = 2n$ into equation number $m = 2n+2$ (with n varying from 0 to $N-1$, where $N = (M-1)/2$), we obtain

$$\frac{\partial f_{2n+1}}{\partial h} + \sum_{l=0}^n k_{2l}^n f_{2l} = \sum_{l=0}^n P_{2l}^n, \quad (5.2.4)$$

where

$$k_{2l}^n = \frac{V_{nl} b_{2l}}{a_{2l}}, \quad P_{2l}^n = \frac{V_{nl} d_{2l}}{a_{2l}}, \quad V_{nl} = (-1)^{n-l} \prod_{i=l}^{n-1} \frac{c_{2i+2}}{a_{2i+2}},$$

and $V_{00} = 1$ (Khazanov 1979). Similarly, substituting the derivatives of the even expansion coefficients from equation number $m = 2n + 1$ into equation number $m = 2n - 1$ and varying n in reverse order, yields

$$\frac{\partial f_{2n}}{\partial h} + \sum_{l=n}^N k_{2l+1}^n f_{2l+1} = \sum_{l=n}^N P_{2l+1}^n, \quad (5.2.5)$$

where

$$k_{2l+1}^n = \frac{W_{nl} b_{2l+1}}{c_{2l+1}}, \quad P_{2l+1}^n = \frac{W_{nl} d_{2l+1}}{c_{2l+1}}, \quad W_{nl} = (-1)^{l-n} \prod_{i=l}^{n+1} \frac{a_{2i-1}}{c_{2i-1}},$$

and $W_{00} = 1$.

Differentiating (5.2.5) with respect to h and substituting the derivative $\partial f_{2l+1} / \partial h$ from (5.2.4) into the resulting equation leads to

$$\frac{\partial^2 f_{2n}}{\partial h^2} + \sum_{l=n}^N \frac{\partial k_{2l+1}^n}{\partial h} f_{2l+1} + \sum_{l=n}^N k_{2l+1}^n \sum_{i=0}^l (P_{2i}^l - k_{2i}^l f_{2i}) = \sum_{l=n}^N \frac{\partial P_{2l+1}^n}{\partial h}. \quad (5.2.6)$$

Separating the term with $l = n$ from the sum on the left-hand side of (5.2.5), we can rewrite this equation in the form

$$f_{2n+1} = -\frac{1}{k_{2n+1}^n} \left[\frac{\partial f_{2n}}{\partial h} + \sum_{l=n+1}^N k_{2l+1}^n f_{2l+1} - \sum_{l=n}^N P_{2l+1}^n \right]. \quad (5.2.7)$$

Finally, substituting (5.2.7) into the corresponding term in the first sum on the left-hand side of (5.2.6), we obtain

$$\frac{\partial}{\partial h} \left(A_{2n} \frac{\partial f_{2n}}{\partial h} \right) - B_{2n} f_{2n} + D_{2n} = 0, \quad (5.2.8)$$

where

$$A_{2n} = 1/k_{2n+1}^n, \quad B_{2n} = A_{2n} \sum_{l=n}^N k_{2l+1}^n k_{2n}^l, \quad D_{2n} = \sum_{l=n+1}^N (\partial/\partial h) (k_{2l+1}^n / k_{2n+1}^n) f_{2l+1} \\ - \sum_{l=n}^N \left[(\partial/\partial h) (P_{2l+1}^n / k_{2l+1}^n) - (k_{2l+1}^n / k_{2n+1}^n) \left(\sum_{i=0, i \neq n}^l k_{2i}^l f_{2i} - \sum_{i=0}^l P_{2i}^l \right) \right].$$

The initial system of equations (5.2.3) has been transformed to a set of second-order differential equations (5.2.8) for the even expansion coefficients and the set of relations (5.2.7) for the odd ones. The equations obtained are characterized by

relatively small correlation, which can be explained by the absence of derivatives of “foreign” functions in them. This allows the method of successive approximation to be applied. The system of equations represented by (5.2.7) and (5.2.8) can be solved by consecutively increasing n from 0 to N . First the even expansion coefficient f_{2n} is determined from (5.2.8) and then the corresponding odd coefficient is determined from (5.2.7). The solution is carried out iteratively, using newly calculated coefficients f as soon as they are available. In other words, coefficients f with n smaller than the current one are obtained from the given iteration while coefficients with n larger than the current one are taken from the previous iteration (Khazanov 1979). We have found solutions obtained in this manner to be stable and to converge quickly.

The superthermal electron energy distribution is obtained by successive calculation of the distribution function for all points of the energy difference grid, varying energy from larger values toward smaller ones, i.e., $E = E_{\max} \rightarrow E_{\min}$. Energy boundary values of the distribution function are determined by assuming that there are no superthermal electrons with high energies, i.e., $f_m(E > E_{\max}) = 0$.

Upper and lower boundary conditions must be specified before (5.2.8) can be solved in the ionosphere. At the lower boundary, $h_l = 100$ km, the pitch-angle distribution of the superthermal electrons is isotropic, i.e., $f_{m|m \neq 0} = 0$, due to frequent collisions of the superthermals with the neutral constituents of the atmosphere. From (5.2.3) it follows that

$$f_m(h_l) = \begin{cases} d_0/B_0, & m = 0 \\ 0, & m = 1, 2, \dots, M \end{cases} \quad (5.2.9)$$

To set the upper boundary conditions, at $h = h_t$, the energy and pitch-angle distribution of superthermal electrons coming into the ionosphere from the plasmasphere must be known; i.e., the quantity $f(E, \mu < 0, h = h_t)$. At midlatitudes, the incoming flux of superthermal electrons depends on conditions for generating photoelectrons in the conjugate ionosphere, as well as on conditions in the plasmasphere. Furthermore, the process of trapping superthermal electrons in the plasmasphere and ultimately filling up the geomagnetic trap depends on the energy distribution of the ionospheric electrons. Therefore, calculations of the distribution function in a given ionosphere should be consistent with those carried out in the plasmasphere and in the conjugate ionosphere. The method for solving the kinetic equation described here can also be applied in the auroral zone, if an appropriate incoming distribution of superthermal electrons is specified at the upper boundary (see Khazanov 1979 for the details).

As an example, we consider conditions at the upper boundary of the ionosphere during equinox, when there is relative symmetry between conjugate hemispheres. In this instance, the pitch-angle distribution of the superthermal electrons at the upper ionospheric boundary is approximated by

$$\begin{aligned} f(E, h = h_t, \mu > 0) &= \overline{f^+}(E) \\ f(E, h = h_t, \mu < 0) &= \overline{f^-}(E). \end{aligned} \quad (5.2.10)$$

(Khazanov et al. 1992) have expressed the incoming distribution function $\bar{f}^-(E)$ as a combination of the distribution of electrons leaving the conjugate ionosphere $\bar{f}^+(E)$ and a trapped distribution $f_b(E, \mu = \mu_{0b})$ at the edge of the loss cone, μ_{0b} is cosine of the pitch angle at the boundary between the free and the trapped zones (see also Sect. 5.5 for more details):

$$\bar{f}^- = f_b + (\bar{f}^+ - f_b)e^{-1/y} \quad (5.2.11)$$

with

$$f_b(E) = \bar{f}^+ \frac{y}{y + 4\delta} + 2\delta^2(y + 4\delta)^{\delta-1} \int_y^{y_{\max}} \frac{\bar{f}^+(y_0)}{(y_0 + 4\delta)^{\delta+1}} dy_0, \quad (5.2.12)$$

where

$$y = \frac{E^2}{E_{n0}^2}, \quad E_{n0}^2 = 2AB(s_t) \int_{-s_t}^{s_t} \frac{n_e(s)}{B(s)} ds, \quad \delta = \left[\ln \left(\frac{1}{1 - \mu_{0b}} \right) \right]^{-1}.$$

The upper limit to the integration in (5.2.12), $y_{\max}$, is determined by the upper bound of the energy interval under consideration, i.e., $E_{\max}$. As $y \rightarrow \infty$ in (5.2.11), no trapping of photoelectrons occurs and $\bar{f}^-(E) = \bar{f}^+(E)$. The other extreme occurs when $y = 0$. In this instance, photoelectrons arising from the conjugate ionosphere are totally trapped, $\bar{f}^-(E) = f_b$. Extensive trapping occurs when $E < \sqrt{2\delta}E_{n0}$.

In order to obtain the pitch-angle distribution of incoming electrons, one must first evaluate the integral in (5.2.12) to obtain $f_b(E)$. Recall that the superthermal distribution function is solved by varying energy from higher values to lower values during a given iteration cycle. Consequently, we divide the integral in (5.2.12) into two parts as follows: from y to $y + \Delta y$ and from $y + \Delta y$ to $y_{\max}$. The second integral is evaluated directly, taking into account that when the distribution function f is calculated at a given energy level $E(y)$, its values at higher energies are known. Approximating the first integral by the a low-order approximation, (5.2.12) can be rewritten as

$$f_b = \alpha_r \bar{f}^+ + \beta_r, \quad (5.2.13)$$

$$\alpha_r = \frac{1}{y + 4\delta} \left[y + \frac{2\delta^2 \Delta y}{y + 4\delta} \right], \quad \beta_r = 2\delta^2(y + 4\delta)^{\delta-1} \int_{y+\Delta y}^{y_{\max}} \frac{\bar{f}^+(y_0)}{(y_0 + 4\delta)^{\delta+1}} dy_0$$

are known functions of energy. Substituting (5.2.13) into (5.2.11), we obtain the following relation between the distribution functions of incoming and outgoing electrons:

$$\bar{f}^- = A\bar{f}^+ + B, \quad (5.2.14)$$

where

$$A = e^{-1/y} + \alpha_r (1 - e^{-1/y}), \quad B = \beta_r (1 - e^{-1/y}).$$

Consider the following expression

$$\begin{aligned} \frac{2}{4n+3} f_{2n+1} &= \int_{-1}^1 f(\mu) P_{2n+1}(\mu) d\mu \\ &= \int_{-1}^0 f(\mu < 0) P_{2n+1}(\mu) d\mu + \int_0^1 f(\mu > 0) P_{2n+1}(\mu) d\mu, \end{aligned}$$

which can be obtained from the orthogonality relations for the Legendre polynomials. Taking into consideration (5.2.10) and (5.2.14), this expression yields

$$f_{2n+1} = \frac{4n+1}{2} \int_0^1 [f^+(\mu)(1-A) - B] P_{2n+1}(\mu) d\mu \quad (5.2.15)$$

at the upper boundary. Expanding the distribution of the outgoing superthermal electrons as a series of Legendre polynomials (see (5.2.2)), (5.2.15) can be written in the form

$$f_{2n+1} = \frac{1-A}{1+A} \sum_{l=0}^N f_{2l} I_{nl} - \frac{B}{1+A} I_{n0}, \quad (5.2.16)$$

where

$$I_{nl} = (4n+3) \int_0^1 P_{2n+1}(\mu) P_{2l}(\mu) d\mu.$$

Finally, the upper-boundary condition is obtained after substitution of the quantity f_{2n+1} from (5.2.7) into (5.2.16):

$$\frac{\partial f_{2n}}{\partial h} + F_n f_{2n} = L_n, \quad (5.2.17)$$

where

$$F_n = k_{2n+1}^n I_{nn} \frac{1-A}{1+A},$$

$$L_n = \sum_{l=n}^N P_{2l+1}^n - \sum_{l=n+1}^N \left[k_{2l+1}^n f_{2l+1} + \frac{1-A}{1+A} I_{nl} k_{2n+1}^n f_{2l} \right]$$

$$- \frac{1-A}{1+A} \sum_{l=0}^{n-1} I_{nl} k_{2n+1}^n f_{2l} + \frac{BI_{n0}}{1+A} k_{2n+1}^n.$$

All of the quantities in (5.2.17) are at the upper boundary altitude $h = h_r$. Included there are values of “foreign” coefficients with number n larger than the current one (the second sum in L_n). These values are obtained from the previous iteration cycle. The coefficients with a smaller number (the last sum in L_n) are obtained from the current iteration.

The parameter f^+ can be determined after computing the distribution of superthermal electrons in the ionosphere. Then, using formulas given in Khazanov et al. (1992) (see also Sect. 5.5), parameters describing the distribution of superthermal electrons in the plasmasphere can be calculated (Polyakov et al. 1979).

5.3 Description of Electron–Electron Collisions in the Lower Ionosphere

5.3.1 Lower Energies

In order to highlight the correct description of electron–electron collision processes, the lower ionosphere altitude ($h < 300$ km) will be considered in this section. The usual description of electron behavior in the Earth’s ionosphere is based on an assumption that electron gas consists of two components – “thermal electrons” and “superthermal electrons” (see also Sect. 4.4 in this book). Although the ambient thermal electrons and the arising photoelectrons (or secondary electrons) are physically undistinguishable, it is possible to consider all electrons with energy $E < 3T_e/2$ as thermal electrons and all electrons with $E \geq E_0$ as photoelectrons (if photoionization of neutral particles takes place). In this case, $E_0 = 10\text{--}15$ eV denotes the mean energy of arising photoelectrons and $3T_e/2 = 0.1\text{--}0.5$ eV is the mean energy of ambient electrons. For intermediate energies $3T_e/2 < E < E_0$, however, such a subdivision is not quite correct (Krinberg 1973), as will be shown in the present section. Considering the solution of the outlined problem in the lower ionosphere, and taking into account only electron–electron collision processes, the following kinetic equation must be solved:

$$S_{ee}(f) + q_e(v) = 0. \quad (5.3.1)$$

Taking into account the isotropy of the electron distribution function in the lower ionosphere, the electron–electron collision term (2.2.12) can be greatly simplified and presented in the following form

$$S_{ee}(E) = An_e \frac{v}{E} \left[\frac{\partial}{\partial E} \left(A_1 f + A_2 E \frac{\partial f}{\partial E} \right) \right], \quad (5.3.2)$$

where dimensionless coefficients $A_{1,2}$ are given by (2.2.13). Dividing the electron distribution function on two parts at some velocity space boundary $v = a$; the distribution function f_T close to Maxwellian, and superthermal electrons with arbitrary distribution function f_e whose structure is defined by nonequilibrium spectrum of the electron source could be introduced as

$$f_T(v) = \begin{cases} f(v) & \text{for } v \leq a, \\ 0 & \text{for } v > a, \end{cases} \quad f_e(v) = \begin{cases} 0 & \text{for } v < a, \\ f(v) & \text{for } v \geq a. \end{cases} \quad (5.3.3)$$

Because of the nonlinearity of (5.3.2), the equations for f_T and f_e are coupled (see (4.4.19) and (4.4.20)). Solutions of these equations must satisfy the conditions $f_T = f_e$ and $\partial f_T / \partial v = \partial f_e / \partial v$ when $v = a$, because of continuity of the distribution function $f(v)$ and its derivative.

Considerable simplification of the problem is possible if the inequalities $E_0 > E_x > T_e$ and $n_e \gg n_e$ take place, where

$$E_x = ma^2/2, \quad n_e = \int f_T dv, \quad n_e = \int f_e dv.$$

Then one can use the hydrodynamic description for the electrons with energy $E < E_x$ and suppose that f_T is Maxwellian distribution function. In the kinetic equation for energetic electrons, $E > E_x$, one can put $S_{ee}(v > a) \approx S_{ee}(f_e f_T)$ and neglect the interaction between energetic electrons. Then considering f_T as a Maxwellian distribution function, from (5.3.2) and (2.2.10), one can obtain

$$\frac{A_2 T_e}{A_1} \frac{\partial^2 f_e}{\partial E^2} + \frac{\partial f_e}{\partial E} + \frac{q_E}{4\pi A n_e} = 0, \quad (5.3.4)$$

where the following notations have been introduced

$$A_1 = \phi(\sqrt{x}) - 2\sqrt{\frac{x}{\pi}} e^{-x}, \quad A_2 = \phi(\sqrt{x}) - 2\sqrt{\frac{x}{\pi}} \left(1 + \frac{3}{2}x \right) e^{-x},$$

$$\phi(y) = \frac{2}{\sqrt{\pi}} \int_0^y e^{-t^2} dt, \quad x = \frac{E_x}{T_e},$$

and the value of $q_E(E) = (4\pi v/m)q(v)$ is defined so that $q_E dE$ is the production rate of photoelectrons with the energy from E to $E + dE$.

Applying the boundary conditions (5.3.3) to the solution of (5.3.4) and taking into account that $f_e \rightarrow 0$ as $E \rightarrow \infty$, the final result can be presented in a form (Krinberg 1973)

$$f_e(E) = \left(M e^{(\eta-1)x} + \frac{\eta}{4\pi A n_e T_e} \int_{E_x}^E q(E') e^{\eta(E'/T_e)} dE' \right) e^{-\eta(E/T_e)}, \quad (5.3.5)$$

in which $M = \frac{n_e}{A_1} \left(\frac{m}{2\pi T_e} \right)^{3/2}$, $\eta = A_1/A_2$, and $q(E) = \int_E^\infty q_E(E') dE''$ represents the production rate of photoelectrons with the energies larger than E . Substituting this solution in the relation of $\partial f_T / \partial v = \partial f_e / \partial v$ when $E = E_x$, the relation for E_x can be obtained (Krinberg 1973)

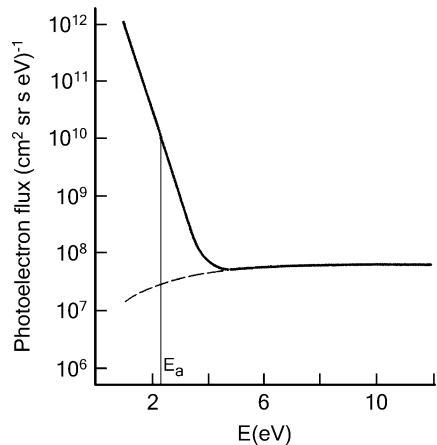
$$x - \frac{3}{4} \ln x = \frac{1}{2} \ln \left(\frac{4}{3\sqrt{\pi}} \frac{q_0}{A_1^2 q(E_x)} D \right), \quad (5.3.6)$$

where $D = 4\pi A n_e M / q_0$, and q_0 is the total production rate.

For the typical daytime ionospheric parameters $n_e = 10^4 - 10^6 \text{ cm}^{-3}$, $T_e = 0.1 - 0.3 \text{ eV}$, $q_0 = 10^2 - 10^4 \text{ cm}^{-3} \text{ s}^{-1}$, and $D = 10^3 - 10^7$, the good approximation for (5.3.6) is $x \approx (\ln D + 3)/2$, which leads to $E_x = 1 - 3 \text{ eV}$, and $\eta - 1 \approx 10^{-3} - 10^1$. Taking into account inequalities $E_0 \gg T_e$ and $\sqrt{D} \gg 1$, and presenting the production rate as $q(E) = q_0 \exp(-E/E_0)$, the solution can be greatly simplified and written as

$$f_e(E) = \frac{q_0}{4\pi A n_e} \left(D e^{(\eta-1)x - \eta(E/T_e)} + e^{-E/E_0} \right). \quad (5.3.7)$$

Fig. 5.4 Electron fluxes calculated according to (5.3.7) with (solid line) and without (dashed line) first term (Krinberg 1973)



In Fig. 5.4, the electron fluxes $\Phi(E) = 2Ef_e/m^2$ are given. These fluxes have been calculated with and without the first term in (5.3.7) for typical ionospheric parameters at an altitude of 300 km: $T_e = 0.25$ eV, $n_e = 5 \times 10^5$ cm⁻³, $q_0 = 10^2$ cm⁻³ s⁻¹, and $E_0 = 10$ eV. It is seen that the calculation according to (5.3.7) gives a sharp flux increase for the small energy values. Neglecting the first term in (5.3.7), describing by the first term in (5.3.4) gives a significant decrease of $\Phi(E)$ for energies less than 5 eV. Based on the result presented here, the second energy derivative in the Coulomb collision operator plays a fundamental role in the determining of low-energy superthermal electron fluxes.

Note, that solution (5.3.7) determined by Krinberg (1973) is based on the linearization of (5.3.2), assuming that the thermal electrons have a Maxwellian distribution function. In Khazanov (1979), this assumption has been removed and the electron–electron collision term has been used in the more general (5.3.2) form. Nevertheless, the final result for the electron distribution was the same. This proves the usefulness of presenting the electron distribution function in the form of (4.4.18), which dramatically simplifies the calculation of the collision moments in hydrodynamic equations (see Sect. 4.4.2 for details).

5.3.2 Fine Structure of Photoelectron Fluxes

Another example of electron–electron collision processes is now presented that demonstrates the inapplicability of (5.3.2) for studies of the so-called fine structure of photoelectron fluxes.

The intense emission lines present in the solar emission spectrum substantially distort the smooth structure of the primary spectrum of photoelectrons, giving rise to a fine structure in them that is easily detected in experiments (Mukai and Hirao 1973) and in appropriate numerical calculations of the full kinetic equation (Khazanov 1979). Thus, the He II 303.8 Å spectral line that, according to Cushman et al. (1975), has $\delta\lambda = 0.12$ Å, produces very narrow energy peaks in the electron source spectrum with a width $\delta E \approx 0.016$ eV. In the case of ionization of oxygen atoms and nitrogen molecules, relevant peaks in the photoelectron energy distribution are within the energy region $22 \leq E \leq 27.2$ eV.

Analytic investigation of the relaxation of photoelectron source spectra, formed by some intense line of the solar spectrum, is important both for understanding the physical processes involved and for a better choice of the integration step in solving the kinetic equation numerically. An analytic study of such a problem was undertaken by Jasperse and Smith (1978) and Krinberg (1978), who examined the narrow peak source relaxation on thermal electrons of the ionospheric plasma, pointed to the important role of Coulomb collisions in the energy distribution deformation, and proposed a method for determining some aeronomic parameters. It should be noted that Jasperse and Smith (1978) and Krinberg

(1978) were insufficiently complete in using Landau's Coulomb collision integral in the form

$$S_{ee}(E) = An_e \frac{v}{E} \left[\frac{\partial}{\partial E} \left(f + T_e \frac{\partial f}{\partial E} \right) \right], \quad (5.3.8)$$

which corresponds the linearized and isotropic part of the Coulomb collision operator (2.2.12).

The Landau collision integral is evaluated from the Boltzmann integral by expanding the distribution function of colliding particles by its Taylor series expansion and by subsequent integration over velocities and scattering angles (Landau 1936; Longmire 1966; Shkarofsky et al. 1966). The integral over the scattering angle χ is taken between the limits $\chi_{\min}$, corresponding to the maximum impact parameter $b_{\max}$, equal to Debye radius, and $\chi_{\max} \sim \pi$, corresponding to $b_{\min}$. In this case, the Coulomb logarithm

$$\ln \Lambda = \ln \left(\sin \frac{\chi}{2} \right) \Big|_{\chi_{\min}}^{\chi_{\max}} \approx \ln \left(\sin^{-1} \frac{\chi_{\min}}{2} \right)$$

is determined mainly by the contribution of small scattering angles. The Taylor series we use allows only for the first three terms of the expansion due to the infinitesimal change of particle momentum (energy). Here, we will not examine the problem regarding the value of derivatives of the distribution functions involved in the expansion. When investigating the fine structure of photoelectron spectra, we are faced with the situation where the distribution function of particles incorporates very steep gradients. In this case, to cut off the series requires an additional condition for the changes in particle momentum or energy during collision, which places an upper limit on scattering angles and therefore modifies the value of the Coulomb logarithm. The maximum scattering angle thus obtained is the upper boundary of applicability of the Coulomb collision integral in the Landau form (5.3.8). As an example, consider the symmetric function $f(E)$ of the photoelectron distribution. Because the energy of photoelectrons greatly exceeds that of thermal electrons colliding with their former energy, after collision their energy is $E' = E - E \sin^2 \chi / 2$. The relevant Taylor series has the form

$$f(E') = f(E) + \frac{df}{dE} \Delta E + \frac{1}{2} \frac{d^2 f}{dE^2} \Delta E^2 + R, \quad (5.3.9)$$

where $\Delta E = -E \sin^2 \chi / 2$ and R is the remainder of the series. For clarity, assume that all terms of the series have the same signs. A necessary condition for limiting the expansion by the first three terms is the smallness of the remainder R of the series, in comparison with the sum of the terms not discarded. The value of the maximum permissible scattering angle χ_0 may thus be determined subject to the equality of the third term of the expansion and the remainder R . The value of χ_0 thus obtained would be somewhat overestimated, making only a minimum estimate of its contribution $\ln(\sin \chi_0 / 2)$ to the Coulomb logarithm (a more exact determination

of the angle χ_0 does not seem to be possible). Thus we determine a minimum value of the influence of the steep gradients of the distribution functions involved in the Landau Coulomb collision integral, causing the Coulomb logarithm to alter because, as we will see later in this section, $\chi_0 < 1$. If we are to take into account the angles $\chi > \chi_0$ (henceforth referred to as “large” angles) we must add to the kinetic equation a collision integral in Boltzmann form, with relevant scattering angles involved.

Linearity of the kinetic equation enables a separate search of its solution for each peak of the source. In the region of local approximation, we have

$$An_e \frac{v}{E} \left[\frac{\partial}{\partial E} \left(f_\alpha + T_e \frac{\partial f_\alpha}{\partial E} \right) \right] + J_B - \frac{v}{E} \sum_n E_\alpha \sigma_n(E_\alpha) n_n f_\alpha + \frac{m}{v} q_\alpha(E) = 0, \quad (5.3.10)$$

where $f_\alpha(E)$ is the photoelectron distribution function in the peak region; $\sigma_\alpha(E_\alpha)$ is the cross-section of inelastic scattering in the middle of the source spectrum at $E = E_\alpha$; n_n is the number density of neutral particles; $q_\alpha(E)$ is the spectrum of the photoelectron source; $A = 2\pi e^4 \ln \Lambda'$, and $\ln \Lambda'$ is the Coulomb logarithm that, in connection with the aforesaid cut-off of “large” scattering angles, takes $\chi > \chi_0$ into account: $\ln \Lambda' = \ln \Lambda + \ln(\sin \chi_0/2)$. Here, in contrast to the previous studies, the Boltzmann collision integral J_B involving the contribution from “large” scattering angles to the process of photoelectron spectrum fine structure formation, is added to (5.3.10).

The first boundary condition for (5.3.10) is

$$f_\alpha \rightarrow 0, \quad E \rightarrow \infty, \quad (5.3.11)$$

and the second one is

$$f_\alpha = 0, \quad E = E_\alpha - \Delta E, \quad (5.3.12)$$

where $\Delta E \approx 2 \text{ eV}$ is a minimum energy loss at inelastic collision, causing the excited atom to pass from its basic state $O(^3P)$ to state $O(^1D)$, as well as the applicability limit of (5.3.10) (Krinberg 1978). The sufficiently narrow peak width of the photoelectron source, connected with the ionization of the neutral atmosphere in the He II line, permits its presentation in (5.3.10) as a delta function

$$q_\alpha(E) = \frac{q_{\alpha 0}}{4\pi} \delta(E - E_\alpha).$$

Equation (5.3.10) may be written in the following operator form

$$\hat{D}(f_\alpha) = -b\delta(x) - \frac{ET_e}{An_e v} \hat{J}(f_\alpha). \quad (5.3.13)$$

Here $\hat{D} = d^2/dx^2 + d/dx - a$, and

$$\hat{J}(f_x) = 2\pi \iint (f'_x f'_e - f_x f_e) \sigma(g, \chi) g \sin \chi d\chi d^3 v_e$$

is the Boltzmann integral operator;

$$f_e = n_e \left(\frac{m}{2\pi T_e} \right)^{3/2} \exp \left(-\frac{mv_e^2}{2T_e} \right)$$

is the distribution function of thermal electrons;

$$\sigma(g, \chi) = e^4 m^{-2} g^{-4} \sin^{-4} \frac{\chi}{2}$$

is the Rutherford cross-section of Coulomb scattering;

$$g = |\mathbf{v} - \mathbf{v}_e|$$

is the relative velocity; and the coefficients entering (5.3.13) are

$$a = \frac{T_e}{An_e} E_x \sigma_n(E_x) n_n, \quad b = \frac{q_{x0} m^2}{8\pi A n_e}, \quad x = \frac{E - E_x}{T_e}.$$

The Boltzmann operator in the right-hand side of (5.3.13) acts as an additional source or losses due to scattering of photoelectrons by “large” angles, which causes them to enter or escape from the region of the spectrum peak. Equation (5.3.13) is an integro-differential equation and its solution is to be tried by successive approximations, following the scheme

$$\hat{D}(f_x^{(n)}) = -b\delta(x) - \frac{ET_e}{An_e v} \hat{J}(f_x^{(n-1)}), \quad n \geq 1. \quad (5.3.14)$$

The zero-order approximation of $f_x^{(0)}$ is defined by the equation, neglecting the contribution of “large” scattering angles $\chi > \chi_0$,

$$\hat{D}(f_x^{(0)}) = -b\delta(x), \quad (5.3.15)$$

whose solution, with the boundary conditions (5.3.11) and (5.3.12) taken into account, is a Green function of the equation

$$f_x^{(0)}(x) = \begin{cases} (C_1 + C_2) \exp(-k_2 x), & x \geq 0 \\ C_1 \exp(k_1 x) + C_2 \exp(-k_2 x), & -\Delta x \leq x \leq 0. \end{cases} \quad (5.3.16)$$

Here

$$C_1 = b/\sqrt{1+4a}, \quad C_2 = C_1 \exp[-(k_1 + k_2)\Delta x], \\ \Delta x = \Delta E/T_e, \quad k_1 = \frac{1}{2}(\sqrt{1+4a} - 1), \quad k_2 = \frac{1}{2}(\sqrt{1+4a} + 1).$$

Before proceeding to deriving a subsequent approximation $f_\alpha^{(1)}$ using $f_\alpha^{(0)}$ inferred previously, we first determine the angle χ_0 . Substituting (5.3.16) into (5.3.9) and, according to the aforesaid, setting the remainder of the series equal to the third term of the series, we obtain the following equation to be valid in the region to the right of the peak maximum ($x > 0$)

$$e^z = z^2 + z + 1, \text{ where } z = \frac{k_2 E_\alpha}{T_e} \sin^2 \frac{\chi_0}{2}.$$

A solution of this transcendental equation for $\sin^2 \frac{\chi_0}{2}$ for the peaks $E_\alpha = 22.0\text{--}27.2\text{ eV}$ at $T_e = 0.1\text{ eV}$ and $\sigma_n n_n \approx 10^{-6}\text{ cm}^{-1}$, $n_e \approx 10^5\text{ cm}^{-3}$ lies within $4.22 \times 10^{-2} - 3.79 \times 10^{-2}$. For $\ln(\sin \chi_0/2)$, this yields values of -3.16 to -3.27 , which reduces the Coulomb logarithm $\ln \Lambda$ by 20–22%. For the region to the left of the source ($x < 0$), the evaluation yields effects of the same order of magnitude. Thus, the applicability of the Landau Coulomb collision integral (5.3.8) in the analysis of photoelectron spectrum fine structure relaxation is limited to the region of scattering angles $\chi_{\min} \leq \chi \leq \chi_0$, which is expressed in terms of the decrease of the Coulomb logarithm value, as determined above. It should be noted that the obtained next approximation, $f_\alpha^{(2)}$, does not produce substantial changes compared with the numbers presented above. It is also interesting to note that the obtained decrease of the Coulomb logarithm value due to a limitation on scattering angles is comparable in magnitude to the effect of polarization losses that cause $\ln \Lambda$ to increase by 25–30% (Schunk and Hays 1971).

We now briefly examine the influence of the decrease of the Coulomb logarithm on using the photoelectron spectrum fine structure in the diagnostics of aeronomic parameters, which are discussed in Chap. 8 of this book. The procedure for determining some aeronomic parameters on the basis of the photoelectron spectrum fine structure, as suggested by Krinberg (1978), permits us to derive electron temperature and solar radiation intensity in the He II 303.8 Å line, using characteristics of photoelectron peaks such as their width ΔE_f and the area δ_{af} .

Because the fine structure peak amplitude does not exceed photoelectron spectrum background by more than two orders of magnitude, the values of ΔE_f and δ_{af} remain virtually unchanged. Therefore, the aforementioned effects would not influence the He II line intensity inferred. According to (Krinberg 1978), the electron temperature T_e is obtainable with the aid of the relation

$$T_e n_e / n_n = b \Delta E_f^2, \text{ where } b = E_\alpha \sigma_n / 4A, \quad A = 2\pi e^4 \ln \Lambda',$$

and the value of the Coulomb logarithm allows for dynamic friction effects (Schunk and Hays 1971) that change its conservative value, $\ln \Lambda = 15$, by 5: $\ln \Lambda' = 20$. Consideration of the scatter angle cut-off $\chi \geq \chi_0$ leads to effects comparable to the influence of the polarization losses and reduces this value to 17. Thus, a decrease of the Coulomb logarithm entails an increase of electron temperature values by 15–20%, as compared with previous results (Krinberg 1978), an electron temperature diagnostics based on photoelectron spectrum fine structure (see Chap. 8 for additional details).

5.4 Pitch-Angle Distribution in the Upper Ionosphere

When solving a simplified variation of the kinetic equation (5.2.1) considered in the preceding section in local approximation (5.3.1), basic attention was devoted to studying the character of the energy spectrum of photoelectrons (highlighting the correct electron–electron collisions description), and it was assumed that, at the altitudes considered, the distribution of superthermal electrons has an isotropic character. With an increase in altitude ($h \sim 200\text{--}250$ km), the role of transport processes increases so that in certain conditions, which will be analyzed in this section, a significant difference in the angular distribution of fast electrons from the spherically symmetrical can occur.

In this section, in order to emphasize the role of electron-neutral collision processes, the energy interval $E > 20$ eV will be considered, where the Coulomb collision process can be neglected. With the main focus on a pitch-angle distribution formation, the kinetic equation (5.4.1) will be written on the assumptions of total nonelastic ionization losses (ignoring the energy cascade toward the thermal electrons), setting $dB/ds = 0$, and with the presence of only one kind of neutral particle; this is valid for heights $h = 200\text{--}700$ km, where only atomic oxygen predominates. As a result, (5.2.1) can be converted to the form (Khazanov 1979)

$$\begin{aligned} \mu \frac{\partial \Phi}{\partial s} = & n_n(s) \int I_{en}(E, \mu_\chi) [\Phi(s, E, \mu') - \Phi(s, E, \mu)] d\mu_\chi d\varepsilon \\ & - n_n(s) \sigma_i(E) \Phi(s, E, \mu) + \frac{q_0(s, E)}{4\pi} \left[1 - \frac{1}{2} P_2(\mu_\chi) P_2(\mu) \right], \end{aligned} \quad (5.4.1)$$

where, instead of the distribution function f , the flux of superthermal electrons Φ has been introduced, $\sigma_i(E)$ is the total inelastic cross-section, and $q_0(s, E)$ is the primary spectrum of photoelectrons from atomic oxygen.

Taking into account the photoelectron exchange between the two magnetically conjugate ionospheres, the kinetic equation (5.4.1) should be written separately for two different ionospheres, I and II, as (Khazanov 1979):

Region I:

$$\begin{aligned} \mu \frac{\partial \Phi^{(1)}}{\partial s} = & n_{01} \exp \left(- \int_0^s \frac{ds}{H_{n1}} \right) \int I_{en}(E, \mu_\chi) [\Phi^{(1)}(\mu') - \Phi^{(1)}(\mu)] d\mu_\chi d\varepsilon \\ & - n_{01} \exp \left(- \int_0^s \frac{ds}{H_{n1}} \right) \sigma_i(E) \Phi^{(1)}(s, E, \mu) \\ & + \frac{1}{4\pi} q_{01} \exp \left(- \int_0^s \frac{ds}{H_{n1}} \right) \left[1 - \frac{1}{2} P_2(\mu_\chi) P_2(\mu) \right]. \end{aligned} \quad (5.4.2)$$

Region II:

$$\begin{aligned} \mu \frac{\partial \Phi^{(2)}}{\partial s} = & n_{02} \exp \left(- \int_s^{s_0} \frac{ds}{H_{n1}} \right) \int I_{en}(E, \mu_\chi) \left[\Phi^{(2)}(\mu') - \Phi^{(2)}(\mu) \right] d\mu_\chi d\varepsilon \\ & - n_{02} \exp \left(- \int_s^{s_0} \frac{ds}{H_{n1}} \right) \sigma_i(E) \Phi^{(2)}(s, E, \mu) \\ & + \frac{1}{4\pi} q_{01} \exp \left(- \int_s^{s_0} \frac{ds}{H_{n1}} \right) \left[1 - \frac{1}{2} P_2(\mu_\chi) P_2(\mu) \right]. \end{aligned}$$

To compute photoelectron fluxes, it was proposed by Khazanov (1979) that expansion in Legendre polynomials could be used

$$\Phi(s, E, \mu) = \sum_m \Phi_m(s, E) P_m(\mu). \quad (5.4.3)$$

Then the system of the following equations for the determination of the coefficients Φ_m can be found (Khazanov 1979)

$$\begin{aligned} \frac{m+1}{2m+3} \frac{\partial \Phi_{m+1}^{(1)}}{\partial y} + \frac{m}{2m+1} \frac{\partial \Phi_{m-1}^{(1)}}{\partial y} + n_{01} H_{n1} \left\{ 2\pi \int_{-1}^1 I_{en} [P_m(\mu_\chi) - 1] d\mu_\chi - \sigma_i \right\} \Phi_m^{(1)} \\ + \frac{q_{01} H_{n1}}{8\pi} (2m+1) \int_{-1}^1 \left[1 - \frac{1}{2} P_2(\mu_\chi) P_2(\mu) \right] P_m(\mu) d\mu = 0, \\ \frac{m+1}{2m+3} \frac{\partial \Phi_{m+1}^{(2)}}{\partial y} + \frac{m}{2m+1} \frac{\partial \Phi_{m-1}^{(2)}}{\partial y} - n_{02} H_{n2} \left\{ 2\pi \int_{-1}^1 I_{en} [P_m(\mu_\chi) - 1] d\mu_\chi - \sigma_i \right\} \Phi_m^{(2)} \\ - \frac{q_{02} H_{n2}}{8\pi} (2m+1) \int_{-1}^1 \left[1 - \frac{1}{2} P_2(\mu_\chi) P_2(\mu) \right] P_m(\mu) d\mu = 0. \end{aligned} \quad (5.4.4)$$

The solution of the system of homogeneous equations (5.4.4) was investigated by Khazanov (1979), and presented in the following form

$$\begin{aligned}
\Phi^{(1)}(y, \mu) &= \sum_{m=0}^M \alpha_m^{(1)} P_m(\mu) \\
&\quad + \sum_{m=0}^M \sum_{k=1}^{(M+1)/2} \left[C_{k1}^{(1)} U_{m,k}(\lambda_k^{(1)}) e^{y/\lambda_k^{(1)}} + C_{k2}^{(1)} U_{m,k}(-\lambda_k^{(1)}) e^{-y/\lambda_k^{(1)}} \right] P_m(\mu), \\
\Phi^{(2)}(y, \mu) &= \sum_{m=0}^M \alpha_m^{(2)} P_m(\mu) \\
&\quad - \sum_{m=0}^M \sum_{k=1}^{(M+1)/2} \left[C_{k1}^{(2)} U_{m,k}(\lambda_k^{(2)}) e^{-z/\lambda_k^{(2)}} + C_{k2}^{(2)} U_{m,k}(-\lambda_k^{(2)}) e^{z/\lambda_k^{(2)}} \right] P_m(\mu).
\end{aligned} \tag{5.4.5}$$

Here the following notations are introduced:

$$\begin{aligned}
y &= \exp \left(- \int_0^s \frac{ds}{H_{n1}} \right); \quad z = \exp \left(- \int_s^{s_0} \frac{ds}{H_{n1}} \right); \\
\alpha_m^{(i)} &= \frac{2m+1}{2} \frac{q_{0i}}{4\pi\sigma_i n_{0i}} \int_{-1}^1 \left[1 - \frac{1}{2} \frac{\sigma_i}{\sigma_i + \sigma_{en}^{(m)}} P_2(\mu_v^{(i)}) P_2(\mu) \right] P_m(\mu) d\mu; \\
\sigma_{en}^{(m)} &= 2\pi \int_{-1}^1 I_{en}(1 - P_m(\mu_\chi)) d\mu_\chi;
\end{aligned}$$

$C_{k(1,2)}^{(i)}$ are the integration constants; and $\lambda_k^{(i)}$ and $U_{m,k}$ are fundamental numbers and fundamentals vectors of the Jacobian matrix whose elements a_m and c_m are defined by the following relationship

$$a_m = \frac{m}{2m+1} \frac{1}{b_{m-1}}, \quad c_m = \frac{m}{2m-1} \frac{1}{b_m}, \quad b_m = n_0 H_n (\sigma_i + \sigma_m).$$

Taking into account that the solution (5.4.5) is limited at $s \rightarrow \pm\infty$ ($y, z \rightarrow \infty$), the integration constants $C_{k1}^{(1)} = C_{k2}^{(2)} \equiv 0$. In the region of the closed magnetic field lines, other integration constants can be found from the conditions of linking the Region I and II solutions at the upper ionospheric boundaries, $y, z \rightarrow 0$:

$$\sum_{m=0}^M \sum_{k=1}^{(M+1)/2} \left[C_{k2}^{(1)} U_{m,k}(-\lambda_k^{(1)}) + C_{k1}^{(2)} U_{m,k}(\lambda_k^{(2)}) \right] P_m(\mu) = \sum_{m=0}^M (\alpha_m^{(2)} - \alpha_m^{(1)}). \tag{5.4.6}$$

In the case of the open magnetic field lines, the integration constants can be found from the condition of the absence of superthermal electron flow downward at altitudes where $y \rightarrow 0$:

$$\Phi^{(-)}|_{y \rightarrow 0} = 2\pi \int_{-1}^0 \Phi \mu d\mu,$$

which, when using one of the solutions (5.4.5) (e.g., for Region I), can be written in a form

$$\sum_{m=0}^{(M-1)/2} \sum_{k=1}^{(M+1)/2} C_{k2}^{(1)} \left[\frac{1}{3} U_{2m+1,k} \delta_{1,k} - U_{2m,k} \left(-\frac{1}{2} \right)^{m+1} \frac{(2m-3)!!}{(m+1)!} \right] = \sum_{m=0}^M \alpha_m^{(1)} I_m,$$

where

$$I_m = \int_0^{-1} \mu P(\mu) d\mu = \begin{cases} 1/2, & m=0 \\ 0, & m=2n+1 \\ \left(-\frac{1}{2} \right)^{n+2} \frac{(2n-1)!!}{(1/2-n)(n+1)!}, & m=2n \end{cases} \quad (5.4.7)$$

In a specific case of symmetrical conditions of illumination in both Regions I and II, the integration constants $C_{k2}^{(2)}$ and $C_{k1}^{(2)}$ are equal to zero, and solutions of (5.4.5) have the following simple form:

$$\Phi^{(i)}(\mu) = \sum_{m=0}^M \alpha_m^i P_m(\mu), \quad i = 1, 2. \quad (5.4.8)$$

In Fig. 5.5, in relative units, examples are given of angular distributions of the photoelectron source (curve 1) and the spectra are established in the collision process of photoelectrons, which are defined by (5.4.8). Here $\mu_v^{(1,2)} = 1$ and curves

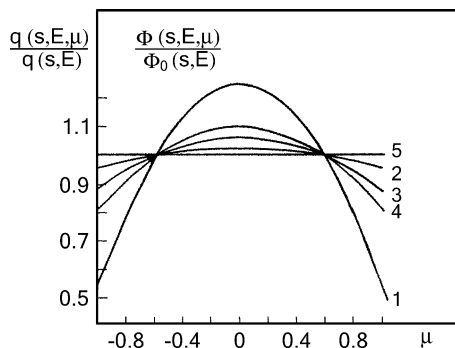


Fig. 5.5 Angular photoelectron distribution with symmetrical conditions of illuminations in the magetoconjugate regions

2–4 apply to energies 20, 50, and 100 eV, respectively. As one sees from the plots presented, the most noticeable differences in solution from the spherically symmetrical (curve 5) is found in the region of large energies where the relative role of elastic collisions leading to photoelectron isotropization drops in comparison with the low energy part of the spectrum.

Figure 5.6 (for conditions $q_{02} = 0$; $\mu_v^{(1,2)} = 1$; and $E = 50$ eV) illustrates an example of forming angular distribution in the hemisphere illuminated by the sun as photoelectrons reach the upper ionospheric boundary. The superthermal electrons shown in the region $\mu = -1 - 0$ are the result of electron scattering by the neutral atmosphere of shaded Region II. Figure 5.7 demonstrates the pitch-angle

Fig. 5.6 Pitch-angle photoelectron distribution at the different altitudes in the illuminated ionosphere. 1 $y = 0$; 2 $y = 0.01$; 3 $y = 0.03$; 4 $y = 0.05$; 5 $y = 0.07$; and 6 $y = 0.1$

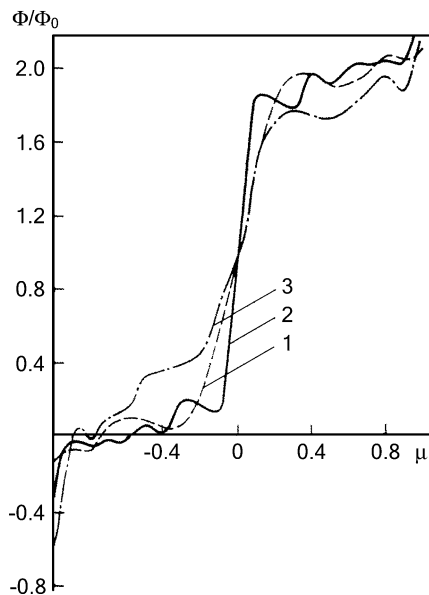
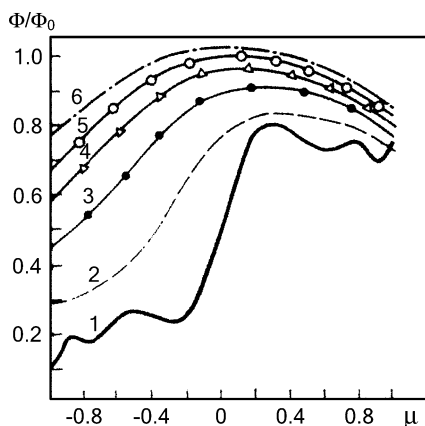


Fig. 5.7 The pitch-angle photoelectron distribution in the polar ionosphere

distribution of photoelectrons at the upper boundary of the polar ionosphere. Here we show the calculation of pitch-angle distribution using a different number of Legendre polynomials, M .

5.5 Transport in the Plasmasphere

The two main processes controlling the behavior of superthermal electrons in the plasmasphere are their motion in the inhomogeneous geomagnetic field and Coulomb collisions with the background thermal plasma. Direct observations have indicated that the motion of superthermal electrons in the plasmasphere is not collisionless (Galperin and Mulyarchik 1966), even though the mean free path of electrons with energies above a few electron volts exceeds substantially the spatial scale of variation of the geomagnetic field and the thermal plasma density. Sanatani and Hanson (1970) and Nagy and Banks (1970) gave some qualitative discussion of electron trapping and the resulting increase in plasmaspheric heating, but the first attempts of quantitative calculations were those of Gastman (1973), Takahashi (1973), and Lejeune and Wormser (1976).

The Boltzmann, kinetic equation for superthermal electrons at altitudes greater than approximately 1,000 km can be written as (Khazanov 1979):

$$\mu \frac{\partial f}{\partial s} + \frac{1 - \mu^2}{2\sigma} \frac{\partial \sigma}{\partial s} \frac{\partial f}{\partial \mu} = \frac{An_e}{E^2} \left\{ E \frac{\partial f}{\partial E} + \frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial f}{\partial \mu} \right] \right\}, \quad (5.5.1)$$

where μ is the cosine of the pitch angle, $f = \Phi/E$ is the distribution function of the superthermal electrons, Φ is the flux of the electrons, E is the energy of the electrons, and $\sigma(s) = B(s_0)/B(s)$, $B(s_0)$, and $B(s)$ are the magnetic field intensities at the boundary between the ionosphere and plasmasphere and a distance s along a given field line, respectively (there is no clear boundary between the ionosphere and plasmasphere, so some appropriate value needs to be selected for a given calculation). The terms on the left-hand side of (5.5.1) describe the change in the distribution of the electrons as they move in an inhomogeneous magnetic field, whereas the terms on the right-hand side correspond to the change in energy and pitch angle due to collisions with thermal electrons and ions.

When there are no collisions present and the source of electrons is the ionosphere, the distribution function at any point along the field line is uniquely determined by the distribution at the foot of the field, at $-s_0$, because of the conservation of the magnetic moment (first adiabatic invariant). The relationship between the cosine of the pitch angle, μ , at a given point, s , along the field line and that at s_0 , written as μ_{s0} , is

$$\mu_{s0} = \sqrt{1 - \sigma(s)(1 - \mu^2)}. \quad (5.5.2)$$

Given that μ_{s0} is real, the range of pitch angles at s is

$$1 - \frac{1}{\sigma(s)} \leq \mu^2 \leq 1. \quad (5.5.3)$$

Inequality (5.5.3) characterizes a family of trajectories in the (s, μ) plane, corresponding to Region I in Fig. 5.8. This figure clearly shows how the pitch angle decreases as the electrons move from the ionosphere toward the magnetic equator and then increases as they move down into the conjugate ionosphere. Particles corresponding to this region can move freely from one hemisphere to the conjugate one, and have been denoted as “precipitating,” “free,” or “fly-through” electrons.

Electrons trapped by the geomagnetic field are defined by the inequality

$$\mu^2 \leq 1 - \frac{1}{\sigma(s)}. \quad (5.5.4)$$

These particles, which correspond to Region II in Fig. 5.8, move in the (s, μ) space in closed circuits, along trajectories L_{II} . The reflection points, s_{ref} , are uniquely determined by the value of the pitch-angle cosine, μ_0 , at the equator ($s = 0$), which is

$$\mu_0 = \frac{\mu}{|\mu|} \sqrt{1 - \frac{\sigma(s)}{\sigma_0} (1 - \mu^2)}, \quad (5.5.5)$$

where $\sigma_0 = \sigma(0) = B(s_0)/B(0)$. The reflection point is determined from the condition $\mu = 0$:

$$\sigma(s_{ref}) = \sigma_0 (1 - \mu_0^2). \quad (5.5.6)$$

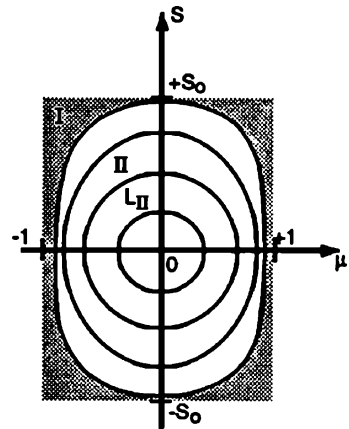


Fig. 5.8 Illustration of the trapped or capture {II} and precipitation “fly-through” {I} zones in the s - μ plane. The *solid lines* are representative trajectories of trapped electrons

In the collisionless case, the parameters of the superthermal electrons in the two zones are independent, because there is no interchange of particles between Regions I and II. The electron population in the trapped region is the result of direct production of superthermal electrons in this zone. In the plasmasphere, these sources are negligible, therefore it is possible to assume that $f_{II} = 0$. In this case, the spatial variations of the thermal electron heating rate due to these superthermal electrons can be written as (Khazanov 1979):

$$Q_e = Q_e(s = \pm s_0) \left[1 - \sqrt{1 - 1/\sigma(s)} \right]. \quad (5.5.7)$$

In the derivation of (5.5.7), it was assumed that the superthermal flux is isotropic at $s = \pm s_0$ and the integration over m took into account inequality (5.5.4).

The above discussion neglected the effects of collisions in the plasmasphere; taking into account collisions produces significantly different results, even when the conventional mean free path of the superthermal electrons $\lambda \sim E^2/An_e$ is greater than the characteristic length of the field tube $H_\sigma \sim \sigma/d\sigma/ds$. This can be demonstrated by looking at the second term on the right-hand side of (5.5.1), which accounts for the pitch-angle diffusion:

$$\frac{An_e}{2E^2} \frac{\partial}{\partial \mu} [(1 - \mu^2) \frac{\partial f}{\partial \mu}] \sim \frac{An_e}{E^2} \frac{f}{\Delta \mu}, \quad (5.5.8)$$

where $\Delta \mu$ is the width of the “fly-through” zone at $s = 0$ ($\sigma = \sigma_0$). The value of $\Delta \mu$ can be written as

$$\Delta \mu = 1 - \mu_{0b}, \quad (5.5.9)$$

where μ_{0b} corresponds to that pitch angle at the equator that results in reflection at $s = s_0$. Equation (5.5.9) can be evaluated by using (5.5.6), and recognizing the fact that μ_{0b} is close to unity, giving:

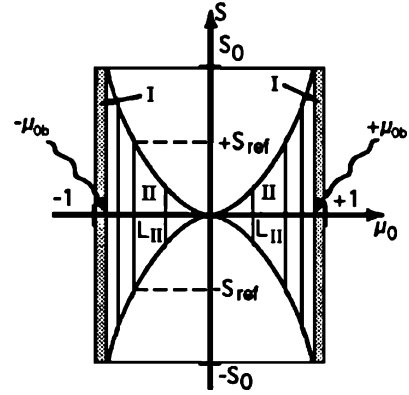
$$\Delta \mu \approx \frac{1}{2\sigma_0}. \quad (5.5.10)$$

Substituting this back into the right-hand side of (5.5.8) shows that pitch-angle diffusion causes changes in the distribution function over a length scale of $\lambda/2\sigma_0 \sim H_\sigma$. Given that $\sigma_0 \gg 1$ for mid- and high-latitude field tubes, pitch-angle diffusion needs to be taken into account even when the conventional mean free path is long compared with the field tube dimension. This can be also seen by recognizing the fact that the pitch-angle width of Region I is very narrow near the equator and thus even small angle deflections will result in electron trapping.

Equation (5.5.1) can be rewritten in terms of μ_0 and s , giving

$$\mu \frac{\partial f}{\partial s} = \frac{An_e}{2E^2} \frac{\sigma}{\sigma_0} \frac{\mu}{\mu_0} \frac{\partial}{\partial \mu_0} \left[\frac{\mu}{\mu_0} (1 - \mu_0^2) \frac{\partial f}{\partial \mu_0} \right] + \frac{An_e}{E} \frac{\partial f}{\partial E}, \quad (5.5.11)$$

Fig. 5.9 Illustration of precipitation and trapped zones and representative trajectories in the s - μ_0 plane



where μ is a function of μ_0 and s . By introducing these new variables, the second term on the left-hand side of (5.5.1) is eliminated. The region over which $f(s, \mu_0)$ is defined in terms of s and μ is shown in Fig. 5.9. In the rest of this section, we solve (5.5.11) by considering the symmetry between the two hemispheres, $f(-s_0 + s, \mu_0) = f(s_0 - s, -\mu_0)$, the long mean free path $\lambda \gg H_\sigma$, and field tubes with $\sigma_0 \gg 1$.

5.5.1 Infinite Trapped Zone Approximation

Krinberg and Matafonov (1978) assumed that superthermal electrons that have scattered from Region I (the “fly-through region”) into Region II (the trapped region) will not scatter back out again. These authors calculated the plasmaspheric transparency, Π , defined as:

$$\Pi(E) = \frac{\int_0^1 \mu f(E, \mu, s_0) d\mu}{\int_0^1 \mu f(E, \mu, -s_0) d\mu} = \frac{\int_{\mu_{ob}}^1 \mu_0 f(E, \mu_0, s_0) d\mu_0}{\int_{\mu_{ob}}^1 \mu_0 f(E, \mu_0, -s_0) d\mu_0}, \quad (5.5.12)$$

where μ_{ob} is pitch-angle cosine at the boundary between the trapped and precipitating electrons. In the above definition of the transparency, only untrapped electrons were considered, thereby neglecting the interaction between the fly-through and trapped electron regions. We shall examine the solution to the kinetic equation (5.5.11), using an infinite capture zone approximation, with the aim of comparing this solution with a more general treatment of the problem to be described in

subsequent sections and to assess the effect of the approximation introduced by the assumption of an infinite volume for Region II.

It is possible to neglect the second term on the right-hand side of (5.5.11) for electrons that are in or near Region I, because pitch-angle diffusion is much more important than energy loss (Khazanov et al. 1981). Therefore, under these conditions, it is relatively easy to show that (5.5.11) can be written as:

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial x} \left(x \frac{\partial f}{\partial x} \right), \quad (5.5.13)$$

where

$$x = 1 - \mu_0, \quad z = \left(E_n^2 / E^2 \right) x_{0b}, \quad E_n^2 = 2A \int_{-s_0}^s \sigma(s') n_e(s') ds'.$$

It was noted before that μ_0 is close to unity, and this was used in obtaining (5.5.13). An examination of (5.5.11) shows that pitch-angle diffusion becomes important over a length scale of λ/σ , requiring that $\sigma \gg 1$ for significant pitch-angle change; this in turn means that $\mu \sim 1$ (see (5.5.3)). The values of μ_{0b} and x_{0b} , the parameters of the boundary between the free and trapped zones, can be determined from (5.5.6) to be:

$$\mu_{0b} = \sqrt{1 - \frac{1}{\sigma_0}} \approx 1 - \frac{1}{2\sigma_0} \quad (5.5.14)$$

and

$$x_{0b} = 1 - \mu_{0b} \approx \frac{1}{2\sigma_0}. \quad (5.5.15)$$

Equation (5.5.13) is solved under the boundary condition

$$f(z=0, x) = f^+(x) \Theta(x_{0b} - x), \quad (5.5.16)$$

where $f^+(x)$ is the distribution function of superthermal electrons, injected into the plasmasphere from the ionospheric level $-s_0$, and Θ is the step function. The other boundary conditions used are:

$$f(z, x=0) \text{ is bounded,} \quad (5.5.17)$$

$$f(z, x \rightarrow \infty) \rightarrow 0. \quad (5.5.18)$$

Boundary condition (5.5.18) requires some explanation, because the domain of definition of x is between 0 and 1. The use of an upper limit of infinity can only be justified in the case of sharp gradients of the distribution function with respect to μ_0 . In the case under consideration here, the region close to $x = 1$ may be regarded as

being infinitely far from the interval in x between 0 and x_{ob} . The assumption that the particles injected from ionospheric levels will concentrate in the μ_o, s plane near their original value of μ_o , does justify this assumption, sometimes referred to as the “infinite capture zone.”

The solution to (5.5.13) can be written as (see Appendix F):

$$f(x, z) = \int_0^{x_{ob}} f^+(\zeta) G(x, \zeta, z) d\zeta, \quad (5.5.19)$$

where the Green function, $G(x, \zeta, z)$, is given by (F9) and z is a dummy variable. As an example, let us consider that electrons are injected at $s = -s_o$ in a direction parallel to the magnetic field, therefore:

$$f^+(\zeta) = \delta(\zeta). \quad (5.5.20)$$

Substitution of (F9) and (5.5.20) into (5.5.19) for the distribution function yields:

$$f(x, z) = \frac{1}{z} \exp\left(-\frac{x}{z}\right). \quad (5.5.21)$$

Equation (5.5.21) indicates that a strong gradient in pitch angle is present at locations where:

$$z = \frac{E_n^2}{2\sigma_0 E^2} \ll 1. \quad (5.5.22)$$

It is inequality (5.5.22) that, in fact, defines the parameters of the field tubes and the energy of superthermal electrons, for which the infinite capture zone approximation holds true. This inequality corresponds to conditions when the mean free path is much longer than the plasma scale height.

Next we determine the expression for plasmaspheric transparency, when $f^+(z)$ is a weak function of z . First we define $\bar{f}(z)$ to be the mean value of $f(x, z)$ in the interval between 0 and x_{ob} for this case of slowly varying f^+ :

$$\bar{f}(z) \equiv \frac{1}{x_{ob}} \int_0^{x_{ob}} f(x, z) dx. \quad (5.5.23)$$

We now substitute (5.5.19) for $f(x, z)$ into (5.5.23):

$$\begin{aligned} \bar{f}(z) &= \frac{1}{x_{ob}} \int_0^{x_{ob}} \int_0^{x_{ob}} f^+(\zeta) G(x, \zeta, z) d\zeta dx \\ &= \bar{f}^+ \frac{1}{x_{ob}} \int_0^{x_{ob}} \int_0^{x_{ob}} G(x, \zeta, z) d\zeta dx. \end{aligned} \quad (5.5.24)$$

Next we use (5.5.12), which defines the transparency, Π , and recognize that between μ_{ob} and 1, $\mu_0 \sim 1$; therefore we can write:

$$\bar{f}^+ \Pi = \int_{\mu_{ob}}^1 \mu_o f(E, \mu_o, s_o) d\mu_o = \int_{\mu_{ob}}^1 f(E, \mu_o, s_o) d\mu_o = \bar{f}(z). \quad (5.5.25)$$

Recalling the relation:

$$\int_0^{x_{ob}} J_o(2k\sqrt{x}) dk = \frac{\sqrt{x_{ob}}}{k} J_1(2k\sqrt{x_{ob}}), \quad (5.5.26)$$

the expression for the transparency can be written as

$$\begin{aligned} \frac{\partial \Psi}{\partial z} &= \frac{\partial}{\partial x} \left(x \frac{\partial \Psi}{\partial x} \right) \Pi(z) = 2 \int_0^\infty \frac{\exp(-k^2 z)}{k} J_1^2(2k\sqrt{x_{ob}}) dk \\ &= 1 - \Phi\left(\frac{3}{2}, 2; \frac{4x_{ob}}{z}\right) \exp\left(-\frac{4x_{ob}}{z}\right), \end{aligned} \quad (5.5.27)$$

where Φ is the degenerate hypergeometric function (Gradshteyn and Ryzhik 1980). In this case, where f^* was assumed to be a weak function of angle, the capture zone approximation is valid only when inequality (5.5.22) is satisfied; this is a limitation on calculating the transparency given by (5.5.27). The numerical values and specific variations described by (5.5.27) are discussed in this section below.

5.5.2 The Distribution Function of Superthermal Electrons in the Plasmasphere

In this section, the transport of superthermal electrons in the plasmasphere is studied, but this time the trapped/capture zone is considered to be finite, that is, scattering back from the trapped (II) into the fly-through (I) zone is taken into account. Let us denote the distribution function of the electrons in zone I as f_I , and obtain a solution for it from (5.5.13), using the initial condition given by (5.5.16). The boundary condition at $x = 0$ ($\mu_0 = \pm 1$) is the boundedness of f_I . The boundary condition at $x = x_{ob}$ is assumed to be the function $f_b(E)$, which is taken to be constant along the trajectory $x = x_{ob}$ (Khazanov 1979). We can write f_I as

$$f_{I(x,z)} = \Psi_{(x,z)} + f_b. \quad (5.5.28)$$

Using (5.5.13), this leads to

$$\frac{\partial \Psi}{\partial z} = \frac{\partial}{\partial x} \left(x \frac{\partial \Psi}{\partial x} \right). \quad (5.5.29)$$

The boundary conditions can be summarized as

$$\Psi(z = 0, x) = f^+(x) - f_b, \quad (5.5.30)$$

$$\Psi(z, x = 0) \text{ bounded}, \quad (5.5.31)$$

$$\Psi(z, x = x_{ob}) = 0. \quad (5.5.32)$$

Using separation of variables to solve (5.5.29) yields:

$$f_I(x, z) = f_b + \sum_{n=1}^{\infty} \exp\left(-\frac{\alpha_n^2 z}{4x_{ob}}\right) J_o(\alpha_n \sqrt{\frac{x}{x_{ob}}}) \times \left\{ \frac{1}{J_1^2(\alpha_n x_{ob})} \int_0^{x_{ob}} f^+(x_o) J_o\left(\alpha_n \sqrt{\frac{x_o}{x_{ob}}}\right) dx_o - f_b \frac{2}{\alpha_n J_1(\alpha_n)} \right\}, \quad (5.5.33)$$

where $J_o(z)$ and $J_1(z)$ are Bessel functions and the α_n s are the eigenvalues of the initial problem, defined by

$$J_o(\alpha_n) = 0; \quad \alpha_n \cong \frac{\pi}{4}(4n - 1); \quad n = 1, 2, \dots \quad (5.5.34)$$

The pitch-angle distribution of ionospheric electrons (generally photoelectrons or backscattered secondary electrons) escaping into the plasmasphere is generally characterized by a weak variation in pitch angle over the interval $x_{ob} < x < 1$ (Khazanov and Gefan 1982). Given this characteristic of f^+ , it can be moved outside the integral in (5.5.33) and this simplifies the expression for f_I to:

$$f_I(z, x) \approx f_b + \sum_{n=1}^{\infty} \exp\left(-\frac{\alpha_n^2 z}{4x_{ob}}\right) J_o(\alpha_n \sqrt{\frac{x}{x_{ob}}}) \frac{2(\bar{f}^+ - f_b)}{\alpha_n J_1(\alpha_n)}, \quad (5.5.35)$$

where:

$$\bar{f}^+ = \frac{1}{x_{ob}} \int_0^{x_{ob}} f^+(x') dx' \quad (5.5.36)$$

is the pitch angle averaged distribution function of the superthermal electrons injected into the plasmasphere from the ionosphere. A similar pitch angle averaging of (5.5.35) for f_I gives:

$$\bar{f}_I(x) = f_b + (\bar{f}^+ - f_b) \sum_{n=1}^{\infty} \frac{4}{\alpha_n^2} \exp\left(-\frac{\alpha_n^2}{4} \frac{z}{x_{ob}}\right). \quad (5.5.37)$$

Given that $\alpha_1^2 \ll \alpha_2^2 \ll \dots$, the main contribution to the infinite sum in (5.5.37) is the first term when z/x_{ob} is large. When z/x_{ob} is small, the contribution from the rest of the terms is appreciable, because:

$$\frac{4}{\alpha_1^2} \approx 0.72 \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{4}{\alpha_n^2} = 1.$$

Therefore in order to get an approximation for arbitrary z/x_{ob} , it is assumed that $4/\alpha_1^2 \sim 1$. Thus (5.5.37) becomes:

$$\bar{f}_I(y_s) \approx f_b + (\bar{f}^+ - f_b) \exp\left(-\frac{1}{y_s}\right), \quad (5.5.38)$$

where:

$$y_s = \frac{E^2}{E_n^2(s)}.$$

Next we look at the solution to (5.5.11) in Region II, the so-called trapped or capture zone. If we assume that $f(s, \mu_o)$ varies only slightly along its trajectory (denoted by L_{II} in Fig. 5.5.2) (Khazanov et al. 1979), integrating (5.5.11) over this trajectory makes the left-hand side of the equation approach zero, because the distribution function is continuous. Therefore the bounce-averaged kinetic equation for this capture zone can be written as (Polyakov et al. 1979)

$$\frac{\partial}{\partial \mu_o} \left\{ a(\mu_o) [1 - \mu_o^2] \frac{\partial \bar{f}_{II}}{\partial \mu_o} \right\} + 2B(\mu_o)E \frac{\partial \bar{f}_{II}}{\partial E} = 0, \quad (5.5.39)$$

where:

$$a(\mu_o) = \int_0^{S_{ref}(\mu_o)} \frac{\sigma(s)}{\sigma_o} \mu n_e(s) ds, \quad B(\mu_o) = \mu_o \int_0^{S_{ref}(\mu_o)} n_e \frac{ds}{\mu},$$

and $\bar{f}_{II}$ is the mean value of $f(s, \mu_o)$ along the trajectory L_{II} . If the ratio $a(\mu_o)/B(\mu_o)$ is assumed to be independent of μ_o , and its value taken to be 0.5, (5.5.39) can be simplified to

$$\frac{\partial}{\partial x} \left(x \frac{\partial \bar{f}_{II}}{\partial x} \right) + 2E \frac{\partial \bar{f}_{II}}{\partial E} = 0. \quad (5.5.40)$$

It has been shown both analytically and numerically that the solutions to (5.5.39) and (5.5.40) are very similar in the region near the boundary between the trapped and fly-through zone (Khazanov et al. 1979).

Next we will obtain the distribution function, f_{II} , in the trapped zone, using (5.5.40). This equation is not appropriate in the region $x \rightarrow 1$ ($\mu_o \rightarrow 0$), but most of the trapped electrons are found in the region near $x = x_{ob} \ll 1$ ($\mu_o = \mu_{ob}$), therefore this limitation will not cause a problem. The boundary conditions to be used to solve (5.5.40) are:

$$f_{II}(x = x_{ob}) = f_b, \quad \left. \frac{\partial f_{II}}{\partial x} \right|_{x=1} = 0, \quad \text{and} \quad f_{II}(E \rightarrow \infty) \rightarrow 0. \quad (5.5.41)$$

We write $f_{II}(x, E) = \Psi(x, E) + f_b$ and the new boundary problem takes the form:

$$\begin{aligned} \frac{\partial}{\partial x} \left(x \frac{\partial \Psi}{\partial x} \right) + \frac{E}{2} \frac{\partial \Psi}{\partial E} + \frac{E}{2} \frac{\partial f_b}{\partial E} &= 0, \\ \Psi(x = x_{ob}) &= 0, \quad \left. \frac{\partial \Psi}{\partial x} \right|_{x=1} = 0, \quad \text{and} \quad \Psi(E \rightarrow \infty) \rightarrow 0. \end{aligned} \quad (5.5.42)$$

The general solution to (5.5.42) is obtained in Appendix G, where it is also shown that a good approximation to the solution is:

$$f_{II}(E, x) = f_b \frac{\ln x}{\ln x_{ob}} + d \left\{ 1 - \frac{\ln x}{\ln x_o} \right\} y^d \int_y^\infty f_b(y') \frac{dy'}{y^{d(1+d)}}, \quad (5.5.43)$$

where $d = [\ln(1/x_{ob})]^{-1}$ and $y = E^2/E_{no}^2$, where y is the same variable as introduced with (5.5.38), with $E_{no} = E_n(s_o)$. The solutions we obtained for the distribution functions in both the trapped (capture) and fly-through (free) regions (5.5.43) and (5.5.38), respectively, contain the arbitrary function $f_b(E)$. Therefore we will now see how an expression for f_b can be obtained.

A continuity equation for the superthermal electrons is obtained by integrating the kinetic equation (5.5.1) with respect to m from -1 to 1 , which is

$$\frac{\partial \sigma \eta_E}{\partial s} = \frac{A n_e}{E} \frac{\partial n_E}{\partial E}, \quad (5.5.44)$$

where

$$\eta_E = \int_{-1}^1 \mu f d\mu \quad \text{and} \quad n_E = \int_{-1}^1 f d\mu.$$

Integrating (5.5.44) with respect to s , from $-s_o$ to $+s_o$, and taking into account the fact that $s_o \gg 1$ and that the main contribution to the integral on the right-hand side comes from the value of $n_E(s)$ at the top of the flux tube, n_{Eo} (Khazanov and Gefan 1982), we get:

$$\eta_E^+ = -\frac{\partial n_{Eo}}{\partial y}, \quad \eta_E^+ = \int_0^1 \mu(f^+ - f^-)d\mu \approx \frac{\bar{f}^+ - \bar{f}^-}{2}, \quad (5.5.45)$$

$$n_{Eo} = 2 \int_0^1 f(s=0)d\mu \approx 2 \int_{x_{ob}}^1 f_{II}(x)dx.$$

In (5.5.45), the rate at which superthermal electrons enter the plasmasphere from the ionosphere, η_E^+ , is the mean of that entering and escaping. Note that in the determination of the total superthermal electron density at the magnetic equator, $s = 0$, it was assumed that the contribution from those in the fly-through region is negligible. Using these various approximations in (5.5.43), the following expression can be obtained for n_{Eo} :

$$n_{Eo} \approx 2dy^d \int_y^\infty f_b \frac{dy'}{y'^{(1+d)}}. \quad (5.5.46)$$

Combining (5.5.38), (5.5.45), and (5.5.46), we can obtain the differential equation involving f_b :

$$\frac{\partial \Phi_o}{\partial y} - \varphi \Phi_o = 2d \frac{\partial \bar{f}^+}{\partial y}, \quad (5.5.47)$$

where

$$\Phi_o = (\bar{f}^+ - f_b)(y\xi + 2d), \quad \xi(y) = \frac{(1 - e^{-1/y})}{2} \quad \text{and} \quad \phi = \frac{d\xi}{y\xi + 2d}.$$

Given that $d \ll 1$ and that the essential variations of $\xi(y)$ take place for $y > 1$, $f(y)$ can be written as $d/(4d + y)$. The expression for f_b can then be written as:

$$f_b = \bar{f}^+ \frac{y}{y + 4d} + \frac{2d^2(4d + y)^d}{y\xi + 2d} \int_y^\infty \frac{\bar{f}^+ dy'}{(y' + 4d)^{d+1}}. \quad (5.5.48)$$

Equations (5.5.38), (5.5.43), and (5.5.48) are the necessary relations for determining the distribution functions of the superthermal electrons in the plasmasphere. However, one should note that in order to obtain the necessary solutions, one needs to know the distribution function of the electrons escaping from the ionosphere, but the self-consistent calculation of that requires the plasmaspheric distribution at the interface. Therefore a self-consistent simultaneous solution for both the ionosphere and the plasmasphere is needed. This can be achieved either by iteratively matching analytic or numerical ionospheric solutions with the analytic solutions described above, or by self-consistent analytic or numerical solutions for both the ionosphere and plasmasphere. The latter is possible, but will not be presented in this section; it

will be the subject of a later discussion. We will now examine the nature of the plasmaspheric electrons using an assumed form for the ionospheric electrons entering the plasmasphere.

5.5.3 Plasmaspheric Transparency and Heating Rate

Let us consider the example in which the electrons moving into the plasmasphere from the ionosphere have an exponential energy distribution [$\sim \exp(-E/E_o)$]. In this case, (5.5.48) becomes

$$f_b = \frac{y + 4d\Delta}{y + 4d} \bar{f}^+, \quad (5.5.49)$$

where $\Delta = 1$ when $E_o \gg E_{no}$ and $\Delta = E_o^2/E_{no}^2$ when $E_o \ll E_{no}$.

We can now write an expression for the plasmaspheric transparency, Π , which we defined earlier (5.5.12) as the ratio of superthermal electrons escaping from the plasmasphere over those entering. In the present approximation, the transparency becomes

$$\Pi = \bar{f}^+ / \bar{f}^- = \frac{2\xi(y + 4d\Delta)}{y + 4d} + \exp(-1/y). \quad (5.5.50)$$

Equation (5.5.50) clearly shows that the transparency approaches unity as y gets large, or, in other words, high-energy electrons pass through the plasmasphere, without significant absorption. The transparency also remains finite at low energies. The solid line in Fig. 5.10 shows the value of the transparency for $d = 0.18$, $\gamma = 5$, and $\Delta = 2.5 \times 10^{-3}$, which can be compared with the dashed line, which corresponds to values obtained from (5.5.50) with the Δ term neglected, and the dash-dot line, which corresponds to the infinite trapped (capture) zone approximation. It can be clearly seen that this infinite trapped zone approximation has only a limited range of applicability, and is inaccurate at low energies. Figure 5.11 shows the

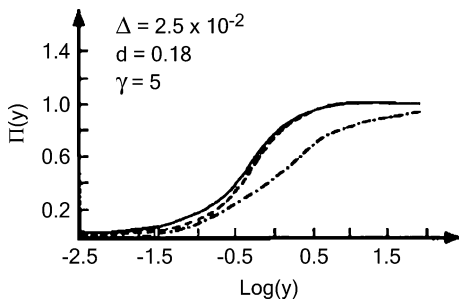


Fig. 5.10 Comparison of calculated transparencies using an infinite (dash-dot curve) and finite trapped (solid curve corresponds to (5.5.50) with finite Δ and dashed curve with $\Delta = 0$) zone approximations

Fig. 5.11 Comparison of calculated transparencies using the approximate analytic expression, given by (5.5.50) (*dashed line*), and the results from a numerical solution of the full kinetic equation, given by (5.5.1) (*solid line*)

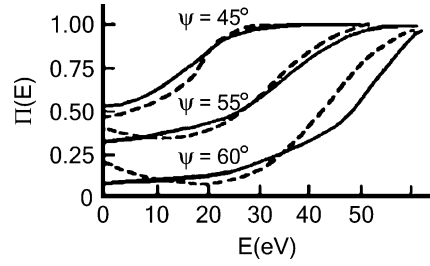


Table 5.1 Parameters used for evaluating (5.5.50)

f	$R_e \times 10^4$ (km)	σ_0	$2d$	E_{n0} (eV)	Δ	E_0 (eV)
45	1.3	12.6	0.79	15	0.30	10
55	2.1	48.7	0.44	30	0.10	10
60	2.7	115.0	0.36	59	0.03	10

calculated transparencies for a number of different geomagnetic latitudes obtained from both the analytic expression given by (5.5.50) and the numerical solution of (5.5.1) (Khazanov et al. 1979). In the calculations, the thermal electrons and ions were assumed to be in diffusive equilibrium along the field line with $n_e(-s_o) = 5 \times 10^3 \text{ cm}^{-3}$ and $T_e + T_i = 7 \times 10^3 \text{ K}$. Table 5.1 gives the various parameters that were used in evaluating the analytic solution given by (5.5.50). Figure 5.11 clearly indicates that the analytic approximation agrees well with the numerical solution of (5.5.1) for the same conditions.

Next we calculate the heating rate resulting from the passage of these super-thermal electrons through the plasmasphere (Khazanov et al. 1979)

$$Q_e = 2\pi A n_e \int_0^\infty \int_{-1}^1 f(E, \mu) d\mu dE. \quad (5.5.51)$$

Using (5.5.38) and (5.5.49), the distribution function for the free electrons can be written as:

$$\bar{f}_I(y) = \bar{f}^+ \left\{ \frac{y + 4d\Delta}{y + 4d} (1 - \exp[-1/y]) + \exp[-1/y] \right\}. \quad (5.5.52)$$

Combining (5.5.43) and (5.5.49), we get the distribution function of the trapped electrons:

$$f_{II} = \bar{f}^+ \frac{y + 4d\Delta}{y + 4d} \times \left\{ \frac{\ln x}{\ln x_{ob}} + 2d\gamma^{2d} y^d (\gamma\sqrt{y})^{-(2d-1)/2} \exp\left[\frac{\gamma\sqrt{y}}{2}\right] W_{-(2d+1)/2, -d}(\gamma\sqrt{y}) \right\}, \quad (5.5.53)$$

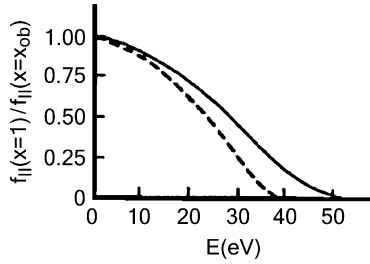


Fig. 5.12 Plot of the ratio of the distribution function at $x = 1$ over that at $x = x_{ob}$ in the trapped zone, as a function of the energy. The *solid lines* correspond to the approximate analytic result given by (G7) and the *dashed line* to the “exact” numerical solution obtained by Khazanov et al. (1979)

where $W_{a,b}$ is the Whittaker function (cf. Gradshteyn and Ryzhik 1980). It can be shown using (5.5.53) that most of the superthermal electrons in the trapped zone, having energies $E \gg E_o$, concentrate near the boundary trajectory, $x = x_{ob}(\mu_o = \mu_{ob})$ (see Fig. 5.12).

Using (5.5.52) and (5.5.53), we can calculate the separate heating rates due to both the free and trapped electrons, Q_{el} and Q_{ell} , respectively, which are:

$$Q_{el} = Q_{-s_o} \frac{1}{\frac{1}{d} \frac{E_o^2}{E_{no}^2} + 1} \left\{ \exp(-\eta_E^+) + \exp(-\eta_E^-) + \frac{1}{d} \frac{E_o^2}{E_{no}^2} \right\} \left[1 - \sqrt{1 - \frac{1}{\sigma}} \right], \quad (5.5.54)$$

$$Q_{ell} = Q_{-s_o} \frac{2E_o^2}{E_{no}^2} \left\{ \sqrt{1 - \frac{1}{\sigma}} - \sqrt{1 - \frac{\sigma}{\sigma_o}} \right\}, \quad (5.5.55)$$

where Q_{-s_o} is the heating rate at the base of the plasmasphere, $\eta_E^{+/-} = 2(E_{+/-}/E_o)$, $E_+^2 = E_n^2$ and $E_-^2 = E_{no}^2 - E_n^2$. Adding (5.5.54) and (5.5.55) together gives the expression for the total heating rate:

$$Q_e = Q_{el} + Q_{ell} = Q_{-s_o} \frac{1}{\frac{1}{d} \frac{E_o^2}{E_{no}^2} + 1} \left\{ \left[\frac{1}{d} \frac{E_o^2}{E_{no}^2} + \exp(-\eta_E^+) + \exp(-\eta_E^-) \right] \left(1 - \sqrt{1 - \frac{1}{\sigma}} \right) + \frac{E_o^2}{E_{no}^2} \left[\sqrt{1 - \frac{1}{\sigma_o}} - \sqrt{1 - \frac{\sigma}{\sigma_o}} \right] \left(\frac{1}{d} \frac{E_o^2}{E_{no}^2} + 1 \right) \right\}. \quad (5.5.56)$$

If we take $\sigma_0 \gg 1$ $S_{ee}(f_\varepsilon, f_\varepsilon)$ (this is easily satisfied for mid- and high-latitude field tubes), for the variation of the thermal electron heating due to the passage of superthermal electrons along a geomagnetic field line, we obtain the simple expression

$$Q_e \approx Q_{-s_o} \left\{ 1 + \frac{1}{d} \frac{E_o^2}{E_{no}^2} \right\}^{-1} \left\{ \frac{E_o^2}{E_{no}^2} \left(\frac{1+d}{d} \right) + 2 \exp(-\eta^+) \right\}. \quad (5.5.57)$$

The above result was obtained for the flux entering the plasmasphere, f^+ , having an exponential form ($\sim \exp\{-E/E_o\}$). It can be shown that if f^+ is given by a delta function ($\delta\{E - E_o\}$) and $d \ll 1$, (5.5.53) and (5.5.57) also hold in the limit of $d \ll 1$. This means that if the exponential approximation for f^+ is not a good one, the input flux can be approximated by a series of delta functions and the corresponding distribution function for the trapped population and heating rate can be “built up” using (5.5.53) and (5.5.57). The approximation of $d \ll 1$ corresponds to a very wide trapping zone, which is satisfied in general.

Finally we can obtain expressions for the integrated heating rates along the field line due to the trapped and fly-through electrons:

$$Q_\sigma = \int_{-s_o}^0 \sigma Q_e ds. \quad (5.5.58)$$

Integrating (5.5.55) we obtain:

$$Q_{\sigma II} \approx Q_{-s_o} \frac{E_o^2}{2A\bar{n}_e}, \quad (5.5.59)$$

where

$$\bar{n}_e = \frac{\int_{-s_o}^0 \sigma n_e ds}{\int_{-s_o}^0 \sigma ds}.$$

In order to calculate $Q_{\sigma I}$, the function

$$\sigma \left[1 - \sqrt{1 - \frac{1}{\sigma}} \right]$$

can be assumed to be equal to 0.5 and the function $\eta_E^+(s)$ written as $Fs^{0.33}$, where:

$$F = \left[\frac{16An_e\sigma_o}{E_o^2} \right]^{1/3}.$$

Under these assumptions, the integrated heating rate due to superthermal electrons in Region I is

$$Q_{\sigma I} \approx Q_{-s_o} \frac{3E_o^2}{16\sigma_o An_e}. \quad (5.5.60)$$

Comparison of (5.5.59) and (5.5.60) indicates that the main contribution to the integrated heating rate comes from the trapped superthermal electrons.

We derived analytic expressions for the distribution functions of superthermal electrons in the plasmasphere. These results were then used to obtain simple expressions for the transparency of the plasmasphere to the passage of these electrons and the resulting heating of the thermal electron population. As expected, the transparency decreases with decreasing energies and the thermal plasma heating is due mainly to the trapped superthermal population. These results will be very useful in future ionospheric model calculations, because they will allow the inclusion, in a quantitative manner, of the energy deposition in the plasmasphere and subsequent heat input into the ionosphere. Additionally, the expression for transparency obtained in this work will make quantitative calculations of the effects of conjugate photoelectrons possible. Clearly a more accurate and self-consistent set of calculations would require the numerical and simultaneous solutions of the electron transport equations in both the ionosphere and plasmasphere. Such calculations will be carried out in Chap. 7. However, the use of the results presented here will, by themselves, lead to better and more quantitative model calculations.

5.6 Thermal Electron Heating Rate

The photoelectrons are the major contributor to the heating rate of the thermal electrons in the ionosphere and plasmasphere. The complex structure of the superthermal electron source and the variety of elementary processes that these electrons undergo do not permit an exact analytical solution of the kinetic equation to be obtained. For this reason, many different numerical models have been created to solve for the transport of superthermal electrons in the ionosphere (e.g., Nagy and Banks 1970; Khazanov 1979; Solomon et al. 1988; Lummerzheim et al. 1989; Strickland et al. 1989; Richards and Torr 1990). However, these models require extensive numerical computations and are not suitable for use in global time-dependent models of the ionosphere and thermosphere. It is common, in the large-scale ionospheric and thermospheric models, to obtain heating rates based on simple parameterizations (e.g., Roble and Ridley 1987; Rasmussen et al. 1988; Schunk and Nagy 2000). A more satisfactory approach would be to use an analytic solution to the kinetic equation in these models, because an analytic solution can be applied in a manner that is consistent with the calculated parameters in the large-scale models. Analytic solutions permit a deeper understanding of the role of the different physical processes taking part in the creation and transport of superthermal electrons (e.g., Nagy and Banks 1970; Krinberg 1978; Khazanov 1979; Khazanov et al. 1994) and allow one to more easily interpret the results of a more complete numerical solution to the kinetic equation.

The purpose of the present section is to develop analytical expressions for the transport of superthermal electrons in the ionosphere, taking into account the possible trapping of photoelectrons in the plasmasphere (Khazanov et al. 1992)

by the geomagnetic field that has been found in Sect. 5.5. Analytical expressions are obtained first by expanding the electron distribution function in a Legendre polynomial series (see Sect. 5.2) and then by solving the resulting equations with a low-order truncation.

To carry out an analytical solution of the kinetic equation (5.2.1), the following simplified assumptions (Khazanov 1979) are valid above approximately 200 km:

1. The neutral atmosphere is isothermal and consists of atomic oxygen with a concentration of $n = n_0 \exp(-h/H_n)$, where H_n is the atomic oxygen scale height.
2. Major inelastic processes include the excitation of the ^1D level of atomic oxygen and the ionization of O . The corresponding cross-sections, σ_1^* (excitation) and σ_2^* (ionization), are approximated by analytical expressions of the form

$$\sigma_1^*(E) = \begin{cases} 0, & E < E_1 \\ \sigma_1 E_1/E, & E \geq E_1 \end{cases}, \quad \sigma_2^*(E) = \begin{cases} 0, & E < E_2 \\ \sigma_2, & E \geq E_2 \end{cases}, \quad (5.6.1)$$

where $E_1 = 2$ eV is the excitation threshold, $E_2 = 15$ eV is the ionization threshold, $\sigma_1 = 3 \times 10^{-17}$ cm², and $\sigma_2 = 10^{-16}$ cm².

3. The elastic cross-section for the scattering of superthermal electrons by atomic oxygen does not depend on energy and is equal to $\sigma_2 = 10^{-15}$ cm².
4. The production rate $q(E)$ of superthermal electrons is assumed to have rotational symmetry and is approximated by the exponential form,

$$q(E) = \frac{q_e}{2EE_0} \exp(-E/E_0), \quad (5.6.2)$$

where $E_0 = 10$ eV is the mean energy of the primary spectrum and q_e is the total ionization rate.

5. The diffusive approximation is used. This is equivalent to truncating the polynomial expansion in (5.2.2) at two coefficients, i.e.,

$$f(\mu, E, s) = f_0(E, s) + f_1(E, s)\mu.$$

In this case, the zero coefficient $f_0(E)$ determines the concentration of superthermal electrons in a given energy interval, while $f_1(E)$ determines the flux. This approximation is valid only for a weak anisotropy of the pitch-angle distribution of superthermal electrons (i.e., $f_0 \gg f_1 \gg f_2 \dots$). This is not true at low energies near the upper boundary of the ionosphere, where the transparency of the plasma-sphere is reduced due to intensive trapping of low-energy superthermal electrons ($E < \sqrt{2\delta}E_{n0}$) (see Khazanov et al. 1992) and $\bar{f}^- \ll \bar{f}^+$ (the downgoing distribution of electrons is much less than the upgoing distribution) is valid. However, we have found in numerical calculations that the quantity $f_0(E)$, and therefore the mean characteristics of the superthermal electron distribution function (such as the

concentration of superthermal electrons and the rate of heating of thermal electrons), differs only slightly in this approximation from a more accurate solution.

Given the assumptions listed above, the system of equations (5.2.3) are:

$$-\frac{\sin I}{3} \frac{\partial f_1}{\partial h} + \frac{n\sigma_1 E_1^2 + An_e}{E} \frac{\partial f_0}{\partial E} + n \left[\sigma_2^*(E_2') \frac{E_2'}{E} f_0(E_2') - \sigma_2^*(E) f_0(E) \right] + \frac{q_e}{2EE_0} e^{-E/E_0} = 0, \quad (5.6.3a)$$

$$\sin I \frac{\partial f_0}{\partial h} = -n\sigma f_1, \quad (5.6.3b)$$

where $E_2' = E + E_2$. In (5.6.3a), the discrete form of the inelastic integral describing the excitation level ^1D is replaced by a differential form in the approximation of continuous energy loss:

$$E_1' \sigma_1^*(E_1') f_0(E_1') - E \sigma_1^*(E) f_0(E) \approx \frac{\partial [\sigma_1^*(E) f_0(E)]}{\partial E} E_1 = \sigma_1 E_1^2 \frac{\partial f_0}{\partial E},$$

where $E_1' = E + E_1$. This substitution is justified by the condition $(\partial f_0 / \partial E) / f_0 \approx 1/E_0 \ll 1/E_1$, which allows us to restrict the expansion of the function $E_1' \sigma_1^*(E_1') f_0(E_1')$ in a Taylor series about E , to only the second term. In (5.6.3b), the contribution of inelastic collisions has been neglected ($\sigma_{1,2}^* \ll \sigma$), as well as the contribution of Coulomb collisions. The influence of Coulomb collisions becomes important for energies smaller than $\sqrt{An_e H_n}$, which, for $N_e \approx 10^5 \text{ cm}^{-3}$, represents 1.5–2.5 eV. Because $\sqrt{AN_e H_n} \ll E_0$, on the average, this influence will be unimportant.

The solution of (5.6.3) will now be carried out separately for the two energy intervals $E \geq E_2$ and $E \leq E_2$, where E_2 is the energy of ionization. Just as with the numerical method described in Sect. 5.2, we start with the higher energy interval and proceed to the lower. Because of the relative strengths of the pertinent cross-sections, certain processes can be neglected in the higher energy range, for instance, the excitation of ^1D . The solution obtained for the higher energies will then be used as an energy boundary condition to obtain a solution in the lower energy interval.

Assuming that $E \geq E_2$ and neglecting Coulomb impact and processes leading to the excitation of the ^1D level of atomic oxygen, the initial system of equations can be transformed to

$$\frac{\partial^2 f_0}{\partial z^2} + \alpha(E) f_0 - \alpha(E + E_2) \frac{E + E_2}{E} f_0(E + E_2) - \frac{e^{-E/E_0}}{E} = 0, \\ f_1 = \sqrt{3} \frac{\partial f_0}{\partial z},$$

where $z = \sqrt{3}nH_n\sigma$; $\alpha = \sigma_2/\sigma$; $A = q_e/2n\sigma E_0 \neq A(z)$. The solution of this system of equations, satisfying the lower boundary conditions (5.2.9), is

$$f_0(h, E) = f_{00} - f_{1\infty} \sqrt{\frac{\sigma}{\sigma^*}} e^{-\sigma' n H_n}, \quad f_1(h, E) = f_{1\infty} e^{-\sigma' n H_n}, \quad (5.6.4)$$

where $f_{00} = q_e e^{-E/E_0} / 2n\sigma^* E E_0$ is the local part of the solution, $\sigma^* = \sigma_2 (1 - e^{-E_2/E_0})$, and $\sigma' = \sqrt{3\sigma^*\sigma}$.

The parameter $f_{1\infty}$ determines the outgoing flux of superthermal electrons into the plasmasphere and is calculated from the upper boundary conditions. To determine the boundary conditions for the case under consideration, we shall proceed from the expression for the transparency Π of the plasmasphere (Khazanov et al. 1992). Then, repeating the procedure preceding the derivation of (5.2.16) and setting $N = 0$, one can obtain

$$f_{1\infty} = \frac{3}{2} \frac{1 - \Pi}{1 + \Pi} f_{0\infty} = \frac{3}{2} \frac{f_{0\infty}}{1 + E^2 / \delta E_{n0}^2}, \quad (5.6.5)$$

where the index ∞ corresponds to the upper boundary level $z \rightarrow 0 (h \rightarrow \infty)$. From (5.6.4) and (5.6.5), it follows that

$$f_{0\infty} = f_{00} \frac{E^2 + \delta E_{n0}^2}{E^2 + \delta E_{n0}^2 \left(3\sqrt{\sigma/\sigma^*}/2 + 1 \right)},$$

$$f_{1\infty} = \frac{3}{2} f_{00} \frac{\delta E_{n0}^2}{E^2 + \delta E_{n0}^2 \left(3\sqrt{\sigma/\sigma^*}/2 + 1 \right)}.$$

It can be seen that an outgoing flux of superthermal electrons into the plasmasphere leads to a significant decrease in the zero expansion coefficient and therefore in the concentration of superthermal electrons in regions above the outgoing level h_0 , where $n(h = h_0) = 1/\sigma'H$. Due to the large value of the ratio σ/σ^* , the influence of this process is significant even when relatively little trapping of superthermal electrons occurs in the plasmasphere ($E^2 \gg \delta E_{n0}^2$).

Let us calculate the secondary ionization rate of neutral oxygen by superthermal electrons. Because secondary ionization is important at altitudes lower than 300 km, we use the local approximation of (5.2.3), $f_0(h, E) = f_{00}$, to calculate Δq_e . This leads to

$$\Delta q_e = 2n\sigma_2 \int_{E_2}^{\infty} E f_0 dE = q_e \frac{1}{e^{E_2/E_0} + 1}. \quad (5.6.6)$$

It can be seen that the magnitude of secondary ionization, for the given values of E_0 and E_2 , represents approximately 20% of the total ionization rate. However, in the lower ionosphere (below 140 km), an increase in the mean energy of the initial superthermal electron spectrum and corresponding enhancement of the role of the secondary ionization, occurs, because these altitudes cannot be reached by solar radiation with long wavelengths.

Let us consider the solution of the system of equations (5.6.3) in the region where the energy is less than the ionization energy E_2 . In this instance, and in accordance with (5.6.1), $\sigma_2^*(E_2') = \sigma_2$ and $\sigma_2^*(E) = 0$. The quantity $f_0(E_2')$ is determined from (5.6.4), assuming $E = E_2$. We shall estimate the contribution of the superthermal electrons from the region $E \geq E_2$: the maximum contribution of the term $n\sigma_2^*(E_2')f_0(E_2')E_2'/E$ in (5.6.4a) can be compared with the photoelectron source, $q_e \exp(-E/E_0)/(2EE_0)$, by using the local part of (5.6.4) ($f_{00} \geq f_0$). One can see that this term is insignificant: $(E + E_2) \exp(-E_2/E_0) \ll E_0 [1 - \exp(E_0/E_2)]$. The excitation of ^1D is neglected as well; its influence will be taken into account later. With these assumptions (5.6.4) can be rewritten as

$$\begin{aligned} x \frac{\partial f_1}{\partial x} + \frac{1}{y} \frac{\partial f_0}{\partial y} + A_0 x \frac{e^{-\alpha y}}{y} &= 0, \\ \frac{\partial f_0}{\partial x} &= f_1, \end{aligned} \quad (5.6.7)$$

where $x = n_0 e^{-h/H_n} \sigma H_n / \sin I$; $y = E/E_i$; $E_i^2 = 3An_{e0}H_n$; $\alpha = E_i/E_0$; $A_0 = 3q_e/2n\sigma E_i E_0 \neq A(x)$. The level $h = 0$ ($x = 0$) is chosen from the condition

$$n_0 = n(h = 0) = \frac{\sin I}{\sigma H_n}. \quad (5.6.8)$$

The electron concentration $n_e = n_{e0}$ is approximated here as independent of altitude, because its altitude variations are considerably smaller than the variations of neutral oxygen.

In view of the fact that the superthermal electron source has an exponential character, the region determined by the variable E may be extended toward infinity without any particular decrease in accuracy of the solution. The energy boundary condition then becomes

$$f_0(E \rightarrow \infty) \rightarrow 0. \quad (5.6.9)$$

In addition, assuming that at the upper boundary of the ionosphere ($h \rightarrow \infty$ ($x \rightarrow 0$)) the superthermal electron flux is given (i.e., the quantity $f_{1\infty}(y)$ is known; it will be determined later when the influence of the plasmasphere is taken into consideration), (5.6.7) can be rewritten in the form

$$x \frac{\partial^2 \Psi}{\partial x^2} + \frac{1}{y} \frac{\partial \Psi}{\partial y} + \frac{x}{y} Q(y) = 0,$$

where

$$\Psi(x, y) = f_0(x, y) - x f_{1\infty}(y),$$

$$Q(y) = A_0 e^{-\alpha y} + \frac{\partial f_{1\infty}}{\partial y},$$

and with boundary conditions,

$$\Psi(y \rightarrow \infty) = 0,$$

$$\frac{\partial \Psi}{\partial x}(x \rightarrow 0) = 0.$$

The eigenfunctions $M_k(x)$ of the obtained problem are expressed by Bessel functions: $M_k(x) = \sqrt{x}J_1(2k\sqrt{x})$, with a continuous spectrum of eigenvalues k . When expressed as Green's functions, the solution is

$$\Psi(x, y) = \int_y^\infty dy_0 \int_0^\infty Q(y_0)G(x, y, x_0, y_0)dx_0,$$

where

$$G = [2\sqrt{xx_0}/(y_0^2 - y^2)]I_{-1}[4\sqrt{xx_0}/(y_0^2 - y^2)] \exp[-2(x + x_0)/(y_0^2 - y^2)];$$

and $I_{-1}(t)$ is the modified Bessel function. Let us carry out the integration with respect to x_0 (Gradshteyn and Ryzhik 1980):

$$G'(x, y, y_0) = \int_0^\infty Gdx_0 = \frac{y_0^2 - y^2}{2} \lim_{c \rightarrow \infty} e^{-u} c \Phi(1, c, u). \quad (5.6.10)$$

Here $\Phi(a, c, u)$ is the degenerated hypergeometric function and $u = 2x/(y_0^2 - y^2)$. Evaluating the limit, $\lim_{c \rightarrow \infty} e^{-u} c \Phi = e^{-u} + u$, in (5.6.10) finally yields

$$f_0(x, y) = \frac{A_0}{\alpha} x e^{-\alpha y} + \int_y^\infty Q(y_0) \frac{y_0^2 - y^2}{2} e^{-\frac{2x}{y_0^2 - y^2}} dy_0. \quad (5.6.11)$$

The unknown parameter $f_{1\infty}$ is included in (5.6.11). It can be determined from the results of the calculation of the superthermal electron distribution function in the plasmasphere. To do this, (5.6.11) is evaluated in the region $h \rightarrow \infty (x \rightarrow 0)$, and the resulting expression is differentiated with respect to y . We obtain

$$\frac{\partial f_{0\infty}}{\partial y} - y f_{1\infty} = -\frac{A_0 y}{\alpha} e^{-\alpha y},$$

where $f_{0\infty} = f_0(x = 0, y)$. Substituting $f_{1\infty}$ from (5.6.5), we obtain an equation for $f_{0\infty}$, to which the solution is

$$f_{0\infty} = (1 + \beta y^2)^\gamma \int_y^\infty \frac{A_0}{\alpha} e^{-\alpha y_0} \frac{y_0 dy_0}{(1 + \beta y_0^2)^\gamma}, \quad (5.6.12)$$

where $\beta = E_i/\delta E$; $\gamma = 3/4\beta$; and $\alpha = E_i/E$.

Let us estimate the values of the parameters α , β , and γ included in (5.6.12). On estimating the quantity $E_i^2 \cong 5 \times 10^{-5} n_{e0}$, the value of the electron concentration n_{e0} (assumed in our model to be independent of altitude) should be taken at the level of the outgoing superthermal electrons ($x = 1$) from condition (5.6.8). Because the value of n_e at this level ($h \sim 300$ km; $n_0 \approx 10^8 \text{ cm}^{-3}$) is $(1-5) \times 10^5 \text{ cm}^{-3}$, that of the square of the energy is $E_i^2 = 5 - 25 \text{ eV}$. Comparing this quantity with data from Khazanov et al. (1992), one can see that $\beta \ll 1$, $\alpha \ll 1$, and $\gamma \gg 1$. As a consequence of this, the integral in (5.6.12) may be evaluated approximately. Integrating by parts,

$$f_{0\infty} = \frac{2}{3} \frac{A_0}{\alpha} e^{-\alpha y} (1 + \beta y^2) \left[1 - \frac{2\alpha}{3y} (1 + \beta y^2) \right],$$

$$f_{1\infty} = \frac{A_0}{\alpha} e^{-\alpha y} \left[1 - \frac{2\alpha}{3y} (1 + \beta y^2) \right].$$

From here the function $Q(y)$ in (5.6.11) can be determined as:

$$Q(y) = \frac{2A_0}{3\alpha} e^{-\alpha y} \left(\frac{\alpha}{y} + \frac{1}{y^2} + \alpha\beta y - 3\beta \right). \quad (5.6.13)$$

The function (5.6.13) is less dependent on the argument y function than the exponential multiplier in (5.6.11). Therefore, this multiplier can be approximated by the step-function, $\theta(y_0 - y_b)$, where $y_b = \sqrt{y^2 + 2x}$, and the integration in (5.6.11) can be carried out by replacing the lower limit y with y_b :

$$f_0(x, y) = \frac{A_0}{\alpha} e^{-\alpha y} \left\{ x + \frac{2}{3} e^{-\alpha(y_b - y)} \left[(1 + \beta y^2) \left(1 + \frac{\alpha x}{y_b} \right) \right] \right\}. \quad (5.6.14)$$

The ultimate expressions for f_0 at $y^2 \gg 2x$ and $y^2 \ll 2x$ are:

$$f_0 \Big|_{y^2 \ll 2x} \approx x + \frac{2}{3} e^{-\alpha\sqrt{2x}} (1 + \beta y^2) \left(1 + \alpha\sqrt{\frac{x}{2}} \right), \quad (5.6.15)$$

$$f_0 \Big|_{y^2 \gg 2x} \approx x + \frac{2}{3} e^{-\frac{\alpha x}{y}} (1 + \beta y^2) \left(1 + \alpha\frac{x}{y} \right) \approx x + \frac{2}{3} (1 + \beta y^2). \quad (5.6.16)$$

Expression (5.6.15) can be reduced to the same form as (5.6.17), in view of the fact that significant variations (as a function of the argument x) in the second term in (5.6.15), occur when $x \geq 1/\alpha^2 \gg 1$, i.e., when the magnitude of this term is considerably smaller than that of the first term. Thus, (5.6.11) can be written as

$$f_0(x, y) = \frac{A_0}{\alpha} x e^{-\alpha y} + f_{0\infty}, \quad (5.6.17)$$

with

$$f_{0\infty} = \frac{2}{3} \frac{A}{\alpha} e^{\alpha y} (1 + \beta y^2).$$

Let us consider the influence of the excitation of the ^1D level of atomic oxygen on the form of the distribution function (5.6.17). It is important to take this into account at altitudes where $n\sigma_1 E_1^2 \geq An_e$ or $n \geq 10^{10} \text{ cm}^{-3}$, as follows from (5.6.3). Because this region is located considerably lower than the outgoing level (where $x = 1$), $f_1 \ll f_0$ and losses from the excitation of ^1D need only be taken into consideration for the local part of the solution to (5.6.3), i.e., f_0 . Then instead of the (5.6.7) we obtain the equation,

$$(1 + \alpha x) \frac{\partial f_0}{\partial y} + A_0 x e^{-y} = 0,$$

where $a = n_0 \sigma_1 E_1^2 / AN_e = 3\sigma_1 E_1^2 \sin I / \sigma E_I^2 \ll 1$. The solution of this equation at altitudes where $x \ll 1/a$, corresponds to the local part of (5.6.17). Correcting it yields:

$$f_0(x, y) = \frac{A_0 x}{\alpha(1 + ax)} e^{-\alpha y} + f_{0\infty}. \quad (5.6.18)$$

Let us carry out the evaluation of the heating rate Q_e of thermal electrons in the ionosphere. In accordance with Khazanov (1979), the magnitude of Q_e is determined by the zero coefficient f_0 :

$$Q_e = 2An_e \int_0^\infty f_0 dE.$$

Most heating of the thermal electrons comes from the low-energy portion of the superthermal spectrum; integrating (5.6.4), the result is:

$$Q_e = Q_{eL} + Q_{eNL}, \quad (5.6.19a)$$

where

$$\begin{aligned} Q_{eL} &= E_s q_e, & E_s &= \frac{E_0}{1 + ax}; \\ Q_{eNL} &= a_s n_e, & a_s &= \frac{2J_\infty E_0}{3\sigma H_n n_{e0}} \left(1 + \frac{4E_0^2}{\delta E_n^2} \right). \end{aligned} \quad (5.6.19b)$$

Here, Q_{eL} is the heat generated in the region by superthermal electrons created locally by EUV radiation, Q_{eNL} is the nonlocal heat due to the transport of

superthermal electrons into the region from lower altitudes and from the plasmasphere, and $J_\infty = q_e(x=0)/n(x=0)$ is the ionization frequency. The nonlocal portion of the heating rate is always greater than zero in this region, because of the significant number of photoelectrons that are transported up from the relatively dense atmospheric region below.

The mean energy E_s , locally transmitted to the thermal electrons from the superthermal ones, is smaller than the mean energy of the initial spectrum E_0 . This is a result of the transfer of part of the energy of the superthermal electrons to excite atomic oxygen. As altitude increases ($x \rightarrow 0$), the intensity of this process diminishes and the superthermal electron energy completely transfers to the thermal electrons.

The approximate solution (5.3.19) that describes the energy deposition of the photoelectrons to the thermal electrons has been compared (not shown here) with a numerical solution of this value presented by Khazanov et al. (1994) in their Fig. 8 and agrees with these results with an accuracy of approximately 15–20%.

Let us calculate the magnitude of the column heating rate $Q_{\sigma 0}$ of the thermal electrons in the plasmasphere. This is total amount of thermal energy that is deposited in the plasmasphere by photoelectrons and eventually returned to the ionospheres through heat conduction carried by the thermal electron population. Based on (5.5.59) in the previous section:

$$Q_{\sigma 0} = Q_{-s_0} \frac{E_0^2}{2AN_e},$$

where Q_{-s_0} represents the nonlocal heating rate at the bottom of the plasmasphere (taken from (5.6.19b)), we obtain

$$Q_{\sigma 0} = \frac{1}{3} \frac{J_\infty E_0^3}{\sigma H_n N_{e0} A}. \quad (5.6.20)$$

Expression (5.6.20) represents the total energy flux of the thermal electrons that is deposited to the ionosphere. As we show in Chap. 6 of this book, the usage of the thermal electron energy source presented by (5.6.19) and (5.6.20) in the electron temperature calculations (under geomagnetically quiet conditions), agrees well with T_e measured by Thomson scatter radar.

In conclusion of this section, let us evaluate the distribution function for superthermal electrons and the thermal electron heating rate in the equatorial ionosphere. In this case, the substitution $\partial/\partial s = \sin I \partial/\partial h - (\cos I/R_E) \partial/\partial \varphi$ should be made in (5.6.3) rather than $\partial/\partial s = \sin I \partial/\partial h$, in accordance with Koen (1983) (R_E is the radius of the Earth). The quantity $\partial f_{0,1}/\partial \varphi$ should be neglected as well in comparison with $f_{0,1}$. Then, taking into account that for small values of latitude φ , $\sin I \approx 2\varphi$, and $\cos I \approx 1$, (5.6.3) can be represented by

$$\frac{\partial}{\partial h} \left[\frac{\sin^2 I}{\sigma n} \frac{\partial f_0}{\partial h} \right] - \frac{2}{\sigma n R_0} \frac{\partial f}{\partial h} + \frac{3AN_e + 3n\sigma_1 E_1^2}{E} \frac{\partial f_0}{\partial E} + \frac{3q_e}{2EE_0} e^{-E/E_0} = 0. \quad (5.6.21)$$

Equation (5.6.21) is written for energies $E \leq E_2$; that is why the ionization loss terms are not included in it. Substituting $x = \sigma n_0 \exp(-h/H_n) H_n$, (5.6.21) can be reduced to

$$\frac{\partial}{\partial x} \left(\sin^2 I x^c \frac{\partial f_0}{\partial x} \right) + \frac{x^c}{xy} \frac{\partial f_0}{\partial y} + A_0 \frac{e^{-\alpha y}}{y} = 0. \quad (5.6.22)$$

Here $c = 2H_n/(R_E \sin^2 I)$, whereas the other designations are analogous to those in (5.6.7). In (5.6.22) the excitation of the ^1D level of atomic oxygen was neglected; it can be incorporated later, as shown above.

To solve (5.6.22), we integrate with respect to h from ∞ to h . Assuming that there is no superthermal electron flux at the upper boundary in the equatorial ionosphere, i.e., $(\partial f_0 / \partial h | h \rightarrow \infty) = 0$, we obtain

$$b \frac{\partial f_0}{\partial x} + \frac{1}{y} \frac{\partial f_0}{\partial y} + A_0 \frac{x}{y} e^{-\alpha y} = 0, \quad (5.6.23)$$

where $b = \sin^2 I + 2H_n/R_E$. The solution of (5.6.23) can be written in the form:

$$f_0 = \frac{A_0}{\alpha} x e^{-\alpha y} + \frac{A_0 b}{\alpha^3} e^{-\alpha y} (\alpha y + 1). \quad (5.6.24)$$

This solution consists of two terms, the first of them represents a local solution, whereas the second is a nonlocal one. By correcting the local solution, in the way it was done for (5.6.18), the process of excitation of the ^1D level of atomic oxygen can be taken into account:

$$f_0 = \frac{A_0}{\alpha(1 + ax)} x e^{-\alpha y} + \frac{A_0 b}{\alpha^3} e^{-\alpha y} (\alpha y + 1).$$

Formulas for evaluation of the thermal electron heating rate in the equatorial ionosphere can be obtained by analogy with (5.6.19):

$$Q_e = Q_{eL} + Q_{eNL}, \quad (5.6.25a)$$

where

$$\begin{aligned} Q_{eL} &= E_s q_e, & E_s &= \frac{E_0}{1 + ax}; \\ Q_{eNL} &= a_s n_e, & a_s &= \frac{2J_\infty E_0}{3\sigma H_n n_{e0}} \left(\sin^2 I + \frac{2H_n}{R_E} \right). \end{aligned} \quad (5.6.25b)$$

As before, the coefficient a_s represents the strength of nonlocal heating. Because magnetic field lines are nearly horizontal in the equatorial region ($I \approx 0$) and the

neutral scale height H_n is small in comparison with R_E , a_s in (5.6.25b) is significantly smaller than the corresponding term in (5.6.19b). Thus, nonlocal heating in the equatorial ionosphere is less significant than at higher latitudes. This reflects the morphology of T_e behavior in the upper ionosphere via transition from equatorial toward higher latitudes (Schunk and Nagy 2000).

5.7 The Loss Cone Distribution

In studies of the dynamics of the radiation belt electrons, most attention is, as a rule, devoted to finding the form of the distribution function of the trapped particles and the spectra of the ELF and VLF radiations excited by them (Kennel and Petschek 1966). However, for example, for modeling the magnetosphere–ionosphere interactions, the electron distribution in the loss cone is of basic importance. It is derived in references (Kennel and Petschek 1966; Etcheto et al. 1973) on the basis of a modified equation of pitch-angle diffusion, averaged over the particle bounce oscillations. However, as noted in Davidson and Walt (1977), this approach does not always yield a true description of the particles in the loss cone, and in this range of pitch angles, it is necessary to solve the nonaveraged equation, which we shall do here.

The kinetic equation for outer radiation belt electrons may be represented in the form (Davidson and Walt 1977):

$$\mu v \frac{\partial f}{\partial s} = \frac{B_L}{B} \frac{\partial}{\partial \mu_0} \left[\frac{\mu}{\mu_0} (1 - \mu^2) D \frac{\partial f}{\partial \mu_0} \right] + Q, \quad (5.7.1)$$

where $f(v, \mu_0, s)$ is the distribution function, $\mu = \cos \alpha = v_{\parallel}/v$ is the cosine of the particle pitch angle, $\mu_0 = [1 - (1 - \mu^2)B_L/B]^{1/2}$, B and B_L are the local and equatorial values of the magnetic field at the distance L , Q is the source of the radiation belt particles, and D is the coefficient of pitch-angle diffusion (Kennel and Petschek 1966):

$$D = (\pi e^2 / m^2 c^2) |\mathbf{B}_k|^2 / |v_{\parallel} - \partial \omega / \partial k|. \quad (5.7.2)$$

In the conventional approximation, (5.7.1) describes the interaction of the particles with low-frequency ($\omega \ll \omega_B$) electromagnetic fields propagating along the magnetic field ($\mathbf{k} \parallel \mathbf{B}$) in cyclotron resonance $kv_{\parallel} = \omega - \omega_B$ (here $\omega_B = |eB/mc|$ is the electron gyrofrequency). The particle source Q in the right-hand member of (5.7.1) is assigned in accordance with (Etcheto et al. 1973):

$$Q = \begin{cases} K_p \left(\frac{m}{2\pi E_0} \right)^{3/2} \frac{dn_2}{dt} (\sin \alpha)^p e^{-E/E_0}, & E \geq E_{\min}, \\ 0, & E < E_{\min} \end{cases} \quad (5.7.3)$$

where E_0 is the characteristic electron energy, equal to $\sim 5 \times 10^3 L^{-3}$ keV, $E_{\min} \approx E_0/2$, dn_2/dt is the source intensity, and p is the anisotropy index.

Because the trapped particles play the main role in the generation of the ELF and VLF radiations, the wave spectrum found in (Etcheto et al. 1973) for the source Q given above may be used to describe the distribution in the loss cone. In this case, with the assumption that the wave spectrum is independent of the coordinate along the line of force, we find that the diffusion coefficient (5.7.2) takes the form:

$$D \approx \frac{\sqrt{\pi}(\tau\omega_{B_0})}{2 \ln(1/R)n_{0L}} \frac{dn_2}{dt} K(v, \mu, s), \quad (5.7.4)$$

where

$$K(v, \mu, s) = \frac{v_g U_L}{|v_{||} - v_g|} \frac{(1 + 2x_L)}{2x_L^2} \exp(-U_L^2/x_L), \quad (5.7.5)$$

and

$$x_L = \omega/\omega_{B_L}, \quad v_g = \partial\omega/\partial k, \quad v_0 = \sqrt{2E_0},$$

$$v_{0L} = c\omega_{B_L}/\omega_{0L}, \quad U_L = v_{0L}/v_0, \quad \omega_0 = \sqrt{4\pi en_0/m},$$

n_0 is the concentration of cold plasma, and R is the coefficient of wave reflection from the ionosphere.

On the basis of the dispersion equation for ELF and VLF waves:

$$\frac{k^2 c^2}{\omega^2} = \omega_0^2 \omega^{-1} / (\omega_B - \omega),$$

and the condition for resonance (5.7.5) converts to the expression

$$K(v, \mu, s) = \frac{v^2 \mu^2}{v_0 v_{0L}} \left(\frac{B_L}{B}\right)^4 \frac{n_0}{n_{0L}} \exp\left(-\mu^2 \frac{v^2}{v_0^2} \frac{B_L^3}{B^3} \frac{n_0}{n_{0L}}\right). \quad (5.7.6)$$

From this it follows that the most effective scattering of electrons by waves occurs in the equatorial region of the magnetosphere. Therefore, in (5.7.1) and (5.7.6) we can set approximately $\mu_0 = \mu = 1$. Hence, the kinetic equation for the particles in the loss cone takes the form:

$$\frac{\partial f}{\partial s} = D^* \frac{\partial}{\partial \mu_0} \left[(1 - \mu^2) \frac{\partial f}{\partial \mu_0} \right], \quad (5.7.7)$$

where $D^* = B_L D(v, \mu = 1, s)/(Bv)$. The source (5.7.3) does not enter directly into (5.7.7) (for the most part it provides electrons with large pitch angles), but it is represented implicitly by the coefficient of diffusion (5.7.4). It should also be noted that (5.7.7) is valid only for heights $h \geq 200$ km, where particle collisions can be ignored (Davidson and Walt 1977).

To solve this problem, we shall use a frame of reference along the force line in which the points $s = 0$ and $s = 2s_L$ represent its endpoints at $h = 200$ km; particles with $\mu > 0$ move upward with $s = 0$. Having changed variables in (5.7.7) as:

$$x = 1 - \mu_0 \quad \text{and} \quad y = 2 \int_0^s D^*(v, s') ds',$$

and taking into account the smallness of the parameter x , (5.7.7) can be written as

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial x} \left(x \frac{\partial f}{\partial x} \right). \quad (5.7.8)$$

A similar equation was examined in Khazanov et al. (1981). As in Khazanov et al. (1981), we shall seek a solution of (5.7.8) with initial conditions $f(x, y = 0) = f^+(x)$, where $f^+(x)$ corresponds to the distribution function of electrons moving upward at point $s = 0$. As boundary conditions, the boundedness of $f(x, y)$ with $x = 0$ and its value at the loss cone $f(x = x_{ob}, y) = f_b(E)$, which can be considered to be approximately independent of s ; here $x_{ob} = 1 - \mu_{0b}$ and $\mu_{0b} = \sqrt{1 - L^{-3}}$. In this case, the distribution function in the loss cone has the form (see also Sect. 5.5 for details):

$$f(x, y) = f_b - 2 \sum_{n=1}^{\infty} \left[\exp\left(-\frac{\alpha_n^2}{4x_{ob}}\right) J_0\left(\alpha_n \sqrt{\frac{x}{x_{ob}}}\right) \frac{f_b - f_n^+}{\alpha_n J_1(\alpha_n)} \right], \quad (5.7.9)$$

where

$$f_n^+ = \frac{\alpha_n}{2x_{ob} J_1(\alpha_n)} \int_0^{x_{ob}} f^+(x) J_0\left(\alpha_n \sqrt{\frac{x}{x_{ob}}}\right) dx,$$

$J_0(x)$ and $J_1(x)$ are Bessel functions, and $\alpha_n \approx [(4n - 1)/4]\pi$ ($n = 1, 2, 3, \dots$) represents the spectrum of the eigenvalues determined from the condition $J_0(\alpha_n) = 0$.

It is known from experiments and numerical calculations that, at low velocities, owing to atmospheric absorption, the electron flux moving upward is much smaller than the flux of precipitating particles (Davidson and Walt 1977). This allows the use of the complete absorption approximation when $f_n^+ = 0$. In this approximation,

the distribution of precipitating electrons is considered at ionosphere heights, namely at point $s = 2s_L$ where $y = 2y_L$ (by virtue of the symmetry of the problem, an analogous picture of precipitation will also be observed at point $s = 0$). In the case of a dipole magnetic field, it can be shown that, according to (5.7.4) and (5.7.6):

$$y_L \approx 2D_L \int_0^{s_L} \left(\frac{B_L}{B} \right)^5 \frac{ds}{v} \approx 0.4D_L R_E L / v, \quad (5.7.10)$$

$$D_L = \frac{\sqrt{\pi} \tau \omega_{BL} v^3 n_{0L}^{-1}}{2 \ln(1/R) v_0 v_{0L}} \frac{dn_2}{dt} \exp(-v^2/v_0^2),$$

where D_L is the coefficient of diffusion at the equator.

Denoting $f(x, y = 2y_L)$ as $f^-(x)$, from (5.7.9) and (5.7.10) can be found

$$f^-(x) \approx f_b \left[1 - 2 \sum_{n=1}^{\infty} \exp\left(-\frac{\alpha_n^2 D_L R_E L}{5 v x_{ob}}\right) \frac{J_0\left(\alpha_n \sqrt{\frac{x}{x_{ob}}}\right)}{\alpha_0 J_1(\alpha_n)} \right]. \quad (5.7.11)$$

Let us analyze (5.7.11). It is evident that the pitch-angle distribution tends toward isotropy with increasing D_L , and the equality $f^-(x) = f_b$ is in practice observed with $D_L \geq 10/(T_0 L^3)$ ($T_0/2 = 2LR_E/v$ approximately corresponds to the transit times of particles along the magnetic field line). From this, keeping in mind the dependence of D_L on energy, we find, with $v_0 \approx 10^{10} \text{ cm s}^{-1}$ and $R = 0.1$ (see Etcheto et al. 1973), that isotropy of the distribution $f^-(x)$ of electrons with energy $E \leq E_0$ should be observed with source intensities $dn_2/dt \geq 30L^{-8} \text{ cm}^{-3} \text{ s}^{-1}$, so that for $E \geq E_0$, higher values of dn_2/dt are required. Further, using the relation $dn_2/dt \approx 2.5L^{-1}R_E^{-1}J_p(E \geq E_0)$, we find from (Etcheto et al. 1973) that the indicated source intensities create fluxes of precipitating electrons $J_p(E \geq E_0) \geq 7 \cdot 10^9 L^{-7} \text{ cm}^{-2} \text{ s}^{-1}$. In particular with $L = 5$, the distribution with $E \leq E_0$ becomes isotropic with $dn_2/dt \geq 10^{-4} \text{ cm}^{-3} \text{ s}^{-1}$ and $J_p(E \geq E_0) \geq 10^5 \text{ cm}^{-2} \text{ s}^{-1}$. Note that these computed fluxes occur within the outer radiation belt.

Let us illustrate the results of the analysis at $L = 5$. Figure 5.13 shows the dependence of the pitch-angle distribution of the precipitating electrons $f^-(x)/f_b(E)$ with $E = E_0$ on the equatorial diffusion mode, characterized by the parameter $Z = D_L/T_0$, for values $Z = 10^{-3}$, 5×10^{-3} , 10^{-2} , and 10^{-1} corresponding to the source intensities $dn_2/dt = 10^{-6}$, 5×10^{-5} , 10^{-5} , and $10^{-4} \text{ cm}^{-3} \text{ s}^{-1}$. The parameter λ is introduced for convenience and is related to x by the ratio $\lambda = 10x/x_{ob}$.

Figure 5.14 shows the function $f^-(x)/f_b(E)$ for particle energies $E_0/2$, E_0 , and $2E_0$, with a constant source energy $dn_2/dt = 10^{-5} \text{ cm}^{-3} \text{ s}^{-1}$.

Fig. 5.13 The pitch-angle distribution of the precipitating electrons

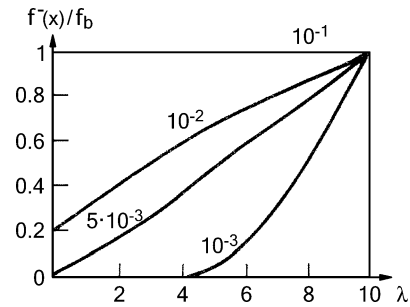
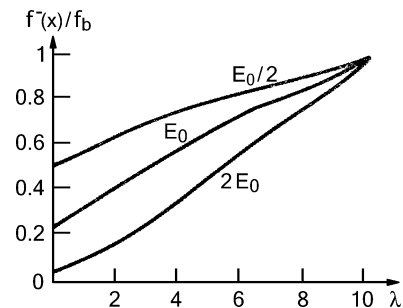


Fig. 5.14 The pitch-angle distribution of the precipitating electrons as the function of E_o



Knowing the intensity of the whistler mode in the inner magnetosphere, the analytical expression (5.7.11) can be coupled with the global energetic electron calculation presented in Chap. 7, and provide the electron flux precipitation on a global scale.

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Chapter 6

Analysis of Cold Plasma Transport

The core plasma of the inner magnetosphere is cold and, potentially, could be handled based on the solutions of the variety of hydrodynamic equations discussed in Chap. 4. The cold plasma of the ionosphere and plasmasphere is the mediator of wave–particle interaction processes and provides the extremely important coupling element between the terrestrial ring current and Earth’s radiation belts. The different parts of the inner magnetosphere also strongly couple to each other and create an additional complexity in the analysis of the numerical models of this system. The use of different sets of transport equations to describe the cold plasma distribution can also lead to different results and create an additional complexity in the interpretation of the role of the physical processes that control the behavior of the core plasma in the ionosphere–magnetosphere system. These difficulties illustrate why approximate analytic solutions of cold plasma transport phenomena are very useful, i.e., because they help us gain physical insight into how the system responds to varying sources of mass, momentum, and energy and also to various external boundary conditions. They also provide a convenient method to test the validity of complicated numerical models.

6.1 Convective Plasma Motion in the Magnetosphere

The solar wind is the primary generator of plasma motion in the magnetosphere. This motion (convection) is formed by the large-scale electric field induced by the interaction of the solar wind with Earth’s magnetosphere (e.g., Stern 1975). The structure of the inner magnetosphere is defined in many respects by this convective plasma motion. For example, the shape and location of the outer boundary of the plasmasphere (the plasmopause) is controlled by magnetospheric convection (Chappell 1972). This control is most evident during magnetically disturbed periods when the convection electric field is intensified, thereby resulting in an erosion of the plasmasphere in the nightside as the plasmopause is shifted toward the Earth.

Nishida (1966) and Brice (1967) have hypothesized that the plasmapause is formed by the combined influence of a large-scale field directed from dawn to dusk in the equatorial plane and a corotation field induced by Earth's rotation. According to this hypothesis, the plasmapause is the interface between two convective regions: inside the plasmapause, plasma is confined within closed force tubes that revolve around the Earth; outside the plasmapause, plasma is confined to force tubes, which are eventually open. Despite some refinements (e.g., Lemaire 1975), this hypothesis describes the general features of the shape of the plasmapause, including a dawn–dusk asymmetry.

In some theoretical works (Marubashi and Grebowsky 1976; Marubashi 1979; Murphy et al. 1980; Klimenko and Namgaladze 1980; Koen et al. 1980; Rasmussen and Schunk 1990; Bailey and Sellek 1990), the three-dimensional structure of the plasmasphere was simulated, taking into consideration magnetospheric convection. Largely because the volume of a convecting magnetic force tube varies, convective transport was shown to affect substantially the altitude distribution of electron density in the plasmasphere. This redistribution was found to be most evident near the plasmapause.

In general, theoretical study of the effects of magnetospheric convection on the dynamics and energetics of the plasmasphere is sufficiently difficult that some simplification is often necessary. In the works cited above, for example, the plasma concentration was calculated numerically for prescribed distributions of charged-particle temperatures. Another approach is to consider the two-dimensional problem created by examining plasma parameters only in the equatorial plane of the magnetosphere (e.g., Rasmussen et al. 1993). In many cases, in solving such a two-dimensional problem, it is possible to carry out the analysis analytically. Such an analytical approach is used in the present chapter.

6.2 Density Distribution

6.2.1 *Model of the Electric Field*

The external electric field $\mathbf{E}$ is the main factor defining convective motions of low-energy (thermal) plasma. The perpendicular component $\mathbf{u}_\perp$ is given by the electric drift $\mathbf{u}_\perp = \mathbf{u}_E = c\mathbf{E} \times \mathbf{B}/B^2$ (where B is the magnetic field strength and c is the speed of light), because other types of drifts may be disregarded in the case of low-energy particles (Lehnert 1964). The total electric field may be represented in solar-magnetospheric coordinates as a superposition of the cross-tail field $\mathbf{E}_c = -\nabla\Phi_c$ (directed from dawn to dusk) and the corotation field $\mathbf{E}_{cr} = -\nabla\Phi_{cr}$ induced by the Earth's rotation (Nishida 1966; Brice 1967; Kavanagh et al. 1968). Volland (1973, 1975) and Stern (1975) suggested that $\Phi_c \propto L^a \sin \lambda$, where L is the geocentric distance expressed in Earth radii R_E in the equatorial plane and λ is the longitudinal angle in the direction of Earth's rotation and measured from midnight. The parameter

a characterizes the degree of screening that prevents the convection electric field from penetrating beyond the plasmapause. Thus,

$$\Phi_c = E_0 R_E L^a \sin \lambda, \quad (6.2.1)$$

where E_0 is the electric field intensity. It is noted that this expression has been used elsewhere to calculate magnetospheric particle dynamics (see, e.g., the monographs of Roederer 1970; Nishida 1978 and references therein). The total electric potential is completed by the addition of the corotation potential,

$$\Phi_{cr} = \frac{-\Omega_E B_E R_E^2}{cL}, \quad (6.2.2)$$

where $\Omega_E \approx 7.27 \times 10^{-5} \text{s}^{-1}$ is the angular frequency of Earth's rotation and $B_E \approx 3.1 \times 10^{-5} \text{T}$ is the magnetic field at Earth's surface at the equator.

Thermal plasma in the equatorial plane is constrained to follow equipotential surfaces. An expression describing these surfaces can be obtained from (6.2.1) and (6.2.2) (Ejiri et al. 1978; Kaye and Kivelson 1981):

$$\left(\frac{L}{L_{st}}\right)^{a+1} \sin \lambda - K(a+1) \frac{L}{L_{st}} + a = 0, \quad (6.2.3a)$$

where K is the characteristic of the trajectory. There are both closed and open trajectories for $K \geq 1$, but only open trajectories for $K < 1$. In the stationary case, the last-closed equipotential, for which $K = 1$, must coincide uniquely with the position of the plasmasphere (see Fig. 6.1a, b). The case for which $a = 1$ (uniform magnetospheric field) may occur for periods of convection up to several hours after the onset of a magnetic storm, when screening of the magnetospheric field is

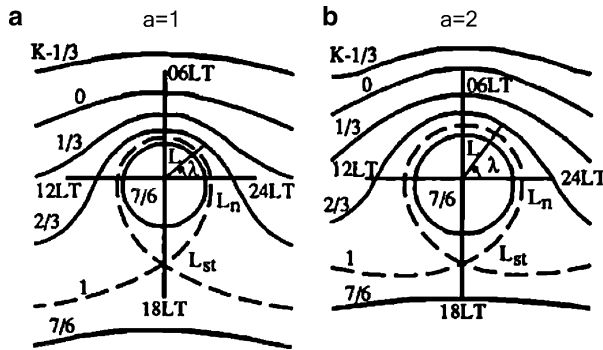


Fig. 6.1 A convective trajectory in the equatorial plane of the magnetosphere (solid line) according to (6.2.4). The location of the plasmapause is described by the dashed line

reduced by particles in the plasma sheet; at other times $a = 2$ (Nishida 1978). Below we consider the case for which $a = 1$. Then (6.2.3) can be presented:

$$L^2 \sin \lambda + \frac{L_{st}^2}{L_n} L - L_{st}^2 = 0, \quad (6.2.3b)$$

where L_n is the intersection of the equipotential surface with the noon–midnight meridian and where $L_{st} = [\Omega_E B_E R_E / (cE_0)]^{1/2}$ is the location along the dusk meridian where the drift velocity vanishes (the stagnation point of the convective flow). The solution to (6.2.3b),

$$L(\lambda) = \frac{L_{st}}{\sin \lambda} \left(\sqrt{\delta^2 + \sin^2 \lambda} - \delta \right), \quad (6.2.4)$$

where

$$\delta = \frac{L_{st}}{2L_n}$$

describes the position along an arbitrary elongated convection trajectory (see Fig. 6.1). The convection electric field gives rise to a dawn–dusk asymmetry in the drift trajectories, thereby resulting in a bulge in the dusk sector of the plasmasphere. Near the Earth, it is seen that the trajectories are nearly circular as a result of the dominance of the corotation potential. As the parameter L_n increases, the dawn–dusk asymmetry becomes more and more pronounced until the outermost closed trajectory is reached (the curve labeled L_{st} in Fig. 6.1). This trajectory is often referred to as the last-closed equipotential and is defined by (6.2.4) with $\delta = 1$.

Before completing this section, we present the operator that describes the effects of perpendicular transport in the continuity equation. For the electric field model defined above, this operator is

$$(\mathbf{u}_E \cdot \nabla) = \Omega_E \left[1 + \sin \lambda \frac{L^2(\lambda)}{L_{st}^2} \right] \frac{\partial}{\partial \lambda}, \quad (6.2.5)$$

where $L(\lambda)$ is described by (6.2.4).

Chapters 7, 9 and 10 provide additional details regarding different empirical magnetospheric electric field models, their influence on the inner magnetosphere particle dynamics, and ionosphere–magnetosphere coupling on a global scale.

6.2.2 Effects of Perpendicular Transport on Equatorial Density

The spatial distribution of plasma concentration in the plasmasphere is described by the continuity equation

$$\frac{dn}{dt} + B \frac{\partial}{\partial s} \left(\frac{nu_{\parallel}}{B} \right) = -n \nabla \cdot \mathbf{u}_E, \quad (6.2.6)$$

where $d/dt = \partial/\partial t + (\mathbf{u}_E \cdot \nabla)$ is the Lagrangian derivative in the coordinate system moving with the velocity $\mathbf{u}_E$ along a convective trajectory and s is the

coordinate along the geomagnetic field line. Note that in (6.2.6) there are no sources or sinks of plasma, density levels in the plasmasphere are completely dominated by transport. The second term on the left-hand side of (6.2.6) describes the effects of transport parallel to the field line whereas the term on the right-hand side describes the effects of perpendicular motions. In the case of convective motion perpendicular to the field line, the volume of a geomagnetic force tube varies, thereby giving rise to a redistribution of plasma. Depending on the sign of $\mathbf{u}_E \cdot \nabla$, perpendicular transport may either increase or decrease the value of n .

A timescale τ_c for variations in plasma density due to convection can be obtained from (6.2.6):

$$\tau_c = \frac{1}{\nabla \cdot \mathbf{u}_E}, \quad (6.2.7)$$

where

$$\nabla \cdot \mathbf{u}_E = -3\Omega_E \frac{L^2}{L_{st}^2} \cos \lambda \quad (6.2.8)$$

for the electric field model assumed here. The shortest timescales occur near noon and midnight. Assuming $L_{st} = 8$ and $L_n = 4.9$, a timescale of a little more than 3 h is obtained. This is much shorter than timescales for refilling of the plasmasphere, which are of the order of several days (Park 1974; Khazanov et al. 1984; Rasmussen et al. 1993).

The timescale τ_c is important because this is the timescale for changes in plasma density due to the change in flux tube volume. These changes occur even if the electric field is assumed constant and plasma moves on closed drift paths. Variations in the convection electric field would tend to enhance the density variations discussed here.

We shall consider plasmaspheric altitudes where hydrogen ions constitute the major charged component. By using the potential nature of the geomagnetic and electric fields, the continuity equation (6.2.6) can be rewritten as (Oraevskii et al. 1985; Gombosi and Rasmussen 1991)

$$\frac{d}{dt} \left(\frac{n}{B^2} \right) + B \frac{\partial}{\partial s} \left(\frac{nu_{||}}{B} \right) = 0. \quad (6.2.9)$$

In this form, the continuity equation can be used to analyze the combined effect of perpendicular and parallel motions on plasma density in the plasmasphere. If parallel transport is ignored ($u_{||} = 0$), then from (6.2.9) it is seen that the following invariant will be conserved (Lehnert 1964; Oraevskii et al. 1985):

$$\frac{d}{dt} \left(\frac{n}{B^2} \right) = 0, \quad (6.2.10)$$

which corresponds to concentration variations of

$$n \propto B^2 \quad (6.2.11)$$

along convective trajectories. In the case of plasma drifts from the dusk sector to the dawn sector, for example, the motion is toward regions of enhanced magnetic field strength (decreasing force tube geomagnetic volume), which corresponds to an increasing concentration. In the case of dawn-to-dusk motion, the situation reverses. In the equatorial plane of the geomagnetic dipole, we have $B \propto L^{-3}$ and the concentration along convective trajectories varies as

$$n \propto L^{-6}. \quad (6.2.12)$$

Let plasma density variations be estimated quantitatively using (6.2.4) and (6.2.12). Table 6.1 presents the relative variations of n during dusk-to-dawn traverses for different initial locations, assuming that the parameter L_{st} is 8. It is seen that, if parallel transport is ignored, $\mathbf{E} \times \mathbf{B}$ convection gives rise to substantial variations in plasma density. These variations are increased as the convective trajectories approach the last-closed equipotential (i.e., as L_n , increases). The variations reach their maximum at the last-closed equipotential where the variation in density is $n(0600 \text{ MLT})/n(1800 \text{ MLT}) \approx 200$. It should be noted that this value is independent of the parameter L_{st} for the electric field model adopted here.

Even if a tube of plasma is originally in hydrostatic equilibrium, any motion perpendicular to the magnetic field will tend to disturb this equilibrium distribution because of changes in flux tube area, which is proportional to $1/B$. Over time, plasma transport along magnetic field lines will bring the plasma distributions back into equilibrium. This redistribution of plasma along a field line occurs via an ion-acoustic wave. Thus a timescale, $\tau_{||}$, for the plasma to reach hydrostatic equilibrium can be estimated from the amount of time it takes for an ion-acoustic wave to travel the length

$$l_B = LR_E \left[Q_0 \cos \theta_0 + \frac{1}{\sqrt{3}} \log \left(\sqrt{3} \cos \theta_0 + Q_0 \right) \right] \quad (6.2.13)$$

of a field line, i.e.,

$$\tau_{||} \approx \frac{l_B}{v_s}, \quad (6.2.14)$$

Table 6.1 Relative variations in plasma density assuming no transport parallel to $\mathbf{B}$ (see (6.2.12)), during convection from dusk to dawn

L_n/L_{st}	2.0/8	2.5/8	3.0/8	3.5/8	3.9/8	4.0/8
$n(600 \text{ LT})/n(1800 \text{ LT})$	2.1	3.4	6.2	15.0	57.0	198.0

where the ion-acoustic velocity is approximately $v_s \approx \sqrt{K_B T_e / m_i}$, $Q_0 = \sqrt{1 + 3 \cos^2 \theta}$, K_B is Boltzmann's constant, T_e is electron temperature, m_i is ion mass, and θ_0 is colatitude at the foot of the field line. For $L = 4$ and $T_e = 5,000$ K, then $v_s \approx 6.4 \text{ km s}^{-1}$, $l_B \approx 9.4 R_E$, and $\tau_{||} \approx 2.5$ h. This is comparable to the 3-h timescale for density changes due to the perpendicular transport alone. It is obvious that parallel motions cannot be neglected and (6.2.11) must be reviewed as an upper limit to the density variations that occur during perpendicular drifts.

The effects of parallel motions are next considered with symmetric conditions assumed in the conjugate ionospheres. In this instance, $u_{||}$, $\partial n / \partial s$, and $\partial B / \partial s$ all vanish in the equatorial plane. However, $\partial u_{||} / \partial s \neq 0$. Thus the continuity equation (6.2.9) takes the form

$$\frac{d}{dt} \left(\frac{n}{B^2} \right) = - \left(\frac{n}{B^2} \right) \frac{\partial u_{||}}{\partial s} \quad (6.2.15)$$

in the equatorial plane of the magnetosphere. During the daytime (or when perpendicular drifts are away from Earth), parallel flows are predominantly away from the ionosphere and into the plasmasphere; at nighttime, the direction of the flow is reversed (Khazanov et al. 1984; Guiter et al. 1991). Therefore the sign of the derivative $\partial u_{||} / \partial s$ is positive at night and negative during the day. Thus the effect of parallel motions on plasmaspheric density is opposite that of perpendicular drifts, and variations in n are decreased in comparison with the variations defined by (6.2.11) and (6.2.12).

Disregarding effects associated with substorms, we seek plasmaspheric density distributions corresponding to steady-state convection in the inner magnetosphere. Taking into account (6.2.5), the solution to (6.2.15) takes the form

$$\frac{n}{B^2} = \frac{n_0}{B_0^2} \exp \left\{ - \frac{1}{\Omega_E} \int_{\lambda_0}^{\lambda} \frac{\partial u_{||} / \partial s}{1 + \sin \lambda L^2(\lambda) / L_{st}^2} d\lambda \right\}, \quad (6.2.16)$$

where n_0 and B_0 are the plasma density and the magnetic field, respectively, at the origin of the reference frame with longitude $\lambda = \lambda_0$. The exponential factor in (6.2.16) characterizes the effect of parallel transport on the distribution of plasma. Because variations in the derivative $\partial u_{||} / \partial s$ are small (except during sunrise and sunset or at other times when the derivative reverses sign) compared with the rest of the terms in the integrand, $\partial u_{||} / \partial s$ can be taken out from within the integral. Thus assuming that $\lambda_0 = -\pi/2$, we obtain after integration

$$\frac{n}{B^2} = \frac{n_0}{B_0^2} \exp \left\{ - \frac{1}{\Omega_E} \frac{\partial u_{||}}{\partial s} \left[\frac{\pi}{4} + \frac{\lambda}{2} + \frac{\delta}{\sqrt{1 + \delta^2}} F(\phi, k) \right] \right\}, \quad (6.2.17)$$

where $F(\phi, k)$ is an elliptic integral of the first kind:

$$F(\phi, k) = \int_0^\phi \frac{d\phi}{\sqrt{1 - k^2 \sin^2 \phi}}, \quad \phi = \sin^{-1} \sqrt{\frac{(1 + \delta^2)(1 + \sin \lambda)}{2(\delta^2 + \sin \lambda)}}, \quad k = \sqrt{\frac{2}{(1 + \delta^2)}}.$$

If (6.2.17) is used to make numerical estimates, assuming $L_{st} = 8$, $\partial u_{||}/\partial s = 2 - 5 \times 10^{-5} \text{ s}^{-1}$ (Khazanov et al. 1984), and $L_n = 3.5$, for example, parallel flows reduce n along drifts paths by a factor ranging from 2.6 to 10.0. As L_n increases, variations of n along convective trajectories are enhanced.

The net effect of this analysis is that variations in n along a convective trajectory should be expressed as

$$n \propto B^\alpha \quad (6.2.18)$$

rather than (6.2.11). The parameter α characterizes the competition between the effect of parallel flows into and out of the plasmasphere with the effect of expansion or compression of a flux tube due to perpendicular transport. The value of α varies from $4/3 \leq \alpha \leq 2$. The lower bound was obtained by assuming that the total content N of a flux tube is approximately constant, i.e., $dN/dt \approx 0$. This assumption is realistic because of the slow rate of exchange of mass between the ionosphere and the plasmasphere, i.e., $\tau_{||} \sim \tau_c \ll \tau_{ref}$, where τ_{ref} is the timescale for refilling or emptying of the plasmasphere through fluxes at the base of the field line. If perpendicular drifts are slow enough so that plasma density is nearly in hydrostatic equilibrium, $\tau_{||} \ll \tau_c$, it can be shown that

$$N \approx n_{eq} V(L) B_E,$$

where

$$V(L) = \frac{32R_E L^4}{35B_E} \sqrt{1 - L^{-1}} \left[1 + \frac{1}{2}L^{-1} + \frac{3}{8}L^{-2} + \frac{5}{16}L^{-3} \right] \quad (6.2.19)$$

is the volume per unit magnetic flux of a tube of plasma and n_{eq} is the plasma density near the equatorial plane (Rasmussen et al. 1993). Because $dN/dt \approx 0$,

$$(1/n_{eq}) dn_{eq}/dt \approx -(1/V) dV/dt \approx -4(1/L) dL/dt \cong (4/3)(dB/dt)/B.$$

Thus $\alpha \approx 4/3$ for slow perpendicular drifts, i.e., when $\tau_{||} \ll \tau_c \ll \tau_{ref}$. We have already seen that $\alpha = 2$ in the other extreme, $\tau_c \ll \tau_{||} \ll \tau_{ref}$. This case corresponds to totally neglecting parallel transport and involves the largest variations of n along convective trajectories.

6.2.3 Plasma Distribution Along a Field Line

The effect of perpendicular transport on the distribution of plasma along a field line is now considered. As in the previous section, we assume steady convection in the inner magnetosphere and consider only those altitudes where H^+ is the dominant ion. It is convenient to write the continuity equation (6.2.9) in the following form,

$$\frac{1}{B} \frac{\partial}{\partial s} \left(\frac{\Phi}{B} \right) = q_c, \quad (6.2.20)$$

where $\Phi = nu_{\parallel}$ is the flux of hydrogen ions parallel to B and

$$q_c = -(\mathbf{u}_E \nabla) \left(\frac{n}{B^2} \right) \quad (6.2.21)$$

represents the effects of perpendicular transport, namely, those effects due to a change in the volume of a flux tube. From (6.2.20) it is seen that the rate of variation of n/B^2 along a convective trajectory must affect the distribution of plasma along that field line.

Let the flux Φ in (6.2.20) be presented in the diffusive form

$$\Phi = -D \left(\frac{\partial n}{\partial s} - \frac{n}{H} \right), \quad (6.2.22)$$

where D is the ambipolar diffusion coefficient, $H = K_B T_p / (m_i g_{\parallel})$ is the plasma scale height, $T_p = T_e + T_i$ is the plasma temperature, which, for simplicity, is assumed to be constant,

$$g_{\parallel} = \frac{2g_E}{L^2} \frac{\cos \theta}{\sin^4 \theta \sqrt{1 + 3 \cos^2 \theta}}$$

is the projection of the acceleration due to gravity onto a dipole field line, $g_E \approx 980 \text{ cm s}^{-2}$ is the gravitation acceleration at Earth's surface, and θ is colatitude. A solution to (6.2.20) is sought by the method of successive approximation using the following pattern:

$$\frac{1}{B} \frac{\partial}{\partial s} \left(\frac{\Phi^{(k)}}{B} \right) = q_c^{(k)}, \quad (6.2.23)$$

where the values of the term $q_c^{(k)}$ are calculated using the previous approximation of plasma density, i.e., $q_c^{(k)} = q_c(n^{(k-1)})$ and k is the iteration index showing the approximation order.

The zero approximation $n^{(0)}$ is obtained by assuming q_c is zero in (6.2.20). At this level of approximation, the effects of perpendicular transport are neglected and the diffusive-equilibrium distribution of plasma along a dipole field line is

$$n^{(0)} = n_0 \exp \left[\frac{\gamma}{L} \left(\frac{1}{\sin^2 \theta} - \frac{1}{\sin^2 \theta_0} \right) \right] \quad (6.2.24)$$

(e.g., Konikov and Khazanov 1982c; Rasmussen et al. 1993), where $\gamma = m_i g_E R_E / (K_B T_p) \approx 7.55 \times 10^3 / T_p$ (T_p is expressed in K). The zero subscript corresponds to values at the reference altitude h_0 . Subsequent approximations will allow for deviations from the diffusive-equilibrium distribution (6.2.24) due to the convection electric field.

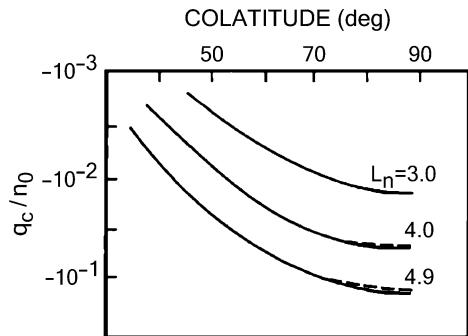
To find $n^{(1)}$, the zero approximation (6.2.24) is used to approximate the convection term represented by (6.2.24). Substituting (6.2.24) into (6.2.21) and applying (6.2.5), we obtain

$$q_c^{(1)} = \frac{n^{(0)}}{B^2} \frac{\Omega_E}{2L_{st}} [6L - \gamma(\sin^{-2} \theta - \sin^{-2} \theta_0)] \left(1 + \frac{L^2}{L_{st}^2} \sin \lambda \right) \times \frac{\cos \lambda}{\sqrt{\delta^2 + \sin \lambda}}. \quad (6.2.25)$$

It is seen that the sign of the term $q_c^{(1)}$ can reverse. During the daytime ($\pi/2 < \lambda < 3\pi/2$), for example, $q_c^{(1)} < 0$ and the effect of perpendicular appears as a loss term in the continuity equation due to an increase in the volume of a flux tube as plasma drifts from dawn to dusk.

Figure 6.2 shows values of $q_c^{(1)}$ along various field lines corresponding to the parameter L_n , at 1400 LT. Here, $|q_c^{(1)}|$ increases equatorward along geomagnetic field lines and as the location of the flux tube trajectory approaches the plasma-pause. This behavior arises because the greatest variations in the volume of the convecting flux tube occur near the equatorial plane and near the plasmopause.

Fig. 6.2 The strength of the convective source (6.2.21) along magnetic field lines for various trajectories at 1400 LT and for $L_{st} = 10$, $h_0 = 1,500$ km, and $T_p = 7,000$ K. The solid line is the first-order approximation, $q_c^{(1)}$, in (6.2.25) and the dashed line is the second-order approximation, $q_c^{(2)}$



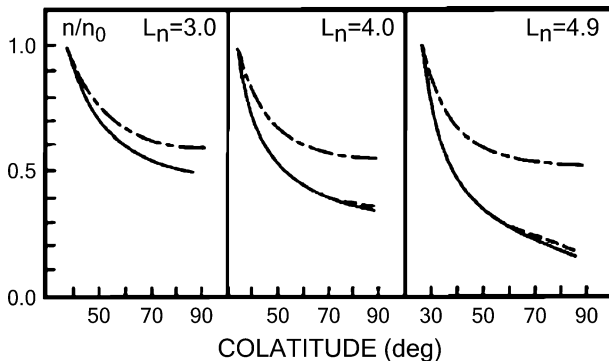


Fig. 6.3 Profiles of the relative distribution of plasma density along magnetic field lines for different drift trajectories at 1400 LT. The *dash-dotted line* is the diffusive-equilibrium distribution $n^{(0)}$ described by (6.2.24), the *solid line* is the approximation $n^{(1)}$ described by (6.2.28), and the *dashed line* is the approximation $n^{(2)}$

The equation for the next approximation $n^{(1)}$ is

$$\frac{\partial n^{(1)}}{\partial \theta} + G(\theta)n^{(1)} = F(\theta), \quad (6.2.26)$$

where

$$G(\theta) = \frac{2\gamma}{L} \frac{\cos \theta}{\sin^3 \theta},$$

and

$$F(\theta) = (LR_E)^2 D^{-1} B \sqrt{1 + 3 \cos^2 \theta} \sin \theta \int_{\theta_0}^{\theta} q_c^{(0)} B \sqrt{1 + 3 \cos^2 \theta} \sin \theta d\theta.$$

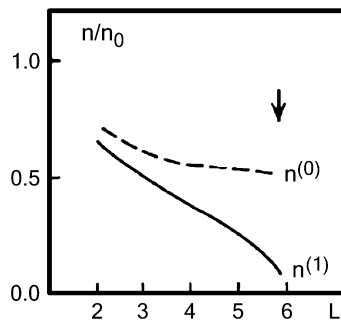
This is a linear inhomogeneous first-order differential equation. The general form of the solution for (6.2.26) is

$$n^{(1)} = \left[n_0 + \int_{\theta_0}^{\theta} F \exp \left(\int_{\theta_0}^{\theta} G d\theta \right) d\theta \right] \exp \left(- \int_{\theta_0}^{\theta} G d\theta \right). \quad (6.2.27)$$

Considering (6.2.24), the solution to (6.2.27) may be expressed as

$$n^{(1)} = n^{(0)} \left[1 + \int_{\theta_0}^{\theta} \frac{F}{n^{(0)}} d\theta \right]. \quad (6.2.28)$$

Fig. 6.4 Equatorial profiles of the relative distribution of plasma density. The *dashed line* is the distribution $n^{(0)}$ without convection, while the *solid line* includes the effects of convection in the first approximation $n^{(1)}$. The *arrow* indicates the location of the plasmopause



The integral containing q_c in (6.2.28) characterizes the deviation of plasma density from diffusive equilibrium as a flux tube drifts around the Earth. Figure 6.3 shows the distribution of density along a field line for various approximations. It is seen that $n^{(1)}$ decreases more rapidly along a magnetic field line than does $n^{(0)}$, leading to substantial deviations near the equatorial plane. As L_n increases, deviations from the diffusive equilibrium are enhanced because the effects of perpendicular transport are relatively stronger near the plasmopause. It is noted that differences between $n^{(1)}$ and $n^{(0)}$ rise from 20% at $L_n = 3$ to almost a factor of 4 at $L_n = 4.9$.

To calculate the next approximation, $n^{(2)}$, it is necessary to find the value of $q_c^{(2)} = q_c(n^{(1)})$. When this is done, equations analogous to (6.2.25) and (6.2.26) are obtained. Calculations show that the differences between $q_c^{(2)}$ and $q_c^{(1)}$ do not exceed 3–4% depending on the parameter L_n . Thus differences between $n^{(2)}$ and the first-order approximation $n^{(1)}$ do not exceed 4% and we limit the remainder of the discussion to the first-order approximation.

Figure 6.4 illustrates the effects of perpendicular transport on the equatorial profile of plasma density. Contrary to the diffusive-equilibrium profile, which shows a gradual and slow decrease as L increases, convection leads to a more rapid decrease in plasma concentration. As the plasmopause is approached, the density decrease is more rapid.

Plasmaspheric research has undergone an amazing revitalization since 1998 and is systematically described by Liemohn (2006). This is due to the emergence of new data sets (from IMAGE EUV and RPI), new data analysis techniques (deconvolving ground-based magnetometer data for field line mass content, and total electron content from GPS receivers), and new modeling capabilities (especially the coupling of plasmaspheric models with ring current models), which are described in Chaps. 9 and 10 of this book.

In terms of global plasmaspheric morphology, the IMAGE EUV database has made a significant contribution to the field. This has been summarized in several recent reviews (e.g., Sandel et al. 2003; Burch 2005). Of key importance is the visualization of the plasmaspheric drainage plume (e.g., Burch et al. 2001; Sandel et al. 2003), showing its connection to the main plasmasphere throughout the storm

sequence, including the way it narrows and eventually wraps itself around the plasmasphere. New features of the plasmopause have been identified, such as shoulders and fingers and notches channels (Sandel et al. 2003). In combination with modeling, explanations have been given for many of these features, such as overshielding as the cause of dawnside shoulders (Goldstein et al. 2002), electric field propagation fronts as the reason for undulations (Goldstein et al. 2004), and small-scale electric potential extrema as the source of deep notches (Liemohn 2006).

The IMAGE RPI instrument results have greatly contributed to our understanding of the distribution of cold plasma along inner magnetospheric field lines. In active sounding mode, radio pulses can be ducted along field lines inside the plasmasphere, and a return signal can be processed to reveal the instantaneous density structure along the field line. The density profiles during quiet times or storms do not follow diffusive equilibrium, but rather are “flatter” near the equator, with less latitudinal variation (at low latitudes) than theoretically predicted (Reinisch et al. 2004). The paper by Liemohn (2006) provides additional details of these plasmaspheric research advances.

6.3 Effects of Magnetospheric Convection on Temperature

The convection electric field causes plasma to drift into regions of differing magnetic field intensity. The adiabatic effects induced by the resulting compression or expansion of the plasma give rise to variations in the internal energy of the plasma. In this case, variations in internal energy are different for the perpendicular degrees of freedom than for the parallel degrees of freedom. This is one possible mechanism for forming an anisotropic distribution of energy for the thermal population in the plasmasphere.

Plasma temperature is described using generalized hydrodynamic equations for anisotropic plasmas (e.g., Barakat and Schunk 1982; Konikov and Khazanov 1982a; Gombosi and Rasmussen 1991). These equations can be presented in the following form, which is convenient for analyzing the effects of perpendicular transport:

$$\frac{d}{dt} \left(\frac{T_{\perp}}{B} \right) + \frac{B}{n} \frac{\partial}{\partial s} \left(\frac{S^{\perp}}{B^2} \right) = \frac{1}{nB} (Q_{\perp} - L_{\perp}), \quad (6.3.1)$$

$$\frac{d}{dt} \left(\frac{T_{\parallel} B^2}{n^2} \right) + \frac{B^2}{n^3} B \left[\frac{\partial}{\partial s} \left(\frac{S^{\parallel}}{B} \right) + \frac{2S^{\perp}}{B} \frac{\partial B}{\partial s} \right] = \frac{B^2}{n^3} (Q_{\parallel} - L_{\parallel}), \quad (6.3.2)$$

where $T_{\perp}$ and $T_{\parallel}$ are, respectively, the perpendicular and parallel temperatures (the particle species index has been omitted for simplicity), $S_{\perp}$ and $S_{\parallel}$ are the parallel components of the heat flux vectors of perpendicular and parallel random energies,

$Q_{\perp,\parallel}$ are external sources of thermal energy, and $L_{\perp,\parallel}$ represents energy lost due to particle collisions. In (6.3.1) and (6.3.2), it is assumed that the magnetic field is sufficiently large that contributions from the viscosity tensor and the perpendicular components of the heat flux vectors can be neglected.

Equations (6.3.1) and (6.3.2) are the CGL adiabatic relations (Chew et al. 1956) supplemented with terms allowing for heat fluxes, external energy sources, and particle collisions. The first terms in these equations describe adiabatic energy variations for the perpendicular and parallel degrees of freedom due to convective transport. The collision terms characterize the relaxation of the temperature anisotropy and the exchange of energy between the various plasma components. In the single-ion approximation, (6.3.1) and (6.3.2) constitute a set of four differential equations that must be supplemented with four additional equations for the heat fluxes (two equations each for the electron and ion components). The solution for the resulting set of equations is sufficiently difficult to obtain that we analyze below only certain aspects of the effect of perpendicular drifts on plasma temperature in the plasmasphere.

6.3.1 Adiabatic Variations

As in a previous section, we consider only the two-dimensional problem in the equatorial plane of the magnetosphere, assuming symmetric conditions in magnetically conjugate regions. In this case, (6.3.1) and (6.3.2) are simplified somewhat:

$$\frac{d}{dt} \left(\frac{T_{\perp}}{B} \right) + \frac{B}{n} \frac{\partial}{\partial s} \left(\frac{S^{\perp}}{B^2} \right) = \frac{1}{nB} (Q_{\perp} - L_{\perp}), \quad (6.3.3)$$

$$\frac{d}{dt} \left(\frac{T_{\parallel} B^2}{n^2} \right) + \frac{B^2}{n^3} \frac{\partial S^{\parallel}}{\partial s} = \frac{B^2}{n^3} (Q_{\parallel} - L_{\parallel}). \quad (6.3.4)$$

The adiabatic CGL relations can be obtained from (6.3.3) and (6.3.4) by disregarding energy sources, heat fluxes, and collisions. These relations imply that

$$T_{\perp} \propto B \quad (6.3.5)$$

and

$$T_{\parallel} \propto \frac{n^2}{B^2} \quad (6.3.6)$$

along convective trajectories in the equatorial plane. Contrary to the perpendicular temperature, variations in $T_{\parallel}$ depend on variations in plasma density. If the variations in n are given by (6.2.18), then

$$T_{\parallel} \propto B^{2(\alpha-1)}. \quad (6.3.7)$$

Assuming a dipole for Earth's magnetic field, we obtain

$$T_{\perp} \propto L^{-3}, \quad (6.3.8)$$

$$T_{\parallel} \propto L^{-6(\alpha-1)}, \quad (6.3.9)$$

where the radial location $L = L(\lambda)$ along a convection path is given by (6.2.4).

From these relations, it is seen that the adiabatic energy variations are independent of particle species, so the respective temperature variations would be the same for electrons and ions. However, we show later that because of the large electron heat conductivity, this adiabatic analysis is only relevant for the ion species.

Because the convective trajectories are symmetric with respect to the dawn–dusk meridian (see Fig. 6.1), the adiabatic temperature variations will also be symmetric with respect to this plane. Therefore we analyze temperature distributions in the nightside only. For example, in the case of convection from dusk toward midnight, plasma drifts into a region of increasing magnetic field and the perpendicular temperature rises. The behavior of the parallel temperature depends on the value of α , which characterizes the variations in n along the trajectory. The $T_{\parallel}$ variations reach their maximum at $\alpha = 2$ (the value that corresponds to disregarding the transport of plasma along field lines and leads to the largest variations in density). For $\alpha = 4/3$, $T_{\parallel} \propto L^{-2}$, and $T_{\parallel}$ varies less rapidly than does the perpendicular temperature.

The temperature anisotropy can be obtained from (6.3.5) and (6.3.7):

$$\frac{T_{\perp}}{T_{\parallel}} \propto B^{3-2\alpha}, \quad (6.3.10)$$

whence it follows that for $\alpha = 3/2$, the temperature anisotropy remains constant along a convective trajectory. For example, if the energy distribution is isotropic at one of the points, it will remain isotropic along the entire drift path. This occurs because the adiabatic effects leading to a temperature anisotropy are neutralized by the variations in plasma density $n \propto B^{3/2}$. These features are illustrated in Figs. 6.5 and 6.6, where temperature ratios $T_{\perp}/T_{\perp 0}$ and $T_{\parallel}/T_{\parallel 0}$ are shown along various trajectories. Here, $T_{\perp 0}$ and $T_{\parallel 0}$ are temperatures at 1800 LT. It is seen that, under a dusk-to-dawn plasma drift, both the perpendicular and parallel temperatures rise with the parallel temperature variations depending on the density variations. The variations in both temperatures reach a maximum at the plasmopause of $T_{\perp}(0600)/T_{\perp 0} = (\sqrt{2} - 1)^{-3} \approx 14$ and $T_{\parallel}(0600)/T_{\parallel 0} = (\sqrt{2} - 1)^{-6(\alpha-1)}$.

Fig. 6.5 (a) Relative variations in perpendicular temperature during adiabatic dusk-to-dawn convection. (b) Radial location of the drift paths shown in (a) (in units of R_E). $L = 4$ reflects the location of the plasmopause at 2400 LT

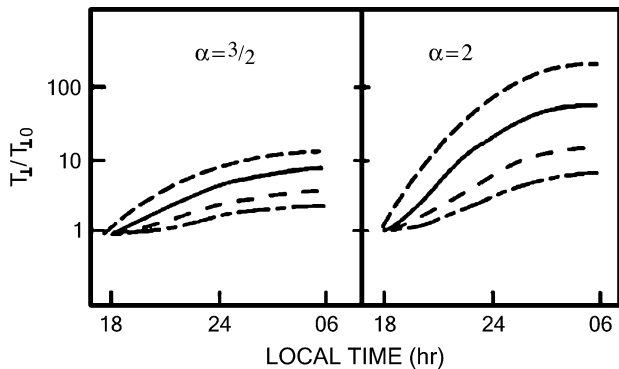
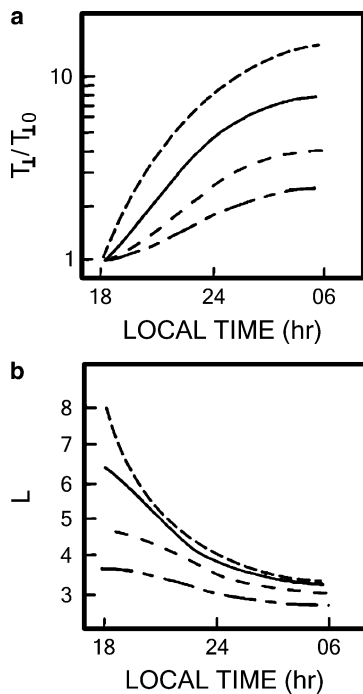


Fig. 6.6 Relative variations in parallel temperature during adiabatic dusk-to-dawn convection. The parameter α describes the rate of density variations with magnetic field. The notation is the same as in Fig. 6.5

Variations in the total particle temperature $T = (2T_{\perp} + T_{\parallel})/3$ are now examined. Equations (6.3.5) and (6.3.7) yield

$$\frac{T}{T_0} = \frac{1}{1 + 2a_0} \left[2a_0 \frac{B}{B_0} + \left(\frac{B}{B_0} \right)^{2(\alpha-1)} \right], \quad (6.3.11)$$

where $a_0 = T_{\perp 0}/T_{\parallel 0}$ characterizes the temperature anisotropy at 1800 LT. The total temperature not only depends on variations in the magnetic field (or equivalently, on L shell), but also depends on the temperature anisotropy; only for $\alpha = 3/2$ is the total temperature independent of the temperature anisotropy.

Let the variations in temperature described by (6.3.11) be compared with variations of the temperature $T^{(0)}$, defined by assuming an isotropic energy distribution. From standard hydrodynamics, it is well known that $T^{(0)}/n^{2/3}$ is constant under adiabatic energy variations. Thus from (6.2.18), we obtain

$$T^{(0)} \propto B^{2\alpha/3} \propto L^{-2\alpha}. \quad (6.3.12)$$

It is seen that the variations in T and $T^{(0)}$ are different. This occurs because even with an initial isotropic distribution ($a_0 = 1$), an anisotropy will eventually develop (see (6.3.10)) unless $\alpha = 3/2$. Thus, if anisotropies are present, standard isotropic hydrodynamics equations will not describe the correct temperature variations.

6.3.2 Heat Conduction

The adiabatic temperature variations described above are reduced by heat conduction. In addition, temperature anisotropies will be reduced by collisional effects. To find the contributions of the competing factors resulting in plasmaspheric temperatures, the characteristic timescales of the various processes are now examined. (The characteristic time for adiabatic temperature variations is taken to be the time over which the value of the variation changes by a factor of e .)

First the variations in total temperature are considered. From (6.3.11) and from the plots presented, it is seen that the latter timescale depends on trajectory location L_n and on α through the density variations described by (6.2.18). As α decreases from 2 to 4/3 and assuming that $a_0 = 1$, the convection time τ_{c_T} for $L_n = 3.5$ varies from $\tau_{c_T} = 1.7 \times 10^4$ to 3×10^4 s, while $\tau_{c_T} = 9.9 \times 10^3$ to 1.5×10^4 s for $L_n = 3.9$. As the drift trajectories approach the plasmapause, the adiabatic variations are enhanced, resulting in a decrease of τ_{c_T} .

The characteristic time for heat conduction is

$$\tau_T = \frac{3nK_B H_T^2}{2\chi}, \quad (6.3.13)$$

where χ is the heat conductivity and H_T is the characteristic length scale for field-aligned temperature variations. In the presence of a temperature anisotropy, the heat conductivity is a function of the temperature anisotropy and can be expressed as

$$\chi_r = \chi_r^o(T_r)K(a_r), \quad (6.3.14)$$

where $\chi_r^o(T_r)$ is the standard heat conductivity of particle species r calculated assuming an isotropic energy distribution and $K(a_r)$ is a function a factor that allows for the effect of a temperature anisotropy (see Sect. 4.7 for details). For $T_{\parallel} > T_{\perp}$ (or $a < 1$), we have $K(a_r) > 1$ and the temperature anisotropy gives rise to an enhanced transfer of thermal energy due to heat conduction. For $T_{\parallel} > T_{\perp}$ we have $K(a_r) < 1$ and the situation reverses, namely, the anisotropy reduces heat conductivity (for more information, see Oraevskii et al. 1985).

Substituting (6.3.14) into (6.3.13) and considering that $H_T \approx LR_E$ near the equatorial plane, we obtain

$$\tau_{T_r} = \frac{3nK_B(LR_E)^2}{2\chi_r^o(T_r)K(a_r)}. \quad (6.3.15)$$

Assuming a fully ionized plasma ($e + H^+$), the heat conductivities are

$$\begin{aligned} \chi_e^o(T_e) &\approx 7.7 \times 10^5 T_e^{5/2}, \\ \chi_i^o(T_i) &\approx 4.6 \times 10^4 T_i^{5/2} \end{aligned} \quad (6.3.16)$$

in units of eV/(cmsK) (Banks and Kockarts 1973). Because of the large $\chi_e^o(T_e)$ value, heat conduction is too strong for the adiabatic assumption to be valid for the electron population. A temperature increase associated with a compression of the plasma, for instance, will result in a transfer of a substantial portion of this energy to the ionosphere. Therefore the electron temperature in the plasmasphere is sensitive to the ionospheric boundaries. This fact has been confirmed by using (6.3.15) to show that $\tau_{T_e} < \tau_{c_T}$. In addition, photoelectron heating of plasmaspheric electrons is important (Khazanov et al. 1992a) and this breaks the adiabatic assumption as well.

6.3.2.1 Electron Heat Conduction

An interesting result can be obtained by assuming that all of the variations in thermal energy resulting from a compression or expansion of the electron gas are transferred to the ionosphere. Assuming that the electron energy distribution is isotropic and that there are no sources or sinks of thermal energy in the plasmasphere, it can be shown that

$$\frac{3}{2} n B^{4/3} \frac{d}{dt} \left(\frac{K_B T_e}{B^{4/3}} \right) = B \frac{\partial}{\partial s} \left(\frac{S_e}{B} \right), \quad (6.3.17)$$

where S_e is the electron heat flux. This equation can be integrated between conjugate ionospheres in order to obtain an expression describing the conservation of the total thermal energy of the electron gas. Because of high thermal conductivity, T_e is nearly constant in the plasmasphere. In order to estimate the electron heat flux S_e , at the upper boundary of the ionosphere (due only to magnetospheric convection), it is

assumed that T_e remains constant in time (this implies that all of the energy from the work done in compressing the electron gas is transferred to the ionosphere). After integrating (6.3.17), an estimate for the maximum electron heat flux at the ionosphere due to convection can be obtained:

$$S_e = -3NK_B T_e \frac{1}{L} \frac{dN}{dt}, \quad (6.3.18)$$

where the approximate $V(L) \propto L^4$ dependence (see (6.2.19)) has been used, N is total tube content, and a positive heat flux corresponds to a flux of energy into the ionosphere. Assuming $K_B T_e \sim 1/3 \text{ eV}$, $N \sim 4 \times 10^{13} \text{ cm}^{-2}$ (Park 1974), and $(1/L)dL/dt \sim -1/(2 \times 10^4) \text{ s}^{-1}$ as plasma drifts toward Earth in the late evening sector, (6.3.18) implies $S_e \sim 2 \times 10^9 \text{ eV cm}^{-2} \text{ s}^{-1}$. This is a relatively large heat flux, of the same order of magnitude as the heat flux produced when photoelectrons collide with plasmaspheric thermal electrons.

This estimate of the electron heat flux implies that high electron temperatures (relative to nighttime values) should be seen after sunset as heat flows in from the plasmasphere. This effect is consistent with the observations of Richards et al. (1995), who found frequent increases in electron temperature occurring after sunset over Millstone Hill. They have estimated that a heat flux of the order of $3.7 \times 10^9 \text{ eV cm}^{-2} \text{ s}^{-1}$ is required in order to obtain the late evening temperature profiles seen in the Millstone Hill radar data. The heat flux required by the measurements is compatible with the electron heat flux due to convection estimated above.

6.3.2.2 Ion Heat Conduction

Ion heat conduction is much less than that for electrons, so the situation may arise when ion heat conduction has no time to reduce the adiabatic temperature variations discussed above. The characteristic timescale for ion heat conduction is now examined for the $L_n = 3.9$ trajectory. Substituting $n \approx 10^3 \text{ cm}^{-3}$, $L \approx 6$, $T_i = 6,000 \text{ K}$, and $a_i = 1$ in (6.3.15), we obtain $\tau_{T_i} \approx 1.5 \times 10^4 \text{ s}$. The τ_{T_i} values decrease with increasing temperature, so, for example, the characteristic time is $\tau_{T_i} \approx 7.3 \times 10^3 \text{ s}$ at $T_i = 8,000 \text{ K}$. It is seen that τ_{T_i} is comparable to or even larger than τ_{c_T} , thereby indicating that convection has a substantial effect on ion temperatures in the plasmasphere.

6.4 Plasma Electron Temperature Anisotropy

As was discussed in Sect. 4.7, the anisotropic plasma hydrodynamics makes use of two heat fluxes representing the transfer of longitudinal and transverse (with respect to external magnetic field) energies to describe thermal-energy transfer. This is a

considerably more complex treatment than isotropic plasma hydrodynamics in which only one heat flux component is considered. The present section analyzes the thermal electron heat flux structure within the framework of anisotropic hydrodynamics and examines the effect of the anisotropic heat fluxes on the thermal plasma distribution both in the ionosphere and in the magnetically connected plasmasphere.

6.4.1 *The Effect of Anisotropy on Electron Temperature in the Plasmasphere*

The energy transfer by heat fluxes is one of the principal processes that govern the behavior of charged-particle temperatures in the outer ionosphere and in the plasmasphere. From the results presented in Sect. 4.7, it follows that the heat conductivities depend strongly on temperature anisotropies. In turn, the dependence must affect the value of heat fluxes and, eventually, the behavior of particle temperatures. It is of interest in this connection to analyze the effect of the charged-particle energy distribution anisotropy on the charged-particle temperatures. Such an analysis is made below for the electron component of the plasma.

The analytic approach, used in the following analysis, necessitates some simplifying assumptions. A stable quasisteady-state distribution of electron temperature is assumed and the convective transfer of thermal energy is disregarded. Considering the total electron temperature $T_e = (2T_{e\perp} + T_{e\parallel})/3$, (6.3.1) and (6.3.2) are used to infer the relevant expression for the heat flux (Khazanov et al. 1996)

$$\nabla \cdot \mathbf{S}_e = Q_e - L_{ei}, \quad (6.4.1)$$

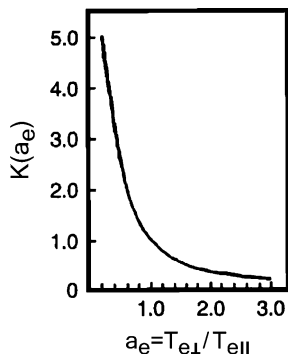
where $\mathbf{S}_e = \mathbf{S}_e^\perp + (1/2)\mathbf{S}_e^\parallel$ is the total heat flux in the electrons. The collision term consists of two components, of which the component Q_e describes the source of thermal electron heating by suprathermal electrons, whereas the component L_{ei} describes the energy loss in electron–ion collisions.

If the total temperature is used in (4.7.15) and (4.7.16), the heat flux S_e will depend on the T_e gradient and on the gradient of the ratio $a_e = T_{e\perp}/T_{e\parallel}$. The electron temperature ratio a_e is assumed to be coordinate independent and should be regarded as an external parameter of the problem. This property of the electron temperature ratio makes it possible to functionally separate the dependences of heat flux on T_e and on a_e and to present the heat flux as

$$S_e = -\chi_e^o(T_e)K(a_e)\frac{\partial T_e}{\partial z}, \quad (6.4.2)$$

where $\chi_e^o(T_e)$ is a commonly adopted standard heat conductivity calculated under an isotropic electron energy distribution. Under the approximation of a fully ionized

Fig. 6.7 The functional factor $K(a_e)$, describing the dependence of the electron heat conductivity on temperature anisotropy, which appears on the right-hand side of (6.4.2) for the total flux of thermal energy S_e



plasma $\chi_e^o(T_e) = \beta_e T_e^{5/2}$, where $\beta_e = 7.7 \times 10^5 \text{ eV cm}^{-1} \text{ s}^{-1} \text{ K}^{-7/2}$. The dimensionless functional factor $K(a_e)$ describes the dependence of heat conductance on temperature anisotropy. The dimensionless functional factor $K(a_e)$ describes the dependence of heat conductance on the temperature anisotropy (see Fig. 6.7). The resultant expression proves to be fairly tedious and therefore is presented only in graphical form (Khazanov et al. 1996). As indicated, the temperature anisotropy may result in a significant change of the heat conductivity compared with the isotropic case ($a_e = 1$). To quantitatively estimate the effect of this factor on electron temperature, (6.4.1) must be solved making allowance for (6.4.2). A further simplification results by assuming that $T_e \approx T_i$ at the plasmaspheric levels and, thus, the electron energy loss in electron-ion collisions can be neglected ($L_{ei} \approx 0$) in (6.4.1), as well, resulting in the expression

$$\beta_e K(a_e) B \frac{\partial}{\partial z} \left(\frac{T_e^{5/2}}{B} \frac{\partial T_e}{\partial z} \right) + Q_e = 0. \quad (6.4.3)$$

With such a formulation of the problem, the electron temperature is inferred from the balance between the heating by suprathermal electrons and the cooling by the field-aligned energy leakage to the underlying ionosphere due to heat conduction. The heating source in (6.4.3) may be approximately presented as $Q_e = a n_e$, where a is a constant (Khazanov 1979). The electron number density variations along a geomagnetic field line will be assumed to be proportional to magnetic field intensity, namely, $n_e(z) = B(z)(n_{eo}/B_o)$, where n_{eo} and B_o are, respectively, electron number density and geomagnetic field intensity at the base of the plasmasphere ($z = z_0$). The magnetic field is assumed to be dipolar:

$$B = (B_E/L^3)(1 + 3 \sin^2 \varphi)^{1/2} / \cos^6 \varphi,$$

where φ is the latitude, B_E is the magnetic field intensity on the Earth's surface in the equatorial plane ($\varphi = 0$); and L is the McIlwain parameter.

Equation (6.4.3) will be more amenable to being solved if the field-aligned coordinate z is replaced by latitude φ . Bearing in mind that $dz = R_E L (1 + 3\sin^2 \varphi)^{1/2} \cos \varphi d\varphi$ in dipolar coordinates, (6.4.3) takes the form

$$\frac{\partial}{\partial \varphi} \left(\frac{\cos^5 \varphi}{1 + 3 \sin^2 \varphi} \frac{\partial T_e^{7/2}}{\partial \varphi} \right) + \frac{A}{K(a_e)} (1 + 3 \sin^2 \varphi)^{1/2} \cos \varphi = 0, \quad (6.4.4)$$

where R_E is the Earth's radius; φ_o is the latitude of the lower boundary located at altitude h_o ; $\cos^2 \varphi_o = (R_E + h_o)/(R_E L)$; and

$$A = \frac{7 \alpha n_{eo} (R_E L)^2 \cos^6 \varphi_o}{2 \beta_e (1 + 3 \sin^2 \varphi_o)^{1/2}}.$$

Boundary conditions must be prescribed. The boundary conditions in magnetically conjugate regions will be assumed to be symmetric. This assumption corresponds to vanishing of the heat flux (6.4.2) or of the derivative of electron temperature in the plane of the geomagnetic equator as indicated by

$$\left. \frac{\partial T_e}{\partial \varphi} \right|_{\varphi=0} = 0. \quad (6.4.5)$$

The value of electron temperature at the lower boundary of the plasmasphere is another boundary condition, given below as

$$T_e|_{\varphi=\varphi_o} = T_{eo}. \quad (6.4.6)$$

The solution for (6.4.4) under the boundary conditions (6.4.5) and (6.4.6) is

$$T_e^{7/2}(\varphi) = T_{eo}^{7/2} + \frac{A}{2K(a_e)} [\mathfrak{S}(\varphi_o) - \mathfrak{S}(\varphi)]. \quad (6.4.7)$$

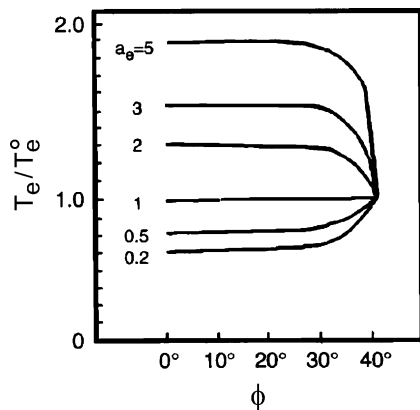
The function $\mathfrak{S}(\varphi)$ entering the solution (6.4.7) is given by the expression

$$\mathfrak{S}(\varphi) = \int_0^\varphi \frac{1 + 3 \sin^2 \varphi}{\cos^5 \varphi} \left[(1 + 3 \sin^2 \varphi)^{1/2} \sin \varphi + \frac{1}{\sqrt{3}} \ln \left(\sqrt{3} \sin \varphi + (1 + 3 \sin^2 \varphi)^{1/2} \right) \right] d\varphi.$$

The second term in (6.4.7) is proportional to the integrated heating of the plasmaspheric reservoir by suprathermal electrons and also contains the factor $1/K(a_e)$, which defines the dependence of the electron temperature distribution on the energy distribution anisotropy. The magnitude of the affect of the anisotropy on T_e will increase as the contribution from the second term of (6.4.7) increases.

The effect of the temperature anisotropy on T_e can conveniently be analyzed by comparing T_e with the temperature (T_e^o) value inferred from (6.4.7) in the isotropic

Fig. 6.8 Electron temperature distribution, relative to the isotropic case, for specified values of the anisotropy parameter a_e along the $L = 2$ geomagnetic field line, $T_{eo} = 3,000$ K, $n_{eo} = 2 \times 10^4 \text{ cm}^{-3}$, $h_0 = 1,000$ km



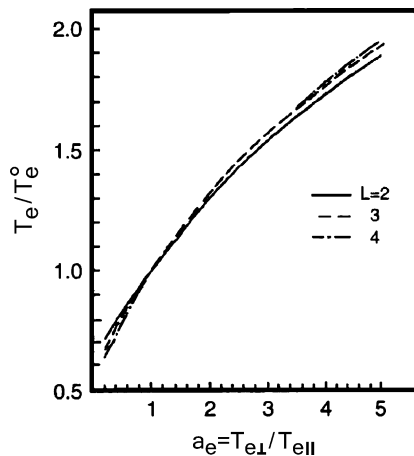
approximation ($a_e = 1$). Figure 6.8 shows the dependence of the relative distribution of electron temperature, T_e/T_e^o , on the anisotropy parameter a_e along the $L = 2$ geomagnetic field line. The dependence has been calculated at $T_{eo} = 3,000$ K, $n_{eo} = 2 \times 10^4 \text{ cm}^{-3}$, $h_o = 1,000$ km, and $a = 4.0 \times 10^{-4} \text{ eV cm}^3 \text{ s}^{-1}$ using (6.4.7). The energy distribution anisotropy leads to an increase in temperature, for $a_e > 1$ ($T_{e\perp} > T_{e\parallel}$), and to a decrease in temperature, for $a_e < 1$ ($T_{e\perp} < T_{e\parallel}$), compared with the isotropic case. Such a behavioral trend is accounted for by the dependence of heat conductivity on temperature anisotropy (see Fig. 6.7) and is associated with a respective change in the heat flux (6.4.2). At $a_e > 1$, for example, the heat conductance falls compared with the isotropic case, thereby reducing the plasmasphere-to-ionosphere transport of thermal energy and, hence, raising the electron temperature in the plasmasphere. The situation reverses at $a_e < 1$, namely, the heat conductivity increases and, hence, the heat influx to the ionosphere rises, thereby reducing the plasmaspheric temperature.

The solution indicates that the magnitude of the temperature anisotropy effect peaks at the top of the field line (in the geomagnetic equatorial plane), where the second term of (6.4.7) reaches its maximum. Figure 6.9 shows the dependence of the relative electron temperature distribution on anisotropy in the equatorial plane using the same values of parameters as in Fig. 6.8. It is seen that the difference between T_e and T_e^o rises as the deviation of the energy distribution from the isotropic pattern increases. The effect of the anisotropy increases with increasing L -shell due to the rise in the integral heating of the plasmasphere that accompanies the enhancement in the flux tube volume.

6.4.2 Electron Temperature Anisotropy Estimations

As was shown in previous sections, the temperature anisotropy affects the heat flux and electron temperature significantly. In this section we consider the processes

Fig. 6.9 Electron temperature, relative to the isotropic case, versus the anisotropy parameter a_e in the geomagnetic equator plane at selected L-shells



involved in the formation of the electron temperature anisotropy in the upper ionosphere and plasmasphere.

According to Oraevskii et al. (1985), the transport equation for the transverse and longitudinal energies of the magnetized plasma constituents can be written as follows, if we ignore the transverse components of the heat fluxes and velocities,

$$n \frac{dT_{\perp}}{dt} + B \frac{\partial}{\partial z} \left(\frac{S^{\perp}}{B} \right) - \frac{S^{\perp}}{B} \frac{\partial B}{\partial z} = Q^{\perp} - L^{\perp}, \quad (6.4.8)$$

$$n \frac{dT_{\parallel}}{dt} + B \frac{\partial}{\partial z} \left(\frac{S^{\parallel}}{B} \right) - \frac{2S^{\perp}}{B} \frac{\partial B}{\partial z} = Q^{\parallel} - L^{\parallel}, \quad (6.4.9)$$

where n , $T_{\parallel}$, and $T_{\perp}$ are the density, longitudinal temperature, and transverse temperature of some arbitrary charged component; Q and L are the influx and loss of energy in the indicated degree of freedom, $S^{\perp}$ and $S^{\parallel}$ are the components of the heat fluxes of the transverse and longitudinal energies along the magnetic field, and the coordinate z is measured along a line of force of the geomagnetic field $\mathbf{B}$. Expressions are given in Oraevskii et al. (1985) for the fluxes $S^{\perp}$ and $S^{\parallel}$ in a partially ionized plasma for the conditions prevailing in the Earth's upper atmosphere.

To pursue the analysis, it is convenient to switch from equations for $T_{\perp}$ and $T_{\parallel}$ to an equation for the degree of anisotropy of the temperatures, $X = T_{\perp}/T_{\parallel} - 1$:

$$\begin{aligned} n \frac{d(XT_{\parallel})}{dt} + B \frac{\partial}{\partial z} \left[\left(S^{\perp} - S^{\parallel} \right) / B \right] - 3 \frac{S^{\perp}}{B} \frac{\partial B}{\partial z} \\ = \left(Q^{\perp} - Q^{\parallel} \right) - \left(L^{\perp} - L^{\parallel} \right). \end{aligned} \quad (6.4.10)$$

It is convenient to combine the terms involving heat fluxes with the quantity $Q^\perp - Q^\parallel$ and treat them as one energy source term shaping the anisotropic plasma energy distribution. Equation (6.4.10) then becomes

$$n \frac{d(XT_\parallel)}{dt} = Q_x - L_x, \quad (6.4.11)$$

where

$$\dot{Q}_x = Q^\perp - Q^\parallel + \frac{3S^\perp}{B} \frac{\partial B}{\partial z} + B \frac{\partial}{\partial z} \left(\frac{S^\parallel - S^\perp}{B} \right), \quad L_x = L^\perp - L^\parallel.$$

Equation (6.4.11) describes the behavior in space and time of the temperature anisotropy of an arbitrary charged component as a result of the action of the sources (Q_x), shaping this anisotropy and the collision relaxation described by the term L_x .

To investigate the anisotropy under steady-state conditions, the time derivative in (6.4.11) is omitted and X is determined from the source-sink balance condition:

$$Q_x = L_x. \quad (6.4.12)$$

In this form, (6.4.12) corresponds to the peculiar quasilocal approximation for X (nonlocal terms also enter Q_x).

Under conditions that exist in the ionosphere and plasmasphere, the relaxation of the electron-temperature anisotropy occurs as a result of Coulomb (electron–electron and electron–ion) collisions. Equation (6.4.12) may then be written (Konikov and Khazanov 1982c, 1984) as

$$\sqrt{1 + 2X_e/3} [K_{200}(X_e) - K_{002}(X_e)] = 27.7 \left(\sqrt{T_e} / n_e^2 \right) (Q_{x_e}), \quad (6.4.13)$$

where $T_e = (2T_\perp + T_\parallel)/3$; Q_x is expressed in $\text{eV cm}^{-1} \text{ s}^{-1}$, T_e in K, and n_e in cm^{-3} .

Equation (6.4.13) permits the known values of Q_x , N_e , and T_e to be used in finding the value of temperature anisotropy. The anisotropy sources enter Q_x in an additive manner, so we can quantitatively analyze the produced temperature anisotropy for each of the sources separately, thereby facilitating comparison of the effectiveness of different mechanisms.

6.4.2.1 Heat Fluxes

The asymmetry of (6.4.8) and (6.4.9) with respect to the interchange of $S^\perp$ and $S^\parallel$ reflects the fact that a change in the thermal energy of a plasma due to transport of this energy by heat fluxes is different for the longitudinal and transverse degrees of freedom. In other words, heat fluxes in a plasma cause the energy distribution of

the plasma to become more anisotropic. This effect is described quantitatively by the terms containing the heat fluxes in (6.4.10) and (6.4.11) for X. In order to evaluate heat fluxes as a source of temperature anisotropy, all other sources that produce anisotropies in the energy distribution are neglected. Q_x can then be written as

$$Q_x = B \frac{\partial}{\partial z} \left(\frac{S^{\parallel} - S^{\perp}}{B} \right) + \frac{3S^{\perp}}{B} \frac{\partial B}{\partial z}. \quad (6.4.14)$$

According to the calculations of Khazanov et al. (1984), the first term on the right-hand side is negligible in comparison with the second term except in a narrow region near the geomagnetic equator, where the derivation $\partial B/\partial s$ tends toward zero. Outside this equatorial region, Q_x can be approximated as

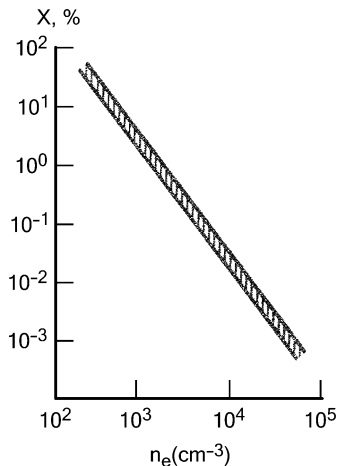
$$Q_x \approx S^{\perp}/L_B, \quad (6.4.15)$$

where $L_B = B(\partial B/\partial z)^{-1}$ is the characteristic dimension for the changes in the geomagnetic field.

If the heat flux is directed along the geomagnetic field, $Q_x > 0$, the temperature anisotropy caused by this source is positive, $T_{\perp} > T_{\parallel}$. The reason for this result is that, in the region from which energy is transported by the heat fluxes, the longitudinal energy flux $S^{\parallel}$ is greater than the transverse-energy flux $S^{\perp}$. The anisotropy in the heat fluxes results in a faster decrease in $T_{\parallel}$ than in $T_{\perp}$ (see the previous section and Khazanov et al. 1984). Because $S_e^{\perp}/v_e \approx S_i^{\perp}/v_i$, at $T_e \approx T_i$, it can be inferred that the heat fluxes introduce equal degrees of anisotropy in the energy distributions of the electrons and ions in the present simplified example. Therefore, the analysis is restricted to the electrons.

Working from (6.4.13), the temperature anisotropy is estimated assuming an arbitrary distribution of the electron density. Consequently, without limiting the analysis to a specific n_e profile, the behavior of an electron-temperature anisotropy as a function of the absolute value of the plasma density can be explored. Figure 6.10 shows the range of values of the electron-temperature anisotropy, calculated from (6.4.13) with the help of (6.4.15), for typical conditions in the Earth's ionosphere and plasmasphere given by $T_e = (2-5) \times 10^3$ K, $L_B \approx 10^9$ cm and $S_e^{\perp} \approx 3 \times 10^9$ eV cm⁻² s⁻¹. For the low-latitude and mid-latitude parts of the plasmasphere ($L \leq 3$, where $n_e \geq (1.5-2) \times 10^3$ cm⁻³), the temperature anisotropy caused by the heat fluxes is weak, less than 1%. This result can be credited to the large values of the relaxation terms, $L_x \propto n_e^2$ at these plasma densities. In contrast, for regions in the outer plasmasphere with $L \geq 3$, where the plasma density may drop below 10^3 cm⁻³ because of magnetospheric substorms (Park 1974; Park et al. 1978), the anisotropy reaches substantial levels, $\sim 20-50\%$. The nonuniform heat transfer in the geomagnetic field at these L values gives rise to an anisotropic energy distribution of the charged particles in the outer plasmasphere where the low electron densities slow the relaxation.

Fig. 6.10 Dependence of X_e on plasma density n_e when the source of anisotropy is the nonuniform character of parallel and perpendicular energy transfer in the geomagnetic field. Parameters adopted for this figure are $T_e = 2,000\text{--}5,000$ K, $H_B = 10^9$, and $S_e = 10^9$ eV $\text{cm}^{-2} \text{s}^{-1}$



As mentioned previously, the change in energy of the plasma owing to transport by heat flows occurs at different rates for the longitudinal and transverse degrees of freedom (Oraevskii et al. 1985). In other words, the presence of heat flows in the plasma leads to the development of anisotropy in the plasma energy distribution. The heat flow in the longitudinal degree of freedom exceeds the corresponding flow in the transverse degree of freedom (Oraevskii et al. 1985). Therefore, in regions from which there is an outflow of thermal energy, $T_{||}$ decreases more rapidly than does $T_{\perp}$, and a positive temperature anisotropy $X_e > 0$ ($T_{\perp} > T_{||}$) is formed. This situation is reversed where a thermal energy inflow occurs, and the anisotropy is negative: $T_{\perp} < T_{||}$.

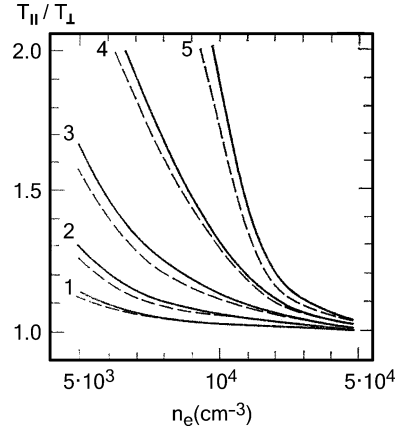
The latter situation is realized in the height region considered here. The heat flow into the outer ionosphere along the lines of force of the geomagnetic field results in the formation of a negative electron temperature anisotropy, which is consistent with the experimental data of Clark et al. (1973) and Oyama et al. (1985). The source Q_x in this case can be described as presented by Konikov and Khazanov (1982c) and by Oraevskii et al. (1985) as:

$$Q_x \approx -|\nabla_{||} S_e| \approx -S_e/H_s, \quad (6.4.16)$$

where S_e is the longitudinal heat flow in the electrons, and H_s is the spatial scale of the changes in heat flow along the magnetic field. To find the magnitude of the anisotropy, the source (6.4.16) must be substituted into (6.4.13), which is solved for X_e (or $T_{||}/T_{\perp}$). The numerical solution of this equation as a function of plasma concentration, for various values of heat flow and electron temperature, is shown in Fig. 6.11.

The quantity 10^{11} eV $\text{cm}^{-2} \text{s}^{-1}$ was selected as the upper value of the heat flow. As demonstrated in Gorbachev et al. (1987), this heat flow may enter the subauroral ionosphere from higher-altitude regions of the plasmasphere, where there is effective interaction (heating) between the cold plasma and the magnetosphere ring current. The trends toward an increase in the anisotropy with increasing heat flow,

Fig. 6.11 The electron temperature anisotropy as a function of the plasma density found from the solution of (6.4.18) with source given by (6.4.19). The numbers 1–5 labeling the curves correspond to the heat flows $S_e = 10^9, 5 \times 10^9, 10^{10}, 5 \times 10^{10},$ and $10^{11} \text{ eV cm}^{-2} \text{ s}^{-1}$; the *solid lines* represent $T_e = 4,000 \text{ K}$ and the *broken lines* represent $T_e = 3,000 \text{ K}$



and a decrease in the anisotropy with increasing plasma concentration are well-illustrated in the Fig. 6.11. The drop in the magnitude of the anisotropy with electron density is due to the rise in Coulomb relaxation. The calculations show that heat flows $S_e \geq 5 \times 10^9 \text{ eV cm}^{-2} \text{ s}^{-1}$ may cause a marked anisotropy of the electron temperature in the presence of electron density $n_e \leq 2 \times 10^4 \text{ cm}^{-3}$. On transition from daytime to nighttime conditions, the present model predicts a growing electron-temperature anisotropy owing to the decrease in plasma concentration (weakening of collisional relaxation). This prediction is in agreement with observational data.

To summarize, the calculations have shown that the heat flow in the auroral and subauroral upper ionosphere can lead to the formation of electron temperature anisotropy, which agrees in sign and magnitude with the measurements (Clark et al. 1973; Oyama et al. 1985); therefore, the heat flow may be considered as one probable mechanism of plasma anisotropy in this ionospheric region.

6.4.2.2 Superthermal Electrons

The nonuniformity of the rate at which energy in the various degrees of freedom is transferred to thermal electrons as they interact with superthermal electrons results from the anisotropy of the superthermal electron distribution function. According to Konikov and Khazanov (1981), the source Q_x in (6.4.13) is

$$Q_x = A_e n_e \int f_2(E) dE, \quad (6.4.17)$$

where

$$A_e = 16\pi^2 e^2 \ln \Lambda / (5m_e^2), \quad f_2(E) = \frac{5}{2} \int_{-1}^1 f_e(E, \mu) P_n(\mu) d\mu$$

is the second coefficient in the expansion of the superthermal electron distribution function f_e in Legendre polynomials $P_n(\mu)$, μ is the cosine of the pitch angle, and E is the energy of the superthermal electrons.

Calculation of f_2 , incorporating the ionospheric and plasmaspheric transport of superthermal electrons, yields $Q_x/n_e \leq (5 - 7) \times 10^{-4} \text{ eV s}^{-1}$ (Khazanov 1979). Substitution of this source into (6.4.13) shows that the maximum anisotropy of the electron temperature caused by the superthermal electrons does not exceed 0.1–0.3%. The superthermal electrons thus cannot cause any significant anisotropy in the energy distribution of the thermal electrons and can be ignored as a cause of anisotropy in the thermal plasma when compared with, for example, anisotropic heat fluxes.

6.4.2.3 External Longitudinal Electric Fields

Indirect experimental evidence based on an analysis of the UT changes in the S_q geomagnetic variations (Wagner et al. 1979) and experiments on the injection of barium clouds suggests that longitudinal electric fields may exist in the mid-latitude parts of the ionosphere and the plasmasphere. According to estimates, the strength of these fields is $E_{\parallel} \leq 10^{-8} \text{ V cm}^{-1}$. Similar figures follow from the three-dimensional theory of S_q variations and an analysis of the fields $E_{\parallel}$ produced by the streams of superthermal electrons (Konikov and Khazanov 1984). The reasons for the appearance of these fields will not be discussed here; the fact that they exist in the ionosphere and the plasmasphere may be important in the formation of temperature anisotropy.

The external longitudinal electric field gives rise to a source $Q_x = -2\sigma_o E_{\parallel}^2$ in the equation for X_e , where

$$\sigma_o = \sqrt{2/(\pi m_e)} T_e^{3/2} \left[4e^2 \times \ln \Lambda K_{002} (1 + 2X_e/3)^{3/2} \right]^{-1}$$

is the longitudinal conductivity (Chodura and Pohl 1971). Equation (6.4.13) then becomes

$$(1 + 2X_e/3)^2 K_{002} (K_{200} - K_{002}) = 1.20 \times 10^{15} (T_e/n_e)^2 E_{\parallel}^2, \quad (6.4.18)$$

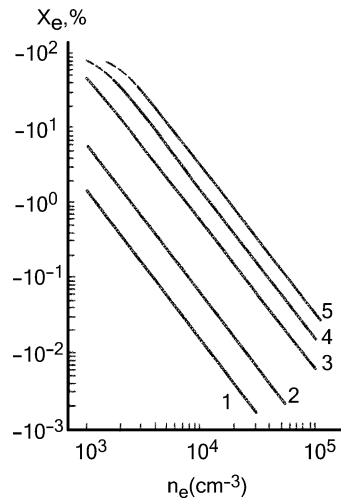
where $E_{\parallel}$ is in volts per centimeter. In the case of a slight anisotropy, $X_e \ll 1$, (6.4.18) simplifies to:

$$X_e \approx -6.72 \times 10^{15} (T_e/n_e)^2 E_{\parallel}^2. \quad (6.4.19)$$

These relations are valid if the fields in them do not exceed the critical field E_D that leads to electron runaway and that is given by the Dreicer criterion (Trubnikov 1965):

$$E_D = 0.2 \times 4\pi n_e e^3 \ln \Lambda / T_e = 10^{-9} n_e / T_e (\text{V cm}^{-1}).$$

Fig. 6.12 The electron temperature anisotropy X_e for selected values of parallel electric field $E_{\parallel}$ plotted versus plasma density



Adopting characteristic values of electron density and temperature leads to critical fields in the range $E_D \leq 10^{-8} \text{ V cm}^{-1}$ in the mid-latitude ionosphere and plasmasphere. The effects of parallel electric fields on the electron-temperature anisotropy are therefore explored by substituting electric field values such that $E_{\parallel} < E_D$ into (6.4.18) and (6.4.19).

Figure 6.12 shows values of X_e calculated from (6.4.18) and (6.4.19) as a function of the plasma density for various fields $E_{\parallel}$. It is apparent from the figure that even weak fields, $\sim 10^{-10} \text{ V cm}^{-1}$, can cause a significant anisotropy (2–8%) in the plasmasphere. Stronger longitudinal electric fields, $E_{\parallel} \geq 10^{-9} \text{ V cm}^{-1}$, cause a high degree of anisotropy in the plasmasphere, $X_e > 40\text{--}50\%$, and an anisotropy of 10–20% in the outer ionosphere; these conclusions agree in order of magnitude with experimental data (Clark et al. 1973; Oyama et al. 1985). This agreement suggests that longitudinal electric fields may play an extremely important role in shaping the anisotropy of the electron temperature at both plasmaspheric altitudes and in the ionosphere.

6.4.2.4 Wave–Particle Interactions: Cherenkov Resonance

Collective wave–particle effects are seldom treated in simulations that are based on the solution of the transport equations even though observational evidence supports the importance of these interactions as an energy source for the thermal plasma. Various spacecraft observations have confirmed the occurrence of ion cyclotron (IC) waves in the plasmasphere–ring current overlap region (Young et al. 1981; Roux et al. 1982; Fraser 1985; Perraut 1982). These waves are believed to be generated as a result of the free energy contained in the ring current ion distribution (Cornwall 1965, 1966; Kennel and Petschek 1966). The IC-wave interactions with

charged particles may substantially affect the behavior of thermal plasma parameters. Therefore, hydrodynamic simulations, relevant to the plasmaspheric region, should be based on a set of hydrodynamic equations that include the effects of the wave–particle-type collective interactions where applicable.

The Grad method was used by Konikov et al. (1989) and Gamayunov et al. (1991) to obtain a set of hydrodynamic equations allowing for the scattering of thermal electrons from IC-waves in the region of plasmasphere–ring current interaction. The collision terms in this set of transport equations describe the variations of macroscopic quantities due to Cherenkov attenuation of IC-waves on thermal electrons. These terms were calculated for Alfvén (A-) and fast magnetosonic (FMS-) waves using a quasilinear approximation. In particular, the attenuation of the waves was shown to substantially heat the cold electron component in the outer plasmasphere.

The heating of thermal electrons during Cherenkov resonance with oblique IC-waves results from work done on the electrons by the component of the wave electric field aligned along the external magnetic field. The work results in an increase of the electron energy associated with the longitudinal degree of freedom. The preferential heating of the thermal electrons in the longitudinal direction during collisionless IC-wave attenuation facilitates the formation of an anisotropic thermal electron energy distribution.

In the case of thermal electron–IC-wave interactions, $Q_{\perp}$ is zero. This results in $Q_x = -Q_{\parallel} < 0$, where $Q_{\parallel}$ is defined by Konikov (1990) and Gamayunov et al. (1991). Thus, the IC-wave energy absorption in the plasmasphere–ring current interaction region gives rise to negative anisotropy of electron temperature, namely, $X < 0$ or $T_{\parallel} > T_{\perp}$.

Figure 6.13 displays the electron temperature anisotropy values obtained by solving (6.4.13) as a function of electron temperature for selected values of the electron density. The longitudinal heat source (due to simultaneous heating by the A- and FMS-waves), used to obtain Fig. 6.13, is plotted in Fig. 6.14 as a function of parallel electron temperature for selected plasma densities. The increase in the heat source with plasma concentration seen in Fig. 6.14 was dictated by the requirement that the electron temperatures remain within the range of observed values.

Fig. 6.13 The electron temperature anisotropy $T_{\parallel}/T_{\perp}$ arising from a longitudinal heat source (due to simultaneous heating by A- and FMS-waves) as a function of $T_{\parallel}$ at different values of n_e . The longitudinal heat source, used in this calculation, is displayed in Fig. 6.14

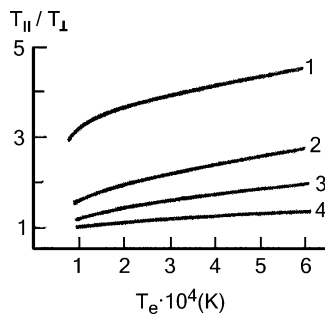
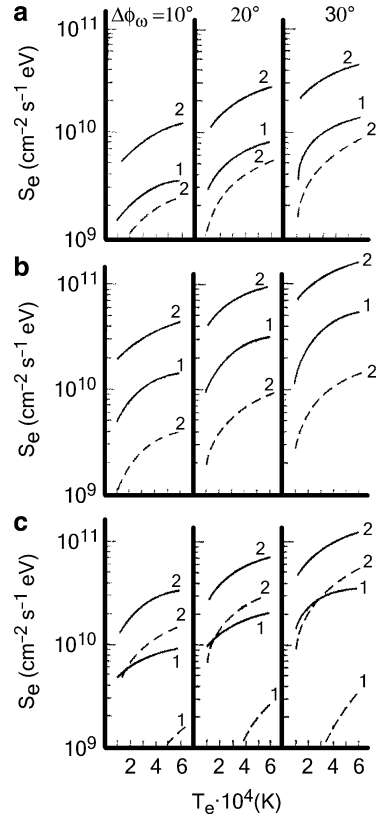


Fig. 6.14 Values of the longitudinal heat source (due to simultaneous heating by A- and FMS-waves) plotted versus plasma density. The electron temperature anisotropy that results from this source is plotted in Fig. 6.13



A general trend in the anisotropy to decrease with increasing plasma concentration is apparent in Fig. 6.13. This trend occurs despite the simultaneous increase in the longitudinal heating with increasing plasma concentration shown in Fig. 6.14. The reasons for such behavior are as follows. The value of the anisotropy is defined by the balance between the anisotropization processes on the one hand, and the collisional relaxation on the other. The Coulomb relaxation rate increases more rapidly with plasma concentration ($\propto n_e^2$) than Q_x , eventually resulting in a decrease of anisotropy as n_e increases. An increase is observed in the anisotropy as the parallel heat source and resulting temperature are increased (Fig. 6.13). This increase in anisotropy has two sources. The enhancement in the parallel heat source makes an obvious contribution to the anisotropy of the electron energy distribution. In addition, a decrease in the Coulomb relaxation rate that is proportional to $T_e^{-1/2}$ allows the increased anisotropy to be maintained more effectively against collisional relaxation as the electron temperature increases.

As the plasma concentration decreases below 10 cm^{-3} , however, the character of the dependence of $T_{\parallel}/T_{\perp}$ on the electron temperature and corresponding parallel heat source changes from a monotonic increase with increasing temperature to a

single maximum with a drop off at both higher and lower temperatures. This trend arises from the $Q_{\parallel}$ behavior feature examined by Konikov (1991) in his Fig. 4 and Sects. 5.2 and 6, i.e., from a decrease of the heating effectiveness with increasing m at low n_e values. The strong control of the source-produced temperature anisotropy over the effectiveness of the heat source itself (via modified heat conductances) allows a feedback loop to be setup wherein the heat source becomes self-regulating. This is seen most clearly in Figs. 6.13 and 6.14 in the case of $n_e = 5 \text{ cm}^{-3}$, when the distributions $\mu(T_{\parallel})$ and $Q(T_{\parallel})$ exhibit respective peaks. At $n_e > 10 \text{ cm}^{-3}$, the peaks are located outside the examined temperature variation range (at $T_{\parallel} > 10^5 \text{ K}$).

A number of factors can modify the electron-temperature anisotropy produced by a local heat source, such as wave–particle collective effects. Although these factors were not included in estimations of the IC-wave anisotropy effects, the significance of their influence is evaluated here. The heating of the thermal electrons by the waves is accompanied by a heat flux in the electron gas that carries energy away from the interaction region along the Earth’s magnetic field lines to the underlying ionosphere. Therefore, the value of the anisotropy is defined not only by the collisional relaxation, taken into account in (6.4.13), but also by the energy outflow (cooling) due to the heat flux. The details of this cooling and the resultant modifications to the IC-wave heating process are examined briefly below.

If the vector components $S_{\perp}^{\perp}$ and $S_{\perp}^{\parallel}$ transverse to the magnetic field are neglected, then, according to (6.4.14), the source of anisotropy resulting from the heat fluxes is of the form (Konikov and Khazanov 1984; Oraevskii et al. 1985)

$$Q_x = B_o \frac{\partial}{\partial z} \left(\frac{S_{\parallel}^{\parallel} - S_{\parallel}^{\perp}}{B_o} \right) + \frac{3S_{\parallel}^{\perp}}{B_o} \frac{\partial B_o}{\partial z}, \quad (6.4.20)$$

where z is a coordinate in the direction of the Earth’s magnetic field. In the near-equatorial region, we have $\partial B_o / \partial z \rightarrow 0$, so (6.4.20) gets simplified:

$$Q_x = \frac{\partial}{\partial z} (S_{\parallel}^{\parallel} - S_{\parallel}^{\perp}) \cong \frac{\partial S_e}{\partial z}, \quad (6.4.21)$$

where $S_e = S_{\perp}^{\perp} + S_{\parallel}^{\parallel} / 2$ is the total heat flux. In the given case, $Q_x > 0$ and the heat flux produces a positive anisotropy of electron temperature ($X > 0$, or $T_{\parallel} < T_{\perp}$) in the plasmaspheric energy source region. This is due to the efficient transport of parallel energy into the underlying ionosphere by heat conduction. The resultant sign of the anisotropy is opposite to that arising from the source region heating by IC-waves. Therefore, the heat flux results in a decrease of the temperature anisotropy produced by the collisionless wave attenuation. For this reason, the values of $T_{\parallel} / T_{\perp}$ presented in Fig. 6.13 must be considered an upper limit of the electron temperature anisotropy in the plasmasphere–ring current interaction region due to wave–particle collective effects.

Estimating the anisotropy source due to the heat fluxes, given by (6.4.21), using the calculations of S_e made in Konikov et al. (1989), indicates that its value is of the

same order as the anisotropy source resulting from the wave–particle interactions. Thus, the heat flux is, to a great extent, capable of diminishing the effect of anisotropization of the thermal electron energy distribution as a result of the thermal electron–IC-wave interactions. Any final estimate of the value of the anisotropy must take into account both the local heat sources due to the wave–particle interactions and the transport of this heat away from the source region and cannot be made based solely on a solution to (6.4.13) with Q_x specified by the wave–particle effects. This problem requires that the transport equations (6.4.8) and (6.4.9) for the transverse and longitudinal heat energies be jointly solved or the complete set of anisotropic hydrodynamic equations be integrated (Oraevskii et al. 1985).

In the same manner that heat fluxes can alter the anisotropy produced by a local anisotropic heat source, plasma convection in response to the large-scale magnetospheric electric field can affect the final value of the electron-temperature anisotropy attributed to a particular local source. The adiabatic effects accompanying the plasma transport along the convection trajectories give rise to a negative anisotropy on the dayside of the plasmasphere, and to a positive anisotropy on the nightside (Gorbachev et al. 1983, 1984). The value of the anisotropy source due to the plasma convection is estimated to be $\sim 6 \times 10^{-4} - 10^{-2} \text{ eV cm}^{-3} \text{ s}^{-1}$, assuming a range of plasma parameters typical of the outer plasmasphere. These values of the anisotropy source term are significantly less than the values of the IC-wave source term given by (6.4.21). Therefore, the contribution of convection, compared with IC-wave effects, can be neglected.

6.4.2.5 Magnetospheric Convection

To examine the role of magnetospheric convection in the temperature anisotropy formation, we have to start from the more general equations (6.3.1) and (6.3.2). In this case, similar to manipulations with (6.4.8) and (6.4.9), the source for the formation of temperature anisotropy can be presented as (Khazanov et al. 1994)

$$Q_x = \frac{1}{nB} \left(1 + \frac{2}{3}X \right) \left[Q_{\perp} - (1+X)Q_{\parallel} + (1+X) \frac{\partial S^{\parallel}}{\partial s} - \frac{\partial S^{\perp}}{\partial s} \right] + (1+X) \frac{n^2}{B^3} \frac{d}{dt} \left(\frac{B^3}{n^2} \right). \quad (6.4.22)$$

Equation (6.4.11) then becomes

$$\frac{dX}{dt} = Q_x - L_x, \quad (6.4.23)$$

where

$$L_x = \frac{1}{nB} \left(1 + \frac{2}{3}X \right) [L_{\perp} - (1+X)L_{\parallel}]$$

describes the relaxation of the anisotropy due to particle collisions.

To obtain an approximate estimate of the anisotropy, the quasilocal approximation is assumed (Q_X contains nonlocal terms, as can be seen in (6.4.22)):

$$Q_X - L_X = 0. \quad (6.4.24)$$

According to (6.4.24), the value of the anisotropy is defined by a balance between the sources of anisotropy and the loss (relaxation due to collisions). Because the sources enter (6.4.22) in an additive manner, each of the sources can be analyzed separately and their effectiveness compared with each other.

To examine the effects of convection only, the terms in the square brackets in (6.4.22) are omitted. Substituting the equation describing the variations in density, (6.2.18), into (6.4.22) and allowing for (6.2.4) and (6.2.5), we obtain

$$Q_{X_s} = \frac{2}{3} \Omega_E (1 + X_s) (3 - 2\alpha) \left(1 + \sin \lambda \frac{L^2}{L_{st}^2} \right) \frac{L}{L_{st}} \frac{\cos \lambda}{\sqrt{\delta^2 + \sin \lambda}}. \quad (6.4.25)$$

The sign of the convective source (6.4.25) is defined by the variations in density (through the parameter α) and by local time (the angle λ). At $\alpha < 3/2$ and during the night ($\cos \lambda > 0$), the source $Q_X > 0$ and convection gives rise to a positive temperature anisotropy, $X > 0$ ($T_\perp > T_\parallel$), whereas, during the day, the anisotropy is negative and $T_\perp < T_\parallel$. For $\alpha > 3/2$, the pattern reverses, namely, the anisotropy is negative at night and positive during the day.

This behavior is also illustrated by (6.3.10), which was inferred from the CGL equations. Equation (6.3.10) characterizes the different rates of adiabatic variations for the perpendicular and parallel temperatures under convective plasma motions. At night, the drifts are toward a region of stronger magnetic field and the perpendicular temperature (for $\alpha < 3/2$) rises compared with the parallel temperature, so a positive anisotropy is formed. For $\alpha > 3/2$, a negative anisotropy is formed at night.

The collisional terms $L_\perp$ and $L_\parallel$ exhibit a complicated dependence on X (Chodura and Pohl 1971; Demars and Schunk 1979; Konikov and Khazanov 1982a; Oraevskii et al. 1985). Therefore, to make the quantitative estimates presented below easier to grasp, we consider the weak-anisotropy approximation, $|X| < 1$. For a one-component plasma containing hydrogen ions,

$$L_{X_e} \approx 1 \times 10^2 \frac{n}{T_e^{3/2}} X_e \quad (6.4.26)$$

$$L_{X_i} \approx \frac{n}{T_i^{3/2}} X_i, \quad (6.4.27)$$

where the linear approximation has been used (Oraevskii et al. 1985) and L_X is expressed in s^{-1} , n in cm^{-3} , and T in K. Because there is little dependence in

(6.4.25) on particle species, it follows from (6.4.26) and (6.4.27) that the electron temperature anisotropy at $T_e \approx T_i$ is much more rapidly damped (by two orders of magnitude) than is the ion temperature anisotropy. From (6.4.23) and (6.4.27), the timescale for damping of ion temperature anisotropies is approximately $T_i^{3/2}/n \approx 1$ h for $T_i = 5,000$ K and $n = 100 \text{ cm}^{-3}$. Thus, convective drifts become an important source for ion temperature anisotropies when plasmaspheric densities are less than approximately 100 cm^{-3} .

Let the ion temperature be estimated in terms of (6.4.27) at midnight ($\lambda = 0$) and noon ($\lambda = \pi$). From (6.4.25) and (6.4.27), we obtain

$$X_i \approx \pm 2 \times 10^{-4} \frac{T_i^{3/2}}{n} (3 - 2\alpha) \left(\frac{L_n}{L_{st}} \right)^2, \quad (6.4.28)$$

where the plus and minus signs correspond to midnight and noon, respectively. Substituting $n \approx 3 \times 10^2 \text{ cm}^{-3}$, $T_i \approx 10^4$ K, $L_n/L_{st} \approx 0.5$, and $\alpha = 3/2$ in (6.4.28), we find that $X_i \approx \mp 15\%$. As seen from (6.4.28), $X_i = 0$ at $\alpha = 3/2$ and the anisotropy reverses sign for $\alpha < 3/2$.

Higher values of temperature anisotropy may be obtained by varying the plasma parameters. It should be noted, however, that the resulting values of X_i must remain within the applicable range of (6.4.27), which is only valid for low values of the degree of anisotropy. Otherwise the total expressions for the collision terms must be used (Chodura and Pohl 1971; Oraevskii et al. 1985; Khazanov et al. 1996). This is especially important when the combined effect of magnetospheric convection and wave–particle interaction heating are considered, such as by Khazanov et al. (1995) for the simulation of plasma anisotropy of the outer plasmasphere.

As seen from (6.4.22), the heat fluxes also give rise to an anisotropy because of the different rates at which internal energy is transferred by heat fluxes for the parallel and perpendicular degrees of freedom (Konikov and Khazanov 1982b; Oraevskii et al. 1985). The value of the anisotropy source Q_X is highly sensitive to differences in the gradients in the parallel and perpendicular fluxes, which, in turn, may be obtained by numerically solving the complicated three-dimensional problem of the ionosphere–plasmasphere interaction. Such a procedure is beyond the scope of the present work, so we only make a qualitative remark. Plasmaspheric heat fluxes have been shown to give rise to an anisotropy with $T_\perp > T_\parallel$ (Konikov and Khazanov 1982b). For $T_e \approx T_i$, the heat fluxes give identical degrees of anisotropy for both the electron and the ion energy distributions. Therefore, at times when the convective source (6.4.25) is positive, the contribution of the latter will be increased due to the effect of the heat fluxes, thereby enhancing the temperature anisotropy. This corresponds to the nighttime period for $\alpha < 3/2$ and to the daytime period for $\alpha > 3/2$. If the convective source is negative, the heat fluxes will tend to reduce the temperature anisotropy brought about by convection. In this case, the sign of the anisotropy may reverse, depending on the relative strengths of the various sources of anisotropy.

6.4.3 Discussion

The preceding sections clearly illustrate the impact of electron temperature anisotropy on thermal conductivity and on the resulting magnitude and altitude distribution of the full electron temperature. These analytical results, though simplified, give important physical insight into the consequences of anisotropic conductivity and how these consequences may be manifested in numerical models that are more complicated.

There are a number of interesting predictions of the simplified model developed here that have implications for data analysis and the interpretation of the results of isotropic hydrodynamic models. The solution for the variation of electron-temperature anisotropy along the geomagnetic field line, given by (6.4.7), clearly indicates a maximum in the anisotropy near the equatorial plane, where the second term on the right-hand side of (6.4.7) reaches its peak value. Figure 6.9 is a plot of the total electron temperature T_e in the equatorial plane (relative to its value in the isotropic case T_e^0) as a function of the electron-temperature anisotropy a_e . Figure 6.8, generated assuming the same thermal plasma parameters as in Fig. 6.9, displays the field-aligned distribution of T_e (relative to T_e^0) for selected values of a_e . It is seen that the difference between T_e and T_e^0 increases with increasing electron-temperature anisotropy. The larger the temperature anisotropy, the more likely that isotropic hydrodynamic models will seriously under- or over-predict the total electron temperature resulting from a given heat source. Further, the difference between T_e and T_e^0 becomes greater as one moves outward in L-shell, for a fixed value of the anisotropy. The immediate cause of the latter behavior is that the heat fluxes, responsible for the anisotropy, tend to increase with increasing L value. These heat flows are generated to transport heat from the plasmasphere into the underlying ionosphere and are proportional to the magnitude of the plasmaspheric heat source, which is limited by the heat capacity of the plasmaspheric plasma. The heat capacity of the plasmasphere is related to the flux tube volume, which increases with increasing L value. Thus the heat flows become greater with L value exerting increasing influence on the thermal plasma parameters. As discussed below, this predicted behavior is consistent with trends in the observational data.

To apply these concepts further to the interpretation of observations data and isotropic hydrodynamic model results, it is necessary first to clearly outline the physical processes that control the temperature anisotropy in the region of closed field lines. Models predict that anisotropic heat conduction, in this region, is responsible for a reversal in the temperature anisotropy from $T_{\perp} > T_{\parallel}$ to $T_{\parallel} > T_{\perp}$ when moving from high to low altitudes along corotating field lines. These model results will be discussed below. The reasons for this are easily understood. In the corotating region, the dominant energy source for the ionospheric plasma is at high altitudes. Parallel thermal energy is preferentially removed from the plasmasphere and deposited in the ionosphere because thermal conductivity is enhanced parallel to the magnetic field. This results in lower parallel compared with perpendicular temperatures ($T_{\perp} > T_{\parallel}$) in the source region. In the magnetically connected topside

ionosphere, the heat flux in the parallel direction is not efficiently transferred into the perpendicular direction until altitudes are sufficiently low that collisional coupling between the two directions becomes important. Therefore, the high-altitude heat source will result in higher parallel compared with perpendicular temperatures ($T_{\parallel} > T_{\perp}$) in the topside ionosphere.

Observations have confirmed the predominance of electron temperature anisotropies with $T_{\parallel} > T_{\perp}$ (Clark et al. 1973; Oyama and Abe 1987) at altitudes below 1,000 km. The number of observed cases of $T_{\parallel} > T_{\perp}$ increases with geomagnetic latitude for latitudes below 60° and decreases with increasing altitude below 865 km (Oyama and Abe 1987). The increase in occurrence frequency toward higher magnetic latitude may reflect the enhancement in anisotropy that results in response to the rising integral heating as the flux tube volume increases in the outer plasmasphere. This effect is clearly illustrated for the anisotropy at high altitudes in Fig. 6.9. The diminishing occurrence frequency with increasing altitude may be an indication of the predicted reversal in the sense of the electron temperature anisotropy. In addition, Oyama and Schlegel (1988) report on observations of an electron temperature anisotropy ($T_{\parallel} > T_{\perp}$) in the ionosphere on auroral field lines. The physics of auroral arcs implies the presence of parallel electric fields at high altitudes on auroral field lines. Consideration of parallel electric fields as a source of anisotropy in (Sect. 6.4.2, using a simplified analytical model, predicts a significant electron temperature anisotropy at ionospheric and plasmaspheric heights in the same sense as that reported in Oyama and Schlegel (1988). Abe et al. (1990) described anisotropies of the electron distribution that were observed during the first 5 months of operation of the Akebono satellite. They found anisotropies with $T_{\parallel} > T_{\perp}$ at high latitudes in agreement with previous satellite observations; however, at equatorial latitudes, anisotropies with both senses were observed, depending on the geophysical conditions. In the equatorial ionosphere, magnetospheric heat fluxes (which result in ionospheric $T_{\parallel} > T_{\perp}$) do not dominate the energetics of the thermal plasma. The sense of the anisotropy then must be strongly influenced by local heat sources (see discussion in (Sect. 6.4.2); which likely vary in relative importance with altitude and geomagnetic location.

Konikov and Khazanov (1981) and Khazanov et al. (1984) modeled plasmaspheric flux tubes from 100 km in one hemisphere to 100 km in the conjugate hemisphere, taking into account the above inhomogeneity in the thermal conductivity and assuming only a photoelectron energy source. Their model results clearly showed this reversal in anisotropy between high and low-altitude portions of the flux tube, although only the high-altitude ($> 1,000$ km) portion of their model was displayed and discussed in Khazanov et al. (1984). Demars and Schunk (1987) modeled conditions in the vicinity of the plasmapause region in the 16-moment approximation considering anisotropic heat flows as lower boundary conditions along with electron and proton temperatures and densities at 1,500 km altitude. Temperature anisotropy in the electron plasma over most of the high-altitude region (plasmaspheric heights) was in the direction $T_{\perp} > T_{\parallel}$, in agreement with Khazanov et al. (1984).

Watanabe et al. (1989) developed a two-dimensional numerical solution of the Boltzmann equation appropriate for the topside F-region to investigate the electron temperature anisotropy that develops in the presence of electric fields and heat fluxes. Collisional relaxation of the anisotropic thermal electron distribution, resulting from electron–electron and electron–oxygen ion Coulomb collisions, was also included. This numerical solution simulated the variation of the thermal electron distribution function with pitch angle rather than the more simplified representation in terms of moments that was used in the simulations of Konikov and Khazanov (1981), Khazanov et al. (1984), and Demars and Schunk (1987). As a result of the increased numerical complexity, the simulation was carried out only for conditions appropriate to the topside ionosphere, not along the entire flux tube, as in the hydrodynamic models. In agreement with the hydrodynamic simulations, their results demonstrated the development of anisotropy in the thermal electron energy distribution in the presence of an electric field or heat flux, which increased with increasing electron energy. They predict an anisotropy that is more pronounced at high altitudes due to longer Coulomb relaxation times and stronger heat fluxes and/or electric fields than exist at lower altitudes.

In contrast to the electron-temperature anisotropy, which is determined mainly by differences in longitudinal and transverse to the field line thermal conductivities, the ion temperature anisotropy is strongly influenced by adiabatic effects associated with longitudinal plasma motions in the flux tube in addition to anisotropic thermal conductivities. Modeling results of Khazanov et al. (1984) indicate that significant ion temperature anisotropy exists only for 2–3 days after the start of refilling. This initial ion temperature anisotropy is driven by longitudinal plasma motions. The authors, however, include no ion heat source at high altitudes; collisions with thermal electrons are the sole energy source for the ions. When a significant ion heat flux is considered by Demars and Schunk (1987), ion temperature anisotropies with $T_{\perp} > T_{\parallel}$ are found over most of the high-altitude region.

The above discussion indicates that the most likely temperature anisotropy to occur in the closed field line region in the presence of a high-altitude heat source for the electrons and the ions (i.e., the ring current) is one that reverses from $T_{\perp} > T_{\parallel}$ at high altitudes in the plasmasphere to $T_{\parallel} > T_{\perp}$ in the magnetically connected topside ionosphere. In this region of the ionosphere, temperature anisotropies are commonly neglected in models; standard hydrodynamic models are solved only for the full plasma temperature assuming $T_{\perp} = T_{\parallel}$. This neglect of temperature anisotropies in the models may cause significant disagreements between modeled and observed plasma temperatures. As indicated in Fig. 6.11, for a fixed heat source, plasma temperature anisotropies modify the thermal conductivity and produce full electron temperature values that differ markedly from the isotropic case. When the temperature anisotropy is such that the perpendicular temperature is greater than the parallel temperature, the full electron temperatures are in excess of those that occur in the isotropic case. Conversely, model electron temperatures are below isotropic values when the parallel temperature is greater than the perpendicular temperature. Therefore, an isotropic hydrodynamic model of the plasmasphere will underestimate plasma temperatures at high altitude and overestimate them at low

altitudes for a given high-altitude heat source if significant anisotropy of the kind described above ($T_{\perp} > T_{\parallel}$ at high altitudes reversing to $T_{\parallel} > T_{\perp}$ at low altitudes) exists.

A potential example of this effect follows and illustrates how analytic treatments can be useful in helping to interpret the results of numerical models that are more complicated. Recently, comparisons were made between the combined Dynamics Explorer (DE) 1 and 2 observations of electron and ion temperatures for three selected nearly coincident satellite passes through the plasmasphere and results of the field line interhemispheric plasma (FLIP) model. The FLIP model solves isotropic hydrodynamic equations along closed field lines from the footpoint in one hemisphere to the footpoint in the conjugate hemisphere. Consistent disagreement between modeled and observed electron and ion temperatures occurred (Horwitz et al. 1990) for two of these passes. The 100% photoelectron trapping and heat deposition to the plasmasphere was assumed in the FLIP model to force reasonable agreement between measured and modeled plasma temperatures at high altitudes. The FLIP model fairly consistently predicted ion temperatures that were too low in the plasmasphere and electron temperatures that were too high in the topside ionosphere compared with observed values. These discrepancies are of the correct sign to be explained by the temperature anisotropy structure described above. The results of the present analytical treatment suggests that improper representation of the plasma temperature anisotropy should be considered as a potential source of error in comparisons between models and observations of the thermal populations on plasmaspheric field lines and may account for some or all of the discrepancies between plasma temperatures predicted by the FLIP model and observed values. The other possibility is that an important heat source is not represented by the FLIP model.

6.5 Heating and Cooling of the Plasmasphere

The terrestrial ionosphere/plasmasphere system is a very complex one, but significant progress has been made during the last two decades in developing comprehensive theoretical models to describe its behavior (e.g., Polyakov et al. 1982; Roble et al. 1987; Schunk et al. 1986; Guiter and Gombosi 1990; Liemohn et al. 1997a; Schunk and Nagy 1978). These major models are all based on extensive numerical approaches. However, simple analytical solutions are still very useful because they usually allow much better insight into the dependence of the system response on a wide spectrum of relevant parameters. Analytic solutions are also useful in testing numerical model calculations.

In this section, we solve the well-known and commonly used energy balance equation for thermal electrons in the upper ionosphere and plasmasphere. We obtain simple analytic expressions for the electron temperature and heat inflows into the ionosphere, as well as characteristic heating and cooling times. These questions have been considered before, using numerical models, but our analytic

expressions allow a simple and, to some degree, more helpful insight into the processes controlling the electron temperatures in these regions.

6.5.1 *Electron Energy Equation and Sources of Heating*

The theoretical basis necessary to calculate the electron temperatures in the ionosphere and plasmasphere is well known (e.g., Banks and Kockarts 1973; Schunk and Nagy 1978; Khazanov 1979). The electron energy equation, appropriate for the case of no field-aligned currents and altitudes greater than approximately 200 km, simplifies to:

$$\frac{3}{2} k n_e \frac{\partial T_e}{\partial t} = B \frac{\partial}{\partial s} \left\{ \frac{\beta_e T_e^{5/2}}{B} \frac{\partial T_e}{\partial s} \right\} - L_{ei} - L_{en} + Q_e, \quad (6.5.1)$$

where T_e is the electron temperature, B is the magnetic field, s is the distance along the magnetic field line, k is the Boltzmann constant, n_e is the electron density, β_e is the appropriate constant associated with the thermal conductivity, L_{ei} and L_{en} are the electron–ion and electron–neutral loss rates, respectively, and Q_e is the electron heating rate.

The main sources of ionospheric electron heating are collisions between superthermal (low-energy photoelectrons or secondary electrons created by precipitating high-energy electrons) and thermal electrons. Photoelectron production, due to EUV and X-ray solar radiation, peaks between approximately 100 and 200 km, which is also the altitude range where precipitating electrons deposit most of their energy. These superthermal electrons move along the geomagnetic field lines, changing direction and losing energy via collisions with neutral and charged particles (e.g., Nagy and Banks 1970; Banks et al. 1974).

Superthermal electrons escaping from the ionosphere experience small-angle scattering when they move through the plasmasphere, due to Coulomb interactions with the ambient thermal plasma. As a result of this scattering process, some of the electrons are scattered out of the loss cone and become trapped. Sanatani and Hanson (1970) and Nagy and Banks (1970) gave some qualitative discussion of electron trapping and the resulting increase in plasmaspheric heating, but the first attempts of quantitative calculations were those of Gastman (1973), Takahashi (1973), and Lejeune and Wormser (1976).

Khazanov et al. (1992) (see also Sect. 5.6 in this book for details), using the appropriate transport equations for superthermal electrons, obtained the following simple analytic expression for the thermal electron heating rate in ionosphere and plasmasphere:

$$Q_e(s) = E_s q(s) + a_s n_e(s), \quad (6.5.2)$$

where E_s is mean energy gained locally by the thermal electrons from the superthermal ones, $q(s)$ is the total ionization rate, and a_s is the strength of nonlocal heating.

Table 6.2 Integrated column heating rates

$\Theta_{\text{oe}}^{\text{pe}}$ (eV cm ⁻² s ⁻¹)	$\Theta_{\text{oe}}^{\text{rc}}$ (eV cm ⁻² s ⁻¹)	$\Theta_{\text{oe}}^{\text{w}}$ (eV cm ⁻² s ⁻¹)	$\Theta_{\text{oe}}^{\text{J}}$ (eV cm ⁻² s ⁻¹)
10 ⁹ –10 ¹⁰	10 ⁸ –10 ¹⁰	10 ⁹ –10 ¹¹	10 ⁹ –10 ¹⁰

There are a number of other, potentially important, processes besides photoelectrons heating the thermal plasma in the plasmasphere. There are numerous indirect indications of such heating processes, such as high ion and electron temperatures (at times exceeding 10,000 K), enhanced heavy ion (O⁺ and O²⁺) densities at high altitudes, and electron temperature enhancements with associated SAR arc emissions in the ionosphere (Chandler et al. 1988).

Cole (1965) proposed that Coulomb collisions of the thermal electrons with energetic ring current protons will result in energy deposition in the outer plasmasphere and that this energy will be transported down into the ionosphere via heat conduction, resulting in enhanced temperatures and 6,300 Å oxygen airglow emissions, known as SAR arcs. Kozyra et al. (1987) generalized this mechanism to the multi-component case of ring current ions, and showed that most of the energy transfer is the result of Coulomb collisions with heavy ring current ions.

Cornwall et al (1971) suggested that ion cyclotron waves generated by the interaction of ring current ions and thermal plasma near the plasmopause are the likely mechanism for the energy transfer. There has been no definitive confirmation of this suggestion to date. In the outer plasmasphere, electromagnetic ion cyclotron waves, Alfvén (A-) and fast magnetosonic (FMS-) waves can interact effectively with the thermal electrons via Cerenkov resonance. The rate of this heating was calculated (Khazanov and Chernov 1988; Konikov et al. 1989) using quasilinear theory.

Krymskiy (1990) proposed that during the intensification of magnetospheric convection, the azimuthal component of the convection field, which penetrates to the plasmasphere boundary, forms an intense meridional electric field in a narrow zone of subauroral ionosphere, near the projection of the plasmopause. This electric field generates an azimuthal current in the boundary layer via an analog of the Cowling conductivity. The estimates of Krymskiy (1990) indicate that Joule dissipation of this current can enhance the electron temperature in the plasmasphere, and the heat flow along the magnetic field lines into the ionosphere is sufficient for generating SAR arcs.

Table 6.2 shows the range of the expected integrated heating rates of the plasmasphere by photoelectrons (Khazanov and Gefan 1982), the ring current (Kozyra et al. 1987), ion cyclotron waves (Chernov et al. 1990) and Joule heating (Krymskiy 1990).

6.5.2 Electron Temperatures in the Upper Ionosphere

Let us consider the solution to (6.5.1) in the topside F region below approximately 1,000 km. The divergence of the magnetic field can be neglected within this small

altitude range. Furthermore, in the upper ionosphere, the time rate of change of the electron temperatures is controlled by the downward heat flow, which has a relatively long time constant. Therefore pseudo steady-state conditions are present at these altitudes and the time derivative in the energy equation (6.5.1) can be neglected at these altitudes. The two main energy loss processes are Coulomb collisions with the ions and “fine structure” cooling by inelastic collisions with atomic oxygen, the major neutral constituent (cf. Schunk and Nagy 1978). Using the expression for the electron heating rate given by (6.5.2), the electron energy equation can be written as:

$$\frac{\partial}{\partial s} \left\{ \beta_e T_e^{5/2} \frac{\partial T_e}{\partial s} \right\} - \frac{\gamma_{ei} n_e^2}{T_e^{3/2}} \{T_e - T_i\} - \frac{\gamma_{en} n_n n_e}{T_n} \{T_e - T_n\} + E_s q + a_s n_e = 0, \quad (6.5.3)$$

where γ_{ei} and γ_{en} are the appropriate constants for the electron–ion and electron–neutral loss rates, and n_n and T_n are the neutral densities and temperatures, respectively. In writing (6.5.3), a simplified form for the fine structure cooling rate was assumed. An analytical solution of this equation was obtained, assuming that the altitude variation of the electron density is given by an exponential. The details of the solution were discussed by Khazanov (1979) and will not be repeated here. The limiting expression obtained for the variation of the electron temperature in the upper ionosphere is:

$$T_e^{7/2} = T_n T_{eo}^{5/2} + \frac{7H_n^2}{2\beta_{eo}} \{E_s q_{eo} + a_s n_{eo}\} - \frac{7\Phi_{tb}}{2\kappa_{eo}} \{s - s_o\}, \quad (6.5.4)$$

where T_{eo} , q_{eo} , and n_{eo} are the electron temperature, the electron production rate, and the electron density at the level, s_o , respectively; H_n is the neutral atomic oxygen scale height; and φ_{tb} is the electron heat flux at the upper boundary of ionosphere. The effects of electron energy losses to the ions and neutrals show up in the first term on the right-hand side of (6.5.4). When the heating and loss terms become negligible, the first term becomes $T_{eo}^{7/2}$ and the second term vanishes; (6.5.4) then reduces to the simple and well-known expression for electron temperature variations controlled by thermal conduction only (e.g., Banks and Kockarts 1973).

The solution for the ionospheric electron temperature, (6.5.4), depends on the heat flux from the plasmasphere, F_{tb} , which is an unknown parameter. In order to establish the value of this heat flux, we must obtain a solution to the plasmaspheric energy balance.

6.5.3 Plasmaspheric Solution

The electron energy equation in the plasmasphere can be obtained by neglecting the cooling terms in (6.5.1), giving:

$$\frac{3}{2}k n_e A \frac{\partial T_e}{\partial t} = \frac{\partial}{\partial s} \left\{ A \beta_e T_e^{5/2} \frac{\partial T_e}{\partial s} \right\} + A Q_e, \quad (6.5.5)$$

where $A(s)$ is the cross section of the magnetic tube. If we assume that the thermal heat flux is zero at the equator (this would be the situation if there is symmetry between the two conjugate hemispheres), we get, integrating (6.5.5) with respect to s from the magnetic equator (apex), s_a , to an arbitrary position, s , along the field line, the following relation:

$$\frac{3}{2}k \frac{n_e(s)V(s)}{A(s)} \frac{\partial T_e(s)}{\partial t} = -\frac{2}{7}\beta_e \frac{\partial \{T_e(s)\}^{7/2}}{\partial s} + Q_e(s) \frac{V(s)}{A(s)}, \quad (6.5.6)$$

where $V(s) = \int_{s_a}^s A(s')ds'$ is the volume of the magnetic tube. In order to obtain (6.5.6), it was assumed that the s variations of n_e , T_e , and Q_e are much weaker than that of $A(s)$.

Let us now integrate (6.5.6) with respect to s from $s = s_o$ to s (recall that s_o is the lower boundary of the ionospheric solution)

$$\frac{3}{2}k n_e(s)I(s) \frac{\partial T_e(s)}{\partial t} + \frac{2}{7}\beta_e \{T_e^{7/2}(s) - T_{eo}^{7/2}\} = Q_e(s)I(s), \quad (6.5.7)$$

where

$$I(s) = \int_{s_o}^s \frac{V(s')}{A(s')} ds'.$$

Let us now use (6.5.7) to write an expression for T_e at the equator, T_{ea} , assuming that:

$$T_{eo}^{7/2} \ll T_{ea}^{7/2}. \quad (6.5.8)$$

Given (6.5.8), we can rewrite (6.5.7) as:

$$\frac{\partial T_{ea}}{\partial t} + \alpha T_{ea}^{7/2} - \beta T_{ea} = 0, \quad (6.5.9)$$

where

$$\alpha = \frac{4\beta_e}{21k n_{ea} I_a}; \quad \beta = \frac{2Q_{ea} I_a}{3k n_{ea} T_{ea}^s}; \quad T_{ea}^s = \left\{ \frac{7Q_{ea} I_a}{2\beta_e} \right\}^{2/7}, \quad (6.5.10)$$

and T_{ea}^s is the steady state solution of (6.5.7) for the equator. The subscripts refer to values of variables at the apex of the field line or, in other words, at the magnetic equator. Finally, in writing (6.5.9) it was assumed that $Q_{ea} \rightarrow Q_{ea}\{T_{ea}/T_{ea}^s\}$.

Table 6.3 Steady-state dayside electron temperature

$\Theta_{oa} \text{ (eV cm}^{-2}\text{s}^{-1}\text{)}$	10^9	10^{10}	10^{11}
$T_{ea}^s \text{ (}^\circ\text{K)}$	2,820	5,445	10,510

Numerical solutions obtained without this substitution/assumption have indicated that the time taken to reach steady state is not changed to any significant degree; this is so because the second term is more important than the third in establishing the characteristic time of the system. The steady-state temperatures, T_{ea}^s , corresponding to a range of column heating rates is given in Table 6.3.

The solution of (6.5.9) for the case when $\beta \neq 0$ is:

$$T_{ea}(t) = \frac{T_{ea}^s T_{em}}{\left\{ T_{em}^{5/2} [1 - e^{-t/\tau_d}] + T_{ea}^{s5/2} e^{-t/\tau_d} \right\}^{2/5}}, \quad (6.5.11)$$

where T_{ea}^m is the initial electron temperature at the equator (e.g., in the morning if solar heating is assumed to be the only heat source) and:

$$\tau_d = \frac{6n_{ea}T_{ea}^s}{5Q_{ea}} \quad (6.5.12)$$

is the characteristic heating time of the plasmasphere.

The solution of (6.5.9) for $\beta = 0$ (no heating) can be written as:

$$T_{ea}(t) = \frac{T_{ea}^s}{\{1 + t/\tau_n\}^{2/5}}, \quad (6.5.13)$$

where

$$\tau_n = \frac{21n_{ea}I_a}{4\beta_e T_{ea}^{s5/2}} \quad (6.5.14)$$

is the characteristic cooling time of the plasmasphere. It needs to be restated that these solution are only valid if inequality (6.5.8) is satisfied.

6.5.4 Analysis

Equation (6.5.11) describes the heating of the plasmaspheric electron gas, from its initial value, T_{ea}^m , to the steady-state value, T_{ea}^s . This rate of transition is characterized by the time constant t_d . It is convenient to express this parameter in terms of the integrated heating rate of plasmasphere, Q_{oa} ($Q_{ea} = Q_{oa}/(V_o/A_o)$, where V_o and A_o

are the values of V and A at $s = s_o$). Equation (6.5.12) for the characteristic time of plasmaspheric can be rewritten as:

$$\tau_d = \frac{6N}{5\Theta_{oa}} T_{ea}^s, \quad (6.5.15)$$

where $N = n_{ea} V_o / A_o$, the total electron content per unit area of the geomagnetic field line from s_o to the equator.

Using this plasmaspheric solution, we can now calculate the heat flux at the upper boundary of the ionosphere. In order to do this, let us write (6.5.6) for conditions at the upper boundary of the ionosphere:

$$\Phi_{tb} = -\frac{2\beta_e A_b}{7} \frac{\partial T_{eb}^{7/2}}{\partial s} = \frac{3}{2} n_{eb} V_b \frac{\partial T_{eb}}{\partial t} - Q_{eb} V_b, \quad (6.5.16)$$

where the subscript b refers to variables at the assumed boundary between the ionosphere and plasmasphere ($\sim 1,000$ km). Next we make the following reasonable assumptions:

$$T_{eb} \approx T_{ea}; \quad n_{eb} \approx n_{ea}; \quad V_b \approx V_o; \quad \Theta_{eb} \approx \Theta_{eo}. \quad (6.5.17)$$

Using the approximations given by (6.5.17) and using (6.5.11) to evaluate the time derivative of T_{ea} , we get the following expression for the heat inflow into the ionosphere:

$$\Phi_{tb}^d(t) = -\Theta_{oa} \left\{ 1 - \frac{T_{ea}^m T_{ea}^s e^{-t/\tau_d}}{\left[T_{ea}^{m5/2} \{1 - e^{-t/\tau_d}\} + T_{ea}^{s5/2} e^{-t/\tau_d} \right]^{2/5}} \right\}. \quad (6.5.18)$$

In the limit $\frac{t}{\tau_d} \gg 1$ (18) leads to:

$$\Phi_{tb}^d(t) = -\Theta_{oa}, \quad (6.5.19)$$

which is the expected limit; in steady state, the energy flow out of the plasmasphere is equal to the volume energy deposition in the plasmasphere.

In a similar manner we can obtain, using (6.5.13) and (6.5.16), the following expression for the downward heat flow at the ionosphere–plasmasphere boundary for the case of no energy deposition in the plasmasphere (e.g., nighttime conditions when solar heating is the only energy source):

$$\Phi_{tb}^n(t) = -\frac{\Theta_{oa}}{\left\{ 1 + \frac{t}{\tau_n} \right\}^{7/5}}. \quad (6.5.20)$$

Note that in the limit of $t/\tau_n \rightarrow \infty$, the flux does go to zero, as expected.

During magnetically quiet conditions, the only significant heating is by the photoelectrons, which leads to a weak latitudinal variation in electron temperatures. During magnetic storms, electron temperature enhancements and related SAR arcs are observed near the ionospheric projection of the plasmopause (cf. Rees and Roble 1975; Kozyra et al. 1987). It is now well accepted that these temperature enhancements are the result of energy deposition in the plasmasphere, near the plasmopause. A variety of different mechanisms have been proposed to account for this heating of the plasma. These various proposed mechanisms were briefly summarized in the previous section.

The range of plausible heating rates were shown in Table 6.2, ranging from approximately 10^9 to 10^{11} eV cm⁻² s⁻¹. In Table 6.4, the characteristic heating times of the plasmasphere are presented, for different L values, corresponding to heating rates of 10^{10} [τ_d^1] and 10^{11} [τ_d^1] eV cm⁻² s⁻¹, respectively. These results, obtained by using (6.5.12), show that the response times are approximately 1 h or less, a relatively short time compared with the duration of a typical storm.

Let us examine the cooling of the plasmasphere and the underlying ionosphere. The question of the maintenance of elevated nighttime ionospheric electron temperatures has been of interest since the early 1960s (e.g., Geisler and Bowhill 1965; Nagy and Bauer 1968). It is now well established that, at mid-latitudes, under magnetically quiet conditions, the energy stored in the plasmasphere is transported down into the ionosphere, via thermal conduction, during the night, elevating the electron temperatures above those of the background neutral gas. Expressions (6.5.13) and (6.5.20) are important quantitative relations describing the behavior of the electron temperatures at night. They provide a simple tool to evaluate the nightside ionospheric electron temperature variations, and also provide a clear insight into the reasons for the latitude dependence of the temperatures. For example, the integrated column electron density in the plasmasphere, N, increases with geomagnetic latitude and, under otherwise equal conditions, the rate of drop in plasmaspheric and ionospheric T_e decreases. This predicted behavior of nighttime T_e agrees well with observations (cf. Schunk and Nagy 1978). Table 6.5 shows the calculated characteristic cooling times of the plasmasphere for equinox midnight conditions at different L-shells. The ratio of the plasmaspheric electron temperature at midnight, T_e^{mn}, to the steady-state daytime temperature, T_{ea}^s = 4,500K, which is

Table 6.4 Heating time constants

L	2	3	4
τ_d^1	9×10^2	3.6×10^3	1.4×10^4
τ_d^2	1.8×10^2	7.2×10^2	2.8×10^3

Table 6.5 Cooling time constants

L	2	3	4
$\tau_n(s)$	1.1×10^4	5.5×10^4	1.92×10^6
T_{ea}^{mn} / T_{ea}^s	0.52	0.87	0.91

based on photoelectron heating only, are also indicated. These calculations were carried out assuming that $Q_{oa} = 5 \times 10^9 \text{ eV cm}^{-2} \text{ s}^{-1}$. The corresponding calculated electron temperature at $s = s_o$ is 2,500 K. (Note that these values satisfy inequality (6.5.8).)

Finally we compare our analytic expressions with some observed values (Khazanov et al. 1992b). We use the results of radar backscatter observations made at Millstone Hill. The specific data we are using are averages corresponding to 815 km, taken at the equinox, during magnetically quiet conditions. In Fig. 6.15, we show the measured and calculated (6.5.11) normalized electron temperature rise. We chose the value of 1,760 K for T_e^m based on the data. The curve shown in Fig. 6.15 corresponds to the value of $N/\Theta_{oa} = 2.15 \times 10^3 \text{ seV}^{-1}$; using a reasonable estimate of $2 \times 10^{13} \text{ cm}^{-2}$ for the column content implies an integrated heating rate of $8 \times 10^9 \text{ eV cm}^{-2} \text{ s}^{-1}$, which, in turn, is also of the right magnitude comparable to the more sophisticated numerical studies (Khazanov 1979). In Fig. 6.16, we show a normalized electron temperature decrease as measured and as predicted by (6.5.13). The value adapted from the data for T_{ea}^s is 3,350 K and we used $1.1 \times 10^{22} \text{ cm}^{-1}$ for $n_{ea}I_a$. Using this limited database, we found very good agreement between the observed and calculated electron temperature values, giving us added confidence in the validity of our analytical results.

Fig. 6.15 Comparison of calculated and measured temperature variations for equinox daytime conditions: plasmasphere heating

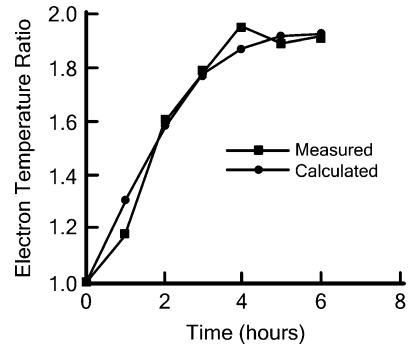
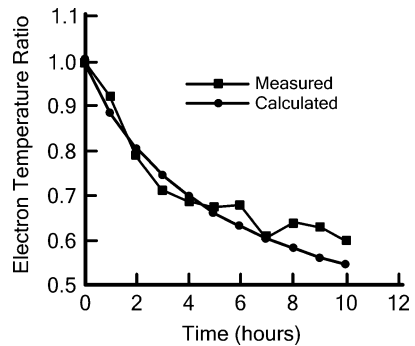


Fig. 6.16 Comparison of calculated and measured temperature variations for equinox daytime conditions: plasmasphere cooling



6.6 Polar Wind

6.6.1 *Historical Remarks*

The polar wind, ion upwelling events, and plasmaspheric refilling are major sources of mass, momentum, and energy for the entire magnetosphere. As the plasma flows up and out of the topside ionosphere, the flow conditions change from subsonic to supersonic, from collision-dominated to collisionless, and from O^+ dominance to H^+ dominance. In the collisionless regime, the ion velocity distributions become highly non-Maxwellian, and the coupling between various plasma species occurs through the development of a self-consistent potential.

The reason for the formation of a self-consistent potential in the collisionless plasma is quite clear. High-mobility electrons tend to overtake ions. As a result, the electric neutrality of the plasma is violated and an electric field appears, which constrains the electrons, forcing them, on average, to travel together with the ions. This field also significantly affects the motion of the ions by accelerating them. The electric force, eE , is proportional to the electron temperature, T_e , and has to be combined with the gravitational force, mg , to determine the pressure distribution of the ions.

Photoelectrons, which form due to ionization of the atmosphere by solar radiation, can alter the self-consistent potential in the space plasma. The presence of the enhanced high-velocity tail in the electron distribution will increase the number of fast ions. Due to enhanced ion acceleration in the expanding plasma, an initial superthermal electron distribution function may be changed.

The polar wind concept was introduced by Banks and Holzer (1968) and Axford (1968). Axford also suggested that the lighter ions must be dragged away from the upper atmosphere by the escaping photoelectron flux. Lemaire (1972) pointed out that the additional electrostatic force acting on the ions due to the escaping photoelectrons can accelerate the ions to higher velocities as long as the photoelectron flux is larger than the escape flux of the thermal electrons. This idea was developed further by Barakat and Schunk (1984) by including precipitating hot electrons of magnetospheric origin in the polar cap region. Several tutorials by Schunk (1986, 1988a, b) and the review by Lemaire et al. (2007) provide a complete picture of the historical development of polar wind studies.

One decade after the effect of photoelectrons on the polar wind flow had been first pointed out, its influence on the field-aligned electric field was rediscovered based on the observed “anomalous” photoelectron distributions over the polar cap (Winningham and Gurgiolo 1982). Another decade later, the effect of the photoelectrons driving the polar wind was inferred from the observed day/night asymmetry in the outflow of thermal O^+ in the polar cap (e.g., Horita et al. 1993), and the availability of reliable measurements of thermal electron anisotropies (e.g., Yau et al. 1995) from the Akebono satellite. Photoelectrons quickly became a hot topic in theoretical polar wind studies. It was then reemphasized that large fluxes of photoelectrons would be able to increase the electric potential difference between

the exobase and the magnetosphere well above the standard value of 2–3 V corresponding to the polar wind in the absence of any photoelectron flux.

Increasing the electrostatic potential difference between the exobase and infinity lowers the critical escape velocity of the O^+ ions, reduces the altitude where their potential energy has a maximum, and decreases the value of the potential barrier that the ions must overcome in order to escape to infinity.

The first generation of exospheric models were developed for cases where the field-aligned distribution of the total potential energy is either (1) a monotonically increasing function of altitude (for the electrons), or (2) a monotonically decreasing function of height (for H^+ or He^+ ions). It was not developed for the case in which the total potential energy has a maximum at a given altitude above the exobase. This is, for instance, the case for the O^+ ions. The presence of such a maximum in the field-aligned distribution of the total potential energy of the O^+ ions was noted by Lemaire and Scherer (1972), but it was not viewed as a key issue. Such a maximum adds a larger number of possible classes of particle orbits in the magnetic, gravitational, and electrostatic fields. In their exospheric PW models where the total potential drop was less than 2 V, this maximum occurs beyond 20,000 km, where the O^+ ions density has become a negligibly minor constituent compared with the H^+ ions, in their polar wind model (see Fig. 1 in Lemaire and Scherer 1972). Beyond such extreme altitudes in the exospheric models of Lemaire–Scherer, the O^+ density was set equal to a vanishingly small constant density, because, at those altitudes, O^+ has become a negligible term in the quasineutrality equation, and therefore no longer affects the electric potential distribution. Unless the electrostatic potential difference between the ionosphere and the magnetosphere significantly exceeds 2–3 V, the effect on the exospheric distributions of H^+ and He^+ was presumed to be negligible.

Chiu and Schulz (1978) point out that, in principle, this was an inconsistency in the first generation of exospheric polar wind models where the potential energy of the O^+ ions is handled as a monotonically increasing function of altitude, and a monotonically decreasing function of height for the H^+ ions. Chiu and Schulz (1978) clearly posed the problem of “accessibility” of some classes of ballistic particles, and introduced the accessibility conditions:

$$d\varphi/dB > 0 \quad (6.6.1)$$

$$d^2\varphi/dB^2 \leq 0. \quad (6.6.2)$$

These constraints (discussed in detail in Chap. 5) simply that the electrostatic potential must increase smoothly from the point of minimum B, with a smoothly decreasing gradient. These constraints fail to be satisfied in some exospheric models, especially when photoelectrons or magnetospheric populations coexist in magnetic flux tubes with cold ionospheric particles. Although Chiu and Schulz (1978) correctly pointed out this deficiency, they did not, however, propose any method to resolve this issue.

Barakat and Schunk (1984) investigated the effects of photoelectrons and elevated temperature of ambient electrons, using a semi-kinetic polar wind model. It was concluded that a nonmonotonic potential barrier (for the O^+) forms. This necessitates the simultaneous solution of the coupled equations for a range of altitudes. They briefly outlined the new requirements and solved it in a self-consistent manner. It was found that the escape flux of O^+ was significantly influenced when the maximum O^+ potential occurred at altitudes less than or approximately three Earth radii.

Khazanov et al. (1998) extended the number of classes of particle orbits in velocity space, and worked out extended analytical expressions for the moments of the VDF when the total potential energy is not a monotonically increasing function of height. They also proposed an iterative method to determine the field-aligned electric potential in cases in which (6.6.1) and (6.6.2) are not satisfied.

Because these accessibility conditions are sometimes violated in Lemaire–Scherer exospheric models, it was relevant to develop a more comprehensive formalism for more general exospheric models. Khazanov et al. (1998) presented general solutions to the Vlasov equation for free-flowing plasma along magnetic field lines using Liouville’s theorem, allowing for an arbitrary potential structure including nonmonotonicities violating Chiu and Schulz’s generalized constraints.

Instead of the Chiu and Schulz’s constraints imposed on φ by the inequalities (6.6.1) and (6.6.2), the following new constraints were introduced by Khazanov et al. (1998) in their Appendix B (see also Sect. 5.1 of this book)

$$d\Pi/dB \text{ must be monotonic} \quad (6.6.3)$$

$$d^2\Pi/dB^2 > 0 \text{ if } d\Pi/dB < 0, \quad (6.6.4)$$

where $\Pi(B)$ is a generalized potential energy that takes into account the gravitational potential, a tentative ponderomotive potential, and the centrifugal potential, Ψ_G , Ψ_{PM} , and Ψ_C , respectively, for each particle species of mass m and electric charge Ze (see Appendix H).

Note that these new constraints are on the derivatives of the potential energy with respect to the magnetic field intensity, not on distance or altitude. The new analytical expressions for the moments of the VDF have been formulated in term of Π for each individual PW particle species (H^+ , O^+ , e^-). The generalized expressions for the densities, particle flux, pressure tensor components, and energy flux in this third generation of exospheric models have been derived by these authors for different truncated VDF at the exobase: bi-Maxwellian and bi-Lorentzian functions. In their Appendix D, they check also that the corresponding moments used in the second-generation models are recovered from these more general formulae when the accessibility constraints (6.6.3) and (6.6.4) are not violated.

6.6.2 General Relations

In this section, we consider the calculation of the upper limit of the potential drop that should be expected in the polar wind due to the presence of photoelectrons. The exclusion of collisional processes leads to easier escape of polar wind particles and therefore to the formation of the largest possible field-aligned electrostatic potential drop over the polar cap. This argument is consistent with the discussion of Tam et al. (1995) and should only be valid above the region of the source where transport is the dominant process in polar wind formation. In this section, a kinetic approach is used for the O^+ , H^+ , and photoelectron (p) distributions, and a fluid approach to describe the thermal electrons (e) and self-consistent electric field ($E_{||}$).

The steady-state collisionless kinetic equation can be presented in the following form:

$$\mu v \frac{\partial f_z}{\partial s} - \frac{1 - \mu^2}{2B} \frac{\partial B}{\partial s} v \frac{\partial f_z}{\partial \mu} + \left(\frac{e_z}{m_z} E_{||} - g \right) \left(\mu \frac{\partial f_a}{\partial v} + \frac{1 - \mu^2}{v} \frac{\partial f_a}{\partial \mu} \right) = 0, \quad (6.6.5)$$

where v is velocity, μ is the cosine of pitch angle, $f_z = f_z(v, \mu, s)$ is the distribution function of species z , s is the distance along the geomagnetic field line, B is the geomagnetic field, and g is the gravitational acceleration.

The most delicate part of the polar wind problem in the presence of photoelectrons is photoelectron–thermal electron interactions. In the absence of collisions, the electron plasma component is coupled with O^+ and H^+ ions based on the quasineutrality and currentless conditions through the self-consistent electric field. In a steady-state collisionless plasma, mass and energy conservation equations for the total electron component can be presented in the form

$$\frac{\partial}{\partial s} \left(\frac{nu}{B} \right) = 0 \quad (6.6.6)$$

$$B \frac{\partial}{\partial s} \left(\frac{Q}{B} \right) - enE_{||}u = 0, \quad (6.6.7)$$

where n , u , and Q are the moments of the electron distribution function

$$\begin{aligned} n(s) &= \int_v f(v, \mu, s) d^3 \mathbf{v}; \quad u(s) = \frac{1}{n(s)} \int_v \mu v f(v, \mu, s) d^3 \mathbf{v}; \\ Q(s) &= \frac{m}{2} \int_v \mu v^3 f(v, \mu, s) d^3 \mathbf{v}. \end{aligned} \quad (6.6.8)$$

Here $d^3 \mathbf{v} = 2\pi v^2 dv d\mu$ is the volume in the velocity space. The term with gravitation in (6.6.7) can be omitted for electrons.

The electron population in (6.6.8) can be separated on two parts (see Sect. 4.4 for details): thermal electrons, with distribution function f_e ; and photoelectrons, with distribution function f_p . Accordingly, the mass and energy conservation equations can be presented as

$$\frac{\partial}{\partial s} \left(\frac{n_e u_e + n_p u_p}{B} \right) = 0 \quad (6.6.9)$$

$$B \frac{\partial}{\partial s} \left(\frac{Q_e + Q_p}{B} \right) - e E_{\parallel} (n_e u_e + n_p u_p) = 0. \quad (6.6.10)$$

Using the kinetic equation (6.6.5) for the photoelectron distribution function ($\alpha = p$), we can find mass and energy conservation for this portion and exclude the photoelectron component from (6.6.9) and (6.6.10). This leads to the set of thermal electron mass and energy equations

$$\frac{\partial}{\partial s} \left(\frac{n_e u_e}{B} \right) = 0 \quad (6.6.11)$$

$$B \frac{\partial}{\partial s} \left(\frac{Q_e}{B} \right) - e n_e E_{\parallel} u_e = 0. \quad (6.6.12)$$

As we pointed out before, these moment equations should be coupled with O^+ , H^+ , and photoelectrons based on the quasineutrality and currentless conditions

$$\begin{aligned} n_e(s) + n_p(s) &= n_{O^+}(s) + n_{H^+}(s) \\ n_e(s)u_e(s) + n_p(s)u_p(s) &= n_{O^+}(s)u_{O^+}(s) + n_{H^+}(s)u_{H^+}(s), \end{aligned} \quad (6.6.13)$$

where densities and fluxes in (6.6.13) for O^+ , H^+ , and photoelectrons must be found based on the solution of the kinetic equation (6.6.5).

6.6.3 The Solution of the Kinetic Equation

The solution of the steady-state collisionless kinetic equation for polar wind applications has been established and discussed in great detail in several papers by Lemaire and Scherer (1970, 1971, 1972). They classify the polar wind particles into four categories (Lemaire and Scherer 1972), (1) ballistic particles that cannot escape; (2) particles with enough energy to escape to interplanetary space; (3) trapped particles with two mirror points along the field line; and (4) incoming particles from the outermost regions.

For the purposes of this section, it is assumed that the incoming particles are absent. This assumption leads to an electrostatic potential distribution with no possibility of trapping particles, and so this population is also omitted.

Because the plasma is considered to be collisionless, the time-independent distribution function in (6.6.5) depends only on the particle's total energy

$$E = \frac{m_\alpha v^2}{2} + e_\alpha U(s) - m_\alpha \Psi(s) \quad (6.6.14)$$

and magnetic moment

$$M = \frac{m_\alpha v_\perp^2}{2B} \quad (6.6.15)$$

The potential differences $U(s)$ and $\Psi(s)$ can be related to the parallel electric and gravitational fields from the following relations

$$U(s) = \int_0^s E_\parallel(s') ds'; \quad \Psi(s) = \int_0^s g(s') ds'. \quad (6.6.16)$$

The particle distribution is constant along any particle trajectory, which is characterized by the constants of motion; therefore, the distribution function at any point along the geomagnetic field can be calculated provided it is known at some boundary in the ionosphere above the region of the source. For convenience, we will transform our distribution function and integration variables from (v, μ) to the constants of motion (E, M) . The particle density and flux for species α can then be written as

$$n_\alpha = \frac{\pi B}{2} \left(\frac{2}{m_\alpha} \right)^{3/2} \iint dE dM \frac{f_\alpha(M, E)}{(E - e_\alpha U + m_\alpha \Psi - MB)^{1/2}}, \quad (6.6.17)$$

$$n_\alpha u_\alpha = \frac{\pi B}{2} \left(\frac{2}{m_\alpha} \right)^2 \iint f_\alpha(M, E) dE dM$$

which is similar to that found by Whipple (1977) and used by Miller and Khazanov (1993) to calculate a steady-state solution for the self-consistent electrostatic potential due to trapped plasma in the magnetosphere.

The particle density at arbitrary positions along the magnetic field can now be easily determined provided the distribution function is known at the lower, ionospheric boundary. The region of determination for the integrals (limits of integration) in (6.6.17) is obtained by considering the condition of reflection, which occurs for $v_\parallel = 0$, and is given by

$$v_\parallel = \left(\frac{2}{m_\alpha} \right)^{1/2} (E - e_\alpha U + m_\alpha \Psi - MB)^{1/2} = 0, \quad (6.6.18)$$

which is a function of the total energy of the particle, the local electric and gravitational potentials and magnetic field, and the sign and the mass of the charged

particle. Equation (6.6.18) depends only on the final position s' and is a necessary but not sufficient condition to guarantee accessibility of an ionospheric particle to s' . For particles to gain access to s' , they must not be reflected at intermediate positions ($0 < s < s'$) along the magnetic field. According to (6.6.18), particles with energy

$$E \geq e_\alpha U(s') - m_\alpha \Psi(s') + M_\alpha B(s') \quad (6.6.19)$$

reach s' and contribute to the particle density $n_\alpha(s')$. Particles having

$$E < e_\alpha U(s') - m_\alpha \Psi(s') + M_\alpha B(s') \quad (6.6.20)$$

are reflected before reaching s' . Using these as limits of integration in (6.6.17), the particle density and flux for species α can be written as

$$\begin{aligned} n_\alpha &= \frac{\pi B}{2} \left(\frac{2}{m_\alpha} \right)^{3/2} \int_{M=0}^{\infty} \int_{E=e_\alpha U(s) - m_\alpha \Psi(s) + M_\alpha B(s)}^{\infty} dE dM \frac{f_\alpha(M, E)}{(E - e_\alpha U + m_\alpha \Psi - MB)^{1/2}}, \\ n_\alpha u_\alpha &= \frac{\pi B}{2} \left(\frac{2}{m_\alpha} \right)^{3/2} \int_{M=0}^{\infty} \int_{E=e_\alpha U(s) - m_\alpha \Psi(s) + M_\alpha B(s)}^{\infty} f_\alpha(M, E) dE dM \end{aligned} \quad (6.6.21)$$

and depend on the electric and gravitational potentials, magnetic field, and ionospheric distribution function.

6.6.4 Moments of the Velocity Distribution Function

In order to calculate the hydrodynamic moments (6.6.8), the distribution function at the lower boundary of the simulation domain must be specified. For the sake of comparing our results with Tam et al. (1995), we start at 500 km (the low-altitude boundary) with upper-half Maxwellian ion and photoelectron distributions. It must be noticed, however, that analytical and numerical analyses for the polar wind (Khoyloo et al. 1991) indicate that the adoption of a Maxwellian for an expanding collisionless plasma in the diverging geomagnetic field is inconsistent and causes a discontinuity in the kinetic solutions. Additionally, choosing the lower boundary above the plasma source region introduces additional uncertainties into the problem and predetermines the solution above the lower boundary.

In our calculations, we separated particles that started from the lower boundary and reached point s on the field line (part of the escaping population discussed in Lemaire and Scherer 1972) from particles that have been reflected below this point (a component of the ballistic particles). Accordingly, we call them the transient and reflected populations. Depending on the electrostatic $e_\alpha U(s)$ and gravitational $m_\alpha \Psi(s)$ energies of polar wind particles, two cases should be considered separately (1) $e_\alpha U(s) > m_\alpha \Psi(s)$ and (2) $e_\alpha U(s) < m_\alpha \Psi(s)$.

Case 1 has only a transient population and the following expressions for the density and particle fluxes

$$n_{\alpha}^{tr}(s) = n_{0\alpha} e^{z_{\alpha}} \left\{ 1 - \Phi\left(z_{\alpha}^{1/2}\right) - y \exp\left(\frac{B}{B_0} \frac{z_{\alpha}}{y^2}\right) \left[1 - \Phi\left(\frac{z_{\alpha}^{1/2}}{y}\right) \right] \right\} \quad (6.6.22)$$

$$n_{\alpha}^{tr}(s) u_{\alpha}^{tr}(s) = n_{0\alpha} u_{0\alpha} \frac{B(s)}{B_0}$$

Here, subscript “0” refers to the lower boundary level; Φ is the error function, $\Phi(x) = (2/\sqrt{\pi}) \int_0^x e^{-t^2} dt$; and the z and y parameters are defined as

$$z_{\alpha} = \left| \frac{e_{\alpha} U(s) - m_{\alpha} \Psi(s)}{T_{\alpha}} \right| y = \left[\frac{B_0 - B(s)}{B_0} \right]^{1/2}$$

where T_{α} is the characteristic temperature (energy) of the Maxwellian for species α at the base. The total density and particle flux that should be used in the quasineutrality and currentless conditions (6.6.9), in this case, are

$$n_{\alpha}(s) = n_{\alpha}^{tr}(s) \quad \text{and} \quad n_{\alpha}(s) u_{\alpha}(s) = n_{\alpha}^{tr}(s) u_{\alpha}^{tr}(s). \quad (6.6.23)$$

Case 2 includes both the transient and reflected particles, and the following expressions for the density and particle fluxes can be found to be

$$n_{\alpha}^{tr}(s) = n_{0\alpha} e^{-z_{\alpha}} \left[1 - y \exp\left(-\frac{B}{B_0} \frac{z_{\alpha}}{y^2}\right) \right] \quad (6.6.24)$$

$$n_{\alpha}^{tr}(s) u_{\alpha}^{tr}(s) = n_{0\alpha} u_{0\alpha} e^{-z_{\alpha}} \left[1 - y^2 \exp\left(-\frac{B}{B_0} \frac{z_{\alpha}}{y^2}\right) \right]$$

and

$$n_{\alpha}^{ref}(s) = n_{0\alpha} \left\{ \Phi\left(z_{\alpha}^{1/2}\right) - \frac{2}{\sqrt{\pi}} \left(\frac{B_0}{B}\right)^{1/2} y e^{-z_{\alpha}} F\left[\left(\frac{B}{B_0} \frac{z_{\alpha}}{y^2}\right)^{1/2}\right] \right\} \quad (6.6.25)$$

$$n_{\alpha}^{ref}(s) u_{\alpha}^{ref}(s) = n_{0\alpha} u_{0\alpha} \frac{B_0}{B} \left[y^2 \exp\left(-\frac{z_{\alpha}}{y^2}\right) - e^{-z_{\alpha}} \right]$$

where F is Dawson’s integral, $F(x) = e^{-x^2} \int_0^x e^{y^2} dy$.

As was pointed out above, the reflected population in (6.6.25) corresponds to the particles that have been reflected below point s back to the base. These quantities are needed at point s , though, and the changing cross-sectional area of the flux tube must be taken into account. Therefore, the total density and particle flux that should be used in the quasineutrality and currentless conditions (6.6.13) are

$$n_{\alpha}(s) = n_{\alpha}^{tr}(s) + \frac{B}{B_0} \left[n_{\alpha}^{ref}(s^{ub}) - n_{\alpha}^{ref}(s) \right] \quad (6.6.26)$$

$$n_{\alpha}(s) u_{\alpha}(s) = n_{\alpha}^{tr}(s) u_{\alpha}^{tr}(s) - \frac{B}{B_0} \left[n_{\alpha}^{ref}(s^{ub}) u_{\alpha}^{ref}(s^{ub}) - n_{\alpha}^{ref}(s) u_{\alpha}^{ref}(s) \right]$$

Here, we add to the transient particles that reached point s , the component of particles reflected between point s and the upper level of the simulation domain, s^{ub} . The flux equation in (6.6.26) can be rewritten in more simple form as

$$n_{\alpha}^{tr}(s)u_{\alpha}^{tr}(s) = n_{0\alpha}u_{0\alpha}\frac{B}{B_{sub}}e^{-z_{\alpha}(s^{ub})}\left[1 - y_{sub}^2 \exp\left(-\frac{B_{sub}}{B_0}\frac{z_{\alpha}(s^{ub})}{y_{sub}^2}\right)\right]$$

where y_{sub} is taken at the upper boundary level.

6.6.5 Thermal Electron Fluid Equations

In the previous section, the analytical expressions for the ion and photoelectron moments in the self-consistent coupling with the thermal electron fluid equations (6.6.11) and (6.6.12) are found as functions of the magnetic and gravitational fields and the self-consistent electrostatic potential. Now we should close the loop and find the corresponding expression for the density of the thermal electrons. The implicit relation for the density of the thermal electrons as a function of the self-consistent electrostatic field can be found from the thermal electron momentum equation

$$-\frac{\partial(n_e T_e)}{\partial s} - \frac{1}{A} \frac{\partial(A m_e n_e u_e^2)}{\partial s} - enE_{||} = 0, \quad (6.6.27)$$

where A is the cross-sectional area of the flux tube. Simple integration of this equation leads to the following expression for the thermal electron density

$$n_e(s) = \frac{1}{T_e(s) + m_e u_e^2(s)} \left\{ n_{e0}(T_{e0} + m_e u_{e0}^2) - \int_0^s n_e(s') \left[\frac{\partial U(s')}{\partial s'} + \frac{A'}{A} m_e u_e^2(s') \right] ds' \right\}. \quad (6.6.28)$$

Now, using the condition of quasineutrality in (6.6.13), an iterative procedure can be used to find the distribution of the self-consistent potential in the polar wind plasma. Only one parameter in (6.6.28) still remains unknown – the electron temperature, $T_e(s)$. To find it, let us present (6.6.14) in the conventional form for the electron temperature by separating the total thermal electron heat flux Q_e into the thermal energy flux Q_e^T and the convective particle motion. Then, using (6.6.10) and considering the thermal energy flux as a thermal conductivity flux according to Banks and Kockarts (1973),

$$Q_e^T = -\chi_0 T_e^{5/2} \frac{\partial T_e}{\partial s}, \quad (6.6.29)$$

the equation for the thermal electron temperature can be written in the form

$$B \frac{\partial}{\partial s} \left\{ \frac{n_e u_e}{B} \left(\frac{5}{2} T_e + \frac{m_e u_e^2}{2} \right) - \frac{\chi_0 T_e^{5/2}}{B} \frac{\partial T_e}{\partial s} - e(U^{sub} - U) \frac{n_e u_e}{B} \right\} = 0. \quad (6.6.30)$$

Here, U^{sub} is potential difference at the upper level of the simulation domain, and χ_0 is the appropriate constant associated with the thermal conductivity. To calculate the thermal electron particle flux in (6.6.30), $n_e u_e$, the currentless condition in (6.6.13) should be used.

Equation (6.6.30) can be integrated twice with respect to s and presented as

$$T_e^{7/2} = T_{e0}^{7/2} + \frac{7}{2\chi_0} \int_0^s B \left\{ \frac{n_e u_e}{B} \left(\frac{5}{2} T_e + \frac{m_e u_e^2}{2} \right) - \left[\frac{n_e u_e}{B} \left(\frac{5}{2} T_e + \frac{m_e u_e^2}{2} \right) \right]_{sub} \right. \\ \left. + \left[\frac{\chi_0 T_e^{5/2}}{B} \frac{\partial T_e}{\partial s} \right]_{sub} - e(U^{sub} - U) \frac{n_e u_e}{B} \right\} ds \quad (6.6.31)$$

The square brackets in (6.6.31) are taken at the upper boundary of the simulation domain.

It should be pointed out here that in order to provide an analytical description of the electron temperature, we greatly simplified the calculation of the electron thermal flux (6.6.30) by using Spitzer's expression for the conductivity coefficient. This would only be true at altitudes less than 3,000 km (Demars and Schunk 1989). However, it was shown by Gorbachev et al. (1991) and Olsen and Leer (1996) that this assumption should not be greatly different from a more complicated description, and the model shows good agreement with the polar wind measurements of Abe et al. (1993), which indirectly demonstrates the validity of this assumption (see next section for the details).

Thus our polar wind model uses the kinetic approach to calculate the distributions of photoelectrons and the O^+ and H^+ ions in the polar wind plasma as an explicit analytical function of the self-consistent electrostatic potential and an implicit analytical solution of the fluid equations (6.6.28) and (6.6.31) to calculate the distribution of the thermal electrons and their temperature, which also contains the self-consistent electrostatic potential. Due to the quasineutrality and currentless conditions (6.6.13), all polar wind plasma parameters and components are coupled and can be determined based on an iterative scheme presented by Khazanov et al. (1997).

Khazanov et al. (1997) found that for low-altitude boundary (500 km) temperatures of 2,000 K for ions and 2,500 K for electrons, varying the photoelectron content at the base produced significant changes in the electrostatic potential and thermal electron temperature. For $n_{p0} = 0.01\%$, the potential drop is 2.87 V and the temperature reaches 3,600 K; for $n_{p0} = 0.10\%$, ϕ_0 is 6.47 V with a high-altitude temperature of 8,800 K; and for $n_{p0} = 1.00\%$, $\phi_0 = 7.25$ V and the temperature rises to 17,300 K. Increasing the low-altitude characteristic energies of the electron species increases the potential and electron temperature, whereas increasing the ion energies decreases the potential. Electron temperatures increase steadily with

photoelectron content, always having the same tendency to rise quickly at low altitudes due to the electrostatic coupling and then reach an asymptotic temperature by $2 R_E$. The results showed little influence on the location of the upper boundary, changing less than 5% in T_e and less than 1% in ϕ when the boundary is moved from $5 R_E$ to $3 R_E$. Inclusion of a source term to represent heating of the thermal electrons through Coulomb collisions with the photoelectrons increases the temperature by less than 1%. In the ion results, a clear separation occurs between a classic polar wind at low photoelectron concentrations and an O^+ -dominated polar wind for high photoelectron densities. Readers should refer to the details of polar wind kinetic studies in the original work by Khazanov et al. (1997) and to the comprehensive review on this topic by Lemaire et al. (2007). Below, we present some quantitative results of this section in comparison with the generalized kinetic polar wind kinetic model.

6.6.6 The Necessity of a Generalized Model

The generalized constraints presented in (Sect. 6.6.1 and (6.1.3) and (6.1.4) must be satisfied for every space plasma population in the calculation throughout the entire spatial domain. Note that the constraints (6.1.3) and (6.1.4) are on the derivative of the potential energy with respect to the magnetic field strength, not on distance. The relationship between distance and magnetic field is shown in Fig. 6.17 for a typical high-latitude field line and an inner magnetospheric field line (equatorial plane geocentric distance of $4 R_E$, $L = 4$). In the following discussion, note that the reference point for ionospheric outflows is the ionospheric boundary, whereas the reference point for trapped and precipitating populations is the magnetospheric boundary.

6.6.6.1 Violation of $d\Pi/dB$ Monotonicity

Let us begin with the requirement of monotonicity for the first derivative of the potential energy with respect to the magnetic field. Consider, for example, oxygen

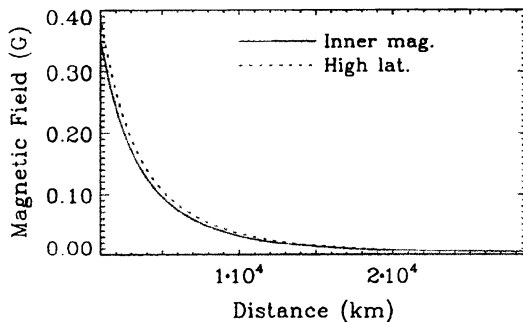


Fig. 6.17 Geomagnetic field strength as a function of distance for a high-latitude field line (solid curve) and an $L = 4$ inner magnetospheric field line (dotted curve)

ions outflowing from the polar cap ionosphere. The ambipolar electric force is usually less than the gravitational force on these particles, causing them to decelerate as they flow up the field line. For a moderate ambipolar field, this situation will generate a total potential energy distribution that satisfies the generalization of (6.6.1) and (6.6.2). However, if superthermal electrons dominate at high altitudes, the ambipolar force can surpass gravity, causing the total potential to decrease with altitude. Calculations based on the generalized model of Sect. 5.1 indicate that photoelectron concentrations of 0.03% can cause the O^+ ions to violate this criterion (Liemohn et al. 1997b). Other forces could also cause this kind of change in the slope, such as centrifugal acceleration due to convection of the field line across the polar cap or ponderomotive force effects from low-frequency waves. These effects increase with altitude (Li and Temerin 1993; Horwitz et al. 1994) and quite easily surpass the gravitational effects, even for modest convection and wave strengths. Figure 6.18 shows a calculation of the total potential for O^+ with the ponderomotive (PM) or centrifugal (C) force added (see Appendix H) for various electric field strengths (perpendicular wave electric field of 10–150 $mV\ m^{-1}$ for Ψ_{PM} and convection electric fields of 10–100 $mV\ m^{-1}$ for Ψ_C at 500 km). Here the total potential energy Π is calculated using the electrostatic and gravitational potential energies from results from Khazanov et al. (1997), with the inclusion of either $m\Psi_{PM}$ from (H4) for the ponderomotive potential energy (Fig. 6.18a) or $m\Psi_C$ from (H8) for the centrifugal potential energy (Fig. 6.18b). Because Ψ_{PM} includes the mass density of the plasma, a rigorous calculation of this potential energy should be done only self-consistently, but this plot still shows the dominant influence of these potential energies, even for moderate electric fields.

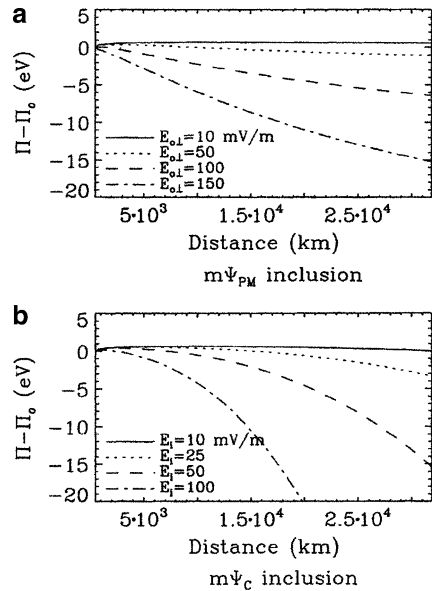


Fig. 6.18 Total potential energies from Khazanov et al. (1997) with the inclusion of (a) ponderomotive force potential energies for the given wave electric fields and (b) centrifugal force potential energies for the given convection electric fields

Violation of the monotonicity of the first derivative for the case of ionospheric outflow is illustrated in Fig. 6.19, showing the total potential energy difference for a decelerated species as a function of distance and magnetic field in Fig. 6.19a, b, respectively. The potential is defined so $\Pi(s_\infty) = 0$. The point s_1 is at the maximum of the potential energy barrier, and s_2 is somewhere beyond this barrier. Figure 6.19c shows the filled region in E - μ space for the point s_2 . Remember that the distribution function is nonzero above the $s_0 v_{||} = 0$ line, and everything to the right and below this line is defined as empty. So, although the permitted region at s_2 is above the $s_2 v_{||} = 0$ line, the distribution function is zero for part of this region (the striped region in Fig. 6.19c). This empty region represents a source cone distribution where velocity space beyond the pitch angles that map to the ionosphere is empty. Because of the potential energy barrier maximum at s_1 , however, all of the particles below the $s_1 v_{||} = 0$ line have been reflected, and thus the region between the s_1 and s_2 lines is also an empty region in the distribution function at s_2 . This new empty region is caused by the acceleration of the particles due to the potential drop from s_1 to s_2 , and the potential energy distribution no longer satisfies the conditions for validity of the previous models.

This violation of monotonicity can occur similarly for the inner magnetospheric case. This situation can occur if some other process besides the electric field dominates the potential structure near the ionosphere, perhaps gravity. If a change in the slope of Π occurs in this case, a change in the filled region for the decelerated particles will occur, shown in Fig. 6.20. Before s_1 , the filled region is simply the area above the $v_{||} = 0$ line for that altitude because the particles are decelerated by the potential energy difference and reflected by the increasing magnetic field. At some s_2 beyond s_1 , however, an empty region now appears near $v = 0$ as they are accelerated through the potential energy drop. Note that there was no empty region for the decelerated species for the previous models because the s_2 line is completely above the s_0 line.

Fig. 6.19 The violation of $d\Pi/dB$ monotonicity along a high-latitude field line for outflowing decelerated particles, showing (a) Π versus s , (b) Π versus B , and (c) E - μ space showing the filled region (dotted area), the empty region included in the previous models (striped area), and the new empty region (shaded area) at spatial point s_2

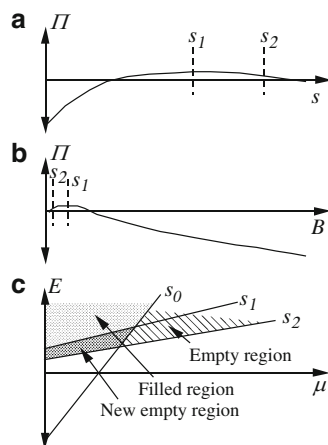


Fig. 6.20 The violation of $d\Pi/dB$ monotonicity along an inner magnetospheric field line for decelerated particles. Plots and shading are the same as in Fig. 6.19

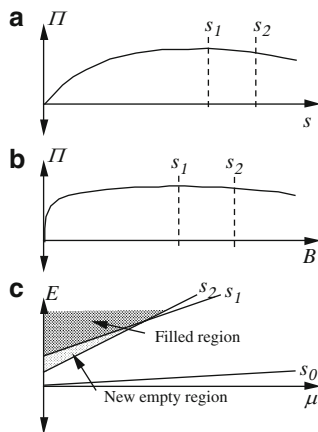
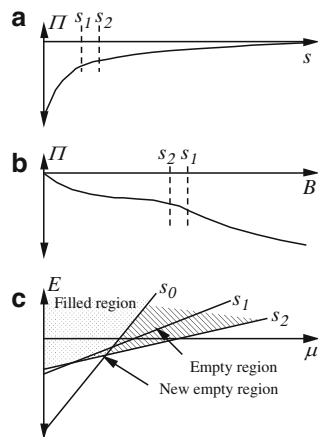


Fig. 6.21 The violation of $d^2\Pi/dB^2$ monotonicity along a high-latitude field line for outflowing accelerated particles. Plots and shading are the same as in Fig. 6.19



6.6.6.2 Violation of $d^2\Pi/dB^2 > 0$ If $d\Pi/dB < 0$

A more frequent violation of the constraints is the violation of the constraint on the second derivative. For high-latitude regions, this can occur quite easily for the decelerated species if the dominant ion changes from oxygen to hydrogen somewhere along the field line. This causes a change in the ambipolar electric field, so the total potential energy will have two characteristic slopes with a knee at the changeover (Fig. 6.21a). Calculation of the second derivative with distance from Fig. 6.21a is monotonic. However, the second derivative with respect to B is not (Fig. 6.21b); the knee in Fig. 6.21a transforms into two inflection points in

Fig. 6.21b. When this occurs, the integrations in E - μ space at some s_2 between these inflection points will resemble that shown in Fig. 6.21c. As in Fig. 6.19c, the possible region of integration is above the $s_2 v_{\parallel} = 0$ line, but the ionospheric source limits the distribution function to a field-aligned cone, creating an empty region (striped area). Because B is now changing faster than Π , a new empty region is also created (shaded area). The particles in this part of E - μ space have been repelled below s_1 and therefore should not be included in the moment calculation at s_2 .

The situation can also arise for the hot plasma of the inner magnetosphere. If a process near the ionosphere causes the potential energy distribution to have two characteristic slopes with respect to s , and the knee is near the ionosphere (as shown in Fig. 6.22a), then Π versus B for the accelerated electrons will have two inflection points and the monotonicity of the second derivative will be violated (as seen in Fig. 6.22b). Figure 6.22c shows the various regions in E - μ space at point s_2 . The drop in the potential energy is greater than the increase in the magnetic field, and the intersections of the $v_{\parallel} = 0$ lines for s_2 and s_1 with the s_0 line are no longer moving monotonically to the left, creating a new empty region (shaded area) between the s_2 and s_1 lines. The striped region is the low-energy hole in the distribution function at s_2 created by the potential energy drop from s_0 to s_2 .

If $d^2\Pi/dB^2 < 0$ and $d\Pi/dB < 0$ for the entire field line, then every single spatial point from the reference altitude to the endpoint will introduce a new line segment to the definition of the filled region from that point on. In the limit of infinite spatial points, these new line segments will form a smooth curve. Figure 6.23a shows this case for a decelerated population in the ionospheric outflow scenario, and Fig. 6.23b shows it for an accelerated population in the trapped plasma scenario. Although some of the new empty regions in E - μ space can be quite small, their proximity to the low-energy range means that they could cause a significant change in the moment calculations, especially the density calculation.

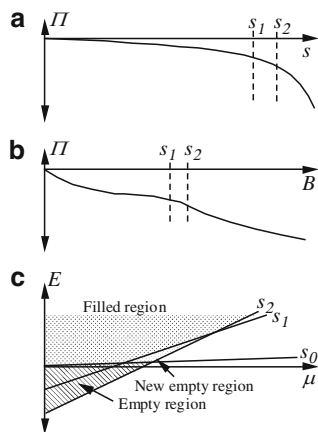
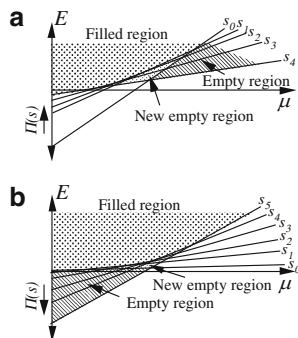


Fig. 6.22 The violation of $d^2\Pi/dB^2$ monotonicity along an inner magnetospheric field line for accelerated particles. Plots and shading are the same as in Fig. 6.19

Fig. 6.23 Violation of the sign difference for the first and second derivatives of Π with respect to B for (a) a high-latitude field line for a decelerated species and (b) an inner magnetospheric field line for an accelerated species. Definitions of the shading are the same as in Fig. 6.19. Note that each spatial point will insert another line into the definition of the filled region all the way to s_∞



6.6.7 Applications of the General Formulation

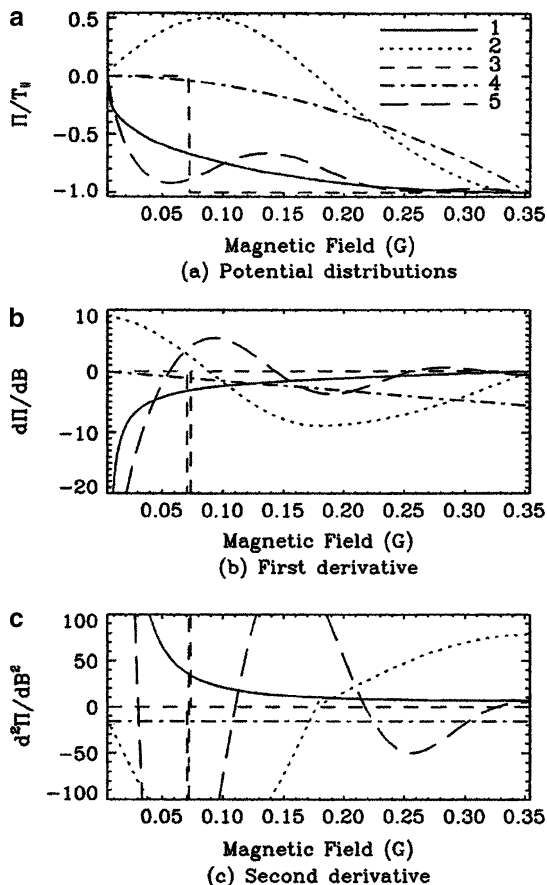
To quantitatively demonstrate the influence of the change in the integration region that this new model specifies, results are shown here for two scenarios: precipitation of a magnetospherically trapped hot population and outflow from the high-latitude ionosphere.

6.6.7.1 Precipitation

The quantitative effects of including other nonlocal $v_{||} = 0$ lines besides that of the reference point can be readily seen by considering magnetospheric precipitation. Magnetospheric precipitation was included in the studies of Lemaire and Scherer (1973), Knight (1973), Whipple (1977), Chiu and Schulz (1978), Stasiewicz (1985), Washimi and Katanuma (1986), Miller and Khazanov (1993), and Wilson et al. (1997), and the resulting formulas are all very similar in that the density or flux at the ionospheric boundary is determined by the total potential drop and magnetic field change between the spatial domain endpoints.

To demonstrate the importance of the generalized formulation, let us calculate the density and flux of the accelerated electrons reaching the ionospheric boundary using the new formulas for several different potential energy distributions between the two endpoints. Five potential energy structures will be considered here, and they are shown in Fig. 6.24a. All five distributions have the same total potential energy drop of $\Pi/T_{||} = 1$, but the shapes are quite different, and so it is expected that the general formulation will calculate different results for each one. A choice of $\Pi/T_{||} = 1$ is quite reasonable for magnetospheric precipitation, because the electrostatic potential difference is typically of similar magnitude to the process responsible for its creation, in this case, the kinetic energy of the trapped population (Miller and Khazanov 1993). In Fig. 6.24, curve 1 corresponds to a potential energy that satisfies the generalized constraints, whereas curves 2–5 violate these constraints in some way. Such total potential energy structures could be created by

Fig. 6.24 (a) Potential energy distributions for magnetospherically trapped population with a net acceleration toward the ionosphere. All five curves have the same final potential energy drop equal to the population's parallel temperature. Also shown are (b) their first derivatives with respect to the magnetic field B and (c) their second derivatives with respect to B



incorporating the additional potential energy terms discussed in Appendix H. Figure 6.24b, c show the first and second derivative of Π with respect to B . To satisfy the generalized constraints, the corresponding curve in Fig. 6.24b must be monotonic, and if this curve is less than zero, then the corresponding curve in Fig. 6.24c must be greater than zero everywhere. Of the five curves drawn, it is clear that only curve 1 satisfies the criteria.

Each of these curves can be used to obtain a filled region for the precipitating part of a trapped population. For this study, the $v_{\parallel} = 0$ lines of 500 equidistant spatial points along an $L = 4$ field line were calculated to determine the filled region for integration at the ionospheric endpoint. The magnetic field distribution used is given in Fig. 6.17, and the anisotropy was taken to be $A = -0.5$ (so $T_{\perp} = 2T_{\parallel}$). The resulting density, flux, and number of integration regions ($\alpha_{\max}$) for curves 1–5 in Fig. 6.24 are presented in Table 6.6. The bump in curve 2 near the ionosphere caused the density to drop nearly in half, and the smooth curve 4 had nearly every spatial point contributing to its ionospheric integration region. These lines form

Table 6.6 Results for hot plasma precipitation

Curve	α_{max}	n/n_o	φ/φ_o
1	2	0.78	1.98
2	28	0.40	1.28
3	2	0.70	1.87
4	479	0.64	1.85
5	2	0.78	1.98

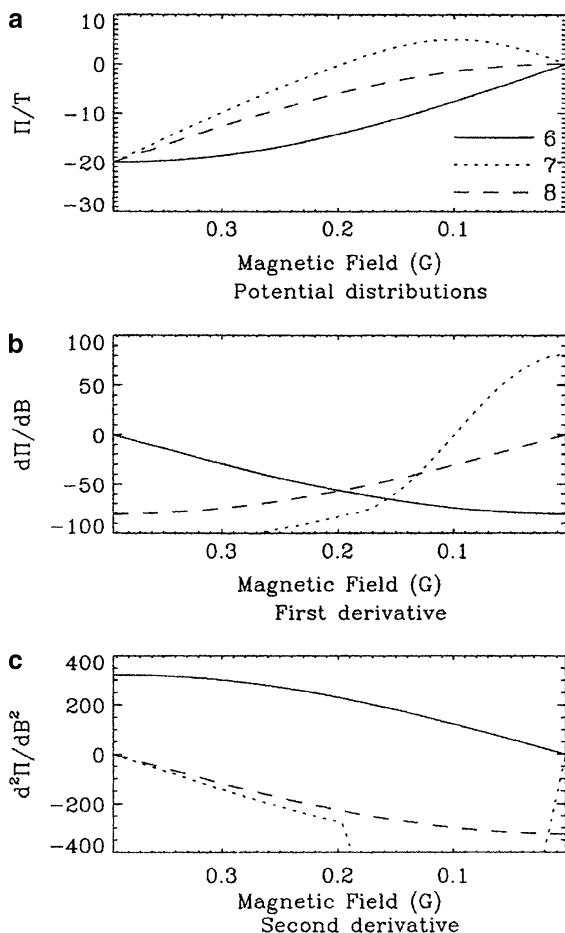
a hyperbola above the simple two-line region of curve 1. The two lines of curve 3 are not the same as the two lines of curve 1; the reference point $v_{||} = 0$ line has been replaced by the line for the spatial point just before the potential drop. Incidentally, curve 5 has exactly the same ionospheric solution as curve 1. This is because all of the violations along the field line happen to be obscured by the ionospheric $v_{||} = 0$ line. However, the moment calculations at most points between the two endpoints will be quite different from those of curve 1, and most of the integration regions for curve 5 at an arbitrary s will have more than two defining lines.

It should be noted that these differences in density and flux are caused by the changes in the integration region due to the various shapes of the field-aligned potential energy structures. All five results have the same total potential energy drop, and so these potential energy distributions would yield identical moment results using the previous models.

6.6.7.2 Ionospheric Outflow

The total potential energy drop used in Sect. (6.6.7.1 is equal to the parallel temperature of the plasma population. This is reasonable for calculations of trapped hot plasmas because the plasma kinetic energy is the primary generator of the field-aligned potential difference (Miller and Khazanov 1993). For the case of ionospheric outflows, however, the potential energy difference can be much greater than the plasma temperature because a primary generator of the potential energy difference is gravity. For example, Khazanov et al. (1997) calculated a total potential barrier for polar wind oxygen ions ranging from 2 to 6 eV, depending on the photoelectron concentration, while the ion temperature was only 0.17 eV at the reference point. These results were a combination of the gravitational potential energy (an 8-eV barrier) and the electrostatic potential energy (a 2- to 6-eV drop). Also, Fig. 6.18 shows that other potential energies can cause the total potential energy difference at high latitudes to be huge compared with the plasma temperature. Therefore a total potential difference of 20 times the plasma temperature will be used for the ionospheric outflow case, which is a conservative estimate of the total potential energy difference experienced by outflowing ions (cf. Khazanov et al. 1997; Wilson et al. 1997). Two cases will be considered: a net potential energy barrier and a net potential energy drop. Three potential energy distributions along a high-latitude field line are shown in Fig. 6.25a for the case of a net deceleration of

Fig. 6.25 (a) Potential energy distributions for an ionospheric outflow population with a net potential barrier. Also shown are (b) their first derivatives with respect to the magnetic field B and (c) their second derivatives with respect to B



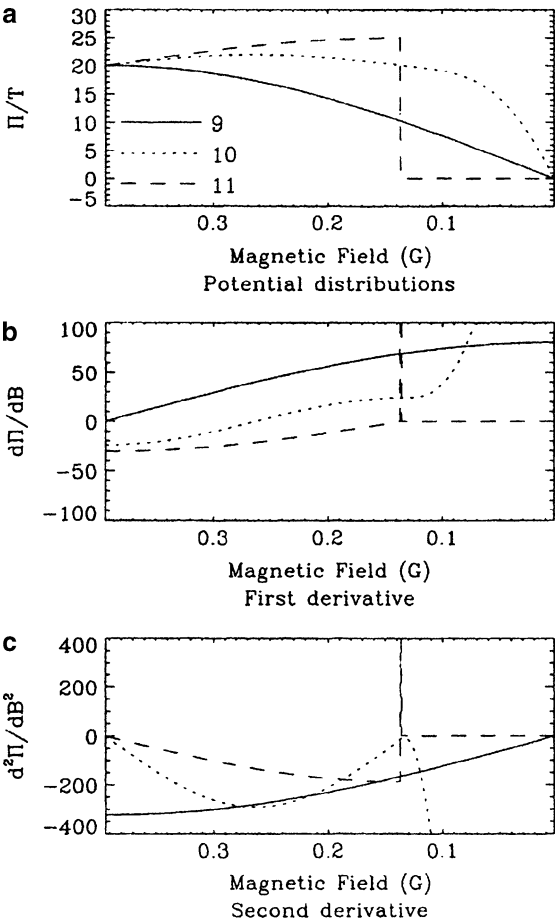
the particles. As is seen in Fig. 6.25b, c, among the three curves, only curve 6 satisfies the generalized constraints.

To obtain calculations of the escaping particles for this decelerated population, 500 points were used along the high-latitude field line shown in Fig. 6.17 for an isotropic reference altitude distribution ($A = 0$). The resulting density, flux, and number of integration regions ($\alpha_{\max}$) for the curves in Fig. 6.25 are presented in Table 6.7. It is clear that the potential energy structure as well as the total potential difference are important factors in calculating the moments of the distribution function. Curve 8, while smooth and monotonic, has a hyperbola for a filled region (similar to curve 4 of Fig. 6.24), and its density is almost 4 times less than that of curve 6. Because the particles subjected to curve 7 encountered a maximum barrier of $\Pi/T = 25$, and those making it past this are then accelerated, this curve yields a density that is over 3 orders of magnitude less than that of curve 6. Note that using

Table 6.7 Results for decelerated polar wind outflow

Curve	α_{max}	n/n_o	φ/φ_o
6	2	2.6×10^{-10}	2.6×10^{-10}
7	304	7.2×10^{-14}	3.3×10^{-13}
8	485	7.2×10^{-11}	1.1×10^{-10}

Fig. 6.26 (a) Potential energy distributions for an ionospheric outflow population with a net potential drop. Also shown are (b) their first derivatives with respect to the magnetic field B and (c) their second derivatives with respect to B



the previous models to calculate these moments would yield identical results for curves 6–8.

The case of net acceleration is given in Fig. 6.26, showing three potential energy curves and their first and second derivatives with respect to B . A potential energy distribution like curves 9 and 10 would be quite easily created with the inclusion of either the ponderomotive or centrifugal force, as seen Fig. 6.18, and one such as curve 11 is analogous to the potential barrier formation discussed by Wilson et al. (1997).

Table 6.8 Results for accelerated polar wind outflow

Curve	α_{max}	n/n_o	φ/φ_o
9	1	7.7×10^{-4}	6.4×10^{-3}
10	109	1.4×10^{-4}	1.2×10^{-3}
11	235	1.1×10^{-5}	1.0×10^{-4}

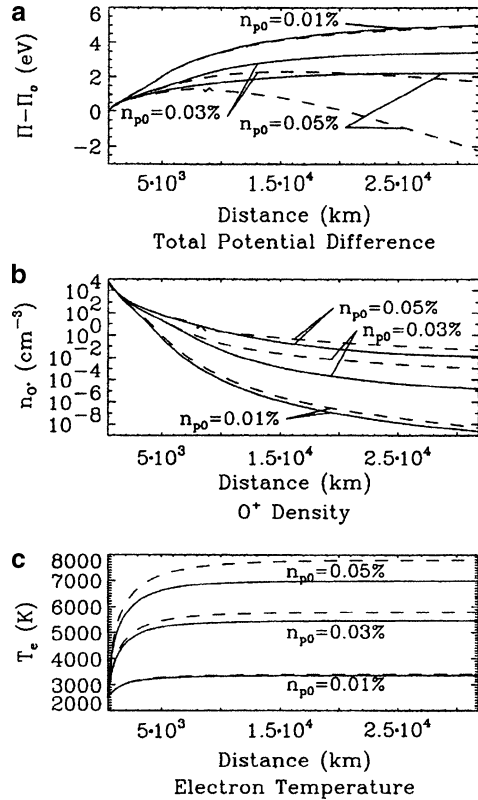
A similar calculation as that for curves 6–8 yields the results shown in Table 6.8. The density and flux for curve 9 are substantially higher than for the other two curves, approaching two orders of magnitude difference. The integration region for curve 9 is defined by only one $v_{||} = 0$ line, that at the base of the ionosphere. Note that these densities and fluxes are much higher than the results for curves 6–8 because of the difference in net potential energy.

6.6.7.3 Self-Consistent Polar Wind Outflows

The final results to be discussed are self-consistent calculations of the generalized model for polar wind outflow. These will be done by comparing results for this model with similar results from Khazanov et al. (1997). Boundary conditions similar to those described by Khazanov et al. (1997) were applied to the new model, including a fluid description for the thermal electrons by solving the momentum and energy equations for the thermal electron density and temperature, respectively, as discussed in that paper. Three levels of n_{p0} , the photoelectron content at the base of the calculation (500-km altitude), are used for these calculations: $n_{p0} = 0.01, 0.03$, and 0.05% . These three values were chosen because they represent realistic values of the photoelectron content in the upper ionosphere, as calculated by Richards et al. (1994) and Liemohn and Khazanov (1995). A photoelectron concentration of up to 1.0% was used by Khazanov et al. (1997) for comparison with the results of Tam et al. (1995) and also to show the maximal influence of photoelectrons that should be expected under any conditions in the high-latitude region. An initial guess for the electrostatic potential energy distribution is made, and the model converges to a steady-state solution through successive iterations of the field-aligned calculation.

The comparative results between the two models are presented in Fig. 6.27. It can be seen that the total potential energy difference for O^+ changes substantially between the two models for higher levels of n_{p0} . In fact, the result changes from a net deceleration to a net acceleration of O^+ . This difference is mainly due to the fact that the new model introduces a hole in the O^+ distribution function at high altitudes, above the altitude of the maximum potential barrier. The previous model filled in this hole, causing the calculation to converge to a different potential energy distribution above this peak barrier. Owing to the dependence of the solution at a given spatial point on those above and below it, the iteration process yields different results for the two models along the entire field line for higher photoelectron concentrations. This change in the potential energy curves has the effect of

Fig. 6.27 Comparison of the results from Khazanov et al. (1997) (solid lines) with those of the new model in the isotropic Maxwellian case (dashed lines). Three values of n_{po} , the photoelectron content at the base of the run (500 km), are shown along the field line for (a) total potential difference (electrostatic and gravitational) for O^+ , (b) O^+ density, and (c) thermal electron temperature

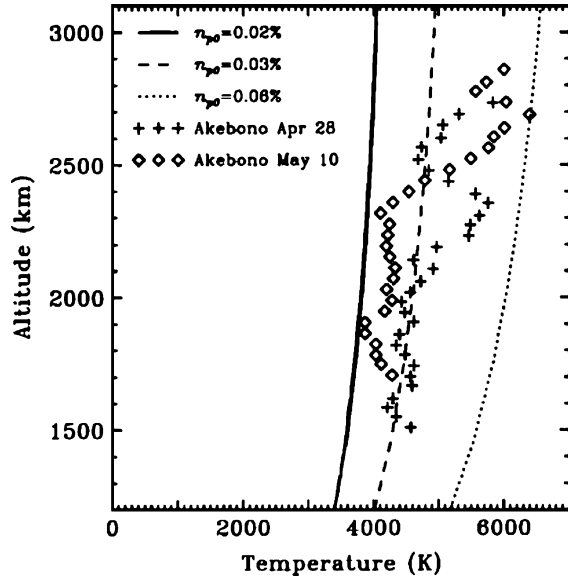


increasing the density by an order of magnitude in the high-altitude region. Also shown is the thermal electron temperature calculated by the two models. The same energy equation was used in the two models, yet T_e is increased in the new model due to the change in the electrostatic potential energy distribution along the field line.

From these results, it can be stated that the validity of the previous models to the scenario of photoelectron-driven polar wind outflows extends up to moderate but not high photoelectron concentrations. As stated above, 0.05% is a high concentration of photoelectrons in the high-latitude upper ionosphere, but it is clear that the low-energy hole in the O^+ distribution region created by the complexity of the potential energy distribution plays a crucial role in the development of the solution and should be taken into account.

The kinetic model results presented above reproduce the Akebono temperature profiles of Abe et al. (1993) very well in the low-altitude range. Figure 6.28 shows this comparison. The data is from April 28 and May 10, 1991, as the satellite passed from the dayside to the nightside at progressively higher altitudes. Data fluctuations were presumed to be latitudinal or local time variations in the ionospheric

Fig. 6.28 Comparison of results for several values of n_{p0} with data from Abe et al. (1993), from April 28 and May 10, 1991



conditions. Shown with the data are three curves at different levels of n_{p0} (0.02, 0.03, and 0.06%), which are reasonable numbers for the photoelectron content based on calculations from the Liemohn and Khazanov (1995) model for the conditions of an illuminated polar cap. This comparison shows that this data can be reproduced by this model for realistic estimates of the photoelectron content at the base.

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Chapter 7

Kinetic Theory of Superthermal Electron Transport

It is important to develop a comprehensive kinetic theory of superthermal electron transport that is equally valid in the ionosphere and in the plasmasphere, and that self-consistently couples the conjugate magnetospheric hemispheres. It can be shown that such a self-consistent approach produces significant changes in the superthermal electron distributions compared with “pure” ionospheric or plasmaspheric calculations. In order to develop such an approach, it is necessary to consider a numerical solution of the kinetic equation for the superthermal electrons that is equally valid for the entire geomagnetic field line, including the transport of these electrons around the globe. Such an approach is presented in this chapter in a rather general form that includes relativistic electrons and highly non-steady state conditions like, for example, artificial electron beam injections from spacecraft.

7.1 Superthermal Electron Studies

A distinctive feature of the ionospheric plasma is a presence of superthermal electrons in the energy range of 1–500 eV. The superthermal electron population is formed due to ionization of neutral atmospheric atoms and molecules by photoionization (photoelectrons) or by impact ionization from energetic particles of magnetospheric origin (secondary electrons). Superthermal electrons play a very important role in a large number of ionospheric and plasmaspheric processes. A fairly detailed knowledge of the superthermal electron distribution function is required when modeling upper atmospheric phenomena, such as the heating of thermal plasma, optical emissions, ionization processes, plasma instabilities, or plasma wave generation.

Several methods have been used to calculate superthermal electron fluxes in the ionosphere (the differential flux is easily related to the distribution function). The simplest approach is a local equilibrium approximation (e.g., Victor et al. 1976), in which the photoelectron flux is determined by balancing local production and loss mechanisms. This approximation is only valid at altitudes that are sufficiently low for transport effects to be neglected. For altitudes above 250 km, a more

sophisticated approach must be applied that can account for the electron transport as well as for the interaction of electrons with other constituents. During the last few decades, many models have been developed to describe superthermal electron transport and energy degradation processes (non-elastic collisions) of photoelectrons (cf. Banks and Nagy 1970; Mantas 1975; Oran and Strickland 1978; Prather et al. 1978; Lejeune 1979; Torr et al. 1990; Link 1992; Khazanov et al. 1992) as well as the transport of auroral (secondary) electrons (Banks et al. 1974; Mantas and Walker 1976; Strickland et al. 1976; Khazanov 1979; Stamnes 1980; Gefan et al. 1985; Porter et al. 1987; Lummerzheim et al. 1989). Except for Khazanov (1979) and Torr et al. (1990), these calculations were based on the separate treatments of superthermal electrons in the ionosphere and in the plasmasphere (for a comprehensive review, see Khazanov et al. 1994).

Khazanov and Gefan (1982) and Khazanov et al. (1994) developed a comprehensive model that is equally valid in the ionosphere and in the plasmasphere and self-consistently couples the two hemispheres. Khazanov et al. (1994) have also shown that this self-consistent approach produces significant changes in the superthermal electron distributions compared with “pure” ionospheric or plasmaspheric calculations. In the next of few sections of this book, a systematic kinetic approach is presented to study ionosphere–magnetosphere transport of superthermal electrons.

7.2 Ionosphere–Plasmasphere Superthermal Electrons Coupling

7.2.1 Mathematical Formulation

The starting point of superthermal electron ionosphere–plasmasphere coupling study with the energies $E > 1\text{--}2$ eV is the kinetic equation (2.5.2), with some additional simplification of collision terms. The collision term simplifications are described in some detail in Appendix I, and the corresponding kinetic equation can be expressed as

$$\begin{aligned}
 & \frac{\beta}{\sqrt{E}} \frac{\partial \Phi}{\partial t} + \mu \frac{\partial \Phi}{\partial s} \\
 &= -\frac{1-\mu^2}{2} \left(\frac{1}{B} \frac{\partial B}{\partial s} - \frac{F}{E} \right) \frac{\partial \Phi}{\partial \mu} + (An_e - EF\mu) \frac{\partial}{\partial E} \left(\frac{\Phi}{E} \right) \\
 &+ \frac{1}{2} \left(\frac{An_e}{E^2} + \sum_n n_n \sigma_n^{(1)} \right) \frac{\partial}{\partial \mu} \left[(1-\mu^2) \frac{\partial \Phi}{\partial \mu} \right] + \sum_n n_n \sum_{j,k>j} E_{jk} \frac{\partial}{\partial E} (\sigma_{en}^{jk} \Phi) \\
 &+ \sum_n n_n \left\{ \frac{\partial}{\partial E} [\sigma_{en}^+ \Phi + (E_n^+ + \langle E \rangle_n)] + \int_{2E+E_n^+}^{\infty} I_{en}^+(E', E) \Phi_0(E') dE' \right\} + q(E, \mu, s).
 \end{aligned} \tag{7.2.1}$$

The method described below is based on a solution to the kinetic equation along the entire length of a closed magnetic field line, that is, simultaneously for the two

conjugate ionospheres and the plasmasphere. We call this the unified approach. It allows the determination of the distribution in energy and pitch angle of superthermal electrons along the complete length of the field line, thereby avoiding the introduction of artificial boundaries between the ionosphere and magnetosphere and, consequently, avoids problems introduced by the uncertainty of these boundary conditions. In addition, it automatically accounts for back-scattered electrons in the atmosphere and plasmasphere, and avoids splitting photoelectrons into a loss cone and a trapped population. The method is not limited to specific situations such as conjugated sunrise or symmetrical illumination of hemispheres, but is equally applicable to arbitrary illumination conditions.

Along a closed magnetic field line in the plasmasphere, the magnetic field is strongly inhomogeneous, while collisional diffusion terms are small due to the long mean free path. In order to decrease undesirable computational effects associated with approximation errors of the derivatives $\partial/\partial s$ and $\partial/\partial \mu$, it is convenient to change variables from (μ, s) to (μ_0, s) , where

$$\mu_0 = \frac{\mu}{|\mu|} \sqrt{1 - \frac{B_0}{B}} (1 - \mu^2), \quad (7.2.2)$$

with B_0 and μ_0 denoting the magnetic field and the cosine of the pitch angle at the magnetic equator of the flux tube. This change of variables is desirable because $\Phi(\mu_0, s)$ now is a slowly varying function with s (μ_0 is the adiabatic invariant). With no external forces, (7.2.1) becomes

$$\begin{aligned} \frac{\beta}{\sqrt{E}} \frac{\partial \Phi}{\partial t} + \mu \frac{\partial \Phi}{\partial s} = & \frac{1}{2} \left(\frac{An_e}{E^2} + \sum_n n_n \sigma_n^{(1)} \right) \\ & \times \left[\frac{(1 - \mu^2)(\mu_0^2 - 1 + B_0/B)}{\mu_0^2} \frac{\partial^2 \Phi}{\partial \mu_0^2} + \frac{\mu_0^2 - 2\mu_0^4 + 1 - (1 + \mu_0^2)B_0/B}{\mu_0} \frac{\partial \Phi}{\partial \mu_0} \right] \\ & + An_e \frac{\partial}{\partial E} \left(\frac{\Phi}{E} \right) + \sum_n n_n \sum_{j,k>j} E_{jk} \frac{\partial}{\partial E} (\sigma_{en}^{jk} \Phi) \\ & + \sum_n n_n \left\{ \frac{\partial}{\partial E} [\sigma_{en}^+ \Phi(E_n^+ + \langle E \rangle_n)] + \int_{2E+E_n^+}^{\infty} I_{en}^+(E', E) \Phi_0(E') dE' \right\} \\ & + q(E, \mu, s). \end{aligned} \quad (7.2.3)$$

The new variable μ_0 takes on values in the range $|\mu_{0B}| < |\mu_0| < 1$, where the lower boundary $\mu_{0B} = \sqrt{1 - B_0/B(s)}$ is a function of s . A specific feature of the algorithm used to solve (7.2.3) is that the number of grid points in μ_0 varies with s because the definition interval varies with s . In the magnetosphere, where the inhomogeneity of the magnetic field causes μ_{0B} to vary significantly, the number of steps in μ_0 is increased by two for each step in s toward the magnetic equator. This scheme is illustrated in Fig. 7.1, which shows the adopted grid as function of s and $\theta_0 = \cos^{-1} \mu_0$. Because the magnetic field changes little in the ionosphere, while at

the same time the step size in s is smaller in this region, the values of μ_{0B} adjacent altitude grid points would be undesirably close if this scheme was extended all the way to the base of the ionosphere. Instead, we assume that the magnetic field is homogeneous below some altitude s_1 (typically 800–1,000 km) so that μ_{0B} is constant in the ionosphere.

In Fig. 7.1, dashed lines indicate the loss cone boundary (LCB) $\mu_0 = \pm\sqrt{1 - B_0/B(s_1)}$. The number of grid points in the loss cone is constant for the complete field line. The trapping region is defined by $\sqrt{1 - B_0/B(s)} < |\mu_0| < \sqrt{1 - B_0/B(s_1)}$ and the loss cone region by $\sqrt{1 - B_0/B(s_1)} < |\mu_0| < 1$. The boundary conditions in s are imposed at conjugate points of the ionosphere, $-s_2$ and $+s_2$ at 90-km altitude using the local approximation ($\partial/\partial s = 0$).

Using the numerical technique developed by Khazanov et al. (1979, 1993a, 1994), (7.2.3) has been solved for these new variables ($t, E, \mu_0(\mu, s), s$) under the following initial and boundary conditions:

$$\Phi(t = 0, s, \mu_0, E) = \Psi_0(s, \mu_0, E) \quad (7.2.4)$$

$$\begin{aligned} \Phi(t, s = -s_1, \mu_0, E) &= \Psi^-(t, \mu_0, E), \quad -1 \leq \mu_0 \leq -\sqrt{1 - B_0/B(-s_1)}; \\ \Phi(t, s = s_1, \mu_0, E) &= \Psi^+(t, \mu_0, E), \quad \sqrt{1 - B_0/B(s_1)} \leq \mu_0 \leq 1; \end{aligned} \quad (7.2.5)$$

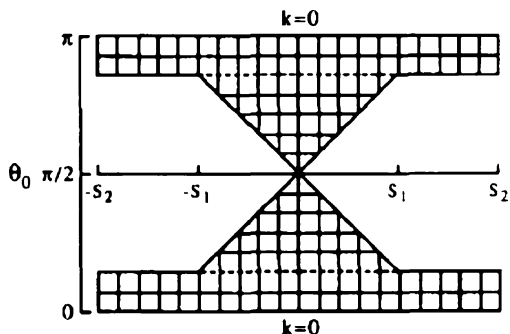
$$\frac{\partial \Phi(\theta_0 = 0, \pi)}{\partial \theta_0} = 0; \quad \theta_0 = \cos^{-1} \mu_0, \quad (7.2.6)$$

$$\Phi^+(t, s, \mu_0 = \mu_{0b}, E) = \Phi^-(t, s, \mu_0 = -\mu_{0b}, E), \quad (7.2.7)$$

$$\frac{\partial \Phi^+}{\partial \theta_0}(t, s, \mu_0 = \mu_{0b}, E) = -\frac{\partial \Phi^-}{\partial \theta_0}(t, s, \mu_0 = -\mu_{0b}, E), \quad (7.2.8)$$

$$\Phi(t, s, \mu_0, E = 0; E = E_{max}) = 0, \quad (7.2.9)$$

Fig. 7.1 The region in (θ_0, s) where Φ is defined. The dashed lines indicate the loss cone boundary, $\pm s_1$ define the boundaries between the two conjugate ionospheres and the magnetosphere, and $\pm s_2$ define the lower boundary in the ionosphere



where Ψ_0 is the initial distribution in the flux tube (in the refilling case, which will be discussed later, Ψ_0 is taken to be zero everywhere); $\Psi^\pm$ are the low altitude (in our case, 90 km) boundary fluxes that were calculated from (7.2.3) for the condition of local equilibrium; and θ_0 is the equatorial pitch angle. The structure of the kinetic equation (7.2.3) and its solution was discussed in great detail by Khazanov et al. (1993b, 1994).

To perform the calculations, we used (in most of the cases presented below) the following input for our superthermal electron model. Solar EUV and X-ray radiation spectra were obtained using the Hinteregger (1981) model, whereas neutral thermospheric densities and temperatures were given by MSIS-90 (Hedin 1991). The electron profile in the ionosphere was calculated based on the IRI model (Bilitza 1990) and extended in the plasmasphere region using the assumption that the electron thermal density distribution in the plasmasphere is proportional to the geomagnetic field. To represent geomagnetically quiet times, we used the distribution $n_e \sim B$, which corresponds to a filled plasmasphere (Newberry et al. 1989). The case $n_e \sim B^2$ will indicate some intermediate step that will occur during plasmaspheric refilling (Khazanov et al. 1984). Photoabsorption and photoionization cross-sections for O, O₂, and N₂ were taken from Fennelly and Torr (1992). Partial photoionization cross-sections for O₂ and N₂ were obtained from Conway (1988), while partial photoionization cross-sections of Bell and Stafford (1992) were adopted for atomic oxygen. Cross-sections for elastic collisions, state-specific excitation, and ionization were taken from Solomon et al. (1988). All of the calculations were performed for a local time of noon at equinox, with $F_{10.7}$ and $\langle F_{10.7} \rangle$ values of 150, chosen so the atmospheric conditions are symmetric and the solar radiation is at an average intensity level.

One feature is that the model described above does not assume anything about the macroscopic magnetic field. The model is capable of handling an arbitrary geometry for the geomagnetic field: open or closed field lines; dipole or a more complex description; tilting of the geomagnetic axis; or any other configuration. For the results shown here, a dipole magnetic field was oriented symmetrically about the equator. For most of the results, symmetric atmospheric illumination was used, meaning that the conjugate atmospheres had the same density profiles and photoelectron production rates. However, a few nonsymmetric atmospheres were used, where the only difference is that the photoelectron production rate is set to zero in one of the atmospheres (referred to as dark).

Another feature of the model is that it can handle an arbitrary grid with respect to energy, distance, or pitch angle to facilitate efficient computation. For our applications, the step sizes with respect to energy, distance, and pitch angle were determined through successive trials of the code. The energy grid will determine the resolution of the fine structure in the superthermal electron spectrum, such as the photoelectron source peaks in the 20–30 eV range due to He II-304 Å. An energy step of 1 eV was used for the present section. The range of energies calculated is from 1 to 150 eV, and the fluxes at energies above 150 eV were assumed to be zero (not for all cases in this book). The spatial grid was determined by first choosing a starting grid so the steps are less than the local mean free paths, and then decreasing

the step size until the solution no longer changed significantly. It was found that 34 grid points in each ionosphere is sufficient and does not need to be altered for different L shells. Because of the lengthening of the magnetic flux tube, however, the number of plasmaspheric steps increases with L shell, ranging from 90 points for $L = 2$ up to 190 points for $L = 6$. The pitch-angle grid, like the spatial grid, has a variable step size. Because the flux does not change much through the loss cone, five to ten points is sufficient in this region for any L shell. In contrast, a large gradient exists in the transition region between the loss cone and the trapped zone, which depends on L shell, and so the total pitch-angle grid points in the equatorial trapped zone (0° – 180°) ranges from 200 for $L = 2$ to 300 for $L = 6$. The number of spatial and pitch-angle grid points is the optimum number for this numerical technique; less points change the results and more points are unnecessary.

The time step was chosen to be less than the characteristic timescales of all the processes in (7.2.3). For example, in the collision-dominated region, the time step must be much less than the time between elastic collisions, and in the transport-dominated region, the time step must satisfy the Courant stability condition, $\Delta t \leq \Delta s/v$, where v is the superthermal electron speed.

The numerical technique includes an iterative process due to the interplay between the upward- and downward-flowing particles at 90° . Thus, at each time step and each energy, the program iterates over the spatial and pitch-angle calculations until the fluxes converge within a preset ratio to the previous iteration. This convergence factor is taken to be 3% in the present calculations.

Our numerical technique is similar to the techniques used by Khazanov et al. (1993a, 1994). In order to check the validity of this numerical scheme, Khazanov et al. (1994) compared their results for “pure” plasmaspheric transport with analytical solutions by Khazanov et al. (1992) and Monte Carlo simulations by Takahashi (1973), and found good agreement in all cases. Also, in Khazanov et al. (1994), ionospheric comparisons showed good agreement with DE 2 data and other ionospheric models of superthermal electrons. Additionally, for the steady-state calculation, we checked the energy conservation of our scheme, finding that the energy is conserved within the convergence criteria of 3%.

7.2.2 *Steady-State Solution*

It is useful to start our discussion with the case of steady-state ionosphere–plasma-sphere superthermal electron coupling. The steady-state solution was obtained by making the time step infinite, and tests concluded that this method of jumping to the steady state produces the same results as stepping slowly in time toward the convergent solution. Steady-state results from the model for $L = 3$ with the thermal density in the plasmasphere proportional to the magnetic field are shown in Figs. 7.2 and 7.3. Figure 7.2a is the omnidirectional flux from the base of the plasmasphere ($h = 800$ km) to the equator with symmetric conditions of illumination. The decrease at low energies is due to Coulomb collisions with the thermal

plasma. The maximum in the spectrum near 20 eV is from the photoelectron source in that region, and the 20–30 eV range still contains some of the fine structure of the peaks due to the He II-304 Å resonance line. At several energies, the photoelectron production rate has a sharp decrease, and the spectrum shows these decreases in the 60–70 eV range and again near 110 eV. Except for these drops, the spectrum above 30 eV is smooth because there are no significant production peaks in this energy range and Coulomb collisions are slow because the cross-section decreases proportional to E^{-2} . There are two reasons for the drop in omnidirectional flux toward the

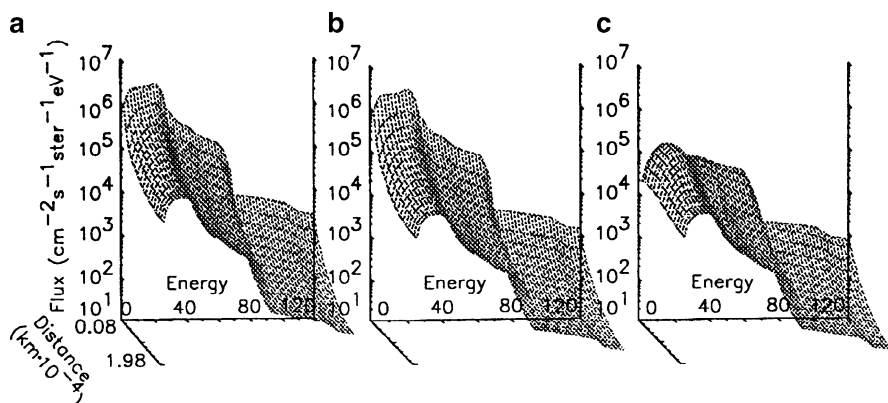


Fig. 7.2 Steady-state omnidirectional fluxes from the top of the ionosphere to the equatorial plane at $L = 3$ with $n_e \sim B$, for (a) symmetric conditions of illumination and for nonsymmetric conditions for the (b) sunlit hemisphere and (c) the dark hemisphere

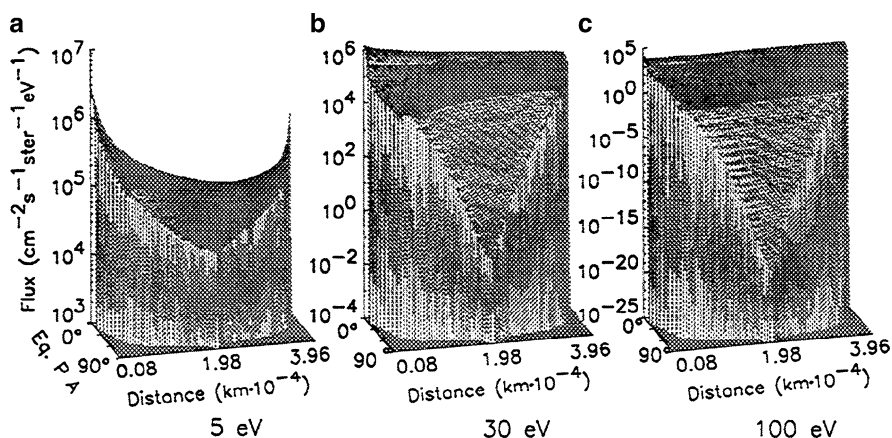


Fig. 7.3 Steady-state pitch-angle distributions in the plasmasphere of upward-flowing electrons for symmetric conditions of illumination at $L = 3$ with $n_e \sim B$ for (a) 5 eV, (b) 30 eV, and (c) 100 eV

equator: Coulomb collisions are transferring energy to the thermal plasma and the magnetic flux tube cross-sectional area increases.

Figure 7.2b, c show the energy spectrum for nonsymmetric illumination of an $L = 3$ flux tube. Figure 7.2b shows omnidirectional fluxes from the sunlit ionosphere to the equatorial plane, and the other half of the plasmasphere, from the equator to the dark ionosphere, is shown in Fig. 7.2c, so the curve at the equator (1.98×10^4 km) is the same for the two plots. These results demonstrate the difference in plasmaspheric flux depending on the ionospheric conditions; the fluxes near the sunlit atmospheres in Fig. 7.2a, b have a similar energy spectrum, but the fluxes near the dark atmosphere in Fig. 7.2c are lower than those in Fig. 7.2a by almost an order of magnitude for low energies and by a factor of 2 for high energies ($E > 70$ eV). For the energy grid used for these calculations, most of the structure in the 20–30 eV range due to the He II-304 Å resonance line has disappeared near the dark atmosphere. This occurs not only because there is no photoelectron source in the connected ionosphere but also because Coulomb collisions have smoothed the energy distribution as the particles move through the plasmasphere from the sunlit hemisphere. The fine structure of these peaks would be more pronounced in the plasmaspheric spectrum if the energy resolution was higher. However, even for higher resolution grids, Coulomb collisions will still smooth away part of the fine structure. Because Coulomb collisions at this altitude are primarily with the thermal plasma, the amount of smoothing that the spectrum experiences through the plasmasphere may be an indication of the integral content of the flux tube.

Figure 7.3 contains a series of pitch-angle distributions through the plasmasphere for an $L = 3$ flux tube with symmetric conditions of illumination. The equatorial plane is located at the middle of each plot, with the two ionospheres connecting to each end of the plot. The distribution is plotted for equatorial pitch angles, and so the areas outside the region of existence of the solution (see Fig. 1 of Khazanov et al. 1993a) are shown as a flat surface at the bottom of the scale. Also, each plot is only half of the pitch-angle distribution (0° – 90°), because the upward fluxes a given distance from its source ionosphere are identical to the downward fluxes that are the same distance from its source ionosphere. The loss cone region is located from 0° to 10° . The first surface, Fig. 7.3a, shows 5-eV fluxes, the second surface, Fig. 7.3b, shows 30-eV fluxes, and the third surface, Fig. 7.3c, shows 100-eV fluxes.

The 5-eV particles, Fig. 7.3a, are strongly influenced by Coulomb interactions with the thermal plasma. The distribution function has a drop of more than an order of magnitude in the loss cone flux level from the ionosphere boundary to the equator because of Coulomb collisions, and the trapped zone shows an order of magnitude decrease from 10° to 90° at the equator due to Coulomb scattering. Notice that the 5-eV loss cone fluxes increase as the electrons approach the conjugate ionosphere. This increase is due to downward-flowing electrons, streaming out of the conjugate ionosphere, backscattering into the upward flowing stream due to collisions with the thermal plasma.

In Fig. 7.3b, c for 30 and 100 eV, respectively, Coulomb interactions are less effective because the cross-section is proportional to E^{-2} , and this is seen in the loss cone and trapped zone fluxes in these two plots. The 30-eV surface plot shows a loss cone flux decrease of close to an order of magnitude through the entire plasmasphere, not after half of the plasmasphere, as seen in the 5-eV fluxes of Fig. 7.3a, and the 100-eV surface plot has a smaller drop in the loss cone, decreasing by a factor of 2 between the conjugate ionospheres. The trapped zones of Fig. 7.3b, c show huge drops in flux, for 30 eV, the drop is 7 orders of magnitude, and 100 eV drops 20 orders of magnitude. Also notice that neither of these energies shows an increase in the flux as the particles move toward the conjugate ionosphere. This is because electrons at these higher energies backscatter very slowly and therefore the downward-flowing electrons do not noticeably contribute to the upward-streaming fluxes.

7.2.3 *Nonsteady-State Solution*

A nonsteady-state calculation is needed to understand the superthermal electron behavior when the source changes rapidly, such as sunrise or sunset conditions, auroral precipitation events, artificial injection of energetic electrons from the spacecraft, or plasmaspheric refilling. During the refilling process, for instance, the streaming of electrons into the plasmasphere creates an electric field and couples the superthermal electrons to the ions. As the plasma flows up and out of the topside ionosphere, the flow conditions change from subsonic to supersonic, from collision-dominated to collisionless, and from O^+ dominance to H^+ dominance. In the collisionless regime, the ion velocity distributions become highly non-Maxwellian and the coupling between various plasma species occurs through the development of a self-consistent potential. Superthermal electrons, which form due to ionization of the atmosphere by solar radiation or by impacts from energetic particles of magnetospheric origin, can redistribute the self-consistent potential in the plasma. The presence of the enhanced high-velocity tail in the electron distribution will increase the number of fast ions. Because of enhanced ion acceleration in an expanding plasma, the initial superthermal electron distribution function may be changed. We also think that only a nonsteady-state superthermal electron transport model can properly handle the early stages of the plasmaspheric refilling process. We coupled our superthermal electron model with a transport model of the thermal plasma in the ionosphere and plasmasphere and discussed the issue of the self-consistent potential formation in a paper by Liemohn et al. (1997b). For the purposes of this section, we excluded the electric field from (7.2.1) and assumed that the thermal plasma density is quasineutral and constant with time.

Even without the electric field, numerical solutions of the kinetic equation given by (7.2.1) are technically very complicated because the characteristic timescales at various altitudes along the field line and for specific energies can be quite different.

An important timescale to consider is the elastic scattering timescale, τ_c , because it is the fastest collisional timescale. This timescale depends on the elastic collision frequency and the particle speed, which are functions of energy. It also depends on the neutral atmosphere and thermal plasma densities, which are functions of position along the field line. Therefore τ_c is different for each energy and for each altitude. It can range from a tenth of a second to an hour (see Table 7.1). For instance, at $L = 2$, 5-eV electrons can have an elastic scattering timescale of 0.1 s at 300 km, while the timescale for 100-eV electrons is 1,200 s at the equatorial plane.

The huge spread of characteristic timescales makes it difficult to choose a time step that will treat each of these regions and energies appropriately. A time step of 0.01 s is appropriate for the lower ionosphere, but convergence in the plasmasphere would be very slow. A step of 10 s is appropriate for the plasmasphere, but then the ionospheric fluxes converge rapidly and the time-dependent features there cannot be seen. It is possible, however, to avoid this problem by switching to a dimensionless time t_D . This was done by dividing both sides of the kinetic equation (7.2.1) by the total scattering term, yielding the time-dependent transfer equation, similar to the approaches used by Strickland et al. (1976), Oran and Strickland (1978), and Link (1992) (see, e.g., equation (2) of Strickland et al. 1976). The advantage is that times are compressed or expanded so the calculation is performed with the same dimensionless timescale for all altitudes and energies, while the drawback is that the results can become difficult to interpret because of the differences in real-time equivalents for a given dimensionless time. Several tests were conducted to determine the correct time step for the calculations. It was also determined that the two methods (real time or dimensionless time) produce equivalent results for a given time.

To simulate the refilling process correctly, the proper initial and boundary conditions must be used. This includes not only superthermal electron conditions, but also the thermal plasma distribution. Unfortunately, these conditions are not yet definitively known. For the nonsteady-state results presented here, the calculations begin with no superthermal electrons in the flux tube. There is experimental (Carpenter and Park 1973; Park et al. 1978; Corcuff et al. 1972) and theoretical (Khazanov et al. 1984) evidence that, during geomagnetic disturbances, the equatorial thermal plasma can decrease to levels as low as 10^{-1} to 1 cm^{-3} . A nonsteady-state thermal plasma transport model in the ionosphere and plasmasphere should be used to simulate this density decrease and the subsequent refilling, but the coupling of a thermal plasma model with our superthermal electron model is beyond the scope of this presentation and will be discussed in an upcoming publication.

Table 7.1 Elastic collision timescales for $L = 2$

Altitude (km)	$E = 5 \text{ eV}$	$E = 30 \text{ eV}$	$E = 100 \text{ eV}$
300	0.10	1.5	9.0
500	0.32	4.7	29
800	1.6	23	140
10,000 (eq.)	13	200	1,200

Nevertheless, there are several ways of modeling this plasmaspheric density decrease. One method is to use a different density distribution in the plasmasphere, such as $n_e \sim B^2$ rather than $n_e \sim B$. Another method would be to decrease the ionospheric densities by dividing the IRI model results by some factor, and thus reduce the plasmaspheric thermal density. Still another would be to use the IRI correction method proposed by Buonsanto (1989) to scale the topside ionosphere density down. Buonsanto (1989) showed that incoherent scatter radar observations of the thermal density in the ionosphere are lower than IRI densities at altitudes above the F_2 peak, and he provides a simple correction to the IRI model. Kozyra et al. (1990) uses this thinner F_2 layer model when calculating the solar cycle variations of stable auroral red arcs. By choosing the correct scaling factors, this method can be used to maintain the IRI model F_2 peak but decrease the density above this altitude to a target topside ionosphere density. Two of these methods will be demonstrated in the results of this section.

The first results presented use the IRI densities in the ionosphere with the plasmaspheric densities proportional to the square of the magnetic field. Upward-flowing fluxes are shown for 30 eV in Fig. 7.4 for four altitudes (300 km: Fig. 7.4a, 500 km: Fig. 7.4b, 800 km: Fig. 7.4c, and 10,000 km: Fig. 7.4d) along an $L = 2$ field line. The first altitude is near the thermal electron density peak, the next altitude is in the transition region of the ionosphere, the third altitude is the top of the ionosphere, and the last altitude is at the equatorial plane in the plasmasphere. The dimensionless times presented are at $t_D = 0.1, 0.4, 0.7, 1.0, 2.0$, and the steady-state flux values. The equivalent real times for $t_D = 1.0$ are presented in Table 7.1.

In Fig. 7.4a, each time presented shows a uniform distribution because the elastic scattering with the thermal plasma and the neutral particles is very efficient. The fluxes in Fig. 7.4b show a decrease at large pitch angles, especially in the steady-state results. This is because the path length to traverse a given field-aligned

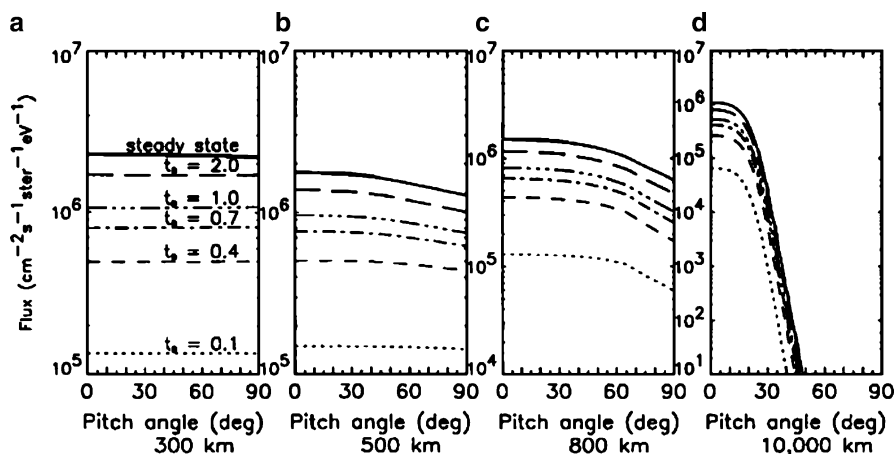


Fig. 7.4 The 30-eV upward-flowing pitch-angle distributions for $L = 2$ with $n_e \sim B^2$, with dimensionless times given in the figure and real-time equivalents given in Table 7.1

distance is longer for a large pitch-angle electron compared with an electron with a smaller pitch angle, and so these longer-path particles undergo more collisions. This decrease at larger pitch angles is also seen at 800 km (Fig. 7.4c), even at $t_D = 0.1$. Once in the plasmasphere, however, the focusing of the electrons due to the changing magnetic field strength creates a maximum in the pitch-angle distribution in the loss cone, which is 20° at the equator for $L = 2$. Coulomb collisions with the thermal plasma scatters electrons into the trapped zone, though, and Fig. 7.4d shows a gradual decrease with pitch angle of two orders of magnitude from the loss cone to 90° . Notice that the loss cone fluxes are continually decreasing, dropping by a factor of 2 from the F_2 peak to the equatorial plane.

It should be noted that, for this thermal density distribution, none of the results at $t_D = 2.0$ have reached the steady-state results in Fig. 7.4, indicating that another elastic scattering timescale or more is needed for convergence.

Time-dependent field-aligned omnidirectional flux spectra in the plasmasphere are shown in Fig. 7.5. These calculations also used the IRI densities in the ionosphere with $n_e \sim B^2$ in the plasmasphere. The nonsteady-state fluxes are presented for the dimensionless times $t_D = 0.1$ (Fig. 7.5a), 0.5 (Fig. 7.5b), 1.0 (Fig. 7.5c), and 2.0 (Fig. 7.5d), with the steady-state spectrum also plotted lightly on each of the figures so the progression of the solution can be seen more clearly. Note that for a given dimensionless time, each grid point corresponds to a different real time due to the energy and spatial dependence of τ_c , and so even though it appears that the lower energies are taking longer to reach steady state, their real times are actually smaller than the high-energy times. The smaller flux values at low energies are due to the increased effect of Coulomb collisions as energy decreases. As the solution develops, these lower-energy fluxes increase not only due to the ionospheric source of particles, but also from the cascading of electrons from higher energies. The peaks in the 20–30 eV range are due to photoionization by the He II-304 Å resonance line. The differences in energy thresholds for the ionization states of the neutral species create a range of structures in the photoelectron spectrum. Notice that the first few time steps show more of the fine structure than the steady-state results. This is because Coulomb collisions smooth the distribution, and this smoothing continues with time. The energy grid will also determine the resolution of the fine structure of the spectrum. The sharp drop in the flux around 70 eV, as discussed before, is due to a corresponding decrease in the primary photoelectron spectrum. Because the $t_D = 2.0$ fluxes are not at the steady-state levels yet, as was the case in Fig. 7.5, a dimensionless time calculation must be conducted for perhaps three or more characteristic elastic scattering timescales to reach the steady-state solution. This means the fluxes in the trapped zone require several elastic collisions before the trapped zone is filled.

Using a different method to achieve a depleted thermal plasma density distribution, the IRI model was decreased above the F_2 peak following the Buonsanto (1989) method with different scaling factors to make the thermal plasma density at the topside ionosphere 545 cm^{-3} , and then $n_e \sim B$ was used in the plasmasphere with $L = 3$. This brings the equatorial thermal plasma density down to 17 cm^{-3} , which is still within the depleted density range (Carpenter and Park 1973; Park et al.

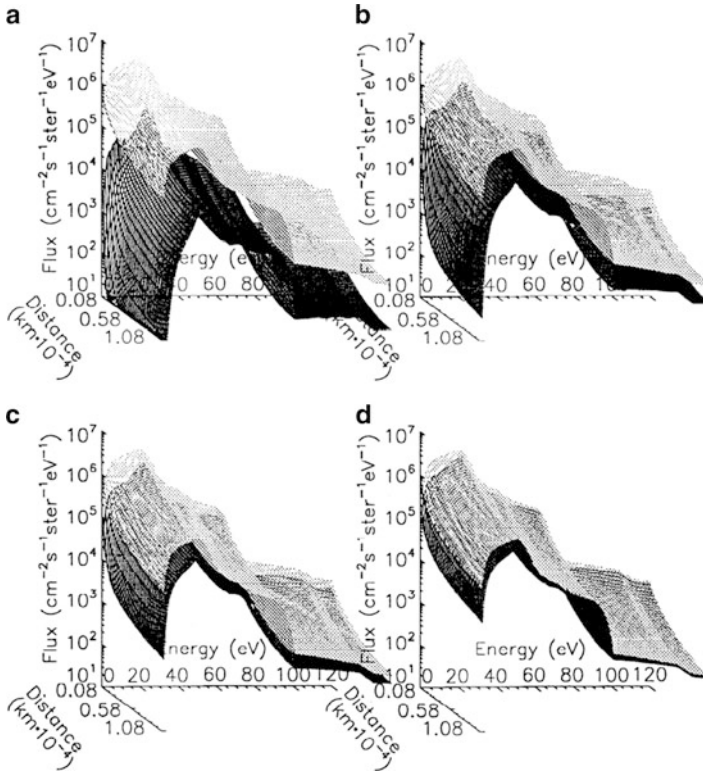
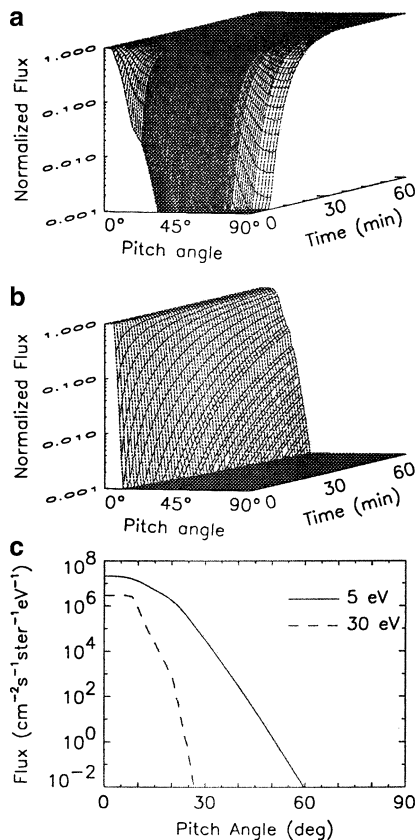


Fig. 7.5 Omnidirectional fluxes for $L = 2$ with $n_e \sim B^2$ from the top of the ionosphere to the equatorial plane at (a) $t_D = 0.1$, (b) $t_D = 0.5$, (c) $t_D = 1.0$, and (d) $t_D = 2.0$, with the steady-state spectrum lightly superimposed on each plot

1978; Corcuff et al. 1972; Khazanov et al. 1984). This is also the same distribution used in Fig. 3 of Khazanov et al. (1993a), where the refilling process was modeled using a plasmaspheric transport model with boundary conditions at 1,000 km. Figure 7.6a, b show normalized equatorial pitch-angle distributions from this calculation for 5 and 30 eV, respectively. This was a dimensional time calculation conducted for 1 h of refilling. The steady-state equatorial distributions to which these results were normalized are given in Fig. 7.6c. Note that the steady-state distributions continue below the presented flux range. Notice that the loss cone fluxes reach steady state in less than a minute. For this greatly depleted case, however, the 5-eV trapped zone fluxes take longer than 30 min to reach steady state and the 30-eV fluxes are still far from the convergent levels after an hour of refilling. Therefore, the superthermal electron plasmaspheric refilling process could take hours and a time-dependent model must be used. Also, the low-energy flux levels are dependent on the high-energy fluxes because of the energy cascading due to Coulomb collisions, so the 5-eV fluxes will continue to adjust closer to the

Fig. 7.6 Equatorial pitch-angle distributions for (a) 30 eV and (b) 5 eV for $L = 3$ with an equatorial thermal plasma density of 17 cm^{-3} , with the (c) fluxes normalized to the steady-state distribution at each pitch angle



convergent values until the higher-energy fluxes reach steady state with the high-energy refilling timescale.

7.2.4 Plasmaspheric Transparency

One important aspect of the superthermal electron problem is their transport through the plasmasphere. The superthermal electrons escaping from the ionosphere experience small-angle scattering when traveling through the plasmasphere as a result of the Coulomb interaction with the thermal plasma. Because of this scattering, some of the superthermal electrons are scattered outside of the loss cone and undergo magnetic reflection: i.e., they become trapped. Sanatani and Hanson (1970) and Nagy and Banks (1970) presented qualitative discussions of electron trapping and the resulting increase in plasmaspheric heating, but the first attempts of quantitative calculations were those of Gastman (1973), Takahashi (1973), Swartz et al. (1975), Lejeune and Wormser (1976), and Khazanov et al. (1977).

By studying the ionosphere and plasmasphere as one system rather than two separate ones, substantial corrections are introduced in the values and characteristics of key parameters describing photoelectron fluxes (Khazanov et al. 1994). This is most clearly exemplified in the concept of the plasmaspheric transparency, which was introduced by Takahashi (1973) as the ratio of the number of particles leaving one end of a magnetic flux tube to the number of particles entering at the other end,

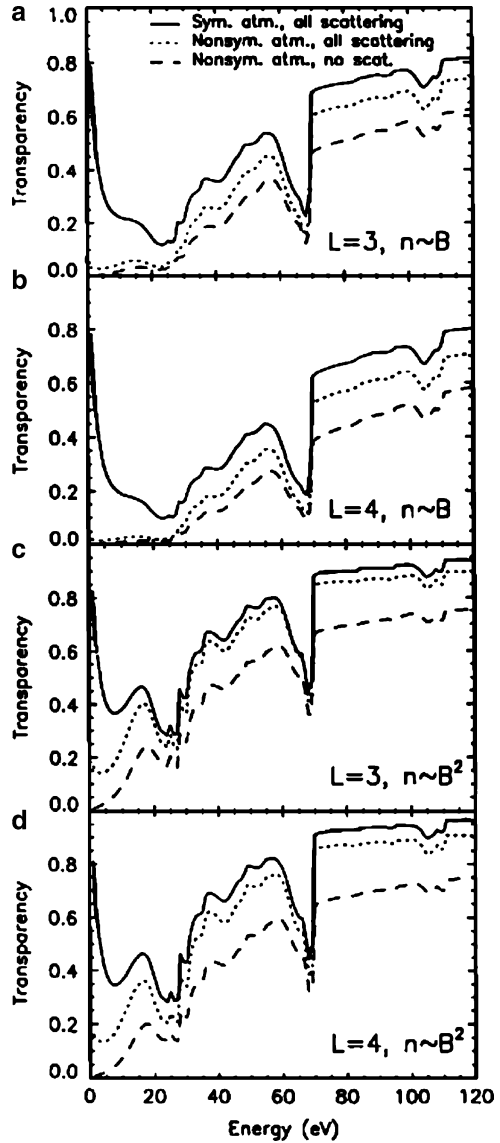
$$T(E) = \frac{\int_{\mu_{0B}}^1 \mu_0 \Phi(E, \mu_0, s_1) d\mu_0}{\int_{\mu_{0B}}^1 \mu_0 \Phi(E, \mu_0, -s_1) d\mu_0}, \quad (7.2.10)$$

where s_1 is the ionosphere–plasmasphere boundary altitude, taken to be 800 km in the present section. On the basis of the results of the “pure” plasmaspheric transport, it was shown that the transparency depends mainly on a single parameter proportional to the Coulomb cross-section and to the total content of the thermal electrons in the magnetic force tube (Takahashi 1973; Lejeune and Wormser 1976; Krinberg and Matafonov 1978; Khazanov et al. 1992). The fact that processes are included such as diffusion across the LCB, reflection in the ionosphere due to elastic collisions with neutral and charged particles, and the redistribution of the flux at low energies due to interactions with neutral and charged particles, makes the term “transparency” acquire its conventional meaning (7.2.10) (Khazanov et al. 1994).

Figure 7.7 shows the calculation of plasmaspheric transparency (7.2.10) based on the results of our new superthermal electron model. The transparency is presented as a function of energy for $L = 3$ and $L = 4$ for the different conditions of illumination, scattering processes in the conjugate hemispheres, and thermal plasma density descriptions along geomagnetic field lines. Figure 7.7a, b show the transparencies that were obtained from steady-state solutions of the coupled code using plasmaspheric thermal density proportional to the magnetic field, and Fig. 7.7c, d present the results for $n_e \sim B^2$. In each plot, the solid curve is the transparency for symmetric illumination with all scattering included for the entire field line, the dotted curve is for nonsymmetric illumination, and the dashed curve is for nonsymmetric illumination with scattering removed from the dark ionospheric region, so no particles escape from this ionosphere. The second and third transparencies are defined, for a given energy, as the number of electrons penetrating into the dark ionosphere divided by the number of electrons leaving the sunlit ionosphere.

As was mentioned before, “pure” plasmaspheric superthermal electron transport predicts the energy dependence of the plasmaspheric transparency to be primarily proportional to the Coulomb cross-section and to the total content of the thermal electrons in the magnetic flux tube. In this case, we should expect continuous increasing of plasmaspheric transparency from zero to one as energy increases.

Fig. 7.7 Plasmaspheric transparencies for (a) $L = 3$ with $n_e \sim B$, (b) $L = 4$ with $n_e \sim B$, (c) $L = 3$ with $n_e \sim B^2$, and (d) $L = 4$ with $n_e \sim B^2$.



This does not reflect the results of the calculations presented in Fig. 7.7, because they show considerable energy structure, and do not follow a simple increase with energy. These features are the result of the energy dependence of the superthermal electron flux created by photoionization and by collisional interactions between superthermal electrons and neutral particles in the ionosphere. As this flux propagates through the plasmasphere, particles undergo Coulomb collisions at a rate dependent on $\partial\Phi/\partial E$ as seen from the Coulomb collision operator, and thus some

structure is introduced into the transparency. The trough near 65 eV is due to the decrease in the photoelectron production spectrum at this energy. The increase of $T(E)$ with decreasing energy seen below ≈ 30 eV is caused by the formation of a plateau-like part of the photoelectron distribution function in the upper ionosphere and plasmasphere. The Coulomb collision term (2.2.2) is small in this situation because it contains the term $\partial\Phi/\partial E$. The exact shape of the function at low energies, and therefore the location of the minimum in $T(E)$, depends on the total content of the thermal particles in the flux tube (Khazanov et al. 1992). Another effect that influences $T(E)$ at small energies is the trapped electron population, which diffuse down to low energies and subsequently into the loss cone, thereby increasing the value of the transparency. It is clear that, in this case, $T(E)$ characterizes diffusion from the trapped region into the loss cone, rather than the transmitting capability of the flux tube (Khazanov et al. 1994). Notice the difference between Fig. 7.7a, b and Fig. 7.7c, d. When the total content in the flux tube is decreased (Fig. 7.7c, d), they become apparent when they are not damped by the thermal plasma, such as the formation at 15 eV and the two peaks at 25 and 30 eV.

There is also an L shell dependence in the transparency. Comparing Fig. 7.7a with b and Fig. 7.7c with d reveals that the higher L shells, with a larger amount of thermal plasma, have lower transparencies. To stress the dependence of the fine structure on ionospheric processes, plasmaspheric transparency (7.2.10) as a function of energy and L shell is shown in Fig. 7.8. When this approximation is used, the fine structure almost disappears and the transparency is smoother than with the exact description of the loss processes. Notice that the 10–50 eV range contains only a trace of the structure evident in Fig. 7.7, and the trough near 65 eV is not as pronounced. As with the previous figure, there is a gradual decrease in the transparency with increasing L shell corresponding to the increase in the total thermal content in the flux tube.

It is clear that plasmaspheric transparency is a complicated function. It depends not only on Coulomb collisions with the thermal plasma particles in the plasmasphere, but also on the primary photoelectron energy source, collisions with neutral

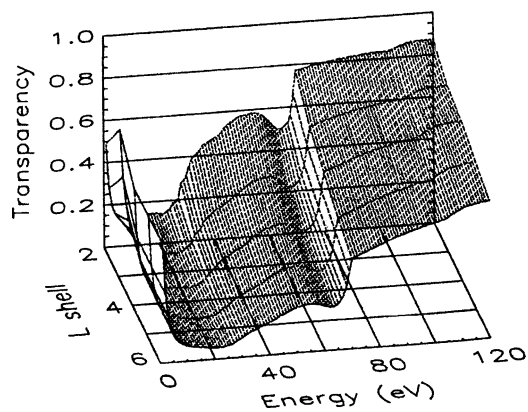


Fig. 7.8 Plasmaspheric transparency for several L shells

and charged particles in the atmosphere and ionosphere, and diffusion across the LCB. Such dependencies, however, can be explained by the simple phenomenological model of plasmaspheric transparency derived by Khazanov and Gefan (1982) and Khazanov et al. (1994). For the purposes of subsequent analysis, let us briefly present this phenomenological model, with some useful modifications introduced by Gefan (1985).

Let us introduce the following quantities: ε is the fraction of superthermal electron energy lost in the plasmasphere; β is the part of superthermal electron energy returned from the plasmasphere due to the electron scattering in the loss cone; $\alpha = 1 - \varepsilon - \beta$ is the “pure” part of plasmaspheric transparency; and A_1 and A_2 are the albedos of the ionosphere in the conjugate hemispheres. By taking into account multiple reflection in the ionosphere and plasmasphere, the unidirectional fluxes P_1^+ , P_2^+ at the boundaries of the ionosphere and plasmasphere (see Fig. 7.9) can be presented in the form

$$P_1^+ = (P_{01}^+ + P_{02}^- \alpha A_1) \left[(1 - \alpha^2 A_1 A_2)^{-1} + \beta A_1 / (1 - \beta A_1) \right], \quad (7.2.11)$$

$$P_2^- = (P_{02}^- + P_{01}^+ \alpha A_2) \left[(1 - \alpha^2 A_1 A_2)^{-1} + \beta A_2 / (1 - \beta A_2) \right], \quad (7.2.12)$$

$$P_2^+ = \alpha P_1^+ + \beta P_2^-, \quad P_1^- = \alpha P_2^- + \beta P_1^+. \quad (7.2.13)$$

Here P_{01}^+ and P_{02}^- are the fluxes entering the plasmasphere. These particles are generated in the ionosphere by photoionization or impact ionization.

Using (7.2.11) and (7.2.12), we can now derive an expression for the plasmaspheric transparency $T = P_2^+ / P_1^+$ and consider the following cases:

Symmetrical conditions of illumination in the conjugate hemispheres ($P_{01}^+ = P_{02}^-$ and $A_1 = A_2 = A$):

$$T_1 = \alpha + \beta. \quad (7.2.14)$$

No source of superthermal electrons in region 2 ($P_{02}^- = 0$):

$$T_2 = \alpha(1 + \beta A). \quad (7.2.15)$$

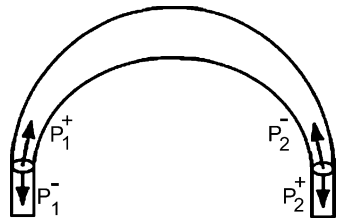


Fig. 7.9 Flow schematic in a plasmaspheric flux tube

No backscattering in the unilluminated ionosphere ($P_{02}^- = 0$ and $A_2 = 0$):

$$T_3 = \alpha. \quad (7.2.16)$$

Thus (7.2.14)–(7.2.16) show that the quantity $T(E)$ is not identical to the “pure” part of the plasmaspheric transparency α . Also, this phenomenological model explains the behavior of the transparency as a function of illumination and scattering processes in the conjugate hemispheres shown in Fig. 7.7. T_1 corresponds to the solid curves where both ionospheres are sunlit and all scattering processes have been included. T_2 corresponds to the dotted curves, which have nonsymmetric conditions of illumination and include all of the scattering processes. T_3 pairs up with the dashed curves of nonsymmetric illumination with no scattering processes included in the dark ionosphere. It is clear that $T_1 > T_2 > T_3$ in all four situations presented in Fig. 7.7, and, from these transparencies, the energy deposition processes of the plasmasphere can be examined.

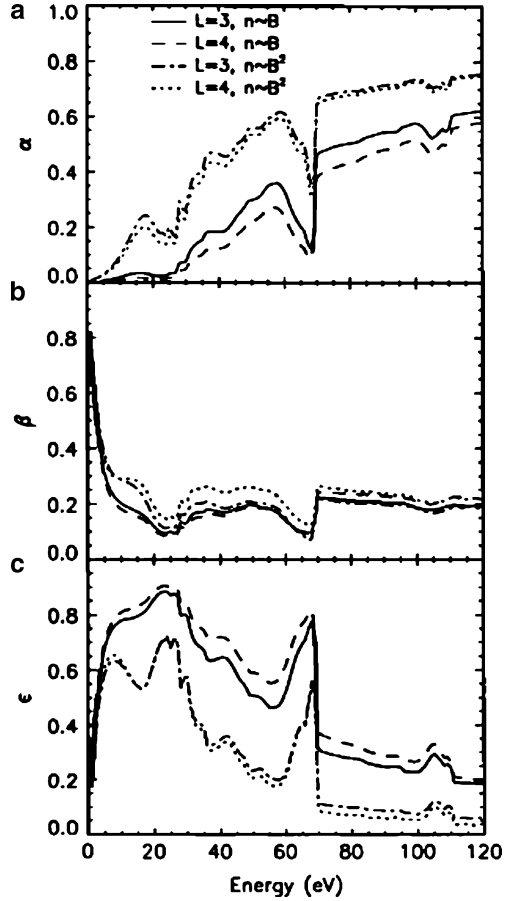
7.2.5 Plasmaspheric Energy Interplay

Let us consider the superthermal electron energy redistribution in the plasmasphere system. This problem can be solved based on the calculation of the parameters $\alpha(E)$, $\beta(E)$, and $\varepsilon(E)$. The portion of particles traveling from one ionosphere to the conjugate ionosphere is described by the “pure” plasmaspheric transparency α ; the part of superthermal electron energy reflected from the plasmasphere to the source ionosphere is β ; and the fraction of energy lost in the plasmasphere is given by ε . To perform these calculations, we have used the values of $T(E)$, defined by (7.2.10), from our new model for the two different cases that correspond to (7.2.14) and (7.2.16) and are shown in Fig. 7.7 as solid and dashed curves, respectively. Solving the system of two algebraic equations and using the relation $\alpha + \varepsilon + \beta = 1$, we find the unknowns α , β , and ε .

Figure 7.10 demonstrates the results of such calculations for $L = 3$ and $L = 4$ as a function of energy. Figure 7.10a is a plot of α , which was defined as the “pure” transparency, and represents the ability of a particle with a given energy to penetrate from one ionosphere through the plasmasphere into the conjugate ionosphere. The rise in α with energy is due to the decrease in the Coulomb cross-section with energy. The formations of the fine structure in the 10–50 eV range are from photoelectron peaks that have undergone collisional changes through the plasmasphere, and the troughs at higher energies are also from structures in the photoelectron source that have been altered due to inelastic Coulomb collisions.

Particles that return to the initial ionosphere after being scattered in the plasmasphere are described by Fig. 7.10b. Because the only plasmaspheric elastic scattering process included in the model is Coulomb scattering, β should, in general, have the same energy dependence as the Coulomb cross-section. While the electrons are

Fig. 7.10 The plasmaspheric coefficients (a) α , (b) β , and (c) ε determined from the transparencies in Fig. 7.7



elastically scattering, they are also cascading in energy due to inelastic collisions, and this effect is seen in β as an additional rise at low energies. Notice that there is a decrease in the 60–70 eV range, as in Fig. 7.10a, and this is again due to the structure of the energy spectrum at that energy.

The quantity β can also be related to Fig. 7.3. Notice in Fig. 7.3a that the flux increases as the electrons approach the conjugate hemisphere. This feature is due to downward-flowing electrons backscattering into the upward-flowing stream. These reflected particles contribute to β because they are returned to their source ionosphere.

Figure 7.10c shows the fraction of energy deposited into the plasmasphere for each electron energy. The general shape of ε is a decrease with increasing energy because the Coulomb cross-section is proportional to E^{-2} , but considerable structure is also present. This is because the efficiency of energy deposition is also proportional to $\partial\Phi/\partial E$, and so the cascading of energy by the superthermal electrons is increased where there is a large decrease in flux over a short energy range. The energy lost by the superthermal electrons is transferred to the thermal plasma. This

absorption is responsible for the peaks in the 60–70 eV and 100–110 eV ranges and also for the peaks below 50 eV. The decrease in ε below 20 eV is due to a decrease of the flux of low-energy superthermal electrons caused by collisions with the thermal plasma. This decrease in the flux below 20 eV can also be seen in Figs. 7.2 and 7.4.

The calculations presented in Fig. 7.10 show the contribution of each energy to the plasmaspheric energy interplay. Integrating these three parameters from 1 to 150 eV, the energy range of the simulation, gives the portion of the total energy penetrating from the source ionosphere to the conjugate ionosphere, $\langle\alpha\rangle$; the part of the total energy reflected in the plasmasphere back to the source region, $\langle\beta\rangle$; and the energy absorbed in the plasmasphere, $\langle\varepsilon\rangle$. These values are given in Table 7.2.

As can be seen from these calculations, more of the energy from the source region is deposited into the conjugate ionosphere than back into the source ionosphere, that is, $\langle\alpha\rangle$ is greater than $\langle\beta\rangle$ in every case. Also, the portion of the energy from the source region deposited to the plasmasphere is greater than the backscattered portion, that is, $\langle\varepsilon\rangle$ is greater than $\langle\beta\rangle$. The relationship between $\langle\alpha\rangle$ and $\langle\varepsilon\rangle$, however, depends on the thermal plasma description and L shell. When $n_e \sim B$, about half of the energy is deposited into the plasmasphere while less than a third reaches the conjugate ionosphere for $L = 3$ and $L = 4$. When $n_e \sim B^2$, the percentages for $\langle\alpha\rangle$ and $\langle\varepsilon\rangle$ reverse, so approximately half flows into the conjugate ionosphere while less than 30% is absorbed by the plasmaspheric thermal plasma. The relationship with L shell is less pronounced, and the change is usually 5% or less between $L = 3$ and $L = 4$.

Comparing the results of Table 7.2 and Fig. 7.10 reveals the energy ranges that contribute the most to each parameter. Figure 7.10a shows that energies above 50 eV are the largest contributors to $\langle\alpha\rangle$. It can be seen from Fig. 7.10b that energies below 20 eV are the most likely to be reflected back to their source ionosphere. From Fig. 7.10c, it can be concluded that superthermal electrons with energies between 10 and 70 eV are the largest contributors of energy to the plasmaspheric thermal plasma.

It is worthwhile to compare $\langle\varepsilon\rangle$, the portion of energy absorbed in the plasmasphere, with the calculations of Newberry et al. (1989). In that study, a comparison was made between data from the retarding ion mass spectrometer (RIMS) on the Dynamics Explorer 1 (DE 1) satellite and the field line interhemispheric plasma (FLIP) model. The FLIP model included a phenomenological factor (trapping factor) to represent the amount of energy lost to the plasmasphere from the photoelectrons, and they concluded that good agreement is achieved between the

Table 7.2 Plasmaspheric energy deposition coefficients

	$\langle\alpha\rangle$	$\langle\beta\rangle$	$\langle\varepsilon\rangle$
$L = 3, n_e \sim B$	0.316	0.192	0.491
$L = 4, n_e \sim B$	0.265	0.195	0.540
$L = 3, n_e \sim B^2$	0.504	0.206	0.290
$L = 4, n_e \sim B^2$	0.483	0.243	0.274

calculated and measured ion temperatures when $\sim 55\%$ of the total photoelectron flux is trapped in the plasmasphere. We conducted a similar study with our model and used the thermal electron profile in the ionosphere and plasmasphere from Newberry et al. (1989), and we found that $\langle \epsilon \rangle = 0.53$. This shows that our calculations are in very good agreement with phenomenological modeling and measurements of the thermal structure of the plasmasphere.

7.2.6 Comparison with Observations

The true test of any numerical model, however, is how well it can reproduce observations and explain the processes occurring in the plasma responsible for the formation of the distribution functions. Because in situ measurements of superthermal electron distributions have primarily been conducted in the ionosphere, we will limit our presentation to the three ionospheric spatial regions mentioned above: the collision-dominated region, the transition region, and the transport-dominated region (Khazanov et al. 1998).

The Atmospheric Explorer (AE) satellites offer a plentiful supply of superthermal electron energy spectra. The electron spectrometers on the AE satellites are ideal for comparison because of the fine spectral resolution achieved in the low-energy range. Although the data is spin averaged, this is not a big problem because the satellites flew through the low to middle ionosphere, where the distribution function is nearly isotropic. Figure 7.11 shows omnidirectional fluxes from the AE-E satellite and model results for similar conditions. The data in this plot is

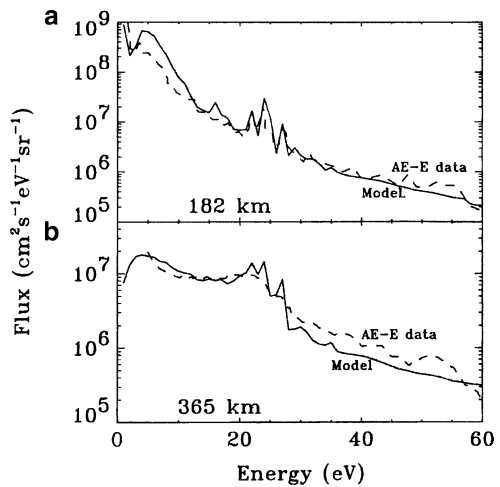


Fig. 7.11 Comparison of model results (*solid lines*) with AE-E data (*dashed lines*) at (a) 182-km and (b) 365-km altitude on day 355 of 1975. The satellite data is reproduced from Doering et al. (1976)

reproduced from Doering et al. (1976), for day 355 of 1975 at 182- and 365-km altitudes. This first altitude is in the region where collisions with neutrals dominate the formation of the distribution function, and local production and loss mechanisms are the major processes in the calculation. The second altitude is in the transition region, where transport is starting to have a significant influence on the distribution. The solar zenith angles for the two spectra are 50° and 37° , respectively. Because AE-E flew in a nearly equatorial orbit, the model comparisons are made at 0° geographic latitude, choosing an appropriate morningside local time with the given solar zenith (because the data collection occurred there; Doering et al. 1976). As above, initial profiles for the thermal plasma are taken from the IRI model and atmospheric parameters are taken from the MSIS model.

In Fig. 7.11a, the spectra agree closely for most of the energy range. The model predicts a slightly higher flux in the 5–15 eV range, but this difference is less than a factor of 2. Figure 7.11b also shows good agreement, with the model predicting more definition in the 20–30 eV range and lower fluxes above 30 eV by a factor of less than 2. These differences could be explained by uncertainties in the experimental data, differences in the neutral atmosphere or ionospheric plasma profiles, or uncertainties in the collisional cross-sections used in the model. The larger fluxes at low energy and the increased definition of the production peaks in the model results indicate that the thermal plasma density from IRI is probably lower than the actual densities; a higher plasma density would act to smooth out these features of the distribution function. It is thought that this difference is not due to detector resolution, because $\Delta E/E$ was 2.5% and the production peaks clearly appear in the low-altitude measurements. The comparison does show, however, that the model accurately calculates the main features of the photoelectron spectrum in the local equilibrium and transition regions of the ionosphere.

At higher altitudes, the pitch-angle dependence of the distribution function becomes more important. This makes a comparison with AE data less informative in the upper ionosphere and plasmasphere. Therefore, a direct comparison with data for this region should be made with DE 2 results from the Low Altitude Plasma Instrument (LAPI). LAPI had less energy resolution than the AE electron spectrometers (with a $\Delta E/E$ of 32%), but had a much narrower field of view and allows for pitch-angle distribution comparisons. Figure 7.12 shows this comparison for the 104th day of 1982 at a local time of 9.3 h and altitude of 690 km (data from R.E. Erlandson, private communication; from Winningham et al. 1989). Note that the pitch-angle distribution is defined by the data, with 0° being downstream particles and 180° being the upflowing electron fluxes. The distributions are shown at energies of (a) 5 eV, (b) 9 eV, (c) 15 eV, (d) 27 eV, and (e) 48 eV. Notice that the model compares reasonably well for most of the cases shown. There is quite a bit of disagreement in the 5-eV results, as well as part of the 9-eV results, but this could be due to spacecraft charging effects or other processes not included in the model. The trends in the data distributions of the other energies are reflected in the model results, and the magnitudes of the model results are not far from the measured values (within a factor of 2).

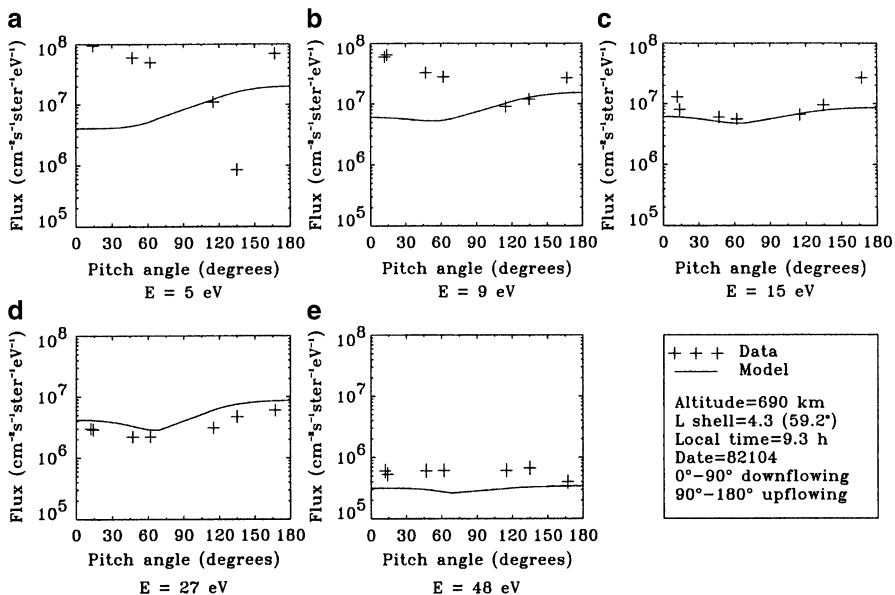


Fig. 7.12 Comparison of model results with DE 2 LAPI pitch-angle distributions. Data courtesy of R.E. Erlandson

7.3 Global Suprathermal Electron Transport

In the previous section, for calculation of suprathermal electrons, it was assumed that the flux tube was corotating with the Earth, so that cross-field line drift effects could be neglected. Khazanov et al. (1996), for the first time, carried out superthermal electron simulations on a global scale showing the influence of these cross-field drift effects on the development of the high-energy PE distribution function. This study showed that this population requires many hours to reach a steady-state level in the magnetosphere.

Another source of suprathermal electrons in the inner magnetosphere is the plasma sheet. Due to the sunward flow of plasma from the dawn-to-dusk electric field in the magnetosphere, plasma sheet electrons (PSEs) and ions are continually pushed toward the Earth, regulated by geomagnetic activity. Alfvén and Fälthammar (1963) calculated drift patterns for these particles, showing the classic tear-drop boundary between closed and open trajectories (i.e., trajectories that return to their starting point and those that do not). This separatrix is often called the Alfvén boundary, and while the magnetosphere is never constant enough for the steady-state assumption of this boundary to be valid, it is a very powerful tool for qualitatively describing the shape and motion of particles in near-Earth space. The motion of the PSEs in from the tail, around this boundary, and out on the dayside is well-known and regularly observed at geosynchronous orbit (DeForest and McIlwain 1971; Eather et al. 1976; Hultqvist et al. 1981; Arnoldy 1986; McComas

et al. 1993; Birn et al. 1997). Enhancements in geomagnetic activity cause this boundary to move in closer to the Earth, and there have been numerous observational and theoretical studies on the motion of electrons in this injection front toward the Earth near local midnight (e.g., Roederer 1970; Barfield et al. 1977; Ejiri 1978; Ejiri et al. 1978, 1980; Kaye and Kivelson 1979; Moore et al. 1981; Moore and Arnoldy 1982; Arnoldy and Moore 1983; Kerns et al. 1994; Burke et al. 1995; Liemohn et al. 1998; Khazanov et al. 1977). Also, Stern (1975) expanded the convection electric field strength description of Volland (1973), and Ejiri (1978) formalized a technique for bounce-averaging quantities along the field line. Averaging the distribution function along the field line (with appropriate mapping to conserve the first adiabatic invariant) is valid if the bounce period is much shorter than any drift or collisional timescale. This assumption is usually true for electrons greater than a few tens of electron volts. While the injected PSE distribution is usually isotropic (e.g., Barfield et al. 1977; Baumjohann et al. 1989; Christon et al. 1989), it has sometimes been noted that the pitch-angle distribution of sub-kiloelectron volt electrons in the injection front form a possible source cone distribution, and that high-energy electrons can have a peak at 40° pitch angle or less (Moore and Arnoldy 1982; Koons and Fennell 1983; Arnoldy 1986). A strong increase in the intensity of the low-energy fluxes during injections has also been observed (Ejiri et al. 1980). One observation of injected PSEs showed the formation of a banded energy structure in the captured electrons (Burke et al. 1995). This event was simulated with a time-dependent, bounce-averaged model by Liemohn et al. (1998), determining that the bands in energy are a natural consequence of captured electrons due to the supercorotation of high-energy electrons caused by magnetic gradient-curvature drift. The bands only formed when the injected PSEs were contained inside the Alfvén boundary for a prolonged period. The increase in intensity in the sub-kiloelectron volt energy range was also shown to be a natural occurrence of the compression of the flux tubes on the nightside.

The theoretical studies of this nonthermal electron population in the inner magnetosphere have so far not addressed the combination of the sources described above. The present section provides a quantitative description of superthermal electron distribution function formation on a global scale in the presence of both PE and PSE source populations for various geomagnetic conditions. In particular, the extension of the bounce-averaged energy range down to several electron volts is examined, allowing for an accurate comparison of not only the relative intensities but also the energy deposition to the thermal plasma for the two populations. The observations of captured PSEs by CRRES (Burke et al. 1995) will also be further examined with this combined distribution function.

7.3.1 *The Field-Aligned and Bounce-Averaged Models*

The model for this section to calculate the superthermal electron distribution function in the ionosphere–plasmasphere system is based on coupling two of our

existing models: our field-aligned and bounce-averaged transport codes. These two models complement each other and offer a unique possibility for simulating superthermal electron motion on a global scale.

The first model that has been discussed in [Sect. 7.2](#) simulates superthermal electron transport through a flux tube in the geomagnetic field. It calculates the time-dependent superthermal electron distribution function, f , as a function of time, distance along the field line, energy, and pitch angle from the gyration-averaged kinetic equation (Khazanov et al. [1994](#)):

$$\frac{\beta}{\sqrt{E}} \frac{\partial \Phi}{\partial t} + \mu \frac{\partial \Phi}{\partial s} - \frac{1 - \mu^2}{2} \left(\frac{1}{B} \frac{\partial B}{\partial s} - \frac{F}{E} \right) \frac{\partial \Phi}{\partial \mu} + EF\mu \frac{\partial}{\partial E} \left(\frac{\Phi}{E} \right) = Q + \bar{S}, \quad (7.3.1)$$

where $\Phi = 2Ef/m^2$ is the superthermal electron flux; t is time; s is the distance along the field line; E is the particle energy; and μ is the cosine of the pitch angle. The inhomogeneity of the geomagnetic field, B , is included, as well as other forces, such as electric fields, in F . Q is the superthermal electron source term and $\bar{S}$ includes the collision integrals, representing interactions with thermal electrons and ions, scattering with neutral particles, and wave-particle interactions.

Initially, this model was developed for transport in the plasmasphere to investigate the refilling and depletion timescales of the trapped zone (Khazanov et al. [1993a](#)), with boundary fluxes needed at the ionospheric interfaces. The ionospheres were then incorporated (Khazanov and Liemohn [1995](#), [1998](#); Liemohn and Khazanov [1995](#)), including elastic and inelastic collisions with neutral particles, for a spatially self-consistent calculation along a flux tube. In these studies, the concept of plasmaspheric transparency was addressed, showing that this quantity is highly energy dependent and that a unified approach is needed. It was also coupled with the time-dependent, field-aligned, hydrodynamic thermal plasma model of Guiter et al. ([1995](#)) to determine the coupling effects between these populations (Liemohn et al. [1997a](#); Liemohn and Khazanov [1998](#)), particularly the influence of a self-consistent field-aligned electric field.

The second model of superthermal electron transport calculates the distribution on a global scale throughout the subauroral magnetosphere. When the flight time along the magnetic field line, τ_b , is very short compared with the collisional timescales, which is the case for higher-energy superthermal electrons, it is possible to average the particle fluxes along the field line over a magnetic mirror bounce period to eliminate the need for a field-aligned calculation. By including particle drifts across field lines, superthermal electron fluxes for the entire inner magnetosphere can be determined by solving the bounce-averaged kinetic equation (Khazanov et al. [1996](#)):

$$\frac{\partial \langle f \rangle}{\partial t} + \langle \mathbf{V}_D \rangle \frac{\partial \langle f \rangle}{\partial \mathbf{R}_\perp} + \left\langle \frac{dE}{dt} \right\rangle \frac{\partial \langle f \rangle}{\partial E} + \left\langle \frac{d\mu_0}{dt} \right\rangle \frac{\partial \langle f \rangle}{\partial \mu_0} = \left\langle \frac{\delta f}{\delta t} \right\rangle_{cc} - \frac{\langle f \rangle}{0.5\tau_b}, \quad (7.3.2)$$

where $\mathbf{R}_\perp$ includes the spatial directions R and φ perpendicular to the magnetic field, μ_0 is the cosine of the pitch angle at the magnetic equator, and $\langle \xi \rangle$ denotes

averaging ξ over a bounce period along the field line. The spatial extent of this model encircles the globe at radial distances from $L = 1.75$ out to $L = 6.5$, providing a complimentary solution of the kinetic equation to the first model throughout the subauroral magnetosphere. Currently, the only collisional process is Coulomb interactions with the thermal plasma, and atmospheric precipitation is treated as a loss.

The formation of the high-energy PE distribution ($E \geq 50$ eV) was examined with this model (Khazanov et al. 1996), showing that cross-field convection is an important process, particularly the transport of electrons through the nightside into the dawn sector. This model has also been used to investigate the formation of the banded electron structure observed by Burke et al. (1995) as PSEs were injected into the inner magnetosphere (Liemohn et al. 1998). That study focused on the capture of the electron cloud and the formation of the bands; this section will examine the contribution of PEs to the observed distributions.

7.3.2 *Combined Global Model*

As was mentioned above, the simulation of the superthermal electron population is based on two major source regions: the ionosphere (mostly PEs and secondary electrons) and the magnetotail (PSEs). Because the PEs and secondary electrons are generated in the atmosphere, their plasmaspheric velocity space distribution has a source cone at pitch angles that map to the ionospheres. The PE and secondary electron trapped population is built up by scattering processes changing the pitch angle of these source cone electrons. The timescale for the formation of the distribution function in the source cone is much faster than that for the trapped zone because the former is controlled by field-aligned transport and the latter by scattering (Khazanov et al. 1993a; Liemohn et al. 1997a). It was also shown by Khazanov et al. (1996) that cross-field drift effects are significant to the development of this trapped zone population. Because of these vastly different timescales, it is possible to separate the solution into two components: a field-aligned calculation to determine the source function and the plasmaspheric source cone distribution; and a bounce-averaged calculation for the trapped zone.

The PSEs typically begin with an isotropic pitch-angle distribution in the near-Earth plasma sheet (Christon et al. 1989). As the plasma convects around the nightside, it moves closer to the Earth, experiencing a stronger magnetic field where a greater amount of the pitch-angle domain is connected to the ionosphere. This part of velocity space is a loss cone, and particles will undergo severe energy loss upon entering the upper atmosphere. Again, this process is much faster than the convection or scattering of particles into the loss cone, and so the calculation can be separated into two parts: a bounce-averaged calculation for the development of the trapped zone fluxes and a field-aligned calculation for the loss cone and ionospheric regions, including the production of secondary electrons.

This section combines these two separate calculations into a single consistent approach. The two models described above present a unique possibility in modeling the nonsteady-state development of the superthermal electron distribution function, providing complementary calculations for a feedback loop of loss cone-trapped zone distribution functions. The field-aligned model is used to calculate the PE distribution function in the plasmaspheric source cone, and these results are used as a boundary condition on the dayside in the bounce-averaged model. This allows for the calculation of the initial calculation of the PE source terms in the ionosphere and their self-consistent propagation between the conjugate footpoints through the inner magnetosphere with the first model, and then using these flux intensities in the second model to simulate the cross-field drift effects on the PE distribution function. It also allows for the calculation of the PE influence to the nightside plasmasphere.

Conversely, the bounce-averaged model is used to calculate a boundary condition for the field-aligned model. Here, precipitation effects of high-energy superthermal electrons on the dayside and all energies on the nightside (where there is no ionospheric source) are investigated, including thermal plasma heating and secondary production in the ionosphere. These field-aligned model results of back-scattering and secondary electron production are then incorporated back into the bounce-averaged model. Iteration of this process yields a self-consistent calculation between the two approaches. The time step in each model is chosen sufficiently small so that the time evolution controls the iteration process. The combination of these two models in this way yields a fully three-dimensional spatial calculation of the superthermal electron distribution function in near-Earth space.

Khazanov et al. (1992) discussed an analytical approach to self-consistently coupling the superthermal electron solutions in the loss cone and in the trapped zone. In that study, a boundary flux injected into the plasmasphere from the topside ionosphere determined the distribution in the fly-through zone using a field-aligned transport equation, and a bounce-averaged kinetic equation was used in the trapped zone. The solutions of these two equations were matched at the pitch angle of the loss cone-trapped zone boundary. This solution was used to calculate the transparency of the plasmasphere for the superthermal electrons and from this the heating rates to the thermal plasma. These results, however, are dependent on the functional form of the assumed distribution at the ionospheric boundary. The model in the present section avoids this restriction because it contains a unified approach to the spatial regions by coupling the field-aligned and the bounce-averaged transport models. Also, this combined model is time dependent, so the development of the distribution function can be examined.

7.3.3 Numerical Implementation

Each code maintains its own numerical implementation because the only interface is the exchange of results at appropriate times. Below is a brief outline of the

modeling procedure for each approach. A comprehensive description has been given previously for both the field-aligned model (Khazanov et al. 1994; Khazanov and Liemohn 1995; Liemohn et al. 1997a) and for the bounce-averaged model (Fok et al. 1993; Jordanova et al. 1996; Khazanov et al. 1996).

The field-aligned code includes the calculation of the distribution in the magnetosphere, where pitch-angle diffusion is the dominant process for scattering particles into and out of the trapped zone. Therefore, the numerical scheme of this model is a second-order pitch-angle diffusion scheme at each point in time, energy, and distance, with fully implicit differences for the derivatives in these three variables. In order to decrease undesirable computational effects associated with approximation errors of the derivatives $\partial/\partial s$ and $\partial/\partial\mu$, it is convenient to change variables from (μ, s) to (μ_0, s) (Khazanov et al. 1979, 1994), where

$$\mu_0 = \frac{\mu}{|\mu|} \sqrt{1 - \frac{B_0}{B(s)}(1 - \mu^2)}. \quad (7.3.3)$$

Because μ_0 comes from the first adiabatic invariant, Φ is now a slowly varying function with s (Khazanov et al. 1993a). The pitch-angle grid is nonuniform, allowing more grid points near the edge of the loss cone where the distribution function rapidly changes, typically with 100 grid points per directional hemisphere. Due to its fully implicit nature, this code can handle an arbitrary time step, making it suitable for investigating the fast initial processes as well as the slow trapped flux build-up. It has been shown that a jump to steady state by setting $\Delta t \rightarrow \infty$ yields the same results as a slow progression in time (Khazanov and Liemohn 1995). Also, Liemohn and Khazanov (1995) presented a calculation of energy conservation and numerical diffusion for this model, finding that the energy conservation is within the expected numerical error of the model.

The bounce-averaged global model uses a combination of second-order high-resolution methods for the advection operators and a second-order diffusion scheme for pitch-angle scattering. The time step here must be chosen small enough to satisfy the Courant–Friedrichs–Lewy condition at each phase space point,

$$|v_{\text{CFL}}| = \left| \frac{a(R, \varphi, E, \mu_0, t) \Delta t}{\Delta x} \right| \leq 1, \quad (7.3.4)$$

where Δx is the local step in variable x (where x is R , φ , E , or μ_0) and a is the corresponding velocity multiplier. The time step is therefore usually less than 10 s for superthermal electron simulations, and $\Delta t = 5$ s was used throughout these runs. The numerical diffusion and dispersion of this technique has been discussed by LeVeque (1992) and Fok et al. (1993), and this will be examined again here in the results section.

The field-aligned and bounce-averaged model use the same energy grid. For this study, it is a geometrically increasing energy step to provide a smooth transition from low to high energies. This also allows for many points at low energies to

capture the fine structure in this region, while simultaneously supplying a reasonable number of grid points at high energies. The number of energy steps ranged from 65 to 75 for the simulations presented below.

The linearized Fokker–Planck collision operator will be used for the Coulomb interactions in (7.3.1) and (7.3.2) (Hinton 1983a, b). Because the simulations can be separated out by the source terms, that is, into distinct PE and PSE calculations, results from each population will be individually presented and discussed, followed by a discussion of the combined distribution from both sources. First, however, a critical issue must be addressed: the proper low-energy limit of this model.

7.3.4 The Low-Energy Limit

Traditionally, bounce-averaging of the kinetic equation is only applicable down to energies with a bounce period much smaller than the collisional timescales, typically above several tens of electron volts for electrons. However, once a quasi-steady-state flux level is achieved in the loss (source) cone, a bounce-averaged approximation can be applied for much lower energies, even for electrons of a few electron volts energy. The validity of this statement will be shown below. Phenomenologically, this extension is justified because of the very different timescales for distribution function development in the loss cone and trapped zone, and we use separate models to calculate the fluxes in these two regions of velocity space.

7.3.4.1 Desired Low-Energy Limit

The issue here is whether the model can be used for calculating the superthermal electron distribution function down to the thermal electron energy range. It is therefore necessary to determine this desired low-energy limit by comparing the superthermal electron energy spectra with that of a typical thermal electron distribution. Figure 7.13 shows fluxes for these two populations (thermal and nonthermal electrons) at two altitudes along an $L = 4$ field line. The superthermal spectra is from a typical photoelectron source, and the fluxes have been averaged over pitch angle for comparison with the thermal plasma spectra, which is assumed to be an isotropic Maxwellian,

$$\phi_{te} = \frac{n_0 E}{\sqrt{2m(\pi E_0)^3}} e^{-\frac{E}{E_0}}. \quad (7.3.5)$$

The values of n_0 and E_0 are given in Table 7.3. The intersection energies E_{int} are 2.8 eV at 800 km and 3.9 eV at the top of the field line.

Another method of determining the desired lower-energy limit of the superthermal electron energy range is by comparing the transport timescale τ_s to the collisional timescale τ_{ee} (the Maxwellization time). The energy where these two

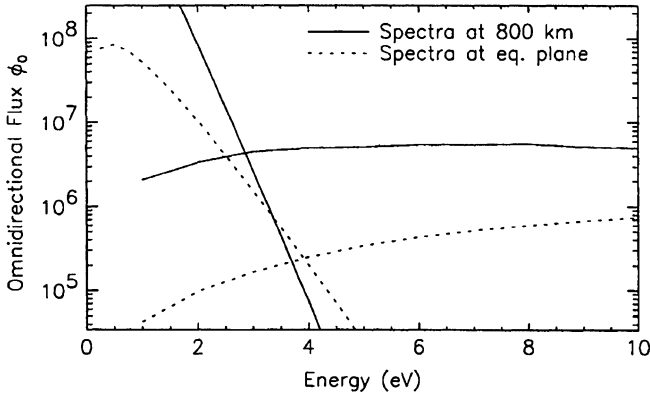


Fig. 7.13 Omnidirectional fluxes for a Maxwellian thermal plasma and modeled photoelectrons at two altitudes along an $L = 4$ flux tube. Thermal plasma parameters are given in Table 7.3

Table 7.3 Determining the desired low-energy limit

	n (cm $^{-3}$)	E_0 (K)	H_e (cm)	E_{int} (eV)	E_b (eV)
800 km	3.1×10^4	3,000	6.9×10^7	2.8	2.3
Equatorial plane	360	5,000	1.4×10^{10}	3.9	3.7

quantities equate is where the distribution changes from “thermal” to “nonthermal” because the electrons are collision dominated below this energy and transport dominated above it. The timescales can be defined as

$$\tau_s = \frac{H_e}{\bar{v}} = \frac{n_e}{\bar{v}|dn_e/ds|}, \quad \tau_{ee} = \frac{1}{\bar{v}n_e\sigma_e} = \frac{E^2}{A\bar{v}n_e}, \quad (7.3.6)$$

where H_e is the scale height of the electron distribution, n_e is the total electron density, and $A = 2\pi e^4 \ln \Lambda = 2.6 \times 10^{-12} \text{ cm}^2 \text{ eV}^2$. Thus, the energy boundary E_b has the form

$$E_b = \sqrt{An_e H_e}. \quad (7.3.7)$$

For the same $L = 4$ field line used in Fig. 7.13, E_b is 2.3 eV at 800 km and 3.7 eV at the equatorial plane. The relevant quantities for this calculation are given in Table 7.3. Knowing this, it is possible to analyze (7.3.1) and (7.3.2) to determine whether the model is valid down to this energy limit.

7.3.4.2 Calculation of the Limit

From the results of the previous section, it would be desirable to use the combined model down to energies of a few electron volts to include the entire superthermal

electron range in the calculation. Although (7.3.1) is valid down to these energies, this is below the traditional validity range for (7.3.2). The bounce-averaged approximation can only be used when the distribution is a slowly varying function of field-aligned distance. During the initial stages of transport through a flux tube, this is clearly not the case, but once this initial front of particles has traversed the flux tube, the variation along the field line is greatly reduced (Liemohn et al. 1997a). It is in this limit that the validity of the bounce-averaged equation (7.3.2) will be explored.

The boundary in velocity space between the trapped zone and the loss (or source) cone is invariably where the largest gradients in the flux occur (e.g., Khazanov et al. 1992, 1993a). It is also at this pitch angle that the distribution function from the field-aligned kinetic equation should match the results of the bounce-averaged kinetic equation. It should be possible, then, to combine (7.3.1) and (7.3.2) to calculate a single distribution function f_b at this pitch angle that satisfies both equations at this pitch angle. Note that such a match can only be done if the variation of f_b along the field line is small. Therefore, the validity of matching these equations is connected to the applicability of the bounce-averaged equation at the edge of the loss cone.

The field-aligned kinetic equation can be written as a steady-state relation between the field-aligned changes in f and the Coulomb scattering terms,

$$\mu \frac{\partial f}{\partial s} = \frac{An_e}{E} \frac{\partial f}{\partial E} + \frac{An_e B_0}{2E^2 B_i} \frac{\mu}{\mu_0} \frac{\partial}{\partial \mu_0} \left[\frac{\mu}{\mu_0} (1 - \mu_0^2) \frac{\partial f}{\partial \mu_0} \right], \quad (7.3.8)$$

where the subscripts i and 0 refer to the ionosphere–plasmasphere interface (where the loss cone is defined) and the equatorial plane, respectively. In steady state, the bounce-averaged kinetic equation in the trapped zone reduces to (e.g., Khazanov et al. 1992)

$$\frac{\partial}{\partial \mu_0} \left[a(\mu_0) (1 - \mu_0^2) \frac{\partial \langle f \rangle}{\partial \mu_0} \right] + 2b(\mu_0) E \frac{\partial \langle f \rangle}{\partial E} = 0, \quad (7.3.9)$$

where a and b are coefficients from the integral over a bounce period,

$$a = \oint \frac{B_0}{B} n_e \mu ds; \quad b = \oint \frac{n_e}{\mu} ds.$$

Note that (7.3.9) can be derived from (7.3.8) if f is a slowly varying function of s (Khazanov et al. 1979). It is expected that these two equations yield the same distribution at the trapped zone boundary, $f(\mu_{0b}) = \langle f(\mu_{0b}) \rangle = f_b$. Note that although the pitch angle μ of a particle changes as it moves along the field line, its equatorial pitch angle μ_0 is a constant (except for changes due to collisional scattering). The goal is to combine (7.3.8) and (7.3.9) into one equation, and this can be done by realizing that the variation near μ_{0b} of $\partial f / \partial \mu_0$ is much bigger than

the variation of a or μ/μ_0 . So, assuming that at μ_{0b} we can write $\mu/\mu_0 \approx d$ and $a(\mu_0) \approx a$, then (7.3.9) can be rewritten as

$$\mu \frac{\partial}{\partial \mu_0} \left[(1 - \mu_0^2) \frac{\partial f_b}{\partial \mu_0} \right] = -\frac{2bE}{a} \frac{\partial f_b}{\partial E} \quad (7.3.10)$$

and this can be included in (7.3.8) to yield

$$\mu \frac{\partial f_b}{\partial s} = \frac{An_e}{E} \frac{\partial f_b}{\partial E} - \frac{An_e}{E} \frac{B_0}{B_i} \frac{bd^2}{a} \frac{\partial f_b}{\partial E}. \quad (7.3.11)$$

Now we can solve (7.3.11) for f_b in order to determine its variation along the field line. For the sake of simplicity, the energy variation of the distribution function in the low-energy superthermal range can usually be parameterized as an exponential, $f(E) \propto \exp(-E/E_0)$, and so (7.3.11) reduces to an ordinary differential equation,

$$\frac{df_b}{f_b} = \frac{An_e}{\mu EE_0} \left[\frac{B_0}{B_i} \left(\frac{\mu}{\mu_0} \right)^2 \frac{b}{a} - 1 \right] ds \quad (7.3.12)$$

that can be easily solved analytically (Khazanov et al. 1998).

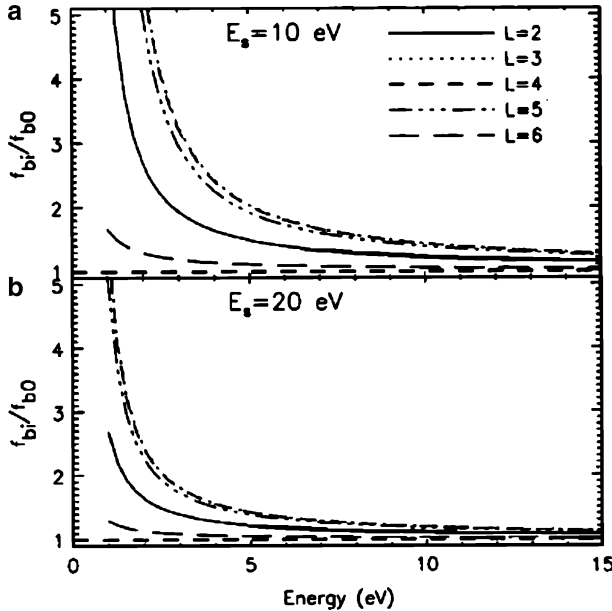


Fig. 7.14 Ratios of topside ionosphere values to magnetic equator values of the electron distribution function at the loss cone-trapped zone boundary at various L shells. Flux ratios are from the solution of (7.3.12), with $E_0 = 10 \text{ eV}$ (*top panel*) and 20 eV (*bottom panel*). The relevant parameters are given in Table 7.4. A heavy dashed line at unity is shown for reference

Table 7.4 Loss cone boundary f ratios at 4 eV

L	B_i/B_0 (deg)	α_{0b}	μ_{0b}	n_{e0} (cm $^{-3}$)	f_{bi}/f_{b0}	
					$E_0 = 10$ eV	$E_0 = 20$ eV
2	8.53	20.0	0.940	2.3×10^3	1.7	1.3
3	32.1	10.2	0.984	1.0×10^3	2.4	1.6
4	79.8	6.4	0.994	360	2.4	1.6
5	160.0	4.5	0.997	180	2.3	1.5
6	281.0	3.4	0.998	15	1.1	1.1

The solution of (7.3.12) is shown in Fig. 7.14 for $E_0 = 10$ and 20 eV at several L values. These choices for E_0 are typical mean values of superthermal electrons in the low-energy range. The distribution function ratio is the value of f at the LCB at 800 km to that at the equatorial plane. The thermal plasma densities used in these calculations are steady-state values at local noon from the Rasmussen et al. (1993) model, and these are given in Table 7.4 along with the magnetic field ratio and the equatorial pitch angle of the loss cone-trapped zone boundary α_{0b} (and its cosine μ_{0b}). Note that the f_b ratios are quite reasonable right down to the desired low-energy limit. Also listed in Table 7.4 are the ratios at 4 eV. They are well below the corresponding magnetic field ratios, indicating that the decrease along the field line due to collisions is rather insignificant compared with the changes due to the inhomogeneous magnetic field. It is clear from this analysis that a bounce-averaged approach is valid down to very low superthermal electron energies. The L shell dependence of these ratio curves is due to the direct relationship between the thermal plasma density and the solution to (7.3.12).

It should be noted that ratios presented in Fig. 7.14 are quite consistent with the PE fluxes shown in Fig. 7.13. Although the ratios are much smaller than the differences between the fluxes in Fig. 7.13, this is largely because Fig. 7.13 shows omnidirectional (pitch-angle averaged) values, which includes the relatively empty trapped zone, while Fig. 7.14 only considers the LCB. The trapped zone constitutes most of the pitch-angle range at the equatorial plane, lowering the omnidirectional fluxes at this altitude, whereas, by definition, there is no trapped zone at 800 km (the topside ionosphere).

This extension allows for the calculation of the total energy spectrum of the nonthermal electron population as well as an accurate determination of the energy deposition rate from the PE and PSE populations to the thermal plasma presented in (Khazanov et al. 1998).

7.3.5 Photoelectron Distribution Function Formation

As discussed above, the linearity of (7.3.1) and (7.3.2) affirms that the distribution functions from the various source terms can be calculated independently, and then summed at the end to show the combined result. The PE source will be discussed first, followed by the PSE source term.

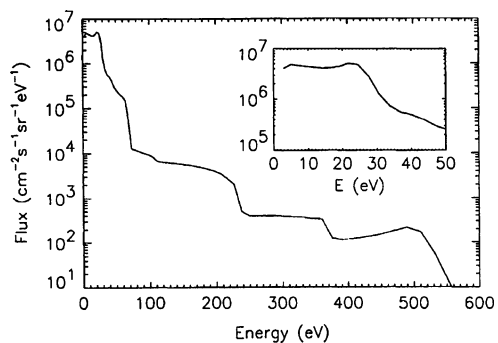
7.3.5.1 The Photoelectron Source Spectrum

An important feature of the PE source spectrum is the presence of spikes due to specific emission lines in the solar EUV spectrum. A prominent line is the He II 30.4-nm line. With an energy of 40 eV, this strong line produces a series of peaks in the 20–30 eV electron energy range due to the various ionization states of the atmospheric constituents.

Another important feature of the photoelectron source spectrum is the presence of the Auger electron peaks. This is due to the double ionization of atmospheric particles by high-energy photons. These photons initially produce a free electron from an inner electron shell of the neutral. Then an electron from the outermost shell transitions down to fill the hole, releasing a photon equal to this transition energy. This photon, however, does not escape but instead produces another free electron from the outermost shell. This second electron is known as an Auger electron, and has an energy of 300–500 eV, depending on the particle it originated from. There have been only a few modeling efforts to include this population (Avakyan et al. 1977; Winningham et al. 1989), showing that the inclusion of Auger electrons is necessary to obtain good agreement with observations while maintaining reasonable soft X-ray solar fluxes. For the present section, which focuses on the comparison of PE fluxes with PSE fluxes in the inner magnetosphere, Auger electrons are a critical component of the results.

As discussed above, the field-aligned model provides a source term for the bounce-averaged model at the LCB on the dayside for the PEs. For the cases to be examined here, a moderately high solar activity level will be used along with various geomagnetic activity levels. The PE source is mostly a function of solar activity level, and largely independent of geomagnetic activity. Therefore, the PE source term being supplied to the bounce-averaged model is essentially the same for the various simulations. A typical LCB spectrum is shown in Fig. 7.15. The low-energy range below 5 eV is sloped upward due to Coulomb collisional damping, the production peaks are essentially washed out into a single increase in flux near 25 eV, and the Auger peaks at 300 and 500 eV are quite distinguishable. Note that there is actually fine structure present in the Auger production peaks (see Chap. 2 resulting

Fig. 7.15 Typical photoelectron spectrum from the field-aligned code used as an input condition for the bounce-averaged model at the loss cone-trapped zone boundary



from the individual ionization states of the atmospheric constituents, and the resolution of this fine structure (and that in the 20–30 eV range, Fig. 7.11) is highly dependent on the energy step of the model. The choice of a geometrically increasing energy step for these calculations smooths out the peaks, especially in the high-energy range. Choosing a ΔE of much less than 1 eV is necessary to accurately resolve each peak (cf. Jasperse and Smith 1978), and the stability of these peaks have been debated as a source of plasma waves (Khazanov 1979). In this chapter, because we are interested in the global formation and evolution of the distribution function rather than the possible instabilities (see Chap. 8 for details), a reasonably large energy step was chosen.

7.3.5.2 Photoelectrons When $Kp = 1$

The combined model presented above now makes this type of time dependent, spatially three-dimensional simulation possible for all superthermal electron sources. Because the PE source term is largely independent of geomagnetic activity, a simulation with constant $Kp = 1$ will be discussed in detail, and then other simulations will be compared with these results.

The development of the source cone region of velocity space on the dayside has been shown in previous sections. Here we will focus on the formation of the trapped zone flux population. The time development of the 90° pitch angle energy spectra is shown in Fig. 7.16 for three L values and six magnetic local times. Note that each subplot has a separate flux range to show the maximum dynamic range of the results at each spatial location. Here and throughout the remaining figures, the differential number flux is given in units of $(\text{eV cm}^2 \text{ s sr})^{-1}$. Because the source is at small pitch angles, the development of the distribution at 90° should be a maximum timescale for PEs. Note the fast build-up of the fluxes on the dayside. A steady-state distribution has been reached for most of the PE energy range after 3 h. This case has relatively high thermal plasma densities, accelerating the scattering of PEs into the trapped zone. On the nightside, the PE fluxes degrade as they move around from dusk to dawn. The peak near 100 eV is due to the source dropping off at higher energies and Coulomb collisions more efficiently absorbing the energy of the slower PEs. The $L = 5$ fluxes are much higher than the $L = 3$ fluxes at MLT = 03. This is not only because of the decreased thermal plasma density at the larger L values, but also because of magnetospheric convection compressing the flux tubes from larger L shells. Thus it appears the flux at this L shell remains constant (and even grows) around the nightside. At MLT = 06, this narrow L shell band of PE fluxes contributes to the dayside fluxes in the high-energy trapped zone, creating a bulge in the distribution function. This bulge is seen even at MLT = 12. By 24 h of simulation time, the fluxes have essentially reached a steady-state level.

Steady-state flux values at several pitch angles are shown in Fig. 7.17 for nine spatial locations in the inner magnetosphere as in Fig. 7.16. On the dayside, it is

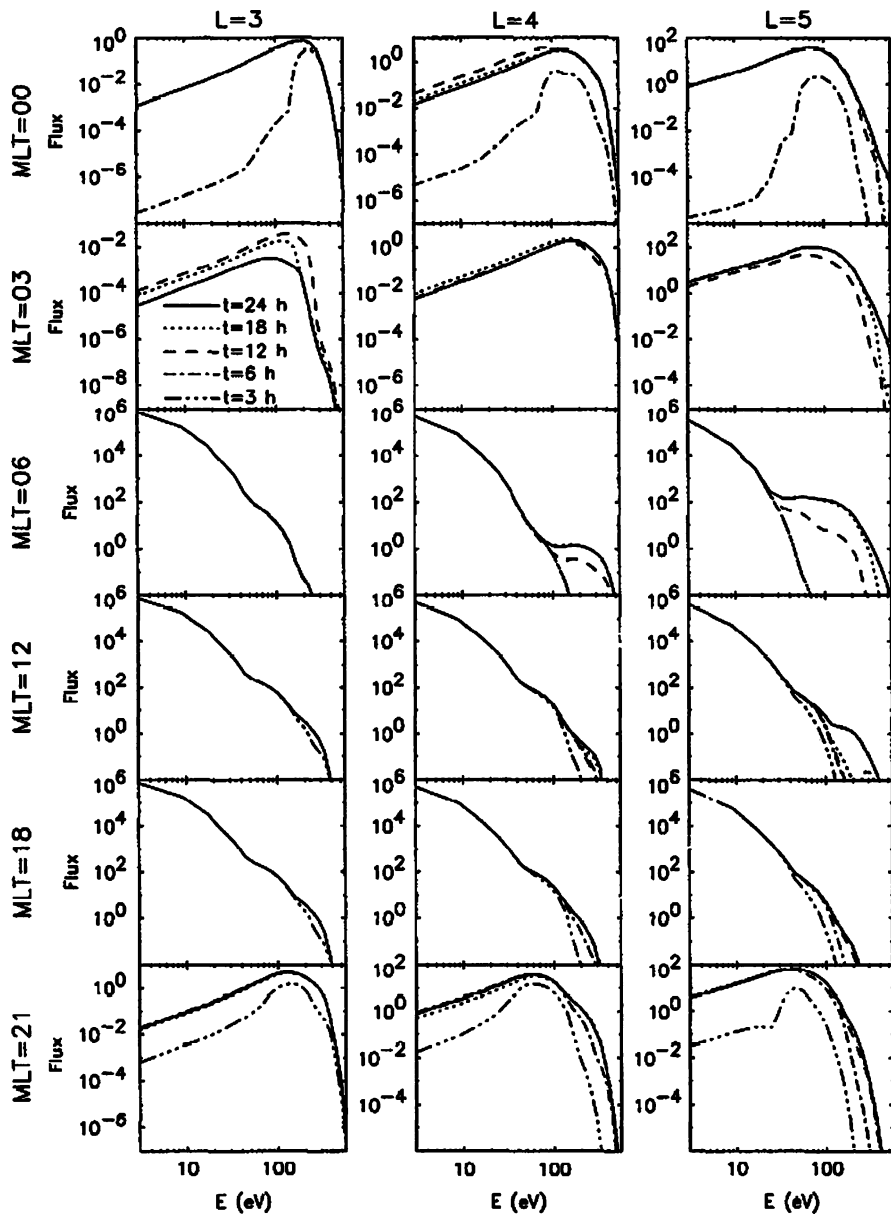


Fig. 7.16 Time development of the 90° pitch-angle energy spectra at various spatial locations in the inner magnetosphere. Simulation times are in hours from photoelectron source turn-on in the dayside ionosphere, with constant $Kp = 1$ geomagnetic activity. Note that each subplot has its own dynamic range. Here and elsewhere, the axis label “flux” refers to differential number flux in $(\text{eV cm}^2 \text{ s sr})^{-1}$

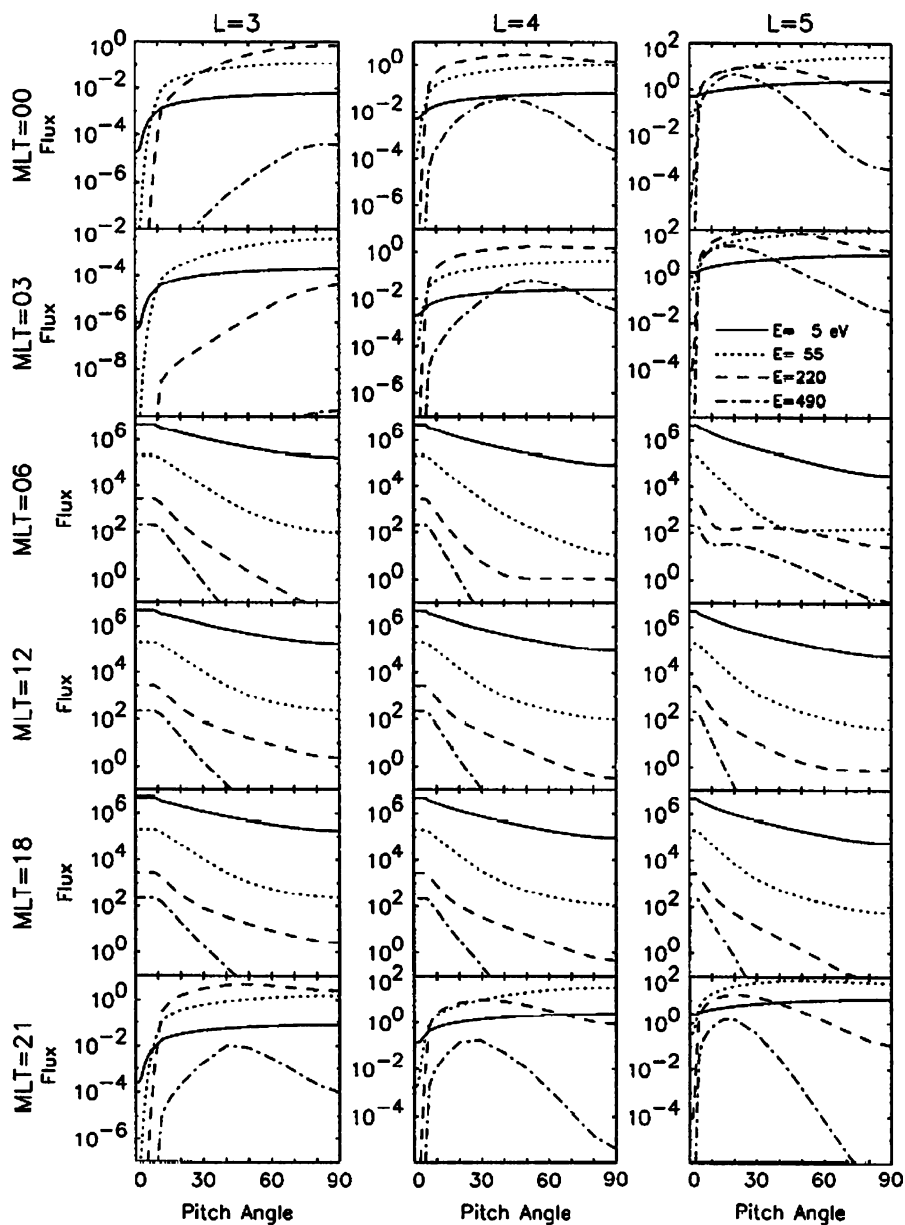


Fig. 7.17 Energy dependence of the steady-state pitch-angle distributions at various spatial locations for a photoelectron source with constant $Kp = 1$. Note that each subplot has its own dynamic range

clear that the trapped zone for low-energy electrons is filled in more than higher energies. On the nightside, the lower-energy fluxes are nearly isotropic in the trapped zone. This is not the case for the higher energies. In the 220- and 490-eV nightside distributions, there is a peak near 30° – 40° . This is largely due to Coulomb interactions. At small pitch angles, scattering depletes the loss cone and nearby trapped zone. At large pitch angles, scattering from the source cone on the dayside is not able to fill the loss cone any more than this, because of the competing processes of Coulomb energy decay and spatial drift. This is different from the source cone distributions observed by Moore and Arnoldy (1982), which are due to the secondary electrons from precipitating plasma sheet particles, a source not included here. Also, the bulge from the particles converting through the nightside is clearly seen at MLT = 06, contributing only to the high-energy trapped zone fluxes.

As was shown by Khazanov et al. (1996, 1998), magnetospheric convection plays a large role in the development of the PE distribution. It creates a high-energy bulge in a narrow L band on the nightside that propagates into the dawn sector, and it acts to deplete the trapped zone fluxes on the dayside by sweeping the plasma away before they can build up. Magnetic gradient-curvature drift is not important for PEs, but Coulomb collisions are very important. This interaction fills in the trapped zone, allowing for a PE population on the nightside, and it removes PEs from the inner plasmasphere on the nightside. High geomagnetic activity completely changes the distribution of PEs throughout the inner magnetosphere, preventing them from moving to the nightside and removing them from the dayside as well.

7.3.6 *Injection of Plasma Sheet Electrons*

For PSEs, the various drift terms play an even more important role in the distribution function formation in the inner magnetosphere because the source of this population is outside of the simulation domain. For this reason, a real Kp history will be used for the baseline case and then other simulations will be compared with this one to examine the effects of the various processes.

Liemohn et al. (1998) discussed the necessity for careful determination of where the Alfvén boundary lies in relation to the outer boundary of the simulation domain. The plasma travels along closed drift paths inside of the Alfvén boundary and along open drift paths outside of it, and this separatrix is highly dependent on geomagnetic activity, as well as energy, pitch angle, and location. The location in φ of where the Alfvén boundary crosses the outer boundary of the simulation at $6.6 R_E$, $\varphi_B(E, \mu_o, Kp)$, must be accurately determined, with injected plasma sheet particles entering along the open trajectories (dawnward of φ_B) and corotating plasma distributions from the dayside filling closed trajectories (duskward of φ_B). As in Liemohn et al. (1998), φ_B is taken from a look-up table so that the appropriate

boundary flux is taken for each energy and pitch angle at each azimuthal angle for each time step.

The models can assume any initial or boundary condition. For the PSEs, the injection flux is assumed to be an isotropic kappa distribution,

$$f = n \left(\frac{m}{2\pi\kappa E_0} \right)^{3/2} \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa - 1/2)} \left(1 - \frac{E}{\kappa E_0} \right)^{-\kappa-1}, \quad (7.3.13)$$

where n is the density, m is the particle mass, and Γ is the gamma function. Although the injected electrons sometimes have a characteristic energy up to a few keV (DeForest and McIlwain 1971; Eather et al. 1976; Evans and Moore 1979; Borovsky et al. 1997), Christon et al. (1989) found that the best-fit values for electrons from ISEE 1 measurements during quiet times are $\kappa = 6$ and $E_0 = 200$ eV, and these values are used in the present section. The density of the injected electrons is taken to be 0.1 cm^{-3} (cf. Huang and Frank 1986; Baumjohann et al. 1989; Christon et al. 1989; Birn et al. 1997). At the beginning of each simulation (UT = 00 of 1/25/98), it is assumed that there are no PSEs inside of the simulation domain.

The geomagnetic activity level is a critical element in electron transport in the inner magnetosphere. Thus the choice of Kp history is very important. Several simulations will hold the activity level fixed at a low ($Kp = 1$) or high ($Kp = 6$) value, but these are simply illustrative cases. For a realistic simulation, the simulations use the Kp history of late January 1991. During this time period, the low-energy plasma analyzer (LEPA) instrument on board the CRRES satellite detected several bands of magnetically trapped 0.1- to 30-keV electrons inside the plasma-pause. Liemohn et al. (1998) examined the formation of this banded structure in the inner magnetosphere, concluding that these bands are a natural occurrence when PSEs are trapped along closed drift paths in the inner magnetosphere by a sudden decrease of geomagnetic activity. Further simulation results during this time period are presented, and the possibility of photoelectron additions to the low-energy band of the trapped population causing its mean energy to decrease nonadiabatically (as discussed by Burke et al. 1995) are investigated.

Using the Kp history for late January 1991 in Fig. 7.18, the PSE injection into the inner magnetosphere can be simulated for the CRRES observations of the banded electron structure event. Figure 7.19 shows a comparison at $L = 4$ and MLT = 00 of three simulations at several times. The simulations are described in Table 7.5. All three have the same Kp history, but they have different combinations of included processes. Here it is seen that Coulomb collisions form the low-energy tail of the captured cloud. Without Coulomb collisions (Sim 2), there are no electrons below 200 eV at this spatial location due to the Fermi acceleration experienced during the Earthward convection. This low-energy tail forms at the expense of the peak intensity level of the lowest energy band. This is particularly evident in the last subplot, because the low-energy peak is not degrading in time or space. With the removal of magnetic gradient-curvature drift as well, there is no banded structure

Fig. 7.18 Kp history during the CRRES observations of the banded electron structures

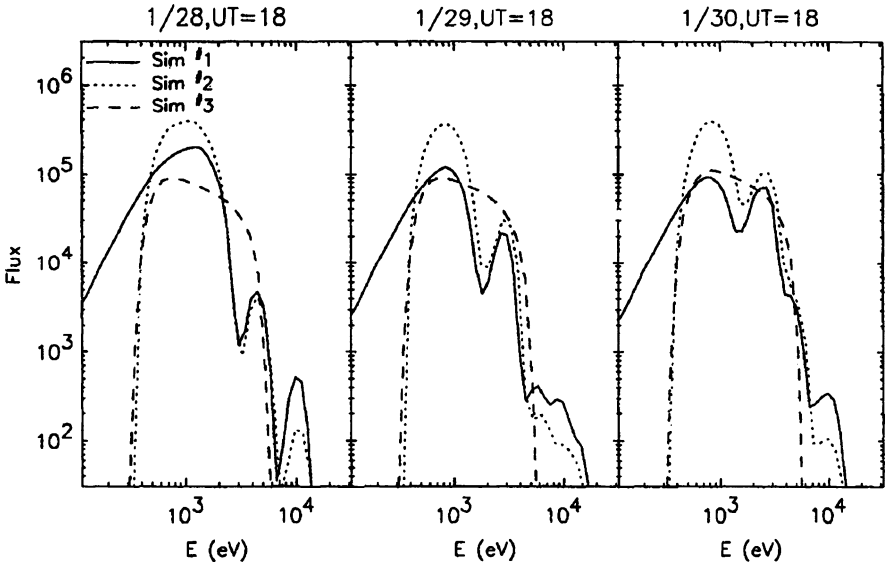
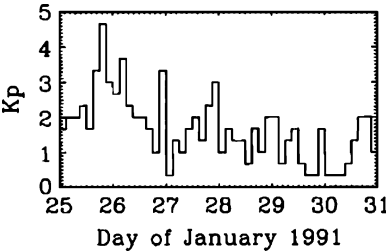


Fig. 7.19 Comparison of plasma sheet electron energy spectra at $L = 4$, $MLT = 00$ for three different simulations at various times. All three have the real Kp history, but Sim #1 has all processes included, Sim #2 has no Coulomb collisions, and Sim #3 has no Coulomb collisions or gradient-curvature drift (*dashed line*)

Table 7.5 PSE comparison simulations

Sim No.	Description
1	Real Kp , all processes
2	Real Kp , no Coulomb collisions
3	Real Kp , no CC or G-C drifts
4	Real up to $t = 48$ then $Kp = 1$, all processes
5	$Kp = 6$ always, all processes

formation at all, and Sim 3 simply shows an accelerated kappa distribution. The cloud is still captured in this simulation, but there is no degradation or distortion of the fluxes.

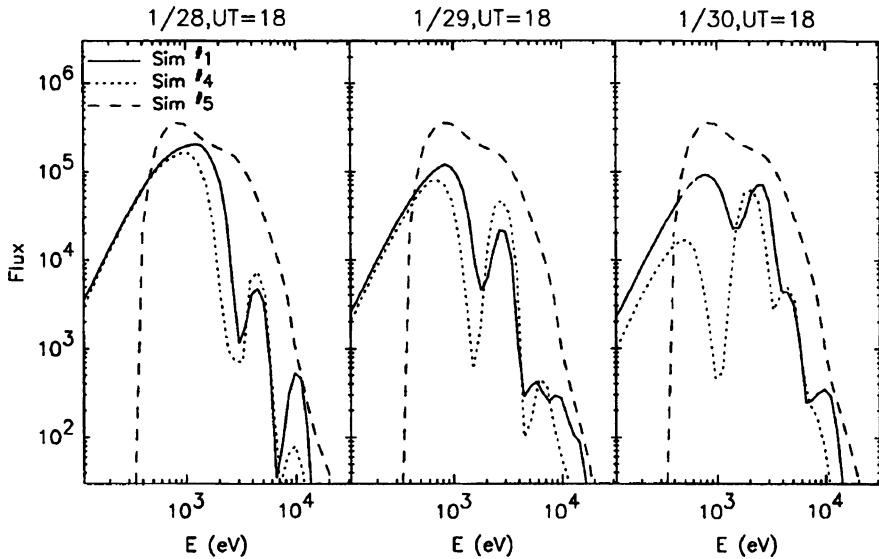


Fig. 7.20 Comparison of plasma sheet electron energy spectra at $L = 4$, $MLT = 00$ for three different Kp history simulations at various times. All three include all processes, but Sim #1 used the real Kp history, Sim #4 used the real Kp history through 1/26 and then switched to constant $Kp = 1$, and Sim #5 used constant $Kp = 6$

Figure 7.20 shows a comparison of Sim 1 with two more cases. Here, the three cases have all processes included but have different Kp histories. Sim 4 is the case without the third Kp spike, so all of the captured electrons are from the first two spikes. It is clear that this final Kp pulse only adds to the existing cloud and is not the sole source of the captured electrons. The differences between Sim 1 and Sim 4 in the last subplot is mainly due to differences in the thermal plasma due to the differing Kp histories. Sim 5, however, is completely different. The constantly high activity level pushes in the kappa distribution, and then constantly replenishes it. This is different from Sim 2 in that these particles are freshly injected instead of captured and undamped. There is no capture in Sim 5 because the activity is constant; a relaxation is necessary to capture a cloud of PSEs.

7.3.7 The Combined Electron Distribution Function

The combination of the two sources described above gives new insight into the development of the total superthermal electron distribution in the inner magnetosphere. Also, a new feature of this model is the ability to calculate the low-energy range of the superthermal electron distribution with a bounce-averaged technique,

allowing for an accurate calculation of the energy deposition to the thermal plasma on a global scale. These two features are discussed in this section.

7.3.7.1 Heating the Thermal Plasma

When energetic plasma is streaming through a background thermal plasma, the energetic particles will lose energy to the thermal plasma through Coulomb collisions. This results in an energy deposition from the energetic particles to the low-energy populations. The instantaneous volume heating rate Q is derived by integrating the Fokker–Planck collision term of Coulomb interactions over velocity space, and is a function of the superthermal electron omnidirectional flux and the thermal plasma density (Liemohn and Khazanov 1995). Of particular interest is the heat flux into the ionosphere P_e , obtained by integrating Q along the field line,

$$P_e = \int_{s_0}^{s_i} Q \frac{B_i}{B_0} ds, \quad (7.3.14)$$

where the subscripts i and 0 refer to the ionospheric footpoint and equatorial plane crossing, respectively. This represents the energy flux into the ionosphere from the energetic population. Modeling studies have shown that variations in P_e have great effects on the thermal, compositional, and optical structures in the ionosphere (Chandler et al. 1988; Kozyra et al. 1990; Comfort et al. 1995).

Using (7.3.14), the energy deposition to the thermal plasma from the various superthermal electron populations can be calculated. Only results for the thermal electrons are shown here because most of the energy loss of the superthermal electrons is transferred to this population. It is a straightforward calculation, however, to produce results for the other thermal plasma species as well.

Integrated electron heating rates from (7.3.14) are shown in Fig. 7.21 (it is a straightforward calculation to produce results for the other thermal plasma species as well). The five columns are defined as follows: heating from PEs; heating from PSEs; summed PE and PSE heating (sum of the first two columns); summed PE and PSE heating with five times the PSE density at the boundary as in the previous two columns (so $n_{\text{PSE}} = 0.5 \text{ cm}^{-3}$ in the boundary kappa distribution); and summed PE and PSE heating with $n_{\text{PSE}} = 3.0 \text{ cm}^{-3}$ at the boundary (30 times larger). This increase in density is simply a multiplier because of the linearity of (7.3.1) and (7.3.2), assuming that the injected distribution is held constant throughout the simulation. These last two columns are shown to represent injection from a dense plasma sheet (Birn et al. 1997; Borovsky et al. 1997).

The first row shows heating after steady-state flux levels have been reached with constant $Kp = 1$. Notice the strong dayside heating from the PEs with just a narrow band of heating around the nightside near $L = 5$. Heating from the PSEs is limited to a strip around the dawnside along the outer boundary. This heating from the plasma sheet is so much weaker than the PE heating that it hardly changes the dawn sector

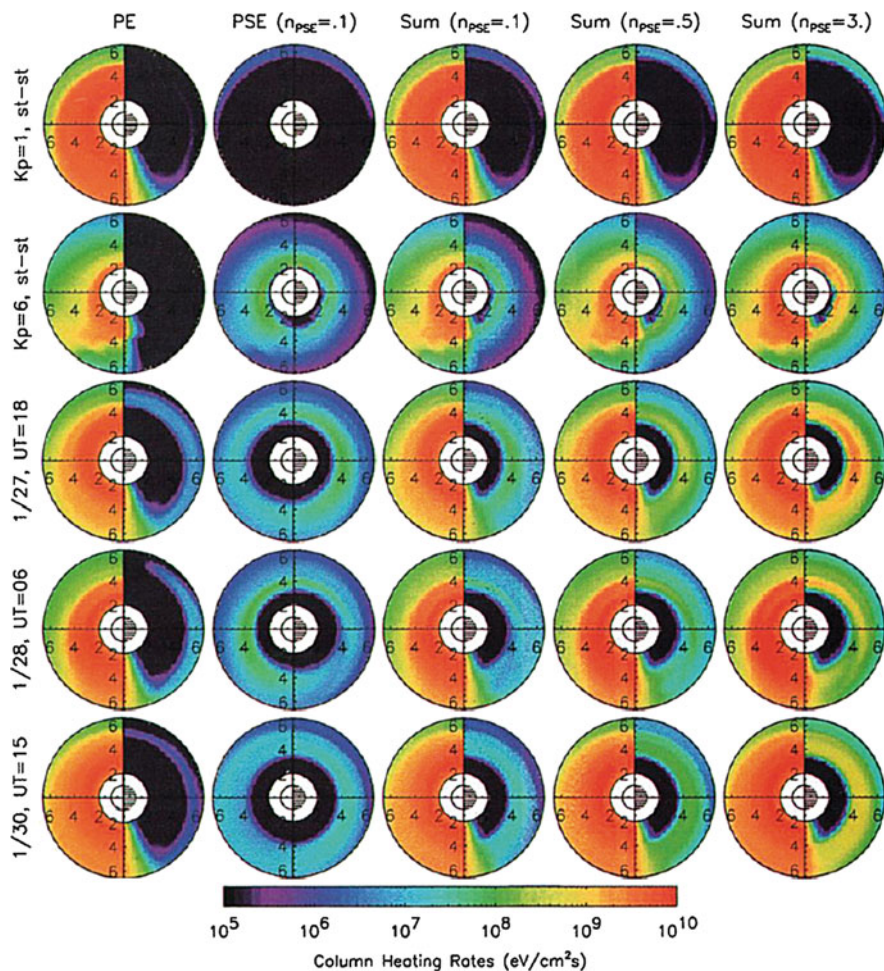


Fig. 7.21 Flux tube-integrated energy deposition rates to the thermal electrons for photoelectrons, plasma sheet electrons, and the combined distribution for three plasma sheet densities. The first row is constant $Kp = 1$ at steady state, the second row is constant $Kp = 6$ at steady state, and the last three rows are at times during late January 1991

of the summed plots from that of the PE results. It is interesting to note that most of the nightside is insignificantly heated by superthermal electrons for this case.

The second row shows results for steady-state superthermal electron fluxes after constant $Kp = 6$. The PE heating is greatly diminished from the $Kp = 1$ values, with essentially no PE heating on the nightside. By contrast, however, the PSE energy deposition has increased dramatically throughout the inner magnetosphere. Keep in mind that these values are proportional to the thermal plasma density, so the decrease at large radial distances is primarily due to decreases in this density. This is also true for the deposition in the afternoon sector from both populations. The

dependence of P_e on the thermal electron temperature is negligible because the characteristic energy of the superthermal electrons is much greater than T_e .

The bottom three rows of Fig. 7.21 are results using the Kp history during the CRRES banded structure observations in late January 1991. The third row is during the final injection of plasma sheet particles when the peak of the cloud is at local midnight, the fourth row is 12 h later, and the final row is more than 2 days after this, chosen so the cloud peak is again at local midnight. In the third row, significant heating is seen around the nightside due to PEs, in fact, in all three times, the nightside heating is more than the steady-state heating when Kp is held at 1. This is mostly due to differences in the thermal plasma density between the two simulations. The PSE heating rates are less than the $Kp = 6$ values, and are not so close to the Earth, but are still substantial, particularly with a denser source population. Twelve hours later, the cloud is on the dayside and far less of an influence because the PE heating is so much stronger there. In the fifth row, the band is degrading but still significantly contributing to heating the thermal plasma on the nightside, and is certainly still more important than PEs in this region.

7.3.7.2 Flux Comparisons

The heating rates in the previous section describe the composite influence of the superthermal electrons on the thermal plasma. It is also interesting to examine the combined distribution function in the superthermal energy range to understand how these bulk influences are generated. This will be done for the January 1991 results. As seen above, the $Kp = 1$ results have no PSEs in most of the simulation domain and the steady-state $Kp = 6$ case is somewhat unrealistic (shown as an extreme case). Therefore, these results will concentrate on a comparison of the distributions of PEs and PSEs during the CRRES banded structure observations.

Figure 7.22 shows energy spectra for the two source populations at various spatial locations for the third row of Fig. 7.21, during the final Kp spike and PSE capture. The spatial locations are chosen to be in the nightside peak of the cloud (first row) and the nightside PE band (second band), and two analogous points on the dayside. Also, two equatorial pitch angles are shown: 90° and the LCB. Row 1 clearly shows the PSE dominance of the distribution throughout the pitch-angle range, with fluxes several orders of magnitude larger than the PE fluxes at this location. In the narrow PE band on the nightside, however, PEs dominate the low-energy range of the spectrum, even at 90° . On the dayside, however, the intersection shifts to higher energies because of the proximity of the PE source, reaching 400 eV near the LCB at $L = 4$. In every case, however, the PSEs dominate the high-energy part of the spectrum, often with positive gradients in the distribution at the intersection and elsewhere. It can be seen that rotating the PSE results by 12 h will make the energy of the PSE–PE intersection decrease in the dayside plots, and will also make the PEs more comparable to the PSEs on the nightside. As time continues, the PSE fluxes will degrade (no more injections), allowing more of the PE distribution to dominate the combined flux function.

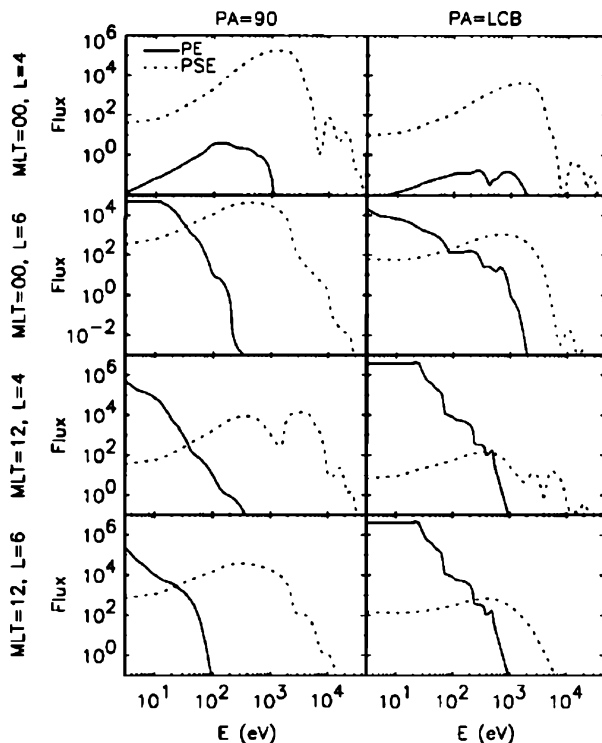


Fig. 7.22 Energy spectra for photoelectrons (*solid line*) and plasma sheet electrons (*dotted line*) at various pitch angles and spatial locations on 1/27/91 at UT = 1,800. This time corresponds to the third row of Fig. 7.21

The combined distribution function of superthermal electrons from these two sources is shown in Fig. 7.23 throughout the CRRES observations at the four spatial locations discussed in Fig. 7.22 (again at 90° and the LCB). Here, the two populations have been summed into a single distribution. In the first column, PSEs form the nightside and PEs for the dayside distributions, except for the high-energy bulge at 90° at $L = 6$. After this, the distribution becomes mixed, except in the first row, which is always dominated by PSEs. The second row shows the PE distribution at low energies transitioning into the PSE distribution. The third column, however, shows no PE rise at low energies because Kp remained sufficiently high to prevent them from reaching $L = 6$ at midnight. The third and fourth rows also show the PEs at low energies and the PSEs at high energies. There is a substantial number of spikes forming in the two $L = 4$ rows at high energies. This is the banded structure forming inside of the Alfvén boundary after the capture of the PSEs, as the magnetic drifts cause the electrons at these energies to have a drift period shorter than the corotation period, and they eventually lap the low-energy electrons. This is not seen for most of the $L = 6$ results because this is typically beyond the Alfvén

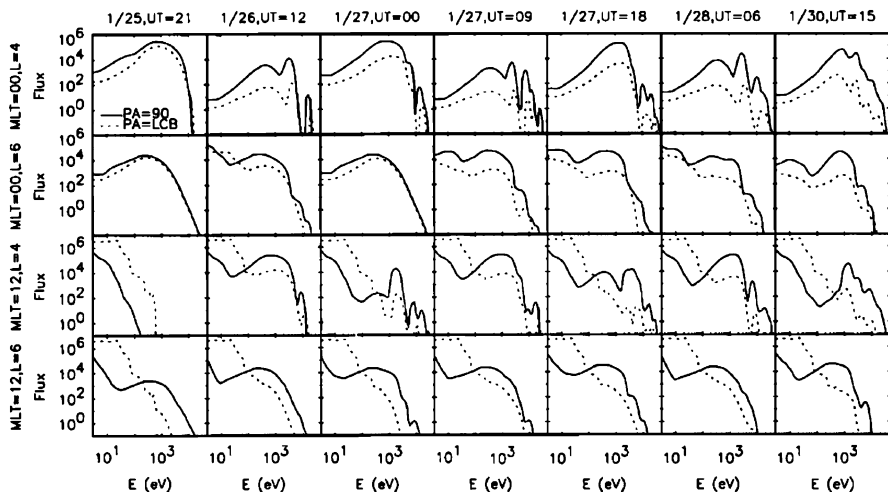


Fig. 7.23 Time development of the combined-source superthermal electron distribution function at several spatial locations and pitch angles

boundary. Only after extended quiet times would this radial distance develop the banded structure seen closer in (Khazanov et al. 1998).

Finally, kinetic modeling results are analyzed by Khazanov and Liemohn (2002) to examine the transport of photoelectrons through the nightside inner magnetosphere. Two sources are considered, those on the dayside from direct solar illumination, and those at all local times from light scattered by the upper atmosphere and geocorona. A natural filter exists on the nightside for the dayside photoelectrons. Coulomb collisions erode the distribution at low energies and low L shells, and magnetospheric convection compresses the electrons as they drift toward dawn. It is shown that, for low activity levels, a band of photoelectrons forms between $L = 4$ and 6 that extends throughout the nightside local times and into the morning sector. For the scattered light photoelectrons, the trapped zone throughout the nightside is populated with electrons of $E < 30$ eV. At high L shells near dawn, convective compression on the nightside yields an accelerated population with electrons at energies up to twice the ionospheric energy maximum (i.e., 1,200 eV for dayside photoelectrons and 60 eV for scattered light electrons). Modeled energy and pitch-angle distributions are presented to show the features of these populations (Khazanov and Liemohn 2002).

7.4 Artificial Relativistic Electrons Injection

In the late 1980s the mildly relativistic (1–10 MeV) electron beam linear accelerators (LINACs) had been reduced in size to the point where they could be feasibly flown on board spacecrafts, suborbital rockets, and balloons. The dynamics of these electron beams injected into the atmosphere from the ionosphere was then studied

by Banks et al. (1987, 1990). Neglecting magnetic field effects, it was determined that collisional processes caused significant radial expansion of the beam. Neubert et al. (1996) showed, using both Monte Carlo simulations and an analytical description based on the envelope equation (Humphries 1990), that a background magnetic field could significantly reduce beam expansion. Habash-Krause (1998) extended this work to much higher energies (1–100 MeV) and currents (1–10 kA), exploring beam stability issues and a variety of inelastic collision processes to determine bremsstrahlung, X-ray, and optical emission rates. In these studies, however, the authors were focusing only on downward beam propagation from altitudes of 60 km (Neubert et al. 1996) and 350 km (Habash-Krause 1998).

7.4.1 Interhemispheric Transport

In this section, we study a single beam pulse propagation injected upward at 700 km using a Boltzmann kinetic equation model (Khazanov and Liemohn 1995) that has been extended to include relativistic effects. This model allows for study of the time-dependent behavior of the beam flux and its secondaries in the ionosphere–magnetosphere system. For these initial calculations, only the relativistic primary electrons are considered in the model, concentrating on the collisional damping of the beam particles. Although wave–particle interactions are often important in relativistic electron transport, this section focuses on the collisional damping of the beam, which is the dominant process when the beam is stable to plasma wave generation.

7.4.1.1 Kinetic Equation

The kinetic equation, which includes the effects of elastic and inelastic collisions with both neutral and charged particles, transport in the inhomogeneous geomagnetic field, and particle sources (Khazanov and Liemohn 1995), can be generalized and adopted for relativistic electrons (Beliaev and Budker 1956; Hamilton et al. 1990; McTiernan and Petrosian 1990; Khazanov et al. 1999a),

$$\begin{aligned} & \frac{\varepsilon}{c\sqrt{\varepsilon^2 - 1}} \frac{\partial \Phi}{\partial t} + \mu \frac{\partial \Phi}{\partial s} - \frac{1 - \mu^2}{2B} \frac{\partial B}{\partial s} \frac{\partial \Phi}{\partial \mu} \\ &= \frac{AS_E}{10} \frac{\partial}{\partial E} \left[\frac{\varepsilon^2 \Phi}{E(\varepsilon + 1)} \right] + \frac{AS_\mu}{20} \left(\frac{\varepsilon}{E(\varepsilon + 1)} \right)^2 \frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial \Phi}{\partial \mu} \right], \end{aligned} \quad (7.4.1)$$

where $\Phi = \Phi(t, E, \mu, s)$ is the differential flux of electrons; μ is the pitch-angle cosine; E is the electron energy; $\varepsilon = 1 + E/(mc^2)$; m is the electron mass; s is the coordinate parallel to the geomagnetic field B ; c is the speed of light, and $A = 2\pi e^4 \ln \Lambda$. The distribution function of relativistic particles injected in an

ionosphere–magnetosphere plasma changes due to Coulomb scattering with thermal electrons and ions, elastic and inelastic interaction with neutral particles and ions, and bremsstrahlung and cyclotron radiation (Petrosian 1985). The input of the two last processes to the evolution of a relativistic beam with a mean energy less than 10 MeV is small and the corresponding terms have been omitted in (7.4.1).

The collisional coefficients are contained in the S variables. The term S_E represents the relativistic electron energy loss due to collisions with free electrons, S_{ee}^E ; neutrals, S_{eN}^E (per atom); and oxygen ions, $S_{eO^+}^E$. The momentum exchange is presented by the term S_μ and includes the interaction with free electrons, S_{ee}^μ ; hydrogen and oxygen ions, $S_{eH^+}^\mu$, $S_{eO^+}^\mu$; and neutrals, S_{eN}^μ (per atom). The expressions for Coulomb scattering and ionization losses are well known (Rossi and Olbert 1970; Landau and Lifshitz 1981) and S_E and S_μ can be presented as:

$$S_E = \sum_{\alpha=e, O^+, N} S_{e\alpha}^E, \quad S_\mu = \sum_{\alpha=e, O^+, H^+, N} S_{e\alpha}^\mu, \quad (7.4.2)$$

where

$$S_{ee}^E = n_e \ln \left(2 \times 10^{13} \left[\frac{\varepsilon^2 - 1}{\sum_{\alpha=e, i} n_\alpha / T_\alpha} \right]^{\frac{1}{2}} \right), \quad S_{eO^+}^E = 7 \frac{n_{O^+}}{n_e} S_{ee}^E \quad (7.4.3a)$$

$$S_{eN}^E = \sum_{\alpha=O, N} n_\alpha Z_\alpha \left[\ln \left(1.4 \times 10^{10} \frac{\varepsilon^3}{Z_\alpha^2} \right) - 2.9 \right]$$

and

$$S_{ee}^\mu = S_{ee}^E, \quad S_{eH^+}^\mu = \frac{n_{H^+}}{n_e} S_{ee}^\mu, \quad S_{eO^+}^\mu = \frac{71}{7} S_{eO^+}^E \quad (7.4.3b)$$

$$S_{eN}^\mu = \sum_{\alpha=O, N} n_\alpha Z_\alpha (Z_\alpha + 1) \ln \left[140 \sqrt{\varepsilon^2 - 1} / Z_\alpha^{\frac{1}{2}} \right].$$

Here n_α , T_α , and Z_α are the density, temperature, and charge of particles of species α (electrons, ions, and neutral atoms of oxygen and nitrogen). All quantities in (7.4.3) are in Gaussian cgs units except energy and temperature, which are in electron volts.

In order to decrease undesirable computational effects associated with approximation errors of the derivatives $\partial/\partial s$ and $\partial/\partial \mu$, it is convenient to change variables from (s, μ) to (s, μ_0) (Khazanov et al. 1979, 1993a), with μ_0 denoting the cosine of the pitch angle at the magnetic equator of the flux tube. This change of variables

is desirable because $\varphi(s, \mu_0)$ now is a slowly varying function with s (μ_0 is the adiabatic invariant). In this case, (7.4.1) becomes

$$\begin{aligned} \frac{\varepsilon}{c\sqrt{\varepsilon^2 - 1}} \frac{\partial \Phi}{\partial t} + \mu \frac{\partial \Phi}{\partial s} = \frac{AS_E}{10} \frac{\partial}{\partial E} \left[\frac{\varepsilon^2 \Phi}{E(\varepsilon + 1)} \right] + \frac{AS_\mu}{20} \left(\frac{\varepsilon}{E(\varepsilon + 1)} \right)^2 \\ \times \left[\frac{(1 - \mu^2)(\mu_0^2 - 1 + B_0/B)}{\mu_0^2} \frac{\partial^2 \Phi}{\partial \mu_0^2} + \frac{\mu_0^2 - 2\mu_0^4 + 1 - (1 + \mu_0^2)B_0/B}{\mu_0} \frac{\partial \Phi}{\partial \mu_0} \right] \end{aligned} \quad (7.4.4)$$

and can be solved numerically as described below, with appropriate initial and boundary conditions (such as those of Khazanov and Liemohn 1995 and also described in the previous section).

Equation (7.4.4) is solved with a generalized multi-stream approach taking into account energy degradation, pitch-angle focusing, pitch-angle diffusion, and field-aligned transport. With the following finite-difference approximations,

$$\frac{\partial \Phi}{\partial t} = \frac{\Phi_n - \Phi_{n-1}}{\Delta t}, \quad \frac{\partial \Phi}{\partial E} = \frac{\Phi_j - \Phi_{j+1}}{E_j - E_{j+1}} \quad (7.4.5a)$$

$$\frac{\partial \Phi}{\partial s} = \left\{ \frac{\Phi_i - \Phi_{i-\sigma}}{s_i - s_{i-\sigma}}, \sigma = \frac{\mu_0}{|\mu_0|} \right\}, \quad (7.4.5b)$$

where the subscripts n , i , and j are the time, space, and energy grid step indices, (7.4.4) is reduced to

$$A \frac{\partial^2 \Phi}{\partial \mu_0^2} + B \frac{\partial \Phi}{\partial \mu_0} + C \Phi = D. \quad (7.4.6)$$

Further replacement of μ_0 by $\theta_0 = \cos^{-1} \mu_0$ and the use of

$$\begin{aligned} \frac{\partial \Phi}{\partial \theta_0} = \frac{\delta_1 \Phi_{k+1} - (\delta_1 + \delta_2) \Phi_k + (\delta_1 + \delta_2) \Phi_k - \delta_2 \Phi_{k-1}}{\delta_2(\delta_1 + \delta_2)} + \frac{(\delta_1 + \delta_2) \Phi_k - \delta_2 \Phi_{k-1}}{\delta_1(\delta_1 + \delta_2)} \\ \frac{\partial^2 \Phi}{\partial \theta_0^2} = \frac{2\delta_1 \Phi_{k+1} - 2(\delta_1 + \delta_2) \Phi_k + 2\delta_2 \Phi_{k-1}}{\delta_1 \delta_2 (\delta_1 + \delta_2)} \end{aligned} \quad (7.4.7)$$

yields a tridiagonal matrix that is readily solved. In (7.4.7), $\delta_1 = \theta_{0,k} - \theta_{0,k-1}$ and $\delta_2 = \theta_{0,k+1} - \theta_{0,k}$. At each energy step, the calculation is swept in both directions along the field line, solving (7.4.6) for each hemisphere of flow ($\mu_0 > 0$ and $\mu_0 < 0$), sharing information at the $\mu = 0$ boundary and iterating until the energy level has converged at all points in the s - μ_0 plane. More details of this derivation are found in Khazanov et al. (1979, 1993a) and Gefan and Khazanov (1990). To avoid large numerical diffusion and to obtain second-order accuracy in the time and space steps

in (7.4.4), the two-step Lax–Wendroff method was used (Lax and Wendroff 1960), which time (space) centers the integration by defining intermediate values of the dependent variables at the half-time (space) steps $t^{n+1/2}$ ($s^{n+1/2}$). After each main step, the intermediate values of Φ are discarded and form no part of the solution (Potter 1973) and thus were omitted in (7.4.5) for simplicity.

7.4.1.2 Ionospheric Results

For the results below, the model was run with nine energy steps, 0.25-MeV wide, each centered at 5 MeV, injected at 700 km at pitch angles between 40° and 90° (equatorial pitch angles of 11.6° – 18.3°). The injection was assumed to be Gaussian with a width of 0.5 MeV. It was run with a time step of 10^{-4} s for an $L = 2$ field line (ionospheric latitude near 45° , with an equatorial plane crossing at $2 R_E$ geocentric). The spatial domain of the simulation extends from 90 km to 90 km in the conjugate ionospheres, with particles traveling below these altitudes lost to the upper atmosphere. The magnetic field is allowed to vary along s above 200 km, making this altitude the defining point for the LCB ($\mu_{0,lc} = 16.5^\circ$). The injection lasted one time step, and then the beam was allowed to decay with time.

Although a standard LINAC has a pulsed beam with an injection lasting several microseconds and a duty cycle around a millisecond, this simulation is still insightful because the fluxes can be scaled to be the same strength as a single LINAC pulse injection. The results will therefore be presented in normalized units, showing the relative intensity drop of the injection as a function of time and bounce.

Figure 7.24 shows the evolution the total number of particles in the beam, defined as

$$N_{tot} \propto \int \frac{B_i}{B} ds \int d\mu \int \frac{\Phi}{\sqrt{E}} dE. \quad (7.4.8)$$

The solid line is the simulation with scattering and losses, whereas the dotted line shows the results from a run with no scattering or loss mechanisms included

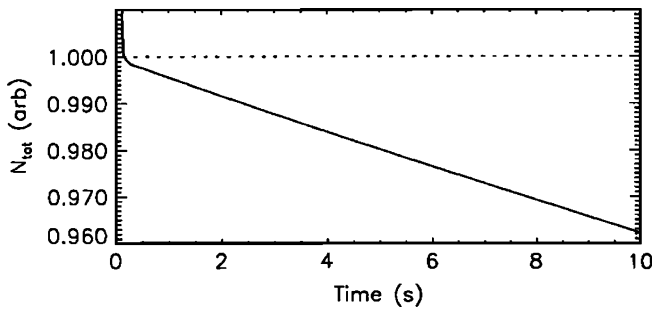


Fig. 7.24 Total beam particle evolution with (solid line) and without (dotted line) collisional interactions

(except loss to altitudes below 200 km). This means that the dotted line result is a test of the particle conservation due to the spatial derivative. Note that the values are normalized to the steady-state level of the no-loss simulation, and that the y axis is condensed around this value. It is seen that after the initial loss of particles at the end of the first traversal due to the injection pitch-angle range spanning part of the loss cone, this line shows no significant loss of particles. The other line shows the effect of scattering on the total number of particles. This curve is expected to have an exponential decline to it from the form of the collision terms, and so an e-folding time for the beam particle count can be calculated as 265 s. Comparing results without either pitch-angle scattering or energy loss, it was found that the scattering losses for an injected relativistic electron beam are less than the energy losses. The same tendency was obtained for solar energetic particles (Petrosian 1985).

Shown in Fig. 7.25 is a trace of the beam maximum location as a function of space and time for the first second of simulation, following the maximum in the 5-MeV intensity for two pitch angles, $\theta_0 = 16.6^\circ$ and 18.2° . The smaller pitch angle magnetically mirrors near 200 km, the boundary of the homogeneous B field regime, and the larger one reflects near the injection altitude of 700 km, so these two curves delimit the portion of the injection that mirrors above the lower boundary (the tapped injection region). It is seen that the bounce period for this beam along an $L = 2$ field line is just about 0.19 s. It is clearly seen in Fig. 7.25 that the bounce period for a particle with fixed energy is a decreasing function of pitch angle.

Equatorial pitch-angle distributions are shown in Fig. 7.26. The solid line presents the injected pitch-angle distribution. After the first few bounces (10 or so), the solution obtains a distribution (the dotted line) whose time variation is purely a decrease in magnitude. The larger tail toward smaller pitch angles is due to the larger scattering processes in the thermosphere and ionosphere. Also, in order to negate the effect of the spatial spread of the beam (seen in Fig. 7.25), the fluxes were summed along the flux tube for each μ_0 grid point, which yielded the proper flux for the beam maximum (another test of particle conservation).

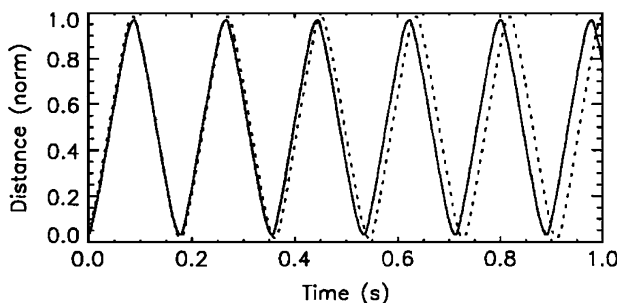


Fig. 7.25 Location of the maximum intensity for 5 MeV at two equatorial pitch-angle values, $\theta_0 = 18.2^\circ$ (solid line) $\theta_0 = 16.6^\circ$ (dotted line), for several bounces of the beam. Distance is normalized to the length of the field line ($3.5 R_E$)

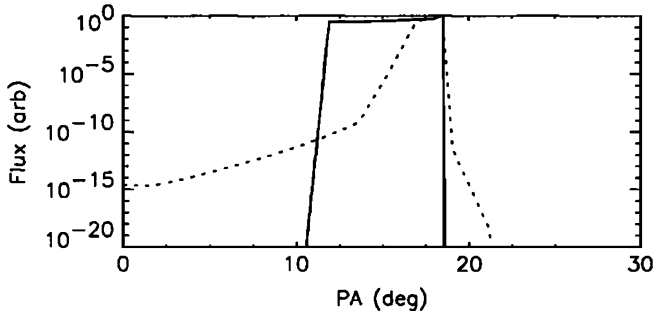


Fig. 7.26 Equatorial pitch-angle distributions at 5 MeV for the initially injected profile (*solid line*) and the quasi-steady-state shape of the distribution after ~ 10 bounces (*dotted line*)

7.4.1.3 Stability Issues

The simulations presented here do not include the effects of plasma waves. This approach is correct if the beam pulse width is below the growth times of the instabilities (Fisher et al. 1988). The types of excited modes and their evolution depend on the experimental design and processing. Therefore we present below only the estimations of the growth rate for some “hydrodynamic” instabilities (the worse case) to outline approximately the pulse width for which the beam energy and momentum degradation arises mainly due to scattering with the background environment. With the choice of beam current 0.1 A, along with the energy, pulse width, and divergence mentioned above (which are very reasonable for the LINAC experiments), we discuss here the two-stream, ion hose, resistive ion hose, and filamentation instabilities.

The beam plasma density n_b can be estimated using the equilibrium radius approach of Humphries (1990). The corresponding beam radius, r_b , is 10^2 – 10^3 m, a few times less than the Larmour radius of the beam particles. In our simulation domain, n_b varies in the range 10^3 – 10^5 m $^{-3}$. For times larger than the period of ambient plasma oscillation and a beam density less than the density of the core plasma, the beam is completely space charge neutralized. The exact degree of the current neutralization is not essential for the beam under consideration here. The two-stream instability growth rate for such a beam density is (Davidson 1990)

$$(\text{Im } \omega)_{\text{max}} \approx \frac{4\pi n_0 e^2 c^2}{\varepsilon} \left(\frac{n_b}{n_0} \right)^{1/3} \approx 10^4 \text{ s}^{-1}. \quad (7.4.9)$$

For the ion hose instability (Buchanan 1987), the time for wave growth is

$$\tau \approx \frac{3r_b}{c} \left[\frac{17\varepsilon n_b}{I(kA)n_0 mc^2} \right]^{1/2} \approx 10^{-6} - 10^{-5} \text{ s}, \quad (7.4.10)$$

where I is the beam current in kiloamps. The growth time for this instability is much less than in the previous case, but it should be noted that this instability “will saturate at a very low level” (Buchanan 1987) and can be ignored here. The expression used above is obtained for an unmagnetized plasma, but the results are of the same order of magnitude for the magnetized case (O’Brien 1989). The filamentation instability arises usually when the beam–plasma system is current neutralized (Davidson 1990). The threshold condition for this instability, even supposing that complete current neutralization is not satisfied, is

$$1 < \frac{E}{\varepsilon} \sqrt{\frac{\omega_{pb}}{\omega_{cb}}} \approx 10^{-3}, \quad (7.4.11)$$

and the beam is stable to filamentation. The resistive hose instability is reported to be the most serious macroinstability in a resistive plasma (Siambis 1992). In our case, it should be checked for low altitudes. Habash-Krause (1998) found that the growth time for this altitude is of the order 0.1 s. It can be supposed therefore that for a pulse width of the order $<10^{-4}$ s, the main sources of beam degradation are collisional.

The main result of this analysis is that a relativistic electron beam injected upward will, after any initial losses to the lower atmosphere (depending on the injection pitch angles), very slowly dampen (due primarily to energy losses) with a timescale of hundreds of seconds ($\tau = 265$ s for the configuration discussed above). The trapped part oscillates between the mirror points and longitudinally drifts with constant L shell. In our case, the drift period is approximately 600 s and comparable to the beam e-folding time. Due to the low altitude of injection, only a small fraction of the injected particles are trapped with very large pitch angles. As a consequence, the trapped particles spend a relatively long time in the region of high neutral density. That is why a shorter beam lifetime was found here compared with the case in which electrons are injected into the inner radiation zone by a nuclear explosion (Schulz and Lanzerotti 1974). In the latter case, most electrons apparently have an initial equatorial pitch angle between zero and 50° . For the beam under consideration, it is between 12° and 18° . The difference in the initial conditions makes a comparison between these two cases difficult, but it is reasonable for the beam considered here to have a smaller decay time. In our calculations, the atmospheric model was assumed to be longitudinally independent and therefore the drift term in the kinetic equation (7.4.1) was omitted, along with electric fields and magnetic disturbances. The role of all these processes in the relativistic beam evolution will be determined by the specific condition of the experiment and can be easily taken into account based on our newly developed global model (Khazanov et al. 1998).

As we pointed out in the introduction, the previous studies of relativistic beam injection were focusing only on downward beam propagation from altitudes of 60 km (Neubert et al. 1996) and 350 km (Habash-Krause 1998). The scenario of our simulation is different and addresses the issue of a single beam pulse injected upward at 700 km. Additionally, we used a completely different technique in our

study. A propagation of a point-launched relativistic electron beam is studied using a Boltzmann kinetic equation model (Khazanov and Liemohn 1995) that has been extended to include relativistic effects. Nevertheless, we did compare the energy losses of relativistic electrons below 350 km predicted from our model with the Habash-Krause (1998) calculations and found very good agreement between the two approaches.

7.4.2 *Global Transport*

In the previous section, relativistic electron beam injection simulation results were presented from a interhemispheric transport model, with a spatial domain reaching from 90 km to 90 km in the conjugate ionospheres. A single beam pulse was injected upward at 700 km during the first time step (10^{-4} s) and allowed to scatter and decay through collisional interactions with neutral particles and the core plasma. Owing to the small time step required, only the first 10 s after injection were considered. For beam stability, the maximum pulse duration should be less than the characteristic time of the wave-beam instabilities. These times were estimated, and the assumed pulse length is well within this limit. The obtained results clarified the role of energy losses and pitch-angle diffusion for beam decay on a single field line. For an $L = 2$ field line, for example, they found a few percent loss during the first 10 s after injection, extrapolating the e-folding time of a 5 MeV beam to be 265 s, which is much bigger than the bounce period (0.19 s) and comparable to the drift period around the Earth.

This section provides the theoretical background and further development of investigations into relativistic electron dynamics in near-Earth space plasma, focusing on their artificial injection from the upper ionosphere into the magnetosphere. The motivation for this work is the lack of magnetospheric simulations of relativistic electron beam injections. Published radiation belt model results that average over local time are unable to address this issue because of the point source nature in both space and velocity of an artificially injected beam. However, the evolution of such beams is important as a source of inner magnetospheric radiation belt particles as well as determining the possible effects of injected particles on satellites in near-Earth space. Also, the point-source nature of this study is an extreme case of a limited-spatial-extent source function for naturally occurring radiation sources, such as storm and substorm injections of high-energy electrons, which are usually quite localized. As discussed below, interesting features arise in the distribution function due to the localization of the source in phase space. Finally, after injection, these particles become part of the inner magnetospheric radiation environment, and are observationally indistinguishable. Therefore a solid understanding of their evolution is necessary to extract the influence of this population from the measurements.

This section also extends our time-dependent global electron model (Khazanov et al. 1996) to the relativistic energy range to simulate the evolution of beam decay

for much longer times after injection than that of Khazanov et al. (1999b). This new relativistic electron transport model takes into account not only collisional processes with charged and neutral particles but also scattering due to wave activity such as hiss, lightning, and very low-frequency transmitter signals. The role and relative importance of these waves in modifying the beam distribution function and in causing additional loss of relativistic electrons is examined. Although this section uses this relativistic electron transport model to examine the evolution of particles from an anthropogenic source, it is equally well suited for studying phenomena with natural sources, particularly the radiation belts. More discussion of this applicability is given at the end of the chapter.

7.4.2.1 Bounce-Averaged Relativistic Electron Kinetic Equation

The relativistic electron kinetic equation averaged over gyrophase, which includes the effects of elastic and inelastic collisions with both neutral and charged particles and transport in the inhomogeneous geomagnetic field, was adopted for relativistic electrons by Khazanov et al. (1999b). It is easy to generalize this equation in order to include drift motion and wave–particle interaction. The kinetic equation for the distribution function of the guiding centers, $f = f(t, \mathbf{r}_\perp, \varphi, E, \mu, s)$, in this case, can be presented as Khazanov et al. (2000a)

$$\begin{aligned} & \frac{\partial f}{\partial t} + \dot{\mathbf{r}}_\perp \nabla_\perp f + \sqrt{\gamma^2 - 1} \frac{c}{\gamma} \left(\mu \frac{\partial f}{\partial s} - \frac{1 - \mu^2}{2} \frac{1}{B} \frac{\partial B}{\partial s} \frac{\partial f}{\partial \mu} \right) \\ &= \frac{2\eta}{mc\gamma\sqrt{\gamma^2 - 1}} \frac{\partial}{\partial E} (\gamma^2 S_E f) + \frac{\eta\gamma S_\mu}{m^2 c^3 (\gamma^2 - 1)^{3/2}} \\ & \quad \times \frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial f}{\partial \mu} \right] + \frac{1}{\gamma^2} \frac{\partial}{\partial \mu} \left[(1 - \mu^2) D_{\mu\mu} \frac{\partial f}{\partial \mu} \right], \end{aligned} \quad (7.4.12)$$

where

$$\gamma = 1 + \frac{E}{mc^2}, \quad \eta = \frac{e^4}{4\pi\epsilon_0^2},$$

and μ is the pitch-angle cosine; c is the light velocity; E , e , and m are the electron kinetic energy, charge, and nonrelativistic mass; γ is the Lorentz factor; ϵ_0 is the permittivity of free space; s is the distance along the geomagnetic field line; and $\dot{\mathbf{r}}_\perp$ is the electron drift velocity in the plane normal to the magnetic field B . The right-hand side of (7.4.12) describes the collisions of relativistic electrons with core plasma and their interaction with waves. The term S_E represents the relativistic electron energy loss due to collisions with free electrons, $S_{E,ee}$; neutrals, $S_{E,eN}$ (per atom); and oxygen ions, S_{E,eO^+} . The momentum exchange is presented by the term S_μ and includes the interaction with free electrons, $S_{\mu,ee}$; hydrogen and oxygen ions,

S_{μ,eH^+} and S_{μ,eO^+} ; and neutrals, $S_{\mu,eN}$ (per atom). The expressions for Coulomb scattering and ionization losses are well known (Rossi and Olbert 1970; Landau and Lifshitz 1981, 1982) and can be presented as

$$\begin{aligned}
 S_E &= S_{E,ee} + S_{E,eO^+} + S_{E,eN}, \quad S_{E,eO^+} = 7 \frac{n_{O^+}}{n_e} S_{E,ee} \\
 S_{E,ee} &= n_e \ln \left[2 \times 10^{13} \left(\frac{\varepsilon^2 - 1}{\sum_{\alpha=e,i} n_{\alpha}/T_{\alpha}} \right)^{\frac{1}{2}} \right] \\
 S_{E,eN} &= \sum_{\alpha=O,N,H} n_{\alpha} Z_{\alpha} \left[\ln \left(1.4 \times 10^{10} \frac{\varepsilon^3}{Z_{\alpha}^2} \right) - 2.9 \right] \\
 S_{\mu} &= S_{\mu,ee} + S_{\mu,eN} + S_{\mu,eH^+} + S_{\mu,eO^+}, \quad S_{\mu,ee} = S_{E,ee} \\
 S_{\mu,eH^+} &= \frac{n_{H^+}}{n_e} S_{E,ee}, \quad S_{\mu,eO^+} = \frac{71}{7} S_{E,eO^+} \\
 S_{\mu,eN} &= \sum_{\alpha=O,N,H} n_{\alpha} Z_{\alpha} (Z_{\alpha} + 1) \ln \left[140 \sqrt{\varepsilon^2 - 1} / Z_{\alpha}^{\frac{1}{2}} \right]. \tag{7.4.13}
 \end{aligned}$$

Here n_{α} , T_{α} , and Z_{α} are the density, temperature, and charge of particles of species α (electrons, ions, and neutral atoms of hydrogen, oxygen, and nitrogen). All quantities in (7.4.14) are in Gaussian cgs units except the energy and temperature, expressed in electron volts.

The wave-particle interaction is represented by the last term of the right-hand side of (7.4.12) and has been described according to the relativistic generalization of the Lyons (1974) method of Albert (1999). For relativistic electrons with energies to be considered in this section ($E < 10$ MeV), only pitch-angle diffusion needs to be taken into account for these interactions.

The bounce-averaged kinetic equation can be easily obtained from (7.4.12) in the usual way (Lyons and Williams 1984; Khazanov et al. 1996), noting that, for the relativistic case, the averaging procedure remains the same, and can be presented as

$$\begin{aligned}
 \frac{\partial Q}{\partial t} + \frac{\partial}{\partial \varphi} \left(\left\langle \frac{\partial \varphi}{\partial t} \right\rangle Q \right) &= \frac{\sqrt{2}}{c\gamma \sqrt{m(\gamma^2 - 1)}} \frac{\partial}{\partial E} \left[\sqrt{E} \left\langle \left(\frac{dE}{dt} \right)_{cc} \right\rangle Q \right] + \frac{1}{h\mu_0} \\
 &\times \frac{\partial}{\partial \mu_0} \left[\langle D \rangle h\mu_0 \frac{\partial Q}{\partial \mu_0} \right] - \frac{Q}{\tau_{atm}}. \tag{7.4.14}
 \end{aligned}$$

Here

$$\left\langle \frac{d\varphi}{dt} \right\rangle = - \frac{3mc^2 R_0}{2eM_E} \frac{(\gamma^2 - 1)}{\gamma} \left[1 - \frac{I(\mu_0)}{6h(\mu_0)} \right]$$

$$\begin{aligned}
I(\mu_0) &= \frac{1}{R_0} \int_{s_m}^{s_{m'}} \sqrt{1 - B(s)/B_m} ds \\
\langle \chi \rangle &= \frac{\int_{s_m}^{s_{m'}} \frac{\chi(s) ds}{\sqrt{1 - B(s)/B_m}}}{\int_{s_m}^{s_{m'}} \frac{ds}{\sqrt{1 - B(s)/B_m}}} \\
\tau_{atm} &= \begin{cases} \frac{2R_0 \gamma h}{c \sqrt{\gamma^2 - 1}} & \alpha_0 < \alpha_{0LC} \\ \infty & \alpha_0 > \alpha_{0LC} \end{cases} \\
\mu_0 = \cos \alpha_0, \quad h(\mu_0) &= \frac{1}{2R_0} \int_{s_m}^{s_{m'}} \frac{ds}{\sqrt{1 - B(s)/B_m}},
\end{aligned}$$

where R_0 is the radial distance in the equatorial plane, φ is the geomagnetic east longitude (magnetic local time [MLT]), α_0 is the equatorial pitch angle, α_{0LC} is the loss cone boundary, subscripts m and m' mark the conjugate mirror points on the field line, M_E represents the Earth dipole moment, and the expression for azimuthal bounce-averaged drift velocity, $\langle d\varphi/dt \rangle$ (Hamlin et al. 1961; Schulz and Lanzerotti 1974), is written in SI units. The diffusion coefficients in (7.4.14) are simply related to (7.4.13),

$$\begin{aligned}
\left\langle \left(\frac{dE}{dt} \right)_{cc} \right\rangle &= \frac{\eta \gamma^2}{\sqrt{2mE}} \sum_{\alpha} \langle S_{e\alpha}^E \rangle \\
\langle D \rangle &= \langle D_{cc} \rangle + \langle D_{\mu\mu} \rangle \\
\langle D_{cc} \rangle &= \frac{\eta \gamma}{m^2 c^3 (\gamma^2 - 1)^{1.5}} \frac{1 - \mu_0^2}{2\mu_0^2} \sum_{\alpha} \left\langle \mu^2 S_{e\alpha}^{\mu} \frac{B_0}{B(s)} \right\rangle \\
\langle D_{\mu\mu} \rangle &= \frac{1}{\gamma^2} \frac{1 - \mu_0^2}{\mu_0^2} \left\langle \mu^2 D_{\mu\mu} \frac{B_0}{B(s)} \right\rangle.
\end{aligned} \tag{7.4.15}$$

The wave diffusion coefficient $D_{\mu\mu}$ is found according to the method of Albert (1999).

The averaged distribution function in (7.4.14), $Q(t, \varphi, \mu_0, E)$, is associated with the number of particles with kinetic energy and cosine of equatorial pitch angle between E and $E + dE$, and μ_0 and $\mu_0 + d\mu_0$, whose gyrocenters are distributed on a given magnetic field line on the interval $s_m < s < s_{m'}$ at time t as

$$\frac{dn}{R_0 d\varphi} = 8\pi c m^2 h \gamma \sqrt{\gamma^2 - 1} Q \mu_0 dE d\mu_0. \tag{7.4.16}$$

It should be noticed here that in (7.4.14) we neglected radial diffusion caused by fluctuations in the large-scale magnetospheric magnetic and electric fields. The particle redistribution caused by this diffusion results in additional density diminution and changes in particle loss rate due to the dependence of the loss cone and the diffusion coefficients on L shell. In the case under consideration, these two effects lead to minor corrections. The loss cone broadening for smaller L shells is negligible compared with the rate of particles loss on a fixed L shell. The pitch-angle diffusion coefficients, even for times longer than those simulated here, are still not changed due to radial diffusion. It can be shown that the density reduction is determined primarily by the difference of the beam particles' azimuthal drift velocities until the time when the beam is homogeneously spread around the L shell. The role of the radial diffusion on density reduction as well as loss cone broadening for times greater than this drift spreading is estimated below for the chosen beam parameters.

7.4.2.2 Scenario of Injection and Model Parameters

The configuration of the injected relativistic electron beam is the same as that discussed by Khazanov et al. (1999b). It assumes an upward injection from a 700-km altitude in the ionosphere. One notable difference from the Khazanov et al. (1999b) setup is the inclusion of an initial condition in azimuth. An initial Gaussian spread in MLT of 0.25 h about local noon is assumed to provide the numerical stability by avoiding sharp gradients in the variable of greatest advective drift. It should be noted, however, that the size of the initial MLT spread only leads to small changes in the process of redistribution of the relativistic electrons along its L shell and does not change the integral content of the particles in the geomagnetic trap and their energy interplay in velocity space.

The velocity space initial condition is a Gaussian in energy applied uniformly across the pitch-angle range. The Gaussian energy spectrum is centered on 5 MeV with a 0.5-MeV width. The pitch-angle range of the injection is an isotropic distribution from 40° to 90° at the 700-km injection altitude. This injection range represents a typical satellite-based LINAC beam pulse in the upper ionosphere. Note that the pitch-angle range extends well inside of the loss cone (taken to be the pitch angle that mirrors at 200-km altitude). Because the numerical model is bounce-averaged, the initial condition is taken after one bounce, so the particles injected into the loss cone are not even included in the simulation. Thus, the equatorial pitch-angle range of nonzero initial value is a cone bounded at the lower end by the pitch angle that mirrors in the conjugate ionospheres at 200 km and at the upper end by the pitch angle that mirrors at 700-km altitude. These pitch-angle ranges for $L = 2, 3$, and 4 are shown in Fig. 7.27, and the exact values are listed in Table 7.6. The top panel in Fig. 7.27 shows both the equatorial plane and 700-km altitude pitch-angle ranges of the initial condition. The extended regions enclosed by dotted lines indicate the rest of the pitch-angle injection range, but these are the pitch angles that map directly to the lower atmosphere and empty their

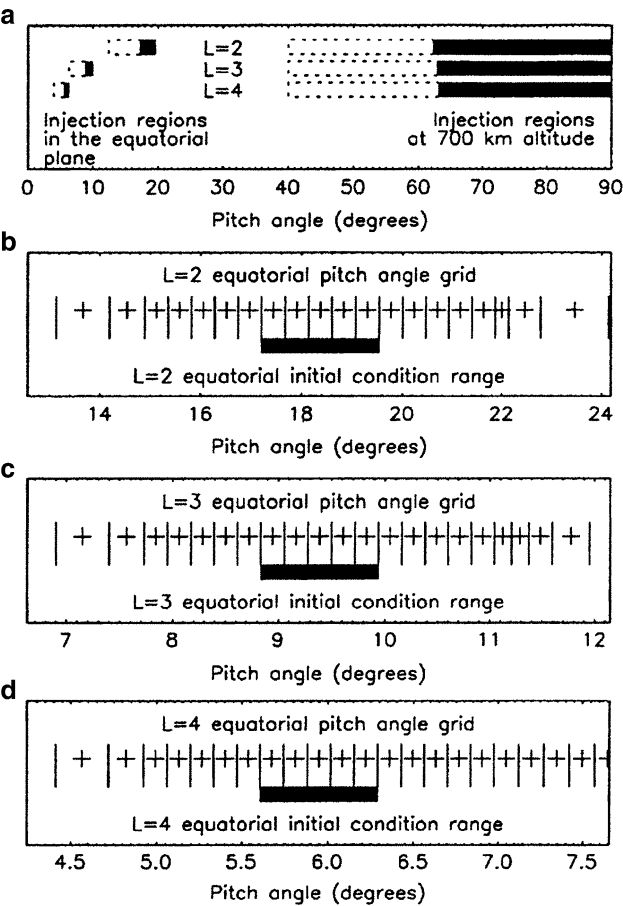


Fig. 7.27 Nonzero initial condition pitch-angle regions at $L = 2, 3$, and 4 . (a) The regions for all three L values at the equatorial plane and at 700 km. The *solid blocks* indicate the regions of filled pitch-angle space after one bounce, and the *dotted-line-enclosed regions* are the rest of the injection width that empty after one bounce. (b–d) The initial condition region for each L along with the equatorial pitch-angle grid (*plus signs* are cell centers, *vertical lines* are cell boundaries)

Table 7.6 Mirror point pitch angles at various altitudes			
L	Equatorial pitch angle		Pitch angle at 700 km
	$\alpha_m(200 \text{ km})$	$\alpha_m(700 \text{ km})$	$\alpha_m(200 \text{ km})$
2	17.21°	19.54°	62.24°
3	8.83°	9.94°	62.85°
4	5.61°	6.29°	63.09°

contents after one bounce. The lower three panels show these initial nonzero ranges along with the equatorial pitch-angle grid in its vicinity at each L value (discussed below).

To examine the evolution of the beam under various geophysical conditions, simulations were conducted at $L = 2, 3$, and 4 ($41.8^\circ, 52.5^\circ$, and 58.2° magnetic latitude at 700 km). These choices span the mid-latitude range where wave activity is significant and where anthropogenic injections of relativistic electrons is most common. Magnetic field related parameters for these L values are listed in Table 7.7. These values will be important when discussing the variation of the wave interaction for different L shells. For reference, Table 7.8 lists the bounce periods for energies and pitch angles within the initial condition domain. These values are also used in the calculation for the atmospheric loss term in (7.4.14). Table 7.9 lists the drift periods for the same energies and pitch angles. It is useful to see how the drift period varies as a function of energy (fast variation), pitch angle (slow variation), and L (fast variation). These differences will be evident in the numerical results as a systematic elongation of the beam as higher energies outpace lower energies and larger pitch angles outpace smaller ones. This is further discussed in Sect. 7.5.4.

The collisional operators on the right-hand side of (7.4.14) require information about the neutral atmosphere and thermal plasma. For these background quantities, the standard models of MSIS-90 (Hedin 1991) and IRI-90 (Bilitza 1990) are used at spring equinox of 1991. This represents a solar maximum condition when collisional losses are expected to be greatest. The equatorial plane thermal plasma densities are taken from a quiet time result from the plasmaspheric model of Rasmussen et al. (1993), assuming no O^+ beyond 1,000-km altitude (only e^- and H^+).

Table 7.7 Magnetic field-related parameters

L	B_0 (G)	$B(200\text{ km})$ (G)	$\tau_{g,0}$ (s)
2	4.00×10^{-2}	0.457	8.93×10^{-6}
3	1.19×10^{-2}	0.503	3.00×10^{-5}
4	5.00×10^{-3}	0.524	7.14×10^{-5}

Table 7.8 Bounce periods in the injection ranges

$E/E_{eq,PA}$ (MeV)	$L = 2$			$L = 3$			$L = 4$		
	17.44°	18.37°	19.30°	8.94°	9.39°	9.83°	5.68°	5.95°	6.22°
4.0	0.1914	0.1897	0.1881	0.3122	0.3108	0.3093	0.4314	0.4301	0.4288
5.0	0.1910	0.1893	0.1877	0.3115	0.3101	0.3087	0.4305	0.4292	0.4278
6.0	0.1907	0.1889	0.1875	0.3111	0.3097	0.3083	0.4300	0.4286	0.4273

Periods are in seconds

Table 7.9 Drift periods in the injection ranges

$E/E_{eq,PA}$ (MeV)	$L = 2$			$L = 3$			$L = 4$		
	17.44°	18.37°	19.30°	8.94°	9.39°	9.83°	5.68°	5.95°	6.22°
4.0	735.9	731.7	727.5	521.3	519.5	517.6	402.5	401.4	400.4
5.0	599.8	596.3	592.9	424.9	423.4	421.9	328.0	327.2	326.3
6.0	506.5	503.5	500.6	358.8	357.5	356.2	277.0	276.2	275.5

Periods are in seconds

Of importance for these calculations is the total number of atoms of each species with which the beam particles will collide. Therefore column densities above 200 km, defined as

$$N = \int_{s_{200}}^{s_{top}} \frac{n(s)B_{s_{200}}}{B(s)} ds$$

are listed in Table 7.10 for the neutral and charged species at various spatial points. The first column of Table 7.10 gives the column density of oxygen atoms and is thus a count equal to the line integral of the atomic oxygen density plus twice the molecular oxygen density. A similar description is true for other columns. Note that a large fraction of the particles contributing to these values are concentrated close to 200 km. Also listed in Table 7.10 are the corresponding equatorial electron densities.

Four wave types are included in the wave–particle diffusion coefficient calculation: plasmaspheric hiss; lightning-generated whistlers; and two bands of VLF transmitter signals. All are right-hand polarized whistler-mode waves. The chosen characteristics of these waves are the average values given by Abel and Thorne (1998), as summarized in Table 7.11. In Table 7.11 (and elsewhere) the subscripts m and w refer to the mean value and spread of the distributions, respectively. The Gaussian distributions in both frequency and propagation angle θ have lower and upper cutoffs of $\zeta_{LC} = \zeta_m - \zeta_w$ and $\zeta_{UC} = \zeta_m + 2\zeta_w$, respectively, where $\zeta = f$ or x ($x = \tan \theta$). The wave intensity for the VLF signals are a complicated function of L , with a peak near $L = 1.8$, a lesser peak near $L = 3$, and sharp drop-offs to zero below $L = 1.3$ and above $L = 3$. The occurrence rate is a measure of how often a particle would see a wave of this type and is a scaling factor for the magnitude of the diffusion coefficient. The occurrence rate used for hiss was 10% for $L = 2$ and 50% for $L = 3$ and 4. Because the waves are not self-consistently calculated and a particular formalism has been used, there are additional constraints on the wave parameters,

$$\begin{aligned} \omega < \Omega_e, \quad \frac{\omega_{pe}^2}{\Omega_e^2} > \frac{\omega}{\Omega_e}, \quad |\omega - \Omega_e| > 10|k_{||}(\omega)|v_{Te} \\ 1 - \frac{\omega^2}{\Omega_p\Omega_e} - \frac{\sin^2\theta}{2} + \left[\frac{\sin^4\theta}{4} + \frac{\omega^2}{\Omega_p^2} \left(1 - \frac{m_e}{m_p} \right)^2 \cos^2\theta \right] > 0, \end{aligned} \quad (7.4.17)$$

where $v_{Te} = (T_e/m_e)^{1/2}$ is the assumed thermal speed of the core electrons. The first two conditions in (7.4.17) are satisfied everywhere, whereas the second two conditions are satisfied everywhere except near the equator at larger L values for higher frequencies. When (7.4.17) is violated, the wave is assumed not to be present and therefore no interaction occurs.

These background particle and wave characteristics are used to calculate pitch-angle scattering and energy loss coefficients for use in (7.4.14). A representative

Table 7.10 Background particle parameters

L	MLT	N_O (cm ⁻²)	N_N (cm ⁻²)	N_H (cm ⁻²)	N_{e^-} (cm ⁻²)	N_{O^+} (cm ⁻²)	N_{H^+} (cm ⁻²)	n_{eq} (cm ⁻³)
2	0000	7.08×10^{15}	5.14×10^{15}	1.07×10^{14}	6.29×10^{12}	2.27×10^{12}	3.85×10^{12}	1.81×10^3
2	1200	9.26×10^{15}	7.10×10^{15}	6.06×10^{13}	1.33×10^{13}	7.19×10^{12}	5.20×10^{12}	2.16×10^3
3	0000	7.26×10^{15}	6.26×10^{15}	9.41×10^{13}	5.39×10^{12}	1.49×10^{12}	3.79×10^{12}	9.02×10^2
3	1200	9.05×10^{15}	7.75×10^{15}	5.67×10^{13}	1.15×10^{13}	5.76×10^{12}	5.11×10^{12}	7.62×10^2
4	0000	7.32×10^{15}	6.91×10^{15}	8.72×10^{13}	3.58×10^{12}	1.17×10^{12}	2.32×10^{12}	1.80×10^2
4	1200	8.87×10^{15}	8.10×10^{15}	5.63×10^{13}	9.38×10^{12}	5.00×10^{12}	3.81×10^{12}	1.80×10^2

Table 7.11 Wave parameters

	Plasmaspheric hiss	Lightning whistlers	VLF1 signal	VLF2 signal
f_m (Hz)	550	4,500	1.71×10^4	2.23×10^4
f_w (Hz)	300	2,000	50	50
θ_m (°)	45	45	45	45
θ_w (°)	22.5	22.5	22.5	22.5
B_w (pT)	10	10	[0,18]	[0,19]
Occurrence (%)	[10,50]	3	2.4	2.4

From Abel and Thorne (1998)

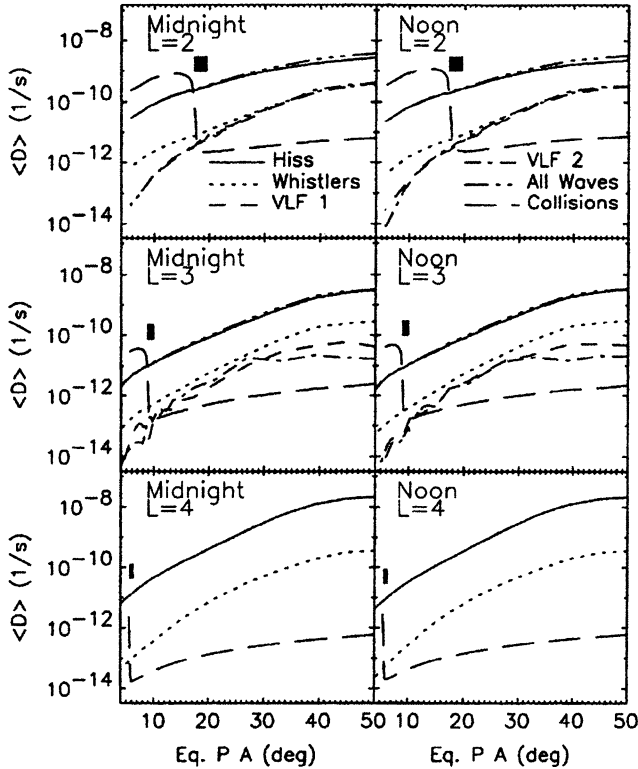


Fig. 7.28 Magnitudes of the bounce-averaged pitch-angle diffusion coefficients in (7.4.15) versus equatorial pitch angle at $L = 2, 3$, and 4 and at MLT = midnight and noon for 5-MeV electrons interacting with hiss (solid line), lightning whistlers (dotted line), the VLF1 band (dashed line), the VLF2 band (dash-dot line), all waves (dash-dot-dot line), and particles (long dashed line). The small black boxes indicate the pitch-angle injection range

sample of the diffusion coefficients is shown in Fig. 7.28 at several spatial locations. The scattering rates from (7.4.15) for each type of wave as well as that for particle collisions are presented, as well as the initial condition pitch-angle range for reference. It is clear that collisional scattering is only effective for pitch angles

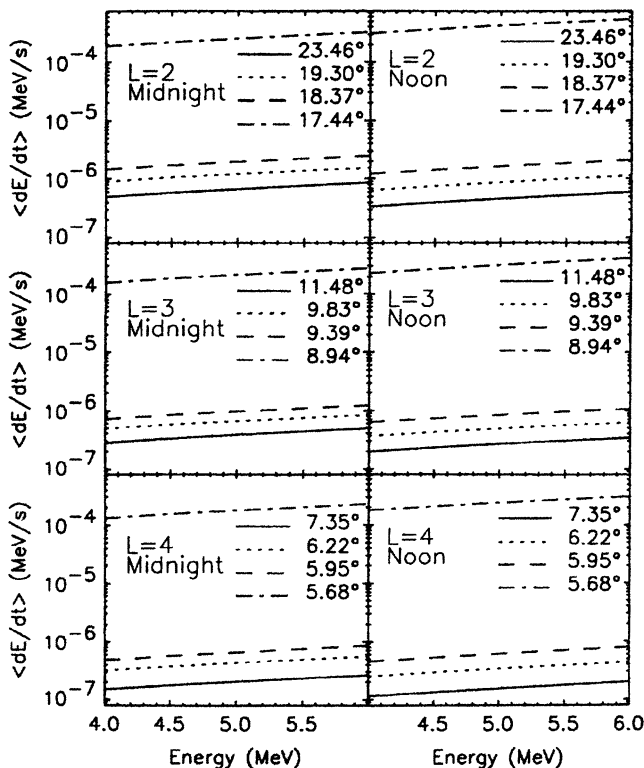


Fig. 7.29 Magnitudes of the bounce-averaged energy loss coefficients in (7.4.15) for several equatorial pitch angles at $L = 2, 3$, and 4 and at MLT = noon and midnight

that reach the atmosphere, with this coefficient rising by two orders of magnitude over its plasmaspheric value. The wave coefficients are dominated everywhere by the interaction with hiss, and drop significantly at lower pitch angles. However, the scattering rate at the inner edge of the injection range is roughly equal for collisions and waves at $L = 2$ and 3 , and they are within a factor of 2 at $L = 4$.

A sample of the collisional energy loss rates is given in Fig. 7.29 at several spatial locations and several pitch angles near the injection regions. Again, note the dramatic increase (of $2+$ orders of magnitude) in loss rate for pitch angles mirroring at 200 km. The dashed line is at the center of the initial injection range; yet the loss rate is minor compared with that at the loss cone edge. This is because there is such an abundance of scattering targets right near the base of the simulation domain. Figures 7.28 and 7.29 are discussed in more detail below.

7.4.2.3 Numerical Implementation

The numerical scheme used to solve (7.4.14) is a combination of advection schemes and diffusion techniques to obtain a second-order accurate result. It is the same

basic technique used in inner magnetospheric modeling studies of photoelectrons (Khazanov et al. 1996, 1998), plasma sheet electrons (Liemohn et al. 1998; Khazanov et al. 1998), and ring current ions (Fok et al. 1993). The method of fractional steps (Yanenko 1971) is used to separate (7.4.14) into a series of equations, setting each operator equal to the time derivative operator. The azimuthal drift and energy loss terms are solved using a high-resolution method that combines the second-order Lax–Wendroff scheme with the first-order upwind scheme via a superbee flux limiter (LeVeque 1992). In this way, the limiter chooses the second-order scheme when the function is smooth, gradually converting to the first-order scheme near sharp gradients. Thus it uses the numerical scheme most suited to the solution of the problem. A second-order accurate Crank–Nicolson scheme is used for the solution of the diffusion operators (Anderson et al. 1984). Each of these equations is solved once, then again in reverse order to reduce systematic error and maintain second-order accuracy.

The numerical grid is chosen to resolve the beam structure yet minimize the calculation requirements. The energy grid is divided into nine cells 0.25-MeV wide, the lowest one centered on 4 MeV and the highest one centered on 6 MeV. The pitch-angle grid is L shell dependent, and the grids near the beam injection ranges are shown in Fig. 7.27. Five grid cells exactly span the injection region, with cell boundaries at the two edges of the injection range. Grid cells to the left of the injection are in the loss cone and will be rapidly depleted during the simulation through atmospheric precipitation, whereas the cells to the right of the injection are in the trapped zone and mirror above 700 km. The MLT step size is chosen to be 0.1 MLT hours wide (240 cells around the Earth). As a result of this MLT grid choice and the values of the drift velocities (Table 7.9), Δt is chosen to be 1.5, 1, and 0.75 s for $L = 2, 3$, and 4, respectively, in order to maintain the numerical stability of the calculation.

To demonstrate the accuracy of this numerical scheme, a solution is calculated for transport only (i.e., a single advection operator) with the given initial condition of a 15 min spread Gaussian distribution. Shown in Fig. 7.30 are the MLT distributions after 0, 1, and 2 drift periods for the chosen energies and pitch angles (τ_d given in Table 7.9). After an initial reconfiguration of the Gaussian distribution into a distribution with a flatter, broader maximum, very little change occurs due to the numerical scheme. Note that this is the fastest advection in the calculation, moving at nearly one grid cell per time step; energy loss is several orders of magnitude slower, as seen in Fig. 7.29, and thus far less numerical dispersion will appear for that variable. Also note that the total particle count (integrating over all phase space) remains the same to six significant digits during these test simulations (discussed further in the next section).

In order to examine the global evolution of an injected relativistic electron beam in the inner magnetosphere, simulations were conducted using the above-mentioned initial and background conditions for $L = 2, 3$, and 4. Equation (7.4.14) was solved for the bounce and gyration averaged distribution function of the relativistic electrons as a function of time, azimuth (MLT), energy, and equatorial pitch angle. Because the kinetic equation is linear, the results presented below are given in normalized units and can be scaled to match any beam injection intensity.

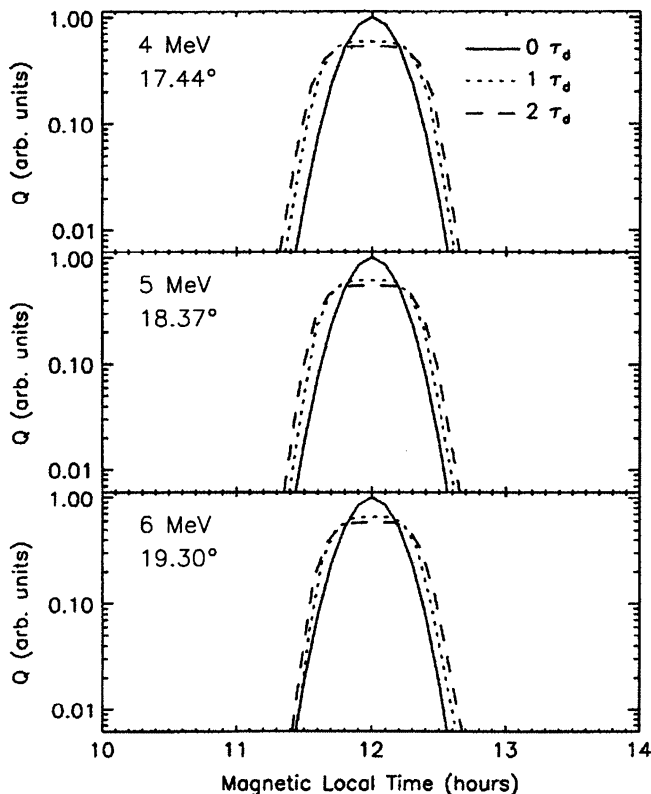


Fig. 7.30 Test of the accuracy of the numerical advection scheme (no collisions included, azimuthal drift only). Plotted in each frame is the initial distribution (*solid line*), the calculated distribution after one drift period (*dotted line*), and after two periods (*dashed line*)

7.4.2.4 Beam Distribution Function

Figure 7.31 shows the distribution function versus energy and MLT for the pitch-angle cell at the midpoint of the initial condition region. These results are for simulations with all processes included (drift, collisions, and wave interactions). The contours are spaced every half order of magnitude, and the normalization is scaled to the maximum value at $t = 0$. As the simulation progresses, the beam splits into a series of packets. This is a numerical effect from choosing only nine energy grid cells. The straight dotted line through the packets marks the drift pattern at each time (shown on the first two rows only). With infinitely many energy grid cells, the result at $t > 0$ would look similar to the result at $t = 0$ but tilted along this dotted line (and accordingly spread from collisions). After 20 min (top right panel, 2 drift periods at 5 MeV), the 6 MeV electrons are about to lap the 4 MeV electrons, and a banded structure appears in the energy spectra. That is, the spectra in Fig. 7.31 develops alternating strips of high and low intensity, resembling horizontal bands

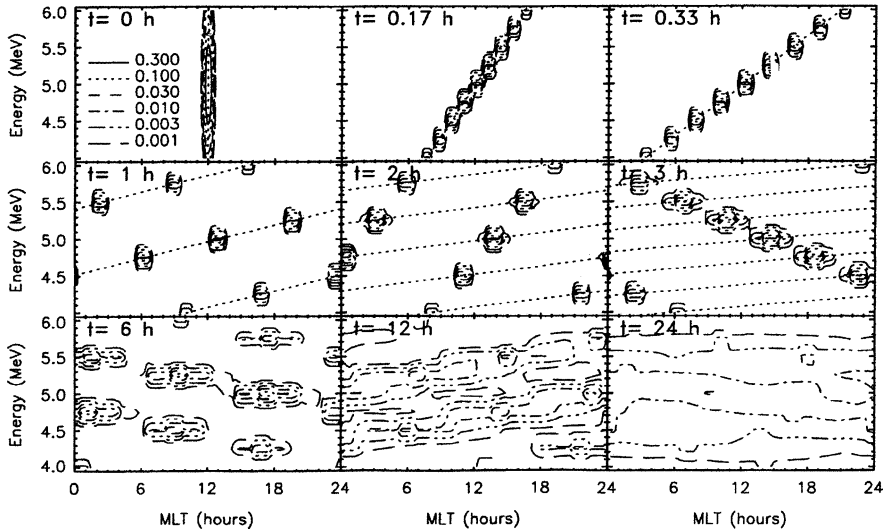


Fig. 7.31 The beam distribution as a function of energy and MLT for 18.4° and $L = 2$ at various times

similar to those seen by Burke et al. (1995) in the CRRES electron measurements in the kiloelectron volt energy range (see also, Liemohn et al. 1998). Although the simulation results still have a beaded-string appearance along the dotted line, the reality (replicated by interpolating between the beads along the dotted line) is a multiply spiked energy spectra at all local times (banded shell). After a few hours, the overlap is severe. The packets are also spreading due to collisions and scattering. For example, the packets at 6 h overlap to such a degree that the bands in energy are no longer discrete (upon interpolating them along the dotted lines). As the simulation progresses, the packets eventually disappear (despite the coarse energy grid) and the beam becomes (numerically and physically) an energetically smooth shell of relativistic electrons at the L value of injection. The characteristic time of this process strongly depends on the L shell, as seen in Table 7.9, with injections at larger L changing into an unbanded shell proportionally faster.

Figure 7.32 shows the pitch angle versus MLT distributions at 5 MeV for the same simulation as in Fig. 7.31. These results are also normalized to the peak value of the initial distribution ($t = 0$). The initial condition shows the narrow Gaussian in MLT and the uniform distribution across the initial condition pitch-angle range. Because the drift differential across the pitch-angle injection range is much less than that for the energy range, the beam numerically does not split into packets, and appears as a tilted version of the initial distribution. In addition, the pitch-angle grid has many more points (for numerical accuracy in the injection range). An interesting feature develops in addition to this tilting effect: a leading population forms right at the edge of the 200-km loss cone that lengthens, broadens, and intensifies with time. This phenomenon arises due to collisional energy losses cascading

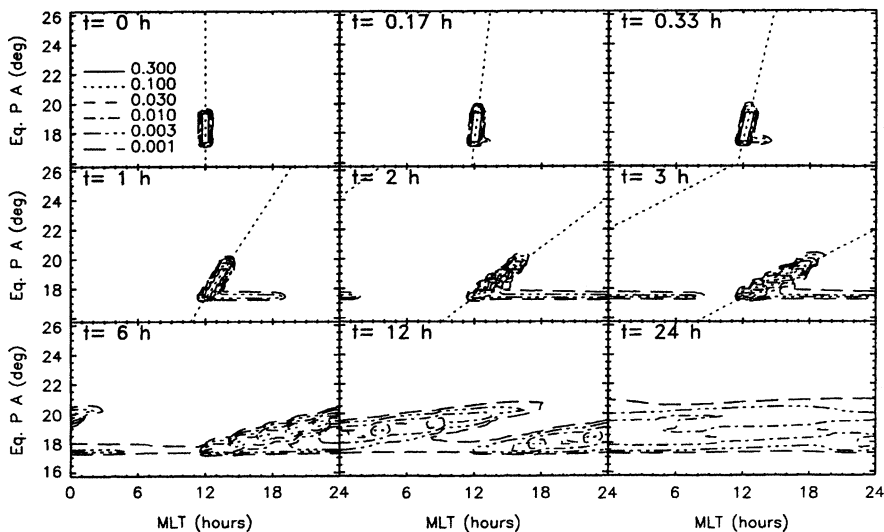


Fig. 7.32 The beam distribution as a function of equatorial pitch angle and MLT for 5 MeV and $L = 2$ at various times

electrons from higher energies. This degradation is strongest right near the edge of the loss cone, where the electrons mirror deep in the atmosphere and encounter a large number of scattering targets. This was seen in Fig. 7.29 as the energy loss rate increased by two orders of magnitude across the beam injection pitch-angle range. As the higher-energy electrons continue to outpace the 5-MeV electrons shown, they continue to deposit decayed electrons at local times farther and farther ahead of the beam location. At larger L values, collisional losses are weaker and thus this leading edge population is less pronounced. The beam, meanwhile, is diffusing in pitch angle (and thus in MLT via the drift differential), eventually becoming a broad band of electrons at all MLT. As the solution shows, this band has more than doubled in size after 1 day from the initial injection width. Because of the smallness of the MLT drift differential for pitch angles, however, significant structure still exists in this band, with an oblique ridge as a distinctive remnant of the original injection configuration. The pitch-angle distributions for the simulations at $L = 3$ and 4 are very similar to these results because the drift differentials are similar to those at $L = 2$.

Because of the coarseness of the contour levels, it is difficult to distinguish individual features in the distribution function from the plots described above. Therefore Figs. 7.33 and 7.34 are line plot slices through the panels of Figs. 7.31 and 7.32 for $L = 2, 3$, and 4. All of these plots are normalized to the distribution function maximum value at $t = 0$. Figure 7.33 shows energy spectra at the MLT of the 5 MeV peak (different at each t and L). After 1 h, a series of spikes are present across the energy range. This is seen in Fig. 7.31 by examining a vertical slice through the $t = 1$ h result at MLT = 12.9 h. The 5-MeV packet is the middle band,

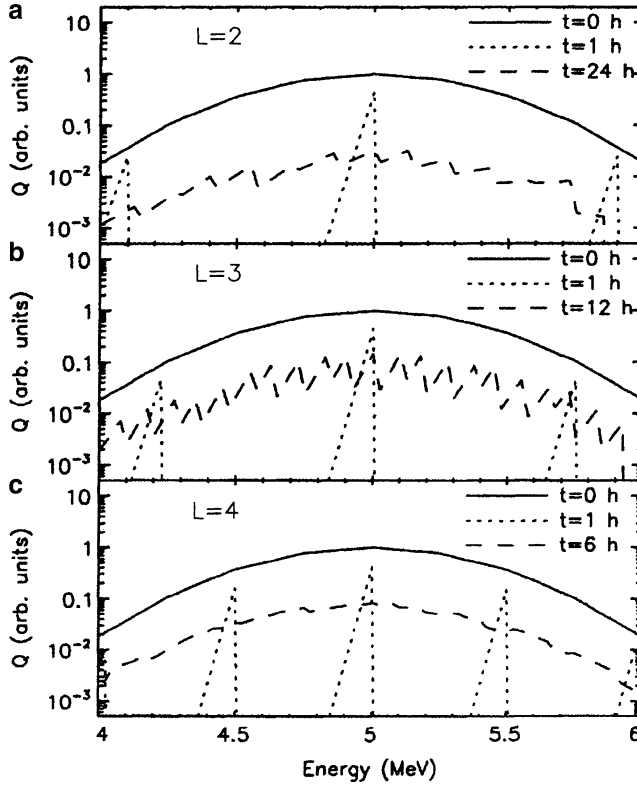


Fig. 7.33 Energy spectra at the beam peak for (a) $L = 2$, (b) $L = 3$, and (c) $L = 4$ at various times after injection. These spectra are from the central pitch angle of the initial injection at the MLT of the peak (different for each L and t value). Note that the *dashed line* is at a different time for each panel

but there are also high- and low-energy bands because the 6-MeV electrons have twice lapped the 4-MeV electrons at this time. In these spiked (banded) energy spectra, the collisional decay of these electrons to lower energies is noticeable, because the peaks have a tail in that direction. As time progresses, these peaks compress together and more peaks are added as the lapping due to differential drift continues. After 24 h, the 4- to 6-MeV beam has lapped itself 52 times (therefore there are 53 peaks) and the bands are no longer distinguishable. At this point, the energy spectrum has regained its original shape, only now it is spread over all MLT instead of being confined to a narrow MLT range. In the late-time spectra, a difference is evident between the intensities at 6 and 4 MeV. This difference is due more to the collisional losses of beam particles as they cascade to lower energies and less due to the energy dependence of the diffusion coefficients. The $L = 3$ results are similar to the $L = 2$ results. The $t = 1$ h plot has three spikes, but they are closer together than the $L = 2$ spikes at this time. This is because the faster drift period at this L value (see Table 7.4) causes the higher energies to lap the lower

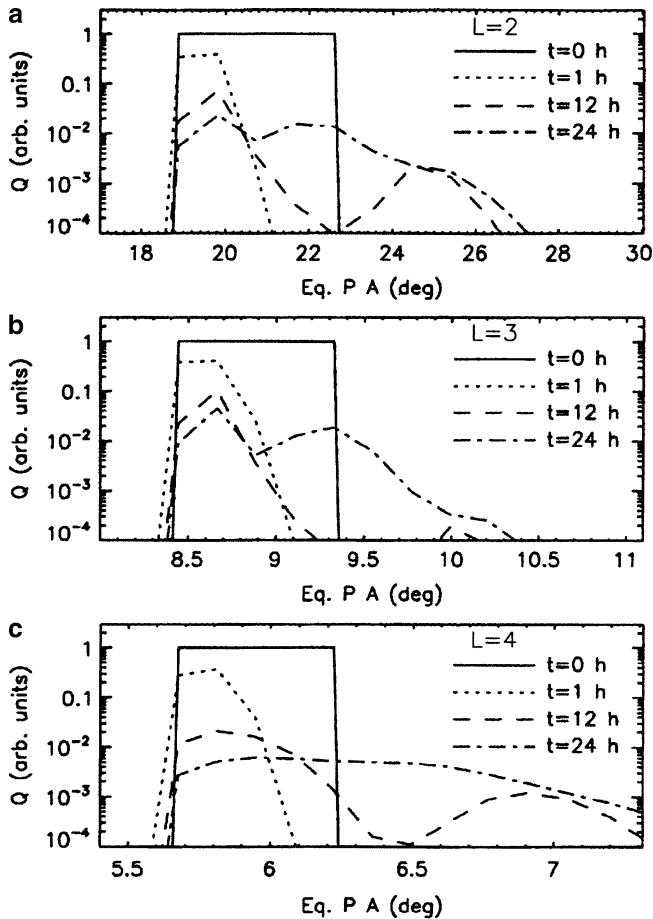


Fig. 7.34 Pitch-angle distributions at the beam peak for (a) $L = 2$, (b) $L = 3$, and (c) $L = 4$ at various times after injection. These spectra are for $E = 5$ MeV at the MLT of the peak (different for each L and t value). Note that the pitch-angle scales are different for each panel

energies more quickly than lower L values. At $t = 12$ h, the beam has nearly regained its original Gaussian energy distribution, only now it is a shell around the Earth. At $L = 4$, this nonspiked shell distribution is seen at $t = 6$ h, at which time, the beam has lapped itself 24 times already.

Pitch-angle distributions for $L = 2, 3$, and 4 are presented in Fig. 7.34. These distributions were chosen for 5 MeV at the MLT of the peak value (again, different for each t and L). Note that the pitch-angle scales are different for each plot. The initial condition is a block, but it does not remain so for long. After 1 h, the larger pitch angles have moved ahead and are therefore not included in the spectra. By $t = 12$ h, however, the smaller pitch angles have been depleted through scattering into the loss cone and the peak of the beam is shifting to larger pitch angles.

The beam is also lapping itself (in pitch angle) at this time, and two peaks are clearly seen in the pitch-angle distribution. Note that this overlap took less than 1 h in the energy spectra, but the drift differential is far less in pitch angle, so it took much longer. However, it does eventually occur, and after 24 h, much of the structure is smoothed away at the location of the beam maximum. One final feature to note is that the diffusion into the trapped zone is clearly seen in these spectra. Because magnetospheric loss processes are very slow, particles in the trapped zone will persist for days and slowly spread throughout the trapped zone. Also note that the loss cone at larger L is more populated than the loss cone at smaller L . There are two reasons for this: (1) the bounce period increases with L so depletion is slower and (2) the source region is smaller so it is easier to scatter into the loss cone.

7.4.2.5 The Integral Content

With so much differential spreading over MLT, it is difficult to extract the magnitude of the beam loss as a function of time from plots of the distribution function. A useful quantity to examine therefore is the total number of particles in the beam. This quantity is found by integrating the distribution function over all phase space at a given time. Figure 7.35 presents this quantity for various simulations. Figure 7.35a shows results at $L = 2$ for azimuthal transport (drift) only, drift plus collisions, drift plus wave interactions, and all processes included. All of the simulations, however, include atmospheric precipitation for those particles reaching the

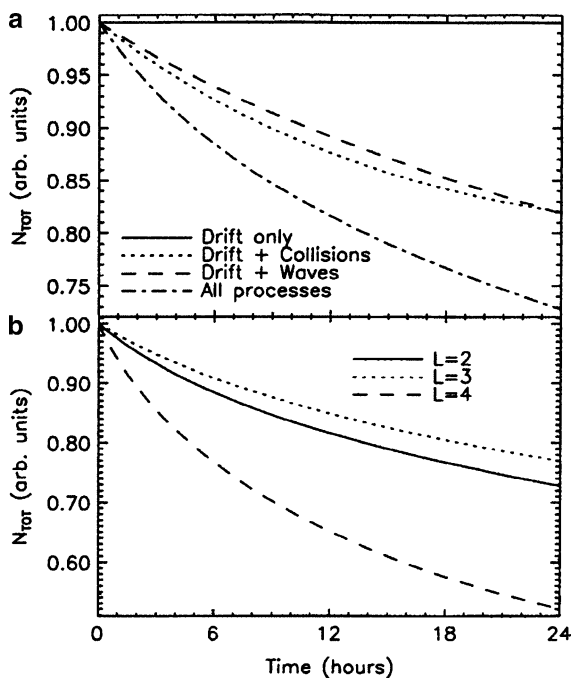


Fig. 7.35 Evolution of the normalized total number of particles of the beam at $L = 2$ (a) for several simulations with various processes included and (b) at several L values with all processes included

loss cone. First, the simulation for drift only solidly maintains its original number of particles. This is encouraging, because the scattering and loss operators were not omitted, but rather calculated with coefficients of zero (meaning numerical errors could still have surfaced but did not). This indicates that any artificial (numerically generated) sources and losses are totally insignificant in the computed results. This is also seen in energy–MLT and pitch angle–MLT plots similar to Figs. 7.31 and 7.32 for this simulation (not displayed, however), which show the simulation maintaining the original functional form of the beam, just tilted due to differential drift. Second, Fig. 7.35a shows that collisions are more important at this L shell than waves for removing electrons from the beam during the first 24 h. Waves are a significant process, though, and the calculated total loss rates cannot be achieved without this mechanism. Third, a definite reduction in the decay rate of the total beam content with time is evident. The particle count drops quickly during the first few hours, then slower at later times. This is because the initial rate is caused by scattering those particles that are initially right at the loss cone edge, whereas the later rate is a function of the scattering efficiency at larger pitch angles (compare with Fig. 7.28). This is especially true comparing the results with and without wave interactions (dotted and dashed lines). Collisions are more efficient than waves at removing particles near the loss cone edge, and thus this particle count (dotted line) is initially lower. However, waves are better at scattering the electrons from deeper in the trapped zone to the loss cone, and so this particle count (dashed line) eventually drops below the collisional simulation, right near 24 h. Figure 7.36 shows these two simulations over a 2-day simulation interval. It is clear that wave scattering dominates the late-stage loss rate of beam particles after the initial loss of those injected near the edge of the loss cone. It should be noted that while these simulations were carried out for 2 days to illustrate this switching of the dominant mechanism, the other runs will not be shown to this time because neglected processes, such as radial spreading, become important at these longer timescales and should be taken into account for an accurate calculation of the real beam evolution beyond 1 day.

Figure 7.35b shows the total particle count for the simulations at $L = 2, 3$, and 4 (all processes included). Here it is interesting to note the variation in the loss rate

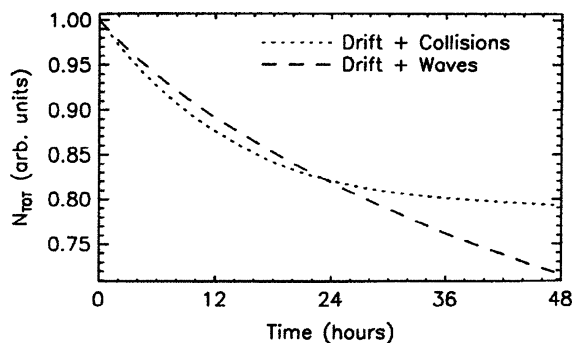


Fig. 7.36 Evolution of the normalized total number of particles of the beam at $L = 2$ for simulations with collisions but without waves (dotted line) and with waves but without collisions (dashed line)

as a function of L . The total particle count drops to 73% of its original value after 24 h at $L = 2$. For $L = 3$, it only drops to 77%. At $L = 4$, however, it drops to 52% of its original particle count after 24 h. The decay rates at $t = 24$ h for the three simulations are 0.82, 0.70, and 1.5% per hour (percent of remaining particles, that is), and they are all slowly decreasing with time. Compare these late-stage loss rates to the loss rates 1 h after injection: 2.4, 1.8, and 7.9% per hour for the three L values, respectively. These results can be explained by reexamining Figs. 7.28 and 7.29. The coefficients in those plots can be roughly converted into timescales of particle energy loss or scattering through an E or μ_0 interval,

$$\tau_E = \frac{\Delta E}{\langle dE/dt \rangle}, \quad \tau_{\mu_0} = \frac{(\Delta\mu_0)^2}{\langle D \rangle}, \quad (7.4.18)$$

where ΔE and $\Delta\mu_0$ are the E and μ_0 extents the particles must move through to be removed from the system (here $\Delta E = 1$ MeV and $\Delta\mu_0 =$ width from given cell center to the edge of the loss cone). These loss timescale estimates are shown in Fig. 7.37 for each loss process. Right at the loss cone edge, it should take ~ 0.8 , 1, and 1.4 h for an electron to lose 1 MeV of energy for $L = 2, 3$, and 4, respectively. Note, however, that it takes roughly two orders of magnitude longer at the next pitch-angle cell because of the reduction in collisional targets (these electrons mirror higher in the atmosphere). Using the scattering coefficients from Fig. 7.28 and the pitch-angle grid from Fig. 7.27, the scattering timescale at the loss cone edge is ~ 4.5 , 6.5, and 1.5 h for $L = 2, 3$, and 4, respectively. The dramatic drop for $L = 4$ is because the width of the pitch-angle injection shrinks with L faster than the diffusion coefficient. This is similar to the result of Khazanov et al. (1992), who found that the timescale for transferring particles to the trapped zone from a source cone distribution decreases with the size of the source cone. The scattering loss timescales monotonically increase across the injection range, as expected with the increase in pitch-angle difference to the loss cone edge. Note that these timescales are only estimations at a single energy and pitch angle, and not global decay rates.

Figure 7.35b can now be viewed in terms of the relative magnitudes of these loss rates. The initial loss rate is highly dependent on the scattering rate at the edge of the loss cone, because the initial condition is a sharp gradient next to the precipitation zone. The order of the initial loss rates of the L shells matches our scattering timescale estimations. The late-stage loss rate is caused by a combination of scattering and decay losses and also significantly depends on the timescales away from the loss cone edge. The difference between the calculated loss rates after 1 day for $L = 3$ and $L = 2$ corresponds to the timescales in Fig. 7.37, as both τ values are bigger at the larger L value and thus the beam loses its particles slower. Particles at $L = 4$, however, have a much smaller diffusive loss timescale than the other L values. This clear reduction in τ_{μ_0} outweighs the slight increase in τ_E , resulting in a faster late-stage loss rate at this L value.

It is also interesting to examine a plot of the total particle count as a function of MLT, given in Fig. 7.38. These results are normalized to the peak initial value of

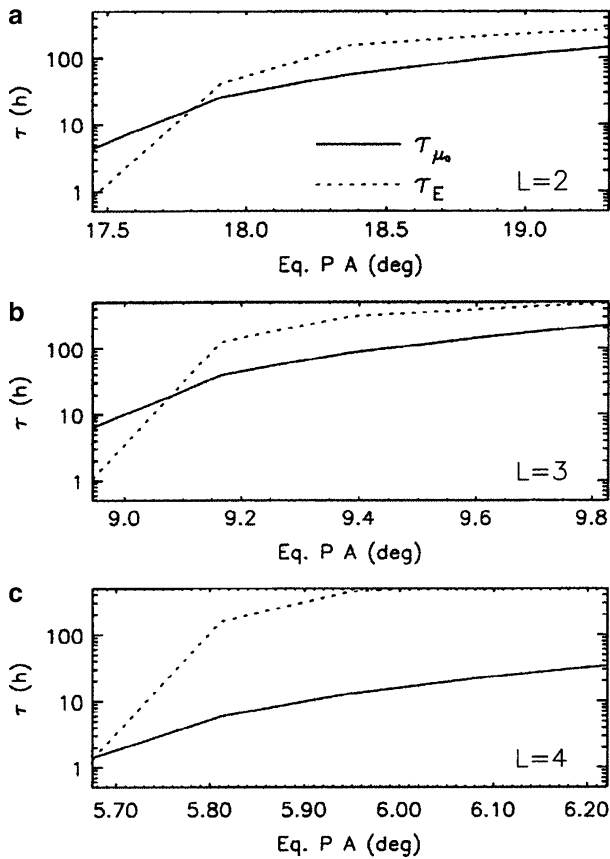
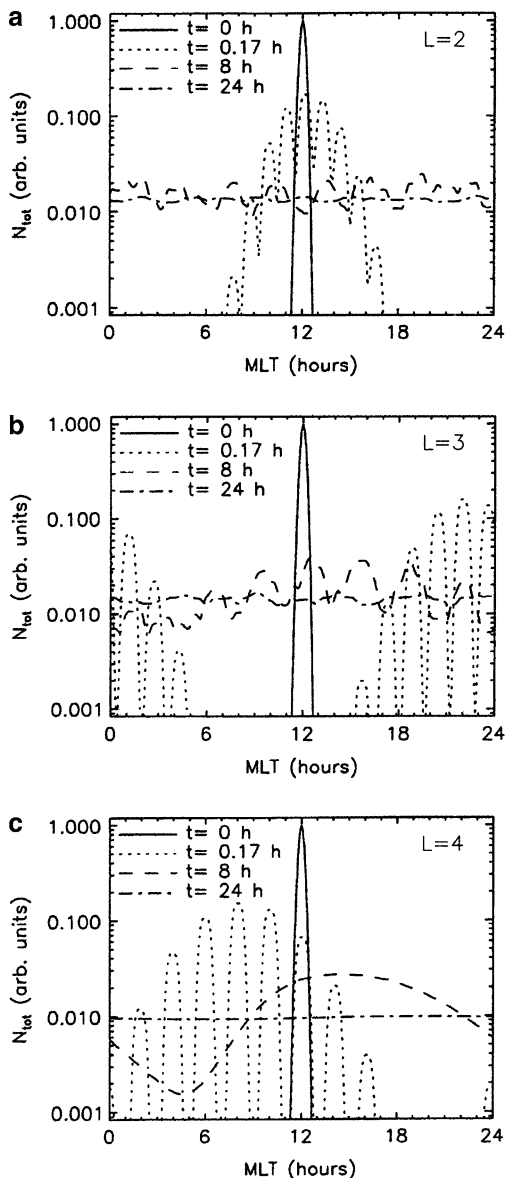


Fig. 7.37 Estimates from (7.4.18) of the loss timescale for a 5-MeV electron for (a) $L = 2$, (b) $L = 3$, and (c) $L = 4$

this integral content, and thus a comparison with Fig. 7.35 must take into account the MLT grid cell width and initial condition functional form. The results after 10 min show the location of the energy packets. Note that these values would be smoothed if a finer energy grid were chosen. However, the results after 8 h are within an order of magnitude across MLT, and the results after 24 h are nearly flat across MLT. This is because the differential drift, combined with the energy loss and pitch-angle diffusion, has smeared the beam into a shell about the Earth. This does not contradict the $t = 24$ h results of Figs. 7.31 and 7.32, which still showed some structure. The shell has structure, but the structure essentially cancels itself out in this integral content value. For example, while the 5-MeV pitch angle versus MLT distribution has a band of high intensity through it, this band stretches through almost all MLT. In addition, other energies will have similar bands offset in MLT from this one (and at different obliqueness), further contributing to the MLT uniformity of the integral content.

Fig. 7.38 Total particle count as a function of MLT at various times for (a) $L = 2$, (b) $L = 3$, and (c) $L = 4$



7.4.2.6 Further Discussion

A few assumptions were used in this section and should be discussed to clarify the conditions for which the model is valid and accurate. A key restriction is related to the length of the beam impulse. Because we are not taking into account the beam-generated waves, the pulse duration should be less than the characteristic time of

the instability with the maximum growth rate. This question was discussed in detail for the problem under consideration in Khazanov et al. (1999b), and it was found that the chosen beam impulse is well within this restriction. There are also several other assumptions about the beam and geophysical configuration. One is that the wave parameters used in the calculation are long-term averages that have been uniformly applied in MLT. Variations in the actual intensities of these waves can be quite large, and local time asymmetries will also have an impact on the results. Additionally, the waves were treated as an externally imposed scattering mechanism, and no self-consistency was attempted in the calculation. It was assumed that the LCBs are constant with MLT and hemisphere, which is not the case in a realistic geomagnetic field configuration. The atmospheres and thermal plasma values were also assumed to be hemispherically symmetric (although they did include MLT dependence). Finally, the only cross-field drift considered in these simulations was the magnetic gradient-curvature drift, which dominates at these energies, and corotation and convection electric fields were omitted. This limits the drift to the azimuthal direction only, making L a constant of motion.

Now let us estimate the role of beam radial spreading, which was omitted in (7.4.14). Our approach is the same as that discussed by Lyons and Williams (1984). For lower L values, the diffusion due to the fluctuations in the geomagnetic field is less than that due to the fluctuations in the magnetospheric electric potential field. Therefore, we make estimations for an electric field of strength $\sim 0.1 \text{ mV m}^{-1}$. The radial diffusion coefficient for the beam parameters specified above is on the order of $10^{-5} R_E^2 \text{ day}^{-1}$. Taking into account that the e-folding time is of the order of 2–3 days, we can compare the role of different effects on the beam parameters on this timescale. The radial spread of the beam particles during this time for $L = 2$ is $\sim 2 \times 10^{-3} R_E$, and therefore does not change the diffusion coefficients and energy loss rates. For particles displaced due to radial diffusion toward smaller L shells, the loss cone size will increase. In our case, the L displacement during the e-folding time is $\sim 10^{-2} R_E$. Such displacement leads correspondingly to a loss cone cosine growth of the magnitude $\sim 10^{-3}$ (Schulz and Lanzerotti 1974). This value represents 7% of the region initially occupied by the trapped part of the beam for $L = 2$. Therefore radial diffusion should be taken into account only in the density reduction calculation. Radial diffusion becomes an important process only after the MLT spreading of the beam is completed. After this spreading, the density reduction is caused by the beam particle losses and radial diffusion, which becomes a significant process. The radial diffusion coefficient differs by less than a factor of 2 over the energy range 4–6 MeV and the related density reduction can be easily estimated.

A final note of discussion is on the comparison of total loss rates determined in this section with those of our previous study (Khazanov et al. 1999b), where two stages of beam decay were discussed: the initial loss of particles after the first pass through the plasmasphere into the conjugate atmosphere; followed by the initial scattering decay rate of the trapped beam particles. A timescale of 265 s was stated for an e-folding of the beam intensity. However, because of the very small time step required to resolve the field-aligned structure of the beam, those simulations were only conducted for the first 10 s after injection, and did not take into account the

cross-field line interplay. As indicated above, the loss rate is initially quite high but then dramatically decreases during the first few hours after injection. In fact, the decay rates for all simulations are still declining after 24 h. Thus the Khazanov et al. (1999a) value is a calculation of the initial decay rate of the beam, while this section is examining a different time regime: the continuation of this initial decay of the trapped population and its gradual decrease with time into a very slow decay rate. It should also be mentioned that because of the injected beams' proximity to the loss cone, these calculated decay rates for the total beam particle count are much faster than radiation belt decay rates, which are typically calculated for scattering from 90° equatorial pitch angle into the loss cone. Here scattering through only a few degrees or less of equatorial pitch angle is necessary to remove a particle from the calculation.

In order to examine the global evolution of an artificially injected relativistic electron beam, the bounce-averaged relativistic kinetic equation was solved for the electron distribution function for various scenarios. It was found that the beam quickly spreads in MLT due to differential drift rates. This, combined with energy loss and pitch-angle diffusion, eventually transforms the beam into a fairly uniform shell around the Earth. It was determined that collisional interactions are initially more important than wave interactions, but eventually drop in significance and the loss rate is dominated by wave scattering. It was also found that the beam total particle loss rate is a complicated function of L and initial condition parameters, and for the chosen conditions, a beam at $L = 4$ drops much faster than an injected beam at $L = 2$ or 3. These loss rates are ~ 0.7 – 1.5% per hour 1 day after injection (as a percent of the remaining beam particles), with the beam total particle counts at 73, 77, and 52% of the initial count at this time for the three L values. This rather strange L dependence of the particle counts was explained by examining the loss mechanisms and injection conditions. The loss of particles is initially 2–5 times faster than the rate after 1 day, but this decay rate decreases throughout the simulation. Also during the first day after injection, the beam transforms from a point source into a uniform shell around the planet. Other interesting features in the distribution function were also examined, such as leading edge development in the pitch angle–MLT plots, banded structures in the energy and pitch-angle spectra, and features of the beam decay.

There are at least four stages in the evolution of an injected beam. The first is the loss of particles after the first interhemispheric traversal, where the particles injected into the loss cone are immediately lost to the lower thermosphere and mesosphere. The next stage is the initial loss of particles injected right next to the loss cone pitch angle via collisional scattering in the upper atmosphere. These first two stages were discussed by Khazanov et al. (1999a). The next stage is the still rapid but now slower loss of particles from upper atmospheric collisions accompanied by differential drift spreading of the beam around the Earth and a slow diffusion into the trapped zone. The fourth stage is the slower loss process of particles from deeper in the injection region, coincident with the transformation of the beam into a continuous and nearly uniform shell around the Earth. These two stages were discussed in this section. Other stages that could exist beyond these

ones include shell broadening by radial diffusion and the approach to an asymptotic decay rate.

Although this section focused on the dynamics of relativistic electrons from an anthropogenic source, the model is fully capable of examining the dynamics of particles from natural sources. This relevance is particularly true for the radiation belts, and a general interpretation of this investigation is the study of the dynamics of radiation belt electrons near the loss cone edge. This model is also well suited for radiation belt modeling because of its ability to resolve magnetic local time effects in the distribution function. The acceleration mechanisms responsible for energizing the source electron population up into the relativistic range is a subject of considerable debate within the space physics community. The location and timing of this source population within the inner magnetosphere during a storm or sub-storm is crucial to their eventual energization. As shown in this section, a spatially localized injection can take many hours, perhaps a day, to transform into a smooth shell of particles around the Earth, and Liemohn et al. (1998) showed that a banded energy structure can persist for many days in an electron population of lower energy. During this developmental time, when the distribution is azimuthally asymmetric, is when the critical processes impart their energy to the electrons. Furthermore, these processes (such as collisional, wave, electric field, and magnetic field interactions) often have large local time dependencies, and an interaction fully resolved local time could reveal influences to the electrons that would not necessarily be observed in an azimuthally averaged interaction. Therefore we think that spatial azimuthal resolution of the relativistic electron distribution function is important, and this model has the ability to perform such a calculation.

7.4.2.7 The Role of Initial Pitch-Angle Distribution

The simulations presented in the previous sections are all for ionospheric injection of the electron beam. As a result, the initial equatorial pitch-angle distribution has a 1° – 2° width that borders the loss cone. For example, the trapped part of the beam injected at the altitude 700 km in the pitch-angle range between 60° and 90° at $L = 2$ occupies pitch angles of 17.5° – 19.6° in the equatorial plane. This range is even smaller for larger L shells (Khazanov et al. 1999b). The entire beam mirrors in the upper atmosphere, where losses from Coulomb collisions dominate the scattering process. In spite of the fact that the bounce-averaged wave diffusion coefficient drops off at small equatorial pitch angles, the inclusion of collisional scattering and loss significantly reduces the lifetime of the beam particles compared with the standard radiation belt lifetime calculation, where wave scattering is more essential.

To illustrate this, several new simulations were conducted using the model of Khazanov et al. (1999a). Figure 7.39 shows the total particle count for three simulations with the initial injection on an $L = 3$ field line, (1) an isotropic injection at the equator; (2) an injection limited to the pitch-angle range of 80° – 90° at the equator; and (3) an injection limited to the pitch-angle range of 9° – 10° at the equator (the case from Khazanov et al. 1999a). As can be seen, the timescales for

beam particle loss are vastly dissimilar when different parts of the trapped zone are filled, and essentially no particle loss occurs in the anisotropic equatorial injection (case 2). The combination of two factors makes the ionospheric injections have much faster losses (as a percentage of total particle number) than the magnetospheric injection of cases 1 and 2. These two factors are a narrow injection region and its proximity to the loss cone.

Figure 7.40 clarifies the role of these factors by showing the evolution of the equatorial pitch-angle distribution for case 2. These distributions are integrated over energy and local time to remove the changes in intensity due to the different drift speeds around the Earth (and therefore show the total scattering rate). Without the energy and local time integrations, the intensity would vary dramatically depending on the chosen location of the presented distribution (see Khazanov et al. 1999a for further explanation). As can be seen, no particles are anywhere near the loss cone, even after 10 days, and thus the particles from such a beam injection would linger in the magnetosphere for timescales similar to those for the radiation belts. Note that there is a slight decrease in the distribution function within the 80° – 90° range, but this deviation from the unity cannot be seen on this scale.

Fig. 7.39 Total beam content results for various injection configurations at $L = 3$, normalized to their initial beam contents. The *solid* curve is for isotropic injection at the equatorial plane, the *dotted* curve is for injection only in the 80° – 90° pitch-angle range at the equatorial plane, and the *dashed* curve is for injection from the upper ionosphere (9° – 10° pitch-angle range at the equatorial plane)

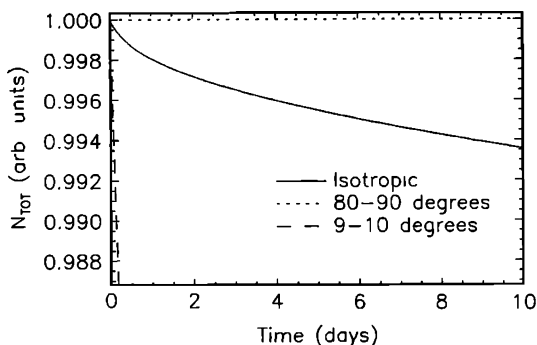
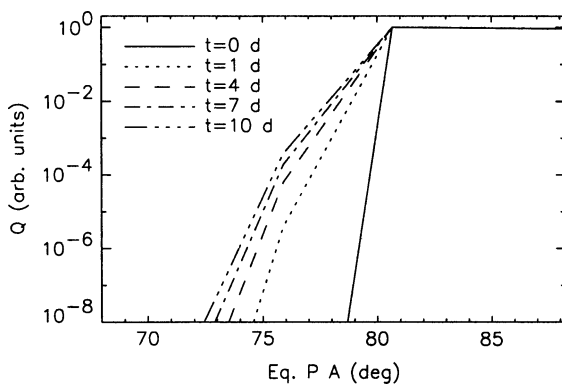


Fig. 7.40 Time development of the pitch-angle distribution for relativistic beam injection in the 80° – 90° pitch-angle range at the equatorial plane, showing the slow propagation of the particles through the trapped zone



Therefore the results of Khazanov et al. (1999a) are consistent with the results from other inner magnetospheric relativistic electron studies and complement the previous studies by focusing on the narrow range of pitch angles nearest the loss cone. In fact, relativistic beam injections and the simulation of such experiments can be used as controlled tracer experiments to investigate the natural radiation environment. The model is well adapted to the analyses of the radiation belt fine structure evolution and the dynamics of various regions of velocity space.

7.5 Magnetospheric Convection Electric Field Dynamics and Storm-Time Particle Energization

7.5.1 *Electric Fields Models*

When an explosive solar flare leads to an interplanetary coronal mass ejection (ICME) directed at the Earth, large disturbances in the geomagnetic environment around the planet ensue (Gosling 1993). One aspect of the resulting magnetic storm, as it is called, is the formation of an intense torus of current around the Earth. This net-westward-flowing current, peaked a few planetary radii above the surface in the equatorial plane, grows for several hours (perhaps a day) during the passage of the ICME. At some point, usually associated with the peak energy input rate (Wu and Lepping 2002), this current reaches its maximum level and begins to decline back to its nominal prestorm level. This decay can be rapid or slow, depending on the dynamics of the particles in the magnetosphere and the ionic composition of the current carriers (Hamilton et al. 1988; Takahashi et al. 1990; Kozyra et al. 1998; Liemohn et al. 1999, 2001).

This so-called ring current is formed by charged particles being injected into the inner magnetospheric region due to enhanced convective forces (e.g., Alfvén and Fälthammar 1963; Lyons and Williams 1984). Because the magnetic field strength monotonically increases and the field line length monotonically decreases as one moves closer to the Earth, this inward convection leads to particle acceleration. This energization results from conservation of both the first and second adiabatic invariants. As the particles enter the dipole-dominated magnetic field near the Earth, they begin to azimuthally drift according to the gradient, curvature, and magnetization forces. The net azimuthal flow from these drift terms is charge dependent, however, with the ions traveling westward and the electrons eastward around the planet. This differential flow results in a westward current, known as the ring current around the Earth.

The formation of the storm-time ring current has been extensively studied over the years. As computational resources increase, bigger and more sophisticated numerical models describing the flow of particles through near-Earth space have been created, allowing for a closer examination of physics involved in this process. Several large-scale ring current codes exist that are presently being used to increase

our understanding of the acceleration of particles in the inner magnetosphere (see the recent reviews by Wolf and Spiro 1997; Chen et al. 1997; Daglis and Kozyra 2002). This includes the identification of partial ring current dominance during the main and early recovery phases, the realization that dayside magnetopause flow out is a major loss process for ring current particles, and a quantitative assessment of the importance of wave–particle interactions on the ion distribution in localized regions (particularly the afternoon sector).

One of the largest uncertainties associated with modeling the injection of the ring current is the description of the large-scale electric field. Created by the interaction of the solar wind magnetic field with the magnetosphere, this field provides the convective force to push the particles into the inner magnetosphere. Its dynamical configuration in this region is still a subject of debate. Until recently, in situ measurements were the only means of diagnosing the magnetospheric particle distribution function. From these particle data, several empirical electric field models have been created, such as that by Nishida (1966) and Brice (1967), Volland (1973) and Stern (1975), and McIlwain (1974, 1986). While satellites such as CRRES have produced statistical compilations of the inner magnetospheric electric field (Rowland and Wygant 1998), the fields are highly variable (up to 6 mV m^{-1}) in the nightside at $2 < L < 6$ for large Kp . Ionospheric observations such as those made by the DMSP satellite (e.g., Burke et al. 1998; Anderson et al. 2001) and ground-based radars (e.g., Foster et al. 1986; Yeh et al. 1991) also show high variability in the subauroral nightside during active times. That said, statistical models are still one of the best options for describing the near-Earth electric fields. They have been used both successfully and unsuccessfully to reproduce the observed features of the storm-time ring current (e.g., Chen et al. 1993, 1999; Kozyra et al. 1998, 2002; Jordanova et al. 1997, 2001; Liemohn et al. 2001, 2002; Khazanov et al. 2003, 2004a). Only recently (from the IMAGE satellite) have global snapshots of the plasma morphology in the inner magnetosphere been obtained with a quality suitable for detailed, quantitative analysis (Burch et al. 2001), and progress in extracting the electric field from these images is slow.

Because the field lines of the inner magnetosphere pass through the mid-latitude ionosphere, the convection patterns here are often useful to help explain the dynamics at the higher altitudes. Using the statistical model of Weimer (1996), Jordanova et al. (2001) showed that this technique produces better comparisons against in situ data than do results using the Volland–Stern electric field model. Another ionospheric convection description is obtained through the assimilative mapping of ionospheric electrodynamics (AMIE) technique (Richmond and Kamide 1988). This approach uses any available ionospheric data for a given time to produce an instantaneous convection pattern. Boonsiriseth et al. (2001) discussed a semiempirical method of mapping these convection patterns out from the ionosphere into the inner magnetosphere, finding that the AMIE model produces small-scale structures in the potential pattern that are not seen in either the Volland–Stern or Weimer models. However, an additional “penetration” electric field (e.g., Ridley and Liemohn 2002) must also be included to match the observed electric field magnitudes on the dusk-side low- L region during storm main phases. Chen et al. (2003) showed that the

AMIE-derived equipotentials concentrate in the evening sector during the main phases of storms, providing a channel for rapid transport of plasma sheet ions into $L \sim 3\text{--}4$ (even $L \sim 2$ for extremely large events). These studies indicate that even though the cross-magnetospheric potential difference is similar for different models, the morphology of the potential pattern plays a crucial role in the formation of the storm-time ring current.

In this section, several electric field models will be used in conjunction with a ring current transport model to examine the differences in the net acceleration to the particles. By examining not only the electric field patterns in the inner magnetosphere but also the resulting ion and electron energy spectra, it is possible to identify the physical mechanisms behind the energization process. In addition, the consequences of these differences in acceleration will be discussed, revealing the importance of an accurate description of the large-scale electric field in the inner magnetosphere. This section will focus on a particular magnetic storm, that of 4 May 1998. This was a very large storm in which the ring current developed quite rapidly, and so it is a good candidate for investigating particle energization and flow. Three different electric field models will be compared: Volland–Stern, Weimer, and AMIE.

7.5.2 Model Description

We simulate the hot magnetospheric plasma dynamics by solving the bounce-averaged kinetic equation for the phase space distribution function, Q , of the RC ion species (H^+ and O^+), and electrons

$$\begin{aligned} \frac{\partial Q_\alpha}{\partial t} + \frac{1}{R_o^2} \frac{\partial}{\partial R_o} \left(R_o^2 \left\langle \frac{dR_o}{dt} \right\rangle Q_\alpha \right) + \frac{\partial}{\partial \varphi} \left(\left\langle \frac{d\varphi}{dt} \right\rangle Q_\alpha \right) \\ + \frac{1}{\gamma \sqrt{\gamma^2 - 1}} \frac{\partial}{\partial E} \left(\gamma \sqrt{\gamma^2 - 1} \left\langle \frac{dE}{dt} \right\rangle Q_\alpha \right) \\ + \frac{1}{f(\mu_o)\mu_o} \frac{\partial}{\partial \mu_o} \left(f(\mu_o)\mu_o \left\langle \frac{d\mu_o}{dt} \right\rangle Q_\alpha \right) = \left\langle \frac{\delta Q_\alpha}{\delta t} \right\rangle_{collis}, \quad \alpha = e, H^+ \end{aligned} \quad (7.5.1)$$

as a function of position in the magnetic equatorial plane (R_o, φ); kinetic energy and the cosine of the equatorial pitch angle (E, μ_o); and time t . Note that the equation is written in relativistic form, where

$$\gamma = 1 + \frac{E}{mc^2}$$

and all of the coefficients in (7.5.1) are converted accordingly (Khazanov et al. 1999a, 2004a). All of the bounce-averaged quantities in (7.5.1) are denoted with $\langle \cdots \rangle$. On the left-hand side, the bracketed terms are the perpendicular flow

speeds, and include all electric and magnetic drifts. These velocities, one for each independent variable in the calculation, will be determined from a number of electric field descriptions, to be discussed below. The $f(\mu_0)$ coefficient is a bounce-averaging term that ranges from 0.74 to 1.38 (Ejiri 1978). The term on the right-hand side of (7.5.1) includes losses from charge exchange (only for ions), Coulomb collisions (pitch-angle scattering and energy degradation terms), and precipitation at low altitudes (assumed lost at 800-km altitude along each field line). Each of the drift and loss terms is dependent on spatial location, energy, and pitch angle, and can be different for each species. Charge exchange and atmospheric loss are both treated as attenuation multipliers, while Coulomb collisions are described through an energy advection term (loss only) and a pitch-angle diffusion term. The formulations for these processes are the same as those discussed in Jordanova et al. (1996). For the electrons, pitch-angle diffusion into the loss cone resulting from interactions with the whistler mode waves in the plasmasphere is taken into account via attenuation factors (Lyons et al. 1972). The source of the particles is flow through the nightside outer simulation domain boundary (12 R_E downtail). Loss through the dayside magnetopause is also taken into account, allowing free outflow of the RC electrons and ions from the simulation domain. For further details about the solution of the kinetic equation (7.5.1), please see Jordanova et al. (1996, 1997), Khazanov et al. (1998, 1999a, 2004a), and Liemohn et al. (2001).

In the calculations below, several methods will be used to calculate the electric fields in the inner magnetosphere. One will be the analytical formulation of the Volland–Stern potential (Volland 1973; Stern 1975), assuming a shielding factor of 2 (nominal shielding near the Earth) and driven by the 3-h Kp index (Maynard and Chen 1975). The pattern is rotated 2 h of local time westward to better match the large-scale convection patterns of the other two models. Another electric field will be an ionosphere-to-magnetosphere mapping of the Weimer-96 model (Weimer 1996), driven by upstream solar wind measurements (1-min time cadence). A final electric field model will be an ionosphere-to-magnetosphere mapping of the AMIE inversion (Richmond and Kamide 1988) of ground-based (magnetometer and radar) and ionospheric (DMSP satellite) data for this interval (also with a 1-min time cadence).

Background and boundary conditions are also an important consideration for these simulations. The code uses a Rairden et al. (1986) geocoronal description for the charge exchange coefficients, and a Rasmussen et al. (1993) plasmaspheric model for the Coulomb collision coefficients. A static but disturbed Tsyganenko and Stern (1996) magnetic field is used for these calculations, so the results can focus on the influences of the electric field on the ion and electron distribution functions. All of the runs use kappa distributions applied at the outer simulation boundary (12 R_E downtail) for the source populations of the ions and electrons,

$$\Phi(\mathbf{r}, E) = n(\mathbf{r}) \frac{\Gamma(\kappa + 1)}{(\pi\kappa)^{3/2} \Gamma(\kappa - 1/2)} \left(\frac{1}{2m_o E_o} \right)^{1/2} \frac{E}{E_o} \left(1 + \frac{E}{\kappa E_o} \right)^{-\kappa-1}, \quad (7.5.2)$$

where the differential number flux Φ is related to the distribution function by $\Phi = 2EQ/m^2$. The coefficients are set by upstream solar wind measurements. A kappa distribution is thought to be reasonable because of the isotropization and acceleration of the plasma by the large amplitude plasma waves and magnetic fluctuations in the tail (e.g., Ma and Summers 1999).

7.5.3 The 1–7 May 1998 Storm

The ICME that hit the magnetosphere on 4 May 1998 was a compound stream of a large magnetic cloud interacting with a trailing high-speed stream. Several studies have examined the solar and heliospheric aspects of this event (Gloeckler et al. 1999; Skoug et al. 1999; Farrugia et al. 2002), and details of the solar wind and interplanetary magnetic field can be found therein. It is actually this stream that hits early on 4 May 1998, after the cloud had passed by geospace on 2–3 May 1998. Figure 7.41 shows the geophysical response to this activity. The Dst index shown in Fig. 7.41a is an hourly value of the globally averaged mid-latitude magnetic perturbation in the north–south direction. In Fig. 7.41b, Kp is a 3-h cadence global activity index related to the amount of fluctuation in auroral-zone ground-based magnetometer records. See Mayaud (1980) for further information about these indices.

Both of these indices are highly disturbed during the early hours of 4 May 1998. Kp is registered on a quasi-logarithmic scale with a range of 0–9, and it reaches 9 (second highest value) for one interval (3–6 UT). The Dst index plummets from an already disturbed value of roughly -62 to -205 nT in only 3 h, indicating the presence of a huge ring current. This index then recovers very quickly and is back up to -125 nT only 5 h later. Note that during this fast decay of the ring current, Kp is still elevated, indicating the presence of strong magnetospheric convection.

A useful quantity to consider for each of the three electric field descriptions to be discussed in this section is the total cross-tail potential difference. The basic configuration of the magnetospheric convection pattern is a dawn-to-dusk-oriented potential drop (i.e., electric field), which drifts the plasma sunward. By finding the maximum and minimum values of the potential, as specified by these various techniques, it is possible to obtain a quantitative assessment of the strength of the flow in the inner magnetosphere. The lower three panels of Fig. 7.41 show this cross-tail potential difference for the three field models. It is seen that the Weimer-96 potential difference spikes up to more than 300 kV during the main phase injection early on 4 May 1998. Similarly, the AMIE potential difference also has a peak value at this time of around 250 kV. The Volland–Stern potential drop, however, is much lower, with a peak value of just over 100 kV. This is partly due to the extent of the magnetosphere during this interval, because the Volland–Stern potential is directly proportional to the distance away from Earth. Hence a larger magnetosphere results in a bigger potential difference than would a smaller magnetosphere.

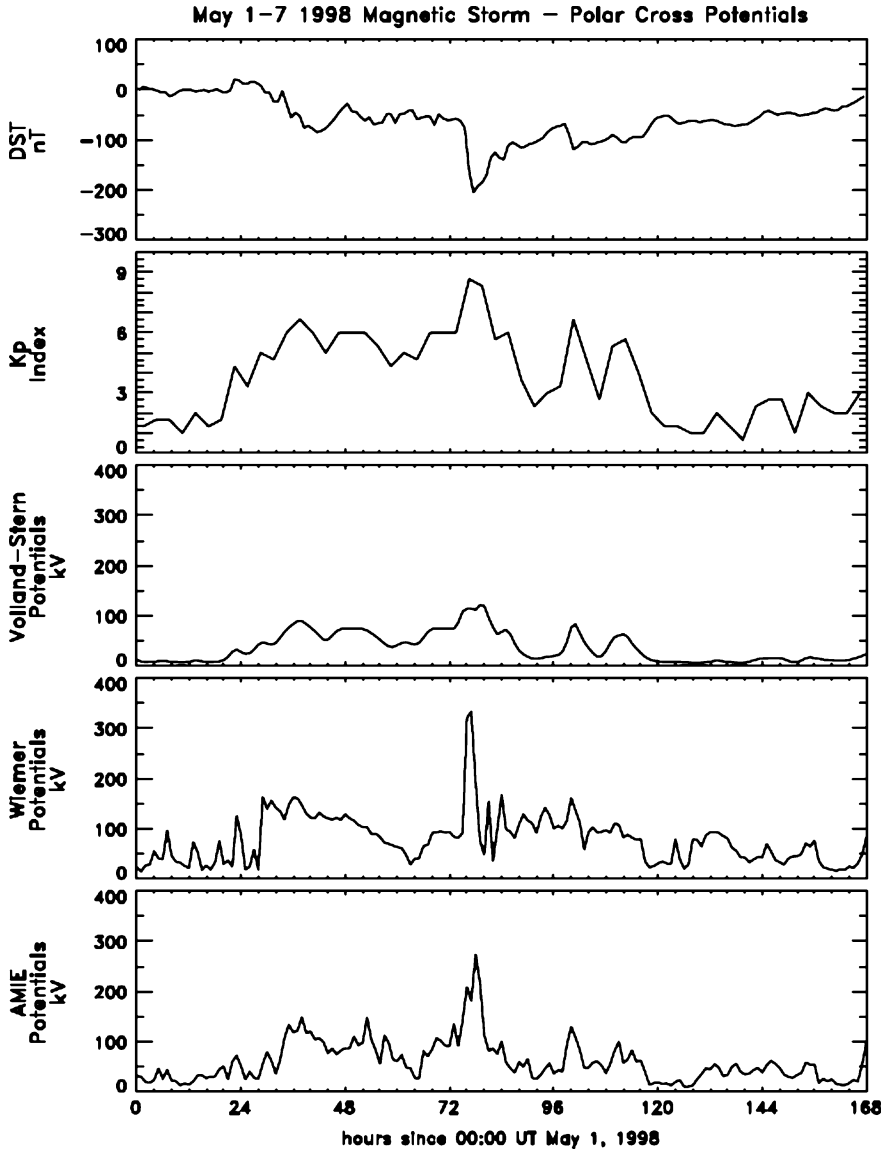


Fig. 7.41 Geophysical quantities during the 4 May 1998 magnetic storm. The first two rows are the Dst and Kp global indices. The next three panels show the total cross-tail potential differences during the 4 May 1998 storm from the Volland–Stern model, the Weimer-96 model, and the AMIE model

7.5.4 Results

This section focuses on the formation of the particle distribution function in the inner magnetosphere assuming various descriptions for the large-scale electric

field. As mentioned above, three field models will be used, (1) the Volland–Stern empirical model, driven by the 3-h Kp index (interpolated each time step); (2) the Weimer-96 empirical model, driven by the 1-min cadence solar wind and interplanetary magnetic field data; and (3) the AMIE model, driven by 1-min resolution ground-based magnetometer data.

In order to isolate the effects of the electric field configuration, all three simulations are run with exactly the same initial and boundary conditions for the inner magnetospheric plasma populations. The source term for the plasma is the nightside plasma sheet, which is defined by (7.5.2) above with $\kappa = 3$ and the parameters n and E_0 specified by the upstream solar wind

$$\begin{aligned} n(t) \text{ (cm}^{-3}\text{)} &= 0.025N_{sw}(t - 3h) + 0.395 \\ E_0(t) \text{ (keV)} &= 0.02V_{sw}(t - 3h) - 3.0, \end{aligned} \quad (7.5.3)$$

as determined from data (Borovsky et al. 1998; Ebihara and Ejiri 2000). In (7.5.3), the units are cm^{-3} for N_{sw} and km s^{-1} for V_{sw} . The initial conditions are those specified by Sheldon and Hamilton (1993) for the quiet ring current.

All of the runs were begun at 00 UT on 1 May 1998 and carried out through the end of 7 May 1998. In this section, only a small portion of these simulation results are examined, namely those around 00 UT on 4 May 1998 when the Dst index plummeted toward its minimum value (i.e., during the main phase injection of the storm-time ring current). It is during this period that the most intense electric fields are present in the inner magnetosphere, making it an ideal interval to study the acceleration of plasma in near-Earth space.

7.5.4.1 Electric Potentials

Figures 7.42–7.44 show electric equipotential contour plots in a region around the Earth ($\pm 12 R_E$ in the x – y plane). The eight panels of each figure are consecutive hourly snapshots of the convection pattern. Each figure shows the patterns for the three electric field descriptions, as listed. These times are during the main phase of the storm on 4 May 1998 (01 UT to 08 UT on that day). Because the electric field is directly proportional to the gradient of these potentials, the relative spacing of the contour lines (drawn every 6 kV) show the relative strength of the electric field as a function of space and time. Note that the corotation electric field has been added to all of these plots. Therefore, the equipotential contour lines are the instantaneous cold-plasma drift trajectories through this region. Higher-energy particles will, of course, also have the energy, pitch angle, and charge-dependent effect of the magnetic gradient-curvature drift superimposed on these trajectories. Even so, it is useful to examine these plots to understand the flow of the plasma sheet particles into the inner magnetosphere during this event. If the electric field is strong enough, the convective drift can dominate the gradient-curvature drift, even for high-energy ions and electrons. The discussion below therefore focuses on the morphology of

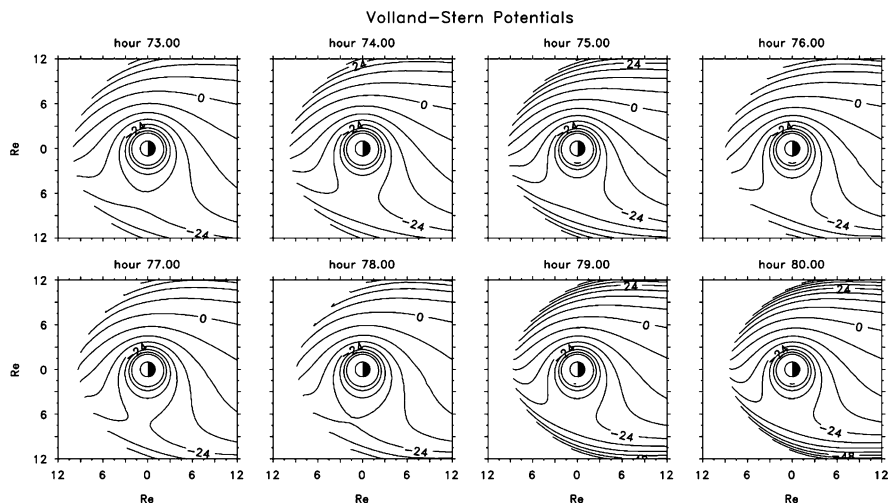


Fig. 7.42 Electric potential isocontour plots for the Volland–Stern electric field model at eight times during the main phase of the 4 May 1998 storm. Contours are shown every 6 kV, and are not drawn inside of $2 R_E$ or beyond the magnetopause, as specified in the model. Note that corotation is included in the potential pattern

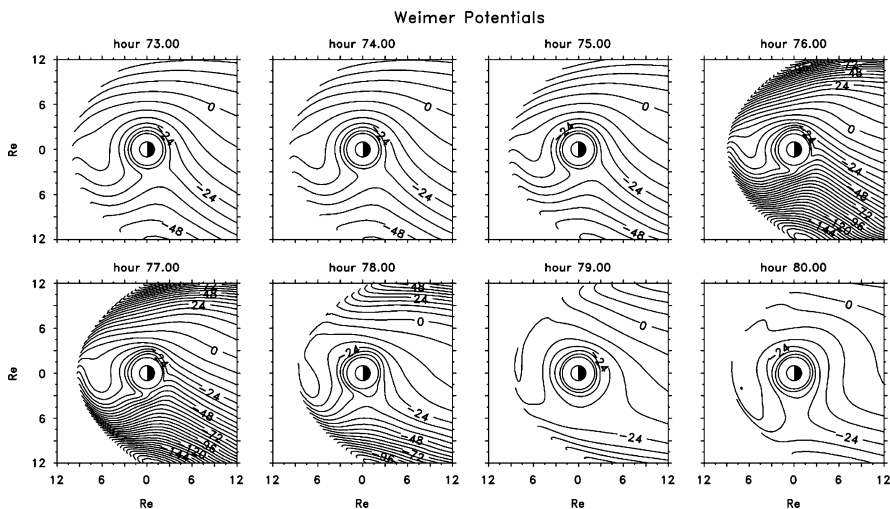


Fig. 7.43 Electric potential isocontour plots like Fig. 7.42, except for the Weimer-96 electric field model. Note that corotation is included in the potential pattern

the potential patterns, especially on regions of intense electric fields (i.e., closely spaced equipotential contours). Such regions will be places of high-speed plasma motion along these equipotential lines, and so they will be referred to as “flow

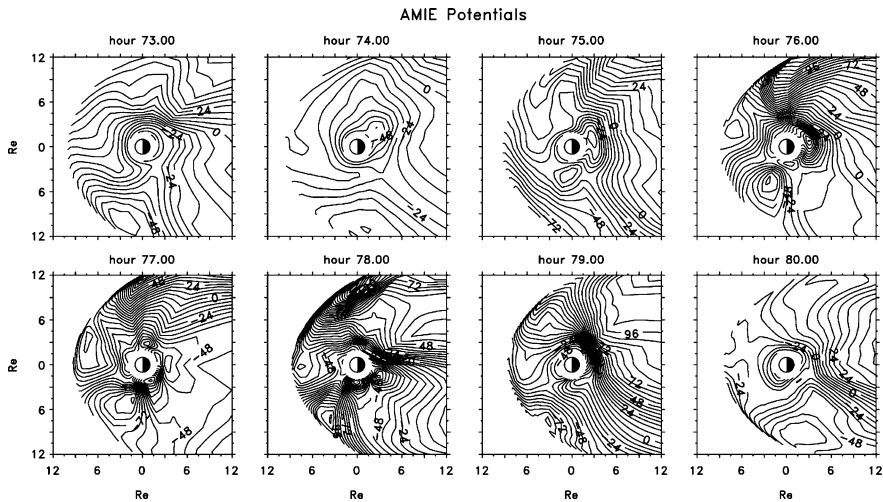


Fig. 7.44 Electric potential isocontour plots like Fig. 7.42, except for the AMIE electric field model. Note that corotation is included in the potential pattern

channels” hereinafter, similar to the Chen et al. (2003) study. In fact, Chen et al. (2003) show drift paths instead of equipotentials for these flow channels in the inner magnetosphere, and so it is useful to compare those results with the ones presented here.

Take a look at Fig. 7.42, which presents the patterns for the Volland–Stern electric field description. They all look very similar to each other in their general morphology. The region near the Earth where the corotation potential dominates (a minimum at the Earth, so it is an inward electric field and thus an eastward drift) expands and contracts with the waning and waxing of the cross-tail potential difference of the large-scale convection field (maximum at dawn, so it is a dawn-to-dusk electric field and thus a sunward drift). This is the classic description of magnetospheric convection, with a teardrop-shaped separatrix dividing the closed trajectories near the planet and the open trajectories farther away. The cold-plasma drift trajectories preferentially flow around the dawnside of the Earth. In all of these plots, the stagnation point is well defined and always located at approximately $6 R_E$ just sunward of dusk (not exactly at dusk because the pattern has been rotated westward slightly).

Now consider Fig. 7.43, containing the patterns for the Weimer-96 electric field description. Even though the same corotation potential has been added to both sets of plots, the Weimer-96 contours are quite a bit different than the Volland–Stern patterns. For one, the stagnation point is rotated toward midnight (postdusk rather than predusk), and the separatrix is much closer to the Earth. The electric fields, as evidenced by the concentration of equipotential contours in a given region, are also larger beyond the separatrix for this field description. Of particular interest for this section are the intense electric fields in the postdusk region, where the

equipotentials are closely spaced and point right at Earth. This phenomenon, which will be dubbed a flow channel, is clearly seen at hours 76 (04 UT) and 77 (05 UT). While the convective electric fields in the Volland–Stern field never exceed 1 mV m^{-1} inside of a $8 R_E$ radius from Earth, in the Weimer-96 model they are able to reach values of 1.7 mV m^{-1} in this flow channel.

In Fig. 7.44 (those for the AMIE electric field description), the convection patterns are even more exotic. Flow channels form and disappear regularly during this interval, with the most intense one at hour 78 (06 UT) directed right along the midnight meridian. Here, electric fields in excess of 3 mV m^{-1} exist in a narrow band stretching from the $12 R_E$ outer boundary of the plot in to the $2 R_E$ inner boundary of the contour traces. These fields are capable of moving plasma from one end of this channel to the other in less than an hour. That is, with electric and magnetic field strengths of 3 mV m^{-1} and 100 nT (a nominal field strength near geosynchronous orbit), respectively, the drift speed is $17 R_E/\text{h}$. So, even though these channels are short-lived in any specific location (the particular one at hour 78 (06 UT) is not present in the midnight sector at either hour 77 (05 UT) or 79 (07 UT), for instance), they are still capable of injecting massive amounts of plasma into the inner magnetosphere, as shown below.

In comparing the cross-tail potential differences given in Fig. 7.41 with the patterns shown in Figs. 7.42–7.44, it is seen that the Weimer-96 model produced the largest cross-tail potential difference while the AMIE model produced the largest electric fields. Because the energization of the plasma is directly related to how far in the plasma is injected into the inner magnetosphere, it is really the electric field that matters for this study. Therefore, while it is useful to consider the cross-tail potential difference as a general indicator of the convective strength at a given time, it is really the morphology of the potential pattern that yields the strong flow channels capable of accelerating the plasma sheet particles up to ring current energies.

7.5.4.2 Particle Energy Spectra

To illustrate the effect of these flow channels on the particles, the pitch angle-averaged energy spectra of differential number flux will be examined. Figure 7.45 presents the results for protons at hour 74 (02 UT), a time near the start of the main phase of the 4 May 1998 storm. Even here, before the main storm, there are significant differences in the flux levels between the various simulations. This is because there was a smaller storm on 2 May 1998 that is still persisting (Dst was still below -50 nT at the start of 4 May 1998). At larger radial distances, the three models produce nearly identical spectra because this is essentially the outer boundary condition, particularly true at $\text{MLT} = 00$. At smaller radial distances, the differences become clear. The AMIE fields are capable of creating an energetic population deep in the inner magnetosphere (flux peak near 100 keV) that is several orders of magnitude larger than that from the other field models. The peak at the highest energy bin (near 400 keV) in the Volland–Stern and Weimer-96 field results

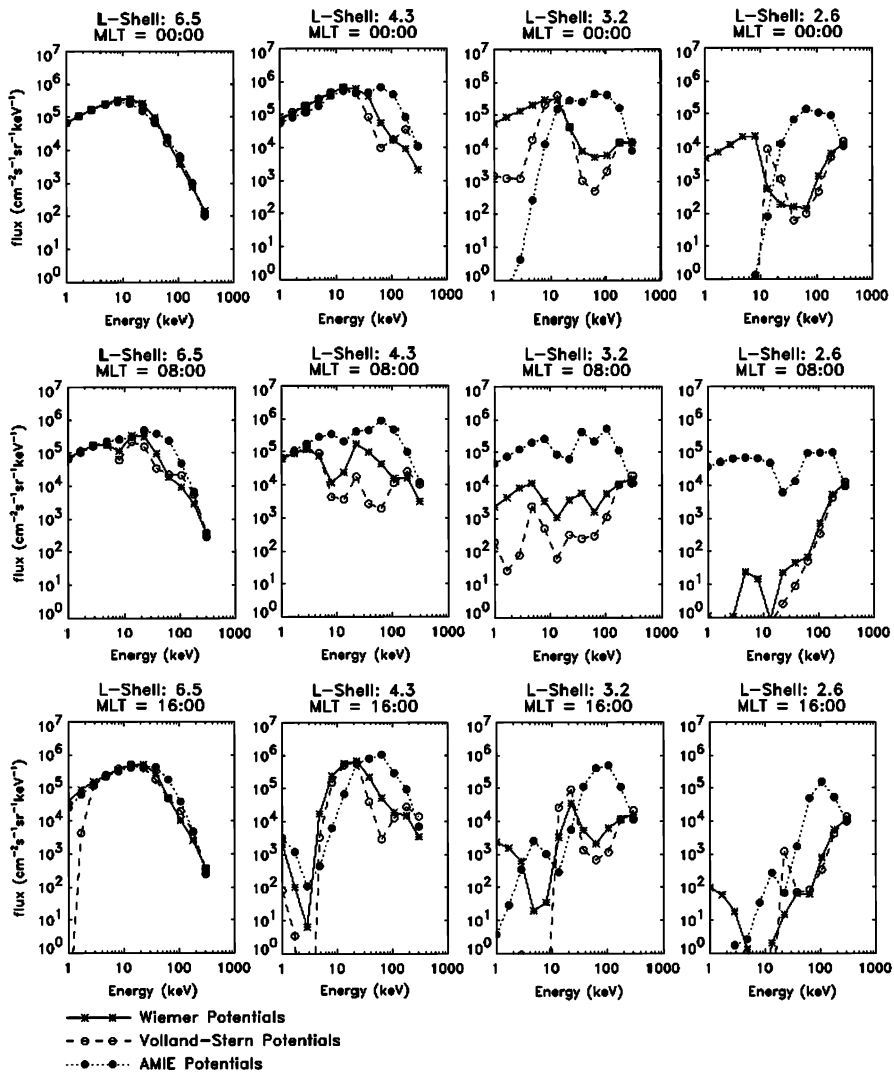


Fig. 7.45 Pitch angle-averaged number flux energy spectra of protons at hour 74 (02 UT, start of the main phase), shown at three local times (rows) and at four radial distances (columns)

are remnants of the specified initial condition. Very little new protons have been injected, thus far in the storm sequence, by either of these two field models. The AMIE potentials, however, even with modest cross-tail potential differences of under 150 kV, have been able to generate a substantial new ring current inside of 4 R_E . The Weimer-96 fields, even though they have had larger cross-tail potential differences earlier in the sequence, have been able to only generate a small ring current in this region.

Taking these fluxes as a reference point, now consider the fluxes plotted in Fig. 7.46. These are results at hour 80 (08 UT, 6 h later), after the main phase of the storm (just past the minimum in Dst). The results at $L = 6.5$ and 4.3 are quite similar to those in Fig. 7.45. This is because these radial distances are, in general, on open drift paths even for moderate convection, and thus even the small storm on 2 May 1998 can populate this region with fresh plasma sheet ions. At $L = 4.3$, MLT = 16, however, the lower-energy part of the spectra is filled in for the AMIE field results, while the other models still show the standard “accessibility gap” in the spectra. This minimum at a few kiloelectron volts is due to the interplay between

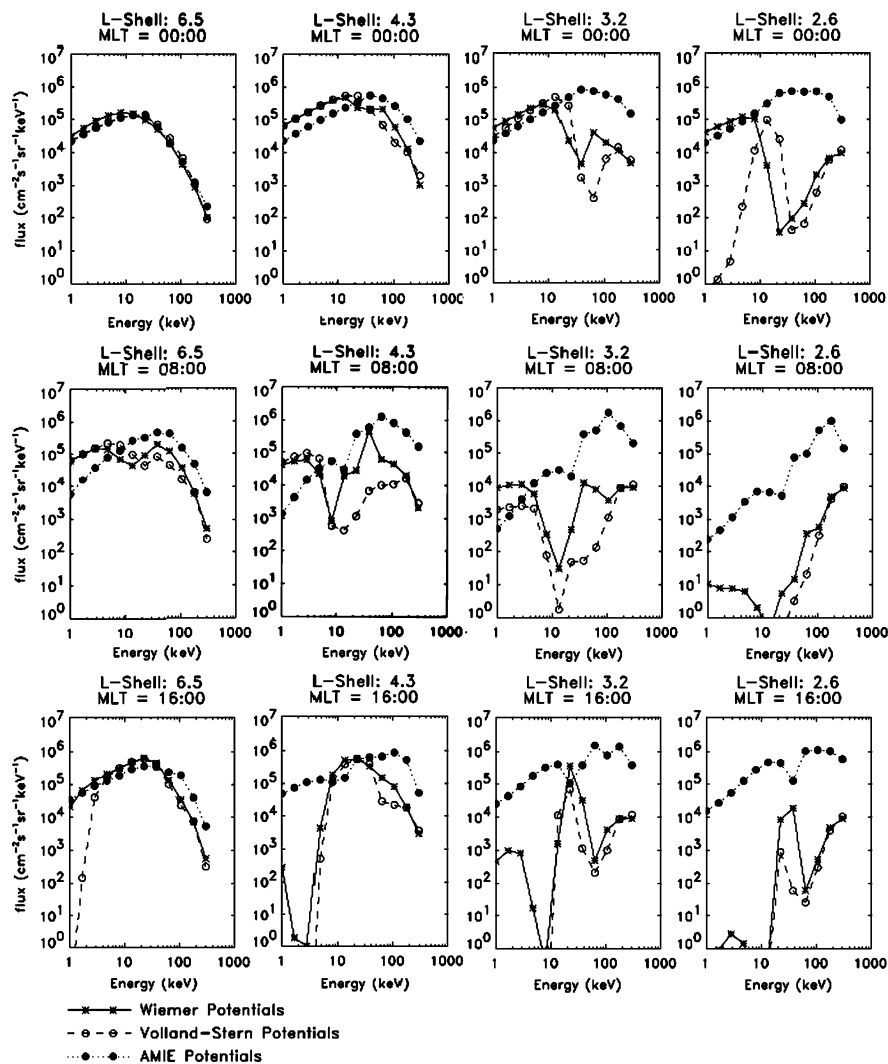


Fig. 7.46 Same as Fig. 7.45 except for protons at hour 80 (08 UT, end of the main phase)

the corotation, convective, and gradient-curvature drifts for protons. Corotation is eastward while gradient-curvature is westward (convection is basically sunward). Ions of a few kiloelectron volts have gradient-curvature drifts on the order of the corotation drifts around $L = 3$ – 5 , and so these ions stagnate on the nightside and are slowly lost due to collisional processes (mainly charge exchange and Coulomb interactions). That this minimum is not evident in the AMIE field results indicates that, at some point on the nightside, the convective drift was strong enough to overcome this stagnation and push the ions in toward the Earth.

At smaller L values (3.2 and 2.6), the differences between Figs. 7.46 and 7.45 become more apparent. At $L = 3.2$, the low-energy range of the spectrum is enhanced for all of the field models at MLT = 00 but only for the AMIE field run at MLT = 16. The sharp peak near 20 keV at MLT = 16 in the Weimer-96 and Volland–Stern field results are from convective access during the main phase. However, it is seen that neither of these models were capable of producing the high-energy population that the AMIE field results show. At $L = 2.6$, the differences between the AMIE results and the other model results are even more dramatic. Again, there is a peak near 30 keV in the Weimer-96 and Volland–Stern field results at MLT = 16, but these do not reach the AMIE flux levels, and neither the low- nor high-energy tails of the spectrum are even close to the AMIE field flux levels. In comparing Figs. 7.45 and 7.46, it is seen that the AMIE fields during the main phase on 4 May 1998 increased the high-energy flux levels by at least an order of magnitude at the deepest L shells. This energization is primarily due to the rapid injection of plasma sheet ions via the flow channels and their intense electric fields.

A similar analysis can be done for the electron flux spectra. These particles have an eastward gradient-curvature drift, and so this term is in the same sense as corotation. Therefore, stagnation will only occur where these two drifts are equal and opposite to the convective drift, which generally occurs in the dusk sector (if at all). The net influence is that the PSEs will all flow eastward during injections, in contrast to ions that might flow either direction, depending on their energy. It is therefore more difficult to inject electrons deep into the inner magnetosphere, especially the high-energy electrons (for which the gradient-curvature drift term is large). A very intense electric field is needed to bring these particles inside of $L = 4$.

Figure 7.47 shows the electron spectra from the three simulations at hour 74 (02 UT). As with the ions, the $L = 6.5$ fluxes are very similar between the models and are close to the outer boundary flux values. Deeper in L shell and at the sub-100-keV energies, it is seen that the Weimer-96 model produces consistently higher flux levels than the Volland–Stern field simulation. In turn, the AMIE model yields higher flux levels than either of the other two in this range. Again, it is not the total potential difference that determines the flux levels but the maximal localized electric fields that preferentially bring in the plasma sheet electrons to the inner magnetosphere. At the smaller radial distances, the Volland–Stern results are a good indication of the prescribed initial condition values.

Figure 7.48 shows electron flux results at hour 80 (08 UT), after the main phase of the storm. The changes are subtler than those seen in the ion results, but interesting

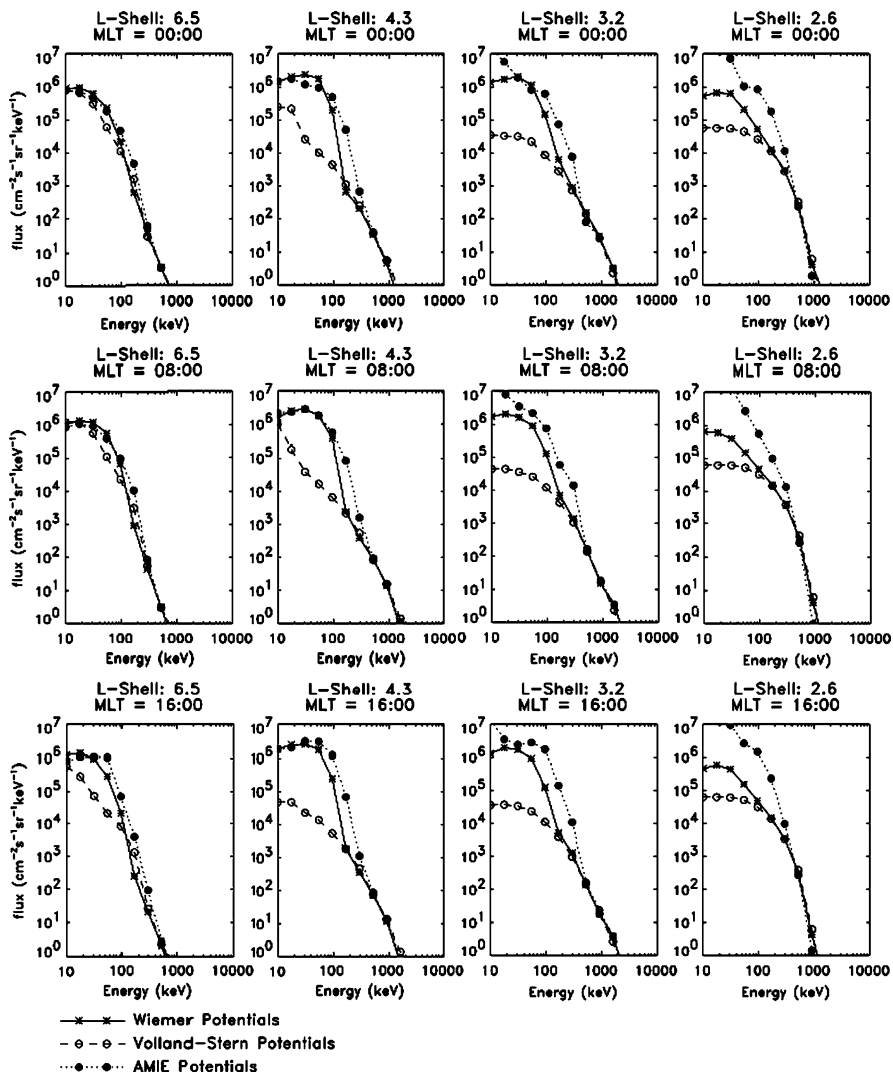


Fig. 7.47 Same as Fig. 7.45 except for electrons at hour 74 (02 UT, start of the main phase)

features should be pointed out. Consider, for instance, $L = 4.3$. Here the Weimer-96 and AMIE potentials yield enhanced fluxes in the high-energy tail of the distribution compared with their prestorm fluxes, while the Volland-Stern model produced almost no change at all (except in the lowest energy channels shown). This difference is from convective access of these particles into the inner magnetosphere. How far in did they penetrate? Examining the results at $L = 3.2$ and $L = 2.6$ shows that the fluxes are either at or slightly below their prestorm values. Therefore, the inward penetration

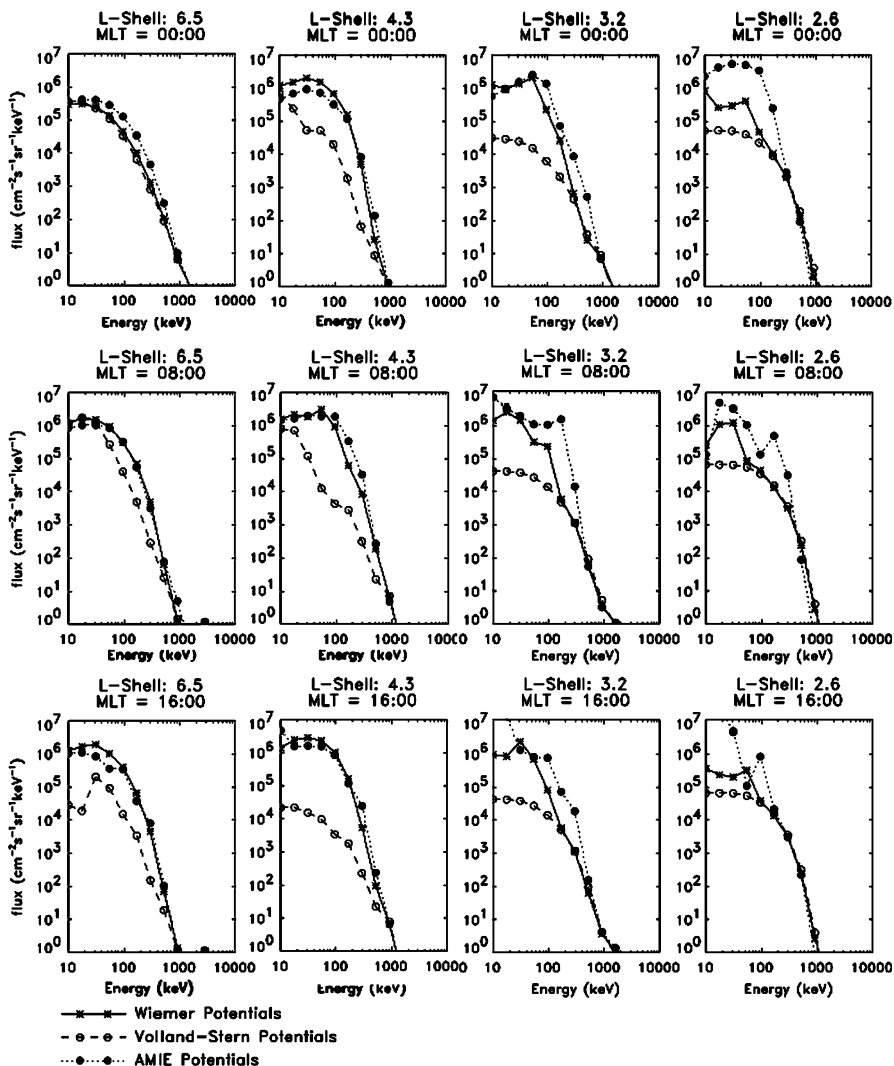


Fig. 7.48 Same as Fig. 7.45 except for electrons at hour 80 (08 UT, end of the main phase)

of electrons due to the flow channels was much less critical than that for the ions. Note that, because of differences in the characteristic energy and number density between the hot ions and hot electrons, most of the total energy in the inner magnetosphere is carried by the hot ions rather than the hot electrons, and therefore it is the energization of the ions that plays the vital role in forming a strong storm-time ring current near the Earth. That said, the sub-100-keV electrons that are brought in by these flow channels form the seed population for the radiation belts. Through resonant interactions between plasma waves and the drift period of these electrons,

additional acceleration can result, yielding a poststorm radiation belt enhancement, as is often seen (e.g., McIlwain 1996 and references therein).

7.5.4.3 Rapid Electric Field Fluctuations

In Khazanov et al. (2004b), simulations are conducted to investigate the influence of rapid electric field fluctuations on electron energization in the inner magnetosphere based on the AMIE technique. Simulations for four different magnetic storms were run, namely those that occurred on May 15, 1997, May 4, 1998, September 25, 1998, and October 19, 1998. Specifically, they examine the formation of high-energy (10–1,000 keV) electrons in the inner magnetosphere during these storm events with relativistic radiation belt transport code that we have been presented above.

The only difference between the numerical experiments was in the convection electric field description. Even here, they are not so different because they all use electric potential values from the AMIE technique, which ingests ground-based and satellite measurements to obtain a best-fit potential pattern for a given instant. The first run used AMIE patterns with a 1-min cadence. The magnetometer data, which dominates the AMIE assimilation (especially at subauroral latitudes where there is no radar data), has a 1-min cadence. The radar data (2-min resolution) and satellite data (~ 20 min per polar passage) are weighted in time to spread their influence over the window of observation. Furthermore, Ridley et al. (1998, 1999) discussed the validity of this time resolution of AMIE potential patterns, finding that 1-min cadence patterns contain real magnetospheric variations. The second through fifth runs used AMIE patterns with cadences of 5, 10, 20, and 180 min, respectively.

It was found that the global electric field pattern should be updated with a cadence of 5 min or less in order to properly simulate the enhancement of the 10- to 1,000-keV electrons in the $L = 3$ –5 range. This is the energy range and spatial region for the seed population of the MeV electrons in the outer zone radiation belt. This study showed that E-field fluctuations are critical to the formation of this seed population during magnetic storms. These electrons have drift periods close to the fluctuation period of the global potential pattern, and therefore they experience energization. A model that does not take into account these rapid fluctuations of the inner magnetospheric electric field will underestimate the fluxes of these electrons. Differences were particularly large (up to several orders of magnitude) relative to simulation results with a 3-h time cadence, analogous to a Kp -driven electric field model.

This study shows that by varying the global structure of the electric field at a high time cadence, specifically 5 min or less between potential pattern updates, a stochastic motion of the particle drift paths occurs. These random changes result in radial diffusion and, because the particle source is at the outer simulation boundary, the net effect is an inward propagation of particles and thus an energization. The traditional approach for achieving this effect, using a radial diffusion operator with a diffusion coefficient proportional to the electric field variability (e.g., Chen et al. 1993, 1994; Sheldon 1994; Beutier and Boscher 1995; Bourdarie

et al. 1996, 1997), is equally valid. This study simply shows that the same stochastic transport and energization can be obtained if the simulation includes a rapidly fluctuating, realistic electric field description. It also quantifies the rapidness with which it much change in order to accurately replicate this diffusion process.

To investigate a rapidly fluctuating electric field dependence, the September 1998 simulation was rerun with a different set of boundary conditions. Specifically, the CPCP and LANL electron moments were averaged over 3-h bins (Khazanov et al. 2004b). This choice of an averaging timescale is to approximate a Kp -driven simulation, which has a 3-h time cadence. The CPCP and electron boundary conditions still change every time step in the new simulation, but the values are now interpolated from these 3-h averages, rather than from the high time resolution input files. A comparison of the results from the two simulations is given in Figs. 7.13 and 7.14. These two plots are shown for main (Fig. 7.49) and recovery (Fig. 7.50) phases of the storm. It is seen that the fluxes are up to three orders of magnitude higher in the high-time-cadence simulation results, especially at the high-energy (10–1,000 keV) end of the spectrum. This is due to the fluctuations in the electric field resonating with the drift periods of these electrons, preferentially accelerating them via a net inward radial drift. This enhancement is seen clearly in the 100s of kiloelectron volt energy range at this radial distance ($L = 4.5$), which is

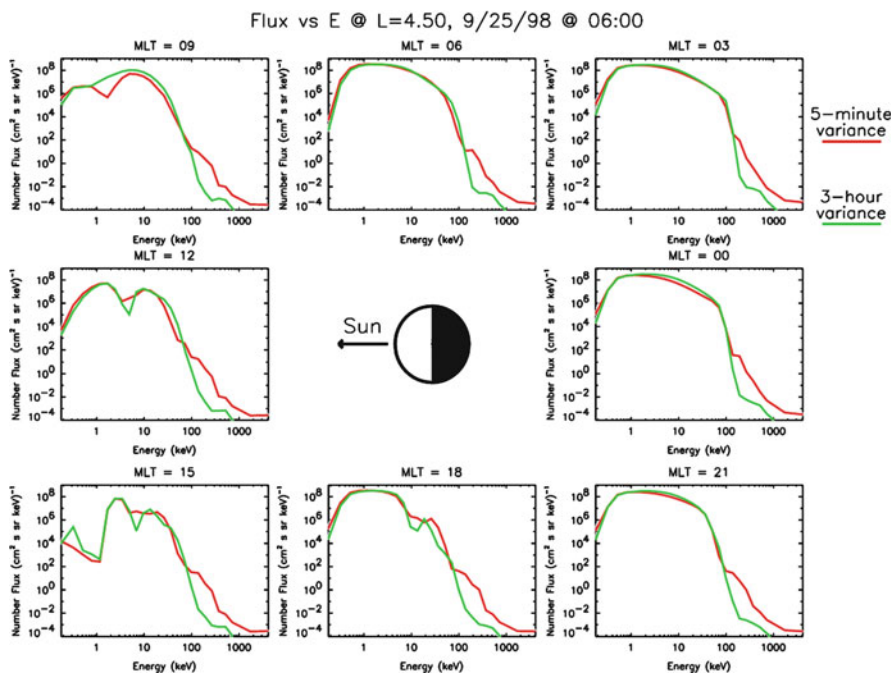


Fig. 7.49 Pitch angle-averaged energy spectra at 8 MLT times around the Earth at $L = 4.5$ during the September 1998 storm for the two time-cadence options run for the September 1998 storm. The results are for $L = 4.5$ near the peak of the storm

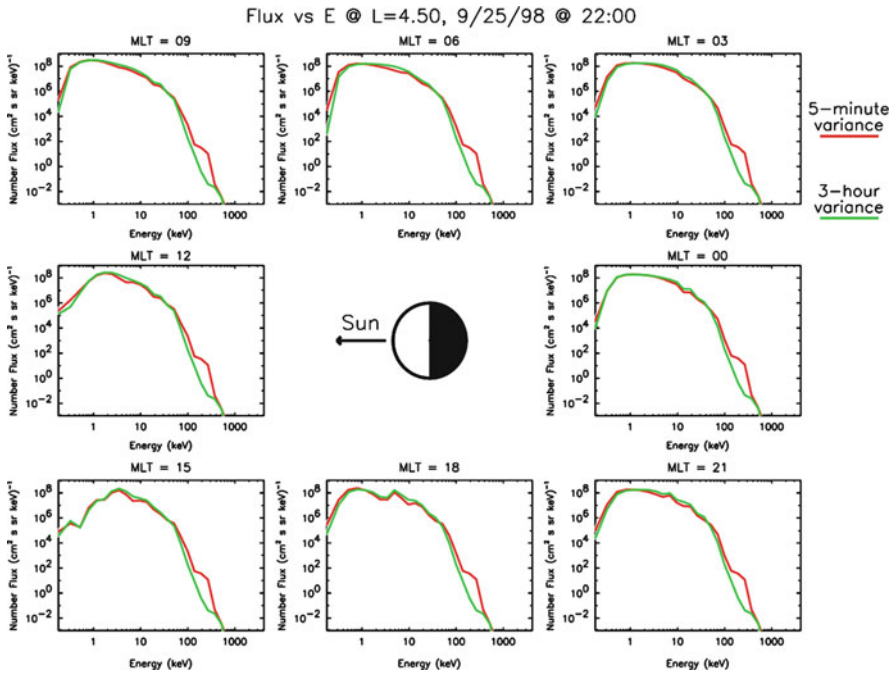


Fig. 7.50 Same as Fig. 7.49 except that the chosen time is in the recovery phase of the storm

exactly where a seed population is needed for the production of outer zone radiation belt relativistic electrons.

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Chapter 8

Kinetic Superthermal Electron Instabilities in the Ionosphere

As shown in Chap. 7 and discussed in this chapter, superthermal electrons depart from a Maxwellian distribution. These departures can lead to plasma instabilities that can influence and further alter the electron distribution function, affecting, to a considerable extent, the energy balance of the Earth's ionospheric plasma. The problem of plasma stability relative to the excitation of electrostatic oscillations is also of considerable interest in the study of the ionosphere, using, for example, incoherent backscatter. In this chapter we consider the instabilities associated with the presence of photoelectrons and secondary electrons in the ionosphere.

8.1 The Generation of Plasma Oscillations by Photoelectrons

8.1.1 Dielectric Permittivity of Plasma

The dielectric permittivity tensor of a magnetoactive plasma was presented in Chap. 3 in (3.4.16) and (3.4.17). Considering electrostatic perturbations only, with the magnetic field much smaller than the electric field, and using the corresponding limit of such oscillations (see (3.5.9)), the dielectric tensor can be given by the expression (Akhiezer et al. 1975)

$$\varepsilon(\omega, \mathbf{k}) = 1 + \sum_{\alpha=e,i} \sum_{l=-\infty}^{\infty} \frac{2\pi\omega_{p\alpha}}{k^2 n_{0\alpha}} \int_{-\infty}^{\infty} dv_{\parallel} \int_0^{\infty} dv_{\perp} J_n^2 \left(\frac{k_{\perp} v_{\perp}}{\omega_{B\alpha}} \right) \frac{\left\{ k_{\parallel} v_{\perp} \frac{\partial}{\partial v_{\parallel}} + n \omega_{B\alpha} \frac{\partial}{\partial v_{\perp}} \right\} f_{\alpha}(\mathbf{v})}{\omega - k_{\parallel} v_{\parallel} - n \omega_{B\alpha}}. \quad (8.1.1)$$

Here

$$\omega_{p\alpha} = \sqrt{\frac{4\pi e_{\alpha}^2 n_{\alpha}}{m_{\alpha}}}, \quad \omega_{B\alpha} = \frac{e_{\alpha} B}{m_{\alpha} c}, \quad e_{\alpha}, m_{\alpha}, n_{0\alpha}$$

are, respectively, plasma frequencies, gyrofrequencies, charge, mass and density of electrons and ions (in what follows, where this does not cause misunderstanding, we omit the indices e of the designations, referring to the electron component ($\omega_{pe} \equiv \omega_p, \omega_{B\alpha} \equiv \omega_B$, etc.)); $J_n(x)$ is Bessel's function of the first kind of order n ; $v_{||}, v_{\perp}$ are the velocity components of the particles with respect to the magnetic field direction; $k_{||}, k_{\perp}$ are the corresponding wave vector components; and $f_{\alpha}(\mathbf{v})$ is the distribution function of electrons or ions. Let the distribution function of electrons be represented as the sum of the distribution functions of thermal electrons $f_e(\mathbf{v})$ and the distribution function of superthermal electrons $f(\mathbf{v})$. It will be assumed that the ions and thermal electrons of the plasma have an equilibrium Maxwell's distribution.

The expression (see Chap. 3 for details)

$$\varepsilon(\omega, \mathbf{k}) = 0 \quad (8.1.2)$$

in the general form defines the real and imaginary parts of the frequency of plasma oscillations. Since the thermal electron density n_e significantly exceeds the superthermal electron density n_e (Khazanov 1979), $\text{Re } \omega$ is determined only by the contribution of thermal electrons and ions and can be derived from (3.5.8) (see also Ginzburg and Rukhadze 1975)

$$1 + \sum_{\alpha=e,i} \frac{1}{k^2 r_{\alpha}^2} \left[1 - \sum_{-\infty}^{\infty} \frac{\omega}{\omega - n\omega_{B\alpha}} I_n(z_{\alpha}) e^{-z_{\alpha}} W\left(\frac{\omega - n\omega_{B\alpha}}{k_{||} v_{\alpha}}\right) \right] = 0. \quad (8.1.3)$$

Here $z_{\alpha} = k^2 \rho_{\alpha}^2$, and ρ_{α} , r_{α} , and v_{α} are the Larmor radius, the Debye radius, and the thermal velocity of electrons or ions, respectively; I_n is a modified Bessel's function; and W is the Kramp function (Ginzburg and Rukhadze 1975) that was defined in Sect. 3.5. In the cold-plasma approximation presented in Sect. 3.6 ($v_{\alpha} = 0$), the solutions of (8.1.3) have the form

$$\omega_1 = \begin{cases} \omega_p \left(1 + \frac{1}{2} \frac{\omega_B^2}{\omega_p^2} \sin^2 \theta \right) & \text{if } \omega_p \gg \omega_B \\ \omega_B \left(1 + \frac{1}{2} \frac{\omega_p^2}{\omega_B^2} \sin^2 \theta \right) & \text{if } \omega_B \gg \omega_p \end{cases} \quad (8.1.4)$$

$$\omega_2 = \left[\frac{\omega_p^2 \omega_B^2}{\omega_p^2 + \omega_B^2} \left(\cos^2 \theta + \frac{m}{M} \right) \right]^{1/2} \quad (8.1.5)$$

$$\omega_3 = \frac{\omega_{Bi} \omega_p \cos \theta}{\sqrt{\omega_{pi}^2 + \omega_{Bi}^2}}, \cos^2 \theta \ll m/M. \quad (8.1.6)$$

Here m/M is the ratio of electron to ion mass.

Taking into account the thermal motions of the plasma electrons and ions gives rise to corrections that depend on k in (8.1.4)–(8.1.6). Moreover, new branches of plasma oscillations appear, namely, these at the harmonics of the electron–cyclotron $n\omega_{Be}$, and the ion–cyclotron $n\omega_{Bi}$.

In a sufficiently dense plasma, it is necessary to take into account particle collisions. If collisions of electrons with the charged and neutral plasma components in a model BGK form are taken into account (Bhatnagar et al. 1954), then, for high-frequency oscillations, the dispersion equation has the form (Ginzburg and Rukhadze 1975)

$$1 + \frac{1}{k^2 r_e^2} \frac{1 - \sum_{-\infty}^{\infty} \frac{\omega + iv_e}{\omega - n\omega_{B\alpha} + iv_e} I_n(z_e) e^{-z_e} W\left(\frac{\omega - n\omega_{Be}}{k_{||} v_e}\right)}{1 - \sum_{-\infty}^{\infty} \frac{iv_e}{\omega - n\omega_{B\alpha} + iv_e} I_n(z_e) e^{-z_e} W\left(\frac{\omega - n\omega_{Be}}{k_{||} v_e}\right)} = 0. \quad (8.1.7)$$

Here v_e is the effective collision frequency of electrons with charged and neutral plasma components.

A quantitative characteristic of the plasma oscillation instability is the value of the growth rate that has been derived in Sect. 3.7 (see also Ginzburg and Rukhadze 1975)

$$\gamma(\omega, \mathbf{k}) = - \frac{\text{Im } \varepsilon(\omega, \mathbf{k})}{\frac{\partial}{\partial \omega} \text{Re } \varepsilon(\omega, \mathbf{k}) \big|_{\omega_r = \text{Re } \omega(\mathbf{k})}}. \quad (8.1.8)$$

From (8.1.3) one can determine all branches of plasma oscillations, and from (8.1.8) one can estimate the degree of instability of these oscillations.

8.1.2 Cherenkov Resonance

The anisotropy of the photoelectron flux caused by the escape of the electrons from the ion production region along geomagnetic field lines is a potential cause of a plasma instability. This issue was investigated for the first time in Vershinin et al. (1979) in connection with an interpretation of VLF emission observations in the range of 8–20 kHz. The analysis was carried out on the basis of a model ejection distribution function without considering collision processes in the escape region. In Matafonov and Vlasov (1984), it was concluded that escaping photoelectrons cannot excite potential plasma oscillations. That conclusion was based on the results reported in Tashchilin and Krinberg (1979) and Matafonov (1979), where it was shown that the photoelectron flux has a distribution function monotonically decreasing for all pitch angles. Indeed, the possibility of the plasma beam instability is determined by the nonmonotonicity of a one-dimensional (integrated over two

variables) distribution function. Therefore, the instability of the plasma oscillations can occur, in particular, in an inhomogeneous magnetic field even in the case of a monotonic energy distribution of photoelectrons. It should also be noted that a correct calculation of the distribution function of photoelectrons in the topside ionosphere is possible only in the case of a self-consistent treatment of their transport along magnetic field lines, as discussed in Chap. 7. The approach used in Matafonov and Vlasov (1984), however, is based on a separate treatment of the ionospheric and plasmaspheric portions of the motion of the photoelectrons (Tashchilin and Krinberg 1979; Matafonov 1979). Such a procedure does, of course (as showed in Chap. 7), distort the real physical picture. Besides, in Tashchilin and Krinberg (1979) the magnetic field in the ionosphere is assumed homogeneous. Such an approximation is in conflict with the concept set forth in Vershinin et al. (1979) that the inhomogeneity of the magnetic field in the topside ionosphere is responsible for the electron flux anisotropy.

In what follows, the question concerning the stability of plasma with respect to the excitation of oscillations by an anisotropic photoelectron flux is considered on the basis of a self-consistent approach that implies a numerical solution of the full kinetic equation along the entire length of the magnetic field lines, as discussed in Chap. 7.

As it is known, in a plasma-beam system, maximum growth rates are represented by growth rates for oscillations excited by the Cherenkov resonance with a wave vector parallel to the external magnetic field. For Langmuir oscillations, the maximum growth rate is defined as (Akhiezer et al. 1975)

$$\gamma = \frac{\pi^2 v_{res}^2 \omega_p}{n_0} \left. \frac{\partial F(v_{||})}{\partial v_{||}} \right|_{v_{||}=v_{res}=\omega_r/k_{||}}. \quad (8.1.9)$$

Here

$$F(v_{||}) = \int_0^{v_{\perp}^2 \max} dv_{\perp}^2 f(v_{\perp}, v_{||}) \quad (8.1.10)$$

and $f(v_{\perp}, v_{||})$ is the electron distribution function. Let us demonstrate that, as an electron flux, with a distribution decreasing monotonically in energy travels along diverging magnetic field lines (as is the case in the ionosphere above 300 km), a maximum appears on the function $F(v_{||})$. Indeed, by virtue of conservation of the first adiabatic invariant, there occurs focusing of the electrons in the pitch-angle cone $\mu_b \leq \mu \leq 1$, where $\mu_b = \sqrt{1 - B/B_1}$; B_1 is the magnetic field at the point of origin of the escape; and B is the local magnetic field. The maximum value of the transverse electron velocity is defined as

$$v_{\perp \max}^2 = \frac{\beta}{1 - \beta} v_{||}^2, \quad \beta = B/B_1. \quad (8.1.11)$$

Assuming that, at the point with magnetic field B_1 , the photoelectron flux has an isotropic and monotonic distribution function

$$f(v_{\perp}, v_{\parallel}) = k \exp\left(-\frac{v_{\parallel}^2 + v_{\perp}^2}{v_0^2}\right) \quad (8.1.12)$$

for a one-dimensional distribution function, we obtain

$$F(v_{\parallel}) = kv_0^2 e^{-\frac{v_{\parallel}^2}{v_0^2}} \left(1 - \frac{\beta}{1-\beta} e^{\frac{v_{\parallel}^2}{v_0^2}}\right). \quad (8.1.13)$$

The function (8.1.13) has a maximum at the point

$$v_m^2 = v_0^2 \left(\frac{1}{\beta} - 1\right) \ln(1 - \beta)^{-1} \quad (8.1.14)$$

and a maximum growth rate

$$\gamma_{\max} = k \frac{\pi^2 \omega_r}{n_0} v_0^3 \sqrt{1 - \beta}. \quad (8.1.15)$$

Thus, in a collisionless plasma, superthermal electrons escaping along diverging inhomogeneous magnetic field lines, even if their initial distribution is a monotonically decreasing one in energy, brings the system to an unstable state.

The above derivation of the growth rate is based on a number of crude approximations. It is assumed that, above a certain level, the photoelectrons move without collisions only under the action of an inhomogeneous magnetic field. It is also ignored that, in regions where the electron energy is small, the distribution function depends strongly on the thermal electron density and temperature. Moreover, for closed magnetic field lines, it is necessary to consider the influence of the flux from the magneto-conjugate point and that a fraction of the geomagnetically trapped photoelectrons have reflection points in the region of escape under consideration. To answer the question of the possibility of instability, an accurate calculation of the photoelectron distribution function is needed in this case. Such solutions were obtained in Chap. 7 by only considering photoelectron collision processes. Because the kinetic equation is solved with variables: energy and pitch angle (see kinetic equation (2.5.2)), it is necessary to transfer in (8.1.9) from $(v_{\perp}, v_{\parallel})$ to (E, μ) . As a result, we have

$$\gamma_1 = \frac{3.4 \times 10^{-8} \pi^2 \omega_r E_{res}}{n_0} \left[\frac{1}{2} \int_{E_{res}}^{\infty} \frac{1}{E^{3/2}} \frac{\partial \Phi}{\partial \mu} dE - \frac{\Phi(E_{res}, \mu=1)}{\sqrt{E_{res}}} - \frac{\Phi_M(E_{res})}{\sqrt{E_{res}}} \right]. \quad (8.1.16)$$

Here $\Phi(E, \mu)$ is the spectrum of photoelectrons related to the distribution function by the relationship

$$\Phi(E, \mu) = \frac{2E}{m_e^2} f(v_\perp, v_\parallel); \quad (8.1.17)$$

Φ_M is the spectrum of thermal electrons; and $E_{res} = mv_{res}^2/2$. We use the dimensions $[\Phi] = \text{cm}^{-2}\text{eV}^{-1}\text{ster}^{-1}\text{s}^{-1}$; $[E, E_{res}] = \text{eV}$; $[n_0] = \text{cm}^{-3}$; $[\gamma] = \text{s}^{-1}$.

Maximum growth rates were determined by considering all possible values of E_{res} . The analysis showed that, in the case in which both magneto-conjugate ionospheres are illuminated by the Sun, the ionospheric plasma is stable at all upper ionospheric heights ($\gamma < 0$). In this case the collisionless Landau damping (see Chap. 3) is always larger than the excitation of potential oscillations by the anisotropic part of the spectrum (the first and second terms in square brackets of (8.1.16)). In the case of an asymmetric illumination, a plasma instability at topside ionospheric heights (in the illuminated region) can take place on time intervals comparable with the time of transition of the electron gas temperature from nightside to dayside values. For mid-latitudes, this is approximately half an hour.

Let us consider in greater detail the instability mechanism. The point here is that the plasma at topside ionospheric heights is rather dense, and collisions prevent the formation of a photoelectron beam by the inhomogeneous magnetic field. A different formation mechanism for nonmonotonicity of the energy distribution function of photoelectrons “works” there, which is associated with the pitch-angle diffusion of escaping photoelectrons. Indeed, since the cross section of Coulomb scattering is inversely proportional to the square of the particle’s energy, the degree of an isotropization of the low-energy electrons is larger than that of the high-energy electrons. As a result, in the longitudinal direction $\mu = 1$ in the interval of small energy values (as shown by numerical calculations at $E < 3 \text{ eV}$), a portion with a positive slope of the electron distribution function forms. This is illustrated in Fig. 8.1, where the electron distribution function (at 1,000-km height, at $n_e = 10^4 \text{ cm}^{-3}$) is represented as the superposition of the Maxwellian and non-Maxwellian electron distribution functions at two different values of the temperature T_e . The nonmonotonic part persists, as the calculations show, up to $T_e \approx 2,500 \text{ K}$. Clearly, such a character of the energy distribution of electrons, which travel strictly along the magnetic field line, affects the character of the one-dimensional function $F(v_\parallel)$, although this relationship is not quite unambiguous. Quantitative analysis of the mechanisms concerned appears to be possible only in the case of a numerical solution of the kinetic equation (2.5.2). Table 8.1 presents the values of growth rates calculated by (8.1.16).

In the calculations, it was assumed that the magneto-conjugate ionosphere is not illuminated. The first row of the table gives the values of growth rates calculated without taking into account collisions above 300 km as well as without taking into account the influence of the plasmasphere and the magneto-conjugate ionosphere (the absence of a downward flux of suprathermal electrons on the upper boundary of the ionosphere). Consequently, in this case of the calculation, the approximations were simulated, which forms the basis for the derivation of (8.1.15). It is interesting to compare the numerical calculation with the analytical estimate of (8.1.15). By specifying the parameters of the photoelectron spectrum k and v_0 in accordance

Fig. 8.1 Nonmonotonicity of the energy distribution function of photoelectrons.
1 Maxwellian electrons,
2 non-Maxwellian electrons,
3 superposition

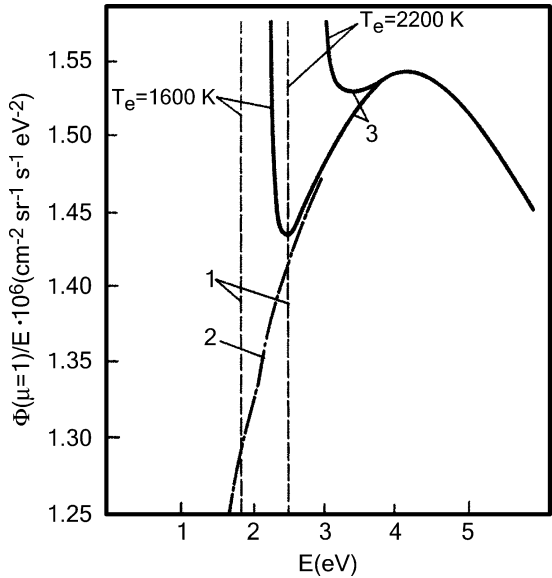


Table 8.1 Values of growth rates calculated by (8.1.16)

Calculation condition	Values of γ_1 (s^{-1}) at T_e (K^0)			
	1,600	1,800	2,000	2,200
Analytically estimated in accordance with (8.1.15)	$+10^3$	$+10^3$	$+9.6 \times 10^2$	$+8.4 \times 10^2$
Including all factors	$+10^2$	$+9.1 \times 10^1$	$+5.7 \times 10^1$	-7.0×10^1
Case of a homogeneous magnetic field	-1.2×10^2	-1.2×10^2	-1.6×10^2	-2.8×10^2
Neglecting pitch-angle diffusion	-10^2	-1.1×10^2	-1.4×10^2	-2.7×10^2

with the results of numerical calculation, at $\beta = 0.7$ (between 300- and 1,000-km heights) and $n_0 = 10^4 \text{ cm}^{-3}$ at 1,000-km height, from (8.1.15) we obtain $\gamma_{\max} \approx 2.5 \times 10^3 s^{-1}$, which is somewhat higher than the value from Table 8.1 at small T_e when it is possible to neglect the distortion of the superthermal electron spectrum.

Taking into consideration that (8.1.15) is an estimate of a “maximum,” the agreement of analytical and numerical results can be regarded as satisfactory.

The next rows of Table 8.1 present the values calculated for the case of a combined account of the ionosphere–plasmasphere transport of photoelectrons that was discussed in Chap. 7. As is evident at the background electron temperature $T_e < 2,500 \text{ K}$, typical of the outer ionosphere in the early morning hours (Khazanov 1979), the upward flux of photoelectrons is capable of generating unstable plasma oscillations at heights $h \sim 500\text{--}1,000 \text{ km}$ ($\gamma > 0$). The lower boundary of this region is determined by the formation of a beam distribution of escaping photoelectrons, and the upper boundary is determined by a rise of electron temperature.

Table 8.2 Growth rates of Langmuir oscillations

E_{res} (eV)	Values of growth rates of Langmuir oscillations γ_l (s^{-1}) at height (km)						
	600	720	1,000	1,150	1,350	1,600	2,000
1.3	-5.6×10^2	-6.4×10^2	-3.9×10^2	-3.0×10^2	-2.3×10^2	-2.1×10^2	-1.8×10^2
1.6	-7.9×10^2	2.0×10^{-1}	4.0×10^2	5.2×10^2	6.0×10^2	6.3×10^2	6.4×10^2
2.0	-9.9×10^1	-7.5×10^0	4.6×10^2	6.3×10^2	7.7×10^2	8.6×10^2	9.1×10^2
2.5	-2.5×10^2	-1.7×10^2	1.9×10^2	4.1×10^2	6.3×10^2	8.0×10^2	9.4×10^2
	<i>Values of $\gamma_{st}(\text{s}^{-1})$</i>						
—	7.5×10^1	3.7×10^1	9.0×10^0	8.5×10^0	7.5×10^0	6.0×10^0	3.0×10^0

It should be noted that the appearance of values of $\gamma > 0$ is a consequence of the combined action of the two above-analyzed transformation mechanisms for the distribution of escaping electrons (focusing of the flux in an inhomogeneous magnetic field and the effective pitch-angle diffusion of low-energy photoelectrons), because neither of them individually is sufficient for the formation of an unstable distribution (the third and fourth rows of Table 8.1). Note also that the most favorable conditions for the production of the plasma instability in the topside ionosphere, as the calculations indicate, occur near the main ionospheric trough but not within the trough itself.

In the polar ionosphere where the geomagnetic field lines are open, and where there occurs simply an escape of the photoelectrons to open tubes with an inhomogeneous magnetic field, a plasma instability occurs at heights $h \geq 500$ km. Table 8.2 gives the growth rates of Langmuir oscillations for the conditions $\varphi = 70^\circ\text{N}$, $F10.7 = 150$, equinox, $t = 12^h$ LT, and $n_0(250 \text{ km}) = 3 \times 10^5 \text{ cm}^{-3}$. Growth rates were determined from (8.1.16) by taking into account the distribution function of thermal electrons, for which the effective temperature at heights above 1,000 km was taken as equal to 2,000 K. Here we also show the dependence of γ_1 versus E_{res} . One can see that maximum values of γ_1 almost always correspond to the condition $E_{res} = 2 \text{ eV}$. Table 8.2 gives also the values of collisional damping of Langmuir waves $\gamma_{st} \approx \nu_e/2$ at the same heights. Note that, at heights corresponding to the lower part of the F-region, photoelectron fluxes (Oran and Strickland 1978) directed downward (earthward) also exist. However, a strong collisional damping, caused mainly by the scattering of electrons off neutral components of ionospheric plasma, totally suppresses the generation of electrostatic oscillations by these fluxes $\gamma < \gamma_{st} \approx \nu_e/2$.

In summarizing the above discussion, we can draw the conclusion that photoelectron fluxes are able to give rise to a plasma instability in the topside ionosphere in middle (in the morning hours) and high latitudes, which, in turn, can be the cause of the generation of electromagnetic VLF signals recorded by ground-based receivers. This agrees with observations of the morning maximum of low-frequency noise emissions from the topside ionosphere (Vershinin et al. 1979). Also, a new type of VLF emissions, called Discrete Plasmaspheric Emissions, have been observed at $L = 4$ by DE-1, ISEE-1 and ISIS-2 (Paulsen and Inan 1988), and by Interkosmos (Boskova et al. 1986). These emissions are mainly associated with the early morning local time sector. We believe that the possible explanation of these events is the instability of photoelectron fluxes in the outer ionosphere and plasmasphere.

8.1.3 Cyclotron Resonance

Let us consider a nonmonotonic but isotropic electron distribution function. Such a situation occurs in the ionosphere at 100- to 300-km heights, where the photoelectron spectrum shows a number of peaks around the energy of approximately 25 eV,

caused by the resonance structure of the ion production processes, and a minimum at energies of approximately 3 eV associated with an effective energy loss by photoelectrons to the excitation of vibration levels of molecular nitrogen (see Chap. 7 for details). It is known that a plasma with an isotropic distribution of particles is stable for disturbances that propagate along magnetic field lines. For “skew” waves, the imaginary part of the dielectric permittivity can be derived from (8.1.1) and presented as (Gefan et al. 1985a)

$$\text{Im}\varepsilon = \frac{2\pi\omega_p^2\omega_r}{|k_{||}|k^2n_0} \sum_n \int_{E_n} dE J_n^2 \left(\frac{k_{\perp}}{\omega_B} \sqrt{\frac{2(E - E_n)}{m}} \right) \frac{d}{dE} f(E), \quad (8.1.18)$$

where

$$E_n = \frac{m(\omega_r - n\omega_B)^2}{2k_{||}^2} \quad (8.1.19)$$

and ω_r is one of the eigen frequencies of plasma oscillations. Here it is taken into consideration that $f(\mathbf{v}) = f(v_{||}^2 + v_{\perp}^2)$, and therefore $\partial f / \partial v_{\perp}^2 = \partial f / \partial v_{||}^2$.

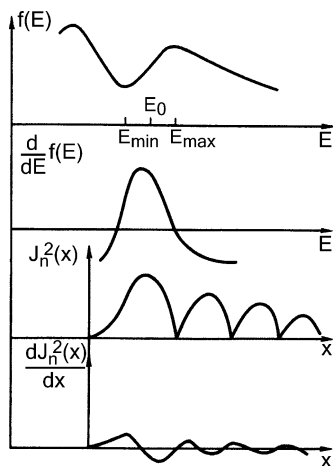
As is evident from (8.1.18), the value and the sign of $\text{Im}\varepsilon$ depends on the relative position on the axis E of the integrand functions, i.e., on the relation between the components of the wave vector components $k_{\perp}$ and $k_{||}$. Obviously, the growth rate γ of the oscillations will be positive and maximal if the portion of the positive derivative of the electron distribution function coincides with the first maximum of the function $J_n^2(x)$ as shown in Fig. 8.2. As is evident from (8.1.8) and (8.1.18), a decrease of $k_{||}$ directly leads to an increase of γ . On the other hand, when $k_{||} \rightarrow 0$, there is an increase of $E_n, f(E_n)$ and the integrand decreases. Therefore, a maximum of growth rate is reached when the difference $\omega_r - n\omega_B$ is minimal. Optimal conditions are realized for the high-frequency upper-hybrid branch $\omega_r \approx \omega_1$ when $k_{\perp} \gg k_{||}$ when the distribution of the waves is quasitransverse. In this case the equation for $\text{Re } \varepsilon$ simplifies substantially and, in the collisionless case, has the form (Zheleznyakov 1977)

$$\text{Re } \varepsilon = 1 - \sum_{n=1}^{\infty} \frac{2n^2\omega_p^2 I_n(z) e^{-z}}{z(\omega_r^2 - n^2\omega_B^2)}. \quad (8.1.20)$$

In the limit of $z \ll 1$, the equation defines high-frequency waves ($\omega > \omega_B$) at the upper hybrid frequency $\omega_1 = \sqrt{\omega_p^2 + \omega_B^2}$ and an infinite set of electron-cyclotron harmonics $n\omega_B$. For values of $z \leq 1$, the solution to the equation $\text{Re } \varepsilon = 0$ is a continuation of the above-mentioned branches of plasma eigen oscillations for which, however, ω_r may deviate from ω_1 and $n\omega_B$.

By solving the equation $\text{Re } \varepsilon = 0$ ($\text{Re } \varepsilon$ is determined from (8.1.20)) and using (8.1.18) and (8.1.19) for particular ionospheric parameters, one can determine ω_r

Fig. 8.2 The conditions for optimal phasing of the functions $f(E)$ and $dJ_n^2(x)/dx$, under which growth rates of the electrostatic oscillations of plasma are largest



and γ . In (Ivanov et al. 1980), numerical growth rate calculations for electrostatic eigen oscillations were carried out for the height range 100–300 km. In this case, use was made of the distribution function of photoelectrons from (Khazanov 1979) where the kinetic equation for photoelectrons was solved numerically considering all possible collisions. These spectra, as shown in Chap. 7, are in good agreement with both theoretical (Khazanov and Liemohn 1998) and experimental (Mukai and Hirao 1973; Lee et al. 1978) spectra. At optimal values of the wave vector ($k_{\perp} \geq 10k_{\parallel}$, $k \approx 0.5 \text{ cm}^{-1}$) and under typical conditions ($n_0 \approx 1.7 - 2.5 \times 10^5 \text{ cm}^{-3}$; $T_e \approx 0.09 - 0.11 \text{ eV}$; $\omega_B = 7 \times 10^6$; $\varphi = 45^\circ$; $t = 12^h \text{ LT}$; $F_{10.7} = 80 - 150$), the maximum values of γ do not exceed $3 \times 10^2 \text{ s}^{-1}$ (Ivanov et al. 1980). Such values of γ were obtained only for the upper-hybrid mode when it is excited by the non-monotonicity of the photoelectron spectrum in the region around 3 eV; the electron–cyclotron harmonics have smaller growth rates.

Growth rates of plasma oscillations, caused by the nonmonotonicity of the photoelectron spectrum at 15- to 30-eV energies, do not exceed approximately 10 s^{-1} . It should be noted that an ionospheric plasma instability, caused by the nonmonotonicity of the photoelectron distribution near 3 eV, was pointed out in Blomberg (1975), where growth rates of the order of $10^3 - 10^4 \text{ s}^{-1}$ were obtained. This result was, however, obtained under the assumption of a cold background plasma $T_e = 0$.

Because, at the heights (100–300 km) under consideration, the ionospheric plasma is essentially a collisional one, a true instability is possible only if the value of the growth rate γ is larger than the value of the collisional damping γ_{st} of the potential plasma oscillations. From (8.1.7) one can readily determine the collisional damping for the upper-hybrid mode (Ginzburg and Rukhadze 1975)

$$\gamma_{st} = \nu_e/2. \quad (8.1.21)$$

For 180- to 200-km altitudes, where values of γ are maximal, $\gamma_{st} \approx 4 \times 10^2 \text{s}^{-1}$ (Gurevich 1978) and $\gamma < \gamma_{st}$. Thus, the oscillations in the ionosphere under consideration actually are on the instability threshold, and their increase in time in the presence of certain variations in geophysical and heliophysical conditions is, in principle, possible. Thus, an instability is possible when solar activity increases, $F_{10.7} > 150$, and when at these heights the density of molecular nitrogen increases, which leads to an increase in nonmonotonicity of the photoelectron spectrum. This instability also is possible in the high-latitude ionosphere where, in addition to photoionization, there occurs an impact ionization by precipitating 1- to 10-keV electrons. These results are discussed in the following sections.

Note that the heights between 180 and 200 km provide the best conditions for the development of the instability because, at lower heights, there is an abrupt increase in the rate of electron collisions with neutrals, and, at higher altitudes, the nonmonotonicity of the photoelectron distribution function disappears. As far as the branch of potential plasma oscillations with frequency ω_2 (from (8.1.15)) is concerned, the largest values of γ_{st} , as shown in (Ivanov et al. 1982b), do not exceed 10s^{-1} ($\theta \approx 80^\circ$, $k \approx 7 \times 10^{-2} \text{cm}^{-1}$), and this rules out the possibility of its excitation because the collisional damping of this branch $\gamma_{st} \approx \nu_e$ (Ginzburg and Rukhadze 1975) has, at heights of approximately 200 km, the value of $4 \times 10^2 \text{s}^{-1}$.

8.1.4 Double Resonance

A plasma instability caused by the nonmonotonicity of the electron distribution function (an instability of the nonresonance type) can occur in the case of strictly transverse propagation of electron–cyclotron waves with respect to B_0 (Korablev 1967).

When $k_{\parallel} = 0$, the general dispersion equation (8.1.2) can be written as

$$1 - \sum_{n=1}^{\infty} \frac{2n^2 \omega_p^2 I_n(z) e^{-z}}{z(\omega_r^2 - n^2 \omega_B^2)} + \frac{2\pi \omega_p^2 \omega}{nk^2} \sum_{n=-\infty}^{\infty} \int dv_{\parallel} dv_{\perp} \frac{\partial f}{\partial v_{\perp}} \frac{J_n^2\left(\frac{k_{\perp} v_{\perp}}{\omega_B}\right)}{\omega - n\omega_B} = 0. \quad (8.1.22)$$

By representing $\omega = n\omega_B + i\gamma_n$, after some manipulation similar to that done in (Korablev 1967), (8.1.22) can be solved for γ_n :

$$\gamma_n = -in\omega_B \left\{ \frac{1}{4} \left[\frac{\omega_B^2(n^2 - 1)z}{4n^2 \omega_p^2 I_n(z) e^{-z}} - \frac{n^2 - 1}{n^2} \right]^2 + \frac{\omega_B^2(n^2 - 1)z}{4n^2 \omega_p^2 I_n(z) e^{-z}} \times \left[\frac{\omega_p^2}{\omega_B^2} \frac{I_n(z)}{z} e^{-z} + \frac{2\pi \omega_p^2}{n_0 k^2} \int_0^{\infty} dv f(v) J_{2n}\left(2 \frac{kv}{\omega_B}\right) \right] \right\}^{\frac{1}{2}}. \quad (8.1.23)$$

A necessary condition for instability is that the expression under the radical be negative. Such wave numbers are optimal when the first term under the radical goes to zero. At small z , this condition corresponds to a double plasma resonance when

the upper-hybrid frequency ω_1 is close to one of the harmonics of the electron gyrofrequency $n\omega_B$. In this case, (8.1.23) takes the form:

$$\gamma_n = -in\omega_B \frac{n^2 - 1}{n^2 \sqrt{2}} \sqrt{\frac{I_n(z)}{z} e^{-z} - \frac{2\pi\omega_B^2}{n_0 k^2} \int_0^\infty dv f(v) J_{2n}\left(2 \frac{kv}{\omega_B}\right)}. \quad (8.1.24)$$

An analysis of such instability in the ionospheric plasma was carried out by Kaladze and Krinberg (1978), where the electron distribution function was represented by a thermal Maxwellian plus one or a number of narrow photoelectron peaks around 20–30 eV. In this case it was found that, under certain conditions in the ionospheric plasma, a nonresonance instability can occur with very large growth rates γ_n , of the order of 10^4 s^{-1} . In addition to a number of peaks, however, the photoelectron distribution function shows considerable background (see Figs. 7.11 and 8.3a). The distribution function of this background is formed by photoelectrons that have experienced inelastic collisions with neutral ionospheric components.

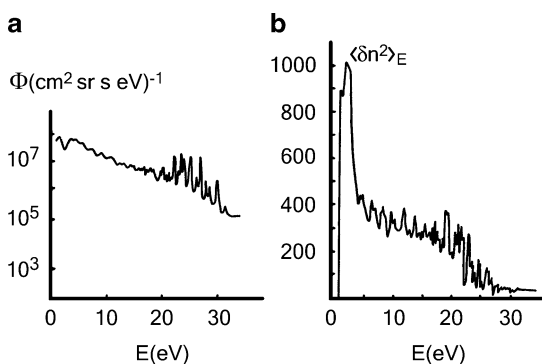
Numerical calculations (Ivanov et al. 1982a) on the basis of photoelectron spectra (Khazanov 1979) showed that an instability of this type in the dayside ionosphere at heights of the lower part of the F-region is not realized. Indeed, let the photoelectron distribution function be represented as the sum of a Maxwellian (with density n_e and mean energy E_e) and one peak with a distribution of the form (Kaladze and Krinberg 1978):

$$f(v) = \frac{n'}{v^2} \delta(v - v_0), \quad (8.1.25)$$

where n' is the peak photoelectron density ($n' \leq 0.1n_e$). Using the asymptotic distribution

$$J_{2x}(x) = \sqrt{\frac{2}{\pi x}} \cos(x - n\pi - \pi/4),$$

Fig. 8.3 The spectrum of high-frequency electron density fluctuations at $h = 200$ km as obtained by taking into account characteristics of the energy distribution of photoelectrons ($E = m_e \omega_p^2 / (2k_{\parallel}^2)$)



one can find that the largest negative value of $J_{2x}(x)$ is

$$J_{2n} \approx -\frac{1}{\pi} \sqrt{\frac{2}{n+1}} \text{ if } 2 \frac{kv}{\omega_B} \approx (n+1)\pi.$$

Let the peak lie at the energy $E_0 = 25 \text{ eV}$ ($v_0 \approx 3 \times 10^8 \text{ cms}^{-1}$). In this case, for $\omega_B = 7 \times 10^6$, we get $k = n \times 3 \times 10^{-2} \text{ cm}^{-1}$. Let us determine the values at which the instability appears. To do this, one has to solve the inequality

$$e^{-z_e} \frac{I(z_e)}{z_e} < \frac{12}{n^2 \sqrt{n} \pi^2} \frac{n'}{n_e}. \quad (8.1.26)$$

Because $z_e = k^2 v_e^2 / \omega_B^2 \geq 1$, then, for $n = 3 - 10$, the inequality (8.1.26) is satisfied for $E_e < 1 \text{ eV}$, whereas at these heights $E_e \geq 3 \text{ eV}$. Thus, the collisionless damping due to the background photoelectrons inhibits the instability. On the other hand, an instability of the nonresonance type $\omega_1 = n\omega_B$ appears to be able to occur in the morning hours at heights around 200 km on timescales shorter than the timescales of inelastic collisions of the resulting photoelectrons with ionospheric neutrals ($\sim 1 \text{ min}$).

The possibility that oscillations can be excited by the nonmonotonicity of the photoelectron distribution in the energy range 2–4 eV caused by the excitation of vibration levels of molecular nitrogen was investigated by Basu et al. (1982), where the possibility of a weak instability ($\gamma_n \geq \gamma_{st}$) was stated at the heights 100–130 km.

Calculations of Gefan et al. (1985c), using the photoelectron spectra from Khazanov (1980), showed that, at ionospheric heights above 120 km, excitation growth rates of the oscillations with $k_{||} = 0$ are less than the corresponding collisionless damping (the first term in (8.1.24)). At lower altitudes, the excitation of oscillations is dominated by collisional damping whose magnitude, with due regard for pair collisions in the kinetic equation in the BGK form, is determined by $\gamma_{st} \approx \nu_e$. The difference between the results obtained by Gefan et al. (1985c) and Basu et al. (1982) is attributed to Basu et al. (1982) using values for the electron gas temperature that were too low (virtually, nighttime profiles) and defining the collisional damping as $\gamma_{st} \approx \nu_e/4$.

8.2 High-Frequency Fluctuations in the Ionospheric Plasma

8.2.1 Fluctuations of a Non-equilibrium Stable Plasma

At present the theory of fluctuations in a stable plasma is rather well developed and, in principle, enables a full description of the motion of electrons in equilibrium and weakly non-equilibrium plasmas (Akhiezer et al. 1975).

Let us consider electron density fluctuations of a magnetized collisional plasma under specific ionospheric conditions. Equilibrium is established between the electron distribution in velocity space and the distribution of plasma eigen oscillations in wave vector space, which permits, on the basis of fluctuation–dissipation theory, their quantitative correlation to be inferred.

Using the kinetic equation for photoelectrons with the collision integral in the BGK form (Bhatnagar et al. 1954), and by representing the electron distribution function as the superposition of a Maxwellian f_e (thermal electrons) function and an arbitrary f (photoelectrons), one can obtain the fluctuation spectrum in the form (Akhiezer et al. 1975)

$$\langle \delta n^2 \rangle_{k\omega} = \frac{2\pi}{|\varepsilon_e|^2} \sum_n \int d\mathbf{v} J_n^2(x) f(\mathbf{v}) \delta(\omega - k_{\parallel} v_{\parallel} - n\omega_B) - \frac{2v_e}{|\varepsilon_e|^2} \Psi, \quad (8.2.1)$$

where $x = k_{\perp} v_{\perp} / \omega_B$;

$$\Psi = \sum_n \sum_{n'} \frac{I_n(z_e) e^{-z_e}}{\omega - n\omega_B} W(\beta) \left\{ 1 - \frac{\omega I_{n'}(z_e) e^{-z_e}}{\omega - n'\omega_B} W(\beta') \right\}. \quad (8.2.2)$$

From (8.2.1) it is evident that the fluctuation spectrum depends on the electron distribution function. Note that in the denominator of (8.2.1) the imaginary part ε_e , defined by (8.1.18), depends on the electron distribution function.

Let us consider potential oscillations that propagate along the magnetic field with $\omega/k_{\parallel} \gg v_e$. In this case (8.2.1) simplifies substantially to become

$$\langle \delta n^2 \rangle_{k\omega} = \frac{1}{|\varepsilon_e|^2} \left\{ 2\pi \int d\mathbf{v} f(\mathbf{v}) \delta(\omega - \mathbf{k}\mathbf{v}) + \frac{2v_e}{\omega^2} n_0 \frac{k^2 v_e^2}{\omega^2} \right\}. \quad (8.2.3)$$

In a plasma with an isotropic distribution function (e.g., at $h < 300$ km in the ionosphere), high-frequency density fluctuations are governed by the formula

$$\langle \delta n^2 \rangle_{k\omega} = \frac{2\pi^2 \frac{m_e}{k} \int_{E_*}^{\infty} dE \frac{\Phi(E)}{E} + \frac{2v_e n_e}{\omega^2} \frac{k^2 v_e^2}{\omega^2}}{\left(1 - \frac{\omega_p^2}{\omega^2} \right)^2 + \left(\frac{v_e}{\omega_p} + \pi^2 \frac{m_e^2 \omega \omega_p^2}{k^3 n_0} \frac{\Phi(E_*)}{E_*} \right)^2}. \quad (8.2.4)$$

Here $\Phi(E)$ is the electron spectrum related to the distribution function by the relationship $\Phi(E) = 2Ef(\mathbf{v})/m^2$; $E_* = m_e v_*^2/2$; $v_* = \omega_p/k$. The largest contribution to (8.2.4) is from eigen oscillations of the ionospheric plasma with frequencies $\omega = \pm\omega_p$; therefore, by availing ourselves of the relationship

$$\lim \frac{\text{Im} \varepsilon_e}{|\text{Re} \varepsilon_e|^2 + |\text{Im} \varepsilon_e|^2} = \pi \delta(\text{Re} \varepsilon_e), \quad (8.2.5)$$

when $\text{Re} \varepsilon_e \gg \text{Im} \varepsilon_e$ ($k^2 r_D^2 \ll 1$), and using $\langle \delta n^2 \rangle_k = \int d\omega \langle \delta n^2 \rangle_{k\omega} / 2\pi$, one can obtain a spectrum of density fluctuations in the form

$$\langle \delta n^2 \rangle_k = \frac{\pi^2 \frac{m_e \omega_p}{2k} \int_{E_*}^{\infty} dE \frac{\Phi(E)}{E}}{\frac{v_e}{\omega_p} + \pi^2 \frac{m_e^2 \omega_p^3}{k^3 n_0} \frac{\Phi(E_*)}{E_*}}. \quad (8.2.6)$$

When $k^2 r_D^2 \ll 1$, i.e., when $E_* \gg T_e$, the main contribution to $\langle \delta n^2 \rangle_k$ is made by photoelectrons, and $\Phi(E)$ in (8.2.6) is a spectrum of the photoelectrons. The quantity

$$\pi^2 \frac{m_e^2 \omega_p^3}{k^3 n_0} \frac{\Phi(E_*)}{E_*}$$

in the denominator is proportional to the collisionless Landau damping of plasma waves in an isotropic plasma.

In the region $kr_D \geq 1$, it is only necessary to take into account thermal electrons ($f_T \gg f_e$) and particle-particle collisions; therefore, from (8.2.4) in the region of eigen frequencies $\omega \approx \omega_p$, it follows that

$$\langle \delta n^2 \rangle_{k\omega_p} \approx \frac{2}{v_e} k^2 r_D^2$$

and $\langle \delta n^2 \rangle_{k\omega} \sim v_e$ when $\omega \neq \omega_p$. The last relationship shows that it is the collisions (close particle encounters) that are the fluctuations with respect to a homogeneous (on average) spatial distribution of particles.

If the photoelectron spectrum is represented by a Maxwellian distribution with density n_e and mean energy E_e , then from (8.2.6) we obtain

$$\langle \delta n^2 \rangle_k^e = \frac{1}{2} n_0 \frac{E_e}{E_*} \quad \text{when } E_* \gg E_e.$$

Then, as in the case of the equilibrium plasma, from (8.2.6) one can obtain

$$\langle \delta n^2 \rangle_k^T = \frac{1}{2} n_0 \frac{T_e}{E_*}, \quad E_* \gg T_e. \quad (8.2.7)$$

At ionospheric heights below 300 km, where $E_e \approx 3 - 20 \text{ eV}$ and $T_e \approx 0.1 - 0.3 \text{ eV}$, the level of electron density fluctuations, as follows from the comparison of (8.2.6) with (8.2.7), can exceed the thermal level by two orders of magnitude. The existence of an increased level of high-frequency oscillations of

ionospheric plasma in the presence of photoelectrons was pointed out by Perkins and Salpeter (1965), where it was shown that the plasma non-equilibrium condition can account for the observed enhancement (up to two orders of magnitude as compared with the expected enhancement) in intensity of the electron component of the reflected signal when sounding the daytime ionosphere by incoherent scatter facilities. In Perkins and Salpeter (1965), the non-equilibrium distribution function of electrons in the dayside ionosphere was specified such that the relationship

$$\frac{mv_*}{E_e(E_*)} = \frac{\partial}{\partial v_*} [\ln f(v_*)] \quad (8.2.8)$$

was satisfied. In what follows, instead of the designation of the mean effective electron energy $T_p(E_*)$ adopted in Perkins and Salpeter (1965), we will introduce $E_e(E_*)$.

In an equilibrium plasma, we have $E_e = T_e$. In Yugvenson and Perkins (1968), Cicerone and Bowhill (1971), and in the references cited later, for example, Cicerone (1974) and Sheffield (1978), under the assumption of (8.2.8), incoherent scatter facilities were used to investigate the relationship $\langle \delta n^2 \rangle_k^e / \langle \delta n^2 \rangle_k^T = E_e(E_*)/T_e$ in the dayside ionospheric plasma. It was shown that the dependence of E_e on E_* , i.e., on k , traces the distribution function of photoelectrons and defines their mean energy. Figure 8.3a shows a photoelectron spectrum with fine structure for 200-km altitude simulated by Khazanov (1979) with 0.2-eV energy resolution and, based on this, calculated high-frequency fluctuations of electron density presented in Fig. 8.3b. The spectrum of electron density fluctuations was obtained without assuming the relationship (8.2.8), but on the condition that the distribution fluctuation can be divided into parts separately describing the thermal electrons and the photoelectrons. It is evident that the features of the energy distribution of the photoelectrons are reflected in the distribution of the fluctuations. Such a relation encourages one to hope that the high-frequency ionospheric electron density fluctuations measured by incoherent scatter facilities can serve as a source of information about the angular and energy distribution of photoelectrons.

Thus, in the stable ionospheric plasma in the presence of photoelectrons, firstly, there is a substantial increase of the level of high-frequency fluctuations in the region of wave numbers $kr_D \ll 1$ and, secondly, the fluctuation spectrum itself has a complicated structure caused by the peculiarities of the superthermal electron distribution.

8.2.2 Coulomb Scattering in a Weakly Non-equilibrium Plasma

Usually, the Coulomb relaxation of superthermal electron spectra in the ionosphere is represented on the basis of a linearized collision integral in the Landau form

(see Chap. 2 for details). For example, for the description of photoelectron scattering by thermal electrons, this collision term can be written as

$$S_{ee}(E) = A_1 n_e \frac{v}{E} \left[\frac{\partial}{\partial E} \left(f + T_e \frac{\partial f}{\partial E} \right) \right] + A_2 n_e \frac{v}{2E^2} \frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial f}{\partial \mu} \right], \quad (8.2.9)$$

where $A_{1,2} = 2\pi e^4 \ln \Lambda_{1,2}$ ($\ln \Lambda$ is a Coulomb logarithm). The first term in (8.2.9) corresponds to energy losses of superthermal electrons, and the second term corresponds to pitch-angle scattering. The quantities $\ln \Lambda_{1,2}$, involved in $A_{1,2}$, are usually considered equal and are calculated by taking into account energy losses to the Cherenkov emission of plasma waves and quantum effects (Schunk and Hays 1971). The estimates made in Swartz et al. (1971), show that taking the polarization losses into account leads to an increase of the value of $\ln \Lambda$ by approximately 30%. Such a treatment is not complete, however, because a superthermal electron is represented as a solitary trial particle. In fact, superthermal electrons form an ensemble of particles. Therefore, in addition to taking into account the polarization friction that renormalizes the value of A_1 , it is also necessary to take into account the diffusion of superthermal electrons in velocity space caused by the scattering due to non-equilibrium, although stable, electron density fluctuations. Thus, there arises the question of renormalizing the value of A_2 .

Note that the friction and electron diffusion processes in a non-equilibrium plasma were considered by Akhiezer et al. (1975), and the diffusion in the turbulent plasma of fast electrons was treated by Alexandrov et al. (1988). In the stable plasma, the diffusion tensor has the form

$$D_{ij} = 2e^4 \int d\mathbf{p}' f(\mathbf{v}') \int d\mathbf{k} \frac{k_i k_j \delta(\omega - \mathbf{k}\mathbf{v}')}{k^2 |\varepsilon_e(\omega, \mathbf{k})|^2} \Big|_{\omega=\mathbf{k}\mathbf{v}}, \quad (8.2.10)$$

where $\mathbf{p} = m\mathbf{v}$, f is the electron distribution function, and ε_e is the plasma dielectric permittivity; and the imaginary part of ε_e depends on the electron distribution function. Unlike electron-ion (or electron-electron) collisions, where integration over k is performed from $k_{\min} = k_D = 1/r_D$ to $k_{\max} \approx mv^2/2e^2$ determined by the minimal distance between colliding particles, it is necessary to integrate over wave numbers from 0 to k_D in order to take into account the interaction of the electrons with the plasma eigen oscillations. For eigen oscillations of a plasma $\text{Re } \varepsilon_e(\omega_p, \mathbf{k}) = 0$, in the transparent region $\text{Im } \varepsilon_e \rightarrow 0 (k^2 r_D^2 < 1)$, the relationship

$$\frac{1}{|\varepsilon_e|^2} = \frac{1}{\text{Im } \varepsilon_e} \frac{\text{Im } \varepsilon_e}{|\text{Re } \varepsilon_e|^2 + |\text{Im } \varepsilon_e|^2} \approx \frac{1}{\text{Im } \varepsilon_e} \delta(\text{Re } \varepsilon_e)$$

is satisfied, where

$$\text{Re } \varepsilon_e = 1 - \frac{\omega_p^2}{\omega^2}, \quad \delta(\text{Re } \varepsilon_e) = \frac{\omega_p}{2} \{ \delta(\omega - \omega_p) + \delta(\omega + \omega_p) \}.$$

The tangential component of the diffusion coefficient assumes the form

$$D_t = \frac{8\pi^3 e^4}{m^2} \int_0^{k_D} dk \int_{-1}^1 d\mu (1 - \mu^2) \times \frac{\int d\mathbf{v}' (f_e + f) \delta(\mathbf{k}\mathbf{v} - \mathbf{k}\mathbf{v}')}{\frac{4\pi e}{mk^2} \left\{ \frac{\omega_p}{v_e^2} \int d\mathbf{v}' f_e \delta(\mathbf{k}\mathbf{v} - \mathbf{k}\mathbf{v}') - \int d\mathbf{v}' \delta(\mathbf{k}\mathbf{v} - \mathbf{k}\mathbf{v}') \mathbf{k} \frac{\partial}{\partial \mathbf{v}'} f \right\}} \quad (8.2.11)$$

If we now introduce the collision frequency as $\nu = D_t/\nu^2$, then in the ionospheric plasma the frequency of electron–plasmon collisions ν_{ep} can be expressed in terms of the frequency of electron–electron collisions ν_{ee} and the spectrum of suprathermal electrons $\Phi(E, \mu)$ as:

$$\nu_{ep} = \frac{\pi}{2} \frac{I(\nu)}{\ln \Lambda} \nu_{ee}, \quad (8.2.12)$$

where

$$I(\nu) = \int_{v_e^2/\nu^2}^1 dx \left(1 - \frac{v_e^2}{v_x^2} \right) \times \frac{\frac{n_e}{v_e^3} e^{-\frac{1}{2x}} + \frac{\sqrt{2\pi}m}{v_e^2} \int_{E_*}^{\infty} dE \frac{\Phi(E, E_*)}{E}}{\frac{n_e}{v_e^3} e^{-\frac{1}{2x}} + \sqrt{\frac{\pi}{2}} \frac{n_e}{v_e^3} x^{3/2} - \sqrt{2\pi}m^2 \frac{\partial}{\partial E_x} \int_{E_*}^{\infty} dE \frac{\Phi(E, E_x)}{E}}. \quad (8.2.13)$$

Here μ (pitch-angle cosine) is designated as $\sqrt{E_*/E}$, and the designation $x = k^2/k_e^2 = T_e/E_*$ is introduced. The distribution function of thermal electrons f_e was considered Maxwellian with density n_e and mean energy $T_e = mv_e^2/2$. From (8.2.12) follows a well-known result, implying that in the equilibrium plasma ν_{ep} is smaller than ν_{ee} by a factor of $2 \ln \Lambda/3$, i.e., it makes up approximately 10% of ν_{ee} ($\ln \Lambda = 15\nu$) (Artsimovich and Sagdeev 1979). When calculating the value of ν_{ep} in the “real” ionospheric plasma, use was made of the results of Khazanov (1979), where photoelectron spectra were obtained numerically for different geophysical and heliophysical conditions. It was found that, at heights corresponding to the lower part of the ionospheric F-region, ν_{ep} makes up 20–30% of ν_{ee} . Because, in this region, electron collisions occur significantly more rarely as compared with collisions between electrons and neutral particles, photoelectron–plasmon interactions can be neglected. A different situation occurs in the topside ionosphere (300–1,000 km) and the plasmasphere, where the transport of the photoelectrons is essential and their mean energy increases significantly ($E_0 = 20\text{eV}$). As calculations indicate (Trukhan 1983), ν_{ep} in these regions make up 30–50% of ν_{ee} , and the contribution of

the interaction of photoelectrons with plasma waves to the energy dissipation processes becomes essential.

Thus, in applications that include calculating the photoelectron distribution function in the ionosphere and the plasmasphere, it is necessary to take into account the interaction of the photoelectrons with the plasma waves they generate, which ultimately implies renormalizing the Coulomb logarithm in the kinetic equation for photoelectrons.

Another important element of the theory of Coulomb scattering of superthermal electrons is the question as to whether the representation of their energy losses, as expressed in the first term of (8.2.9) is applicable in problems investigating the relaxation of the fine structure of their spectra. This question was discussed in a Chap. 5

8.2.3 *Unstable Plasma Fluctuations*

The problem of determining the fluctuation level in a non-equilibrium unstable plasma, or in a plasma, on the stability boundary, becomes significantly complicated because a rapid transfer of energy from plasma particles to waves occurs. In order to describe this process, it is necessary to simultaneously solve several integro-differential equations, which are nonlinear, with rather intense interactions between the excited waves. The ionospheric plasma in the presence of superthermal electrons of energies 1–100 eV (photoelectrons, secondary electrons), however, is a weakly non-equilibrium one; therefore, the interaction of the waves there is unimportant. In terms of the theory of quasilinear relaxation (see Chap. 3 for details), one can make, without solving the unwieldy equations, some simple estimates of the fluctuation level in the ionospheric plasma for different geophysical situations. Such estimations are based on the assumption that taking into account wave–particle interactions does not appear to substantially modify a complete solution of the kinetic equation for superthermal electrons obtained by including collisional mechanisms for electron relaxation. Wave processes will only affect regions of the resonance electron spectrum, resulting in a smoothing out of the nonmonotonicity and a decrease in anisotropy of the electron distribution function in corresponding energy ranges.

The system of equations that governs the quasilinear relaxation process in a magnetic field, discussed in Chap. 3, can be presented in the following form for high-frequency electrostatic oscillations (Akhiezer et al. 1975):

$$\frac{\partial f}{\partial t} = \frac{\pi}{2} \frac{e^2}{m^2} \sum_{n=-\infty}^{\infty} \int dk_{\perp}^2 \hat{R} \frac{|\varphi_k|^2 J_n^2 \left(\frac{k_{\perp} v_{\perp}}{\omega_B} \right)}{\left(v_{\parallel} - \frac{\partial \omega_r}{\partial k_{\parallel}} \right)} \hat{R} f \quad (8.2.14)$$

$$\frac{\partial |\varphi_k|^2}{\partial t} = (2\gamma - \nu_e) |\varphi_k|^2 \quad (8.2.15)$$

$$\gamma = -\frac{1}{2}v_e + \frac{\pi}{2} \frac{\omega_p^3}{k^2 |k_{\parallel}| n_0} \sum_n \int dv_{\perp}^2 J_n^2 \left(\frac{k_{\perp} v_{\perp}}{\omega_B} \right) \hat{R} f \quad (8.2.16)$$

where

$$\hat{R} = k_{\parallel} \frac{\partial}{\partial v_{\parallel}} + \frac{n\omega_B}{v_{\perp}} \frac{\partial}{\partial v_{\perp}}.$$

In the ionosphere, as a consequence of a permanently operating source of electrons of energies 1–100 eV, the quasilinear process is a stationary one. Thus, for example, in the lower ionosphere, where the main instability mechanism for plasma oscillations is associated with the nonmonotonicity of the distribution function of fast electrons, the kinetic equation of superthermal electrons that includes the process of quasilinear diffusion can be represented as (Khazanov 1979):

$$S_{ee} + S_{ep} + S_{en} + q = 0 \quad (8.2.17)$$

$$\varepsilon_k = \frac{k^2 |\varphi_k|^2}{4\pi} \quad (8.2.18)$$

$$\frac{1}{2}v_e = -\frac{\pi^2 \omega_p^4 m^{3/2}}{\sqrt{8} k_{\perp} k_{\parallel} n_0 \omega_B} \int_{E_n}^{\infty} dE \frac{\bar{\Phi}(E)}{E \sqrt{E - E_n}} \frac{\partial J_n^2(x)}{\partial x}. \quad (8.2.19)$$

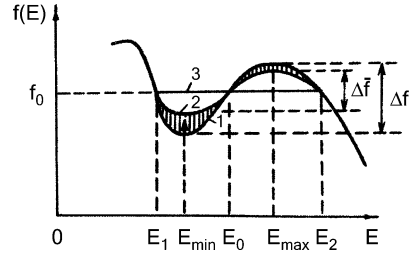
Here

$$S_{ep} = \frac{\partial}{\partial E} \left[D(E, E_n) \frac{\partial}{\partial E} \left(\frac{\Phi}{E} \right) \right] \quad (8.2.20)$$

$$D = \pi \frac{m^{3/2} \omega_p^4}{64 \sqrt{2} n_0} \int dk_{\perp}^2 \frac{\varepsilon_k J_n^2(x)}{k^2 \left| \sqrt{E_n} - \sqrt{\frac{m}{2}} \frac{\partial \omega}{\partial k_{\parallel}} \right|}. \quad (8.2.21)$$

Here ε_k is the spectral density of electromagnetic energy of the oscillations. Expression (8.2.19) was obtained from (8.2.16) for an isotropic distribution function of superthermal electrons in the lower ionosphere. In an absence of a source of fast electrons, the quasilinear relaxation of the nonmonotonic distribution function tends to suppress the instability and “deforms” the nonmonotonicity of the distribution to a plateau state. It will be assumed that scattering of the electrons by waves leads to a decrease in nonmonotonicity of the distribution from state 1 to state 2 with respect to a certain plateau 3 (Fig. 8.4). If the maximum and minimum positions of the spectrum nonmonotonicity are known, one can determine a certain

Fig. 8.4 Quasilinear relaxation of the distribution function of resonance electrons under the assumption of the invariable position of its extreme points. 1 and 2 consecutive states of the distribution function, 3 plateau (final state)



energy E_0 , for which the wave amplitude with a corresponding phase velocity will be considered unchanged, both with and without regard to the quasilinear relaxation. Then, from the law of conservation of particles, we obtain

$$\int_{E_{\min}}^{E_0} dE \frac{\Phi_0 - \Phi(E)}{\sqrt{E}} = \int_{E_0}^{E_{\max}} dE \frac{\Phi(E) - \Phi_0}{\sqrt{E}}. \quad (8.2.22)$$

Now we look at the expression for maximum linear growth rate of the oscillations $\gamma_{\max}$. This expression is determined by the spectrum of superthermal electrons, which does not take into account electron–wave interactions, and (8.2.19) that involves the smoothed spectrum of superthermal electrons determined from the balance of collisions of electrons with particles and waves (we denote it by $\bar{\Phi}(E)$). We have:

$$\gamma_{\max} = -\frac{1}{2}v_e - \frac{\pi^2 \omega_p^4 m^{3/2}}{\sqrt{8} k_{\perp} k_{\parallel} n_0 \omega_B} \int_{E_n}^{\infty} dE \frac{\Phi(E)}{E \sqrt{E - E_n}} \frac{\partial J_n^2(x)}{\partial x} \quad (8.2.23)$$

$$0 = -\frac{1}{2}v_e - \frac{\pi^2 \omega_p^4 m^{3/2}}{\sqrt{8} k_{\perp} k_{\parallel} n_0 \omega_B} \int_{E_n}^{\infty} dE \frac{\bar{\Phi}(E)}{E \sqrt{E - E_n}} \frac{\partial J_n^2(x)}{\partial x}. \quad (8.2.24)$$

If the expressions for $\Phi(E)$ and $\bar{\Phi}(E)$ are represented by parabolas and if it is assumed that the quasilinear relaxation mechanism leads only to a change in amplitudes of the spectrum minimum and maximum, without changing their position in energies, then from the system (8.2.23) and (8.2.24) it is easy to obtain the relationships between the values in maxima and minima of the spectra with and without considering wave processes:

$$\frac{\Delta \bar{\Phi}(E_{\min})}{\Delta \Phi(E_{\min})} = \frac{\Delta \bar{\Phi}(E_{\max})}{\Delta \Phi(E_{\max})} = \frac{\frac{1}{2}v_e}{\frac{1}{2}v_e + \gamma_{\max}}, \quad (8.2.25)$$

where $\Delta \Phi(E_{\max})v$ and $\Delta \Phi(E_{\min})$ are determined from the expressions

$$\Delta\Phi(E) = \Phi(E) - \Phi_0$$

$$\Delta\Phi(E) = \begin{cases} \Delta\Phi(E_{\max}) \frac{(E - E_0)(E - E_2)}{(E_{\max} - E_0)(E_{\max} - E_2)} & \text{if } E \in E_0 - E_2 \\ -\Delta\Phi(E_{\min}) \frac{(E - E_0)(E - E_1)}{(E_{\min} - E_0)(E_{\min} - E_1)} & \text{if } E \in E_1 - E_0 \end{cases}. \quad (8.2.26)$$

It should be noted that an approximation of $\Phi(E)$ could be made using an arbitrary function with no change in the final result presented by (8.2.25) (Trukhan 1983). Using (8.2.25), one can estimate the action of the quasilinear relaxation on the spectrum of superthermal electrons and find the corresponding level of ionospheric plasma oscillations. The energy density transferred to the waves from the resonant electrons can be obtained from the expression

$$W = 4\pi \sqrt{\frac{m}{2}} \int_{E_1}^{E_2} dE \sqrt{E} [\Phi(E) - \bar{\Phi}(E)]. \quad (8.2.27)$$

Using (8.2.27), one can, in turn, estimate values of effective collision frequencies with plasma oscillations (Tsytovich 1967)

$$\nu_{ep} \approx \frac{\pi}{4} \omega_p \frac{W}{n_0 T_e} \left(\frac{v_e}{v} \right)^2 \frac{v_{ph}}{v}, \quad (8.2.28)$$

which, by several times, exceeds the Coulomb collisional frequency that can be presented in a similar way as

$$\nu_{ee} \approx \frac{\pi}{4} \omega_p \frac{W_T}{n_0 T_e} \left(\frac{v_e}{v} \right)^3 \ln \Lambda. \quad (8.2.29)$$

Moreover, it is easy to obtain an estimate of the value of additional heating ($Q = \nu W$) of ionospheric plasma in the instability region. Thus, by knowing the calculated spectrum of superthermal electrons (considering only particle–particle interactions), using (8.2.25), (8.2.22), and (8.2.27), one can estimate the wave–particle interaction plasma heating. This estimation will be performed in the next section of this chapter.

8.3 Relaxation of Electron Fluxes in the Auroral Ionosphere

8.3.1 The Origin of Electron Beams

The auroral ionosphere is characterized by the presence of a current system that is the result of the interaction of the Earth's magnetosphere with the solar wind. Currents act to provide the electromagnetic coupling between the magnetosphere

and the ionosphere (Lyons and Williams 1984). Depending on the direction of the current in the auroral ionosphere, it is possible to identify regions where charge transfer processes are different. These are the regions of downward and upward field-aligned currents as well as the region of closing transverse currents. Transverse currents flow at E-layer ionospheric heights and are due mainly to the motion of ions with respect to magnetized electrons. The region of upward field-aligned current is characterized by a large-scale structure ($L > 100$ km) where current densities are relatively small ($j_{\parallel} \approx 1 \mu\text{A m}^{-2}$) as well as by a small-scale structure ($L < 10$ km) where the field-aligned current density reaches $j_{\parallel} \approx 10^2 \mu\text{A m}^{-2}$ (Volokitin et al. 1984). Small-scale field-aligned currents are associated with the existence of strong field-aligned electric fields with the strength of up to $E_{\parallel} \approx 10^2 \text{ mV m}^{-1}$, at both ionospheric heights and at altitudes of $5 - 8 \times 10^3$ km (double layers) (Liperovsky and Pudovkin 1983). It has been generally recognized to date that an acceleration of electrons due to these fields occurs, leading to the formation of electron beams with 1- to 10-keV energies that are responsible for the formation of discrete auroral forms.

The processes of beam formation in the downward current region are less well studied. Here relatively weaker electric fields accelerate, in the antiearthward direction, secondary electrons that are produced by impact ionization. The downward current region is characterized by a softer spectrum of precipitating magnetospheric electrons with energy $E < 1$ keV. In this region, the downward current is caused by the imbalance of upward-directed secondary electron fluxes and precipitating magnetospheric electrons.

The regions of downward and upward currents are spatially separated, are largely elongated in latitude, and, taken together, form, at ionospheric heights, the nighttime oval of diffuse polar auroras with embedded polar auroral arcs.

Let us consider in greater detail the formation mechanisms for beam electron distributions in the auroral ionosphere in regions of downward and upwards currents. The formation mechanisms for strong electric fields in the upward current regions are not well understood to date. However, it is possible (without specifying the source of the field-aligned electric field and without particularizing the potential distribution along a geomagnetic field line) to obtain the connection of the density of the field-aligned current of electrons from the magnetosphere with the full potential difference V (see Sect. 5.1 for details). If the electron distribution in the magnetosphere is Maxwellian with density n_M ($n_M \approx 1 \text{ cm}^{-3}$) and thermal energy T_M ($T_M \approx 1 \text{ keV}$), and, if the current of ionospheric electrons is neglected, then the field-aligned current density can be expressed as (Lyons and Williams 1984)

$$j_{\parallel} = en_M \left(\frac{T_M}{2\pi m} \right)^{1/2} \frac{B_i}{B_m} \left[1 - \left(1 - \frac{B_m}{B_i} \right) \exp \left(- \frac{eV_{\parallel}}{T_M \left(\frac{B_i}{B_m} - 1 \right)} \right) \right]. \quad (8.3.1)$$

Here B_i and B_m are the geomagnetic field strengths along a field line in the ionosphere and the magnetosphere, respectively. One can see that the electric field acts to lower the points of magnetic reflection of the magnetospheric electrons, and

the beam effect is produced. At values of $V_{\parallel} = 0$, and typical values of $B_i/B_m \approx 30$, the value of field-aligned current density $j_{\parallel} \approx 10^{-6} \text{ A m}^{-2}$ and, at $V_{\parallel} = 10 \text{ kV}$, $j_{\parallel}$ reaches a value of the order of $3 \times 10^{-5} \text{ A m}^{-2}$, which compares with maximum values of $j_{\parallel}$ usually observed over discrete polar auroras. Since the acceleration in the field-aligned electric field increases the parallel component of the electron velocity and does not have an effect on the perpendicular component, the distribution of precipitating electrons can be approximated by the function

$$f = \frac{n_M}{(2\pi\Delta u)^{3/2}} e^{-\frac{(v_{\parallel} - u)^2}{\Delta u^2} - \frac{v_{\perp}^2}{\Delta u^2}}, \quad (8.3.2)$$

where the directed velocity in the beam $u = \sqrt{2eV_{\parallel}/m}$, and the thermal spread in the beam is $\Delta u = \sqrt{T_M/m}$. Typical values of the parameters of such beams are $u \approx 3 \times 10^9 \text{ cm s}^{-1}$, $u \geq 3\Delta u$, and $n_M \approx 0.1 - 1 \text{ cm}^{-3}$.

Depending on the characteristics of the precipitated initial beam (8.3.2), there can be plasma instability. Then, these precipitating electrons can lose their energy not only in collisions with charged and neutral particles, as described by (2.5.2), but also in “collisions” with beam-excited plasma waves. Such processes are studied in detail in the book by Mishin et al. (1989), with applications to artificial and natural plasma–electron beam phenomena.

Gefan et al. (1985b) studied the applicability of a collisional description of the interaction of down-flowing auroral electrons with the ionosphere plasma, and developed the criteria for such an approach. It was shown that the Cherenkov instability of such electron beams can be stabilized by the inhomogeneities of the plasma density and the geomagnetic field. This clarifies the facts of penetration into the auroral ionospheric F-region of unstable electron beams with $E_b \sim 1 \text{ keV}$ (a so-called inverted V structure) (Lyons and Williams 1984), whose collective energy dissipation near the altitude of maximum electron density of the F2-region seems to cause a number of observed effects (Mishin et al. 1990), and extends the applicability of the collisional description of the interaction of inflowing auroral electrons with the ionospheric plasma. Below, only precipitated fluxes that are initially stable and described based on (2.5.2) will be used to calculate secondary electrons. Remarkably, as will be shown below and discussed in detail, initially stable precipitated electron beams produce (via collisional processes!) an unstable secondary electron distribution function.

The main current carriers in the region of downward current are secondary electrons. Here the secondary electrons are accelerated upward by field-aligned electric fields, which leads to the formation of a quasibeam distribution. Let us consider in greater detail the formation mechanism for this beam in the upper ionosphere.

The main factors that influence the motion of the escaping electrons are the inhomogeneity of the magnetic field along a field line with a scale length $H_B = B/(dB/ds) \approx 2.5 \times 10^8 \text{ cm}$; Coulomb collisions with the background plasma particles, with a free path length $\lambda \approx E^2/(An_e)$, where E is the energy of the escaping electrons, and $A = 2.6 \times 10^{-12} \text{ eV cm}^2$; and the acceleration of the electrons by the

potential difference with the scale length $H_{E_{\parallel}} \approx E/(eE_{\parallel})$, where $E_{\parallel}$ is the longitudinal component of the electric field.

A comparison of the above spatial scales shows that, for typical values of $n_e \approx 10^4 \text{ cm}^{-3}$, 10-eV superthermal electrons are in the regime of collisionless escape ($H_B \ll \lambda$). For lower-energy electrons, H_B and λ are of the same order of magnitude. In the presence of an electric field $E_{\parallel} \geq 10^{-2} \text{ mV m}^{-1}$, however, the relationship $H_{E_{\parallel}} \ll \lambda$ is satisfied, and the collisionless approximation is valid throughout the interval of superthermal energies. When $E_{\parallel} \approx 10^{-3} \text{ mV m}^{-1}$, the spatial scales of H_B , λ , and $H_{E_{\parallel}}$ are comparable and, to determine the electron distribution function, it is necessary to solve the kinetic equation (2.5.2) by taking account of all of the above-mentioned factors.

Consider the collisionless transformation of a secondary electron flux escaping from the ionosphere. Let, at a certain level s_1 , the flux in the upper hemisphere have an isotropic and monotonic distribution function

$$f(v_{\perp}, v_{\parallel})|_{s_1} = f_0 \exp\left(\frac{v_{\parallel}^2 + v_{\perp}^2}{v_0^2}\right).$$

In the absence of collisions, the invariants of electron motion are $v_{\perp}^2/B = \text{const}$, and $E - w = \text{const}$, where w is the energy increment due to the electrostatic acceleration above the level s_1 . As a consequence, the flux is collimated into a so-called “escape cone”

$$v_{\perp}^2 \leq v_{\perp \text{max}}^2 = \frac{\beta}{1 - \beta} \left(v_{\parallel}^2 - \frac{2w}{m} \right), \quad \beta = \frac{B(s)}{B(s_1)}.$$

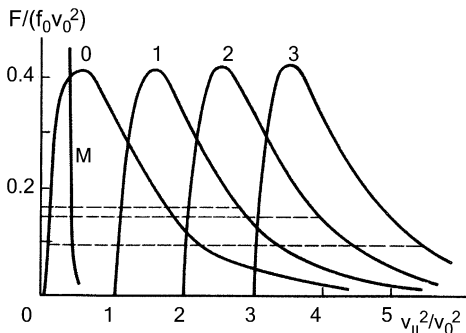
The electron distribution function in longitudinal velocities assumes the form

$$\begin{aligned} F(v_{\parallel})|_s &= \int_0^{v_{\perp \text{max}}^2} dv_{\perp}^2 f(v_{\perp}, v_{\parallel})|_s \\ &= \begin{cases} f_0 v_0^2 e^{-\frac{(v_{\parallel}^2 - 2w/m)}{v_0^2}} \left[1 - e^{-\frac{(v_{\parallel}^2 - 2w/m)\beta}{v_0^2(1-\beta)}} \right] & \text{when } v_{\parallel}^2 > 2\frac{w}{m} \\ 0 & \text{when } v_{\parallel}^2 \leq 2\frac{w}{m} \end{cases}. \end{aligned} \quad (8.3.3)$$

The extent of the electrostatic acceleration region along the magnetic field line is approximately 1,000 km (ionospheric heights) with a change in magnetic field $\beta \approx 0.7$. Figure 8.5 shows the dimensionless function $F(v_{\parallel})/(f_0 v_0^2)$ on the upper boundary of acceleration for different ratios between the electrostatic increment

Fig. 8.5 The electron distribution function in longitudinal velocities in the presence of a potential difference along the magnetic field line for $\beta = 0.7$.

Numbers near the curves correspond to chosen values of w/E_0 , M is the Maxwellian function for thermal electrons at $n/n_e = 0.001$ (n is the secondary electron density) and $E_0/T_e = 2.5$



of energy w and the characteristic energy of the secondary electron spectrum $E_0 = mv_0^2/2$. The positive slope ($\partial F/\partial v_{||}^2 > 0$) of the curves on the portion $2w/m < v_{||}^2 < v_m^2$, where

$$v_m^2 = 2\frac{w}{m} + v_0^2 \left(\frac{1}{\beta} - 1 \right) \ln(1 - \beta)^{-1}$$

provides the possibility of the excitation of plasma oscillations by a group of resonant secondary electrons. Plasma oscillations that are able to interact effectively with electron beams have the frequencies ω_1 and ω_2 . The corresponding growth rates are defined by the expression (Akhiezer et al. 1975)

$$\gamma_{1,2} = \frac{2\pi^2 v_{res}^3 \omega_p^2 \cos^2 \theta}{\omega_{1,2}^2 n |\partial \text{Re } \epsilon_e / \partial \omega|_{\omega_{1,2}}} \int_0^\infty dv_\perp^2 \frac{\partial f}{\partial v_{||}^2} J_0^2 \left(\frac{k_\perp v_\perp}{\omega_B} \right), \quad (8.3.4)$$

where v_{res} is determined by the Cherenkov resonance condition

$$kv_{res} \cos \theta = \omega_{1,2}. \quad (8.3.5)$$

For angles satisfying $\cos \theta > \sqrt{m/M}$ for branch ω_2 , and $\cos \theta > \frac{\omega_p}{\omega_B} \frac{v_0}{v_m}$ for branch ω_1 , taking into account J_0^2 in (8.3.4) is unimportant, which permits us to pass to the function $F(v_{||})$ introduced above. Thus, the growth rate of Langmuir oscillations ($\omega \approx \omega_p$) at $\cos \theta = 1$ is

$$\gamma_1 = \pi^2 v_{res}^3 \omega_p n^{-1} \left. \frac{\partial F}{\partial v_{||}^2} \right|_{v_{res}}. \quad (8.3.6)$$

For estimations, one may put

$$\frac{\partial F}{\partial v_{||}^2} \approx \frac{F(v_m)}{v_{||}^2 - 2\frac{w}{m}} = f_0 \left(\frac{\beta}{1 - \beta} \right)^2 \frac{(1 - \beta)^{1/\beta}}{\ln(1 - \beta)^{-1}}. \quad (8.3.7)$$

If the density of secondary electrons is taken as $n \approx 10^1 - 10^2 \text{ cm}^{-3}$, then, for $w \geq 10 \text{ eV}$ ($E_{\parallel} \geq 10^{-2} \text{ mV m}^{-1}$), $\gamma \geq 10^4 \text{ s}^{-1}$, which greatly exceeds the frequency of electron-ion collisions ν_e , suggesting that the excitation of plasma oscillations is, in principle, possible.

At $E_{\parallel} \geq 10^{-2} \text{ mV m}^{-1}$ one cannot neglect the influence of Coulomb collisions on the motion of electrons with energy $E \leq 10 \text{ eV}$, and it is necessary to solve the kinetic equation (2.5.2) with the electric field taken into account. The method and the results of solving (2.5.2) were discussed in Chap. 7. Based on this approach, Fig. 8.6 shows the result of the initially stable electron flux precipitation obtained by Gefan et al. (1985a). Below, we present the calculations of growth rates (8.3.6) from the solution of (2.5.2) with the upper boundary condition

$$\Phi|_{s=s_{\max}, \mu < 0} \sim E \exp\left(-\frac{E}{E_b}\right), \quad E_b = 10^3 \text{ eV}. \quad (8.3.8)$$

Such a spectrum of precipitating electrons is characteristic for diffuse auroral zones (Meng 1976; Valchuk et al. 1979). The calculations used a standard electron density profile with the density $n_e = 10^4 \text{ cm}^{-3}$ at 1,000-km altitude. Tables 8.3 and 8.4 present maximum values of growth rates at 1,000-km altitude obtained by suitable selection of ν_{res} . Table 8.3 gives the calculated γ_1 for $E_{\parallel} = 0$ for different values of energy flux of precipitating electrons P_E with the energy spectrum (8.3.8) and background electron temperature T_e .

The destabilizing role of the field-aligned electric field that accelerates the secondary electrons is shown in Table 8.4, which presents the results of calculations of γ_1 for $P_E = 1 \text{ erg cm}^{-2} \text{ s}^{-1} \text{ ster}^{-1}$ for different values of $E_{\parallel}$ (the field-aligned field was specified to be homogeneous). One can see that, with increasing $E_{\parallel}$, the condition for the excitation of plasma oscillations improves: when $E_{\parallel} = 5 \times 10^{-3} \text{ mV m}^{-1}$, the dependence of γ on T_e virtually disappears because the nonmonotonicity from the function $F(v_{\parallel})$ is shifted rather far from the region of thermal velocities.

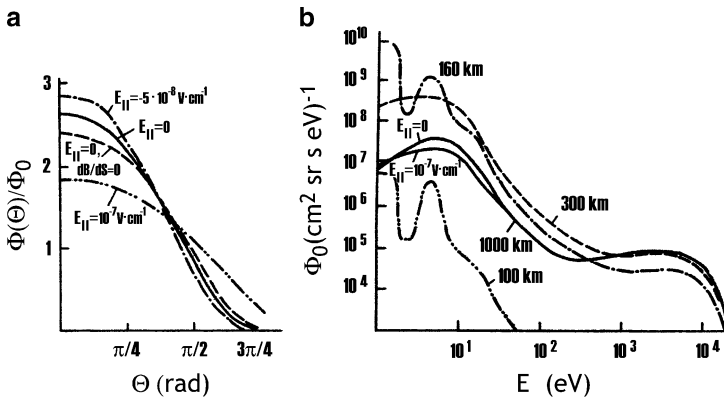


Fig. 8.6 (a) Angular distribution of escaping secondary electrons depending on the magnetic field model and the magnitude of longitudinal electric field. (b) The omnidirectional electron flux

Table 8.3 Calculated γ_1 for $E_{||} = 0$ for different values of energy flux of precipitating electrons P_E with the energy spectrum (8.3.8) and background electron temperature T_e

$P_E(\text{erg} \times \text{cm}^{-2} \times \text{s}^{-1} \times \text{ster}^{-1})$	Values of γ_1 (s^{-1}) at T_e (K^0)			
	2,000	2,500	3,000	3,500
0.142	5.17×10^2	-3.24×10^3	-1.83×10^4	-5.8×10^4
1.0	5.27×10^3	1.84×10^3	-1.37×10^4	-5.47×10^4
10.0	5.54×10^4	5.17×10^4	3.65×10^4	-3.76×10^4

Table 8.4 The results of calculations of γ_1 for $P_E = 1 \text{ erg} \times \text{cm}^{-2} \times \text{s}^{-1} \times \text{ster}^{-1}$, for different values of $E_{||}$ and T_e

$E_{ } (\text{mV m}^{-1})$	Values of γ_1 (s^{-1}) at T_e (K^0)			
	4,000	5,000	6,000	7,000
0	-1.33×10^5	-4.15×10^5	-8.33×10^5	-1.3×10^6
0.0025	1.56×10^4	1.2×10^4	-3.2×10^3	-4.35×10^4
0.005	2.88×10^4	2.88×10^4	2.88×10^4	2.7×10^4

Let us now estimate the rate of energy transfer from the escaping flux of the secondary electrons to the upper ionospheric plasma. We will consider that, as a result of interactions with plasma oscillations, the unstable beam relaxes via a quasilinear relaxation until a plateau-shaped section is formed in the function $F(v_{||})$ (Fig. 8.5, broken lines). The location on the abscissa, to the left of which is formed the section $F(v_{||}) = \text{const}$, is determined by the condition of the conservation of resonance electrons, which for the function (8.3.3) yields

$$\sqrt{\tilde{\kappa}} e^{-\tilde{\kappa}} \left[1 - a \exp\left(-\frac{\tilde{\kappa}\beta}{1-\beta}\right) \right] = \frac{\sqrt{\pi}}{2} \left\{ 1 - \Phi(\sqrt{y}) - a\sqrt{1-\beta} [1 - \Phi(\sqrt{\kappa})] \right\},$$

where $\tilde{\kappa} = \tilde{v}_{||}^2/v_0^2$ is the boundary of the plateau sections, Φ is the integral error function, $\kappa = W/E_0$, $y = W/E_0(1-\beta)$, and $a = \exp[W\beta/E_0(1-\beta)]$. From this algebraic equation, the magnitude of $\tilde{\kappa}$ may be determined numerically (the values of $\tilde{\kappa}$ for some ratios of W/E_0 are given in Fig. 8.5).

The energy evolution during the beam relaxation process is determined as

$$P = \pi \int_0^{\tilde{v}_{||}} dv_{||} v_{||} \frac{m_e v_{||}^2}{2} [F(v_{||}) - F(\tilde{v}_{||})]$$

From this expression, it follows that for $W \approx 10 - 30 \text{ eV}$, $E_0 \approx 10 - 15 \text{ eV}$, and the typical intensity of the secondary electron flux, $P \approx 10^{-9} - 10^{-8} \text{ W m}^{-2}$, and the wave energy density of the order of 10^2 eV cm^{-3} .

8.3.2 Peculiarities of the Plasma Noise Distribution in the Auroral Ionosphere

Consider the spectral characteristics of plasma waves excited by electron beams in the auroral ionosphere. Because the ionospheric plasma is inhomogeneous (especially in the high-latitude region), an instability will occur if the amplification of a wave packet (Andrianov et al. 1977), as it is carried away from the instability region,

$$\Gamma = \int dl \frac{(\gamma - \gamma_{st})}{v_g}, \quad (8.3.9)$$

exceeds $\ln \Lambda/2 \approx 10$. Here $v_g = d\omega/d\mathbf{k}$ is the group velocity, and l is the group path of the packet over the instability region. In this case the plasma fluctuation level W significantly exceeds the equilibrium level $W_T \approx T_e/r_D^3$; $W = W_T e^\Gamma$.

Let the electron distribution in the beam be represented by (8.3.2). Consider the stability of two branches of plasma oscillations separately in a dense ($\omega_p > \omega_B$) and rarefied ($\omega_p < \omega_B$) plasma. The group velocity for the branches $\omega_{1,2}$ is defined as:

$$\frac{d\omega_1}{dk} \approx u \cos \theta \left\{ 9 \frac{v_e^4}{u^4} + r^2 \cos^2 \theta \sin^2 \theta \right\}^{1/2} \quad (8.3.10)$$

$$\frac{d\omega_2}{dk} \approx u \left\{ 9 \frac{v_e^4}{u^4} \left(\cos^2 \theta + \frac{m}{M} \right) + \sin^2 \theta \frac{\cos^2 \theta}{\left(\cos^2 \theta + \frac{m}{M} \right)} \right\}^{1/2}. \quad (8.3.11)$$

Here $u \approx \omega_{1,2}/k_{||}$, $r = \min(\omega_p/\omega_B, \omega_B/\omega_p)$.

The packet's group path is determined from the conservation of the waves' frequency and from the resonance condition with the beam $\Delta k_{||}/k_{||} \approx \Delta u/u$, and is defined as

$$\Delta l \approx 2H_p \frac{\Delta u}{u} \cos^2 \theta \left\{ 3 \frac{v_e^2}{u^2} - r^2 \sin^2 \theta \right\} \quad (8.3.12)$$

for ω_1 and

$$\Delta l \approx 2H_p \frac{\Delta u}{u} \cos^2 \theta \left\{ 3 \frac{v_e^2}{u^2} \left(\cos^2 \theta + \frac{m}{M} \right) + \sin^2 \theta \right\} \quad (8.3.13)$$

for ω_2 .

Here H_p is a typical plasma inhomogeneity. It is convenient to introduce typical scales of inhomogeneity across and along the geomagnetic field $H_\perp$ and $H_{||}$, respectively. In this case H_p is expressed as

$$H_p = \frac{H_\perp H_{||}}{H_\perp \cos \theta \pm H_{||} \sin \theta}. \quad (8.3.14)$$

The choice of the sign in the denominator depends on the variations of $H_{\perp}$ and $H_{\parallel}$. From (8.3.14), it is evident that, in a certain direction, determined by the condition $tg\theta = H_{\perp}/H_{\parallel}$, the value of H_p can be significant.

Linear growth rates of the excitation of plasma eigen-oscillations $\omega_{1,2}$ by an electron beam (8.3.2) have the form:

$$\gamma(\omega_1, \theta) = \begin{cases} \gamma_b \cos^2 \theta & \text{when } \omega_p > \omega_B \\ \gamma_b \left(\frac{\omega_p}{\omega_B}\right)^3 \sin^2 \theta \cos^2 \theta & \text{when } \omega_B > \omega_p \end{cases} \quad (8.3.15)$$

$$\gamma(\omega_2, \theta) = \begin{cases} \gamma_b \frac{\omega_B}{\omega_p} \frac{\cos^2 \theta \sin^2 \theta}{\sqrt{\cos^2 \theta + \frac{m}{M}}} & \text{when } \omega_p > \omega_B \\ \gamma_b \frac{\cos^2 \theta}{\sqrt{\cos^2 \theta + \frac{m}{M}}} & \text{when } \omega_B > \omega_p \end{cases} \quad (8.3.16)$$

$$\gamma_b \equiv \sqrt{\frac{\pi}{8e}} \omega_p \frac{n_b}{n} \left(\frac{u}{\Delta u}\right)^2. \quad (8.3.17)$$

It is taken into consideration that $I_0(z_b)e^{-z_b} \approx 1$ at the angles satisfying $\cos \theta > \frac{\omega_p}{\omega_B} \frac{\Delta u}{u}$ for the branch ω_1 and $\cos \theta > \sqrt{m/M}$ for the branch ω_2 .

Thus, the amplification is defined as:

$$\Gamma(\omega_1, \theta) = \gamma_b H_B \frac{\Delta u}{u^2} \left(\frac{\omega_p}{\omega_B}\right)^3 \cos^2 \theta \sin^3 \theta \quad (8.3.18)$$

$$\Gamma(\omega_2, \theta) = \begin{cases} 2\gamma_b H_p \frac{\Delta u}{u^2} \frac{\cos^2 \theta}{\sqrt{\cos^2 \theta + \frac{m}{M}}} & \text{when } \Delta l < L_b \\ \gamma_b \frac{L_b \sqrt{\cos^2 \theta + \frac{m}{M}}}{u \sin \theta} & \text{when } \Delta l > L_b \end{cases} \quad (8.3.19)$$

when $\omega_B > \omega_p$, and

$$\Gamma(\omega_1, \theta) = \begin{cases} 2\gamma_b H_p \frac{\omega_B}{\omega_p} \frac{\Delta u}{u^2} \cos^2 \theta \sin \theta & \text{when } \Delta l < L_b \\ \gamma_b \frac{L_b \omega_p}{u \omega_B \sin \theta} & \text{when } \Delta l > L_b \end{cases} \quad (8.3.20)$$

$$\Gamma(\omega_2, \theta) = \begin{cases} \gamma_b H_B \frac{\Delta u}{u^2} \frac{\omega_B}{\omega_p} \frac{\cos^2 \theta \sin^3 \theta}{\sqrt{\cos^2 \theta + \frac{m}{M}}} & \text{when } \Delta l < L_b \\ \gamma_b \frac{L_b}{u} \frac{\omega_B}{\omega_p} & \text{when } \Delta l > L_b \end{cases} \quad (8.3.21)$$

when $\omega_p > \omega_B$.

Here H_B is a typical geomagnetic field inhomogeneity scale length ($H_B \approx 2 \times 10^3$ km). It is necessary to include thermal corrections in (8.3.18)–(8.3.21) for angles close to 0° . Note that, when $\theta \rightarrow 0^\circ$, the expression for Γ has the form

$$\Gamma(\omega, \theta) = 2\gamma_b H_p \frac{\Delta u}{u^2} \quad (8.3.22)$$

for both ω_1 when $\omega_p > \omega_B$ and ω_2 when $\omega_B > \omega_p$.

Figure 8.7 gives a schematic representation of the angular dependence $\Gamma_{1,2}(\theta) \equiv \Gamma(\omega_{1,2}, \theta)$. One can see that $\Gamma(\omega_2, \theta) > \Gamma(\omega_1, \theta)$ when $\omega_B > \omega_p$ for all angles θ . When $\omega_p > \omega_B$, $\Gamma(\omega_1, \theta) > \Gamma(\omega_2, \theta)$ except for angles close to $\pi/2$. It is also evident that the wave amplification of the dominant branches is largest along the magnetic field direction. Therefore, the beam mainly excites the high-frequency part of the plasma wave spectra. Moreover, the dominant branch shows a further maximum in the angular distribution of Γ . The presence of two maxima is due to the group velocity and the group path of the wave packet being determined by both the thermal corrections to $\omega_{1,2}$ and the anisotropy introduced by the magnetic field. The position of the minimum in the angular distribution of Γ is determined from the condition

$$\sqrt{3} \frac{v_e}{u} \approx r \sin \theta_t. \quad (8.3.23)$$

In (8.3.18)–(8.3.21), and also in Fig. 8.7, it is taken into consideration that if the group path Δl of a wave packet, which is in resonance with the beam, exceeds the size of spatial localization of the beam L_b , then the value of Γ (plasma wave intensity) must decrease in a certain interval $\theta_1 - \theta_2$ (dashed line). Figure 8.7 presents the angular distribution of Γ for the case when $\gamma(\theta) < \gamma_{st}(\theta_v)$ (vertical dash-dotted line).

One can see that the electron beam does virtually not excite waves that propagate nearly normal to the geomagnetic field direction. Let us estimate the values of Γ for plasma waves near the lower-hybrid resonance ($\theta_0 = \arccos \sqrt{m/M}$), whose intensity in the ionosphere usually is rather high.

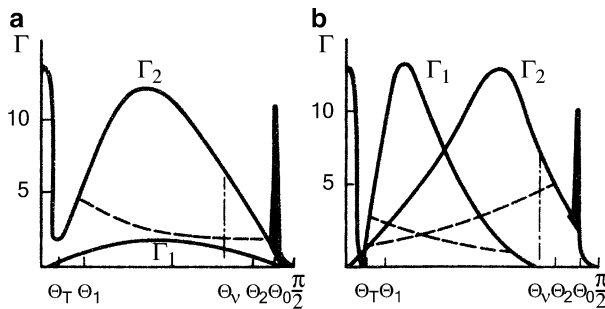


Fig. 8.7 The angular dependence of growth rates Γ_1 and Γ_2 of branches (a) $\omega_B > \omega_p$ and (b) $\omega_p > \omega_B$ of plasma oscillations

For parameters of a precipitating electron beam (in the upward current region) and the magnetospheric plasma ($h > 1,000$ km), $H_p \approx 1000$ km, $n_b \approx 0.1 - 1$ cm $^{-3}$, $\Delta u \approx 10^9$ cm s $^{-1}$, $u \approx 3 \times 10^9$ cm s $^{-1}$, $n \approx 10^2 - 10^3$ cm $^{-3}$, $\sqrt{m/M} \approx 1/40$ (hydrogen ions), $\Gamma_2(\theta_0)$ can reach 10. At topside ionospheric heights (300–1,000 km), where $H_p \approx 200$ km, $n \approx 10^4 - 10^5$ cm $^{-3}$, and $\sqrt{m/M} \approx 1/170$ (oxygen ions), for the same beam parameters, $\Gamma_2(\theta_0) \leq 1$. At these altitudes, however, a wave channel is possible for a wave packet near the lower-hybrid resonance. The presence of this wave channel leads to an absolute beam instability due to multiple returns of the wave packet to the amplifier in a region occupied by the beam (Trakhtengerts and Shapaev 1977). It is easy to estimate that, while traveling along a quiet auroral arc (and in this case L_b is largest, typically greater than 300 km), the packet performs less than 30 oscillations, which is sufficient for the development of the instability ($\Gamma \approx 10$). In Fig. 8.7, such an increase of Γ is marked by dark peaks.

Thus, in the region of upward current at topside ionospheric heights and above, waves propagating along the geomagnetic field are the first to be enhanced. A significant enhancement of the plasma waves is also possible near lower-hybrid resonance, however, which may be the reason for the appearance of a VLF emission such as auroral hiss (LaBelle et al. 1998).

8.3.3 *Instabilities of the Secondary Electrons in the Region of Their Production*

Until the present, the question of the stability of the ionospheric plasma and the flux of secondary electrons produced as a result of impact ionization, in a collision-dominated region, has not been considered. This issue is of importance, however, because secondary electrons, with mean energy of several electron volts, largely determine the energy regime of the polar ionosphere. As calculations show (see Fig. 8.6), spectra of secondary electrons have a number of features that can cause ionospheric plasma instabilities. Such features include primarily the non-monotonicity of the energy distribution of electrons of 2- to 4-eV energies brought about by effective energy losses to the oscillatory excitation of molecular nitrogen. In Gefan et al. (1985b), by analogy with Ivanov et al. (1980), a plasma stability analysis is made for high-latitude ionospheric conditions when the superthermal component is represented by secondary electrons. It is supposed that precipitating electron fluxes (initially stable) by themselves do not cause the plasma instability. The boundary conditions for solving the kinetic equation (2.5.2) were specified at 1,000- to 2,000-km altitudes in the form of beams with the spectrum $\Phi(E) = AE \exp(-E/E_b)$, and the parameters A and E_b were varied in such a way that precipitating electron fluxes have values typical of diffuse auroral zones (Valchuk et al. 1979; Lui et al. 1977).

Table 8.5 presents the growth rates of oscillation excited at frequencies close to the upper-hybrid frequency $\omega_1 \approx \sqrt{\omega_B^2 + \omega_p^2}$ at lower ionospheric F-region heights,

Table 8.5 The growth rates of secondary electrons

h (km)	Values of γ (s^{-1}) depending on beam energy ($E_b(\text{keV})/P$ ($\text{cm}^{-2} \times s^{-1}$)/ $P_E(\text{erg} \times \text{cm}^{-2} \times s^{-1})$), $A = 88.5$			γ_{st} (s^{-1})
	$0.5/7 \times 10^7/0.1$	$1/2 \times 10^8/1$	$3/2, 5 \times 10^9/24$	
125	1.3	3.7×10^1	2.5×10^3	3.5×10^3
140	1.3	2.1×10^2	6.3×10^3	2.3×10^3
160	6.2×10^1	6.2×10^2	1.1×10^4	5×10^2
180	1.2×10^2	8.8×10^2	1.1×10^4	4.2×10^2
200	7.4×10^1	4.5×10^2	4.3×10^3	3×10^2
220	-1.4×10^1	-8.2×10^3	-8.1×10^3	3.5×10^2

Note: The italicized values of γ represent the conditions $\gamma > 0$, and $\gamma > \gamma_{st}$ (plasma unstable)

obtained through a numerical calculation for different energies of precipitation. The wave vector components of such oscillations have the values $k_{\perp} \approx 0.5 \text{ cm}^{-1}$ and $k_{\perp} \geq 10k_{\parallel}$.

Table 8.5 also gives the corresponding values of the decrements of collisional damping γ_{st} under consideration. From the table it is evident that $\gamma > \gamma_{st}$ (the plasma is unstable) for fluxes $P \geq 2 \times 10^8 \text{ cm}^{-2} \text{ s}^{-1}$ and energy fluxes $P_E \geq 1 \text{ erg cm}^{-2} \text{ s}^{-1}$. It should be noted that the intensity of secondary electrons and γ depend on the “hardness” of the precipitating electron spectra, i.e., on the quantity characterizing the energy spread of the particles in the beam, E_b . With increasing “hardness” the growth rate γ decreases, which is associated with the displacement of the ion production maximum to lower heights. At heights $h < 300 \text{ km}$, plasma oscillations with frequencies at multiples of the electron gyrofrequency $\omega = n\omega_B$, with $k_{\perp} > k_{\parallel}$, are also unstable. Their growth rates, however, are smaller than those given in the table. The excitation of oscillations at these heights under conditions of double resonance ($k_{\parallel} = 0$) was found to be not effective, like in the mid-latitude ionosphere (see Sect. 8.1).

Thus, high-energy stable electron precipitation can be accompanied by a plasma instability at 160- to 200-km altitudes caused by secondary electrons. Such an instability must be stabilized by nonlinear wave processes and, partly, by the plasma inhomogeneity. The inhomogeneity-induced stabilization is due to the escape of waves with group velocity

$$\frac{d\omega}{dk_{\parallel}} = \left(3k_{\parallel} r_D^2 - \frac{k_{\parallel}}{k_{\perp}^2} \frac{\omega_B^2}{\omega_1^2} \right) \omega. \quad (8.3.24)$$

Is it assumed that in the direction normal to the geomagnetic field direction, the plasma is homogeneous. Using the Hamilton equation for a convecting wave packet, one can readily estimate the rate of energy pumping along the geomagnetic field line:

$$\gamma_H \approx \frac{\ln \Lambda}{4} \frac{\omega_1}{H_p \Delta k_{\parallel}}. \quad (8.3.25)$$

The value of $\Delta k_{||} = k_{||} \Delta E_{res} / 2E_{res}$ is determined by the character of the non-monotonicity of the secondary electron spectrum. For a height $h = 180$ km when $H_p > 50$ km and $\omega_1 = 2 \times 10^7 \text{ s}^{-1}$, we obtain $\gamma_H = 4 \times 10^2 \text{ s}^{-1}$. Such values of γ_H are comparable with the collisional damping of the plasma oscillations. From the expression for group velocity (8.3.24), it is evident that at a height where the condition $k_{\perp}^2 R_e^2 = 1/3 (R_e = v_e / \omega_B)$ is satisfied and, hence $d\omega/dk_{||} = 0$, the instability can have an absolute character.

Let us assess the possible geophysical effects of an ionospheric plasma instability caused by secondary electrons. At heights $h < 300$ km, where the distribution of secondary electrons is isotropic and the instability is due to the nonmonotonic character of their distribution function, the effect of nonlinear relaxation, as shown in Sect. 8.3.2, can be evaluated by (8.2.25). For example, for a precipitation spectrum with an isotropic downward pitch-angle distribution when $E_b = 3 \text{ keV}$ and $A = 88.5$, the maximum growth rate is $\gamma_{\max} = 4.3 \times 10^3$ for an altitude of 200 km when $E_{res} = 2.8 \text{ eV}$ ($k_{\perp} \approx 0.5 \text{ cm}^{-1}$), and $\gamma_{st} \approx v_e/2 = 2 \times 10^2 \text{ s}^{-1}$. Quasilinear relaxation, as follows from (8.2.25), will cause the depth of the “pit” on the distribution function of secondary electrons to decrease by 15 times, i.e., virtually to a plateau. In this case, the energy density $W \approx 10 \text{ eV cm}^{-3}$ is approximately three to four times the energy density of thermal fluctuations $W_T = nT_e/N_D$, where $N_D = nr_D^3$ ($n \approx 2 \times 10^5 \text{ cm}^{-3}$, $T_e \approx 0.1 \text{ eV}$). However, the energy density of thermal fluctuations is determined by all fluctuations in the k interval from 0 to r_D^{-1} , whereas W corresponds to a small volume of wave numbers $2\pi\Delta k_{||}(\Delta k_{\perp})^2$, where

$$\Delta k_{||,\perp} = k_{||,\perp} \frac{\Delta E_{res}}{2E_{res}}.$$

The ratio of the density fluctuation level in the unstable plasma to the thermal level can be obtained by using the Poisson equation:

$$\frac{\langle E_{||}^2 \rangle_k}{8\pi} = \frac{2\pi^2 e}{k^2} \langle \delta n^2 \rangle_k$$

to be

$$\frac{\langle \delta n^2 \rangle_k}{\langle \delta n^2 \rangle_k^T} \approx \frac{8\pi^2 W}{T_e \Delta k_{||} (\Delta k_{\perp})^2} \leq 10^6. \quad (8.3.26)$$

The energy of plasma oscillations that has been transferred from the resonant electrons, in turn, goes into additional heating of the thermal plasma. The effectiveness of such heating is proportional to $(v_{ei} + v_{en})W \approx 8 \times 10^3 \text{ eV cm}^{-3} \text{ s}^{-1}$. The heating of the thermal plasma under the same conditions due to collisions is approximately $2 \times 10^3 \text{ eV cm}^{-3} \text{ s}^{-1}$. One can see that the effectiveness of plasma heating, at heights where the instability is realized, can increase several times, which must lead to an anomalous increase in temperature. Rees et al. (1977) presented the results of both rocket and satellite measurements during a diffuse

electron precipitation with an energy flux $P_E \approx 7 \text{ erg cm}^{-2}\text{s}^{-1}$ (in our example, $P_E \approx 20 \text{ erg cm}^{-2}\text{s}^{-1}$), as well as noting a small (of order 20–30%) increase (compared with calculations) in the electron gas temperature in the height range of approximately 200 km. This deviation can be easily explained both qualitatively and quantitatively in terms of the plasma instability caused by secondary electrons. Note that the instability concerned can occur at heights of approximately 200 km in both the downward and upward current region.

Kelley and Earle (1988) have reported on oscillations that have characteristics similar to those predicted by the above theoretical study. These authors observe waves at frequencies between the local upper hybrid and plasma frequencies in the upper E-region of the morning auroral oval. No theoretical explanation yet exists, but Kelley and Earle (1988) speculate that these waves may be the manifestation of the plasma instability process due to the superthermal electron flux, which accompanies collisional ionization in the auroral E region.

Oyama and Hirao (1979) and Oyama et al. (1983) reported on rocket data from the daytime, mid-latitude ionosphere. They found a broad layer of weakly enhanced thermal electron energy tails that started appearing at 135 km, became most intense around 150 km, and gradually vanished at an altitude of approximately 210 km. These observations once again indicate the presence of plasma instabilities and wave–particle interactions in the E- and lower F-region ionosphere.

Consider a plasma stability in the topside ionosphere in the downward current region. Here, as was shown in the preceding section, the distribution function of secondary electrons acquires a quasibeam character. Excitation growth rates of plasma waves here are not less and are, sometimes, larger than those for precipitating electron beams. Thus, in the region of downward current, the instability ($\Gamma > 10$), which is caused by secondary electron beams escaping from the ionosphere, must occur. Because the region under consideration is less constrained in space (the transverse size can reach hundreds of kilometers) than is the region of upward current, a predominant enhancement of wave intensity will occur both along the geomagnetic field and at a certain angle to it. This angle is determined from the condition for maximum in (8.3.14). The amplification of the wave packet for this angle can be significant. An intense excitation by secondary electrons of plasma waves with frequencies $\omega_2(\theta)$, with a characteristic dependence of the enhancement Γ on the angle, can be the cause of the appearance of a VLF emission known as saucers (James 1976) that propagates in the antiearthward direction. With a rough equality of the particle fluxes, the energy flux of precipitating electrons is, as a minimum, an order of magnitude larger than the energy flux of secondary electrons. This difference is reflected in the intensity difference of saucers and auroral hiss. The difference in energetics also has an effect on the mechanisms for stabilizing the instabilities. Thus, the energetics of precipitating electrons admits the possibility of stabilization of the beam instability, by nonlinear mechanisms, characteristic for turbulent plasma. An intense induced scattering of the excited waves due to thermal ions is possible (Mishin et al. 1990).

For plasma waves excited by secondary electron beams, nonlinear mechanisms of wave energy transfer over the spectrum do not play a role. Thus, for example, the rate of induced scattering due to both ions and electrons in a thermal plasma by these waves does not exceed the value 10^2s^{-1} , which is much less than growth rates of beam instability. The stabilization of such an instability proceeds on account of the quasilinear relaxation of secondary electron beams. For typical conditions of the auroral ionosphere, the energy density of plasma waves, excited by a beam of secondary electrons, as found in the previous section, is $W \approx 10^2 \text{ eV cm}^{-3}$. One of the geophysical manifestations of the high level of plasma noise is the intense heating of the instability region $Q = \nu_e W \approx 10^3 \text{ eV cm}^{-3} \text{s}^{-1}$ that exceeds by far the heating due to collisions in the plasma $Q \approx 10^1 \text{ eV cm}^{-3} \text{s}^{-1}$ (Khazanov 1979).

Note also the fundamental differences in the character of the distribution of the waves excited by a beam of precipitating and secondary electrons. The emission that is generated by precipitating electrons is directed earthward while secondary electrons excite waves directed antiearthward. The minimum in the spectrum of predominant branches, excited by precipitating electrons, lies at angles $\theta_T \approx 1^\circ$, while for the waves excited by secondary electrons $\theta_T \approx 30^\circ$.

For the region of upward current, as a result the beam of precipitating electrons being located in space, the plasma noise distribution at angles $\theta_T < \theta < \theta_0$ is strongly “cut off” (dashed line in Fig. 8.7). The intensity distribution of plasma noise in the downward current region is not so critical to the localization size of the secondary electron beam. The plasma noise intensity here has high values almost for all θ . The branch ω_2 , excited by secondary electrons, forms wideband noise (Maggs 1976).

8.4 Some Radio Physical Effects of Superthermal Electrons

At present, radio wave incoherent scatter (RIS) facilities, along with Earth-orbiting satellites, provide the most informative ionospheric diagnostic tools. The techniques developed to date make it possible to measure many parameters and characteristics of the ionospheric plasma. Nonetheless, the question of expanding the diagnostic capabilities of RIS stations is important. The last few decades (see references in Djuth et al. 1994; Bhatt et al. 2008) witnessed a significantly increased interest to plasma line measurements in the RIS spectrum because this line is directly associated with electron density fluctuations in the ionospheric plasma which, in turn, are determined by the distribution of superthermal electrons. This fact induces us to hope that the RIS method can be used to reconstruct some distinctive features of the energy and angular distribution of photoelectrons.

Acquisition and analysis of such characteristics can, firstly, provide additional information about ionospheric parameters and, secondly, specify the spectra of photoelectrons, which is important in the study of the ionospheric energy regime.

8.4.1 *The RIS Spectrum in the Presence of Photoelectrons: Diagnostic of Aeronomical Parameters*

In (Carlson et al. 1977), a method was suggested to study the processes that form the energy distribution of photoelectrons by comparing theoretically calculated and experimentally measured fluctuation spectra from incoherent scatter facilities for the same heliophysical and geophysical conditions. In this case, a theoretical spectrum is determined by the energy distribution of photoelectrons, which is obtained by solving the kinetic equation for electrons (2.5.2). It is of interest to expand the possibility of such investigations through the study of fine structure (calculated and measured with good resolution in energy) of the spectrum of plasma waves generated by photoelectrons with energies 15–32 eV, for which the distribution function has a number of peaks caused by the resonance character of the ion production processes.

Consider high-frequency fluctuations in electron density in the daytime ionosphere at lower F-region heights. This region is of interest because of the main energy release of solar ultraviolet, and the energy distribution of photoelectrons has a number of distinctive features.

For the sake of simplicity, we shall limit our attention to considering fluctuations with wave vectors directed along the geomagnetic field. In this case, for lower F-region heights where the photoelectron distribution is isotropic, the fluctuation spectrum can be represented as (Ivanov et al. 1979)

$$\langle \delta n^2 \rangle_k = n_0 \frac{\frac{v_e}{\omega_r} \frac{k^2 v_e^2}{\omega_p^2} + \pi^2 \frac{\omega_r m}{k} \int_{E_*}^{\infty} dE \frac{\Phi(E)}{E}}{\frac{v_e}{\omega_p} + \pi^2 \frac{m \omega_r \omega_p^2}{k^3 n_0} \frac{\Phi(E_*)}{E_*}}, \quad (8.4.1)$$

where $\Phi(E)$ is the photoelectron spectrum and $E_* = m\omega_p^2/2k^2$. For the case of backscatter, between the wave vector $\mathbf{k}_0$ ($k_0 = \omega_0/c$) of the sounding VHF signal and the vector $\mathbf{k}$, there exists a simple relation $2\mathbf{k}_0 = -\mathbf{k}$; therefore, E_* can be rewritten as

$$E_* = \frac{mc^2}{8} \frac{\omega_r^2}{\omega_0^2}, \quad (8.4.2)$$

where ω_0 is the frequency of the sounding signal. The eigen frequency of the electrostatic oscillations ω_r close to the plasma frequency is determined from the solution of the equation

$$1 + \frac{1 - \varphi\left(\frac{\omega_r}{\sqrt{2}kv_e}\right)}{k^2 r_e^2} = 0, \quad (8.4.3)$$

where

$$\varphi(z) = 2ze^{-z^2} \int_0^{\infty} e^{t^2} dt.$$

From (8.4.1) it is evident that the intensity distribution of plasma waves is associated with the energy distribution of photoelectrons, and the latter's peculiarities find their reflection in the fluctuation spectrum (see Fig. 8.3). In this case, as follows from (8.4.1), the dependence has an inversely proportional character because the integral in the numerator is a weakly varying function E_* . Such a dependence makes it possible to suggest a method of obtaining information about the relative composition of the Earth's neutral atmosphere at lower F-region heights using ground-based incoherent scatter facilities. In fact, comparative analysis based on identifying peaks from the ionization of molecular nitrogen N_2 and oxygen atoms O by the helium line HeII 304 Å makes it possible to determine their relative number density.

As is known (see Chap. 7 for details, and Fig. 8.3), peaks of 22.3, 23.7, and 27.2 eV are produced as a result of the ionization of O ; in this case, oxygen ions are in the states ^2P , ^2D , and ^4S , respectively. Peaks at 22, 24, and 25.2 eV result from the ionization of N_2 . The molecular nitrogen ions are in the states $B^2\Sigma_u^+$, $A^2\Pi_u^+$, and $X^2\Sigma_g^+$, respectively. Smaller peaks are caused by the ionization of molecular oxygen, whose density at these heights is an order of magnitude lower than the N_2 and O density, as well as by the ionization of N_2 and O by the line HeII 256 Å, whose intensity is an order of magnitude lower than the HeII 304 Å intensity. In order to measure the ratio of N_2 and O densities, it is necessary, by knowing their ionization profiles, to compare the intensities, for example, of peaks on 25.2 and 27.2 eV (minima on the fluctuation spectrum). Here there seem to be two possible methods of experimental implementation of the proposed technique. Namely, measuring, at the same height, the plasma line spectrum by varying the radar frequency. In this case, measurements in the energy range from 24 to 28 eV require varying the frequency of the sounding signal ω_0 less than by 10% ($\Delta E_*/E_* = 2\Delta\omega_0/\omega_0$).

Application of broadband sounding signals is possible. In this case, the receiver must be provided with a system of filters with good frequency–time resolution. An alternative is to measure the plasma line spectrum at a constant frequency ω_0 from different heights (from different delays). In this case, the change is due to height variations in electron density. For real ionospheric conditions, the height range required for measuring the spectrum in the energy range 24–28 eV is 8–16 km ($\Delta E_* \sim \Delta n_0$), which is comparable with typical scales of density variation of N_2 and O . Therefore, such a method is used to measure the ratio of N_2 and O densities at fixed (but displaced with respect to each other) heights. To achieve a reliable measurement of the ratio of N_2 and O densities, it is desirable to have the resolution accuracy in height less than 1 km.

Note that photoelectron spectra can be peaked more strongly at energies between 15 and 30 eV as compared with the spectrum presented in Fig. 8.3a ($\Delta E = 0.2\text{eV}$). Indeed, the width of the peaks in the formation of the photoelectron distribution is smeared from the value 0.016 eV, determined by the width of the helium line, to a value corresponding to the mean energy of thermal electrons T_e (see Sect. 5.3.2). Because, for heights around 200 km, $T_e \approx 0.1\text{eV}$, a smaller peak width, and hence their larger amplitude, can ease the diagnostics by the proposed method.

On the other hand, measurements of the peak widths of spectrum fine structures, as discussed by Krinberg (1978) and Konikov and Khazanov (1985), can provide information about the HeII 304 Å intensity, and electron temperature in the ionospheric plasma. Because the plasma line width is of the order of 100 kHz (Vidal-Madjar et al. 1975), for a radar frequency of approximately 500 MHz, it will correspond to the energy range on the photoelectron spectrum around 25 eV only of approximately 0.02 eV. This induces us to hope that the photoelectron spectrum width can be measured rather reliably, and that the value of T_e can thereby be determined. However, one should keep in mind that, in order to determine the correct value of T_e , the Coulomb logarithm should be considered as discussed in Chap. 5, in Sect. 5.3.2 (see also Konikov and Khazanov 1985 for additional details). Note that the heights between 160 and 230 km seem to be optimal for such measurements because at higher altitudes the peaks are smeared by Coulomb collisions, and the N_2 density decreases rapidly, whereas below these heights the effective collision frequency increases, which leads (as is apparent from (8.4.1)) to a decrease in nonmonotonicity of the electron density fluctuation spectrum in the ionospheric plasma. Figure 8.8 taken from (Carlson et al. 1977) gives the plasma line intensities in the incoherent scatter spectrum in terms of kT_p (see (8.2.8)), measured at heights from 300 to 500 km at a constant radar frequency ω_0 . Unfortunately, experimental points show a strong spread near energies over 10 eV and do not cover the energy in excess of 24 eV (this was beyond the scope of the investigation); moreover, the heights do not correspond to those where fine structure peaks are maximal; nevertheless, one can see that there is fine structure of

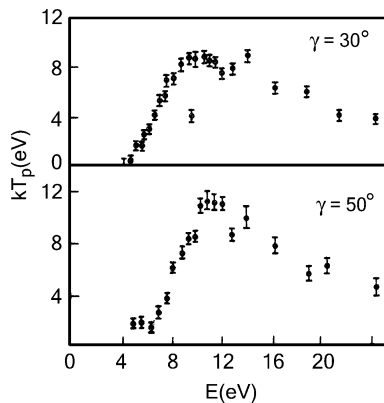


Fig. 8.8 The plasma line intensity distribution in the incoherent scatter spectrum as measured at a constant radar frequency ω_0 (Carlson et al. 1977) for two values of angle γ between $\mathbf{k}_0$ and $\mathbf{B}$

the plasma line in the energy region over 10 eV. In Fig. 8.8, γ is the angle between the wave vector $\mathbf{k}_0$ and $\mathbf{B}$.

Taking the magnetic field into account does not complicate the interpretation of the results because there remains a connection between the fast varying (in E_*) values of the functions $\langle \delta n^2 \rangle_k$ and $\Phi(E_*)$ in the energy range 15–30 eV. However, it is important to take into account the thermal electron distribution in the numerator and the denominator of (8.2.1) when $(\omega_r - n\omega_B)/k_{||} \approx v_e$.

The fluctuation spectrum in a magnetoactive plasma will have the form

$$\langle \delta n^2 \rangle_{k_{\perp}, k_{||}} = \frac{F_T + \sum_n \frac{2\pi^2 m}{|k_{||}|} \int_{E_n}^{\infty} dE \frac{\Phi(E)}{E} J_n^2(x)}{\frac{v_e}{\omega_p} + \frac{\omega_p^2}{k^2 v_e^2 n_0} F_T + \frac{\pi^2 \omega_r \omega_p^2}{|k_{||}| k^2 n_0} \frac{\Phi(E_0)}{E_0} - \frac{\pi^2 \omega_r \omega_p^2}{|k_{||}| k^2 n_0} \sum_n \int_{E_n}^{\infty} dE \frac{\Phi(E)}{E} \frac{\partial}{\partial E} J_n^2(x)}, \quad (8.4.4)$$

where $F_T(k_{\perp}, k_{||}, \omega_r)$ is the distribution function of thermal electrons that has the form (Salpeter 1961)

$$F_T = \sqrt{\frac{\pi}{2}} \frac{n_0}{|k_{||}| v_e} \sum_n e^{-\left| \frac{\omega_r - n\omega_B}{\sqrt{2} k v_e} \right|^2} I_n(z_e) e^{-z_e}. \quad (8.4.5)$$

Here

$$x = \frac{k_{\perp}}{\omega_B} \sqrt{\frac{2(E - E_n)}{m}}, \quad E_n = \frac{m|\omega_r - n\omega_B|^2}{2k_{||}^2}, \quad z_e = \frac{k_{\perp}^2 v_e^2}{\omega_B^2},$$

When the resonance condition $(\omega_r - n\omega_B)/k_{||} \approx v_e$ is satisfied, the thermal electron distribution function is much larger than the photoelectron distribution function. In this case the fluctuation spectrum decreases to an equilibrium one. Under typical ionospheric conditions, the relationship $(\omega_r - n\omega_B)/k_{||} \approx v_e$ for different n is satisfied in narrow height ranges separated in height by more than 50 km. Therefore, it is always possible to find a height range for measuring the ratio of N_2 and O densities where there exists a simple connection between $\langle \delta n^2 \rangle_k$ and Φ defined by (8.4.1).

When measuring ionospheric plasma fluctuations using incoherent scatter facilities, it is also necessary to consider multiple plasma lines in the incoherent scatter spectrum caused by variety of high-frequency eigen oscillations in the magnetoactive plasma. All of them are, to a certain extent, enhanced by the presence of photoelectrons. Measurements of the number density of the main neutral species of the ionosphere at lower F-region heights, their variations during the day, and their seasonal behavior are of independent interest because

such data obtained in rocket experiments are scarce, and satellite data refer to higher altitudes. Moreover, by obtaining such data, it would become possible to correct models of neutral components used when calculating photoelectron spectra, and to monitor solar activity in the ultraviolet range when choosing an optimal model.

Indeed, in Carlson et al. (1977), it was shown that plasma line spectra, provided that all energy dissipation mechanisms for photoelectrons are known, can provide solar ultraviolet intensities in the line HeII from the fine structure of the photoelectron spectrum, with an energy resolution better than 0.5 eV. It was shown that the areas of the peaks produced by the ionization of atomic oxygen *O* at 250- to 300-km altitudes do not depend on the *O* density and directly can be used to determine the intensity of HeII 304 Å. Because the high-frequency electron density fluctuation spectrum, as follows from comparison of Fig. 8.3a, b, is determined by the energy distribution of photoelectrons, the time variations in the HeII line intensity can be measured by incoherent scatter facilities. Thus, in order to obtain a profile of the peak with an energy resolution of 0.2 eV, for example, for a peak on 27.2 eV covering an interval of approximately 2 eV, it is necessary to take (with a radar frequency $f_0 = \omega_0/2\pi \approx 500$ MHz (Arecibo, Millstone Hill)) the plasma line intensity from the interval (bin) of approximately 10 km ($h < 300$ km) with a height resolution of 1 km.

In addition to acquiring relative characteristics of ionospheric plasma, incoherent scatter facilities appear to be able to provide their absolute values as well. Thus, in (Cicerone and Bowhill 1971), a technique was suggested for reconstructing absolute values of photoelectron spectra. To obtain the energy distribution of photoelectrons, it is necessary to solve the differential equation that can be easily obtained from (8.4.1)

$$f'(E_*) + f(E_*) \left\{ \frac{\langle \delta n^2 \rangle'_{E_*}}{\langle \delta n^2 \rangle_{E_*}} + \frac{c}{a \langle \delta n^2 \rangle_{E_*}} \right\} + \frac{b}{a} \frac{\langle \delta n^2 \rangle'_{E_*}}{\langle \delta n^2 \rangle_{E_*}} = 0, \quad (8.4.6)$$

where

$$f(E_*) = \frac{m^2}{2E_*} \Phi(E_*), \quad \langle \delta n^2 \rangle'_{E_*} = \frac{d}{dE_*} \langle \delta n^2 \rangle_{E_*}, \quad c = \frac{2\pi^2 \omega_r}{|k|m}, \quad b = \frac{\nu}{\omega_r}, \quad a = 2\pi^2 \frac{\omega_r \omega_p^2}{|k|^3 n_0}.$$

The coefficients in (8.4.6), as functions of E_* , are specified from experiment. The integration constant is determined (Cicerone and Bowhill 1971) from the condition that the distribution function of photoelectrons is equal to the distribution function of thermal electrons at low energies, where the measured fluctuation spectrum decreases to the thermal level $\langle \delta n^2 \rangle_k^\varphi = \langle \delta n^2 \rangle_k^T = 1/2n_0(kr_D)^2$. The thermal level of the fluctuations, and also the distribution function of thermal electrons, depend on the parameters n_0 and T_e and are determined during concurrent experiments measuring the ion line of the incoherent scatter spectrum. An alternate choice of the integration constant is also possible, namely by measuring background fluctuations

outside the plasma line at a fixed frequency, for example $\omega' = \omega_0 + \omega \geq \omega_0 + \omega_p$. In this case, the value of

$$\langle \delta n^2 \rangle_k = \frac{\pi^2 \frac{\omega_r m}{|k|} \int_{E_*}^{\infty} dE \frac{\Phi}{E}}{\left(1 - \frac{\omega_r^2}{\omega^2}\right)} \quad (8.4.7)$$

is measured, and then, from (8.4.1) and (8.4.7), the value of $\Phi(E_*)$ is determined at energies higher than 20 eV, where the quantity v_e/ω_r in the numerator of (8.4.1) can be neglected. In Cicerone and Bowhill (1971), by comparing reconstructed spectra of photoelectrons with those obtained theoretically for the same heliophysical and geophysical conditions, it was shown that obtaining absolute values of the photoelectron spectrum from measurements of electron density fluctuations in the ionospheric plasma is possible with an error less than 30%. Figure 8.9 presents the photoelectron spectra, taken from Abren and Carlson (1977) and reconstructed by using the method described above from radar measurements at Arecibo. One can see that the photoelectron spectra have typical values as well as fine structure in the energy range over 15 eV.

These methods can, possibly, be used not only to determine time variations but also absolute values of ultraviolet intensity in both the continuum emission spectrum and the HeII line. Indeed, the maximum heights of peaks on the photoelectron spectrum, for example at 25.2 and 27.2 eV, as follows from Chap. 5, have the form

$$f_0^N = \frac{m^2}{8\pi E_*} \frac{N_n P_n \sigma_n^v(E_*)}{\sum_n N_n \sigma_n(E_*)} \frac{F_\Sigma}{2\delta E_*}, \quad (8.4.8)$$

where $E_* = 25.2$ or 27.2 eV; N_n is the density of N_2 or O ; σ_n^v is the ionization cross-section of O or N_2 ionization; σ_n is the cross-section of inelastic collisions with N_2 or O , respectively; δE_* is the peak half-width; P_n represents the known probabilities of ion production in different quantum states; and F_Σ is the helium line intensity.

From the system of equations (8.4.8) for two peaks, one can determine

$$F_\Sigma = \frac{S_1 + S_2}{P_{N_2} \frac{\sigma_{N_2}^v}{\sigma_{N_2}} + P_O \frac{\sigma_O^v}{\sigma_O} \frac{f_O^0}{f_{N_2}^0}} \left(1 + \frac{f_O^0}{f_{N_2}^0}\right), \quad (8.4.9)$$

where $S_1 = f_O^0 \cdot 2\delta E_*$ and $S_2 = f_{N_2}^0 \cdot 2\delta E_*$. Thus, by measuring the heights and areas of the peaks, one can obtain intensity values of HeII 304 Å and the ratio of N_2 to O densities. Measuring absolute values of N_2 and O densities seems to be possible if our consideration includes the nonmonotonicity of the photoelectron spectrum in the energy range 2–4 eV caused by the excitation of vibration levels of

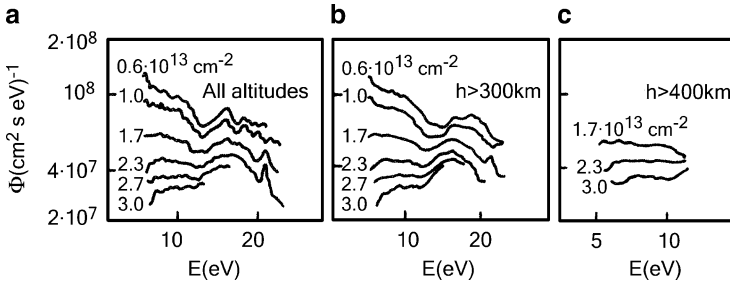


Fig. 8.9 The photoelectron spectra as obtained in (Abren and Carlson 1977) from plasma line intensity measurements by the Arecibo radar

N_2 . The excitation cross section of N_2 is known from laboratory experiments; therefore, by knowing the photoelectron spectrum and determining the “pit” depth at 2.2 eV, it is possible to directly obtain the N_2 density.

Often, incoherent scatter stations include several transmitters operating at different carrier frequencies; therefore, it is possible to take different parts of the photoelectron spectrum from about the same heights. Thus, suitably staged experiments, combined with careful analysis and interpretation of experimental incoherent scatter data on the plasma line are able to give information about many new plasma parameters of the Earth’s atmosphere.

Note in conclusion that when measuring ionospheric plasma fluctuations with wave vectors $\mathbf{k}$ directed at an angle to the geomagnetic field, the spectrum of the reflected incoherent scatter signal shows several plasma lines caused by the variety of eigen oscillations ($\omega_1, \omega_r, n\omega_B$) in the magnetoactive plasma. During the daytime, all of the plasma lines are, to a certain extent, intensified by the presence in the plasma of photoelectrons and can be recorded reliably. Determination of the position of plasma lines in the incoherent scatter spectrum can also serve as an additional source of information about ionospheric parameters (Ivanov et al. 1981). Indeed, the frequencies of potential eigen oscillations of a plasma are determined from the dispersion equation $\text{Re } \varepsilon = 0$, which depends on such parameters as $\omega_p, \omega_B, k_{||}, k_{\perp}, T_e$. By measuring the position of several plasma lines, i.e., by determining the plasma eigen frequencies ω_r , it is possible to construct a system of equations $\omega(\omega_p, \omega_B, T_e, \cos \theta)$ from which these parameters can be determined.

Since the method is based only on measuring the positions of the maxima of the plasma and cyclotron lines, and on Doppler shifts, one may frequently expect highly accurate measurement of such parameters as the electron temperature and density as well as the magnitude of the local geomagnetic field. Thus, in Hugfors and Lehtinen (1981), using the method of measuring the plasma line position, the mean energies of thermal electrons T_e were determined. To determine T_e , use was made of the upper-hybrid frequency for which, when $\omega_p > \omega_B$ and $k^2 r_D^2 \ll 1$,

$$\omega_1^2 = \omega_p^2 + \omega_B^2 \sin^2 \theta + 3\omega_p^2 k^2 r_D^2. \quad (8.4.10)$$

The values of ω_p , ω_B , $\cos \theta$ were determined from ion line measurements of the incoherent scatter spectrum. It was shown that such a method of obtaining values of T_e is much more accurate, and more operative, compared with the conventional method of determining T_e from measurement of the ion line shape. A similar method of determining T_e is also suggested in Tereshchenko (1982). The proposed method of measuring the positions of several plasma lines, combined with numerical analysis of the dispersion equation, will make it possible to obtain a set of ionospheric parameters n_0 , T_e , ω_B , irrespective of their determinations from the main ion component of the incoherent scatter spectrum, which must improve the reliability and accuracy of obtaining such data.

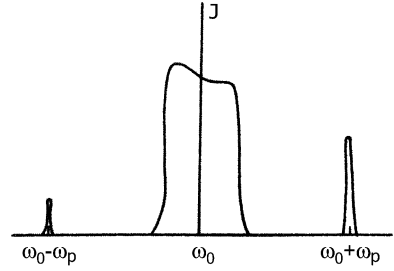
8.4.2 Ground-Based Diagnostic of Photoelectron Fluxes

Since photoelectron free path lengths are comparable with the irregularity scales of the Earth's upper atmosphere, they can leave the region of photo production along geomagnetic field lines. At upper ionospheric heights ($h > 300$ km), photoelectron fluxes with a mean energy of 10–20 eV are directed to the magneto-conjugate ionosphere. The knowledge of angular distributions of such photoelectron fluxes is required for calculating heating rates of the plasmasphere, whose accumulated energy is known to determine the values of n_0 and T_e of the nightside ionosphere, as well as for a quantitative description of the exchange of ionization and energy of the magneto-conjugate ionospheres.

In applied problems of modeling of ionospheric processes in the lower part of the F-region (heights below 300 km), the influence of the conjugate ionosphere is often neglected and attention is confined to specifying electron fluxes at topside ionospheric heights taken from separate rocket or satellite measurements. Regular measurement of photoelectron fluxes by ground-based radio physical facilities would permit these problems to be solved correctly. Let us consider the possibility of such measurements to obtain quantitative links that permit us to reconstruct the angular distribution of photoelectrons from incoherent scatter data (Ivanov et al. 1982a).

As is known, incoherent scatter spectra have two plasma lines, namely a Stokes and an anti-Stokes line, with frequencies shifted relative to the sounding frequency by $\pm \omega_p$. Figure 8.10 is a schematic representation of an incoherent scatter spectrum. The plasma line on $\omega_0 - \omega_p$ is associated with high-frequency plasma waves, propagating upward along geomagnetic flux tubes, enhanced by photoelectrons traveling upward. The line $\omega_0 + \omega_p$, accordingly, is associated with photoelectrons traveling downward. The difference in enhancement of these lines suggests the presence of directed electron fluxes. Since incoherent

Fig. 8.10 The spectrum of the incoherent scatter signal



scatter facilities can only measure two these lines $\omega_0 + \omega_p$ and $\omega_0 - \omega_p$ (we denote them by $\langle \delta n^2 \rangle_k^+$ and $\langle \delta n^2 \rangle_k^-$), there is a constraint on the possibility of reconstructing the photoelectron distribution function (Lejeune 1979). Indeed, if the photoelectron spectrum is represented as an expansion in Legendre polynomials (see Chap. 5), one can only determine the first two expansion terms from experiment. However, these two terms virtually determine the photoelectron distribution at heights $h > 300$ km because the even expansion terms do not contribute at $\mathbf{k}$, $\mathbf{k}_0 \parallel \mathbf{B}$, while the odd terms with $n \geq 3$ are already small compared with $n = 1$.

The spectrum of high-frequency electron density fluctuations in this case will have the form ($k_\perp = 0$)

$$\langle \delta n^2 \rangle_k = \frac{\frac{\pi^2 m^2}{k} \int_{E_*}^{\infty} dE \frac{\Phi_0}{E} + \frac{\pi^2 m^2}{k} \sqrt{E_*} \text{sign } \omega_r \int_{E_*}^{\infty} dE \frac{\Phi_1}{E^{3/2}}}{\frac{v_e}{\omega_p} + 2 \frac{\pi^2 m^2}{k} E_* \left\{ \frac{\Phi_0(E_*) + \Phi_1(E_*)}{\sqrt{E_*}} - \frac{1}{2} \int_{E_*}^{\infty} dE \frac{\Phi_1}{E^{3/2}} \right\}} \Bigg|_{\omega_r = \pm \omega_p} \quad (8.4.11)$$

Here $\Phi = \Phi_0 + \Phi_1 \sqrt{E_*/E} \text{sign } \omega_r$, $v_e \sim n_0/T_e^{3/2}$, $E_* = m\omega_p^2/2k^2$, n_0, T_e are determined from ion component measurements (central line in Fig. 8.10). Thus, if there are data on the plasma lines, then, from the system of integral equations,

$$\begin{aligned} \frac{1}{2} \left\{ \langle \delta n^2 \rangle_k^+ + \langle \delta n^2 \rangle_k^- \right\} &= \frac{\pi^2 \frac{m^2}{k} \int_{E_*}^{\infty} dE \frac{\Phi_0}{E}}{\text{Im} \varepsilon_e(\Phi_0, \Phi_1)}, \\ \frac{1}{2} \left\{ \langle \delta n^2 \rangle_k^+ - \langle \delta n^2 \rangle_k^- \right\} &= \frac{\pi^2 \frac{m^2}{k} \sqrt{E_*} \int_{E_*}^{\infty} dE \frac{\Phi_1}{E^{3/2}}}{\text{Im} \varepsilon_e(\Phi_0, \Phi_1)}, \end{aligned} \quad (8.4.12)$$

where

$$\text{Im}\varepsilon_e(\Phi_0, \Phi_1) = \frac{v_e}{\omega_p} + 2 \frac{\pi^2 m^2}{k} E_* \left\{ \frac{\Phi_0(E_*) + \Phi_1(E_*)}{\sqrt{E_*}} - \frac{1}{2} \int_{E_*}^{\infty} E \frac{\Phi_1}{E^{3/2}} \right\}; \quad (8.4.13)$$

one can determine $\Phi_0(E_*)$ and $\Phi_1(E_*)$, which define the isotropic and directed parts of the photoelectron spectrum.

Reconstructing photoelectron spectra is possible if plasma-line intensities are taken as delay functions. It is possible to show that slight changes in plasma frequency ω_p (of the order of 10%) correspond to variations of energy E_* by several electron volts, which makes it possible to determine the integral part of the spectrum for both $\Phi_0(E_*)$ and $\Phi_1(E_*)$. Since the incoherent scatter method provides good height resolution for obtaining $\Phi_0(E_*)$ and $\Phi_1(E_*)$, it is necessary to take the plasma line intensity from different (though closely spaced) heights (different ω_p). Using the results reported in Polyakov et al. (1979), where $\Phi_0(E)$ and $\Phi_1(E)$ were obtained for daytime mid-latitude ionospheric conditions, one can obtain the expected relative differences of plasma line amplitudes in the incoherent scatter spectrum for the height $h = 300$ km with $n_0 = 4 \times 10^5 \text{ cm}^{-3}$. We obtain $\Delta \langle \delta n^2 \rangle_k / \langle \delta n^2 \rangle_k \approx 0.1$, where $\Delta \langle \delta n^2 \rangle_k = \langle \delta n^2 \rangle_k^+ - \langle \delta n^2 \rangle_k^-$; here k corresponds to the value of $E_* = 10 \text{ eV}$. One can see that the photoelectron flux causes a small difference in plasma-line amplitudes, of the order of 10%. At 500-km altitude, this difference reaches 50% for energies $E_* = 15 \text{ eV}$. In Cicerone (1974) and Sheffield (1978), the incoherent scatter method was applied to investigate the anisotropy of photoelectron spectra. Their data agree with the estimates presented above. The above experimental technique for recording the directed part of photoelectron spectra is applicable if the ionospheric plasma is stable, when it is possible to use the linear theory of plasma density fluctuations and there exist simple functional correlations between $\Phi(E)$ and $\langle \delta n^2 \rangle_k$. As shown above, in the mid-latitude ionosphere, such a situation is realized almost always.

Photoelectron fluxes at 160- to 100-km altitudes are directed downward and are caused, as shown in Lejeune and Kofman (1977), by the presence of a sharp anisotropy in the direction ahead of regions of electron scattering due to oxygen atoms. For energies less than 4 eV (mean photoelectron energies), the anisotropy of the photoelectron spectra Φ^-/Φ^+ can be of the order of 2 (Lejeune and Kofman 1977), where $\Phi^\pm$ represents photoelectron fluxes directed upward and downward, respectively. Such values, as shown in Lejeune and Kofman (1977), account for the 20–50% plasma line differences measured at Arecibo at those heights.

8.4.3 Generation of VLF Emission

Electromagnetic emissions in the frequency range $f = 1\text{--}100$ kHz (including VLF emissions) serves as one of the indicators of “disturbance” of the near-terrestrial

space environment (Raspov and Kleimenova 1977; Omura et al. 1990). In the deep magnetosphere, in the region of closed geomagnetic field lines, the generation of VLF emissions is associated mainly with the cyclotron instability in the Earth's radiation belts (Bespalov and Trakhtengerts 1986; Omura et al. 1990). At the same time, there exists a good correlation between VLF emission bursts at ionospheric heights with photoelectron fluxes between magneto-conjugate regions and with precipitating auroral electrons (see, Raspov and Kleimenova 1977). The observed emission intensity ($10^{-16} - 10^{-15} \text{ W m}^{-2} \text{ Hz}^{-1}$) exceeds by far both the equilibrium and weakly non-equilibrium levels determined by the stable motion of superthermal electrons.

It is easy to demonstrate that the latter do not directly generate the VLF emissions. Indeed, from the resonance condition $v_{res} = (\omega - n\omega_B)/k_{\parallel}$ (where $\omega \approx (kc/\omega_p)^2 \omega_B \cos \theta$ is the frequency of the VLF waves), it follows that the velocity of the electrons must satisfy the obviously not satisfiable inequality ($n = -1$)

$$v_{res} > c \frac{\omega_B}{\omega_p} \geq 0.3c. \quad (8.4.14)$$

Liperovsky and Pudovkin (1983) attributed the VLF emissions to the transformation of plasma waves excited by superthermal electron fluxes ($E < 300 \text{ eV}$) into electromagnetic waves due to plasma irregularities both of natural origin and produced by nonlinear effects. Above it was shown that superthermal electron fluxes, under certain situations, effectively excite plasma oscillations at topside ionospheric heights. On the other hand, oscillations in the region of lower-hybrid resonance (LHR) are a continuous extension of the whistler branch to a region of large values of the refractive index. Therefore, the emission can leave the generation region, by transforming to transverse VLF waves due to large-scale horizontal gradients of electron density (e.g., in the region of the mid-latitude trough). Conversion of plasma oscillations to electromagnetic waves occurs with their frequency remaining the same; hence, taking into account the frequency range of VLF emissions, it is necessary to consider the excitation of plasma oscillations near the lower-hybrid resonance, i.e., when $\theta > \theta_0 = \arccos \sqrt{m/M}$. In this region of angles, the excitation rate of the oscillations by an electron beam is significantly smaller than $\gamma_{\max}$, defined by (8.1.9), and can be inferior, in magnitude, to the collisional damping (Alexandrov et al. 1988)

$$\gamma_{st} = \frac{v_e}{2} \left(1 + \frac{\omega_p^2}{\omega_p^2 + \omega_B^2} \sin^2 \theta \right).$$

Indeed, from numerical calculations (Gefan et al. 1986), it follows that when $n \geq 10^4 \text{ cm}^{-3}$, there exists a region of angles $\theta > 85^\circ$, where the growth rate $\gamma = \gamma_{\max} \cos \theta$ ($\gamma_{\max}$ is determined from Table 8.1) becomes less than γ_{st} . This situation corresponds to the excitation of oscillations only with frequencies $f > 100 \text{ kHz}$. In the main ionospheric trough, the electron density can reach very

low values, which substantially reduces the role of collisions. Thus, when $n \leq 10^3 \text{ cm}^{-3}$, the typical geomagnetic field inhomogeneity scale is much less than the free path length of resonant photoelectrons. This result leads to the more well-defined “beam-like” distribution of photoelectrons (see the first line in Table 8.1), which, combined with the decrease of $\gamma_{st} \sim v_e$, is responsible for the appearance of low frequencies in the spectrum of excited oscillations.

The intensity of the excited waves at angles $\theta \geq \theta_0$ is determined by the gain coefficient (8.3.9)

$$\Gamma \approx 2H_p \cdot \gamma_{\max} \cdot \frac{\Delta v_{res}}{v_{res}^2} \sqrt{\frac{m}{M}}.$$

The values of $\gamma_{\max}$, Δv_{res} , v_{res} depend in a complicated manner on electron density, temperature, and illumination conditions, and can be determined by solving the kinetic equation for photoelectrons. Numerical calculations (Gefan et al. 1986) show that the conditions for a substantial amplification ($\Gamma \geq 10$) of waves near the lower-hybrid resonance are satisfied only in regions of low electron density, $n_e \leq 10^3 \text{ cm}^{-3}$. It should also be noted that the generation conditions for plasma noise near LHR improve significantly in the presence of the LHR waveguide (Trakhtengerts and Shapaev 1977). The level of plasma waves can be estimated from calculated distribution functions (Gefan et al. 1985c) if it is assumed that an unstable beam with the distribution function $F(v_{\parallel})$ (8.1.10), as a result of the interaction with the oscillations, relaxes to a plateau state. Estimates in (Gefan et al. 1986) give values of the energy flux, converted to oscillations, of the order of 10^{-9} Wm^{-2} , which corresponds to a spectral density of $10^{-15} \text{ Wm}^{-2} \text{ Hz}^{-1}$. Note that such a level of plasma waves is able to ensure the observed intensity of a VLF emission burst in the early morning hours (Vershinin 1975), with a transformation efficiency of the order of unity.

The same also applies to auroral precipitation. Thus, although the formation of the required transverse electron density gradients is, in principle, possible during precipitation (see, e.g., Volokitin et al. 1984), it is also of interest to consider the VLF generation mechanism of auroral hiss (Raspopov and Kleimenova 1977), which is also effective in a transversally homogeneous case. This mechanism may be provided by the process of merging ion-sound oscillations excited in a nonisothermal plasma of the PT layer. This nonlinear process was first considered in interpreting turbulent plasma heating experiments. Its effectiveness in active electron beam-injection experiments was pointed out in Mishin et al. (1990).

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Chapter 9

Kinetic Theory of Ring Current and Electromagnetic Ion Cyclotron Waves: Fundamentals

The inner magnetospheric plasma is a very unique composition of different plasma particles and waves. Among these plasma particles and waves are ring current (RC) particles and electromagnetic ion cyclotron (EMIC) waves. The RC is the source of free energy for the EMIC wave excitation provided by a temperature anisotropy of RC ions, which develops naturally during inward $\mathbf{E} \times \mathbf{B}$ convection from the plasma sheet. The cold plasmasphere, which is under the strong influence of the magnetospheric electric field, strongly mediates the RC–EMIC waves-coupling process, and ultimately becomes part of the particle and energy interplay. On the other hand, there is a strong influence of the RC on the inner magnetospheric electric and magnetic field configurations and these configurations, in turn, are important to RC dynamics. Therefore, one of the biggest needs for inner magnetospheric research is the continued progression toward a coupled, interconnected system, with the inclusion of nonlinear feedback mechanisms between the plasma populations, the electric and magnetic fields, and plasma waves.

9.1 General Overview

9.1.1 Terrestrial Ring Current

The ring current–inner radiation belt energy source is very important in the energetics of the thermal plasma environment in the subauroral, mid-latitude, and even equatorial ionosphere. The energy stored in this region is comparable to that stored in the particle reservoir in the plasma sheet. The slow release (timescales of hours to days) of this energy via charge-exchange, Coulomb drag, and wave–particle interaction processes produces very different effects on the ionospheric thermal plasma background than the dramatic rapid releases of energy from the magnetotail into the auroral regions. The observational evidence for this interaction from low-altitude satellites and ground-based observations is irrefutable. Satellites have documented elevated electron temperatures associated with regions of overlap between the ring current and outer plasmasphere (cf. Büchner et al. 1983; Kozyra et al. 1986; Brace et al. 1988). Characteristic zones of anisotropic proton precipitation, with energies

typical of the ring current and radiation belts, appear in the magnetically connected subauroral ionosphere (Lundblad et al. 1978). Emissions characteristic of heavy neutral particle precipitation are routinely observed from the ground (Tinsley 1979, 1981). These emissions have been used to estimate precipitating fluxes (Ishimoto et al. 1992a, b). Heavy particle precipitation from ring current decay processes is thought to be important in maintaining the nightside low-altitude ionosphere and may be involved in the production of intermediate layers (thin layers of ionization occurring in the nightside E-F1 region) (Kelley 1989). However, the details of the physical processes involved in the interaction between the ring current particles injected earthward and the thermal populations of the plasmasphere and geocorona have yet to be worked out. Until recently, an incomplete knowledge of the composition and species-dependent pitch angle and energy characteristics of the ring current over most of its energy range prevented detailed investigations of the energy transfer processes. Furthermore, ring current models lacked information on the magnetic activity dependent sources of ring current plasma.

The satellite missions (e.g., AMPTE/CCE, CRRES) have begun to supply important information in these areas. Observations by the AMPTE satellite, summarized in a review article by (Gloeckler and Hamilton 1987), indicate that the ring current contains a significant fraction of heavy ionospheric ions throughout its entire energy range and that this heavy ion percentage varies with magnetic activity. In fact, O^+ can dominate the ring current during the main phase of a large magnetic storm (Hamilton et al. 1988). Young et al. (1982) demonstrated a solar cycle dependence in the heavy ion content of the plasma near geosynchronous orbit, with a larger percentage of heavy ions during solar maximum than during solar minimum, and, since this plasma is the source for the ring current, a similar solar cycle dependence is expected in the ring current composition. The average energy of the ring current varies with magnetic activity as well, because ring current ions are injected into lower altitude regions, as the severity of magnetic storms increases, with an accompanying enhancement in the energization achieved, via betatron and Fermi acceleration, in combination with other nonadiabatic processes. Major magnetic storms produce ring current populations with peak energy densities occurring near 85–120 keV, depending on species (Hamilton et al. 1988); whereas, storms that are more moderate produce ring currents with lower characteristic energies of the order of ~ 40 –60 keV (Gloeckler et al. 1985). Decay processes for the ring current ions (i.e., charge exchange, Coulomb drag, and wave scattering) depend heavily on the composition and energy characteristics of the ring current. Figure 9.1, taken from Kozyra and Nagy (1991), summarizes the considerably more complex picture of ring current decay processes that has evolved during the last few decades, as satellites with increasing measurement capabilities have flown through the ring current region. The biggest need for inner magnetospheric research, including the ring current, is the continued progression toward a coupled, interconnected understanding of this region, especially the nonlinear feedback mechanisms between the plasma populations, the electric and magnetic fields, and plasma waves. The last topic is the major focus of this chapter.

As was mentioned by Liemohn (2006) in his review paper, our knowledge of the ring current has also undergone a major transformation during the course of the

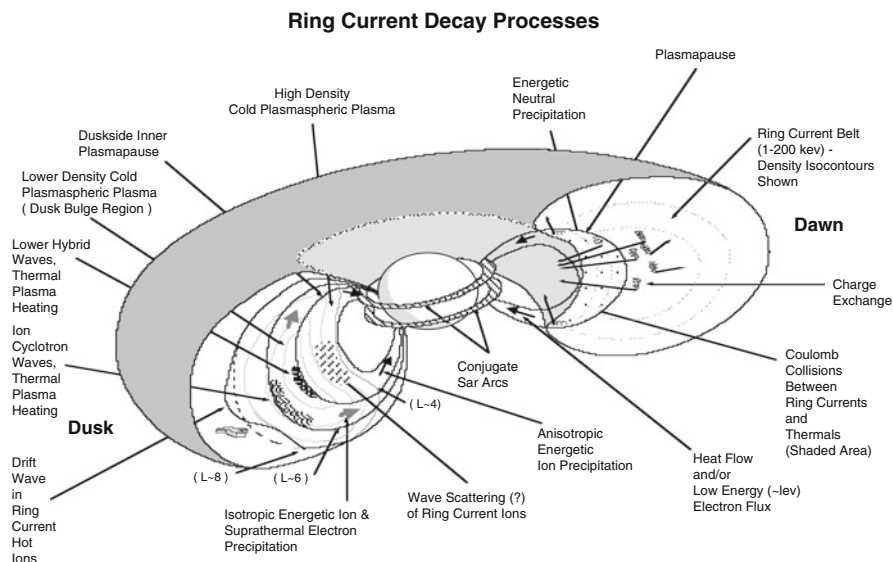


Fig. 9.1 Ring current decay process presented by Kozyra and Nagy (1991) (courtesy of J.U. Kozyra and A.F. Nagy, University of Michigan)

IM/S Campaign. As with the plasmasphere, this progress is directly related to the availability of new data sets (such as IMAGE HENA and MENA), new data analysis techniques (such as inversions of these images), and new modeling applications (such as coupling between ring current codes and electric and magnetic field models). Major advancements have occurred in the areas of ring current morphology, feedback with the electric and magnetic fields, connection to the plasma sheet, and interactions with plasma waves.

9.1.2 Electromagnetic Ion Cyclotron Waves

Electromagnetic ion cyclotron (EMIC) waves are a common and important feature of the Earth's magnetosphere. The source of free energy for wave excitation is provided by the temperature anisotropy of ring current (RC) ions, which naturally develops during inward convection from the plasma sheet. The EMIC waves have frequencies below the proton gyrofrequency, and they are excited mainly in the vicinity of the magnetic equator with a quasi-field-aligned wave normal angle (Cornwall 1965). These waves were observed in the inner (LaBelle et al. 1988; Erlandson and Ukhorskiy 2001) and outer (Anderson et al. 1992a, b) magnetosphere, at geostationary orbit (Young et al. 1981; Mauk 1982), at high latitudes (Erlandson et al. 1990), and at ionospheric altitudes (Iyemori and Hayashi 1989; Bräysy et al. 1998).

Feedback from EMIC waves causes nonadiabatic pitch-angle scattering of the RC ions (mainly protons) and their loss to the atmosphere, which leads to the decay of RC (Cornwall et al. 1970). This is especially important during the main phase of storms, when RC decay is possible with a timescale of around an hour or shorter (Gonzalez et al. 1989). During the main phase of major storms, RC O^+ may dominate (Hamilton et al. 1988; Daglis 1997). These ions cause damping of the He^+ -mode EMIC waves, which may be very important for RC evolution during the main phase of the greatest storms (Thorne and Horne 1994, 1997). Obliquely propagating EMIC waves interact well with thermal plasmaspheric electrons due to Landau resonance (Thorne and Horne 1992; Khazanov et al. 2007). Subsequent transport of the dissipating wave energy into the ionosphere causes an ionospheric temperature enhancement (Gurgiolo et al. 2005). This wave dissipation is a mechanism proposed to explain stable auroral red arc emissions present during the recovery phase of storms (Cornwall et al. 1971; Kozyra et al. 1997b). Measurements taken aboard the Prognoz satellites revealed a so-called “hot zone” near the plasmapause, where the temperature of plasmaspheric ions can reach tens of thousands of degrees (Bezrukikh and Gringauz 1976; Gringauz 1983, 1985). Nonlinear induced scattering of EMIC waves by thermal protons (Galeev 1975) was used in the RC–plasmasphere interaction model by Gorbachev et al. (1992) and Khazanov et al. (1994, 1995) in order to account for these observations. An extended analysis of thermal/suprathermal ion heating by EMIC waves in the outer magnetosphere was presented by Anderson and Fuselier (1994), Fuselier and Anderson (1996), and Horne and Thorne (1997).

Relativistic electrons (≥ 1 MeV) in the outer radiation belt can also strongly interact with EMIC waves (Thorne and Kennel 1971; Lyons and Thorne 1972). Data from balloon-borne X-ray instruments provides indirect but strong evidence that EMIC waves cause precipitation of outer-zone relativistic electrons (Foat et al. 1998; Lorentzen et al. 2000). These observations stimulated theoretical and statistical studies, which demonstrated that EMIC wave-induced pitch-angle diffusion of megaelectron volt electrons can operate in the strong diffusion limit with a timescale of several hours to a day (Summers and Thorne 2003; Albert 2003; Meredith et al. 2003). This scattering mechanism is now considered to be one of the most important means for relativistic electron loss during the initial and main phases of storm.

All of the above clearly demonstrates that EMIC waves strongly interact with electrons and ions of energies ranging from ~ 1 eV to ~ 10 MeV, and that these waves strongly affect the dynamics of resonant RC ions, thermal electrons and ions, and the outer radiation belt relativistic electrons. The effect of these interactions is nonadiabatic particle heating and/or pitch-angle scattering, and loss to the atmosphere.

The rate of ion and electron scattering/heating in the Earth’s magnetosphere is not only controlled by the wave intensity–spatial–temporal distribution but also strongly depends on the spectral distribution of the wave power. Unfortunately, there are still very few satellite-based studies of EMIC waves, especially during the main phase of magnetic storms, and currently available observational information regarding EMIC wave power spectral density (mainly from the AMPTE/CCE and CRRES satellites) is poor (Engebretson et al. 2008). Ideally, a combination of theoretical models and available and reliable data should be used to obtain the

power spectral density of EMIC waves on a global magnetospheric scale throughout the different storm phases. To the best of our knowledge, there is only one model that is able to self-consistently simulate a spatial, temporal, and spectral distribution of EMIC waves on a global magnetospheric scale during the different storm phases (Gamayunov and Khazanov 2008). This model is based on first principles, and explicitly includes the wave generation/damping, propagation, refraction, reflection, and tunneling in a multi-ion magnetospheric plasma. The He^+ -mode EMIC wave simulations based on this model have showed that the equatorial wave normal angles can be distributed in the source region, i.e., in the region of small wave normal angles, and also in the entire wave region, including those near 90° . The occurrences of the oblique and field-aligned wave normal angle distributions appear to be nearly equal, with a slight dominance of oblique events (Khazanov and Gamayunov 2007a). This theoretical prediction is supported by a large data set of the observed wave ellipticity (Anderson et al. 1992b; Fraser and Nguyen 2001; Meredith et al. 2003). The observation of a significant number of linearly polarized events near the equator suggests that waves are often highly oblique there. Using the more reliable wave step polarization technique, Anderson et al. (1996) and Denton et al. (1996) analyzed data from the AMPTE/CCE spacecraft and presented the first analysis of near linearly polarized waves for which the polarization properties were determined. They found a significant number of wave intervals with a wave normal angle $\theta > 70^\circ$, the highest θ ever reported. Compared with field-aligned waves, such highly oblique wave normal angle distributions can dramatically change the effectiveness (by an order of magnitude or more) of both the wave-induced RC proton precipitation (Khazanov et al. 2007) and relativistic electron scattering (Glauert and Horne 2005; Khazanov and Gamayunov 2007a). Strong sensitivity of the scattering rates to the wave spectral characteristics, and the wide distribution of EMIC wave normal angles observed in the magnetosphere suggests that in order to use EMIC waves for heating and/or scattering of the magnetospheric particles in a model, the wave spectral distribution will require special care, and should be properly established.

The resulting EMIC wave power spectral density depends on the RC and cold plasma characteristics. On the other hand, the convective patterns of both RC ions and the cold plasmaspheric plasma are controlled by the magnetospheric electric field, determining the conditions for the interaction of RC and EMIC waves. Therefore, this electric field is one of the most crucial elements necessary to properly determine the wave power spectral density. The region 2 field-aligned currents (FACs) couple the magnetosphere and ionosphere. This large-scale coupling determines and maintains a self-consistent dynamic of the electric field and RC (Vasyliunas 1970; Jaggi and Wolf 1973; Garner et al. 2004; Fok et al. 2001; Khazanov et al. 2003b; Liemohn et al. 2004). A self-consistent simulation of the magnetosphere–ionosphere system should provide, at least in principle, the most accurate theoretical electric field. The EMIC waves resulting in the magnetosphere are not only a passive element in the coupled RC-ionosphere system but also may influence the electrodynamics of coupling. During storm times, the wave-induced RC proton precipitation not only changes the FAC distribution, but can potentially

modify the conductance and/or the neutral gas velocity in the ionosphere–thermosphere system (Galand et al. 2001; Galand and Richmond 2001; Fang et al. 2007a, b). Both of these characteristics are crucial elements in the magnetosphere–ionosphere electrodynamics. Such wave-induced modification can be especially important equatorward of the low-latitude edges of the electron and proton auroral ovals, where the wave-induced RC ion precipitation may be a dominant energy source. In addition, electrons and protons do not interact in the same way with the atmosphere. One should keep in mind that energetic protons ionize more efficiently than electrons do because their energy loss for each produced electron is smaller than that of energetic electrons (Galand et al. 1999). Therefore, even if the proton energy flux is smaller compared with the electron flux, the response of the atmosphere to protons can be significant. The above arguments suggest that a self-consistent model of the magnetospheric electric field, RC, plasmasphere, and EMIC waves is needed to properly model wave spectral distribution and to improve the modeling of the large-scale magnetosphere–ionosphere electrodynamics.

In the following sections, we present the computational model that is a result of coupling two RC models. The first model deals with the large-scale magnetosphere–ionosphere electrodynamic coupling and provides a self-consistent description of RC ions and the magnetospheric electric field (Liemohn et al. 2001, 2004; Ridley and Liemohn 2002; Khazanov et al. 2003b). The second model is governed by a coupled system of the RC kinetic equation and the wave kinetic equation. This model self-consistently treats a mesoscale electrodynamic coupling of RC and EMIC waves, and determines the evolution of the EMIC wave power spectral density (Khazanov et al. 2006, 2007). The RC–EMIC wave model explicitly includes the wave growth/damping, propagation, refraction, reflection, and tunneling in a multi-ion magnetospheric plasma. Although RC ions and EMIC waves in the second model are treated self-consistently, the electric field is externally specified. So far, the above two models were used independently. As such, the main purpose of this chapter is to present a self-consistent model of the magnetospheric electric field, RC, plasmasphere, and EMIC waves along with initial results from the model simulations. The results presented in this section were obtained from simulations of the 2–4 May 1998 geomagnetic storm, that we previously analyzed using an analytical formulation of the Volland–Stern electric field (Khazanov et al. 2006, 2007).

9.2 EMIC Wave Generation and Propagation in Magnetosphere

9.2.1 EMIC Wave Modes in Multicomponent Plasma

Observations have shown that the simple, one-ion theory for EMIC waves is not enough, because both cold and hot heavy ions are present in the magnetosphere, He^+ and O^+ being the most important. The heavy ions in the cold plasma can have

profound effects on the generation and propagation of EMIC waves (Young et al. 1981; Gomberoff and Neira 1983). The most notable new feature is the formation of stop bands above the heavy ion gyrofrequencies. Following the treatment presented by Gomberoff and Neira (1983) for cold heavy ions, Kozyra et al. (1984) added the hot heavy ion populations (He^+ and O^+) in the equations. In many respects, hot heavy ions modify the wave growth in the same way as the cold population, especially when the ion densities of the two populations are comparable, as in the plasmaspheric bulge region or just outside the plasmasphere.

The growth rates of the EMIC waves calculated by Khazanov et al. (2003a) are presented in Fig. 9.2. For reference we also provide the EMIC wave refractive indices in the appropriate cold plasmas. Each EMIC wave mode is shown by a distinctive color, and letters L and R mark the left-hand and the right-hand polarized branches of the waves, respectively. Results in Fig. 9.2a are obtained for a core plasma of electrons, 77% of H^+ , 20% of He^+ , and 3% of O^+ , which is in the range of 10–30% for He^+ and 1–5% for O^+ , following observations by Young et al. (1977) and Horwitz et al. (1981) (see also the theoretical paper by Kozyra et al. 1984). Pure electron–proton plasma is considered for obtaining the results in

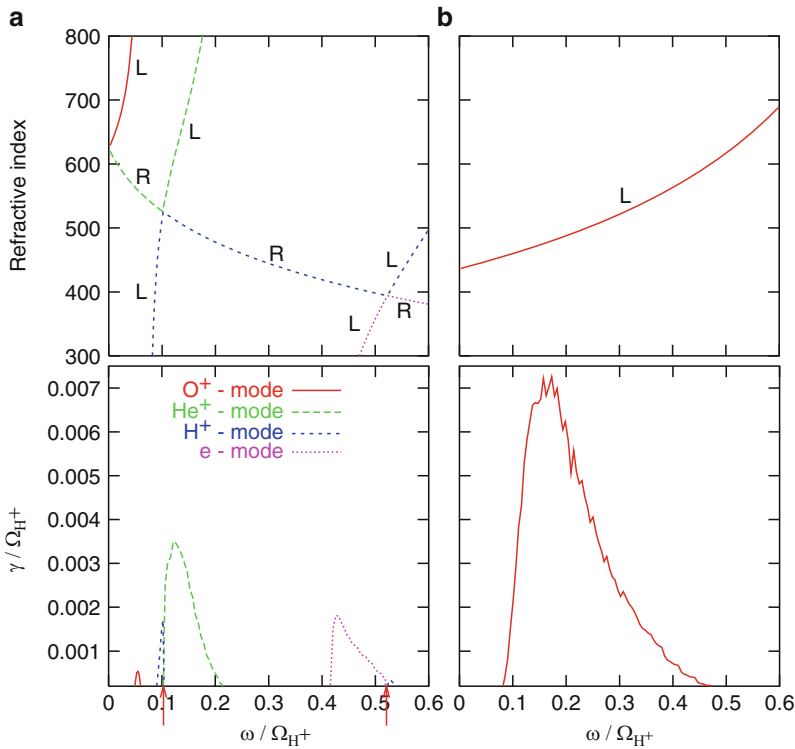


Fig. 9.2 Equatorial refractive indices and growth rates for the quasi-field-aligned ($\theta = 2.25^\circ$) EMIC waves at 0000 UT on 1 May 1998. $L = 4.25$, $\text{MLT} = 17$, plasmaspheric electron density is 165 cm^{-3} , the Earth's magnetic field is 405 nT, and RC ions are energetic protons

Fig. 9.2b. In both cases, the RC is entirely made up of energetic H^+ . In order to calculate the EMIC wave growth rates, we used an initial phase space distribution function of the RC protons, and this function is shown in Fig. 9.3. In a multi-ion core plasma, all the EMIC wave modes are named in accordance with the gyro-frequencies to which the field-aligned modes approach for infinite wave numbers, i.e., there are Ω_{O^+} , Ω_{He^+} , Ω_{H^+} , $|\Omega_e|$ for O^+ -, He^+ -, H^+ -, and e -modes, respectively. The refractive indices in Fig. 9.2 are presented for oblique wave propagation. In this case, the dispersive curves in a multi-ion thermal plasma do not cross over and do not even touch each other (see Fig. 9.2a where $\theta = 2.25^\circ$), but, along the dispersive curves, the wave polarizations are changing, crossing so-called crossover frequencies (labeled by the red arrows at the bottom of Fig. 9.2a). The situation is changing for a field-aligned wave propagation; with decreasing the wave normal angle, θ , the dispersive curves firstly touch each other and then, with further wave normal angle decreasing, cross over (at crossover frequencies) with dramatic reconstruction of the dispersive curves. Note that, in the cold plasma approximation, the dispersive curves cross over only for the wave normal angles $\theta = 0$ and $\theta = \pi$. In the case of a field-aligned wave propagation, the wave polarizations are conserved along each of the dispersive curves, and we get the well-known left-hand polarized bands of the EMIC wave (O^+ , He^+ , and H^+ bands) and fast magnetosonic wave with the right-hand polarization (it is easy to see these bands in Fig. 9.2a if one does not pay attention to the line colors). In a multi-ion thermal plasma, the peak of the growth rate takes place for the He^+ -mode inside the frequency region in which the EMIC waves are left-hand polarized. The growth rate peak for the e -mode is approximately two times less than for the He^+ -mode and again takes place in a region of the “left-hand polarization” of the dispersive curve. Growth rates of O^+ - and H^+ -modes are negligible in comparison with the other two modes. The growth rate profile in the e -p thermal plasma is wider and the peak is two times higher than for the He^+ -mode (compare Fig. 9.2a and b).

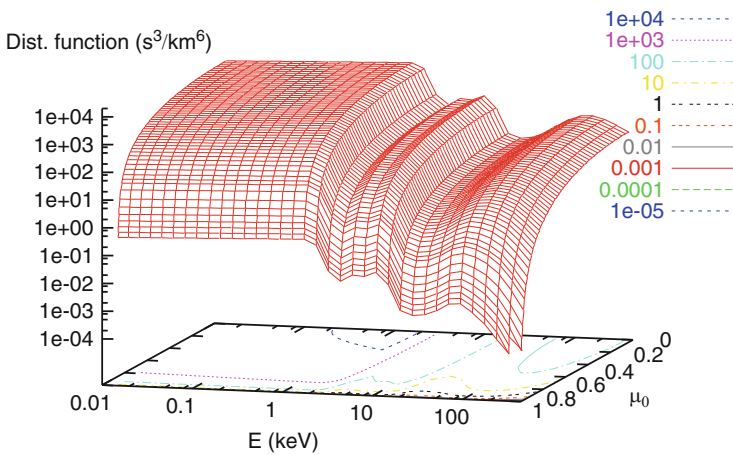


Fig. 9.3 Equatorial RC proton distribution function at 0000 UT on 1 May 1998, $L = 4.25$, $MLT = 17$

Figure 9.4 demonstrates the bounce-averaged pitch-angle diffusion coefficients characterizing 10.8 keV RC proton scattering rates on the EMIC waves of different modes. In order to avoid an influence of PSD structure on diffusion coefficients, the PSD is set up to the “white noise,” i.e., for each wave mode, $\langle B_{\omega,\theta}^2 \rangle$ is a constant inside an available (ω, θ) -region (for e -mode, only frequency range $\Omega_{\text{He}^+} - \Omega_{\text{H}^+}$ is considered, and there are $\Omega_{\text{O}^+} - \Omega_{\text{H}^+}$, $0 - \Omega_{\text{He}^+}$, $0 - \Omega_{\text{O}^+}$ frequency ranges for H^+ -, He^+ -, and O^+ -modes, respectively). It follows from Fig. 9.4 that diffusion of RC ions with the equatorial pitch angles less than approximately 60° is much faster due to interaction with the e -mode than with other modes, and the combined consideration of Figs. 9.2a and 9.4 leads to the following conclusion. While the EMIC waves grow slower in the e -mode than in the He^+ -mode, and, as a consequence, less-intense waves may be observed in the e -mode than in the He^+ -mode, the resulting diffusion coefficient may stay bigger for the e -mode than for the He^+ -mode. Note that we also calculated the diffusion coefficient (not shown) for the case of the e -mode, considering only the frequency range where the e -mode grows, $0.4\Omega_{\text{H}^+} - 0.5\Omega_{\text{H}^+}$. The result is almost the same as in Fig. 9.4, with the exception of no resonances for the pitch angles greater than approximately 55° . Thus, for a correct description of the RC–EMIC wave interaction, both e - and He^+ -modes of EMIC waves should be used. Although the above speculations seem reasonable, it is obvious that in order to check the validity of the conclusion regarding the e -mode, the RC–EMIC wave simulation including both He^+ - and e -modes are required.

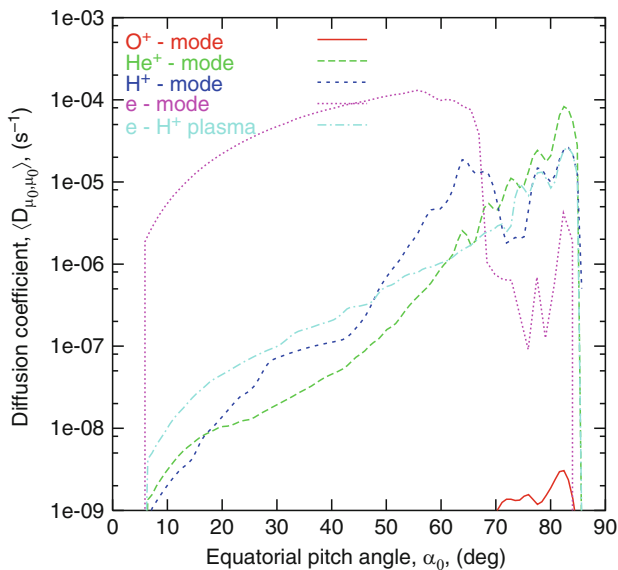


Fig. 9.4 Bounce-averaged pitch-angle diffusion coefficients for 10.8 keV RC protons interacting with EMIC waves at 0000 UT on 1 May 1998. $L = 4.25$, $\text{MLT} = 17$, boundary of the loss cone pitch angles is 5.1° , power spectral density is “white noise” and expressed as $\langle B_{\omega,\theta}^2 \rangle = 1 nT^2 / (\text{rad}^2 \text{ Hz})$

9.2.2 Propagation in the Magnetosphere

In order to adequately take into account wave propagation and refraction in a multi-ion magnetosphere, we explicitly include the ray-tracing equations in a self-consistent RC–EMIC waves model that is presented in the following sections, and use the general form of the wave kinetic equation.

The general set of ray-tracing equations (the so-called approximation of “geometric optics”) is well-known and may be found in many plasma physics monographs (see, e.g., Stix 1992). These equations were originally derived by Haselgrove (1954) and Haselgrove and Haselgrove (1960) by using the method of Hamilton and have subsequently been widely used (e.g., Yabroff 1961; Kimura 1966; Rauch and Roux 1982; Horne 1989). In the present section, in order to solve the ray-tracing equations, we use a dipole approximation for a geomagnetic field, and, in spite of our MLT-dependent plasma density model, we ignore a slight longitudinal drift of the waves during propagation (see, e.g., Bespalov and Trakhtengerts 1986). Under these assumptions, the complete three-dimensional (3D) ray-tracing equation set is reduced to a plane geometry. To write down these equations and carry out their subsequent integration, we use two coordinate systems, as shown in Fig. 9.5. The Earth-centered polar coordinate system is used to characterize any point P that lies on the ray trajectory by length of the radius vector, r , and magnetic latitude, λ . The wave vector $\mathbf{k}$ is described by two components, k_r and k_λ , in a local Cartesian coordinate system centered on the current point P , and with its axes oriented along the radius vector and the magnetic latitude direction, respectively (see Fig. 9.5). Using the selected coordinate systems and assuming the medium is stationary

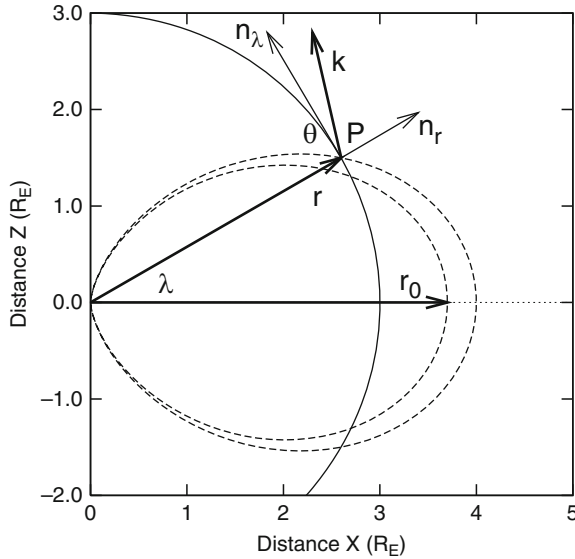


Fig. 9.5 The coordinate systems used to integrate ray-tracing equations is shown

(in other words, the characteristic changing timescale for the plasmaspheric thermal plasma is much greater than the characteristic wave propagation time), the resulting system of ray-tracing equations can be written as (compare with Haselgrove 1954; Haselgrove and Haselgrove 1960; Yabroff 1961; Kimura 1966; Burtis 1973)

$$\frac{dr}{dt} = -\frac{\left(\frac{\partial G}{\partial \mathbf{k}}\right)_r}{\frac{\partial G}{\partial \omega}} \equiv v_{g,r}, \quad (9.2.1)$$

$$\frac{d\lambda}{dt} = -\frac{\left(\frac{\partial G}{\partial \mathbf{k}}\right)_\lambda}{\frac{\partial G}{\partial \omega}} \equiv v_{g,\lambda}, \quad (9.2.2)$$

$$\frac{dk_r}{dt} = k_\lambda \frac{d\lambda}{dt} + \frac{\left(\frac{\partial G}{\partial \mathbf{r}}\right)_r}{\frac{\partial G}{\partial \omega}}, \quad (9.2.3)$$

$$\frac{dk_\lambda}{dt} = -\frac{k_\lambda}{r} \frac{dr}{dt} + \frac{\left(\frac{\partial G}{\partial \mathbf{r}}\right)_\lambda}{\frac{\partial G}{\partial \omega}}. \quad (9.2.4)$$

In these equations, the following notations are introduced: t is the time, ω is the angular wave frequency, $v_{g,r} \equiv v_{g,r}(\mathbf{k}, \mathbf{r})$ and $v_{g,\lambda} \equiv v_{g,\lambda}(\mathbf{k}, \mathbf{r})$ are the components of wave group velocity parallel to radius vector and to magnetic latitude direction, respectively, $G \equiv G(\omega, \mathbf{k}, \mathbf{r})$ is a function that only has roots for EMIC eigenmodes, i.e., $G = 0$ at any point along EMIC wave phase trajectories. In this study, we use the following form for the function G :

$$G = \left(\frac{kc}{\omega}\right)^2 - N^2(\omega, \mathbf{k}, \mathbf{r}), \quad (9.2.5)$$

where N is the refractive index of the EMIC wave modes. Because the main dispersion effects for EMIC waves can be described in the cold plasma approximation, the differences between the raypaths for hot and cold plasmas should be minor (this was also stated by Horne and Thorne 1993 after performing ray-tracing simulations). This fact allows us to use the cold plasma approximation in order to derive the refractive index N , and its general expression can be found, e.g., in the work of Stix (1992) and Lyons and Williams (1984). In spite of the fact that all RC ions (mainly H^+ , O^+ , and He^+ ions) are “hot,” the major EMIC wave dispersion properties can still be described by the cold plasma approximation (of course we are talking here about the real part of N). Thus, the “cold” refractive index depends on the influence of both the thermal and RC plasmas. In order to calculate this refractive index, we assume that the core plasmaspheric plasma consists not only of H^+ ions but also of He^+ and O^+ ions. The thermal plasma density models used are described below.

The system of Hamilton's equations (9.2.1)–(9.2.4), including the dispersion relation (9.2.5), are integrated using the Adams predictor–corrector method that has been previously used in many ray-tracing simulations (e.g., Yabroff 1961; Kimura 1966; Burtis 1973; Rauch and Roux 1982; Horne 1989). In this method, the length of integration step, Δt , is automatically adjusted in order to keep the desired tolerance between predicted and corrected values. Usually, we set $e_{rel} = 10^{-5}$ as the relative error for all variables, and $e_{abs} = 10^{-4}$ degrees as the absolute error for calculation of latitude, but we also prevent Δt from being greater than $\tau \approx 10^{-2}$ s during integration. In each τ seconds, substituting the results of integration in local dispersion relation (9.2.5), we calculate the wave frequency, ω_τ . If the relative error is big enough, namely $|\omega_\tau - \omega|/\omega > 5 \cdot e_{rel}$, then the wave number, k , found from integration, is corrected as suggested by Yabroff (1961). Khazanov et al. (2006) tested this code in order to reproduce previously published results for EMIC wave propagation by Rauch and Roux (1982) and Horne and Thorne (1993). Here we show only some selected results of this study in the media with electrons and H^+ , He^+ , O^+ ion species (see Khazanov et al. 2006 for the details), and present the propagation-only He^+ -mode.

Figures 9.6–9.8 demonstrates the propagation characteristics (phase trajectories) of the He^+ -mode in the frequency range $\Omega_{O^+} < \omega < \Omega_{He^+}$. Figure 9.6 shows the

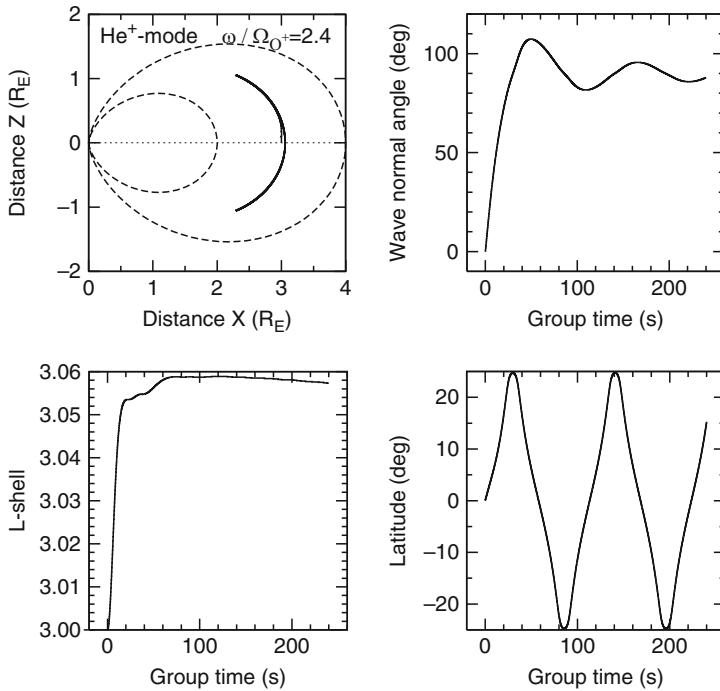


Fig. 9.6 Raypath, wave normal angle, L shell, and latitude for He^+ -mode propagating in e - H^+ - He^+ - O^+ thermal plasma are shown. The ray is launched from the equator with $\omega/\Omega_{O^+} = 2.4$, $\theta = 0^\circ$, and the wave is left-hand polarized

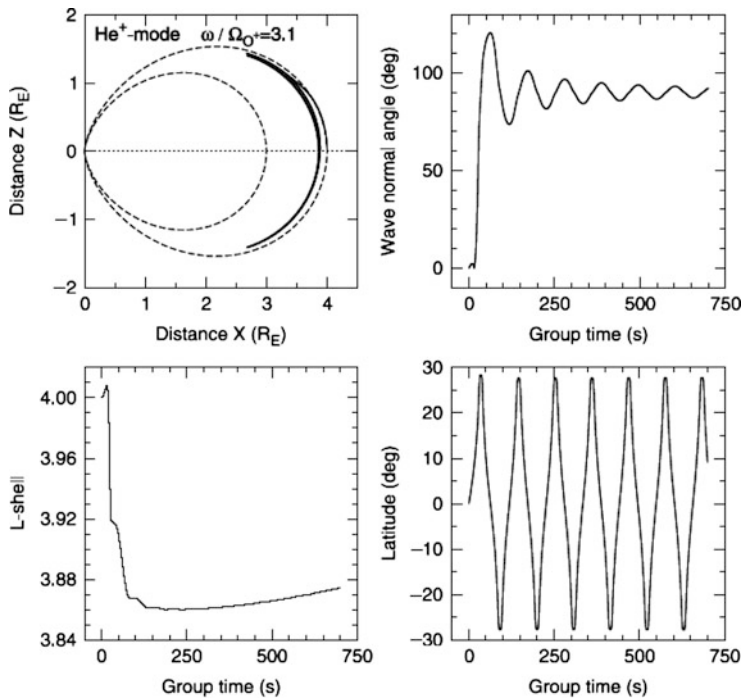


Fig. 9.7 Same as Fig. 9.6, except the ray is launched from $L = 4$ with $\omega/\Omega_{O^+} = 3.1$

results for a ray launched in the left-hand polarized mode from the geomagnetic equator at $L = 3$ with $\theta = 0$ and $\omega/\Omega_{O^+} = 2.4$. As one can see in this plot, the ray bounces between reflection points, and essentially stays on the same magnetic field line. In this case, the wave normal angle rapidly increases due to dominance of the magnetic field gradient and curvature over the density gradient. Figure 9.7 demonstrates the results for the left-hand polarized mode launched outside the plasmapause at $L = 4$ with $\theta = 0$ and $\omega/\Omega_{O^+} = 3.1$. The effect of the steep plasmapause density gradient on ray propagation is presented in Fig. 9.8. We launch the left-hand polarized mode from the geomagnetic equator at $L = 3.8$ with $\theta = 0$ and $\omega/\Omega_{O^+} = 2.4$. In agreement with the results of the previous studies, after the first reflection, the ray moves inward toward the Earth and crosses the equator at $L = 3.4$ with 180° .

9.2.3 Tunneling Effects

The wave kinetic equation of EMIC waves that is presented in this chapter treats waves as quasiparticles, and, according to ray-tracing simulations based on the system (9.2.1)–(9.2.4), these quasiparticles bounce between surfaces of the bi-ion

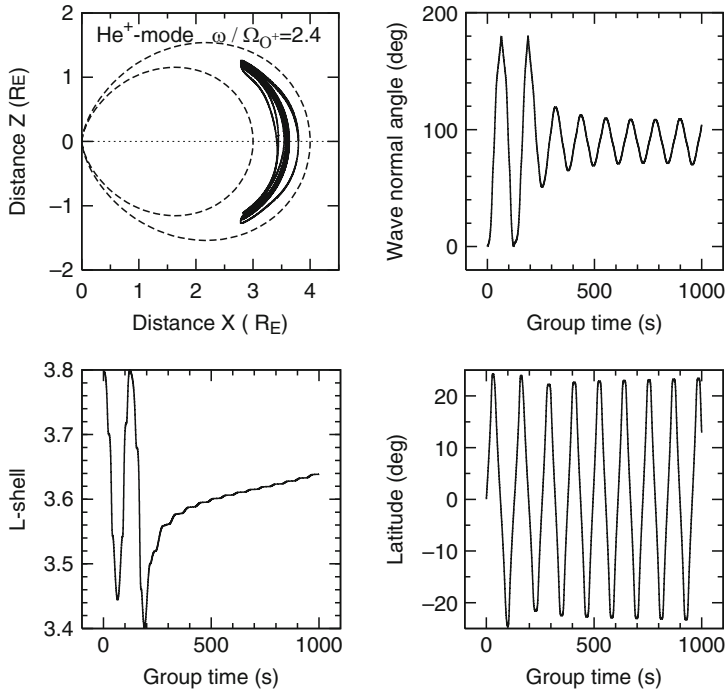


Fig. 9.8 Same as Fig. 9.6, except the ray is launched from $L = 3.8$ with $\omega/\Omega_{O^+} = 2.4$

hybrid frequency in opposite hemispheres. It is clear that a “geometric optics” approximation is not valid near the reflection points, and a full wave description is required to analyze wave propagation in these regions (see, e.g., Stix 1992). In order to explain simultaneous measurements of EMIC waves by the GEOS spacecrafts and in the vicinity of their magnetic footprints, Perraut et al. (1984) have demonstrated that, for EMIC waves of class III, generated at the geomagnetic equator, some part of the wave energy can tunnel through the region of reflection in $H^+ - He^+$ thermal plasma, and these waves can be detected on the ground. We generalize and quantify the effect of wave tunneling for the case under consideration in the present study, namely, for the He^+ -mode of EMIC waves propagating in a multispecies ($e^-H^+ - He^+ - O^+$) magnetosphere.

In order to obtain the wave equation that describes EMIC wave propagation in the vicinity of the reflection point, we follow Perraut et al. (1984) and/or Rauch and Roux (1982). Using the cold plasma approximation, we can present the dispersion equation in the vicinity of the reflection point as $k_{\parallel}^2 + \Phi(z) = 0$, where $k_{\parallel}$ is a component of the wave vector parallel to the external magnetic field, z is a coordinate along the geomagnetic field line, and function $\Phi(z)$ can be found in the work of Perraut et al. (1984) and Rauch and Roux (1982). This dispersion equation is obtained from Maxwell’s equations after Fourier analysis. Thus, the correct wave

equation near the reflection point can easily be obtained by substituting operator $-\partial^2/\partial z^2$ in the above dispersion equation instead of $k_{\parallel}^2$. This substitution leads to the following equation:

$$\frac{\partial^2 E}{\partial z^2} - \Phi(z)E = 0, \quad (9.2.6)$$

where E is a transverse electric field of EMIC waves. The component of the wave vector parallel to the external magnetic field should vanish at the reflection point, z_1 , i.e., $\Phi(z_1) = 0$. Thus, in the vicinity of this point, we can present $\Phi(z)$ in the form:

$$\Phi(z) \approx (z - z_1)\Phi'(z_1) = (z - z_1)\frac{\omega^2}{c^2} \left(-\frac{\partial S}{\partial z} \right)_{z_1}. \quad (9.2.7)$$

In (9.2.7), c is the speed of light, function S is specified below, and we took into account the fact that z_1 is very close to the point where the wave frequency is equal to the O^+ – He^+ bi-ion hybrid frequency, i.e., $S(z_1) \approx 0$. It follows from (9.2.7) that a linear combination of the Airy functions is a solution for (9.2.6) (see, e.g., Abramowitz and Stegun 1964). Taking into account that the wave electric field, E , should evanesce for $z > z_1$, we can write down the particular solution as

$$E(z) = Ai\left((z - z_1)(\Phi'(z_1))^{1/3}\right) \quad (9.2.8)$$

and easily obtain the transmission coefficient for wave amplitude, T . At a distance $\delta z = z_2 - z_1 > 0$, this coefficient is

$$T = \frac{E(z_2)}{E(z_1)}. \quad (9.2.9)$$

Note that the wave power transmission factor is just T^2 in the case of adiabatic variance for plasma parameters.

As we stated above, coordinate z_1 can be found from the following equation:

$$\omega^2 = \omega_{bi}^2(z_1) = \frac{\Omega_p^2(z_1)}{256(\eta_1 + 0.25\eta_2 + 0.0625\eta_3)} \times \left(\eta_1 + 0.25\eta_2 + \eta_3 + \frac{15\eta_1\eta_2}{\eta_1 + 4.25\eta_2 + \eta_3} \right), \quad (9.2.10)$$

where ω_{bi} is the O^+ – He^+ bi-ion hybrid frequency, Ω_p is a local proton gyrofrequency, and η_1, η_2, η_3 ($\eta_1 + \eta_2 + \eta_3 = 1$) are the relative densities for plasmaspheric H^+ , He^+ , and O^+ ions, respectively. In order to find coordinate z_2 , we follow Perraut et al. (1984) and use the equation

$$\omega = \Omega_{O^+}(z_2), \quad (9.2.11)$$

where Ω_{O^+} is an oxygen gyrofrequency. Further, for the cold plasma approximation, function S introduced in (9.2.7) has the form of

$$S = -\frac{\omega_{pe}^2}{|\Omega_e|\Omega_p} \left(\frac{\eta_1}{y^2 - 1} + \frac{4\eta_2}{(4y^2) - 1} + \frac{16\eta_3}{(16y)^2 - 1} \right), \quad (9.2.12)$$

where $\omega_{pe}^2 = 4\pi n_e e^2 / m_e$ is the squared electron plasma frequency, $\Omega_e = eB/m_e c$ is a local electron gyrofrequency, and $y = \omega/\Omega_p$ is the normalized wave frequency.

For simplicity, we use a magnetic model, $n_e \sim B$, for the cold plasma density, consider η_1, η_2, η_3 to be coordinate independent, and use parabolic approximation for the geomagnetic field B near the equator. The transmission factor T that has been found by Khazanov et al. (2006) depends on the L shell, equatorial electron density, and normalized wave frequency, but, most dramatically, T depends on a fraction of O^+ ions that determines the width of the stop zone. All of these dependences are illustrated in Fig. 9.9. Note that we use the same equatorial electron densities for both $L = 4$ and $L = 5$ in order to demonstrate “pure” L shell dependence. We can see from Fig. 9.9 that the smaller the EMIC wave frequency, the larger the transmission factor. This result is in agreement with the conclusion of Perraut et al. (1984) that “the transmission through the stop zone induced by the presence of He^+ ions in the magnetospheric plasma is a low frequency bypass filter.” Dependence of the transmission factor on plasma density just reflects the fact that the larger the plasma density the smaller the EMIC wavelength and, as a consequence, the less wave energy that can be transmitted through the stop zone. The effect of an increase in the transmission coefficient by decreasing the L shell can reveal itself during the main and early recovery phases of

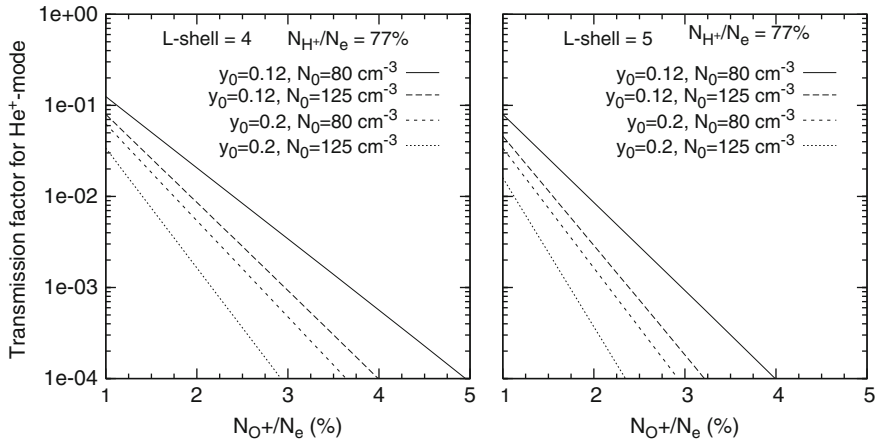


Fig. 9.9 Transmission factors for He^+ -mode of EMIC waves versus relative abundance of plasmaspheric O^+ ions are shown. Results from combinations of two equatorial densities and normalized wave frequencies are presented for $L = 4$ and $L = 5$ (Khazanov et al. 2006)

a geomagnetic storm, when the wave generation regions move closer to Earth. The plasmaspheric thermal O^+ ion abundance is approximately 1–5% in quiet geomagnetic conditions (Horwitz et al. 1981), and we present all of the results in Fig. 9.9 for this range. It is obvious that the amount of O^+ ions controls the width of another stop zone, to which the transmission coefficient will be very sensitive. The growth rate for the He^+ -mode has maximum at a y_0 of approximately 0.12 (Khazanov et al. 2003a), and, as follows from Fig. 9.9, the transmission factor $T \approx 10\%$ for $L = 4$, $y_0 = 0.12$, and $\eta_3 = 1\%$. In the following sections, we assume that thermal plasmaspheric plasma consists of H^+ , He^+ , and O^+ ions with ratios to electron content of 0.77, 0.20, and 0.03, respectively. For $y_0 = 0.12$, $L = 4$, and $\eta_3 = 0.03$, the transmission factor is $T \approx 0.003$, as follows from Fig. 9.9. Thus, only a minor portion of the EMIC wave energy (about T^2) can tunnel across the reflection region. This fact allows us to ignore the effect of tunneling in the wave kinetic equation for the He^+ -mode presented in the Sect. 9.4.

In conclusion of this section, we note the following:

1. In addition to the tunneling of the He^+ -mode, the equations obtained above also allow us to analyze the tunneling of the H^+ -mode of EMIC waves through the He^+-H^+ stop zone (note that the O^+ -mode of EMIC waves reflects from the ionosphere altitudes).
2. The component of the wave vector parallel to the external magnetic field comes to zero at the reflection surface, and EMIC wave energy is not absorbed through resonant processes (see, e.g., Rauch and Roux 1982; Stix 1992). Thus, only reflection and transmission are possible in the vicinity of the surface.

9.3 Bouncing EMIC Waves Versus Unidirectional Propagation

EMIC waves represent the substantial part of the wave spectrum in the near-Earth plasma. At the Earth's surface, they are observed in a different morphological forms. One of the most common types of these waves are Pc1 geomagnetic pulsations ("pearls," 0.2–0.5 Hz). For many decades, the governing picture of Pc1 pearl generation was based on the motion of a wave packet bouncing between the conjugate ionospheres along the magnetic field line. This picture was considered consistent with characteristic period of Pc1 pulses as detected by ground-based stations. Moreover, it was reported that the wave-intensity envelopes at conjugate ground-based sites were approximately in "antiphase" (e.g., Gendrin and Troitskaya 1965; Obayashi 1965; Saito 1969). This fact implied the existence of only one wave packet in the absence of its symmetric counterpart. Doubts about the validity of the BWP concept are mainly based on several arguments. One of them is that the repetition period of pulsations as obtained from satellite data is the same as on the ground. This result was obtained, e.g., by Erlandson et al. (1996), who compared Viking observations with ground-based measurements at the Finnish magnetometer network, and by Mursula et al. (2001), who used data from Polar and the same ground-based sites.

Distinguishing between the wave packets bouncing between the off-equatorial magnetic latitudes, corresponding to the bi-ion hybrid frequencies in conjugate hemispheres, and unidirectional wave propagation may be done by analyzing the spatial–temporal distribution of the EMIC wave Poynting vector derived from observations (Erlandson et al. 1990, 1992; Fraser et al. 1992, 1996; LaBelle and Treumann 1992; Mursula et al. 2001; Loto’aniu et al. 2005). The most comprehensive statistical study based on 248 EMIC events was conducted by Loto’aniu et al. (2005), which is the focus of our analysis in this section. To estimate a field-aligned component of the Poynting vector for EMIC waves in the Pc1 frequency range, Loto’aniu et al. used three components of the wave magnetic field and two components of the electric field measured by CRRES. The observational data set covered approximately 10 months in the period 7 September 1990–9 October 1991, where most events were at higher L-shells and in the late afternoon MLT sector. Of the 248 events, 81 (33%) and 167 (67%) occurred above and below 11° |MLat|, respectively. Loto’aniu et al. reported bidirectional wave energy propagation, both away and toward the equator, for 26% of the events observed below 11° |MLat|, and unidirectional energy propagation away from the equator for all events outside $\pm 11^\circ$ of the equator. The observed wave energy propagation was independent of event frequency or wave branch.

First, we interpret the Loto’aniu et al. results using two qualitatively different physical models. The first wave model assumes an ion cyclotron instability region located near the equator with no reflecting boundaries off the equator. The second model is the same as the first except waves are reflecting at the off-equatorial magnetic latitudes corresponding to the bi-ion hybrid frequencies in conjugate hemispheres. Second, we present an analysis of the observed wave ellipticity, which allows indirect discrimination between the first and second wave models. Finally, we present some additional arguments in support of the BWP concept of EMIC waves propagation.

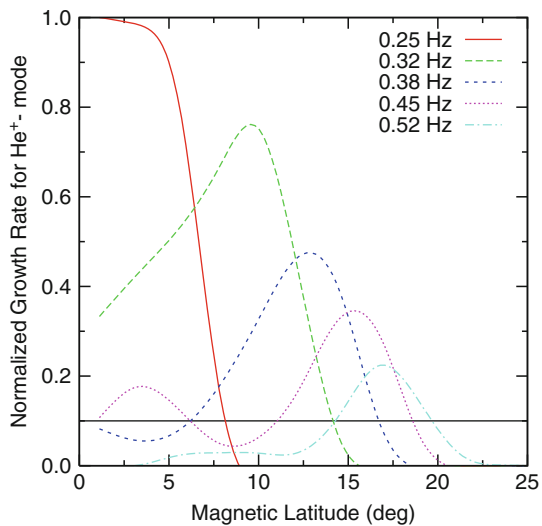
9.3.1 *Poynting Flux Analysis: No Reflection*

It is obvious that, in order for the first wave model to be applicable, the wave growth rate should be high enough to ensure wave amplification up to observable amplitudes during one pass through the unstable region. In the framework of this model, the seed fluctuations, irregularly emitted in one or both directions, will be amplified inside the instability region, and propagate either parallel or antiparallel to the magnetic field with equal probability. If the growth rate is high enough to allow an equatorward wave packet to amplify to observable levels before it reaches the equator, one would anticipate an irregularly bidirectional wave propagation inside some zone smaller than the equatorial instability region, but unidirectional wave propagation away from the equator otherwise. Note that, even inside this zone, bidirectional wave propagation will be only seen on an irregular basis but, on average, there is a unidirectional energy propagation away from the equator (see Fig. 4 of Loto’aniu et al. 2005). This is because the waves propagating away from

the equator in both hemispheres, on average, have higher energy than the waves propagating toward the equator. Let us now estimate a latitudinal extent of the near equatorial instability region. To do so, we plot in Fig. 9.10 the growth rate versus latitude for the He^+ -mode EMIC waves. All of the results shown are obtained from our RC–EMIC wave model that will be shown in the next few sections at $L = 6$ and $\text{MLT} = 15$ for hour 48 after 0000 UT on 1 May 1998. Depending on wave frequency, the growth rate has maximum at different magnetic latitudes ranging from 0° to 17° , where we assume that the boundary of the ion cyclotron instability region is located somewhere between 10° and 20° |MLat|. This estimate is consistent with calculations by Thorne and Horne (1997), so we use their results for the wave amplification rate. From their Fig. 3b, one can obtain a wave gain rate of 5 dB per degree inside the near equatorial instability region. This suggests that a path-integrated gain of 50 dB may be attained during the equatorward wave propagation from 20° to 10° . This is enough to observe bidirectional wave propagation inside $\pm 10^\circ$ of the equator, and unidirectional propagation outside this zone. This estimate matches the $|\text{MLat}| = 11^\circ$ boundary, which separates bidirectional and unidirectional wave propagation in the Loto’aniu et al. (2005) statistical results.

Loto’aniu et al. (2005) found bidirectional wave energy propagation for only 26% of all events below 11° |MLat|. To interpret this, we may assume that 60% of 167 wave packets observed below 11° |MLat| could not reach an observable amplitude during their propagation from the edge of instability region to the equator. Indeed, in the case when all the equatorward packets can be amplified by reaching $|\text{MLat}| = 11^\circ$ (actually the specific satellite location during observation, which is in the range $0^\circ \leq |\text{MLat}| < 11^\circ$), on average, we anticipate to find an oppositely directed partner for each wave packet observed inside $\pm 11^\circ$ with an expectation probability of $2/3$ (assuming that wave energy propagates equally backward and forward, or in both directions at the same time). In the opposite case when all the equatorward packets are not able to gain enough energy before they reach the satellite, the

Fig. 9.10 The normalized growth rate versus magnetic latitude for the field-aligned propagating He^+ -mode EMIC waves. The RC is assumed to be entirely made up of energetic protons, and the thermal plasma consists of the cold electrons, 77% H^+ , 20% He^+ , and 3% O^+ . All of the results are taken from our simulation at $L = 6$, $\text{MLT} = 15$ for hour 48 after 0000 UT on 1 May 1998. The 10% level is marked by a horizontal line



probability of bidirectional events is 0. Therefore the 26% may be explained as a result of 40:60 split between these two extremes.

The above consideration shows that the first wave model is consistent with the Loto'aniu et al. statistical results. However, inconsistency remains with the wave packet periodicity reported by Loto'aniu et al. for some bidirectional events observed below 11° |MLat|. Because wave packet periodicity was not initially included in the first wave model, some modification to the model is required. (Recall that, in order to keep a qualitative difference between the first and second wave models, we are not allowed to introduce any reflecting surfaces in the first model.) Probably the simplest way to modify the model is to add ULF waves, which potentially are able to produce bidirectional repetition (e.g., Mursula et al. 2001). Unfortunately, we cannot avoid the inconsistency by simply adding ULF waves to the first model. Indeed, Fig. 3b of Loto'aniu et al. (2005) shows bidirectional wave propagation with a well-defined repetitive structure but, on average, the simultaneous compressional Pc5 wave amplitudes were less than 0.3 nT. Therefore it is unlikely that bidirectional repetitive patterns are due to a modulation of the plasma characteristics by Pc5. It is possible, at least in principle, that periodic packets are a result of periodic wave generation, which is controlled by a group of resonant hot ions bouncing between the mirror points in the conjugate hemispheres (Erlandson et al. 1992). However, this mechanism requires anomalous phase bunching of the ions, and it is unlikely that such spatially bunched bouncing ions can exist over time (Fraser et al. 1996). Therefore we conclude that the first wave model is not consistent with a dynamical wave packet periodicity observed by Loto'aniu et al. (2005).

9.3.2 Poynting Flux Analysis: With Reflection

Now we reanalyze the Loto'aniu et al. (2005) results using the second wave model. Note that the reflection coefficient in the second model should be greater than zero, otherwise the second model is identical to the first model. In order to avoid unnecessary complications, we first simplify the second model as much as possible (1) only one quasimonochromatic EMIC wave packet propagates along a field line, bouncing between the off-equatorial magnetic latitudes corresponding to the bi-ion hybrid frequencies in conjugate hemispheres; (2) the field-aligned group velocity is assumed to be the Alfvén velocity, and to be coordinate independent; (3) the wave packet has duration ΔT , length ΔL , and a boxcar profile; and (4) wave growth inside the near equatorial instability region is neglected, so wave energy may only decrease due to imperfect reflection (the reflection coefficient may include all possible loss mechanisms including wave tunneling/transformation, and resonant energy absorption by the thermal and hot plasmas). A sketch of the second wave model is shown in Fig. 9.11. The coordinate z is measured along the field line with zero at the equator, the bi-ion reflecting surfaces are placed at $\pm l$, and a star marks the satellite location (see Fig. 9.11a). Assuming an ideal instrument, which can instantly provide the Poynting vector of EMIC waves for a specified frequency, we plotted in Fig. 9.11b, c a field-aligned component of the net Poynting vector as

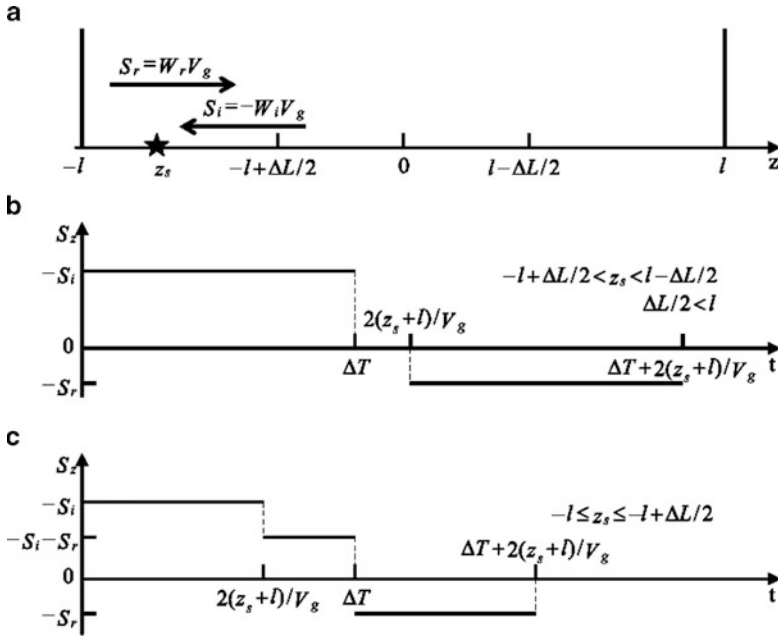


Fig. 9.11 (a) Sketch of the second wave model (see text). The coordinate z is measured along the field line with zero at the equator. The bi-ion reflecting surfaces are located at $\pm l$, and a star marks satellite with coordinate z_s . The field-aligned wave group velocity and component of the incident (reflected) Poynting vector are V_g and S_i (S_r), respectively. The incident (reflected) wave density is W_i (W_r). (b, c) The field-aligned component of the net Poynting vector versus time as observed by perfect instrument at two satellite locations. The wave packet duration and length are ΔT and ΔL , respectively

observed at two satellite locations. Recall that we assumed a quasimonochromatic wave packet, so the instantaneous measurements of the electric and magnetic wave fields allow us to calculate the Poynting flux for the known frequency. The only difference between measurements at these two locations is a partial, or total, overlap between the incident and reflected portions of the wave packet, observed in Fig. 9.11b and lacking in Fig. 9.11c. Because the wave reflection coefficient, $R = S_r/S_i$, is always less than 1, it follows from Fig. 9.11c that a duration of the equatorward (reflected) wave signal decreases when the satellite approaches the reflection surface, while a duration of wave packet propagating away from the equator does not depend on z_s , being always ΔT . In the limiting case, $z_s = -l$, the equatorward signal disappears and only a poleward propagating signal may be observed. This preliminary consideration suggests that a finite time needed to obtain a single measurement of the Poynting vector may be a serious issue for detecting the equatorward wave propagation.

The measurements of the electric and magnetic fields, taken at the instrument sampling rate, permit calculation of the total Poynting vector only. They provide no information regarding frequency-dependent effects, which are crucial for EMIC

waves in a multi-ion magnetosphere. For example, it is possible for high-frequency EMIC waves to reflect at the off-equatorial latitude where their frequency is close to the local bi-ion hybrid frequency. Particularly in $H^+-He^+-O^+$ plasma, the He^+ -mode and H^+ -mode EMIC waves may reflect at the corresponding bi-ion latitudes while the O^+ -mode EMIC waves can propagate to the Earth (Khazanov et al. 2006). Moreover, depending on the wave normal angle, even He^+ -mode and H^+ -mode can pass freely through the reflecting zone. Therefore to study the frequency and wave normal angle-dependent effects in wave propagation, the signal should be decomposed into constituent quasimonochromatic components. Fast Fourier transform (FFT) analysis has become the conventional method for this purpose. The time window Δt_{FFT} , used in the FFT analysis, is determined by the desired frequency resolution, which is the reciprocal of the window length. To achieve acceptable frequency resolution, time segments of one to several minutes or even much longer are typically used (e.g., Fraser 1985; Ishida et al. 1987; Loto'aniu et al. 2005).

The FFT analysis implies that the following combination of the electric and magnetic field has to be evaluated in order to estimate the Poynting flux at any particular time t :

$$\begin{aligned} \frac{\delta \mathbf{E}(t, \omega)}{\Delta t_{FFT}} \times \frac{\delta \mathbf{B}(t, \omega)}{\Delta t_{FFT}} &= \frac{1}{(\Delta t_{FFT})^2} \int_{t-\Delta t_{FFT}/2}^{t+\Delta t_{FFT}/2} \delta \mathbf{E}(t_1) \exp(i\omega t_1) dt_1 \\ &\times \int_{t-\Delta t_{FFT}/2}^{t+\Delta t_{FFT}/2} \delta \mathbf{B}(t_2) \exp(i\omega t_2) dt_2. \end{aligned} \quad (9.3.1)$$

Both the electric and magnetic field signals may be decomposed as a superposition of components with different frequencies having slowly varying amplitudes as functions of time. This implies that both integrals in (9.3.1) are maximized for the resonant components, and we may take into account only these components in the total signals $\delta \mathbf{E}(t_1)$ and $\delta \mathbf{B}(t_2)$. In other words, we may substitute $\delta \mathbf{E}(t_1) \sim \delta \mathbf{E}_\omega(t_1) \exp(-i\omega t_1)$ and $\delta \mathbf{B}(t_2) \sim \delta \mathbf{B}_\omega(t_2) \exp(-i\omega t_2)$ in (9.3.1), where $\delta \mathbf{E}_\omega(t_1)$ and $\delta \mathbf{B}_\omega(t_2)$ are the slowly varying amplitudes of the field components corresponding to ω . This gives

$$\begin{aligned} \frac{\delta \mathbf{E}(t, \omega)}{\Delta t_{FFT}} \times \frac{\delta \mathbf{B}(t, \omega)}{\Delta t_{FFT}} &= \frac{1}{(\Delta t_{FFT})^2} \int_{t-\Delta t_{FFT}/2}^{t+\Delta t_{FFT}/2} \delta \mathbf{E}_\omega(t_1) \exp(i\omega t_1) dt_1 \\ &\times \int_{t-\Delta t_{FFT}/2}^{t+\Delta t_{FFT}/2} \delta \mathbf{B}_\omega(t_2) \exp(i\omega t_2) dt_2 \\ &\approx \frac{1}{\Delta t_{FFT}} \int_{t-\Delta t_{FFT}/2}^{t+\Delta t_{FFT}/2} \delta \mathbf{E}_\omega(t_1) \delta \mathbf{B}_\omega(t_1) dt_1. \end{aligned} \quad (9.3.2)$$

Equation (9.3.2) means that, in order to obtain the Poynting flux at time t , we have to average the signals shown in Fig. 9.11b, c over a window Δt_{FFT} centered on t . The Poynting flux estimate is reliable only if we obtain a slowly varying result while shifting the FFT time window over an interval longer than Δt_{FFT} . In other words, the result should be slowly varying during the observational time $\Delta t_{obs} = \Delta t_{FFT} + \tau$, where $\tau \geq 0$. For example, Loto'aniu et al. (2005) used $\Delta t_{FFT} = 50$ s, but the EMIC wave measurements were identified as a reliable event if both the electric and magnetic signals were continuously observed for at least $\Delta t_{obs} = 60$ s. Keeping in mind these two facts, it follows immediately from Fig. 9.11b, c that the equatorward Poynting flux only can be reliable if Δt_{FFT} does not exceed some limiting window length Δt_{max} . In other words, one is able to observe both the waves propagating away from and toward the equator if $\Delta t_{FFT} < \Delta t_{max}$, but only the wave packet propagating away from the equator otherwise. In the case $\tau = 0$, referring to Fig. 9.11b, c, we can find this limiting window from the equation

$$\int_{\Delta T + 2(z_s + l)/V_g - \Delta t_{max}}^{\Delta T + 2(z_s + l)/V_g} S_z(t) dt = 0,$$

which gives (recall that $0 < R < 1$)

$$\frac{\Delta t_{max}}{\Delta T} = \frac{2(z_s + l)}{\Delta L(1 - R)} \quad \text{when} \quad 2(z_s + l)/\Delta L \leq (1 - R) \quad (9.3.3)$$

and

$$\frac{\Delta t_{max}}{\Delta T} = \frac{2(z_s + l)}{\Delta L} + R \quad \text{when} \quad 2(z_s + l)/\Delta L > (1 - R). \quad (9.3.4)$$

The case $\tau \neq 0$ does not introduce any new physics in the model but rather makes analysis more complicated. It is for this reason that we examine (9.3.3) and (9.3.4), despite $\tau = 10$ s in the paper of Loto'aniu et al. (2005). Note that requirement $\tau \neq 0$ decreases Δt_{max} for any satellite location, on average, by $1.5\tau - 2\tau$ compared with (9.3.3) and (9.3.4).

Figure 9.12 shows $\Delta t_{max}/\Delta T$ versus $2(z_s + l)/\Delta L$ for different reflection coefficients, where all of the specified parameters, such as ΔT , ΔL , and Δt_{FFT} , are taken from the paper by Loto'aniu et al. (2005). For a given $\Delta t_{FFT} = 50$ s, the intersection between the horizontal and colored lines in Fig. 9.12 gives the “critical” satellite location that separates the low-latitude region (in which both the poleward and equatorward Poynting vectors may be reliably measured) from a high-latitude “blind” region (where only a wave packet propagating away from the equator may be registered). If $R = 0.01$, for example, the equatorward Poynting flux may only be measured if $\Delta l = (z_s + l) > 2.2 R_E$ for all three wave packets (“min”, “max”, and “av”) shown in Fig. 9.12 (for the “av” wave packet, e.g., substituting

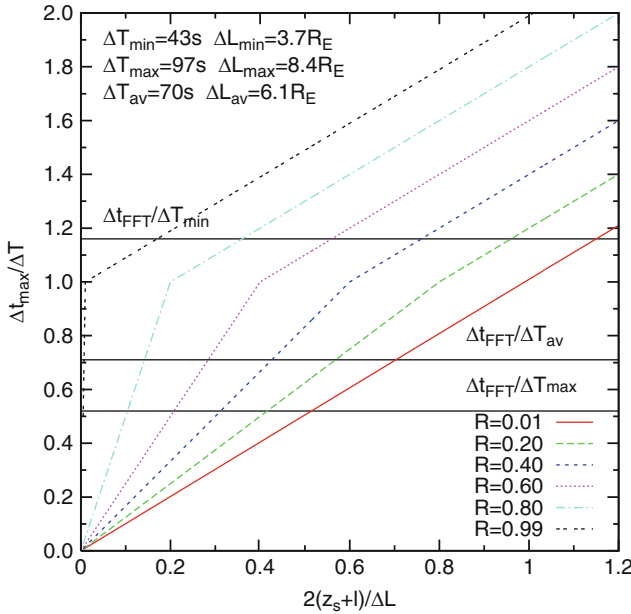


Fig. 9.12 Normalized Δt_{max} versus distance from the reflecting surface as specified by (9.3.3) and (9.3.4)

$\Delta t_{max} = \Delta t_{FFT} = 50$ s, $\Delta T = \Delta T_{av} = 70$ s, and $\Delta L = \Delta L_{av} = 6.1 R_E$ in (9.3.3), we get the “critical” satellite location $\Delta l_{av} = (z_s + l) = 2.2 R_E$. On the other hand, the bi-ion reflecting latitudes, whose location depends on EMIC wave frequency and ion composition, are typically located at $|\text{MLat}| \approx 15^\circ\text{--}30^\circ$ (e.g., Horne and Thorne 1993; Khazanov et al. 2006; Loto’aniu et al. 2005). At $L = 6$, for example, these suggest that the satellite should be located inside of $11^\circ |\text{MLat}|$ in order to “see” the reflected signal because the near equatorial field-aligned distance $2.2 R_E$ corresponds to a latitudinal extent of approximately 20° . In such a case, it is impossible to discriminate between the first and second wave models using the statistics of Loto’aniu et al. (2005). In order to do that, we should be able to measure the reflected signal outside the near equatorial instability region. The ability to detect a small reflected signal outside $|\text{MLat}| \approx 10^\circ\text{--}20^\circ$ is also crucial to resolve whether wave packets are bouncing between the off-equatorial bi-ion magnetic latitudes or whether there is only unidirectional wave propagation. Indeed, it was shown that wave amplification of 20–40 dB (factor of 10–100 in energy) may be attainable for the He^+ -mode EMIC waves (Hu and Fraser 1994). In this case, a small reflected signal may be reamplified while propagating through the near equatorial instability region and be observed by the satellite.

For the range of reflection coefficients given in Fig. 9.12, the “blind” region extends over $\Delta l = 0.3\text{--}2.2 R_E$ and $\Delta l = 0.02\text{--}2.2 R_E$ in the case of the “min” and “max” wave packet, respectively. In the absence of any information regarding reflection coefficients, we average (9.3.3) and (9.3.4) over the entire possible

range of the reflection coefficients, obtaining $\Delta l \approx 1.2 R_E$ for both wave packets (the same result can be obtained by substituting $R = 0.5$ in (9.3.3) and (9.3.4)). Note that the Alfvén velocity increases toward the reflecting latitudes, suggesting an even greater wave packet length is possible than the estimate used and, as a result, a greater Δl ($\tau = 10$ s also increases $\langle \Delta l \rangle$). Thus, on average, the equatorward/reflected wave packet cannot be observed if the satellite is located closer to the reflection point than $1.2 R_E$, and only the packet propagating away from the equator may be observed there. In terms of latitude, taking $L = 6$ as a typical L-shell, we have 11° as a width of the “blind” zone. Then, assuming $|\text{MLat}| \approx 22^\circ$ as a typical (an average) bi-ion reflecting latitude, we obtain that, on average, bidirectional wave energy propagation, both away from and toward the equator, should be observed below $11^\circ |\text{MLat}|$, and only unidirectional energy propagation away from the equator outside $\pm 11^\circ$ of the equator.

The fact that Loto’aniu et al. (2005) found bidirectional wave energy propagation for only 26% of all events below $11^\circ |\text{MLat}|$ may be interpreted similarly to what we did in the framework of the first wave model. Indeed, in the case when all the equatorward packets can be observed before they reach the actual satellite location, which is in the range $0^\circ \leq |\text{MLat}| < 11^\circ$, we expect an oppositely directed packet for each wave packet registered inside $\pm 11^\circ$ with a probability of 1. In the opposite case when a combination of reflection coefficient, wave packet duration/length, and latitude of the reflecting surface does not allow the equatorward wave packets to be observed before they reach the satellite, the probability of a pair of bidirectional events is 0. Therefore the observed 26% may be explained as a result of an appropriate splitting between these two extreme conditions. Thus, the analysis shows that the Loto’aniu et al. (2005) statistical results are consistent with the second wave model.

Although the second wave model is consistent with the Loto’aniu et al. (2005) statistics, the low-frequency EMIC branch cannot be treated in the framework of this model. Indeed, there is no bi-ion hybrid frequency for the O^+ -mode EMIC waves in the $H^+ - He^+ - O^+$ plasma, and this wave mode can freely propagate to the Earth. Due to the following energy absorption in the ionosphere, it is possible to have unidirectional propagation for this mode (e.g., Erlandson et al. 1992), and this mode may be treated in the framework of the first wave model. In Sect. 9.3.1, it was shown that the first wave model is consistent with the Loto’aniu et al. statistics if the wave growth rate is high enough. Therefore the combination of the first and second wave models can explain why the observed wave energy propagation was independent of the wave branch in the Loto’aniu et al. statistical results.

Let us now interpret the dynamical periodicity observed by Loto’aniu et al. (2005) for some bidirectional events below $11^\circ |\text{MLat}|$. The wave packet oscillation is an internal feature of the second model, and all we are going to do at this point is to estimate the double-hop bounce period using this model, and then compare it with the experimental value. The wave packet transit time depends on the field-aligned distance between the off-equatorial bi-ion reflecting surfaces and wave group velocity, which depends on the ion composition and wave frequency. Unfortunately, most of this information is not available. For that reason, let us first

estimate the double-hop bounce period using a constant Alfvén velocity of 550 km s^{-1} reported by Loto'aniu et al. for the events shown in their Fig. 3b. Note that the Alfvén velocity was estimated assuming an electron–proton plasma. For $L = 6$, taking $|\text{MLat}| = 22^\circ$ as a typical bi-ion reflecting latitude, we get a theoretical bounce period of 107 s, which is 1.7 times smaller than 180 s, the average double-hop bounce period observed by Loto'aniu et al. This is not surprising because the field-aligned group velocity for EMIC waves is only the Alfvén velocity in the case of an electron–proton plasma at the zero frequency limit. Both the inclusion of heavy ions and a nonzero frequency correction will decrease the group velocity (e.g., Fraser 1972), resulting in the increase of bounce period. For example, Fraser et al. (1989) found that the group propagation time was two times greater than the Alfvén time for the He^+ -mode EMIC waves. Similar results were also reported by Arnoldy et al. (2005) and Erlandson et al. (1992). The factor two is more than enough to reconcile the above theoretical estimate with the observed double-hop bounce period. Therefore we conclude that the second wave model is consistent with the dynamical wave packet periodicity observed by Loto'aniu et al. (2005).

The above quantitative analysis, although simplified, clearly demonstrates that a finite time needed to obtain a single measurement of the Poynting vector can be a serious problem in detecting the reflected/equatorward waves. Using instantaneous measurements of the electric and magnetic wave field (e.g., Erlandson et al. 1990, 1992; Fraser et al. 1996; Mursula et al. 2001), we can reliably track the dynamics of the total Poynting flux but the contribution of the different frequencies is unclear. On the other hand, applying the FFT analysis to natural signal (e.g., Fraser et al. 1992; LaBelle and Treumann 1992; Loto'aniu et al. 2005), we are able to resolve frequencies, but the extracted Poynting flux dynamics may be completely wrong for a particular frequency. There is a closely related problem in quantitative determination of the wave polarization and minimum variance direction for EMIC waves in the Earth's magnetosphere (Anderson et al. 1996). In an attempt to resolve that problem, Anderson et al. developed a new approach, the “wave step technique”, which operates on timescales of a few wave periods. Despite the working with very short time windows, it achieves good frequency precision. It is possible, therefore, that one can achieve a better time–frequency balance in obtaining the EMIC wave Poynting vector by using the wave step technique. Another possible way to circumvent the problem may be to remove the selection criterion that qualitatively similar results should be obtained over intervals not less than Δt_{FFT} . This approach, although unreliable, will permit analysis of much shorter periods of the equatorward signal, indicating the data set intervals that require special attention.

9.3.3 EMIC Wave Ellipticity Argument

It is very difficult to locally generate highly oblique EMIC waves because the growth rate mainly maximizes for field-aligned waves. Moreover, the strongest generation takes place in the near equatorial region (see Fig. 9.10). This suggests

that the greatest wave amplification takes place for the wave packets that have nearly field-aligned wave normals over the entire near-equatorial unstable region. As a result, in the framework of the first wave model, where only a unidirectional wave propagation is permitted, one would anticipate seeing mainly quasi-field-aligned EMIC waves near the equator. Indeed, ray-tracing calculations by Thorne and Horne (1997) and Horne and Thorne (1997) showed that, to attain the greatest wave amplification, the wave normal angle should be confined to less than 10° during wave propagation over the entire near equatorial region. On the other hand, wave reflection, included in the second wave model, results in the wave energy outflow to the region of higher wave normal angles (e.g., Horne and Thorne 1993; Khazanov et al. 2006). This makes it possible to observe a mixture of oblique and field-aligned wave normals in the near equatorial zone. Therefore analysis of the wave normal angle distributions near the equator may discriminate between the first and second wave model. Instead of analyzing wave normals, we may analyze the EMIC wave ellipticity, which is related to the wave normal angle. That is, if the plasma properties and wave frequency are specified, ellipticity determines wave normal angle, and vice versa for a plane EMIC wave. This is very helpful because there are very few satellite-based studies of EMIC waves that report wave normal angles, but great data sets of the wave ellipticity from AMPTE/CCE and CRRES (Anderson et al. 1992b; Fraser and Nguyen 2001). The wave ellipticity is defined in the plane perpendicular to the external magnetic field as the ratio of the minor to the major axis of the wave polarization ellipse being -1 for left circular, 0 for linear, and $+1$ for right circular polarization. The model-based expectations may be reformulated in the terms of ellipticity as follows: in the case of the first wave model, the ellipticity is expected to be close to -1 in the near equatorial region, whereas, for model two, the near equatorial ellipticity is expected to be distributed between -1 and some number around 0 .

The CRRES statistics show that EMIC events near the magnetic equator are evenly distributed from left-hand polarized to linearly polarized with some right-hand polarized admixture (Fraser and Nguyen 2001; Meredith et al. 2003). The statistical results from AMPTE/CCE are qualitatively the same in the prenoon–dusk MLT sector whereas waves at dawn are mostly linear polarized at all sampled latitudes, including the equator (Anderson et al. 1992b). It was emphasized by Anderson et al. (1992b) and Fraser and Nguyen (2001) that the observation of a significant number of linearly polarized events near the equator cannot be explained by polarization reversal from left-handed through linear to right-handed at the crossover frequency, as suggested by Young et al. (1981). (Recall that if the Young et al. mechanism takes place, then the quasi-field-aligned waves can have a linear polarization.) Therefore the observed linear polarization inside the near equatorial zone suggests that waves should be often highly oblique. Using the more reliable wave step technique, Anderson et al. (1996) and Denton et al. (1996) analyzed data from the AMPTE/CCE spacecraft and presented the first results for near-linearly polarized EMIC events. They indeed found a significant number of wave intervals where the wave normal angle was greater than 70° . The CRRES and AMPTE/CCE observations cannot be reconciled with the first wave model but have

a natural explanation in the framework of the second model. The global EMIC wave simulation based on the second wave model (Khazanov and Gamayunov 2007b; Khazanov et al. 2007) showed that occurrences of the oblique and field-aligned wave normal angles are nearly equal near the equator, with a slight dominance of oblique events consistent with observations.

We summarize the results of our analysis presented above as follows (1) both the first and second wave models are equally consistent with the statistical results reported by Loto'aniu et al. (2005); (2) the first wave model is not consistent with the wave packet periodicity observed by Loto'aniu et al. (2005), whereas the second wave model is; (3) a combination of the first and second wave models can explain why the observed wave energy propagation was independent of the wave branch in the Loto'aniu et al. (2005) statistical results; and (4) the first wave model is not consistent with the wave ellipticity observed in the near equatorial region, whereas the second wave model is.

9.3.4 Convective Instability Argument

The absolute instability is not realized in the case of Pc1 pulsations (Troitskaya and Guglielmi 1967; Gomberoff and Neira 1983). Guglielmi et al. (2001) also discussed this question and supported the earlier results that, for a smooth distribution function like two-temperature Maxwellian, the conditions for absolute instability are far from being reachable. In this case, the maximum amplitude of a wave packet amplified from noise is $\exp(\Gamma_{\max})$ times above the noise level, where Γ is the one-path gain and the subscript “max” refers to the maximum value. It is assumed (e.g., Horne and Thorne 1994) that a gain of $\Gamma \geq 10-15$ is sufficient for the noise to grow to the observable wave amplitudes. Horne and Thorne (1994) found that such conditions can be realized in the outer magnetosphere (at $L \sim 7$) for the energetic-proton densities $N_h \approx 1 \text{ cm}^{-3}$. For typical conditions of pearl generation, i.e., for low-disturbance periods and near the plasmopause, the gain is usually several times smaller ($\Gamma \leq 10$) (Demekhov 2007). In this case, reflection is needed for a wave packet to develop, and the well-known condition for wave generation can be written in the form

$$\Gamma_{tot}(\omega) = 2\Gamma(\omega) + \ln|R_+(\omega)R_-(\omega)| \geq 0,$$

where ω is the wave frequency, and $R_+(\omega)$ and $R_-(\omega)$ are the effective reflection coefficients of conjugate reflection regions. As an obvious but sometimes forgotten consequence, the generation starts at a frequency where Γ_{tot} reaches a maximum (Demekhov 2007). Hence, in the case in which a wave packet corresponds to a minimum ionospheric reflection, this reflection can be significant only if, by chance, the magnetospheric gain Γ has a very sharp frequency peak coincident with the reflection minimum. Other possibilities are that (1) the waves are generated at a different field line, (2) the instability is absolute, or (3) reflection takes place above the ionosphere (Guglielmi et al. 2001).

Indeed, the reflection regions can be not only the ionospheres but also, e.g., heavy-ion gyroresonance regions, as we have seen in Sect. 9.2 of this chapter. It was noted (Fraser et al. 1996; Guglielmi et al. 2001) that, in the latter case, for a satellite detector below the reflection points, the wave Poynting flux is obviously downward and the repetition period is the same as on the ground. Moreover, the wave-bounce period is smaller compared with inter-ionospheric propagation, which seems to be easier fit to some observations (e.g., Mursula et al. 2001). This attractive mechanism requires more refined study and comparison with observations. In particular, the resonant cyclotron absorption of waves by heavy ions increases their thermal spread, which can significantly decrease the opacity of the resonance region (Suvorov and Trakhtengerts 1987).

An alternative explanation for not observing upward wave Poynting flux is because it is below the observational threshold. That explanation was suggested by Demekhov (2007). Loto'aniu et al. (2005) reported downward Poynting flux in the range of $1.3\text{--}10\ \mu\text{W m}^{-2}\text{ Hz}^{-1}$ with uncertainty of $0.1\ \mu\text{W m}^{-2}\text{ Hz}^{-1}$. The implication is that reflection of 1% of the wave energy would not have been observable. Given 1% reflection, a one-pass integrated gain $\Gamma > 2.3$ would be needed for the CRRES instrument to observe the reflected wave. This gain is quite realistic, and measurements that are more sensitive appear to be needed to confirm the presence or absence of reflected Pc1 waves. Yet another explanation for not observing upward directed waves is because the upward wave is masked by the stronger down-going wave (Demekhov 2007). Pearl wave packet durations are close to the repetition period, meaning that the packet length can be even longer than the field-line length between reflection points. In this case, the reflected portion of the wave packet may often be present at the same time as the large-amplitude downward waves. Off the equator and toward the reflection point, the larger wave will dominate, obscuring observation of the weaker upward reflected wave. Near the equator, wave amplification will take place along with a smaller wave group velocity and it will be more likely that waves in both directions will be observed with similar amplitude. Although a quantitative testing of these scenarios is required, the arguments presented by Demekhov (2007) seem reasonable.

In the conclusion to this section, let us discuss the “paradoxical behavior” (as has been pointed out by Guglielmi and Kangas (2007)) of Pc1 pulsation that is relevant to the issue of the magnetospheric propagation and tunneling of the EMIC waves. This paradox is related with the negative correlation of the Pc1 number occurrence with the solar activity. As mentioned by Guglielmi and Kangas (2007), a result is paradoxical in the following sense that Pc1 oscillations are entirely a magnetospheric phenomenon. They are excited on closed magnetic shells and propagate along geomagnetic field lines arriving on the ground in the magnetically conjugated regions. Pc1 wave fields are located at great distances from the interplanetary medium (tens to hundreds of wavelengths). Thus, it is physically impossible that the wave activity depended directly on the parameters of a medium located far beyond the wave field.

Recall from the previous sections that Pc1s are excited as the EMIC waves in the equatorial zone of the magnetosphere, and that there exist two opacity bands on the

propagation path of EMIC waves from one hemisphere to the other. The bands widen with increasing relative concentration of oxygen ions O^+/H^+ , and the tunneling coefficient depends exponentially on the band width, and, as we showed above, an admixture of oxygen ions has a dramatic impact on the propagation of EMIC waves in the magnetosphere. Now, if we bear in mind that at the minimum of the solar-activity cycle, the concentration of O^+ in the magnetosphere is much lower than at the maximum of the cycle (Young et al. 1982), the answer regarding Pc1 solar activity dependence becomes clear: Pc1 waves are observed most often in years when the relative concentration of oxygen ions along the paths of wave propagation is strongly reduced, i.e., during the solar minimum. This picture allows us to understand qualitatively the fact that the Pc1 occurrence rate varies in the antiphase with the 11-year variation of the sunspot number.

Now we point out another issue that has been discussed by Guglielmi and Kangas (2007): direct correlation of Pc1 with solar wind plasma density. According to the Cluster satellite observations (Lundin et al. 2003), the H^+ ions penetrate sporadically from solar wind into the magnetosphere in the form of so-called plasma transfer events (PTEs). It is quite clear that the O^+ does not change in the period of such intrusions. Thus, PTEs result in the change of ion composition of the magnetospheric plasma, or more precisely, PTEs are accompanied by a drop in the ratio O^+/H^+ . According to the discussion above, this favors Pc1 propagation.

Thus, the two remarkable properties of Pc1 waves, namely, the direct correlation of Pc1 with solar wind plasma density and inverse correlation of Pc1 with sunspot number, can be explained by the variations in the relative contents of the O^+/H^+ ions in the magnetosphere.

What is the main conclusion regarding bouncing EMIC waves versus unidirectional propagation? In the frame of the standard model, the Pc1 wave packet oscillates along the geomagnetic field line and this packet is periodically reflected from the ionosphere at the conjugate points (e.g., Nishida 1978). An alternative version of the magnetospheric reflection model was proposed by Guglielmi et al. (2001) with the aim of solving the difficult problem associated with existence of the opacity bands in the multicomponent magnetospheric plasma. It is obvious that one or another type of reflection is needed, because the oscillatory system responsible for the appearance of Pc1 wave packets represents a feedback amplifier (Guglielmi and Kangas 2007), and, based on the many evidences presented in this section, one may conclude in a favor of bouncing EMIC waves between the points of reflection versus the unidirectional propagation ones. Some clarification in this question is still needed.

9.4 Governing Equations

9.4.1 Wave Kinetic Equation

In order to obtain the equation that describes EMIC wave energy evolution, let us start from the wave kinetic equation (e.g., Stix 1992). If the background medium

is stationary (in other words, the characteristic timescale for the changing of plasmaspheric thermal plasma is much greater than the wave energy evolution time), this equation has the form

$$\frac{\partial W(t, \mathbf{r}, \mathbf{k})}{\partial t} + \frac{\partial \omega(\mathbf{r}, \mathbf{k})}{\partial \mathbf{k}} \cdot \frac{\partial W}{\partial \mathbf{r}} - \frac{\partial \omega}{\partial \mathbf{r}} \cdot \frac{\partial W}{\partial \mathbf{k}} = 2\gamma(t, \mathbf{r}, \mathbf{k}) \cdot W. \quad (9.4.1)$$

In this equation, $W(t, \mathbf{r}, \mathbf{k})$ is the EMIC wave power spectral density and $\gamma(t, \mathbf{r}, \mathbf{k})$ includes both the energy source due to wave interaction with RC ions and the energy sink due to absorption by thermal and hot plasmas. Note that in spite of the quasistationarity of the background plasma, the RC itself can vary. In practice it cannot change the dispersive properties of the medium, but may lead to modification of γ , and we specify this explicit time dependence in γ . As we can see, characteristics of the left-hand side of (9.4.1) are solutions of the ray-tracing equations (9.2.1)–(9.2.4), and $W(t, \mathbf{r}, \mathbf{k})$ is just transferred along the corresponding ray phase trajectories with simultaneous growth/damping depending on the sign of γ along these trajectories.

In $e\text{--}H^+\text{--}He^+\text{--}O^+$ plasma, if EMIC wave generation is caused by the hot RC protons with positive temperature anisotropy, the most intense wave generation takes place for the He^+ -mode in the frequency range $\Omega_{O^+} < \omega < \Omega_{He^+}$ (see Sect. 9.2.1 for details). (Actually only left-hand polarized waves grow, and the corresponding wave frequencies should be in the range between the cross-over frequency and Ω_{He^+} if the wave normal angle is $\theta \neq 0$.) In the present section, we consider only this EMIC wave mode. The typical ray trajectories for this wave mode are illustrated in Figs. 9.6–9.8. Note that we do not present the evolution of wave numbers, k , in Figs. 9.6–9.8, because they can be easily found from the local dispersion equation, $G=0$, by using the function (9.2.5). Ray-tracing simulations for the He^+ -mode of EMIC waves demonstrate that (1) these waves are well guided along the magnetic field line, and they experience “fast” quasiperiodical bouncing between the surfaces, which are very close to the surfaces of $O^+\text{--}He^+$ bi-ion hybrid frequency in the opposite hemispheres, (2) the wave normal angle is oscillating about $\theta = \pi/2$ and progressively, but “slowly,” goes to 90° , and (3) the ray L shell coordinate is also “slowly” drifting. The ray phase path for the He^+ -mode in the vicinity of the plasmopause is illustrated in Fig. 9.8, and the typical timescales for “fast” and “slow” motions that we introduced above may be estimated as $\tau_{fast} \sim 10^2$ s and $\tau_{slow} \sim 10^3$ s. Another timescale that characterizes the wave evolution is a typical growth time, and this time may be evaluated as $\tau_{growth} = 1/\gamma \sim 10^3$ s. The stated time hierarchy gives us a clue that the bounce-averaged approximation that we used in Khazanov et al. (2002, 2003a, 2006) is still an appropriate approach in order to integrate the wave kinetic equation (9.4.1). See the previous section for a detailed discussion.

The general solution for (9.4.1) may be obtained by integrating this equation along its characteristics and can be written in the form of

$$W(t, \mathbf{r}, \mathbf{k}) = W \left(\mathbf{r} - \int_0^t \dot{\mathbf{r}}(\mathbf{r}(t'), \mathbf{k}(t')) dt', t = 0, \mathbf{k} - \int_0^t \dot{\mathbf{k}}(\mathbf{r}(t'), \mathbf{k}(t')) dt' \right) \\ \times \exp \left(2 \int_0^t \Gamma(\mathbf{R}_0, \tau, \mathbf{K}_0) d\tau \right), \quad (9.4.2)$$

$$\Gamma(\mathbf{R}_0, \tau, \mathbf{K}_0) = \gamma \left(\mathbf{R}_0 + \int_0^\tau \dot{\mathbf{r}}(\mathbf{r}(t'), \mathbf{k}(t')) dt', \tau, \mathbf{K}_0 + \int_0^\tau \dot{\mathbf{k}}(\mathbf{r}(t'), \mathbf{k}(t')) dt' \right), \quad (9.4.3)$$

$$\mathbf{R}_0 = \mathbf{r} - \int_0^t \dot{\mathbf{r}}(\mathbf{r}(t'), \mathbf{k}(t')) dt', \quad \mathbf{K}_0 = \mathbf{k} - \int_0^t \dot{\mathbf{k}}(\mathbf{r}(t'), \mathbf{k}(t')) dt', \quad (9.4.4)$$

where $\mathbf{r}(t) \equiv \mathbf{r}(t, \mathbf{R}_0, \mathbf{K}_0)$ and $\mathbf{k}(t) \equiv \mathbf{k}(t, \mathbf{R}_0, \mathbf{K}_0)$ are solutions of the ray-tracing equations (9.2.1)–(9.2.4), $\dot{\mathbf{r}} = d\mathbf{r}/dt = \mathbf{v}_g = \partial\omega/\partial\mathbf{k}$, $\dot{\mathbf{k}} = d\mathbf{k}/dt = -\partial\omega/\partial\mathbf{r}$, $\mathbf{R}_0$ and $\mathbf{K}_0$ are the initial radius vector and the normal wave vector, respectively. Argument $t = 0$ in the right-hand side of (9.4.2) emphasizes the fact that W is an initial power spectral density distribution for the EMIC waves. For the purpose of the present study, we do not need to resolve (9.4.1) on the timescale less than τ_{fast} . Then for any particular $\mathbf{r}$ and $\mathbf{k}$, this fact allows us to average both sides of (9.4.2) over the “fast” wave bounce period, T_g ,

$$\langle W(t, \mathbf{r}, \mathbf{K}) \rangle = \frac{1}{T_g} \int_t^{t+T_g} W(\mathbf{R}_0 + \mathbf{r}(t'), t, \mathbf{K}_0 + \mathbf{k}(t')) dt' = \langle W(t, \mathbf{R}_0, \mathbf{K}_0) \rangle, \quad (9.4.5)$$

$$T_g \equiv T_g(\mathbf{R}_0, \mathbf{K}_0) = \oint \frac{d\lambda}{d\lambda/dt}. \quad (9.4.6)$$

The averaging in (9.4.5) is performed along the ray phase trajectory. It is obvious that $\langle W(t, \mathbf{r}, \mathbf{k}) \rangle$ does not depend on a coordinate along the ray phase path, and now we are able to track power spectral density in the equatorial plane only. In the equatorial plane, the wave raypath has coordinates $\mathbf{r}_0$, $\mathbf{k}_0$, and their “slow” time change is described by

$$\frac{d\mathbf{r}_0}{dt} = \dot{\mathbf{r}}_0 = \frac{1}{T_g(\mathbf{r}_0, \mathbf{k}_0)} \int_0^{T_g} \dot{\mathbf{r}}(t) dt = \frac{1}{T_g(\mathbf{r}_0, \mathbf{k}_0)} \oint \dot{\mathbf{r}}(\lambda) \frac{d\lambda}{d\lambda/dt}, \quad (9.4.7)$$

$$\frac{d\mathbf{k}_0}{dt} = \dot{\mathbf{k}}_0 = \frac{1}{T_g(\mathbf{r}_0, \mathbf{k}_0)} \int_0^{T_g} \dot{\mathbf{k}}(t) dt = \frac{1}{T_g(\mathbf{r}_0, \mathbf{k}_0)} \oint \dot{\mathbf{k}}(\lambda) \frac{d\lambda}{d\lambda/dt}. \quad (9.4.8)$$

In order to write down (9.4.7) and (9.4.8) explicitly, we have to (1) find solutions for the system of ray-tracing equations (9.2.1)–(9.2.4), (2) substitute these solutions into the right-hand side of (9.2.1)–(9.2.4), and (3) average the result over a “fast” time period, T_g . Note that (9.2.1)–(9.2.4) are written in the coordinate systems specified in Sect. 9.2, and only for the sake of brevity we write (9.4.7) and (9.4.8) in a vector form. Further, keeping in mind that $t \gg T_g$, after straightforward manipulations, the system (9.4.2), (9.4.3) takes the form of

$$\begin{aligned} \langle W(t, \mathbf{r}_0, \mathbf{k}_0) \rangle &= \left\langle W \left(\mathbf{r}_0 - \int_0^t \dot{\mathbf{r}}_0(t') dt', t = 0, \mathbf{k}_0 - \int_0^t \dot{\mathbf{k}}_0(t') dt' \right) \right\rangle \\ &\times \exp \left(2 \int_0^t \langle \Gamma(\bar{\mathbf{R}}_0, \tau, \bar{\mathbf{K}}_0) \rangle d\tau \right), \end{aligned} \quad (9.4.9)$$

$$\begin{aligned} \langle \Gamma(\bar{\mathbf{R}}_0, \tau, \bar{\mathbf{K}}_0) \rangle &= \left\langle \gamma \left(\bar{\mathbf{R}}_0 + \int_0^{\tau-T_g} \dot{\mathbf{r}}_0(t') dt', \tau, \bar{\mathbf{K}}_0 + \int_0^{\tau-T_g} \dot{\mathbf{k}}_0(t') dt' \right) \right\rangle \\ &\approx \left\langle \gamma \left(\bar{\mathbf{R}}_0 + \int_0^{\tau} \dot{\mathbf{r}}_0(t') dt', \tau, \bar{\mathbf{K}}_0 + \int_0^{\tau} \dot{\mathbf{k}}_0(t') dt' \right) \right\rangle, \end{aligned} \quad (9.4.10)$$

$$\langle \gamma(\bar{\mathbf{R}}_0, \tau, \bar{\mathbf{K}}_0) \rangle = \frac{1}{T_g(\bar{\mathbf{R}}_0, \bar{\mathbf{K}}_0)} \int_0^{T_g} \gamma \left(\bar{\mathbf{R}}_0 + \int_0^t \dot{\mathbf{r}}(t') dt', \tau, \bar{\mathbf{K}}_0 + \int_0^t \dot{\mathbf{k}}(t') dt' \right) dt, \quad (9.4.11)$$

$$\bar{\mathbf{R}}_0 = \mathbf{r}_0 - \int_0^t \dot{\mathbf{r}}_0(t') dt', \quad \bar{\mathbf{K}}_0 = \mathbf{k}_0 - \int_0^t \dot{\mathbf{k}}_0(t') dt', \quad (9.4.12)$$

where $\bar{\mathbf{R}}_0$ and $\bar{\mathbf{K}}_0$ are equatorial initial values for the radius vector and wave normal vector, respectively.

Solution (9.4.9)–(9.4.12) completely describes the evolution of the bounce-averaged EMIC wave power spectral density if all of the plasma properties and initial distribution, W , are specified. An averaged wave power spectral density, $\langle W \rangle$, should satisfy a transport equation similar to the differential equation (9.4.1), and this transport equation has the following form:

$$\frac{\partial \langle W(t, \mathbf{r}_0, \mathbf{k}_0) \rangle}{\partial t} + \dot{\mathbf{r}}_0 \cdot \frac{\partial \langle W \rangle}{\partial \mathbf{r}_0} + \dot{\mathbf{k}}_0 \cdot \frac{\partial \langle W \rangle}{\partial \mathbf{k}_0} = 2 \langle \gamma(t, \mathbf{r}_0, \mathbf{k}_0) \rangle \cdot \langle W \rangle, \quad (9.4.13)$$

where $\dot{\mathbf{r}}_0 = \partial\omega(\mathbf{r}_0, \mathbf{k}_0)/\partial\mathbf{k}_0$, $\dot{\mathbf{k}}_0 = -\partial\omega(\mathbf{r}_0, \mathbf{k}_0)/\partial\mathbf{r}_0$, and the timescale for (9.4.13) is in the range $\tau_{fast} \ll \Delta t \ll \tau_{slow}$. Substituting the solution (9.4.9)–(9.4.12) in (9.4.13), it is easy to verify that indeed the left-hand side in the equation is equal to the right-hand side. Note that (9.4.13) can also be obtained by applying a corresponding bounce averaging procedure to the original wave kinetic equation (9.4.1).

It is more convenient to characterize the EMIC wave power spectral density by frequency and equatorial wave normal angle instead of the wave vector, $\mathbf{k}_0$, (see, e.g., Lyons and Williams 1984; Khazanov et al. 2002, 2003a). The transformation into (ω, θ_0) is straightforward, and we write down only the resulting form of (9.4.13) in these new variables,

$$\frac{\partial\langle W \rangle}{\partial t} + \dot{r}_0 \cdot \frac{\partial\langle W \rangle}{\partial r_0} + \dot{\theta}_0 \cdot \frac{\partial\langle W \rangle}{\partial \theta_0} = 2\langle \gamma(t, r_0, \varphi, \omega, \theta_0) \rangle \cdot \langle W(t, r_0, \varphi, \omega, \theta_0) \rangle, \quad (9.4.14)$$

where

$$\dot{\theta}_0 = \frac{d\theta_0}{dt} = \frac{1}{T_g(r_0, \varphi, \omega, \theta_0)} \oint \left(k_\lambda \frac{dk_r}{dt} - k_r \frac{dk_\lambda}{dt} \right) \frac{d\lambda}{k^2 d\lambda/dt}. \quad (9.4.15)$$

In (9.4.14) and (9.4.15), we took into account the fact that vector $\mathbf{r}_0$ has only one component for 2D ray propagation, and also emphasized the parametric dependency for the functions on geomagnetic longitude, φ . For the simplicity of the discussion here, in the EMIC wave equation (9.4.14), we omitted two effects, tunneling of the EMIC waves across the stop bands (see Sect. 9.2.3 for the details) and reflection from the ionosphere (Khazanov et al. 2003b). Depending on the heavy ion content in the magnetosphere, and the character of EMIC wave bounce along the field line (see the end of Sect. 9.3.4), both these factors are normally taken into account in the calculations presented below.

9.4.2 Ring Current Particle Transport

We simulate RC dynamics by solving simultaneously the bounce-averaged kinetic equation for the phase space distribution function, F , of the RC species (see chapter 8 for the details), and bounce-averaged kinetic wave equation for wave spectral power, B^2 , of EMIC waves (Khazanov et al. 2006)

$$\begin{aligned} \frac{\partial F}{\partial t} + \frac{1}{r_o^2} \frac{\partial}{\partial r_o} \left(r_o^2 \left\langle \frac{dr_o}{dt} \right\rangle F \right) + \frac{\partial}{\partial \varphi} \left(\left\langle \frac{d\varphi}{dt} \right\rangle F \right) + \frac{1}{\sqrt{E}} \frac{\partial}{\partial E} \left(\sqrt{E} \left\langle \frac{dE}{dt} \right\rangle F \right) \\ + \frac{1}{f(\mu_o)\mu_o} \frac{\partial}{\partial \mu_o} \left(f(\mu_o)\mu_o \left\langle \frac{d\mu_o}{dt} \right\rangle F \right) = \left\langle \frac{\delta F}{\delta t} \right\rangle_{collis} \end{aligned} \quad (9.4.16)$$

$$\frac{\partial \langle B^2(t, r_0, \varphi, \omega, \theta_0) \rangle}{\partial t} + \dot{\theta}_0 \cdot \frac{\partial \langle B^2 \rangle}{\partial \theta_0} = 2\langle \gamma(t, r_0, \varphi, \omega, \theta_0) \rangle \cdot \langle B^2 \rangle. \quad (9.4.17)$$

The RC distribution function is the function of position in the magnetic equatorial plane (r_0, φ); kinetic energy and the cosine of the equatorial pitch angle (E, μ_0); and time t . In the left-hand side of the RC particle equation, all of the bounce-averaged drift velocities are denoted as $\langle \cdots \rangle$. The term on the right-hand side of (9.4.16) includes losses from charge exchange, Coulomb collisions, ion-wave scattering, and precipitation at low altitudes. Loss through the dayside magnetopause is taken into account, allowing free outflow of RC ions from the simulation domain. (For more details regarding (9.4.16), see Khazanov et al. 2003a and references therein.) The ion-wave collisional term, included in the right-hand side of (9.4.16), is a function of the EMIC wave power spectral density (see Lyons and Williams 1984; Khazanov et al. 2003a for details) that is obtained from the wave kinetic equation (9.4.17).

The bounce-averaged kinetic wave equation for wave spectral power (9.4.17) is written for the magnetic energy of EMIC waves and neglects the small term that describes the radial wave drift. We implement the solution of (9.4.16) and (9.4.17) along with the system of ray tracing (9.2.1)–(9.2.4). The ion-wave collisional term, included in the right-hand side of (9.4.16), is a function of EMIC wave power spectral density that is described by the wave kinetic equation (9.4.17). On the other hand, $\langle \gamma(t, r_0, \varphi, \omega, \theta_0) \rangle$ in (9.4.17) is a function of the phase space distribution function, F (Lyons and Williams 1984; Khazanov et al. 2003a). Thus, the system (9.4.16) and (9.4.17) self-consistently describes the interacting RC and EMIC waves in a quasilinear approximation. Note that, in order to describe wave–particle interactions, we have to know the off-equatorial power spectral density distribution for the EMIC waves, and this distribution can be easily mapped from the magnetic equator with ray tracing (9.2.1)–(9.2.4) solutions.

9.4.3 *Coupling with the Magnetospheric Electric Field*

The resulting EMIC wave power spectral density depends on the RC and cold plasma characteristics. On the other hand, the convective patterns of both RC ions and the cold plasmaspheric plasma are controlled by the magnetospheric electric field, determining the conditions for the interaction of RC and EMIC waves. Therefore, this electric field is one of the most crucial elements necessary to properly determine the wave power spectral density. The region 2 field-aligned currents (FACs) couple the magnetosphere and ionosphere. This large-scale coupling determines and maintains a self-consistent dynamic of the electric field and RC (Vasyliunas 1970; Jaggi and Wolf 1973; Garner et al. 2004; Fok et al. 2001; Khazanov et al. 2003b; Liemohn et al. 2004). A self-consistent simulation of the magnetosphere–ionosphere system should provide, at least in principle, the most accurate theoretical electric field. The EMIC waves resulting in the magnetosphere are not only a passive element in the coupled RC–ionosphere system, but also may influence the electrodynamics of coupling. During storm times, the wave-induced RC proton precipitation not only changes the FAC distribution, but can potentially

modify the conductance and/or the neutral gas velocity in the ionosphere–thermosphere system (Galand et al. 2001; Galand and Richmond 2001; Fang et al. 2007a, b). Both of these characteristics are crucial elements in the magnetosphere–ionosphere electrodynamics. Such wave-induced modification can be especially important equatorward of the low-latitude edges of the electron and proton auroral ovals, where the wave-induced RC ion precipitation may be a dominant energy source. In addition, electrons and protons do not interact in the same way with the atmosphere. One should keep in mind that energetic protons ionize more efficiently than electrons do because their energy loss for each produced electron is smaller than that of energetic electrons (Galand et al. 1999). Therefore, even if the proton energy flux is smaller compared with the electron flux, the response of the atmosphere to protons can be significant. The above arguments suggest that a self-consistent model of the magnetospheric electric field, RC, plasmasphere, and EMIC waves is needed to properly model wave spectral distribution and to improve the modeling of the large-scale magnetosphere–ionosphere electrodynamics.

Now we are going to take into account the magnetosphere–ionosphere coupling by self-consistently treating the current closure between RC and the ionosphere. Vasyliunas (1970) mathematically formulated a self-consistent model of the magnetosphere–ionosphere coupling by providing the basic equations governing the system. He outlined a logical chain of the model as follows (1) the magnetospheric electric field determines the distribution of RC ions and electrons and, particularly, the total plasma pressure at any point; (2) from the plasma pressure gradients, the electric current perpendicular to the magnetic field can be calculated; (3) because the total current density should have zero divergence under magnetospheric conditions, the divergence of the perpendicular current density must be canceled by the divergence of FAC density, and so the divergence of the perpendicular current integrated along the entire field line gives the total FAC flowing into/out of the conjugate ionospheres; (4) from the requirement that FAC is closed by the horizontal ohmic currents in the ionosphere, the distribution of the electric potential in the ionosphere can be found; and (5) the ionospheric potential can be mapped back into the magnetosphere along geomagnetic field lines, and the requirement that this “new” magnetospheric electric field agrees with the “initial” magnetospheric field closes the magnetosphere–ionosphere system description.

To quantify the above logical chain, Vasyliunas (1970) used the following equations:

$$\mathbf{J}_\perp(v_0, \varphi, s) = \frac{\mathbf{B}}{B^2} \times \left(\nabla P_\perp + \frac{P_\parallel - P_\perp}{B^2} (\mathbf{B} \cdot \nabla) \mathbf{B} \right) \quad (9.4.18)$$

$$J_{\parallel,i}(\lambda(r_0), \varphi) = -B_i(\lambda(r_0), \varphi) \int_{s_S}^{s_N} \frac{\nabla \mathbf{J}_\perp}{B(r_0, \varphi, s)} ds \quad (9.4.19)$$

$$\nabla \mathbf{I}_i = j_{\parallel,i} \sin \chi, \quad \mathbf{I}_i = \Sigma \left(-\nabla \Phi_i + \frac{\mathbf{V}_n}{c} \times \mathbf{B} \right), \quad (9.4.20)$$

where $P_{\perp}$ and $P_{\parallel}$ are the total plasma pressure (we neglect the electron pressure in the current study) perpendicular and parallel to the external magnetic field $\mathbf{B}$, respectively, and $\mathbf{J}_{\perp}$ is the perpendicular current density. The FAC density at the ionospheric level is $J_{\parallel,i}$ (positive for current flowing into the ionosphere), B_i is the magnetic field in the ionosphere, and integration in (9.4.19) is done along the entire magnetic field line between foot points s_S and s_N . The coordinates $(\lambda(r_0), \varphi)$ are the corresponding ionospheric latitude and MLT for the magnetic field line crossing the equatorial plane at (r_0, φ) (assuming that φ is the same at the equator and at the ionospheric altitude). In (9.4.20), $\mathbf{I}_i$ and Σ are the height-integrated horizontal ionospheric current density and conductivity tensor, respectively, and χ is an inclination of the magnetic field (dip angle). The electric potential at the ionosphere level is Φ_i , and v_n is the velocity of the neutral gas in the ionosphere. Following many previous studies, in the present section, we assume that the neutral gas corotates with the Earth and neglect the potential drop between the ionosphere and the equatorial magnetosphere (e.g., Ebihara et al. 2004). Finally, it should be noted that, in general, (9.4.20) is written for the northern and southern ionospheres with the corresponding FAC $j_{\parallel,i}$, whereas (9.4.19) gives only the total FAC flowing into/out of the conjugate ionospheres but the obvious equation $J_{\parallel,i} = j_{\parallel,i}(s_S) + j_{\parallel,i}(s_N)$ is held.

The set of (9.4.16), (9.4.17), (9.2.1)–(9.2.4), and (9.4.18)–(9.4.20) drives the RC, the EMIC waves, and the magnetospheric electric field in a self-consistent manner if all the initial and boundary conditions are specified and the ionospheric Hall and Pedersen conductances are known. A block diagram of the self-consistent coupling of the RC, EMIC waves, plasmasphere, and ionosphere is presented in Fig. 9.13. The system characteristics in orange boxes are externally specified, and the dashed lines connect the model elements, which are currently not linked.

9.4.4 Simulation Scenarios

9.4.4.1 Coupling with Plasmas

The geomagnetic field used in the present section is taken to be a dipole field. It is a reasonable approximation for the present study because the most important results are obtained from simulations of the 2–3 May 1998 period ($D_{st} = 106$ nT) when the Earth's magnetic field was only slightly disturbed in the inner magnetosphere (e.g., Tsyganenko et al. 2003). The convection electric field is calculated self-consistently as described in the previous section, and the total electric field includes the magnetospheric convection and corotation field. The equatorial cold electron density, n_e , is obtained from the dynamic global core plasma model of Ober et al. (1997). This model is basically the same as the time-dependent model of Rasmussen et al. (1993), which was used in the previous studies, except the Ober et al. model is linked with a self-consistent electric field obtained from the system (9.4.16),

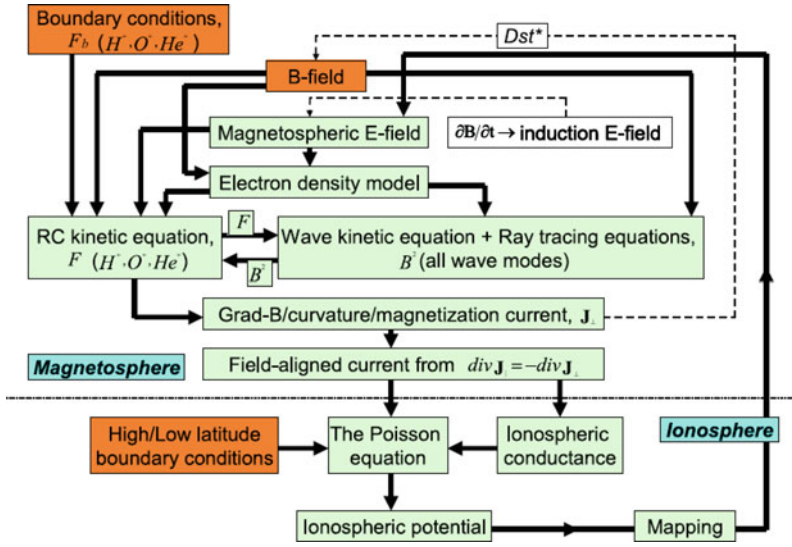


Fig. 9.13 The block diagram of the RC, EMIC waves, plasmasphere, and ionosphere coupling in our model. The system characteristics in *orange boxes* are externally specified, and the *dashed lines* connect the model elements that are currently not linked

(9.4.17), (9.2.1)–(9.2.4), and (9.4.18)–(9.4.20), whereas the Rasmussen et al. model is driven by the Volland–Stern convection field (Volland 1973; Stern 1975) with Kp parameterization. Thus, the cold plasma density dynamics are also electrically self-consistent in our global RC–EMIC wave model. This is extremely important for a correct description of the EMIC wave generation/damping and propagation. In order to model the EMIC wave propagation and interaction with RC, we also need to know the density distribution in the meridional plane. In the present work, we use a magnetic field model for the meridional density distribution, i.e., $n_e \sim B$, because a more sophisticated analytical model by Angerami and Thomas (1964) used in our previous studies (e.g., Khazanov et al. 2006) was found to give nearly the same results. The meridional model is then adjusted to the equatorial density model. Thus, the resulting plasmaspheric model provides a 3D spatial distribution of the electron density. Besides electrons, the cold magnetospheric plasma is assumed to consist of 77% H^+ , 20% He^+ , and 3% O^+ , which are in the range of 10–30% for He^+ and 1–5% for O^+ following the observations by Young et al. (1977) and Horwitz et al. (1981). Geocoronal neutral hydrogen number densities, needed to calculate loss due to charge exchange, are obtained from the spherically symmetric model of Chamberlain (1963), with its parameters given by Rairden et al. (1986).

During the main phase of major storms, RC O^+ may dominate (e.g., Hamilton et al. 1988; Daglis 1997) and, as a result, contribute to strong damping of the He^+ -mode EMIC waves (Thorne and Horne 1997). Although there is no doubt that, in principle, this process is important, let us evaluate the validity of excluding the

He^+ -mode damping by RC O^+ in the 2–4 May 1998 storm simulation. Using the RC kinetic model of Jordanova et al. (1998), Farrugia et al. (2003) found that, during the main phase of the 4 May 1998 storm, the energy density of RC H^+ is greater than twice that of O^+ at all MLTs, and the contribution of He^+ to the RC energy content is negligible. This implies that the RC O^+ content does not exceed 30% during the main phase of this storm. This estimate was obtained from a global simulation, which did not include oxygen band waves. On the other hand, Bräysy et al. (1998) observed a very asymmetric O^+ RC during the main phase of the 2–8 April 1993 storm, which may suggest that a majority of the RC oxygen ions get lost before they reach the dusk MLT sector. This result is difficult to explain in terms of charge exchange and Coulomb scattering, and suggests that the production of EMIC waves contributes significantly to RC O^+ decay during the main and early recovery phases. In other words, due to the generation of the O^+ -mode EMIC waves, most RC O^+ might precipitate before reaching the dusk MLT sector (Bräysy et al. 1998). Therefore, to estimate the RC O^+ content correctly, the O^+ -mode should be included in the simulation, and it is likely that Farrugia et al. (2003) overestimated the RC O^+ content during 4 May 1998. Moreover, the calculations of Thorne and Horne (1997) clearly demonstrated that even the RC O^+ percentage noted above cannot significantly suppress the He^+ -mode amplification, and only slightly influences the resulting growth; inclusion of 26% O^+ in the RC population causes the net wave gain to decrease by only 20%. In addition, the most important results shown in the present section are obtained from simulations of the 2–3 May 1998 period, i.e., the first main ($D_{\text{st}} = 106$ nT) and recovery phases of the May 1998 large storm, when the RC O^+ content should be even smaller than the Farrugia et al. estimate for 4 May 1998. It is for these reasons that we chose to exclude RC O^+ in the present simulations, and to assume that the RC is entirely comprised of energetic protons.

9.4.4.2 Ionospheric Coupling Loop

Equation (9.4.20) must be solved taking into account the contributions from both the northern and southern ionosphere. In the present study, because we assume the magnetic field lines to be equipotentials, the northern and southern ionospheres can just be replaced by an effective single ionosphere with $\Sigma = \Sigma_S + \Sigma_N$, and total FAC $J_{\parallel,i}$ flowing into/out of it. After the resulting equation is solved, and Φ_i is found, we can easily calculate the FACs $j_{\parallel,i}(s_S)$ and $j_{\parallel,i}(s_N)$ flowing into/out the southern and northern ionosphere.

The ionospheric Hall and Pedersen conductances in our model are not calculated self-consistently but rather specified by empirical models. The resulting conductance arises from four sources (1) the solar extreme ultraviolet (EUV), (2) scattered solar EUV on both sides of the terminator, (3) starlight, and (4) auroral particle precipitation. The direct solar conductance is controlled by the solar zenith angle and the solar UV and EUV radiations, which correlate with the solar radio flux index $F_{10.7}$. In the present study, we use the empirical model of Moen and Brekke

(1993) for determining direct solar conductance. The scattered solar EUV and starlight conductance models are taken from the study of Rasmussen and Schunk (1987). In order to specify the conductance from auroral precipitation, we use either the Hardy et al. (1987) statistical model or an empirical relationship between the FACs and the local Hall and Pedersen conductance established by Ridley et al. (2001, 2004). The Hardy et al. model is compiled from the electron precipitation patterns obtained by the DMSP satellites and gives the Hall and Pedersen conductance as a function of MLT and magnetic latitude for seven levels of activity as measured by Kp. The Ridley et al. relationship was derived using the assimilative mapping of ionospheric electrodynamics (AMIE) technique (Richmond and Kamide 1988). The AMIE technique was run at a 1-min cadence for the entire month of January 1997, using 154 magnetometers. This resulted in almost 45,000 2D maps of the Hall and Pedersen conductances and FAC. The conductance was derived from the Ahn et al. (1998) formulation, which relates ground-based magnetic perturbations to the Hall and Pedersen conductances. The Ridley et al. analysis showed an exponential relationship between the local FAC and the conductance (see Amm 1996; Goodman 1995):

$$\Sigma = \Sigma_0 e^{-A|j_{||,i}|}, \quad (9.4.21)$$

where the constants Σ_0 and A are independent of the magnitude of $j_{||,i}$, but depend on location and whether the current is upward or downward. Although the Ridley et al. relationship is entirely empirical and not based on first principles, by using it, we introduce into the model a degree of self-consistency between the ionospheric conductance and FAC. This is a principle modification because a self-consistent description of the ionospheric conductance makes (9.4.20) nonlinear compared with the case of statistical conductance model. For a previous use of the Ridley et al. relationship in the RC simulation, see Liemohn et al. (2005).

To conclude this section, we note that the numerical implementations used to solve (9.4.16), (9.4.17), and (9.2.1)–(9.2.4) are described in detail in (Khazanov et al. 2003a, 2006), and, to solve (9.4.20), a preconditioned gradient reduction resolution (GMRES) solver is used (Ridley et al. 2004). The GMRES method is robust enough to handle a wide variety of FAC and conductance patterns.

9.4.4.3 Initial and Boundary Conditions

The initial RC distribution is constructed from the statistically derived quiet-time RC proton energy distribution of Sheldon and Hamilton (1993) and the pitch-angle characteristics of Garcia and Spjeldvik (1985). The nightside boundary condition for (9.4.16) is imposed at the geostationary distance, and it is obtained using flux measurements from the Magnetospheric Plasma Analyzer (Bame et al. 1993) and the Synchronous Orbit Particle Analyzer (Belian et al. 1992) instruments on the geosynchronous LANL satellites during the modeled event. Then, according to Young et al. (1982) and Liemohn et al. (1999), we divide the total flux measured at geostationary orbit between the RC H^+ , O^+ , and He^+ depending on geomagnetic

and solar activity as measured by Kp and $F_{10.7}$ indices. Only the H^+ flux is used as a boundary condition in the simulation.

In the present section, the poleward boundary for (9.4.20) is taken at magnetic latitude $\lambda = 69^\circ$. On this boundary, we specify the electric potential using either the Weimer (1996) statistical model (hereinafter the W96 model), which is driven by the interplanetary magnetic field (IMF) B_Y , B_Z components and solar wind velocity, or the convection model of Volland and Stern (Volland 1973; Stern 1975) with Kp parameterization given by Maynard and Chen (1975) and shielding factor of 2 (hereinafter the VS model). The second boundary condition is specified at $\lambda = 30^\circ$, and we use either the W96 model or the VS model, both of which give the potential close to zero at that latitude. It should be noted that the result of calculation is insensitive to the choice of the lower boundary condition, as demonstrated by Wolf (1970). Thus, the magnetospheric electric field is calculated self-consistently in the domain $30^\circ < \lambda < 69^\circ$. At the same time, we should emphasize that, compared with RC, the cold electron density is modeled in a more extended domain of $L \leq 10$, and, in order to specify the electric field in the entire $L \leq 10$ region, we use either the W96 or the VS model for the magnetic latitudes above $\lambda = 69^\circ$.

The initial RC, plasmasphere, and EMIC wave distributions are derived independently and, moreover, they have nothing to do with a particular state of the magnetosphere/plasmasphere system during a simulated event. Only the boundary conditions provided by the LANL satellites can be considered as data reflecting a particular geomagnetic situation (and, to a certain extent, the used ionospheric conductance model and an imposed cross-polar cap potential drop). Therefore, before the simulation of a particular geomagnetic event can occur, we first must find an appropriate initial state for the RC, electric field, plasmasphere, and EMIC waves that is self-consistent and reflects the particular geomagnetic situation. To obtain the self-consistent initial distributions for the entire system, we first prepared the plasmasphere by running the Ober model for 20 quiet days. Then, at 0000 UT on 1 May 1998, a simulation of (9.4.16), (9.4.17), (9.2.1)–(9.2.4), and (9.4.18)–(9.4.21) was started using all of the controlling parameters and the initial/boundary conditions along with a background noise level for the He^+ -mode EMIC waves (e.g., Akhiezer et al. 1975). We ran the model code for 24 h to achieve a quasi-self-consistent state for the system. Note that 24 h has nothing to do with the typical time for wave amplification and instead reflects the minimum time needed to adjust the RC and waves to each other and to the real prehistory of a storm. The self-consistent modeling of the May 1998 storm period was started at 0000 UT on 2 May (24 h after 1 May 0000 UT) using solutions of (9.4.16), (9.4.17), and the cold plasma distribution at 2400 UT on 1 May as the initial conditions for further simulation.

9.4.4.4 Interplanetary and Geomagnetic Drivers

The ionospheric boundary condition in our simulations is driven either by IMF B_Y , B_Z components and solar wind velocity (the W96 model) or the 3-h Kp index

(the VS model). The Hardy et al. (1987) ionospheric conductance model is driven by K_p . All of these driving parameters are shown in Fig. 9.14 during the 2–4 May 1998 period. Interplanetary data are obtained from the Magnetic Field Investigation (Lepping et al. 1995) and the Solar Wind Experiment (Ogilvie et al. 1995) instruments aboard the WIND satellite. The interplanetary configuration of 1–5 May 1998 consists of a coronal mass ejection (CME) interacting with a trailing faster stream (Farrugia et al. 2003). The CME drives an interplanetary shock observed by the instruments aboard the WIND spacecraft at approximately 2220 UT on 1 May. Three episodes of the large negative IMF B_Z component were monitored. The first episode started at 0330 UT on 2 May (27.5 h after 1 May, 0000 UT), the second at 0230 UT on 4 May (74.5 h after 1 May, 0000 UT), and the third (not shown) at 0200 UT on 5 May (98 h after 1 May, 0000 UT). These caused a “triple-dip” storm with the minimums $D_{st} = -106$ nT, $D_{st} = -272$ nT, and $D_{st} = -153$ nT (not shown). The planetary K_p index reached maximum values of $K_p \approx 7^-$ and $K_p \approx 9^-$ at the times when D_{st} minimums were recorded.

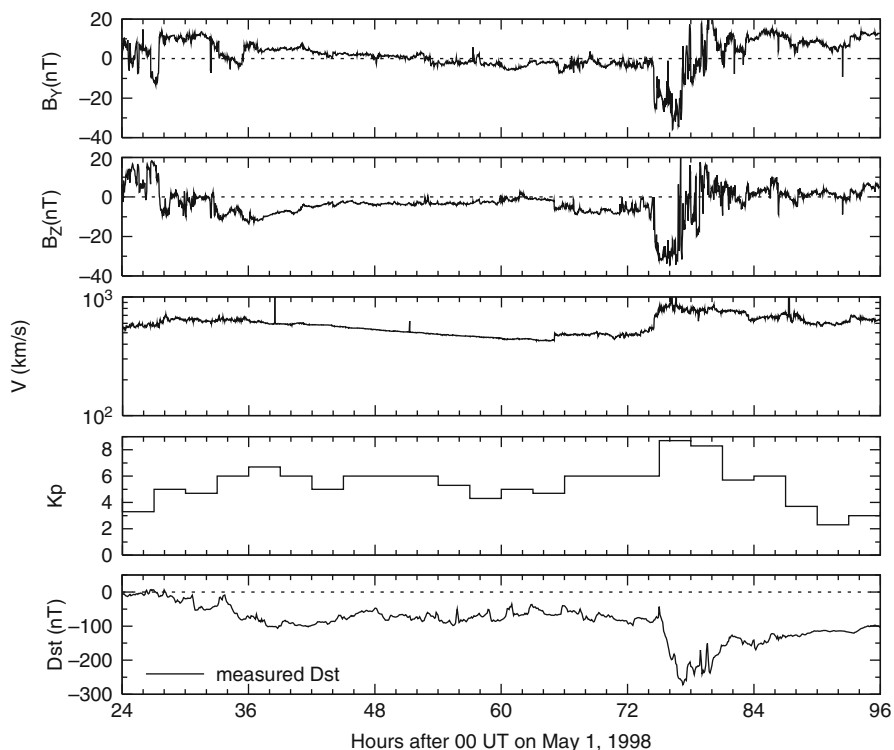


Fig. 9.14 The interplanetary and geomagnetic characteristics during 2–4 May 1998. The interplanetary magnetic field GSM B_Y and B_Z components, the solar wind velocity, 3-h K_p index, and the measured D_{st} index. The hours shown are counted from 0000 UT on 1 May 1998

9.5 The EMIC Wave Normal Angle Distribution

In order to demonstrate the effects of EMIC wave propagation and refraction on the wave energy distributions and evolution, we select two time intervals during the storm. The first interval takes place on 2 May, from 2400 UT to 4800 UT after 1 May, 0000 UT, and covers the period of the first D_{st} dip. The second one from 7200 UT to 8600 UT after 1 May, 0000 UT represents the period of largest D_{st} decrease on 4 May. Below, we present some of our new results and compare them with results based on a different EMIC wave treatment that was adopted in earlier studies, and discuss experimental investigations of these waves.

9.5.1 Prediction from Theory

In comparison with our earlier published results (Khazanov et al. 2003a), in the present section, we consider the case of the He^+ -mode in multi-ion thermal plasma. The equatorial (MLT, L shell) distributions of the squared wave magnetic field,

$$B_w^2(r_0, \varphi, t) = \int_{\omega_{\min}}^{\omega_{\max}} d\omega \int_0^{\pi} d\theta_0 B^2(r_0, \varphi, t, \omega, \theta_0)$$

are shown in Fig. 9.15 for the He^+ -mode of the EMIC waves. These results are obtained by using our old model (Khazanov et al. 2003a), in which the wave propagation along geomagnetic field lines has been assumed and wave refraction has been ignored. The thermal plasma consists of electrons, 77% of H^+ , 20% of He^+ , and 3% of O^+ . White lines in the figure show the contours of equatorial plasma-spheric electron density, and all of the specified hours are counted from 0000 UT on

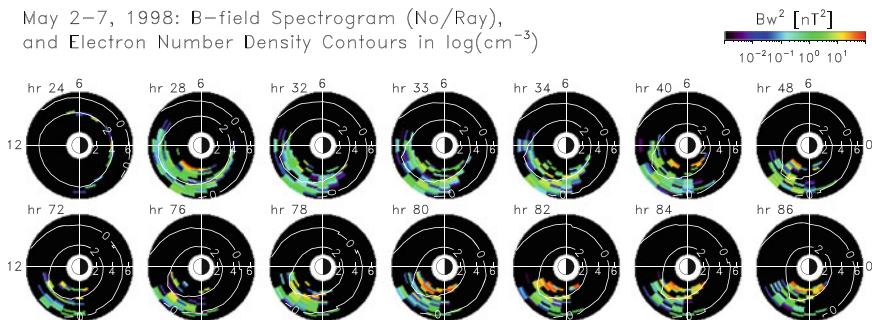


Fig. 9.15 Snapshots of the equatorial (MLT, L shell) distributions of squared wave magnetic field for the He^+ -mode are shown. The results are obtained by using our previous model (Khazanov et al. 2003a) in which wave propagation along geomagnetic field lines has been assumed, and wave refraction has been ignored

1 May. It follows from Fig. 9.15 that at 24 h after 1 May, 0000 UT, the wave activity is mainly concentrated in the night MLT sector, and the wave active zones are very well organized by the plasmopause location. Later, on 2 and 4 May, we observe wave activity predominantly in the postnoon–premidnight MLT sector, with a very wide radial extension. It should be noted, however, that results in Fig. 9.15 can only be treated as rough distributions of wave power, and we provide them only for reference and comparison purposes.

The new results obtained from simulation based on the developed system of governing (9.4.16) and (9.4.17) are presented in Fig. 9.16. There is a very impressive qualitative difference between the EMIC wave energy distributions in Fig. 9.16 and in Fig. 9.15. In comparison with the results in Fig. 9.15, the B field distributions in Fig. 9.16 are highly organized by location of the plasmopause gradient, and a maximum of $B^2 \approx 50 \text{ nT}^2$ is observed in the dusk MLT sector shortly after absolute D_{st} minimum; MLT = 17, $L = 3.5$, at 78 h after 1 May, 0000 UT. Analyzing the results of ray-tracing simulations, Horne and Thorne (1993) have shown that, at the plasmopause, the density gradient counteracts refraction caused by the magnetic field gradient and curvature. As a result, net refraction is suppressed, and the He^+ -mode grows preferably at the plasmopause. Comparing the wave magnetic field distributions in Fig. 9.16 with the density distributions, we can clearly observe this effect in the obtained wave (MLT, L shell) distributions (Khazanov et al. 2006).

In conclusion of this section, let us compare the presented global wave distributions with the results of another global model where a different EMIC wave description has been adopted. Jordanova et al. (1997, 2001) have incorporated in their model the results of simulations of wave propagation and amplification based on the HOTRAY ray-tracing code (Horne 1989). They have found that the plasmopause density gradient has a major impact on the path-integrated wave gain for the He^+ -mode. Further, in order to construct a global model of convective wave gain, these authors (1) approximate phase space distribution functions from their model with bi-Maxwellian distributions, and calculate field-aligned convective growth rates according to Kozyra et al. (1984), (2) integrate these growth rates

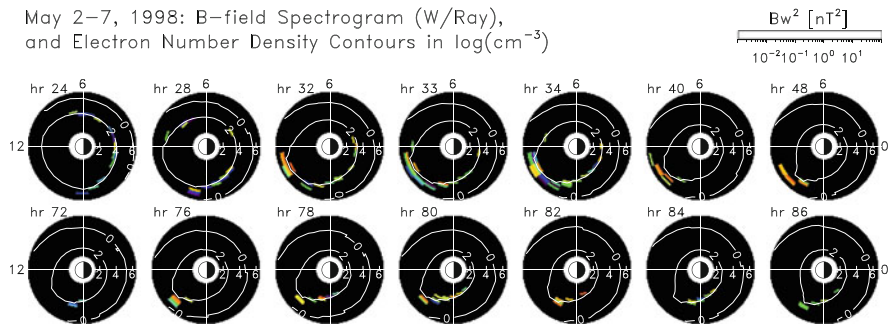


Fig. 9.16 Same as Fig. 9.15, except B field distributions are obtained by solving (9.4.17), i.e., wave propagation and refraction are explicitly taken into account in the wave kinetic equation

along geomagnetic field lines over ± 5 latitude range, and (3) multiply the results of integration by a factor of 2 at the plasmapause, and by a factor of 1.5 at 0.25 RE distance on both sides of the plasmapause in order to obtain agreement with HOTRAY amplification. After that, a Gaussian shape for the EMIC wave power spectral density has been assumed in both wave frequency and wave normal angle, then, based on a statistical study, they have related the calculated wave gains to the EMIC wave amplitudes, and have taken into account wave-induced RC scattering inside of a ± 10 magnetic latitude range. Thus, this model (for more details see Jordanova et al. 1997, 2001 and references therein, who basically had adapted the same approach) takes into account some features of the wave propagation and refraction and therefore should be compared with results from our developed model.

Using the same RC initial and boundary conditions as we set in our model, we started the model by Jordanova et al. (2001) at 0000 UT on 1 May 1998. The selected equatorial snapshots of the squared wave magnetic field distributions are presented in Fig. 9.17 for the He^+ -mode of EMIC waves. We indeed find that EMIC wave activity is preferably localized in the postnoon–premidnight MLT sector, and it is partly controlled by the plasmapause location. Nevertheless, in comparison with the results in Fig. 9.16, this wave activity occupies equatorial zones that are more extended, and we do not observe in Fig. 9.17 that the plasmapause density gradient has a major impact on the wave excitation during the studied storm. Overall, this version of global RC model by Jordanova et al. (1997, 2001) produces more extended and less intense equatorial wave distributions during the May 1998 storm period than found by our more complete treatment of RC–EMIC wave interaction.

It is well-known that quasilinear wave–particle interaction strongly depends on the wave/particle characteristics, particularly on the wave power spectral density, which itself is determined by a self-consistent evolution of the wave–particle system. In most studies, to describe a quasilinear interaction of EMIC waves and RC ions, a Gaussian approximation to the shape of power spectral density is assumed (see, e.g., Lyons 1974; Jordanova et al. 2001). On the other hand, as

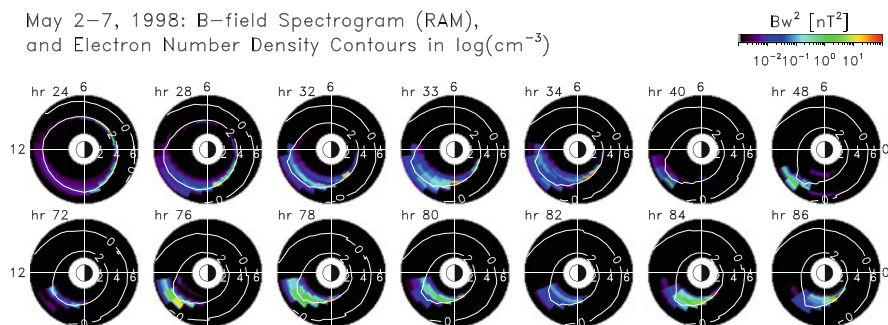


Fig. 9.17 Same as Fig. 9.15, except B field distributions are obtained from a simulation using the model as described by Jordanova et al. (2001)

follows from the theoretical study by Khazanov et al. (2003a), the Gaussian distribution is not always a good fit to the form of wave power spectral density. In this paper Khazanov et al. (2003a) have assumed that the wave distributions are quasi-field-aligned, and distributions over wave normal angles are entirely concentrated in the regions of $\theta < 27^\circ$ for forward and of $153^\circ < \theta < 180^\circ$ for backward propagation, respectively. Thus, they have integrated the power spectral density over wave normal angle and have only analyzed the shapes of the resulting frequency spectra. Now we have quite a different situation because wave propagation and refraction are rigorously taken into account. We next discuss the consequence this causes in the power spectral density distributions over wave normal angle.

As follows from the results of Sect. 9.2, the He^+ -mode of EMIC waves bounces between the surfaces of the $\text{O}^+ - \text{He}^+$ bi-ion hybrid frequency in opposite hemispheres, and the wave normal vector becomes more oblique when the wave packets approach these surfaces ($\theta_0 = 90^\circ$ at the reflection point). Thus, for the specified wave frequency, there is a quasistationary distribution of wave normal angles (and wave normal numbers) between the equatorial plane and reflective surfaces. In comparison with (2) from Khazanov et al. (2003a), the left-hand side in (9.4.17) is different, and this fact may lead to strong modification of the wave power spectral density distribution. If we put $\theta_0 = 0$ in (9.4.17), then the wave normal angle distribution will be truly stationary (of course, in the case of a stationary medium), and the equatorial EMIC wave power spectral density will be concentrated in the region of “relatively small” equatorial wave normal angles (we mean that the region about 90° will be empty). In reality, $\dot{\theta}_0 \neq 0$, and $\dot{\theta}_0$ is positive. This leads to energy outflow from the region of small wave normal angles to $\theta_0 = 90^\circ$ (see the second term in the left-hand side of (9.4.17)). The EMIC wave growth rate maximizes for $\theta_0 = 0^\circ$ and electron Landau damping has a peak for θ_0 close to 90° . Note that Landau damping disappears at $\theta_0 = 90^\circ$. Then, the resulting power density distribution over wave normal angle depends on the ratios between rates of wave growth (mostly in the region of small θ_0), Landau damping (mostly at large θ_0), and $\dot{\theta}_0/\theta_0$.

In Fig. 9.18 we present energy distributions over equatorial wave normal angle for the He^+ -mode of EMIC waves. All of the squared magnetic field spectra are

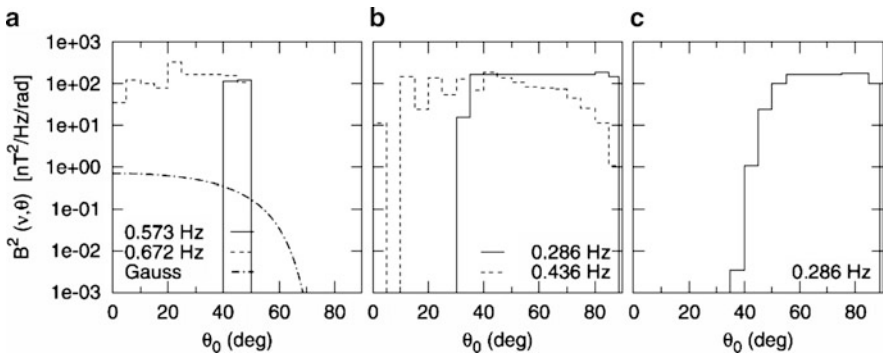


Fig. 9.18 Equatorial spectral distributions for He^+ -mode of EMIC waves are shown

shown in the postnoon–dusk MLT sector at 48 h after 0000 UT on 1 May 1998. We also present in Fig. 9.18a the unnormalized Gaussian distribution from Jordanova et al. (2001), which has a maximum at $\theta_0 = 0^\circ$, and it is only valid in the region of $0^\circ < \theta_0 < 45^\circ$ (dash-dotted line). Figure 9.18a demonstrates a typical quasi-field-aligned wave normal angle distribution for $\nu = 0.672$ Hz. In this case, the wave growth rate in the region of small θ_0 is larger than the outflow rate to the region of greater θ_0 . Diametrically opposed cases are presented in Fig. 9.18b for $\nu = 0.286$ Hz and in Fig. 9.18c, where EMIC wave energy is concentrated in the region of large θ_0 . The dashed line ($\nu = 0.436$ Hz) in Fig. 9.18b represents an intermediate case. Although power spectral density in the latter case drops dramatically for $\theta_0 > 40^\circ$, there is still a very large $B^2(\nu, \theta)$, and we observe spread distribution in the entire wave normal angle region. Almost no presented power spectral density distributions appear as Gaussian functions, and most important is that EMIC wave energy can occupy not only the region of generation, i.e., the region of small θ_0 , but the entire wave normal angle region and even the region near $\theta_0 = \pi/2$. The latter, for example, is extremely crucial for energy transfer to thermal plasmaspheric electrons by resonant Landau damping, and subsequent downward heat transport and excitation of stable auroral red arcs (Cornwall et al. 1971). We consider this effect in the next sections.

To conclude this section, it should be stressed that, from a simulation of the May 1998 storm, Khazanov et al. (2006) found that the equatorial He^+ -mode energy distributions are not Gaussian over the equatorial wave normal angle, θ_0 , and that the wave energy can occupy not only the source region, i.e., the region of small wave normal angles, but all wave normal angles, including those near 90° . This is caused by energy outflow from the region of small wave normal angles to $\theta_0 = \pi/2$, which is due to the wave bouncing between surfaces of the bi-ion hybrid frequency in opposite hemispheres. Figure 9.18 shows spectra at one time and at three spatial points only, but it would be interesting to see the wave normal angle distributions on global spatial and temporal scales. In order to provide such a global view during the May 1998 storm, we calculate the average equatorial wave normal angle,

$$\langle \theta_0(r_0, \varphi, t) \rangle = \frac{\int_{\omega_{\min}}^{\omega_{\max}} d\omega \int_0^{\pi} d\theta_0 B^2(r_0, \varphi, t, \omega, \theta_0) \theta_0}{\int_{\omega_{\min}}^{\omega_{\max}} d\omega \int_0^{\pi} d\theta_0 B^2(r_0, \varphi, t, \omega, \theta_0)},$$

using the results from Khazanov et al. (2006, Fig. 6), where r_0 , φ , t , ω , θ_0 , and B are the radial distance in the magnetic equatorial plane, MLT, time, wave frequency, equatorial wave normal angle, and the wave magnetic field. Results are presented in Fig. 9.19. To give a measure of the wave spectrum widths, Khazanov and Gamayunov (2007a), in their Fig. 9.5, also calculated the wave normal angle standard deviation.

As follows from Fig. 9.19, the highly oblique waves with $\langle \theta_0 \rangle > 50^\circ$ are mainly observed in the noon–dusk MLT sector for high L-shells (in the plasmaspheric

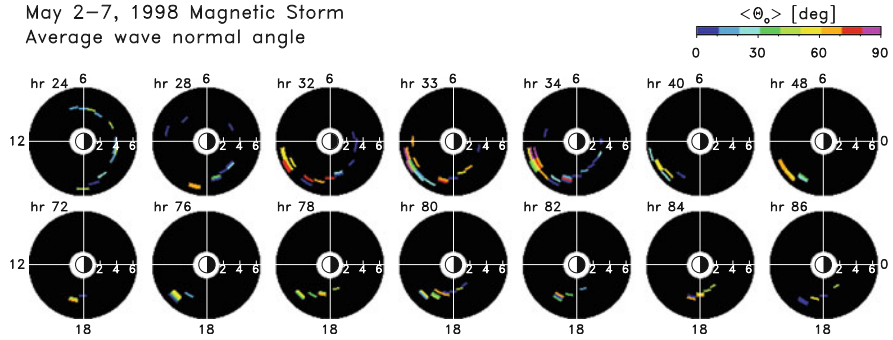


Fig. 9.19 Average equatorial wave normal angle for the He^+ -mode EMIC waves during the May 1998 event. The only L-MLT locations shown are where $B^2 \geq 10^{-2} \text{ nT}^2$. The specified hours are counted from 0000 UT on 1 May 1998

drainage plume), and an extremely oblique wave propagation with $\langle \theta_0 \rangle > 80^\circ$ is found in hour 33 ($L = 6.25$, $\text{MLT} = 14$) and 34 ($L = 5.75$, $\text{MLT} = 13$) snapshots. Although events with $\langle \theta_0 \rangle < 50^\circ$ are not well-separated spatially from oblique waves, there is a tendency for them to be localized preferentially along the more narrow nightside plasmapause (compare Fig. 9.19 with the density distribution in Fig. 7 of Khazanov et al. 2006), especially for field-aligned events with $\langle \theta_0 \rangle < 30^\circ$ (see the first row in Fig. 9.19). The occurrences of the oblique and field-aligned wave normal angle distributions appear to be nearly equal during the May 1998 storm. It is interesting to note that among 830 EMIC events observed on the Combined Release and Radiation Effects Satellite and analyzed by Meredith et al. (2003), 414 events had ellipticity $|\varepsilon| < 0.3$, i.e., were likely oblique.

The theoretical results presented here clearly demonstrate that storm-time EMIC wave normal angle distributions are highly variable both in space and time, and that equatorial distributions range from field-aligned distributions through highly oblique distributions, which are in qualitative agreement with the results of Anderson et al. (1996), Denton et al. (1996), and Meredith et al. (2003) that are discussed in the following section.

9.5.2 Comparison with Observations

In order to estimate the wave normal angle, the minimum variance direction is found from the wave observations. For a plane EMIC wave, the magnetic fluctuation, $\delta \mathbf{B}$, and wave vector $\mathbf{k}$ are related by $\mathbf{k} \cdot \delta \mathbf{B} = 0$. Thus, the fluctuation $\delta \mathbf{B}$ is entirely in the plane perpendicular to $\mathbf{k}$, and the minimum variance direction $\mathbf{e}_{\min}$ is parallel to $\mathbf{k}$. In this case the angle between $\mathbf{e}_{\min}$ and the external magnetic field ($\mathbf{B}_0$), $\theta_{\min}$, gives the angle between $\mathbf{k}$ and $\mathbf{B}_0$, θ . Fraser (1985) and Ishida et al. (1987) found that $\theta_{\min}$ was generally less than 30° , and, for most waves, $\theta_{\min} < 15^\circ$. Then, assuming that the observed waves could be represented by a single plane mode, they related the derived angle to the wave normal angle as $\theta = \theta_{\min}$.

Another important spectral wave characteristic is ellipticity, ε , which is closely related to θ . For a plane EMIC wave, ε determines θ , and vice versa, if the plasma properties and wave frequency are specified. The ellipticity is defined as the ratio of the minor to the major axis of the wave polarization ellipse in the plane perpendicular to $\mathbf{B}_0$ with $\varepsilon = -1$ for left circular, $\varepsilon = 0$ for linear, and $\varepsilon = +1$ for right circular polarization. The EMIC waves observed near the equator are mainly linear or left-hand polarized with some admixture of the right-hand polarization (Anderson et al. 1992b; Fraser and Nguyen 2001; Meredith et al. 2003; Ishida et al. 1987). There is a clear tendency for the polarization to become more linear with increasing magnetic latitude. The observation of a significant number of linear polarized events occurring near the equator cannot be explained by the polarization reversal from left-handed through linear to right-handed at the crossover frequency, as suggested by Young et al. (1981), and is intriguing because of small $\theta_{\min}$ (Meredith et al. 2003); waves should be highly oblique for $\varepsilon \approx 0$, which is inconsistent with the reported θ (actually $\theta_{\min}$) and ε .

Let us now outline the two outstanding data analysis problems which, in our opinion, are closely related to the above inconsistency and should be resolved first in order to extract the correct wave polarization properties from the observations. Fast Fourier transform (FFT) analysis has become the conventional method for quantitative determination of the wave polarization and minimum variance direction (Means 1972; Arthur et al. 1976). Fourier analysis implicitly assumes that the analyzed signal is a superposition of components with different frequencies and that during the analyzed time segment each component has no random phase variations (stationary signal in both time and orientation for vector signal) (Anderson et al. 1996). For example, FFT analysis applied to a series of wave packets with the same frequency but with arbitrary relative phases will produce a “broad” range of frequencies. Anderson et al. (1996) showed that when the magnetic fluctuations are not stationary in time and, specifically, when the axes of the wave polarization ellipse fluctuate in azimuth, then the FFT analysis of the minimum variance direction and polarization are unreliable. The reason is that the time window for analysis contains numerous randomly fluctuating wave packets. The time window, in turn, is determined by the desired frequency resolution, which is the reciprocal of the window length. To achieve acceptable frequency resolution, time segments of several minutes or even much longer are typically used (Fraser 1985; Ishida et al. 1987).

Analyzing 46 EMIC events, each 30–60 min long, from 44 different days, Anderson et al. (1996) found that polarization parameters vary over a time period of a few wave periods. This allows them to conclude that significant polarization axis fluctuations are a common feature of EMIC waves and hence that nonstationarity effects are a general property of waves in magnetosphere. The stationarity timescales are too short for standard FFT analysis, and, to address this problem, Anderson et al. (1996) developed a minimum variance technique that operates on timescales of a few wave periods. They called this technique a “wave step,” and showed how to determine which method, FFT and/or wave step, is best for a given data set. Note that despite using very short time windows, the wave step procedure

achieves good frequency precision (Anderson et al. 1996). Compared with the wave packet technique, the decomposition of a nonstationary signal using the traditional FFT analysis can yield a dramatic underestimate of the minimum variance polar angle (often more than 45°) and an overestimate of $|\epsilon|$. The maximum disagreement occurs for linear polarization. This is a significant problem because the minimum variance direction determines the EMIC wave normal vector orientation, which is crucial for resolving major outstanding questions of the EMIC wave generation, propagation, and damping. Using the more reliable wave step polarization results, Anderson et al. (1996) presented the first analysis of nearly linear polarized waves for which the polarization properties have been determined. They found a significant number of wave intervals with $\theta_{\min} > 70^\circ$, the highest $\theta_{\min}$ ever reported.

However, it should be noted that $\theta_{\min} \neq 0$ if $\delta\mathbf{B}$ is due to a superposition of plane waves with different azimuthal angles (Anderson et al. 1996; Hoppe et al. 1982). A quantitative analysis of the effects of superposition on the observed wave polarization properties has been presented by Denton et al. (1996). Using data from the AMPTE/CCE spacecraft, Denton et al. (1996) made a detailed comparison between the observed polarization properties of EMIC waves and those predicted by theory, where the theoretical linear wave properties were based on the plasma parameters observed during EMIC events, calculated using the linear dispersion code XWHAMP (Schwarz and Denton 1991). Denton et al. (1996) analyzed the ellipticity, the ratio of parallel (along $\mathbf{B}_0$) magnetic fluctuation δB_z to the major axis component of the elliptical perturbation in the perpendicular plane δB_{major} , and the phase angle $\phi_{z\text{--}major}$ between δB_z and δB_{major} . They found that the observed polarization properties are inconsistent with the assumption that the resultant observed waves are from a single plane wave. Namely, (1) the observed ellipticity (ϵ^{obs}) data plotted versus $\theta_{\min}$ are at great variance to the theoretical curves, (2) although $\delta B_z / \delta B_{\text{major}} = \epsilon^{\text{obs}} \tan \theta_{\min}$ if the single wave assumption is valid, the observed distribution of $\delta B_z / \delta B_{\text{major}}$ appears to be relatively independent of $\epsilon^{\text{obs}} \tan \theta_{\min}$, and (3) the distribution of $\phi_{z\text{--}major}$, although peaked around 90° (which is consistent with the single wave assumption for guided mode), is often quite broad. In order to explain the discrepancies, Denton et al. (1996) developed a simple model with two constituent waves in various azimuthal orientations and temporal phase relations. They showed that the distribution of observed polarization properties can be well accounted for as resulting from a superposition of more than one plane wave, and furthermore, the required constituent waves have properties consistent with linear dispersion theory. When there is a superposition of waves, the instantaneously observed polarization characteristics do not reliably reflect the constituent wave properties and the minimum variance direction cannot be associated with wave vector. Denton et al. (1996) therefore concluded that wave polarization analysis, which assumes that the observed fluctuations are due to the single plane wave, is not valid. Particularly, they noted that determination of wave vector orientation by means of minimum variance analysis is especially susceptible to error, because even the median value of $\theta_{\min}$ gives an unreliable estimate to θ .

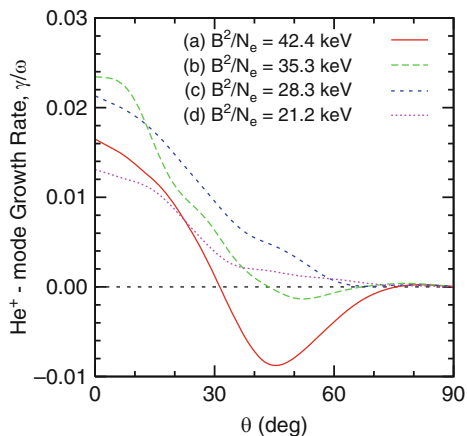
The effects of wave superposition on the observed polarization characteristics are generally as large or larger than the variations between parameters associated with linear wave dispersion. Nevertheless, although individual resultant wave properties can be quite different from those of the constituent waves, the entire distribution from an ensemble of resultant waves can have some properties in common with the constituent waves. Assuming that both constituent waves have the same ellipticity, ε^C , and wave normal angle, θ^C , Denton et al. (1996) showed that the median value of the resultant ellipticity $\bar{\varepsilon}^R$ is equal to the constituent ellipticity ε^C . Similarly, they showed that the median value $\delta B_z^R / \delta B_{major}^R$ is close to $\bar{\varepsilon}^R \tan \theta^C$. In this way they inferred the essential polarization properties of constituent waves from the observations. For example, for the 1985-018 EMIC wave event, they found $\varepsilon^C = 0.07$ and $\theta^C = 77^\circ$, which is consistent with theoretical wave linear properties based on the plasma parameters observed during the event.

In general, the observed EMIC waves have more than two constituent waves. Thus, even if the correct FFT and/or wave step method is used, there still exists an uncertainty that has to be resolved in order to extract the correct polarization properties from observations. (We should emphasize that the simple model of Denton et al. 1996 has been remarkably successful at qualitatively explaining the distribution of the observed polarization parameters.) Thus, combinations of reliable data and theoretical models should be used in order to obtain the power spectral density of EMIC waves over the entire outer RB throughout the different storm phases.

9.6 The Effect of Ring Current H^+ Density

The effective proton temperatures transverse to ($T_\perp$) and along ($T_\parallel$) the field line comply with an inequality $T_\perp > T_\parallel$ in many space plasma regimes. If the temperature anisotropy ($A = T_\perp / T_\parallel - 1$) of RC ions exceeds some positive threshold, EMIC waves can be unstable (Kennel and Petschek 1966; Cornwall et al. 1970). The instability growth rate strongly depends on the so-called proton characteristic energy for cyclotron interaction, which is the energy of the external magnetic field per one particle, i.e., $E_c = B^2 / (8\pi N_e)$ (Kennel and Petschek 1966). Thus, the local growth rate of EMIC waves should be particularly sensitive to the local plasma density. Assuming that RC is entirely made up of energetic H^+ , Fig. 9.20 shows that the equatorial growth/damping rate for the He^+ -mode EMIC waves depends on plasma density. Note that the growth/damping rates in Fig. 9.20 are due to the RC-EMIC wave interaction only, and the wave absorption by thermal plasma is omitted (of course, this is included in our global simulations). All of the results in Fig. 9.20 are obtained for the wave frequency $\nu = 0.475$ Hz and case (a) is just taken from our modeling at $L = 5.25$, $MLT = 15$, and hour 48 ($N_e = N_0 = 68.3 \text{ cm}^{-3}$; $B = 215.3 \text{ nT}$) without any modification. To produce the results (b),

Fig. 9.20 The equatorial growth/damping rate versus wave normal angle for the He^+ -mode EMIC waves



(c), and (d), we only renormalized the local plasma density as $N_e = 1.2N_0$, $N_e = 1.5N_0$, and $N_e = 2N_0$, respectively. As follows from Fig. 9.20, transition from case (a) to case (b) increases the peak growth rate by a factor of 1.4, extends the region of growth, and makes the wave damping negligible. The further plasma density increase eliminates the region of wave damping. To understand the dependencies shown in Fig. 9.20, let us recall that the wave growth rate depends on plasma density as $\gamma \sim \exp(-1/N_e)/\sqrt{N_e}$, i.e., it has a maximum for some particular N_e if the other plasma and all the wave parameters are fixed (e.g., Kennel and Petschek 1966). Therefore, although the characteristic energy decreases with plasma density, the growth rate can either increase or decrease, depending on the wave normal angle (the rest of the parameters are fixed in Fig. 9.20). For a particular wave normal angle, it depends on whether we move to the growth rate maximum with a density increase or whether we move from the maximum. A strong sensitivity of the wave growth rate to plasma density was a key point for the study by Gamayunov and Khazanov (2008). In this paper they included RC H^+ in the real part of the EMIC wave dispersion relation and studied its effect on wave distribution during the storm.

It is well known that plasma density is one of the most important characteristics controlling EMIC wave generation, and RC ions are usually included in the total density model in order to study the wave dispersive properties (e.g., Kozyra et al. 1984; Gamayunov et al. 1993). Despite that, an assumption that the total plasma density is dominated by thermal plasma was made in previous efforts to model EMIC waves on a global magnetospheric scale (e.g., Kozyra et al. 1997a; Jordanova et al. 2001; Khazanov et al. 2006). In other words, RC ions were only included in the imaginary part of the wave dispersion relation (in growth rate) but omitted in the real part of this relation. This assumption is especially severe outside the plasma-pause, where the RC density is comparable to or even dominates the thermal plasma density. Consequently, RC ions in the real part of the wave dispersion relation should affect the EMIC wave generation and its global distribution during the storm

(Gamayunov and Khazanov 2008). This claim is strongly supported by the observations. Recent satellite observations of EMIC waves in the Pc1–2 frequency range (Engebretson et al. 2007) showed that, although the temperature anisotropy of energetic (>10 keV) RC protons was high during the entire 22 November 2003 perigee pass (in the dawn MLT sector at $L \approx 4.5$), EMIC waves were observed only in conjunction with intensification of the ion fluxes below 1 keV by more than an order of magnitude. This suggests that the suprathermal plasma (≤ 1 keV) plays an important role in the destabilization of the more energetic RC and/or plasma sheet ions such that high-energy anisotropic RC and/or plasma sheet proton distributions appear to be a necessary but not sufficient condition for occurrence of EMIC waves (see Gamayunov and Khazanov 2008 for more details). Similar results were obtained for Pc1–2 events in the dayside outer magnetosphere by Engebretson et al. (2002) and Arnoldy et al. (2005). These observations provide clear evidence that both the cold plasmaspheric plasma and suprathermal ions injected from the plasma sheet control EMIC wave excitation. Consequently, an inclusion of RC H^+ in the real part of EMIC wave dispersion relation is needed for a more realistic wave simulation on the global magnetospheric scale.

To demonstrate how an inclusion of RC H^+ in the real part of the wave dispersion relation affects the He^+ -mode EMIC waves, we simulated the 2–3 May 1998 storm. The equatorial distributions of the squared wave magnetic field are shown in Fig. 9.21 for the He^+ -mode EMIC waves. The distributions in the first row are obtained from the simulation when RC H^+ is only treated as a source of free energy for the EMIC wave generation and omitted in the real part of the wave dispersion relation. The second row is for the case in which RC H^+ is taken into account in both the real and imaginary parts of the wave dispersion relation. There are clearly more waves in the second row compared in Fig. 9.21. The reason for that is an increase of the local growth rate caused by an increase of the total density (see Fig. 9.20). This suggests (see Gamayunov and Khazanov 2008) that the density

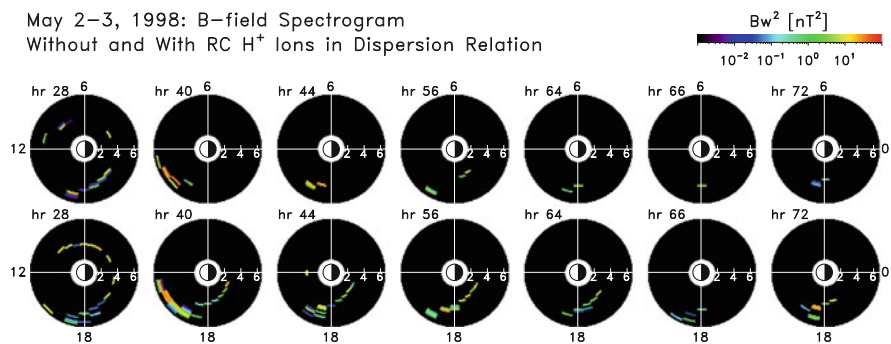


Fig. 9.21 The equatorial distribution of the squared wave magnetic field of the He^+ -mode EMIC waves during 2–3 May 1998. The first row shows the case when RC H^+ is only treated as a source of free energy to generate waves and omitted in the real part of the wave dispersion relation. The second row demonstrates distributions when RC H^+ is taken into account in both the real and imaginary parts of the wave dispersion relation

gradient becomes less effective in counteracting the wave refraction caused by the magnetic field gradient/curvature. Therefore the much greater wave activity observed in the second row of Fig. 9.21 is due to an increase of the local growth rate that overshoots the increase of the wave refraction.

The main findings from these simulations presented by Gamayunov and Khazanov (2008) can be summarized as follows:

1. RC H^+ contributes only a few percent to the total plasma density near the inner edge of the plasmasphere boundary layer but can dominate outside the plasma-pause (except for a dayside plasmaspheric drainage plume). Approximately 90% of the RC H^+ density in the dawn MLT sector is formed by the suprathermal ions with $E \leq 2$ keV, whereas a major contribution at dusk comes from the 10–100 keV ions allowing not more than 10–20% for the suprathermal ions.
2. Including RC H^+ in the real part of the wave dispersion relation increases the local growth rate, leading to a dramatic change in the wave global patterns. The “new” waves are generated not only on the plasmopause, as expected from previous global simulations, but also inside and outside the plasmopause, consistent with the observations.

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Chapter 10

Kinetic Theory of Ring Current and Electromagnetic Ion Cyclotron Waves: Applications

Feedback from the RC and EMIC waves to the ionosphere–magnetosphere coupled system is tremendous. The RC energy source is very important to the energetics of the thermal plasma environment in the subauroral, the mid-latitude and even the equatorial ionosphere. The energy stored in this region is comparable to that stored in the particle reservoir in the plasma sheet. The slow release (timescales of hours to days) of this energy via charge-exchange, Coulomb drag, and wave–particle interaction processes produces very different effects on the ionospheric thermal plasma background than the dramatic rapid releases of energy from the magnetotail into the auroral regions. Space observation shows that EMIC wave-induced pitch-angle diffusion of megaelectron volt electrons can operate in the strong diffusion limit with a timescale of several hours to a day. This scattering mechanism is now considered to be one of the most important means of relativistic electron loss during the initial and main phases of a magnetic storm. It essentially couples the research of the outer radiation belt with studies of the RC, EMIC waves, and plasmasphere systems.

10.1 Wave-Induced Precipitated RC Fluxes

One of the most pronounced manifestations of the RC–EMIC wave interaction is a scattering of the RC ions into the loss cone. This process leads to decay of the RC (see, e.g., Cornwall et al. 1970), especially during the main phases of the storms when the RC decay time of approximately 1 h or less is possible (Gonzalez et al. 1989). In order to demonstrate further the role of the EMIC wave propagation and refraction, we obtain a global history of the RC proton precipitating flux,

$$J_{lc} = \frac{1}{\Omega_{lc}} \int_{E_1}^{E_2} dE \int_{\mu_{lc}}^1 d\mu_0 j, \quad \Omega_{lc} = \int_{\mu_{lc}}^1 d\mu_0 \quad (10.1.1)$$

for the May 1998 storm. In (10.1.1), μ_{lc} is the cosine of the equatorial pitch angle at the boundary of loss cone and j is the equatorial ion differential flux. For the presentation below, we selected exactly the same time intervals as in Chap. 9; the first interval takes place on 2 May, from 24 to 48 h after 1 May, 0000 UT, and the second interval is from 72 to 86 h after 1 May, 0000 UT. For reference purposes, we first present the results from the model with no wave–particle interaction (Khazanov et al. 2006). In Fig. 10.1, we show the selected snapshots of the precipitating fluxes integrated over the energy range 1–50 keV. These fluxes result only from magnetospheric convection of the RC ions and their scattering due to Coulomb collisions with thermal plasma. The most intense precipitating fluxes do not exceed $10^6 (\text{cm}^2 \text{s sr})^{-1}$ and are observed in the postmidnight–dawn MLT sector for $L > 4.25$ during the early recovery phase, starting at 80 h after 1 May, 0000 UT.

The precipitating fluxes obtained from a simulation based on the system of governing equations derived in Chap. 9 are presented in Fig. 10.2. First of all, we

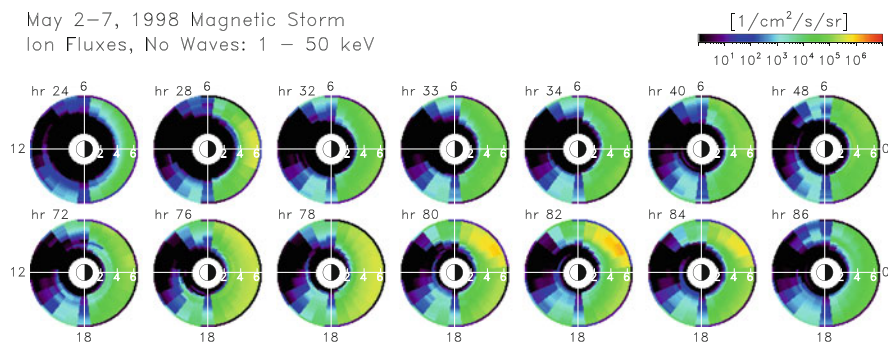


Fig. 10.1 Ring current (RC) proton precipitating fluxes averaged over the equatorial pitch-angle loss cone and integrated over the energy range 1–50 keV. The results are obtained from the model with no wave–particle interaction. All specified hours are counted from 0000 UT on 1 May 1998

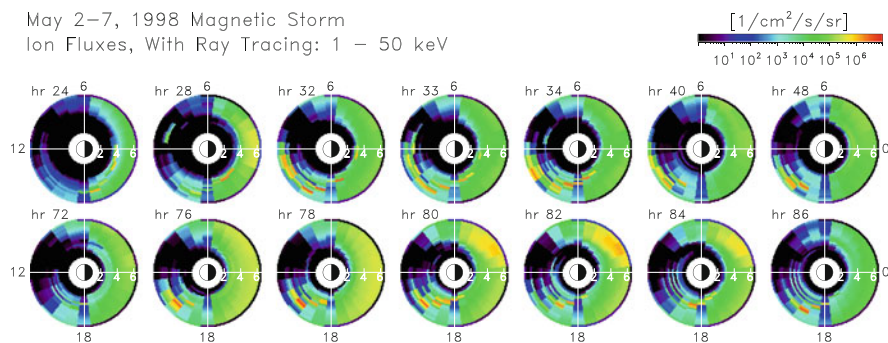


Fig. 10.2 Same as Fig. 10.1, except the complete system of governing equations from Chap. 9 is used for the simulation

observe that the wave-induced precipitating fluxes exhibit quite a lot of fine structure and are highly organized by location of the plasmopause gradient (compare with the equatorial wave distributions in Fig. 9.16 of Chap. 9). In agreement with previous modeling results (Jordanova et al. 2001), the wave-induced precipitating fluxes are more intense in comparison with the results in Fig. 10.1, and the strongest fluxes of approximately $4 \times 10^6 (\text{cm}^2 \text{s sr})^{-1}$ are observed during the main and early recovery phases of the storm. The very interesting and probably more important conclusion can be derived by comparing Fig. 10.2 with Fig. 9.16 in Chap. 9; in a number of cases, the most intense precipitating fluxes are not simply connected to the most intense EMIC waves. For example, the strongest precipitating flux of $4 \times 10^6 (\text{cm}^2 \text{s sr})^{-1}$ is observed at $L = 5.25$, $\text{MLT} = 16$ at 86 h after 1 May, 0000 UT, but there is quite moderate EMIC wave intensity of only $B_w^2 = 2.7 \text{ nT}^2$.

Let us consider this feature in detail by analyzing the data from three (MLT, L-shell) points in the snapshot at 48 h: point a, where $L = 5.25$, $\text{MLT} = 16$; point b, where $L = 5.75$, $\text{MLT} = 15$; and point c, where $L = 5.75$, $\text{MLT} = 14$. The precipitating fluxes and the EMIC wave power densities in these points are listed in Table 10.1.

The number densities for the 1- 50-keV range RC protons are practically the same in all of these points; they range from 1.1 to 1.4 cm^{-3} . Thus, the observed differences between precipitating fluxes are mainly due to differences in the RC–EMIC wave diffusion rates, which depend on the EMIC wave power spectral density. The simple analysis of the magnitudes of the precipitating fluxes and the EMIC wave intensities can not explain these differences; for example, $B_{w,(a)}/B_{w,(c)} = 1.3$ and $J_{lc,(a)}/J_{lc,(c)} = 17.7$, but, on the other hand, the ratio $B_{w,(b)}/B_{w,(a)} = 1.2$ is practically the same as $B_{w,(a)}/B_{w,(c)}$ but $J_{lc,(b)}/J_{lc,(a)} = 0.4$ is far less than 1. At the same time, as we have demonstrated in Chap. 9 (see Fig. 9.18), the EMIC wave power spectral density distributions over the equatorial wave normal angle are essentially different for cases A–C. In case A, the EMIC wave energy is entirely concentrated in the region of generation, i.e., in the region of small θ_0 . As a consequence, we observe the most intense RC proton precipitating fluxes (the EMIC wave growth rates are maximized for a field-aligned wave propagation that is caused by most efficient wave–particle interaction). In case C, however, because the wave energy is in the region of large θ_0 only, the smallest RC precipitating fluxes are produced. In the intermediate case B, although the wave power $B_{w,(b)}^2$ has the greatest amplitude, only the wave energy concentrated in a

Table 10.1 Wave-induced RC proton precipitating fluxes, the equatorial EMIC wave power densities, and the “wave” energy deposition rates to thermal plasmaspheric electrons

Point coordinates	Our model			Ram, Kozyra et al. (1997a)	
	$J_{lc} (\text{cm}^2 \text{s sr})^{-1}$	$B_w^2 (\text{nT}^2)$	$P_{w,e} (\text{eV} (\text{cm}^2 \text{s})^{-1})$	$J_{lc} (\text{cm}^2 \text{s sr})^{-1}$	$B_w^2 (\text{nT}^2)$
(a) $L = 5.25$, $\text{MLT} = 16$	2.5×10^6	28.6	4.8×10^9	2.7×10^5	0.2
(b) $L = 5.75$, $\text{MLT} = 15$	1.1×10^6	41.6	2.0×10^{10}	4.1×10^6	0.7
(c) $L = 5.75$, $\text{MLT} = 14$	1.4×10^5	16.3	1.2×10^{10}	2.2×10^6	0.7

quasi-field-aligned region can effectively scatter the RC protons, and we compute a flux that is more than twice less than in case A. Thus, the characteristic of the EMIC wave power spectral density distribution over θ_0 is an extremely crucial factor for the effectiveness of the RC ion scattering (see additional results by Khazanov et al. 2006 and compare their precipitated fluxes with other global RC models).

It is very clearly demonstrated above that the EMIC wave power spectral density distribution over the wave normal angle can extremely impact the effectiveness of the RC loss cone ion scattering. At the same time, we considered only three spatial points in one snapshot, and it is very interesting now to see the wave power spectral density distributions on the global spatial and temporal scales. Figure 9.19, presented in the previous chapter, clearly demonstrates the calculated EMIC wave normal angle distribution asymmetry at the selected snapshots during the May 1998 storm. The quasi-field-aligned, highly oblique, and intermediate wave distributions are represented in the spectrograms. Thus, all not quasi-field-aligned distributions shown in Fig. 9.18 of Chap. 9 are not unusual distributions and are widely observed in the results of our simulation, and also in a EMIC wave data presented in Sect. 9.19.

10.2 Heating of Thermal Plasmaspheric Electrons and Subauroral Ionospheric Temperature Enhancement

10.2.1 EMIC Wave and Coulomb Heating of Thermal Plasmaspheric Electrons

The EMIC waves generated by the RC ions not only cause RC ion scattering into the loss cone but also effectively transfer energy to thermal plasmaspheric electrons due to resonant Landau damping. The total energy deposition rate to the thermal electrons can be obtained by integrating the local EMIC wave energy deposition rate along each geomagnetic field line from the equator to the ionosphere altitude. Assuming that no other energy sources or sinks exist along the geomagnetic field line above the ionosphere, the resulting heat flux at the ionospheric level can be calculated as

$$P_{w,e} = -B_{iono} \int_0^{s_1} \frac{ds}{B(s)} \int_{\omega_1}^{\omega_2} d\omega \int_0^\pi d\theta \frac{B^2(\omega, \theta, s)}{4\pi} \gamma_e(\omega, \theta, s). \quad (10.2.1)$$

In (10.2.1), s is a coordinate along geomagnetic field line, θ is the wave normal angle, $B^2(\omega, \theta, s)$ is the squared spectral magnetic field of the He^+ -mode of EMIC waves, $\gamma_e(\omega, \theta, s)$ is the Landau damping rate of the He^+ -mode of EMIC waves due to interaction with thermal plasmaspheric electrons, and the ratio of geomagnetic

field amplitudes at the ionosphere altitude and at the current position, $B_{iono}/B(s)$, takes into account the change in the magnetic tube cross-section along the field line. The Landau damping rate strongly depends on the wave normal angle, and it is very sensitive to the ratio of the parallel phase velocity to the electron thermal velocity. The dependencies of the Landau damping rate on the wave normal angle are presented in Fig. 10.3 for different sets of the plasma parameters. In order to calculate these damping rates, we assume the plasma consists of the Maxwellian electrons, and cold ions; 77% of H^+ , 20% of He^+ , and 3% of O^+ . The Landau damping rate is essentially equal to zero for small wave normal angles, and it has reasonably large values only for highly oblique wave propagation, so, in Fig. 10.3, we provide the wave normal angle ranges only where $|\gamma_e|/\omega \geq 10^{-4}$. The parallel phase velocity for the He^+ -mode of EMIC waves is very close to the Alfvén velocity, which is $v_A = 4.4 \times 10^7 \text{ cm s}^{-1}$ for Fig. 10.3a, b. On the other hand, the damping rate maximizes for the ratio of the parallel phase velocity to the electron thermal velocity of approximately 1.2, i.e., $v_A/v_{Te} = 1.2$ (see, e.g., Cornwall et al. 1971). This number corresponds to the electron temperature $T_e = 0.4 \text{ eV}$, and we

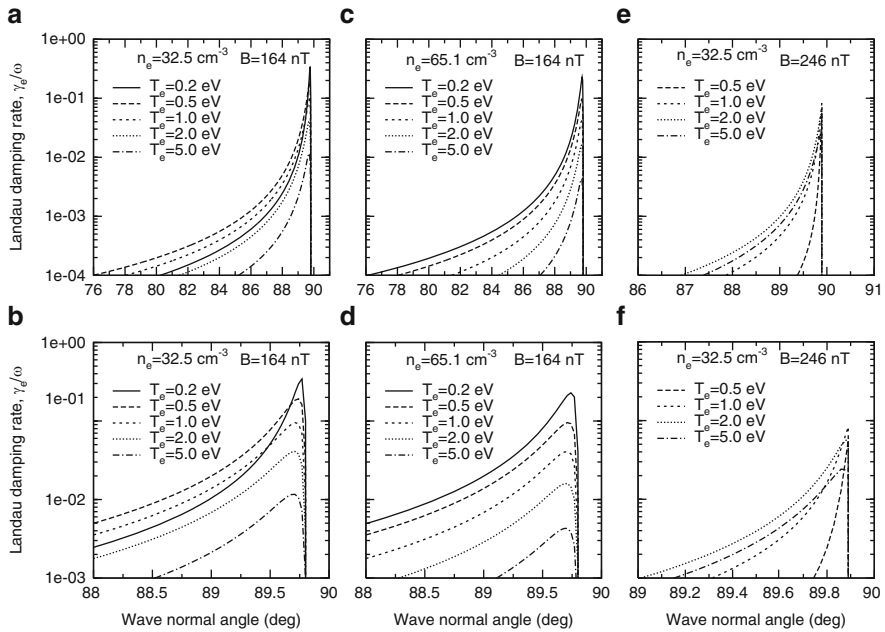


Fig. 10.3 The electron Landau damping rates versus the wave normal angle for the He^+ -mode of EMIC waves. The plasma is assumed to consist of the Maxwellian electrons and cold ions (77% of H^+ , 20% of He^+ , and 3% of O^+). The second row in the figure represents the corresponding results from the first row but in the vicinity of the damping rate maximum only. All of the results are calculated for the wave frequency $\nu = \omega/2\pi = 0.286 \text{ Hz}$. (a, b) The electron number density and magnetic field are taken from our global model at location $L = 5.75$, $MLT = 14$, at 48 h after 1 May 1998, 0000 UT (nominal case). (c, d) The doubled electron number density of the nominal case. (e, f) The one and half of the magnetic field amplitude of the nominal case

observe that the Landau damping rate in Fig. 10.3b is maximal for $T_e = 0.5$ eV (not shown, the damping rate for $T_e = 0.4$ eV only slightly exceeds the rate for $T_e = 0.5$ eV at its maximum). The further electron temperature increase leads to a decrease of the damping rate. In the case when $v_A/v_{Te} \gg 1.2$, the Landau damping rate decreases exponentially. Note that all of the results in Fig. 10.3c–f can be understood if it is kept in mind that $v_A \sim B/\sqrt{n_e}$.

The thermal plasmaspheric plasma in the presented results is treated independently from the self-consistent dynamics of the RC–EMIC wave system (for details about the core plasma density model, see Chap. 9). This means, particularly, that we do not take into account a change of the background electron temperature due to resonant Landau damping of the EMIC waves. In order to check the sensitivity of the “wave” energy deposition rate to this restriction, we made four runs with four different thermal electron temperatures of 0.5, 1.0, 2.0, and 5.0 eV. On the whole, the energy deposition rate is maximized for electron temperatures in the range of 1–2 eV, and is less intense for the case of $T_e = 0.5$ eV than for $T_e = 5.0$ eV. Considering the global EMIC wave energy distribution, wave-induced RC precipitating flux, and energy deposition rate, we did not find any essential differences between these four cases, and the “wave” energy deposition rate presented below will relate to the case of $T_e = 1$ eV.

For reference and comparison purposes, we also calculate the energy deposition rate to the thermal electrons due to Coulomb collisions with RC ions. According to Young et al. (1982), the total ion flux, measured at geostationary orbit, can be divided between the RC H^+ , O^+ , and He^+ depending on geomagnetic and solar activity as it is measured by Kp and $F_{10.7}$ indices. For the 2–7 May 1998 storm period, the geosynchronous ion fractions are shown in Fig. 10.4. The RC He^+

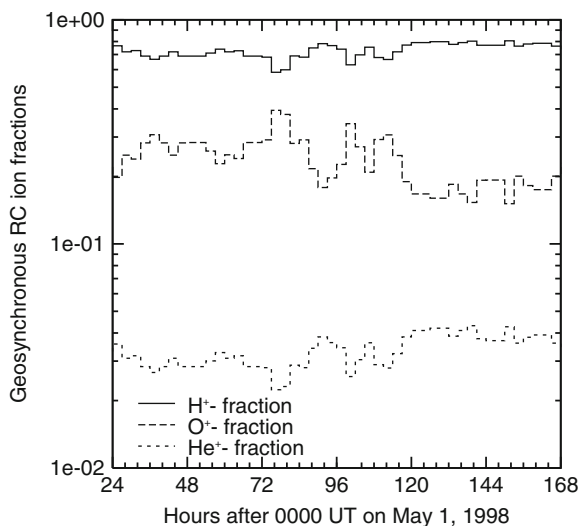


Fig. 10.4 Fractions of the RC ions at geosynchronous distance for the 2–7 May 1998 storm period

fraction does not exceed 4% during this event, and the RC O^+ ions mostly populate 20–40% of the RC content. The latter allows us to assume that the RC is entirely made up of energetic H^+ and O^+ ions, and to ignore the RC He^+ in calculation of the Coulomb collision energy deposition rate to the thermal electrons. The energy deposition rate to the thermal electrons due to Coulomb interaction with the RC ions can be written in the same manner as (10.2.1). Considering the RC H^+ and O^+ , multiplying their Coulomb drag energy loss terms by the energy, and integrating over the entire phase space volume and along the geomagnetic field line, we get

$$P_{RC,e} = -B_{iono} \int_0^{s_1} \frac{ds}{B(s)} \int d\mathbf{p} E \left(\frac{\delta F}{\delta t} \right)_{RC,e} = \int_0^{s_1} ds \int G \bar{j} dE, \quad RC = H^+, O^+, \quad (10.2.2)$$

where an explicit form for the term $(\delta F / \delta t)_{RC,e}$ can be derived from the Coulomb collision term (2.2.2) (see, e.g., in the work of Khazanov et al. 2003a), $\bar{j}$ is the omnidirectional flux of the RC ions, and G is the Coulomb collisional energy loss per unit length experienced by the selected RC ion in a specified background plasma.

As was demonstrated by Khazanov et al. (2007), both the “wave” and the Coulomb collisional energy sources for the thermal plasmaspheric electrons depend on electron temperature; the wave-induced energy deposition rate is maximized for electron temperatures in the range of 1–2 eV, the collisional energy source has a maximum for electron temperature of approximately 0.5 eV, and both of these energy sources gradually decrease with further temperature enhancement. At the same time, we do not treat the thermal plasma self-consistently in the present section, i.e., we do not take into account a change of the background electron temperature due to interaction with waves and RC. Thus, we have to specify the electron temperature in order to compare the “wave” and “collisional” energy sources. Because the Coulomb collisional energy source can readily act as the primer to heat plasmaspheric electrons (Thorne and Horne 1992; Kozyra et al. 1987), below we use $T_e = 1$ eV as an electron temperature.

From the same simulation, based on the system of governing equations derived in Chap. 9, we obtained the energy deposition rates to the thermal plasmaspheric electrons as they are described by (10.2.1) and (10.2.2). The selected snapshots are presented in Fig. 10.5 for the two energy sources discussed above; the heat fluxes caused by the EMIC wave energy absorption due to Landau resonance, and the heat fluxes due to Coulomb energy degradation of the RC H^+ and O^+ ions. The energy deposition rates caused by the resonant interaction of the EMIC waves and thermal plasmaspheric electrons are presented in the first and second rows. The (MLT, L-shell) localizations of the energy flux from this channel are practically coincident with the corresponding EMIC wave energy distributions (compare with Fig. 6 in the paper by Khazanov et al. 2006). There are spot-like patches that are mostly localized in the vicinity of the 10^2 cm^{-3} electron number density contour in the plasmasphere boundary layer. The most intense energy deposition rates are

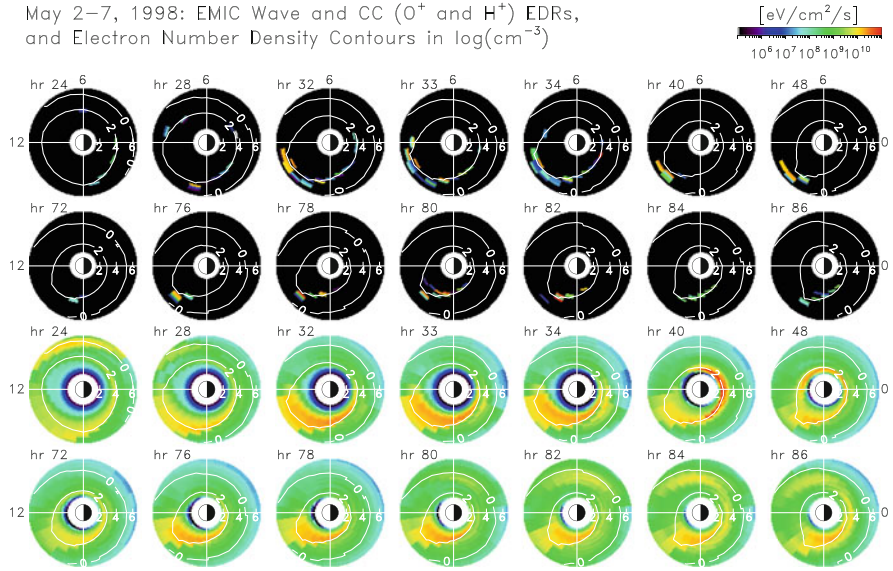


Fig. 10.5 Energy deposition rates to the thermal plasmaspheric electrons at the topside ionospheric altitude

observed in the postnoon–premidnight MLT sector, and maximize at the magnitude of $7.8 \times 10^{10} \text{ eV}(\text{cm}^2\text{s})^{-1}$ at $L = 3.25$, MLT = 22 at 34 h after 1 May, 0000 UT.

The third and fourth rows in Fig. 10.5 are the snapshots of the energy deposition rates into the thermal plasmaspheric electrons due to Coulomb energy degradation of the RC H^+ and O^+ ions. Coulomb collisions occur throughout the plasmasphere–RC overlap region. As a result, this energy source is smooth in intensity, located well earthward of the plasmopause, and spatially separated from the “wave” energy source. Although the intense energy deposition rates are mostly observed in the postnoon–premidnight MLT sector, sometimes this kind of energy source can be effective everywhere, encircling almost the entire globe (see, e.g., the hours 40 and 48 in Fig. 10.5). The greatest energy deposition rate of approximately $10^{11} \text{ eV}(\text{cm}^2\text{s})^{-1}$ is observed at hour 40, and the typical energy deposition rates are approximately of $2 \times 10^{10} \text{ eV}(\text{cm}^2\text{s})^{-1}$, and found during two periods, 32–48 h and 76–86 h after 1 May, 0000 UT.

10.2.2 *Electron Heating Events in Subauroral Topside Ionosphere: Qualitative Comparison with Observations*

The overlap of the RC with the outer plasmasphere plays a major role in the storm-time-related electron temperature elevations in the subauroral topside

ionosphere. One of the major deficiencies in the study of the coupling of the RC to the ionosphere is the lack of concurrent and conjugate magnetospheric and ionospheric plasma measurements over complete anomalous heating events. Recently, Gurgiolo et al. (2005) presented a global view of the RC–plasmasphere system during the 18–19 June 2001 storm. In order to have concurrent and conjugate plasma measurements from the plasmasphere, the RC, and the topside ionosphere, Gurgiolo et al. (2005) combined data from the IMAGE and DMSP satellites. IMAGE has been used (1) to obtain the plasmaspheric He^+ ion density in the geomagnetic equatorial plane (EUV) and (2) to obtain global 2D distributions of the RC ion fluxes in the geomagnetic equatorial plane (MENA and HENA). DMSP satellites (F-12, F-13, and F-15) have been used to obtain electron temperature and precipitating electron and ion fluxes along the satellite tracks at a nominal altitude of 830 km. Gurgiolo et al. (2005) analyzed the relationships of the topside heating of the subauroral ambient electrons with the plasmasphere and RC conditions. They found that subauroral heating occurs within the plasmasphere–RC overlap region and can be separated into two classifications, inner and outer heating events. The inner events take place well earthward of the plasmapause (>0.75 RE in the equatorial plane) and generally occur in the dawn MLT sector. The outer events occur in the plasmasphere boundary layer within 0.75 RE of the equatorial plasmapause and are more prevalent in the dusk MLT sector. Note that sometimes both inner and outer events are observed in the evening MLT sector.

Our theoretical results presented in Fig. 10.5 very clearly demonstrate spatial separation between two energy sources for thermal plasmaspheric electrons. The “wave” source produces intense thermal fluxes mainly in the postnoon–premidnight plasmasphere boundary layer, a radially narrow region but extended in MLT. On the other hand, the “Coulomb” source is acting well earthward of the plasmapause and can be found in both postnoon–premidnight and morning MLT sectors. (Although the radial extension of the “Coulomb” source usually exceeds 2 RE in the postnoon MLT sector, there is less than 1 RE for the heating radial extension in the dawn MLT sector.) This spatial structure of the thermal electron heating caused by interaction of the RC with the plasmasphere is supported by concurrent and conjugate plasma measurements from the plasmasphere, the RC, and the topside ionosphere (Gurgiolo et al. 2005). At the same time, we should note that, in the present chapter, we do not discuss the mechanisms of energy transfer from the high-altitude energy sources into the ionosphere. (The heat conduction and/or low-energy downward electron fluxes are now widely accepted as the main energy transport mechanisms.) Thus, it is not a fact that calculated energy deposition rates are the real thermal fluxes that can be observed at the ionosphere level. These theoretical numbers give only the maximal values of the heat fluxes that could be potentially transferred downward to the ionosphere from the specified energy sources.

The observed heating is mostly localized in the equatorial radial direction (or in a latitudinal direction as it observed by DMSP satellites), and, interpreting the satellite data, Gurgiolo et al. (2005) have come to the conclusion that “while

Coulomb collisions may act as a heat source in the equatorial plasmasphere, collisional heat conduction is not the source of the heat flux into the ionosphere,” and some alternate mechanism(s) should act to transfer the heat into the upper ionosphere. They have tried to explain the observed inner and outer heating events by drawing in the wave instabilities. Although they had difficulties in explaining the heat flux driver for the inner heating events, they convincingly argued that the EMIC wave instability is driving the heat flux into the ionosphere for the outer heating events. This is exactly the result we obtained from our global self-consistent RC–EMIC wave model. Note that there is an extremely important fact we should keep in mind; despite the fact that the “wave” energy source is less intense globally than the “Coulomb” source, the EMIC waves scatter the thermal electrons into the loss cone (even if these electrons are heating due to Coulomb collisions with RC), so these heated electrons can precipitate into the ionosphere. At the same time, we should emphasize that Gurgiolo et al. (2005) encountered one difficulty connected with using the EMIC wave instability as a heat driver for the outer heating events. Namely, they found “in no instance in this entire storm is ion precipitation observed in conjunction with a subauroral temperature enhancement. Ion precipitation is expected to accompany heating produced in the damping of ion cyclotron waves (Cornwall et al. 1971).” We believe that this difficulty can be naturally resolved in the frame of the RC–EMIC wave theoretical model that is presented in Chap. 9.

As we demonstrated in Chap. 9, the EMIC wave power density distribution over wave normal angle can occupy not only the region of generation, i.e., the region of small θ_0 , but the entire wave normal angle region, and even the region near $\theta_0 = \pi/2$ only. The intensities of the RC ion precipitating fluxes depend on the intensities of the EMIC waves in the region of small θ_0 , and the magnitudes of energy deposition rates to thermal electrons mostly depend on the intensities of the oblique EMIC waves. Selecting the same spatial points a–c as we did in Chap. 9, and analyzing the data presented above, in Table 10.1, we list the values for the wave-induced energy deposition rates to the thermal plasmaspheric electrons in these points.

At point b, the wave power, $B_{w,(b)}^2$, has the greatest amplitude, the EMIC wave energy is near evenly distributed in the entire wave normal angle region (see Fig. 9.18b in Chap. 9), and, as a result, we observe the greatest energy deposition rate in this point. Let us now consider the points a and c. First, for these points, the ratio $B_{w,(a)}^2/B_{w,(c)}^2 = 1.8$ is held. Second, although the core plasma density, n_{e0} , in point a is approximately 30% greater than in point c, and, for the dipole model, the ratio of the magnetic fields in these points is $B_{(a)}/B_{(c)} = 1.3$ (γ_e depends on n_{e0} and B , see Khazanov et al. 2007 for details), the Landau damping rates are almost the same in these points. Thus, we can expect the energy deposition rate in point (a) to be greater than in point c. Despite these facts, the energy deposition rate in point a is more than two times less than in point (c) ($P_{w,e(a)}/P_{w,e(c)} = 0.4$) because the EMIC wave power spectral density distributions over θ_0 are diametrically opposite in these two points (see Fig. 9.18 in Chap. 9). As a consequence, we observe the similar intensities of the energy deposition rates in these points, but there is a

dramatic difference between the RC proton precipitating fluxes, $J_{lc,(a)}/J_{lc,(c)} = 17.7$. Thus, the satellite measurements (Gurgiolo et al. 2005) and our theoretical results allow us to argue that the subauroral temperature enhancement in the outer heating events (1) is driven by thermal electron energy absorption due to Landau resonance with EMIC waves and (2) these events are not necessary to be accompanied by the elevated RC ion precipitations.

10.2.3 Relationship of Wave Heating to SAR Arcs

In conclusion of this section, let us point out the possible relation of the Landau resonance between the EMIC waves and thermal plasmaspheric electrons to the energy source to drive excitation of stable auroral red (SAR) arcs. Although the spacecraft and ground-based measurements along with statistical and theoretical studies provide strong evidence in support of the importance of the Coulomb energy degradation of the RC ions as a SAR arc energy source (Kozyra et al. 1987), the role of plasma waves in SAR arc formation is now unclear and remains a controversial issue (Kozyra et al. 1997b). The newest SAR arc observations, with instrumentation that allows enhanced spatial and temporal resolution, have revealed surprising spot-like patches of enhanced emissions that move along the length of the SAR arc (Kozyra et al. 1997b). No explanation for these small-scale structures yet exists. As we demonstrated above, the EMIC waves produce intense electron thermal fluxes in the MLT extended and radially narrow region in the postnoon–premidnight plasmasphere boundary layer. There are spot-like patches of the enhanced heat fluxes, and the most intense energy deposition rate during the studied event reaches the value of $10^{11} \text{ eV}(\text{cm}^2 \text{ s})^{-1}$ at $L = 3.25$, MLT = 22 at 34 h after 1 May, 0000 UT. The first and the second rows in Fig. 10.6 present the total energy deposition rates caused by the EMIC wave energy absorption and the Coulomb energy degradation of the RC H^+ and O^+ ions. The third and the fourth rows are the heat fluxes due to Coulomb energy degradation of the RC H^+ and O^+ ions only. In many snapshots in this figure, we can clearly observe the spots of the enhanced energy depositions to thermal electrons. The theoretically obtained great thermal fluxes and their spot-like spatial structure make this “wave” energy source a possible mechanism to drive the above-mentioned SAR arc feature. The spot-like spatial structure produced by this “wave” mechanism depends on the structure of the region of enhanced density gradient in the plasmasphere boundary layer, and multiple plasmopause density drops are often observed (Thorne and Horne 1992). On the other hand, if Coulomb collisions are responsible for the observed spot-like patches of the enhanced heat fluxes, dense plasma peaks and/or higher-density structures in the RC itself are required. Certainly, additional measurements and theoretical investigations are required in order to explain these small-scale structures in the SAR arcs.

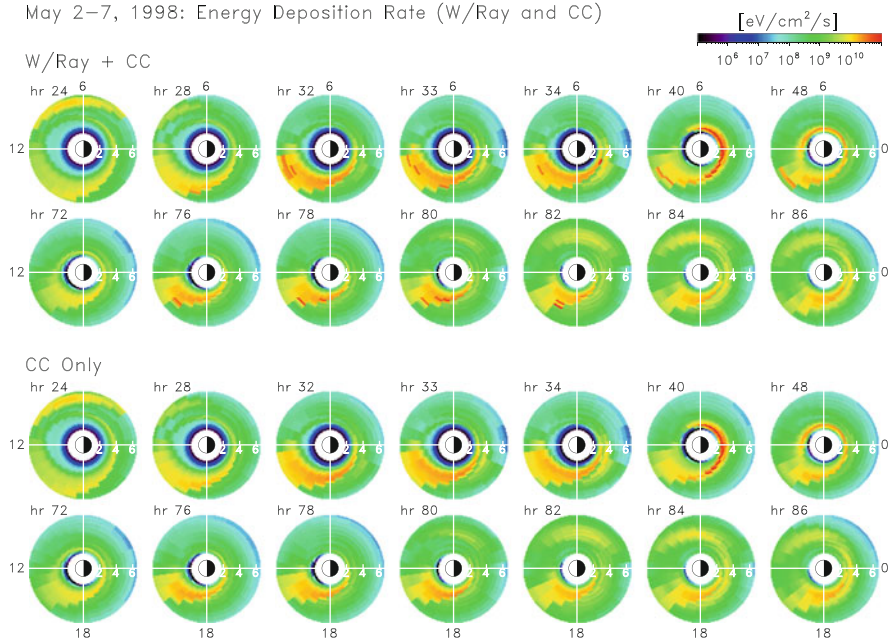


Fig. 10.6 Energy deposition rates to the thermal plasmaspheric electrons at the topside ionospheric altitude. All of the heat fluxes are obtained for $T_e = 1$ eV and are mapped back in the equatorial plane along the dipole field lines. The first and second rows show the total deposition rates caused by the EMIC wave energy absorption and the Coulomb energy degradation of the RC H^+ and O^+ ions. The third and fourth rows are the heat fluxes due to Coulomb energy degradation of the RC H^+ and O^+ ions only. All of the specified hours are counted from 0000 UT on 1 May 1998

10.3 Associated Global Inner Magnetosphere Environment

10.3.1 Magnetospheric Electric Field

The convective patterns of both the RC ions and the cold plasmaspheric plasma are controlled by the magnetospheric electric field, thereby determining the conditions for interaction of RC ions and EMIC waves. Therefore, this electric field is one of the most crucial elements in simulating the wave power spectral density on a global magnetospheric scale. As we pointed out in Chap. 9, self-consistent simulation of the magnetosphere–ionosphere system should provide, at least in principle, the most accurate theoretical electric field (Vasyliunas 1970; Jaggi and Wolf 1973). The need for a self-consistent model of the magnetospheric electric field, RC, plasmasphere, and EMIC waves is evident. We have incorporated the large-scale magnetosphere–ionosphere electrodynamic coupling in our previous self-consistent model of interacting RC ions and EMIC waves, as was described in Chap. 9. The resulting computational model treats self-consistently not only EMIC waves and RC ions but also the magnetospheric electric field, RC, and plasmasphere.

A few runs of this model were performed by Gamayunov et al. (2009) to get a qualitative assessment of the effects of the high latitude ionospheric boundary condition and the ionospheric conductance. The results presented here were obtained from simulations of the 2–4 May 1998 geomagnetic storm (mostly the 2–3 May period). We performed three simulations that differ by the electric potential specified at the high-latitude ionospheric boundary (we used the W96 model and the VS model with Kp parameterization), and/or the ionospheric conductance from auroral precipitation (using the Hardy et al. 1987 conductance model and the Ridley et al. relationship between the FACs and the conductance; see Sect. 9.4.4 for details). The following three combinations were used in the simulations presented by Gamayunov et al. (2009): (1) the VS model and the Hardy et al. (1987) model; (2) the W96 model and the Hardy et al. (1987) model; and (3) the W96 model and the Ridley et al. relationship. In addition, one more simulation was done: (4) the W96 model and the Hardy et al. (1987) model applying a 20-min window as the time needed to reestablish a new potential pattern in the magnetosphere. The RC has been simulated inside geostationary orbit only, and the high latitude ionospheric boundary has been placed near the ionospheric projection of this orbit.

To provide information about magnetospheric electric field coming from the self-consistent simulations, we selected six snapshots of the equatorial electric field patterns from 2 May, and one snapshot at hour 77 (0500 UT on 4 May). The corresponding electric potential contours are shown in Fig. 10.7. The view is over the North Pole with local noon to the left. We present results for three runs. The equipotentials from a simulation with the VS model at the high-latitude ionospheric boundary and the Hardy et al. (1987) conductance are shown in Fig. 10.7 (top). The other two runs are performed with the W96 model applied at $\lambda = 69^\circ$, and differ only by the conductance model assumed. Figure 10.7 (middle) shows results for the Hardy et al. (1987) conductance model, while Fig. 10.7 (bottom) is for a case when the Ridley et al. empirical relationship between the FAC and conductance is used. The potential configurations in Fig. 10.7 are similar to those from the Rice Convection Model (e.g., Garner et al. 2004). Overall, qualitatively, the same large-scale potential distributions are in all three models, presented in Fig. 10.7, with a well-defined, large-scale, dawn-to-dusk electric field. Despite this, the potential patterns reveal large differences in both the magnitude of the potential and the shape of the contours. This suggests a difference in the fine structure of the electric field distribution because this field is proportional to the gradient of the potential.

One obvious feature observed in Fig. 10.7 is a significantly enhanced electric field in the region $L \approx 3\text{--}4$ in the dusk–postmidnight MLT sector at hour 77 (and, not shown, at hour 76). This radially narrow intensification of the radial electric field (poleward electric field in the ionosphere) creates a westward flow channel, mainly in the dusk-to-midnight MLT sector, while a region of westward (antisunward) convection is also observed in the postmidnight sector equatorward of $L = 3$ (see Fig. 10.7).

This westward flow channel has come to be called the subauroral polarization stream (SAPS) (Foster and Burke 2002; Foster and Vo 2002). The SAPS effect arises from the region 2 FACs, which flow down into the subauroral ionosphere and close the region 1 FACs through the poleward Pedersen currents. Because of the

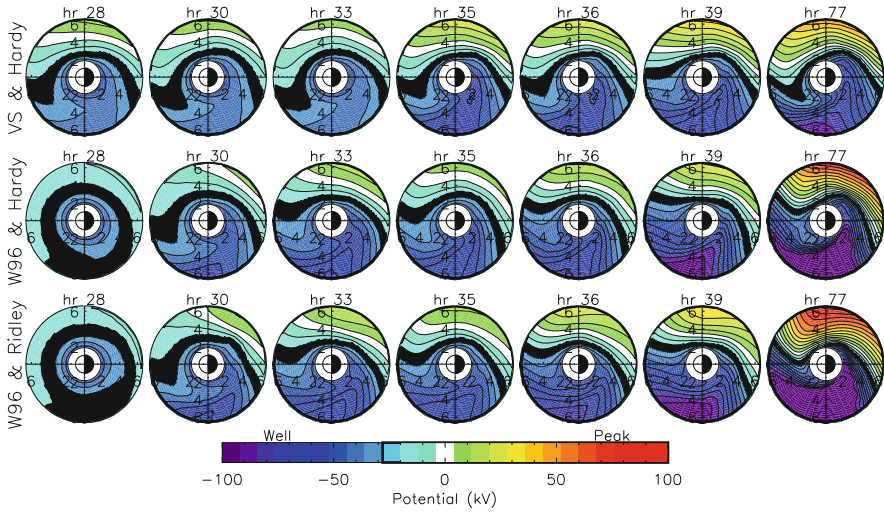


Fig. 10.7 The equatorial potential contours in the inner magnetosphere. The view is over the North Pole with local noon to the left. All of the indicated hours are counted from 0000 UT on 1 May 1998. *Top*: results from a simulation with the VS model at the high-latitude ionospheric boundary and the Hardy et al. (1987) conductance model. *Middle*: simulation with the W96 model at $\lambda = 69^\circ$ and the Hardy et al. (1987) conductance model. *Bottom*: the same as in this figure (*middle*) except that the Ridley et al. empirical relationship between the FAC and the local Hall/Pedersen conductance is used. Equipotentials are drawn every 8 kV

low conductance at subauroral latitudes, the Pedersen current generates an intense poleward electric field between the region 2 FAC and the low-latitude edge of the auroral particle precipitation (Southwood and Wolf 1978; Anderson et al. 1991, 1993; Ridley and Liemohn 2002; Khazanov et al. 2003b; Mishin and Burke 2005).

Although the SAPS localization is correctly predicted by our model, it is likely that the SAPS electric field is overestimated for the W96 boundary condition (see Gamayunov et al. 2009 for the details). Indeed, from the statistical model based on the electric field data measured by the Akebono/EFD instrument, Nishimura et al. (2007) derived the equatorial E_Y electric field component in the dusk SAPS region to be 6 mV m^{-1} during the main phase of storm. It should be noted, however, that the SAPS electric field can sometimes reach more than 10 mV m^{-1} during the main phase of geomagnetic storms (Shinbori et al. 2004), and the CPCP drop derived by Nishimura et al. (2007) is 180 kV, whereas, in our simulation, it is 300 kV. The measurements taken by the double-probe electric field instrument on board the CRRES spacecraft show a similar electric field magnitude (Wygant et al. 1998). There are at least two reasons that may lead to an overestimation of the SAPS electric field in our simulation:

1. Because the W96 model was constructed from data with IMF under 10 nT, this model essentially overestimates the CPCP drop during the 4 May event when IMF was approximately 40 nT (e.g., Burke et al. 1998).

2. In the present simulations, we did not take into account the FACs beyond geostationary orbit, which may contribute essentially to the shielding of mid-latitudes from a high-latitude driving convection field; the effect of FAC is proportional to the volume of the magnetic flux tube, and from the estimate by Vasyliunas (1972) the effect of FAC at $L = 6.6$ is approximately 20% of the FAC effect at $L = 10$. Both of these issues should be addressed in future studies.

10.3.2 *Plasmasphere*

The plasmopause, and/or dayside plume, and/or detached plasma are the favorable regions for EMIC wave generation in the inner magnetosphere. This is because the density gradient there is enhanced and counteracts refraction caused by the magnetic field gradient and curvature (e.g., Horne and Thorne 1993; Fraser et al. 2005; Khazanov et al. 2006). As a result, the net refraction is suppressed at the plasmopause/plume edge, allowing wave packets to spend more time in the phase region of amplification. Thus, the cold plasma distribution is extremely crucial for EMIC wave excitation. Both the convection and the corotation electric fields control the cold plasma dynamics. As such, we first discuss the snapshots of the total electric potential obtained from our simulations. Figure 10.7 shows the resulting equipotential contours, which also coincide with the instantaneous cold plasma flow. The most striking reconfiguration of the potential is observed in Fig. 10.7 (middle and bottom) in the 28- and 30-h snapshots. Starting at hour 28, the CPCP drop increases by approximately 100 kV during 1 h for the W96 convection model. The strong convection causes a shrinking of the closed equipotential contours as shown in Fig. 10.7 (there is stronger shrinking during hour 29). Later, an extremely developed SAPS is observed at hours 76–77 (see Gamayunov et al. 2009 for the details), and the overshielding electric field (negative E_Y) following a decrease of the CPCP difference in the W96 model is found in the inner magnetosphere at hour 79 (not shown).

Figure 10.8 shows the selected distributions of the equatorial cold plasma density for three self-consistent simulations. For each run, the plasmasphere was first prepared by running the Ober code for 20 quiet days. Then, starting at 0000 UT on 1 May 1998, we solved (9.4.16), (9.4.17), (9.2.1)–(9.2.4), and (9.4.18)–(9.4.20) using the initial and boundary conditions and the time series for all controlling parameters (see Chap. 9). For the VS model (Fig. 10.7, top), a broad dayside plume is formed a few hours before hour 28. Subsequently, up to hour 39, gradual intensification of the convection causes nightside plasmaspheric erosion and the plume narrowing in the MLT extent. The latter takes place mostly in the eastward flank of the plume, where the convection and corotation fields reinforce each other, while the duskside plume edge remains roughly stationary (Spasojević et al. 2003; Goldstein et al. 2005). During the following storm progression, the magnetospheric convection field driven by the VS potential drop remains relatively high, and the convection patterns are relatively steady (3-h cadence). Compared with Fig. 10.8 (middle and bottom), these result in the most eroded and shrunken plasmasphere at

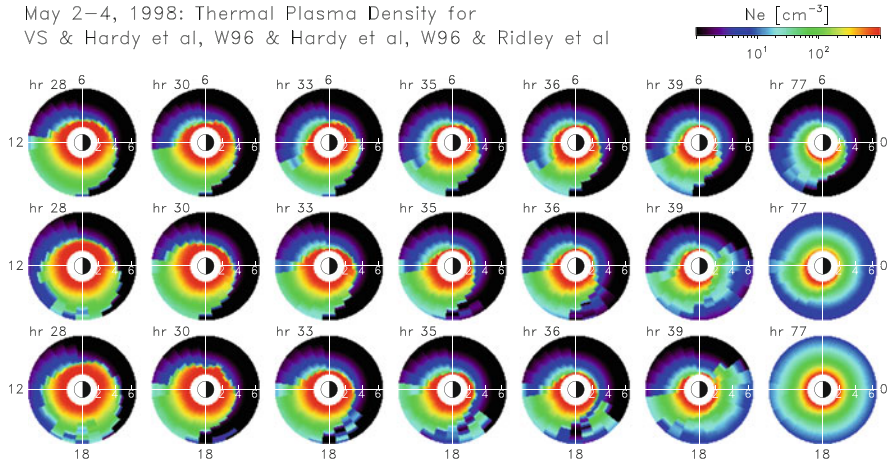


Fig. 10.8 The equatorial cold plasma density distributions from three self-consistent simulations. *Top*: results from a simulation with the VS model at the high-latitude ionospheric boundary and the Hardy et al. (1987) conductance model. *Middle*: simulation with the W96 model at $\lambda = 69^\circ$ and the Hardy et al. (1987) conductance model. *Bottom*: the same as in this figure (*middle*), except that the Ridley et al. empirical relationship between the FAC and conductance is used

hour 77 with a well-defined nightside plasmopause (compare these results with Fig. 7 in the work of Khazanov et al. 2006, where the entire plasmasphere was driven by the analytical formulation of the VS potential).

Cold plasma density distributions in Fig. 10.8 (middle and bottom) are qualitatively similar to each other, but exhibit quite a bit of difference compared with distributions in Fig. 10.8 (top). At hour 28, the plasmasphere is well populated, and the plasmopause is well defined. Starting at hour 28, an increase of the CPCP drop by 100 kV during 1 h causes formation of the plume by hour 29 (not shown), and the presented snapshots at hour 30 are close to those at hour 29. One of the most distinguishable features observed in Fig. 10.8 (middle and bottom) is the presence of a cold plasma on the nightside.

The findings from our initial self-consistent inner magnetosphere model can be summarized as follows (Gamayunov et al. 2009):

1. Although the poleward boundary for the ionospheric potential is specified at the projection of geostationary orbit in most models (probably except for the Rice Convection Model), we are not able to specify well the ionospheric potential there. Indeed, the existing models of ionospheric electric potential (like the AMIE technique (Richmond and Kamide 1988), the Weimer (1996, 2001) and the Boyle et al. (1997) models) are much more reliable at high latitudes and give a poor representation of the potential and its significant variation in the inner magnetosphere (Foster and Vo 2002). In addition, the effect of FACs is proportional to the volume of the magnetic flux tube, and so this effect at $L = 6.6$ is approximately 20% of the FAC effect at $L = 10$, suggesting that FACs beyond geostationary orbit may produce a major shielding of mid-latitudes

from a high-latitude driving field. Thus, the region beyond geostationary orbit should be included in the magnetosphere–ionosphere coupling. An extension of the simulation domain, at least to $\lambda = 72^\circ$, is vital for a truly self-consistent modeling of the magnetosphere–ionosphere coupling.

2. Compared with the case of the Hardy et al. (1987) model, the Ridley et al. empirical relationship between the FAC and conductance produces quite a bit of difference in the potential distribution and, overall, stronger convection at the subauroral latitudes (see Gamayunov et al. 2009 for the details). This difference strongly affects the cold plasma distribution, RC precipitation pattern, and EMIC waves. More importantly, a self-consistent description of the ionospheric conductance makes (9.4.20) nonlinear compared with the case of a statistical conductance model. This is a principle point requiring that a self-consistent model, based on first principles, of the ionospheric conductance should be incorporated into a simulation of the magnetosphere–ionosphere coupling.
3. A fine density structure in the plasmasphere boundary layer, plume, detached plasma, etc. controls the wave propagation. This fine structure may be a more crucial factor in controlling the generation of EMIC waves, than just the intensity/distribution of the RC and the local plasma density. There is very large difference between the wave activity, whereas the density distributions in Fig. 10.8 (middle and bottom) do not differ so dramatically. This suggests that to model the EMIC wave distribution and wave spectral properties accurately, the plasmasphere should be simulated self-consistently because its fine structure requires as much care as that of the RC.
4. It is shown that the effect of a finite time needed to reestablish a new potential pattern throughout the ionosphere and to communicate between the ionosphere and the equatorial magnetosphere is important. This effect was ignored in all previous simulations but it should be taken into account to model a self-consistent electric field properly.

Concluding this section, we would like to emphasize that, in order to make significant progress in developing a truly self-consistent model of the electric field, we need to considerably improve our ability to accurately specify the electric field at high latitudes and ionospheric conductance. Without this ability, we will not be able to accurately specify EMIC wave spectra in the inner magnetosphere and correctly describe the wave-induced heating and/or scattering of the magnetospheric particles.

10.4 Relativistic Electrons Scattering by EMIC Waves

10.4.1 Introduction

The flux of outer-zone relativistic electrons (above 1 MeV) is extremely variable during geomagnetic storms. The competition between loss and acceleration, both of

which are enhanced during storm periods, determines the resulting relativistic electron flux level in the Earth's outer radiation belt (RB) (e.g., Summers et al. 2004; Reeves et al. 2003; Green et al. 2004). During the main phase, the relativistic electron flux may decrease by up to two or three orders of magnitude. Analyzing 256 geomagnetic storms during the period 1989–2000, Reeves et al. (2003) found that 53% of the storms lead to higher flux levels during the storm recovery phase in comparison with prestorm levels, 28% produce no change, and 19% lead to net decrease in flux levels. The large electron flux decrease during the main storm phase is usually associated with either the D_{st} effect, when the relativistic electrons adiabatically respond to the inflation of the magnetic field lines caused by the formation of a partial RC (Kim and Chan 1997), and/or the drift out the magnetopause boundary (Li et al. 1997), and/or the nonadiabatic scattering into the loss cone due to cyclotron interaction with EMIC waves (Thorne and Kennel 1971; Lyons and Thorne 1972; Summers and Thorne 2003; Albert 2003; Thorne et al. 2005).

Precipitation of the outer RB electrons due to resonant pitch-angle scattering by EMIC waves is considered to be one of the more important loss mechanisms, so in the present section, we concentrate on this process only. This mechanism was suggested in early theoretical studies decades ago (Thorne and Kennel 1971; Lyons and Thorne 1972), however, direct experimental evidence of EMIC wave-induced relativistic electron precipitation is scanty because of a lack of concurrent measurements of low-altitude precipitating electrons and magnetically conjugate equatorial waves. Relatively recently, data from balloon-borne X-ray instruments provided indirect but strong evidence for EMIC wave-induced loss of outer-zone relativistic electrons in the late afternoon–dusk MLT sector (Foat et al. 1998; Lorentzen et al. 2000; Millan et al. 2002). These observations stimulated theoretical and statistical studies that demonstrated that this mechanism for MeV electron pitch-angle diffusion can operate at the strong diffusion limit, and can compete with relativistic electron depletion caused by the D_{st} effect during the initial and main phases of a storm (Summers and Thorne 2003; Albert 2003; Meredith et al. 2003; Loto'aniu et al. 2006).

Although the effectiveness of relativistic electron scattering by EMIC waves depends strongly on the wave spectral properties, unrealistic assumptions regarding the wave angular spread were made in earlier theoretical studies. That is, only field-aligned or quasi-field-aligned EMIC waves were considered as a driver for relativistic electron precipitation (except Glauert and Horne 2005, where a calculation for prescribed oblique wave distributions was presented for the H^+ -mode). At the same time, there is growing experimental (Anderson et al. 1996; Denton et al. 1996) and theoretical (Khazanov et al. 2006, 2007) evidence that EMIC waves can be highly oblique; EMIC waves occur not only in the source region, i.e., at small wave normal angles, but also in the entire region, even near 90° . This can dramatically change the effectiveness of relativistic electron scattering by EMIC waves. In the present section, we calculate the pitch-angle diffusion coefficients using the wave normal distributions provided by our self-consistent RC–EMIC wave model (Khazanov et al. 2006), and quantify the effect of oblique EMIC waves on outer RB relativistic electron scattering.

10.4.2 Pitch-Angle Diffusion Coefficients: Model Calculations

In our relativistic electron studies, we use the relativistic form of the diffusion coefficient from Khazanov et al. (2003a). The recent extensive statistical analysis of EMIC events by Meredith et al. (2003) showed that in approximately 11% of the observations, the minimum electron resonant energy fell below 2 MeV, and that most of these cases were associated with wave frequencies just below the helium gyrofrequency. Thus, in what follows, we take into account only the He^+ -mode of EMIC waves. Although the model by Khazanov et al. (2006) provides self-consistent spectra for the He^+ -mode, in order to eliminate an unnecessary complication, the analysis in this section is done for prescribed wave spectra and plasma parameters.

First, a Gaussian frequency spectrum,

$$B^2(\omega) \sim \exp\left\{-\frac{(\omega - \omega_m)^2}{\delta\omega^2}\right\}, \quad \omega_{LC} \leq \omega \leq \omega_{UC} \quad (10.4.1)$$

is assumed, where, following Summers and Thorne (2003) and/or Albert (2003), $\omega_{LC} = \omega_m - \delta\omega$, $\omega_{UC} = \omega_m + \delta\omega$, $\omega_m = 3\Omega_{\text{O}^+}$, and $\delta\omega = 0.5\Omega_{\text{O}^+}$, where Ω_{O^+} is the gyrofrequency of O^+ . Second, the wave normal angle distribution is assumed to be a constant inside a specified region and zero otherwise. Below we consider the following three cases,

$$\begin{aligned} \text{CaseA : } & 0^\circ \leq \theta < 30^\circ, \quad 150^\circ < \theta \leq 180^\circ \\ \text{CaseB : } & 30^\circ \leq \theta < 60^\circ, \quad 120^\circ < \theta \leq 150^\circ, \\ \text{CaseC : } & 60^\circ \leq \theta \leq 89^\circ, \quad 91^\circ \leq \theta \leq 120^\circ \end{aligned} \quad (10.4.2)$$

which allow us to model field-aligned, intermediate and highly oblique wave spectra. Note that the diffusion coefficient is a linear functional of the wave spectral intensity, and the sum of cases A–C describe a situation when EMIC wave energy is evenly distributed in the entire wave normal angle region, $0^\circ \leq \theta \leq 180^\circ$ (we excluded the region near 90° because of the Landau damping by thermal electrons; e.g., Thorne and Horne 1992; Khazanov et al. 2007). For benchmark purposes, we also calculate the diffusion coefficients for a Gaussian distribution over $\tan\theta$ ($0^\circ \leq \theta \leq 15^\circ$), which has been used by Albert (2003). In each case, the wave amplitude is normalized to ensure

$$\int_{\omega_{LC}}^{\omega_{UC}} d\omega \int_0^\pi d\theta B^2(\omega, \theta) = 1 \text{ nT}^2. \quad (10.4.3)$$

Finally, to specify the ion content, we follow Summers and Thorne (2003), Albert (2003), Meredith et al. (2003), Loto'aniu et al. (2006), and just prescribe the storm-time ion composition to be 70% H^+ , 20% He^+ , and 10% O^+ .

10.4.2.1 Equatorial Coefficient

To consider the effect of the wave normal angle distribution on the effectiveness of relativistic electron scattering by EMIC waves, we first calculate the local pitch-angle diffusion coefficient. Results of our calculation are presented in Fig. 10.9. The first row shows the local (equatorial) pitch-angle diffusion coefficients, and the second row shows the corresponding resonant numbers averaged with the following weights:

$$\langle n(E, \alpha) \rangle = \frac{\sum_n n \int_{\omega_{LC}}^{\omega_{UC}} d\omega \int_0^\pi d\theta D_{\alpha\alpha}^n(\omega, \theta, E, \alpha)}{\sum_n \int_{\omega_{LC}}^{\omega_{UC}} d\omega \int_0^\pi d\theta D_{\alpha\alpha}^n(\omega, \theta, E, \alpha)}, \quad (10.4.4)$$

where E and α are the electron kinetic energy and pitch angle, and $D_{\alpha\alpha}^n(\omega, \theta, E, \alpha)$ is the partial pitch-angle diffusion coefficient. Note that the resonances $\pm n$ come together because the ω -term can be omitted in the quasilinear resonance condition, $\omega - k_{\parallel}v_{\parallel} - n\Omega_e/\gamma = 0$ (e.g., Summers and Thorne 2003), and the wave spectra are symmetric around $\theta = 90^\circ$. The “Gauss” lines in Fig. 10.9 show the result of a Gaussian distribution over θ , and reproduce the equatorial diffusion coefficients of Albert (2003, Fig. 6).

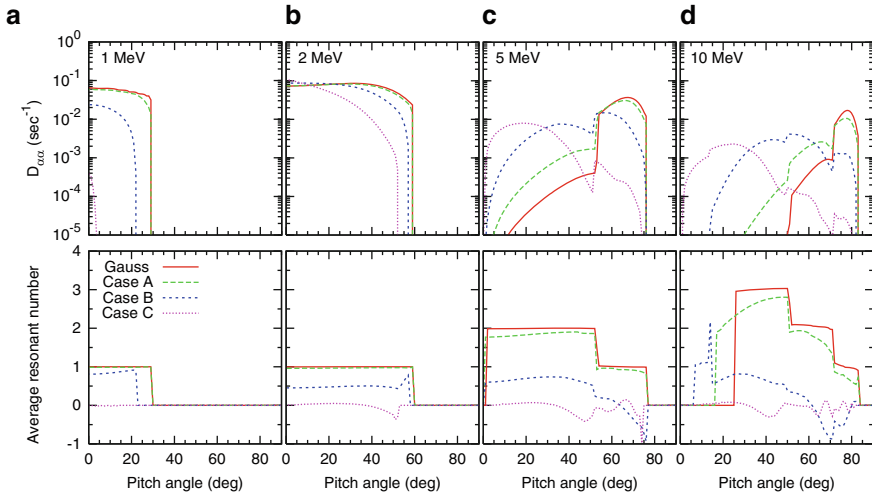


Fig. 10.9 Equatorial diffusion coefficients versus equatorial pitch angle for scattering relativistic electrons by the He^+ -mode of electromagnetic ion cyclotron (EMIC) waves. The wave spectrum parameters and ion content are given in the text, $L = 4$, and $(\omega_{pe}/\Omega_e)^2 = 10^3$, where ω_{pe} and Ω_e are the electron plasma frequency and gyrofrequency (without Lorentz factor), respectively. The curve “Gauss” is obtained for a wave normal angle distribution adopted by Albert (2003). The second row shows the corresponding average resonant numbers (see the text for definition)

For all energies, case A is slightly less than “Gauss” if only $|n| = 1$ resonances operate, but in the region of $|n| > 1$ it is approximately five times greater than “Gauss” (Fig. 10.9c, d). These dependencies are in good agreement with the previous results of Albert (2003, Fig. 10). For both “Gauss” and case A, as follows from the second row in the Fig. 10.9, the contribution from $n < 0$ is negligible compared with the contribution from $n > 0$, especially for lower electron energies (see Fig. 10.9a, b). Cases B and C further increase the EMIC wave normal angle, which further suppress the resonances $|n| = 1$, and shrink the region of pitch angles subject to diffusion for low energies (see Fig. 10.9a, b). At the same time, they increase by orders of magnitude the contribution from $|n| > 1$, which operate for greater electron energies, and increase the pitch-angle region subject to diffusion (see Fig. 10.9c, d). The growing contribution of resonances with $n < 0$ is more pronounced in Cases B and C because EMIC waves become more elliptically polarized with the increase in the wave normal angle. The above results are in good qualitative agreement with the results by Glauert and Horne (2005) obtained for the H^+ -mode of EMIC waves.

Overall, compared with field-aligned waves, the intermediate and highly oblique wave distributions decrease the pitch-angle range subject to diffusion, and reduce the scattering rate by orders of magnitude for low-energy electrons ($E < 2$ MeV) when only principal $|n| = 1$ resonances operate. For greater electron energies (see Fig. 10.9c, d), the $|n| = 1$ resonances operate only in a narrow region at large pitch angles, and despite their greatest contribution for the field-aligned waves, cannot support electron diffusion into the loss cone. In this case, the oblique waves with $|n| > 1$ resonances are more effective, and extend the range of pitch-angle diffusion down to the loss cone.

10.4.2.2 Bounce-Averaged Coefficient

To calculate the bounce-averaged diffusion coefficients, we use all of the plasma/wave parameters used in Sect. 10.4.2.1, and, in addition, a dipole magnetic field model, and the meridional density distribution from (Khazanov et al. 2006). We further assume that the EMIC waves are distributed latitudinally along the entire magnetic field line, and the wave spectra are equatorial. Results of our calculation are presented in Fig. 10.10 in which the “Gauss” lines reproduce the bounce-averaged diffusion coefficients from Albert (2003, Fig. 6).

In all considered cases, the bounce averaging does not change the shape of the diffusion coefficients for energies 1 and 2 MeV (compare Figs. 10.9a, b and 10.10a, b) but simply reduces the pitch-angle diffusion rates by approximately an order of magnitude. For energies 5 and 10 MeV, the peak values of the coefficients in Fig. 10.10c, d are lower by approximately a factor of 3 than in Fig. 10.9c, d. However, the bounce-averaged results for $E > 2$ MeV differ qualitatively from the local coefficients for all wave normal distributions in Fig. 10.9c, d. Owing to significant scattering at higher latitudes, the bounce-averaged diffusion coefficients are extended down to the equatorial loss cone compared with the equatorial results.

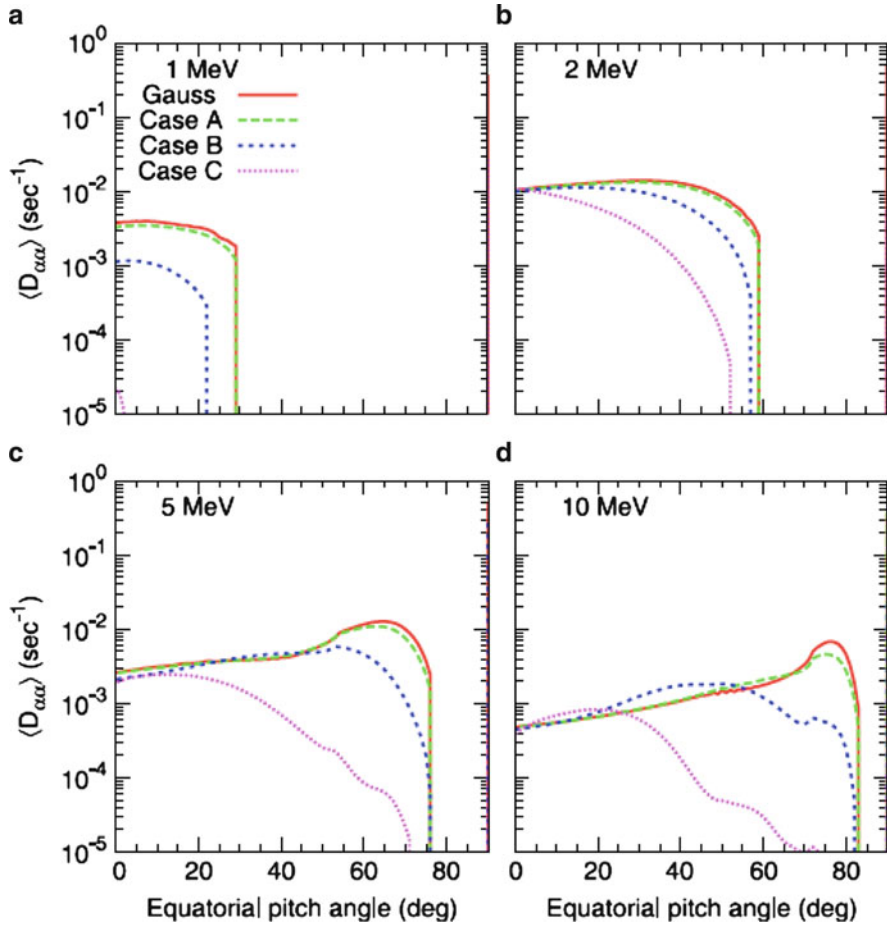


Fig. 10.10 Bounce-averaged diffusion coefficients versus equatorial pitch angle for scattering relativistic electrons by the He^+ -mode of EMIC waves. All of the plasma/wave parameters are the same as in Fig. 10.9, the magnetic field is a dipole, and the waves are confined to mirror points

The bounce-averaged results in Fig. 10.10 demonstrate clearly the effect of EMIC wave normal angle distribution on relativistic electron scattering.

Recently, Shprits et al. (2006) showed that electron lifetimes are most sensitive to the value of the pitch-angle scattering rate near the edge of the equatorial loss cone. Usually, that value is used to estimate the electron loss timescale (e.g., Summers et al. 2007). Considering Fig. 10.10a, b, we can see that the intermediate and highly oblique wave distributions reduce the scattering rate near the loss cone by up to orders of magnitude because only principal $|n| = 1$ resonances operate. For higher electron energies (Fig. 10.10c, d), when $|n| > 1$ resonances start to operate, the pitch-angle scattering near the edge of the equatorial loss cone depends only slightly on the wave normal angle distribution, resulting in nearly the same

diffusion rate for all cases. In other words, there is an electron energy depending on specified plasma/wave parameters, which separates the lower and higher energy regions with different wave scattering properties. The field-aligned wave normal angle distribution leads to a significant overestimate of the diffusion rate compared with oblique waves in the lower-energy region. In the higher-energy region, the scattering rate near the edge of the loss cone almost does not depend on the wave normal angle distribution.

10.4.3 Bounce-Averaged Diffusion Coefficient: Self-Consistent Calculations

10.4.3.1 Wave Normal Angle Distributions for He^+ -Mode of EMIC Waves

To analyze the wave normal angle characteristics, in this section, we use the results from a self-consistent theoretical model of RC and EMIC waves by Khazanov et al. (2006). The model is governed by a set of quasilinear and ray-tracing equations, which explicitly includes the wave generation and damping, propagation, refraction, and reflection/tunneling in a multi-ion magnetospheric plasma. From a simulation of the May 1998 storm, Khazanov et al. (2006) found that the equatorial He^+ -mode energy distributions are not Gaussian over the equatorial wave normal angle, θ_0 , and that the wave energy can occupy not only the source region, i.e., the region of small wave normal angles, but all wave normal angles, including those near 90° . This is caused by energy outflow from the region of small wave normal angles to $\theta_0 = \pi/2$, which is due to the wave bouncing between surfaces of the bion hybrid frequency in opposite hemispheres. Because the EMIC wave growth rate maximizes for the wave normal angle $\theta_0 = 0$, and because electron Landau damping has a peak for θ_0 close to 90° , the resulting wave normal angle distribution depends on ratios between the rates of wave growth (mostly in the region of small θ_0), Landau damping (mostly at large θ_0), and energy outflow rate, $\dot{\theta}_0/\theta_0$. Figure 9.18 shows the energy distribution over the equatorial wave normal angle for the He^+ -mode EMIC waves. All the magnetic field spectra shown are in the postnoon–dusk MLT sector, 48 h after 0000 UT on 1 May 1998. Case A demonstrates a typical quasi-field-aligned wave normal angle distribution, where wave growth rate in the region of small θ_0 dominates the outflow toward greater θ_0 . The diametrically opposite case is given by line *c*, where EMIC wave energy is concentrated in the region of large θ_0 . An intermediate case B corresponds to a situation when all the timescales have the same order of magnitude. Although power spectral density in that case drops for $\theta_0 > 40^\circ$, there is still a very large $B^2(v, \theta_0)$, and we observe a broad distribution in the entire wave normal angle region.

Figure 9.18 shows spectra at one time and at three spatial points only, but it would be interesting to see the wave normal angle distributions on global spatial

and temporal scales. In order to provide such a global view during the May 1998 storm, we calculated the average equatorial wave normal angle presented in Fig. 9.19.

As follows from Fig. 9.19, the highly oblique waves with $\langle\theta_0\rangle > 50^\circ$ are mainly observed in the noon–dusk MLT sector for high L-shells (in the plasmaspheric drainage plume), and an extremely oblique wave propagation with $\langle\theta_0\rangle > 80^\circ$ is found in hour 33 ($L = 6.25$, $MLT = 14$) and 34 ($L = 5.75$, $MLT = 13$) snapshots. Although events with $\langle\theta_0\rangle < 50^\circ$ are not well separated spatially from oblique waves, there is a tendency for them to be localized preferentially along the more narrow nightside plasmopause (compare Fig. 9.19 with the density distribution in Fig. 7 of Khazanov et al. 2006), especially for field-aligned events with $\langle\theta_0\rangle < 30^\circ$ (see the first row in Fig. 9.19). The occurrences of the oblique and field-aligned wave normal angle distributions appear to be nearly equal during the May 1998 storm. It is interesting to note that among 830 EMIC events observed on the Combined Release and Radiation Effects Satellite and analyzed by Meredith et al. (2003), 414 events had ellipticity $|\varepsilon| < 0.3$, i.e., were likely oblique.

The theoretical results clearly demonstrate that storm-time EMIC wave normal angle distributions are highly variable both in space and time, and that equatorial distributions range from field-aligned distributions through highly oblique distributions, which are in qualitative agreement with the results of Anderson et al. (1996), Denton et al. (1996), and Meredith et al. (2003).

10.4.3.2 Diffusion Coefficient

To compare with Sect. 10.4.2, we now calculate the bounce-averaged pitch-angle diffusion coefficients using the plasma and wave parameters from the self-consistent model of Khazanov et al. (2006). In order to calculate the diffusion coefficients, we use the simulation results at 48 h after 0000 UT on 1 May 1998 and select the L-MLT locations presented in Fig. 9.18. Note that Fig. 9.18 shows wave spectra for three frequencies only, with wave energy at lower frequencies in locations $L = 5.25$, $MLT = 16$ and $L = 5.75$, $MLT = 15$. Although these additional spectra have wave normal angle distributions similar to Fig. 9.18, we use the total power spectral densities in the following calculations (the squared magnetic fields are $B_{(a)}^2 = 28.6 \text{ nT}^2$, $B_{(b)}^2 = 41.6 \text{ nT}^2$, and $B_{(c)}^2 = 16.3 \text{ nT}^2$ at those L-MLT locations). The ion composition used by Khazanov et al. (2006), 77% H^+ , 20% He^+ , and 3% O^+ , will be used below. At selected points, the equatorial values of $(\omega_{pe}/\Omega_e)^2$ are in the range 96–146. Thus, we expect the electron minimum resonant energy, which depends on $(\omega_{pe}/\Omega_e)^2$, and, for the He^+ -mode, on concentration of He^+ (Summers and Thorne 2003), to be greater than in Fig. 10.9. The results are shown in Fig. 10.11. While not as impressive as Fig. 10.10, it has the advantage of being self-consistent. As we anticipated, the oblique lowest frequency wave distribution in Fig. 9.18c cannot scatter electrons with energies below 10 MeV, and spectra in Fig. 9.18a, b scatter only the electrons with energies near 5 MeV and above.

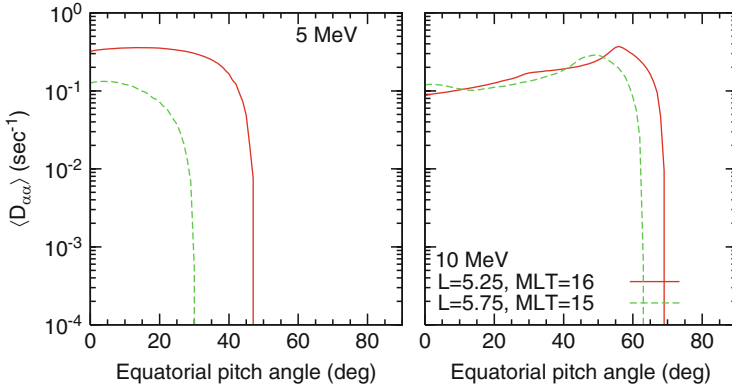


Fig. 10.11 Bounce-averaged diffusion coefficients for relativistic electron scattering by the He^+ -mode of EMIC waves. The wave spectra are taken from the simulation by Khazanov et al. (2006) and shown in Fig. 9.18

In contrast to Fig. 10.10, the difference between the red and green lines in Fig. 10.11 is rather due to different plasma/wave parameters than due to different wave normal angle distributions. Despite that, all the curve shapes in Fig. 10.11 can be readily found in Fig. 10.10. Analyzing Figs. 10.9–10.11 together, we see that in all cases presented in Fig. 10.11, the principal $n = 1$ resonance determines the diffusion coefficients for all of the equatorial pitch angles, except Fig. 10.11b, where $|n| > 1$ resonances slightly dominate around the 30° pitch angle for the red line, and near the loss cone for the green one.

10.4.4 CRRES-Based Calculations

10.4.4.1 Minimum Resonant Energy

Meredith et al. (2003) presented an extensive statistical analysis of more than 800 EMIC events observed on CRRES to establish whether electron scattering can occur at geophysically interesting energies (≤ 2 MeV). In the absence of specific information on the wave-normal angle, the dispersion relation for strictly field-aligned propagating EMIC waves was used to obtain the electron resonant energy. For consistency, Meredith et al. (2003) included only waves with a high ellipticity ($|\varepsilon| \geq 0.3$) in the survey. This yielded a subset of 416 events, the majority of which were identified as L-mode. Considering only the central wave frequency, ω_m , in each wave packet, Meredith et al. (2003) found that in approximately 11% of the observations, the electron minimum resonant energy fell below 2 MeV. These cases were restricted to regions where $\omega_{pe}/\Omega_e > 10$ and were associated with wave frequencies just below the helium or proton gyrofrequencies. More recently, trying to increase the above percentage, Loto'aniu et al. (2006) considered the entire

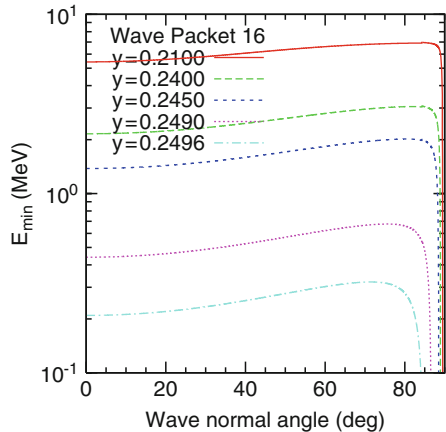
frequency range for each of the 25 EMIC wave packets observed on CRRES during the initial phase of a geomagnetic storm on 11 August 1991. These authors also used the dispersion relation for strictly parallel propagating EMIC waves and found that, in comparison with results using ω_m only, there are three to four times more wave packets that are able to interact with electrons below 2 MeV.

The minimum resonant energy depends on the wave normal angle, and the dependency is stronger in the vicinity of the resonant frequencies where the wave number grows especially fast. Omitting the ω term in a quasilinear resonance condition ($\omega - k_{\parallel}v_{\parallel} - n\Omega_e/\gamma = 0$) and taking $n = 1$, we can obtain the minimum kinetic energy required by electrons for cyclotron resonance interaction with EMIC waves,

$$\frac{E_{min}}{m_e c^2} = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} - 1, \quad \left(\frac{v}{c}\right)^2 = \frac{1}{1 + \cos^2 \theta \left(\frac{kc}{\Omega_e}\right)^2}, \quad (10.4.5)$$

where E_{min} is the minimum kinetic energy, m_e is the electron rest mass, c is the speed of light, and k and v are the wave number and electron velocity. Note that (10.4.5) can be obtained from equation (7) of Summers and Thorne (2003) by omitting the two smallest terms in their equation. To calculate the electron minimum energy, we select the plasma parameters reported by Loto'aniu et al. (2006, wave packet 16), and the results of our calculation are presented in Fig. 10.12. For $\theta = 0^\circ$, as reported in many previous studies (e.g., Summers and Thorne 2003), in order to get lower E_{min} , the required wave frequency has to be closer to the He^+ gyrofrequency (in other words, the wave number should be greater). For most wave-normal angles, increasing the angle slightly also increases the minimum energy, but there is a dramatic decrease of E_{min} in the region near $\theta = 90^\circ$. This transition boundary depends on the wave frequency. Indeed, there is a resonant

Fig. 10.12 Minimum resonant energy versus normal angle of the He^+ -mode EMIC waves. The plasma density and magnetic field are 17 cm^{-3} and 171 nT, taken from the work of Loto'aniu et al. (2006, wave packet 16). The ion composition is 70% H^+ , 20% He^+ , and 10% O^+ , and the normalized wave frequency is defined as $y = \omega/\Omega_{\text{H}^+}$



wave-normal angle (the angle at which the wave number becomes infinite in the “cold plasma” approximation) for any frequency in the range between Ω_{He^+} and the corresponding bi-ion frequency, and this angle is closer to $\theta = 0^\circ$ if the wave frequency is closer to Ω_{He^+} . Because of the wave number increase, the resonant energy decreases dramatically in the vicinity of the resonant wave-normal angle, an effect clearly observed in Fig. 10.12. Thus, in cold plasma, $E_{\min}$ is lower for oblique or highly oblique wave propagation, depending on wave frequency, than for strictly field-aligned propagating EMIC waves. But, of course, the diffusion coefficient for those wave-normal angles should be significant in order to determine the “physically meaningful” $E_{\min}$, and moreover the cyclotron damping in the vicinity of the He^+ gyrofrequency can be very strong (see below).

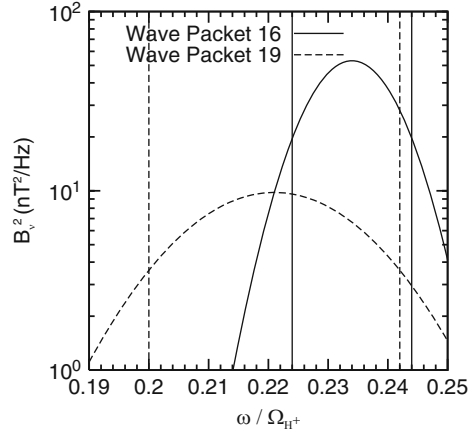
10.4.4.2 Pitch-Angle Diffusion Coefficient

It was demonstrated in Sect. 10.4.4.1 that oblique wave propagation can strongly change the effectiveness of both the local and bounce-averaged relativistic electron scatterings. At the same time, those results were obtained for plasma parameters and wave spectra, which were specified independently. Thus, it is important to consider the effect of using concurrent observational data. In contrast to the previous section, we now calculate the local pitch-angle diffusion coefficients using the data for plasma and wave parameters reported by Loto’aniu et al. (2006).

A long-duration wave event was observed by CRRES on 11 August 1991, in the interval 0500–0700 universal time (UT) (14.4–15.8 MLT), over a magnetic latitude range of -26° to -24° and $L = 6.3$ – 7.6 . CRRES was close to apogee in the plasmatrough, and the electron density varied slowly from 12 to 17 cm^{-3} . A total of 25 EMIC wave packets were identified both below and above the local He^+ gyrofrequency (Loto’aniu et al. 2006). In order to estimate the spectral properties of the wave packets, these authors fitted a Gaussian distribution to the static wave packet transverse power spectral density. Typical FFT data windows and frequency resolutions for the static spectrograms were 100 s and 0.02 Hz, respectively. The Gaussian function fit provided the central frequencies, ω_m , and the spectral semi-bandwidths, $\delta\omega$. The total wave magnetic power, δB^2 , was estimated for each wave packet by summing the power spectral density bins in the range $\omega_m \pm \delta\omega$ and then multiplying the result by $\delta\omega$. Using the full wave spectral range, Loto’aniu et al. (2006) found that electrons with $E \leq 2 \text{ MeV}$ could interact with only three wave packets (16, 17, and 19) if storm-time ion concentration was assumed (70% H^+ , 20% He^+ , and 10% O^+). Those packets were the He^+ -mode EMIC waves, and, for the calculation below, we selected two of them. The associated plasma and wave characteristics are summarized in Table 10.2. Note that to generate this table, we used the definition of full width at half maximum (FWHM) as it was given by Loto’aniu et al., that is, $\text{FWHM} = 2\sqrt{2\ln 2}\delta\omega$, despite the Gaussian fit $\sim \exp\{-(\omega - \omega_m)^2/\delta\omega^2\}$. Of the packets 16, 17, and 19, wave packet 16 has

Table 10.2 Wave packet and local environment properties selected from the work of Loto'aniu et al. (2006)

Wave packet	$y_m = \omega_m / \Omega_{H^+}$	$\delta y = \delta\omega / \Omega_{H^+}$	$y_{LC} = y_m - \delta y$	$y_{UC} = y_m + \delta y$	δB^2 (nT ²)	B_o (nT)	N_e (cm ⁻³)
16	0.23	0.01	0.22	0.24	2.21	170.9	17
19	0.22	0.02	0.20	0.24	0.84	160.2	15

Fig. 10.13 Transverse power spectral densities for wave packets 16 and 19 obtained by Loto'aniu et al. (2006). The solid and dashed vertical lines restrict the frequency range $\omega_m \pm \delta\omega$ for packets 16 and 19, respectively

the most narrow distribution and 19 has the widest distribution, with corresponding power spectral densities presented in Fig. 10.13.

To show the effect of the wave-normal angle distribution on relativistic electron scattering, we use the wave-normal angle distributions from (10.4.2), and in addition, a storm-time ion concentration is assumed. For reference purposes, we also calculate the diffusion coefficients for strictly parallel/antiparallel propagating EMIC waves. For each wave packet, the power spectral density is normalized to the corresponding wave magnetic power δB^2 shown in Table 10.2, and this normalization is kept the same for any particular wave-normal angle distribution (see (10.4.2)). In order to estimate the minimum resonant energy, we use y_{UC} from Table 10.2. For strictly field-aligned wave propagation, as follows from Fig. 10.12, the energy is approximately 2 MeV for both wave packets (we can use Fig. 10.12 for wave packet 19 because ω_{pe}/Ω_e was nearly the same during both). This minimum resonant energy exceeds the values presented by Loto'aniu et al. (2006), especially for wave packet 16; for this packet and a storm-time ion concentration, they obtained $E_{min} = 0.2\text{MeV}$ that, as follows from Fig. 10.12, corresponds to a y_{UC} about 0.2496.

Figure 10.14 shows the results of our calculation for wave packet 16. For strictly parallel wave propagation, the minimum resonant energy is only slightly below 2 MeV, and the diffusion coefficients for field-aligned and intermediate wave

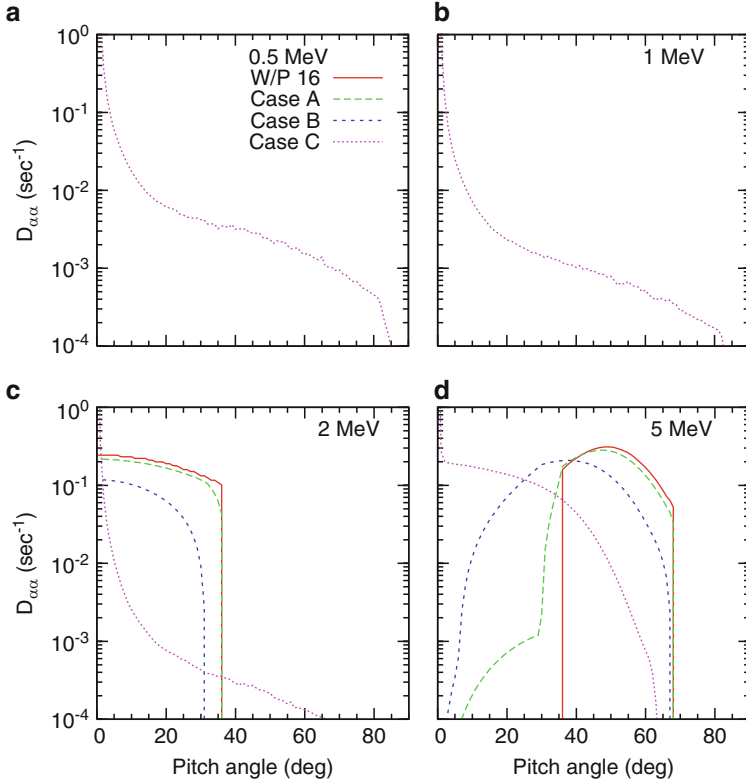


Fig. 10.14 Local pitch-angle diffusion coefficients for wave packet 16. Calculations are based on a storm-time ion composition. “W/P 16” shows the results for strictly parallel–antiparallel propagating He^+ -modes, and Cases A–C are obtained for the corresponding wave-normal angle distribution given by (10.4.2)

propagation are only nonzero in Fig. 10.14c, d. Cases A and B demonstrate results similar to Fig. 10.9b, c. Because y_{UC} is very close to the He^+ gyrofrequency, the minimum resonant energy falls below 1 MeV if the wave-normal angle exceeds 88° , so that case C may potentially scatter such low-energy electrons with an appreciable rate as shown in Fig. 10.14a, b. Another feature of highly oblique waves is clearly observed in Fig. 10.14d, where the range of pitch-angle diffusion is substantially extended down to the loss cone. Although case C exhibits a quite different behavior compared with Fig. 10.9, there is a similarity between the diffusion coefficients in Figs. 10.9c and 10.14d.

The diffusion coefficients for wave packet 19 are shown in Fig. 10.15. Both Fig. 10.15c, d are quite similar and demonstrate qualitatively the same behavior as in Fig. 10.9a, b. As follows from Fig. 10.15a, b, case C practically does not scatter low-energy electrons, mainly because of a lower y_{UC} for wave packet 19 than in Fig. 10.14.

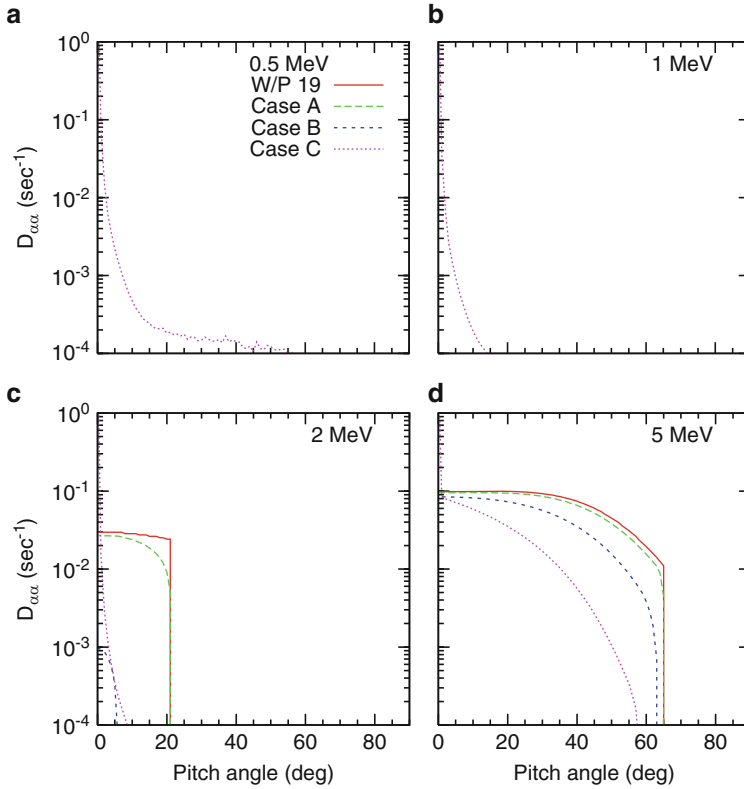


Fig. 10.15 Same as Fig. 10.14, except for wave packet 19

10.4.4.3 Cyclotron Damping Near He^+ Gyrofrequency and Its Consequence for Electron Scattering

As follows from Table 10.2, y_{UC} is very close to the local He^+ gyrofrequency ($\nu_{\text{He}^+} = 0.25$) for both wave packets. In this frequency region, the He^+ -mode experiences strong cyclotron damping due to interaction with thermal He^+ (e.g., Akhiezer et al. 1975). To demonstrate this, Khazanov and Gamayunov (2007a) assumed the He^+ temperature to be $T_{\text{He}^+} = 1\text{eV}$, and calculated the wave damping rate for the storm-time ion composition and plasma parameters observed during wave packets 16 and 19. For example, for a packet 16, the damping rate for y_{LC} has only a narrow peak for $\theta > 89^\circ$, and this region is excluded from the calculation of the diffusion coefficients (see (10.4.2) and Khazanov and Gamayunov 2007b for the details). For y_m , the region of damping near 90° extends slightly below 89° , and, in addition, small damping appears for a field-aligned wave propagation. The situation becomes dramatically different for y_{UC} when the He^+ -mode experiences strong damping in the entire wave-normal angle region; the energy damping time is

$0.5/\gamma_{\text{He}^+} \approx 7s$, which is only four times greater than the wave period. In all cases, substantial damping takes place only if $|y - 0.25| \leq k_{\parallel} v_{\parallel, \text{He}^+} / \Omega_{\text{H}^+}$, where $v_{\parallel, \text{He}^+}$ is the field-aligned temperature of He^+ . Moreover, we use a “cold plasma” approximation (see Chap. 3 for details) in our diffusion coefficient software (as was done by Loto’aniu et al. 2006), some must check the validity of this approximation. Particularly, the inequality

$$|y - 0.25| > \frac{k_{\parallel} v_{\parallel, \text{He}^+}}{\Omega_{\text{H}^+}} = \varepsilon_{th} \quad (10.4.6)$$

should hold.

Inequality (10.4.6) is extremely crucial for the diffusion coefficient calculation because thermal effects should be considered if inequality (10.4.6) is violated, but more importantly, the He^+ -mode damps strongly in the region $|y - 0.25| \leq \varepsilon_{th}$. For wave packets 16 and 19, inequality (10.4.6) is strongly violated in the vicinity of y_{UC} , and waves cannot exist in these frequency regions, which for $T_{\text{He}^+} = 1\text{eV}$ are the ranges $\varepsilon_{th} = 5 \times 10^{-3} - 9 \times 10^{-2}$ and $\varepsilon_{th} = 3 \times 10^{-3} - 6 \times 10^{-2}$, respectively. Using these numbers and Table 10.2, we conclude that, in order to suppress cyclotron damping completely, the He^+ temperature should be decreased at least by 1/80 for wave packet 16, and at least by 1/40 for wave packet 19. Any reasonable change to the temperature assumed in our calculation cannot eliminate the effect and can only influence the frequency range subject to cyclotron damping.

In conclusion, we emphasize that, as we demonstrated above, the He^+ -mode does not experience significant cyclotron damping by thermal He^+ if $y \leq y_m$, and, one should be always careful with the frequency range selection for the diffusion coefficients calculation. This result is consistent with the conclusions of Meredith et al. (2003) regarding the electron minimum resonant energy that were obtained by considering only the central wave packet frequencies, and suggests that the number of EMIC wave packets that are able to interact with electrons below 2 MeV may significantly decrease compared with the estimate of Loto’aniu et al. (2006).

10.5 The Nonlinear Coupling of Electromagnetic Ion Cyclotron and Lower Hybrid Waves in the Ring Current Region

10.5.1 Lower Hybrid Waves

Wave–particle and wave–wave interactions are the crucial elements of magnetosphere and ionosphere plasma dynamics. Such interaction provides a channel of energy redistribution between different plasma populations, and leads to connection between physical processes developing on the different spatial and temporal scales. The low-hybrid waves are particularly interesting for plasma dynamics, because they couple well with both electrons and ions. Various mechanisms for LHW

excitation have been studied as well as the phenomena produced by such waves (Davidson et al. 1977; Sonnerup 1980; Chang and Coppi 1981; Bingham et al. 1984; Gurnett et al. 1984; Ganguli and Palmadesso 1987; Pottelette et al. 1990; Omelchenko et al. 1994). In some cases, the LHW activity in magnetosphere has been observed simultaneously with low-frequency waves (LFWs) (LaBelle et al. 1988; Pottelette et al. 1990; McFadden et al. 1998). Such simultaneous wave activity also has been observed in active ionospheric sounding rocket experiments (Arnoldy 1993; Bale et al. 1998). One reason for this may be that there is a common source for both waves. Another possible explanation is LHW generation due to the LFW activity (Khazanov et al. 1996, 1997a, b).

It is well known in the plasma physics that the LFWs with frequencies $\omega < \Omega_i$ (where Ω_i is the ion cyclotron frequency) could drive a host of high-frequency waves through the drifts of the plasma particles produced by the former waves. LHW excitation is possible due to any transverse electric field with a frequency comparable to the ion cyclotron frequency, such as the fields of ion cyclotron, Alfvén, and fast magnetosonic waves. These waves are frequently observed in the ionosphere and plasmasphere (LaBelle et al. 1988; Pottelette et al. 1990; Erlandson et al. 1990; Fraser et al. 1992; Anderson et al. 1992a, b). The study by Khazanov et al. (1997a) paid attention to the special role of heavy ions in this mechanism due to the fact that the particle drift velocity in the LFW electric field is mass dependent. They showed that the relative drift between the electrons and heavy ions across the ambient magnetic field (from the observed amplitudes of Alfvén waves) is sufficiently large to drive LHWs in magnetospheric plasma. The idea has been adopted to explain some observations in the space plasma (Khazanov et al. 1997b) and active ionospheric sounding rocket experiments (Bale et al. 1998).

In particular, this mechanism of LHW generation has been applied to the ring current region of the magnetosphere (Khazanov et al. 1997a, 2000) and will be presented here in association with EMIC waves that were discussed in the previous sections.

Four shapes of the wave energy spectral density distribution can be identified from the modeling results (Khazanov et al. 2003a), three single-peaked distributions (symmetric, and right and left sides extended); and double-peaked distributions. The last shape was found in 17% of analyzed waveforms and the peaks frequency separation is approximately 40% of the spectrum width. (A more detailed description of the EMIC waves can be found in the paper by Khazanov et al. 2003a.) We will approximate all of the distributions by Gaussian distributions instead of more accurate cubic splines (Khazanov et al. 2003a). Such an approximation overestimates the peak, not more than 20%, and underestimates the width of the spectrum. Because the LHW generation depends on the particles drift velocity and, therefore, is proportional to the square root of the wave energy density, the corresponding 10% overestimation will be neglected in the following analyses as well as the difference in the spectrum width. The difference in the spectrum width is not important because of the low field amplitudes at the wings of the spectrum. Therefore, we assume that the magnetic field energy spectral density of EMIC waves distribution is Gaussian:

$$B_{\omega}^2 = C \exp \left[-\frac{(\omega - \langle \omega \rangle)^2}{2D^2} \right] = C \exp \left[-\frac{(1-x)^2}{2d^2} \right], \quad (10.5.1)$$

where B_{ω}^2 is the wave magnetic field energy spectral density; $\omega, \langle \omega \rangle$ are the current and mean frequencies, and $x = \omega/\langle \omega \rangle, d = D/\langle \omega \rangle$, are the normalized frequency and the standard deviation. The normalized standard deviations for the different shapes of distribution listed above are $d = 0.10, 0.13, 0.14$, and 0.19 . The corresponding ratios of the mean frequency to the hydrogen gyrofrequency $\langle \omega \rangle/\Omega_H$ are $0.51, 0.57, 0.50$, and 0.55 .

10.5.2 Generation of Lower Hybrid Waves

Particles of plasma subjected to the LFW with the electric field perpendicular to the ambient magnetic field are drifting with the mass dependent velocities. The difference of these velocities can cause beam instability in such a system, in particular, the LHW generation. To calculate the particle drift velocity, the LF wave electric field is needed. Because the EMIC waves were calculated in quasilinear approximation, the energy density distribution of these waves is averaged over time, and is large in comparison with the wave period. This time average is equivalent to the average over the statistical ensemble, where different realizations are defined by some set of phase, randomly changing from one realization to another. Although the wave field exhibits randomness on the timescale large in comparison with the wave period, it is still regular on the time intervals smaller or around the period of the EMIC wave (Tsytovich 1970). The frequency of the LHWs is approximately $\sqrt{\Omega_H \Omega_e}$ (Ω_H, Ω_e are the hydrogen and electron gyrofrequencies, respectively), and their period is much smaller than the EMIC waves period. We restrict our analyses by such cases that the LHW's growth rate is also comparable to the frequency of EMIC waves and, therefore, the time of LHW's excitation is comparable to the EMIC waves period. In fact, we will seek such cases that the LHW growth rate is at least five times greater than the EMIC wave frequency. Therefore, for such LH waves, the characteristic time of generation is essentially smaller than the characteristic time at which the stochastic changes of EMIC waves takes place. Under this condition, the EMIC wave field can be considered as regular for the drift velocity calculations. Based on the results presented in previous section, we will assume that the EMIC wave is left-hand polarized, with the wave vector directed along the ambient magnetic field (axis z). Fourier components of the electric and magnetic fields of this wave are:

$$\begin{aligned} \mathbf{E}_{\omega} &= \frac{b_{\omega}}{N_{\omega}} (\mathbf{e}_x - i\mathbf{e}_y) e^{-i\omega t + ik(\omega)z}, \quad \mathbf{B}_{\omega} = b_{\omega} (\mathbf{e}_x - i\mathbf{e}_y) e^{-i\omega t + ik(\omega)z}, \\ N_{\omega}^2 &= 1 - \sum \frac{\omega_{pz}^2}{\omega(\omega - \Omega_{\alpha})}, \quad \omega_{pz}^2 = \frac{4\pi n_{\alpha} e^2}{m_{\alpha}}, \quad \Omega_{\alpha} = \frac{e_{\alpha} B}{m_{\alpha} c}, \quad \alpha = e, H^+, He^+, O^+ \end{aligned} \quad (10.5.2)$$

Here N_{ω}^2 is the plasma refractive index for the EMIC waves, and n_{α} are the plasma component densities. We also will assume, taking into account (10.5.1), that

$$b_{\omega} = A \exp \left[-\frac{(\omega - \langle \omega \rangle)^2}{4D^2} \right], \quad (10.5.3)$$

where the coefficient A is found from the condition:

$$B^2 = \left(\int_{-\infty}^{+\infty} b_{\omega} d\omega \right)^2 \quad (10.5.4)$$

and B^2 is the EMIC wave magnetic field energy density calculated from the numerical model presented in Fig. 10.16. Neglecting by the EMIC wave magnetic field, the relative velocity of an ion species α , with respect to the electrons, can be calculated as (Khazanov et al. 1997a):

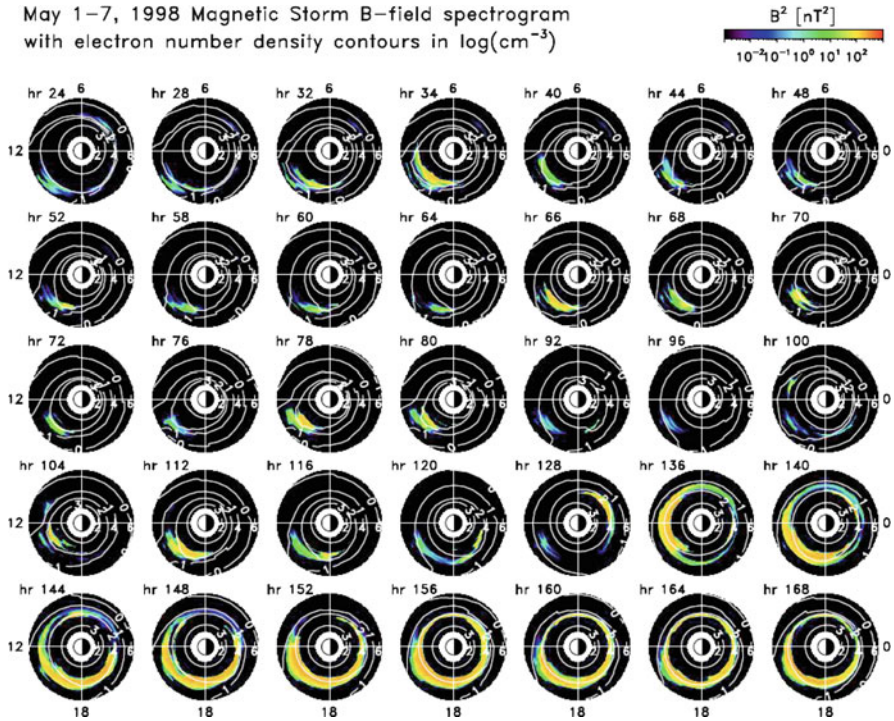


Fig. 10.16 Evolution of the squared EMIC wave magnetic field and core plasma density in (MLT, L-shell)-space during the May 2–7, 1998 storm period

$$\mathbf{u}_\alpha = \frac{e}{m_H \Omega_H} \int_{-\infty}^{+\infty} \frac{\omega \mathbf{E}_\omega}{(\omega - \Omega_\alpha)} d\omega. \quad (10.5.5)$$

In our calculations, we used the electric field two times smaller than given by (10.5.2). Such field reduction according to the estimations essentially overlaps the errors related to the use of different approximations in numerical calculation of EMIC waves magnetic field energy density (Khazanov et al. 2002, 2003a) and the calculation of corresponding electric fields and drift velocities in this work. It can be found (after substitution in (10.5.5) of the EMIC electric field (10.5.2)), that the relative ion velocity for fixed plasma composition depends on parameters: m_H/m_α , $\langle\omega\rangle/\Omega_H$, d , and B^2/n . For a fixed point along the magnetic field line, $z = 0$, the results of the velocity calculation for the hydrogen, helium, and oxygen ions are presented in Fig. 10.17. They are calculated for the Gaussian EMIC wave energy distribution with the normalized standard deviation $d = 0.10$ and the mean frequency $\langle\omega\rangle = 0.51\Omega_H$. The figure presents the ion's velocity due to an initially created set of waves with the initial energy density equal to the average density at a fixed point in space. Because the set of waves includes different frequencies, the ion's velocity amplitude slowly diminishes. Such a slow rate of amplitude change means that the field action on the ion is close to the action of a monochromatic wave. Calculation for the monochromatic wave with the mean

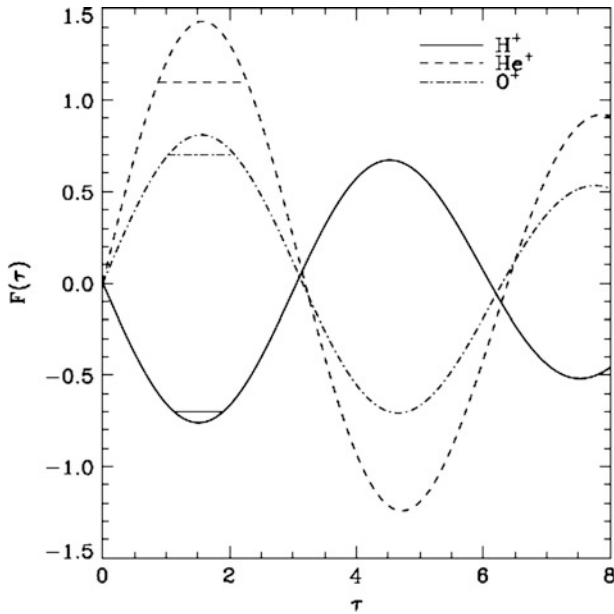


Fig. 10.17 Normalized ion drift velocity $u_{\alpha x}/\sqrt{2}v_{TH} = F(\tau)\sqrt{B^2/n}$ as a function on normalized time $\tau = t\langle v \rangle$

frequency leads to ion velocity amplitudes approximately 15% higher than presented in Fig. 10.17. For all four shapes of EMIC waves energy density distribution, when the Gaussian approximation of the shape is used, the relative ion's velocities, calculated using (10.5.5), are very close. The velocities dependence on the ambient magnetic field through the parameter $\langle\omega\rangle/\Omega_H$ (see (10.5.3)–(10.5.5)) in the domain of interest, where the EMIC activity is high, is also weak. This permits us below to use the velocities averaged over the corresponding parameters $\langle\omega\rangle/\Omega_H$, d with an accuracy of approximately 15%. The main dependence of the ion relative velocity is the dependence on the EMIC waves magnetic field energy and plasma densities and, as can be found from (10.5.5), has the form $u_{\alpha x}/\sqrt{2}v_{TH} = F(\tau)\sqrt{B^2/n}$, where the coefficient of proportionality, $F(\tau)$, presented by Fig. 10.17, is dependent on the m_H/m_α ratio. The dependence on n for the drift velocity is because the electric field of EMIC waves is expressed with the help of the wave magnetic field and the phase velocity, which depends on the plasma density. These results, presenting the ion relative velocity with respect to the electrons, will be used to analyze the excitation of LHW in linear approximation (Akhiezer et al. 1975). The core plasma below is considered to be the multi-component, containing 77% of H^+ , 20% of He^+ , and 3% of O^+ . This thermal plasma composition is used for the calculation of the Coulomb collisions of the ring current ions and EMIC wave propagation in the numerical model (Khazanov et al. 2002, 2003a).

It should be noted that a refined approach to the problem of LHW excitation by the field of the LF wave should be based on the weak turbulence theory, or the parametric approach, if we are neglecting the finiteness of the LF wavelength (Gamayunov et al. 1992). In this study we use the approximate approach, assuming that, during the time of LHW generation, the particles drift velocities can be considered as constant. We also restrict the analyses by the case of unmagnetized ions. Then the dispersion equation for LHW has the form (e.g., Akhiezer et al. 1975; Khazanov et al. 1997a)

$$1 + \frac{\omega_{pe}^2}{k^2 v_{Te}^2} [1 + I_0(x) e^{-x} Z_e(z_e)] + \sum_{\alpha} h \frac{\omega_{p\alpha}^2}{k^2 v_{T\alpha}^2} [1 + z_{\alpha} Z(z_{\alpha})] = 0$$

$$x = \frac{k_{\perp}^2 v_{Te}^2}{\Omega_e^2}; \quad z_e = \frac{\omega}{\sqrt{2} k_z v_{Te}}; \quad z_{\alpha} = \frac{\omega - \mathbf{k}_{\perp} \mathbf{u}_{\alpha}}{\sqrt{2} k v_{T\alpha}}; \quad k_z = k \cos \vartheta, \quad (10.5.6)$$

$$v_T^2 = \frac{T}{m}; \quad \omega_{pe}^2 = \frac{4\pi n e^2}{m_e}; \quad \omega_{p\alpha}^2 = \frac{4\pi n_{\alpha} e^2}{m_{\alpha}}; \quad \alpha = H^+, He^+, O^+$$

where $I_0(x)$ is the modified Bessel function; $Z(z_{\alpha})$ is the plasma dispersion function; and ω and $\vec{k}$ are the frequency and the wave vector of the LH oscillations, respectively; and ϑ is the angle between the wave vector and the ambient magnetic field.

In order to obtain (10.5.6), we assumed that $\Omega_{\alpha} \ll \omega \ll \Omega_e$ and the motion of all ion species is unmagnetized, i.e., $k_{\perp}^2 v_{T\alpha}^2 / \Omega_{\alpha}^2 \gg 1$. As follows from the analyses of (10.5.6), the LHWs, excited in our case, have wavelengths large in comparison with

the Debye length, and the first term, the unit in (10.5.6), can be safely neglected. Equation (10.5.6) can then be split into two, for the real and imaginary parts as:

$$2 + I_0(x)e^{-x} \operatorname{Re}[z_e Z(z_e)] + \sum_{\alpha} c_{\alpha} \operatorname{Re}[z_{\alpha} Z(z_{\alpha})] = 0,$$

$$I_0(x)e^{-x} \operatorname{Im}[z_e Z(z_e)] + \sum_{\alpha} c_{\alpha} \operatorname{Im}[z_{\alpha} Z(z_{\alpha})] = 0 \quad (10.5.7)$$

where c_{α} is the concentration of ion species. Because of the Landau damping on electrons, only LHWs with the wave vector near normally directed to the ambient magnetic field can be excited. Therefore, the difference between k and $k_{\perp}$ in x and z_{α} can be neglected. Equations (10.5.7) are the equations for the real and imaginary parts of the normalized LHWs phase velocity ($\omega/\sqrt{2}k v_{TH}$). For the fixed plasma composition, the system of equations (10.5.7) depends on three parameters: the ratio of the LHW length to the electron gyroradius (x), the angle between the LHW vector and the ambient magnetic field (ϑ), and the ion's velocity in the EMIC wave electric field ($u_{\alpha}/\sqrt{2}v_{TH}$). This last dependence is in fact the dependence on the ratio of the magnetic field energy density of EMIC waves to the plasma density, B^2/n , as it was found from (10.5.5). Parameter B^2/n , which is plotted in Fig. 10.18, is calculated from the data presented in Fig. 10.16 on the global scale. For the local magnitude of this parameter, we need to look for solutions to (10.5.7), depending on the wavelength and the angle between the wave vector and the ambient magnetic field. Growing solutions to these equations exist only for ion velocities comparable to the hydrogen thermal velocity (Khazanov et al. 1997a). That means that the excitation of LHW can be expected only during a part of the EMIC wave period, when the ion drift velocity satisfies this condition. On the other hand, as it was stated above, the dispersion equation (10.5.6) is valid if, during a time of order γ^{-1} (where γ is the growth rate of LHW), the ion velocity can be considered as constant. To satisfy both of these conditions, we will use such an approach, solving the system (10.5.7). From Fig. 10.17, we choose the normalized time interval $\Delta\tau = \pm 0.5$ ($\tau = t\langle\omega\rangle$), around the moment when the velocity is maximum, $\tau \approx 1.5$. During this time interval, the velocity of ions are larger than the velocities at the ends of the interval, which are (see Fig. 10.17) $u_{H/O} = (+/-) \cdot 0.7 v_{TH} \sqrt{2B^2/n}$, $u_{He} = -1.1 \cdot v_{TH} \cdot \sqrt{2B^2/n}$ for hydrogen, oxygen, and helium (B in nT, n in cm^{-3}). The choice of the time interval, $\Delta\tau = \pm 0.5$, is to some degree arbitrary, but it is restricted by two conditions named above: the velocity at the ends of the interval should be large enough compared with the hydrogen thermal velocity to drive the LHWs, and the time interval should be long enough compared with the LHW growth time. As a result of these restrictions, the permitted variations of the time interval are not large, and other possible choices of the time interval lead to the close results. These minimal velocities ($u_{H/O} = (+/-) \cdot 0.7 \cdot v_{TH} \cdot \sqrt{2B^2/n}$, $u_{He} = -1.1 \cdot v_{TH} \cdot \sqrt{2B^2/n}$) will be substituted in the system (10.5.7), and for such LH waves we will seek that their period is at least five times smaller than the time interval $2\Delta\tau$, and that the characteristic growth time, γ^{-1} , is not larger than the same

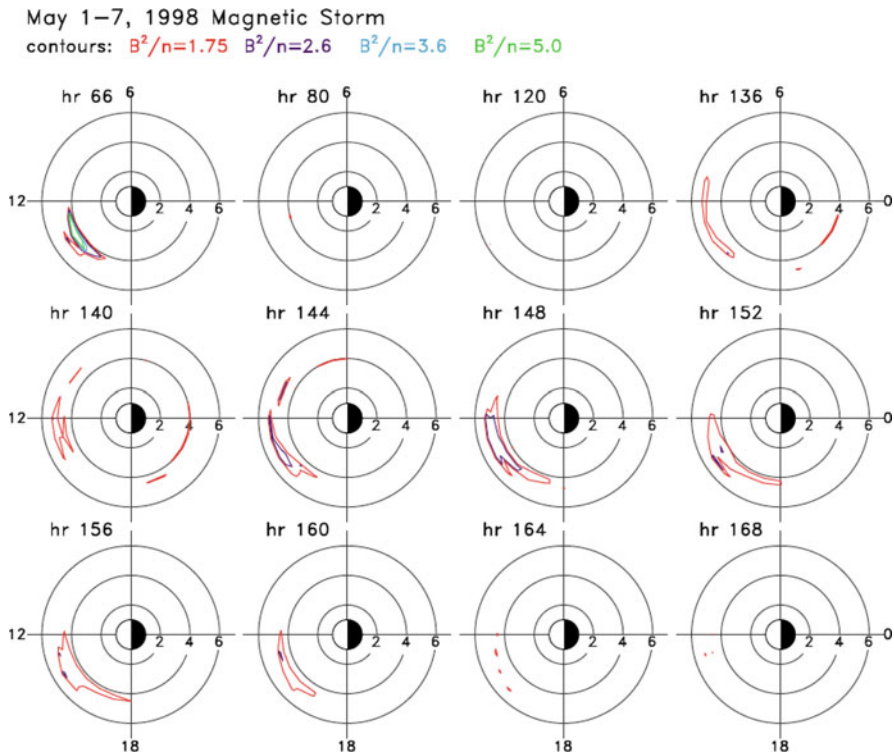


Fig. 10.18 Evolution of the squared EMIC wave electric field $\sim B^2/n$ (normalized) in (MLT, L-shell)-space during the May 2–7, 1998 storm period

time interval. These restrictions, and the assumptions used to obtain the dispersion equations (10.5.6) and (10.5.7), constitute the conditions that we imposed on the solutions of (10.5.7):

$$10\Omega_H \leq \omega \leq 0.1\Omega_e, \quad 10 \frac{\Omega_H^2}{v_{TH}^2} \leq k^2 \leq 0.1 \frac{\omega_{pH}^2}{v_{TH}^2}, \quad \Delta t \equiv \frac{2\Delta\tau}{\langle\omega\rangle} \geq \frac{10\pi}{\omega}, \quad \Delta t \geq \frac{1}{\gamma}. \quad (10.5.8)$$

The first inequality here is imposed because we are looking for LHW. The second condition selects the wavelength larger than the Debye radius and smaller than the ion Larmor radius.

The system of equations (10.5.7) in the domain of parameters defined by inequalities (10.5.8) was solved by Broyden's method on the global scale. Results of these calculations are summarized in Table 10.3. This table presents the main characteristics of the LHW excitation process and the parameters of generated waves that are organized according to the magnitude of the parameter B^2/n . The first column in this table characterizes the EMIC wave electric field as it was

Table 10.3 Main characteristics of the LHWs excitation process

$B^2/n(\text{nT}^2\text{cm}^3)$	$\frac{u_H}{\sqrt{2}v_{TH}}$	$\frac{u_{He}}{\sqrt{2}v_{TH}}$	$\frac{u_O}{\sqrt{2}v_{TH}}$	$\cos \vartheta \sqrt{\frac{m_H}{m_e}}$	$\frac{k^2 v_{Te}^2}{\Omega_e^2}$	$\frac{\text{Re}\omega}{\sqrt{2}v_{TH}}$	$\frac{\text{Im}\omega}{\sqrt{2}v_{TH}}$
1.75	−0.8	1.4	0.8	0.03	0.07–0.11	1.2–1.0	0.20–0.16
				0.3	0.1–0.16	1.1–1.0	0.17–0.13
2.6	−1.1	1.8	1.1	0.03	0.03–0.17	1.4–1.0	0.42–0.13
				0.3	0.03–0.22	1.5–1.0	0.34–0.11
				0.6	0.07–0.25	1.6–1.2	0.21–0.10
				0.7	0.15–0.17	1.5–1.4	0.13
3.6	−1.3	2.1	1.3	0.03	0.02–0.13	1.6–1.1	0.61–0.15
				0.3	0.02–0.17	1.8–1.1	0.50–0.13
				0.8	0.07–0.23	1.9–1.6	0.20–0.11
5.3	−1.6	2.5	1.6	0.03	0.02–0.08	1.9–1.2	0.80–0.20
				0.3	0.02–0.11	2.1–1.3	0.72–0.16
				0.9	0.08–0.23	2.2–1.9	0.31–0.11
				1.1	0.10–0.21	2.3–2.1	0.16–0.11

discussed above. The next three columns present the normalized drift velocities. The coincidence of hydrogen and oxygen drift velocity magnitudes (columns two and four) is accidental (Fig. 10.17), and holds only with the presented accuracy. The next three columns describe the exited LH waves. From this data the wave frequency and wavelength for the listed angles of propagation can be calculated. The last column can be used for the growth rate calculation for a wave with a fixed frequency and wave vector.

The left-hand and right-hand quantities in the last three columns correspond one to another. They change from left to right close to monotonically, but not proportionally. As can be seen from this table, the larger the EMIC electric field, the larger is the domain of the angles of LHW generation, ϑ , their phase velocities, $\text{Re}\omega/\sqrt{2}kv_{TH}$, and the maximum growth rate, $\text{Im}\omega/\sqrt{2}kv_{TH}$. The angles of generation of the LHWs are restricted by Landau damping on the electrons and the waves are generated nearly perpendicular to the ambient magnetic field. The phase velocity is always smaller than the helium drift velocity. That is the relative drift of helium to electrons that drives the instability. The drift of oxygen ions leads to the amplification of longer waves (the drift velocity is larger than the phase velocity), but oxygen absorbs the shorter ones (opposite ratio between these two velocities). In our case the input of oxygen to the LHW generation is small, due to its small content in plasma (3%). It can be found from the dispersion equations (10.5.7) and the data in the Table 10.3, that the hydrogen and helium density perturbations in the LHW are of the opposite signs and partially compensated. The generated wavelengths, λ , are in the range of $10\rho_e \leq \lambda \leq 40\rho_e$ (where ρ_e is the electron gyroradius), and for the larger wavelength, the phase velocity is larger. From the side of shorter waves ($\lambda < 10\rho_e$), damping prevails on the excitation. Longer waves have smaller frequencies, i.e., larger periods. Therefore, our condition that the time interval during which the drift velocity drops below the chosen level should be larger than five wave periods and larger than the growth character time restricts the wavelength from the larger side (10.5.8). The growth rate,

$\text{Im}\omega/\sqrt{2}kv_{TH}$, is restricted from the smaller side by the last time condition from inequalities (10.5.8). It is a growing function of the EMIC electric field ($\sqrt{B^2/n}$); this function is larger for the wave vectors normal to the ambient magnetic field, and the ratio $\text{Im}\omega/\text{Re}\omega$ changes in the range 0.05–0.4. The character of the LHW excitation described above is similar to that obtained for the LHWs, excited by an ion beam propagating across the magnetic field and forming a jet spectrum in k -space (Musher et al. 1978). With the results presented in the Table 10.3, combined with the distribution of the parameter B^2/n from the Fig. 10.18, the LHW excitation on the global scale can be analyzed. As is seen from this figure and the table, the LHWs are mainly excited at the end of the storm period, when the EMIC activity is well developed (Fig. 10.16). The periods of LHWs excitation during the storm directly reflect the dependence on the EMIC wave electric field, which is proportional to the $\sqrt{B^2/n}$ ratio. That is why the LHW excitation takes place, for example, at hour 66, but not at hour 34 (Figs. 10.16 and 10.19). The EMIC waves magnetic fields are close, but the density is approximately 10 times larger at hour 34. As a result, the electric field and, therefore, the drift velocities, are approximately three times smaller in this case. The EMIC waves electric field, with good

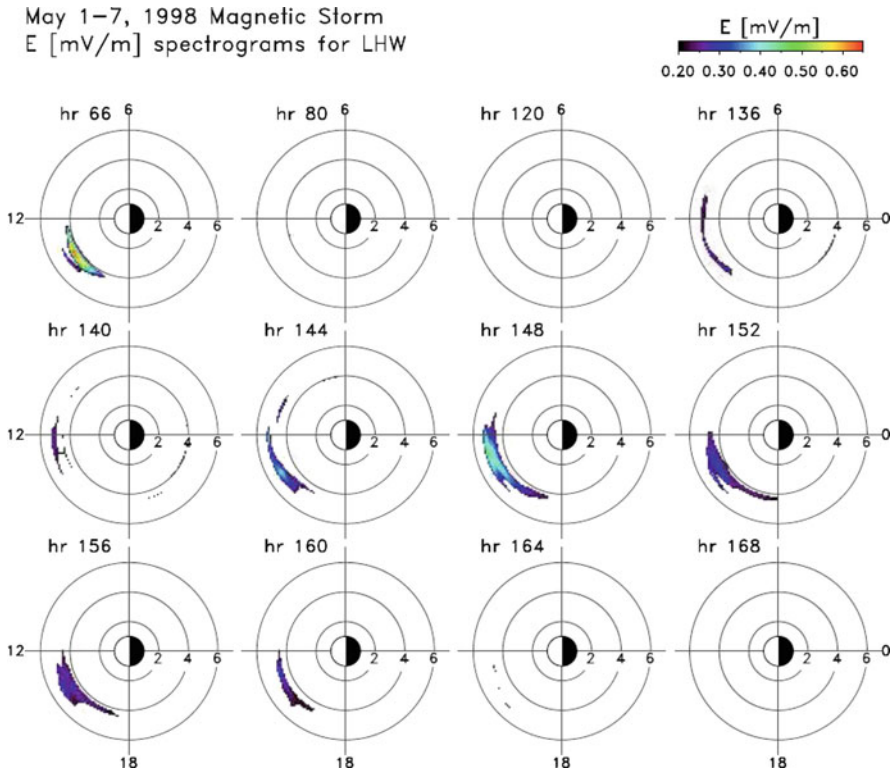


Fig. 10.19 Low-hybrid wave electric field in (MLT, L-shell)-space during the May 2–7, 1998 storm period

accuracy, can be calculated with the help of (10.5.2)–(10.5.4) as $E[\text{mVm}^{-1}] = 280\sqrt{B^2/L^3 n}$, and in the region of interest is 3–6 mV m^{-1} , as can be seen from Fig. 10.16.

10.5.3 Results and Discussion

As presented above, the LHW growth rate permits us to identify the regions where the LHW activity can be expected during the storm evolution (Table 10.3, Fig. 10.18). Using the calculated LHW growth rates, we can also estimate what the expected level of LHW activity is. We will assume that the level of the thermal noise in plasma is the initial level of LH activity. This level is apparently surpassed during the storm and such a choice is a guarantee from an overestimation in what follows. The level of thermal noises in Maxwellian proton–electron plasma can be found in Akhiezer et al. (1975). We will use this result for our estimations in spite of the 20% admixture of helium. Because the initial level of LHW energy density is the multiple before the exponent in the expression for the wave energy density at later moments,

$$E^2(t) = E_{\text{thermal}}^2(t=0)e^{2t \text{Im } \omega}, \quad (10.5.9)$$

this discrepancy (plasma in our case is multicomponent), as well as the use of average parameters below, should not change the estimations essentially. The spectral component of the electric field space correlation function is:

$$\langle E^2 \rangle_{\mathbf{k}} = 8\pi^2 T \frac{\Omega_e^2}{\Omega_e^2 + \omega_{pe}^2}. \quad (10.5.10)$$

Calculating the volumes of the k -space corresponding to the different drift velocities determined by the parameter $\sqrt{B^2/n}$ (Table 10.3), and multiplying this volume on the spectral density (10.5.10), the initial LHW energy densities, $E^2(t=0)$, are found to be:

$\sqrt{B^2/n} \text{ (nT}^2 \text{ cm}^3\text{)}$	1.75	2.6	3.6	5.3	(10.5.11)
$10^{12} E^2(t=0) (\text{mV}^2 \text{ m}^{-2})$	2.3	4.0	5.3	6.6.	

As can be seen from (10.5.11), the initial energy levels are close for our set of drift velocities. Because the LHW spectral energy density (10.5.10) is independent of the wave vector, it also follows from (10.5.11) that the volumes of k -space, in which the LHW is generated, are growing with the energy density of EMIC wave electric field. Now we are able to calculate the LHW energy density at the end of the time interval $2\Delta\tau = 1$, where $\tau = t/\langle\omega\rangle$ is the time normalized on the average EMIC

wave period. Recall that during this time the drift ions velocities exceed their values used in the LHW growth rate calculations and, therefore, at least during this time the generation of LHWs, hold on. To perform the calculations with the help of (10.5.9), we need also the growth rate. The growth rates are k -dependent, as can be seen from Table 10.3. The region of LHW activity in k -space forms a jet, as we have seen above. Approximately, it is a cone with the axis normal to the ambient magnetic field. The main part of the cone volume is located near the outer boundary, $k = k_{\max}$. As it was found in the calculations, the growth rate sharply drops in the narrow vicinity near $k_{\max}$ ($\sim 10\%$ of $k_{\max}$), but remains close to the constant for the essential part of the k -region before this drop. Thus, the growth rate from this main region was used to calculate the LHW energy density and electric field at the end of the time interval $2\Delta\tau \equiv t\langle v \rangle = 1$ from (10.5.9). The results are:

$\sqrt{B^2/n} \text{ (nT}^2 \text{ cm}^3\text{)}$	1.75	2.6	3.6	5.3	(10.5.12)
$E(t = 1/\langle v \rangle) \text{ (mVm}^{-1}\text{)}$	0.002	0.06	0.2	0.6.	

In calculating these LHW electric fields, we neglected their nonlinear damping, which can restrict the growth and saturate the LHW activity on the lower level. As it was found by Musher et al. (1978) for LHWs excited by an ion beam propagating across a magnetic field in electron–proton plasma, the main mechanism, which leads to the saturation of such waves, is induced scattering by ions and electrons. The case in which the beam is formed by a heavy component of plasma due to electric field of a low frequency wave was studied by Khazanov et al. (1997a). They came to the same conclusion, i.e., quasilinear diffusion only slightly diminishes the LHW growth rate and the saturation results from the induced scattering. Therefore, the quasilinear effects will not change the results (10.5.12) dramatically. Following Musher et al. (1978), the damping rate for induced scattering can be presented as follows:

$$\gamma^{sc} = \frac{\omega_{pe}^4}{\omega(\Omega_e^2 + \omega_{pe}^2)} \frac{E^2}{8\pi nT}. \quad (10.5.13)$$

For a quasistationary state, the growth rate of the LHW from the Table 10.3 should be equal to this damping rate. From this condition, we can calculate the saturation levels of LHW activity for different values of the B^2/n , i.e., for different drift velocities. This saturation level for the LHW electric field is in the range 1–2 mV m^{−1}, and exceeds the electric field for all four cases presented in (10.5.12). Therefore, at the end of the time period $2\Delta\tau \equiv \langle \omega \rangle t = 1$, the level of LHW electric fields presented in (10.5.12) can be expected. After this the EMIC wave electric field and the helium drift velocity drop below the level needed for the LHW generation and the LHW activity will be damped during the remaining part of the EMIC wave period. Then the generation will be switched on again if the EMIC waves are still present in this domain. What the resulting quasistationary level of

LHWs will be, if the EMIC activity exists long enough in the region, depends first of all on the damping rate during the remaining part of the EMIC wave period, when the ion's drift is below the level needed for LHW generation. To estimate the LHW damping and following growth at the next period of EMIC wave, we can look at the problem as two initial value problems. Then, at the end of one part of the EMIC waves period, we have the initial LHW characteristics for the next part of the period. For initial perturbation with a fixed wave vector k , the electric field can be found as (Akhiezer et al. 1975):

$$E(t, \kappa) = -i \int \frac{D(t=0, \kappa)}{\omega \varepsilon(\omega, \kappa)} e^{-i\omega t} d\omega \sim C e^{-i\omega(\kappa)t}. \quad (10.5.14)$$

Here, $D(t=0, \kappa)$ is the initial electric displacement, $\varepsilon(\omega, k)$ is the dielectric permittivity, and $\omega(k)$ is the solution of the dispersion equation $\varepsilon(\omega, k) = 0$ for the LHW mode in our case. As can be seen from (10.5.14), the evolution of the LHW electric field for the harmonic with fixed k depends on the $\text{Im}\omega(k)$. Therefore, we need also the damping rates for the wave vectors of the excited LHW activity, which are presented in Table 10.3. We calculated these damping rates and the total growth and damping during one EMIC wave period, $2\pi/\langle\omega\rangle$, i.e.:

$$\Gamma = \delta t \text{Im}\omega(k)|_{\text{growth}} + \left(\frac{2\pi}{\langle\omega\rangle} - \delta t \right) \text{Im}\omega(k)|_{\text{damping}}, \quad (10.5.15)$$

where $\delta t = 2\Delta\tau/\langle\omega\rangle$. It was found that, for all cases when the LHWs are excited, there are always some wave vectors $\mathbf{k}$ for which the LHW activity is enriched during the EMIC wave period. The existence of such $\mathbf{k}$ can be understood from the data presented in the Table 10.3 for the growth rate and the k -dependence of the damping. The growth rate is largest for small $\mathbf{k}$ and decreases for the larger $\mathbf{k}$. The damping for waves with small $\mathbf{k}$ is small and increases for larger $\mathbf{k}$. Therefore, some harmonics with small $\mathbf{k}$ can survive during the damping part of the EMIC wave period, if the angle ϑ (Table 10.3) is such that the Landau damping for electrons is not too strong. Surviving LH harmonics form a narrow jet in k -space and we can neglect the difference between the different harmonics in this jet. Note that (10.5.13) for the scattering rate depends on the total energy density of the LHW electric field in the used approximation. If the EMIC wave acts long enough, then the exact level of the LHW perturbations that survives the damping part of the period are not important for our estimation. These perturbations will grow at least until the induced scattering will not stop this process on some quasistationary level. To calculate this level, we can introduce via (10.5.15) an effective growth rate as the growth rate averaged over the EMIC wave period for the most quickly growing harmonic:

$$\langle\gamma\rangle_{\text{eff}} = \frac{\langle\omega\rangle}{2\pi} \Gamma. \quad (10.5.16)$$

Equating the growth rate (10.5.16) to the damping rate due to induced scattering (10.5.13), we found the LHW electric field for the quasistationary state

$\sqrt{B^2/n} \text{ (nT}^2 \text{ cm}^3\text{)}$	1.75	2.6	3.6	5.3	(10.5.17)
$E \text{ (mV m}^{-1}\text{)}$	0.2	0.3	0.55	0.65.	

We compared these electric fields to the threshold of the modulational instability of the LHW (Musher et al. 1978), which, in our case, can be presented as

$$\frac{E_{tr}^2}{8\pi nT} = \frac{m_e}{m_H} \frac{k^2 v_{Te}^2}{\omega_{pe}^2}. \quad (10.5.18)$$

This threshold is higher than the electric fields given by (10.5.17), but in the last case, (0.65 mV m⁻¹), the threshold is 1.1 mV m⁻¹. Apparently the modulational instability can be excited in some points during the storm period. Therefore, depending on the time during which the EMIC waves exists in the region of LHW generation the LHW electric fields in the range between those listed in (10.5.12) and (10.5.17) can be reached.

We have no observational results for the LHW during this storm period, but we still can compare our results with the data obtained from other events. Data from the May 6, 1982 event measured by the Dynamics Explorer 1 satellite gave the experimental value of LHW electric field 0.003 mV m⁻¹ (Olsen et al. 1987). Another case of LHW activity was observed by AMPTE IRM satellite at April 6, 1985, when the LHW electric field was 0.6 mV m⁻¹ (LaBelle et al. 1988). These data are in the range of electric fields values listed above in (10.5.12) and (10.5.17) and testify that the calculated results are reasonable.

We also estimated the outflow of the energy from LHW to the core plasma. There are two main channels of energy outflow from the LHW. One of them is the induced scattering on ions and electrons. Because the frequency of LHWs is low, the energy is mainly transformed to the electrons (Musher et al. 1978). For the stationary case, the energy flow, Q , can be estimated from the balance between the energy pumped to the LHWs by the helium drift and the energy outflow from this waves to the particles. It follows from this balance (Khazanov et al. 1997a) that:

$$\frac{Q}{nT} \sim \frac{\gamma^2 \omega}{\Omega_e^2}. \quad (10.5.19)$$

For these estimations, we used the effective growth rate (10.5.16). For the two last cases presented in (10.5.17), the energy of particles doubles during an hour. For weaker electric fields, the effect is small. Another channel of particle energization is due to the quasilinear effects. Mainly, the energy is directed to the hydrogen. The admixture of oxygen ions is small (0.03%), as well as their thermal velocities, compared to the phase velocity of the LHWs, and this channel of energy outflow is negligible. Because the waves are generated in the narrow cone around the normal

to the ambient magnetic field, the quasilinear effects for electrons are also small. Details of these calculations can be found in Khazanov et al. (1997a). We estimated this effect for the case that the LHW electric field is 0.55 mV m^{-1} with the help of expressions (23) and (32) from this reference. Kinetic energy gained by hydrogen ion is approximately $K(t) \sim 0.1K(t=0)t^{0.4}$, and doubles during an hour. Because the phase wave velocity in this case is only approximately two times larger than the hydrogen thermal velocity, the portion of particles involved in this interaction is large enough and constitutes $\sim 10\%$ of the hydrogen population.

Results presented above characterize, on the global scale, the morphology, dynamics, and level of LHW activity generated by EMIC waves during the May 2–7, 1998 storm period. The latter waves were calculated in the framework of the self-consistent model for the ring current particles and generated by these particles EMIC waves. Observational data were used as the inputs to the models involved in these calculations. Drift velocities of H^+ , He^+ , and O^+ were then found, and the region of LHW generation on the global scale was calculated. Assuming induced scattering as the saturation mechanism for the LHWs, their quasistationary level was estimated. In these estimations for the processes of induced scattering, we used the electron–proton model. Because of different approximations involved in our calculations, we reduced the electric field of the EMIC waves in the study of LHW generation two times. Analyses of this precaution lead us to conclusion that the region of LHW generation and the level of such activity are apparently underestimated in this study. Obtained under these approximations, results for the LHWs electric field are consistent with the observational data. It should be stressed that to determine the role of the discussed mechanism of LHW generation for the ring current region and separate it from other sources of LH activity, detailed analyses of events based on the measurements of EMIC and LH waves, as well as cold plasma parameters, is required. From another side, to eliminate the restrictions used at the drift velocity calculation and to support our conclusions about the nonlinear stage of LHW evolution, direct modeling (PIC simulation) was performed by Singh and Khazanov (2004) and Singh et al. (2007).

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Concluding Remarks

The inner magnetospheric plasma is a unique composition of different kinds of plasmas and electromagnetic fields. It covers a huge plasma energy range with spatial and time variations of many orders of magnitude. Such a situation requires a kinetic approach as the starting point for a proper theoretical description of these inner magnetospheric plasma phenomena. This is the first book to attempt to outline and provide a kinetic description of the inner magnetospheric plasma.

As demonstrated in this book, the inner magnetosphere is a complicated and dynamic region of geospace coupled to many other parts of the Sun–Earth system and, also, displaying coupling among the different components of the inner magnetosphere. These major plasma components are the: plasmasphere, superthermal electrons (SE), ring current (RC), and radiation belts (RB). Each of these populations has distinct features and contributes in a different way to the dynamic and energetic processes of the inner magnetosphere. Each has a unique coupling mechanism with surrounding plasma and electromagnetic fields, providing a very noticeable contribution to the physics of this region of geospace.

It is generally recognized that there is a need for better understanding of the coupling processes between inner magnetospheric plasma populations, fields, and regions. Self-consistent calculations need to continue to be developed, including global modeling that links the inner magnetosphere to the other regions of geospace. In addition, improvements are needed to obtain the correct description of the different inner magnetosphere populations, keeping in mind the applicability of the different set of transport equations that are involved in the corresponding self-consistent coupling loop. A few examples of such improvements were demonstrated in this book to illustrate possible future advances in the description of a particular inner magnetosphere region.

Plasmasphere. The plasmasphere (together with the ionosphere) is the coldest inner magnetosphere population with a velocity distribution function close to Maxwellian. The ionosphere is the major particle source of this region, and its energy balance is controlled by interactions with SE and the RC. Colliding with the RB and the RC, the plasmasphere serves as a catalyst for plasma wave excitation in the inner magnetosphere and further energy redistribution in the ionosphere–magnetosphere system.

The combination of the different plasmaspheric energy sources, as well as the anisotropy of the energy transport along and across the magnetic field lines, leads to anisotropies of the plasmaspheric electron and ion temperatures. These anisotropies, in turn, produce the plasma density and temperature redistributions along and across magnetic field lines. These effects were clearly demonstrated in the book based on analytical solutions of the different hydrodynamic equations in the case of a simple dipole geomagnetic field geometry. There is no doubt that going to a realistic magnetic configuration, or to a more complicated representation of the electric field and the resulting magnetospheric convection, will preserve all of the effects that have been discussed using analytical representations of magnetic and electric fields.

Interacting with the plasmasphere, the RC is the source of free energy for EMIC waves that dramatically contributes to the energy balance of this region. A knowledge of plasmaspheric characteristics such as the plasma field-aligned distribution and heavy ion composition are absolutely crucial for the determination of EMIC wave intensity and spectral power distributions. On the other hand, EMIC waves combined with SE control the plasmaspheric energy balance in a way that has a strong influence on plasma, the field-aligned distribution, and heavy ion composition in this region. There is no doubt that small-scale structures like shoulders, notches, fingers, etc., discovered by IMAGE spacecraft, can be only understood in a framework of unique inner magnetosphere coupling processes.

Superthermal Electrons. The SE is the major energy contributor to the ionosphere and inner plasmasphere via the Coulomb collision processes. The SE escape from the ionosphere to the plasmasphere is controlled by strong Coulomb coupling with the thermal plasma distribution along the entire magnetic field line. The plasma distribution along the field line, in turn, is controlled by electron and ion temperature distributions that are mostly determined by SE heating of the thermal electrons. We have especially emphasized the proper Coulomb collision description of the SE interaction with the thermal plasma that must be performed using the Landau collision term. This issue was discussed in detail in the book based on the analysis of some cases in which such a description is crucial. Also, some of the peculiarities of the SE distribution function can lead to instabilities causing additional energy and pitch-angle redistributions.

The topics mentioned above illustrate the necessity of including SE formation in the inner-magnetosphere coupling loop. Because of such interconnection, we have developed a comprehensive kinetic theory of SE transport, which is equally valid in the ionosphere and in the plasmasphere, that self-consistently couples the conjugate magnetospheric hemispheres. It has been shown that such a self-consistent approach produces significant changes in the superthermal electron distributions, compared with “pure” ionospheric or plasmaspheric calculations, and must be included in a future ionosphere–magnetosphere modeling network. Such an approach is presented in a rather general form that includes relativistic electrons and highly nonsteady-state conditions like, for example, artificial electron beam injections from spacecraft.

Ring Current. The RC is the biggest energy player in the inner magnetosphere. It is the source of free energy for EMIC wave excitation provided by a temperature

anisotropy of RC ions, which develops naturally during inward $\mathbf{E} \times \mathbf{B}$ convection from the plasma sheet. The cold plasmasphere, which is under the strong influence of the magnetospheric electric field, strongly mediates the RC-EMIC wave-particle-coupling process and ultimately becomes part of the particle and energy interplay. On the other hand, there is a strong influence of the RC on the inner magnetospheric electric and magnetic field configurations, and these configurations, in turn, are important to RC dynamics. Therefore, one of the biggest needs for inner magnetospheric research is the continued progression toward a coupled, interconnected system with the inclusion of nonlinear feedback mechanisms between the plasma populations, the electric and magnetic fields, and plasma waves.

As we clearly demonstrated in this book, EMIC waves strongly interact with electrons and ions of energies ranging from ~ 1 eV to ~ 10 MeV, and these waves strongly affect the dynamics of resonant RC ions, thermal electrons and ions, and the outer RB relativistic electrons. As we found, the rate of ion and electron scattering/heating in the Earth's magnetosphere is not only controlled by the wave intensity-spatial-temporal distribution but also strongly depends on the *spectral distribution of the wave power*. The latter is also a function of the plasmaspheric heavy ion content, and the plasma density and temperature distributions along the magnetic field lines. The above discussion places RC-EMIC wave coupling dynamics in context with inner magnetospheric coupling processes and, ultimately, relates RC studies with plasmaspheric and SE formation processes as well as with outer RB physics.

Appendix A

The characteristic equations (3.9.2) of the kinetic equation (3.9.1) can be presented as

$$\frac{dx_k}{dt} = f_k(x_i, t, \theta_m), \quad \frac{d\theta_n}{dt} = \frac{1}{\varepsilon} \omega_n(x_i, t) + A_n(x_i, t, \theta_m), \quad (\text{A1})$$

with functions f_k, A_n , which are periodic with respect to the arguments θ_m , and ε is a small parameter. Equations (3.9.2) can be presented in this form introducing the angle $\phi = -\int \tilde{\omega} dt$. We seek a substitution of variables like this,

$$x_k = \xi_k + \sum_{r=1} \varepsilon^r g_k^r(\xi_i, t, \alpha_m), \quad \theta_n = \alpha_n + \sum_{s=1} \varepsilon^s q_n^s(\xi_i, t, \alpha_m), \quad (\text{A2})$$

such that the new variables ξ_k, α_m satisfy equations that do not contain the phase θ_m :

$$\frac{d\xi_k}{dt} = \varphi_{ok}(\xi_i, t) + \sum_{r=1} \varepsilon^r \varphi_k^r(\xi_i, t), \quad \frac{d\alpha_n}{dt} = \frac{1}{\varepsilon} \omega_n(\xi_i, t) + \sum_{s=0} \varepsilon^s \Omega_n^s(\xi_i, t). \quad (\text{A3})$$

First, (A2) are substituted into (A1), and then the time derivatives in terms like $\frac{\partial g_k^r}{\partial \xi_i} \frac{d\xi_i}{dt}$ are replaced by (A3). The right-hand side of the (A1) can be expanded in a Taylor series over the small parameter ε . Equating terms of the same order in ε , we can find equations that relate the functions $f_k(\xi_i, t, \alpha_m)$, $A_n(\xi_i, t, \alpha_m)$ and the new functions φ_k, Ω_n, g_k and q_n . For example, the zeroth order equalities are:

$$\varphi_k^0(\xi_i, t) + \frac{dg_k^1}{d\alpha_m} \omega_m = f_k, \quad \Omega_n^0 + \frac{dq_n^1}{d\alpha_m} \omega_m = A_n + \frac{\partial \omega_n}{\partial \xi_i} g_i^1.$$

The periodic functions $f_k(\xi_i, t, \alpha_m)$, $A_n(\xi_i, t, \alpha_m)$ can be written as a sum of harmonics $C_{o,o}(\xi_i, t) + \sum_{v,\mu} C_{v,\mu}(\xi_i, t) \exp i(v\alpha_1 + \mu\alpha_2 + \dots)$. Requiring that g_k and q_n be periodic functions with no constant terms, the equation of given order in ε can be split in two: an angle-dependent part and an angle-independent part. The latter part is found by integration over the period and yields $\varphi_k^0(\xi_i, t) = \bar{f}_k$, $\Omega_n^0 = \bar{A}_n$. Subtracting the result from the initial equation and integrating over α_m we will have $g_k^1 = \frac{f_k}{i\omega_n} \equiv \bar{f}_k$, $q_n^1 = \bar{A}_n + \frac{\partial \omega_n}{\partial \xi_k} \bar{g}_k^1$. More details of these calculations can be found in Morozov and Solov'ev (1966), presented there with an accuracy to the first order in ε . The solution of the obtained system the equations for the unknown variables ξ_k can be presented as

$$\begin{aligned} \frac{d\xi_k}{dt} = & \bar{f}_k + \overline{\sum_{r=1}^{r=2} \frac{\partial f_k}{\partial \xi_n} g_n^r + \frac{\partial f_k}{\partial \alpha_m} q_m^r} + \frac{1}{2} \\ & \times \left(\overline{\frac{\partial^2 f_k}{\partial \xi_n \partial \xi_m} g_n^1 g_m^1} + 2 \overline{\frac{\partial^2 f_k}{\partial \xi_n \partial \alpha_m} g_n^1 q_m^1} + \overline{\frac{\partial^2 f_k}{\partial \alpha_n \partial \alpha_m} q_n^1 q_m^1} \right), \end{aligned} \quad (\text{A4})$$

where the bar notes that the oscillating terms are removed from these expressions and:

$$\begin{aligned} g_k^1 = & \bar{f}_k, \quad q_n^1 = \bar{A}_n + \frac{\partial \omega_n}{\partial \xi_k} \bar{g}_k^1, \quad \Omega_n^0 = \bar{A}_n, \quad g_k^2 = \frac{\partial \bar{g}_k^1}{\partial t} - \bar{f}_i \frac{\partial \bar{g}_k^1}{\partial \xi_i} - \frac{\Omega_n^0}{\omega_n} g_k^1 + \overline{\frac{\partial f_k}{\partial \xi_i} g_i^1} + \overline{\frac{\partial f_k}{\partial \alpha_n} q_n^1} \\ q_n^2 = & -\frac{\partial \bar{q}_n^1}{\partial t} - \bar{f}_i \frac{\partial \bar{q}_n^1}{\partial \xi_i} - \frac{\Omega_m^0}{\omega_m} q_n^1 + \frac{\partial \omega_n}{\partial \xi_i} \bar{g}_i^2 + \overline{\frac{\partial A_n}{\partial \xi_k} g_k^1} + \overline{\frac{\partial A_n}{\partial \alpha_m} q_m^1} \end{aligned} \quad (\text{A5})$$

The operation $\tilde{z}$ applied to trigonometric functions in a multiple periodic system means:

$$\overline{\exp i(v\alpha_1 + \mu\alpha_2 + \dots)} = \frac{\exp i(v\alpha_1 + \mu\alpha_2 + \dots)}{i(v\omega_1 + \mu\omega_2 + \dots)}.$$

It is assumed that there is no resonance in the system, i.e., $v\omega_1 + \mu\omega_2 + \dots \neq 0$.

Kinetic equation (3.9.1), with the use of the chain rule, can now be rewritten in the new variables ξ_k, α_m

$$\frac{\partial f}{\partial t} + \frac{d\xi_k}{dt} \frac{\partial f}{\partial \xi_k} + \frac{d\alpha_m}{dt} \frac{\partial f}{\partial \alpha_m} = 0, \quad (\text{A6})$$

with $\frac{d\xi_k}{dt}, \frac{d\alpha_m}{dt}$ from (A3). As a result, the coefficients in the transformed kinetic equation are independent on the angle variables α_m . It can be proved that the

distribution function f satisfying this equation is also angle independent (Qin et al. 1998). Hence (A6) is angle independent and reduces to the first two terms, with the coefficients $\frac{d\xi_k}{dt}$ calculated with the help of (A4).

The solution of (A6) can be transformed (Bernstein and Catto 1985) to the old variables x_k, θ_m . With accuracy to the second order in ε , the inverse transformation to (A2) is

$$\xi_k = x_k - \varepsilon g_{1k} + \varepsilon^2 \left(-g_{2k} + g_{1l} \frac{\partial g_{1k}}{\partial x_l} + q_{1m} \frac{\partial g_{1k}}{\partial \theta_m} \right), \quad (\text{A7})$$

where g_k and q_n are now functions of old variables x_k, θ_m , and t . Then, with the same accuracy, we have

$$f(\xi_k, t) = f(x_k, t) + \frac{\partial f}{\partial x_k} (\xi_k - x_k) + \varepsilon^2 \frac{\partial^2 f}{\partial x_k \partial x_l} \frac{g_{1k} g_{1l}}{2}, \quad (\text{A8})$$

with $\xi_k - x_k$ from (A7). It is not necessary to distinguish between the initial and the transformed variables in the right-hand side of this expression. This difference is explicitly taken into account in the transformation of the distribution function, with second order accuracy. It should be noted that only if the terms that are linear with respect to the wave energy are taken into account, then (A4) and (A8) are significantly simplified.

Appendix B

Table B.1 The values of function $K_{LMN} = \frac{1}{\pi} \int u^{-3} u_1^L u_2^M u_3^N \exp[-(y_1^2 + y_2^2) - (1+X)u_3^2] d^3u$

L	M	N	K_{LMN}	$K_{LMN}(X < 1)$
2	0	0	$X^{-1}[-1 + (1+X)\varphi(X)]$	$2/3 - 2X/15$
0	0	2	$2X^{-1}[1 - \varphi(X)]$	$2/3 - 2X/5$
2	2	0	$X^{-2}[3 + X + (1+X)(X-3)\varphi(X)]/8$	$2/15 - 4X/105$
2	0	2	$X^{-2}[-3 + (3+X)\varphi(X)]/2$	$2/15 - 4X/35$
0	0	4	$X^{-2}[2 + (1+X)^{-1} - 3\varphi(X)]$	$2/5 - 4X/7$
4	2	0	$X^{-3}[-15 - 4X + 3X^2 + (15 + 9X - 3X^2 + 3X^3)\varphi(X)]/32$	$4(3-X)/105$
2	2	2	$X^{-3}[15 + X + (X^2 - 6X - 15)\varphi(X)]/16$	$4(1-X)/105$
2	0	4	$X^{-3}[-13 - 2(1+X)^{-1} + 3(X+5)\varphi(X)]/4$	$4(3-5X)/105$
0	0	6	$X^{-3}[8 + 9(1+X)^{-1} - 2(1+X)^{-2} - 15\varphi(X)]/2$	$4/7 - 4X/3$

Note: $\varphi(X) = \frac{1}{\sqrt{X}} \arctan \sqrt{X}$

Table B.2 The values of function $K_{a,b} = \int_0^1 t^a (1+t^2X)^{-b} dt$

a	B	$K_{a,b}$	$K_{a,b}(X < 1)$
0	1	$\varphi(X)$	$1 - X/3$
2	1	$X^{-1}[1 - \varphi(X)]$	$1/3 - X/5$
0	3/2	$(1+X)^{-1/2}$	$1 - X/2$
2	3/2	$X^{-1}[\Psi(X) - (1+X)^{-1/2}]$	$1/3 - 3X/10$
0	2	$[\varphi(X) + (1+X)^{-1}]/2$	$1 - 2X/3$
2	2	$X^{-1}[\varphi(X) - (1+X)^{-1}]/2$	$1/3 - 2X/5$
4	2	$X^{-2}[2 + (1+X)^{-1} - 3\varphi(X)]/2$	$1/5 - 2X/7$
0	5/2	$[1 - X(1+X)^{-1}/3](1+X)^{-1/2}$	$1 - 5X/6$
2	5/2	$(1+X)^{-3/2}/3$	$1/3 - X/2$

(continued)

Table B.2 (continued)

a	B	$K_{a,b}$	$K_{a,b}(X < 1)$
4	5/2	$X^{-1} \left[-X^{-1}(1+X)^{-1/2} - (1+X)^{-3/2} / 3 + X^{-1} \Psi(X) \right]$	$1/5 - 5X/14$
0	3	$\left[(1+X)^{-2} + 3(1+X)^{-1} / 2 + 3\varphi(X)/2 \right] / 4$	$1 - X$
2	3	$X^{-1} \left[-(1+X)^{-2} + (1+X)^{-1} / 2 + \varphi(X)/2 \right] / 4$	$1/3 - 3X/5$
4	3	$X^{-2} \left[(1+X)^2 - 5(1+X)^{-1} / 2 + 3\varphi(X)/2 \right] / 4$	$1/5 - 3X/7$
6	3	$X^{-3} \left[1 + 9(1+X)^{-1} / 8 - (1+X)^{-2} / 4 - 15\varphi(X)/8 \right]$	$1/7 - X/3$
0	7/2	$\left[1 - 2X(1+X)^{-1} / 3 + X^2(1+X)^{-2} / 5 \right] (1+X)^{-1/2}$	$1 - 7X/6$
2	7/2	$\left[1/3 - X(1+X)^{-1} / 5 \right] (1+X)^{-3/2}$	$1/3 - 7X/10$
4	7/2	$(1+X)^{-5/2} / 5$	$1/5 - X/2$
6	7/2	$X^{-1} \left[-23(1+X)^{-5/2} / 15 - 7X^{-1}(1+X)^{-5/2} / 3 \right. \\ \left. - X^{-2}(1+X)^{-5/2} + X^{-2} \Psi(X) \right]$	$1/7 - 7X/18$
0	4	$\left[(1+X)^{-3} / 3 + 5(1+X)^{-2} / 12 \right. \\ \left. + 5(1+X)^{-1} / 8 + 5\varphi(X)/8 \right] / 2$	$1 - 4X/3$
2	4	$X^{-1} \left[-(1+X)^{-3} / 3 + (1+X)^{-2} / 12 \right. \\ \left. + (1+X)^{-1} / 8 + \varphi(X)/8 \right] / 2$	$1/3 - 4X/5$
4	4	$X^{-2} \left[(1+X)^{-3} / 3 - 7(1+X)^{-2} / 12 \right. \\ \left. + (1+X)^{-1} / 8 + \varphi(X)/8 \right] / 2$	$1/5 - 4X/7$
6	4	$X^{-2} \left[-5X^{-1}(1+X)^{-1} / 8 - 3(1+X)^{-2} / 4 \right. \\ \left. + X^{-1}(1+X)^{-2} / 3 - X^{-1}(1+X)^{-3} / 3 + 5X^{-1}\varphi(X)/8 \right] / 2$	$1/7 - 4X/9$
0	9/2	$\left[1 - X(1+X)^{-1} + 3X^2(1+X)^{-2} / 5 \right. \\ \left. - X^3(1+X)^{-3} / 7 \right] (1+X)^{-1/2}$	$1 - 3X/2$
2	9/2	$\left[1/3 - 2X(1+X)^{-1} / 5 + X^2(1+X)^{-2} / 7 \right] (1+X)^{-3/2}$	$1/3 - 9X/10$
4	9/2	$\left[1/5 - X(1+X)^{-1} / 7 \right] (1+X)^{-5/2}$	$1/5 - 9X/14$
0	5	$\left[7(1+X)^{-3} / 6 + (1+X)^{-4} + 35(1+X)^{-2} / 24 \right. \\ \left. + 35(1+X)^{-1} / 16 + 35\varphi(X)/16 \right] / 8$	$1 - 5X/3$
2	5	$X^{-1} \left[-(1+X)^{-4} + 5(1+X)^{-2} / 24 + (1+X)^{-3} / 6 \right. \\ \left. + 5(1+X)^{-1} / 16 + 5\varphi(X)/16 \right] / 8$	$1/3 - X$
4	5	$X^{-2} \left[-3(1+X)^{-2} / 2 + (1+X)^{-4} + 3(1+X)^{-1} / 16 \right. \\ \left. + (1+X)^{-2} / 8 + 3\varphi(X)/16 \right] / 8$	$1/5 - 5X/7$
6	5	$X^{-3} \left[5(1+X)^{-1} / 16 - 23(1+X)^{-2} / 24 - (1+X)^{-4} \right. \\ \left. + (8 - 9X)(1+X)^{-3} / 6 + 5\varphi(X)/16 \right] / 8$	$1/7 - 5X/9$

(continued)

Table B.2 (continued)

a	B	$K_{a,b}$	$K_{a,b}(X < 1)$
0	11/2	$(1+X)^{-1/2} \left[1 - 4X(1+X)^{-1} / 3 + 6X^2(1+X)^{-2} / 5 - 4X^3(1+X)^{-3} / 7 + X^4(1+X)^{-4} / 9 \right]$	$1 - 11X/6$
2	11/2	$(1+X)^{-3/2} \left[1/3 - 3X(1+X)^{-1} / 5 + 3X^2(1+X)^{-2} / 7 - X^3(1+X)^{-3} / 9 \right]$	$1/3 - 11X/10$
4	11/2	$(1+X)^{-5/2} \left[1/5 - 2X(1+X)^{-1} / 7 - X^2(1+X)^{-2} / 9 \right]$	$1/5 - 11X/14$
6	11/2	$(1-X)^{-7/2} \left[1/7 - X(1+X)^{-1} / 9 \right]$	$1/7 - 11X/18$
0	6	$\left[21(1+X)^{-3} / 80 + 9(1+X)^{-4} / 40 + (1+X)^{-5} / 5 + 21(1+X)^{-2} / 64 + 63(1+X)^{-1} / 128 + 63\varphi(X)/128 \right] / 2$	$1 - 2X$
2	6	$X^{-1} \left[-(1+X)^{-5} / 5 + 7(1+X)^{-2} / 192 + 7(1+X)^{-3} / 240 + (1+X)^{-4} / 40 + 7(1+X)^{-1} / 128 + 7\varphi(X)/128 \right] / 2$	$1/3 - 6X/5$
4	6	$-X^{-2} \left[-11(1+X)^{-4} / 40 + (1+X)^{-5} / 5 + 3(1+X)^{-1} / 128 + (1+X)^{-2} / 64 + (1+X)^{-3} / 80 + 3\varphi(X)/128 \right] / 2$	$1/5 - 6X/7$
6	6	$X^{-3} \left[3(1+X)^{-1} / 128 + (1+X)^{-2} / 64 - 39(1+X)^{-3} / 240 - 3(3X-4)(1+X)^{-4} / 40 - (1+X)^{-5} / 5 + 3\varphi(X)/128 \right] / 2$	$1/7 - 2X/3$

Note: $\Psi(X) = \frac{1}{\sqrt{X}} \ln(\sqrt{1+X} + \sqrt{X})$

Appendix C

Coefficient values in the expressions of the third-order moments (4.4.13)–(4.4.16) of the Coulomb collision integral:

$$\begin{aligned}
 C_\alpha = & -8\rho_s \left\{ \frac{6b_{r\perp}^2}{\beta_{sr\perp}} (1 - a_{sr}) N_{402} + \left[\frac{12}{\beta_{s\perp} + \beta_{r\perp}} X_{sr} + \frac{18\mu_{sr}b_{r\perp}}{\beta_{s\perp}} \left(\frac{b_{r\perp}}{m_r} - \frac{b_{s\perp}}{m_s} \right) \right] \right. \\
 & \times N_{220} + \frac{2b_{r\perp}^2}{\beta_{sr\perp}} (K_{220} + a_{sr}K_{202}) + \left[\frac{4 - 3a_{sr}}{\beta_{s\perp} + \beta_{r\perp}} - \frac{6\mu_{sr}b_{r\perp}}{\beta_{sr\perp}} \left(\frac{b_{r\perp}}{m_r} - \frac{b_{s\perp}}{m_s} \right) \right] \\
 & \left. \times K_{200} + \frac{a_{sr}}{\beta_{s\perp} + \beta_{r\perp}} K_{002} \right\} \quad (C1)
 \end{aligned}$$

$$\begin{aligned}
 C_\alpha^\perp = & 32b_{r\perp}(\mu_r + 3b_{r\perp})K_{420} + 64\mu_r b_{r\perp}K_{222} + 16[5\mu_r b_{s\perp} \\
 & - 2\mu_r - 6b_{r\perp}(3b_{s\perp} - 1)]K_{220} + 32\mu_r(2b_{s\perp} - 1)K_{202} \\
 & - 8b_{s\perp}(2\mu_r + 9b_{s\perp} - 6)K_{200} - 16\mu_r b_{s\perp}K_{002} \quad (C2)
 \end{aligned}$$

$$\begin{aligned}
 C_\alpha^\parallel = & 32a_{sr}^2 b_{r\perp}(\mu_r - 3b_{r\perp})K_{222} + 32\mu_r a_{sr}^2 b_{r\perp}K_{204} \\
 & - 16a_{sr}b_{r\perp}(\mu_r - 3b_{r\perp})K_{220} + 16a_{sr}(\mu_r a_{sr}b_{s\perp} \\
 & - \mu_r b_{r\perp} - 3a_{sr}b_{s\perp}b_{r\perp})K_{202} + 16\mu_r a_{sr}^2 b_{s\perp}K_{004} \\
 & - 8a_{sr}b_{s\perp}(\mu_r - 3b_{r\perp})K_{200} - 8\mu_r a_{sr}b_{s\perp}K_{002} \quad (C3)
 \end{aligned}$$

$$\begin{aligned}
 D_\alpha^\perp = & 32b_{r\perp}(3b_{r\perp} - \mu_r)K_{420} - 64\mu_r b_{r\perp}K_{222} + 16b_{r\perp}[5\mu_r - 2(8b_{r\perp} - b_{s\perp})] \\
 & K_{220} + 64\mu_r b_{r\perp}K_{202} - 8b_{r\perp}(2\mu_r + 2 - 9b_{r\perp})K_{200} - 16\mu_r b_{r\perp}K_{002} \quad (C4)
 \end{aligned}$$

$$\begin{aligned}
 D_\alpha^\parallel = & -32a_{sr}^2 b_{r\perp}(\mu_r - 3b_{r\perp})K_{222} - 32\mu_r a_{sr}^2 b_{r\perp}K_{204} + 16a_{sr}b_{r\perp}(\mu_r - 3b_{r\perp}) \\
 & K_{220} + 16a_{sr}b_{r\perp}[\mu_r + \mu_r a_{sr} + a_{sr}(b_{s\perp} - 2b_{r\perp})]K_{202} + 16\mu_r a_{sr}^2 b_{r\perp}K_{004} \\
 & - 8a_{sr}b_{r\perp}(\mu_r + b_{s\perp} - 2b_{r\perp})K_{200} - 8\mu_r a_{sr}b_{r\perp}K_{002} \quad (C5)
 \end{aligned}$$

$$\begin{aligned}
R_\alpha = & -2\rho_s \left\{ \frac{6}{\beta_{sr\perp}} \left[\mu_s b_{r\parallel} (2b_{r\perp} + b_{r\parallel}) - \mu_r (b_{r\perp} b_{s\parallel} + b_{r\parallel} b_{s\parallel} + b_{s\perp} b_{r\parallel}) \right. \right. \\
& - \frac{1}{2} a_{sr} \beta_{sr\perp} / (\beta_{s\parallel} + \beta_{r\parallel}) \left. \right] N_{202} - \frac{12}{\beta_{s\parallel} + \beta_{r\parallel}} N_{220} - \frac{8b_{r\parallel}}{\beta_{sr\perp}} (2b_{r\perp} + b_{r\parallel}) N_{402} \\
& - \frac{6a_{sr}}{\beta_{sr\perp}} B_{r\parallel} (2b_{r\perp} + b_{r\parallel}) N_{204} + \frac{2b_{r\parallel}}{\beta_{sr\perp}} \left[[2b_{r\perp} (1 + a_{sr}) + 4b_{r\parallel}] K_{202} + b_{r\parallel} a_{sr} K_{004} \right] \\
& + \left[\frac{4}{\beta_{s\parallel} + \beta_{r\parallel}} - \frac{2b_{r\perp}}{\beta_{sr\perp}} (\mu_s \beta_{r\parallel} - \mu_r \beta_{s\parallel}) \right] K_{200} \\
& + \left. \left[\frac{a_{sr}}{\beta_{s\parallel} + \beta_{r\parallel}} - \frac{2b_{r\parallel}}{\beta_{sr\perp}} (\mu_s \beta_{r\perp} - \mu_r \beta_{s\perp}) - \frac{2b_{r\parallel}}{\beta_{sr\perp}} (\mu_s \beta_{r\parallel} - \mu_r \beta_{s\parallel}) \right] K_{002} \right\} \quad (C6)
\end{aligned}$$

$$\begin{aligned}
R_\alpha^\perp = & -16b_{r\parallel} (2b_{r\perp} + 2\mu_r + b_{r\parallel}) K_{222} + 8b_{r\parallel} (2\mu_r - 4b_{s\perp} + b_{r\parallel} + 2) K_{202} \\
& + 8 \left[\mu_r (b_{s\perp} - b_{r\perp}) - \frac{\beta_{sr\perp}}{\beta_{s\parallel} + \beta_{r\parallel}} \right] K_{220} - 4b_{s\perp} \left(2\mu_r - \frac{\beta_{r\perp}}{\beta_{s\parallel} + \beta_{r\parallel}} \right) K_{200} \\
& + 32\mu_r b_{r\perp} K_{420} + 8b_{r\parallel} b_{s\perp} K_{002} \quad (C7)
\end{aligned}$$

$$\begin{aligned}
R_\alpha^\parallel = & -4a_{sr} [2\mu_r (2b_s - b_{r\parallel} - a_{sr} b_{s\perp}) + (6b_{r\perp} b_{s\parallel} + 6b_{r\parallel} b_{s\parallel} - 2b_{r\perp} - b_{r\parallel})] \\
& K_{202} + 32\mu_r a_{sr}^2 b_{r\perp} K_{222} - 16\mu_r a_{sr} b_{r\perp} K_{220} - 2[2\mu_r a_{sr} b_{s\perp} + b_{s\parallel} (3b_{s\parallel} - 1)] \\
& K_{200} - 4a_{sr} b_{s\perp} (3b_{s\parallel} - 1) K_{002} - 8a_{sr}^2 b_{s\perp} b_{r\parallel} K_{004} - 8a_{sr}^3 b_{r\parallel} \\
& (2b_{r\perp} + 2\mu_r + b_{r\parallel}) K_{204} \quad (C8)
\end{aligned}$$

$$\begin{aligned}
P_\alpha^\perp = & 16b_{r\parallel} (2b_{r\perp} + 2\mu_r + b_{r\parallel}) K_{222} - 8b_{r\parallel} (2\mu_r + 4b_{r\perp} + b_{r\parallel}) K_{202} + 8b_{r\parallel} b_{r\perp} K_{002} \\
& + 8b_{r\perp} \left(8\mu_r + \frac{\beta_{s\perp}}{\beta_{s\parallel} + \beta_{r\parallel}} \right) K_{220} - 4b_{r\perp} \left(2\mu_r + \frac{\beta_{s\perp}}{\beta_{s\parallel} + \beta_{r\parallel}} \right) K_{200} - 32\mu_r b_{r\perp} K_{420} \quad (C9)
\end{aligned}$$

$$\begin{aligned}
P_\alpha^\parallel = & 4a_{sr} [2\mu_r (a_{sr} b_{r\perp} - 3b_{r\parallel}) - b_{r\parallel} (6b_{r\perp} + 5b_{r\parallel} - b_{s\parallel})] K_{202} \\
& + 8a_{sr}^2 b_{r\parallel} (2b_{r\perp} + 2\mu_r + b_{r\parallel}) K_{204} + 16\mu_r a_{sr} b_{r\perp} K_{220} \\
& - 32\mu_r a_{sr}^2 b_{r\perp} K_{222} - 2[2\mu_r a_{sr} b_{r\perp} - b_{r\parallel} (3b_{r\parallel} - 1)] K_{200} \\
& + 12a_{sr} b_{r\perp} b_{r\parallel} K_{002} - 8a_{sr}^2 b_{r\perp} b_{r\parallel} K_{004} \alpha = 1, 2 \quad (C10)
\end{aligned}$$

$$C_3 = -8\rho_s \left\{ \left[\frac{3\mu_{sr} b_{r\perp}}{\beta_{sr\perp}} \left(\frac{b_{r\perp} + b_{r\parallel}}{m_r} - \frac{b_{s\perp} + b_{s\parallel}}{m_s} \right) + \frac{3\mu_{sr} b_{r\parallel}}{\beta_{sr\perp}} \left(\frac{b_{r\perp}}{m_r} - \frac{b_{s\perp}}{m_s} \right) + \frac{3X_{sr}}{\beta_{s\perp} + \beta_{r\perp}} \right] \right\}$$

$$\begin{aligned}
& \times N_{202} + \frac{4X_{sr}}{\beta_{sr\perp}} b_{r\perp} (2b_{r\parallel} + b_{r\perp}) N_{402} + \frac{2b_{r\perp} b_{r\parallel}}{\beta_{sr\perp}} (1 - 2a_{sr}) K_{202} + \frac{4b_{r\perp}^2}{\beta_{sr\perp}} K_{220} \\
& + \left[\frac{1}{\beta_{s\perp} + \beta_{r\perp}} - \frac{\mu_{sr} b_{r\perp}}{\beta_{sr\perp}} \left(\frac{b_{r\perp} + b_{r\parallel}}{m_r} - \frac{b_{s\perp} + b_{s\parallel}}{m_s} \right) \right] K_{200} - \frac{\mu_{sr} b_{r\parallel}}{\beta_{sr\perp}} \left(\frac{b_{r\perp}}{m_r} - \frac{b_{s\perp}}{m_s} \right) K_{002} \Bigg\}
\end{aligned} \tag{C11}$$

$$\begin{aligned}
C_3^\perp = & 64a_{sr} [\mu_r (b_{r\parallel} - 2b_{r\perp}) - b_{r\perp} (2b_{r\parallel} + b_{r\perp})] K_{222} + 32\mu_r a_{sr} b_{r\parallel} K_{204} \\
& + 16 [\mu_r (b_{s\parallel} - a_{sr} b_{r\parallel} - 2a_{sr} b_{s\perp} + 2a_{sr} b_{r\perp}) - a_{sr} (4b_{s\perp} b_{r\perp} + 4b_{s\perp} b_{r\parallel} \\
& - 2b_{r\parallel} - b_{r\perp})] K_{202} + 32b_{s\parallel} (\mu_r - 2b_{r\perp}) K_{220} - 8b_{s\parallel} (\mu_r + 4b_{s\perp} - 2) K_{200} \\
& - 16\mu_r a_{sr} b_{r\parallel} K_{004} - 8 [\mu_r b_{s\parallel} + a_{sr} b_{s\perp} (2b_{s\perp} - 1)] K_{002}
\end{aligned} \tag{C12}$$

$$\begin{aligned}
C_3^\parallel = & \frac{16}{3} a_{sr}^3 [\mu_r (b_r - 2b_{r\perp}) - b_{r\perp} (2b_{r\parallel} + b_{r\perp})] K_{204} - 4a_{sr} b_{s\parallel} (\mu_r - 2b_{r\perp}) K_{200} \\
& + 8a_{sr}^2 [\mu_r (2b_{r\perp} + b_{s\parallel} - b_{r\parallel}) - b_{r\perp} (4b_{s\parallel} - b_{r\perp} - 2)] K_{202} + \frac{16}{3} a_{sr}^3 \mu_r b_{r\parallel} K_{006} \\
& - 4a_{sr} (\mu_r b_{s\parallel} - a_{sr} b_{s\perp} b_{r\perp}) K_{002} + 8a_{sr}^2 \left[\mu_r (b_{s\parallel} - b_{r\parallel}) - \frac{1}{3} a_{sr} b_{s\perp} b_{r\perp} \right] K_{004}
\end{aligned} \tag{C13}$$

$$\begin{aligned}
D_3^\perp = & 64a_{sr} [\mu_r (2b_{r\perp} - b_{r\parallel}) + b_{r\perp} (2b_{r\parallel} + b_{r\perp})] K_{222} - 32\mu_r a_{sr} b_{r\parallel} K_{204} \\
& + 16 [\mu_r (a_{sr} b_{r\parallel} + b_{r\parallel} - 4a_{sr} b_{r\perp}) - a_{sr} b_{r\perp} (4b_{r\parallel} + 3b_{r\perp} - b_{s\perp})] K_{202} \\
& + 16\mu_r a_{sr} b_{r\parallel} K_{004} + 32b_{r\parallel} (\mu_r - 2b_{r\perp}) K_{220} - 8b_{r\parallel} (\mu_r - 4b_{r\perp}) K_{200} \\
& - 8 [\mu_r b_{r\parallel} - a_{sr} b_{r\perp} (2b_{r\perp} - 1)] K_{002}
\end{aligned} \tag{C14}$$

$$\begin{aligned}
D_3^\parallel = & \frac{16}{3} a_{sr}^3 [\mu_r (2b_{r\perp} - b_{r\parallel}) + b_{r\perp} (2b_{r\parallel} + b_{r\perp})] K_{204} - \frac{16}{3} a_{sr}^2 \mu_r b_{r\parallel} K_{006} \\
& + 8a_{sr}^2 [2\mu_r (b_{r\parallel} - b_{r\perp}) - b_{r\perp} (4b_{r\parallel} + b_{r\perp})] K_{202} - 4a_{sr} b_{r\parallel} (\mu_r - 2b_{r\perp}) K_{200} \\
& - 4a_{sr} (\mu_r b_{r\parallel} + a_{sr} b_{s\perp} b_{r\perp}) K_{002} + 8a_{sr}^2 \left(\mu_r b_{r\parallel} + \frac{1}{3} a_{sr} b_{s\perp} b_{r\perp} \right) K_{004}
\end{aligned} \tag{C15}$$

$$\begin{aligned}
R_3 = & -12\rho_s \left\{ \left[\frac{3X_{sr}}{\beta_{s\parallel} + \beta_{r\parallel}} - \frac{6\mu_{sr} b_{r\parallel}}{\beta_{sr\perp}} \left(\frac{b_{r\parallel}}{m_r} - \frac{b_{s\parallel}}{m_s} \right) \right] N_{202} + \right. \\
& \times \left. \frac{6b_{r\parallel}^2 X_{sr}}{\beta_{sr\perp}} N_{204} + \frac{2b_{r\parallel}^2}{\beta_{sr\perp}} K_{202} + \frac{1}{\beta_{s\parallel} + \beta_{r\parallel}} K_{200} \right\}
\end{aligned} \tag{C16}$$

$$\begin{aligned}
R_3^\perp = & 192\mu_r a_{sr} b_{r\parallel} K_{222} + 96\mu_r b_{s\parallel} K_{220} - 24b_{r\parallel} (2a_{sr} \mu_r + 3b_{s\parallel}) K_{202} \\
& - 24\mu_r b_{s\parallel} K_{200} + 36b_{s\parallel} b_{r\parallel} K_{002} + 24a_{sr} b_{r\parallel}^2 K_{004} - 48a_{sr} b_{r\parallel}^2 K_{204}
\end{aligned} \tag{C17}$$

$$\begin{aligned}
R_3^{\parallel} = & 16\mu_r a_{sr}^3 b_{r\parallel} K_{204} + 24\mu_r a_{sr}^2 (b_{s\parallel} - b_{r\parallel}) K_{202} - 12\mu_r a_{sr} b_{s\parallel} K_{200} - 6a_{sr} b_{s\parallel} \\
& (5b_{s\parallel} - 3) K_{002} - 12a_{sr}^2 b_{r\parallel} \left(\frac{10}{3} b_{s\parallel} - 1 \right) K_{004} - 8a_{sr}^3 b_{r\parallel} K_{006}
\end{aligned} \tag{C18}$$

$$\begin{aligned}
P_3^{\perp} = & 24b_{r\parallel} (2a_{sr}\mu_r - 2b_{r\parallel} + b_{s\parallel}) K_{202} - 24\mu_r b_{r\parallel} K_{200} - 192\mu_r a_{sr} b_{r\parallel} K_{222} \\
& + 96\mu_r b_{r\parallel} K_{220} + 12b_{r\parallel} (2b_{r\parallel} - b_{s\parallel}) K_{002} - 24a_{sr} b_{r\parallel}^2 K_{004} + 48a_{sr} b_{r\parallel} K_{204}
\end{aligned} \tag{C19}$$

$$\begin{aligned}
P_3^{\parallel} = & 48\mu_r b_{r\parallel} a_{sr}^2 K_{202} - 12\mu_r a_{sr} b_{r\parallel} K_{200} - 16\mu_r a_{sr}^3 b_{r\parallel} K_{204} + 6a_{sr} b_{r\parallel} \\
& (5b_{r\parallel} - 1) K_{002} - 12a_{sr}^2 b_{r\parallel} \left(3b_{r\parallel} - \frac{1}{3} b_{s\parallel} \right) K_{004} + 8a_{sr}^3 b_{r\parallel}^2 K_{006}
\end{aligned} \tag{C20}$$

$$N_{202} = \frac{1}{3X_{sr}} (a_{sr} K_{002} - K_{200}), \quad X_{sr} = a_{sr} - 1$$

$$N_{220} = \frac{1}{12X_{sr}} [(3a_{sr} - 2)K_{200} - a_{sr}K_{002}]$$

$$N_{004} = \frac{1}{3X_{sr}} [(a_{sr} - 3)K_{002} + 2K_{200}]$$

$$N_{402} = 3N_{222}$$

$$N_{204} = \frac{1}{3X_{sr}} [a_{sr}K_{004} - (2a_{sr} + 1)K_{202} + K_{200} - K_{002}]$$

$$N_{222} = \frac{1}{12X_{sr}} [(5a_{sr} - 2)K_{202} - a_{sr}K_{004} - K_{200} + K_{002}].$$

Appendix D

Below are presented collisional moments calculated in Demars and Schunk (1979) in the 16-moment approximation for the particles interacting according to a generalized law

$$F = K_{st}/r^k, \quad (\text{D1})$$

where K_{st} , k are constants that define the type of interaction; and r is the distance between interacting particles of the kind s or t . Transport scattering cross-section corresponding to (D1) of order l is (Chapman Cowling 1958)

$$\sigma_{st}^{(\ell)} = 2\pi A_\ell(k) (K_{st}/\mu_{st}g^2)^{2/(k-1)} \quad (\text{D2})$$

$$A_\ell(k) = \int_0^\infty (1 - \cos^\ell \chi) \rho_0 d\rho_0. \quad (\text{D3})$$

Here χ is the scattering angle; and ρ_0 is the nondimensional shooting parameter. The quantity $A_\ell(k)$ is a constant coefficient depending on the constraints l and k (on the interaction law).

We will consider here only the moments $J^{(1)}$, $J_{||}^{(2)}$, and $J_{\perp}^{(2)}$, as other moments are very complicated (see more in Demars and Schunk 1979):

$$\begin{aligned} \mathbf{J}_s^{(1)} = & - \sum_t \frac{3v_{st}}{4\pi\Gamma(5/2 - n/2)} \left\{ 2\rho_s (I_{200}\mathbf{u}_\perp + a_{st}I_{002}\mathbf{u}_\parallel) \right. \\ & + 2\beta_{st\perp} \left[(2I_{220} - I_{200}) \left(\mathbf{S}_{s\perp}^\perp - \frac{\rho_s}{\rho_t} \mathbf{S}_{t\perp}^\perp \right) + a_{st} (2I_{202} - I_{002}) \left(\mathbf{S}_{s||}^\perp - \frac{\rho_s}{\rho_t} \mathbf{S}_{t||}^\perp \right) \right] \\ & \left. + \beta_{st||} \left[(2a_{st}I_{202} - I_{200}) \left(\mathbf{S}_{s\perp}^\parallel - \frac{\rho_s}{\rho_t} \mathbf{S}_{t\perp}^\parallel \right) + a_{st} \left(\frac{2}{3}a_{st}I_{004} - I_{002} \right) \left(\mathbf{S}_{s||}^\parallel - \frac{\rho_s}{\rho_t} \mathbf{S}_{t||}^\parallel \right) \right] \right\} \end{aligned} \quad (\text{D4})$$

$$J_{s||}^{(2)} = -\frac{3\rho_s}{2\pi\Gamma(5/2 - n/2)} \times \sum_t \frac{v_{st}}{m_s + m_t} \left[2a_{st}I_{002}(T_{s||} - T_{t||}) + \frac{m_t}{\beta_{st\perp}} \frac{A_2(k)}{A_1(k)} (I_{002} - I_{200}) \right] \quad (D5)$$

$$J_{s\perp}^{(2)} = -\frac{3\rho_s}{2\pi\Gamma(5/2 - n/2)} \times \sum_t \frac{v_{st}}{m_s + m_t} \left[4I_{200}(T_{s\perp} - T_{t\perp}) + \frac{m_t}{\beta_{st\perp}} \frac{A_2(k)}{A_1(k)} (I_{200} - I_{002}) \right]. \quad (D6)$$

Here $n = 4/(k - 1) - 1$; $\Gamma(y)$ is the gamma function;

$$v_{st} = \frac{8\sqrt{2\pi}}{3} A_\ell(k) \Gamma\left(\frac{5-n}{2}\right) \frac{n_t m_t}{n_s + m_t} \left(\frac{K_{st}}{2\mu_{st}}\right)^{\frac{n+1}{2}} \beta_{st||}^{\frac{1}{2}} \beta_{st\perp}^{\frac{n-1}{2}} \quad (D7)$$

is the collision frequency (momentum exchange); and

$$I_{LMN} = \int \frac{y_1^L y_2^M y_3^N}{y^n} \exp[-(y_1^2 + y_2^2 + a_{st} y_3^2)] d^3 y. \quad (D8)$$

These integrals are of the more general form than K_{LMN} introduced in Sect. 3.1 to describe collisions of the charged particles with each other. If we assume $k = 2$ in (D1) ($n = 3$ – the Coulomb interaction), then the integral is I_{LMN} but the factor $1/\pi$ will be included in K_{LMN} (4.4.7). Below is the table of their values that are used here:

L	M	N	I_{LMN}
2	0	0	$\pi \Gamma\left(\frac{5-n}{2}\right) (Y_{0,5}^{(n)} - Y_{2,5}^{(n)})$
0	0	2	$2\pi \Gamma\left(\frac{5-n}{2}\right) Y_{2,5}^{(n)}$
2	2	0	$\frac{\pi}{4} \Gamma\left(\frac{7-n}{2}\right) (Y_{0,7}^{(n)} - 2Y_{2,7}^{(n)} + Y_{4,7}^{(n)})$
2	0	2	$\pi \Gamma\left(\frac{7-n}{2}\right) (Y_{2,7}^{(n)} - Y_{4,7}^{(n)})$
0	0	4	$2\pi \Gamma\left(\frac{7-n}{2}\right) Y_{4,7}^{(n)}$

In the table,

$$Y_{\ell,m}^{(n)} = \int_0^1 t^\ell (1 + X t^2)^{-\frac{m-n}{2}} dt, \quad X = a_{st} - 1.$$

Integrals $Y_{\ell,m}^{(n)}$ can be expressed through hypergeometric functions

$$Y_{\ell,m}^{(n)} = \frac{1}{\ell+1} {}_2F_1\left(\frac{m-n}{2}, \frac{\ell+1}{2}, \frac{\ell+3}{2}, -X\right). \quad (D9)$$

The values of these integrals for certain types of particle interactions are given in Demars and Schunk (1979).

The above moments (D4)–(D6) were calculated in the assumption that the difference between macroscopic velocities of the colliding particles were small compared with thermal velocity (4.4.3). Generalizing these results for arbitrary velocities in Barakat and Schunk (1981) for particles having bi-Maxwellian distribution functions (4.1.7) and interacting according to (D1), corresponding expressions are obtained for the friction force and energy exchange processes:

$$\mathbf{J}_s^{(1)} = -\rho_s \sum_t v_{st} \hat{\mathbf{D}} \cdot \mathbf{u} (D \quad (D10)$$

$$\begin{aligned} J_{s||}^{(2)} &= \rho_s \\ &\times \sum_t \frac{v_{st}}{m_s + m_t} \left[3\Psi_{||}(T_{t||} - T_{s||}) + 2m_t \mathbf{u}_{||} \cdot \hat{\mathbf{D}} \cdot \mathbf{u} + m_t \frac{A_2(k)}{A_1(k)} \left(\frac{\Omega_2(n)}{\beta_{st\perp}} - \frac{2\Omega_1(n)}{\beta_{st||}} \right) \right] \end{aligned} \quad (D11)$$

$$\begin{aligned} J_{s\perp}^{(2)} &= \rho_s \\ &\times \sum_t \frac{v_{st}}{m_s + m_t} \left[3\Psi_{\perp}(T_{t\perp} - T_{s\perp}) + m_t \mathbf{u}_{\perp} \cdot \hat{\mathbf{D}} \cdot \mathbf{u} - \frac{m_t}{2} \frac{A_2(k)}{A_1(k)} \left(\frac{\Omega_2(n)}{\beta_{st\perp}} - \frac{2\Omega_1(n)}{\beta_{st||}} \right) \right], \end{aligned} \quad (D12)$$

where

$$\begin{aligned} \hat{\mathbf{D}} \cdot \mathbf{u} &= \sqrt{\frac{2}{\beta_{st\perp}}} \Lambda(n) \\ \Psi_{ii} &= \frac{4}{3} \left[\Omega_1(n) - \sqrt{\frac{\beta_{st||}}{2}} \mathbf{e}_{||} \cdot \hat{\mathbf{D}} \cdot \mathbf{u} \right] \\ \Psi_{\perp} &= \frac{2}{3} \left[\Omega_2(n) - \sqrt{\frac{\beta_{st\perp}}{2}} \mathbf{e}_{\perp} \cdot \hat{\mathbf{D}} \cdot \mathbf{u} \right] \\ \mathbf{e} &= \sqrt{\beta_{st||}/2} \mathbf{u}_{||} + \sqrt{\beta_{st\perp}/2} \mathbf{u}_{\perp} \end{aligned} \quad (D13)$$

Quantities $\Omega_1(n)$, $\Omega_2(n)$, $\Lambda(n)$ are given by

$$\begin{aligned}\Lambda(n) &= \frac{3a_{st}^{-1/2}}{4\pi\Gamma(\frac{5}{2}-\frac{n}{2})} \left(\frac{2}{\beta_{st\perp}}\right)^{\frac{n-1}{2}} \int \frac{\mathbf{g}}{g^n} \exp[-(\mathbf{X}-\mathbf{e})^2] d\mathbf{X} \\ \Omega_1(n) &= \frac{3a_{st}^{1/2}}{4\pi\Gamma(\frac{5}{2}-\frac{n}{2})} \left(\frac{2}{\beta_{st\perp}}\right)^{\frac{n-2}{2}} \int \frac{g_{II}^2}{g^n} \exp[-(\mathbf{X}-\mathbf{e})^2] d\mathbf{X} \\ \Omega_2(n) &= \frac{3a_{st}^{-1/2}}{4\pi\Gamma(\frac{5}{2}-\frac{n}{2})} \left(\frac{2}{\beta_{st\perp}}\right)^{\frac{n-2}{2}} \int \frac{g_{\perp}^2}{g^n} \exp[-(\mathbf{X}-\mathbf{e})^2] d\mathbf{X}.\end{aligned}\quad (\text{D14})$$

In the limiting case of small directional velocity differences $|\mathbf{e}| \ll 1$, the integrals (D14) are reduced to

$$\begin{aligned}\Lambda(n) &= \frac{3(\beta_{st\perp}/2)^{1/2}}{2\pi\Gamma(5/2-n/2)} (a_{st}I_{002}\mathbf{u}_{||} + I_{200}\mathbf{u}_{\perp}) \\ \Omega_1(n) &= \frac{3a_{st}}{4\pi\Gamma(5/2-n/2)} I_{002} \\ \Omega_2(n) &= \frac{3}{2\pi\Gamma(5/2-n/2)} I_{200}.\end{aligned}\quad (\text{D15})$$

Using these expressions from (D13), we get

$$\begin{aligned}D_{\alpha\beta} &= \frac{3}{2\pi\Gamma(5/2-n/2)} (\tau_{\alpha\beta}a_{st}I_{002} + n_{\alpha\beta}I_{200}) \\ \Psi_{II} &= \frac{a_{st}}{\pi\Gamma(5/2-n/2)} I_{002}, \quad \Psi_{\perp} = \frac{1}{\pi\Gamma(5/2-n/2)} I_{200}.\end{aligned}\quad (\text{D16})$$

Substituting these relations into (D10)–(D12), we arrive to (D4) and (D5).

In the opposite limiting case of large macroscopic velocity differences $|\mathbf{e}| \gg 1$:

$$\begin{aligned}\Lambda(n) &= \frac{3}{4\Gamma(5/2-n/2)} \sqrt{\frac{\pi\beta_{st\perp}}{2a_{st}}} \left(\frac{2}{\beta_{st\perp}u^2}\right)^{n/2} \mathbf{u} \\ \Omega_1(n) &= \frac{3\sqrt{\pi a_{st}}\beta_{st\perp}}{8\Gamma(5/2-n/2)} \left(\frac{2}{\beta_{st\perp}u^2}\right)^{n/2} u_{||}^2 \\ \Omega_2(n) &= \frac{3\sqrt{\pi/a_{st}}\beta_{st\perp}}{8\Gamma(5/2-n/2)} \left(\frac{2}{\beta_{st\perp}u^2}\right)^{n/2} u_{\perp}^2 \\ \Psi_{||} &\approx \Psi_{\perp} \approx 0 \\ D_{\alpha\beta} &= \frac{3\sqrt{\pi/a_{st}}}{4\Gamma(5/2-n/2)} \left(\frac{2}{\beta_{st\perp}u^2}\right)^{n/2} \delta_{\alpha\beta}.\end{aligned}\quad (\text{D17})$$

It should be noted also that, in Barakat and Schunk (1981), simple approximate expressions are obtained for the moments (D10)–(D12) for arbitrary values of the directional velocities for strictly parallel or perpendicular motions of plasma.

Appendix E

$$\hat{D}_1 Q_n \Big|_{a=\frac{k_{\perp} v_{\perp L}}{\omega_B}} = x \left[-A_n T^* + \frac{1+T^*}{2} A_{n-1} - \frac{1-T^*}{2} A_{n+1} \right], \quad (E1)$$

$$\hat{D}_2 Q_n \Big|_{a=\frac{k_{\perp} v_{\perp L}}{\omega_B}} = x [2(x-1)A_n - x(A_{n-1} + A_{n+1})], \quad (E2)$$

$$\begin{aligned} \hat{D}_3 Q_n \Big|_{a=\frac{k_{\perp} v_{\perp L}}{\omega_B}} = & x^2 [T^*(4-3x)A_n + (2(1+T^*) - (2T^*+1)x)A_{n-1} \\ & + (2(1-T^*) - (2T^*-1)x)A_{n+1} - \frac{1+T^*}{2} x A_{n-2} - \frac{1-T^*}{2} x A_{n+2}] ; \end{aligned} \quad (E3)$$

$$\begin{aligned} \hat{D}_1 \left[Q_n \frac{n}{x} + T R_n \right] \Big|_{a=\frac{k_{\perp} v_{\perp L}}{\omega_B}} = & \frac{x}{2} (3|T|^2 - 1) A_n + \frac{1+T^*}{2} [1+T-2xT] A_{n-1} \\ & - \frac{1-T^*}{2} [1-T+2xT] A_{n+1} + \frac{|1+T|^2}{4} x A_{n-2} + \frac{|1-T|^2}{4} x A_{n+2} \Big], \end{aligned} \quad (E4)$$

$$\begin{aligned} \hat{D}_2 \left[Q_n \frac{n}{x} + T R_n \right] \Big|_{a=\frac{k_{\perp} v_{\perp L}}{\omega_B}} = & x \left\{ T(4-3x)A_n + (2T(x-1) + x-2)A_{n-1} \right. \\ & \left. + [2T(x-1) - x+2]A_{n+1} - \frac{1+T}{2} x A_{n-2} + \frac{1-T}{2} x A_{n+2} \right\}, \end{aligned} \quad (E5)$$

$$\begin{aligned}
\hat{D}_3 \left[Q_n \frac{n}{x} + TR_n \right] \Big|_{a=\frac{k_1 v_{\perp} t}{\omega_B}} &= x \left\{ (2-x)(1-5|T|^2)x A_n \right. \\
&+ \left[-2|1+T|^2 + x(1+4T^*+4T+7|T|^2) + \frac{x^2}{4}(1-5T^*-5T-15|T|^2) \right] A_{n-1} \\
&+ \left[-2|1-T|^2 + x(1-4T^*-4T+7|T|^2) + \frac{x^2}{4}(1+5T^*+5T-15|T|^2) \right] A_{n+1} \\
&+ x \left[-2|1+T|^2 + \frac{x^2}{2}(1+2T^*+2T+3|T|^2) \right] A_{n-2} \\
&+ x \left[-2|1-T|^2 + \frac{x^2}{2}(1-2T^*-2T+3|T|^2) \right] A_{n+2} \\
&\left. - \frac{x^2}{4}|1+T|^2 A_{n-3} - \frac{x^2}{4}|1-T|^2 A_{n+3} \right\},
\end{aligned}
\tag{E6}$$

$$\hat{D}_1 = T^* \lambda \frac{\partial}{\partial a} + n; \quad \hat{D}_2 = \lambda^2 \frac{\partial^2}{\partial a^2} + \lambda \frac{\partial}{\partial a} - n^2;$$

and

$$\hat{D}_3 = T^* \left(\lambda^3 \frac{\partial^3}{\partial a^3} + \lambda^2 \frac{\partial^2}{\partial a^2} - (1+n^2) \lambda \frac{\partial}{\partial a} + 2n^2 \right) + n \hat{D}_2.$$

Appendix F

The method of separation of variables is used to obtain a solution to (5.5.13); thus we write:

$$f(x, z) = X(x)Z(z). \quad (\text{F1})$$

This in turn leads to:

$$X(x) = C_{1k}J_o(2k\sqrt{x}) + C_{2k}N_o(2k\sqrt{x}), \quad (\text{F2})$$

$$Z(z) = \exp(-k^2z), \quad (\text{F3})$$

where k is the separation constant and C_{1k} and C_{2k} are constants. J_o and N_o are the zero-order Bessel and Neuman functions. The second boundary condition, (5.5.17) requires that C_{2k} be zero, because $N_o(x)$ goes to infinity as $x \rightarrow \infty$. Thus the solution to (5.5.13) can be written as:

$$f(x, z) = \int_0^\infty C(k) \exp(-k^2z) J_o(2k\sqrt{x}) dk, \quad (\text{F4})$$

where $C(k) = C_{1k}$. Next, using the boundary condition given by (5.5.16), we get:

$$f^+(x) \Theta(x_{ob} - x) = \int_0^\infty C(k) J_o(2k\sqrt{x}) dk. \quad (\text{F5})$$

Recalling the following relation:

$$\int_0^\infty J_o(2k\sqrt{x}) J_o(2p\sqrt{x}) dx = \frac{1}{2k} \delta(k - p), \quad (\text{F6})$$

we can obtain $C(k)$ by first multiplying (F5) by $J_o(2p\sqrt{x})$ and then integrating it over x . This leads to:

$$C(k) = 2k \int_0^{x_{0b}} f^+(\zeta) J_o(2k\sqrt{\zeta}) d\zeta. \quad (F7)$$

Substituting (F7) into (F4) and introducing the Green function yields:

$$G(x, \zeta, z) = 2 \int_0^\infty k \exp(-k^2 z) J_o(2k\sqrt{\zeta}) J_o(2k\sqrt{x}) dk. \quad (F8)$$

Integrating the above given Green function yields:

$$G(x, \zeta, z) = \frac{1}{z} I_o \left(\frac{2\sqrt{x\zeta}}{z} \right) \exp \left[-\frac{x + \zeta}{z} \right], \quad (F9)$$

where I_0 is the modified Bessel function.

Appendix G

The boundary conditions in (5.5.41) are equivalent to a statement on the symmetry condition on the pitch-angle distribution of superthermal electrons in the trapped region $f_{II}(\mu_o) = f_{II}(-\mu_o)$, because the initial (5.5.39) varies as the sign μ_o does. The eigen functions, $M_n(x)$, of the solution to (5.5.42) are given in terms of Bessel and Neuman functions as:

$$M_n(x) = N_1(2K_n)J_o(2K_n\sqrt{x}) - J_1(2K_n)N_o(2K_n\sqrt{x}), \quad (G1)$$

where the K_n s are the eigennumbers ($n = 0, 1, 2, \dots$) determined from the equation $M_n(x_{ob}) = 0$. The system of functions given by (G1) are orthogonal in the interval $x_{ob} < x < 1$. The square of the norm is:

$$A_n = \int_{x_{ob}}^1 [M_n(x)]^2 dx = x_{ob} M_n^1 B_n,$$

where

$$B_n = \frac{1}{\sqrt{x_{ob}}} M_n^0 - M_n^1, \quad (G2)$$

$$M_n^m = J_m(2K_n)N_m(2K_n\sqrt{x_{ob}}) - N_m(2K_n)J_m(2K_n\sqrt{x_{ob}})$$

We can write $\Psi(x, E)$ as an infinite series using $M_n(x)$:

$$\Psi(x, E) = \sum_{n=0}^{\infty} C_n(E) M_n(x). \quad (G3)$$

The expansion coefficients, C_n , are obtained by substituting (G3) into (5.5.42), which gives:

$$\frac{\partial C_n}{\partial E} - \frac{2K_n^2}{E} C_n = -\frac{1}{A_n} \frac{\partial f_b}{\partial E} \int_{x_{ob}}^1 M_n(x) dx. \quad (G4)$$

Using these results, we can write the expression for the distribution function of superthermal electrons in the trapped zone as:

$$f_H = f_b + \frac{1}{\sqrt{x_{ob}}} \int_E^\infty \frac{\partial f_b}{\partial E_o} \sum_{n=0}^\infty \left(\frac{E}{E_o} \right)^{2K_n^2} \frac{M_n(x)}{K_n B_n} dE_o. \quad (G5)$$

Equation (G5) can be simplified if $x_{ob} \ll 1$. Under these conditions, the K_n and C_n coefficients can be written as:

$$\begin{aligned} \frac{1}{K_o^2} &= \ln \frac{1}{\gamma^2 K_o^2 x_{ob}} \approx \ln \frac{1}{x_{ob}}; \\ K_n|_{n=1,2,\dots} &= \frac{\pi}{2} \left[n + \frac{1}{4} + \frac{1}{\pi} \tan^{-1} \frac{\pi}{\ln(1/\gamma^2 K_n^2 x_{ob})} \right] \\ &\approx \frac{\pi}{2} \left(n + \frac{1}{4} \right), \\ C_o(x) &= 1 - \frac{\ln x}{\ln x_{ob}}; \\ C_n(x)|_{n=1,2,\dots} &= \frac{\pi \ln \{ \gamma^2 K_n^2 x_{ob} \}}{K_n \{ \pi^2 + \ln^2 (\gamma^2 K_n^2 x_{ob}) \}} \left[\frac{\pi}{\ln(\gamma^2 K_n^2 x_{ob})} N_o(2K_n \sqrt{x}) - J_o(2K_n \sqrt{x}) \right], \end{aligned} \quad (G6)$$

where γ is the Euler constant. The inequalities given below follow from (G6):

$$K_o < K_n|_{n=1,2,\dots}; \quad C_o(x) > C_n(x)|_{n=1,2,\dots} \quad \text{for} \quad x_{ob} \leq (\gamma^2 K_n^2 e^\pi)^{-1}.$$

Therefore in evaluating (G5) only the first term in the series has to be considered, thus giving:

$$f_H(E, x) = f_b \frac{\ln x}{\ln x_{ob}} + d \left(1 - \frac{\ln x}{\ln x_{ob}} \right) y^d \int_y^\infty f_b(y') \frac{dy'}{y'^{(1+d)}}, \quad (G7)$$

where $d = K_o^2 = (\ln[1/x_{ob}])^{-1}$ and y' is a dummy variable. Here we also introduced the variable $y = E^2/E_{no}^2$, which is consistent with y_s , introduced in connection with (5.5.38); $E_{no} = E_n(s_o)$.

Appendix H: Other Possible Potential Energies

H.1 The Ponderomotive Potential of Alfvén Waves

We follow Guglielmi et al. (1996), where the single-particle approach has been used to evaluate the ponderomotive force of an Alfvén wave. Let B be the background magnetic field and $E_{\perp}$ be the amplitude of the Alfvén wave. Then the field-aligned component of the so-called Miller force in the low-frequency limit $\omega \ll \Omega_i$ acting on a charged particle with mass m can be presented in the form

$$F_M = \frac{m}{4B^2} \frac{\partial E_{\perp}^2}{\partial s}, \quad (\text{H1})$$

where s is geocentric distance and $\partial/\partial s$ is the spatial derivative along the field line. It is useful to note that (H1) can be derived by averaging the Lorentz force over a period of the Alfvén oscillation.

In addition to (H1), the “magnetic pumping” force

$$F_{LH} = -\frac{m}{2B^2} E_{\perp}^2 \frac{\partial \ln B}{\partial s} \quad (\text{H2})$$

exists if the background magnetic field is inhomogeneous (Lundin and Hultqvist 1989), which can be rewritten using the well-known relation

$$E_{\perp} = c_A b \propto B/\rho^{1/4},$$

where $c_A = B/(\mu_0 \rho)^{1/2}$ is the Alfvén speed, $\rho = \sum m_{\alpha} n_{\alpha}$ is the plasma mass density, b is the magnetic wave amplitude, and μ_0 is the permeability of free space (Alfvén and Fälthammar 1963). Combining (H1) and (H2), the total ponderomotive force of Alfvén waves $F_{PF} = F_M + F_{LH}$ can be rearranged into the form (Guglielmi et al. 1996)

$$a_{\text{PM}} = (b^2/8\mu_0\rho)\partial \ln \rho/\partial s. \quad (\text{H3})$$

Here a_{PM} is the ponderomotive acceleration.

The total ponderomotive force (H3) is independent of B and directed toward decreasing ρ , providing the electromagnetic lift of the background plasma. It is proportional to the mass m and has more influence on the acceleration process of heavy ionospheric ions. Because of the mass density dependence, the ponderomotive force has to be found in the process of the self-consistent coupling of all plasma components based on the quasi-neutrality and currentless conditions.

The ponderomotive potential Ψ_{PM} can be defined as $F_{\text{PM}} = -m\nabla\Psi_{\text{PM}}$ and presented for our one-spatial-dimension case in the following form:

$$\Psi_{\text{PM}} = \frac{2\alpha}{\sqrt{\rho}} \quad (\text{H4})$$

and $\alpha = \sqrt{\rho_0}E_{0\perp}^2/(8B_0^2) = \text{const}$; $E_{0\perp} = E_{\perp}(s_0)$, $B_0 = B(s_0)$, and $\rho_0 = \rho(s_0)$ are values at a certain point s_0 on a given field line of the geomagnetic field. This s_0 is a reference point where the low-frequency wave amplitude is known or assumed and does not have to be the same reference point as that for the distribution function. Because (H4) contains the mass density, and this moment calculation depends on the potential structure, a self-consistent calculation along the field line should be performed when including this effect.

H.2 Centrifugal Acceleration

In the centrifugal acceleration given by Horwitz et al. (1994) for meridional convection across the pole, the approximate parallel acceleration along a polar magnetic field line may be presented in the form

$$a_{\text{C}}^{\text{M}} = 1.5 \left(\frac{E_i}{B_i} \right)^2 \left(\frac{s_i^2}{s_i^3} \right), \quad (\text{H5})$$

where E_i , B_i , and s_i refer to the electric and magnetic field magnitudes and geocentric distance along the field line in the ionosphere, respectively. Again, this reference point does not have to be the same reference point as that for the distribution function.

For purely longitudinal convection along a constant L shell, the centrifugal acceleration is

$$a_{\text{C}}^{\text{L}} = a_{\text{C}}^{\text{M}} \left(1 - \frac{s}{s_i L} \right)^{3/2} \left(1 - \frac{3s}{4s_i L} \right)^{5/2}. \quad (\text{H6})$$

Similar to the previous section, we can also introduce a centrifugal potential

$$F_C = -m\nabla\Psi_C. \quad (\text{H7})$$

In the case of meridian convection, this can be presented in the following analytical form in the field-aligned case:

$$\Psi_C^M = -0.5 \left(\frac{E_i}{B_i} \right)^2 \left(\frac{s}{s_i} \right)^3. \quad (\text{H8})$$

The total potential energy for each plasma component that will be presented in the final expressions for the moments of the distribution function can now be written as

$$\Pi = e_\alpha\varphi - m_\alpha(\Psi_G + \Psi_{PM} - \Psi_C). \quad (\text{H9})$$

Here φ is self-consistent electrostatic potential, and Ψ_G , Ψ_{PM} , and Ψ_C are gravitational, ponderomotive, and centrifugal potentials, respectively.

Appendix I

I.1 Coulomb Collisions

When describing Coulomb interactions of electrons with charged particles, the Landau collision integral is often used, but on a linearized form, assuming that the thermal electrons and ions can be described by Maxwellian distribution functions (Shkarofsky et al. 1966)

$$\langle S_{ee} \rangle = An_e \left\{ \frac{\partial}{\partial E} \left[\frac{\Phi}{E} + T_e \frac{\partial}{\partial E} \left(\frac{\Phi}{E} \right) \right] + \frac{1}{4E^2} \frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial \Phi}{\partial \mu} \right] \right\} \quad (\text{I1})$$

$$\langle S_{ei} \rangle = An_i \left\{ \frac{m_e}{m_i} \frac{\partial}{\partial E} \left[\frac{\Phi}{E} + T_i \frac{\partial}{\partial E} \left(\frac{\Phi}{E} \right) \right] + \frac{1}{4E^2} \frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial \Phi}{\partial \mu} \right] \right\}, \quad (\text{I2})$$

where $A = 2\pi e^4 \ln L = 2.6 \times 10^{-12} \text{ eV}^2 \text{ cm}^2$, and $\ln L$ is the Coulomb logarithm. Further, one can show that the term $\partial^2(\Phi/E)/\partial E^2$ is small compared with the other terms in (I1) (Khazanov and Gefan 1982). Neglecting this derivative and the energy loss in collisions with ions ($m_e/m_i \ll 1$) in (I2), (I1) and (I2) becomes

$$\langle S_{ee} \rangle + \langle S_{ei} \rangle = An_e \left\{ \frac{\partial}{\partial E} \left(\frac{\Phi}{E} \right) + \frac{1}{2E^2} \frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial \Phi}{\partial \mu} \right] \right\}, \quad (\text{I3})$$

where n_e is the density of the thermal electrons. Arriving at (I3), the condition of plasma quasineutrality ($n_e = n_i$) has been used.

I.2 Elastic Collisions

The term describing elastic collisions of electrons with neutral particles is (Gurevich 1978)

$$\begin{aligned} \langle S_{ea} \rangle = & 2n_\alpha \frac{m_e}{m_\alpha} \frac{\partial}{\partial E} \left\{ \sigma_\alpha^{(1)} E^2 \left[\frac{\Phi}{E} + T_n \frac{\partial}{\partial E} \left(\frac{\Phi}{E} \right) \right] \right\} \\ & + n_\alpha \int I_\alpha(E, \chi) [\Phi(E, \mu') - \Phi(E, \mu)] d\Omega, \end{aligned} \quad (\text{I4})$$

where the scattering ($\mu' \rightarrow \mu$) is described by the scattering angles (χ, η) , with $\mu' = \cos \chi \cos \theta + \sin \theta \sin \chi \cos \eta$ and the solid angle $d\Omega = \sin \chi d\chi d\eta$. The density of neutral particles of species α is n_α , the transport cross-section is $\sigma_\alpha^{(1)}(E) = \int I_\alpha(E, \chi) (1 - \cos \chi) d\Omega$, and the differential elastic cross-section is $I_\alpha(E, \chi) d\Omega$.

A convenient approximation of (I4) is

$$\langle S_{ex} \rangle = \frac{n_\alpha \sigma_\alpha^{(1)}}{2} \frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial \Phi}{\partial \mu} \right]. \quad (\text{I5})$$

Strictly speaking, (I5) is physically justified only for high-energy electrons when the interaction with the *Coulomb field* of a scattering center is of primary importance. Nonetheless, we found that, when solving the kinetic equation, the replacement of the isotropic type of elastic scattering (I4) by the Coulomb one (I5) does not result in great differences in the determination of the flux at energies $E > 1$ eV. Lejeune (1979) showed that the error caused by the replacement of isotropic scattering with forward peak scattering of incident electrons with energies of a few tens of electron volts constitutes 20–30%. It should be emphasized that forward peak scattering leads to an increase of the secondary electron production rate, which to some extent compensates for the underestimation of the flux when the approximation of continual energy loss is applied as shown below. Thus, (I5) may be used when solving (7.2.1) throughout the complete energy range of interest. At this point, we note that the introduction of (I5) is not necessary for solving the kinetic equation. It is introduced only to increase the speed of the computational algorithm.

I.3 Inelastic Collisions

Expressions describing inelastic collisions with neutral particles are also derived using a number of well-known approximations. Let us assume that the dissipating electron does not change direction (if forward scattered) and, if ionization occurs, that the secondary electron is emitted perpendicular to the velocity of the initial electron before the collision (Mott and Massey 1965). Then $\langle S_{e\alpha}^* \rangle$ and $\langle S_{e\alpha}^+ \rangle$ can be written in the form (Mantas 1973; Khazanov 1979)

$$\langle S_{e\alpha}^* \rangle = n_\alpha \sum_j \left[\sigma_{\alpha j}^* (E + E_{\alpha j}^*) \Phi(E + E_{\alpha j}^*, \mu) - \sigma_{\alpha j}^*(E) \Phi(E, \mu) \right] \quad (\text{I6})$$

$$\begin{aligned} \langle S_{e\alpha}^+ \rangle = & n_\alpha \int_{E+E_\alpha^+}^{2E+E_\alpha^+} I_\alpha^+(E', E' - E - E_\alpha^+) \Phi(E', \mu) dE' + \frac{n_\alpha}{2\pi} \\ & \int_{2E+E_\alpha^+}^{\infty} I_2^+(E', E) \int_0^{2\pi} \Phi\left(E', \sqrt{(1-\mu^2)} \cos \eta\right) d\eta dE' - n_\alpha \sigma_\alpha^+(E) \Phi(E, \mu) \quad (17) \end{aligned}$$

where σ_{ij}^* is the total cross-section of scattering bringing a neutral particle into an excited state characterized by a threshold energy E_{ij}^* ; and the ionization energy is E_α^+ . We also have

$$\langle \sigma_{e\alpha}^+ \rangle = \int_0^{(E-E_\alpha^+)/2} I_\alpha^+(E, E_2) dE_2,$$

which is the total cross-section of ionization by an electron with energy E . $I_\alpha^+(E, E_2)$ is the appropriate differential cross-section, and E_2 is the energy of a secondary electron.

To further simplify (16) and (17), we use the continuous energy loss approximation (Hoegy et al. 1965). This approximation is valid for high-energy electrons, for which the energy lost during a collision is small in comparison with the original electron energy. Although this condition is not always fulfilled for suprathermal electrons, there is some experience of successfully using the continuous loss approximation at certain energy ranges when calculating photoelectron spectra in the ionosphere (Khazanov 1979). Problems are encountered only in regions with a pronounced fine structure of the energy spectrum, in particular, near the photoionization peaks at $E = 22\text{--}27$ eV.

Using a Taylor series expansion of functions with argument in the form $E + dE$, (16) and (17) become

$$\langle S_{e\alpha}^* \rangle = n_\alpha \sum_j E_{ij}^* \frac{\partial}{\partial E} \left[\sigma_{ij}^*(E) \Phi(E, \mu, s) \right] \quad (18)$$

$$\begin{aligned} \langle S_{e\alpha}^+ \rangle = & n_\alpha \\ & \times \left\{ \int_{2E+E_\alpha^+}^{\infty} I_\alpha^+(E', E) \Phi_0(E', s) dE' + \frac{\partial}{\partial E} \left[\sigma_\alpha^+(E) \Phi(E, \mu, s) (E_\alpha^+ + \langle E \rangle_\alpha) \right] \right\}, \quad (19) \end{aligned}$$

where

$$\langle E \rangle_\alpha = \frac{1}{\sigma_\alpha^+} \int_0^{(E-E_\alpha^+)/2} E_2 I_\alpha^+(E, E_2) dE_2$$

is the average energy of secondary electrons. The second term on the right-hand side of (I9) defines the electron production rate due to collisional ionization of atmospheric species, assuming the distribution function is isotropic

$$\Phi_0(E, s) = \frac{1}{2} \int_{-1}^1 \Phi(E, \mu, s) d\mu. \quad (\text{I10})$$

Let us evaluate the accuracy of the approximation of continuous loss (I8–I9) in a low-energy region, where a major part is played by secondary electrons. The approximation is based on the two first terms in the expansion:

$$\zeta(E + \langle E^* \rangle) = \sum_{n=0}^{\infty} \frac{\langle E^{*n} \rangle}{n!} \frac{\partial^n \zeta}{\partial E^n}.$$

Here $\zeta = \langle \sigma \rangle \Phi$, and $\langle \sigma \rangle$ and $\langle E^* \rangle$ are some effective values of the inelastic scattering cross-section and energy loss. Noting that ζ , in the local approximation, is defined by the energy spectrum of the electron production source, the uncertainty introduced by the approximation of continuous loss must be of the order of

$$\varepsilon = \frac{\langle E^* \rangle}{2E_0}, \quad (\text{I11})$$

where E_0 is the characteristic energy of the secondary electron production spectrum. Both E^* and E_0 decrease with decreasing E , whereas ε nowhere exceeds 30%.

A more accurate evaluation of the error introduced by assuming continuous energy loss can be found by comparison with electron spectra calculated without this assumption. This was done for the case of photoelectrons in the lower and upper ionosphere (Polyakov et al. 1976). It was concluded that significant error was introduced only in the range from 22 to 27 eV, where the electron spectrum is sharply peaked due to ionization of the atmosphere by solar radiation (Krinberg et al. 1974). However, the secondary electron spectrum, produced by impact ionization, does not have these peaks, which decreases the significance of the total error introduced. The sharp minimum in the electron distribution often observed at $E < 4$ eV does not affect the accuracy of the approximation because it is caused by the large value of the cross-section of oscillatory excitation of molecular nitrogen, rather than by a nonmonotonicity in the energy spectrum of production. The resulting $\zeta(E)$ function, therefore, is a fairly smooth one.

Again we point out that the assumption of continuous energy loss over the whole energy range is not a necessity, but a convenient way of reducing the complexity of

the numerical algorithm. Calculations showing the fine structure at the low-energy portion of the suprathermal spectrum using (I6) and (I7) have been presented in Polyakov et al. (1976).

The terms $\langle S_{zi}^* \rangle$ and $\langle S_{ei}^- \rangle$ can generally be neglected because of the prevalence of neutral particles over ions as well as the small value of the cross-section of dissociative recombination for suprathermal electrons.

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