

International Cryogenics Monograph Series

Ben-Zion Maytal
John M. Pfotenhauer

Miniature Joule-Thomson Cryocooling

Principles and Practice

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Principles and Practice

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*to my wife **Hana***
Ben-Zion Maytal

*to my wife **Nadine***
John Pfotenhauer

Preface

This book is the first in English, being entirely dedicated to the topic of *Miniature Joule-Thomson Cryocooling*. The sole previous book on the same subject, *Throttle Type Microrefrigerators*, was published in Russian in Moscow 35 years ago, by Suslov et al. Various books on the general topic of cryogenics and even those focused on cryocoolers include at most not more than a single chapter on Miniature Joule-Thomson cryocooling. The authors of this book have been motivated by the conviction that the subject deserves an updated, broader and deeper treatise.

The five parts of the present book include nine chapters, arranged according to a detailed list of content, and an index.

Part I, comprising Chap. 1, attempts to portray all cryocoolers from a common basis and focuses on the uniqueness of miniature Joule-Thomson cryocooling from this perspective. In fact, the common basis can aptly serve as a first and preliminary lesson whenever one introduces the subject of cryocoolers.

Part II combines Chaps. 2, 3, and 4, to comprise the *theoretical foundation* of the subject. Chapter 2 focuses on the Joule-Thomson effect in both differential and integral forms, as well as the inversion of the same effect. Chapter 3 discusses the thermodynamic principles behind the Linde-Hampson liquefaction process. Chapter 4 identifies the *real gas properties* that are key parameters of candidate coolants for Joule-Thomson cryocooling. The deviation of “real gas” properties from the ideal gas model is an alternative expression of the Joule-Thomson effect that drives the Joule-Thomson cryocooling process.

Part III includes Chaps. 5, 6, and 7 and deals with the *practical and application aspects* of the subject. Chapter 5 scans the variety of operating modes including continuous, staged, fast cooldown, and hybrid. Chapter 6 focuses strictly on details of construction and configuration. Chapter 7 deals thoroughly with aspects of the transient behavior during the cooldown of Joule-Thomson cryocoolers. However, cooldown is also discussed in Chap. 5 in the context of the fast cooldown mode of cryocooling.

Parts II and III of the book correspond with the traditional process for pure gases. These sections address Joule-Thomson cryocooling in the manner used by *Linde* and *Hampson*, but focuses on the small-scale, miniature version operated in an open cycle and at elevated pressure.

Part IV, which is Chap. 8, is entirely dedicated to the use of *mixed coolants* for Joule-Thomson cryocooling. Mixed coolants enable new possibilities that are not attainable with pure coolants. Chapter 8 highlights both the theoretical topics and the practical issues for this type of Joule-Thomson cryocooler, either with or without phase separators, and therefore it is bigger than the others.

Part V, the ninth and last chapter, gathers various *special topics* of general significance and relevance that do not fit well under the headings of the first eight chapters, such as gas purity, choked flow rates of real gases, modeling of cryocoolers, cryosurgical devices, and warming via the Joule-Thomson effect.

The field of cryocoolers as a branch of cryogenics is continuously growing and developing. Joule-Thomson cryocooling, defined by the Linde-Hampson process, has a special position within this group. It uniquely depends upon the real gas properties of the coolants, that is, their deviation from the ideal gas model. These aspects may attract not only people who are directly involved in miniature Joule-Thomson cryocooling but also those possessing a general interest in the disciplines of thermodynamics and cryogenics.

A detailed list of references, chapter by chapter, provides a broad literature survey; it consists of more than 1,200 relevant articles in addition to more than 450 related *patents*. Patents expose a variety of ideas and practical engineering experience, and therefore frequently unfold important details of construction.

Various topics are explored in a *chronological perspective* (such as the inversion of the Joule-Thomson effect, the integral inversion curve, mixed gas cryocooling, and flow regulating mechanisms).

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Nomenclature

a	Attraction parameter in van der Waals' equation of state, N mole^{-1} Cross sectional area (of an orifice), m^2
$\bar{a}$	Normalized area of an orifice in a closed cycle cryocooler, Eq 8.22
A	Area, m^2
b	Repulsion parameters in van der Waals' equation of state, $\text{m}^3 \text{mol}^{-1}$
B, C, D	Second, third, fourth etc. virial coefficient of an equation of state expanded by density, Appendix A2.3
B', C', D'	Second, third, fourth etc. virial coefficient of an equation of state expanded by pressure, Appendix A2.3
c	Speed of sound, m s^{-1} Elasticity coefficient (of a spring), N m^{-1} Concentration, mol m^{-3}
c_P	Isobaric specific heat capacity, $\text{J mol}^{-1} \text{K}^{-1}$
c_V	Isochoric specific heat capacity, $\text{J mol}^{-1} \text{K}^{-1}$
c_{PO}	Isobaric specific heat capacity at zero pressure, $\text{J mol}^{-1} \text{K}^{-1}$
c_{V0}	Isochoric specific heat capacity at zero pressure, $\text{J mol}^{-1} \text{K}^{-1}$
C	Total heat capacity, J Cost of cooling or of liquefying a cryogen, J/J or J mol^{-1}
$\dot{C}$	Capacity rate, $\dot{n} c_P$, J s^{-1}
COP	Coefficient of performance, Eq 1.74
D, d	Outer and inner diameters, m
d	Volumetric displacement of a compressor, swept volume, m^3
d^{HYD}	Hydraulic diameter, $4A/p$
d_{HELIX}	Diameter of the helix of a coiled tube, m
f	Fanning friction factor, Eqs 3.160 and 3.161 Frequency, s^{-1}
F	Force, N
FOM	Figure of merit, $\text{COP}/\text{COP}_{\text{CARNOT}}$
G	Mass flux, $\dot{n}/A$, $\text{mol m}^{-2} \text{s}^{-1}$
g	Gravitational acceleration, 9.81 m s^{-2} Gibbs free energy, $h - T s$
h	Specific enthalpy, J mole^{-1} Convective heat transfer coefficient, $\text{W m}^{-2} \text{K}^{-1}$
h^0	Specific stagnation enthalpy, $h + \rho V^2/2$, J mol^{-1}
h_G	Specific enthalpy of saturated gas, J mol^{-1}

h_F	Specific enthalpy of saturated liquid, J mol^{-1}
h^R	Residual specific enthalpy, based on common (P, T) , J mol^{-1}
h^{IG}	Ideal gas specific enthalpy, J mole^{-1}
Δh_T	Specific enthalpy change at constant temperate, (integral isothermal Joule-Thomson effect), specific cooling content of gas, J mol^{-1}
Δh_T^{MAX}	Maximum Δh_T value along the differential inversion curve, J mol^{-1}
h^{MIX}	Specific enthalpy of a mixture, J mol^{-1}
Δh^{MIX}	Specific mixing enthalpy, J mole^{-1}
δh_D	Downstream recuperated enthalpy, $h_5 - h_4$, J mol^{-1}
δh_U	Up stream recuperated enthalpy, $h_2 - h_1$, J mol^{-1}
H	Total enthalpy, J
i	Ineffectiveness of a heat exchanger, $1 - \varepsilon$
I	Irreversibility, the loss of potential useful work, J
	Inefficiency, $1 - \eta$
I_M	Interchanger's magnification index (Sect. 1.4)
k	Permeability of a porous media, Eq. 6.9, m^2
	Boltzmann's constant, R/N , J K^{-1}
K_1	Discharge coefficient, $\dot{n}/P$, Eq. 9.12, $\text{mol s}^{-1} \text{Pa}^{-1}$
K_2	Discharge coefficient, $\dot{n}/\sqrt{P}$, Eq. 9.13, $\text{mol s}^{-1} \text{Pa}^{-1/2}$
K_T	Isothermal compressibility, $\rho^{-1} (\partial \rho / \partial P)_T$ or $-v^{-1} (\partial v / \partial P)_T$, Pa^{-1}
L	Latent heat of evaporation, $h_G - h_F$, J mole^{-1}
L^*	Critical length for reaching speed of sound, m
l	Length, m
m	Mass, kg
$\dot{m}$	Mass flow rate, recuperated flow rate of a cryocooler, kg s^{-1}
M	Molar mass, kg mol^{-1}
Ma	Mach number, V/c
n	Number of moles
$\dot{n}$	Molar flow rate, mole s^{-1}
n_{LIQ}	The mass of liquefied coolant in the cryocooler, mol
$\dot{n}_{FR}$	Warm ("free") non recuperated molar flow rate, mol s^{-1}
$\dot{n}_{RE}$	Recuperated flow rate of a cryocooler, mole s^{-1}
N	Avogadro's number (of molecules in a single mole), $6.023 \cdot 10^{23}$
NTU	Number of Heat Transfer Units of a heat exchanger, $U A / \dot{C}^{MIN}$
Nu	Nusselt Number, $h d / \lambda$
p	Perimeter, m
P	Pressure, Pa
P_C	Critical pressure, Pa
P_S	Saturation pressure, Pa
P_{TR}	Triple point pressure, Pa
P^*	Regulated pressure, Pa
q	Heat flux, W m^{-2}
Q	Heat, J
Q_V	Volumetric cooling content (per unit volume of displaced coolant), $\dot{Q}/\dot{V}$, J m^{-3}
$\dot{Q}$	Cooling capacity or heat load at steady state operation, W
	Rate of heat transfer, W
$\dot{Q}_{LOAD}$	Heat load, W

r	Capacity rate ratio of heat exchanging streams, $\dot{C}^{MIN} / \dot{C}^{MAX} \leq 1$
R	Universal gas constant, $8.314 \text{ J mol}^{-1} \text{ K}^{-1}$
Re	Reynolds number, $V d / \nu$
s	Specific entropy, $\text{J mole}^{-1} \text{ K}^{-1}$
s^r	Specific residual entropy based on common (ρ, T) , $\text{J mol}^{-1} \text{ K}^{-1}$
S	Total entropy, J K^{-1}
	Surface area parallel to the direction of flow, m^2
SP	Splitting ratio of a liquefier, Sect. 3.8.2.3
t	Time, s
t_{CD}	Cooldown period, s
T	Absolute temperature, K
T_{BOIL}	Normal boiling temperature, K
T_{BOYLE}	Temperature of Boyle, satisfying $B(T) = 0$, K
T_C	Critical temperature, K
T_{CD}	Cooldown temperature from which an isenthalpic expansion generates the first liquid yield (Sects. 3.2.1.6 and 4.4.1), K
T_D	Temperature of the state where the differential inversion curve intersects the saturation line, K, (Sect. 2.4.4)
T_{EU}	Temperature associated with eutectic composition, K
T_{INV}	Inversion temperature of the Joule-Thomson effect, K
T_M	Normal melting temperature, K
TND	Cooldown tendency of gases, $ \Delta T_h / \Delta T_{CD}$, (Sect. 7.4.4)
T_2, T_3	Temperatures at the inlet and outlet of the Joule-Thomson valve of a Linde-Hampson device, K
T_4	Temperature at the (returning) inlet to recuperator's low pressure channel of a Linde-Hampson device, K
T_{PINCH}	Pinch point temperature of a heat exchanger, K
T_{PRE}	Precooling temperature of a final stage, K
T_{TR}	Triple point temperature, K
T_S	Saturation temperature, K
T_λ	Lambda point, super fluidity temperature of transition, K
ΔT	Temperature difference (between the two streams of the heat exchanger), K
ΔT_{CD}	Cool down temperature range, $T_{AMB} - T_{CD}$, Sect. 4.4.1, K
ΔT_h	Integral isenthalpic Joule-Thomson effect, K
ΔT_{LMTD}	Logarithmic mean temperature difference, Eq. 3.92, K
u	Internal energy, J
U	Total heat transfer coefficient, $\text{W m}^{-2} \text{ K}^{-1}$
	Total internal energy, J
UA	Thermal conductance between the channels of a counter flow heat exchanger, W K^{-1}
V	Velocity, m s^{-1}
	Volume, m^3
$\dot{V}$	Rate of volumetric displacement (of a compressor), $\text{m}^3 \text{ s}^{-1}$
ν	Specific volume, $1/\rho$, $\text{m}^3 \text{ mol}^{-1}$
ν^R	Residual volume, $\nu - \nu^{IG}$, $\text{m}^3 \text{ mol}^{-1}$
w	Specific loss of availability due to irreversibility, I/n , J mol^{-1}
W	Work, J
W^{LOST}	Work or availability loss associated with irreversibility, J

x	Axial longitudinal position coordinate along the helical finned tube heat exchanger, m Quality of a two phase fluid, mass fraction of the vapor phase
X	Longitudinal coordinate along the heat exchanger, m Composition, molar fraction
y	Liquefaction yield, mass fraction of a stream
Y	Relative compressibility, $R T (\partial \rho / \partial P)_T$, Sect. 4.7
z	Molar fraction of a component in a mixture
Z	Compressibility, $P v / (R T)$

Greek Symbols

α	Thermal diffusivity, $\lambda / (\rho c_P)$, $\text{m}^2 \text{s}^{-1}$ Coefficient of thermal expansion, $\Delta l / (l \cdot \Delta T)$, K^{-1}
α_C	Acentricity factor of Riedel, Eq. A2.35
β	Thermal expansivity, $-\rho^{-1} (\partial \rho / \partial T)_P$ or $v^{-1} (\partial v / \partial T)_P$
ε	Effectiveness of a heat exchanger Eddy viscosity (of turbulent flow), N s m^{-2}
Φ	Reduced specific density, ρ / ρ_C Dissipation function, Sect. 3.12.2
Γ	The ratio of real gas choked mass flux and the ideal gas, G / G^{IG}
ϕ	Isothermal Joule-Thomson coefficient, $(\partial h / \partial P)_T$, $\text{m}^3 \text{mole}^{-1}$
η	Efficiency of a heat exchanger Joule's free expansion coefficient by pressure, $(\partial T / \partial P)_u$, K Pa^{-1}
η^*	Joule's free expansion coefficient by volume, $(\partial T / \partial v)_u$, K mol m^{-3}
η_{EX}	The efficiency of the isentropic expander
η_f	Thermal efficiency of a fin
η_V	Volumetric efficiency of a compressor
η_I	First Law efficiency
η_{II}	Second Law efficiency, exergy efficiency
θ	Ratio of low and high counter flow inlet temperatures, $T_{L,IN} / T_{H,IN}$
κ	Specific heat ratio, c_P / c_V
λ	Thermal conductivity, $\text{W m}^{-1} \text{K}^{-1}$
μ	Adiabatic Joule-Thomson coefficient, $(\partial T / \partial P)_h$, K Pa^{-1} Dynamic viscosity, N s m^{-2} Chemical potential, J mol^{-1}
ν	Kinematic viscosity, μ / ρ , $\text{m}^2 \text{s}^{-1}$
Π	Reduced pressure, P / P_C
ρ	Density, $1/v$, mol m^{-3}
τ	Time constant, thermal or hydrodynamic delay, s Shear stress, Pa
Θ	Reduced temperature, T / T_C
Σ	Scaling parameter of a recuperator, $\delta h / \Delta h_T$, Sect. 3.7.6
ω	Acentricity factor of Pitzer, Eq. A2.14
ξ	Fraction of maximum integral isothermal Joule-Thomson effect, $\Delta h_T / \Delta h_T^{MAX}$, Sect. 4.3.7.2
ζ	Satellite cooler injected mass fraction, Sect. 3.1.2, Fig. 3.1

Subscripts

0	Zero pressure Stagnation conditions Initial
1, 2, 3, 4, 5	Thermodynamic states at the generalized model of cryocoolers in Fig. 1.2 and the Linde-Hampson cycle in Figs. 3.3, 3.4, and 3.5
AMB	Ambient
BOIL	Boiling
C	Critical state
CARNOT	Ideal reversible Carnot cycle
CD	Cooldown
CUT	Cut off pressure
C.V.	Control volume
D	Downstream, low pressure expanded stream of a counter flow heat exchanger
DEWAR	Dewar
EFF	Effective value
EVP	Evaporator
EU	Eutectic composition
F	Saturated liquid state Final
G	State of saturated gas
H	High, temperature or pressure, hot stream
HE	Heat exchanger
HELIX	Helix
IN	Inlet stream
INV	Joule-Thomson inversion state
JT	Joule-Thomson
L	Low, temperature or pressure, cold stream
OPT	Optimum
OUT	Outlet stream
PRE	Pre-cooling
PINCH	Pinch point along the heat exchanger
S	Saturation conditions
ST	Standard conditions
REV	Reversible
TUBE	Tube
TR	Triple point
U	Up stream, high pressure incoming stream of a counter flow heat exchanger

Superscripts

(1)	Cooler
(2)	Liquefier
(A)	Referring to component A of a the mixture
<i>i</i>	The <i>i</i> -th component of a mixture The <i>i</i> -th stage of a staged cryocooler

CD	Cooldown
HYD	Hydraulic
IG	Ideal gas
MIN	Minimum value
MAX	Maximum value
MIX	Referring to the mixture
MIXING	Referring to the mixing process
r	Residual property, deviation from the ideal gas at the common (ρ , T)
R	Residual property, deviation from the ideal gas at the common (P , T)
REF	Reference value
SS	Steady state

Abbreviations

AC	Alternating current, referred to pulsating flow
CD	Cooldown
CFD	Computational Fluid Dynamics
C.V.	Control volume
DC	Direct current, referred to continuous flow
D.I.C.	Differential inversion curve
EOS	Equation of state
GRS	Gas refrigerant supply
I.I.C.	Integral inversion curve
JT	or J-T Joule-Thomson
Lit	Liter
LRS	Liquid refrigerant supply
MTBF	Mean time between failures
NG	Natural gas
PPM	Parts per million, by volume
SLPM	Standard liters per minute
μm	Micro meter as a measure of length, 10^{-6} m

General Notation

$[X]$	The unit of the entity X ; for instance, $[F]=\text{N}$
$\equiv$	Identity, used for definition
$\bar{X}$	Average of the variable X
$\overline{AB}$	Length of a section starting at point A and ending at B
$\dot{X}$	Time derivative of X
$\sigma(X)$	Standard deviation (variance) of the distribution of the variable (X)
$\left(\frac{\partial y}{\partial x}\right)_z$	Partial derivative of the variable y by x while preserving z

*It is simple to reduce the temperature by about 25 K below ambient, but it is much harder to reduce the temperature by about 250 K. One may find a single phenomenon that reduces the temperature by 25 K, such as the expansion of a compressed gas or the thermoelectric effect. However, there is no single physical effect that alone can take us 250 K below ambient. Nevertheless, the devices under the heading of cryocoolers do so. This means that cryocoolers have the capability of amplifying or accumulating a relatively small temperature drop into a large temperature reduction below ambient. This fundamental feature is the essence of cryocoolers. The temporal development of this magnifying evolution is the cooldown process. **Dr. Charles William Siemens**¹ in 1857, in England, focused precisely on this issue in his invention of the “Refrigeration Apparatus”, there teaching how,*

“....the principle of the Invention is adapted to produce an accumulated effect or an indefinite reduction of temperature....”

This is the essence of cryocooling, and Dr. Charles William Siemens may legitimately be honored as the inventor of the fundamental and common principle of cryocoolers.

(The original patent of Dr. Siemens is detailed in Appendix [A1](#).)

1.1 The Generalized Model of Cryocoolers

1.1.1 The “Interchanging” Process

Siemens’ contribution to the field of cryocoolers has been discussed by multiple authors of the history of refrigeration and cryocooling.^{2,3,4,5,6} He is often acknowledged for inventing the counter flow heat exchanger. On July 28, 1857 he completed a critical analysis of Dr. John Gorrie’s⁷ open cycle cold air machine for making ice. Published only 25 years after the cold air machine was introduced,⁸ Siemens’ analysis recommended that a counterflow heat exchanger be employed between the compressor and the expander, in order to avoid the waste of energy associated with the exhaust stream of cold air from the machine. Apparently inspired by this analysis, on July 29, 1857, Siemens filed a provisional protection for his “Refrigerating Apparatus”. Indeed, he describes in his invention¹ a device of two counter flowing and thermally interacting streams that exchange heat in what he calls “an interchanger”,

The air thus cooled.... is then conducted through an **interchanger** or apparatus by which it is made **to cool the compressed air, which enters the interchanger in the opposite direction**.

However, the two streams of this interchanger are arranged in a different manner, and operate with different boundary conditions than those of the standard counter flow heat exchanger. In contrast to the standard counterflow heat exchanger in which the inlet conditions remain constant in time, the temperature difference between the inlet conditions

of the counterflowing streams of Siemens’ interchanger increases with time. In view of this distinct and significant function the term ‘interchanger’ is maintained throughout this text in order to distinguish it from the standard counterflow heat exchanger.

Figure 1.1 highlights the distinction between Siemens’ interchanger and a “traditional” counter flow heat exchanger. The upper part of the figure qualitatively displays the evolution of the temperature distribution as a function of the longitudinal coordinate, X , or the corresponding heat transfer area, A , along a traditional counterflow heat exchanger. Following the notation in this figure, the hot stream enters the heat exchanger at the warm end ($X = 0$) at the temperature $T_{H, IN}$. It is cooled while interacting with a cold stream flowing in opposite direction. This colder stream enters at the opposite, cold end ($X = l$) at the temperature $T_{L, IN}$. The exit temperatures of both streams are $T_{H, OUT}$ and $T_{L, OUT}$, respectively. Two parameters are relevant to this description. The entire temperature span,

$$\Delta T_{IN-IN} = T_{H, IN} - T_{L, IN} \quad (1.1)$$

and the cold end temperature difference,

$$DT = T_{H, OUT} - T_{L, IN} \quad (1.2)$$

In the usual counter flow heat exchanger, the two inlet temperatures, $T_{H, IN}$ and $T_{L, IN}$ are defined and fixed, so that one may say, ΔT_{IN-IN} is both imposed and preserved. As time progresses, the value of DT decreases while the

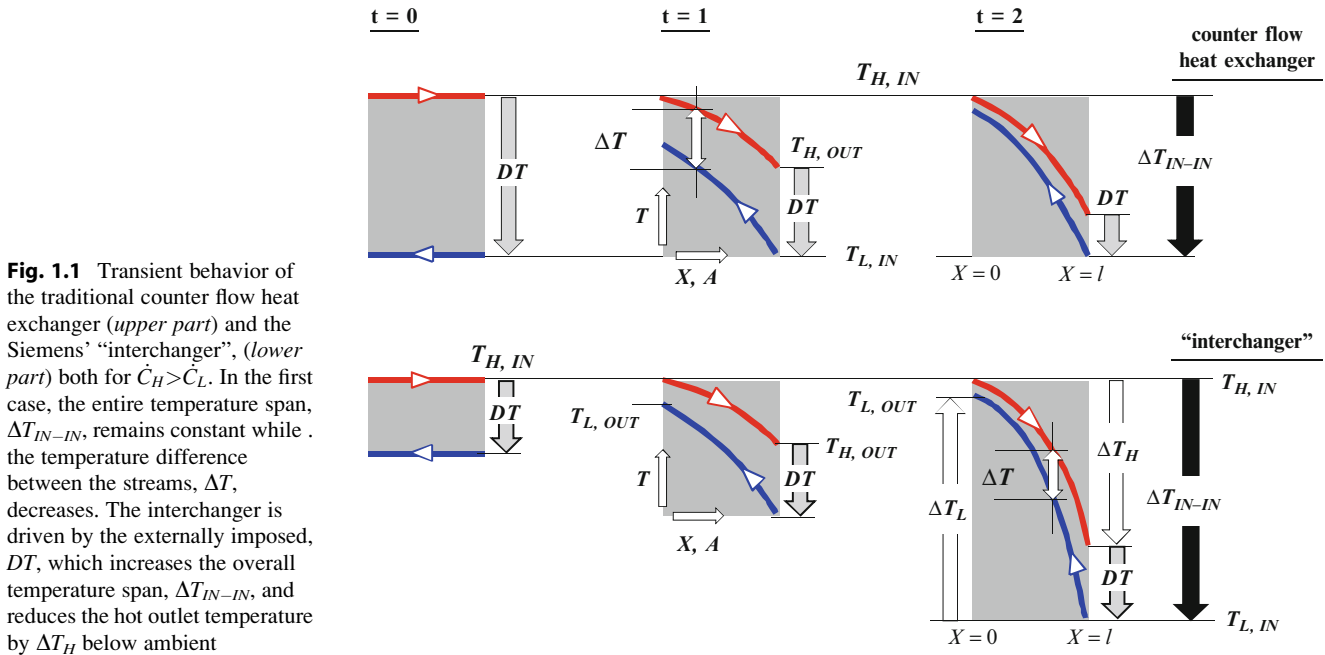
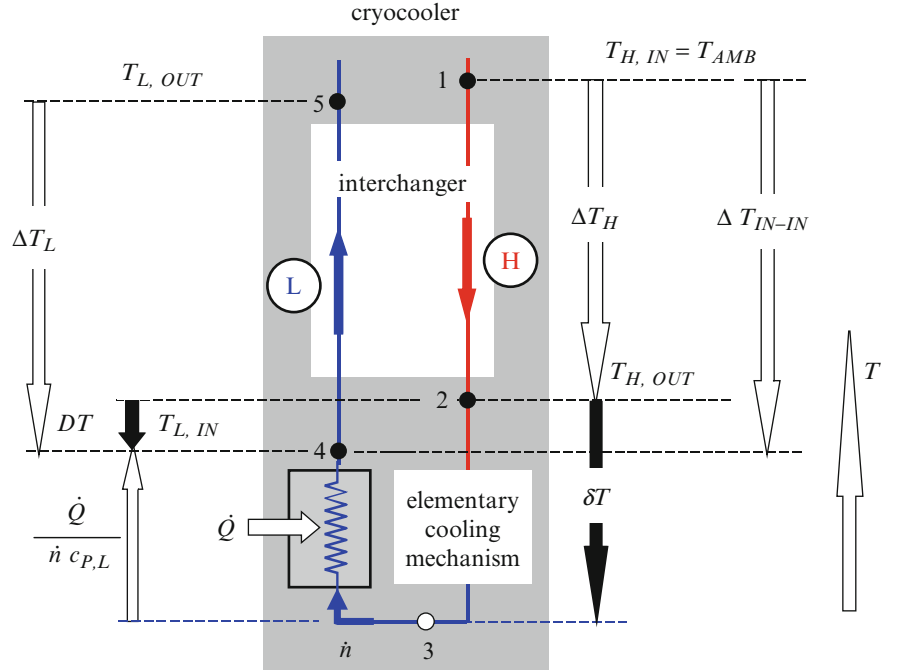


Fig. 1.1 Transient behavior of the traditional counter flow heat exchanger (upper part) and the Siemens’ “interchanger”, (lower part) both for $\dot{C}_H > \dot{C}_L$. In the first case, the entire temperature span, ΔT_{IN-IN} , remains constant while the temperature difference between the streams, ΔT , decreases. The interchanger is driven by the externally imposed, DT , which increases the overall temperature span, ΔT_{IN-IN} , and reduces the hot outlet temperature by ΔT_H below ambient

Fig. 1.2 Layout of the conceptual model of cryocoolers inspired by Siemens' invention of the Provisional Specification 2064 UK, (1857)



temperature difference between the streams, ΔT , decreases from moment $t = 0$ to $t = 1$ and finally reaches steady state at $t = 2$. The value of DT becomes as small as is allowed by the effectiveness of the heat exchanger.

Next, let us examine the other boundary conditions, characterizing the “interchanging” mode as shown in the lower part of Fig. 1.1. The cold stream does not enter at a fixed temperature. Rather, the value of DT is externally imposed (instead of changing as time progresses). For simplicity, DT is preserved and constant. In this way, as the warmer stream is cooled by the opposing colder incoming stream, the consequent self sustained positive feedback mechanism further decreases both $T_{H, OUT}$ and $T_{L, IN}$, further and further below ambient conditions, by the respective amounts of ΔT_H and ΔT_{IN-IN} .

$$\Delta T_H = T_{AMB} - T_{H, OUT} = T_{H, IN} - T_{H, OUT} \quad (1.3)$$

The growth of ΔT_H is bounded only by the effectiveness of the heat exchanger process.

The distinction between a counter flow heat exchanger and an interchanger may be summarized as follows: The former uses ΔT_{IN-IN} as an externally imposed constant input and the value of DT is the corresponding output, decreasing during the transient response. The interchanger employs DT as an externally imposed constant (for the simplicity of this presentation) input, while the value of ΔT_{IN-IN} is the output that develops as a transient response. The increase of ΔT_{IN-IN} is the essence of cryocoolers and the origin of their value.

Note that at the end of the transient phases of the interchanger and the counter flow heat exchanger, both produce similar temperature distributions. Both obey the same effectiveness relation for a counter flow heat exchanger. In other words, an interchanger may be distinguished from a counter flow heat exchanger only by observing the transient evolution of its temperature profile.

1.1.2 The Conceptual Model of Cryocoolers

1.1.2.1 The Essential Constituents

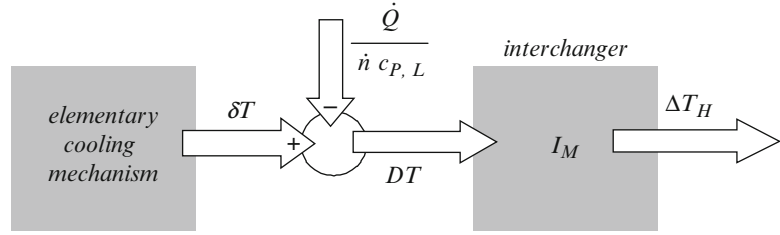
The essential constituents of any cryocooler⁹ are the elementary cooling mechanism and the interchanger. A schematic representation of their combined configuration is shown in Fig. 1.2 (along with a definition of states 1 through 5), and the functional description of the corresponding cryocooler is shown in Fig. 1.3.

The Elementary Cooling Mechanism

The elementary cooling mechanism, generates a limited and relatively small temperature reduction, δT , of the coolant as it passes through that component: the coolant enters at T_2 and leaves at the lower T_3 . The effect of δT is somewhat attenuated by the instantaneous heat load, $\dot{Q}$, applied at the cold end because $\dot{Q}$ elevates the temperature of the returning stream, so that,

$$T_4 - T_3 = \frac{\dot{Q}}{\dot{n} c_{P, L}}. \quad (1.4)$$

Fig. 1.3 Functional diagram of the conceptual model of cryocoolers



Here $\dot{n}$ is the flow rate of the coolant and $c_{P,L}$ is its specific heat capacity at the low temperature end of the cryocooler. Hence

$$DT = \delta T - \frac{\dot{Q}}{\dot{n} c_{P,L}} = T_2 - T_4 \quad (1.5)$$

The value of DT is imposed on the interchanger. Cooling is maintained as long as $DT > 0$, which consequently requires that,

$$\delta T > \frac{\dot{Q}}{\dot{n} c_{P,L}} \quad (1.6)$$

In the case where $\dot{Q}$ is absorbed by the boiling of a liquefied cryogen, then $\dot{n} c_{P,L} \rightarrow \infty$, $DT = \delta T$, and $T_3 = T_4$.

The Interchanger

The interchanger, driven by the small temperature drop DT , generates the overall temperature reduction, ΔT_H , that is significantly larger than δT . The temperature reduction is a self-sustained mechanism, resulting from the two counter-flowing streams of a coolant, which are thermally interacting with each other. The counter flowing media, that is the coolant, deserves special attention since it is an integral part of the interchanger.

1.1.2.2 The Coolant

The coolant is the intermediating agent between the elementary cooling mechanism and the interchanger. It provides at least one of the following two functions:

A Media that Undergoes a Thermodynamic Transition

A media that undergoes a thermodynamic transition, thereby producing the δT . Through the elementary cooling mechanism at the cold end of the interchanger, the warm stream of the coolant undergoes a thermodynamic change, preserving its enthalpy, entropy or chemical potential. The returning stream at a reduced temperature (by DT) might also have a lower pressure and density or a new composition.

A Convective Fluid

A convective fluid that serves as the flowing media of the interacting streams. It is the agent for convective heat

transfer between the streams, to sustain the process of interchanging. This fluid does not undergo any thermodynamic transition except that its temperature is reduced by δT due to an external element such as an adiabatic demagnetization or another closed cycle cryocooler. Some examples are presented in Sect. 1.3.6.4.

In principle, these two functions are independent. Indeed, in some less common cases the “coolant” only serves the second function.

Ideal Gas Coolants Versus Real Gas Properties

For the first kind of coolants, an isentropic transition generates a change (decrease) of temperature, δT , even if the coolant is an ideal gas and therefore these are often referred to as *ideal gas cryocoolers*. Isenthalpic transitions of coolants do not generate any cooling ($\delta T \neq 0$) if they behave as ideal gases. Therefore cryocoolers relying on isenthalpic transitions are often referred to as *real gas properties cryocoolers*.

1.1.2.3 The Cooldown Process of Cryocoolers

Most of the thermodynamic descriptions of cryocooling focus on the *being*, that is, on their steady state operation.^{6,10,11} The conceptual model of cryocoolers that include the characteristics of its elementary cooling mechanism and the “interchanger” throw additional light on the *becoming*, that is, on the mechanism of cooldown or any transition from one steady state to another. In order to provide a complete description of a cryocooler it is essential to explore its mechanism of reaching the low temperatures, as well as estimate its attainable temperatures. During the cooldown process the values of δT and also $\dot{Q}$ may change as a function of the instantaneous temperature, $T_{H, OUT}$.

1.1.2.4 Comments

ΔT_{IN-IN} Versus ΔT_H

When cryocooling is accomplished by liquefaction of coolants, as with the Joule-Thomson type, the value of T_3 reaches the boiling point towards the end of the cooldown and $T_3 = T_4$. Then, ΔT_{IN-IN} , that is $(T_1 - T_4)$, becomes fixed even though ΔT_H continues to grow. In such a case, it is preferred to characterize the reduction of temperature in terms of an increase in ΔT_H rather than a growth of ΔT_{IN-IN} .

Isothermal Absorption of Heat Load

The elementary cooling from state 2 to 3 and the absorption of $\dot{Q}$, from state 3 to 4, may occur simultaneously. In such a case, states 2, 3 and 4 are almost at the same temperature; DT is very small, and heat is absorbed almost isothermally with low irreversibility (as in the Stirling cryocooler).

1.1.3 The Magnification Index of the Interchanger, I_M

1.1.3.1 Definition

Regarding DT as an input signal and the entire temperature reduction, ΔT_{IN-IN} , as the output signal, then the gain of the operation can be expressed as,

$$I_M = \frac{\Delta T_{IN-IN}}{DT} = \frac{T_1 - T_4}{DT} = \frac{\Delta T_H}{DT} + 1 \quad \text{or} \quad \Delta T_H = (I_M - 1)DT \quad (1.7)$$

The mechanism of the process does not include any stabilizing process that would counteract further or unlimited growth that is, a positive feedback mode. However, in practice it is bound by the size of the heat transfer area. (other practical counteracting factors are discussed in Sect. 1.3.3.5). Hence, the actual value of I_M is determined by the *effectiveness*, ε , of the heat exchanger as broadly discussed in literature^{12,13} which is dependent on the *capacity rates* of the cold and hot streams, correspondingly defined by,

$$\begin{aligned} \dot{C}_L &= \dot{n}_L c_{P,L} = (\dot{n} c_P)_L \quad \text{and} \\ \dot{C}_H &= \dot{n}_H c_{P,H} = (\dot{n} c_P)_H \end{aligned} \quad (1.8)$$

It will be assumed here that each of these is constant, or alternatively, that the constant values are regarded as representing the average values along the entire channel. In the case of cryocoolers in contrast to liquefiers, the two streams share the same flow rate, $\dot{n}$. The quantity I_M has a different definition depending whether the hot stream capacity rate is larger or smaller than that of the cold stream.

1.1.3.2 Hot Stream with Minimum Capacity Rate

When $\dot{C}_H \leq \dot{C}_L$, and a pinch point occurs at the cold end, the *effectiveness* of a counter flow heat exchanger assumes the form,

$$\varepsilon = \frac{\Delta T_H}{T_{IN-IN}} = \frac{T_1 - T_2}{T_1 - T_4} \quad (1.9)$$

Following the notation of the lower part of Fig. 1.1, the effectiveness becomes,

$$\varepsilon = \frac{\Delta T_H}{\Delta T_H + DT} = 1 - \frac{DT}{\Delta T_H + DT} = 1 - i \quad (1.10)$$

or

$$I_M = \frac{1}{1 - \varepsilon} = \frac{1}{i} = \frac{\Delta T_{IN-IN}}{DT} \quad \text{or} \quad \varepsilon = 1 - \frac{1}{I_M} \quad (1.11)$$

where i is the ineffectiveness of the interchanger and equals $1 - \varepsilon$.

Siemens¹ provides a clear statement regarding the “indefinite reduction of temperature”: The value of I_M is limited by the ineffectiveness of the interchanger but independent of the extent of the imbalance. To the same extent that one increases the effectiveness, approaching unity, one obtains a deeper and deeper cooldown.

1.1.3.3 Cold Stream with Minimum Capacity Rate

The situation when $\dot{C}_H > \dot{C}_L$ represents the more common case for cryogenic applications and particularly cryocoolers. Such a condition is characterized by hot pressurized streams, carrying higher heat capacity than the cold stream. The thermal energy (“interchanged enthalpy”) that is exchanged between the two streams is,

$$\begin{aligned} \dot{C}_L (T_5 - T_4) &\equiv \dot{C}_L \Delta T_L = \dot{C}_H \Delta T_H \\ &\equiv \dot{C}_H (T_1 - T_2) \end{aligned} \quad (1.12)$$

In this case, when the pinch point is at the warm end, the definition of *effectiveness* becomes,

$$\varepsilon = \frac{\Delta T_L}{T_{IN-IN}} = \frac{T_5 - T_4}{T_1 - T_4} \quad (1.13)$$

thus,

$$\varepsilon = \frac{(\dot{C}_H / \dot{C}_L) \Delta T_H}{\Delta T_H + DT} \quad (1.14)$$

Combining Eqs. 1.14 and 1.7 leads to,

$$I_M = \frac{1}{1 - \varepsilon (\dot{C}_L / \dot{C}_H)} \quad \text{or} \quad \varepsilon = \frac{\dot{C}_H}{\dot{C}_L} \left(1 - \frac{1}{I_M} \right) \quad (1.15)$$

In addition to being dependent on ε , the value of I_M is also a function of the extent of capacity rate ratio, $\dot{C}_L / \dot{C}_H < 1$. In this case, the description provided by Siemens was incomplete, since the “interchanger” cannot sustain an “indefinite” growth of I_M . Even if the effectiveness approaches unity, the magnification index cannot exceed,

$$\begin{aligned} I_M^{MAX} &= \frac{1}{1 - (\dot{C}_L / \dot{C}_H)} \quad \text{or} \\ \left(\frac{\Delta T_H}{DT} \right)^{MAX} &= \frac{1}{(\dot{C}_H / \dot{C}_L) - 1} \end{aligned} \quad (1.16)$$

The existence of a lower limit of I_M , occurring when $\dot{C}_H > \dot{C}_L$ may be appreciated by recalling that the colder stream drives the cooling process that produces ΔT_H . The temperature change in the cold stream, ΔT_L , evokes a smaller temperature change in the warmer stream, ΔT_H , as defined by Eq. 1.12. Thus, in this case, the strength of the positive feedback mechanism is weakened, while in case when $\dot{C}_H < \dot{C}_L$ it grows continuously. Nevertheless, in all cases, $I_M > 1$.

1.1.3.4 The Unified Expression

An alternate single expression for the effectiveness that applies in both cases is:

$$\varepsilon = \frac{\dot{C}_H}{\dot{C}^{MIN}} \cdot \frac{\Delta T_H}{\Delta T_H + DT} = \frac{\dot{C}_L}{\dot{C}^{MIN}} \cdot \frac{\Delta T_L}{\Delta T_H + DT} \quad (1.17)$$

so, for the magnification index,

$$I_M = \frac{1}{1 - \varepsilon \left(\dot{C}^{MIN} / \dot{C}_H \right)} \geq 1 \quad (1.18)$$

The magnification index I_M reflects the impact of the interchanger's effectiveness, ε on the cold end temperature.

1.1.3.5 Example

Assuming an effectiveness of $\varepsilon = 0.95$,

If $\dot{C}_H / \dot{C}_L \leq 1$ then the magnification index $I_M = 20$.

If $\dot{C}_H / \dot{C}_L = 1.2 > 1$ then the magnification index $I_M = 4.8$ and $I_M^{MAX} = 6$.

For an elementary temperature reduction of $\delta T = 10$ K, we may theoretically expect in the first case to cool below ambient conditions by 200 K, while in the other case, the cooling is only 48 K.

1.1.3.6 The Ideal Case with $\varepsilon \rightarrow 1$ and $DT \rightarrow 0$

When $\dot{C}_H \leq \dot{C}_L$, by combining Eqs. 1.7 and 1.11, we arrive at,

$$\begin{aligned} \lim_{\substack{\varepsilon \rightarrow 1 \\ DT \rightarrow 0}} (\Delta T_{IN-IN}) &= \lim_{\substack{\varepsilon \rightarrow 1 \\ DT \rightarrow 0}} (I_M DT) \\ &= \lim_{\substack{\varepsilon \rightarrow 1 \\ DT \rightarrow 0}} \left(\frac{\Delta T_{IN-IN}}{DT} DT \right) \\ &= \Delta T_{IN-IN} \end{aligned} \quad (1.19)$$

Thus, by the same extent that DT becomes smaller, it is possible to correspondingly increase the magnitude of I_M ,

thereby increasing the effectiveness, ε (to approach unity) so that any desirable value of ΔT_{IN-IN} would become attainable. However, when $\dot{C}_H > \dot{C}_L$, by Eqs. 1.7 and 1.16, we have

$$\begin{aligned} \lim_{\substack{\varepsilon \rightarrow 1 \\ DT \rightarrow 0}} (\Delta T_{IN-IN}) &= \lim_{\substack{\varepsilon \rightarrow 1 \\ DT \rightarrow 0}} (I_M DT) \\ &= \lim_{\substack{\varepsilon \rightarrow 1 \\ DT \rightarrow 0}} (I_M^{MAX} DT) = 0 \end{aligned} \quad (1.20)$$

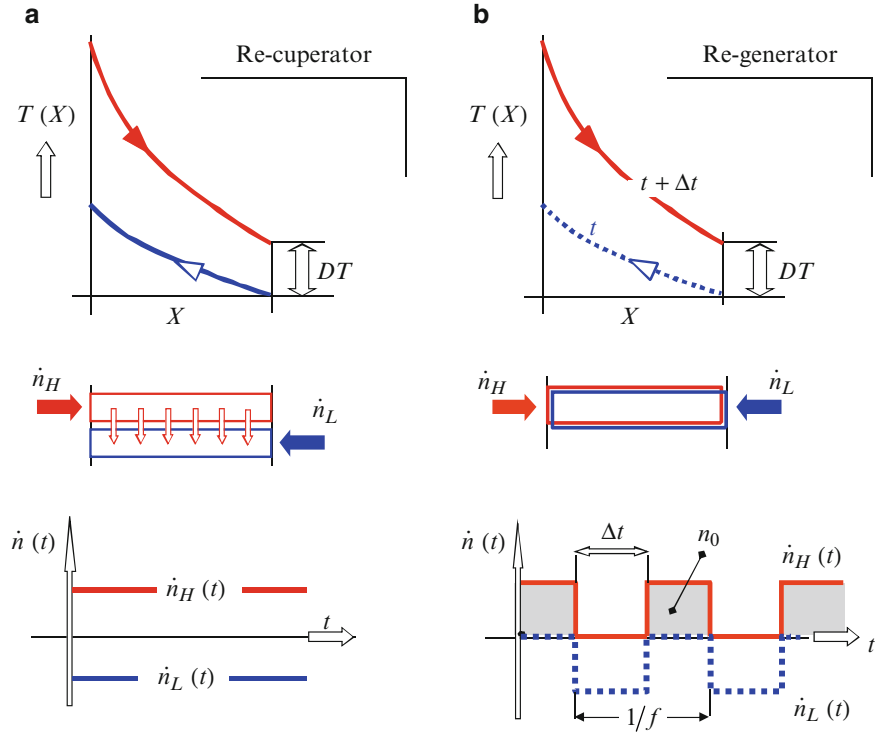
which means that this case is dominated by the unlimited shrinking of DT ; since the magnification index is bounded, it is not possible to accumulate a deep temperature drop.

1.1.4 Implementation of Interchangers

1.1.4.1 Recuperators and Regenerators

The two types of heat exchange processes shown in Fig. 1.4, although not typically grouped together, may nevertheless both be classified as counter flow heat exchangers. The first includes two separate interacting channels, each carrying a continuously flowing stream in opposite directions. This type of heat exchanger is generally referred to in the literature as a *recuperator*. The second kind of heat exchanger incorporates a thermal *storage mechanism* and is regarded in the literature as a *regenerator*. In this case, the two streams exchange heat flow in opposite directions, but share a single flow channel made of porous material with a high heat capacity. To accomplish such a task, a periodic time-sharing is required. That is, the first stream flows through the porous media throughout a given short time interval, Δt . It then stops and is followed by a second, counterflowing stream, through the same porous channel during the successive time interval, Δt . In this way, the heat capacity of the porous bulk serves as an intermediary between the two streams; it stores the cooling effect of the first stream at one moment and transmits it to the second stream at the next moment. Also, when the cold and hot streams alternately flow through the same channel, any local temperature of the wall and gas undergoes a time dependent oscillation. If the regenerator is stationary, it is generally accompanied by a displacer, whose motion is judiciously synchronized with a reciprocating expander. The displacer drives the flow and alternates its direction through the regenerator. Another method of producing the cyclic behavior incorporates a dynamic regenerator that moves through the gaseous bulk thereby inducing an alternating flow of gas relative to the regenerator.

Fig. 1.4 Two basic kinds of counter flow heat exchangers in a balanced case where $\dot{C}_H = \dot{C}_L$; (a) the double-channel and continuous-flow recuperator, and (b) the single channel storage type, with alternately flowing streams. The frequency of the flow cycles is f



1.1.4.2 DC and AC Cryocoolers

For purposes of classification, one frequently finds the distinction between recuperative cryocoolers (such as the Linde-Hampson, Claude and Brayton cycles) and regenerative cryocoolers (such as the Solvay, Stirling, Gifford-McMahon and Pulse tube cycles) in the literature.^{14,15,16,17}

The recuperator is associated with a continuous (or DC) flow interchange and a generated δT that is steady. A regenerator is associated with a δT that is periodically generated as a gas cyclically expands and flows, or pulses, through the regenerator. Each pulse in one direction is followed by a counter-flowing pulse of the same size in the opposite direction, thereby generating the AC flow. Hence, the flow rates of the regenerative interchanger are balanced. In general, the capacity rates of the two types of interchangers are quite similar. The cyclic flow at a frequency f (a period of $1/f$, in Fig. 1.4), with a molar mass n_0 flowing in one direction, is equivalent to a constant average flow rate,

$$\bar{n} = \bar{n}_H = \bar{n}_L = n_0 f \quad (1.21)$$

The two types of interchangers differ significantly with respect to construction and configuration. Nevertheless, one should note the following thought regarding the conceptual model of cryocoolers: **All designs** utilize the same crucial magnification effect associated with a counterflow heat exchanger and thereby **function as a Siemens' "inter-changer"**. The AC flow cryocoolers accomplish the task of

interchanging with a regenerator while the DC flow cryocoolers interchange via a recuperator. *Cryogenic Regenerative Heat Exchangers* are analyzed and discussed in the book by Ackermann¹⁸ bearing that title.

Both the recuperator- and regenerator-type "interchangers" were mentioned in Siemens' provision¹:

The construction of the interchanger may be varied or two or more regenerators or respirators filled with wire gauze or metallic plates, or other suitable materials may be employed alternately for the compressed and expanded air.

Consequently, the performance of any cryocooler is strongly dependent on its interchanger, as discussed below.

1.2 Characteristics of Interchangers

The parameter, $\dot{C}_L/\dot{C}_H$, influences the distribution of temperatures along the heat exchanging stream, the temperature difference between the streams, ΔT , and the magnification index, I_M . For simplicity in the following discussion, the streams are assumed to have constant capacity rates, $\dot{C}_H$ and $\dot{C}_L$. Alternatively, one may observe that the results obtained below remain locally valid along a limited region where only minor changes of the capacity rates occur. The value of ΔT is relevant for the interchanger because of its impact on (a) the size, and, (b) the irreversibility or entropy generation.

The performance of the interchanger will be formulated, in Sects. 1.2.1 through 1.2.4, in terms of its behavior as a recuperator; this is also the upper limit of performance for any regenerative interchanger.

1.2.1 The Temperature Domain

Any longitudinal increment, dx , of an interchanger corresponds to two increments of temperature along each of the opposite streams, dT_H and dT_L , so that,

$$d\dot{Q} = \dot{C}_L dT_L = \dot{C}_H dT_H \quad (1.22)$$

The absence of a minus sign in this equation is due to the opposite reference directions of $\dot{C}_L$ and $\dot{C}_H$. Thus, the relative differential change of temperature may be expressed as,

$$\frac{dT_H}{dT_L} = \frac{\dot{C}_L}{\dot{C}_H} \quad (1.23)$$

The temperature difference, ΔT , between the streams as shown in Fig. 1.1 can be given as:

$$\frac{d(T_H - T_L)}{dT_L} = \frac{d(\Delta T)}{dT_L} = \frac{\dot{C}_L}{\dot{C}_H} - 1 \quad (1.24)$$

and,

$$\frac{d(\Delta T)}{dT_H} = 1 - \frac{\dot{C}_H}{\dot{C}_L} \quad (1.25)$$

The distribution of ΔT along the heat exchanger is influenced by the capacity rates relations as follow:

If $\dot{C}_L/\dot{C}_H < 1$, values of ΔT increase toward the cold end and approach zero toward the warm end pinch point,

If $\dot{C}_L/\dot{C}_H > 1$, values of ΔT increase towards the warm end and approach zero toward the cold end pinch point, and,

If $\dot{C}_L/\dot{C}_H = 1$, the balanced case exists, where ΔT stays constant.

In order to remain consistent with the Second Law of Thermodynamics, ΔT is always positive and cannot change its sign along the heat exchanger.

The temperatures of the two streams T_H and T_L are **linearly** interdependent. Thus, ΔT is also a linear function of each of the two temperatures, as illustrated elsewhere.^{12, 13} This simple interrelation in the temperature domain is not preserved in the (longitudinal) spatial domain as discussed in the next section.

1.2.2 The Longitudinal Domain

Any longitudinal increment, dX , of an interchanger corresponds with two increments of heat transfer area, dA_H and dA_L along the two interchanging streams (as with dT_H and dT_L). The value of dA_H and dA_L may be equal. However, for the Hampson type of interchanger in a Joule-Thomson cryocooler, dA_L is larger. Since X , dA_H and dA_L are monotonically interdependent, each of these may be used the longitudinal extent of the interchanger.

1.2.2.1 The Dimensionless Longitude, NTU

The amount of heat that is exchanged between the two streams is driven by ΔT so that,

$$d\dot{Q} = \dot{C}_H dT_H = \Delta T \cdot U_H \cdot dA_H = \Delta T \cdot d(UA) \quad (1.26)$$

$$d\dot{Q} = \dot{C}_L dT_L = \Delta T \cdot U_L \cdot dA_L = \Delta T \cdot d(UA) \quad (1.27)$$

where U_L and U_H are the local overall heat transfer coefficients, so determined to describe the local *thermal conductance*, $d(UA)$, of the dX element,

$$U_H \cdot dA_H = d(UA) = U_L \cdot dA_L \quad (1.28)$$

obtained by

$$\frac{1}{d(UA)} = \frac{1}{h_H \cdot dA_H} + \frac{1}{h_L \cdot dA_L} \quad (1.29)$$

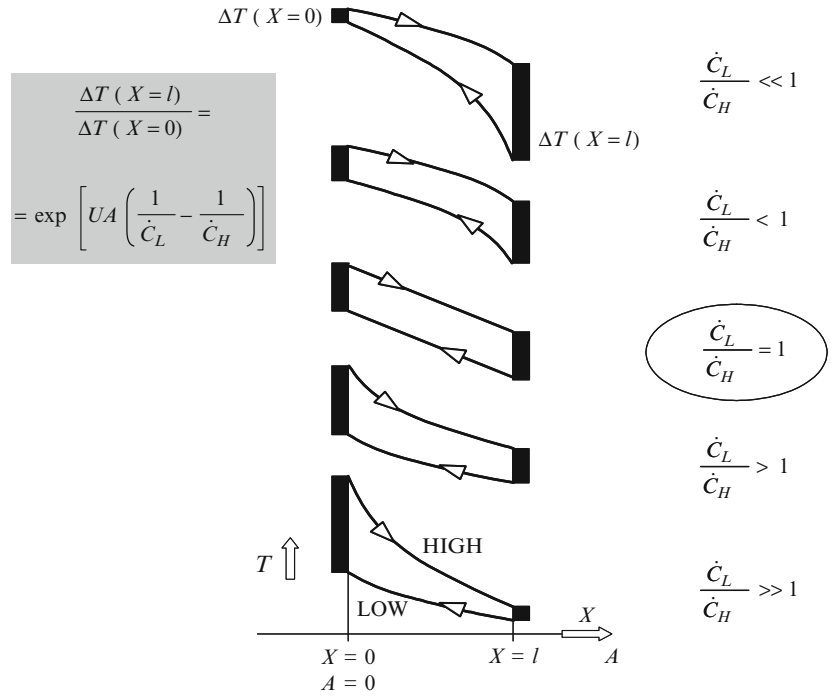
through the convective heat transfer coefficients, h_D and h_U , of both channels. Note here that the thermal resistance of the wall separating the two flow streams is assumed to be negligible, as well as conduction along that wall.

Rearranging Eqs. 1.26, 1.27, and 1.28 and integrating along any of the streams, (L) and (H), while assuming a constant value of $\dot{C}$ for each stream, leads to the definition of the *Number of Thermal Units*, referenced separately to each channel, NTU_H and NTU_L ,

$$\begin{aligned} \int_{(H)} \frac{dT_H}{\Delta T(T_H)} &= \int_{(H)} \frac{d(UA)}{\dot{C}_H} = \frac{1}{\dot{C}_H} \int_{(H)} d(UA) = \frac{UA}{\dot{C}_H} \\ &\equiv NTU_H \end{aligned} \quad (1.30)$$

$$\begin{aligned} \int_{(L)} \frac{dT_L}{\Delta T(T_L)} &= \int_{(H)} \frac{d(UA)}{\dot{C}_L} = \frac{1}{\dot{C}_L} \int_{(L)} d(UA) = \frac{UA}{\dot{C}_L} \\ &\equiv NTU_L \end{aligned} \quad (1.31)$$

Fig. 1.5 The evolution of longitudinal temperature profiles of a counter flow heat exchanger as a function of the heat capacity ratio of the streams



Where UA is the *total thermal conductance* of the entire interchanger. Note that ΔT is dependent on the longitudinal position X and common to both streams. However, the functions $\Delta T(T_H)$ and $\Delta T(T_L)$ are not identical and the above two integrals are not the same; their ratio is $\dot{C}_L/\dot{C}_H$. Subtracting Eq. 1.31 from 1.30 and integrating along the heat exchanger leads to,

$$\begin{aligned} \frac{\Delta T(UA)}{\Delta T(UA=0)} &= \frac{\Delta T(X=l)}{\Delta T(X=0)} = \exp(NTU_H - NTU_L) \\ &= \exp \left(\frac{UA}{\dot{C}_L} - \frac{UA}{\dot{C}_H} \right) \end{aligned} \quad (1.32)$$

where UA also serves as a measure of the distance between sections 1 and 2 of the interchanger. Hence, the evolution of ΔT along the interchanger is an exponential function of the longitudinal position expressed in terms of UA . For constant but non necessarily equal values of $\dot{C}_L$ and $\dot{C}_H$, the value of ΔT is an exponential function of X . It increases when $\dot{C}_H > \dot{C}_L$ and decreases for $\dot{C}_H < \dot{C}_L$, and illustrated in Fig. 1.5.

The NTU may be interpreted as the dimensionless longitude, measure of the heat exchange area or thermal conductance.

1.2.2.2 The Curvature of the Temperature Profiles

From Eqs. 1.26 and 1.27 we have,

$$\frac{dT_L}{dA_L} = \frac{U_L}{\dot{C}_L} \Delta T \quad \text{and} \quad \frac{dT_H}{dA_H} = \frac{U_H}{\dot{C}_H} \Delta T \quad (1.33)$$

The shape of both profiles $T_L(A)$ and $T_H(A)$, depends on the distribution of ΔT along the heat exchanger:

If $\dot{C}_L/\dot{C}_H < 1$, the profiles are *convex*, since dT_L/dA and dT_H/dA like ΔT , start (at the warm end) with small values and grow toward the cold end.

If $\dot{C}_L/\dot{C}_H > 1$, the profiles are *concave*, since dT_L/dA and dT_H/dA like ΔT starts (at the warm end) with large values and shrinks towards the cold end.

If $\dot{C}_L/\dot{C}_H = 1$, the profiles are linear, since ΔT stays constant.

Due to the linear dependence of ΔT on temperature, the longitudinal distributions of T_H and T_L are **exponential**, in the same way that ΔT is exponential as determined by Eq. 1.33.

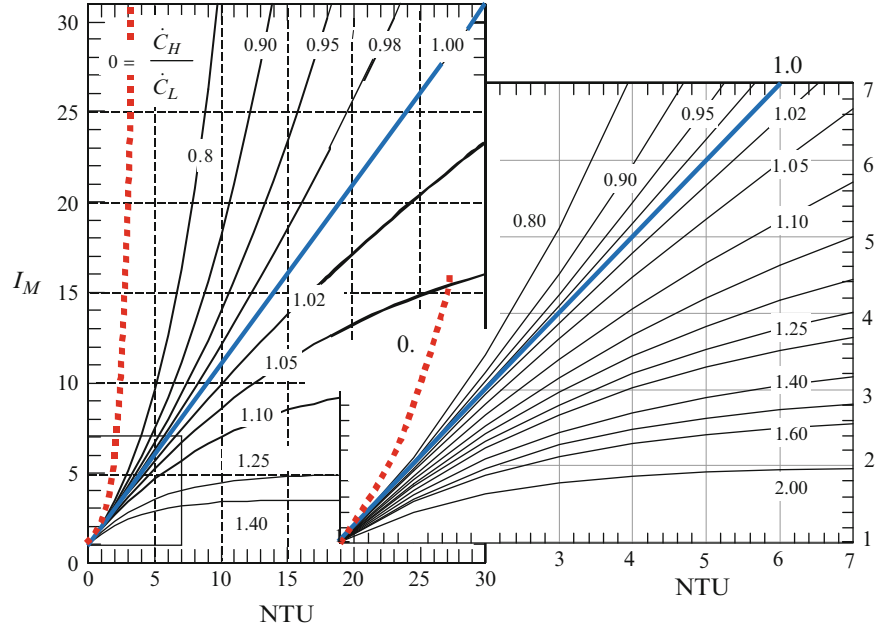
Figure 1.5 scans the evolution of temperature profiles as a function of the parameter $\dot{C}_L/\dot{C}_H$, transitioning from concave to convex through the linear profile. In the general case, each profile might include a concave and a convex section if $(\dot{C}_L/\dot{C}_H - 1)$ changes its sign.

Four different longitudinal profiles are possible, in general, through all combinations of the following two parameters: (a) increase/decrease of ΔT as T approaches lower values, and, (b) convex/concave profiles of temperature. However, from the arguments given above, only two possibilities are viable:

- (a) Convex profiles with ΔT increasing toward the cold end (for $\dot{C}_L/\dot{C}_H < 1$).
- (b) Concave profiles with ΔT decreasing toward the cold end (for $\dot{C}_L/\dot{C}_H > 1$).

A convex profile with ΔT decreasing toward the cold end is not possible.

Fig. 1.6 The magnification index as a function of NTU for various values of $\dot{C}_H/\dot{C}_L$ values



1.2.3 Dependence of I_M on the Size of the Interchanger

1.2.3.1 Formulation

The definition of the effectiveness of a counter flow heat exchanger, ε , depends on the location of the pinch point, that is, where the temperature profiles of the counter flowing streams closely approach each other, as indeed is reflected by Eq. 1.10 and by Eq. 1.14. However, when choosing the larger NTU, so that,

$$\begin{aligned} \text{NTU} &= \text{MAX}(\text{NTU}_L, \text{NTU}_H) \quad \text{or} \\ \text{NTU} &= \frac{UA}{\dot{C}^{\text{MIN}}} \end{aligned} \quad (1.34)$$

and when one includes the parameter,

$$r = \frac{\dot{C}^{\text{MIN}}}{\dot{C}^{\text{MAX}}} \leq 1 \quad (1.35)$$

where,

$$\begin{aligned} \dot{C}^{\text{MIN}} &= \text{MIN}(\dot{C}_L, \dot{C}_H) \quad \text{and} \\ \dot{C}^{\text{MAX}} &= \text{MAX}(\dot{C}_L, \dot{C}_H) \end{aligned} \quad (1.36)$$

then, a single expression describes both cases, regardless of the location of the pinch point, or equivalently, whether $\dot{C}_H$ is larger or smaller than $\dot{C}_L$. The ε -NTU relation is,

$$\varepsilon = \frac{1 - e^{-\text{NTU}(1-r)}}{1 - re^{-\text{NTU}(1-r)}} \quad (1.37)$$

The expressions of I_M can be explicited by the ε -NTU relation for the two domains of $\dot{C}_L$ versus $\dot{C}_H$,

$$I_M(\dot{C}_H > \dot{C}_L) = \frac{1 - re^{-\text{NTU}(1-r)}}{1 - r} \quad (1.38)$$

the growth of which as function of NTU is bounded by an upper value, according to Eq. 1.16, and,

$$I_M(\dot{C}_H \leq \dot{C}_L) = \frac{e^{\text{NTU}(1-r)} - r}{1 - r} \quad (1.39)$$

the growth of which is not bounded. This behavior is displayed graphically in Fig. 1.6. The two domains of $\dot{C}_L$ versus $\dot{C}_H$ have a unified formulation,

$$(I_M)^{-1} = 1 - \frac{1 - e^{-\text{NTU}(1-r)}}{1 - re^{-\text{NTU}(1-r)}} \cdot \left(\frac{\dot{C}^{\text{MIN}}}{\dot{C}_H} \right) \leq 1 \quad (1.40)$$

1.2.3.2 The Extreme Behavior

The sharpest growth of I_M as a function of the size parameter, NTU, belongs to the case where $\dot{C}_L \rightarrow \infty$. This case occurs for example when the returning stream of the interchanger boils. By Eq. 1.39 with $\dot{C}_L$ larger than $\dot{C}_H$, one obtains,

$$I_M = e^{\text{NTU}} \quad @ \quad \dot{C}_H/\dot{C}_L = 0 \quad (1.41)$$

which is a sharply increasing function. Since boiling is accompanied by a relatively large value of U , even with a small area, A such that NTU equals 2, the magnification index I_M grows to a value of 7.4. The significance of this

occurrence for Joule-Thomson cryocooling is discussed later, in Sect. 7.2.3.1.

The opposite case, when $\dot{C}_H \rightarrow \infty$, as for example when the incoming stream is condensing, produces no magnification. That is,

$$I_M \rightarrow 1 \quad @ \quad \dot{C}_H / \dot{C}_L \rightarrow 0 \quad (1.42)$$

1.2.3.3 The Case of Balanced Capacity Rates

The case of balanced capacity rates, $\dot{C}_H = \dot{C}_L$, with the effectiveness of,

$$\varepsilon = \frac{\text{NTU}}{1 + \text{NTU}} \quad \text{or} \quad \text{NTU} = \frac{\varepsilon}{1 - \varepsilon} \quad (1.43)$$

results in a magnification index with the simple form,

$$I_M(\dot{C}_H = \dot{C}_L) = 1 + \text{NTU} \quad \text{or} \quad \frac{\Delta T_H}{\delta T} = \text{NTU} \quad (1.44)$$

1.2.3.4 Remarks

1. One should recognize that I_M is dependent on the parameter $\dot{C}_H / \dot{C}_L$, which is not the same as r . The value of $\dot{C}_H / \dot{C}_L$ can be larger or smaller than unity. However, the capacity rate ratio r , as defined by Eq. 1.35 is always less than or equal to unity. This point is demonstrated as well by the general expression for I_M given in Eq. 1.18 where I_M includes the parameter $\dot{C}^{MIN} / \dot{C}_H$ but not $\dot{C}^{MIN} / \dot{C}^{MAX}$.
2. Note that this model expresses the efficiency solely in terms of the thermal conductance. One may find expressions for ε -NTU relationships in the literature that are more developed than the basic one given in Eq. 1.37, and that account for various real effects such as axial conduction,^{19,20,21} real gas properties,²¹ parasitic heat leak,^{21,22,23} and pressure drop in the channels.²⁴ Each of these mechanisms is responsible for an additional loss of effectiveness and therefore decrease the magnitude of I_M .

1.2.4 Entropy Generation

1.2.4.1 Formulation

The generation of entropy associated with the local exchange of heat, dQ , between the counter flowing streams across a finite temperature difference, ΔT , is given by,

$$\begin{aligned} ds &= \frac{dQ}{T_L} - \frac{dQ}{T_H} = \frac{\Delta T}{(T_H - \Delta T)T_H} dQ \\ &= \frac{\Delta T}{(T_L + \Delta T)T_L} dQ \end{aligned} \quad (1.45)$$

The quantity dQ may be integrated along any one of the streams. Integrating along the cold stream, the total generation of entropy would be,

$$\Delta s = \int_{T_{L,IN}}^{T_{L,OUT}} \frac{c_{P,L} \cdot \Delta T}{(T_L + \Delta T)T_L} dT_L \quad (1.46)$$

1.2.4.2 Ideal Gas Counter-Flow Heat Exchanger

The entropy generated over any temperature span ($T_{H,IN}$, $T_{L,IN}$) is obtained^{25,26,27,28} by a closed form expression,

$$\begin{aligned} \frac{\dot{S}}{\dot{C}^{MIN}} &= r \ln \left[1 + \varepsilon \left(\frac{1}{\theta} - 1 \right) \right] + \ln [1 - r \varepsilon (1 - \theta)] \\ \text{where } \theta &= \frac{T_{L,IN}}{T_{H,IN}} \end{aligned} \quad (1.47)$$

This unified expression is valid for both cases of whether $\dot{C}_L$ is smaller or larger than $\dot{C}_H$. However, in one case it is normalized by $\dot{C}_L$ and in the other, by $\dot{C}_H$. The parameter ε for each case is defined by Eq. 1.9 or 1.13. The sources of irreversibility are the temperature imbalance of the streams and the ineffectiveness of the recuperator, if one neglects the dissipative pressure drop in the channels. For $\varepsilon = 1$ and $r = 1$, and for any θ , no entropy is generated; $\dot{S} = 0$. The value of $\dot{S}$ monotonically increases as a function of θ but it peaks as function of ε ^{29,30} at a value of $\varepsilon = 1/(1 + r)$.

1.2.4.3 Cryocoolers' Interchanging Process

Uniquely to an interchanger (in contrast to a counter flow heat exchanger), the quantities ε and θ are not independent; a larger value of ε decreases the value of $T_{L,IN}$, and thereby decreases the value of θ . The change of ε that induces a corresponding change of θ , is the essence of a cryocoolers' interchanging process. The interchanging process is driven by DT . Since the anchor temperature of an interchanger is $T_{H,IN}$, it is reasonable to specify δT as a fraction of $T_{H,IN}$, in the form,

$$\delta = DT / T_{H,IN} \quad (1.48)$$

Any set of I_M and δ values determine a pair of corresponding values for ε and θ . Following Eq. 1.7,

$$I_M \delta = 1 - \theta \quad (1.49)$$

Determining ε from Eqs. 1.11 and 1.15, and employing Eqs. 1.48 and 1.49, the expression for $\dot{S}$ from Eq. 1.47 in terms of I_M and δ for the two regimes becomes,

$$\frac{\dot{S}}{\dot{C}_H} = r \ln \left(1 + \delta \frac{I_M - 1}{1 - \delta I_M} \right) + \ln [1 - r \delta (I_M - 1)] \text{ for } \dot{C}_H \leq \dot{C}_L \quad (1.50)$$

$$\frac{\dot{S}}{\dot{C}_L} = r \ln \left(1 + \frac{\delta}{r} \cdot \frac{I_M - 1}{1 - \delta I_M} \right) + \ln [1 - \delta (I_M - 1)] \text{ for } \dot{C}_H > \dot{C}_L \quad (1.51)$$

1.2.4.4 Optimization Under Finite Size Constraint

Grassmann and Kopp³¹ examined a group of heat exchangers all sharing a similar size, which as noted above could be characterized as having a similar NTU. (Other kinds of constraints are discussed elsewhere^{32,33}). In such a case, if one refers to the colder stream, then by the integration of Eq. 1.27 with a constant $c_{P,L}$, the constraint of a constant NTU_L becomes,

$$\frac{A_L}{\dot{n} \cdot c_{P,L}} = \int_{T_{L,IN}}^{T_{L,OUT}} \frac{dT_L}{\Delta T \cdot U_L(T_L)} = \text{Const} \quad (1.52)$$

The heat transfer coefficient, U_L , is allowed to be temperature dependent. By variational calculus³⁴ one finds that the entropy generation determined by Eq. 1.46 is minimized when the ΔT along the heat exchanger is “nearly” proportional to the absolute temperature, in the form,

$$\left[\begin{array}{l} @U_L = U_L(T_L) \\ c_{P,L} = \text{Const} \end{array} \right] \frac{\sqrt{U_L a} - a}{U_L - \sqrt{U_L a}} T_L = \Delta T = \frac{\sqrt{U_L a} - a}{U_L - a} T_H \quad (1.53)$$

The parameter a is a constant (with the same units as U_L) that is determined through the constrain of preserving A_L by Eq. 1.52. For $U_L = \text{Const}$ and $c_{P,L} = \text{Const}$, following Quack et al.³⁵ the optimum temperature profiles are associated with a linear decay of ΔT towards the cold end (the T_H is proportional to T_L),

$$\begin{aligned} \Delta T &= \frac{\ln (T_{L,OUT}/T_{L,IN})}{\text{NTU}_L} T_L \\ &= \frac{\ln (T_{L,OUT}/T_{L,IN})}{\text{NTU}_L + \ln (T_{L,OUT}/T_{L,IN})} T_H \end{aligned} \quad (1.54)$$

The associated entropy generation in this case is,

$$\Delta s = c_{P,L} \frac{[\ln (T_{L,OUT}/T_{L,IN})]^2}{\text{NTU}_L + \ln (T_{L,OUT}/T_{L,IN})} \quad (1.55)$$

1.2.5 Regenerative Versus Recuperative Interchanging

Regenerative interchanging is based on energy storage and periodic time-sharing of the regenerating matrix (dA_H and dA_L are identical). Consequently, the effectiveness of regenerative interchanging approaches but cannot exceed the limit of recuperative interchanging even under extremely ideal conditions such as:

- (a) The matrix specific heat capacity, $c_P \rightarrow \infty$, thereby eliminating any restriction on the energy storage process, and,
- (b) No delay in the transient response associated with flow switching as the frequency, $f \rightarrow \infty$.

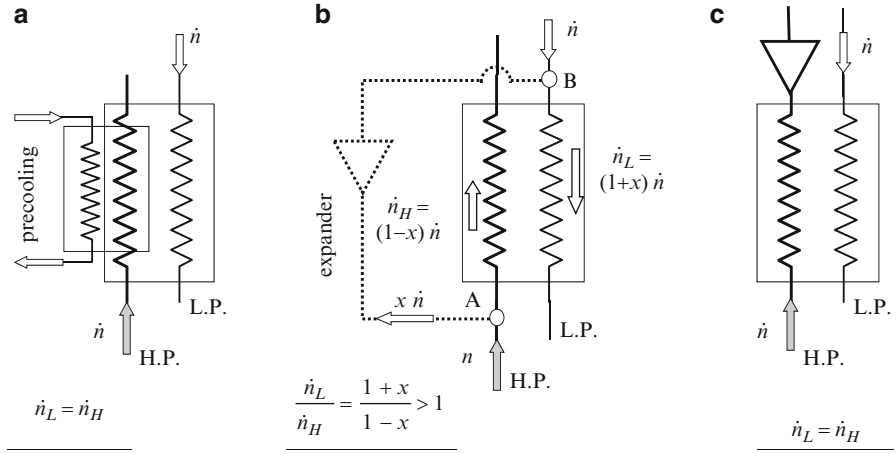
In addition to the parameters $\dot{C}_H/\dot{C}_L$ and NTU of recuperation, regenerator have an additional non dimensional parameter, which accounts for finite values of c_P^{REG} and f , called the *matrix capacity rate ratio*, r^{REG} ,

$$r^{REG} = m^{REG} c_P^{REG} f / C^{MIN} \quad (1.56)$$

Here m^{REG} is the mass of the regenerator. Coppage and London³⁶ obtained the effectiveness of a regenerative interchanger, $\varepsilon = \varepsilon(\text{NTU}, r, r^{REG})$, for the balanced streams case, $r = 1$. The result is applicable for AC types of cryocoolers, which generally use helium at moderate pressures. Shah³⁷ extended this analysis for $r < 1$. Alternative analyses are provided elsewhere by Hausen³⁸ or Jacob.³⁹ In these, the description of $\varepsilon(\text{NTU}, r, r^{REG})$ is obtained in the form of a numerical solution for the transient heat transfer equation. There are no closed form solutions such as we have for recuperative interchanging. A numerical code for optimizing a regenerator was developed by Gary, Radebaugh and Daney.^{40,41} Minimizing the entropy generation in regenerators was discussed elsewhere.^{42,43}

In contrast to recuperators, and because of the finite value of r^{REG} the degradation of ε may be compensated by increasing the value of NTU through the “density” (compactness) of heat transfer area within the regenerator’s porous matrix.

Fig. 1.7 Three types of enhanced interchanging, by (a) precooling, (b) parallel isentropic expander, and (c) serial isentropic expander (H.P. and L.P. represent high and low pressure)



One should observe that the regenerative interchanging of cryocoolers is actually an “auto-interchanging”, since the same amount of gas interchanges with itself in the two successive stages, before and after an expansion.

1.2.6 Enhanced Interchanging

1.2.6.1 A Preferred Condition

A preferred condition for an interchanger is one for which $\dot{C}_H < \dot{C}_L$, or even better, one for which $\dot{C}_H$ is much smaller than $\dot{C}_L$. For a given heat transfer area and effectiveness, such a condition creates the potential for reaching lower temperatures because the magnification coefficient (Sect. 1.1.3.3) is larger. The temperature distribution associated with this condition also produces a smaller value of ΔT at the lower temperatures – a condition that is consistent with a lower generation of entropy, following Sect. 1.2.4.4.

1.2.6.2 Sub-optimum Interchanging

In addition to an insufficient NTU, three other factors can impede realization of the preferred, or optimum, interchanging condition where $\dot{C}_H < \dot{C}_L$ and thereby also limit the value of I_M :

The Most Common Factor

The most common factor is simply when $c_{P,L} \leq c_{P,H}$. In general the higher pressure stream will have an equal or larger specific heat capacity than the returning stream. Quite “rarely”, one encounters the situation where $c_{P,L} > c_{P,H}$ such as with helium and hydrogen at low temperatures, as will be introduced in Sect. 1.3.4.3 and displayed in Fig. 1.10. At higher temperatures, the values of $c_{P,H}$ and $c_{P,L}$, for helium are very similar, thus ΔT remains almost constant along the interchanger. A dilution

refrigerator also has $c_{P,L} > c_{P,H}$ at sub K temperatures as discussed in Sect. 1.3.5.7.

Interchanging in Liquefiers Where $\dot{n}_L \leq \dot{n}_H$

Cryocoolers that liquefy gases fulfill their intended function through the evaporation of the liquefied portion. Liquefiers, however, separate and remove the liquid portion out of the evaporator and the machine. The fraction, y , of the low pressure stream that is extracted from the evaporator, causes $\dot{n}_L$ to be only $(1-y)\dot{n}_H$, so that $\dot{n}_L < \dot{n}_H$. The unbalanced flow condition is accompanied by an increased ΔT (as shown later in Fig. 3.20) and thus a higher entropy generation.

Low Temperature Degradation

Low temperature degradation of a regenerator arises due to the decrease of its matrix c_P (in keeping with the Third Law of the thermodynamics⁴⁴). This feature reduces the effectiveness of regenerative interchangers and limits the value of I_M , especially when approaching liquid helium temperatures.

1.2.6.3 Factors That Enhance Interchanging

Factors that enhance interchanging are frequently utilized to compensate for conditions that would otherwise suppress interchanging. Figure 1.7 displays three types of enhanced interchangers. Versions (b) and (c) combine expanders with the interchanging process, a common practice for larger scale liquefiers and cryocoolers.

Precooling

Precooling is the first strategy shown in Fig. 1.7a. This approach is discussed in detail in Sect. 5.4. Lowering the temperature of the warmer stream with an external counterflowing stream may be regarded as assisting the low pressure stream in its task of extracting heat from the

higher pressure stream. This approach therefore enhances the effectiveness of the interchanging process. Two-staged and hybrid Joule-Thomson cryocoolers (Sects. 5.4 and 5.6) also fit in this category.

Split Flow Isentropic Expansion

In version (b), of Fig. 1.7, a substantial fraction, x , (typically about 0.6) of the high-pressure flow is diverted from the main stream (at junction A), and undergoes a near-isentropic expansion, cooling down and returning to the low pressure channel (at junction B). As a result, the high pressure flow $\dot{n}_H$, is reduced down to $(1 - x)\dot{n}$, a value smaller than $\dot{n}_L$, enabling the condition where $\dot{C}_H < \dot{C}_L$ even though $c_{p,L} < c_{p,H}$ (an approach optimized by Bejan⁴⁵). Note also that the isentropic expansion is typically accompanied by an extraction of work, and a temperature reduction that is similar to that obtained by precooling. In the case of liquefiers this flow splitting compensates for the inherent reduction of its low pressure flow rate $\dot{n}_L$. Claude's liquefier employs a section of this type (b) between two sections of standard counter flow heat exchangers. Kapitza used such a section between the cold end of his liquefier and the standard counter flow heat exchanger at the warm end. Heylandt reversed the order of the same two sections. Collins incorporated two such sections between three standard heat exchanger sections (see Sect. 3.1.4.3). An optimal series of such sections was discussed by Knudsen and Ganni.⁴⁶

Serial Isentropic Expansion

Version (c) configures the expanders in series with the heat exchanger. The points of isentropic expansion along the high pressure stream are accompanied by temperature reductions along this warm stream. The effect of this approach is similar to that of precooling (a) and desirably suppresses the growth of ΔT towards the cold end. This idea was originally proposed by Grassmann and Kopp³¹ and further explored for the purpose of improving helium coolers by Voth,⁴⁷ Kundig,⁴⁸ Siegler and Quack⁴⁹ and Quack et al.³⁵ and for improving neon coolers by Fredrich, Haberstroh, and Quack.⁵⁰ Such a configuration was analyzed by Maytal⁵¹ for an open cycle small Joule-Thomson cryocooler.

Hybrid Interchanging

It is advantageous for the AC type of cryocoolers to employ regenerators at higher temperatures, but utilize recuperative interchanging at lower temperatures such as the liquid helium range. In 1980, Daney⁵² proposed harnessing a couple of three-stage AC cryocoolers synchronized to operate at 180° out of phase with respect to each other. The superposition of the out-of-phase flow rates in the final stages produced a DC flow. Using this approach, a single recuperator was able to exchange heat between the two AC cryocoolers

at the low temperature range. Similarly, Will, and deWaele et al.^{53,54,55} combined two out-of-phase pulse-tube cryocoolers and utilized a single recuperator to effectively replace the two regenerators that would otherwise be required for the two pulse-tubes.⁵⁶

The Boreas cryocooler^{57,58,59,60,61} is a three-stage AC type cryocooler with hybrid interchanging so arranged that:

The technique achieves high heat-exchange effectiveness over the entire temperature range from room temperature down to liquid-helium temperature by using counterflow heat exchange almost exclusively in the colder stage and using a combination of both counterflow and regenerative heat exchange in the upper stage.

1.3 Real Cryocoolers in View of the Generalized Model

1.3.1 From Siemens to Linde and Hampson

The elementary cooling mechanism in the original Provisional Specification of Siemens¹ was an expansion, as he describes,

air is allowed to expand in a cylinder or engine of a suitable construction by which its temperature is lowered.

Here cooling is a result of subtracting mechanical work from the gas, and Siemens suggested that it be recovered “to assist in working the pump” (the compressor). The cooling resulting from the gas expansion is represented by elementary δT or DT in terms of the above discussion and the lower part of Fig. 1.1.

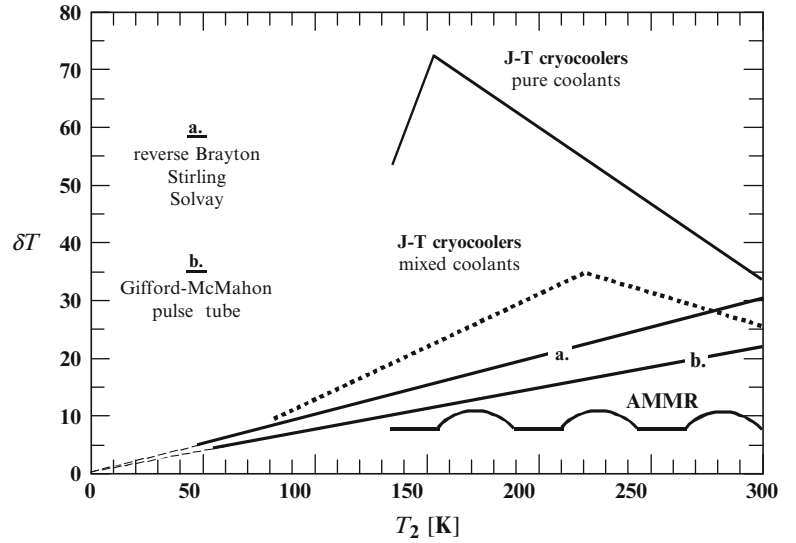
In 1860, Kirk^{11,62} built the first Stirling cooling system. Its elementary cooling mechanism was the same as proposed by Siemens and the interchanging function was accomplished by a regenerator as will be discussed later.

Similarly, Linde in Germany, attempted to build cryocoolers with expanders like Siemens and Kirk, however, he faced problems associated with their lubrication.³ Perhaps this reason motivated his idea to utilize the Joule-Thomson⁶³ effect, thereby eliminating any moving pistons. He substituted Siemens' expander with a *throttle* as the elementary cooling mechanism.^{64,65,66} Hampson⁶⁷ did the same in England almost at the same time, both in 1895. In *The Quest for Absolute Zero*, Mendelssohn² concluded that:

With Siemens' cooling cycle and his heat exchanger....it was only a question of time before somebody would get the idea to substitute for the expansion engine a Joule-Thomson plug.

The idea of Linde and Hampson is simple and elegant, eliminating any moving mechanism at the cold end. In spite of this simplicity, it is interesting to note the elapse of about four decades from the discovery of Joule and Thomson before Linde and Hampson put it to work.

Fig. 1.8 Typical values to demonstrate the trends of the elementary temperature drop, δT adiabatically, imposed at the interchanger's cold end, as a function of the instantaneous cooldown temperature, T_2 , at the inlet to the elementary cooling mechanism



1.3.2 The Elementary Cooling Mechanisms and Their Characteristics, $\delta T(T)$

A stream at the temperature of T_2 enters the elementary cooling mechanism and undergoes the temperature reduction of δT . The cooling mechanism is characterized by the temperature dependence of $\delta T(T_2)$ at any given operating conditions. The candidate mechanisms for cooling, that can be harnessed in a continuously operating cryocooler, are quite few in number and are reviewed below. The most commonly used are, isentropic and isenthalpic expansions, separation and mixing of phases, and adiabatic demagnetization. Their typical behavior is qualitatively displayed in Fig. 1.8.

1.3.2.1 Continuous Isentropic Expansion

Continuous isentropic expansion is traditionally realized by allowing a stream to expand through a loaded turbine⁶⁸ and extracting work. This is the elementary cooling mechanism of the reverse Brayton cryocooler. Recent attempts were made to build an electrostatic isentropic expander using MEMS technology.⁶⁹ The isentropic pressure drop from P_{IN} to P_{OUT} , is accompanied by a drop of δT below the inlet temperature T_2 , satisfying the condition,

$$\delta T = \alpha T_2 = \alpha (T_1 - \Delta T_H) \quad (1.57)$$

where,

$$\begin{aligned} \alpha &= \left[1 - \left(\frac{P_L}{P_H} \right)^{(\kappa-1)/\kappa} \right] \eta_{EX} \\ &= \left[1 - \left(\frac{P_4}{P_2} \right)^{(\kappa-1)/\kappa} \right] \eta_{EX} \end{aligned} \quad (1.58)$$

The term η_{EX} represents the efficiency of the isentropic expander, indicating the fraction $\Delta h / \Delta h_s$ of the actual enthalpy reduction, Δh , (which is also the work production) out of the ideal isentropic enthalpy reduction, Δh_s .

The actual enthalpy reduction manifests itself in the form of a temperature reduction (δT). Hence, the temperature drop produced by such an expansion is proportional to the instantaneous temperature, T_2 at the inlet to the expander. One may deduce that at lower temperatures, the associated temperature reduction decreases in magnitude, being driven by a smaller δT . As a quantitative example of this type of cooling, helium when expanding through a pressure ratio of 1.5 and with $\eta_{EX} = 0.7$, has a value of $\alpha = 0.105$.

Since they are driven by a continuous flow, reverse Brayton cryocoolers are typically referred to in the literature as DC flow systems. In contrast, AC flow cryocoolers, as discussed below, produce alternating compression and discharge of the same chamber.

1.3.2.2 Series of Isentropic Expansions with Work Extraction

The standard example used for isentropic expansion incorporates a piston in a rigid cylinder filled with gas. The gas is allowed to expand against an external load and perform work. The extracted work results in a temperature reduction of the gas. Such a process is in fact utilized in the Stirling, Solvay and Vuilleumier cryocoolers. The same cylinder is successively compressed and expanded, and the same gas flows into and out of the expansion chamber as an alternating current. These cryocoolers are identified as AC flow coolers. The temperature dependence of δT follows the same formulation as given in Sect. 1.3.2.1).

1.3.2.3 Series of Blow Down (Isentropic) Expansions

When compressed gas in a chamber discharges, the remaining gas in the chamber experiences an isentropic expansion. One may regard the remaining gas as performing work by pushing on the gas that exits the chamber. (One part of the gas performs work on another part, but no net work is extracted from the entire bulk of the gas). The temperature of the expelled gas also decreases gradually, as the discharge process proceeds. The remaining gas, as a bulk, after reaching the final lowest temperature is forced by a displacer to flow into the interchanger. This mechanism is employed in Gifford-McMahon (GM) and pulse tube cryocoolers, these also being AC flow coolers. The δT in this case remains proportional with T_2 as in Eq. 1.57, however, the proportionality ratio is smaller than α by Eq. 1.58. In other words, for a given pressures ratio, the δT in this case is smaller than in the case of isentropic expansion with work extraction (Sect. 1.3.2.4), where the entire content of the chamber reaches the lowest temperature.

1.3.2.4 The Joule-Thomson Expansion Valve

The Joule-Thomson expansion valve, also fed by a DC flow, provides the elementary cooling mechanism for a Linde-Hampson cryocooler or liquefier, with pure or mixed coolants. The expansion valve decreases the pressure adiabatically without any output of work thus preserving the enthalpy of the steam. As a real gas effect, the expanded stream undergoes a temperature change of ΔT_h (discussed in Sect. 4.5), which is the adiabatic integral Joule-Thomson effect. Below the inversion temperature (discussed in Sects. 2.4, 2.5, and 2.6), this expansion is accompanied by a temperature reduction. For example, a 30 MPa stream of argon at 295 K has a δT , which is the $|\Delta T_h|$, of 84.2 K. For nitrogen at 25 MPa and 200 K the $|\Delta T_h|$ is 78.4 K.

At lower temperatures, the value of δT for pure gases becomes larger, in contrast to the trend for isentropic expansion. Hence, as temperatures are decreased, the cooling mechanism becomes stronger. The temperature dependence of δT may be approximated in a linear form,

$$\delta T(T_2) = -\Delta T_h = B - \gamma T_2 = B - \gamma (T_1 - \Delta T_h) \quad (1.59)$$

where B is a constant and γ represents the rate of growth of δT as the temperature decreases below T_{AMB} . As will be shown in Sect. 4.5.5.2, Eq. 4.72, γ may be expressed as,

$$\gamma = -\frac{\partial}{\partial T} \delta T = \frac{\partial}{\partial T} \Delta T_h = \frac{c_P - c_{PO}}{c_{PO}} = \frac{1}{r} - 1 \quad (1.60)$$

Since the high and low pressure streams share a common flow rate, the ratio of the heat capacities is the same as the

capacity rate ratio, r . Representative values for r , as listed in Table 3.4 are, 0.68 for nitrogen and 0.55 for argon; the corresponding values of γ are, 0.47 and 0.8. After reaching the fixed boiling point of the gas, δT decreases.

For mixed gases (as will be discussed and displayed in Fig. 8.3), δT may decrease at lower temperatures, resembling the behavior of an isentropic expansion. It is also possible that the temperature dependence may be non-monotonic.

1.3.2.5 The Injector

The injector, also fed by a DC flow, replaces the throttle in some applications, especially for liquefaction of helium⁷⁰ and below temperatures of 4 K, as discussed in Sect. 5.6.5.1. Expansion through an injector is not purely isenthalpic and is less dissipative than expansion through a throttle valve.

1.3.2.6 Adiabatic Demagnetization

Adiabatic demagnetization of certain materials generates cooling. The Active Magnetic Regenerative Refrigerator, (AMRR)⁷¹ employs a series of magnetic materials with successively lower Curie temperatures. The δT of each one is fairly constant over a narrow temperature range. Consequently, δT over the entire cooldown range is almost constant, but exhibits some small fluctuations.

1.3.2.7 Phase Separation of ^3He - ^4He Mixtures

Below 0.87 K mixtures of ^3He - ^4He separate into a ^3He -rich phase and a ^3He -dilute phase, so that evaporation of ^3He molecules from the -rich to -dilute phase produces cooling, that in adiabatic conditions generates a temperature drop. This evaporative cooling mechanism is utilized in the dilution refrigerator.⁷² In continuous operation these cryocoolers can cool to as low as 5 mK while pumping heat to about 600 mK. The δT shrinks drastically at lower temperatures, starting at about 450 mK and shrinking to only a few mK at the lowest temperatures.

1.3.2.8 Mixing Two Separate Streams of ^3He and ^4He

Mixing two separate streams of ^3He and ^4He at a temperature below 2.5 K has an endothermic effect. At the cold end of a recuperator, in an adiabatic mixing chamber, the temperature of the exiting mixture is reduced by δT . This elementary cooling mechanism is an alternative to the phase separation of ^3He - ^4He mixture as proposed for space applications by Benoit.^{73,74,75}

1.3.2.9 A Vortex Tube

A Vortex tube is a continuous flow element, that separates the incoming stream into a colder exhaust stream and a warmer exhaust stream. It generates a δT of about 10–30 K at room temperature, and seems to decrease at lower temperatures. However, as explained in Sect. 1.3.5.9 no cryocooling application based on this cooling mechanism has been reported.

1.3.2.10 Thermoacoustic Expanders

Thermoacoustic expanders have been proposed and tested^{76,77,78,79} but not yet incorporated into a complete cryocooler.

1.3.3 The Lowest Attainable Stable Temperature of Cryocooling

1.3.3.1 Formulation

In an adiabatic condition, such that $DT = \delta T$, and $T_3 = T_4$, any of the fundamental cooling mechanisms will produce a larger temperature drop than in the case when a heat load is applied, and therefore the associated cryocooler may reach a lower temperature. For any given constant value of δT , the lowest attainable temperature is only a function of I_M , by definition, through Eq. 1.7. However, in practice, as discussed in Sect. 1.3.2, δT is a function of the temperature at the inlet to the elementary cooling mechanism, so that, $\delta T = \delta T(T_2)$. In the general case with a heat load,

$$DT = T_2 - T_4 = \delta T(T_2) - \frac{\dot{Q}}{\dot{n} c_{P,L}} = DT(T_2, \dot{Q}) \quad (1.61)$$

Hence, in order to obtain T_4 one should find T_2 . The value of T_2 associated with stable cryocooling is determined by matching the characteristic of the elementary cooling mechanism, $\delta T(T_2)$, with the interchanger's magnification capability, so that,

$$T_1 - T_2 = \Delta T_H = (I_M - 1) \cdot DT(T_2, \dot{Q}) \quad (1.62)$$

The solution of $T_2(I_M, \dot{Q})$ depends on two parameters, I_M and $\dot{Q}$.

Remark: Theoretically, there may be cases where Eq. 1.62 has no solution for T_2 . Such a situation would imply no stable temperature for cryocooling; a search in temperature space would continue going lower and lower until some other constraint, such as the Second Law, would be encountered (as later discussed in Sect. 1.3.3.5).

Solutions for T_2 are explored below for two types of δT (T_2).

1.3.3.2 Monotonically Decreasing δT

At any instantaneous cooldown temperature, the adiabatic case ($\delta T = DT$), combined with Eqs. 1.57 and 1.7, leads to the following expression for the total temperature difference between the two entry points of the interchanger:

$$\Delta T_{IN-IN} = T_1 - T_4 = \frac{\alpha T_1 I_M}{1 + \alpha (I_M - 1)} \quad (1.63)$$

Note that any small value of α may be compensated for by a sufficiently large value of I_M , in order to reach a desirable value of ΔT_{IN-IN} . If δT would remain constant during the cooldown at its initial value of αT_1 , the numerator would represent the corresponding value of ΔT_{IN-IN} for the given value of I_M . However, as in the case of an isentropic expansion, δT shrinks as the temperature drops. The denominator in Eq. 1.63, which is larger than unity, reduces the quantity $\alpha T_1 I_M$ due to the decrease of δT during the cooldown. The above solution, combining the characteristics of both the elementary cooling mechanism and of the interchanger, is graphically displayed as an “intersection” of Eqs. 1.57 and 1.7 in Fig. 1.9. A non-conditional solution always exists which is a steady state point of cryocooling for any values of I_M and $\dot{Q}$. The black circle represents the lowest achievable temperature, under adiabatic conditions, and the white circle represents the conditions achievable with a heat load. The figure also demonstrates that an interchanger with a smaller magnification index, $I_M^* < I_M$, can only achieve a smaller value of ΔT_H .

For helium expanding through a pressure ratio of 1.5 and with $\eta_{EX} = 0.7$, Eq. 1.58 gives $\alpha = 0.105$. For reverse Brayton cryocoolers the two recuperating streams typically exhibit well balanced capacity rates. Assuming an interchanger with an ineffectiveness of $i = 0.01$ and therefore a magnification index of $I_M = 100$, the lowest achievable temperature would be $T_{AMB}/12.7$; starting at room temperature, the lowest achievable temperature would be about 23 K. For similar conditions but operating with hydrogen, α would be 0.076 and the lowest temperature would be, $T_{AMB}/9.22$ or 32 K.

In terms of NTU, Eqs. 1.57 and 1.44, combined with the conditions $\dot{C}_H = \dot{C}_L$ and adiabatic operation, leads to the expression,

$$\begin{aligned} \frac{T_2}{T_1} &= (1 + \alpha \text{NTU})^{-1} \\ &= \left\{ 1 + \text{NTU} \eta_{EX} \left[1 - (P_L/P_H)^{(\kappa-1)/\kappa} \right] \right\}^{-1} @ \dot{Q} = 0 \end{aligned} \quad (1.64)$$

1.3.3.4 The Minimum Value of δT When $\dot{C}_H > \dot{C}_L$

In this case, the value of I_M is bounded by Eq. 1.16. Therefore, in order to reach a desired value of $T_{L, IN}$, one should provide a sufficiently large value of δT . Combining Eq. 1.49 with the relationship one obtains from Eq. 1.16: $I_M = 1/(1-r)$, we arrive at the expression

$$\delta \geq (1 - \vartheta)(1 - r) \quad \text{or} \quad \delta T \geq (T_1 - T_4)(1 - r) \quad (1.69)$$

For example, consider a nitrogen-charged Joule-Thomson cryocooler with $T_1 = 300$ K and T_4 as the normal boiling point of 77.3 K. At the two pressures of 20 and 0.1 MPa, the average capacity rate ratio is $r = 0.65$. Therefore, the value of δT by Eq. 1.69 has to be at least 78 K in order to reach the normal boiling point. This condition corresponds to the highest value of $I_M = 1/(1-r) = 2.86$. A smaller value of I_M would not be sufficient in order to reach the normal boiling point, a condition that will be subsequently referred to as “under recuperated”, in Sect. 3.3.3.

1.3.3.5 Other Limits to the Lowest Attainable Temperature

Various additional factors, not represented in the above formulation, may also limit the lowest attainable temperature and therefore reduce its prescriptive power:

Thermal Losses

Thermal losses such as longitudinal thermal conduction or an external parasitic heat load along the cryocooler (see Sect. 1.2.3.4, remark 2) degrade the interchanger’s effectiveness and suppress the magnification index, thus reducing the possibility for deeper cooling.

The Second Law

The Second Law of the thermodynamics may set a limit on the lowest attainable temperature, on the basis of entropy balance (for real gas properties), as discussed later in Sects. 1.4.2.3 and 1.4.3.3.

A Pinch Point

A pinch point in the interchanging process limiting any further reduction of temperature, may occur due to the temperature dependence of heat capacities. The present model assumed constant capacity rates for each channel. Such assumptions do not enable a prediction of pinch points.

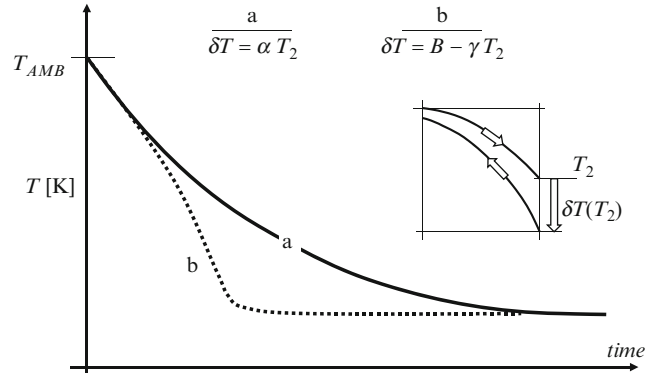


Fig. 1.10 Qualitative display of cooldown curves for cryocoolers with different characteristics, $\delta T(T_2)$ associated with their elementary cooling mechanism. (a) is the type associated with Stirling cryocoolers where δT decreases at lower temperatures, and (b) represents a Joule-Thomson cryocooler, with an increasing value of δT . The two patterns do not generally have the same time scale; pattern (b) is typically much faster

1.3.4 The Shape of Cooldown Curves

The elementary cooling mechanism δT that drives the interchanging process during cooldown, as illustrated in the lower part of Fig. 1.1, is primarily characterized by its magnitude. One might expect that an elementary cooling mechanism with a larger value of δT will reach the target cooldown temperature faster. In addition, the temperature dependence of δT is relevant to the character of the cooldown process.

When the magnitude of δT shrinks as the cooldown proceeds, such as in the case of an isentropic expansion as described by Eq. 1.57, the positive feedback mechanism weakens and the progress of the cooldown process is gradually attenuated. The rate of cooldown decreases and the shape of the cooldown curve takes on a concave form as illustrated in Fig. 1.10a.

In the case of Joule-Thomson cryocooling with pure gases, the value of δT at lower temperatures grows as described by Eq. 1.59, hence the driving potential for further lowering the temperature also grows. Therefore, one may expect a convex shape for the cooldown curve as shown in Fig. 1.10b. Such a temperature dependence for δT may be one of the reasons for the more rapid cooldown, in general, of Joule-Thomson cryocoolers in comparison to other types with an isentropic expansion.

1.3.5 The Relationship Between I_M and δT for Various Cryocoolers

According to Eqs. 1.11 and 1.15, the magnitude of I_M depends solely on $\dot{C}_H/\dot{C}_L$ and the effectiveness of the interchanger, ε , but is independent of δT . Nevertheless, in any

Table 1.1 Comparison of various single stage cryocoolers. (The AC cycles are marked by gray). The up and down arrows indicate whether δT increases or decreases as T gets lower. (H - magnetic field, c - concentration, μ - chemical potential)

Cryocooler	Effect of elementary temperature drop		T_H/T_L	Inter-changer	typical δT [K]	$\dot{C}_H/\dot{C}_L$
Linde-Hampson	Isenthalpic pressure drop	$\left(\frac{\Delta T}{\Delta P}\right)_h$	$\frac{300}{80} = 3.7$	re-cuperator	40 $\uparrow$ to 90 $\uparrow$	1.4
Joule-Thomson mixed-coolants					25 $\uparrow$ to 5 $\uparrow$	1
satellite cooler (helium)			$\frac{300}{4} = 75$		1 $\uparrow$ to 3 $\uparrow$	0.9
reverse Brayton	Work extraction		$\frac{300}{40} = 7.5$	re-generator	40 $\downarrow$ to 3 $\downarrow$	1
Stirling, Solvay, Vuilleumier	Work extraction (AC)	$\left(\frac{\Delta T}{\Delta P}\right)_s$			40 $\downarrow$ to 3 $\downarrow$	
Gifford-McMahon, Pulse tube	Chamber discharge					
Active magnetic recuperative refrigeration	Adiabatic demagnetization	$\left(\frac{\Delta T}{\Delta H}\right)_s$	$\frac{300}{80} = 3.7$		10 $\pm$ 5	
Dilution refrigerator	^3He - ^4He mixture phases separation	$\left(\frac{\Delta T}{\Delta c}\right)_\mu$	$\frac{0.600}{0.005} = 120$	recuperator	.450 $\downarrow$ to .005 $\downarrow$	0.13

viable and practically proven cryocooler, the magnitude of δT is relevant and may be related with I_M in the following respect: a larger value of δT could accommodate a smaller value of I_M , and vice versa. Table 1.1 compares some real cryocoolers on the basis of their capacity ratio $\dot{C}_H/\dot{C}_L$ which reflects the value of I_M , versus δT , and their typical temperature span ratio, T_H/T_L .

1.3.5.1 The Joule-Thomson Cryocooler with Pure Coolants (Except He and H₂)

As will be described in Chap. 3, the Linde-Hampson cycle incorporates an “over-balanced” interchanger, that is $\dot{C}_H/\dot{C}_L > 1$. Although its flow rates are balanced, $\dot{n}_L = \dot{n}_H$, the average heat capacity of the high pressure stream $\bar{c}_{p,H}$ is larger than that of the low pressure stream, $\bar{c}_{p,L}$. Consequently, the typical capacity rate ratio is about 1.4 and therefore, the associated magnification factor is relatively low. Following Fig. 1.6, with an NTU of 10 to 20 the corresponding values of I_M are only 3.22 and 3.50. These low values are compensated for by the relatively large values of δT that also increase as the cooldown progresses.

1.3.5.2 Joule-Thomson Cryocoolers with Mixed Coolants (discussed in Chap. 8)

In various portions of the heat exchanger with a mixed coolant, two phase flow conditions can exist. Thus, it is

possible for the capacity rates to approach the balanced case, or even produce the condition where $\dot{C}_H/\dot{C}_L < 1$.

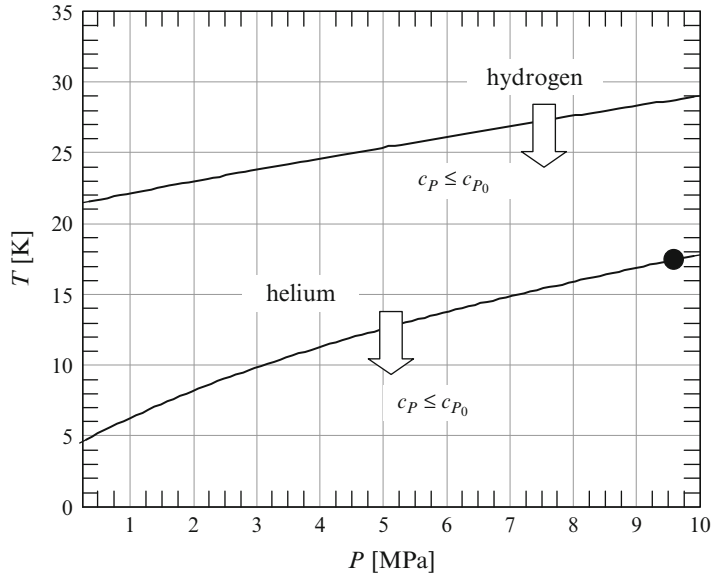
1.3.5.3 Joule-Thomson Cryocoolers of He and H₂

These two pure gases present unique features with respect to their heat capacities. In the temperature range of about 20 K and prior to liquefaction the high pressure stream has a lower capacity rate than the low pressure stream, $\dot{C}_H < \dot{C}_L$. The same fact is true for D₂ and He-3. Figure 1.11 focuses on this feature, displaying curves of $c_p = c_{p0}$, and thereby identifies the maximum temperature for each pressure that still satisfies the condition $\dot{C}_H < \dot{C}_L$.

1.3.5.4 Satellite Cryocoolers That Liquefy Helium (see also Sects. 3.1.2.3 and 3.1.2.4)

Joule-Thomson cryocooling systems that provide a steady supply of liquid helium into an evaporator create the situation where $\dot{n}_L > \dot{n}_H$. The liquid in the return stream elevates the capacity rate of that stream so that the system becomes under-balanced, $\dot{C}_H/\dot{C}_L < 1$. The resulting large magnification factor I_M , enables operation over a broad temperature range, as large as about 75 K. This configuration produces the opposite capacity ratio compared to that of a liquefier where a liquid portion is extracted.

Fig. 1.11 Curves of $c_P = c_{P0}$; at the temperature range below these curves the interchangers of the Joule-Thomson cryocoolers are characterized by $\dot{C}_H < \dot{C}_L$. The black point represents the maximum (pressure) state along the integral inversion curve of helium (later discussed through Fig. 2.16)



1.3.5.5 Stirling, Gifford-McMahon, Pulse Tube and Reverse Brayton Cryocoolers

Most cryocoolers operate with a nearly balanced interchanger, a condition that is the most demanding case, in terms of size or heat transfer area, for achieving a given effectiveness. However, the balanced flow also induces a large magnification index. For example, with an NTU of about 10, the magnification index I_M is 11. The large value of I_M compensates for the relatively small value of δT , that shrinks as the interchanging process proceeds. The elementary isentropic cooling mechanism is typically associated with expansion through a relatively low pressure ratio (below 2). These cryocoolers therefore operate with only a minor difference between the specific heat capacities of the two streams and their associated capacity rates.

1.3.5.6 The Active Magnetic Regenerative Refrigerator (AMRR)

The circulating fluid through the interchanger of an AMRR is not active in the thermodynamic sense. Its coolant is passive as explained in Sect. 1.1.1.4. The pressure and density of the warm and cold pressure stream remain nearly the same. Therefore, such an interchanger is well balanced, with $\dot{C}_H = \dot{C}_L$.

1.3.5.7 The Dilution Refrigerator

The dilution refrigerator presents the unique case of a cryocooler that is significantly “under balanced” to the extent that the ratio $\dot{C}_H/\dot{C}_L$ is as low as about 0.13. The associated value of I_M for an NTU of about 10 is as large as 6,900. The large magnification index enables deep cooling covering a wide temperature span ($T_H/T_L = 120$) in spite of the small value of δT .

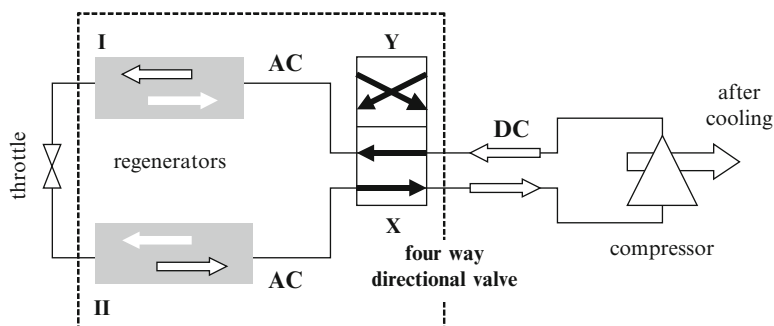
1.3.5.8 The Mixing Refrigerator

The mixing refrigerator generates its cooling mechanism, δT , by mixing two separate streams of ^3He and ^4He at the cold end as explained in Sect. 1.3.2.8. The entry channel of the interchanger includes two separate conduits; one carrying pure ^3He and a parallel one carrying ^4He , which are mixed in the mixing chamber at the cold end. A single and colder stream returns and simultaneously cools the two incoming streams. The ratio $\dot{C}_H/\dot{C}_L$ is similar to that of the dilution refrigerator.

1.3.5.9 A Non-viable Cryocooler Due to Inherently Poor Interchanging

By similar argumentation, one may explain why the elementary cooling mechanism associated with a vortex tube does not provide a viable option for building a cryocooler. The vortex tube can indeed provide a reasonable δT of about 20 K. However, the tube produces a flow-splitting ratio of about 0.25. This means that only one quarter of the incoming stream experiences a temperature drop and could therefore be utilized as the counter flow-stream of the interchanger. The capacity ratio, $\dot{C}_H/\dot{C}_L$ is therefore about 4, and the associated magnification factor I_M is only about 1.33. Indeed, no report has been found (by the authors of this book) of any experiment utilizing a vortex tube to reach temperatures below even 200 K. Nellis et al.⁸⁰ analyzed various cases of pure coolants while Cao, Gong et al.⁸¹ proposed and analyzed a mixed gas auto-cascade using a vortex tube as the final expander instead of a throttle valve. Liu, Gong, et al.⁸² demonstrated experimentally that the separation effect of a coolant below 80 K can produce temperatures of both the warmer and the colder streams that are below the inlet temperature.

Fig. 1.12 A Linde-Hampson closed cycle where the function of recuperation is accomplished by two regenerators as built and reported by Carbonell et al.⁸⁶



1.3.6 Special Examples of Interchangers

1.3.6.1 Continuous Flow Interchanging Using Two “Opposing” Regenerators

Since the isenthalpic expansion process associated with the Linde-Hampson machines consumes a continuous flow of gas, it is natural for these systems to incorporate a recuperative type of interchanger. However, various interesting exceptions to this choice have surfaced through the years. For example, Kapitza⁸³ in 1939 built a small air liquefier, enhanced by an expansion turbine, in a similar manner to that of Claude. Although his liquefier is a continuous flow device, the warmer section of the interchanger is configured with two parallel regenerators. The counter flowing streams are switched from one regenerator to another by two synchronized valves, one at the colder end and the other at the warmer. The matrix construction of the regenerators and the alternating flow direction through each one, enable the removal of frozen impurities such as water and carbon dioxide from the fresh air intake, thereby combining decontamination with cooling. This type of reversing regenerator arrangement for counter flow heat exchanging is also discussed elsewhere.^{12,62,84,85}

Carbonell et al.⁸⁶ used a similar idea in 1972 to build a closed cycle Joule-Thomson cryocooler with mixed refrigerants as shown in Fig. 1.12. The regenerators transform the DC flow of the compressor into an alternating flow (AC). This is accomplished by an electrically operated four-way directional valve that periodically changes its position from X to Y and vice versa. At state X, regenerator I is pressurized and the outlet of regenerator II is exhausted to the suction port of the compressor. Switching to the other state (Y) of the valve exchanges the roles of the two regenerators and the warm pressurized stream floods the colder regenerator, II. The alternating flows in the two regenerators are out of phase. Consequently, the system included in the dashed rectangle is supplied by a DC flow and serves the function of a single interchanger. This interesting example is further discussed in Sect. 8.3.8.5 on mixed-coolant Joule-Thomson cryocooling.

An example of an opposite configuration is given by Will and deWaele^{53, 54, 55} who combined two out-of-phase pulse tubes in such a way that they were able to replace the conventional two-regenerator arrangement with a single common recuperator.

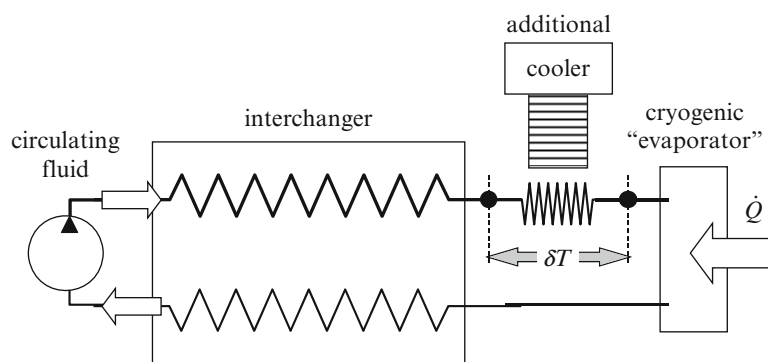
1.3.6.2 Combining a Periodic Expander with a Recuperative Interchanger

In contrast to the configuration of a DC compressor coupled to a regenerative interchanger described in the previous paragraph, Dehne’s patent⁸⁷ presents a system where a periodic expansion device is coupled with a recuperator. The configuration is unique since a regenerator is typically used as an interchanger along with periodic expansion devices. This AC cooler is also unique in that the pressure reservoir drives an open system. The reciprocating piston simultaneously functions as a valve for switching the flow by alternating the connections to the recuperators. The net effect of the reciprocating piston is to rectify the AC flow. The interchanging streams flowing in the two separated channels are periodic and phase shifted by 180 degrees. The pulses are strongly attenuated due to the compressibility of the flowing media. The elementary cooling mechanism results when the piston expands against the opposing constant force generated by the pressure supplied from the reservoir.

1.3.6.3 Pulse Tube Expander Interchanged by a Recuperator

Liang et al.^{76, 77} studied a pulse tube that used a switching valve to generate the elementary cooling effect. The valve was located at the cold end of a recuperative interchanger, and the system was driven by a DC compressor. Although the configuration might be described as an AC version of a Brayton cryocooler or a DC version of Solvay’s cryocooler, the expander in these systems extract work at their cold end. The pulse tube expander’s elementary cooling mechanism on the other hand, behaves like the isentropic discharge volume explained in Sect. 1.3.3.2.

Fig. 1.13 Convective fluid interchanging assisted by an additional cryocooler which generates the elementary temperature drop, δT



1.3.6.4 Interchanger Combined with Convective Cooling

For a few unique cryocoolers the origin of the elementary cooling mechanism, δT , does not involve a thermodynamic transition of the coolant itself (such as throttling or isentropic expansion) but rather utilizes an external device or an additional cryocooler. Figure 1.13 displays a schematic of this idea. A more common example of the same idea is the Active Magnetic Regenerative Refrigerator (Sects. 1.3.2.5 and 1.3.4.6) that is based on the cooling effect of the adiabatic demagnetization of (solid) elements. The fluid is used for interchanging only; at low pressures the capacity rates of the interchanging streams are fairly well balanced with a relatively high value of I_M . Therefore, even if δT decreases at lower temperatures, there is still a cryocooling effect for further lowering the temperature (see Sect. 1.1.3.4). Various other reports utilize different additional cryocoolers in order to generating an elementary cooling effect:

A Gifford-McMahon (GM) or Stirling Cryocooler

Van der Bosch et al.^{88,89,90,91} in 1994 combined a loop of helium with a two stage GM cryocooler. They referred to it as “closed cycle gas flow system” the purpose of which was “cooling high Tc d.c. SQUID magnetometers”. A stream of helium gained its δT at the cold end through a thermal interface with both stages of the GM cryocooler. The interchanger was a tube in tube, Linde style of heat exchanger. At steady state the value of δT was about 60 K and the system reached a temperature below 50 K. Troell et al.⁹² built a similar neon based system in 1994 with a flexible interface to the SDUID.

Another version with a similar approach, described by Hughes, Nellis, and Pfotenhauer^{93,94} in 2004, combines a GM cryocooler with a recuperative closed loop interchanger. In this case, the circulating fluid is a mixture of nitrogen and hydrocarbons (a candidate coolant for Joule-Thomson cryocooling) and the arrangement is used primarily to measure the mixture’s heat transfer coefficient at various temperatures. The pressure of both streams remains close to the charging pressure of the circulating closed loop and

may be set at any desirable value. Note however that the GM cryocooler by itself was not able to cool down the stream as low as was achieved by the interchanging arrangement.

Here we can clearly recognize the central benefit of adding a closed-cycle gas-flow interchanger to a cryocooler: the cryocooler’s performance is enhanced. If the cryocooler would pump heat directly from the cooled object, it would induce a larger heat load, and elevate the attainable temperature. The additional interchanger accommodates a relatively small value of δT (due to its large I_M) thereby imposing a smaller heat load on the cryocooler and enabling the cryocooler to reach a lower temperature. Note, that the entire closed cycle gas flow system cannot cool below the lowest no-load temperature of the cryocooler. One may regard the configuration as an external amplifier. The interchanger of the GM cryocooler itself limited its performance, but by adding the convective interchanging loop, the effective interchanger magnification factor for the GM cryocooler is enhanced and it reaches a lower temperature.

These systems can also be used to transmit the cooling capacity of a cryocooler to a distant location.

Unger⁹⁵ and Corey⁹⁶ used the same principle to liquefy a stream of air from the atmosphere with a GM or Stirling cryocooler.. They intended to build a small-scale device for extracting oxygen out of air.

A Mixed Coolant Closed Cycle Joule-Thomson Cryocooler

A mixed coolant closed cycle Joule-Thomson cryocooler was proposed by Apparao, Podtcherniaev, Flynn et al.⁹⁷ for cooling another closed loop of nitrogen or helium of a chilling system down to about 125 K.

The Thermoelectric Elements

The thermoelectric elements, proposed by Hingst⁹⁸ and later by Hed^{99,100} are combined with the interchanging loop of a convecting fluid although not for the cryogenic range of temperatures. Here, interchanging is accomplished as a fluid flows along the cold side of a series of thermoelectric elements and then returns along the cold side of the

interchanger. A series of adjacent elements is located along the flow path of the fluid. McGrew¹⁰¹ proposed utilizing thin film thermoelectric elements arranged in a longitudinal series but interchanged with counter flowing currents of electricity. The arrangement is expected to provide a solid state, miniature cryocooler capable of lifting 2 mW at 77 K.

1.3.6.5 Interchanging Mass Transfer

Although not exactly the same as the above described processes of interchanging, some mechanisms of mass transfer are closely related to the thermal interchanging process:

Separation of Isotopes

Mass diffusion between two counter flowing streams, is driven by a concentration gradient instead of a temperature gradient. The separation tube discovered and developed by Clusius and Dickel between 1938 and 1950, and described by Jost¹⁰² operates on the basis of mass diffusion. It is a vertical glass tube of few meters long with a small radial concentration gradient at its bottom that drives a demixing process that is magnified by a free convection counter flow. The process generates a large vertical concentration gradient where the lighter molecules are carried inward and upward and the heavier molecules, outward and downward. This invention is very efficient at separating extremely similar chemical compounds such as isotopes of H-³⁵Cl from H-³⁷Cl, or ⁸⁴Kr from ⁸⁶Kr or hexafluoride of ²³⁵U from that of ²³⁸U.

Counter Current Exchange: A Principle of Biology

“Counter current exchange. A principle of biology”, Scholander¹⁰³ summarizes a similar behavior found in a broad array of biological system as a:

means for producing high concentration by gentle, stepwise summation of small primary concentration effect . . .

In these systems the high concentration is accomplished by bundles of small parallel capillaries operating with a counter current exchange, so that the index of magnification may reach values as high as 3,000. Examples of such systems include swim bladders, gills of fishes, placenta, renal functioning and as well heat exchange in mammals and birds.

1.3.7 Refrigerator Versus Cryocooler

Based on the above idea that a cryocooler is comprised of combination of a elementary cooling mechanism and an interchanger with a magnification index, one may distinguish between refrigerators and cryocoolers, by noting that a refrigerator satisfies the condition,

$$\Delta T_H = DT \quad \text{or} \quad I_M = 1 \quad (1.70)$$

In other words, a refrigerator does not utilize an interchanger, and the elementary cooling effect is sufficient to reach the desired temperature. Such a description, for example, fits the vapor compression cycle as established by Perkins⁶² already in 1834. The lowest attainable temperature is achieved as the result of a single effect: a quantity of compressed liquid coolant at ambient temperature is exposed to atmospheric pressure. It evaporates and immediately reaches its saturation temperature, that is below ambient, typically around -30°C . To close the cycle, the expanded vapor is recompressed and re-condensed into a liquid (after being cooled to ambient), thus perpetuating a steady operation.

In this sense, the single shot sub-Kelvin magnetic refrigerator is not a cryocooler but a refrigerator that operates in the cryogenic range of temperatures. However, what is called the dilution refrigerator is, in this perspective, a cryocooler since it involves a significant interchanging process.

Note that it is not helpful to make the opposite claim. That is, even if a cooling system includes a recuperator but does not reach temperatures in the cryogenic range, it should continue to be called a refrigerator. Such an example is the vapor compression refrigerator with a subcooling heat exchanger.

1.4 Second Law Considerations

1.4.1 Performance of Cryocoolers

1.4.1.1 The Thermodynamic Presentation of Cryocoolers

Clausius' statement of the Second Law of Thermodynamics,^{104,105} does not allow spontaneous heat flow from a lower to a higher temperature. Indeed, any refrigerator that continuously pumps heat (at the rate of $\dot{Q}_L$ from a lower temperature T_L) through a cyclic process consumes work (at a rate of $\dot{W}$) as illustrated in Fig. 1.14.

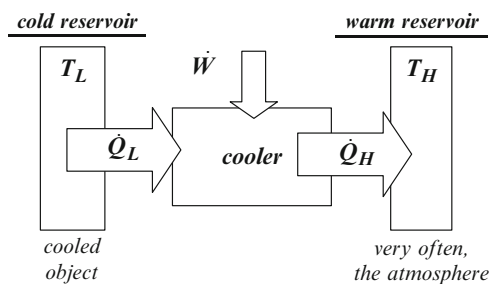


Fig. 1.14 A generalized representation of a cooling system interacting with two thermal reservoirs; the heat source is the cooled object and the heat sink is the cooling system for the compressor

For a cryocooler, the cold reservoir, or the cooled object, may also be identified as the heat source. The warm reservoir, the heat sink, is frequently the atmosphere (the environment that cools the compressor). At steady state, the incoming energy is balanced by the rejected energy,

$$\dot{Q}_L + \dot{W} = \dot{Q}_H \quad (1.71)$$

For an ideal reversible cycle, the amount of heat $\dot{Q}_H$ that is continuously rejected to a higher temperature reservoir, T_H , also satisfies the Second Law balance of entropy flow,

$$\frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H} + \dot{S} = 0 \quad (1.72)$$

where $\dot{S}$ is the irreversible rate of entropy generation inside the cryocooler.

1.4.1.2 The Sites of Entropy Generation

Two types of ‘ideal’ coolers can be distinguished according to the origin of the entropy generation $\dot{S}$, within the framework of finite time thermodynamics.^{106,107} These are: (a) the endo-reversible cryocoolers^{108,109,110} (or heat engines¹¹¹) where all irreversibilities are associated only with the flow of heat $\dot{Q}_L$ and $\dot{Q}_H$, driven by the finite temperature differences between the device and the reservoirs, while the “core” cooler (or heat engine) is an ideal Carnot device with $\dot{S} = 0$, and, (b) the exo-reversible (multi stage) cryocoolers,^{112,113} that absorb $\dot{Q}_L$ and release $\dot{Q}_H$ reversibly at constant T_L and T_H , so that $\dot{S}$ is only generated (in each stage) internally.

In keeping with the conceptual model of cryocoolers presented above (Sect. 1.1.2) the internal generation of entropy may occur in the elementary cooling mechanism (ECM) and/or in the interchanger (INT),

$$\dot{S} = \dot{S}_{ECM} + \dot{S}_{INT} \quad (1.73)$$

For cryocoolers such as the reverse Brayton, Stirling, or Gifford-McMahon cycles, their elementary cooling mechanisms have the potential of approaching $\dot{S}_{ECM} = 0$, in the ideal case. Such is not the case for Joule-Thomson cryocoolers, since $\Delta s(h = \text{const}) > 0$. For all real cryocoolers, $\dot{S}_{INT} > 0$, as discussed in Sect. 1.2.4. However, since the values of $\dot{C}_H/\dot{C}_L$ and ΔT (between the streams) for a JT cryocooler are larger than for other cryocoolers, the values of $\dot{S}_{INT}$ and thus the total entropy $\dot{S}$ generated by the JT cryocooler is larger.

1.4.1.3 The Coefficient of Performance, COP

The Coefficient of Performance, COP, of a refrigerator, sometimes referred to³² as the First Law efficiency, η_I , is defined¹² by

$$\text{COP} = \dot{Q}_L / \dot{W} = \eta_I \quad (1.74)$$

and may be larger or smaller than unity.

According to Eqs. 1.71 and 1.72 the highest COP value is associated with a reversible Carnot cycle for which $\dot{S} = 0$; its associated reversible work, $\dot{W}_{REV}$, is also the minimum required work, $\dot{W}^{MIN}$, so that,

$$\begin{aligned} \text{COP}_{\text{CARNOT}} &= \frac{\dot{Q}_L}{\dot{W}^{MIN}} = \eta_I^{MAX} = \frac{T_L}{T_H - T_L} \\ &= \frac{1}{T_H/T_L - 1} \end{aligned} \quad (1.75)$$

This upper limit of performance becomes smaller as one cools to lower temperatures, T_L . The Carnot COP defines the possible range of COP values for cryocoolers operating between T_H and T_L . Jacobs showed¹¹⁴ that in the case where the heat load is not absorbed at a constant temperature, the actual $\text{COP}_{\text{CARNOT}}$ is smaller than the amount defined by the above expression.

For a given cooling capacity, $\dot{Q}_L$, the Carnot COP determines the minimum necessary work,

$$\dot{W}^{MIN} = \dot{Q}_L / \text{COP}_{\text{CARNOT}} \quad (1.76)$$

that is also the availability¹¹⁵ associated with absorbing $\dot{Q}_L$ at T_L . In any other real process, Eqs. 1.71 and 1.72 define the actual, and larger, amount of work as

$$\dot{W} = \frac{\dot{Q}_L}{\text{COP}_{\text{CARNOT}}} + T_H \dot{S} = \dot{W}^{MIN} + \dot{W}^{LOST} \quad (1.77)$$

and

$$\frac{1}{\text{COP}} - \frac{1}{\text{COP}_{\text{CARNOT}}} = \frac{T_H \dot{S}}{\dot{Q}_L} = \frac{\dot{W}^{LOST}}{\dot{Q}_L} \quad (1.78)$$

1.4.1.4 The Figure of Merit, FOM

The Figure of Merit, FOM, of a refrigerator, also known as the *Second Law efficiency*, η_{II} , is defined as the fraction,

$$\text{FOM} = \frac{\dot{W}^{MIN}}{\dot{W}} = \frac{\dot{W}^{MIN}}{\dot{W}^{MIN} + \dot{W}^{LOST}} \quad (1.79)$$

and

$$\begin{aligned} \text{FOM} &= \left(1 + \frac{\dot{W}^{LOST}}{\dot{Q}_L} \text{COP}_{\text{CARNOT}} \right)^{-1} = 1 - \frac{T_H \dot{S}}{\dot{W}} \\ &= 1 - \frac{\dot{W}^{LOST}}{\dot{W}} \leq 1 \end{aligned} \quad (1.80)$$

where $\dot{W}$ is the actual power. From Eqs. 1.74 and 1.75 one obtains

$$\begin{aligned} \text{FOM} &= \frac{\dot{W}^{\text{MIN}}}{\dot{W}} = \frac{\dot{Q}_L / \text{COP}_{\text{CARNOT}}}{\dot{W}} = \frac{\text{COP}}{\text{COP}_{\text{CARNOT}}} \\ &= \frac{\eta_I}{\eta_I^{\text{MAX}}} = \eta_{II} \leq 1 \end{aligned} \quad (1.81)$$

This comparison of the COP for any cryocooler with that of the reversible Carnot cycle provides an indication of the extent of irreversibility in the actual cycle. Through the use of Eq. 1.7, an alternate expression for FOM can be given in terms of I_M and DT ,

$$\text{FOM} = \text{COP} \left(\frac{T_H}{T_L} - 1 \right) = \text{COP} \left(\frac{T_H/DT}{I_M} - 1 \right)^{-1} \quad (1.82)$$

Corollaries:

- Due to the larger amounts of generated entropy $\dot{S}$, the COP and FOM of JT cryocoolers are smaller than those of other cryocoolers with similar cooling capacity, as determined by Eqs. 1.78 and 1.80.
- For a group of cryocoolers sharing the same COP, the one that reaches the lowest cold end temperature, T_L due to the largest value of I_M , has the highest FOM, generates the least irreversibility, and is most efficient in view of the Second Law. (See Sect. 3.13 with particular relevance to Joule-Thomson cryocoolers).

Long term surveys over a large variety and large quantity of cryocoolers^{116,117,118,119} show that cryocoolers with larger cooling capacity are more efficient and have larger values of FOM. In addition, cryocoolers generally achieve lower values of COP and FOM as the cryocooling temperature, T_L , becomes lower. One may accurately interpret this as the result of a larger “effort” required to accomplish “deeper” cooling.

1.4.2 The “Real Gas Properties” Group of Cryocoolers

1.4.2.1 Description

These cryocoolers are driven solely by the “real gas properties”, of the coolant as represented in Fig. 1.15. Here we exclude coolers that utilize the extraction of (external) useful work as a cooling mechanism. The model for these cryocoolers includes a high-pressure inlet stream and a low-pressure outlet stream. An ideal case is assumed so that, $T_{IN} = T_{OUT} = T_H$. By continuity, both streams share the same flow rate, $\dot{n}$. Joule-Thomson cryocoolers operating with pure coolants (Chap. 3) or with mixed coolants (Chap. 8) belong to this group.

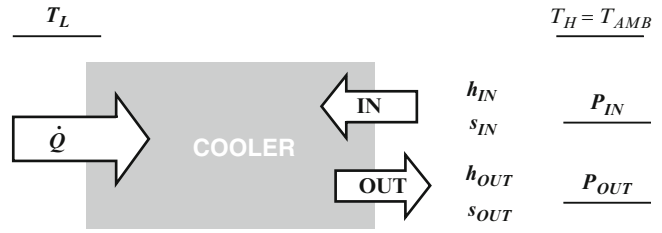


Fig. 1.15 The general model of “real gas properties” cryocoolers having inlet and outlet streams at the same temperature, T_H , that is also the ambient temperature T_{AMB} , and the same flow rate but at different pressures

1.4.2.2 The First Law

The First Law applied to the cooler in Fig. 1.15 as a system, determines the cryocooling capacity solely by the parameters at the warm end,

$$\dot{Q}_L = \dot{n} (h_{OUT} - h_{IN}) \equiv \dot{n} \Delta h_T \quad (1.83)$$

Since the enthalpy of an ideal gas is independent of pressure, it would not generate any cooling effect. The expression “real gas properties” cryocoolers highlights the fact that the magnitude of the cooling effect is a function of the coolant’s deviation from ideal gas behavior.

1.4.2.3 The Second Law

The Second Law determines the lowest attainable temperature through the balance of entropy flow,

$$\frac{\dot{Q}_L}{T_L} + \dot{S} + \dot{n} s_{IN} - \dot{n} s_{OUT} = 0 \quad (1.84)$$

that, combined with Eq. 1.83 gives,

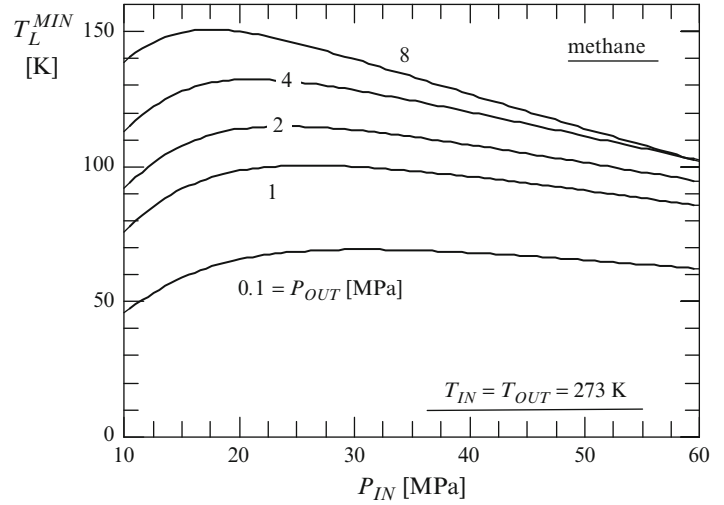
$$T_L = \frac{\Delta h_T}{\Delta s_T - \dot{S}/\dot{n}} \quad (1.85)$$

The lowest attainable temperature is obtained when $\dot{S} = 0$ and therefore it has to be higher than,

$$T_L \geq \Delta h_T / \Delta s_T \equiv T_L^{\text{MIN}} \quad (1.86)$$

Figure 1.16 displays the limiting values, T_L^{MIN} , for methane at the ambient temperature of 273 K as function of P_{IN} (from 10 to 60 MPa) for various values of P_{OUT} (0.1, 1, 2, 4, 8 MPa). For instance, at 20 and 8 MPa inlet and outlet pressures of methane the lowest attainable temperature is 150 K. For P_{OUT} of 0.1 MPa the Linde-Hampson cryocooler reaches the liquefaction temperature of methane and the temperature will be clamped at the value of 111.7 K.

Fig. 1.16 The lowest attainable temperature by a “real gas properties” cryocooler, as function of the inlet and outlet pressures, employing methane as a coolant



However, the analysis above simply considering the Second Law allows the temperature to go even lower, down to about 70 K.

1.4.2.4 The COP and FOM

The minimum work required in order to transform the exhaust gas from P_{OUT} to P_{IN} at a constant temperature T_H , would be,

$$\begin{aligned} \dot{W}/\dot{n} &= T_H(s_{OUT} - s_{IN}) - (h_{OUT} - h_{IN}) \\ &= T_H \Delta s_T - \Delta h_T \end{aligned} \quad (1.87)$$

as later shown in Eq. 3.167. Thus by combining Eqs. 1.74 and 1.86 one obtains,

$$\begin{aligned} \text{COP} &= \frac{\dot{Q}_L}{\dot{W}} = \frac{\Delta h_T}{T_H \Delta s_T - \Delta h_T} = \left(T_H \frac{\Delta s_T}{\Delta h_T} - 1 \right)^{-1} \\ &= \left(\frac{T_H}{T_L^{MIN}} - 1 \right)^{-1} \end{aligned} \quad (1.88)$$

In practice, the COP is smaller since the input work is larger than the minimum value associated with isothermal compression. For the same inlet and outlet conditions at the warm end, this group of cryocoolers share a common value of COP. However, each of the cryocoolers still has a degree of freedom with respect to the low temperature T_L it achieves. Hence, because of the different values of T_L that are possible, the COP does not provide a distinguishing identifier among all these cryocoolers. Notice however, that for the same group of cryocoolers, Eqs. 1.75 and 1.88 give,

$$\text{FOM} = \frac{(T_H/T_L) - 1}{(T_H/T_L^{MIN}) - 1} = \frac{1 - (T_L/T_H)}{(T_H/T_L^{MIN}) - (T_L/T_H)} \quad (1.89)$$

Thus, the FOM does depend on the achieved value of T_L . In this regard, T_L^{MIN} defines the limiting value of T_L : as T_L approaches T_L^{MIN} or $T_L/T_L^{MIN} \rightarrow 1$, the FOM approaches unity.

1.4.3 The Ideal Gas Group of Cryocoolers

1.4.3.1 Description

Figure 1.15 also depicts the operation of ideal gas cryocoolers, but in contrast to the previous group, these type of cryocoolers employ ideal isentropic expanders ($\eta_{EX} = 1$) and work is extracted across the system boundary of the cryocoolers. At the relevant conditions of operation, the working fluid obeys the ideal gas model. The temperature at the inlet to the isentropic expander is very close to the temperature T_L of the cooled object. Stirling coolers typify this group of cryocoolers.

1.4.3.2 The COP and FOM

Since the enthalpy of an ideal gas depends solely on temperature, when $T_{IN} = T_{OUT} = T_H$,

$$h_{IN} = h_{OUT} \neq f(P_H) \quad (1.90)$$

Therefore, by the First Law, the cooling power equals the extracted work,

$$\dot{Q}_L = \dot{W}_{OUT} \quad (1.91)$$

Using the ideal gas expressions associated with isothermal work for the low temperature,

$$\dot{W}_{OUT} = \dot{n} R T_L \ln (P_H/P_L) \quad (1.92)$$

and high temperature processes in the cycle,

$$\dot{W}_{IN} = \dot{n} R T_H \ln (P_H/P_L) \quad (1.93)$$

The expression for the net work of the entire cycle results as,

$$\begin{aligned} \dot{W} &= \dot{W}_{NET} = \dot{W}_{IN} - \dot{W}_{OUT} \\ &= \dot{n} R (T_H - T_L) \ln (P_H/P_L) \end{aligned} \quad (1.94)$$

Combining Eqs. 1.91, 1.92, and 1.94 gives the COP,

$$\text{COP} = \frac{\dot{Q}_L}{\dot{W}} = \frac{T_L}{T_H - T_L} = \text{COP}_{\text{CARNOT}} \quad (1.95)$$

Thus, theoretically, cryocoolers of this group have the potential of approaching the ideal Carnot efficiency, with FOM = 1.

1.4.3.3 The Lowest Attainable Temperature Defined by the Second Law

For the ideal case when $T_{IN} = T_{OUT} = T_H$, the lowest attainable temperature is obtained from Eq. 1.84 with $\dot{S} = 0$,

$$\frac{\dot{Q}_L/\dot{n}}{T_L} \leq R \ln \left(\frac{P_H}{P_L} \right) \quad \text{or} \quad T_L \geq \frac{\dot{Q}_L/\dot{n}}{R \ln(P_H/P_L)} \equiv T_L^{\text{MIN}} \quad (1.96)$$

1.4.3.4 The Finite Lowest Attainable Temperature

It is still possible to define a lowest temperature associated with the condition $\dot{Q} = 0$, but only for a non ideal cryocooler that has a finite temperature difference, ΔT , at the warm end,

$$\Delta T = T_{IN} - T_{OUT} \quad (1.97)$$

Assuming an ideal gas interchanger with balanced capacity rates, Eqs. 1.9 and 1.43 give,

$$\begin{aligned} \varepsilon &= \frac{T_1 - T_2}{T_1 - T_4} = 1 - \frac{\Delta T}{T_1 - T_2 + \Delta T} \quad \text{or} \\ \Delta T &= (T_1 - T_2) \frac{1 - \varepsilon}{\varepsilon} = \frac{T_1 - T_2}{\text{NTU}} \end{aligned} \quad (1.98)$$

Then the energy balance for the entire cryocooler can be expressed as,

$$\begin{aligned} \dot{Q} &= \dot{W}_{OUT} - \dot{n} (h_{IN} - h_{OUT}) = \dot{W}_{OUT} - \dot{n} \Delta T c_P \\ &= \dot{n} \eta_{EX} R T_2 \ln (P_H/P_L) - \dot{n} \frac{T_1 - T_2}{\text{NTU}} c_P \end{aligned} \quad (1.99)$$

Solving for T_2 when $\dot{Q} = 0$ one obtains the lowest possible value of T_2 ,

$$\begin{aligned} T_2 &= \frac{T_1}{1 + \text{NTU} \eta_{EX} \left(1 - \frac{1}{\kappa}\right) \ln (P_2/P_3)} \\ &= T_L^{\text{MIN}} (\dot{Q} = 0) \end{aligned} \quad (1.100)$$

Such an approach was reported elsewhere.¹²⁰ One should compare this expression with the exact prediction previously derived in Eq. 1.64. The two are quite close for moderate compression ratios (but still practical relevance) such as $P_2/P_3 < 2$.

1.5 A Comparison of Joule-Thomson with Other Coolers

1.5.1 Introduction

The variety of cryocoolers is broad indeed, and their evolution is reviewed from time to time in periodic literature,^{14,15,17,117,121,122,123,124,125,126,127,128,129} reports,^{116,130} in handbooks,^{131,132} in connection with superconductive devices^{10,133,134,135,136,137} or for infrared systems.^{138,139,140,141,142} Space applications¹⁴³ and up to date advances of research and development are reported in the series of *Cryocoolers*,¹⁴⁴ and in the bibliography.^{6,145,146,147} One of the earliest reviews, appearing in 1967 and titled, “Miniature refrigeration systems – a review”,¹⁴⁸ belongs to Dr. P.H. Parkinson of the Royal Radar Establishment. Perhaps it is more than just a coincidence that he was also a design pioneer of miniature Joule-Thomson cryocoolers. Hansen¹⁴⁹ provides a comprehensive review of Joule-Thomson cooling technology.

The next section explores the distinctive features of Joule Thomson cryocoolers and the associated niche applications where they provide advantageous features, or at least can be recommended as a competing technology. The discussion also presents an approximate boundary for the unique features of Joule-Thomson coolers placing their preferred regime within the range provided by the whole family of cryocoolers.

1.5.2 Characteristics

1.5.2.1 Cryocooling Via a Boiling Bath of Cryogen

Cryocooling via a boiling bath of cryogen is utilized only in the Joule-Thomson cryocoolers as the method of extracting heat from the cooled object. Unlike other cryocoolers, Joule-Thomson devices liquefy the coolant and primarily use the latent heat rather than the sensible heat of the coolant. This cooling mechanism enables:

High Heat Flux

Heat can be removed from the cooled object within a small space. Thus, the diameter of the “cold finger” per a given cooling power can be reduced, in comparison to other cryocoolers that do not liquefy the coolant.

High Temperature Stability

The temperature of cryocooling, that is set by the boiling point of the coolant, has a natural tendency to remain stable, resulting in a low thermal noise level.¹⁵⁰ The temperature remains the same at zero heat load as at the maximum supportable heat load, as long as the ambient pressure does not change.

Cooling Large and Irregularly Shaped Objects

The interior space of the cooled object, by accommodating the two-phase boiling cryogen, becomes the evaporator of the cycle. Thus, a state of homogeneous cooling extends even to its remote corners.¹⁵¹ Alternative cryocoolers generally do not provide liquefaction (unless they include a Joule-Thomson expansion in their final stage). Rather, they typically include a metal cylindrical cold finger (filled with cold gas), and cooling has to be distributed by thermal conduction. Such a cooling process generates a higher temperature gradient between the cooled object and the coolant at steady state.

1.5.2.2 Compact and Light Weight “Cold Finger”

Due to the relatively large δT of the JT effect, that also grows as the temperature decreases, the JT cryocooler accommodates a smaller value of I_M for the interchanger, thereby allowing a lower effectiveness, lower NTU, and smaller dimensions.

1.5.2.3 The Open Cycle Mode of Operation

Except for dewars holding an amount of liquid cryogen, the open cycle mode of operation is only viable with Joule-Thomson cryocoolers. Furthermore, the cooling is driven by a pressurized gas reservoir at room temperature, and may be supplied for up to a few hours, all the while provided by a device whose size remains practical and tolerable.

No Moving Parts in the Entire Cooling System

The compressor is left behind at the vessel charging facility.

Reliable Operation After a Long Storage Period

A pressure vessel may remain sealed for a long time on the shelf.

No Heat Rejection at Ambient Temperature

According to the Second Law of Thermodynamics, any closed cycle cooling system absorbing heat from the cooled object must reject heat to a second reservoir, typically the ambient surroundings. An open cycle operation is not

limited by such a restriction, thereby eliminating the necessity for exchanging heat with the environment.

A System That Becomes More Compact as the Cooling Duration Shortens

The size of the pressure vessel for a JT cooler is a direct function of the required cooling duration. Consequently, for very short cooling demands, the open cycle system sharply shrinks into a very compact configuration, significantly smaller than other cryocoolers. This feature may be best appreciated by comparing it to the situation with other closed cycle cryocoolers. These are electrically powered and generally include a compression unit that remains the same size for long or short term operation. In fact, the fixed compressor size might be advantageous for longer runs. However, note that autonomous systems will require an electrical battery whose size will correspond with the duration of operation.

1.5.2.4 Rapid Cooldown: The Ultimate Advantage

This unique feature is unachievable by other cryocoolers. For example, JT coolers may produce liquid nitrogen temperature within a few seconds. The temperature of liquid krypton (of about 120 K) is reached immediately, at the moment of blow down. For comparison, the cooldown process with other cryocoolers will require at least a few minutes. Thus, the combination of long-term stand by and “immediate readiness” may be satisfied only with the aid of Joule-Thomson cryocooling.

It should be clarified that an open system based on a rechargeable liquid nitrogen dewar is excluded from this comparison because of its short shelf life due to boil-off and the typically bulky construction of a dewar.

If one considers the cooldown process as a quasi-static process, then the instantaneous cooling power as function of the instantaneously temperature for Joule-Thomson cryocoolers increases as the cooldown proceeds. In contrast, the instantaneous cooling power of closed cycle cryocoolers based on an isentropic expansion (such as Stirling or pulse tube cryocoolers), decreases as the temperature is reduced.¹⁵² As a result the cooldown period is prolonged relative to a JT cooler starting with a similar cooling power.

1.5.2.5 A Very Low Level of Vibrations at the Cold End

The cold finger of a Joule-Thomson cryocooler lacks any moving or vibrating elements. Furthermore, in the case of a closed cycle operation, the compressor may be mounted significantly far away from the cold finger, so that its vibrations are mechanically decoupled from the cold end. For an open cycle system, the compressor is eliminated completely.

1.5.2.6 Cryocooling of a Gimbaled Payload

Since the cold finger is driven by flow through a pressurized tube that can be both flexible and long, the Joule-Thomson cryocooler becomes a favorite candidate for mounting on delicate mechanisms such as gimbals.^{151,153,154} An alternative strategy based on a precooled single-phase gaseous loop and ultra flexible tubing is described elsewhere.^{155,156}

1.5.2.7 Ease of Integration

Joule-Thomson cryocoolers provide an advantage of flexible integration at the system level due to the possibility of remotely locating the pressure source, whether it is a compressor or a pressurized vessel.

1.5.2.8 Ease of Distributing the Cooling Power

Joule-Thomson cryocoolers enable a single and remote compression unit to feed a multiplicity of distant cold fingers as discussed elsewhere for a set of spaceborne sensors.^{157,158} Kittel¹⁵⁹ and Salerno¹⁶⁰ describe the requirements of a distributed cooling system for both ground- and space-based cryogenic propellant depot. Kittel comments:

Regenerative coolers produce cooling in a small volume. Recuperative coolers produce a flow of cold gas. This flow can be piped considerable distances with only a small effect on performance.

1.5.2.9 Closed Cycle Joule-Thomson Cryocoolers

Open cycle Joule-Thomson cryocoolers typically employ pure coolants, such as nitrogen, argon or air, at elevated pressures in the 20–70 MPa range. This configuration enables a very rapid cooldown, typically less than one minute. Due to the initial high density, a large amount of gas can be stored in a given vessel, thereby providing a longer run time. Other cryocoolers typically operate in a lower pressure range, providing a practical advantage for containment and compression. Mixed gases have been extensively studied as new candidate coolants for closed cycle Joule-Thomson cryocoolers, and these enable significantly reduced operating pressures in the 1.2–4 MPa range. Therefore, it is possible to build closed cycle Joule-Thomson cryocoolers on the basis of a much simpler and more reliable compressor, similar to those used with the other type of cryocoolers. The topic of mixed gas Joule-Thomson cryocoolers is broadly discussed in Chap. 8. Section 8.9.1 focuses on a detailed comparison with the Stirling type closed cycle cryocoolers.

1.5.3 Drawbacks

1.5.3.1 Requirement for High Purity Gas

One of the primary difficulties associated with Joule-Thomson cryocoolers is clogging. The topic is discussed

thoroughly in Chap. 9. In short, contamination of some gases at a level even as low as ten parts per million can result in solid formation that will clog the delicate expansion valve of a Joule-Thomson cryocooler. Such a problem can threaten both the immediate and long-term operation of the cryocooler. Providing and maintaining such a level of cleanliness at the system level requires special attention.

1.5.3.2 Inferior Thermodynamic Efficiency

In comparison with other types of cryocoolers, Joule-Thomson cryocoolers would not be recommended for applications where the thermodynamic coefficient of performance or the cost of energy plays a central factor. The efficiency of Joule-Thomson coolers is significantly improved by employing mixed gas coolants.

1.5.3.3 A Requirement of High Pressure for Open Cycle Operation

High-pressure systems in general present more complicated handling challenges than low pressure systems, especially with regard to safety and leakage.

1.5.3.4 A Higher Compression Ratio

As a result of the higher pressures associated with pure-gas closed cycle Joule-Thomson cryocoolers the compression unit must provide a high compression ratio. Cryocoolers that utilize an isentropic expansion for the elementary cooling mechanism, such as the reverse Brayton, Stirling, Gifford-McMahon, or Pulse Tube coolers, tolerate compression ratios below 2, while the Joule-Thomson system typically requires compression ratios in the range of 5–10. The absolute high pressure in both cases is comparable.

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Part II

Theoretical Aspects

2.1 Introduction

James Joule and **William Thomson (Lord Kelvin)** discovered the effect bearing their names and measured it at low pressures for air, nitrogen, oxygen, carbon dioxide, hydrogen and mixtures of air and carbon dioxide. Since “...the thermal effects experienced by air in rushing through small apertures” were discovered,¹ about 16 decades ago, the effect has been continuously examined, measured and studied. This elementary effect is the driving force that initiates, empowers and perpetuates the operation of the Linde-Hampson machines, the Joule-Thomson cryocoolers, liquefiers and refrigerators that utilize the throttling effect.

The early experiments of Joule and Thomson stimulated the modeling approach for this process as one of constant enthalpy in order to contrast real gas properties with the ideal gas equation of state. In addition, their focus on the “irregular” warming of hydrogen as it expands led to the comprehension of the inversion phenomena, realizing that each gas undergoes this kind of “irregularity” at its own particular conditions.

On the macroscopic level, the Joule-Thomson effect is a measure of deviation from the *ideal gas* model:

$$Pv = RT \quad \text{or} \quad \frac{Pv}{RT} \equiv Z = 1 \quad (2.1)$$

attributable to Robert Boyle (1627–1691), Jacques Charles (1746–1823) and Joseph Gay-Lussac (1778–1850) and relating the pressure, specific volume and temperature. Here R is a universal constant for all gases. Such a gas is incapable of undergoing any (non vanishing) Joule-Thomson effect. The discovery of the Joule-Thomson effect was not consistent with this equation of state. It had a huge impact, promoting numerous studies to establish a universal equation of state that would enhance or replace the ideal gas equation. Any new equation of state had to provide a better prediction of the Joule Thomson effect as an expression of the *real gas* properties.

The very early proposals were of Rankine in 1854 and of Joule and Thomson themselves, in 1862,

$$Pv = RT + \text{Const} \frac{P}{T^2} \quad (2.2)$$

Various more advanced and subsequent equations of state are discussed in Appendix A2. One may find a comprehensive survey describing the chronology of equations of state development elsewhere.²

On the microscopic level, the Joule-Thomson effect manifests the existence of intermolecular repulsion and attraction forces for real (non-ideal) fluids as explained by Hirschfelder, Ewell and Roebuck,³ in 1938:

The imperfectness of a gas is due to the attraction and repulsion of the molecules. Both the Joule-Thomson and the virial coefficients are direct measures of these forces and both become zero for a perfect gas.

Thermodynamic states that are dominated by intermolecular attraction or repulsion will experience a respective temperature decrease or increase, after the Joule-Thomson expansion. Under the assumption of no intermolecular forces, also referred to as the van der Waals forces, the kinetic theory of gases leads to the ideal gas equation of state. Van der Waals equation of state is presented in Appendix A2.1.

In this context, the following point is perhaps appropriate. Refrigeration cycles that utilize the Joule-Thomson effect require compression of the working gas. The necessary input energy is frequently regarded as an investment for running the cooler. Note that refrigeration is only possible in such cycles to the extent that the refrigerants deviate from the ideal gas model and not solely by the extent of invested energy. For example, it is possible by compressing to a very high pressure and thereby investing a large amount of energy, that the Joule-Thomson effect may vanish or even become negative, resulting in no cooling at all. Thus, any survey of the Joule-Thomson effect remains incomplete

without a thorough discussion of the associated inversion phenomenon.

Many alternative cryocooling methods are not based on the deviation from the ideal gas model, often referred to as real gas behavior. The Stirling cryocooler, and the associated pulse tube and Gifford-McMahon cycles, can be formulated and predicted in terms of ideal gas features with reasonable accuracy. Using real gas properties would enable refinement and additional accuracy, but would not dictate the feasibility of these cycles.

2.2 The Joule-Thomson Coefficient

2.2.1 The Differential Coefficients

2.2.1.1 The Single Phase Domain

The *adiabatic Joule-Thomson coefficient* is a differential measure of the Joule-Thomson effect: the change of temperature under isenthalpic ($h = \text{Const}$) expansion per small pressure drop, hence defined by:

$$\mu = \left(\frac{\partial T}{\partial P} \right)_h = \mu(P, T) \quad (2.3)$$

For an ideal gas, the enthalpy is solely a function of temperature and independent of pressure. Hence, at constant enthalpy, a change of pressure cannot produce a change in temperature, therefore $\mu = 0$.

For a real gas, the dependence of enthalpy on pressure is another reflection of the Joule-Thomson effect. Thus, the *isothermal Joule-Thomson coefficient* used as an alternative measure, expresses the curvature of isenthalps in the (T, P) plane,

$$\phi = \left(\frac{\partial h}{\partial P} \right)_T = \left(\frac{\partial u}{\partial P} \right)_T + \left[\frac{\partial}{\partial P} (Pv) \right]_T \quad (2.4)$$

And

$$\begin{aligned} \left(\frac{\partial \phi}{\partial T} \right)_P &= \left[\frac{\partial}{\partial T} \left(\frac{\partial h}{\partial P} \right)_T \right]_P = \left[\frac{\partial}{\partial P} \left(\frac{\partial h}{\partial T} \right)_P \right]_T \\ &= \left[\frac{\partial}{\partial P} c_P \right]_T \end{aligned} \quad (2.5)$$

by the definition of specific isobaric heat capacity:

$$(\partial h / \partial T)_P = c_P \quad (2.6)$$

The two coefficients are related by:

$$\begin{aligned} \phi &\equiv \left(\frac{\partial h}{\partial P} \right)_T \equiv - \left(\frac{\partial h}{\partial T} \right)_P \left(\frac{\partial T}{\partial P} \right)_h = -c_P \mu \\ &= \phi(P, T) \end{aligned} \quad (2.7)$$

The units of ϕ are like those of the molar specific volume v , m^3/mol , while μ is measured in K/Pa .

The sign of μ (opposite to ϕ) may be positive or negative reflecting differential cooling or heating. The transition from cooling to heating occurs at the *inversion states*, which deserve a separate discussion (Sect. 2.4).

2.2.1.2 The Two Phase Domain of a Pure Substance

The two phase domain of a pure substance is characterized by the fact that the isotherms are also the isobars. Therefore, μ becomes dT/dP which is the slope of the saturation line in the (T, P) plane. The specific heat capacity in this domain approaches infinity and so does ϕ

2.2.2 The Integral Effect

Sometimes the *integral adiabatic Joule-Thomson effect*, measured in units of absolute temperature (Kelvin), is utilized for an expansion from P down to atmospheric pressure by integrating the differential effects:

$$\left[\int_P^{0.1 \text{ MPa}} \mu \, dP \right]_s \equiv \Delta T_h(P, T) \quad (2.8)$$

Similarly, the *integral isothermal Joule-Thomson effect*:

$$\left[\int_{0.1 \text{ MPa}}^P \phi \cdot dP \right]_T \equiv \Delta h_T(P, T) \quad (2.9)$$

is useful and has units of specific enthalpy, J/mol .

Both integrals of the Joule-Thomson coefficients, Δh_T and ΔT_h are directly and highly relevant to practical operation of Joule-Thomson cryocoolers. Their relationship is discussed in detail in Sect. 4.5, especially in Sect. 4.5.2.1.

2.2.3 Derivatives Through the Equations of State

The Joule-Thomson coefficients may be derived through any given equation of state, $f(T, P, v) = 0$.

2.2.3.1 In Terms of Volume Derivatives

$$\mu = -\frac{\phi}{c_P} = \frac{1}{c_P} \left[T \left(\frac{\partial v}{\partial T} \right)_P - v \right] = \frac{T^2}{c_P} \left[\frac{\partial (v/T)}{\partial T} \right]_P \quad (2.10)$$

$$\left(\frac{\partial \phi}{\partial T}\right)_p = -T \left(\frac{\partial^2 v}{\partial T^2}\right)_p \quad (2.11)$$

2.2.3.2 In Terms of Pressure Derivatives

$$\mu = \frac{1}{\rho c_P} \left[\frac{T \left(\frac{\partial P}{\partial T}\right)_\rho}{\rho \left(\frac{\partial P}{\partial \rho}\right)_T} - 1 \right] = \frac{1}{\rho c_P} \left[\frac{\left(\frac{\partial P}{\partial \ln T}\right)_\rho}{\left(\frac{\partial P}{\partial \ln \rho}\right)_T} - 1 \right] \quad (2.12)$$

$$\mu = \frac{T \left(\frac{\partial P}{\partial T}\right)_\rho - \rho \left(\frac{\partial P}{\partial \rho}\right)_T}{c_V \rho^2 \left(\frac{\partial P}{\partial \rho}\right)_T + T \left[\left(\frac{\partial P}{\partial T}\right)_\rho\right]^2} \quad (2.13)$$

2.2.3.3 In Terms of Compressibility, Z

In terms of compressibility, Z, in the (P, T) plane,

$$\mu = \frac{R T^2}{P c_P} \left(\frac{\partial Z}{\partial T}\right)_P = \frac{v}{c_P} \left(\frac{\partial \ln Z}{\partial \ln T}\right)_P \quad (2.14)$$

or in the (ρ, T) plane:

$$\mu = \frac{1}{\rho \cdot c_P} \left[\frac{T \left(\frac{\partial Z}{\partial T}\right)_\rho - \rho \left(\frac{\partial Z}{\partial \rho}\right)_T}{Z + \rho \left(\frac{\partial Z}{\partial \rho}\right)_T} - 1 \right] \quad (2.15)$$

2.2.3.4 In Terms of the Product Pv

In Terms of the Product Pv,

$$\mu = \frac{1}{\rho c_P} \left\{ T \left[\frac{\partial}{\partial T} (Pv) \right]_P - Pv \right\} \quad (2.16)$$

2.2.3.5 In Terms of the Residual Volume

$$v^R = v - v^G = v - \frac{RT}{P} \quad (2.17)$$

we obtain:

$$\mu = \frac{1}{c_P} \left[T \left(\frac{\partial v^R}{\partial T}\right)_P - v^R \right] \quad (2.18)$$

2.2.3.6 In Terms of Heat Capacities

$$\mu = \frac{1}{c_P} \left[\frac{c_P - c_V}{\left(\frac{\partial P}{\partial T}\right)_\rho} - \frac{1}{\rho} \right] \quad (2.19)$$

2.2.3.7 In Terms of the Virial Coefficients

In terms of the virial coefficients of the virial equation of state expanded by density (Appendix A2.3),

$$\mu = -\frac{1}{c_P} \frac{(B - T \frac{dB}{dT}) + \rho(2C - T \frac{dC}{dT}) + \rho^2(3D - T \frac{dD}{dT}) + \dots}{1 + 2B\rho + 3C\rho^2 + 4D\rho^3 + \dots} \quad (2.20)$$

For virial expansion by pressure (Appendix A2.3),

$$\mu = \frac{RT^2}{c_P} \cdot \left[\frac{dB'}{dT} + P \frac{dC'}{dT} + P^2 \frac{dD'}{dT} + \dots \right] \quad (2.21)$$

2.2.3.8 In Terms of the Intermolecular Forces

On the microscopic level, states that are dominated by attractive intermolecular forces reduce their temperature after the Joule-Thomson expansion thus, $\mu > 0$. Extending the distance between the molecules and overcoming the attractive forces consumes (“internal work”) part of their kinetic (thermal) energy, which means, lowering the temperature. Oppositely, states of closer distance between molecules experience repulsive forces. The separation of molecules due to the Joule-Thomson expansion in this case, is accompanied by a gain of kinetic energy, which means, an increase of temperature and $\mu < 0$.

The second virial coefficient relates with the potential energy of interaction $E(r)$ of two molecules separated by a distance r ,

$$B = 2\pi N \int_0^\infty r^2 \left\{ 1 - \exp\left[-\frac{E(r)}{kT}\right] \right\} dr \quad (2.22)$$

where N is Avogadro’s number and k is Boltzman’s constant. The quantity $E(r)$ is determined empirically; its gradient corresponds with forces of repulsion at small values of r and attraction at intermediate values of r . The other virial coefficients may be also derived quantitatively through any proposed function $E(r)$.

2.2.3.9 The van der Waals Gas

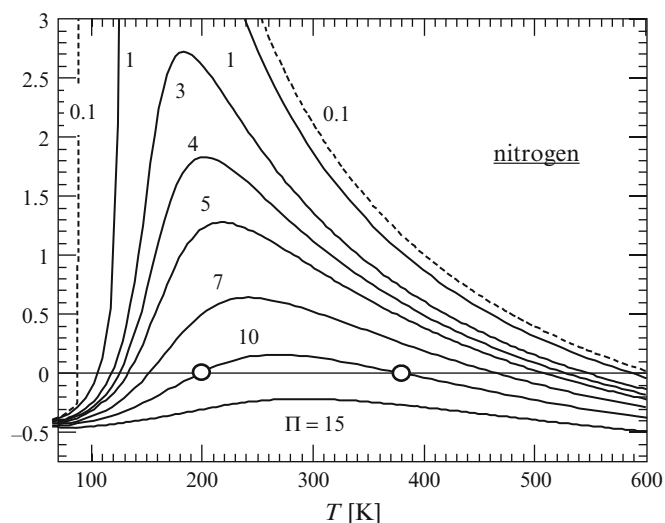
The van der Waals gas (Appendix A2.1),

$$\mu = -\frac{b}{c_P} \cdot \frac{\frac{27/4}{T/T_C} \cdot \left(1 - \frac{b}{v}\right)^2 - 1}{\frac{27/4}{T/T_C} \cdot \frac{b}{v} \cdot \left(1 - \frac{b}{v}\right)^2 - 1} \quad b = \frac{v_C}{3} \quad (2.23)$$

Complementary references^{4,5,6} are recommended for determining μ through various equations of state.

Fig. 2.1 The adiabatic Joule-Thomson coefficient of nitrogen as function of temperature along some lines of constant reduced pressure; the highest values belong to lower pressures

$$\left[\frac{\mu}{\text{K}} \right] \left[\frac{\text{K}}{\text{MPa}} \right]$$



2.2.3.10 The Principle of Corresponding States

The equation of state (formulated in terms of reduced coordinates) is the ultimate manifestation of the principle of corresponding states. The equation of state enables one to determine ϕ , which is $-\mu \cdot c_p$. However, equations of state are not capable of specifying c_p themselves. Therefore, the equation of state alone, without additional information, is not sufficient for obtaining μ . Francis and Luckhurst⁷ state that the principle of corresponding states is indeed applicable for organizing data for values of ϕ but not for values of μ .

2.2.4 The Zero Pressure Joule-Thomson Coefficient

The existence of a non-vanishing zero pressure coefficient, μ_0 , attracted attention through experiments and theoretical studies. As the pressure and densities approach zero, the intermolecular forces become less significant, and the ideal gas model and its associated equation of state should have higher predictive power. Consequently, one might expect that at low pressure, the Joule-Thomson coefficient approaches zero. Nonetheless, the value of μ_0 , even when extrapolated completely to zero pressure, exhibits a non-zero value. Furthermore, μ increases with decreasing pressure, as is demonstrated in Fig. 2.1, where μ is displayed as a function of temperature for nitrogen in the form of constant reduced pressure curves, up to a value of $\Pi = 15$. The highest values of μ occur at zero pressure. Alternatively, Fig. 2.2 displays μ as a function of pressure for reduced temperatures from $\Theta = 1$ to 2. As a supplement, Fig. 2.3 displays reduced temperatures from $\Theta = 2$ to 5. The separation of the two figures is for clarity as the constant temperature curves do intersect. One may observe that μ reaches finite values, μ_0 , as P approaches zero.

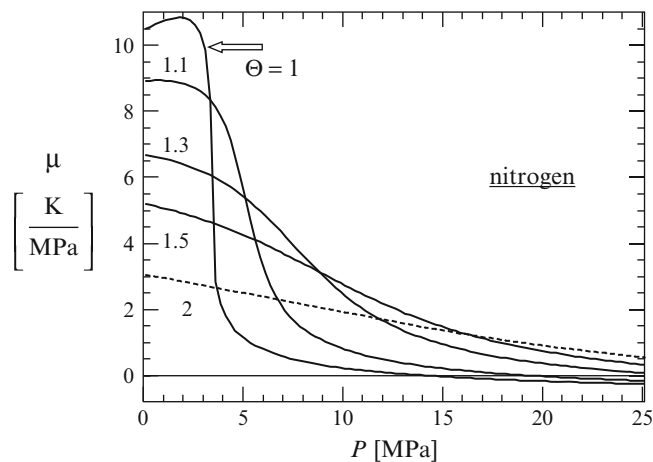


Fig. 2.2 The adiabatic Joule-Thomson coefficient of nitrogen as function of pressure along some lines of constant reduced temperature between 1 and 2

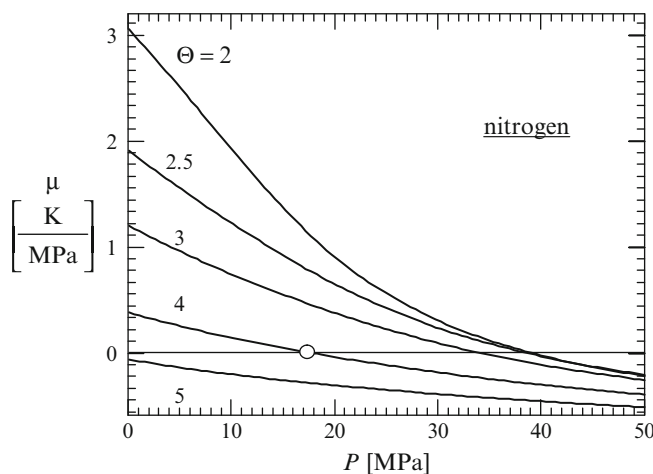


Fig. 2.3 The adiabatic Joule-Thomson coefficient of nitrogen as function of pressure along some lines of constant reduced temperature between 2 and 5

The simple real gas equation of state, that of van der Waals, predicts a non-zero value of μ , that is,

$$\mu_0 = \frac{1}{c_{P0}} \left(\frac{2a}{RT} - b \right) \quad (2.24)$$

Similarly, the isothermal Joule-Thomson coefficient at zero pressure, in terms of v , is given by:

$$\frac{\phi_0}{v_C} = -\frac{\mu_0 c_{P0}}{v_C} = \frac{\frac{v}{v_C} \left(1 - 2 \frac{v}{v_C} \right)}{\frac{2}{3} - \frac{v}{v_C}} \quad (2.25)$$

and more explicitly in a temperature dependent non-dimensional form:

$$\frac{\mu_0 c_{P0}}{v_C} = \frac{1}{3} \left(\frac{27/4}{T/T_C} - 1 \right) \quad (2.26)$$

In terms of the second virial coefficient of the equation of state expanded by density,

$$\mu_0 = \frac{T^2}{c_{P0}} \frac{d}{dT} \left(\frac{B}{T} \right) \quad (2.27)$$

and

$$\phi_0 = B - T \cdot \frac{dB}{dT} \quad (2.28)$$

As discussed later, there is only a single temperature where μ_0 vanishes.

On the microscopic level,

$$\mu_0 = \frac{2\pi N}{c_{P0}} \int_0^\infty r^2 \left\{ 1 - \left[1 - \frac{E(r)}{kT} \right] \exp \left[-\frac{E(r)}{kT} \right] \right\} dr \quad (2.29)$$

Hirschfelder, Ewell and Roebuck³ used this relation to calibrate $E(r)$ in terms of measured values of μ_0 . This approach is discussed by McLure.⁸ Vortmeyer⁹ interpreted μ_0 measurements of gas mixtures.¹⁰ Gustafson employed the intermolecular potential of Lennard-Jones^{11,12} to determine μ_0 and ϕ_0 for noble gases and mixtures. Nain and Aziz¹³ compared various intermolecular potentials for predicting μ_0 and the inversion temperatures of noble gases.

Since the principle of corresponding states is applicable⁷ for ϕ but not for μ , a generalized correlation was proposed for ϕ_0 ,

of pure gases and mixtures, based on the intermolecular potential and supported by experimental evidence, such as,¹¹

$$\frac{\phi_0}{v_C} = 0.290 - \frac{1.133}{T/T_C} - \frac{2.784}{(T/T_C)^2} \quad (2.30)$$

2.2.5 Speed of Sound and the Joule-Thomson Coefficient

Sheludyakov¹⁴ related the Joule-Thomson coefficient with the speed of sound, c , which is given by,¹⁵

$$c^2 = \left(\frac{\partial P}{\partial \rho} \right)_s = -v^2 \cdot \frac{c_P}{c_V} \cdot \left(\frac{\partial P}{\partial v} \right)_T \quad (2.31)$$

Applying:

$$c_P - c_V = -T \frac{\left[\left(\frac{\partial P}{\partial T} \right)_v \right]^2}{\left(\frac{\partial P}{\partial v} \right)_T} \quad (2.32)$$

one obtains:

$$\mu = \frac{1}{c \cdot \rho \cdot c_P} \left[T \cdot \frac{c_P}{c_V} (c_P - c_V) \right]^{1/2} - \frac{1}{\rho \cdot c_P} \quad (2.33)$$

or

$$\phi \rho = \frac{1}{c} \left[T \frac{c_P}{c_V} (c_P - c_V) \right]^{1/2} - 1 \quad (2.34)$$

For an ideal gas:

$$c_P - c_V = R \quad (2.35)$$

$$\frac{c_P}{c_V} = \kappa \quad (2.36)$$

$$c^{IG} = (\kappa R T)^{1/2} \quad (2.37)$$

thus μ becomes identically zero.

2.2.6 The Joule-Thomson Coefficient of Mixtures

Let us examine a mixture at (T, P) with enthalpy h^{MLX} . Under the same conditions, let the enthalpies of each component, marked by i , be h^i . Along the enthalpy coordinate,

the difference between the mixture and its components, weighted by the individual concentrations, z^i , is the *mixing enthalpy*, Δh^{MIX} (sometimes referred as the *excess enthalpy*),

$$\Delta h^{MIX} = h^{MIX} - \sum h^i \cdot z^i \quad (2.38)$$

Similarly, one may examine any other property of the mixture, as for example, the molar volume, v , and discuss the associated *mixing volume*, which is,

$$\Delta v^{MIX} = v^{MIX} - \sum v^i \cdot z^i \quad (2.39)$$

or the *mixing isothermal Joule-Thomson coefficient*, ϕ , which is:

$$\Delta \phi^{MIX} \equiv \phi^{MIX} - \sum \phi^i \cdot z^i \quad (2.40)$$

This should be interpreted as the excess of ϕ (gain or loss) due to the mixing effect of the components.

Following Wisniak et al.,¹⁶

$$\Delta \phi^{MIX} = \Delta v^{MIX} - T \left[\frac{\partial}{\partial P} (\Delta v^{MIX}) \right]_T \quad (2.41)$$

$$\Delta \phi^{MIX} = \left[\frac{\partial}{\partial P} (\Delta h^{MIX}) \right]_T \quad (2.42)$$

Gustafson¹¹ and Wisniak et al.^{16,17} discussed the possibility of obtaining a synergy of the Joule-Thomson differential and integral effects by mixing various components. They analyzed this possibility through the van der Waals, Redlich-Kwong, Soave and Peng-Robinson equations of state. Their conclusion was that for binary mixtures of components A and B, a synergetic effect is possible when the following relationship between critical temperatures and specific volumes are simultaneously satisfied:

$$T_C^{(A)} > T_C^{(MIX)} > T_C^{(B)} \quad (2.43)$$

$$V_C^{(A)} < V_C^{(MIX)} < V_C^{(B)} \quad (2.44)$$

These two conditions are in conflict. Practically, most of the reported binary mixtures do not satisfy the second condition, thus, the opportunities for the synergetic effect are rare. If instead of the second condition, the opposite is satisfied, $V_C^{(A)} > V_C^{(MIX)} > V_C^{(B)}$, synergism is possible but under other restricting conditions.

Sobanski and Kozak¹⁸ examined the influence of composition and pressure on the differential Joule-Thomson effect for mixtures such as: R114-NH₃, N₂-R170, R114-SO₂ and the breathing mixtures of O₂-He. Searching for optimum

values of μ , they found that these are possible in cases where there are large differences in the critical temperatures of the components. In addition, they noted that the specific heat is a central factor influencing the occurrence of extremes in μ .

2.2.7 Miscellaneous

2.2.7.1 Thermal Expansivity and the Joule-Thomson Coefficient

In terms of the *thermal expansivity* of the fluid:

$$\beta \equiv \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_P = \frac{1}{T} + \left(\frac{\partial \ln Z}{\partial T} \right)_P \quad (2.45)$$

the Joule-Thomson differential coefficients become:

$$\mu = \frac{v}{c_P} (\beta T - 1) \quad \text{or} \quad \phi = v(1 - \beta T) \quad (2.46)$$

For an ideal gas ($Z = 1$):

$$\beta^{IG} = \frac{1}{T} \quad (2.47)$$

which eliminates the Joule-Thomson effect. If the thermal expansivity remains below this value, $\beta < 1/T$, the Joule-Thomson expansion generates a warming effect, $\mu < 0$.

Equation 2.45 may be rearranged to express the residual thermal expansivity (in comparison to an ideal gas at the same temperature),

$$\beta^R \equiv \beta - \beta^{IG} = \left(\frac{\partial \ln Z}{\partial T} \right)_P \quad (2.48)$$

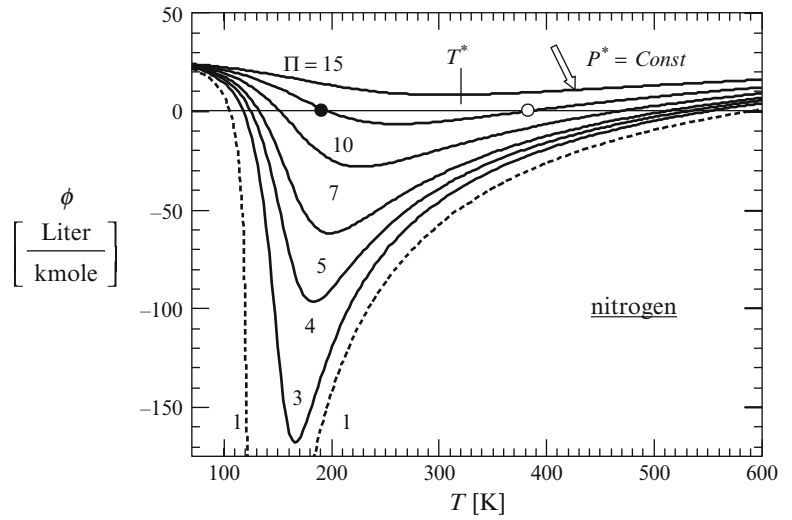
In general, dense fluids (liquids) exhibit small values of thermal expansivity, so that,

$$\mu = -v/c_P \quad @ \quad \beta T \ll 1 \quad (2.49)$$

a result indicating that dense fluids warm up under isenthalpic expansion. However, values of v for a dense phase are small and therefore the negative value of μ is small. Fundamentally, such a fluid will warm up only under a differential pressure drop in an adiabatic tube.

The liquid phase of helium, in its form as a superfluid, He-II, is a unique example of a stream that cools down as a result of a pressure drop.^{19,20} Below the Lambda (transition) point of 2.17 K, this fluid exhibits a negative thermal expansivity $\beta < 0$. Consequently, μ cannot be negative for $T < 2.17$ K.

Fig. 2.4 The isothermal Joule-Thomson coefficient, $\phi = (\partial h / \partial p)_T$, as function of temperature along lines of constant reduced pressure; at any given P^* the line peaks at the associated temperature T^*



In general, the thermal expansivity is positive, $\beta > 0$, since fluids (and solids) tend to expand when the temperature increases and therefore:

$$\varphi < v \quad @ \quad \beta > 0 \quad (2.50)$$

For a van der Waals gas, the locus of the minima rests along the line of constant specific volume,¹⁷ at the value of the critical state, $v^* = v_C$, and thus obeys the equation:

$$\Pi^* = 4\Theta^* - 3 \quad (2.54)$$

2.2.7.2 Entropy and the Joule-Thomson Coefficient

An isenthalpic pressure drop (by throttling) is always associated with irreversibility. The extent of entropy generation accompanying the Joule-Thomson effect is given by:

$$\left(\frac{\partial s}{\partial P} \right)_h = -\frac{1}{\rho \cdot T} \quad (2.51)$$

The isothermal Joule-Thomson coefficient, given in terms of entropy, is:

$$\phi = -\mu c_P = T \left(\frac{\partial s}{\partial P} \right)_T + v \quad (2.52)$$

2.2.7.3 Minimum of the Isothermal Joule-Thomson Coefficients

Figure 2.4 displays values of ϕ for nitrogen as a function of temperature along curves of constant pressure. Any curve associated with the constant pressure P^* has a minimum at a temperature T^* . The extremum (P^* , T^*) satisfies the condition,

$$(\partial \phi / \partial T)_P = 0 \quad (2.53)$$

2.2.7.4 The Volumetric Joule-Thomson Coefficient

The volumetric Joule-Thomson coefficient measures the change of temperature with specific volume rather than with pressure. It is related to the traditional Joule-Thomson coefficient in the form,

$$\left(\frac{\partial T}{\partial v} \right)_h = -\frac{P \cdot \mu}{\mu \cdot c_V + v} \quad (2.55)$$

This combination of parameters changes its sign (that is, it undergoes an inversion) at the same states where μ changes its sign.

One might also observe that the change of specific volume under an isenthalpic pressure change is related to the traditional Joule-Thomson coefficient by

$$\left(\frac{\partial v}{\partial P} \right)_h = -\mu \cdot \frac{c_V}{P} - \frac{v}{P} \quad (2.56)$$

2.2.7.5 The Joule-Thomson Effect and Magnetism

Atroshchenko et al.²¹ studied an isenthalpically expanding stream of a van der Waals gas, which at the same time crosses a border between two domains of different magnetic field intensities; the magnetization of the gas obeys Curie's Law. Tropinin²² discussed the Joule-Thomson effect in paramagnetism.

2.3 Measurements of the Joule-Thomson Effect

2.3.1 Experiments

There are few traditional versions for the experimental determination of the Joule-Thomson effect.

2.3.1.1 The Adiabatic Expansion

The adiabatic expansion method is the earliest and the most commonly applied. It directly targets the definition of the Joule-Thomson coefficient, μ . In principle, it is similar to the original “porous plug” experiment conducted by Joule and Thomson. In this approach, compressed gas experiences an isenthalpic expansion through a small (differential) change of pressure and the temperature is measured at the outlet. A correction is required to account for temperature changes due to changes in kinetic energy.²³ The controlled parameters of this experiment are the inlet pressure and temperature. Such data may be arranged to determine μ over the relevant (P , T) domain.

Air was one of the earliest and most extensively studied gases.^{1,23,24,25,26,27} Other pure gases being characterized in similar experiments included: helium,^{28,29} argon,³⁰ pentane and paraffinic naphtha,³¹ nitrogen,^{32,33,34,35,36,37} n-butane and n-pentane,³⁸ isobutane,³⁹ propane,⁴⁰ ethane,^{33,35,41} methane,^{33,37,42} carbon dioxide,^{25,43} natural gases,⁴⁴ hydrogen,^{45,46} deuterium,⁴⁷ oxygen,²⁷ freon-12⁴⁸ and neon.^{49,50}

Mixed gases have also been studied using the same method. Air was mixed with helium.²⁸ Binary mixtures of nitrogen were prepared with helium,⁵¹ argon, oxygen,^{52,53} hydrogen,^{52,53,54,55} ethane,^{33,35} carbon dioxide⁵⁶ and ternary mixtures with methane and ethane, helium and methane or hydrogen and methane.³⁶ Argon was mixed with helium⁵⁷ and carbon dioxide.^{58,59} Quaternary mixtures, such as methane, argon, nitrogen and hydrogen, have also been studied.⁵⁵

Methane has been mixed with ethane,^{33,60} n-butane⁶¹ propane⁶² and hydrogen.¹⁰ Ethylene has been mixed with hydrogen,¹⁰ propylene with carbon dioxide⁶³ and neon with helium.⁶⁴

Most of the measured expansions are associated with cooling. However, some experiments involved quantum gases and focused on measuring the warming effect.^{45,47,51,57}

The adiabatic Joule-Thomson coefficient as a function of temperature and pressure is displayed in Fig. 2.1. It becomes larger at lower temperatures and pressures. The more dense phase (especially liquid) is characterized by a significantly lower values of μ

2.3.1.2 The Isothermal Expansion

The isothermal expansion method is an alternative method of generating the data of the Joule-Thomson effect. Gas

expands isenthalpically over a narrow range of pressure and is warmed (in case of cooling) at the outlet to reach the temperature of the inlet, thus sustaining the isothermal condition. The heat supply is a measure of Δh^T per unit of dropped pressure, which leads to the coefficient, ϕ . This method was applied for pure gases and for mixtures.^{65,66,67,68,69,70,71,72,73,74,75}

2.3.1.3 Measurement of Compressibility

Measurement of compressibility, or PvT data, often collected by the method of Barnett,⁷⁶ in 1936, may be manipulated to obtain the isothermal Joule-Thomson coefficient, ϕ . Adding information regarding heat capacities (ideal gas values do suffice) allows one to translate it into the adiabatic coefficient, μ .

In some cases, measurements of thermo-physical properties are presented through the second and third virial coefficients as a function of temperature.^{77,78,79} In other cases, the collected PvT data was already manipulated in order to present the Joule-Thomson coefficient itself as for xenon⁸⁰ krypton⁸¹ and fluorine.^{82,83}

2.3.1.4 The Integral Adiabatic Joule-Thomson Effect

The integral adiabatic Joule-Thomson effect, ΔT_h , studied by direct measurement, is a less popular experimental approach. More often, experiments have focused on μ and ϕ . Depending on initial conditions, the net, or integral temperature change resulting from the expansion of the stream to atmospheric pressure may be positive (elevating its temperature) or negative.

Zelmanov,²⁹ Roebuck et al.²⁸ and Bozhko et al.⁸⁴ carried out such measurements for helium. Koeppe⁴⁶ and Johnson et al.^{45,85} performed the same measurements with hydrogen and deuterium.

Measurements of ΔT_h for mixtures have also been reported, mainly for validation of theoretical predictions. Schluender and Zemlin⁸⁶ calculated ΔT_h and compared their results with measurements for mixtures of methane or ethylene with hydrogen. Similarly, Koeppe⁵² focused on the following mixtures: N_2 - O_2 , A - O_2 , A - N_2 , N_2 - H_2 and N_2 - He . Zemlin^{87,88} discussed the relation between the integral Joule-Thomson effect and the heat of mixing, and thereby obtained ΔT_h for CO_2 - C_2H_4 , CO_2 - CH_4 , CH_4 - C_2H_4 , N_2 - CO_2 and N_2 - CO_2 .

2.3.2 Miscellaneous

2.3.2.1 The Critical State Joule-Thomson Coefficient

The critical state Joule-Thomson coefficient was correlated by Hendricks et al.⁸⁹ who showed, on a semi-empirical basis, that it may be approximated by:

$$\mu_C \approx \frac{T_C}{6P_C} \quad (2.57)$$

Hendricks et al. normalized the values of μ by the expression for μ_C as given above for each gas and so obtained the reduced Joule-Thomson coefficients, μ/μ_C , as function of the reduced temperature and pressure, (Θ, Π) . This unified presentation obeys quite well the principle of corresponding states.

2.3.2.2 Joule-Thomson Effect of a Vapor-Gas Mixture

A unique experiment is reported by Thirumaleshvar and Richardson,⁹⁰ who employed a room temperature mixture of nitrogen with water vapor. Upon expansion, only the vapor liquefies due to the supersonic expansion, and the droplets are separated from the main stream at the outlet of the nozzle. Due to the lack of mixing, steady state conditions are not achieved after the expansion. Because a portion of the energy is locked away by liquid water, the remaining gaseous stream undergoes deeper reduction of temperature in comparison to the expansion of pure nitrogen.

2.3.2.3 A Liquid-Liquid Phase Change

A liquid-liquid phase change may also occur under Joule-Thomson expansion. A unique example is evidenced in liquid helium which exists in two phases: the normal helium phase, called He-I, and the super fluid helium phase, He-II, that exists below the Lambda temperature, $T_\lambda(P)$, or more accurately, below the lambda line that separates the two phases. He-II exhibits special cooling features desirable for high field magnets. In some applications, a transformation of He-I into He-II is achieved by expansion through a Joule-Thomson throttle.^{91,92,93} For example, an expanded He-I stream of 100 kPa at 4.2 K down to 0.16 kPa will produce a liquid (He-II) yield of 62 % at 1.8 K.⁹⁴

2.3.2.4 The Joule-Thomson Effect of a Solid-Gas Aerosol

The Joule-Thomson effect of a solid-gas aerosol was measured and analyzed by Rybolt and Pierotti^{95,96,97,98,99} with an expectation of utilizing the effect in power and refrigeration cycles.¹⁰⁰ The surface activity of the powder and its interaction with a surrounding gas are analogous to the van der Waals intermolecular forces. The interaction provides another type of deviation from the ideal gas model thus enabling a Joule-Thomson effect. The study by Rybolt and Pierotti also included an attempt to establish an equation of state for the interacting aerosols.

2.4 Differential Inversion States

2.4.1 Introduction

2.4.1.1 Definition

The inversion states of the Joule-Thomson effect mark the transition from cooling to heating and vice versa under isenthalpic differential expansion. It is the purpose of the present chapter to define this border and to study its nature. In terms of a differential pressure drop the inversion states satisfy the following relation:

$$\mu = (\partial T / \partial P)_h = 0 \quad (2.58)$$

The loci of all such states comprise the *differential inversion curve* (D.I.C.).

2.4.1.2 The Extended Inversion States

However, this definition captures only the states of gradual and continuous transitions from negative to positive (and vice versa) values of μ , passing through zero. Additional inversion states exist where μ changes sign without passing through (but rather by jumping over) values of $\mu = 0$. The *extended inversion states* are located on the lower part of the saturation curve.

2.4.1.3 On the Microscopic Level

On the microscopic level, the inversion states are viable due to the fact that the intermolecular forces as a function of the separating distance between the molecules, change from attraction to repulsion.^{3,8,11,13}

2.4.2 Basic Features of the Differential Inversion Curve in the (P, T) Plane

Figure 2.5 is a qualitative display of the differential inversion curve (D.I.C.), while Fig. 2.6 is an actual, quantitative example for nitrogen. A number of features are noteworthy: (a) By definition, the isenthalps peak on the differential inversion curve as displayed in Fig. 2.6. This figure also shows an integral inversion point, X, which will be discussed in Sect. 2.6.

(b) A highest inversion temperature, T^{MAX} , (point A in both figures) can be identified, and its value is characteristic to each gas. For temperatures above T^{MAX} , no cooling is obtainable under isenthalpic differential expansion at any pressure, rather only heating. Very often, the term *inversion temperature* refers to T^{MAX} . Table 2.1 lists these values for various gases.

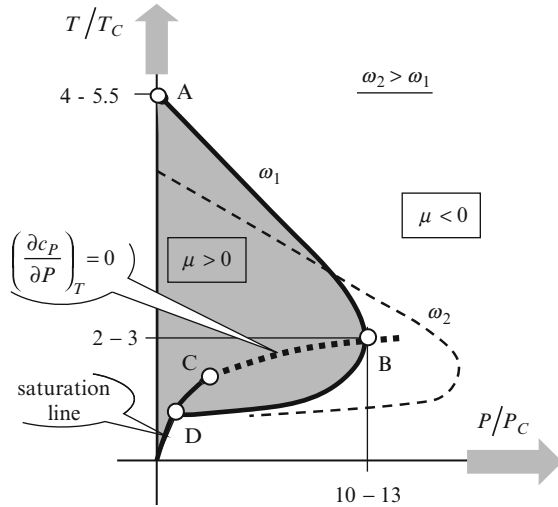


Fig. 2.5 Typical features of the differential inversion curve in the plane of reduced pressure and temperature. Also are demonstrated the effect of gas' acentricity factor, ω , and the trajectory of the extended saturation line, $(\partial c_P / \partial P)_T = 0$

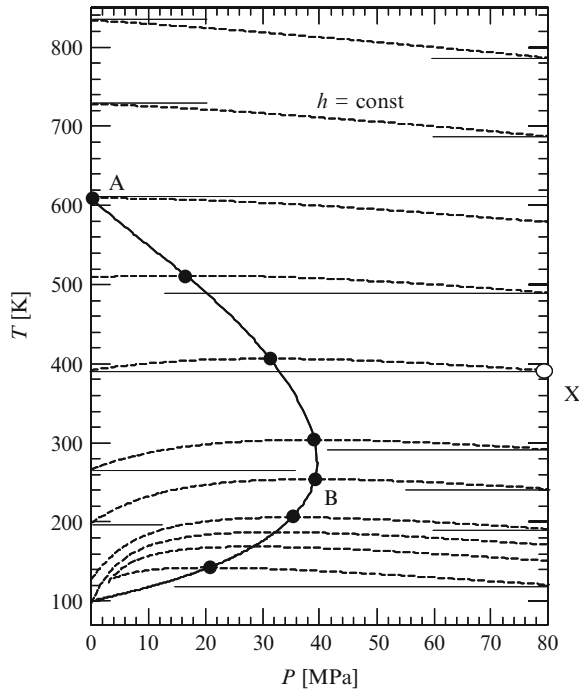


Fig. 2.6 The differential inversion curve of nitrogen, as the locus of the peaks of constant enthalpy trajectories

- (c) Below T^{MAX} , cooling or heating (in the sense of differential expansion) are possible and the inversion between the two effects occurs at a temperature that depends on pressure. For each inversion pressure, there are two inversion temperatures, already derived by Porter^{101,102} in 1906 and 1910, and shown in Fig. 2.5. Two branches of the inversion curve, an upper one (AB) and a lower

Table 2.1 The differential maximum inversion temperatures and pressures (states A and B in Figs. 2.3 and 2.4) of various gases

Gas	Highest inversion temperature		Coordinates of the state of peaking inversion pressure			
	T^{MAX} [K]	Θ^{MAX}	P^{MAX} [MPa]	Π^{MAX}	T [K]	Θ
Helium-3	34	10.2	2.49	21.7	20	6.0
Helium-4	43	8.3	3.92	17.2	23	4.4
Hydrogen	201	6.1	16.22	12.6	98	3.0
Deuterium	203	5.4	19.42	11.6	97	2.5
Neon	220	5.0	29.7	11.1	104.2	2.4
Nitrogen	608	4.8	39.4	11.6	277	2.2
Argon	763	5.1	56.5	11.6	345	2.3
Carbon monoxide	730	4.9	40.0	11.6	292	2.2
Oxygen	757	5.1	59.2	11.7	347	2.2
Methane	1,009	5.3	54.2	11.8	335	2.3
Krypton	1,079	5.1	56.5	11.1	345	3.0
Carbon dioxide	1,275	4.2	91.3	12.3	600	1.9
Xenon	1,486	5.1	67.7	11.6	685	2.4

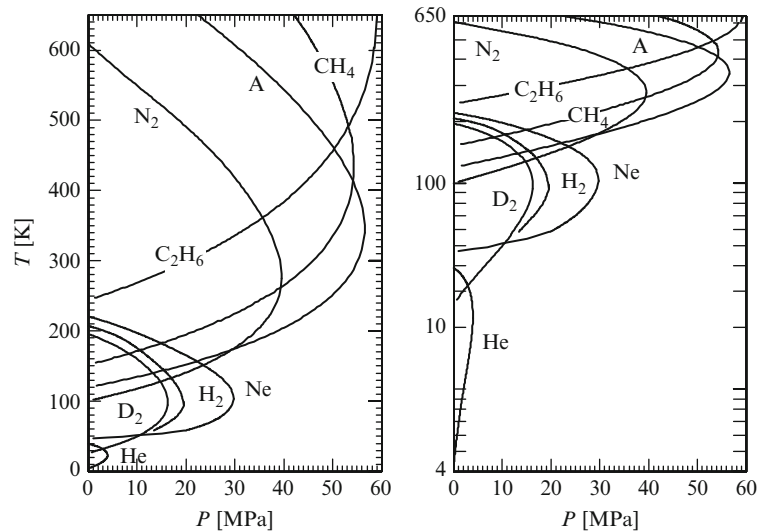
one (BD) can be identified and associated with the higher and lower inversion temperatures. This feature is further evidenced in Fig. 2.1, where μ is displayed as a function of temperature along curves of constant pressure. Each curve displays an extremum. However, only the curves of pressures below Π^{MAX} intersect the $\mu = 0$ line at two different values of temperature.

- (d) The highest inversion temperature for many gases (point A), in reduced terms, is in the range of $\Theta^{MAX} \approx 4.05.5$ (excluding the quantum gases).
- (e) The inversion pressure exhibits an extremum so that any inversion pressure corresponds with two inversion temperatures. The maximum inversion pressure for many gases (point B), is in the range of $\Pi^{MAX} \approx 10$ to 13. The associated reduced temperature associated with Π^{MAX} is about $\Theta \approx 2$ to 3 (excluding the quantum gases) as discussed in the next section. For pressures above P^{MAX} and at any temperature the result of a differential isenthalpic expansion is only a temperature increase. Values of P^{MAX} for some gases are listed in Table 2.1.
- (f) At the peak pressure along the inversion curve, (state B), the following condition is satisfied:

$$\textcircled{a} \quad \Pi = \Pi^{MAX} \quad (\partial c_P / \partial P)_T = 0 \quad (2.59)$$

- (g) Actually, a locus of states exist that fulfill the condition defined by this equation. It is the dashed line CB in Fig. 2.5 that begins at the critical state C and intersects the differential inversion curves at the peak pressure, P^{MAX} , that is, at state B. Therefore, this locus is referred to as the *extended saturation line* or sometimes as the

Fig. 2.7 Differential inversion curves of some gases in the plane of pressure and temperature



*transposed critical line.*¹⁰³ It plays a role in determining the heat transfer coefficient in the vicinity of the critical state.

Figure 2.7 displays differential inversion curves of some gases in the (T, P) plane, once with a linear scale of T and once with a logarithmic scale. In the latter, the lower boiling point gases (especially helium) spread over a range comparable to the higher boiling point gases like nitrogen and argon.

2.4.3 The Inversion States and the Principle of Corresponding States

2.4.3.1 Dependence on Acentricity Factor

Many gases exhibit similar although not identical values of Θ^{MAX} and Π^{MAX} , which means that normalizing solely on the basis of T_C and P_C is incomplete. Indeed, the principle of corresponding states is enhanced by introducing a third parameter, that is the molecular acentricity factor after Pitzer, ω , as discussed in Appendix A2, which relates with Z_C by Eq. (A2.34).

A gas of higher acentricity factor, ω , or lower Z_C exhibits:

1. A lower Θ^{MAX} (point A), and;
2. A larger Π^{MAX} (point B), which occurs at,
3. A lower Θ (Π^{MAX}), as shown in Fig. 2.5. (compare with Sect. 2.5.1.).

The additional information for deriving ω is obtained from the saturation data of the fluid. Colazo et al.¹⁰⁴ came to the conclusion that the equations of state that are based on vapor–liquid–equilibrium data (such as Peng–Robinson or Soave–Redlich–Kwong) do predict accurately the lower wing of the differential inversion curve, but less accurately the upper wing of supercritical temperatures.

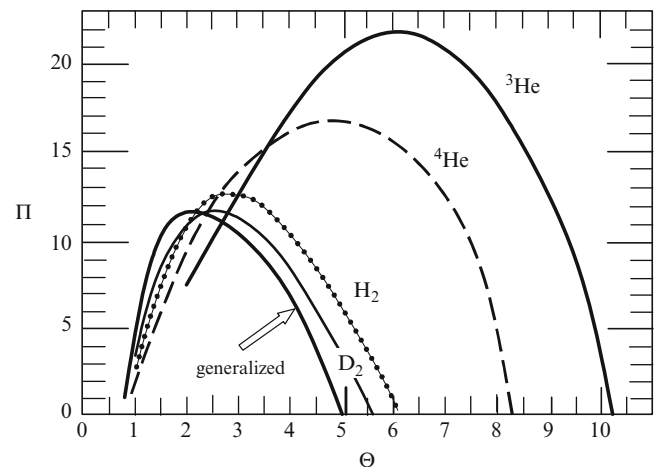


Fig. 2.8 The differential inversion curves of the quantum gases in the plane of reduced pressure and temperature and their deviation from the generalized behavior of gases

Koeppel¹⁰⁵ examined the inversion states as a function of the acentricity factor of Riedel, α_C , by Eq. (A2.36) of Appendix A2. First, he plotted the values of Θ^{MAX} as a function of α_C . In addition, he discussed the distribution of Π for a given Θ , for different gases.

2.4.3.2 The Quantum Gases

^3He , ^4He , D_2 and H_2 do not fit into this frame as displayed in Fig. 2.8. They have negative acentricity factors and higher Θ^{MAX} and Θ (Π^{MAX}) values, which is consistent with the ω dependence trend mentioned in the previous paragraph. However, the Π^{MAX} increases too, which is inconsistent with the general trend of other gases. The isotope ^3He exhibits the largest quantum effect. Neon belongs to the group of quantum gases but its quantum effects are weak

so that it behaves as one of the typical low acentricity gases, presented in Fig. 2.8 by the “generalized” curve. The differential inversion curve of ^3He was derived by Kamatsevich et al.¹⁰⁶ and by Maytal.¹⁰⁷

A gas with a stronger quantum effect covers a larger area (i.e. “more” thermodynamic states) in the plane of reduced coordinates (Π, Θ) with $\mu > 0$. The quantum effect might be regarded as an additional mechanism of deviation from the ideal gas model and possessed only by the quantum gases. Saygin and Sisman¹⁰⁸ showed that the Joule-Thomson effect of a quantum ideal gas does not vanish, as:

At sufficiently low temperatures or high pressures, which appear in cryogenic systems, the quantum nature of the gas becomes important and an ideal gas should be treated as a quantum ideal gas instead of a classical ideal gas. For a quantum ideal gas, enthalpy depends both on temperature and pressure.

Their view might explain the larger domain of thermodynamic states occupied by the differential inversion curves of the quantum gases. The Saygin and Sisman study distinguishes between the Bose and Fermi type of quantum gas particles.

2.4.3.3 The Pseudo-Critical Parameters of the Quantum Gases

At the top of Table 2.1 are listed the values of maximum inversion temperatures and pressures.

Numerous studies were devoted to formulating a broader statement of the principle of corresponding state to include the effects of the quantum gases. Gunn et al.¹⁰⁹ reviewed some of them in the context of inversion temperatures. He proposed a modification of the actual critical temperature and pressure (helium 9.4 K, 0.549 MPa, hydrogen 41.4 K, 1.854 MPa, neon 45.3, 2.705) of these gases, accounting for their quantum effects. Based on these *pseudo-critical-parameters* the associated reduced temperatures and pressures would obey the principle of corresponding states.

2.4.4 Differential Inversion Curve Extension Below the Critical Temperature

- The position of the differential inversion curve in the (T, P) plane and below the critical temperature (point C) in reference to the saturation line was studied experimentally by Roebuck⁴³ in 1942 with CO_2 , and theoretically by Koeppel¹⁰⁵ in 1960. The lower wing of the inversion curve defined by vanishing values of $(\partial T/\partial P)_h$, meets the saturation line at point D (Fig. 2.9) and stops there.
- Practically, state D ranges in reduced temperature, Θ_D , around 0.8, and in reduced pressure, Π_D , of about 0.25, as listed in Table 2.2.

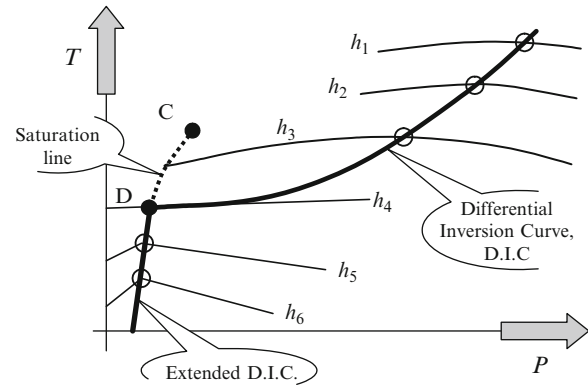


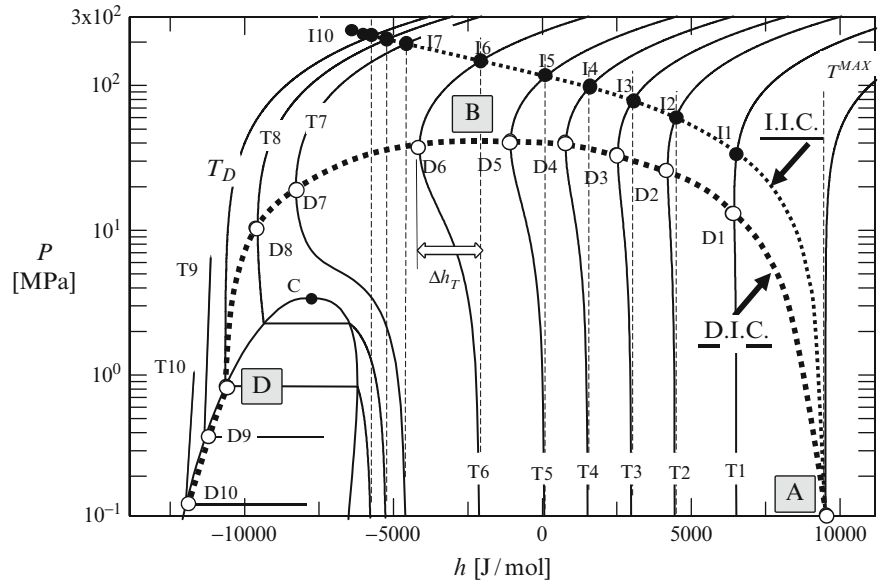
Fig. 2.9 The differential inversion curve below the critical temperature and its intersection with the saturation line

Table 2.2 Values of temperature and pressure of the state where the lower wing of the inversion curve meets the saturation line (point D in Fig. 2.5)

Gas	T_D [K]	Θ_D	P_D [MPa]	Π_D
He-4	4.3	0.830	0.11	0.48
H_2	26.9	0.817	0.48	0.37
D_2	38.3	0.818	0.61	0.36
Ne	44.5	0.819	0.77	0.29
N_2	101.3	0.802	0.85	0.25
CO	106.8	0.804	0.87	0.25
F_2	116.6	0.807	1.34	0.26
A	121.6	0.806	1.32	0.27
O_2	124.6	0.806	1.32	0.26
CH_4	154.0	0.807	1.24	0.27
Kr	168.7	0.806	1.48	0.27
C_2H_4	227.5	0.805	1.12	0.24
C_2H_6	246.2	0.806	1.16	0.24
C_3H_8	297.8	0.804	0.95	0.22
CO_2	304.1	0.801	1.45	0.20
NF_3	321.6	0.793	1.95	0.17
i C_4H_{10}	343.2	0.841	0.81	0.22

- The condition of vanishing $(\partial T/\partial P)_h$ defining the inversion curve manifests itself along any isenthalp in the (T, P) plane, by a change of sign in μ as the isenthalp crosses the inversion curve. This also means that the isenthalpic trajectories peak on the differential inversion curve such as those points marked on the lines h_1 , h_2 and h_3 in Fig. 2.9.
- Below point D, any isenthalp, such as h_5 and h_6 in the same figure, alters the sign of its slope, μ , as it crosses the saturation line (although μ exhibits a discontinuity at the saturation line) and peaks on the saturation line.
- Therefore, the saturation line below T_D plays the role of an *extended inversion curve* sharing with the locus of $\mu = 0$ the essence of the same idea.

Fig. 2.10 A qualitative display of the differential inversion curve (D.I.C.) and the integral inversion curve (I.I.C.) in the plain of (P, h) . It is an almost exact presentation of nitrogen. The set of temperatures (T_2, T_4, T_6, T_7) is (450 K, 350 K, 225 K, 140 K)



- (f) The saturation line in the (T, P) plane is a projection of the two-phase dome, which occupies a planar domain when described in any other two coordinates. Hence, each point on the saturation line represents two states: the saturated liquid and the saturated vapor, of low and high specific volumes, v' and v'' , correspondingly.
- (g) Koeppe¹⁰⁵ pointed out that below T_D the differential inversion curve is actually represented by the saturated liquid states, (T, v') .
- (h) Whenever it becomes significant to distinguish between the two coexisting states of saturation, the T - P plane display remains incomplete. Thus, any explicit description of the subcritical extension of the inversion curve has to take place in coordinates that unfold the entire two-phase dome, such as the P - h and the T - s planes.

It begins at point A at a very low pressure, increases to the highest inversion pressure at point B and decreases to point D on the saturated liquid border of the two phase dome.

The lower isotherms T_D , T8, T9 and T10, are subcritical, thus they cross the two-phase dome, in contrast to the higher and supercritical isotherms, T1 through T7. The isotherm of T_D and below (T9, T10) also peak on the saturated liquid line at points D, D9 and D10, but without satisfying the above vanishing slope condition as fulfilled by the points on the inversion curve. Therefore, the saturation liquid line, starting at point D and down, proceeding through points D9 and D10, is the precise representation (instead of the saturation line, in the T - P plane) of the *extended inversion curve*.

The states marked by I1 through I9 belong to the *integral inversion curve*, and are discussed in Sect. 2.6.

2.4.5 The Differential Inversion Curve (D.I.C.) in Various Coordinate Planes

2.4.5.1 The P - h Plane

The shape of the thermodynamic states defining the inversion curves in the P - h plane is mapped by the aid of isotherms as displayed in Fig. 2.10. The isotherms marked by T1 through T_D to T10, all represent temperatures gradually lower than the maximum inversion temperature, T^{MAX} . All of the isotherms display an extremum in terms of the enthalpy, h . The locus of the extremum points (such as D1, through D10 and D) comprises the *differential inversion curve* (D.I.C.). The equation of this curve in the P - h plane can be expressed as,

$$(\partial h / \partial P)_T = 0 \quad (2.60)$$

2.4.5.2 The T - s Plane

The T - s plane is explored with the aid of isenthalpic curves as shown in Fig. 2.11. With analogy to the coverage of the P - h plane by isotherms (T1 to T6), here a family of isenthalps (h_1 through h_D to h_6), all confined in the domain below the maximum inversion temperature, T^{MAX} (point A), is chosen. Each isenthalp has a maximum temperature, such as states D1, D2, D3 and D, obeying the equation of the *differential inversion curve* in this plane, namely:

$$(\partial T / \partial s)_h = 0 \quad (2.61)$$

As shown in the same figure, the lowest temperature that still fulfills the above equation rests on the saturated liquid line, at state D. On the same border isenthalps H5 and H6 peak at the states D5 and D6, but do not have vanishing

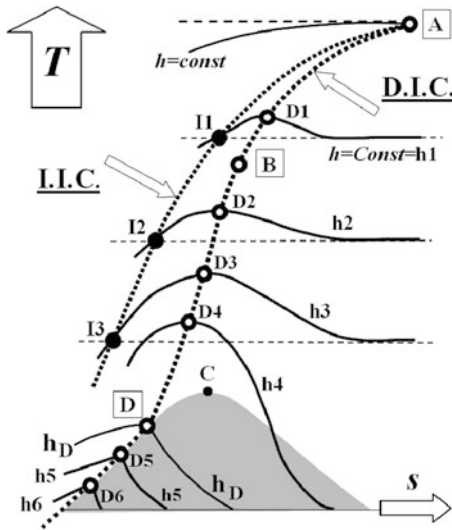


Fig. 2.11 A qualitative display of the differential inversion curve (D.I.C.) and the integral inversion curve (I.I.C.) in the plane of (T, s)

slopes on the inversion curve. The line D-D5-D6 is the *extended inversion curve*.

The differential inversion curve in the T - s plane does not have any graphic visible manifestation of the fact that the inversion pressure has a maximum value (point B), and that for any lower inversion pressure there are two inversion temperatures.

2.4.5.3 The h - s Plane

The h - s plane named after Mollier is another useful diagram. The locus of the minima of isotherms (later discussed in Fig. 3.32), that is where,

$$(\partial h / \partial s)_T = 0 \quad (2.62)$$

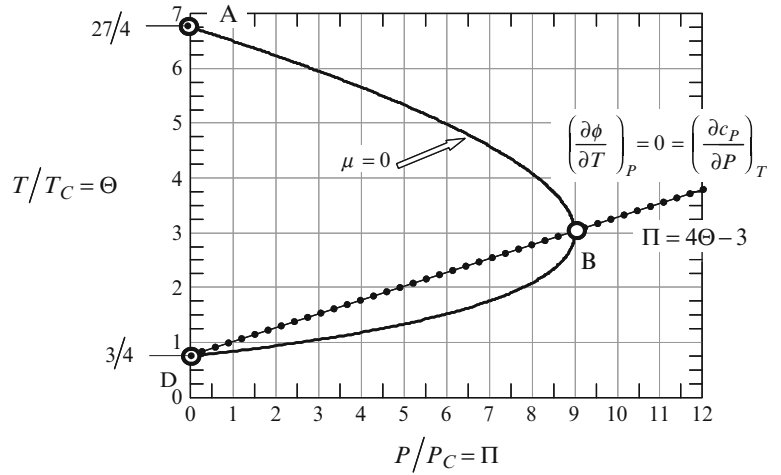
is the *differential inversion curve*. The slope of the isotherms in terms of μ , ϕ and β , is:

$$\begin{aligned} \left(\frac{\partial h}{\partial s} \right)_T &= T \frac{\mu}{\mu_s} = T \frac{1}{1 - v/\phi} = T - \frac{1}{\beta} \\ &= (\beta^{-1})^{IG} - (\beta^{-1}) \equiv -(\beta^{-1})^R \end{aligned} \quad (2.63)$$

2.4.6 Inversion States of Mixtures

- (a) The inversion pressure of a binary mixture, at any given temperature, or the inversion temperature at a given pressure, gradually changes with composition between the two corresponding values of the pure components, as derived

Fig. 2.12 The differential inversion curve of the van der Waals gas in the plane of reduced temperature and pressure



2.4.8.1 The (T, P) Plane

1. The (highest) inversion temperature (point A) is,

$$\Theta^{MAX} = \frac{2a}{Rb} = \frac{27}{4} \quad (2.66)$$

at which point the dimensionless slope is, $d\Theta/d\Pi = -1/4$.

2. The lowest inversion temperature (point D) $\Theta^{MIN} = 3/4$.
3. The maximum inversion pressure of $\Pi^{MAX} = 9$ occurs at the corresponding reduced temperature of $\Theta(\Pi^{MAX}) = 3$.
4. The closed form solution for the differential inversion curve is given by,

$$\Pi = 24\sqrt{3\Theta} - 12\Theta - 27 \quad (2.67)$$

or, in a similarity formulation,

$$3 \cdot \left(1 - \sqrt{\frac{\Theta}{\Theta^{MAX}}}\right) \cdot \left(\sqrt{\frac{\Theta}{\Theta^{MAX}}} - 1\right) = \frac{\Pi}{\Pi^{MAX}} \quad (2.68)$$

2.4.8.2 The Density

The density along the differential inversion curve:

1. The reduced density, Φ , can be expressed as a function of the reduced temperature:

$$\Phi = 3 \cdot \left(1 - \sqrt{\frac{\Theta}{\Theta^{MAX}}}\right) \quad \text{or} \quad \frac{\Theta}{\Theta^{MAX}} = \left(1 - \frac{\Phi}{3}\right)^2 \quad (2.69)$$

It has a monotonic dependence on Φ .

2. Expressed as a function of the reduced pressure, its behavior is not monotonic:

$$\Phi = 1 \pm \sqrt{1 - \frac{\Pi}{\Pi^{MAX}}} \quad \text{or} \quad \frac{\Pi}{\Pi^{MAX}} = 2 \cdot \Phi - \Phi^2 \quad (2.70)$$

The plus sign relates to the lower temperature branch of higher densities and the minus sign to the higher temperature branch and lower densities.

3. At the state of maximum inversion pressure (point B in Fig. 2.5) the specific volume is:

$$\rho(\Pi^{MAX}) = \rho_C \quad (2.71)$$

4. At the lowest state of the differential inversion curve, point D:

$$\rho(\Theta^{MIN}) = 2 \cdot \rho_C \quad (2.72)$$

That is, the density along the differential inversion curve of a van der Waals gas changes from 0 at Φ^{MAX} through ρ_C at Π^{MAX} and up to $2 \cdot \rho_C$ at Φ^{MIN}

2.4.8.3 The Extended Saturation Line

The extended saturation line is defined by,

$$\left(\frac{\partial}{\partial p} c_P\right)_T = 0 \quad (2.73)$$

From Eq. 2.5 we have, that along the extended saturation line,

$$(\partial\phi/\partial T)_P = 0 \quad (2.74)$$

Following Sect. 2.2.7.3, the equation of the line BD for a van der Waals gas becomes,

$$\Pi = 4\Theta - 3 \quad (2.75)$$

It is a straight line, as all isochoric lines of the van der Waals gas are straight. Along the extended saturation line, $\rho = \text{Const} = \rho_C$.

2.4.8.4 The Integral Joule-Thomson Effect

The integral Joule-Thomson effect monotonically increases along the differential inversion curve, towards the lower temperatures, by

$$\left(\frac{\Delta h_T}{RT^{MAX}}\right)^{1/2} = 1 - \left(\frac{\Theta}{\Theta^{MAX}}\right)^{1/2} = \frac{\Phi}{3} \quad (2.76)$$

2.4.8.5 The Quantity $c_P - c_V$

The quantity $c_P - c_V$ monotonically increases along the differential inversion curve, by

$$\left(\frac{c_P - c_V}{R}\right)_{DIC} = \left(\frac{\Theta^{MAX}}{\Theta}\right)^{1/2} = \frac{3}{3 - \Phi} > 1 \quad (2.77)$$

2.4.9 Equation of State Dependence

- (a) The equation of state, $f = (T, P, v) = 0$, or the PvT data, are sufficient for identifying the inversion states without explicit specification of heat capacities. It appears that Witkowski¹¹² in 1898 was first to derive the dependence of the inversion curve on the equation of state,

$$(\partial Z / \partial T)_P = 0 \quad (2.78)$$

$$\left(\frac{\partial v}{\partial T}\right)_P = \frac{v}{T} \quad \text{or} \quad \left(\frac{\partial \ln v}{\partial \ln T}\right)_P = 1 \quad \text{or} \quad \left(\frac{\partial \ln \rho}{\partial \ln T}\right)_P = -1 \quad (2.79)$$

Wisniak¹¹³ used this relation as a criteria for evaluating various equations of state according to their capability to accurately predict the inversion curve.

$$T \left(\frac{\partial P}{\partial T}\right)_\rho = \rho \left(\frac{\partial P}{\partial \rho}\right)_T \quad \text{or} \quad \left(\frac{\partial \rho}{\partial \ln T}\right)_\rho = \left(\frac{\partial P}{\partial \ln \rho}\right)_T \quad (2.80)$$

$$T \left(\frac{\partial Z}{\partial T}\right)_\rho = \rho \left(\frac{\partial Z}{\partial \rho}\right)_T \quad (2.81)$$

- (b) The power to predict the differential inversion curve is a severe test for the accuracy of any equation of state. This was recognized already by Porter.¹⁰¹
- (c) For a virial equation of state (Appendix A2) expanded by pressure, the differential inversion curve would be:

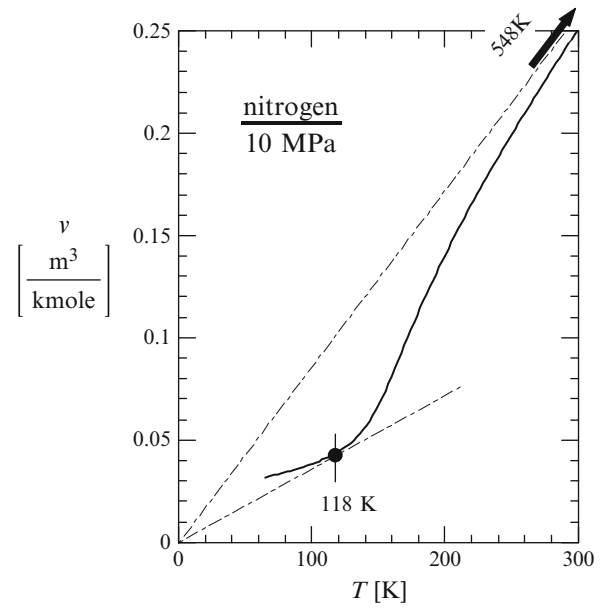


Fig. 2.13 Two differential inversion points (117 K, 548 K) for on a given constant pressure (10 MPa) line, fulfilling $(\partial v / \partial L)$ in the (v, T) plane

$$\frac{dB'}{dT} + \frac{dC'}{dT}P + \frac{dD'}{dT}P^2 + \dots = 0 \quad (2.82)$$

- (d) The maximum inversion temperature (at zero pressure) is related to the second virial coefficient, B , (expanded by density) by,

$$\frac{dB}{dT} = \frac{B}{T} \quad (2.83)$$

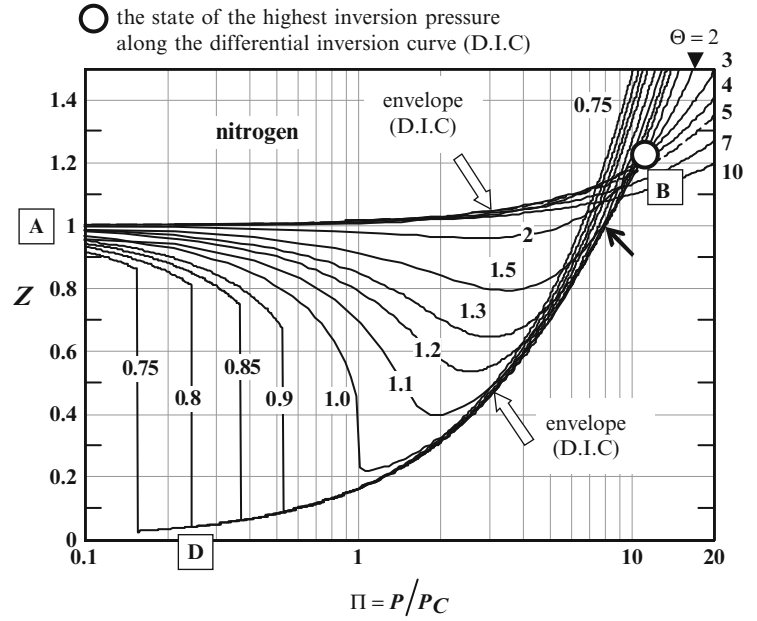
and to B' , (expanded by pressure) by,

$$dB'/dT = 0 \quad (2.84)$$

2.4.10 The Differential Inversion States in the Plane of (v, T)

- (a) A single 10 MPa isobar of nitrogen (which is below the highest inversion pressure) is shown in Fig. 2.13 in the (v, T) plane. The geometrical interpretation of the inversion state in this plane rests on the relationship $(\partial v / \partial T)_P = v/T$. It is as follows: the tangent to the isobar, at an inversion state associated with the same pressure, passes through the origin in this plane. Below Π^{MAX} , each isobar has two such points. Above Π^{MAX} , no tangent passes through the origin.

Fig. 2.14 The differential inversion curve as the envelope of isotherms in the compressibility chart



- (b) The isobar of Π^{MAX} in the (v, T) plane has a single inflexion point, the tangent of which passes through the origin. In 1906 Porter¹⁰¹ formulated the feature of the inversion state of Π^{MAX} according to the relationship,

$$\left(\frac{\partial^2 v}{\partial T^2}\right)_P = 0 \quad (2.85)$$

By the identity:

$$T\left(\frac{\partial^2 v}{\partial T^2}\right)_P = -\left(\frac{\partial c_P}{\partial P}\right)_T \quad (2.86)$$

a second equivalent condition can be identified,

$$\left(\frac{\partial c_P}{\partial P}\right)_T = 0 \quad (2.87)$$

at the maximum inversion pressure.

2.4.11 The Compressibility Plane of $Z(\Pi)$

2.4.11.1 The Envelope of Isotherms in the (Z, Π) Plane

The envelope of isotherms in the (Z, Π) plane is the loci of the differential inversion states in this plane, as demonstrated for nitrogen in Fig. 2.14. Adjacent members of the family of isotherms intersect each other, and it is possible to identify such an envelope. This interpretation is based in the fact that each inversion state fulfills the condition $(\partial Z/\partial T)_P = 0$. Hahle¹¹⁴ noticed the relation between

two intersecting isotherms on this plane and the differential inversion curve. On both sides of any intersection, the term $(\partial Z/\partial T)_P$ changes its sign, while in between, there is a point where it vanishes. In 1971, Miller¹¹⁵ and Bursik¹¹⁶ also studied the inversion states in this plane. The presentation of Z as function of Π was proposed by Nelson and Obert.¹¹⁷

2.4.11.2 Intersections of Adjacent Isotherms in the (Z, Π) Plane

Intersections of adjacent isotherms in the (Z, Π) plane do not occur for temperatures above the highest inversion temperature, T^{MAX} . Since there are no inversion states above T^{MAX} , therefore there are no intersections of the corresponding isotherms.

2.4.11.3 The Isotherms at Zero Pressure and $Z = 1$ Vicinity

The T^{MAX} isotherm passes through point A as $P \rightarrow 0$, where $Z = 1$. Therefore, the envelope of isotherms at state A coincides with the T^{MAX} isotherm in the same plane of (Z, Π) .

Consequently, the T^{MAX} isotherm has to have the highest slope of all isotherms, as demonstrated in Fig. 2.15, namely,

$$\left(\frac{\partial Z}{\partial P}\right)_{T=T^{MAX}} > \left(\frac{\partial Z}{\partial P}\right)_T \quad @ \quad P \rightarrow 0 \quad (2.88)$$

Since,

$$\lim_{P \rightarrow 0} \left(\frac{\partial Z}{\partial P}\right)_T = B'(T) = \frac{B(T)}{RT} = f(T) \quad (2.89)$$

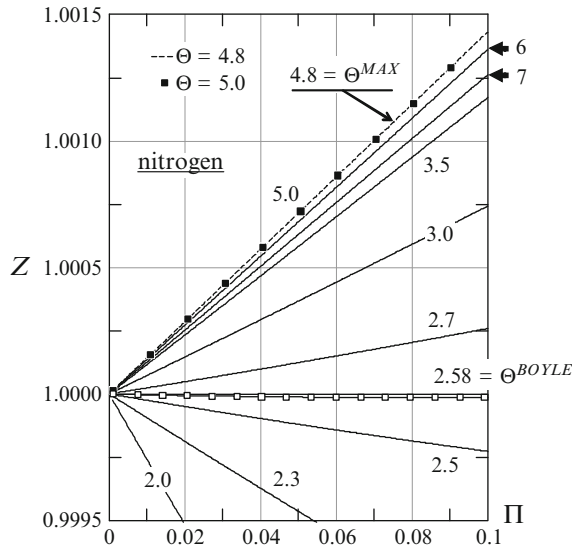


Fig. 2.15 The isotherm of T^{MAX} at $P \rightarrow 0$ has the highest slope, $(\partial Z/\partial P)_T$ in the compressibility plane of (Z, Π)

or

$$\lim_{P \rightarrow 0} \left[\frac{d}{dT} \left(\frac{\partial Z}{\partial P} \right)_T \right] = 0 \quad @ \quad T = T^{MAX} \quad (2.90)$$

namely, $B'(T)$ peaks at $T = T^{MAX}$, which is a test of consistency for any experimentally derived expression for $B'(T)$. That is consistent with Eq. 2.27 which shows that B/T at the temperature of $\mu_0 = 0$, which is T^{MAX} .

This corollary and the previous one of Sect. 2.4.11.2 are a contribution of the inversion states argument to the analysis of the compressibility chart.

For a van der Waals gas,

$$\lim_{P \rightarrow 0} \left(\frac{\partial Z}{\partial \Pi} \right)_\Theta = \frac{1}{108} \frac{\Theta^{MAX}}{\Theta} \left(2 - \frac{\Theta^{MAX}}{\Theta} \right) = f(\Theta) \quad (2.91)$$

One may verify that $f(\Theta)$ reaches a maximum at $\Theta = \Theta^{MAX}$ where $(\partial Z/\partial \Pi)_\Theta = 1/108$. For nitrogen (Fig. 2.15) it is about 2/3 of this prediction.

2.4.11.4 Boyle Temperature and the Maximum Inversion Temperature

The Boyle temperature, T_{BOYLE} , is the one for which:

$$\lim_{P \rightarrow 0} \left(\frac{\partial Z}{\partial P} \right)_T = 0 \quad (2.92)$$

or the temperature at which the second virial coefficients vanish:

$$B(T_{BOYLE}) = B'(T_{BOYLE}) = 0 \quad (2.93)$$

For the van der Waals gas by Eq. 2.91:

$$\Theta_{BOYLE} = \Theta^{MAX}/2 \quad (2.94)$$

Hendricks⁸⁹ examined this ratio for a group of real gases. In 1938 Hirschfelder et al.³ utilizing a model with a square well intermolecular potential, solved the simultaneous pair of conditions,

$$\mu_0(\Theta^{MAX}) = 0 \quad (2.95)$$

$$B(\Theta_{BOYLE}) = 0 \quad (2.96)$$

and obtained:

$$\Theta^{MAX}/\Theta_{BOYLE} = 1.88 \quad (2.97)$$

This relation is obeyed quite well by the real gas nitrogen in Fig. 2.15.

Heyes and Llaguno¹¹⁸ determined Θ_{BOYLE} by the Redlich-Kwong (R-K) and the modified R-K equations of state. They found that Θ^{MAX} was about 2.989 and 2.819 times larger than Θ_{BOYLE} . In addition, they applied the Lennard-Jones intermolecular potential in order to obtain the entire inversion curve. Breitenstein and Lustig¹¹⁹ derived similar relations between Θ_{BOYLE} and Θ^{MAX} for symmetrical multi-center molecules assuming the Lennard-Jones intermolecular potential.

2.4.11.5 Z as a Function of the Inversion Pressure for a van der Waals Gas

Z as a function of the inversion pressure for a van der Waals gas has two separate expressions for the two branches of the inversion curve:

$$\frac{Z}{Z^{MAX}} = \frac{12\Pi}{(3 \pm \sqrt{9 - \Pi})(6 \mp \sqrt{9 - \Pi})^2} \quad (2.98)$$

$$Z^{MAX} = 9/8$$

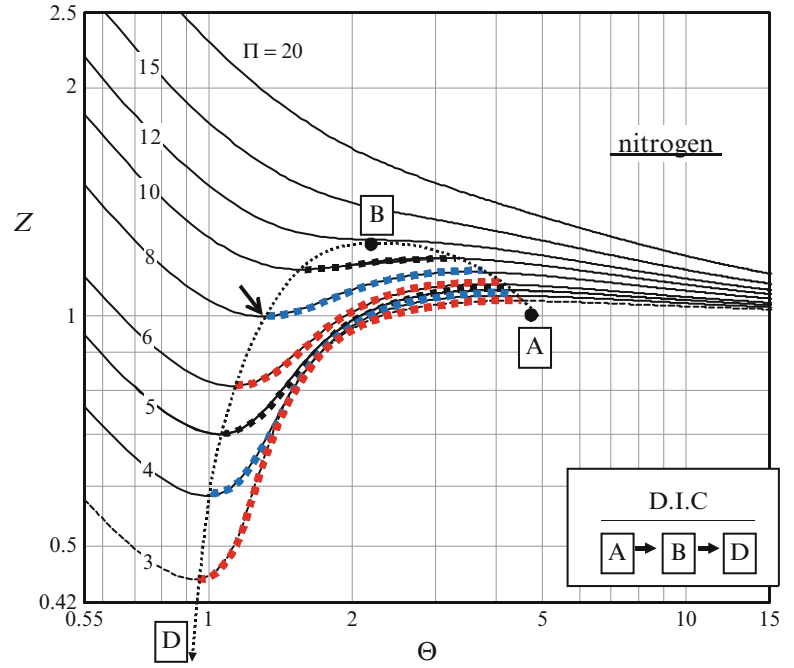
The match of the two branches at the state of Π^{MAX} is discontinuous, as one may observe in Fig. 2.14.

2.4.12 The Compressibility Plane of Z (Θ)

2.4.12.1 The Differential Inversion States

Alternatively, the compressibility may be introduced as function of temperature, $Z(\Theta)$, with pressure as an adjustable parameter (displayed in Fig. 2.16) instead of the traditional presentation of $Z(\Pi)$ for various temperatures (Fig. 2.14). A curve of constant Π may have a section of $(\partial \ln Z / \partial \ln \Theta)_\Pi > 0$, which is the locus of values for which $\mu > 0$. The ends of

Fig. 2.16 The compressibility as a function of temperature as a is used to identify the differential inversion



this section are the two inversion states associated with the same value of Π . Thus, the differential inversion curve is obtained in this plane, explicitly exhibiting states A of highest inversion temperature through state B of the highest inversion pressure and continuing to state D.

2.4.12.2 Z as a Function of the Inversion Temperature

Z as a function of the inversion temperature may be formulated in a closed form expression for a van der Waals gas. In particular, the following relation is found¹²⁰:

$$Z = \frac{3}{2} \cdot \left(\frac{\Theta^{MAX}}{\Theta} \right)^{1/2} - \frac{1}{2} \cdot \left(\frac{\Theta^{MAX}}{\Theta} \right) \quad (2.99)$$

As displayed in Fig. 2.17 it is a continuous expression, which peaks at the state of largest inversion pressure (point B at $\Theta = 3$, $\Pi^{MAX} = 9$, Fig. 2.5). For the van der Waals gas $Z^{MAX} = 9/8$. Hence, along the upper branch (higher temperatures, from state A to B) Z varies from 1 to 9/8. However, along the lower temperatures branch (state B to D in Fig. 2.5) it ranges from 9/8 to zero. This figure compares real gases with the corresponding van der Waals predictions. Note that the temperature scale is normalized by the actual maximum inversion temperature of each gas, T^{MAX} (and not by T_C). For the van der Waals gas, it is $T^{MAX} = (27/4) T_C$ but for nitrogen and argon, the particular highest inversion temperatures are closer to $5T_C$.

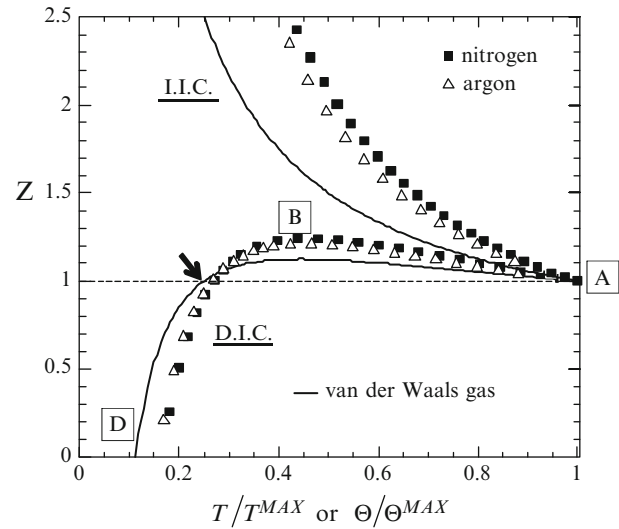


Fig. 2.17 Compressibility along the differential (D.I.C.) and integral (I.I.C.) inversion curves; van der Waals prediction versus real gases¹²⁰

2.4.12.3 The Intersection of the Inversion Curve with the Unit Compressibility Line

The intersection of the inversion curve with the unit compressibility line, as denoted by a black arrow in Figs. 2.14, 2.16, and 2.17 was studied by Holleran.¹²¹ He assumed that for each gas, it occurs at a temperature that is half of the Boyle temperature, as is indeed predicted by the above van der Waals condition of $Z = 1$.

$$Z\left(\Theta = \frac{1}{2}\Theta_{BOYLE}\right) = 1 \quad (2.100)$$

Borsik,¹²² Angus¹²³ and Holleran¹²⁴ further discussed the same issue.

2.5 Empirical Correlations for the Differential Inversion Curve

It is a difficult challenge for any equation of state to precisely predict the differential inversion curve. A relatively small variation of the equation may result in a large change in the differential inversion curve. Therefore, it is helpful to correlate the differential inversion curve in its reduced form, directly through the accumulated measurements of inversion points themselves.

2.5.1 The Correlation of Jacob and the Principle of Corresponding State

The first attempt in this regard was reported by Jacob¹²⁵ already in 1921. It is unique in the sense that it accounts for the difference in the differential inversion curve as a function of the molecular structure of the gas:

$$\Pi = 23.27 - 1.174\left(\frac{\Theta}{Z_C}\right) - \frac{178.6}{(\Theta/Z_C)^2} \quad (2.101)$$

Jacob used the critical state compressibility, Z_C , as an empirical parameter characterizing the molecular structure. The principle of the corresponding states is enhanced by re-scaling of the reduced temperature, Θ , in the form, Θ/Z_C .

The maximum inversion temperature and the slope at this state are proportional to Z_C of the specific gas:

$$\Theta^{MAX} = 19.5065 \cdot Z_C \quad (2.102)$$

and

$$\frac{d\Theta}{d\Pi}(\Theta^{MAX}) = -0.8882 \cdot Z_C \quad (2.103)$$

The maximum inversion pressure, $\Pi^{MAX} = 11.4760$, according to this correlation is independent of Z_C , and occurs at a temperature,

$$\Theta(\Pi^{MAX}) = 6.7258 \cdot Z_C \quad (2.104)$$

Since Z_C is related to ω via Eq. (A2.34) of Appendix A2., one may verify the consistency of these expressions with the discussion in Sect. 2.4.3 and Fig. 2.5.

2.5.2 The Generalized Correlations for Low Acentricity Gases

The subsequent empirical correlations do not include the distinction between the different types of molecules. These are attempts to suggest an average expression for a wide variety of low and moderate acentricity gases. Gunn et al.¹⁰⁹ fit a fifth degree polynomial based on 89 experimental inversion points of low acentricity molecule gases,

$$\begin{aligned} \Pi = & -36.275 + 71.598 \cdot \Theta - 41.567 \cdot \Theta^2 + 11.826 \cdot \Theta^3 \\ & - 1.6721 \cdot \Theta^4 + 0.091197 \cdot \Theta^5 \end{aligned} \quad (2.105)$$

Hendricks et al.⁸⁹ try to improve the accuracy of Gunn's correlation by using a sixth degree polynomial based on numerous inversion points of xenon, krypton, methane, nitrogen, oxygen, fluorine, argon, carbon dioxide, carbon monoxide, helium, hydrogen and neon, in order to obtain a generalized differential inversion curve.

$$\begin{aligned} \Pi = & -32.5209374 + 65.6922312 \cdot \Theta - 39.7384300 \cdot \Theta^2 \\ & + 12.9300299 \cdot \Theta^3 - 2.46176904 \cdot \Theta^4 \\ & + 0.25378553 \cdot \Theta^5 - 0.010986565 \cdot \Theta^6 \end{aligned} \quad (2.106)$$

In addition, Hendricks et al. prepare separate correlations (using sets of 7 coefficients in their polynomials) for the differential inversion curves of: methane, nitrogen, oxygen, argon, carbon dioxide, carbon monoxide and the quantum gases: neon, hydrogen, helium.

Miller's¹¹⁵ proposal is unique by virtue of its simplicity:

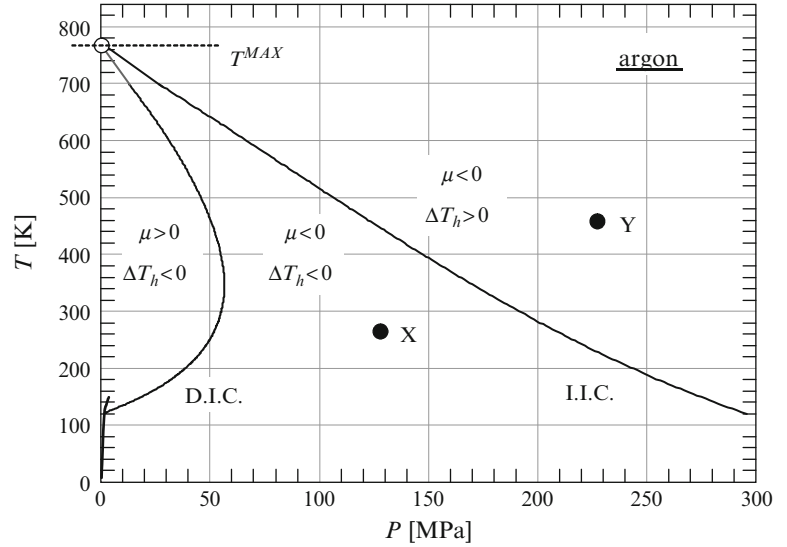
$$\Pi = 24.21 - \frac{18.54}{\Theta} - 0.825 \cdot \Theta^2 \quad (2.107)$$

with $\Theta^{MAX} = 4.98$ and $\Pi^{MAX} = 11.8$ occurring at $\Theta(\Pi^{MAX}) = 2.24$.

2.6 The Integral Inversion States

As introduced in the chronological review in the next section, the early study of the inversion phenomena was based on the integral effect. However, the integral inversion states received significantly less discussion than the differential version. Discussions in the literature that only mention the

Fig. 2.18 The relation between the differential (D.I.C.) and integral (I.I.C.) inversion curves



“inversion” states or curve, are routinely related to the differential inversion case alone.

2.6.1 Formulation

The states defined by a vanishing *integral* isenthalpic Joule-Thomson effect under expansion to 0.1 MPa,

$$\Delta T_h = T(0.1 \text{ MPa}, h) - T(P, h) \quad (2.108)$$

comprise the *integral inversion curve* (I.I.C.). The conjugated integral isothermal Joule-Thomson effect also vanishes on the same curve. Hence, an equivalent formulation of the integral inversion curve is:

$$\Delta h_T = h(0.1 \text{ MPa}, T) - h(P, T) = 0 \quad (2.109)$$

Applying one of the general expressions for Δh_T as Eq. 4.17, one obtains^{126,127}:

$$\frac{Z-1}{T} = \left[\int_{\rho_0}^{\rho} \left(\frac{\partial Z}{\partial T} \right)_{\rho} \cdot \frac{d\rho}{\rho} \right]_T \quad (2.110)$$

where ρ_0 is the density associated with 0.1 MPa. This equation is numerically solved in conjunction with the equation of state,

$$Z = Z(T, \rho) \quad (2.111)$$

2.6.2 Characteristics of the Integral Inversion Curve (I.I.C.)

Figure 2.18 is a real display of the integral inversion curve in comparison to the differential one for argon in the $T - P$ plane.

It is a layout of the differential and integral cooling borders. Note that while the two inversion curves share a common highest (reduced) inversion temperature Θ^{MAX} , the slopes of the two curves at this point are not identical. There are states within the integral cooling region that locally exhibit differential heating. Let us observe the state marked by X in the same figure and at a temperature below Θ^{MAX} . It is characterized by differential warming ($\mu < 0$) and is located outside of the differential inversion curve. However, state X still exhibits cooling in the integral sense, ($\Delta T_h < 0$), being positioned under or below the integral inversion curve, when gas beginning at this state undergoes an isenthalpic expansion from P (at state X) down to 0.1 MPa.

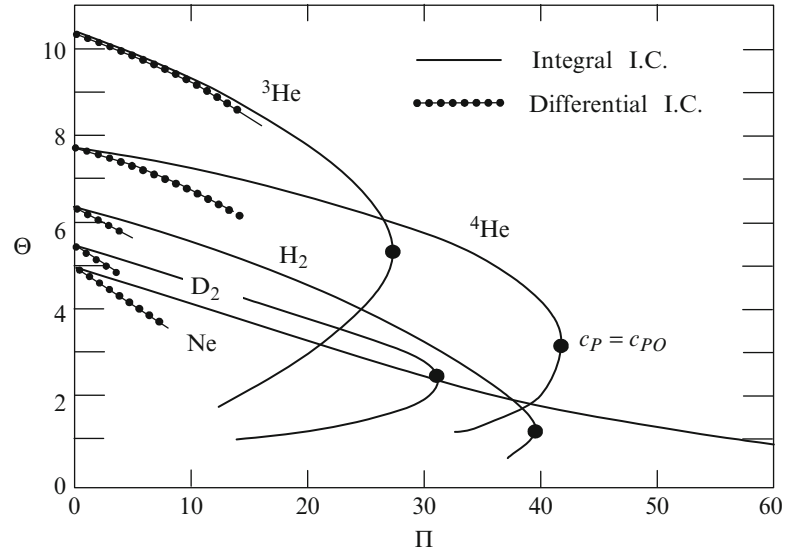
Koepp¹²⁸ was perhaps the first one who explicitly applied the term *integral inversion curve*, in analogy with the differential curve. He showed that the integral inversion curve in the (P, T) plane may exhibit a maximum pressure (which indeed happens only with He-3, He-4, H₂, and D₂ as discussed in the next section) satisfying the condition:

$$c_P = c_{PO} \quad @ \quad P_{INV}^{MAX}(IIC) \quad (2.112)$$

This feature is analogous to the maximum of the differential inversion curve where $(\partial c_P / \partial P)_T = 0$, by Eq. 2.59.

States at temperatures above Θ^{MAX} only undergo warming under an isenthalpic expansion. The quantum gases helium, hydrogen and their isotopes, at room temperatures belong to such a class. However, there are states below Θ^{MAX} where warming is still obtained, such as point Y in Fig. 2.18. Point Y represents the interesting practical application of drilling down to deep natural gas reservoirs, a process that is characterized by high pressure and high temperatures. As reported for such wells in the North Sea^{129,130} typical temperatures inside the reservoir are about 200 °C, which is still below the highest inversion

Fig. 2.19 The integral inversion curves of the quantum gases peak on the pressure scale. The *dotted* lines represent parts of the differential inversion curves



temperature for natural gas. However, the pressure in the reservoir is as high as 110 MPa, and that is above the integral inversion pressure. When the mixture of hydrocarbons is released into the atmosphere, its temperature is typically elevated by about 5–20 °C. As reported, the temperature increase was a surprise.¹³¹ A cooling effect was expected because the temperature of the reservoir was below the maximum inversion temperature associated with the known composition of the natural gas components. However, further analysis of the integral expansion process verified with a reasonable accuracy the warming effect. In other words, the conditions inside the reservoir are located at a state such as point Y in Fig. 2.18.

The I.I.C. was also plotted in previous occasions, in the $P - h$ plane of Fig. 2.10, in the $T - s$ plane of Fig. 2.11 and in the $Z - T$ plane of Fig. 2.17. In the $h - T$ plane, according to Eq. 2.109, the integral inversion states are located on the 0.1 MPa isobar.

2.6.3 The Principle of Corresponding States and the Quantum Gases

Maytal and Van Sciver¹³² correlated the integral inversion curve for low acentricity gases, such as nitrogen, argon, carbon oxide and methane, in terms of reduced properties,

$$\Theta = \Theta^{MAX} - 0.105 \cdot \Pi + 5.644 \cdot 10^{-4} \cdot \Pi^2 \quad (2.113)$$

Maytal and Pfothenauer¹³³ derived the integral inversion curves through the Peng-Robinson equation of state. They correlated its slope at the highest inversion temperature for various gases as a function of the acentricity factor, ω ,

Table 2.3 The peaking states of integral inversion curves of the quantum gases after Maytal and Shavit^{126,127}

	³ He	⁴ He	H ₂	D ₂
Π	27.29	41.80	38.72	32.72
P [MPa]	3.13	9.57	50.92	53.72
Θ	5.26	3.35	1.56	2.35
T [K]	17.46	17.42	51.80	90.00

$$\frac{d\Theta}{d\Pi} = -0.07711 + 0.2282\omega - 0.3577\omega^2 \quad @ \quad \Theta = \Theta^{MAX} \quad (2.114)$$

However, only the quantum gases practically exhibit such an extremum pressure. Other gases at low temperatures and high pressures “penetrate” the solid region before reaching the peak. Maytal and Shavit^{126,127} derived the integral inversion curves for ³He, ⁴He, H₂ and D₂ (Fig. 2.19) and verified Koeppe’s maxima condition, $c_P = c_{P0}$ for ⁴He and H₂. The absolute and reduced values of the *maximum inversion pressures* and the corresponding temperatures are listed in Table 2.3. For example, ³He pressurized above only 3.13 MPa will warm up under any isenthalpic expansion, even if cooled to very low temperatures.

2.6.4 The van der Waals Equation of State and the Integral Inversion Curve

2.6.4.1 The Plane of (P,T)

The integral inversion curve for a van der Waals gas is given in terms of reduced temperature and pressure by

$$\frac{\Pi}{\Pi^{MAX}} = 1 - \left(\frac{\Theta}{\Theta^{MAX}} \right)^2 \quad (2.115)$$

with a maximum integral inversion pressure of $\Pi^{MAX} = 27$. Its slope at the maximum inversion temperature is $d\Theta/d\Pi = -1/8$ which is one half of the value of the slope for the differential inversion curve.

2.6.4.2 The Reduced Density

The reduced density along the integral inversion curve is described by

$$1. \quad \Phi = 3 \left(1 - \frac{\Theta}{\Theta^{MAX}} \right) \quad (2.116)$$

implying that the density changes linearly with the temperature along the curve; between zero and $3\rho_C$.

$$2. \quad \Phi = 3 \left(1 - \sqrt{\frac{\Pi}{\Pi^{MAX}}} \right) \text{ or} \quad \frac{\Pi}{\Pi^{MAX}} = 2 \left(\frac{\Phi}{3} \right) - \left(\frac{\Phi}{3} \right)^2 \quad (2.117)$$

2.6.4.3 Compressibility, Z

Compressibility, Z, along the integral inversion curve can be expressed in terms of temperature or pressure.

1. As a function of temperature it is given by:

$$Z = \frac{1}{2} \left(\frac{\Theta^{MAX}}{\Theta} + 1 \right) > 1 \quad (2.118)$$

The compressibility has a value of at least 1. It is a monotonically increasing function while approaching lower temperatures as displayed in Fig. 2.17.

2. As a function of pressure, the compressibility is expressed as:

$$Z = \frac{1}{2} + \frac{1}{2\sqrt{1 - \Pi/27}} > 1 \quad (2.119)$$

2.6.4.4 The Relationship Between the Differential and Integral Inversion Curves

The relationship between the differential and integral inversion curves as discussed by Maytal¹²⁰:

1. For the same reduced density, the corresponding reduced inversion temperatures of the integral (I.I.C.) and differential (D.I.C.) inversion curves are related by:

$$\Theta^{MAX} \cdot \Theta_{DIC}(\Phi) = \Theta_{IIC}^2(\Phi) \quad (2.120)$$

2. For the same reduced inversion temperatures, the reduced densities along the two curves are related by:

$$\Phi_{IIC}(\Theta) = 2 \cdot \Phi_{DIC}(\Theta) - \frac{1}{3} \cdot \Phi_{DIC}^2(\Theta) \quad (2.121)$$

3. The residual entropy defined on the basis of common values of (ρ, T)

$$s^r = s(\rho, T) - s^{IG}(\rho, T) \quad (2.122)$$

along the differential inversion curve is given by,

$$s_{DIC}^r = \frac{1}{2} R \ln \left(\frac{\Theta}{\Theta^{MAX}} \right) < 0 \quad (2.123)$$

4. The residual entropy at the same reduced temperature, but on the two inversion curves are related by

$$s_{IIC}^r(\Theta) = 2 \cdot s_{DIC}^r(\Theta) \quad (2.124)$$

$$\left(\frac{d\Phi}{d\Pi} \right)_{DIC} = \left(\frac{d\Phi}{d\Pi} \right)_{IIC} = -\frac{1}{18} \quad @ \quad \Theta^{MAX} \quad (2.125)$$

2.6.4.5 The Differential Joule-Thomson Effect

5. The differential Joule-Thomson effect along the integral inversion curve, monotonically increases towards the lower temperatures, by

$$\bar{\Theta} = \frac{\Theta}{\Theta^{MAX}} \frac{1}{3} > \frac{\varphi}{v_C} = \frac{1}{3} \cdot \frac{1 - \bar{\Theta}}{1 + (1 - \bar{\Theta}) \cdot \bar{\Theta}} > 0 \quad (2.126)$$

2.6.4.6 The Values of $c_P - c_V$

The values of $c_P - c_V$ peaks along the integral inversion curve at $\Theta = \Theta^{MAX}/2$,

$$\frac{4}{3} > \left(\frac{c_P - c_V}{R} \right)_{IIC} = \left[\left(\frac{\Theta}{\Theta^{MAX}} \right)^2 - \frac{\Theta}{\Theta^{MAX}} + 1 \right]^{-1} > 1 \quad (2.127)$$

2.7 Chronological Notes on Inversion States

2.7.1 Witkowski, 1898: The Discovery of the Differential Inversion States

The stimulus for studying the inversion states was the outstanding behavior of hydrogen in the experiments of Joule and Thomson, namely that it warmed up while other gases such as air, nitrogen and CO₂ cooled down. Joule and Thomson were not aware of the concept of inversion states and did not consider them. They related the temperature

increase to the nature of hydrogen rather than to the thermodynamic state of hydrogen at room temperature.

Witkowski¹¹² may be acknowledged as the first to recognize, in 1898, the possibility of an inverted Joule-Thomson effect. He calculated for the first time the (maximum) inversion temperatures at atmospheric pressure, through the van der Waals equation of state for air and hydrogen. Berthelot in 1900¹³⁴ obtained the loci of all states that satisfy $\mu = 0$ in reduced notation, through the same equation of state. On an empirical basis, Witkowski recognized that the alternative interpretation of the cooling effect measured by Joule and Thomson, expressed in the form of $A/T-B$, instead of a/T^2 necessitates the existence of an inversion temperature.

2.7.2 van der Waals and Olszewski, 1900: Focusing on Integral Inversion States

The Linde-Hampson liquefaction apparatus^{135,136,137,138} invented in 1895, played a central role and was present in the background of inversion phenomena in the early studies. This apparatus represented a breakthrough in the attempts to liquefy gases and was an object of interest for van der Waals¹³⁹ who commented:

The accurate knowledge of this process has of late proved to be more necessary than before, as Linde has applied this process for obtaining very low temperatures and as in Linde's apparatus this way of expansion is made use of to obtain liquid air.

Air and nitrogen underwent the cooling process in a straightforward manner. The question posed was whether it was impossible to liquefy hydrogen with the same machine. Van der Waals understood that the machine was driven by the integral Joule-Thomson effect. Hence, his first concern was the maximum of

the cooling T_1-T_2

and the fact that “there is a most advantageous value for P_1 in Linde's apparatus” for air liquefaction. His second concern was:

For the points, for which the cooling = 0.

Applying his equation of state, he obtained a reduced form of the *integral inversion curve*.

Olszewski in 1902¹⁴⁰ was the pioneering experimentalist of inversion states. He obtained a single Integral Inversion Point for hydrogen, which was 11.35 MPa and 192.8 K. In 1907,¹⁴¹ he measured the integral inversion temperatures for various pressures of nitrogen (3–15.9 MPa) and air (2–6 MPa) and displayed an integral inversion curve. Unfortunately, something in the experiment went wrong and his results were completely incorrect. Roebuck in 1925²⁸ mentions that the curve goes in the wrong direction.

2.7.3 Porter, 1906: Shifting the Attention to the Differential Inversion States

Up to this point, the main interest was in the integral inversion states. It was Porter in 1906¹⁰¹ who shifted the attention away from the integral inversion curve to the differential one through his words:

Instead of dealing with a **finite** change of pressure, we shall consider the difference of pressure on two sides of the plug as being **infinitesimal**; considerable **simplification results from doing this**.

Indeed, Porter contributed significantly to the theoretical understanding of the differential inversion states and curve. However, it was only in 1962 (to the best of authors' knowledge) that the integral inversion curve was studied again by Koepe.¹²⁸

2.7.4 The Differential Inversion Curve by the EOS

Around 1970, a trend of deriving differential inversion curves through various cubic equations of states began, through the use of numerical methods. These studies compared and examined the broad range of proposed equations of state by their capability to fit, or predict experimentally derived inversion points. Each one attempted to employ a new and more advanced equation of state, such as the van der Waals, modified van der Waals, Dieterici, generalized Dieterici, Redlich-Kwong, Soave-Redlich-Kwong, Martin-Hou, Bettie-Bridgman, Benedict-Webb-Rubin, Truncated Virial, Peng-Robinson, Lee-Kesler, or Perturbed Hard-Chain equation. Miller¹¹⁵ reported a best fit with the Redlich-Kwong equation of state, while Dilay^{142,143} with the Lee-Kesler equation of state. Other contributors are Juris et al.,¹⁴⁴ Feroiu et al.,¹⁴⁵ Geana et al.,¹⁴⁶ and Najjar et al.¹⁴⁷ Colina, Colazo et al.¹⁴⁸ used the data of inversion states for adjusting or calibration of any proposed equation of state.

2.7.5 Molecular Simulation of the Inversion States

Gustafson in 1970^{11,12} applied a molecular approach regarding inversion phenomena. He determined maximum inversion temperatures of binary mixtures of monatomic gases. McClure in 1971⁸ used a square well model for the intermolecular potential and obtained fairly accurate maximum inversion temperatures for CO₂, N₂ and Ar. Heyes and Llaguno¹¹⁸ in 1992 derived the entire differential inversion curve by the Lennard-Jones intermolecular potential. Similar contributions belong to Colina and Muller,^{149,150} Vrabec et al.,¹⁵¹ Chacin et al.¹⁵² and Colina et al.¹⁵³ obtained the

Table 2.4 Chronology of exploring differential and integral inversion phenomena of gases

Remarks	
(a) <u>Underlined</u> are the references which deal with integral inversion states	
(b) Only the first author of each reference is mentioned in this list	
$\Delta T_h = 0$	<u>1900</u> Van der Waals ¹³⁹
H ₂	<u>1902</u> Olszewski, ¹⁴⁰ 1921 Jacob, ¹²⁵ <u>1923</u> Meissner ¹⁵⁶ , 1940 Wooley ¹⁵⁷ , 1956 Koeppe, ⁴⁶ 1960 Koeppe, ¹⁰⁵ 1966 Gunn, ¹⁰⁹ 1972 Hendricks, ⁸⁹ <u>1997</u> Maytal ^{126,127}
N ₂	<u>1907</u> Olszewski, ¹⁴¹ 1910 Porter, ¹⁰² 1921 Jacob, ¹²⁵ 1935 Roebuck, ³² 1961 Din ¹⁵⁸ , 1972 McClure, ⁸ <u>1972</u> Hendricks ⁸⁹
Air	<u>1907</u> Olszewski, ¹⁴¹ 1916 Schultze ¹⁵⁹ , 1917 Jacob ¹⁶⁰ , 1921 Jacob, ¹²⁵ 1925 Roebuck, ²⁶ 1956 Din ¹⁶¹ , 1972 Hendricks, ⁸⁹ <u>1993</u> Najjar, ¹⁴⁷ Colina ¹⁴⁸
CO ₂	<u>1910</u> Porter, ¹⁰² 1921 Jacob, ¹²⁵ 1942 Roebuck, ⁴³ <u>1971</u> McClure, ⁸ 1999 Chacin, ¹⁵² 2002 Colina ¹⁵³
(C ₂ H ₅) ₂ O	<u>1921</u> Jacob, ¹²⁵ <u>1966</u> Novak ¹⁶²
C ₂ H ₂	<u>1921</u> Jacob ¹²⁵
Air-He	<u>1933</u> Roebuck ²⁸
Ar	<u>1934</u> Roebuck, ³⁰ 1960 Koeppe, ¹⁰⁵ 1969 Gosman ¹⁶³ , <u>1972</u> Hendricks ⁸⁹
N ₂ -He	<u>1938</u> Roebuck, ⁵¹ 1939 Hirshfelder, ³ 1959 Koeppe, ⁵² 1965 Pfenning ¹⁶⁴ , <u>1966</u> Gunn ¹⁰⁹
Ar-He	<u>1940</u> Roebuck ⁵⁷
⁴ He	<u>1940</u> Zelmanov, ²⁹ 1960 Koeppe, ¹⁰⁵ 1966 Gunn, ¹⁰⁹ 1972 McCarty ¹⁶⁵ , 1972 Hendricks, ⁸⁹ <u>1997</u> Maytal ^{126,127}
NH ₃	<u>1956</u> Din ¹⁶⁶
CO	<u>1956</u> Din, ¹⁶⁶ 1966 Novak, ¹⁶² <u>1972</u> Hendricks ⁸⁹
C ₃ H ₈	<u>1956</u> Din ¹⁶¹
N ₂ -H ₂	<u>1959</u> Koeppe, ⁵³ <u>1966</u> Gunn. ¹⁰⁹
CH ₄	<u>1961</u> Din, ¹⁵⁸ 1966 Novak, ¹⁶² <u>1972</u> Hendricks ⁸⁹
He-Ne	<u>1967</u> Gladun ⁶⁴
D ₂	<u>1967</u> Prydz ¹⁶⁷ , <u>1997</u> Maytal ¹²⁷
Ne	<u>1966</u> Gunn, ¹⁰⁹ 1968 Runge ¹⁶⁸ , <u>1972</u> Hendricks ⁸⁹
C ₂ H ₄	<u>1966</u> Novak ¹⁶²
C ₂ H ₆	<u>1966</u> Novak ¹⁶²
³ He	<u>1960</u> Koeppe, ¹⁰⁵ 1970 Duant ¹⁶⁹ , 1974 Kraus ¹⁷⁰ , <u>1987</u> Kamatsevich, ¹⁰⁶ 1995 Maytal, ¹⁰⁷ <u>1997</u> Maytal ¹²⁷
F ₂	<u>1970</u> Prydz ^{171,172} , 1972 Hendricks ⁸⁹
O ₂	<u>1972</u> Hendricks ⁸⁹
Kr	<u>1960</u> Koeppe, ¹⁰⁵ <u>1972</u> Hendricks ⁸⁹
Xe	<u>1972</u> Hendricks ⁸⁹
H ₂ S	<u>2005</u> Kristof ¹⁵⁴
two phases	<u>2006</u> Nichita, ¹¹¹ inversion states of hydrocarbon mixtures in the two phase zone
General	<u>2007</u> Maytal, ¹²⁰ the interrelation between the differential and integral inversion curves
N ₂ + CH ₄	<u>2009</u> Vrabec. ¹⁵⁵
CH ₄ + CO ₂ ,	N ₂ + CH ₄ + CO ₂ , CH ₄ + C ₂ H ₆ , N ₂ + CH ₄ + CO ₂

inversion curve of CO₂ and Kristof et al.¹⁵⁴ of H₂S. Vrabec et al.^{151,155} applied this method for mixtures of nitrogen, hydrocarbons and carbon dioxide.

2.7.6 Miscellaneous

- (a) Table 2.4 displays the chronology of inversion phenomena studies. Yet unaddressed are the inversion states for the gas-solid Joule-Thomson effect as explored by Rybolt and Pierotti.^{95,96,97,98,99}
- (b) One could not imagine a more appropriate introduction or closure for any treatise of the Joule-Thomson inversion phenomena than the following remark:
Very often, the Joule-Thomson effect is associated in people's mind with cooling and temperature reduction. The following treatise indicates that the opposite

conclusion is more correct, if we have to choose one of them. Only a limited group of thermodynamic states will produce cooling while the group of states that result in warming is boundless. Thus, **the Joule-Thomson effect is "more" associated with warming than with cooling.** Section 9.5 will deal with some aspects of warming by a Joule-Thomson "cryocooler".

2.8 Joule Expansion

2.8.1 The Joule Coefficient by Pressure

The coefficient of free expansion, through an adiabatic increase of volume known as *Joule's expansion* is called the Joule coefficient. Such an expansion does not involve any interaction of work.

Hausen in 1926¹⁷³ and Roebuck²⁴ in 1930, analyzed this process by its characteristic coefficient,

$$\eta = (\partial T / \partial P)_u \quad (2.128)$$

As with Joule-Thomson expansion, Joule expansion is not accompanied by a temperature change for an ideal gas. In contrast to the Joule-Thomson expansion process, Joule expansion, preserving energy, is not associated with continuous flow through an open system, but is rather realized in a closed system process.

The associated generation of entropy in Joule expansion can be determined from:

$$ds = \frac{1}{T} \cdot d(P \cdot v) \quad (2.129)$$

The Joule coefficient is related to the Joule-Thomson coefficient by:

$$\mu - \eta = (\partial T / \partial P)_{P_v} \quad (2.130)$$

or

$$\eta = \mu + \frac{1}{c_P} \left[\frac{\partial(P \cdot v)}{\partial P} \right]_u = \frac{\mu \cdot c_P + \left[\frac{\partial(P \cdot v)}{\partial P} \right]_T}{c_P - \left[\frac{\partial(P \cdot v)}{\partial T} \right]_P} \quad (2.131)$$

The extrapolation of Joule's coefficient to zero pressure results in a finite (non-zero) value, as formulated by Hirschfelder et al.³:

$$\eta_0 = \mu_0 \frac{c_{P0}}{c_{V0}} + \frac{B}{c_{V0}} = \frac{T}{c_{V0}} \frac{dB}{dT} \quad (2.132)$$

where B is the temperature dependent second virial coefficient of the equation of state expanded by density.

On the microscopic level, Hirschfelder³ derived the zero pressure free expansion coefficient,

$$\eta_0 = \frac{2\pi N}{c_{P0}} \int_0^\infty r^2 \cdot \frac{E(r)}{kT} \cdot \exp\left[-\frac{E(r)}{kT}\right] dr \quad (2.133)$$

where $E(r)$ is the energy of interaction of two molecules separated by a distance r , N is Avogadro's Number, and k is Boltzmann's constant (compare to Eq. 2.29).

For the van der Waals gas:

$$\frac{\mu}{\eta} = 1 - \frac{R/c_P}{1 - b/v} \quad (2.134)$$

2.8.2 The Joule Coefficient by Volume

The free expansion coefficient may be alternatively related to the change of volume instead of pressure. Thus a relationship analogously corresponding to the volumetric JT coefficient of expansion by Eq. 2.55, is sometimes used,

$$\eta^* \equiv \left(\frac{\partial T}{\partial v} \right)_u = -\frac{1}{c_V} \cdot \left[\frac{\partial u}{\partial v} \right]_T \quad (2.135)$$

$$\eta^* = -\frac{1}{c_V} \left[T \left(\frac{\partial P}{\partial T} \right)_v - P \right] = \frac{1}{c_V} \left(\frac{\beta T}{K_T} - P \right) \quad (2.136)$$

where the *isothermal compressibility* is:

$$K_T \equiv -\frac{1}{v} \left(\frac{\partial v}{\partial P} \right)_T = \frac{1}{P} - \left(\frac{\partial \ln Z}{\partial P} \right)_T \quad (2.137)$$

The Joule coefficient may be expressed in terms of density as:

$$\eta^* \equiv -\rho^2 \cdot \left(\frac{\partial T}{\partial \rho} \right)_u = -\frac{1}{c_V} \cdot \left[T \left(\frac{\partial P}{\partial T} \right)_\rho - P \right] \quad (2.138)$$

$$\eta^* = \frac{\rho^2}{c_V} \left(\frac{\partial P}{\partial \rho} \right)_T \left[\phi - \left[\frac{\partial(P/\rho)}{\partial P} \right]_T \right] \quad (2.139)$$

The extrapolated value of η^* to zero pressure is zero. Still, at sufficiently low pressures and non zero low densities (ρ_0), the following relation approximately holds:

$$\eta_0^* = -\frac{R}{c_V} T^2 \rho_0^2 \frac{dB}{dT} \quad (2.140)$$

For the van der Waals gas:

$$\eta^* = -\frac{a}{c_V} \frac{1}{v^2} \quad \text{and} \quad \eta_0^* = -\frac{a}{c_{V0}} \rho_0^2 \quad (2.141)$$

The integral temperature change under Joule's expansion of van der Waals' gas,

$$\Delta T_u \equiv T_2 - T_1 = -\frac{a}{c_V} \left(\frac{1}{v_1} - \frac{1}{v_2} \right) \quad (2.142)$$

While approaching the critical state and at its vicinity, the Joule coefficient may be approximated⁸⁹ by,

$$\eta_C^* \approx \frac{5 P_C}{(c_V)_C} \quad (2.143)$$

2.8.3 The Inversion of the Joule's Effect

The three entities, η_0 , dB/dT and μ_0^* (as pressure approaches zero) change their sign at the same temperature. It is the inversion temperature of Joule expansion and is associated with the peak of $B(T)$. Below this inversion temperature, the free expansion process lowers the temperature of the gas, as is generally known in common practice. However, above it, the temperature of the freely expanded gas increases.

Roebuck²⁸ mentions in 1934 that above 203 K, helium warms up under free expansion. Later on, the transition temperature was derived by Keyes¹⁷⁴ as 195 K and by Keesom¹⁷⁵ as 175 K as discussed elsewhere.¹⁷⁶ For helium it occurs at a reduced temperature as high as approximately $\Theta = 35$. For nitrogen, it occurs at “only” about $\Theta = 17$ but still at as high a temperature^{89,177,178} as 2,140 K.

Boschi-Filho et al.¹⁷⁹ studied the zero pressure inversion of Joule's effect by examining experimental data of the second virial coefficients of simple gases as a basis for utilizing the Lennard-Jonnes potential. They obtain a general relationship between the inversion temperature of the Joule effect, $T_{INV,J}$, and of the Joule-Thomson effect, $T_{INV,J-T}$:

$$T_{INV,J} = 3.911 \cdot T_{INV,J-T} \quad (2.144)$$

In addition, they obtained $T_{INV,J-T}$ values for some simple gases with relatively wide ranges of uncertainties associated with the experimental data: argon 2,000–5,600 K, helium 150–250 K, hydrogen 390–930 K, nitrogen 1,100–3,900 K, neon 480–1,260 K, and xenon 140–2,400 K. The authors mention that the available experimental data for krypton and oxygen would not suffice for deriving $T_{INV,J-T}$.

One should observe that the van der Waals gas does not undergo an inversion of the Joule effect since $B(T)$ for a van der Waals gas does not peak and its derivative with respect to T does not change sign.

2.9 Isentropic Expansion

When work is extracted externally from a given amount of gas, in a reversible manner and without any interaction of heat, it experiences an isentropic expansion.

2.9.1 The Coefficient of Isentropic Expansion

2.9.1.1 The Real and the Ideal Gas

The isentropic expansion processes are characterized by the coefficient,

$$\mu_s = \left(\frac{\partial T}{\partial P} \right)_s \quad (2.145)$$

For an ideal gas, the Joule-Thomson coefficient vanishes, but not the isentropic one, as it becomes:

$$\mu_s^{IG} = \frac{v}{c_p} > 0 \quad (2.146)$$

The ideal gas exhibits a positive isentropic expansion coefficient.

The parameter μ_s^{IG} may be interpreted as follows: any state characterized by (P, v, T) and c_p of a real gas, exhibits a value of μ_s . If this real gas would behave as an ideal gas with the same v and c_p , it would have an isentropic expansion coefficient, μ_s^{IG} instead of μ_s . As such μ_s^{IG} may be used to normalize μ_s so that μ_s/μ_s^{IG} is a measure of the real gas effect, whether larger or smaller than 1. One might expect, that for states that are well represented by the ideal gas model this ratio would approach unity.

2.9.1.2 In Terms of the Thermal Expansivity

In terms of the thermal expansivity, β , the isentropic expansion coefficient may be expressed as:

$$\mu_s = \frac{T}{c_p} \left(\frac{\partial v}{\partial T} \right)_P = \frac{1}{c_p} \left(\frac{\partial v}{\partial \ln T} \right)_P = \frac{v}{c_p} \beta T \quad (2.147)$$

By Eqs. 2.47 and 2.146,

$$\frac{\mu_s}{\mu_s^{IG}} = \beta T = \frac{\beta}{\beta^{IG}} \quad (2.148)$$

This relationship means that $\mu_s > 0$ (always), or that an isentropic expansion always generates a cooling effect, at least as long as $\beta > 0$, which is the most common occurrence. The case of $\beta < 0$ is quite rare. One example is provided by helium in its super fluid form, He-II, below the Lambda point, where $(\partial v / \partial T)_P < 0$. Therefore, the coolers that utilize an isentropic expansion cannot cool helium down below the Lambda point.¹⁸⁰

2.9.1.3 In Terms of Heat Capacities

$$\mu_s = \frac{c_p - c_v}{c_p} \left(\frac{\partial T}{\partial P} \right)_v \quad (2.149)$$

2.9.1.4 In Terms of the Speed of Sound

In terms of the speed of sound, c ,

$$\frac{\mu_s}{\mu_s^{IG}} = \frac{1}{c} \left[T \cdot \frac{c_P}{c_V} \cdot (c_P - c_V) \right]^{1/2} \quad (2.150)$$

or

$$\mu_s = \mu + v/c_P \quad (2.156)$$

2.9.1.5 Derivatives Through the Equations of State

$$\mu_s = \frac{R \cdot T}{P \cdot c_P} \cdot \left[Z + T \cdot \left(\frac{\partial Z}{\partial T} \right)_P \right] \quad \text{or} \quad \frac{\mu_s}{\mu_s^{IG}} = 1 + \left(\frac{\partial \ln Z}{\partial \ln T} \right)_P \quad (2.151)$$

$$\frac{\mu_s}{\mu_s^{IG}} = \frac{\left(\frac{\partial P}{\partial \ln T} \right)_\rho}{\left(\frac{\partial P}{\partial \ln \rho} \right)_T} \quad (2.152)$$

2.9.1.6 For the van der Waals Gas

$$\frac{\mu_s \cdot c_P}{v_C} = \frac{1}{3} \cdot \frac{c_P - c_V}{R} \cdot \left(\frac{v}{b} - 1 \right) \quad \text{or} \quad \frac{\mu_s}{\mu_s^{IG}} = \frac{c_P - c_V}{R} \left(1 - \frac{b}{v} \right) \quad (2.153)$$

The isentropic expansion coefficient is always positive since $v > b$ and $c_P > c_V$. As well, for the same gas,

$$\frac{\mu_s \cdot c_P}{v_C} = \frac{\frac{1}{3} \cdot \frac{v}{b} \cdot \left(1 - \frac{b}{v} \right)}{1 - \frac{27/4}{T/T_C} \cdot \frac{b}{v} \cdot \left(1 - \frac{b}{v} \right)^2} \quad \text{or} \quad \frac{\mu_s}{\mu_s^{IG}} = \frac{1 - \frac{b}{v}}{1 - \frac{27/4}{T/T_C} \cdot \frac{b}{v} \cdot \left(1 - \frac{b}{v} \right)^2} \quad (2.154)$$

2.9.1.7 The Isentropic Expansion Coefficient by Density

The isentropic expansion coefficient by density or volume instead of pressure,

$$\left(\frac{\partial T}{\partial \rho} \right)_s = \frac{R T}{\rho c_V} \left[Z + T \left(\frac{\partial Z}{\partial T} \right)_\rho \right] \quad (2.155)$$

2.9.2 The Relationship Between the Isentropic and Isenthalpic Expansion

2.9.2.1 The Relationship Between μ_s and μ

In 1926 Hausen¹⁸¹ already studied μ_s in relation with the Joule-Thomson coefficient, μ ,

$$\mu = \mu_s - \mu_s^{IG} \equiv \mu_s^r \quad (2.157)$$

which means that the residual of μ_s is the Joule-Thomson coefficient, μ . Note that the residual is μ_s^r and not μ_s^R since the comparison is to the ideal gas state with a common (v, T) and not (P, T) .

Hausen noticed for the two coefficients that μ approaches μ_s near the critical state, since c_P is growing infinitely in that region. However, a similar behavior occurs even for a finite value of C_P when v becomes small, for example at low temperatures and high pressures.

By Eqs. 2.147 and 2.10 we get,

$$\frac{\mu}{\mu_s} = 1 - \frac{v}{T} \left(\frac{\partial T}{\partial v} \right)_P = 1 - \left(\frac{\partial \ln T}{\partial \ln v} \right)_P \quad (2.158)$$

and by Eqs. 2.14 and 2.146,

$$\frac{\mu}{\mu_s^{IG}} = \left(\frac{\partial \ln Z}{\partial \ln T} \right)_P \quad (2.159)$$

Duant¹⁸² discussed this ratio for air. In terms of the speed of sound,

$$\frac{\mu}{\mu_s} = 1 - \frac{c}{[T \cdot (c_P/c_V) \cdot (c_P - c_V)]^{1/2}} \quad (2.160)$$

2.9.2.2 The Role of the Differential Inversion Curve

Inside the inversion curve, where μ is positive, we get by Eq. 2.157, $\mu_s > v/c_P$, and the opposite outside the inversion curve, as illustrated in Fig. 2.20.

Since $\mu_s > 0$, Eq. 2.156 bounds the value of μ , so that,

$$\mu > -v/c_P \quad \text{or} \quad \phi < v \quad (2.161)$$

Inside the D.I.C. this relation is not informative since any positive entity ($\mu > 0$) is always larger than a negative one. However, outside the D.I.C. where $\mu > 0$, this relation bounds the magnitude of (the negative) μ , so that,

$$|\mu| < v/c_P \quad @ \quad \mu < 0 \quad (2.162)$$

For example, for nitrogen at 68 MPa and 65 K, $v/c_P = 0.55$ K/MPa, hence $|\mu|$ has to be smaller: and indeed, $\mu = -0.45$ K/MPa.

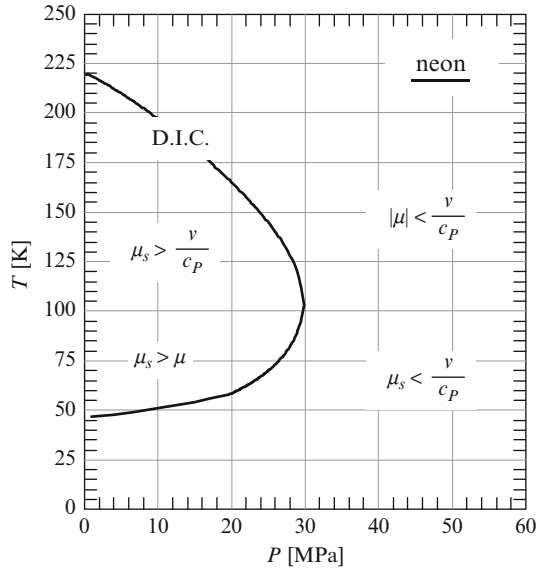


Fig. 2.20 The μ_s on both sides of the D.I.C. Also is shown the Bound of μ at its negative domain. The background is the D.I.C. of neon

In general, by Eq. 2.157 we get,

$$\mu_s > \mu \quad (2.163)$$

This relation is meaningful only inside the D.I.C.: there any one of the two entities orientates the other one. For example, neon at 60 K and 5 MPa yields a value of $\mu = 2.456$ K/MPa. The value of μ_s has to be larger; and indeed, $\mu_s = 4.236$ K/MPa. However, outside the D.I.C. the relation $\mu_s > \mu$ is trivial and useless, since $\mu_s > 0$ and $\mu < 0$; $|\mu|$ and μ_s cannot be correlated.

2.9.2.3 The Integral Effect

The integral effect, in analogy to ΔT_h by Eq. 2.8 would be,

$$\Delta T_s \equiv \left[\int_{P_1}^{P_2} \mu_s dP \right]_s \quad (2.164)$$

From a common initial state (P_1, T_1) and expanding down to the same pressure P_2 , we obtain, that the isentropic temperature drop, ΔT_s , is larger than the isenthalpic one; ΔT_h ; or ΔT_s is more negative, thus,

$$\Delta T_s < \Delta T_h \quad (2.165)$$

since by Eq. 2.163 the isentropic integrand, μ_s , in Eq. 2.164 is unconditionally larger than the isenthalpic one, μ . For example, for nitrogen expanding from the initial state of

300 K, 40 MPa, down to 6 MPa we have $\Delta T_s = -124.2$ K and only $\Delta T_h = -24.2$ K.

For an ideal gas, the initial and final temperatures, T_1 and T_2 are related with the corresponding pressures, $P_1 > P_2$, in the form,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{(\kappa-1)/\kappa} \quad \text{or} \quad \frac{\Delta T_s^{IG}}{T_1} = \frac{T_2 - T_1}{T_1} = \left(\frac{P_2}{P_1} \right)^{(\kappa-1)/\kappa} - 1 < 0 \quad (2.166)$$

For a real gas like nitrogen, at elevated pressures this prediction generally over-estimates the reduction of temperature but under-estimates the temperature reduction for pressures below about 10 MPa, as demonstrated elsewhere.¹⁸³

2.9.3 The Isentropic Expansion and Cryocoolers

The isentropic expansion process is utilized for cryocooling as an elementary mechanism for reducing the temperature. The three typical versions are:

- Expansion against a force, of a confined amount of gas opposing a piston, as utilized in a Stirling cryocooler:
- Expansion against a torque, while a stream flows through a turbine as in the reverse Brayton cycle, and the last one, which is also the earliest one:
- A fast discharge of a pressure vessel: the remaining gas in the vessel undergoes an isentropic expansion and decreases its temperature as in Gifford-McMahon and Pulse-tube cryocoolers.

Simon^{184,185,186,187} in 1932 built a helium liquefier based on the last principle. After being precooled by liquid hydrogen, the rapid discharge of pressurized helium caused the remaining amount of gas in the vessel to turn into liquid. This principle takes us back to the origin of cryogenics. Louis Cailletet in 1877¹⁸⁸ liquefied oxygen for the first time using a similar approach. Attempts to utilize this effect for miniature Joule-Thomson cryocooling is discussed later in Sect. 5.5.5.6.

2.10 Preserving the Stagnation Enthalpy of Flow

The Joule-Thomson effect as discussed in Sect. 2.2 was formally based on the assumption that the expanding stream (adiabatic and with no work production) preserved the enthalpy, h . However, the accurate energy balance should

be based on the total enthalpy or the *stagnation enthalpy*, h^0 , which includes the kinetic energy, thus preserving:

$$h^0 = h + V^2/2 = \text{Const} \quad (2.167)$$

Therefore, the Joule-Thomson model that assumes a constant value of h is a good approximation for “small” velocities, that is when the specific kinetic energy is small compared to the integral Joule-Thomson effect:

$$V^2/2 \ll \Delta h_T \quad (2.168)$$

Arp¹⁸⁹ discussed the general case accounting for any specific kinetic energy and derived an expression that is analogous to the Joule-Thomson coefficient,

$$\left(\frac{\partial T}{\partial P}\right)_{h^0} = \frac{\mu \cdot c_P + V^2 K_T}{c_P + V^2 \beta} \quad (2.169)$$

The above expression includes two parameters, the *thermal expansivity*, β , and the *isothermal compressibility*, K_T . These two parameters indicate the changes of specific volume, which manifests itself in changes of the flowing velocities and densities or kinetic energies. If they are small and consequently the velocities are small, then $\mu = (\partial T/\partial P)_h$ closely approaches $(\partial T/\partial P)_{h^0}$. For the van der Waals gas, β can be obtained through Eqs. 2.148 and 2.154 and by Eq. 2.137,

$$K_T = \frac{1}{54P_C} \cdot \frac{\frac{27/4}{T/T_C} \cdot \frac{v}{b} \cdot \left(1 - \frac{b}{v}\right)^2}{1 - \frac{27/4}{T/T_C} \cdot \frac{b}{v} \cdot \left(1 - \frac{b}{v}\right)^2} \quad (2.170)$$

In general, $(\partial T/\partial P)_{h^0}$ may be positive even if μ vanishes or is zero due to the velocity of the gas. This additional mechanism of temperature reduction is explained on the macroscopic level is viable also for ideal gases, and is discussed in textbooks of compressible gas dynamics.^{15,190,191}

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This chapter is entirely devoted to the traditional Linde-Hampson cryocooling process that employs pure gases at elevated pressures, which are practically at least twice the critical pressure. In terms of the perspective of cryocoolers provided in Sect. 1.2, the interchanger of a Linde-Hampson system is the recuperator, whose cold stream capacity rate, $\dot{C}_L$, is lower than that of the hot stream, $\dot{C}_H$. Hence, the pinch point occurs at the warm end of the recuperator. The present chapter also discusses the Linde-Hampson liquefier, which was the original target of the inventors.^{1,2,3,4} The notation of thermodynamic states along the cycle, 1-2-3-4-5-1, is consistent with that of the unified model of cryocoolers in Fig. 1.2.

This chapter deals with the steady state operation of the Linde-Hampson process. The cool down process, which leads to the steady state performance, will be discussed separately in Chap. 7. The First Law of Thermodynamics is applied to balance the energy flow through different parts (as control volumes) of the system. This enables a formulation of the general performance equation, the yield of liquefaction, the nozzle inlet temperatures for various gases, the driving temperature difference between the recuperating streams, the significance of the heat exchanger's efficiency and sizing for the heat exchanging area. Subsequently, the Second Law of Thermodynamics is applied to evaluate the input power required to sustain a steady state cooling process.

3.1 General Perspective

3.1.1 The Fundamental Elements of the Linde-Hampson Cycle

The Linde-Hampson cryocooling process includes the following functions:

3.1.1.1 Throttling

Throttling expansion as the elementary mechanism of reducing the temperature while preserving enthalpy (that is a real gas effect),

3.1.1.2 Recuperation

Recuperation is accomplished via a counter-flow heat exchange ("interchanging") between a low pressure stream and a high pressure stream. This function generates a positive feedback that amplifies the elementary temperature drop to further lower the temperature, and,

3.1.1.3 Sub-critical Expanded Pressure

Sub-critical expanded pressure at the outlet of the throttle. In this way, the single phase stream entering the throttle turns into a two-phase cryogen stream after expansion.

3.1.1.4 A Supply Pressure that Is Above the Critical Point

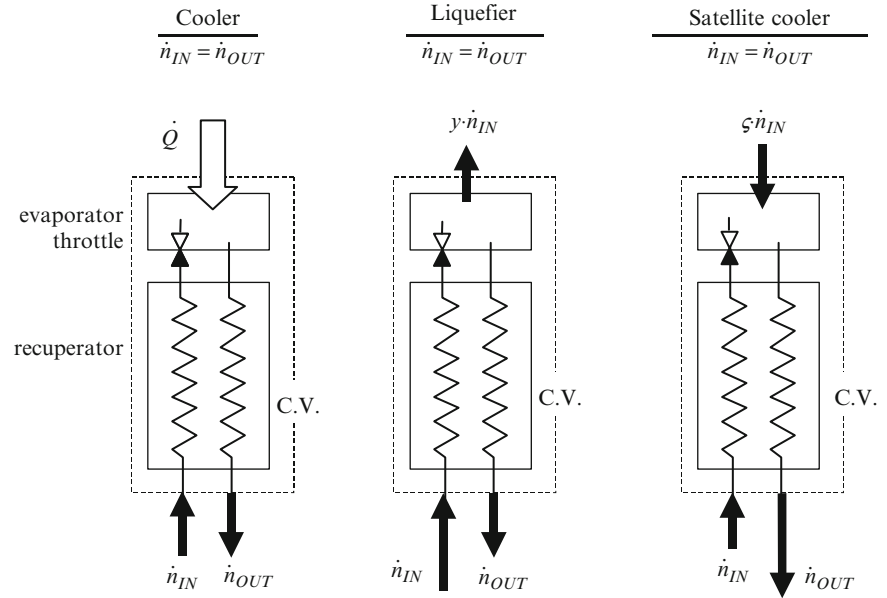
A supply pressure that is above the critical point is utilized in most cases. However, the Linde-Hampson cycle is also viable with a sub-critical supply pressure. Although such a condition is less common (very often arranged with a precooler), in some cases it might even be advantageous as discussed in Sects. 5.6.6.1 and 6.4.5.2.

3.1.1.5 A Phase Transition

A phase transition occurs twice: First, a new phase is generated by the throttling expansion below the critical pressure. Second, this phase disappears in one of the following two manners:

- By absorbing heat at constant temperature, thereby operating as a cryocooler, or,
- By being extracted from the cycle thereby performing the function of a liquefier (or a "solidifier").

Fig. 3.1 Three versions of the Linde-Hampson cycle, distinguished by the flow rate balance. The terms $y\dot{n}_{IN}$ and $\zeta\dot{n}_{IN}$ represent streams of liquid coolant crossing the borders of the control volumes C.V.



3.1.2 Classification by Flow Rate Balance

Three versions of this thermodynamic cycle are displayed in Fig. 3.1, distinguished by the balance of the flow rates before and after expansion:

3.1.2.1 The Cooler

The cooler is characterized by $\dot{n}_{IN} = \dot{n}_{OUT}$. Naturally, this is the most relevant application for the subject of this book. The entire liquefied portion of gas evaporates while absorbing the heat load at the boiling point temperature. High and low-pressure streams share a common circulating flow rate. In this version the cooler may operate as a closed system.

3.1.2.2 The Liquefier

The liquefier has an unbalanced flow rate, $\dot{n}_{IN} > \dot{n}_{OUT}$. This is the process first proposed by Linde and Hampson. Its purpose is the production and supply of liquid cryogen. In this case, the liquefied yield is continuously removed (instead of being evaporated) producing the effect of a phase separation. Hence, the low pressure stream flowing back toward the compressor has a reduced flow rate.

3.1.2.3 The Satellite Cooler

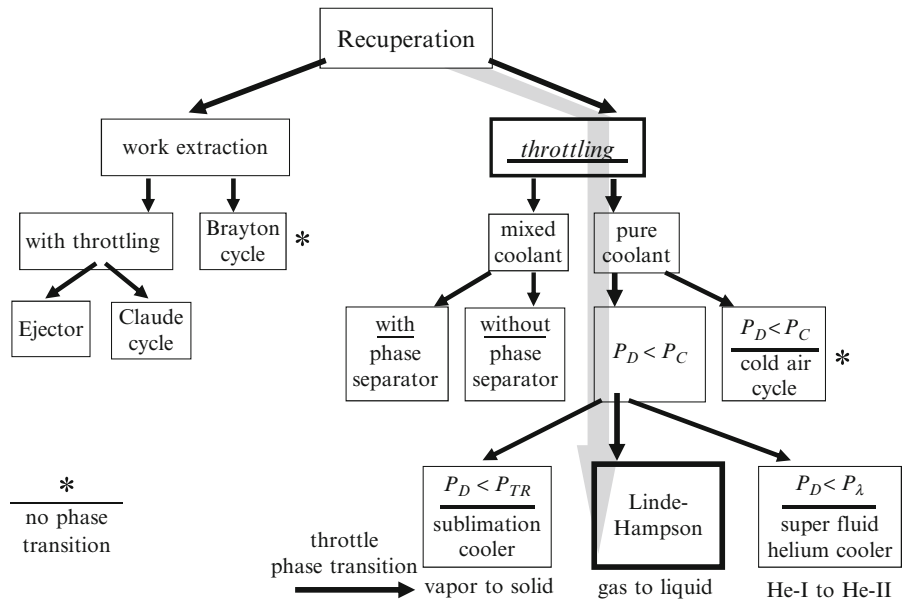
The satellite cooler (or the economizer) has the opposite imbalance of flow rates as compared to the liquefier, $\dot{n}_{IN} < \dot{n}_{OUT}$. The satellite cooler is typically utilized for

large-scale helium cooling systems. Liquid cryogen is injected at the evaporator inducing a larger yield of liquid production. It could be said that the injected liquid flow rate magnifies the total yield. Furthermore, in the case of helium at room temperature, the injection of cryogen takes the place of a precooling stage. Otherwise, the expanded helium would warm up; being above its inversion temperature and no cooling would be produced. All liquid helium evaporates in the evaporator as in a usual cooler. Readers may find a description and analysis of this version elsewhere.^{5,6,7,8,9,10,11,12,13,14,15,16}

3.1.2.4 Remarks

1. The last two versions are inherently open systems even though the flow is circulated by a compressor in a closed loop. This is because a mass flux crosses the borders of these systems, either going out of the system (as in the case of the liquefier) or coming into the system (as in the case of the satellite cooler).
2. The cooler and the liquefier are characterized by $\dot{C}_H > \dot{C}_L$, therefore, both have the pinch point at the warm end.
3. The performance of the cryocooler and the liquefier is determined by the gas conditions at the warm end, namely, the temperature and pressure of the incoming and outgoing streams.
4. The satellite cooler enlarges the flow rate of the stream having a lower specific heat capacity so that $\dot{C}_H < \dot{C}_L$. Therefore, its pinch point shifts to the cold end, somewhere below 20 K when operating with helium. At the warm end there is a substantial temperature difference.

Fig. 3.2 Classification of various cycles that employ a recuperating counter flow heat exchanger



3.1.3 Classification by Phase Transition

3.1.3.1 Vapor–Liquid Phase Transition

Vapor–liquid phase transition characterizes the vast majority of Linde–Hampson cryocooling devices (Fig. 3.2). The expanded down stream pressure, P_D , is indeed below the critical pressure, P_C , but above the triple point pressure, P_{TR} . Cryocooling occurs when the liquid cryogen in the evaporator boils away as it absorbs heat at the boiling temperature, T_{BOIL} , and at the associated saturation pressure, P_D .

3.1.3.2 Vapor–Solid Phase Transition

Vapor–solid phase transition is significantly less common. It can occur if expanded pressure is below the triple point pressure. In such a case, the gaseous stream throttles into the domain of solid. This type of cryocooling is common to space applications using hydrogen as a coolant.¹⁷ The low pressure afforded by outer space maintains the pumping effect below the triple point pressure of hydrogen. Sublimation of the solid cryogen in the “evaporator” (or rather the “sublimator”) drives the cooling process.

Another example of expansion across the vapor–solid transition curve is the method of producing solid carbon dioxide (“dry ice”) by the Linde–Hampson process. The atmospheric pressure is below the triple point of carbon dioxide. Since the solid yield is removed, the flow rates in the recuperator are not balanced and the device (solidifier) is analogous to the common liquefier (Fig. 3.1).

3.1.3.3 Normal (He-I) to Superfluid Helium (He-II) Transition

Normal (He-I) to superfluid helium (He-II) transition by throttling expansion is utilized for helium cryocoolers operating

below the Lambda point as discussed in Sect. 2.3.2.3 and elsewhere.^{18,19,20,21,22,23} Typically, normal liquid helium at 4.2 K and at sub-critical pressure, enters the Joule–Thomson cryocooler, recuperates and expands inside the two-phase dome to about 1.8 K. A portion of the He-I turns into He-II.

3.1.4 Closely Related Recuperative Cycles

For the sake of completeness, a few other cycles are presented that might be regarded as closely related variants of the Linde–Hampson cycle. These are not exactly the traditional cycle as characterized in the previous paragraph, but still preserve the “spirit” and essence of the original concept while employing recuperation: Fig. 3.2 provides an organized view of this extended family. The use of mixed coolants is discussed separately in Chap. 8.

3.1.4.1 Ejector

Ejector expansion may be considered, in comparison to pure throttling expansion, as a kind of enhancement to the original Linde–Hampson cycle. The ejector has three ports, with two corresponding levels of low pressure (see Fig. 5.30), and the pressure drop is not purely isenthalpic as in the case of pure throttling. Nevertheless, no external work is extracted in this cooling cycle, and in this sense, it remains a version of the original Linde–Hampson cycle. Rietdijk^{24,25,26} and Haisma²⁷ proposed replacing the throttle with an ejector to liquefy helium and hydrogen²⁸ and to circulate two phase helium.^{29,30} Such an expansion generates less entropy than an isenthalpic one. Here the expansion occurs in two steps, into two successive evaporators, each of different pressure. Part of the kinetic energy is recovered as work against the pressure gradient; and the coolant is pumped from the low-pressure evaporator to the

high-pressure evaporator, instead of being entirely dissipated. However, this work remains “internal” since it is not extracted across the borders of the cooler.

An ejector expander was utilized in a liquid-helium-temperature Joule-Thomson cryocooler that was coupled to a Stirling precooler. It is discussed in the section addressing hybrid Joule-Thomson cryocoolers (Sect. 5.6.5.1), which provides a natural context for a more detailed introduction of the concept.

3.1.4.2 The Cold Air Cycle

The cold air cycle is completely similar to the Linde-Hampson cycle, however, it does not involve any phase change and does not reach temperatures as low as the Linde-Hampson cycle. The low expansion pressure is higher than the critical pressure, so boiling does not occur and cooling is provided at a non-constant temperature. In this cycle, the compression ratio and the associated work of compression are significantly smaller. Entropy generation under an isenthalpic expansion is more significant below the critical pressure, as observed in Fig. 3.5. Hence, a large entropy generation is avoided by not allowing the fluid to expand below the critical pressure. Consequently, a higher efficiency can be expected. Baehr³¹ and Linge³² analyzed this cycle and mention that Carl Linde was aware of this advantage and applied such a cycle to precool the supply air for a liquefaction machine.

3.1.4.3 Combining the Use of a Recuperator with the Extraction of External Work

Combining the use of a recuperator with the extraction of external work is a temperature reducing mechanism that is independent of real gas properties. Cryocooling may be accomplished in this approach even with an ideal gas coolant. The variety of corresponding cycles may be categorized into two main versions:

The Reverse Brayton Cycle

The reverse Brayton cycle^{33,34} employs a loaded turbine to expand the entire flow as if replacing the throttle of the Linde-Hampson cycle. An analysis of this cycle as a cryogenic refrigerator was published elsewhere.³⁵ It is often regarded as a ideal gas cycle since it does not rely upon the real gas properties.

The Claude Cycle

The Claude cycle^{33,34} may be regarded as a throttle cycle precooled by a Brayton cycle.³⁶ The pressurized stream directed from the compressor of a Claude cycle splits. The primary flow circulates through a recuperator, throttle valve, evaporator and back through the recuperator, cooling until it

eventually liquefies in the evaporator. The secondary flow stream cools by performing external work at constant entropy through an expander. It is then used to pre-cool the primary flow stream. For each operating pressure, an optimum splitting ratio minimizes the consumption of energy. The high pressure side of the Claude cycle remains below the critical pressure. Variations of the Claude cycle developed by Heylandt³⁷ and Kapitza³⁸ operate at super-critical pressure, and low pressure respectively. Each of these variations presents advantages and disadvantages for practical operation. A further evolution developed and built by Professor Collins^{38,39,40} at MIT, with two and more expanders, provides the basis for most of the continuously operating helium liquefiers in use today. Although the throttling effect at room temperature with helium results in a temperature increase, an isentropic expansion with helium always causes cooling. An analysis of a Claude cycle designed to reach the boiling point of helium through the aid of a single expander was published elsewhere.⁴¹ Khalil with McIntosh⁴² and Knudsen with Ganni⁴³ have developed optimized cycles using multi expanders.

3.1.4.4 The Superfluid Joule–Thomson Refrigerator

The superfluid Joule–Thomson refrigerator has been experimentally demonstrated by Brisson and Miller.⁴⁴ They employed a mixture of He-4 with 3% of He-3 to pump heat from 0.68 to 1.2 K. Below 1.2 K the superfluid He-4 (He-II) becomes a conduit for circulating the He-3. The lighter isotope undergoes a throttling process and recuperation analogous to a common Joule-Thomson cryocooler.

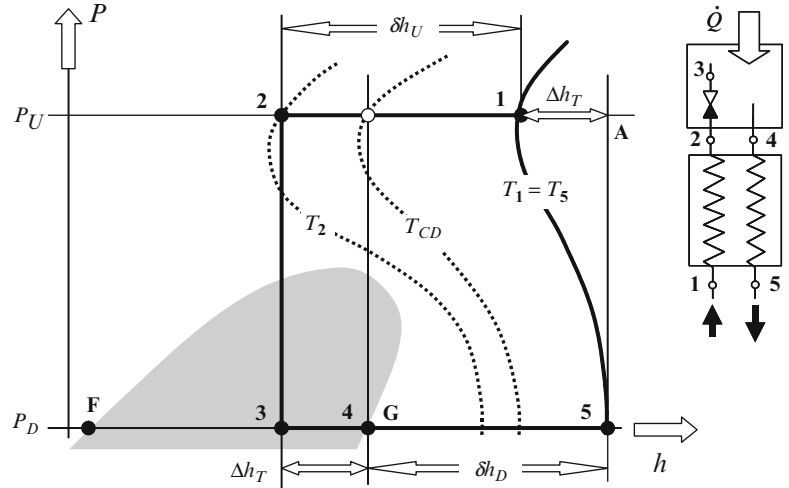
3.2 The Ideal Linde-Hampson Cryocooling Cycle

The entire discussion of this chapter concerning the Linde-Hampson thermodynamic cycle assumes ideal behavior in the following respects: the heat load is balanced by a maximum generation of cooling capacity, and the heat exchanger is perfectly effective, so that there is no temperature difference at the warm end, $T_1 = T_5$. Compression is isothermal. The cooling power is provided solely at the evaporator, and there are no parasitic heat leaks at the heat exchanger. Deviations from the ideal model will be discussed in subsequent chapters.

3.2.1 The P - h Plane

The thermodynamic cycle named after Linde and Hampson comprises five distinct processes: three at constant pressure, (one of them in the two-phase region, so it is also at constant

Fig. 3.3 The ideal Linde-Hampson cycle in the P - h thermodynamic plane (Mollier diagram)



temperature), another at constant temperature and one as a constant enthalpy expansion. Therefore, the coordinates of P and h provide a natural stage for introducing this cycle, as shown in Fig. 3.3. Projecting the same cycle onto the thermodynamic planes that incorporate temperature, such as h - T and T - s diagrams (discussed in Sects. 3.2.2 and 3.2.3) is useful for highlighting the typical temperature differences encountered in the cycle. (See also a projection on the h - s plane in Fig. 3.32, introduced later for an additional purpose). The traces of the individual processes are identified as follows:

3.2.1.1 The High Pressure Isobar

(Section 1–2), $P = \text{Const} = P_U = P_1 = P_2$.

During this process, heat is rejected in the recuperative heat exchanger and the temperature decreases. The state shown as 1 represents the inlet condition to the cryocooler and the high pressure channel of the heat exchanger. Point 2 is the state at the outlet from the heat exchanger and the entrance to the expansion nozzle. Because the trajectory from state 1 to state 2 comprises the upstream portion of the cryocooler, any parameter associated with this section is identified with the subscript, U , throughout this book. The extracted enthalpy,

$$\delta h_U = h_2 - h_1 < 0 \quad (3.1)$$

will be referred as the *recuperated enthalpy* associated with the upstream flow. The corresponding temperature difference along the upstream heat exchanger is,

$$\Delta T_U = T_1 - T_2 \quad (3.2)$$

3.2.1.2 The Isenthalpic Expansion

(Section 2–3), $h = \text{Const} = h_2 = h_3$.

This process is perhaps the most distinguishing feature of the Linde-Hampson machine. The pressure decreases in the throttling device, from the high value of $P_2 = P_U$ down to P_3 , below the critical pressure, and produces a corresponding temperature drop from T_2 to T_3 . The isenthalpic expansion is the integral Joule-Thomson effect that is so central and essential to the operation of this thermodynamic cycle. The trajectory of the process penetrates the two-phase dome, an essential feature of the Linde-Hampson cycle, in contrast to the cold air cycle.³¹ As a result, the single-phase stream flowing into the nozzle (at state 2) becomes, after expansion, a two-phase mixture (state 3). T_3 is the boiling temperature related to the downstream pressure P_3 , which is the saturation pressure. The process terminates at state 3 with coexisting gaseous and liquid phases. The liquefied fraction of the two phase flow depends on the initial state, 2, increasing as T_2 is decreased. The yield of liquefaction is determined, according to the two-phase lever rule, by the ratio of enthalpy section lengths, shown as $\overline{34}/\overline{F4}$ on the p – h diagrams in Fig. 3.3, h – T in Fig. 3.4 and on the T – s diagram in Fig. 3.5.

3.2.1.3 Isothermal Phase Change

(Section 3–4), $T_3 = T_4 = T_{\text{BOIL}}$, $P_3 = P_4 < P_C$.

The entire thermodynamic cycle is intended to support and maintain this section of heat absorption at low temperature from the target object. The goal is accomplished through the process of cryogen boiling in the evaporator. Because the heat absorption is evidenced in the change of phase, there is no associated temperature change and cycle provides a high degree of thermal stability. The constant temperature state becomes possible only when P_3 is lower than the critical pressure. Note that the liquid portion is entirely evaporated and the final state 4 is located on the saturated vapor line (denoted by G).

Fig. 3.4 The ideal Linde-Hampson cycle in the thermodynamic plane of h - T

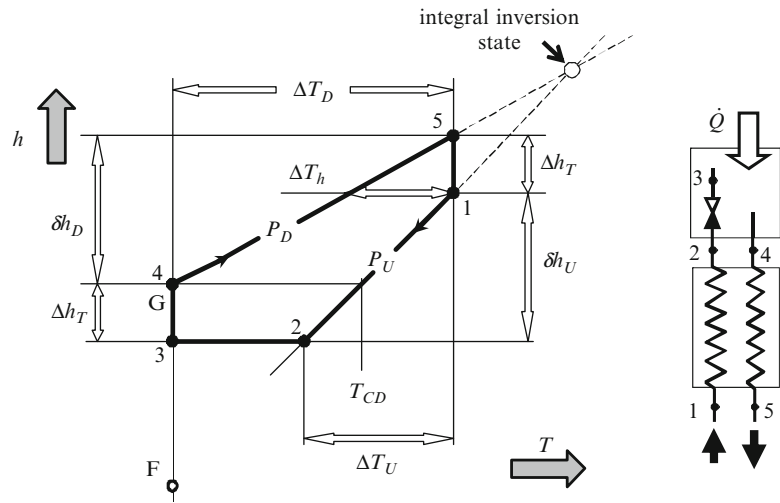
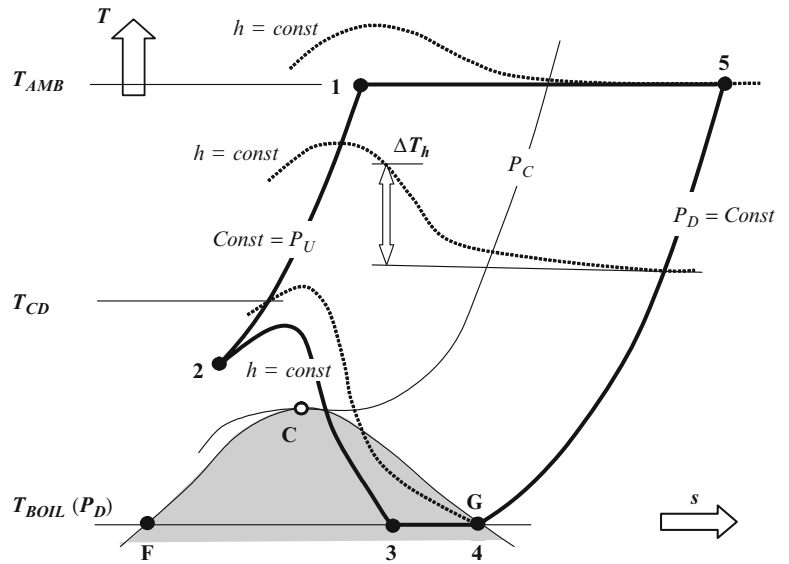


Fig. 3.5 Quantitative presentation of the ideal Linde-Hampson cycle in the T - s plane



3.2.1.4 The Low Pressure Isobar

(Section 4–5), $P = \text{Const} \equiv P_D = P_4 = P_5$.

In the recuperator, heat is absorbed and the temperature rises. This process occurs for the downstream, expanded low pressure; therefore, the subscript D is adapted to characterize any parameter associated with this section. The corresponding increase of enthalpy (lost by the upstream flow) is the other reflection of the *recuperated enthalpy*,

$$\delta h_D \equiv h_5 - h_4 > 0 \quad (3.3)$$

and the associated temperature difference along this stream is,

$$\Delta T_D = T_5 - T_4 \quad (3.4)$$

3.2.1.5 Isothermal Compression

(Section 5–1), $T_1 = T_5 = T_{AMP}$

In the isothermal compression process, the pressure is raised from P_D to P_U . The heat of compression is removed by an external cooling loop in order to maintain the temperature constant at the ambient value, T_{AMB} . Two heat reservoirs are necessary for continuous, closed-cycle cooling as shown in Fig. 1.14. The heat absorbed at the low temperature from the cooled object (the first reservoir), along with the heat of compression from the gas, is rejected at the second reservoir, which is the ambient temperature cooling loop of the compressor. Both reservoirs are necessary in order to maintain a cyclic steady state for the entire cooler. In addition, the condition of $T_1 = T_5$ is fulfilled only by an ideal recuperator.

3.2.1.6 The Cool Down Temperature, T_{CD}

The cool down temperature, T_{CD} , of the nozzle, as shown in Figs. 3.3, 3.4, and 3.5, presents another state of special interest that is discussed in detail in Sect. 4.4. In particular, an upper limit to the temperature of the gas entering the nozzle (Sect. 3.10), can be identified such that after the isenthalpic expansion, the fluid still reaches the boiling point, T_{BOIL} , although only as saturated vapor, and hence with zero yield of liquefied cryogen. The subscript CD, representing **Cool Down**, is used to highlight the fact that when this temperature is first reached at the nozzle, the cool down process terminates, while the yield of liquefaction just begins. T_{CD} is dependent on the pressure of operation, P_U , as shown in Figs. 4.23, 4.24, and 4.25. The curvature of the isotherms in Fig. 3.3, explain why there might be two values of P_U associated with the same T_{CD} value.

3.2.1.7 Remarks

1. It is worth pointing out that the equality of the enthalpy sections in the recuperator $\overline{12} = \overline{45}$ as clearly evidenced on the P - h diagram represents a crucial feature of the cycle. From the conservation of energy, one can observe that the upstream and downstream recuperated enthalpies (or the *extent of recuperation*, δh) are actually the same. That is,

$$\delta h_U + \delta h_D = 0 \quad \text{or} \quad \delta h = |\delta h_U| = |\delta h_D| \quad (3.5)$$

This relationship, formulated in terms of the enthalpies of recuperation, is another reason why it is of advantage to use the P - h plane for presenting the Linde-Hampson cycle. Simply by applying topological arguments, one may deduce the effect of various conditions of operation.

2. As a consequence of the above relationship, one can also readily observe from Fig. 3.3 that $\overline{34} = \overline{A1}$. Employing the identity of an entire cycle,

$$(h_1 - h_2) + (h_2 - h_3) + (h_3 - h_4) + (h_4 - h_5) + (h_5 - h_1) = 0$$

and

$$h_2 = h_3$$

results in the following equality,

$$h_4 - h_3 = h_5 - h_1$$

Namely, the latent heat of evaporation, which is the absorbed heat at the low temperature, equals the warm end enthalpy difference. As discussed later, this equation

could be said to represent the basic equation of the cooler. The same feature cannot be so easily demonstrated if one is analyzing the cycle in the T - s plane.

3. In order for the Linde-Hampson cycle to produce cooling, that is if $h_5 - h_1 > 0$, the isotherms, in the P - h plane cannot be straight and vertical.
4. There is an optimum pressure of operation, as one may deduce by inspecting the shape of the isotherms in the P - h plane. It is the pressure, associated with the peak of the isotherm, that maximizes the length of the section $\overline{34}$ and the yield of the liquefaction.
5. In general, since $P_U > P_C$, the entire high pressure stream including state 2, at the inlet to the nozzle, is of a single phase. However, in the case of a sub-critical pressure supply, $P < P_C$, state 2 and some colder sections of this stream may be in a two phase condition.

3.2.2 The h - T Plane

Alternatively, the same cycle may be presented with the same notation of states (1 through 5) in the h - T plane as shown in Fig. 3.4. The h - T plane is ideally suited for highlighting the integral Joule-Thomson effects. The isenthalpic effect, ΔT_h , is simply the horizontal separation between the P_U and P_D isobars. The isothermal effect, Δh_T , is represented by the vertical separation of the two isobars. Both effects are discussed in detail in Sects. 4.3 and 4.5. The slope of an isobar in this plane is the specific heat capacity, by Eq. 2.6.

The isobar of P_U intersects the $P_D = 0.1$ MPa isobar at an integral inversion state, since at this point $\Delta h_T = 0$. According to Eq. 2.109, the integral inversion curve (I.I.C.) in this plane is defined by the condition,

$$h(P, T) = h(0.1 \text{ MPa}, T) = f(T) \quad @ \text{ I.I.C.} \quad (3.6)$$

and therefore coincides with the 0.1 MPa isobar.

The equivalence of the recuperated enthalpies, $|\delta h_U| = |\delta h_D|$, may also be explicitly expressed as,

$$\left[\int_{T_2}^{T_1} c_{P,U} \cdot dT \right]_{P=P_U} = \left[\int_{T_4}^{T_5} c_{P,D} \cdot dT \right]_{P=P_D} \quad (3.7)$$

For pure coolants it is true (in general) that,

$$c_{P,U} > c_{P,D} \quad (3.8)$$

therefore, the integral equality determines an opposite relation for the associated temperature ranges,

$$\Delta T_U < \Delta T_D \quad (3.9)$$

as is clearly demonstrated in Fig. 3.4. Note that while the enthalpy change through the recuperator is equivalent for the two streams, the temperature change is not.

For mixed coolants, the heat capacities of both streams are much closer to each other; hence the temperature changes, ΔT_U and ΔT_D are also more similar. For helium and hydrogen the heat capacity of the high pressure stream can be smaller than that of the return, or low pressure, stream (Fig. 1.11).

3.2.3 The T - s Plane

Another aspect of the Linde-Hampson cycle may be observed by displaying the processes in the T - s plane as shown in Fig. 3.5. The isenthalpic process moving from state 2 to 3 achieves a maximum, signifying that P_U is higher than the inversion pressure at state 2. While following the isenthalpic trajectories close to ambient temperature, one may observe that an expansion from P_U down to atmospheric pressure is associated with a large production of entropy, but a small decrease in temperature. Expansion from P_U down to P_C results in a larger decrease in temperature for a similar generation of entropy. This observation provides the rationale behind the dual pressure cycle (Sect. 5.4.9) that increases its COP by expanding the majority of the flow from P_U down to an intermediate pressure close to P_C , leaving only a small fraction of the flow to expand down to atmospheric pressure. The cold air cycle³¹ as mentioned in Sect. 3.1 also enjoys the same advantage.

The recuperated enthalpy, δh , of the Linde-Hampson cycle, is not observed explicitly in this plane of thermodynamic states. In this sense, it is less attractive. Therefore, the description in the T - s plane remains incomplete without mentioning that,

$$\delta h_U = \left[\int_{T_1}^{T_2} T \cdot ds \right]_{P=P_U} = \left[\int_{T_5}^{T_4} T \cdot ds \right]_{P=P_D} = -\delta h_D \quad (3.10)$$

3.2.4 The Maximum Specific Cooling Capacity

The general form of the First Law of Thermodynamics for an open system,⁴⁵ is applied to a control volume (C.V.), which includes the entire cryocooler as shown in Fig. 3.1a,

$$\begin{aligned} \frac{d}{dt}(E)_{C.V.} &= \dot{Q} - \dot{W} \\ &+ \sum_{INLET} \dot{n}_{IN} \cdot h_{IN} - \sum_{OUTLET} \dot{n}_{OUT} \cdot h_{OUT} \quad (3.11) \end{aligned}$$

where $\dot{n}$ is the molar flow rate of the coolant. At steady state, the first term, which is the rate of change of the internal energy of the entire control volume, vanishes. For a cryocooler, (in contrast to a liquefier, as discussed later),

$$\dot{n}_{IN} = \dot{n}_{OUT} = \dot{n} \quad (3.12)$$

No work is performed ($\dot{W} = 0$) by an operating Joule-Thomson cryocooler, and the heat exchange with the surrounding, $\dot{Q}$, is defined by the heat load that it absorbs from the cooled object. Consequently, the energy balance reduces to,

$$\dot{Q}/\dot{n} = h_{OUT} - h_{IN} \quad (3.13)$$

The cooling power per unit flow rate, $\dot{Q}/\dot{n}$, is the *specific cooling capacity* of the cryocooler. The term h_{IN} depends on the inlet conditions but the term h_{OUT} is a result of the performance of the cryocooler. The difference between the two increases as h_{OUT} grows. For any given outlet pressure h_{OUT} increases with T_{OUT} . By the Second Law of the thermodynamics, T_{OUT} is bounded by T_{IN} . Therefore, the maximum value of T_{OUT} is given by,

$$T_{OUT} \equiv T_5 = T_1 \equiv T_{IN} \quad (3.14)$$

as is true for the ideal Linde-Hampson cycle described above. The highest specific cooling capacity is then dependent on the temperature and pressures at the inlet and outlet according to,

$$(\dot{Q}/\dot{n})^{MAX} = h_5 - h_1 = h(P_5, T_1) - h(P_1, T_1) \quad (3.15)$$

In this case the cooling potential of the exhaust stream is completely absorbed by or transferred to the incoming stream.

The occurrence of a zero temperature difference at the warm end of the heat exchanger in a Linde-Hampson cryocooler with pure gases is possible in theory. Indeed, since the incoming warmer stream typically has a larger heat capacity rate than that of the cold returning one ($\dot{C}_U > \dot{C}_D$) a pinch-point will exist at the warm end of the heat exchanger. This idea was discussed in Chap. 1.

In general, the coolant expands down to about atmospheric pressure, and we may assume that $P_5 = 0.1$ MPa. The integral isothermal Joule-Thomson effect, a quantity specific to each gas, is defined as

$$\Delta h_T(P, T) \equiv h(0.1 \text{ MPa}, T) - h(P, T) \quad (3.16)$$

It is the difference in enthalpy at a given temperature due to the change of pressure. Furthermore, from Eq. 3.15 we have,

$$(\dot{Q}/\dot{n})^{MAX} = \Delta h_T \quad @ T = T_1 = T_{IN} \quad (3.17)$$

Note that Δh_T may be interpreted as the potential specific cooling content of the gas at the inlet conditions (P_{IN}, T_{IN}). It enables one to determine the minimum necessary gas consumption required to balance any heat load. The features of the thermodynamic function, Δh_T , which play a central role in Joule-Thomson cryocooling, will be broadly discussed in Sect. 4.3.

Remarks:

1. A necessary condition for achieving cooling is, $h_5 > h_1$, namely, the enthalpy of the inlet high pressure stream, has to be less than that of the outlet low pressure stream. Otherwise, as with helium or neon at 300 K, the “cooler” will warm up as is discussed later in Sect. 9.5.
2. For an ideal gas, $\Delta h_T = 0$, and no cooling is attainable. The cooling potential of the Linde-Hampson cycle depends on the extent to which a real gas departs from the ideal gas model, thereby enabling $\Delta h_T > 0$.

3.3 Real Linde-Hampson Cooler Cycles

3.3.1 Introduction

Any realization of a Linde-Hampson cycle has to deal with the following independent parameters:

- (a) The actual applied heat load, $\dot{Q}$,
- (b) The flow rate, $\dot{n}$, that is related to the actual size of the throttle,
- (c) The coolant's inlet conditions, (P_1, T_1), and,
- (d) The extent of recuperation, δh , and ΔT_U , corresponding to the available heat transfer area.

The first three are parameters associated with the operation of a cryocooler. The fourth one corresponds to a feature of its geometry and construction.

These four parameters determine the performance characteristics of a real Linde-Hampson cycle in two regards:

- (a) The temperature difference at the warm end, and,
- (b) The lowest temperature along the high pressure stream, at the inlet to the Joule-Thomson valve.

Each of these characteristics is discussed below.

3.3.2 The Temperature Difference at the Warm End of the Recuperator, ΔT^*

3.3.2.1 Operation with Excess Flow Rate as a Source of ΔT^*

In order to support any given heat load, $\dot{Q}$, the flow rate cannot be lower than,

$$\dot{n}^{MIN} = \dot{Q} / \Delta h_T \quad (3.18)$$

For example, nitrogen at 10 and 40 MPa at 300 K exhibits values of Δh_T of 548 and 1,122 J/mol, respectively. Balancing a heat load of 200 mW would then require corresponding flow rates of $0.87 \cdot 10^{-4}$ and $1.78 \cdot 10^{-4}$ mole/s. In practice it is more common to use units of standard liters (SL) rather than moles as explained later in Sect. 9.2.1.3. Because one mole is equivalent to 22.4 SL, the associated flow rates in this case become 0.24 and 0.49 SLPM (Standard Liters Per Minute).

However, in practice one may apply a larger flow rate than the minimum requirement (for example through the use of a larger orifice) and regard this as an excess flow rate since,

$$\dot{n} > \dot{n}^{MIN} = \dot{Q} / \Delta h_T \quad (3.19)$$

then

$$\dot{Q} / \dot{n} < \Delta h_T. \quad (3.20)$$

The magnitude of this inequality, $(\Delta h_T - \dot{Q} / \dot{n})$, is a *measure of the flow excess*.

Equivalently, the condition may be considered as only partial utilization of the available cooling potential since for a given flow rate $\dot{n} \cdot \Delta h_T$ becomes larger than the actual heat load, $\dot{Q}$. (Such a mode of operation may be desirable for maximizing the rate of cold production, $\dot{Q}$, while sacrificing $\dot{Q} / \dot{n}$. This idea is discussed later in Sect. 3.9.)

The system adjusts itself to a new steady state by reducing the cooling capacity to match the actual heat load. The adjustment is accomplished by reducing the temperature at the outlet state from T_5 to T_5^* (see Fig. 3.6) thereby degrading $h_{OUT} - h_{IN}$ to balance $\dot{Q} / \dot{n}$. The low-pressure stream exits the system with some excess cooling content. In other words,

$$\begin{aligned} h_{OUT} - h_{IN} &= h_5^* - h_1 \equiv \Delta h^* \\ &= h(P_5, T_5^*) - h(P_1, T_1) = \dot{Q} / \dot{n} < \Delta h_T \end{aligned} \quad (3.21)$$

Thus, a non-zero temperature difference is generated and maintained at the warm end of the recuperator,

$$\Delta T^* \equiv T_5 - T_5^* = T_1 - T_5^* > 0 \quad (3.22)$$

3.3.2.2 The Largest ΔT^*

The largest ΔT^* occurs when no heat load is applied, that is, $\dot{Q} = 0$. In this case, by an energy balance, $h_{OUT} = h_{IN}$ and states 1 and 5* share the same enthalpy. If the outlet pressure is 0.1 MPa, the temperature T_5^* will be below T_1 by the amount,

$$\Delta T_h(P, T) = T(0.1 \text{ MPa}, h) - T(P, h) \quad (3.23)$$

3.3.2.4 Recuperator's Lack of Thermal Conductance as a Source of ΔT^*

The value of ΔT^* is determined through the First Law of thermodynamics, by the operating parameters, $\dot{Q}$ and $\dot{n}$, while satisfying Eq. 3.21. One should not be misled and regard it as an indication of the lack of thermal conductance, (UA) , or the ineffectiveness of the recuperator. A Linde-Hampson recuperator may have a non vanishing ΔT^* , depending on the applied $\dot{Q}$ and $\dot{n}$, even with a perfectly effective recuperator of unlimited heat transfer area. Rather, the flow excess produces a non-zero value of ΔT^* , independent of the (UA) . Section 3.7.1.2 below, discusses the flow excess as a source of ineffectiveness for the recuperator.

Next, consider a recuperator whose size is unlimited and operates without any flow excess, so that $\dot{Q}/\dot{n} = \Delta h_T$, and therefore $\Delta T^* = 0$. If one gradually reduces (UA) , then ΔT^* will grow and consequently reduce the cooling $\dot{Q}$ (for a given flow rate, $\dot{n}$). In contrast to the previous case, now it is ΔT^* that produces the excess flow, reducing $\dot{Q}/\dot{n}$ below Δh_T .

3.3.3 The Extent of Recuperation

3.3.3.1 Definition

The above results remain valid regardless of whether the cycle is used for liquefying, for simply cooling to the normal boiling point, or even for cooling to some higher temperature. However, the attainable cold temperature is of central interest in cryocooling. The *extent of recuperation* (or interchanging) is the parameter that determines how low of a temperature the cryocooler actually achieves. It determines the difference, for example between the series of cycles (a) through (d) displayed in Fig. 3.6. All of these cycles are associated with the same $\dot{Q}$, $\dot{n}$, and inlet state 1, and thus share the same outlet state, 5^* , ΔT^* and Δh^* . The four cycles are noted by their intermediate states 1-2a-3a-4a-5*, 1-2b-3b-4b-5*, 1-2*-3*-4*-5*, and 1-2d-3d-4d-5*. These are distinguished by the extent of recuperation, δh , which is the enthalpy extracted from the warm incoming stream and transmitted to the outgoing expanded stream in the recuperator. In general, for any cycle X, (where X is a, b, *, or d), δh is defined by,

$$\delta h^X = h_1^X - h_2^X = h_5^X - h_4^X \quad (3.31)$$

Note that in Fig. 3.6, the extent of recuperation is such that,

$$\delta h^a < \delta h^b < \delta h^* < \delta h^d \quad (3.32)$$

The lowest achievable temperature in each cycle due to the recuperation occurs just prior to the isenthalpic

expansion and is respectively labeled 2a, 2b, 2*, and 2d. As the associated value of δh is increased, the temperature at state 2 decreases. Thus,

$$T_2^a > T_2^b > T_2^* > T_2^d \quad (3.33)$$

The extent of recuperation can also be expressed in terms of the associated temperature reduction, $T_1^X - T_2^X$, which is ΔT_U^X , so that,

$$\Delta T_U^a < \Delta T_U^b < \Delta T_U^* < \Delta T_U^d \quad (3.34)$$

3.3.3.2 The Nominal Extent of Recuperation

The cycle (c) in Fig. 3.6 noted by the states 1-2*-3*-4*-5* has a unique feature referred to as the *nominal extent of recuperation*, and is denoted by δh^* , and the associated ΔT_U^* . For any given $\dot{Q}$ and $\dot{n}$ such a cycle is distinguished by the *minimum extent of recuperation* that is necessary to support cryocooling at the constant boiling temperature, T_{BOIL} , while entirely absorbing the applied heat load $\dot{Q}/\dot{n}$. In other words, the final state, 4*, after the fluid has absorbed the heat coincides precisely with the saturated vapor state, G. Then the specific enthalpy difference between state 3* and state G is exactly $\dot{Q}/\dot{n} \leq \Delta h_T$.

3.3.3.3 Under and Over Recuperated Cycles

The cycles noted by (a) and (b) are under-recuperated. The first one never reaches T_{BOIL} . Cycle (d) is over-recuperated. It cools down “too” low, that is, lower than is necessary for the cryocooling mission so that $T_2^d < T_2^*$. In this case, the returning stream will include vapor and some fraction of liquid, so that some amount of liquid will be observed both in the evaporator and at the entrance to the returning channel (“wet windings”). This liquid portion at the cold end may produce an instability as discussed in Sect. 7.2.3.1 (1).

3.3.3.4 The Extent of Recuperation (δh) and the Magnification Index (I_M)

A larger value of δh means a lower value of T_2 and a larger ΔT_U . The temperature difference ΔT_U has the same meaning as the term ΔT_H , used within the generalized model of cryocoolers as introduced in Sect. 1.1, and Fig. 1.1. According to that model, a larger value of ΔT_H is associated with a larger magnification index, I_M . Hence, δh , that is the interchanged enthalpy, and I_M are related,

$$\begin{aligned} \delta h^X &= \Delta T_U \bar{c}_{P,U} = (I_M - 1) DT \bar{c}_{P,U} \\ &= (I_M - 1) (T_2^X - T_4^X) \bar{c}_{P,U} \end{aligned} \quad (3.35)$$

The cycles (a) to (d) in Fig. 3.6 are distinguished by their corresponding values of I_M . The under recuperated case is a

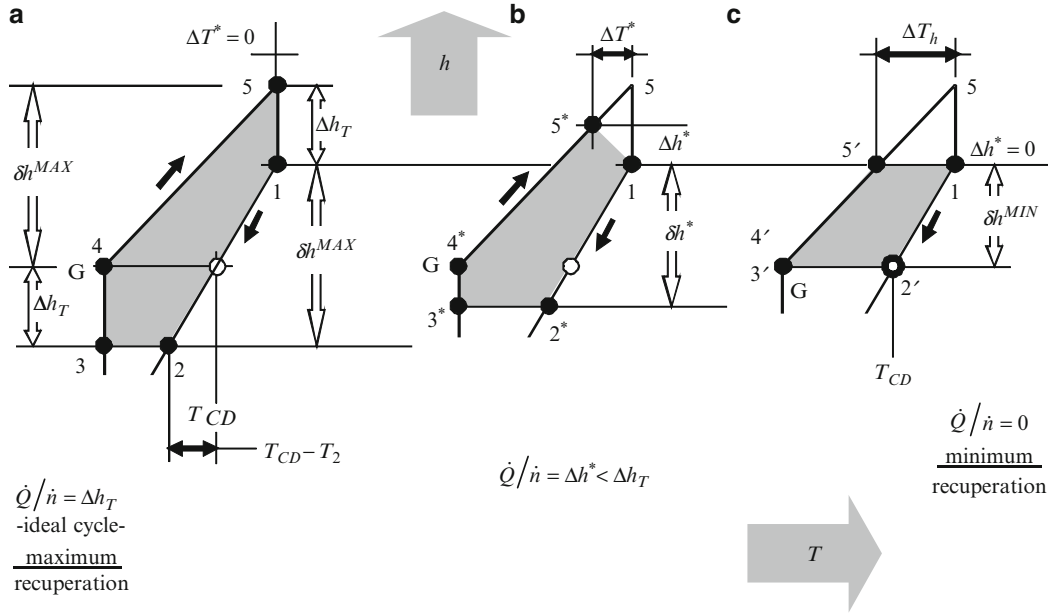


Fig. 3.7 Cycles in the h - T plane of nominal recuperation and different amounts of excess flow (b and c): the ideal cycle (a) does not have any excess flow

result of a magnification index I_M that is “too” small. Note especially the difference between the nominally recuperated and the over recuperated cycles. In the former case, the relative capacity rates are such that $\dot{C}_H > \dot{C}_L$, resulting in a bounded value of I_M . However, the over recuperated cycle is dominated by the evaporation in its cold section; there $\dot{C}_H \ll \dot{C}_L$ and the magnification index I_M may reach high values.

3.4 Cycles Of Nominal Recuperation

As discussed later in Sect. 3.7, the extent of recuperation is restricted by the corresponding heat transfer area (or the thermal conductance, UA) of the recuperator. At least in theory, excess-recuperation does not enhance the performance of cryocoolers. Therefore, cycles of nominal recuperation may serve as a reference model and design goal. Dean and Mann⁴⁶ consider such cycles as a “stable equilibrium condition” (while the under-recuperated cycle as “a possible stable condition”).

Let us examine the category of all cycles characterized as having nominal recuperation. This group of cycles share the same inlet conditions of the warm stream, (P_1, T_1) , and the same inlet state of the counterflowing cold stream that is of a saturated vapor. The extreme temperatures of the group are (T_{BOIL}, T_1) . The individual cycles are distinguished by the outlet temperature of the warm stream, T_5^* , by the extent of nominal recuperation, δh^* , and the associated specific

cooling capacity, Δh^* . Each cycle is then also distinguishable by $\dot{Q}/\dot{n}$, whatever its origin: operation with excess flow or lack of thermal conductance in the recuperator.

The series of states 1-2-3-4-5 as cycle (a) in Fig. 3.7 has the highest nominal extent of recuperation which is,

$$\delta h^{MAX} = h_1 - h_2 = h_5 - h_4 \quad (3.36)$$

In this extreme nominal cycle, $\dot{Q}/\dot{n}$ reaches the highest attainable value of specific cooling capacity, Δh_T , since in this condition $T_5 = T_1$. For this case,

$$\delta h^{MAX} = h_5 - h_G = h_1 - h_2 = h_1 - h_G + \Delta h_T \quad (3.37)$$

This equation determines state 2, prior to expansion through the nozzle, as a function of the starting conditions (P_1, T_1) .

The extent of recuperation represented by case (b), produces a lower cooling capacity. Instead of leaving the heat exchanger at state 5 (so that $T_5 = T_1$) the cold stream exits at state 5* (with enthalpy h_5^*) at the lower temperature $T_5^* < T_1$. The recuperated enthalpy is accordingly smaller,

$$\delta h^* = h_5^* - h_G = h_1 - h_2^* < \delta h^{MAX} \quad (3.38)$$

The correspondingly smaller cooling content in this case is given by,

$$\Delta h^* = h_5^* - h_1 = \Delta h_T - c_{pO} \Delta T^* < \Delta h_T \quad (3.39)$$

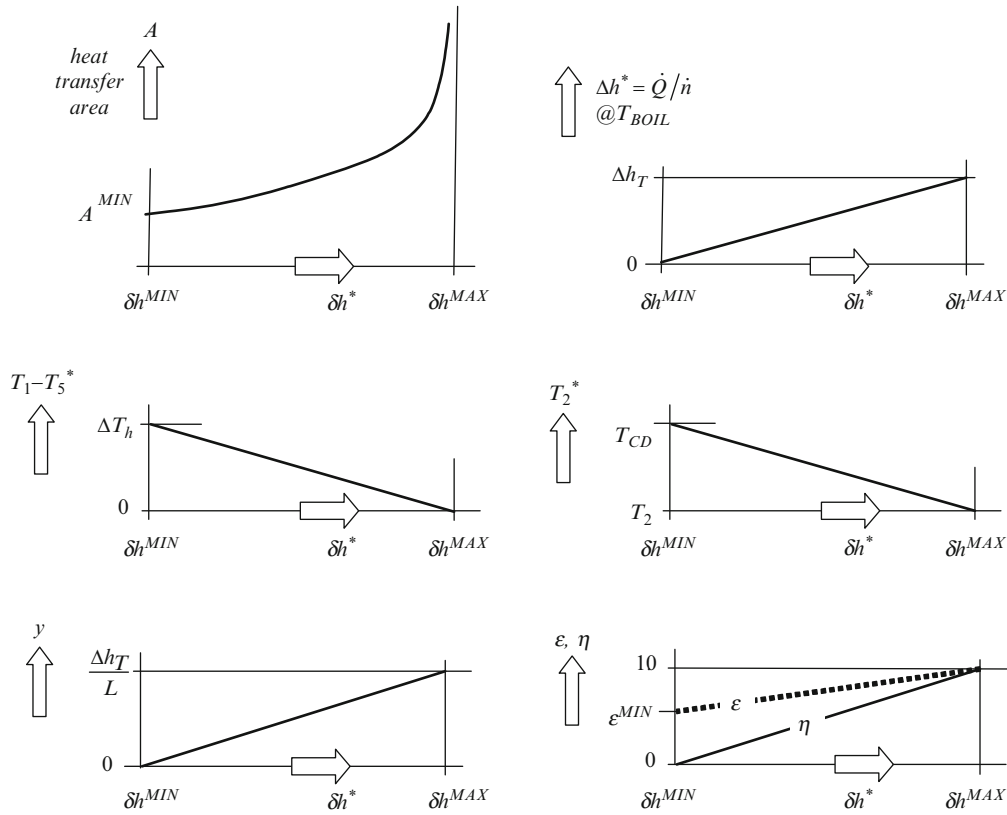


Fig. 3.8 The various performance parameters for the cryocooler in terms of the nominal extents of recuperation, $\delta h^{MIN} < \delta h^* < \delta h^{MAX}$

The smallest extent of nominal recuperation occurs in cycle (c) of Fig. 3.7 and is defined by the outlet state 5' such that the enthalpy h'_5 satisfies the condition,

$$h'_5 = h_1 \quad T'_5 - T_1 = \Delta T_h(T_1) \quad (3.40)$$

In this case the cold end of the cooler still reaches T_{BOIL} but the cooling capacity is zero.

The minimum and maximum values of recuperated enthalpy associated with this heat exchanger, are

$$\delta h^{MIN} = h'_5 - h_G = h_1 - h_G \quad (3.41)$$

$$g h^{MAX} = h_1 - h_2 = \delta h^{MIN} + \Delta h_T \quad (3.42)$$

In general one finds,

$$\delta h^* = h_1 - h_2^* = \delta h^{MIN} + \Delta h^* = \delta h^{MIN} + \dot{Q}/\dot{n} \quad (3.43)$$

As the nominal recuperation extent increases from the minimum value (δh^{MIN}) through the intermediate values (δh^*) and up to the ideal maximum value (δh^{MAX}), the temperature at the inlet to the nozzle is driven lower and lower from T'_2 (which is the limiting temperature T_{CD} , as discussed in Sect. 4.4) down through T_2^* to the lowest value, T_2 . Decreasing T_2^* increases the yield of liquefaction. Figure 3.8 displays the influence of the extent of nominal recuperation,

δh^* , on various performance parameters, some of which will be discussed at a later point.

3.5 Performance of Nominal Recuperators

3.5.1 Introduction

This section discusses the effectiveness, ϵ , and efficiency, η , of nominal recuperators and their dependence on ΔT^* . As described in the next chapter, the effectiveness serves as an indicator of the size of the nominal recuperator in terms of its heat transfer area. The efficiency is a measure of the performance of the cryocoolers.

This section is entirely devoted to recuperators with nominal extent of recuperation. Since all the extents of recuperation discussed in this section are nominal, therefore, for simplicity the adjective “nominal” is omitted.

3.5.2 The Effectiveness, ϵ

3.5.2.1 Definition

The effectiveness of a recuperator is defined as the ratio of the actual recuperated enthalpy to the highest possible recuperation (the latter being determined by the difference of the

enthalpies at the two inlets of the recuperator). In the case where $\dot{C}_H > \dot{C}_L$, as occurs with pure coolants,

$$\varepsilon = \frac{\delta h^*}{\delta h^{MAX}} = \frac{h_5^* - h_G}{h_5 - h_G} \quad (3.44)$$

thus,

$$h_5 - h_5^* = i \delta h^{MAX} \quad (3.45)$$

where,

$$i = 1 - \varepsilon \quad (3.46)$$

is the *ineffectiveness* of the heat-exchanger. Often introduced instead of ε , it appears to be the more “natural” parameter that simplifies the formulation.

A higher effectiveness corresponds to a larger extent of recuperation and a lower temperature at the entrance of the expansion valve, T_2^* .

3.5.2.2 The Degraded Specific Cooling Power

The degraded specific cooling power, corresponding to a less effective heat exchanger is given by,

$$\begin{aligned} \frac{\dot{Q}}{\dot{n}} &= \Delta h^* = h_5^* - h_1 = \Delta h_T - (h_5 - h_5^*) \\ &= \Delta h_T - i \delta h^{MAX} \end{aligned} \quad (3.47)$$

Thus, the extent of the wasted cooling power is characterized by the term $i \delta h^{MAX}$. The degraded specific cooling power can also be expressed in terms of δh^{MIN} :

$$\Delta h^* = \varepsilon \delta h^{MAX} - \delta h^{MIN} = \varepsilon \Delta h_T - (1 - \varepsilon) \delta h^{MIN} \quad (3.48)$$

3.5.2.3 The Minimum Effectiveness, ε^{MIN}

The point at which a cryocooler begins to produce cooling, (that is, $\dot{Q} = 0$ at the boiling point of the flowing cryogen), and at which a liquefier starts to liquefy, as the recuperator size is increased from zero, occurs at the minimum recuperated enthalpy, δh^{MIN} , and the corresponding minimum effectiveness, ε^{MIN} given by: (the same expression applies for both a cooler and a liquefier)

$$\varepsilon^{MIN} = \frac{h_1 - h_G}{h_5 - h_G} = \frac{\delta h^{MIN}}{\delta h^{MAX}} = \frac{\delta h^{MIN}}{\delta h^{MIN} + \Delta h_T} \quad (3.49)$$

where ε^{MIN} depends on the gas and its inlet state, (T_1, P_1) , having a similar peaking pressure dependence as Δh_T .

Some minimum effectiveness values for nitrogen and argon at various pressures are compared in Table 3.1 One

Table 3.1 The exact minimum effectiveness (%), ε^{MIN} , at 300 and 340 K as function of inlet pressure (at 0.1 MPa outlet) compared to the approximation, $1 - \frac{|\Delta T_h|}{T_1 - T_{BOIL}}$

P [MPa]	300 K		340 K
	$1 - \frac{ \Delta T_h }{T_1 - T_{BOIL}}$	ε^{MIN}	ε^{MIN}
Nitrogen			
5	95.59	95.63	97.27
10	91.71	91.79	94.92
20	86.15	86.30	91.59
30	83.43	83.61	90.00
40	82.78	82.96	89.75
50	83.47	83.65	90.44
Argon			
5	91.79	91.89	94.72
10	83.86	84.04	89.71
20	70.55	70.88	81.36
30	61.75	62.16	75.57
40	56.99	57.56	72.17
50	55.09	55.33	70.63

Table 3.2 Minimum effectiveness (%), ε^{MIN} , for some gases at 300 K as a function of pressure

P [MPa]	O ₂	CH ₄	Kr	CF ₄
5	93.87	87.94	77.50	82.61
10	87.92	75.47	52.46	63.30
20	77.87	55.78	9.32	45.72
30	71.11	46.69	0.	40.70
40	67.34	43.46	0.	38.69
50	65.76	43.43	0.	37.80

may readily observe that the lower values systematically belong to argon. For example, compare the maximum allowable ineffectiveness values at (300 K, 40 MPa). For nitrogen, it is about 17% while for argon it more than twice as large, about 41%. One can conclude generally, that even a less effective heat exchanger would suffice for a given job when used with argon, while for the same job using nitrogen it would be completely inadequate, resulting in no yield at all.

Table 3.2 shows values of ε^{MIN} for other gases. The zero value for krypton at 30 MPa and above indicates that the boiling point is achieved without any recuperation, just by the isenthalpic expansion process (as discussed in Sect. 4.6).

The maximum allowable inefficiency is,

$$\begin{aligned} i^{MAX} &\equiv 1 - \varepsilon^{MIN} = \frac{\Delta h_T}{h_5 - h_G} = \frac{\Delta h_T}{\delta h^{MAX}} \\ &= \frac{\Delta h_T}{\delta h^{MAX} + \Delta h_T} \end{aligned} \quad (3.50)$$

Since the specific heat capacity for most cryogenics only varies slightly at the low pressure of 0.1 MPa even down to

temperatures as low as the normal boiling point, we may approximate the enthalpy change associated with δh^{MAX} as:

$$h_5 - h_G \approx c_{PO}(T_1 - T_{BOIL}) \quad (3.51)$$

where c_{PO} is the specific isobaric heat capacity at the low outlet pressure (close to 0.1 MPa) and is quite constant and close to the ideal gas values.

Next, we introduce the relationship, $\Delta h_T = -c_{PO}\Delta T_h$, through Eq. 4.63, that will be discussed later within its natural context in Sect. 4.5.2. Finally, we obtain the relationship,

$$i^{MAX} = 1 - \varepsilon^{MIN} \approx \frac{|\Delta T_h|}{T_1 - T_{BOIL}} \quad (3.52)$$

Table 3.1 shows indeed that this expression closely approximates the exact values of ε^{MIN} , deviating by less than 1.1%.

The numerator of Eq. 3.52 is the largest possible value of ΔT^* . Therefore, if $i > i^{MAX}$ the denominator has to be smaller than $T_1 - T_{BOIL}$. In other words, the lowest reachable temperature has to be above T_{BOIL} , which is indeed the under-recuperated case.

3.5.2.4 The Relationship Between ΔT^* and ε

Through the definition of ε we have that the ineffectiveness is given by,

$$1 - \varepsilon = \frac{h_5 - h_5^*}{h_5 - h_G} \quad (3.53)$$

Regarding again the isobaric specific heat at low pressure as a constant, the ratio of enthalpy differences may then be substituted by a ratio of temperature differences,

$$1 - \varepsilon \approx \frac{T_1 - T_5^*}{T_1 - T_{BOIL}} \quad (3.54)$$

The warm end temperature difference is explicitly proportional to the ineffectiveness of the heat exchanger. Introducing the expression for ε^{MIN} and applying the relationship given by Eq. 3.52 we finally obtain the compact relation,

$$\frac{T_1 - T_5^*}{|\Delta T_h(T_1, P_1)|} = \frac{1 - \varepsilon}{1 - \varepsilon^{MIN}} = \frac{i}{i^{MAX}} \quad (3.55)$$

or

$$T_5^* = T_1 - \frac{i}{i^{MAX}} |\Delta T_h(T_1, P_1)| \quad (3.56)$$

This relationship demonstrates that the maximum viable temperature difference at the warm end is associated with

the highest ineffectiveness, $i = i^{MAX}$, and may be expressed as, $T_1 - T_5^* = |\Delta T_h(P_1, T_1)|$, which is consistent with Sect. 3.3.2.2 and Eq. 3.24.

The ineffectiveness is thus proportional to $T_1 - T_5^*$, but also dependent on the initial temperature T_1 and pressure P_1 .

3.5.3 The Efficiency, η

3.5.3.1 The Definition of Efficiency

The definition of efficiency is based on the relative cooling capacity that can be attained by employing a heat exchanger of nominal recuperation. The efficiency is related to the cooling power of the Joule-Thomson cryocooler by,

$$\dot{Q} = \dot{n}(h_5^* - h_1) \equiv \eta \dot{n} \Delta h_T \quad (3.57)$$

or more explicitly,

$$\eta = \frac{(\dot{Q}/\dot{n})}{(\dot{Q}/\dot{n})^{MAX}} = \frac{\Delta h^*}{\Delta h_T} = \frac{h_5^* - h_1}{\Delta h_T} \quad (3.58)$$

Notice that the highest efficiency, $\eta = 1$, occurs when the numerator equals Δh_T , and $T_1 - T_5^* = 0$. The efficiency vanishes when no cooling power is generated. This parameter is useful in practical calculations of Joule-Thomson cryocoolers, providing a direct and explicit measure of cooling power.

3.5.3.2 The Relationship Between η and ε

Combining Eqs. 3.48 and 3.58 results in the relationship:

$$\eta = \varepsilon - (1 - \varepsilon) \frac{\delta h^{MIN}}{\Delta h_T} = \varepsilon \left(1 + \frac{\delta h^{MIN}}{\Delta h_T} \right) - \frac{\delta h^{MIN}}{\Delta h_T} \quad (3.59)$$

Within the range of combinations of ε and $\delta h^{MIN}/\Delta h_T$ for which $\eta \geq 0$ Eq. 3.59 ensures that $\eta \leq \varepsilon$. The effectiveness and efficiency are equal only when $\eta = \varepsilon = 1$. Substituting the expressions found in Eqs. 3.49 and 3.50 for the minimum effectiveness, $\varepsilon = \varepsilon^{MIN}$, one may verify that indeed the corresponding efficiency becomes $\eta = 0$. The short range of effectiveness values essential for achieving liquefaction, $(\varepsilon^{MIN}, 1)$, is mapped into the efficiency range of $(0, 1)$.

The efficiency, η , and effectiveness, ε , are also simply related in terms of ε^{MIN} by,

$$\eta = \frac{\varepsilon - \varepsilon^{MIN}}{1 - \varepsilon^{MIN}} \quad (3.60)$$

One should note that this relationship remains dependent on the operating conditions of the coolant, (P_1, T_1) , through the value of ε^{MIN} .

Table 3.3 The exact efficiency, $\eta = (h_5^* - h_1)/\Delta h_T$ as a function of the warm end temperature difference, $T_1 - T_5^*$, compared to the approximation, $1 - (T_1 - T_5^*)/\Delta T_h$

$T_1 - T_5^* \text{ [K]}$	Nitrogen (300 K, 30 MPa)		Argon (300 K, 40 MPa)	
	$1 - \frac{T_1 - T_5^*}{ \Delta T_h }$	η	$1 - \frac{T_1 - T_5^*}{ \Delta T_h }$	η
0	1.000	1.000	1.000	1.000
5	0.865	0.865		
10	0.732	0.730	0.891	0.892
15	0.597	0.595		
20	0.462	0.461	0.783	0.783
25	0.327	0.326		
30	0.193	0.191	0.675	0.674
35	0.059	0.055		
37.2 = $ \Delta T_h $	0.000	0.000		
40			0.566	0.566
50			0.458	0.454
60			0.350	0.348
70			0.242	0.239
80			0.133	0.130
90			0.025	0.021
92.3 = $ \Delta T_h $			0.000	0.000

Equation 3.59 readily enables the efficiency to also be expressed in terms of ineffectiveness according to,

$$\eta = 1 - i \frac{\delta h^{MAX}}{\Delta h_T} \quad (3.61)$$

from which we may introduce the *inefficiency*, I , (in analogy to the ineffectiveness, i),

$$I = 1 - \eta \quad (3.62)$$

and obtain a linear relationship between I and i ,

$$I = \frac{i}{i^{MAX}}. \quad (3.63)$$

3.5.3.3 Relationship Between η and ΔT^*

Referring back to Eq. 3.55 we find that the inefficiency may be expressed in terms of warm end temperatures according to:

$$I = \frac{h_5 - h_5^*}{\Delta h_T} \approx \frac{T_1 - T_5^*}{|\Delta T_h|} \quad (3.64)$$

which is consistent with Eq. 3.29. While $T_1 - T_5^*$ varies in the range $(0, \Delta T_h)$, the efficiency η varies from 0 to 1. Table 3.3, listing the corresponding efficiencies for nitrogen and argon demonstrates the linear dependence of η on $T_1 - T_5^*$ which results from the fact that the specific heat capacity of the low pressure stream is relatively constant.

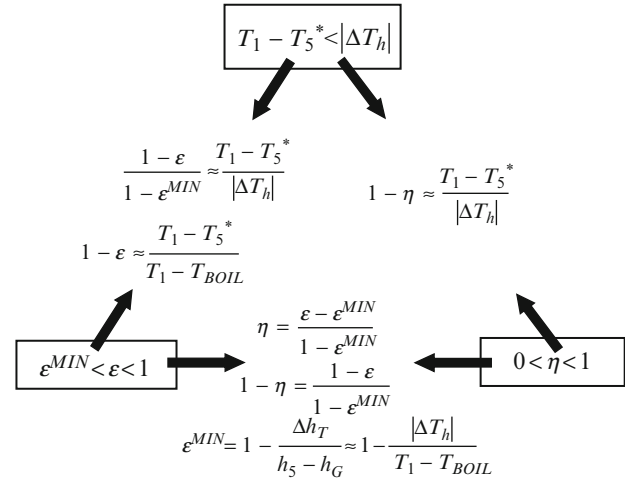


Fig. 3.9 Relationship between the performance parameters η , ε , and $T_1 - T_5^*$ of a nominal recuperator

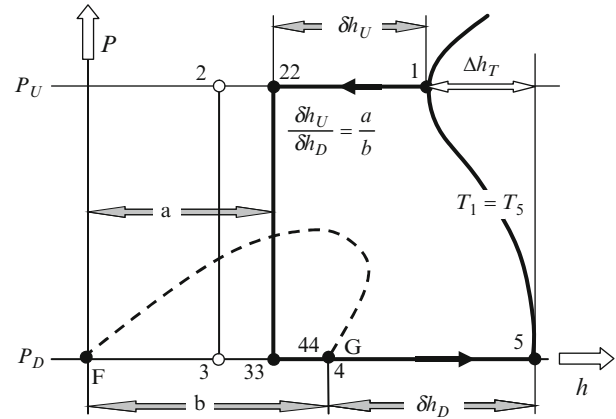


Fig. 3.10 The cycle of a liquefier (1-22-33-4-5) versus the cycle of a cooler (1-2-3-4-5); both of nominal recuperation. Note that Sects. 1–22 and 4–5 are not equal because of the removal of the liquefied fraction

All of the above discussed approximations relating each combination of the three parameters, η , ε and $T_1 - T_5^*$ are summarized in Fig. 3.9.

3.6 The Linde-Hampson Liquefier Cycles

3.6.1 The Ideal Cycle of the Liquefier

3.6.1.1 Liquefiers with Nominal Recuperation

Liquefiers with nominal recuperation are the subject of the present section. Figure 3.10 displays the ideal cycle of the cooler, 1-2-3-4(G)-5, in the P - h plane where the notation is consistent with the previous discussion. It is a nominal cryocooling cycle therefore state 4 coincides with state G. In

the ideal cycle of the liquefier state 2, 3 and 4 of the cryocooler are replaced by states 22, 33 and 44 so the entire cycle becomes 1-22-33-44(G)-5. States 1 and 5 are common to both cycles.

Like a cryocooler, a liquefier may also be over recuperated so that state 4 (in Fig. 3.7) is inside the two-phase dome instead of being on the boarder state, G. In spite of the phase separation at the evaporator, *not all liquefiers are of nominal recuperation*. The amount of liquid phase that is extracted is limited by the First Law. It is not possible to remove the entire liquid content from the evaporator if it is more than this limit, and still maintain a steady state operation. An over recuperated liquefier will have two phase flow in the cold, returning stream.

3.6.1.2 The Misbalance of Flow Rates

Qualitatively, the thermodynamic cycle of a liquefier resembles that of a refrigerator, or cooler. However, the significant and quantitative distinction arises from the fact that a liquefied fraction, y , is extracted in the evaporator from the incoming upstream, $\dot{n}_U$. Consequently, the flow rate of the returning low-pressure stream, $\dot{n}_D$, is smaller than that of the incoming high-pressure stream so that,

$$\dot{n}_D = (1 - y)\dot{n}_U \quad (3.65)$$

The typical misbalance of capacity rates for the two counter flowing streams, $\dot{C}_H > \dot{C}_L$, is even more likely than in case of the cryocooler due to the lower flow rate of the lower specific heat capacity stream. Hence, the pinch point of the liquefier is also located at the warm end so that, $T_1 = T_5$, as in case of the ideal cycle for the cooler.

3.6.1.3 The Yield of Liquefaction

Accounting for the misbalance of the flow rates, the energy balance over the control volume of the entire liquefier shown in the center of Fig. 3.1, determines the ideal yield of liquefaction at steady state operation,

$$y^{(2)} = \frac{\Delta h_T}{h_5 - h_F} = \frac{\Delta h_T}{\delta h^{MAX} + L} \quad (3.66)$$

This is analogous to the maximum specific cooling capacity of the ideal cryocooler cycle.

3.6.1.4 The Span of Specific Enthalpies

The span of specific enthalpies along the two recuperating channels, in contrast to the ideal cryocooler, are not equal,

$$\delta h^{MAX} = h_5 - h_G > h_1 - h_{22} \quad (3.67)$$

however, the actual rate of recuperation in each stream is the same,

$$\frac{d}{dt}(\delta H) = \dot{n}_D(h_5 - h_G) = \dot{n}_U(h_1 - h_{22}) \quad (3.68)$$

3.6.2 The Real Cycle of the Liquefier

As with the case of real nominal cryocooling cycles, real liquefier cycles also exhibit a non vanishing temperature difference at the warm end, $\Delta T^* = T_1 - T_5^*$. The origin of the temperature difference, in the case of a liquefier is not only the ineffectiveness of the recuperator, resulting from insufficient heat transfer area. The same holds true for nominal cryocooler cycles. As discussed for cryocoolers in Sect. 3.3.2, the origin of ΔT^* for liquefiers may also be an operation with excess flow rate, which means, extracting less liquid than the $y^{(2)}$ fraction.

The actual effectiveness for the recuperator of a liquefier, is as defined by Eq. 3.44 for cryocoolers in terms of the lower capacity rate downstream. Therefore, all expressions for the effectiveness, ε , and efficiency, η , as developed for the nominal cryocoolers in the previous Sect. 3.5 hold as well for real liquefiers. The single distinction is that for liquefiers, δh^{MAX} is $h_5 - h_G$, but not $h_1 - h_{22}$, as mentioned by Eq. 3.67.

The cryocooler cycle with minimum effectiveness and zero cooling power, $\dot{Q} = 0$, is exactly the same corresponding cycle for liquefiers that produces zero yield, $y = 0$, since in this case $\dot{n}_D = \dot{n}_U$. Therefore, the liquefier has the same values of ε^{MIN} and δh^{MIN} as the cryocooler and the same maximum value of ΔT^* which is $|\Delta T_h|$ as the cryocooler, by Eq. 3.24.

3.7 Sizing of Nominal Recuperators

The nominal recuperator is a counter flow heat exchanger with fixed and known inlet conditions; the warm inlet is at ambient temperature and elevated pressure, and the counterflowing cold inlet is a low pressure saturated vapor at the corresponding boiling point.

3.7.1 Size Versus Performance

3.7.1.1 Lack of NTU as a Source of Ineffectiveness

Following the discussion in Sect. 1.2.3 and Eqs. 1.34, 1.35, 1.36, and 1.37,

$$\eta = 1 - \frac{(1 - r)/i^{MAX}}{e^{NTU(1-r)} - r} = 1 - I \quad (3.69)$$

Fig. 3.11 The ineffectiveness of a counter flow heat exchanger as function of the non dimensional heat transfer area, NTU, for some values of the capacity rate ratio, r

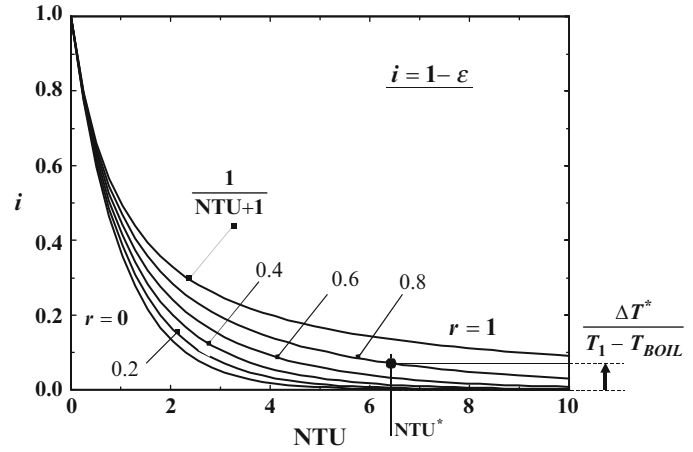
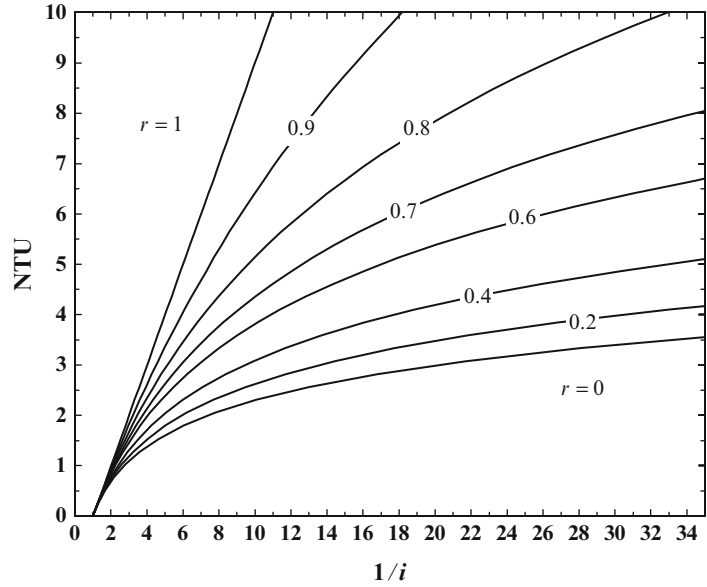


Fig. 3.12 Sizing the counter flow heat transfer area in terms of NTU for any value of (the reciprocal of) the ineffectiveness, for various values of the heat capacity ratio, r . For $r = 1$ the relation in these coordinates is linear



or the ineffectiveness,

$$i = \frac{1-r}{e^{NTU(1-r)} - r} \quad (3.70)$$

Figure 3.11 shows how the ineffectiveness approaches zero as NTU is increased. Applying Eq. 1.30 to the Linde-Hampson nominal recuperator,

$$NTU = \left[\int_{T_{BOIL}}^{T_5^*} \frac{dT_D}{\Delta T(T_D)} \right]_{\text{along the low pressure stream}} = \frac{(UA)}{\dot{n} c_{p0}} \quad (3.71)$$

So the NTU may also be obtained by a given distribution of ΔT as a function of T_D for any given warm end

temperature difference, $T_1 - T_5^*$, (Sect. 3.11.1.2) as integrated in Sect. 3.11.1.3.

An alternate, but equivalent relationship can be provided, giving NTU in terms of the ineffectiveness:

$$NTU = \frac{1}{1-r} \ln \left(\frac{1-r}{i} + r \right) \quad (3.72)$$

As shown in Fig. 3.12, NTU increases monotonically as a logarithmic function of $1/i$. In the vicinity of the balanced case as r approaches unity, (by the aid of L'Hospital rule by differentiating the numerator and denominator),

$$NTU = \lim_{r \rightarrow 1} \left[\frac{1}{1-r} \ln \left(\frac{1-r}{i} + r \right) \right] = \frac{1}{i} - 1 \quad @ \quad r = 1 \quad (3.73)$$

As can be observed in Fig. 3.12, the balanced case imposes the most demanding requirements for NTU in order to obtain a desired small value of ineffectiveness. As r becomes smaller the demand for NTU decreases and the least demanding case is that of zero capacity rate ratio for which,

$$NTU = -\ln(i) \quad @ \quad r = 0 \quad (3.74)$$

In the practice of miniature Joule-Thomson cryocoolers with pure coolants, an NTU of 15 is regarded as a large, while a value of 3 or 4 is small.

The above relations are general and therefore display i in the range of 0–1. However, within the scope of our consideration of Linde-Hampson cryocoolers and liquefiers that reach the boiling point of the coolant, the ineffectiveness ranges from 0 to only i^{MAX} .

3.7.1.2 Excess Flow Operation as a Source of Ineffectiveness

The above ε – NTU relation determines, for any finite NTU, the corresponding ineffectiveness, $i > 0$, and the associated ΔT^* at the warm end. Both i and ΔT^* become larger as the NTU decreases. However, the excess flow operation when $\dot{Q}/\dot{n} < \Delta h_T$, as discussed in Sect. 3.3.2.3, has a similar effect as an ineffectiveness but the origin of this ineffectiveness is not the lack of NTU. The values of ΔT^* and i may also increase due to the larger extent of excess flow, even if $NTU \rightarrow \infty$. This is an *operational ineffectiveness* (OPR), induced in any excess flow operation. Following Eqs. 3.27 and 3.54, we have,

$$i_{OPR} = i(\dot{Q}/\dot{n}) = \frac{\Delta h_T - \dot{Q}/\dot{n}}{c_{P0}(T_1 - T_{BOIL})} = \frac{\Delta T^*}{T_1 - T_{BOIL}} \quad (3.75)$$

Since such a condition is independent of the actual value of NTU, it may be interpreted as an “artificial” ineffectiveness. The “real” or traditional ineffectiveness induced by the lack of thermal conductance, and given by Eq. 3.70,

$$i_{NTU} = i(NTU) = \frac{1-r}{e^{NTU(1-r)} - r} \quad (3.76)$$

determines the *minimum or necessary* size, NTU^* , (as displayed in Fig. 3.11 for $r = 0.8$) for any amount of excess flow ($\Delta h_T - \dot{Q}/\dot{n}$) and the corresponding ΔT^* . This value must fulfill the condition,

$$i_{OPR} = i_{NTU} = i_{NTU}^* \quad (3.77)$$

which, by Eq. 3.72, becomes

$$NTU^* = \frac{1}{1-r} \ln \left(\frac{1-r}{i_{OPR}} + r \right) \quad (3.78)$$

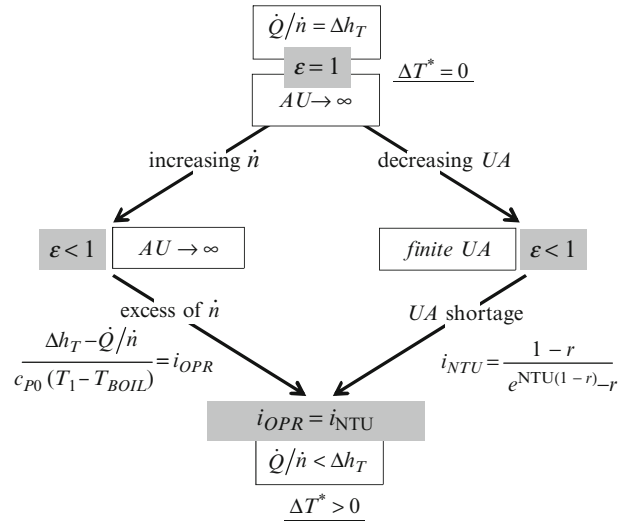


Fig. 3.13 The two sources of ineffectiveness in operation: excess of flow rate (even if there is no shortage of thermal conductance, and shortage of thermal conductance)

Note that a larger size, $NTU > NTU^*$, will not increase $\dot{Q}/\dot{n}$ or reduce ΔT^* . Put another way, operating with excess flow might result in an excess of heat transfer area; the larger heat transfer area will not reduce the ineffectiveness of operation. However, a smaller size, $NTU < NTU^*$, would not suffice to maintain the same corresponding ΔT^* and will increase the value of i_{STC} above i_{OPR} . The two origins of ineffectiveness are illustrated in Fig. 3.13.

3.7.2 The Average Ratio of Capacity Rates

In the practice of cryogenic heat exchangers and especially for Joule-Thomson cryocoolers of nominal recuperation, the lower heat capacity belongs to the cold, or returning flow stream. Since both streams of the Joule-Thomson cryocooler share the same flow rate, and the cold stream heat capacity is well approximated by c_{PO} , we obtain,

$$r = \frac{\bar{c}_{P,D}}{\bar{c}_{P,U}} \approx \frac{c_{PO}}{\bar{c}_{P,U}} \quad (3.79)$$

The average heat capacities of the low- and high-pressure streams, $\bar{c}_{P,D}$ and $\bar{c}_{P,U}$ respectively, are defined on the basis of enthalpy differences. Following the notation of Fig. 3.7b,

$$\bar{c}_{P,U} = \frac{h_1 - h_2^*}{T_1 - T_2^*} \quad (3.80)$$

$$\bar{c}_{P,D} = \frac{h_5^* - h_G}{T_5^* - T_{BOIL}} \quad (3.81)$$

The numerators in these expressions are the same recuperated enthalpy, thus,

Table 3.4 Percent values of $r = \bar{c}_{p,D}/\bar{c}_{p,U}$ at 300 K and outlet pressure 0.1 MPa at various inlet pressures for nitrogen and argon, by Eq. 3.82 with $T_1 = T_5$

P [MPa]	Nitrogen	Argon
10	65.99	56.79
20	64.93	52.04
30	66.46	53.76
40	67.97	56.27
50	69.15	58.44
60	70.03	60.14

$$r = \frac{T_1 - T_2^*}{T_5^* - T_{BOIL}} \quad (3.82)$$

Some representative values of $\bar{c}_{p,D}/\bar{c}_{p,U}$ for nitrogen and argon are listed in Table 3.4. Note that the recuperators with argon are less balanced than those using nitrogen.

3.7.3 Minimum Number of Heat Transfer Units, NTU^{MIN}

Equation 3.72 also enables us to define a relationship between a minimum value of NTU and the corresponding maximum value of ineffectiveness:

$$NTU^{MIN} = \frac{1}{1-r} \ln \left(\frac{1-r}{i^{MAX}} + r \right) \quad (3.83)$$

Applying Eq. 3.82 for the limiting case as described in the diagram of Fig. 3.7c of minimal recuperation where,

$$T_5^* = T_5' = T_1 - |\Delta T_h(T_1, P_1)| \quad (3.84)$$

and

$$T_2^* = T_2' = T_{CD} \quad (3.85)$$

hence,

$$r = \frac{T_1 - T_{CD}}{T_1 - T_{BOIL} - |\Delta T_h|} \quad @ \quad \delta h^* = \delta h^{MIN} \quad (3.86)$$

Equation 3.52 provides i^{MAX} , then,

$$NTU^{MIN} = \left(\frac{T_1 - T_{CD}}{T_{CD} - T_{BOIL} - |\Delta T_h|} + 1 \right) \ln \left(\frac{T_{CD} - T_{BOIL}}{|\Delta T_h|} \right) \quad (3.87)$$

This expression includes only thermodynamic functions of the gas itself, T_{BOIL} , T_{CD} , and ΔT_h and their dependence on the initial state (P_1, T_1) as subsequently discussed in Sects.

Table 3.5 Values of NTU^{MIN} as predicted by Eq. (3.87).

P [MPa]	Nitrogen		Argon	
	300 K	340 K	300 K	340 K
5	7.28	8.66	4.95	5.56
10	4.68	5.57	2.84	3.31
15	3.84	4.53	2.05	2.47
20	3.32	3.96	1.64	2.02
25	3.08	3.75	1.38	1.75
30	2.95	3.57	1.22	1.57
35	2.91	3.57	1.12	1.46
40	2.91	3.61	1.05	1.38
45	2.97	3.67	1.02	1.33
50	3.04	3.83	0.99	1.31
55	3.20	4.02	0.99	1.31
60	3.32	4.27	1.00	1.31

4.4 and 4.5. Values of NTU^{MIN} predicted by the above expression are listed in Table 3.5 for nitrogen and argon at 300 and 340 K for various pressures. The exact values of NTU^{MIN} appear in Table 3.9. These are derived by direct integration as given in Eq. 3.71,

$$NTU^{MIN} = \left[\int_{T_{BOIL}}^{T_5'} \frac{dT_D}{\Delta T} \right]_{\text{along the low pressure stream}} \quad (3.88)$$

using values of ΔT calculated as a function of T_D or T_U in Sect. 3.11.1.3. The procedure of obtaining values of ΔT as a function of T_D is described later in Sect. 3.11.1 and is accompanied with some representative values.

Practically, the sizes of heat exchangers must exceed the minimal values of NTU. Operation at temperatures above ambient requires an even larger heat transfer area. Argon tolerates a smaller heat exchanger than nitrogen. Both ε^{MIN} (or i^{MAX}) and NTU^{MIN} are pressure dependent. Put another way, the corresponding pressure with any NTU^{MIN} is the minimum or *cut off pressure* for that specific heat exchanger. A system operating at a slightly lower pressure will not be able to maintain cooling at T_{BOIL} since it will require a larger NTU. The requirements on NTU are less severe for elevated pressures. At lower pressures in the range of 5–20 MPa the specific heat capacities are more sensitive to temperature changes. Since the approximated expression for NTU^{MIN} by Eq. 3.87 is based on average heat capacities for the two streams (for obtaining r), it is more accurate in situations where these have a weaker temperature dependence along the channels.

One should pay attention to the fact that the NTU is inversely proportional to the flow rate, $\dot{n}$, which in turn is dependent on P . For example, in the case of a fixed orifice, $\dot{n}$ increases with P but not so in the case of a flow demand cryocooler.

Minimum sizes of heat exchangers, in terms of recommended NTU, for rapid cooldown applications with different gases at various conditions of pressure and temperature are discussed by Longworth and Steyert⁴⁷ and Bonney and Stubbs.⁴⁸

In the case when $NTU < NTU^{MIN}$ the cooler is under-recuperated. Then, the value of ΔT^* does not determine ε since the low temperature is not fixed; for the same ΔT^* one obtains different values of ε depending on how much smaller NTU is than NTU^{MIN} or the resultant low temperature.

3.7.4 Flow Rate Dependence of Recuperator's Size

A given heat exchanger, when operated at elevated flow rates, effectively “becomes shorter” because its NTU becomes smaller. For a given recuperator with a specified heat transfer area, the NTU varies inversely with the flow rate.

The ε – NTU relationship may be utilized to predict how the effectiveness, ε , degrades as the flow rate increases, or how $T_1 - T_5^*$ increases, since through the combination of Eqs. 3.64 and 3.69 we have,

$$\frac{T_1 - T_5^*}{|\Delta T_h|} = \frac{(1-r)/i^{MAX}}{e^{(1-r)NTU} - r} \quad (3.89)$$

For example, consider a nitrogen cryocooler at 300 K ambient temperature pressurized at 30 MPa with ΔT_h of -36.9 K. When its flow rate corresponds to a value of $NTU = 10$, the warm end temperature difference is about 2.6 K, the associated ineffectiveness is 0.01, and the efficiency, η , is about 0.94. By doubling the flow rate, namely, $NTU = 5$, the value of $T_1 - T_5^*$ grows to about 15.5 K, i becomes 0.07, and η drops to only 0.57.

In this sense, an “infinite” recuperator of “unlimited” heat transfer area, which enables $T_1 = T_5$, is practically realized by operating with $\dot{n}$ approaching zero rather than by operating with A approaching infinity.

The highest practical flow rate is the one that still enables the cooler to reach T_{BOIL} , but with zero cooling capacity. Such a maximum flow rate corresponds to an associated value of NTU^{MIN} by,

$$\dot{n} \leq \frac{UA}{NTU^{MIN}_{CP}} \equiv \dot{n}^{MAX} \quad (3.90)$$

For the balanced case,

$$\dot{n} \leq \frac{UA}{\left(\frac{1}{i^{MAX}} - 1\right) C_P} \equiv \dot{n}^{MAX} \quad @ \quad r = 1 \quad (3.91)$$

3.7.5 Size Versus Duty

The above paragraphs of Sect. 3.7.1. discussed the size of a recuperator in terms of NTU as a function of $T_1 - T_5^*$ or ε . The present paragraph introduces a complementary approach. It estimates the conductance (UA) of recuperators in terms of their heat transfer rate, that is the rate of recuperation, $\dot{n} \cdot \delta h^*$. Following Eq. 3.31 and as displayed in Fig. 3.7, the rate of recuperation is given by $\dot{n}(h_5^* - h_G)$ and as well by, $\dot{n}(h_1 - h_2^*)$. The temperature difference between the two streams at the warm end of the recuperator of a cryocooler is ΔT^* by Eq. 3.22, and at the cold end, $T_2^* - T_{BOIL}$. For the case in which the two streams have constant specific heats, it is common³⁸ to determine the logarithmic mean temperature difference, defined by

$$\Delta T_{LMTD} = \frac{T_2^* - T_{BOIL} - \Delta T^*}{\ln \frac{T_2^* - T_{BOIL}}{\Delta T^*}} \quad (3.92)$$

in order to obtain the total conductance of the recuperator,

$$UA = \frac{\dot{n} \delta h^*}{\Delta T_{LMTD}} \quad (3.93)$$

By explicating $\dot{n} \cdot \delta h^*$, one gets,

$$\Delta T_U = T_1 - T_2^* = \Delta T_{LMTD} \frac{NTU}{r} \quad (3.94)$$

One may refer to it in view of the unified model of cryocoolers and compare it with Eq. 1.7. For $r = 1$ the ΔT_{LMTD} is the uniform temperature difference between the streams and represents the δT and by Eq. 1.44, the NTU plays the role of $(I_M - 1)$.

3.7.6 Scaling a Recuperators' Size

From the above discussion, one can appreciate that the NTU^{MIN} and the amount of heat exchanging area will correspond, in some way, with the extent of recuperation. Longworth⁴⁹ and Bonney & Stubbs⁴⁸ provide a scale for comparing the necessary heat transfer area, by normalizing the maximum recuperated enthalpy, δh^{MAX} , with the potential specific cooling content, Δh_T , thereby defining the scaling parameter,

$$\Sigma = \frac{\delta h^{MAX}}{\Delta h_T} = 1 + \frac{\delta h^{MIN}}{\Delta h_T} = \frac{h_5 - h_G}{\Delta h_T} = \Sigma(gas, P, T) \quad (3.95)$$

From Eqs. 3.50 and 3.52 we have,

$$\Sigma = \frac{1}{1 - \varepsilon^{MIN}} = \frac{1}{i^{MAX}} = \frac{T_1 - T_{BOIL}}{|\Delta T_h|} \quad (3.96)$$

That is, the scaling factor Σ is the reciprocal of the maximum allowable inefficiency of the heat exchanger. Notice then that a coolant that allows a larger value of i^{MAX} will be able to accommodate a smaller heat exchanger.

For the group of balanced cases, by Eq. 3.73,

$$NTU^{MIN} = \sum -1 = \frac{\delta h^{MIN}}{\Delta h_T} @ r = 1 \quad (3.97)$$

and one may observe that Σ is a linear measure of NTU. Dobak et al.^{50,51} use values of Σ as a criterion for selecting mixtures that will accommodate smaller recuperators. Indeed, the candidate mixtures providing the higher values of efficiency are almost balanced; hence, it is appropriate to use it as a measure. However, in general, the relationship between Σ and the NTU, as obtained through Eq. 3.83, is

$$NTU^{MIN} = \frac{1}{1-r} \ln \left[(1-r) \sum +r \right] \quad (3.98)$$

Therefore, the scale of Σ does not provide a linear means for measuring NTU^{MIN} . Furthermore, it also depends on r . Nevertheless, it may be used for comparison purposes. For example, a meaningful comparison can be made between recuperators even for two systems with differing gases and operating conditions:

	Nitrogen at (30 MPa, 300 K)	Argon (10 MPa, 350 K)
δh^{MAX}	6,572 J/mole	5,503 J/mole
Δh_T	1,077 J/mole	513 J/mole
Σ	6.4	10.7

As indicated by Σ , the heat exchanger of nitrogen is smaller in terms of NTU^{MIN} , although it recuperates 6,572 J/mole while that of argon recuperates only 5,503 J/mole.

Longworth⁴⁹ calculated values of Σ for various gases at operating conditions along the differential inversion curve, which means that for each temperature, the associated optimal pressure with the largest value of Δh_T is applied. For each gas, the values of h_G used in Eq. 3.95 were determined from the lowest possible boiling point, namely, the triple point.

In general, gases with a higher boiling point require smaller heat exchangers. However, krypton, which exhibits a lower boiling point than CF_4 , requires a smaller heat exchanger. The noble gases have a unique position in this sense, requiring less heat transfer area as compared to other gases than would be expected from a comparison of their boiling points. This topic will be discussed later in Sect. 4.9.

Let us examine the significance of the condition defined by $\Sigma = 1$. In this case, by Eq. 3.96, $i^{MAX} = 1$ and therefore $\varepsilon^{MIN} = 0$ and $\delta h^{MIN} = 0$. In other words, this case is associated with no recuperation and no heat exchanger at all. Put another way, this is the case for which the boiling

point is achieved by a direct and single expansion without any recuperative heat exchanging, i.e., $NTU = 0$.

It is also true that when $\Sigma = 1$, the recuperated enthalpy exactly matches the cooling content, $\delta h = \Delta h_T$, a condition that occurs in the limiting case when the cooler's inlet temperature, T_1 , equals T_{CD} . This case is highlighted by the isotherms in the P - h plane of Fig. 3.3. So, that,

If $T_1 > T_{CD}$ then $\delta h \leq \Delta h_T$ $\sum > 1$ and $NTU^{MIN} > 0$, and,
If $T_1 \leq T_{CD}$ then $\delta h \leq \Delta h_T$ $\sum \leq 1$ and $NTU^{MIN} = 0$.

In practice the condition of $T_1 \leq T_{CD}$ may occur in two cases:

- While using a coolant with a somewhat elevated boiling point such as krypton (at the proper pressure) or nitrous oxide, and,
- When precooling a low boiling point coolant, such as if nitrogen is precoolled by CF_4 .

Even in the case when $T_1 \leq T_{CD}$ one may use a recuperator with $NTU > 0$ in order to increase the yield of liquefaction.

3.8 Yield of Liquefaction

Liquefaction occurs when a fraction of the stream that is expanded into the evaporator of the Linde-Hampson cryocooler is in the liquid phase. During steady state operation, the rate of evaporation of the liquefied portion is a direct reflection of the cryocooler's cooling capacity at the corresponding boiling temperature. Thus, the rate of liquid production in a Joule-Thomson cryocooler determines a limit to its cooling power, that is, the rate at which it can absorb heat from a payload. The defining characteristic of a liquefier is its production and supply of liquefied gas. The yield of liquefaction for both the cryocooler and the liquefier is defined by the *liquid* mass fraction, y , of the pressurized incoming flow rate, $\dot{n} = \dot{n}_U$. The terms $y^{(1)}$ and $y^{(2)}$ are used respectively for the liquid mass fractions for a cryocooler and a liquefier. The rate of liquid production in either device is then given by $y \dot{n}$.

3.8.1 The Cryocooler

3.8.1.1 The Ideal Operation

The ideal operation of a cryocooler, that is the one with the highest specific cooling capacity, $(\dot{Q}/\dot{n})^{MAX}$, is also the one that has the highest steady state yield of liquefaction. Such a condition occurs when $T_1 = T_5$. The maximum cooling power can also be expressed in terms of the latent heat of evaporation, L ,

$$L \equiv h_G - h_F \quad (3.99)$$

Table 3.6 Ideal cooler yield of liquefaction, $y^{(1)} = \Delta h_T / L$, (%), with different gases and pressures expanding to $P_5 = 0.1$ MPa

(a)						
$T_1 = T_5 = 300$ K						
P [MPa]	N_2	Ar	O_2	CH_4	Kr	CF_4
5 MPa	5.15	5.65	5.55	9.52	9.56	14.78
10 MPa	9.68	11.12	10.94	19.37	20.22	31.20
20 MPa	16.16	20.29	20.04	34.91	38.57	46.15
30 MPa	19.33	26.36	26.16	42.09	46.74	50.42
40 MPa	20.09	29.64	29.58	44.48	49.48	52.13
50 MPa	19.29	30.96	30.09	44.46	50.01	52.88

(b)				(c)	
$T_1 = T_5 = 100$ K				$T_1 = T_5 = 15$ K	
P [MPa]	H_2	D_2	Ne	P [MPa]	He
5 MPa	14.85	11.64	8.86	1	36.98
10 MPa	24.00	19.00	16.14	2	64.88
20 MPa	26.50	23.82	24.71	3	75.11
30 MPa	17.40	19.91	26.82	6	51.22
40 MPa	3.45	11.82	25.23	9	2.83

through the linear relationship:

$$\dot{Q}^{MAX} = y^{(1)} \dot{n} L = \dot{n} \Delta h_T \quad (3.100)$$

Thus when the cryocooler is operating at steady state,

$$y^{(1)} = \frac{\Delta h_T}{L} \quad (3.101)$$

Some values for the liquefaction yield of different gases are listed in Table 3.6 for $P_5 = 0.1$ MPa. Higher value of P_5 elevates the values of $y^{(1)}$. One may observe that at similar inlet conditions the values are larger for the higher boiling point coolants. For instance, methane generates a yield double that of nitrogen.

3.8.1.2 Operation with Excess Flow

Operation with excess flow is associated with a lower specific cooling power, $\dot{Q}/\dot{n}$, and a yield of liquefaction, $y_e^{(1)}$ or $y_\eta^{(1)}$, that remains below the ideal value $y^{(1)}$, that is,

$$y_\eta^{(1)} = \frac{\Delta h^*}{L} = \eta \frac{\Delta h_T}{L} \eta y^{(1)} \quad (3.102)$$

This relationship is independent of the extent of recuperation, whether nominal or over recuperated. However, if we restrict the discussion only to cryocoolers that have nominal recuperation, then it becomes possible to apply and combine all the previously obtained expressions in Sect. 3.5. that are associated with the effectiveness and efficiency of recuperators, such as Eq. 3.60:

Table 3.7 Liquefaction yield ratio between an ideal cooler and liquefier, (independent on the feed pressure) Eq. (3.105) for $P_5 = 0.1$ MPa

(a) $T_1 = T_5 = 300$ K							
N_2	Ar	O_2	CH_4	Kr	CF_4	C_2H_6	C_3H_8
2.18	1.69	1.91	1.79	1.42	1.69	1.37	1.25

(b) $T_1 = T_5 = 100$ K				(c) $T_1 = T_5 = 15$ K	
H_2	D_2	Ne		He	
2.92	2.54	1.90		3.96	

$$\frac{y_e^{(1)}}{y^{(1)}} = \eta = \frac{\varepsilon - \varepsilon^{MIN}}{1 - \varepsilon^{MIN}} \quad (3.103)$$

Recall that ε^{MIN} is the minimum effectiveness of the recuperator as first introduced in Sect. 3.5.2.3. The extent to which the liquid yield is degraded is related to ε can be obtained by combining Eqs. 3.25, 3.61, 3.101, and 3.102 to give,

$$y^{(1)} - y_e^{(1)} = i \frac{\delta h^{MAX}}{L} = \frac{\Delta T^*}{L/c_{PO}} \quad (3.104)$$

3.8.2 The Liquefier

3.8.2.1 The Ideal Liquefier Versus Cryocooler

Through Eqs. 3.66 and 3.101, for similar initial conditions, the ratio of the yield of a cryocooler to that of a liquefier becomes,

$$\frac{y^{(1)}}{y^{(2)}} = \frac{h_5 - h_F}{L} = \frac{\delta h^{MAX}}{L} + 1 \quad (3.105)$$

Values of this ratio for various gases are listed in Table 3.7 for $P_5 = 0.1$ MPa. These values increase for higher level of P_5 . One may observe several characteristics regarding this ratio:

1. It is independent of the inlet pressure, but dependent on the operating temperature.
2. Its value approaches unity as the, ratio $\delta h^{MAX}/L$, decreases. In general, less recuperation is required for liquefying the higher boiling point gases which have larger L .
3. A cooler produces more liquid than the liquefier under similar operating conditions,

$$y^{(1)} > y^{(2)} \quad (3.106)$$

The last observation results from the fundamental difference between the two systems. The returning flow stream in a liquefier is reduced by the amount of $y^{(2)} \dot{n}$. The recuperative heat exchange process is thereby also reduced and less productive in terms of liquefaction yield.

The difference between the reciprocals of the corresponding y values,

$$\frac{1}{y^{(2)}} - \frac{1}{y^{(1)}} = \frac{\delta h^{MAX}}{\Delta h_T} = \frac{1}{i^{MAX}} \quad (3.107)$$

is pressure dependent through i^{MAX} . As explained in Sect. 3.6.1, the same value of i^{MAX} holds for both cryocooler and liquefier as long as they operate with the same inlet conditions.

3.8.2.2 The Finite Size Recuperator

The reduced yield produced by a liquefier operating with a non-ideal recuperator, that is one with a non-zero ineffectiveness, is noted by $y_\varepsilon^{(2)}$ or by the associated term $y_\eta^{(2)}$. Since *all* liquefiers (that produce some liquid) have **nominal recuperators** it becomes possible to utilize all the expressions of Sect. 3.5. that are associated with ε , η , and $T_1 - T_5^*$.

Using an energy balance, we have,

$$\begin{aligned} y_\varepsilon^{(2)} &= \frac{h_5^* - h_1}{h_5^* - h_F} = \frac{\Delta h^*}{\delta h^* + L} = \frac{\Delta h_T - (1 - \varepsilon) \delta h^{MAX}}{\varepsilon \delta h^{MAX} + L} \\ &= \frac{\Delta h_T - i \delta h^{MAX}}{h_5 - h_F - i \delta h^{MAX}} \end{aligned} \quad (3.108)$$

and in reference to the yield of a cryocooler of the same effectiveness,

$$\frac{y_\varepsilon^{(1)}}{y_\varepsilon^{(2)}} = \frac{\delta h^*}{L} + 1 = \varepsilon \frac{\delta h^{MAX}}{L} + 1 \quad (3.109)$$

namely, the ratio is linearly dependent on ε .

By Eq. 3.108, combined with Eq. 4.63 so that $\Delta h_T / c_{PO} = |\Delta T_h|$,

$$\begin{aligned} y_\eta^{(2)} &= \frac{h_5^* - h_1}{h_5^* - h_F} = \frac{\Delta h_T - c_{PO} \Delta T^*}{h_5^* - h_F - c_{PO} \Delta T^*} = \frac{1 - \frac{c_{PO} \Delta T^*}{\Delta h_T}}{\frac{h_5^* - h_F}{\Delta h_T} - \frac{c_{PO}}{\Delta h_T} \Delta T^*} \\ &= \frac{\left(1 - \frac{\Delta T^*}{|\Delta T_h|}\right) y^{(2)}}{1 - \frac{\Delta T^*}{|\Delta T_h|} y^{(2)}} = \frac{\eta y^{(2)}}{1 - (1 - \eta) y^{(2)}} \end{aligned} \quad (3.110)$$

thus,

$$y_\eta^{(2)} > \eta y^{(2)} \quad (3.111)$$

Notice that Eq. 3.110 for a liquefier is analogous to the relationship given by Eq. 3.102 for a cryocooler.

By Eq. 3.102, combined with the result of Eq. 3.58, which is $h_5^* - h_1 = \eta \Delta h_T$, one obtains the yield of liquefaction in terms of the efficiency, η ,

$$\begin{aligned} \frac{1}{y_\eta^{(2)}} &= \frac{h_5^* - h_F}{h_5^* - h_1} = \frac{h_1 - h_F}{h_5^* - h_1} + 1 = \frac{h_5 - \Delta h_T - h_F}{\eta \Delta h_T} + 1 \\ &= \frac{1}{\eta} \left(\frac{h_5 - h_F}{\Delta h_T} - 1 \right) + 1 = \frac{1}{\eta} \frac{\delta h^{MIN} + L}{\Delta h_T} + 1 \\ &= \frac{Const}{\eta} + 1 \end{aligned} \quad (3.112)$$

3.8.2.3 The Splitting Ratio, SP

The splitting ratio, SP represents the ratio of liquefied flow rate that is removed, $y^{(2)}$, to the gas flow rate that returns to the compressor, $1 - y^{(2)}$. For an ideal liquefier,

$$SP \equiv \frac{y^{(2)}}{1 - y^{(2)}} = \frac{\Delta h_T}{\delta h^{MIN} + L} \quad (3.113)$$

Note that SP is monotonically dependent on $y^{(2)}$. The real liquefier with a recuperator of efficiency η has a degraded splitting ratio,

$$SP_\eta \equiv \frac{y_\eta^{(2)}}{1 - y_\eta^{(2)}} \quad (3.114)$$

By Eq. 3.112, we obtain the relationship,

$$\frac{1}{y_\eta^{(2)}} - 1 = \frac{1}{\eta} \left[\frac{1}{y^{(2)}} - 1 \right] \quad (3.115)$$

which means,

$$SP_\eta = \eta SP \quad (3.116)$$

Hence, in analogy to the cooling capacity of nominal cryocoolers that are degraded by the factor η , the splitting ratio of liquefiers are also degraded by the factor η . The factor $y_\eta^{(2)}$ is itself degraded by η through the function,

$$y_\eta^{(2)} = 1 - \frac{1}{1 + \eta SP} \quad (3.117)$$

3.8.2.4 Sizing a Liquefier's Recuperator

Cryocoolers and liquefiers have both been characterized by the effectiveness of their recuperators. However, since they do not share a common ratio of capacity rates, the associated size of their respective recuperators are not identical even if their respective effectiveness values are the same. Put another way, the same value of ε leads to different values of

NTU because of different respective values of r . The translation of the NTU of the liquefier in order to obtain the heat transfer area, A , has to account for the reduced flow rate.

3.9 Maximizing Production Rates

3.9.1 The Highest Specific Cooling Rate, $\dot{Q}/\dot{n}$, Versus the Highest Cooling Rate, $\dot{Q}$

The highest specific cooling capacity of a cryocooler, $(\dot{Q}/\dot{n})^{MAX}$ and the highest yield of liquefaction of a liquefier, $y^{(2)}$, are achieved when $T_1 - T_S^* = 0$ or when ε and η reach the value of unity without any excess flow rate (Sect. 3.3.2.1). Such a condition requires a recuperator with an infinitely large NTU. However, any real Linde-Hampson machine has a finite size (UA) recuperator. Hence, infinitely large NTU means infinitely small (zero) $\dot{n}$ and therefore, $\dot{Q} = 0$ and $y^{(2)} = 0$. A non zero flow rate $\dot{n}$ will reduce ε and η but will generate a non-zero cooling capacity, that is $\dot{Q} > 0$. The purpose of this section is to identify conditions that maximize the production rate⁵² of cooling, $\dot{Q}$, and liquefaction, $y^{(2)}$, while allowing degradations of $\dot{Q}/\dot{n} < (\dot{Q}/\dot{n})^{MAX}$ and $y_e^{(2)} < y^{(2)}$. For example, it is the larger value of $\dot{Q}$ that accelerates the cooldown process, not $\dot{Q}/\dot{n}$. The results will demonstrate that the optimum conditions are achieved at relatively low efficiencies and conditions of high excess flow. This discussion is a type of finite-time consideration^{53,54} for endo-reversible Carnot engines, being limited by the rates of heat transfer and optimized for their finite output power.

3.9.2 Cryocoolers with Fixed Recuperating Area

We begin with the objective of finding the flow rate for any given size of recuperator, A , which maximizes the cooling power. The potential specific cooling capacity, Δh_T , is dependent solely on the coolant and the operating conditions (P_1, T_1). The actual cooling power of any given size recuperator, A , depends on the actual flow rate, $\dot{n}$, associated with a particular size of an orifice at any given inlet pressure and temperature. However, in that the flow rate impacts the NTU, it also influences the effectiveness of the heat exchanger, ε , and efficiency, η , so that the combined influence of both effects is,

$$\dot{Q} = \dot{n} \eta \Delta h_T = \dot{n} \frac{\varepsilon - \varepsilon^{MIN}}{1 - \varepsilon^{MIN}} \Delta h_T \quad (3.118)$$

Furthermore, ε and $\dot{n}$ are interconnected in an opposing manner. On the one hand, a larger flow rate enhances the heat-transfer-coefficient and as reflected in Eq. 3.118 directly increases the refrigeration power. On the other hand, it indirectly “shrinks” the size of the heat exchanger since now the values of NTU become effectively smaller, being inversely proportional to $\dot{n}$. Therefore, the actual effectiveness decreases and a reduced fraction of the potential cooling capacity is realized. Such conflicting trends result in an optimal flow rate that maximizes $\dot{Q}$.

Since,⁵⁵

$$Nu \propto Re^{0.8} \quad (3.119)$$

therefore,

$$U \propto \dot{n}^{0.8} \quad (3.120)$$

and for a heat exchanger with constant area, A , we have,

$$NTU = \frac{UA}{\dot{n} c_P} \propto \frac{1}{\dot{n}^{0.2}} \quad \text{or} \quad \dot{n} \propto NTU^{-5} \quad (3.121)$$

Recalling the relationships between i , η , and NTU by Eqs. 3.63 and 3.70,

$$\eta = 1 - \frac{i}{i^{MAX}} = 1 - \frac{(1-r)/i^{MAX}}{e^{NTU(1-r)} - r} \quad (3.122)$$

for any given operating conditions, (P_1, T_1),

$$\begin{aligned} \dot{Q} &= \eta \dot{n} \Delta h_T \propto \Delta h_T \frac{1}{NTU^5} \left[1 - \frac{(1-r)/i^{MAX}}{e^{NTU(1-r)} - r} \right] \\ &\equiv \Delta h_T f(NTU) \end{aligned} \quad (3.123)$$

$$\frac{1}{NTU^5} \left[1 - \frac{(1-r)/i^{MAX}}{e^{NTU(1-r)} - r} \right] \equiv f(NTU) \quad (3.124)$$

The function $f(NTU)$ for nitrogen and argon for various operating pressures is plotted in Figs. 3.14 and 3.15. Each pressure has associated parameters of i^{MAX} and r . Hence, $f(NTU)$ at any given pressure is proportional to the cooling rate as a function of NTU and its associated value of $\dot{n}$.

Indeed, it is apparent that there is an optimal value of NTU, and an associated flow rate, that maximizes $f(NTU)$. For example, when operating with nitrogen at 30 MPa and 300 K, it is worthwhile to increase the flow rate so that the value of NTU is decreased to 3.53, and the resulting efficiency of the recuperator is only $\eta = 0.23$ (about a quarter of the ideal value), in order to generate the highest cooling capacity. This is somewhat above the minimum value, that

Fig. 3.14 The function $f(\text{NTU})$ for nitrogen, at various pressures, presented as function of NTU which represents the flow rate, $\dot{n}$. The function $f(\text{NTU})$ is proportional to the cooling rate

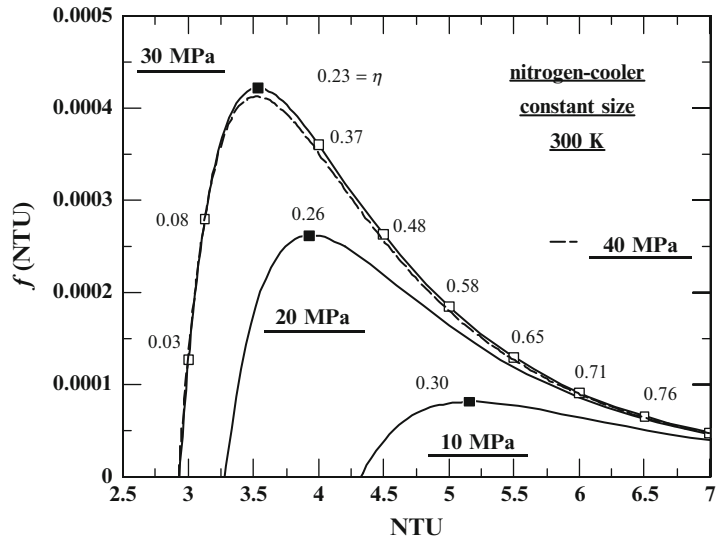
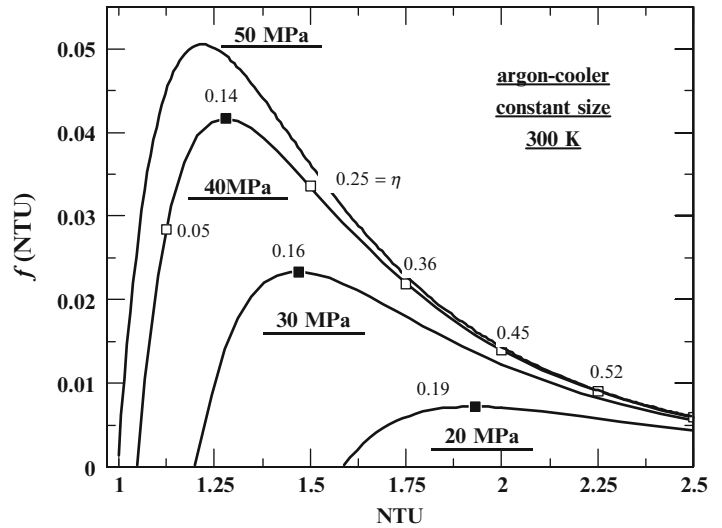


Fig. 3.15 The function $f(\text{NTU})$ for argon, at various pressures, presented as function of NTU which represents the flow rate, $\dot{n}$. The function $f(\text{NTU})$ is proportional with the cooling rate



is $\text{NTU}^{\text{MIN}} = 2.95$ as independently derived in Table 3.5. Similarly, argon at 40 MPa and 300 K, reaches the highest $\dot{Q}$ when the flow rate is large enough to result in an $\text{NTU} = 1.28$ and $\eta = 0.14$. In this case, $\text{NTU}^{\text{MIN}} = 1.05$. The low values of η associated with the largest values of $\dot{Q}$ correspond with large values of $T_1 - T_5^*$, by Eq. 3.64 and later confirmed in Table 3.8.

The $f(\text{NTU})$ functions may be used to compare the optimum cooling capacities at different pressures only after being multiplied by the typical values of Δh_T for each pressure. Then the global optimum for nitrogen is realized at 37.6 MPa with $\text{NTU} = 3.6$ and $\eta = 0.22$. For argon, the global optimum is reached at 53.2 MPa (also very close to the inversion pressure) with $\text{NTU} = 1.23$ and $\eta = 0.13$.

A sharp decrease in flow rate, as occurs in flow regulated cryocoolers (discussed in Sects. 5.3.3 and 6.2), elevates

the values of NTU , η , and $\dot{Q}/\dot{n}$. However, it also decreases the generated cooling capacity $\dot{Q}$ to match the actual heat load.

3.9.3 Cryocoolers with Fixed Flow Rate

The objective of the present section is to investigate the benefit that can be realized by reducing the size of a Hampson type recuperator until the cooling power per unit area of the recuperator is maximized. This means making the most compact heat exchanger for a cryocooler that still provides the necessary cryocooling power. The compactness of the cryocooler may be measured by the parameter $\dot{Q}/(UA)$, which is the cooling power per unit size of the recuperator. We will consider a Hampson type recuperator for which the cross sectional area of the channels of the

recuperating streams is preserved while the length is shortened. For the case of constant $\dot{n}$, U may be regarded as constant and the desired maximization becomes,

$$\frac{\dot{Q}}{U \cdot A} = \frac{\dot{n} \cdot \eta \cdot \Delta h_T}{U \cdot A} \quad (3.125)$$

As discussed later in Sect. 4.5.2, ΔT_h and Δh_T are inter-related through Eq. 4.64 so that, $\Delta T_h = -\Delta h_T / c_{PO}$, we find,

$$\frac{\dot{Q}}{U A} = \frac{\Delta h_T}{c_{PO}} \frac{\eta}{NTU} = -\Delta T_h \frac{\eta}{NTU} \quad (3.126)$$

where ΔT_h is a function of the inlet conditions, (P_1, T_1) and,

$$\frac{\eta(NTU)}{NTU} \equiv g(NTU) \quad (3.127)$$

The function $g(NTU)$ peaks at low values of NTU and efficiency. For nitrogen the highest value of $\dot{Q}/(UA)$ is 4.3 K reached at 37.8 MPa with a recuperator of $NTU = 4.2$ and $\eta = 0.67$. For argon, the highest $\dot{Q}/(UA)$ is 22.3 K reached at 52.5 MPa with a recuperator of $NTU = 2.0$ and $\eta = 0.55$.

3.9.4 Liquefiers with Fixed Recuperating Area

For a given size of recuperator in a liquefier, the optimum performance is achieved by increasing the flow rate to obtain the highest production of liquid, that is $y_\eta^{(2)}$. Recalling from Eq. 3.117 that

$$y_\eta^{(2)} = \frac{\eta SP}{1 + \eta SP} \quad (3.128)$$

and combining this with the result of Eq. 3.121, converts the maximization objective to,

$$y_\eta^{(2)} \dot{n} \propto \frac{y_\eta^{(2)}}{NTU^5} = \frac{\eta SP}{1 + \eta SP} \frac{1}{NTU^5} \equiv j(NTU) \quad (3.129)$$

This target function, $j(NTU)$, depends on η , which includes the capacity ratio r as a parameter as seen in Eq. 3.69. The removal of the liquefied yield contributes to the imbalance of the capacity rates in addition to the difference of the specific heat capacities of the two streams. That is in contrast to the previous cases of the above Sects. 3.9.2 and 3.9.3. There, the sole source of imbalance is the difference of specific capacity rates. In that case, r is a fixed figure that only depends on the supplied pressures. However, in the present case,

$$\begin{aligned} r &= \frac{\dot{C}_D}{\dot{C}_U} = \frac{\dot{n}_D}{\dot{n}_U} \cdot \frac{\bar{c}_{P,D}}{\bar{c}_{P,U}} = \left[1 - y_\eta^{(2)}\right] \frac{\bar{c}_{P,D}}{\bar{c}_{P,U}} \\ &= \frac{1}{1 + \eta \cdot SP} \cdot \frac{\bar{c}_{P,D}}{\bar{c}_{P,U}} \end{aligned} \quad (3.130)$$

Hence, $j(NTU)$ is obtained by Eq. 3.129 in conjunction with Eqs. 3.130 and 3.69. Here too there is an optimum flow rate that maximizes the rate of the liquefier's production of liquid. For instance, at 40 MPa, the optimum flow rate is associated with an NTU of 3.5 and a yield of liquefaction that is only 0.26 of the ideal value determined by Eq. 3.66.

3.10 Nozzle Inlet Temperature

One more temperature is necessary to fully explore the Linde-Hampson cycle and the associated temperature bounds of the recuperator. It is the temperature of the high pressure stream exiting the recuperator prior to the isenthalpic expansion. For ideal recuperators this temperature is represented by T_2 (Figs. 3.3, 3.4, and 3.5) for a cryocooler of nominal recuperation, and by T_{22} (Fig. 3.10) for the liquefier. In any real case of non-vanishing ineffectiveness, the corresponding temperatures are T_2^* (Fig. 3.7) and T_{22}^* . The temperature T_2 reflects how far below ambient the “interchanging” process (in the sense of Sect. 1.1.1) has to take place in order to reach the boiling point and maintain the desired cooling capacity or liquid cryogen production at steady state.

The isenthalpic expansion at the nozzle results in a two phase flow at the outlet so that,

$$h_2 = h_3 = y \cdot h_F + (1 - y) \cdot h_G \quad (3.131)$$

Introducing expressions for the ideal yield, $y^{(1)}$ and $y^{(2)}$ as derived in Eqs. 3.97 and 3.101 leads to,

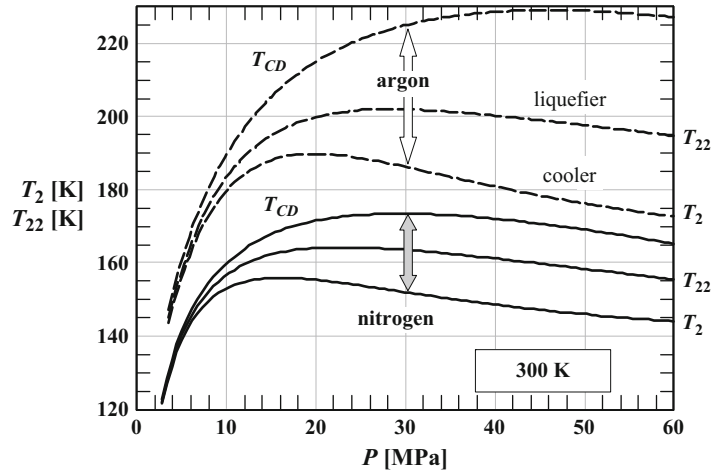
$$h_2 = h(T_2, P_1) = h_G - \Delta h_T(T_1, P_1) \quad (3.132)$$

$$h_{22} = h(T_{22}, P_1) = h_G - \frac{L}{h_5 - h_F} \Delta h_T(T_1, P_1) \quad (3.133)$$

The above equations can be solved to obtain the responding values of T_2 and T_{22} as a function of any given inlet conditions, (T_1, P_1) . Comparing the two equations, one should notice that $L/(h_5 - h_F)$ is the ratio of the yield of liquefaction for a liquefier to that of a cooler, see Eq. 3.102. Since $L = h_G - h_F < h_5 - h_F$, then,

$$h_2 < h_{22} \quad (3.134)$$

Fig. 3.16 Nozzle inlet temperature for an ideal cryocooler (T_2) and an ideal liquefier (T_{22}) as a function of inlet pressure, P , at the inlet temperature of 300 K, for nitrogen and argon



Further, because enthalpy is a monotonically increasing function of temperature at a given pressure (recall that $c_P > 0$), then for any given gas,

$$T_2 < T_{22} \quad (3.135)$$

In other words, the cooler's nozzle inlet temperature is lower than that of a liquefier.

In the case of real cycles, with a non-zero recuperator ineffectiveness, the same equations can be solved for T_2^* and T_{22}^* (instead of T_2 and T_{22}) after replacing Δh_T by Δh^* . Since Δh^* is smaller than Δh_T , h_2 will be smaller than h_2^* so that,

$$T_2^* > T_2 \quad \text{and} \quad T_{22}^* > T_{22} \quad (3.136)$$

Thus, the “interchanging” process produces less of a temperature drop as the ineffectiveness of the “interchanger” is increased.

Figure 3.16 displays the pressure dependent solutions of T_2 for a cooler and T_{22} for a liquefier both operating with nitrogen and argon at $T_1 = 300 \text{ K}$. A third curve for each gas shows the pressure dependence of T_{CD} , that was defined in Sect. 3.2.1.6. and will be fully discussed in Sect. 4.4. T_{CD} is the upper limit of T_2 and T_{22} and corresponds to cycles with zero cryocooling and yield of liquefaction. Therefore, following the notation of Fig. 3.7c,

$$T_2' = T_{CD} \quad (3.137)$$

The extent that T_2 falls below T_{CD} , shown in Fig. 3.7a, may be expressed as,

$$T_{CD} - T_2 = \frac{\Delta h_T}{c_{P,U}} = \frac{c_{PO}}{c_{P,U}} \frac{\Delta h_T}{c_{PO}} = r |\Delta T_h| \quad (3.138)$$

where $|\Delta T_h|$ is a function of (T_1, P_1) . The same relation holds for the liquefier's value of T_{22} but in this case r is smaller, according to Eq. 3.130. The pressure dependence of ΔT_h , as subsequently discussed in Sect. 4.5, and its dependence on the species of gas, throws some additional light on the separation between T_{CD} and T_2 . Similarly,

$$T_{CD} - T_2^* = r (|\Delta T_h| - \Delta T^*) \quad (3.139)$$

Coolers that operate with neon or hydrogen requiring a staged configuration have been studied separately. Such coolers have to be pre-cooled by an additional stage of liquid argon, air or nitrogen, for which reason the inlet temperature must be set as low as $T_1 = 100 \text{ K} = T_5$. Lacking the pre-cooling, they would warm up instead of cool down upon expansion, since room temperature is high above their inversion temperatures. These gases' nozzle inlet values, T_2 , are shown in Fig. 3.17.

Figures 3.18 and 3.19 respectively demonstrate the influence of the recuperator inlet temperature, T_1 , on cooler's nozzle inlet temperature, T_2 for nitrogen and helium cryocoolers.

One may observe the following common aspects of T_2 :

- The temperature T_2 displays a maximum as a function of the operating pressure. It moderately decreases at higher pressures and strongly decreases at lower pressures.
- T_2 decreases monotonically as the cooler's inlet temperature, T_1 decreases. However, the maxima become remarkably sharper at lower values of T_1 .
- Representative values: for a cooler and liquefier of nitrogen at 300 K the maximum temperatures are 165 and 157 K, correspondingly. Similarly, for argon these are 202 and 190 K.

Fig. 3.17 Nozzle inlet temperature for an ideal cryocooler T_2 and an ideal liquefier (T_{22}) as a function of inlet pressure, P , at the inlet temperature of 100 K, for hydrogen and neon

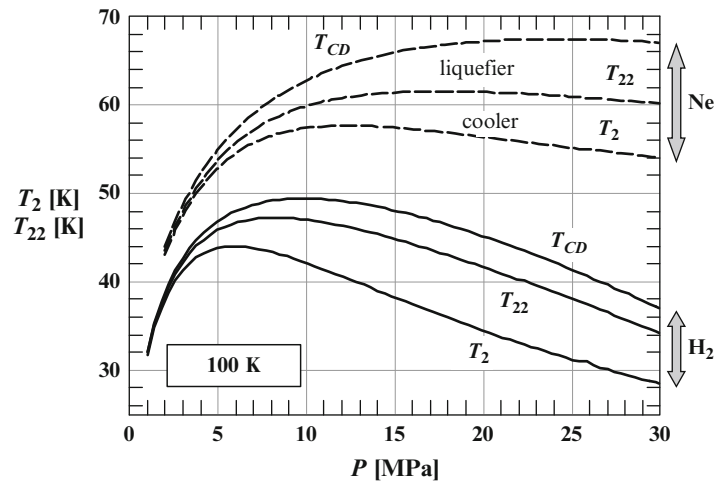


Fig. 3.18 Nozzle inlet temperature for an ideal cryocooler (T_2) as a function of inlet pressure, P , at the various inlet temperatures, for nitrogen

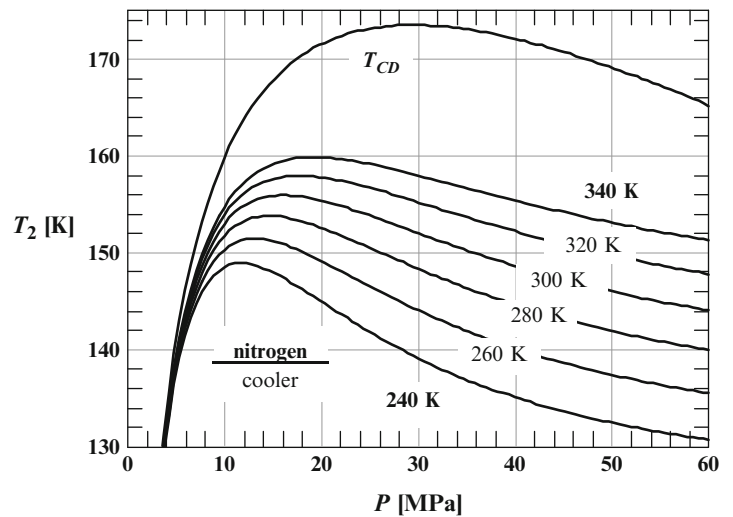
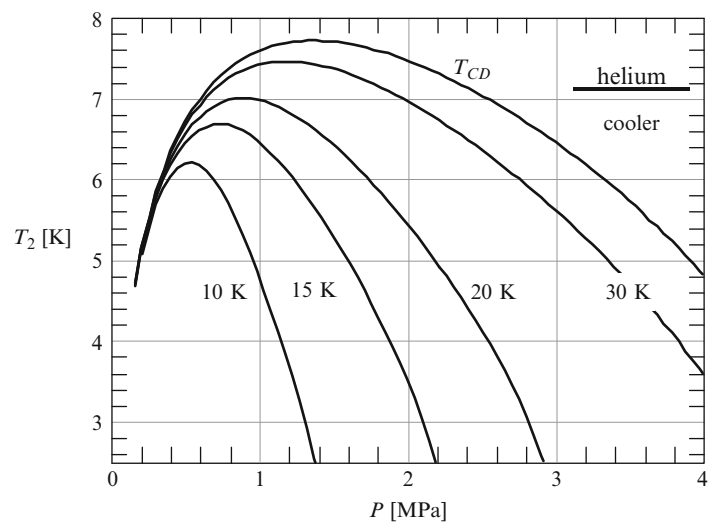


Fig. 3.19 Nozzle inlet temperature for an ideal cryocooler (T_2) as a function of inlet pressure, P , at the various inlet temperatures, for helium



- (d) For argon, T_2 is higher than for nitrogen. One may deduce that argon tolerates a lower recuperator performance. For neon T_2 is higher than for hydrogen.

3.11 Temperature Differences Between the Recuperating Streams

3.11.1 The Coolers

3.11.1.1 The Ideal Cooler

To begin, we choose a control volume, as shown in Fig. 3.20a, that includes only a fraction of the heat exchanger and its cold end. At any specified longitudinal cross section, X , of the heat exchanger, the temperature of the high pressure incoming stream is, $T_U(X)$, and the corresponding temperature of the low pressure out-going stream is, $T_D(X)$. The corresponding enthalpies of the up-stream and down-stream flows are $h_U = h(T_U, P_U)$ and $h_D = h(T_D, P_D)$. This control volume defines an open system, and its energy balance at steady state enables us to determine h_D in terms of h_U . For the ideal case with maximum specific cooling capacity, where $T_1 = T_5$, the relationship becomes,

$$h(T_D, P_D) - h(T_U, P_U) = \Delta h_T \quad (3.140)$$

This equation can be solved implicitly for T_D as a function of T_U for any given P_D and P_U . One may then obtain the temperature difference between the two streams,

$$\Delta T = T_U - T_D \quad (3.141)$$

for any given T_U or for the corresponding T_D . Put another way, at any cross section defined by a certain value of T_U ,

(not at a specified location X , as will be subsequently discussed) one can evaluate the temperature difference $\Delta T = T_U - T_D$ at steady state.

Figure 3.21 displays values of ΔT along the recuperator, once as a function of T_U , and once as a function of T_D for nitrogen and argon nominal coolers and liquefiers. The lowest values of T_U (marked by squares) are the values of T_2 corresponding to the different operating pressures. The lowest values of T_D are T_{BOIL} . The operating conditions of this figure are 8 and 20 MPa for the upstream pressure, a recuperator inlet temperature of 300 K, and a downstream pressure $P_D = 0.101$ MPa. Figure 3.22 displays ΔT values for neon and hydrogen coolers as a function of T_U operating with a recuperator inlet temperature of 100 K. This figure includes values for the over recuperated cryocoolers as well. The values of T_U at the small circles mark the associated values of T_2 for the nominal recuperator. For values of T_U below this point (over-recuperated), ΔT decreases linearly as shown by the dashed lines, since the associated values of T_D are fixed at T_{BOIL} .

3.11.1.2 Cryocoolers Operating with Excess Flow

Cryocoolers operating with excess flow were discussed in Sect. 3.3.2. This condition results in a fractional specific cooling capacity, $\Delta h^* = h_5^* - h_1 < \Delta h_T$, and the energy balance leads to the equation,

$$h(T_D, P_D) - h(T_U, P_U) = \Delta h^* = \Delta h_T - (h_5 - h_5^*) \quad (3.142)$$

Equation 3.142 can be solved to obtain ΔT as a function of T_D for various values of $T_1 - T_5^*$. The family of curves in Fig. 3.23 belongs to nitrogen covering the entire

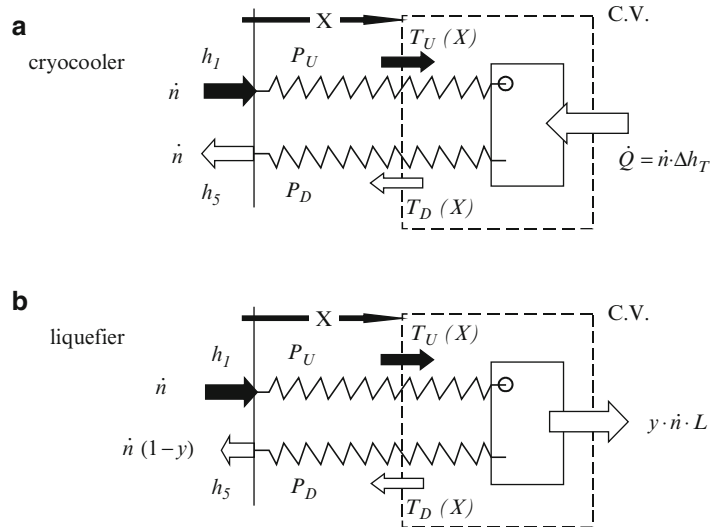


Fig. 3.20 Defined control-volumes for the energy balance in the case of a cooler and a liquefier

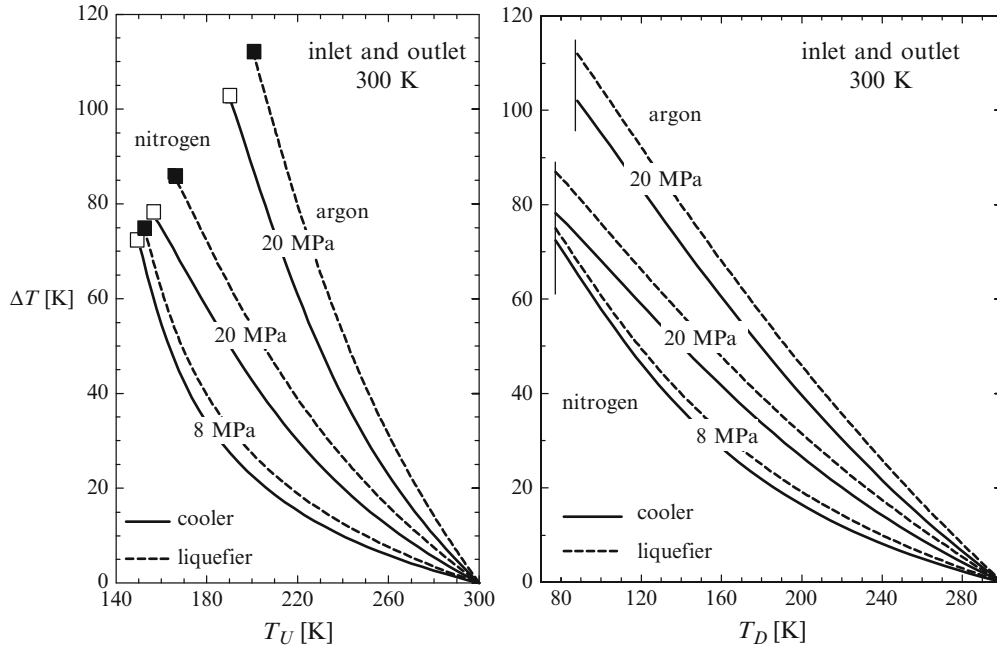


Fig. 3.21 Temperature differences between the recuperating streams along the heat exchanger of an ideal cooler and liquefier of nitrogen and argon

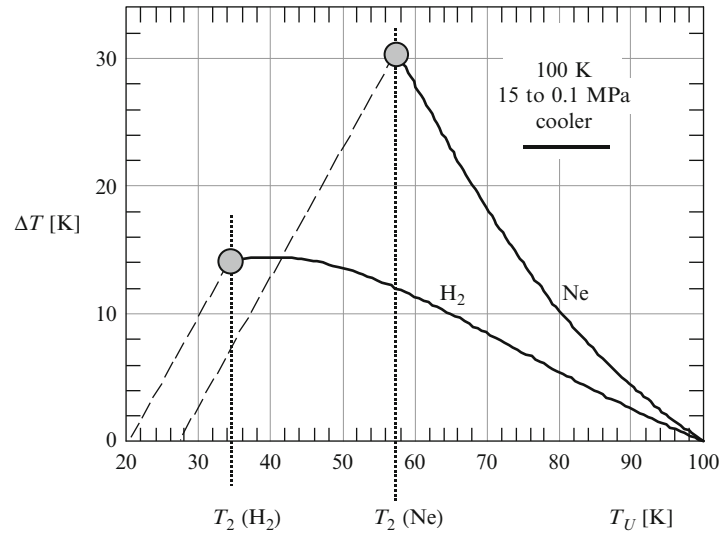


Fig. 3.22 Temperature differences between the recuperating streams along the heat exchanger of an ideal cooler of neon and hydrogen

range of $T_1 - T_5^*$ for a nominal cryocooler: from 0 to $|\Delta T_h| = 38.4$ K. Similarly, Fig. 3.24 displays a family of curves belonging to argon with $T_1 - T_5^*$ ranging from 0 up to $|\Delta T_h| = 91.5$ K.

One should observe that as the cryocooler runs at a larger excess flow rate and a lower value of $\dot{Q}/\dot{n}$, it experiences higher values of ΔT along the recuperator. Indeed, one may interpret lower values of $\dot{Q}/\dot{n}$ as an indication of excess cooling capacity.

In regard to the ideal case when $T_1 = T_5$, the warm end temperature difference, $\Delta T^* = T_1 - T_5^* > 0$, changes the value of ΔT at the cold end. Following Eqs. 3.138 and 3.139,

$$(T_2^* - T_{BOIL}) - (T_2 - T_{BOIL}) = T_2^* - T_2 = r |\Delta T_h| \quad (3.143)$$

The maximum value of $|\Delta T_h|$ at the warm end corresponds to the maximum change of $r |\Delta T_h|$ at the cold end of the counter flow recuperator.

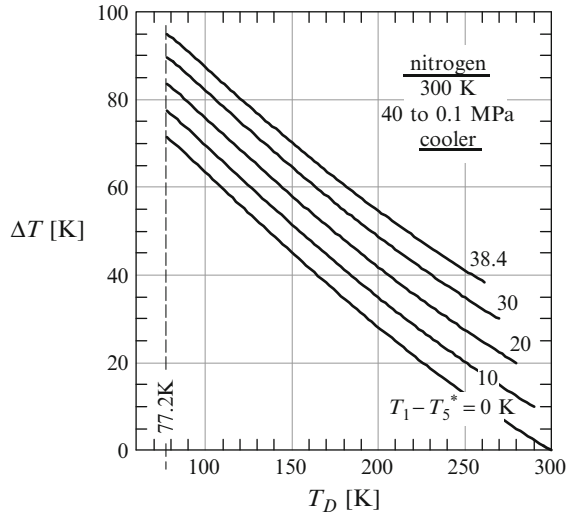


Fig. 3.23 Temperature differences between the recuperating streams along the heat exchanger depending on the outlet temperature T_5^* or the difference at the warm end, $T_1 - T_5^*$ for nitrogen cooler

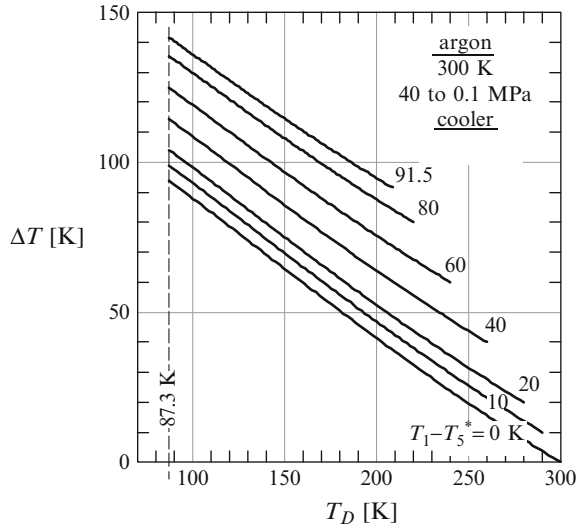


Fig. 3.24 Temperature differences between the recuperating streams along the heat exchanger depending on the outlet temperature T_5^* or the difference at the warm end, $T_1 - T_5^*$ for an argon cooler

3.11.1.3 η as a Function of NTU for Nominal Cryocoolers

After deriving the profiles of ΔT as a function of T_D , it is possible to integrate its reciprocal following Eq. 3.71 and thereby determine the exact values of NTU. The ΔT profile and the associated value of NTU depends on the species of the coolant, the pressure and temperature and the extent of excess flow as reflected by η or $T_1 - T_5^*$. Table 3.8 lists the NTU values for the cases shown in Figs. 3.23 and 3.24, namely, for nitrogen and argon at 40 MPa and 300 K.

The lowest values, NTU^{MIN} , representing the cryocoolers with zero cooling power (at T_{BOIL}), are of special interest.

Table 3.8 The η values as function of NTU for nominal cryocoolers pressurized by nitrogen and argon at 40 MPa and 300 K determined by $\int (\Delta T)^{-1} dT_D$

$T_1 - T_5^*$ [K]	Nitrogen		Argon	
	NTU	η	NTU	η
0.5	18.33	0.98	13.16	0.99
1.0	15.53	0.97	11.13	0.99
2.0	12.81	0.95	9.50	0.98
5.0	9.40	0.87	7.17	0.95
10	6.97	0.74	5.49	0.89
20	4.80	0.48	3.91	0.78
30	3.70	0.22	3.06	0.67
38.4 = $ \Delta T_h $	3.04	0.00		
40			2.50	0.56
50			2.07	0.45
60			1.75	0.34
70			1.49	0.23
80			1.27	0.13
90			1.09	0.01
91.2 = $ \Delta T_h $			1.06	0.00

Table 3.9 The exact NTU^{MIN} values for each operating pressure at the temperatures of 300 and 340 K for nitrogen and argon, as determined by $\int (\Delta T)^{-1} dT_D$

P [MPa]	Nitrogen		Argon	
	300 K	340 K	300 K	340 K
5	10.5	12.7	6.2	7.4
10	5.8	8.7	3.2	3.9
15	4.3	6.5	2.2	2.7
20	3.6	5.5	1.7	2.2
30	3.1	4.7	1.2	1.7
40	3.0	4.6	1.1	1.5
50	3.2	4.8	1.0	1.4
60	4.0	5.4	1.0	1.4

These are associated with the largest values of $T_1 - T_5^*$ which are equal to $|\Delta T_h|$ and depend on the actual operating conditions. The NTU^{MIN} values are obtained by the same integration procedure defined through Eq. 3.71 and are listed in Table 3.9.

3.11.2 The Liquefier

Similarly, let us balance the energy flow through the control volume of a part of an ideal liquefier as shown in Fig. 3.20b, operating with maximum yield. The extracted fraction of liquefied cryogen, $\Delta h_T / (h_5 - h_F)$, as discussed in Sect. 3.8, creates a difference between the up and downstream flow rates, resulting in the expression,

$$h(T_D, P_D) - h(T_U, P_U) = \Delta h_T \frac{h(T_D, P_D) - h_F}{h_5 - h_F} \quad (3.144)$$

Since $h(T_D)$ is smaller than h_5 one finds that the right hand side is smaller than Δh_T , which, according to Eq. 3.140 is the term associated with a cryocooler. In a real case of finite heat exchanger area and non zero ineffectiveness, and when the temperature of the outlet stream, T_5^* , is lower than that of the inlet stream, T_1 , the equation for ΔT becomes,

$$h(T_D, P_D) - h(T_U, P_U) = (h_5^* - h_1) \frac{h(T_D, P_D) - h_F}{h_5 - h_F} \quad (3.145)$$

For any value of T_U , the corresponding value of ΔT associated with the liquefier is larger than that with a cooler. This is demonstrated in Fig. 3.21. One may say that at the same value of T_D , the associated value of T_U is higher in the case of the liquefier because of the weaker recuperation process.

3.11.3 Dependence on Specific Heat Capacities

Figures 3.21 and 3.22 both reflect the fact that as one approaches the nozzle of a nominal cryocooler or liquefier and the values of T_U decrease, the ΔT between the streams increases. This reflects the unbalanced nature of capacity rates of the high and low-pressure streams of a pure coolant. Specific heat capacities of the low-pressure stream are lower than those of the high pressure stream.

Consider the relationship between the specific heat capacities of the two flow streams. Based on the local exchange of heat, the energy balance between the counter-flowing streams is,

$$\dot{n}_U c_{P,U} dT_U = \dot{n}_U c_{P,U} dT_D \quad (3.146)$$

and in the balanced case of a cooler with equal flow rates in both streams, we have

$$\frac{dT_U}{dT_D} = \frac{c_{P,D}}{c_{P,U}} \quad (3.147)$$

or,

$$\frac{d(\Delta T)}{dT_D} = \frac{d(T_U - T_D)}{dT_D} = \frac{c_{P,D}}{c_{P,U}} - 1 \quad (3.148)$$

Since $c_{P,D} < c_{P,U}$, (at least for pure coolants) the right hand side of equation is negative, which means that ΔT increases for lower temperatures.

In a liquefier, the misbalance is even sharper due to the reduced downstream flow rate, so that

$$\frac{d(\Delta T)}{dT_D} = \left[1 - y^{(2)}\right] \cdot \frac{c_{P,D}}{c_{P,U}} - 1 \quad (3.149)$$

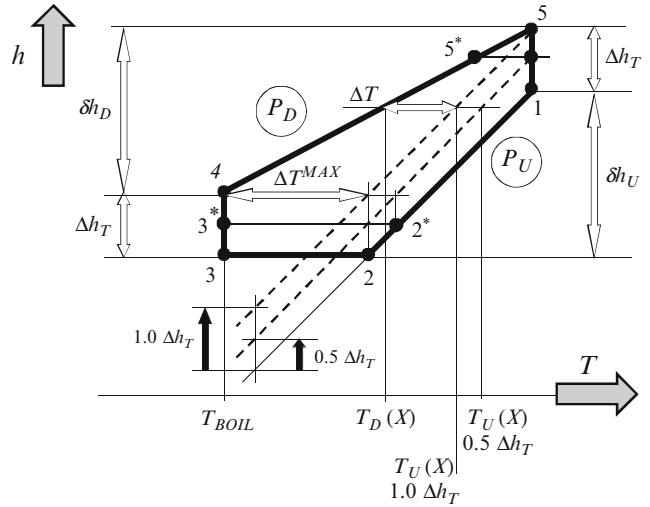


Fig. 3.25 The operating lines for analyzing a cooler are generated in the $h - T$ plane by elevating the high pressure isobar. The two dashed lines are associated with the maximum cooling capacity per unit flow rate, Δh_T , and half of that value

Thus, for the same specific heat ratio $c_{P,D}/c_{P,U}$, the trend of increasing ΔT at lower temperatures becomes even more pronounced as $d(\Delta T)/dT_D$ becomes more negative.

3.11.4 The Operating Line

The operating line provides a convenient graphical means to describe and explore the behavior of ΔT as a function of the temperature along the recuperator of the Linde-Hampson cycle under various modes of operation. Since a negative ΔT violates the second law of the thermodynamics, the operating line may be used to determine the intermediate location and associated temperature where ΔT , the temperature difference between the high and low pressure streams, vanishes; that is the *lowest attainable temperature*. In the traditional Linde-Hampson cycle with pure coolants, the lowest attainable temperature at steady state is always the boiling point, independent of the actual cooling capacity. However, in the general case, as in the case of mixed coolants discussed in Chap. 8, or in the special cases of helium and hydrogen, a pinch point may occur before the boiling point.

3.11.4.1 The Cooler

As introduced by Maytal et al.,⁵⁶ the *operating line* is generated in the (h, T) plane by elevating the high pressure isobar (towards the low pressure isobar) by an amount of Δh^* , which represents the actual specific cooling capacity that is, $\dot{Q}/\dot{n}$. The dashed lines in Fig. 3.25 are the operating lines associated with the maximum cooling capacity per unit flow rate, Δh_T , and half of that value. The horizontal separation between the downstream isobar and the operational line

is the ΔT between the high and low pressure flow streams for the specific value of T_U . Actually, it is the graphical equating of enthalpies that serves to solve Eqs. 3.140 and 3.142.

Some general conclusions that have already drawn are also reflected with the aid of the operating lines:

1. T_1 and T_5 represent the inlet and outlet temperatures to the cryocooler at its warm end for the case of an ideal heat exchanger and one that is utilizing the full cooling capacity. At the warm end, $\Delta T = 0$, and the gas leaves the cryocooler at the same temperature as the inlet.
2. However, when only a fraction of the full cooling power is utilized (see for example, Fig. 3.7b), even with an ideal heat exchanger, the outlet state, 5^* , can never reach the inlet temperature so that $T_5^* < T_1$. In this respect it is important to note that a temperature difference at the warm end may result simply due to operation at fractional cooling capacity and does not always indicate an impaired capability of the heat exchanger. Put another way, if the cooler were operated at the maximum cooling capacity, it still might exhibit $\Delta T = 0$ at the warm end.
3. Furthermore, values of ΔT are larger along the entire heat exchanger when a smaller fraction of the maximum specific cooling capacity, Δh_T , is maintained. This feature is depicted in Fig. 3.25 for a specific value of $T_D(X)$ and the associated values of $T_U(X)$ when $\dot{Q}/\dot{n}$ is at full capacity, ($1.0 \Delta h_T$) and at half capacity, ($0.5 \Delta h_T$).
4. The coldest obtainable temperature at the low pressure outlet of the cooler's heat exchanger,

$$T_5^* = T_5' = T_1 - |\Delta T_h(P_1, T_1)| \quad (3.150)$$

occurs when no heat load is applied (but the boiling temperature is still maintained), as shown in Fig. 3.7c. For example, with nitrogen at 30 MPa and 300 K, the coldest outlet is 263 K. The term, Δh_T , depends on the inlet condition and is shown in Fig. 3.4. ΔT_h , the adiabatic integral Joule-Thomson effect, is very relevant to the understanding of the operation of a Joule-Thomson cryocooler. For the sake of a systematic presentation, it is more fully discussed in its natural context, that occurs in Sect. 4.5.

5. The maximum value of ΔT is obtained at the intersection of the h - T line associated with the low pressure stream, and the saturated vapor enthalpy h_G . If only saturated vapor returns from the evaporator, as in the case of a nominal cryocooler, the highest value of ΔT occurs exactly at the cold end of the heat exchanger. In case of an over recuperated cryocooler, when some liquid cryogen also enters the low-pressure return line, ΔT will peak and decrease towards the nozzle.
6. The local slope of each isobar represent the specific heat capacity, by Eq. 2.6. If both streams would have the same local slopes, and therefore the same specific heat capacities, the operating line would closely approach the low pressure isobar and the ΔT values along the

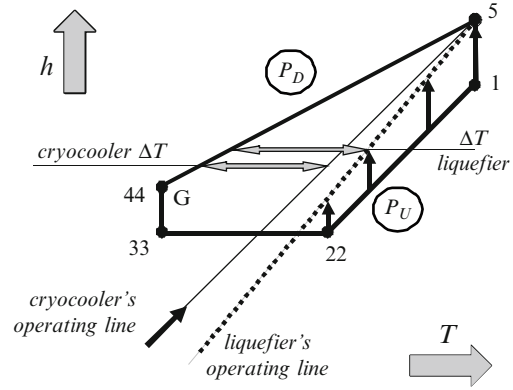


Fig. 3.26 The idea of the operating line is applied to a liquefier as well as to a cooler shown for full cooling capacities. Since the operating line of a liquefier is below that of the cooler, its ΔT profile is higher than that of a cooler. Compare the display of states 22, 33, and 44 with Fig. 3.10

heat exchanger would vanish. A larger difference in heat capacities results in larger values of ΔT , a larger amount of irreversibility, and therefore a lower COP.

7. While pure coolants have larger heat capacities at high pressure, mixtures of gases in the two-phase region may be developed with more similar heat capacities and therefore lower values of ΔT . This issue will be subsequently discussed in Chap. 8.

3.11.4.2 The Liquefier

The *operating line* for a liquefier has to represent Eq. 3.144. Figure 3.26 shows the operating line of a liquefier and of a cooler operated at full cooling capacity. In this case, it is not just an elevation of the entire high-pressure isobar. The extent of elevation changes locally according to the local value of h_D . Because of Eq. 3.144 the elevation remains smaller than Δh_T , while,

$$\frac{h_D - h_F}{h_5 - h_F} \leq 1 \quad (3.151)$$

Since the operating line of the liquefier is below that of the cooler, its ΔT profile is steeper than that of a cooler, as indeed seen in Fig. 3.21. When running the cooler at partial cooling capacity the situation may change: at the warmer region, the cooler has higher values of ΔT , but towards lower temperatures the liquefier exhibits the higher values.

3.11.4.3 Helium and Hydrogen JT Cryocooling

Helium and hydrogen JT cryocooling is unique since these pure gases produce the unusual condition where $c_{P,U} < c_{P,D}$. Figure 3.27 displays in the h - T plane, two helium isobars at 4 and 0.1 MPa. These could represent, for example, a final JT stage to reach 4.2 K with a hybrid cryocooler that is precooled to 20 K by a GM or Stirling cryocooler (as described in Sects. 5.6.3 and 5.6.4). Following Fig. 4.13, these are conditions defining a maximum value of Δh_T ,

Fig. 3.27 Operating lines with full (red dotted line) and 50% (blue dashed line) cooling capacity. The corresponding lowest attainable temperatures are 5 and 6.4 K

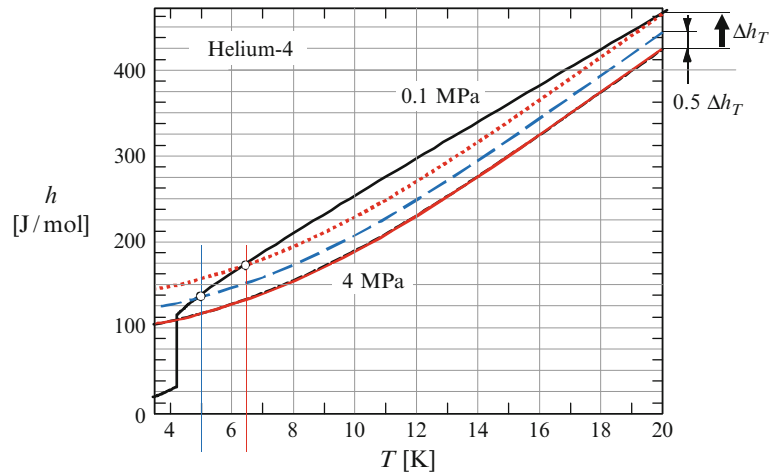
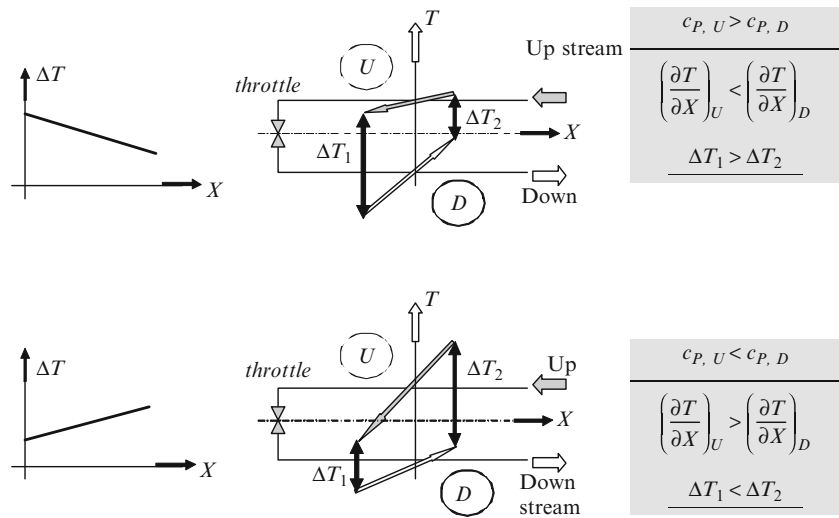


Fig. 3.28 The behavior of $\Delta T = T_U(X) - T_D(X)$ along the heat exchanger whether increasing or decreasing, and its relation with specific heat capacity values



reaching the value of 44 J/mole. The figure shows two operating lines representing a full, and a 50%, heat load. In both cases, the cryocooling is not isothermal at a boiling point; and the lowest temperatures are 5 K and 6.4 K, correspondingly. (See Fig. 5.31 of Sect. 5.6.5.3, showing how a sequential double throttling configuration in such a case might be advantageous).

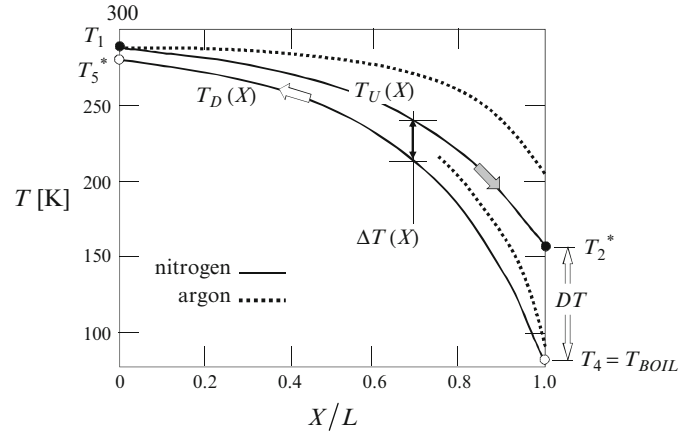
3.11.5 Longitudinal Temperature Distribution

Studying the thermodynamic cycle in the h - T plane has enabled a definition of the temperature difference across the recuperator, $T_U - T_D$, as a function of the local temperature (T_U or T_D). Attention is now focused on the temperature distribution of each stream $T_U(X)$ and $T_D(X)$ along the

linear coordinate, X , of the heat exchanger. The behavior of the temperature difference, $T_U(X) - T_D(X)$, along the heat exchanger, whether increasing or decreasing, and its dependence on c_P values, is highlighted in Fig. 3.28. For a given amount of exchanged heat, the stream with the higher values of c_P exhibits a more moderate temperature gradient $\partial T / \partial X$ than the one with the lower values of c_P and vice versa. The graphical presentation of these trends demonstrates the previously derived results (Sect. 1.2.2.2).

Nevertheless, the task of deriving $T_U(X)$ and $T_D(X)$ requires additional detailed information about the actual sizes, shapes, materials and heat transfer coefficients along the high and low pressure flow channels. Integration of the relevant differential equations that conserves mass, energy and momentum is discussed in Sect. 9.3. However, at this point, it may be useful to evaluate the actual heat capacity of

Fig. 3.29 Qualitative display of the longitudinal temperature distributions, $T_U(X)$ and $T_D(X)$



the cooler, that is, the total amount of heat to be extracted in the process of reaching steady state operation.

Figure 3.29 is a qualitative display of the temperatures distribution along the inlet and outlet streams for nitrogen and argon of a given cooler. These are convex profiles and the gap between $T_U(X)$ and $T_D(X)$ increases downstream towards the nozzle at $X = L$. Obviously, the steady state difference $T_U(X) - T_D(X)$ at any $T_U(X)$ or $T_D(X)$ have to obey the previously predicted values of ΔT through the First Law of Thermodynamics.

while entropy is generated. For an isenthalpic process the change of entropy is,

$$ds = -\frac{d\rho}{\rho \cdot T} \quad @ \quad dh = 0 \quad (3.153)$$

namely, the “local” extent of entropy generation reflects the extent of the pressure drop. Indeed, throttling is inherently characterized by entropy generation. If less entropy is generated than predicted by the above relation, the process does not preserve enthalpy, $dh \neq 0$, and involves work or heat interactions.

It is interesting to mention that $ds > 0$ whenever $dp < 0$ independent of whether the temperature increases or decreases under the isenthalpic expansion. Expansion of a more dense phase generates less entropy. Therefore, the cycle that throttles a liquid, rather than a gaseous phase has the potential for a higher COP.

The JT effect is an expression associated with throttling. However, throttling is possible without the JT effect as in case of an ideal gas, where $\mu = 0$ and $T_{IN} = T_{OUT}$, but entropy is nevertheless generated; following Eq. 3.153 the amount of entropy generated is given by,

$$\Delta s^{IG} = s_{OUT} - s_{IN} = R \ln(P_{IN}/P_{OUT}) \quad (3.154)$$

and is independent of the inlet temperature.

If work is also extracted (externally), the expansion is not isenthalpic, and a larger temperature drop results. The maximum temperature drop for a given change of pressure is obtained with an isentropic expansion. Such an expansion is reversible and extracts the maximum possible useful work between two states. However, work extraction is associated with mechanical devices and moving parts called expanders. Claude’s cycle includes an expander in addition to the throttling element of the Linde-Hampson cycle, thereby reducing the irreversibility and elevating the cycle efficiency. The reverse Brayton cycle (Sect. 3.1.4.3) is similar to the

3.12 The Mechanisms of Throttling

3.12.1 Introduction

The throttling effect is the essential cooling mechanism in Joule-Thomson cryocoolers, and provides the driving mechanism for the liquefaction of coolants in the Linde-Hampson process. Throttling is also utilized in other devices, for example pressure regulators. It deserves an in-depth consideration of its various features.

An adiabatic flow from a high-pressure region expanding to a low-pressure region has the potential to produce work, generate changes in kinetic energy, induce different kinds of potential energy, or a combination of these effects. However, none of these effects accompany the pure throttling of an adiabatic stream. By the First Law of Thermodynamics, such a steady flow undergoing a throttling process preserves enthalpy,

$$h_{IN} = h_{OUT} \quad (3.152)$$

and is referred to as undergoing a Joule-Thomson expansion. Work could have been obtained but was not; hence, the throttling process is associated with a loss of potential *useful work*. It may alternatively be stated that *availability*⁵⁷ is lost

Linde-Hampson machine with respect to its use of a recuperative heat exchanger. However, the throttle is replaced by an isentropic expansion through a work-producing turbine.

The fundamental mechanisms involved in the throttling process of a stream are: viscous interactions, turbulence and shock waves. In principle, the three phenomena are independent, although they share the capability to dissipate the energy of the flowing stream. Practically, a combination of these mechanisms occurs. Pressure is partially reduced by viscous effects and/or turbulence, while the remaining reduction occurs (in general) through shock waves.

3.12.2 The Laminar Regime

Viscous effects dominate the laminar regime. Viscosity, μ , reflects the capability of fluids to sustain a shearing stresses, τ , between adjacent flowing layers. The shearing “effect” is associated with the velocity gradient and stress through the relationship,

$$\tau = \mu \cdot \left(\frac{\partial V}{\partial y} \right) \quad (3.155)$$

where y is the direction perpendicular to the velocity vector, V . In the laminar regime these stresses dominate the flow pattern and induce a momentum transfer in the lateral direction of y , which is significantly larger than the convective momentum transfer in the longitudinal direction of V . The Reynolds Number is a dimensionless measure of the relative magnitude of the two modes of momentum flow,

$$\text{Re} = \frac{V \cdot d}{\nu} = \frac{G \cdot d}{\mu} \quad (3.156)$$

where μ and ν are the dynamic and the kinematic viscosities. If Re remains below a transition value (about 2,000 for a straight tube), the laminar condition prevails.

The local rate of entropy generation is given by,

$$\dot{s} = \frac{\mu}{T} \Phi \quad (3.157)$$

where $\Phi > 0$, is the *dissipation function* formulated in terms of spatial derivatives of the velocity field as one may find elsewhere⁵⁸ and other text books.

The pressure gradient in laminar flow is directly proportional to the velocity, that is,

$$\frac{dP}{dx} \propto V \quad (3.158)$$

In porous channels, the flow is divided into a plurality of streams, the hydraulic diameters of which are so small that

they cause the flow to become laminar. The original porous plug utilized by Joule and Thomson in 1852⁵⁹ thereby produced a laminar flow. Equation 3.158 also explains the linear relationship between the pressure drop and flow rate in Darcy’s Law for a porous media.

3.12.3 The Turbulent Regime

For higher values of Re , flow becomes turbulent: the streamlines are not regular anymore and the velocity fluctuates in direction and magnitude. Turbulence also induces a transverse momentum exchange, in analogy to the effect of viscosity in laminar flow. Hence, one way of describing turbulence is by introducing the *eddy viscosity*, ε , known also as the “turbulent coefficient of viscosity” so that the associated local rate of entropy generation is given by,

$$\dot{s} = \frac{\varepsilon}{T} \Phi \quad (3.159)$$

Turbulence causes the exchange of momentum between adjacent layers of different velocities. The extent of this exchange is specified by ε , which effectively plays a similar role as μ does in the laminar regime.

An alternative method of describing the dissipative mechanism in a turbulent flow is through the wall “friction” factor, f , of Fanning, which is related to the shearing stress on the wall, τ_w and the average velocity, $\bar{V}$ by,

$$f = \frac{\tau_w}{\rho \bar{V}^2 / 2} \quad (3.160)$$

Most of the entropy production for turbulent flow occurs on the wall, and the sheer related friction factor is the dominant source of the pressure gradient for turbulent flow. They are related through the expression,

$$\frac{dP}{dx} = -\frac{4f}{d} \cdot \frac{1}{2} \rho \bar{V}^2 \quad (3.161)$$

The friction factor f depends on Re through the von Karman-Nikuradse relationship for turbulent flow,

$$\frac{1}{\sqrt{4f}} = -0.8 + 0.87 \ln(4f \text{ Re}) \quad (3.162)$$

which may be approximated for the range of $3 \cdot 10^4 < \text{Re} < 10^6$ by,

$$f = 0.046 / \text{Re}^{0.2} \quad (3.163)$$

It is also possible to define the pressure drop observed in the laminar regime in terms of a friction factor by the expression,

$$f = 16/\text{Re} \quad (3.164)$$

While entropy is generated through the entire cross section of flow in the laminar regime, it occurs mainly on the walls in the turbulent regime.

3.12.4 Shock Waves

The remaining contribution to the pressure drop occurring in an expansion nozzle results from shock waves. This dissipative mechanism is independent of viscous effects, and in fact may also occur in non-viscous and ideal gas flows. For an ideal gas compressible flow with an inlet to outlet pressure ratio larger than about two, the flow achieves the sonic velocity at the outlet of the element. A shock wave, or a series of them, results in an abrupt pressure drop to the outlet or ambient condition. The transition from high to low pressure occurs in such a narrow zone that it may be considered as a surface of discontinuity. Flow enters and exits the shock wave while preserving enthalpy. In that the flow cannot exceed the sonic velocity, the valve ‘chokes’ the flow and limits its rate. The mass, or molar, flow rate through a choked valve is then adjustable only by the cross sectional area or the upstream density. One must carefully consider the flow area in a Joule-Thomson nozzle, to ensure that the target flow rate is achievable. Fundamental and complementary reading about sonic and supersonic compressible flows accompanied by shock waves is available elsewhere.^{60,61}

One may determine the Mach Number, Ma_1 , upstream of a normal shock wave that is associated with a given pressure ratio, by the relation,

$$\frac{P_{1,0}}{P_{2,0}} = \frac{1}{(\kappa + 1)^{(\kappa+1)/(\kappa-1)}} (2\kappa \text{Ma}_1^2 - 1)^{1/(\kappa-1)} \times \left(\kappa - 1 + \frac{2}{\text{Ma}_1^2} \right)^{\kappa/(\kappa-1)} \quad (3.165)$$

where $P_{1,0}$ and $P_{2,0}$ are respectively the up- and down-stream total (stagnation) pressures. For example, flow will be accelerated up to $\text{Ma}_1 = 7$ for a pressure ratio of 64.43 if the gas has a specific heat ratio, $\kappa = 1.4$. For a gas with $\kappa = 1.35$ or 1.3 the pressure ratio will be 161 and 345 respectively.

For an ideal gas the entropy generated across a shock wave is related to the ratio of total (or stagnation) pressures by the expression,

$$\frac{\Delta s}{R} = \frac{P_{1,0}}{P_{2,0}} \quad (3.166)$$

resembling Eq. 3.154.

3.13 Second Law of Thermodynamics Considerations

3.13.1 Coefficient of Performance, COP

3.13.1.1 Formulation

Following the description and notation of the Linde-Hampson cycle in Sect. 3.2, and the general presentation of cryocoolers in Sect. 1.4.1 (Fig. 1.14), $\dot{Q}_L$ or $\dot{Q}$, is extracted isothermally at T_{BOIL} along the Sect. 3–4 in Fig. 3.3. The minimum work of isothermal compression, $T_1 = T_5$, which has to be invested for continuous operation of a Linde-Hampson cryocooler, would be,

$$\begin{aligned} \frac{\dot{W}}{\dot{n}} &= \left[\int_{P_5}^{P_1} v dP \right]_{T=T_1} = \left[\int_{P_5}^{P_1} (dh - T ds) \right]_{T=T_1} = \\ &= T_1 (s_5 - s_1) - (h_5 - h_1) = T_1 \Delta s_{T=T_1} - \Delta h_{T=T_1} \end{aligned} \quad (3.167)$$

The benefit in terms of the extracted heat at the low temperature per unit of supplied work, is the *coefficient of performance*, following Eq. 1.74,

$$\begin{aligned} \text{COP} &= \frac{\dot{Q}}{\dot{W}} = \frac{\Delta h_T}{T_1 \Delta s_T - \Delta h_T} \\ &= \left(T_1 / \frac{\Delta h_T}{\Delta s_T} - 1 \right)^{-1} \end{aligned} \quad (3.168)$$

When operating with excess flow (Sect. 3.3.2.1) or with reduced effectiveness of the recuperator, $\varepsilon < 1$ (Sect. 3.5.2), the specific cooling capacity, Δh^* , will be lower than Δh_T and the corresponding COP will be lower than the value predicted by Eq. 3.168.

On the basis of any equation of state, $f(P, v, T) = 0$, and following Eq. 2.21,

$$\Delta h_T = \left[\int_{P_5}^{P_1} \left[T \left(\frac{\partial v}{\partial T} \right)_P - v \right] dP \right]_{T=T_1} \quad (3.169)$$

then, by Eqs. 3.167 and 3.168,

$$\text{COP} = T_1 \left[\int_{P_5}^{P_1} \left(\frac{\partial v}{\partial T} \right)_P dP \right]_{T=T_1} \bigg/ \left[\int_{P_5}^{P_1} v dP \right]_{T=T_1} - 1 \quad (3.170)$$

Fig. 3.30 Pressure dependence of COP at 295 K for some gases. The outlet pressure is 0.1 MPa and $T_1 = T_5$. The maximum values occur at pressures below the inversion pressures (marked by circles)

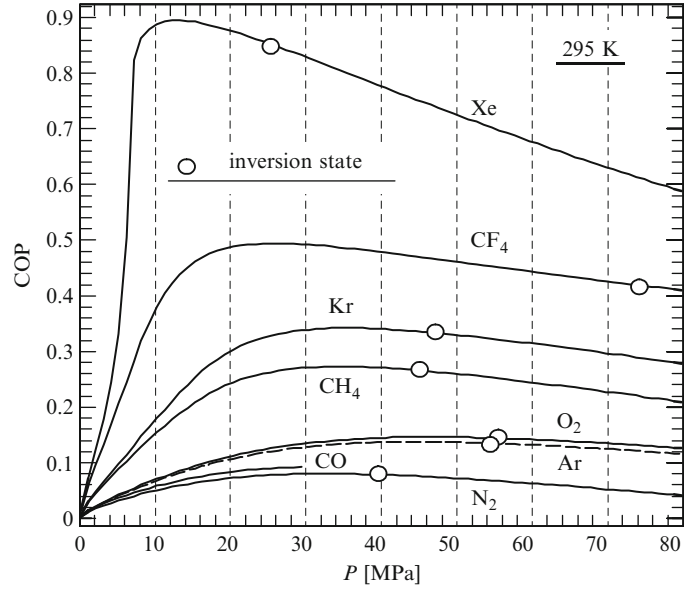
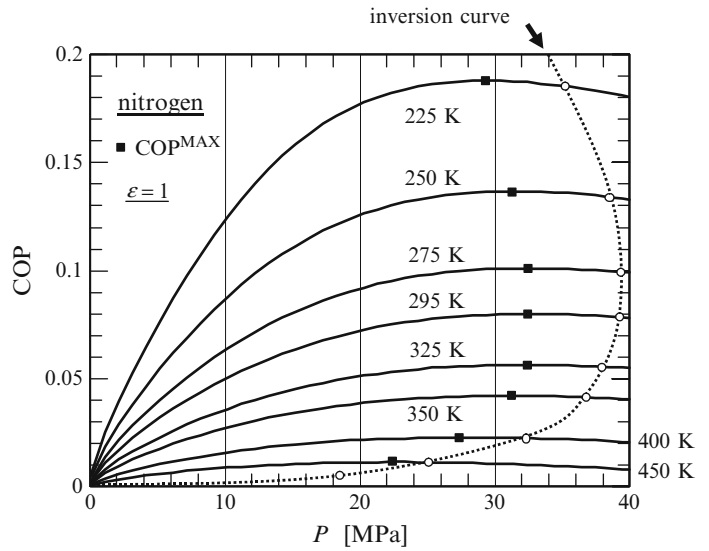


Fig. 3.31 The COP of nitrogen for various temperatures as function of pressure. The inversion curve demonstrates that the pressures of maximum COP are below the corresponding inversion pressures at the same temperatures. The outlet pressure is 0.1 MPa and $T_1 = T_5$



Note: The above formulation and subsequently derived values of COP , are based on the assumption that $\dot{Q}/\dot{n} = (\dot{Q}/\dot{n})^{\text{MAX}} \Delta h_T$, which is valid only for the non practical case of $\dot{Q} = 0$ as explained in Sect. 3.9.1. This implies that any real case will have lower COP values.

3.13.1.2 The Dependence of COP on the Inlet Pressure

The dependence of COP on the inlet pressure, P_1 , for a given outlet state, (P_5, T_5) and a given warm-end temperature $T_1 = T_5$, then, would be,

$$\text{COP} = \left\{ T_1 / \left(\frac{h_5 - h_1(P_1)}{s_5 - s_1(P_1)} \right) - 1 \right\}^{-1} \quad (3.171)$$

Figure 3.30 displays the pressure dependence of COP at 295 K for nitrogen, and some higher boiling point gases. It also includes the associated inversion state for each gas. Figure 3.31 displays the COP of nitrogen at different inlet temperatures as function of pressure and the inversion curve in these coordinates.

While the inversion pressure is associated with the maximum value of Δh_T , the COP peaks at a pressure that

Table 3.10 The highest attainable coefficient of performance of a Joule-Thomson cryocooler for some gases at 295 K. The outlet pressure is 0.1 MPa

Gas	COP ^{MAX}	P (COP ^{MAX}) [MPa]	P_{INV} [MPa]	COP _{CARNOT}	FOM ^{MAX}
Nitrogen	0.078	32.4	39.5	0.355	0.23
Argon	0.137	43.5	54.7	0.427	0.32
Oxygen	0.146	45.5	55.8	0.442	0.33
Methane	0.273	34.4	45.5	0.610	0.45
Krypton	0.342	35.4	47.8	0.685	0.50
R14	0.494	26.3	75.1	0.968	0.51
Xenon	0.890	13.2	25.0	1.271	0.70
Ethylene	0.904	14.2	25.8	1.800	0.50
Ethane	1.240	7.20	18.2	1.670	0.74

maximizes the ratio $\Delta h_T / \Delta s_T$. This means that the COP does not peak along the inversion curve. By approximating Δs_T with the ideal gas expression, that is $R \ln(P_1/P_5)$, then the maximum value of $\Delta h_T / \Delta s_T$, is obtained by setting its derivative to zero, thereby obtaining the condition for P_1 associated with the maximum COP,

$$\frac{\partial h(P_1)}{\partial P_1} \equiv \phi(P_1) = \frac{-\Delta h_T}{P_1 \ln(P_1/P_5)} < 0 \quad (3.172)$$

The condition $\phi(P_1) < 0$, which is equivalent to the condition $\mu(P_1) > 0$, defines the zone inside the differential inversion curve (Fig. 2.5). Hence, the solution for P_1 (at any T_1) is smaller than the corresponding value of $P_{INV}(T_1)$.

For any given temperature, there are two pressures, $P_A < P_{INV}$ and $P_B > P_{INV}$, with the same specific cooling capacity, Δh_T . In between these two pressures, Δh_T is maximized at P_{INV} . Operating at the lower pressure, P_A , requires less compression work, and therefore the associated COP is higher than when operating at P_B . Hence, the maximum coefficient of performance, COP^{MAX}, is associated with the lower pressure, P_A , below the corresponding inversion pressure and designated as P_{11} , so that,

$$P_{11} = P_1(\text{COP}^{\text{MAX}}) < P_{INV}(T_1) \quad (3.173)$$

This relationship is also demonstrated in Figs. 3.30 and 3.31. Notice then that the conditions for maximum refrigeration are not the same as those for minimum cost. The maximum values, COP^{MAX}, and the associated pressures, $P_1(\text{COP}^{\text{MAX}})$ are listed in Table 3.10.

The same idea was also discussed by Hendricks et al.⁶² Dean and Mann^{46,63} present the pressure dependence of COP for a wide range of temperatures for nitrogen, helium and para-hydrogen. They approximate the isothermal work of compression using the ideal gas expression.

3.13.1.3 The Pressure of the Optimum COP

The pressure of the optimum COP was analytically studied by Levrenchenko.⁶⁴ Differentiating Eq. 3.171 with respect to P_1 leads to an equation, the solution of which is the optimal pressure, P_{11} , that maximizes the COP at T_1 ,

$$\left[\left(\frac{\partial h}{\partial s} \right)_T \right]_{@P=P_{11}} = \frac{h(P_5) - h(P_{11})}{s(P_5) - s(P_{11})} = \left(\frac{\Delta h_T}{\Delta s_T} \right)^{\text{MAX}} \quad (3.174)$$

It is interesting to examine the quantity $\Delta h_T / \Delta s_T$ within the h - s plane of a Linde-Hampson cycle. With the aid of isothermal trajectories, Fig. 3.32 displays the graphical interpretation associated with Eq. 3.174. The line extending from state 5, that is tangential to the isotherm T_1 touches the isotherm at the solution state of P_{11} . Hence,

$$\text{COP}^{\text{MAX}} = \left\{ T_1 / \left[\left(\frac{\partial h}{\partial s} \right)_T \right]_{@P=P_{11}} - 1 \right\}^{-1} \quad (3.175)$$

Alternatively, $P_1(\text{COP}^{\text{MAX}})$ may be derived though the formulation that is based on an equation of state. Differentiating Eq. 3.170 one obtains the equation for P_{11} ,

$$v(P_1) \left[\int_{P_5}^{P_{11}} \left(\frac{\partial v}{\partial T} \right)_P dP \right]_{T=T_1} = \left[\frac{\partial v}{\partial T} \right]_{P=P_{11}} \left[\int_{P_5}^{P_{11}} v dP \right]_{T=T_1} \quad (3.176)$$

The associated maximum COP, obtained by also applying Eqs. 2.45 and 2.46, is then,

$$\begin{aligned} \text{COP}^{\text{MAX}}(P_{11}, T_1) &= \left\{ T_1 / \left(\frac{h_5 - h_{11}}{s_5 - s_{11}} \right) - 1 \right\}^{-1} \\ &= T_1 \left[\frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_P \right]_{@P=P_{11}} - 1 \\ &= T_1 \cdot \beta(P_{11}, T_1) - 1 = -\frac{\phi}{v}(P_{11}, T_1) \quad (3.177) \end{aligned}$$

$$\begin{aligned} \left(\frac{\Delta h_T}{\Delta s_T} \right)^{\text{MAX}} &= \frac{h_5 - h_{11}}{s_5 - s_{11}} = \left[\left(\frac{\partial h}{\partial s} \right)_T \right]_{@P=P_{11}} \\ &= T_1 - \frac{1}{\beta(P_{11}, T_1)} = \frac{T_1}{1 - \frac{v}{\phi}(P_{11}, T_1)} \quad (3.178) \end{aligned}$$

which is consistent with Eq. 2.63.

3.13.1.4 Remarks of Consistency

1. On the differential inversion curve (D.I.C.), $T\beta = 1$. Hence, the state (P_{11}, T_1) is not on the D.I.C. since then, from Eq. 3.178, COP^{MAX} would vanish. Put another way,

$$\begin{aligned} \text{COP}^*(T_1) &= \left\{ T_1 / \left[\left(\frac{\partial h}{\partial s} \right)_T \right]_{\substack{P=P^* \\ T=T_1}} - 1 \right\}^{-1} \\ &= \text{COP}_{\text{CARNOT}} = \frac{1}{(T_1/T_L^{\text{MIN}}) - 1} \end{aligned} \quad (3.181)$$

determines the lowest attainable temperature for any choice of T_1 ,

$$T_L^{\text{MIN}} = \left[\left(\frac{\partial h}{\partial s} \right) \right]_{\substack{P=P_{11} \\ T=T_1}} @ \left[\left(\frac{\partial^2 h}{\partial s^2} \right) \right]_{\substack{P=P^* \\ T=T_1}} = 0 \quad (3.182)$$

The associated values are listed in Table 3.11. (Compare with Sect. 1.4.2.3).

3.13.2 The Cost of Refrigeration

The ideal cost of refrigeration is expressed in terms of the minimum supplied work (J) per unit of heat removed at the low temperature,

$$C^{(1)} = \frac{\dot{W}}{\dot{Q}} = \frac{\dot{W}}{y^{(1)} \dot{n} L} = \frac{1}{\text{COP}} = T_1 \frac{s_5 - s_1}{\Delta h_T} - 1 \quad (3.183)$$

For example, using the values in Table 3.10 for COP^{MAX} , one finds that in order to supply 1 W of cooling by nitrogen at its boiling point, 13.2 W of work must be expended.

A liquefier is intended to provide liquid cryogen instead of cooling. Hence, its performance, measured by $C^{(2)}$, is expressed in terms of invested work per liquefied unit (mole) of cryogen, that is,

$$C^{(2)} = \frac{\dot{W}}{y^{(2)} \dot{n}} = (h_5 - h_F) \left(T_1 \frac{s_5 - s_1}{\Delta h_T} - 1 \right) \quad (3.184)$$

Values of $C^{(2)}$ are listed in Table 3.12 for different gases.

Cost of operation may be reduced by staging; whether by dual pressure cycle with a single coolant (Sect. 5.4.9) or by an additional precooling cycle with a higher boiling point coolant (Sect. 5.5.5.2). Mixed coolants also reduce the cost of refrigeration (Chap. 8). In the practice of open cycle Joule-Thomson cryocooling, the cost of cooling is not a major issue.

3.13.3 Figure of Merit, FOM

Any refrigerant may be characterized by its highest coefficient of performance, COP^{MAX} . The COP associated with

Table 3.12 The yield $y^{(2)}$ and cost of liquefaction, $C^{(2)}$, of a liquefier in terms of energy (kJ) per liquefied mole, by an ideal Linde-Hampson process for different gases at 300 K pressurized to 15 and 20 MPa and expanding to 0.1 MPa

Gas	15 MPa		20 MPa	
	$C^{(2)}$ [kJ/mole]	$y^{(2)}$	$C^{(2)}$ [kJ/mole]	$y^{(2)}$
Nitrogen	201.0	0.0622	176.7	0.0750
Carbon monoxide	179.4	0.0715	157.2	0.0840
Argon	127.5	0.0965	106.9	0.1214
Oxygen	139.0	0.0886	116.4	0.1115
Methane	74.7	0.1595	63.4	0.1974
Ethane	17.8	0.5235	18.4	0.5257

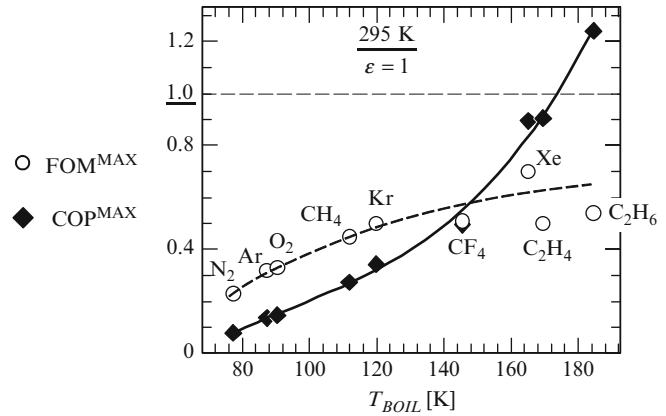


Fig. 3.33 Values of COP^{MAX} and FOM^{MAX} for various gases as a function of their boiling temperatures

the Carnot cycle⁶⁵ provides an upper bound for the value of COP^{MAX} for a real cycle. Following Eq. 1.75,

$$\text{COP}_{\text{CARNOT}} = \frac{T_{\text{BOIL}}}{T_1 - T_{\text{BOIL}}} = \left(\frac{T_1}{T_{\text{BOIL}}} - 1 \right)^{-1} \quad (3.185)$$

Using the *Figure of Merit*, defined by Eq. 1.81, and applying Eqs. 3.168 and 1.86 one obtains,

$$\begin{aligned} \text{FOM} &= \frac{\Delta h_T}{T_1(s_5 - s_1) - \Delta h_T} \cdot \frac{T_1 - T_{\text{BOIL}}}{T_{\text{BOIL}}} \\ &= \frac{(T_1/T_{\text{BOIL}}) - 1}{T_1/T_L^{\text{MIN}} - 1} \end{aligned} \quad (3.186)$$

Figure 3.33 and Table 3.10 display values of COP^{MAX} and the corresponding FOM^{MAX} for various gases as a function of their normal boiling temperature, T_{BOIL} . A general trend may be easily observed: higher boiling point gases exhibit higher coefficients of performance. Derking, ter Brake, et al.,⁶⁶ calculated COP^{MAX} and FOM^{MAX} for a group of gases by adjusting the values of P_D for each gas in order to enable a common boiling temperature (that

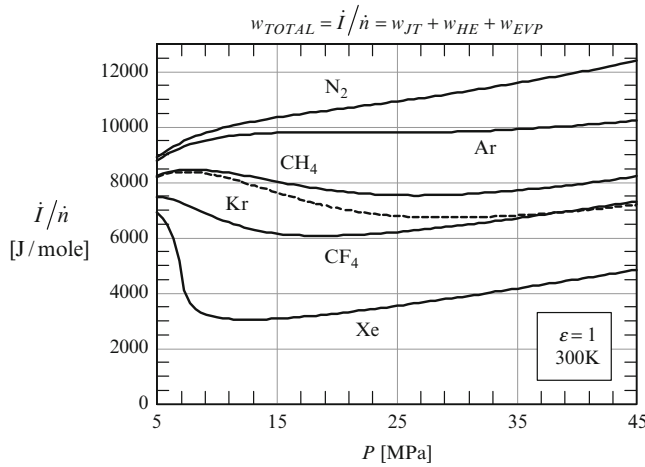


Fig. 3.34 The irreversibility of the entire Linde-Hampson cryocooler as function of pressure for various gases. The outlet pressure is 0.1 MPa

was not their normal boiling temperature). It should be noted that the above observations relate to gases used as coolants in an ideal Linde-Hampson cycle, with an ideal heat exchanger, where the entire cooling content, Δh_T , is completely utilized.

The fact that the FOM is always smaller than unity results from the generation of irreversibility. The entropy generated along a Linde-Hampson cycle (balanced in Sect. 1.4.2.3) is given by,

$$\Delta s = (s_5 - s_1) - \frac{\Delta h_T}{T_{BOIL}} \quad (3.187)$$

The associated irreversibility, obtained from the theorem of Gouy-Stodola⁶⁷ as,

$$\frac{\dot{I}}{\dot{n}} = T_1 \Delta s = T_1 (s_5 - s_1) - \frac{T_1}{T_{BOIL}} \Delta h_T \quad (3.188)$$

where

$$I = W^{LOST} \quad (3.189)$$

signifies the lost work, or, the extent to which the actual work exceeds the ideal minimum work associated with the reversible Carnot cycle.

The quantity $\dot{I}/\dot{n}$ is described in Fig. 3.34 for various gases as a function of the high pressure. The lost work, w_{TOTAL} decreases as the cold end temperature, T_{BOIL} increases. This is consistent with the fact that the FOM is larger for the gases with a higher boiling point T_{BOIL} (Fig. 3.33).

Remark: Fig. 3.6 displays a group of cycles with different degrees of recuperation and different values of T_L ; cycles (a)

and (b) have $T_L > T_{BOIL}$. All these cycles share the same COP, but not the same FOM. From Eq. 1.82 and its following corollary, the cycle that reaches a lower temperature, T_L , has a higher FOM and less irreversibility.

3.13.4 Availability Analysis

The irreversibility, derived by Eq. 3.188 refers to the entire cooler. An analysis of availability (or exergy) enables one to determine the particular irreversibility contribution (of destruction of availability) for each element. The availability function for a stream^{57,68} at any state is defined as,

$$b = h - T_1 s \quad (3.190)$$

The rate of destroying useful work, I_X , by any element, X , per unit flow is the specific irreversibility of the same element and is denoted by,

$$w_X = \dot{I}_X / \dot{n} \quad (3.191)$$

The irreversibilities associated with each component are derived by balancing the availability flow for those components. The notation defining the states along the Linde-Hampson cycle remains consistent with Sect. 3.2. Thus: for the adiabatic Joule-Thomson throttle valve,

$$w_{JT} = b_2 - b_3 \quad (3.192)$$

for the heat exchanger,

$$w_{HE} = b_1 + b_4 - b_2 - b_5 \quad (3.193)$$

for the evaporator,

$$w_{EVP} = b_3 - b_4 + \frac{\dot{Q}}{\dot{n}} \left(1 - \frac{T_1}{T_{BOIL}} \right) \quad (3.194)$$

where the last term is the availability of the added heat, $\dot{Q}$, at the temperature T_{BOIL} . The availability balance of the entire cryocooler gives,

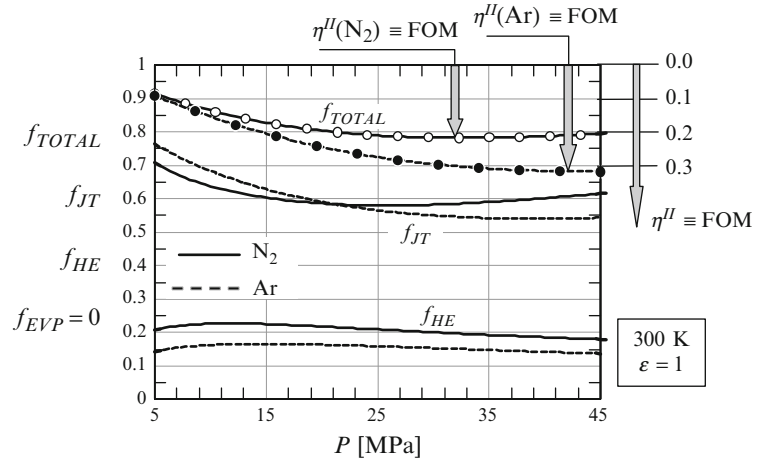
$$w_{TOTAL} = b_1 - b_5 + \frac{\dot{Q}}{\dot{n}} \left(1 - \frac{T_1}{T_{BOIL}} \right) \quad (3.195)$$

One may easily verify that Eq. 3.195 provides the same result as that obtained by summing the specific irreversibilities of the elements,

$$w_{TOTAL} = w_{JT} + w_{HE} + w_{EVP} \quad (3.196)$$

and it agrees with the expression given in Eq. 3.188, and shown in Fig. 3.34.

Fig. 3.35 The relative portions of lost availability with respect to the invested availability as wasted by the various elements of a Linde-Hampson cryocooler, and the FOM, or Second Law efficiency



The invested availability is $b_1 - b_5$. It is also the minimal (reversible) work required for compression from state 5 to state 1, consistent with Eq. 3.167. The relative share of the total irreversibility for each element, expressed as a fraction of the invested availability, (which is the purpose of the present section) is,

$$f_{JT} = \frac{w_{JT}}{b_1 - b_5} f_{HE} = \frac{w_{HE}}{b_1 - b_5} f_{EVP} = \frac{w_{EVP}}{b_1 - b_5} \quad (3.197)$$

The total fraction of wasted availability for all the elements, with respect to the invested availability is,

$$f_{TOTAL} = f_{JT} + f_{HE} + f_{EVP} \quad (3.198)$$

Figure 3.35 displays these fractions as a function of pressure for nitrogen and argon. State 2 is determined by assuming a nominal recuperation (Sect. 3.4). The most remarkable result is the large waste of availability (about 60%) at the Joule-Thomson valve. An additional 20% is lost in the heat exchanger and the remaining 20% provide the useful effect of cryocooling. No irreversibility is associated with the evaporator $f_{EVP} = 0$, in the present model for pure gases since all heat transfer interactions occur at the same constant temperature, that is T_{BOIL} . The relative share of the useful work,

$$\eta_{II} = 1 - f_{TOTAL} \quad (3.199)$$

also known as the *Second Law efficiency* and sometimes referred to as *exergy efficiency*, is also displayed in Fig. 3.35, and is equivalent to the FOM:

$$\begin{aligned} \eta_{II} &= \frac{\dot{Q}}{\dot{n}} \left(\frac{T_1}{T_{BOIL}} - 1 \right) = \frac{\Delta h_T}{T_1(s_5 - s_1) - \Delta h_T} \cdot \frac{T_1 - T_{BOIL}}{T_{BOIL}} \\ &= \frac{COP}{COP_{CARNOT}} = FOM \end{aligned} \quad (3.200)$$

The equivalence of η_{II} as derived through the analysis of availability and FOM is consistent with the general result of Sect. 1.4.1.4 and Eq. 1.81.

One may find a similar analysis for a Linde-Hampson liquefier, including the compressor, elsewhere.³³

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4.1 Introduction

The correct acquaintance with the thermo-physical properties of gases is the primary ingredient in the recipe for successful design and operation of Joule-Thomson cryocoolers. This is particularly true of the class of cryocoolers that utilize the throttle effect with real gas properties, and the extent to which their deviation from those of an ideal gas provides the driving potential for the cryocooling process. The gas as a coolant is much more than simply a working media, as for instance in case of pulse tube cryocoolers, where real gas properties play a minor role. In the service of Joule-Thomson cryocoolers, each particular gas has its own unique characteristics that cannot be replaced without significant impact on performance. Furthermore, the thermo physical features of the selected operating gas directly influence the construction and possible modes of operation.

In this chapter, the relevant thermo-physical properties of gases are formulated and arranged to reflect their suitability as proper coolants for Joule-Thomson cryocooling. For example, the thermo-physical properties of coolants should be considered with respect to their boiling point rather than to their critical parameters T_C , T_P , as is generally considered by the principle of corresponding states. The integral Joule-Thomson effect, by virtue of its significant pressure drop, naturally plays a central role in the analysis and synthesis of any Linde-Hampson machine. Both reflections of the integral effect, the isenthalpic and the isothermal, are discussed and related.

This chapter is primarily focused on pure gases. Mixed coolants deserve a special and separate treatment as given in Chap. 8.

4.2 Temperatures of Phase Transition

As explained in Sect. 3.1.3, heat absorption by a Linde-Hampson cryocooler is associated with a phase transition; in most applications it is the liquid-vapor transition and in less

common applications, it is the solid vapor transition. Therefore, the temperatures of liquefaction and solidification of various gases and their pressure dependence deserve special attention.

4.2.1 Liquefaction

The practice of cryocooling is frequently associated with the boiling point of a gas, since its liquefaction temperature defines the lower limit of the cryocooler's operation. Indeed, the operating temperature is the functional parameter that is most readily identifiable for any running Joule-Thomson cryocooler.¹ It dictates how deep we can cool down the payload. The Linde-Hampson cycle absorbs heat at the temperature of the boiling cryogen. Any specification of a given cryocooling job begins with the temperature of operation.

The list of normal boiling points of pure gases in the cryogenic range is well defined and not too long, as may be seen in Fig. 4.1. Therefore, once the required operating temperature is specified, little choice remains for selecting the pure coolant. Examining the list of available coolants, one may observe the following characteristics:

- (a) There is a wide range of temperatures of about 50 K that is not “covered” by any pure coolant. It is below the normal boiling point for nitrogen (77.35 K), and above that for neon (27.1 K).

(Sobel^{2,3} focused precisely on this non “covered” temperature range, and proposed mixtures that normally boil between 27 and 77 K. These will be discussed later in Sect. 8.10.2. The mixture components are neon, nitrogen and argon as well as various hydrocarbons).

- (b) Below this gap exist the liquid states for the quantum gases, whose maximum inversion temperatures are below room temperatures.
- (c) The lowest normal boiling temperature belongs to ^3He , which is 3.24 K. The use of ^3He instead of ^4He essentially avoids the unique difficulties associated with

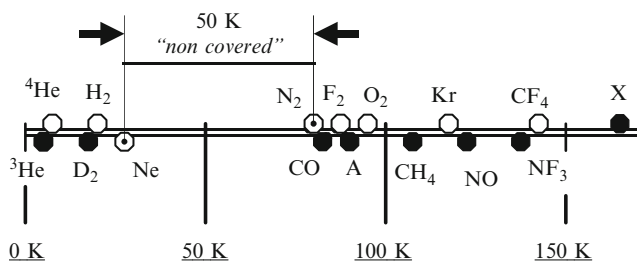


Fig. 4.1 The sequence of the normal boiling points of pure gases; the lowest one is helium-3

superfluid helium. In addition, ^3He has a higher vapor pressure than ^4He , which means that for a given extent of evacuation, a lower temperature can be reached.

Duant et al.⁴ and Wilkes⁵ built Joule-Thomson cryocoolers that liquefy ^3He . By reducing the pressure at the outlet of the cryocooler below 1 atm, they were able to further reduce the boiling point. Uhlig,⁶ Tanaka et al.,⁷ and Sawano et al.⁸ incorporated a ^3He cryocooler as part of a $^3\text{He}/^4\text{He}$ dilution refrigerator without a pumped ^4He stage.

- (d) Above the gap are the many gases whose inversion temperatures are above room temperature. The coldest gas with an inversion point above room temperature is nitrogen. Put another way, the lowest operating temperature that can be achieved through a spontaneous cool-down without precooling (see subsequent discussion in Sect. 5.4) is the normal boiling point of liquid nitrogen. In this regard, it is serendipitous that the coolant exhibiting such a special feature is such a common gas and major constituent of the surrounding atmosphere.

Most of the existing Joule-Thomson cryocoolers operate in the temperature range of liquid nitrogen and argon. For most gases, (at least the relevant candidates for Joule-Thomson cryocooling) the normal boiling point is related to the critical temperature by,

$$\frac{T_{\text{BOIL}}}{T_C} = 0.55 \rightarrow 0.65 \quad (4.1)$$

also known as the Guldberg-Guye rule.

The normal boiling point is only a single state on the vaporization curve, the one associated with the pressure of 101 kPa. There are applications where cryocoolers operate at other ambient pressures. Sometimes, while pumping on the outlet, the pressure is reduced to lower the boiling point and temperature of operation. Usually, a cryocooler also accommodates a backpressure induced by the hydrodynamic restriction impeding the release of the outgoing stream. In this case the operating temperature is elevated above the normal boiling point. Table 4.1 presents the changes in boiling points of some gases as function of the exhaust pressure.

For low acentricity gases such as Ar, Kr, Xe, N_2 , O_2 , CH_4 , CO, the saturation line is very well correlated⁹ by,

$$\ln\left(\frac{P}{P_C}\right) = 5.25\left(1 - \frac{T_C}{T}\right) \quad (4.2)$$

For example, Eq. 4.2 enables one to determine the reduced normal boiling point, T_{BOIL}/T_C in terms of the ambient (or normal) reduced pressure, $1.2 < \Theta < 3$. Through the principle of corresponding states, it may also roughly represent other gases. The pressure ratios between 1/50 and 1/20 correspond with ratios of T_{BOIL}/T_C between 0.57 and 0.63, which are quite consistent with Guldberg-Guye rule of Eq. 4.1.

4.2.2 Solidification

As the highest boiling point is the critical temperature, the lowest boiling point is the triple point, T_{TR} . A further decrease in outlet pressure (as might occur at high altitude) leads to states of sublimation, which is the solid-vapor transition instead of that of liquid-vapor. The lowest obtainable cryocooling temperature by a given coolant is slightly above T_{TR} . When the triple point is reached, the gas solidifies at the outlet of the expansion nozzle. Such a cryocooler will absorb heat from the cooled object by sublimation (solid-vapor transition) instead of evaporation (liquid-vapor transition).

Table 4.2 lists the triple point parameters, T_{TR} and P_{TR} , for various gases. The temperature margin, $T_{\text{BOIL}} - T_{\text{TR}}$, is of practical relevance, since it reflects the potential to lower the operating temperature by pumping on the outlet while still allowing a liquid-vapor transition. It is displayed in Fig. 4.2 as a function of the normal boiling points. The noble gases are characterized by a small temperature margin between their normal boiling point and triple point, and a relatively high triple point pressure. Pitzer¹⁰ noticed that the reduced triple point for the noble gases is 0.55. For O_2 , CF_4 C_3H_8 it is 0.35, 0.39 and 0.23 respectively. In the lower temperature range, hydrogen exhibits a wide margin in comparison with neon. In this sense, nitrogen is also more promising than argon. However, oxygen, whose normal boiling point is so close to that of argon, exhibits a very wide margin, thus enabling, by pumping on the exit, a temperature reduction to 55 K. Indeed, some attempts were reported of operating Joule-Thomson cryocoolers with oxygen^{11,12} or at least including oxygen in a mixture with nitrogen.^{13,14}

4.3 The Integral Isothermal Joule-Thomson Effect, Δh_T

The basic energy balance equation of a cryocooler, $\dot{Q} = \dot{n} \cdot \Delta h_T$, allows Δh_T to be interpreted as the potential specific cooling content of a gas, as derived in Sect. 3.2.4. The isothermal enthalpy change, Δh_T , is solely the result of a

Table 4.1 Saturation (boiling) temperatures of various gases as function of pressure

P [kPa]	Helium	Hydrogen	Neon	Nitrogen	Argon	Methane	Oxygen	Krypton
10	2.49	14.43					72.68	
20	2.88	15.87		65.81		95.1	77.10	
30	3.15	16.84		68.33		98.7	80.05	
40	3.37	17.58		70.30		101.5	82.25	
50	3.55	18.2	24.96	71.93		103.7	84.05	
60	3.71	18.72	25.49	73.28		105.7	85.55	
70	3.85	19.19	25.96	74.46	84.06	107.4	86.95	
80	3.98	19.61	26.35	75.52	85.23	108.9	88.15	116.8
90	4.10	19.99	26.73	76.48	86.30	110.2	89.15	118.3
100	4.21	20.34	27.07	77.35	87.28	111.5	90.15	119.6
110	4.31	20.67	27.39	78.18	88.19	112.8	91.15	120.8
120	4.41	20.98	27.69	78.95	89.03	113.9	91.95	122.0
130	4.50	21.27	27.97	79.67	89.83	114.9	92.75	123.1
140	4.58	21.54	28.23	80.35	90.58	115.9	93.55	124.1
150	4.67	21.80	28.48	80.98	91.30	116.8	94.25	125.0
200	5.03	22.93	29.57	83.78	94.40	120.8	97.35	129.2
250		23.88	30.48	86.09	96.96	123.9	99.95	132.6
300		24.69	31.26	88.08	99.16	126.9	102.15	135.6
350		25.42	31.96	89.84	101.10	129.2	104.15	138.2
400		26.08	32.58	91.42	102.90	131.4	105.95	140.5
450		26.68	33.15	92.86	104.50	133.5	107.55	142.7
500		27.23	33.68	94.19	105.90	135.4	108.95	144.6
1,000		31.36	37.56	103.7	116.6	149.1	119.6	159.1
1,500			40.21	110.4	124.0	158.5	127.0	169.0
2,000			42.27	115.6	129.7	165.9	132.7	176.7
2,500			43.97	119.9	134.6	172.0	137.5	183.1
3,000				123.6	138.7	177.3	141.7	188.7

Table 4.2 Triple point and boiling point temperature and the gap between

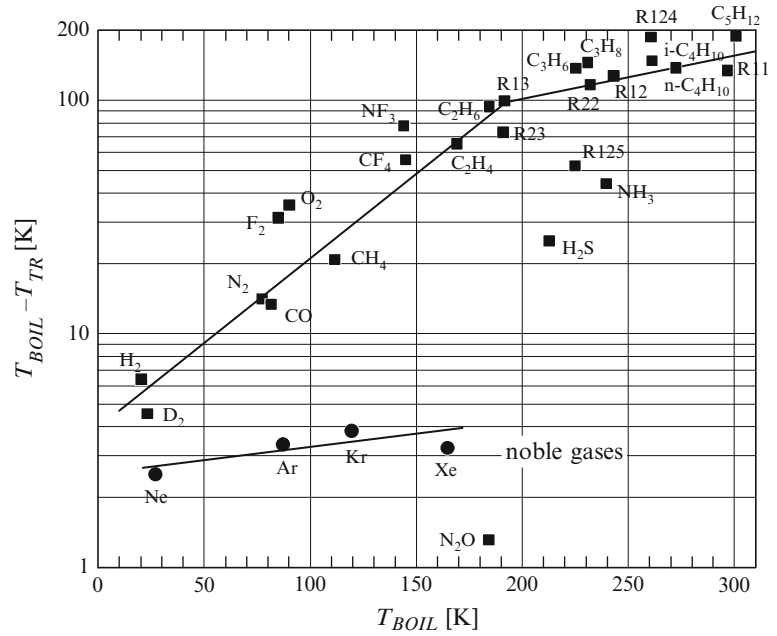
Gas	T_{BOIL} [K]	T_{TR} [K]	P_{TR} [kPa]	$T_{BOIL} - T_{TR}$ [K]
H ₂	20.4	13.8	6.95	6.6
D ₂	23.57	18.72	16.9	4.85
Ne	27.1	24.56	42.7	2.44
N ₂	77.4	63.15	12.3	14.25
CO	81.65	68.15	15.18	13.5
F ₂	84.86	53.48	2.52	31.46
Ar	87.3	83.80	68.0	3.5
O ₂	90.2	54.36	0.15	35.84
CH ₄	111.6	90.68	11.6	20.92
Kr	119.9	115.76	72.2	4.14
NF ₃	144.1	66.36	$1.83 \cdot 10^{-3}$	77.74
CF ₄	145.1	89.3	0.12	56.0
Xe	165.1	161.36	80.5	3.74
C ₂ H ₄	169.4	103.99	0.12	65.41
C ₂ H ₆	184.5	90.35	$1.1 \cdot 10^{-3}$	94.15
C ₃ H ₈	231.1	85.47	$1.7 \cdot 10^{-7}$	145.63
NH ₃	239.8	195.4	6.0	44.4
i-C ₄ H ₁₀	266.90	113.55	$1.9 \cdot 10^{-5}$	147.81
n-C ₄ H ₁₀	272.70	134.86	$6.6 \cdot 10^{-4}$	137.84
CO ₂		216.55	517.0	

change of pressure, and already defined by Eq. 3.16. After the boiling point, it is the property of most significance for the function of Joule-Thomson cryocoolers. Consequently, we will explore it in detail. In particular, its dependence on the thermodynamic state (pressure and temperature), and the variation of its magnitude from one gas to another significantly impacts the operation of Joule-Thomson cryocoolers. The magnitudes of Δh_T will be investigated as a function the range of boiling points afforded by the various working gases.

4.3.1 Residual Enthalpy, h^R , and Δh_T

By rigorously applying Eq. 3.16, Δh_T can be determined at each state point in the thermodynamic plane defined by the variable set (T, P) . One should not be misled to conclude that Δh_T is just a negative function of enthalpy, $h(T, P)$, shifted by the value of $h(0.1 \text{ MPa}, T)$. Note, most significantly that the shifting term is temperature dependent. Therefore, the trajectories of constant Δh_T in the (T, P) plane do not coincide with those of the isenthalps themselves.

Fig. 4.2 Display of the gap between the normal boiling point and the triple point for various gases as function of their normal boiling point; the noble gases have the smallest gap



For an ideal gas, enthalpy is solely a function of temperature and independent of pressure,

$$dh^{IG} = c_P^{IG} dT \quad c_P^{IG} = \text{Const} \quad (4.3)$$

Therefore, it does not exhibit any change resulting from a decrease in pressure, and $\Delta h_T^{IG} = 0$. In this respect, Δh_T is strictly a characteristic of real (non-ideal) gases. Therefore, a real gas equation of state must be used in order to model Δh_T , since the ideal gas equation would be unable to capture the crucial aspects of its nature.

Let us introduce the idea of *residual* (or *departure*) *enthalpy*, measuring the departure from the enthalpy of an ideal gas at the same pressure and temperature (P, T), as is often done for any other properties such as the entropy or internal energy. Hence,

$$h^R(P, T) = h(P, T) - h^{IG}(P, T) \quad (4.4)$$

However, h^{IG} is independent of pressure, therefore,

$$h^{IG}(P, T) = h^{IG}(0.1 \text{ MPa}, T) \quad (4.5)$$

and

$$\Delta h_T(P, T) = -h^R(P, T) = h^{IG} - h \quad (4.6)$$

This equivalence is highlighted because much effort has been devoted to formulate, model and measure the residual properties. Hence, the results obtained may be utilized for the study of Δh_T . The two functions, Δh_T and h^R , have the

same value, but are opposite in sign. Even for discharge pressures higher than 0.1 MPa, the residual enthalpy still approximates Δh_T quite well. Since h^R by definition vanishes for an ideal gas, it is once again demonstrated that in this case Δh_T is identically zero for an ideal gas.

One must keep in mind however, that the isothermal enthalpy difference, Δh_T , and the residual enthalpy, h^R , are derived from completely different physical principles. It is only due to the definition of $P \Delta h_T$ with reference to the low discharge pressure, that the two values coincide.

4.3.2 Pressure Dependence of Δh_T

The pressure dependence of the values of Δh_T for some candidate coolants provides a topic for investigation that is of practical significance. For the purpose of this discussion, it will also be helpful to observe various features of the isotherms in the (P, h) plane shown in Fig. 2.10.

At any given temperature, differentiating Eq. 4.4 provides the pressure dependence of Δh_T ,

$$\left(\frac{\partial}{\partial P} \Delta h_T \right)_T = - \left(\frac{\partial h}{\partial P} \right)_T = -\phi = \mu c_P \quad (4.7)$$

Since

$$\left(\frac{\partial h^{IG}}{\partial P} \right)_T = 0 \quad (4.8)$$

The value of $\left(\frac{\partial}{\partial P} \Delta h_T \right)_T$ changes its trend according to the sign of μ . In particular, Δh_T increases with pressure when

Table 4.3 Pressure of achieving maximum Δh_T for some gases at their typical warm inlet temperatures; the values listed are the corresponding inversion pressures

300 K		85 K	
Nitrogen	39.1 MPa	Neon	28.2 MPa
Argon	54.6 MPa	Hydrogen	15.5 MPa
Oxygen	57.5 MPa		
Methane	45.8 MPa		
Ethane	20.1 MPa	20 K	
Krypton	48.6 MPa	Helium	3.86 MPa

$\mu < 0$, namely, inside the differential inversion curve (D.I.C.). Outside the D.I.C. the Δh_T decreases with pressure when $\mu < 0$. Note, that Δh_T decreases until it is zero, on the integral inversion curve (I.I.C.). Outside the I.I.C. Δh_T is negative and integral Joule Thomson effect does not provide cooling but rather warming.

On the D.I.C.

$$(\partial h / \partial P)_T = -\mu \cdot c_P = 0 \quad (4.9)$$

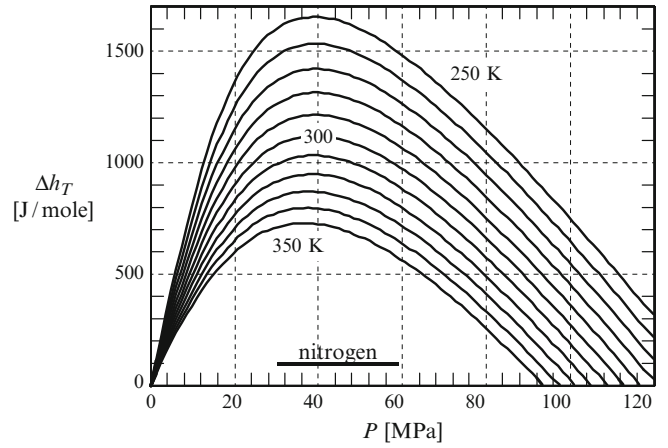
which means,

$$\left(\frac{\partial}{\partial P} \Delta h_T \right)_T = 0 \quad (4.10)$$

The refrigerant's maximum cooling content, at any given temperature, T , is located on the differential inversion curve. An alternative statement would be that any trajectory of constant Δh_T has its maximum on the differential inversion curve. Note that the trajectories of constant Δh_T in the (T, P) plane are not the same as the trajectories of constant enthalpy (as in Fig. 2.6 for nitrogen). Nevertheless, the two groups do share the same set of maximum values. Wooley et al.¹⁵ plotted trajectories of constant Δh_T and h for hydrogen in the vicinity of its inversion curve.

At any given temperature, T , the highest Δh_T , which is Δh_T^{MAX} , occurs at the inversion pressure that corresponds with the given T . For each T there is also another peak value, $\Delta h_T^{MAX}(T)$. The complete temperature dependence of Δh_T^{MAX} is discussed later in Sect. 4.3.6. Upon approaching T_{INV}^{MAX} , the value of Δh_T becomes smaller and smaller until it vanishes.

The vicinity of $\Delta h_T^{MAX}(T)$ is, in general, a fairly broad maximum with regard to pressure changes, as demonstrated in the next section. Indeed, as a general trend and recommendation, Joule-Thomson cryocoolers and liquefiers operate close to the inversion pressure associated with the ambient temperature. Values of the inversion pressure for some gases at 300 K are listed in Table 4.3. The same values

**Fig. 4.3** The pressure dependence of the specific cooling capacity, Δh_T , for nitrogen at various temperatures under expansion from P down to 0.1 MPa

are also listed for the quantum gases at their typical temperatures of operation.

The quantity Δh_T , expressed as $h_5 - h_1$ (see Sect. 3.2.4) is, in principle, also dependent on the low pressure, P_5 through h_5 . Since P_5 (previously assumed as 0.1 MPa) is a small fraction of P_C , the enthalpy at this state, $h_5(P_5)$ was assumed to behave as that of an ideal gas, h^{IG} , and therefore is independent of pressure. Thus, Δh_T is very weakly dependent on the expanded pressure, P_5 . However, the work of compression and the COP are significantly dependent on P_5 .

4.3.3 Examples of the Pressure Dependence of Δh_T

4.3.3.1 The Super Critical Temperature Range, $T > T_C$

Coolants of cryogenic interest exhibit normal boiling points below 150 K. If the relevant range of environmental temperatures is from 250 to 350 K, the associated reduced temperature, ranges from $\Theta = 1.1$ to 3. In this reduced temperature range, the D.I.C. reaches its maximum values at inversion pressures which are about $10 P_C$. Put another way, in this temperature range, the pressure must be elevated to as high as $10 P_C$ in order to approach and enjoy the Δh_T^{MAX} values.

In general, one should operate the cooler in the vicinity of these pressures that vary with each gas according to their associated values of P_C . Figures 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, and 4.9 display this pressure dependence for nitrogen, air, argon, oxygen, methane, krypton and R-14 (CF_4), from 270 K and 280 K up to 340 K and 360 K. Operating at a low pressure such as $2 P_C$ utilizes only a fraction of Δh_T^{MAX} . On the other hand, operating at pressures exceeding $20 P_C$, will have an effect of over-pressurizing and significantly decreasing the values of Δh_T .

Fig. 4.4 The pressure dependence of the specific cooling capacity, Δh_T , for air at various temperatures under expansion from P down to 0.1 MPa

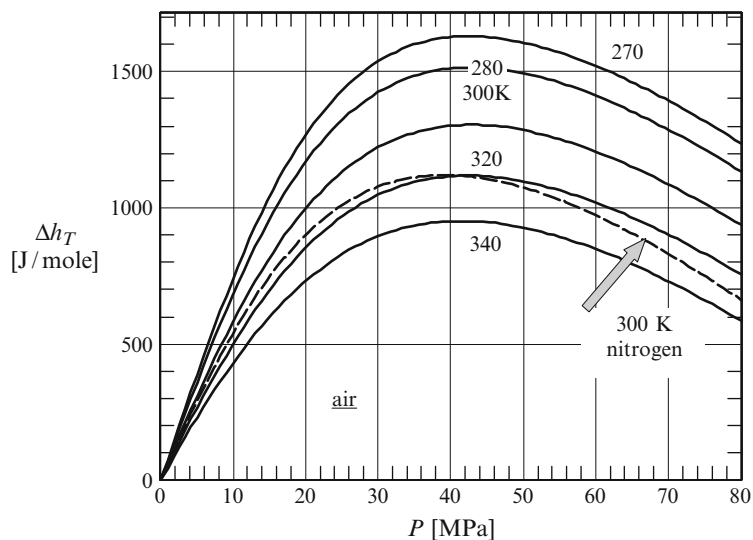


Fig. 4.5 The pressure dependence of the specific cooling capacity, Δh_T , for argon at various temperatures under expansion from P down to 0.1 MPa

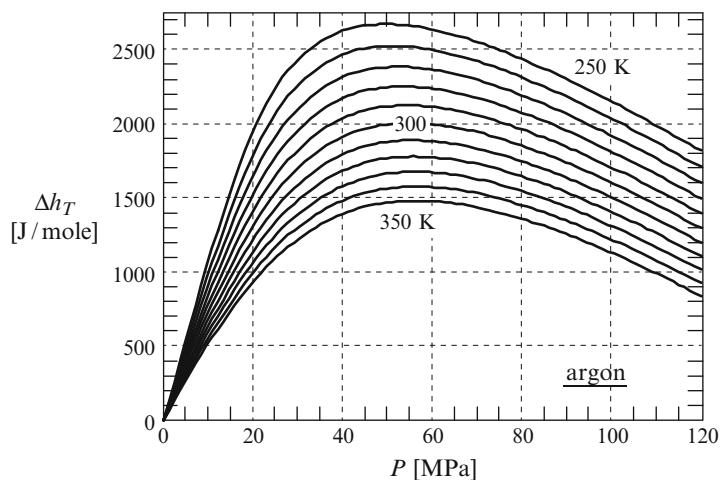


Fig. 4.6 The pressure dependence of the specific cooling capacity, Δh_T , for oxygen at various temperatures under expansion from P down to 0.1 MPa

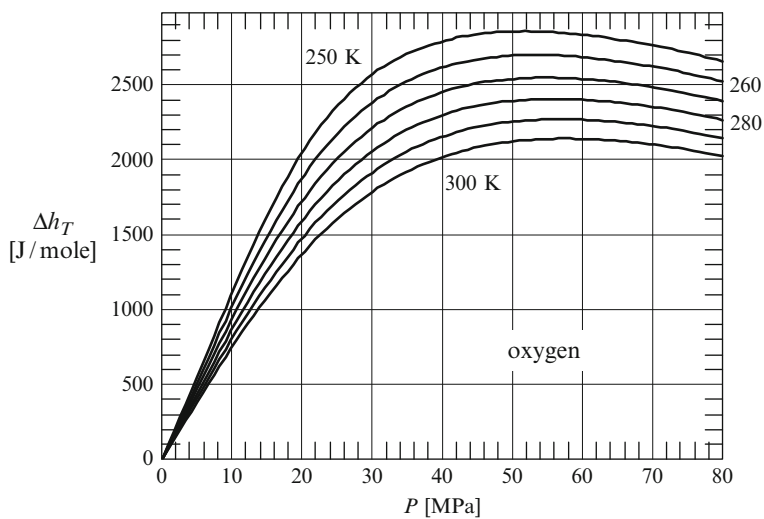


Fig. 4.7 The pressure dependence of the specific cooling capacity, Δh_T , for methane at various temperatures under expansion from P down to 0.1 MPa

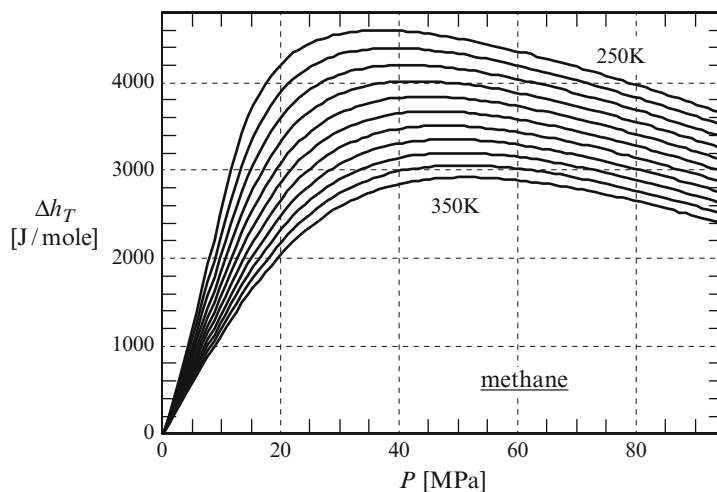


Fig. 4.8 The pressure dependence of the specific cooling capacity, Δh_T , for krypton at various temperatures under expansion from P down to 0.1 MPa

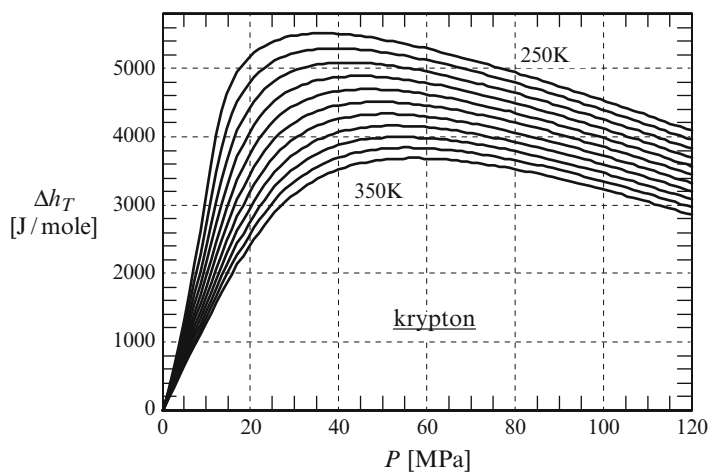


Fig. 4.9 The pressure dependence of the specific cooling capacity, Δh_T , for refrigerant R-14 at various temperatures under expansion from P down to 0.1 MPa

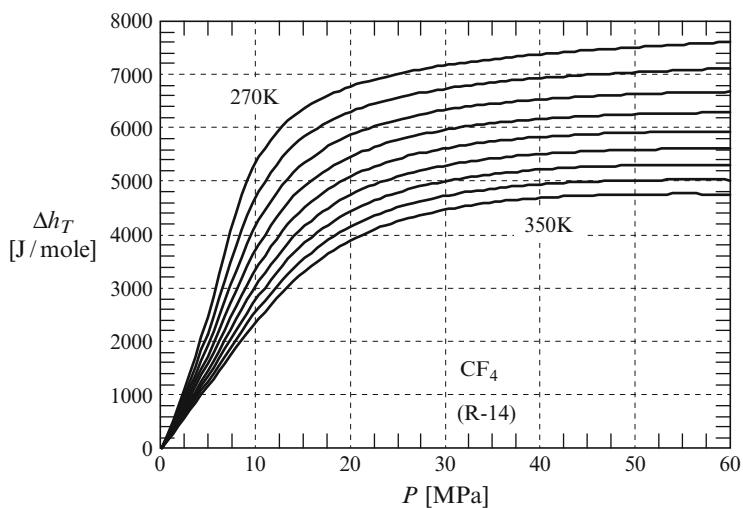


Fig. 4.10 The pressure dependence of the specific cooling capacity, Δh_T , for various refrigerants at room temperature (a) under expansion from P down to 0.1 MPa: (a) expressed in J/mole, (b) expressed in J/g

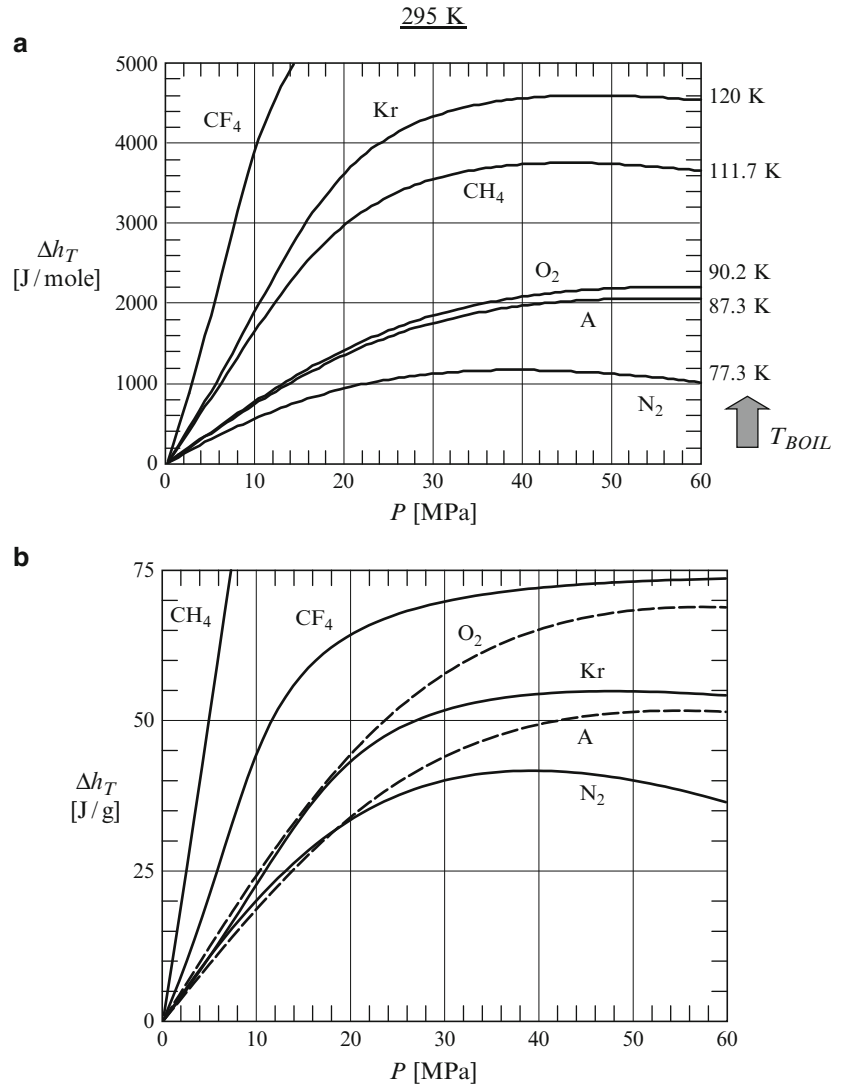


Figure 4.10(a) is a comparative display of Δh_T for these gases at room temperature. As a general trend, gases with a higher normal boiling point provide higher values of Δh_T . However, if Δh_T is expressed per unit mass (J/g) rather than per mole, (J/mole), the molecular weight of the gases plays a role and the comparison takes on a different form, as shown in Fig. 4.10(b). The curves in (b) do intersect and their order is different than in (a). The larger values in (b) belong to methane (and the hydrocarbons).

4.3.3.2 Δh_T in the Low Pressure Range

The slopes of these curves at zero pressure are related to μ_0 . Equation 2.8, when applied at zero pressure, becomes,

$$\left[\left(\frac{\partial}{\partial P} \Delta h_T \right)_T \right]_{P=0} = - \left[\left(\frac{\partial h}{\partial P} \right)_T \right]_{P=0} = \mu_0 c_{P0} \quad (4.11)$$

The left hand side of Eq. 4.11 is the slope of $\Delta h_T(P)$ curves at zero pressure. On the right hand side, μ_0 is the zero pressure Joule-Thomson coefficient, which has been extensively measured and analyzed, and whose importance is discussed in Sect. 2.2.4.

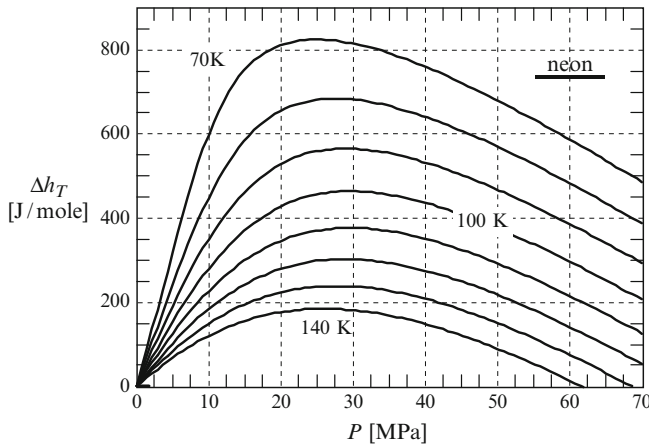
Numerous studies of the Joule-Thomson coefficient focused attention on the zero pressure values. For example, Gustafson¹⁶ and Vortmeyer¹⁷ dedicated their investigations solely on μ_0 . The heat capacity of an ideal gas at zero pressure, c_{P0} , is well known for all gases and may be quite well approximated by the relation,

$$c_{P0} = \frac{\kappa}{\kappa - 1} R \quad (4.12)$$

The combination of Eqs. 4.11 and 4.12 provides an independent consistency check for the zero pressure slope of $\Delta h_T(P)$. For example, Roebuck et al.^{18,19} measured μ_0 for nitrogen and argon at 298 K, and these are summarized in

Table 4.4 Average slope of $\Delta h_T(P)$ in the pressure range of 0–10 MPa

Average $\left(\frac{\partial}{\partial P} \Delta h_T\right)_T$; $\left[\frac{\text{J/mole}}{\text{MPa}}\right]$	300 K	330 K
Nitrogen	54	42
Argon	71	58
Methane	159	126
Krypton	182	143
R-14	369	268

**Fig. 4.11** The pressure dependence of the specific cooling capacity, Δh_T , for neon at various temperatures under expansion from P down to 0.1 MPa

the Smithsonian Physical Tables.²⁰ They report values of μ_0 for nitrogen and argon of 2.217 and 3.720 K/MPa, respectively. Through κ values of 1.4 and 1.667, the associated c_{PO} values are 29.98 and 20.83 J/mole/K and the values of $(\partial \Delta h_T / \partial P)_T$ at zero pressure are 66.4 and 77.5 J mole⁻¹ K⁻¹. Average slope values measured in the relatively lower pressure range, $0 < P < 10$ MPa, for the common gases are listed in Table 4.4.

The average slope determines the linear relation between Δh_T and P and may be used to derive a final, or cut-off, pressure for a charged vessel feeding a Joule-Thomson cryocooler. Maytal²¹ applied it to optimize the size of a fixed orifice, as is also discussed in Sect. 9.2.5.2.

4.3.3.3 The Pressure Dependence of Δh_T for the Quantum Gases

The pressure dependence of Δh_T for the quantum gases is shown in Fig. 4.11 for neon and Fig. 4.12 for hydrogen and deuterium. In Joule-Thomson cryocoolers, these gases are generally precooled by an earlier stage, utilizing, for example, liquid nitrogen or argon (Sect. 5.4.6). Therefore, the temperatures displayed in Figs. 4.10, 4.11, and 4.12 for Δh_T are chosen to be in the vicinity of, and above, the boiling points of nitrogen and argon instead of at room or

environmental temperatures. Similarly, Fig. 4.13 is dedicated to helium. In this case the range of temperatures is associated with precooled by hydrogen or neon.

There are isotherms of Δh_T as a function of P that intersect, as shown in Fig. 4.13 for helium. (For hydrogen and deuterium the intersection occurs at lower temperatures than those in Fig. 4.12). This unique occurrence for helium, hydrogen and their isotopes is related to the fact that their I.I.C.s display maximum values, as discussed in Sect. 2.6.3 and displayed in Fig. 2.19.

The critical pressures of deuterium, hydrogen and helium are less than about half the values of other gases. Therefore, it is quite “easy” to over pressurize the gas so that Δh_T would reach negative values and warming instead of cooling would occur. In terms of the inversion states, an idea introduced in the next section, one could characterize such a condition as having crossed over the contour of the integral inversion states.

The isotopes, hydrogen and deuterium have relatively similar values of T_C and Δh_T . However, the relative difference between the T_C of He-3 and He-4 is large. Indeed, He-3 has a significantly lower value of Δh_T than He-4 at similar conditions, (T, P) as displayed in Fig. 4.13b and based on references.^{22,23}

4.3.3.4 The Sub Critical Temperature Range, $T < T_C$

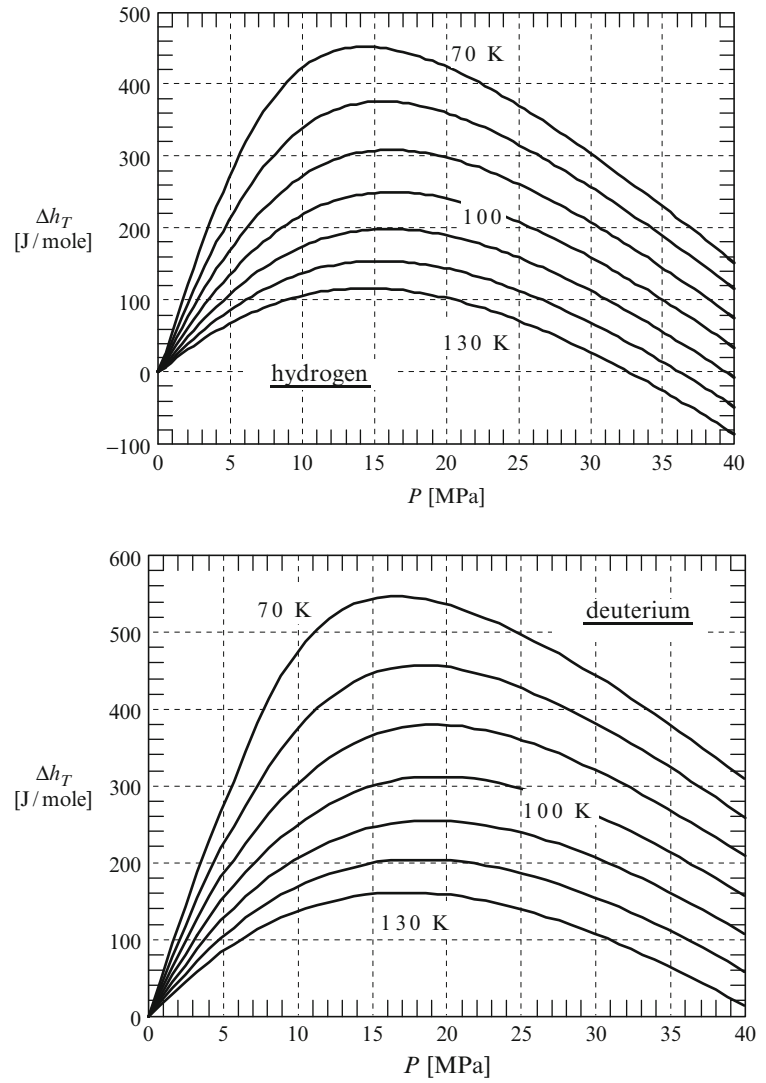
A typical display of this region is shown in Fig. 4.14 for nitrogen. In principle, the $\Delta h_T(P)$ in this range remains similar to that in the supercritical range. However, while the curve of $T = T_C = 126.3$ K is still smooth, the curves associated with subcritical temperatures (120 and 100 K) contain a vertical jump. The jump occurs at the saturation pressure associated with the subcritical temperature of the curve due to evaporation accompanying the pressure drop of the isenthalpic expansion. Figure 4.15 displays Δh_T values of xenon at temperatures in the vicinity of room temperature which is also close to and below $T_C = 289.7$ K.

Below a certain temperature, T_D , as discussed in Sect. 2.4.4, the curves of Δh_T reach their highest value on the saturation curve (Fig. 2.9), serving as an extension of the inversion curve. That is demonstrated for nitrogen ($T_D = 101.3$ K) at the 100 K isotherm in Fig. 4.14. The 120 K isotherm peaks at a higher pressure than the associated saturation pressure.

4.3.4 Deriving Δh_T by the Equations of State

The residual enthalpy, as a very good approximation for Δh_T , may be obtained by integrating the equation of state.^{9,24,25,26,27} For the ease of integration, each version of the state equation has a preferred expression out of the following equivalent general thermodynamic relations. The particular equations of state may be found in Appendix A2.

Fig. 4.12 The pressure dependence of the specific cooling capacity, Δh_T , for hydrogen and deuterium at various temperatures under expansion from P down to 0.01 MPa



4.3.4.1 General Expressions

1. Derivatives of specific volume,

$$h^R = - \left\{ \int_0^P \left[T \left(\frac{\partial v}{\partial T} \right)_P - v \right] dP \right\}_T = -T^2 \left\{ \int_0^P \left[\frac{\partial(v/T)}{\partial T} \right]_P dP \right\}_T \quad (4.13)$$

2. Derivatives of pressure,

$$h^R = \left\{ \int_{\infty}^v \left[v \left(\frac{\partial P}{\partial v} \right)_T + T \left(\frac{\partial P}{\partial T} \right)_v \right] dv \right\}_T \quad (4.14)$$

$$= RT(Z - 1) + \left\{ \int_{\infty}^v \left[T \left(\frac{\partial P}{\partial T} \right)_v - P \right] dv \right\}_T \quad (4.15)$$

3. Derivatives of compressibility,

$$\begin{aligned} h^R &= -RT^2 \left[\int_0^P \left(\frac{\partial Z}{\partial T} \right)_P \frac{dP}{P} \right]_T \\ &= -RT^2 \left\{ \int_0^P \left[\frac{\partial(Z/P)}{\partial T} \right]_P dP \right\}_T \end{aligned} \quad (4.16)$$

$$h^R = -RT^2 \cdot \left[\int_0^{\rho} \left(\frac{\partial Z}{\partial T} \right)_{\rho} \cdot \frac{d\rho}{\rho} \right]_T + RT(Z - 1) = \quad (4.17)$$

$$= RT^2 \left[\int_{\infty}^v \left(\frac{\partial Z}{\partial T} \right)_v \frac{dv}{v} \right]_v - RT + Pv \quad (4.18)$$

Fig. 4.13 The pressure dependence of the specific cooling capacity, Δh_T : (a) for He-4 at various temperatures under expansion from P down to 0.01 MPa, and (b) for He-3 and He-4 at 10 K, under expansion from P down to 0.1 MPa

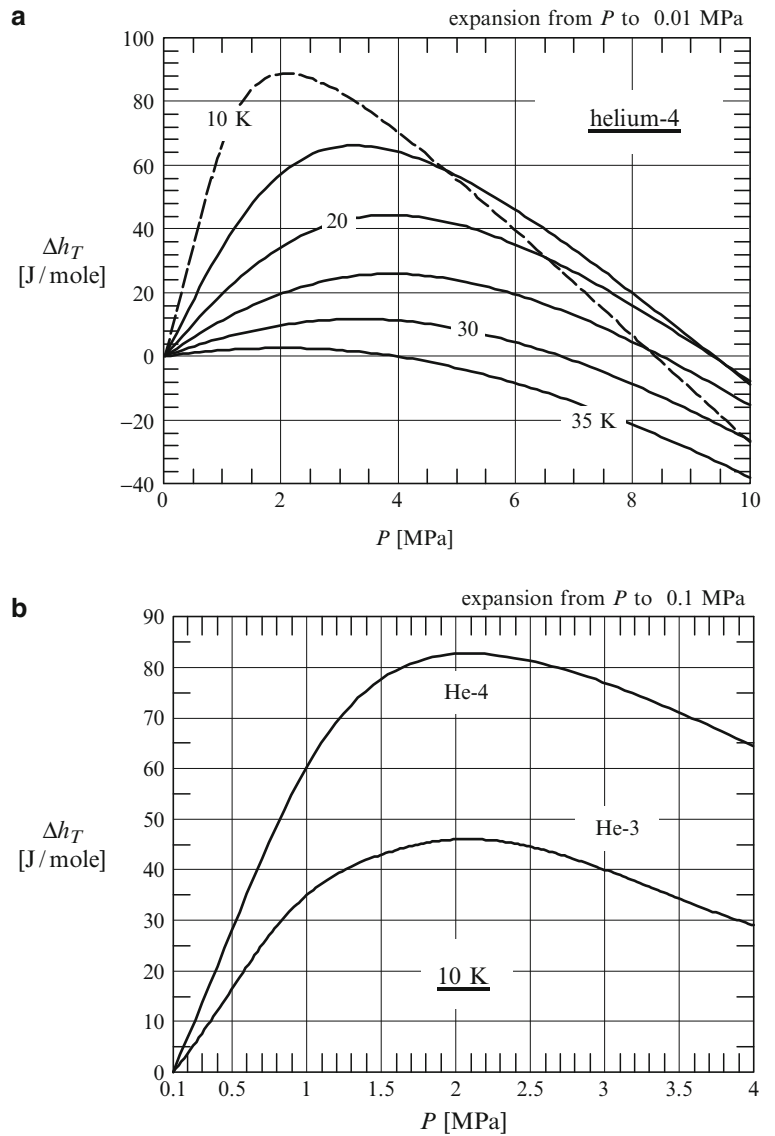


Fig. 4.14 Pressure dependence of the specific cooling content, Δh_T , for nitrogen in the vicinity of the critical pressure and at various sub-critical and super-critical temperatures under expansion from P down to 0.01 MPa

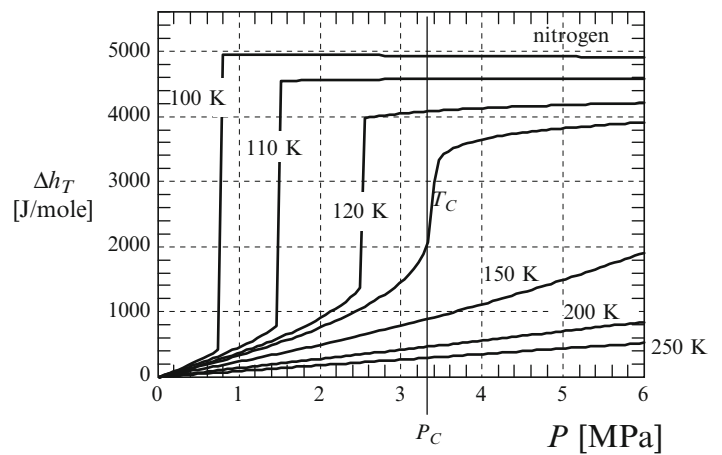
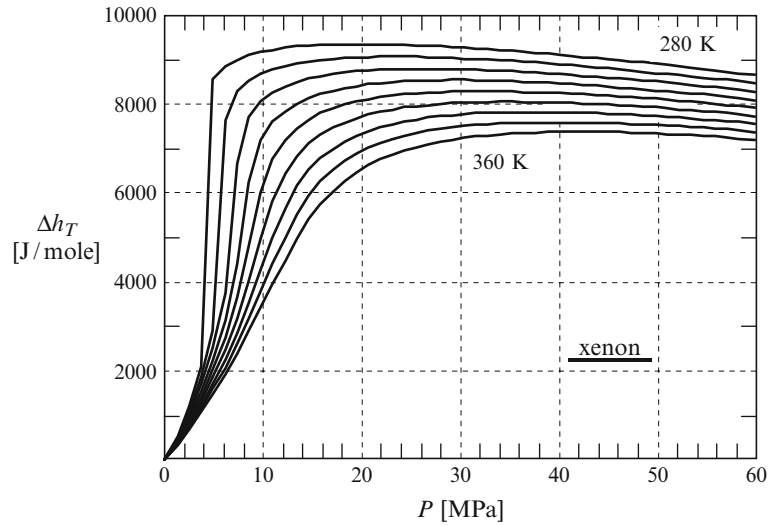


Fig. 4.15 The pressure dependence of the specific cooling capacity, Δh_T , for xenon at various temperatures under expansion from P down to 0.1 MPa $T_C = 289.7$ K



or in its reduced form,

$$\frac{h^R}{R T_C} = -\Theta^2 \left[\int_0^\Pi \left(\frac{\partial Z}{\partial \Theta} \right)_P \frac{d\Pi}{\Pi} \right]_\Theta \quad (4.19)$$

4.3.4.2 The Van der Waals Equation of State

The van der Waals equation of state, enables one to accomplish a closed form integration and obtain a relatively simple expression for the residual enthalpy,

$$-\frac{h^R}{R T_C} = \frac{27}{64} \frac{\Pi}{Z\Theta} - (Z-1)\Theta \quad (4.20)$$

$$h^R = \frac{RTb}{v-b} - \frac{2a}{v} \quad (4.21)$$

$$h^R = -a\rho + \frac{P}{\rho} - RT \quad \text{or} \quad \frac{h^R}{RT} = Z - 1 - \frac{a\rho}{RT} \quad (4.22)$$

4.3.4.3 The Peng-Robinson Equation of State

In contrast to the van der Waals equation of state, this one does not provide a completely closed form solution, but does enable analytical integration of the enthalpy in terms of compressibilities.

$$h^R = RT(Z-1) \frac{T \frac{da(T)}{dT} - a(T)}{2\sqrt{2}b} \cdot \ln \frac{Z + (1+\sqrt{2}) \frac{bP}{RT}}{Z + (1-\sqrt{2}) \frac{bP}{RT}} \quad (4.23)$$

The parameters a and b are defined in Appendix A2.2.

4.3.4.4 The Virial Equation of State Expanded by Pressure

The virial equation of state expanded by pressure,

$$h^R = -PT \left[\frac{dB}{dT} - \frac{B}{T} \right] - \frac{P^2}{2R} \left[\frac{B^2 - C}{T} + \frac{dC}{dT} - 2B \frac{dB}{dT} + \dots \right] \quad (4.24)$$

and in its truncated version,

$$h^R = P \left[B - T \frac{dB}{dT} \right] \quad (4.25)$$

4.3.4.5 The Virial Equation of State Expanded by Density

The virial equation of state expanded by density,

$$h^R = -RT^2 \left[P \frac{dB'}{dT} + \frac{1}{2} P^2 \frac{dC'}{dT} + \dots \right] \quad (4.26)$$

4.3.4.6 The Critical State, $\Delta h_T(T_C, P_C)$

The critical state, $\Delta h_T(T_C, P_C)$, may be determined for the van der Waals gas by the exact solution through Eq. 4.20 with $Z_C = 3/8$ and $\Pi = \Theta = 1$,

$$\Delta h_T(T_C, P_C) = -h^R(T_C, P_C) = \frac{7}{4} RT_C \quad (4.27)$$

The prediction of the Peng-Robinson equation of state (Appendix A2.2) follows from Eq. 4.23,

$$\begin{aligned}\Delta h_T(T_C, P_C) &= -h^R(T_C, P_C) \\ &= -RT_C(Z_C - 1) - \frac{m+1}{2\sqrt{2}} \cdot \frac{a}{b} \\ &\quad \cdot \ln \frac{Z_C + (1 + \sqrt{2})0.078}{Z_C + (1 - \sqrt{2})0.078}\end{aligned}\quad (4.28)$$

and finally,

$$\frac{\Delta h_T(T_C, P_C)}{RT_C} = 2.371 + 1.883 \omega - 0.330 \omega^2 \quad (4.29)$$

For gases of low acentricity factor, the value of $\Delta h_T/(RT_C)$ at the critical state is about 2.4. The values obtained by any numerical code for thermophysical properties like in reference²⁸; for N₂, O₂, CH₄, Ar, Kr, Xe, are somewhat higher although quite close: 2.64, 2.76, 2.46, 2.79, 2.42, 2.45, respectively.

For He and H₂, the predicted values of $\Delta h_T/(RT_C)$ are smaller, 1.60 and 1.95 respectively, due to their negative acentricity factors. The values obtained by the same reference are also smaller: 1.72 and 2.06 respectively.

4.3.5 Temperature Dependence of Δh_T

4.3.5.1 The Role of the Residual Specific Heat Capacity, c_p^R

In the examples in Figs. 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, 4.9, 4.10, 4.11, 4.12, 4.13, 4.14, and 4.15 one may observe $\Delta h_T(P)$ which is also displayed at various temperatures. The values of Δh_T increase as the temperature decreases, namely, a colder gas exhibits a higher cooling potential. The extent that Δh_T changes with temperature at any given pressure is obtained by differentiating Eq. 4.4,

$$\left(\frac{\partial}{\partial T} \Delta h_T\right)_P = -(c_P - c_{PO}) \equiv -c_P^R \quad (4.30)$$

where c_P^R is the residual specific heat capacity. In most cases of pure gases, the higher pressure heat capacity, c_P , is larger than that of the low and expanded pressure, c_{PO} , thus,

$$c_P^R \equiv c_P - c_{PO} > 0 \quad (4.31)$$

and therefore,

$$\left(\frac{\partial}{\partial T} \Delta h_T\right)_P < 0 \quad (4.32)$$

Note the significance of Eq. 4.32: Δh_T increases as T decreases. This point was highlighted by Gosney.²⁹ To the

extent that c_P is larger than c_{PO} , Δh_T will increase, a trend that intensifies at lower temperatures; see geometrical interpretation of Δh_T in Fig. 3.4.

4.3.5.2 Helium and Hydrogen

Actually, (4.31) and (4.32) are valid for all gases and states except some defined states of helium and hydrogen. Helium and hydrogen have states where $c_P - c_{PO} \leq 0$, and therefore, $(\frac{\partial}{\partial T} \Delta h_T)_P > 0$. In this range, the values of Δh_T decrease as T is further decreased. The condition where $c_P = c_{PO}$ was mentioned in Sects. 2.6.2 and 2.6.3 and is associated with the highest pressure of the integral inversion state. However, such a peaking behavior in the integral inversion curve is observed only for helium, hydrogen and their isotopes. Furthermore, these gases have a locus (line) of states where $c_P = c_{PO}$ as displayed in Fig. 1.11 where, the uniqueness of Joule-Thomson cryocoolers operating with helium and hydrogen is highlighted.

Figure 4.16 shows $\Delta h_T(T)$ for helium at 1.5 MPa and hydrogen at 3 MPa, both expanding down to 0.01 MPa. The figure focuses on the two discussed issues:

- (a) For helium and hydrogen, the values of $\Delta h_T(T)$ may decrease at lower temperatures, and,
- (b) The decrease starts at the state of $c_P = c_{PO}$.

4.3.5.3 Expansion into the Two Phase Zone

Values of c_P^R are a function of the pre-expansion state (T, P). Therefore, the relation (4.30) is valid whether the expanded gas remains at a single phase or it undergoes a phase change through the isenthalpic expansion, falling into the two phase region. Put another way, the property Δh_T increases at lower T , both above and below T_{CD} . In Fig. 4.16, for hydrogen at 3 MPa expanding to 0.01 MPa, $T_{CD} = 49.5$ K. The T_{CD} temperature was already introduced in Sect. 3.2.1.6 and is further explored in Sect. 4.4.

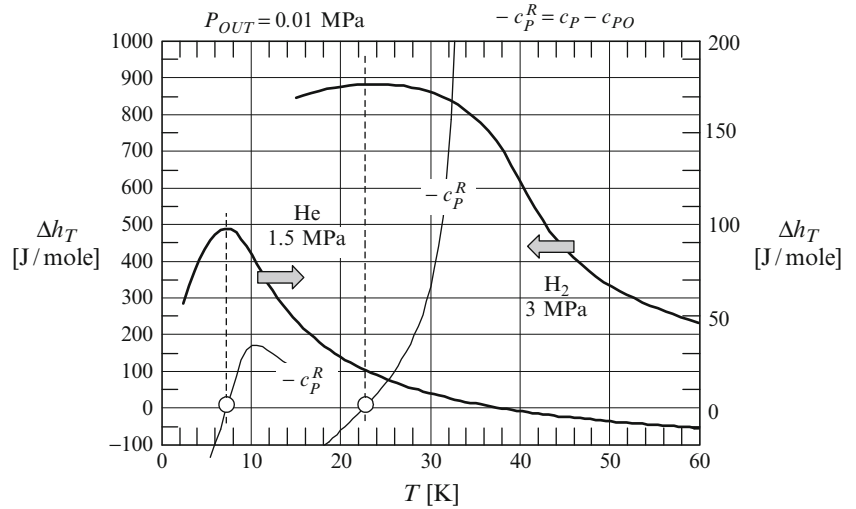
4.3.6 Temperature Dependence of Δh_T^{MAX}

The Δh_T^{MAX} values occur on the differential inversion curve (D.I.C.) and depend solely on the local temperature, T , along the D.I.C. The associated pressure is the inversion pressure that corresponds with T .

4.3.6.1 The Low Temperature Range, $1.2 < \Theta < 3$

The van der Waals equation of state remarkably combines a simple formulation with qualitative predictability of relevant real gas behavior. It was used by Maytal and Van Sciver¹

Fig. 4.16 Δh_T for helium and hydrogen (Note that Δh_T begins to decrease at temperatures below the condition where $c_P = c_{PO}$)



who suggested modifying the expression of Eq. 4.20 by inserting additional (or external) real gas information. At the reduced temperature range of $\Theta = 1.2$ to $\Theta = 3$, the reduced inversion pressures are about $\Pi = 10$. The vicinity of the peaks of Δh_T^{MAX} is quite flat. Therefore, even significant deviations from the associated inversion pressure still result in values close to Δh_T^{MAX} . Compressibility data of real gases in this range of states show that associated values of Z are approximately in the range of $Z = 1.0$ to 1.2 , as shown in Sects. 2.4.11, 2.4.12 and Figs. 2.14 and 2.16. For simplicity, using the unit value of Z results in the relationship,

$$\Delta h_T^{MAX} = \frac{135}{32} R \frac{T_C^2}{T} \quad (4.33)$$

which, enhanced³⁰ by using an average compressibility of $Z = 1.1$, leads to,

$$\Delta h_T^{MAX} = 3.83 R \frac{T_C^2}{T} - 0.1 R T \quad (4.34)$$

or in a reduced form

$$\frac{\Delta h_T^{MAX}}{R T_C} = \frac{3.83}{\Theta} - 0.1 \Theta \quad (4.35)$$

The focus on this temperature range, $1.2 < \Theta < 3$, is relevant for gases that are commonly utilized in Joule-Thomson cryocooling, for these have boiling temperatures below 200 K and inlet temperatures in the range of 250–350 K. A better match between Eq. 4.35 and data is obtained at lower temperatures, $\Theta = 1.2$ to 2 and deviates more as temperatures tend towards $\Theta = 3$. Xu et al.³¹ compared the temperature dependence predicted by the less accurate expression (4.33) with results obtained by a more advanced

Peng-Robinson equation of state and still a nice match is observed.

4.3.6.2 The Entire Inversion Curve Range, $T > T_C$

Another approach for describing the temperature dependence of Δh_T^{MAX} involves absorbing the functional temperature dependence predicted by the van der Waals equation of state along the entire inversion curve, up to Θ_{INV} , and then calibrating the resulting relationship. Combining the residual enthalpy of a van der Waals gas, as defined by Eq. 4.21, with that along the differential inversion curve, one obtains the locus of Δh_T^{MAX} for any gas in the (Θ, Φ) plane that satisfies,

$$\Theta = \frac{3}{4} (3 - \Phi)^2 \quad (4.36)$$

By Eq. 2.75 it leads to the expression of Δh_T^{MAX} values along the inversion curve,³⁰

$$\left(\frac{\Delta h_T^{MAX}}{R \cdot T_C} \right)^{1/2} = \left(\frac{27}{4} \right)^{1/2} - \Theta^{1/2} \quad (4.37)$$

Since the reduced inversion temperature of the van der Waals gas is, $\Theta_{INV} = 27/4$, the above expression may be modified for use with any other gas, in the form,

$$\left(\frac{\Delta h_T^{MAX}}{R \cdot T_C} \right)^{1/2} = \Theta_{INV}^{1/2} - \Theta^{1/2} \quad (4.38)$$

so that for Θ_{INV} the individual inversion temperatures specific to each gas should be inserted, which in fact are closer

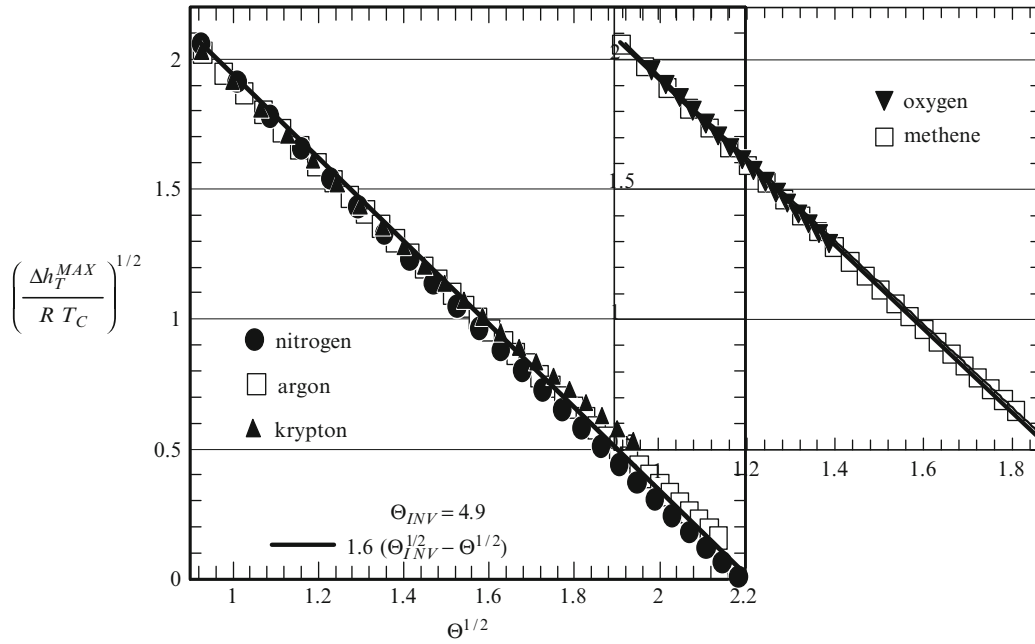


Fig. 4.17 The maximum cooling capacity (as function of P) at any super-critical T as function of T of low acentricity gases

to 5 than to 6.75. As shown in Fig. 4.17, the linear relation between $(\Delta h_T^{\text{MAX}})^{1/2}$ and $\Theta^{1/2}$ is indeed valid for real gases, but a correction of the slope is needed, so that,

$$\left(\frac{\Delta h_T^{\text{MAX}}}{R T_C}\right)^{1/2} = 1.6 \left(\Theta_{\text{INV}}^{1/2} - \Theta^{1/2}\right) \quad (4.39)$$

For the low acentricity gases displayed: $\Theta_{\text{INV}} = 4.9$.

4.3.6.3 The Sub Critical Temperature Range, $T \leq T_C$

It should be mentioned that the lowest inversion temperature predicted for the van der Waals gas is $\Theta_{\text{INV}}^{\text{MIN}} = 3/4$, which may be interpreted as Θ_D (as introduced in Sect. 2.4.4). This may be identified as the limit of validity for the exact relation (4.39) and the modified or corrected relation of (4.40). The values of Δh_T^{MAX} below T_{CD} are achieved along the *extended inversion curve*, on the saturated liquid line as discussed in Sect. 2.4.4 and displayed in Figs. 2.5 and 2.9. All isotherms below the critical temperature cross the two-phase dome and the pressure drop is associated with evaporation.

One may question whether the evaporation associated with expansion alters the temperature dependence of Δh_T^{MAX} . Figures 4.17 and 4.18 focus on the sub-critical range of temperature. One may observe that the same prediction of Eq. 4.39 maintains a reasonable fit for both the subcritical isotherms and those in the supercritical range. Indeed, as evidenced in the P - h plane in

Fig. 2.10, the curvature of the isotherms, and the maximum values of Δh_T , gradually increase as the temperature decreases through the supercritical values from T_1 to T_3 and subsequently display a further gradual increase as the temperature is reduced to sub-critical values, T_8 , T_D , T_9 and T_{10} .

4.3.6.4 The $\Delta h_T^{\text{MAX}}(T_C)$

The $\Delta h_T^{\text{MAX}}(T_C)$ for low acentricity gases, according to Eq. 4.39 is

$$\Delta h_T^{\text{MAX}}(T_C) = 3.77 R T_C \quad (4.40)$$

which is naturally larger than $\Delta h_T(T_C, P_C)$ calculated by Eq. 4.29.

4.3.7 Mapping the Integral Effect, Δh_T

Although one may be aware of the loci of the maximum values of Δh_T , occasionally practical considerations dictate operation at conditions slightly deviating from the optimal states. In such a case, a sensitivity analysis of Δh_T as a function of deviations from the differential inversion curve is as interesting as it is of practical significance. For example, what degradation can be expected from operating at half of the optimal pressure, or what penalty would be realized from over-pressurizing by 25% of the same value? The answer is arranged in two forms of mapping.

Fig. 4.18 The maximum cooling capacity (as function of P) at any sub-critical T as function of T of low acentricity gases

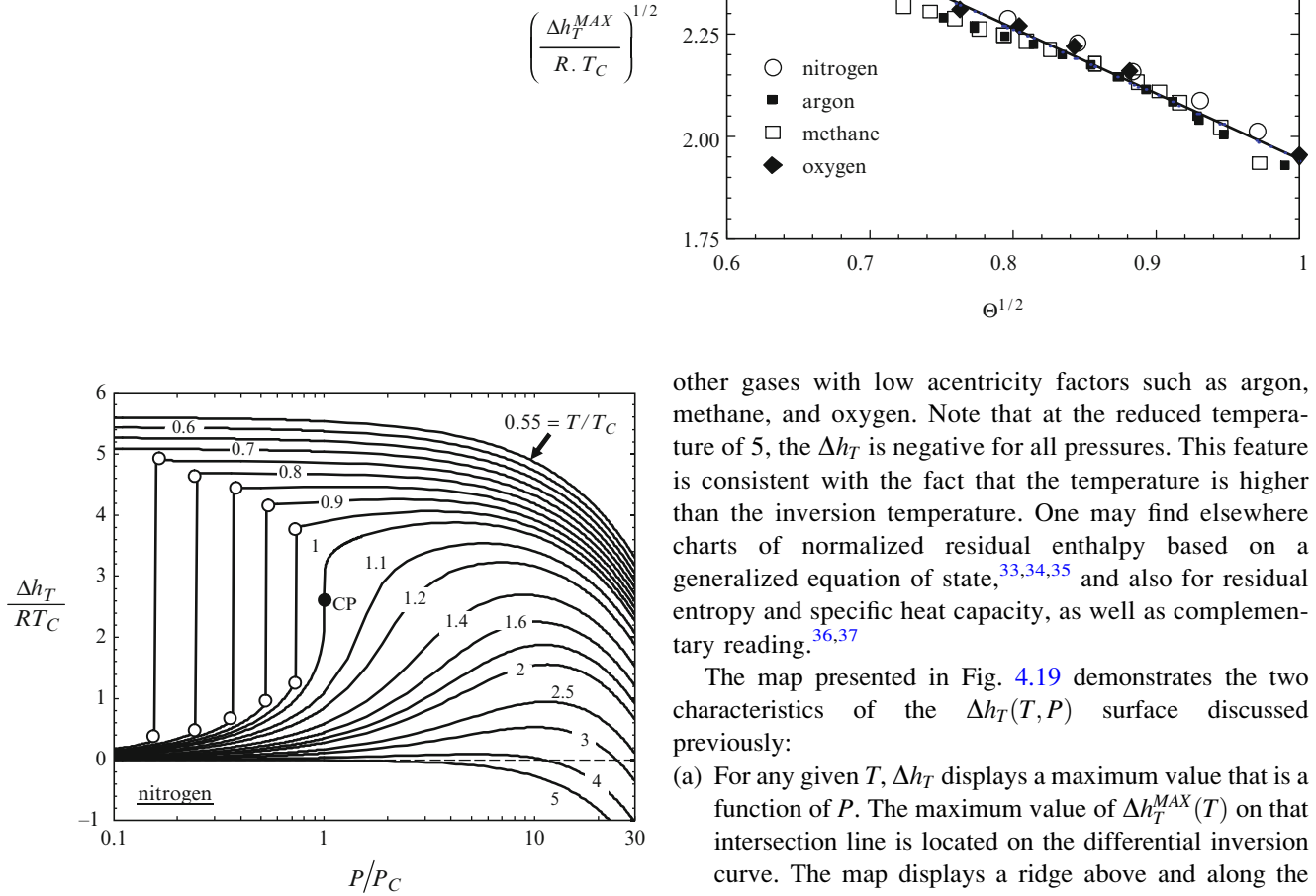


Fig. 4.19 The normalized Δh_T of nitrogen as function of reduced pressure and temperature. It is applicable for gases of low acentricity factors as argon, methane, oxygen. The circles note states on the saturation curve

4.3.7.1 The Absolute Mapping

Motivated by these considerations, a surface can be drawn representing the function Δh_T over the plane of state coordinates, (T, P) . Such a map, may be prepared with the use of any numerical code for thermo-physical properties of pure fluids as for example, those of reference.³²

A plane of constant temperature bisects the surface of $\Delta h_T(T, P)$ along a curved line. Each line describes Δh_T as function of pressure at the given temperature. The series of these lines prepared for nitrogen, in terms of reduced variables, is displayed in Fig. 4.19. The temperature axis is perpendicular to the plane of the figure. The lines are continuous for $T \geq T_C$ and discontinuous for $T < T_C$. The states of discontinuity rest on the saturation line and are marked by circles. The same graph may be applicable for

other gases with low acentricity factors such as argon, methane, and oxygen. Note that at the reduced temperature of 5, the Δh_T is negative for all pressures. This feature is consistent with the fact that the temperature is higher than the inversion temperature. One may find elsewhere charts of normalized residual enthalpy based on a generalized equation of state,^{33,34,35} and also for residual entropy and specific heat capacity, as well as complementary reading.^{36,37}

The map presented in Fig. 4.19 demonstrates the two characteristics of the $\Delta h_T(T, P)$ surface discussed previously:

- For any given T , Δh_T displays a maximum value that is a function of P . The maximum value of $\Delta h_T^{\text{MAX}}(T)$ on that intersection line is located on the differential inversion curve. The map displays a ridge above and along the differential inversion curve, and,
- For constant P , Δh_T becomes larger at lower T .

4.3.7.2 The Relative Mapping

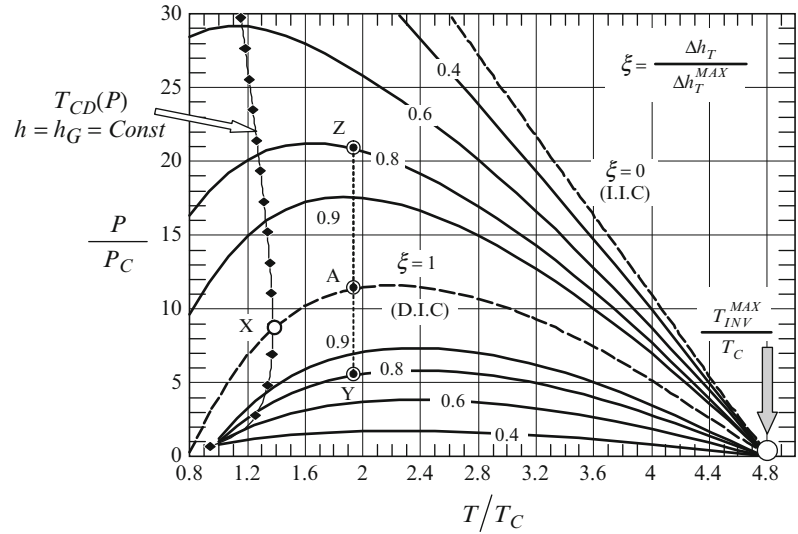
Any state point on the (T, P) plane possessing $\Delta h_T(T, P)$ manifests itself with only a fraction, ξ , of the maximum cooling effect, $\Delta h_T^{\text{MAX}}(T)$, obtainable at the same temperature, so that,

$$\xi = \frac{\Delta h_T}{\Delta h_T^{\text{MAX}}} = \frac{\Delta h_T(T, P)}{\Delta h_T^{\text{MAX}}(T)} = \xi(T, P) \quad (4.41)$$

A type of ‘topographical’ map displaying lines of constant ξ values, is shown in Fig. 4.20 for low acentricity gases like nitrogen, argon, oxygen, or methane. As the separation between the lines is broader the surface of $\Delta h_T(T, P)$ is more flat and vice versa.

The dimensionless form of Fig. 4.20, presented in terms of reduced coordinates, gives it a universal validity in keeping with the principle of corresponding states. The

Fig. 4.20 Mapping by trajectories of constant fractions of $\Delta h_T/\Delta h_T^{MAX}$ for low acentricity gases as nitrogen, oxygen, argon, methane. The highest value, $\xi = 1$, belongs to the differential inversion curve and the zero value represents the integral inversion curve. The state X on the differential inversion curve is of T_{CD}^{MAX}



traditional differential inversion curve is defined by $\xi = 1$. Other loci of specific ξ fractions such as 40%, 60%, 80%, and 90% are displayed on both sides of the ridge. The line of small diamonds in the same figure is a locus of constant enthalpy representing at any pressure the corresponding values of T_{CD} , as will be discussed subsequently in Sect. 4.4.

With the above comments in mind, we are now ready to draw the final picture and conclusions. The surface of Δh_T is quite flat in the vicinity of its maximum, $\xi = 1$. Thus the optimum value of Δh_T is relatively insensitive to deviations in pressure, a feature of significant practical value. Variations from the optimal design pressure have only a minor effect on the performance of a Joule-Thomson cryocooler. An illustrative example can be made in the vicinity of the reduced temperature $\Theta = 2$ on the differential inversion curve (state A). In this case, halving (state Y) or doubling (state Z) the pressure with respect to its optimum value would reduce the associated value of Δh_T by only about 25%. However, the dependence of Δh_T on pressure becomes more sensitive as the temperature approaches Θ_{INV} .

The trajectory of $\xi = 0$ associated with vanishing Δh_T is the Integral Inversion Curve and deserves a separate treatment. A subsequent section addresses this topic alone.

4.3.8 The Space of Coolants: Normal Boiling Point Dependence of $\Delta h_T^{MAX}(T)$

One may ask whether in Joule-Thomson systems it is possible to identify a general trend or correlation between the two most important characteristics of coolants: the boiling point, T_{BOIL} , which represents the temperature of operation, and the cooling potential Δh_T . Should one expect a refrigerant with a

lower boiling temperature (T_{BOIL}) to provide a higher or lower cooling content, at a given temperature and pressure?

For the sake of general validity, the correlation should be based on one of the equations of state. These incorporate the critical temperature, T_C , rather than T_{BOIL} . Hence, we involve an empirical approximation between these two temperatures, that is $T_{BOIL}/T_C \approx 0.6$ (Sect. 4.2.1).

Following the approximate expressions for $1.2 < \Theta < 3$ of Eq. 4.33, we obtain,

$$\Delta h_T^{MAX} = 11.72 R \frac{T_{BOIL}^2}{T} \quad (4.42)$$

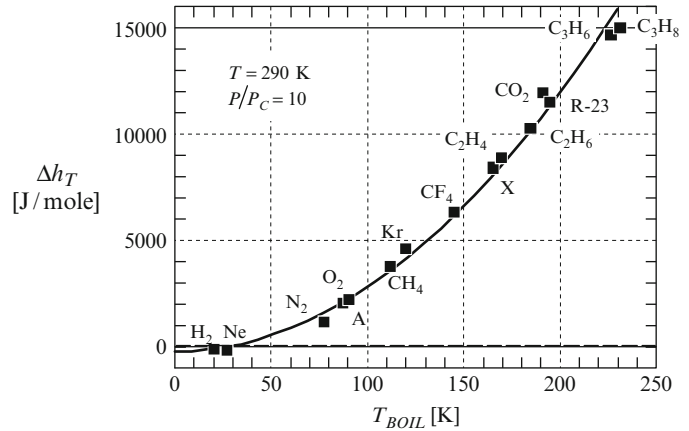
The enhanced expression³⁰ obtained by using an average compressibility of $Z = 1.1$, leads to,

$$\Delta h_T^{MAX} = R T \left(10.65 \frac{T_{BOIL}^2}{T^2} - 0.1 \right) \quad (4.43)$$

The validity of this result at 290 K and $\Pi = 10$ for various gases, is demonstrated in Fig. 4.16. The data displayed in Fig. 4.16 is partly based on measurements³⁸ and in some cases from the more advanced Peng-Robinson equation of state.³⁹ Nevertheless, the reasonable match provides confidence that the van der Waals modified but still simple relation indeed is capable of reflecting real behavior.

Referring back to the central theme of the section, a clear trend emerges from Fig. 4.21; a coolant with a lower boiling point also has less potential for cooling. For a given ambient temperature, T , the Δh_T^{MAX} of coolants increases monotonically with T_{BOIL} . One may observe this trend also with the aid of the Δh_T values mapped in Fig. 4.19. A given T corresponds to a lower reduced temperature, T/T_C , for higher values of T_C and T_{BOIL} . From Fig. 4.19, larger values

Fig. 4.21 The dependence of the specific cooling capacity, Δh_T , on the normal boiling points of gases; the line represents the prediction $\Delta h_T^{MAX} = R T (10.65 T_{BOIL}^2/T^2 - 0.1)$ at the reduced conditions of $\Pi = 10$ and $1.2 < \Theta < 3$ for various gases



of $\Delta h_T^{MAX}/(R T_C)$ and Δh_T^{MAX} correspond with lower values of T/T_C .

To the extent that one is able to compromise the low temperature of operation, one may gain a higher specific cooling capacity. The limiting dependence of Δh_T^{MAX} on the boiling point is restricted solely to pure gases. A given mixture with a particular value of Δh_T^{MAX} may boil over a range of temperatures depending on the conditions of operation of the Linde-Hampson device. There is no simple correlation between T_{BOIL} and Δh_T^{MAX} for a mixture. On the other hand, the advantage that can be gained by using gas mixtures refrigerants for Joule-Thomson cryocooling is well known, and the entirety of Chap. 8 is dedicated to this theme.

4.3.9 Temperature Dependence of Δh_T at a Constant Specific Density Process

The temperature change of a charged pressure vessel resulting from a change in the ambient or surrounding temperature will be accompanied by a pressure change. An increased temperature alone will result in a decreased specific cooling capacity, Δh_T . However, since the process also occurs at a constant volume, the resulting elevated pressure will oppose the decrease of Δh_T . The final consequence is that a higher temperature will degrade the cooling capacity.

For the van der Waals gas following Eq. 4.21, we have the non-dimensional expression,

$$\frac{\Delta h_T}{R T_C} = \frac{9}{4} \Phi - \frac{\Phi}{3 - \Phi} \Theta \quad (4.44)$$

where Φ and Θ are the reduced volume and temperature, respectively. Since v stays constant Eq. 4.44 predicts that the change in Δh_T is proportional with the change of T at any pressure, so that,

$$\frac{\partial}{\partial \Theta} \left[\left(\frac{\Delta h_T}{R T_C} \right) \right]_{\Phi=Const} = - \frac{\Phi}{3 - \Phi} = Const \quad (4.45)$$

or

$$\delta(\Delta h_T)_{\rho=Const} = Const \cdot \delta T \quad (4.46)$$

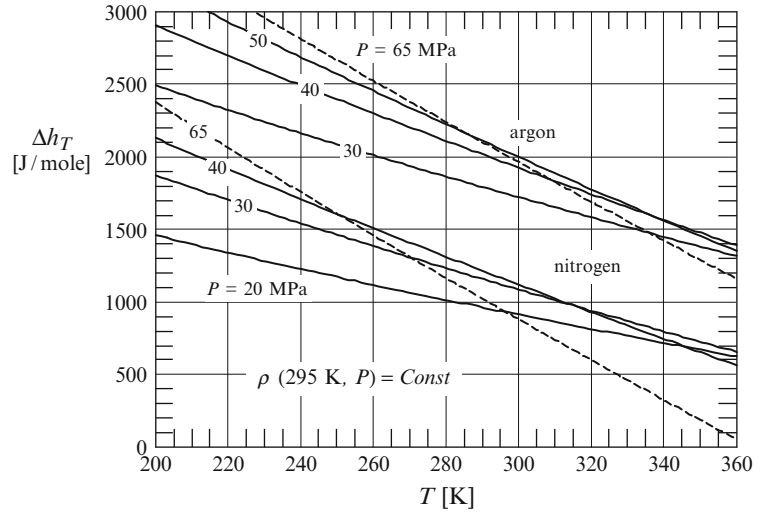
Practically, the linear relationship between Δh_T and T holds quite well, although not at the same slope of $R \Phi/(3 - \Phi)$. For example, the Δh_T of 1,355 J/mol for a bottle of nitrogen at 295 K at the pressure of 20 MPa decreases by 23 to 22 J/mol for each elevation of 5 K (up to 350 K). Similarly, for nitrogen at 295 K and 30 MPa, the Δh_T of 1,122 J/mol increases by about 37–38 J/mol for each 5 K decrease of temperature (down to 260 K).

Figure 4.22 displays the decrease of Δh_T as the temperature increases while preserving the density. The functions are very close to straight lines. The slope of the lines is larger (more negative) as the preserved density is larger. This is consistent with the prediction of the van der Waals equation of state. The density of each line is determined by the charging pressure (noted on each curve) at 295 K.

4.4 Cooldown Temperature, T_{CD}

As the recuperative heat exchange process lowers the temperature of the incoming high-pressure stream down to T_{CD} at the nozzle inlet, the expanded outlet stream reaches the boiling point. The cooldown temperature, T_{CD} , may be used to notify the termination of the cooldown process. Whether the value of T_{CD} for a gas is above or below room temperature (295 K) is of practical interest. For the case when T_{CD} is above room temperature, a pressure drop (or blow-down) even without recuperation allows one to reach the normal boiling point and obtain liquid cryogen.

Fig. 4.22 Trajectories of constant density; the value of the constant density corresponds to 295 K and the (charging) pressure, P , noted on each curve, $\rho(295 \text{ K}, P) = \text{Const}$



4.4.1 Definition

Section 3.2.1.6 (Figs. 3.3, 3.4, and 3.5) introduced the significance of T_{CD} ; it is the highest gas temperature at each pressure, P , from which an isenthalpic expansion still produces a single phase flow stream at the normal boiling point, T_{BOIL} . Fundamentally, T_{CD} is defined by the condition,

$$h(T_{CD}, P) = h(T_{BOIL}, 0.1 \text{ MPa}, x = 1) \equiv h_G \quad (4.47)$$

where x is the quality of the fluid. Therefore,

$$T_{CD} = T_{CD}(h_G, P) \quad (4.48)$$

Note that T_{CD} is dependent on the pressure of operation and the outlet pressure. The associated cooldown temperature gap is

$$\Delta T_{CD} = \begin{cases} T_{AMB} - T_{CD} & @T_{AMB} > T_{CD} \\ 0 & @T_{AMB} < T_{CD} \end{cases} \quad (4.49)$$

In the case when T_{CD} is higher than T_{AMB} there is no temperature gap to achieve liquefaction. Liquefaction is achieved immediately, even without any heat exchanger, simply through the process of a single blow down.

4.4.2 Pressure Dependence of T_{CD}

Figure 4.23 displays the pressure dependent values, $T_{CD}(P)$, for nitrogen, argon and methane. Values for hydrogen and neon are shown in Fig. 4.24, for helium in Fig. 4.25. In general, the curves display a maximum, and any value of T_{CD} may be obtained at two different pressures.

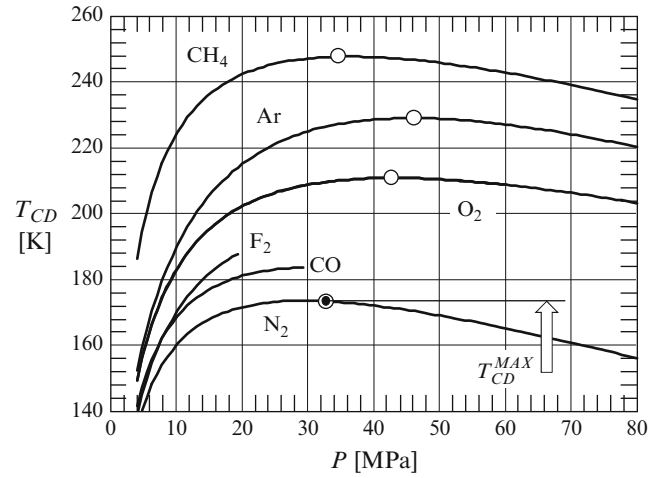


Fig. 4.23 Pressure dependence of T_{CD} for nitrogen, argon, methane, carbon monoxide, and fluorine. Peak values occur on the differential inversion curve

By definition through Eq. 4.47, the trajectory of $T_{CD}(P)$ in the $T - P$ plane is an isenthalp of $h = h_G = \text{Const}$. However, the maximum value of any isenthalp occurs on the differential inversion curve and this also is where the highest value of $T_{CD}(P)$ occurs; see point X in Fig. 4.20. This highest value is unique for each gas and will be noted by T_{CD}^{MAX} . The value of T_{CD}^{MAX} and its associated inversion pressure, P_{INV} , have to simultaneously satisfy the following two equations,

$$h(T_{CD}^{MAX}, P_{INV}) = h_G \quad (4.50)$$

$$\mu(T_{CD}^{MAX}, P_{INV}) = 0 \quad (4.51)$$

However, in the next section we will present an alternate strategy for determining T_{CD}^{MAX} .

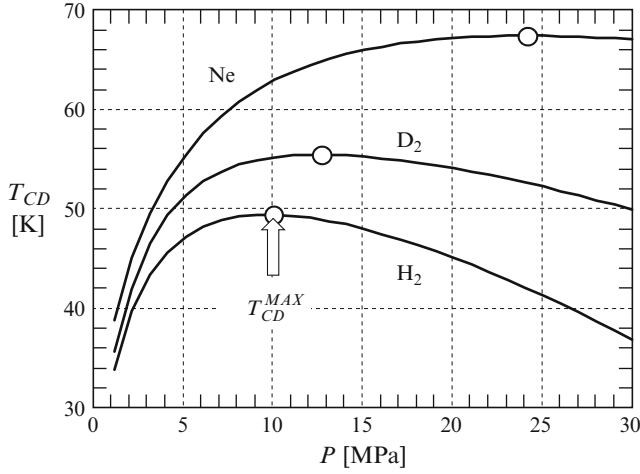


Fig. 4.24 Pressure dependence of T_{CD} for neon and hydrogen; peak values occur on the differential inversion curve

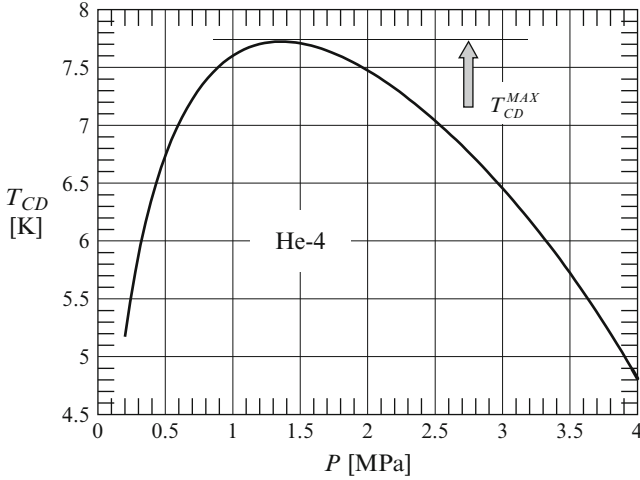


Fig. 4.25 Pressure dependence of T_{CD} for He-4; the peak value occurs on the differential inversion curve

4.4.3 Evaluation of T_{CD}^{MAX}

4.4.3.1 Formulation

Figure 4.26 shows the isotherm T_{CD}^{MAX} in the $P-h$ plane. Point X in this figure and in Fig. 4.20. represents an inversion state with the peak value, Δh_T^{MAX} , corresponding to T_{CD}^{MAX} , and designated as $\Delta h_T^{MAX}(T_{CD}^{MAX})$. The isenthalpic expansion from state X terminates on state Y, located on the saturated vapor border of the two phase dome, (T_{BOIL} , 0.1 MPa, $x = 10$). The associated temperature drop is

$$\Delta T_h(XY) = T_{BOIL} - T_{CD}^{MAX} < 0 \quad (4.52)$$

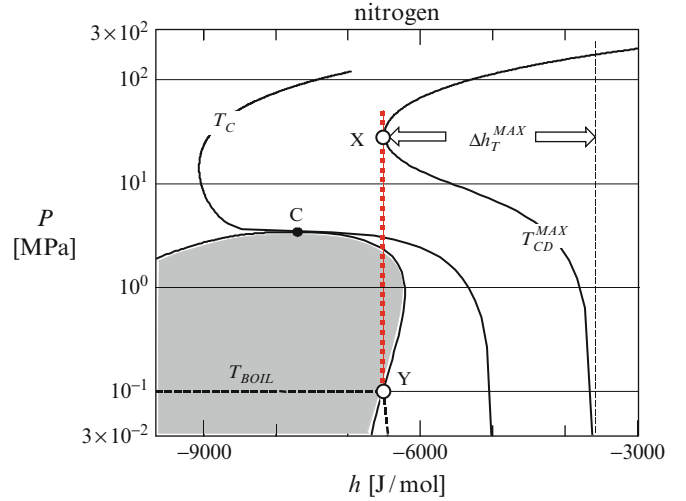


Fig. 4.26 The isotherm T_{CD}^{MAX} in the plane of $P-h$ together with the critical temperature and the normal boiling point isotherms

Later, in Sect. 4.5.2 the relationship between Δh_T and the associated value of ΔT_h is derived for the case where no phase change is involved, so that,

$$\Delta h_T^{MAX}(T_{CD}^{MAX}) = -c_{PO} \cdot \Delta T_h(XY) \quad (4.53)$$

As previously derived by Eq. 4.35, the function Δh_T^{MAX} of an inversion state in the lower range of $1.2 < \Theta < 3$, may be expressed for the present case as,

$$\frac{\Delta h_T^{MAX}}{R \cdot T_C} = 3.83 \frac{T_C}{T_{CD}^{MAX}} - 0.1 \frac{T_{CD}^{MAX}}{T_C} \quad (4.54)$$

The combination of these relations finally leads to the expression,⁴⁰

$$T_{CD}^{MAX} = \frac{\frac{c_{PO}}{R} T_{BOIL} + \left[\left(\frac{c_{PO}}{R} T_{BOIL} \right)^2 + 15.32 T_C^2 \left(\frac{c_{PO}}{R} + 0.1 \right) \right]^{1/2}}{2 \left(\frac{c_{PO}}{R} + 0.1 \right)} \quad (4.55)$$

The convenient capability of this analysis to predict the real T_{CD}^{MAX} values for the various gases through their heat capacities, critical temperatures, and boiling points is examined and demonstrated in Table 4.5.

Since,

$$\frac{c_{PO}}{R} = \frac{\kappa}{\kappa - 1} \quad (4.56)$$

one may derive a very rough but also quite simple expression,

Table 4.5 Predicted cooldown temperatures, T_{CD}^{MAX} , (by Eq. 4.55), in comparison to real values, T_{CD}^{MAX} . Krypton is the gas of lowest T_{BOIL} among all gases of T_{CD}^{MAX} above room temperature

	T_{BOIL} [K]	T_C [K]	c_{PO} [J/(mol K)]	T_{CD}^{MAX} [K]	T_{CD}^{MAX} [K]	$1 - \frac{T_{CD}^{MAX}}{T_{CD}^{MAX}}$
He	4.22	5.2	20.93	7.72	8.64	12.0%
H ₂	20.4	33.2	28.86	49.4	45.7	7.5%
D ₂	23.6	38.3	29.20	55.88	52.55	5.9%
Ne	27.1	44.4	20.76	68.0	68.5	0.7%
N ₂	77.3	126.3	29.00	173.5	173.4	0.2%
CO	81.65	132.7	29.14	183.8	182.11	0.9%
Ar	87.3	150.8	20.87	228.9	229.75	0.4%
O ₂	90.2	154.7	29.43	210.6	208.5	1.0%
CH ₄	111.67	190.7	35.35	247.7	241.6	2.5%
Kr	119.9	210.0	21.0	317.6	317.55	0.0%
NF ₃	144.1	234	53.55	287.6	263.55	8.4%
CF ₄	145.1	227.7	61.15	282.0	252.7	10.4%
Xe	165.1	298.8	21.01	440.9	438.22	0.6%
C ₂ H ₄	169.45	282.5	43.14	353.3	337.52	4.5%
C ₂ H ₆	184.5	305.3	52.83	359.0	342.93	4.5%
C ₃ H ₆	226.2	385.5	62.28	392.0	394.2	0.6%
C ₃ H ₈	231.2	370.6	74.79	391.1	380.5	2.7%
NH ₃	239.9	405.6	36.73	535.0	508.7	5.2%
i-C ₄ H ₁₀	261.6	407.0	98.71	401.8	394.0	1.9%
n-C ₄ H ₁₀	272.6	425.0	100.8	388.3	408.8	5.3%
C ₂ Cl ₂ F ₄	276.7	419.0	114.31	405.0	397.0	2.0%

$$T_{CD}^{MAX} \approx \frac{1}{2}T_{BOIL} + 2\sqrt{\frac{\kappa-1}{\kappa}}T_C \quad (4.57)$$

4.4.3.2 The Space of Gases

Using the approximation that $T_{BOIL} \approx 0.6 T_C$, together with Eqs. 4.55 and 4.56 leads to the simplified expression,

$$\frac{T_{CD}^{MAX}}{T_{BOIL}} = \frac{1}{2} \left(1 + 42.6 \frac{\kappa-1}{\kappa} \right)^{1/2} + \frac{1}{2} \quad (4.58)$$

Although this formulation is less accurate, it provides a better understanding of how T_{CD}^{MAX} changes for the various groups of gases. For $\kappa = 1.2, 1.3, 1.4$ and 1.667 we obtain correspondingly, $T_{CD}^{MAX}/T_{BOIL} = 1.92, 2.14, 2.31$ and 2.62 . The noble gases exhibit relatively larger T_{CD} values. That behavior is demonstrated in Fig. 4.27. For example, let us compare diatomic nitrogen and monatomic argon. The corresponding values of T_{CD}^{MAX} values are 174 and 229 K. In this case although there is only a 10 K difference in the boiling point, there is a 55 K difference in the cooldown temperatures. Furthermore, oxygen exhibits a higher boiling point (90.2 K) than argon (87.3 K), even though T_{CD}^{MAX} of argon is higher. A similar comparison exists between CF₄

and Kr. The lower boiling point belongs to Kr but CF₄ has a higher T_{CD}^{MAX} .

In Table 4.5, the gases are arranged by their respective values of T_{BOIL} . The lowest boiling point that may be obtained by a single Joule-Thomson expansion from room temperature is that of krypton, 119.9 K. The pure gases with lower boiling points (neon, hydrogen and helium) warm up while expanding at room temperature.

4.4.4 The Smallest Cooldown Range, ΔT_{CD}^{MIN}

The smallest ΔT_{CD} range is associated with the largest T_{CD} . Following Eq. 4.49,

$$\Delta T_{CD}^{MIN} = T_{AMB} - T_{CD}^{MAX} \quad (4.59)$$

For the various groups of gases from Eq. 4.58 we have,

$$\begin{aligned} \frac{\Delta T_{CD}^{MIN}}{T_{BOIL}} &= \frac{T_{AMB}}{T_{BOIL}} - \frac{T_{CD}^{MAX}}{T_{BOIL}} \\ &= \frac{T_{AMB}}{T_{BOIL}} - \frac{1}{2} \left(1 + 42.6 \frac{\kappa-1}{\kappa} \right)^{1/2} - \frac{1}{2} \end{aligned} \quad (4.60)$$

A gas with a larger value of κ will have a smaller value of T_{CD}^{MAX} for a given value of T_{BOIL} as observed in Fig. 4.27. Smaller values of ΔT_{CD}^{MIN} are desirable for faster cooldown. If $\Delta T_{CD}^{MIN} = 0$, liquefaction occurs immediately, simply by a single expansion even without any heat exchanger.

Note that T_{CD}^{MAX} occurs along the differential inversion curve at a pressure of about $9 P_C$. That is also the pressure where ΔT_{CD}^{MIN} occurs.

4.5 The Integral Isenthalpic Joule-Thomson Effect, ΔT_h

There are two parameters which distinguish the behavior of ΔT_h from that of the previously discussed Δh_T : the cooldown temperature, $T_{CD}(P)$, and the molecular structure of gases as reflected by c_{PO} . This issue will be explored below.

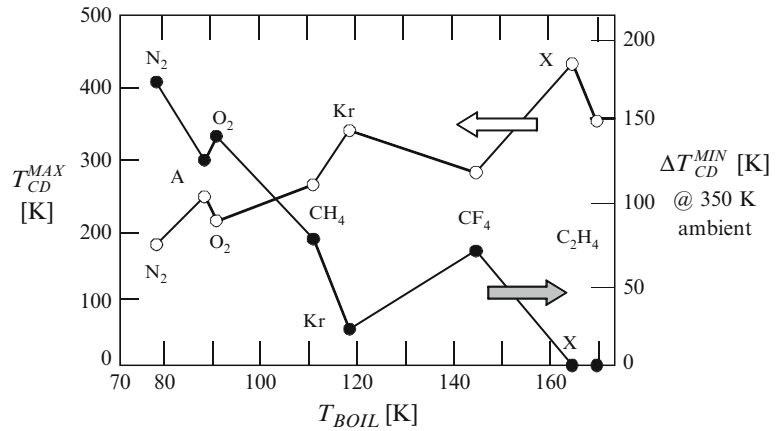
4.5.1 Introduction

4.5.1.1 Definition

While a gas expands isenthalpically, from a pressure P down to ambient, it undergoes a temperature change

$$\begin{aligned} \Delta T_h &= T_{OUT}(0.1 \text{ MPa}) - T_{IN}(P) = \\ &= T(0.1 \text{ MPa}, h) - T(P, h) = \Delta T_h(P, T) \end{aligned} \quad (4.61)$$

Fig. 4.27 The maximum cool down temperature and cooldown range as function of normal boiling point



that is defined as the integral isenthalpic (or adiabatic) Joule-Thomson effect. This effect was introduced in Sect. 2.2.2. in terms of the Joule-Thomson coefficient through Eq. 2.9. The values of T_{IN} and T_{OUT} are the temperatures before and after the isenthalpic expansion. Further discussion will show that ΔT_h is not equivalent to Δh_T .

4.5.1.2 The Two Domains of ΔT_h

The two domains of ΔT_h , each exhibit a different behavior and deserve separate attention. In the (P, T) plane of thermodynamic states, the line of $T_{CD}(P)$ is the common border separating the two domains, as shown in Fig. 4.20. Since the line of $T_{CD}(P)$ is a locus of constant enthalpy, h_G , it is maximized on the differential inversion curve and discussed later in Sect. 4.5.

The Domain of $T > T_{CD}(P)$

The domain of $T > T_{CD}(P)$, to the right of the $T_{CD}(P)$ line in Fig. 4.20 includes the states that remain in a single phase after isenthalpic expansion down to 0.1 MPa. In this domain the value of T_{OUT} depends on (T_{IN}, P) so that ΔT_h is state dependent, by (T, P) or (P, h) .

The Domain of $T < T_{CD}(P)$

The domain of $T < T_{CD}(P)$, to the left of the $T_{CD}(P)$ line in the same figure includes the states that end inside the vapor dome after isenthalpic expansion down to 0.1 MPa and are therefore two-phase. These expansions are associated with a phase transition. The value of T_{OUT} is constant, equal to T_{BOIL} , and does not depend on T_{IN} .

4.5.1.3 The Driving Potential of the Cooling Process, ΔT_h

The driving potential of the cooling process, ΔT_h , serves as the fundamental mechanism causing a temperature drop, as discussed in the unified view of cryocoolers in Sect. 1.1. Such a decrease in temperature occurs in the nozzle of a Joule-Thomson cryocooler at the very first moment after opening

Table 4.6 Values of ΔT_h , at 300 K and 30 MPa of various gases

Gas	ΔT_h [K]	T_{BOIL} [K]
He	+18.4	4.2
H ₂	+11.9	20.4
D ₂	+11.2	23.6
Ne	+11.1	27.1
N ₂	-36.9	77.3
CO	-42.7	81.7
Ar	-81.4	87.3
O ₂	-60.9	90.2
CH ₄	-99.9	111.7
Kr	-180.1	119.9
CF ₄	-107.2	145.1

the valve that connects the cryocooler to a pressure source. The entire cryocooler is at that moment still at ambient temperature. The ΔT_h initiates a positive feedback process, amplifying itself, until finally liquefaction is achieved. It is the fundamental mechanism¹ of any Linde-Hampson machine that enables it to approach and achieve liquefaction of the working gas. As such, it is directly responsible for the rate at which the process of liquefaction proceeds and for the cooldown period of the cryocooler.^{41,42,43}

4.5.1.4 Examples of Various Gases and States

Table 4.6 presents a list of refrigerants, in order of their normal boiling point along with their associated values of, ΔT_h , for fixed initial conditions of temperature and pressure. A gas with a more negative value of ΔT_h exhibits a higher tendency for self-sustained liquefaction. One should notice that the trend of ΔT_h values is not completely consistent with that of the normal boiling points. The positive values of ΔT_h indicate that heating occurs under the isenthalpic expansion.

Furthermore, one may quantitatively relate different states of the same gas or different gases, on the basis of the magnitude of ΔT_h . Such a comparison is accomplished for nitrogen and argon as presented in Table 4.7. For example, nitrogen has to be pressurized to 40 MPa and cooled to

265 K in order to obtain a similar value of ΔT_h as argon provides at 300 K when pressurized only to 15 MPa. Or, at 40 MPa argon has a larger (absolute) value of ΔT_h at about 50 K above room temperature than nitrogen does at the same pressure at about 50 K below room temperature.

4.5.1.5 Chronological Note

Van der Waals⁴⁴ was the first to recognize (early as 1898) the significance of ΔT_h in regard to the improved performance of the recently invented Linde-Hampson apparatus. For any given temperature, T_1 , he sought the corresponding pressure, P_1 , of “...such a value that the cooling has its maximum value; or in other words, there is a most advantageous value for P_1 in Linde’s apparatus”. Van der Waals reference to “the cooling” captured the same idea as the isenthalpic temperature drop, ΔT_h .

Van der Waals put to work his proposed equation of state, subsequently named after him, to find for all values of T_1 , the corresponding values of P_1 , which comprised the loci of states (T_1, P_1) characterized by a maximum value of ΔT_h . He did

Table 4.7 Values of ΔT_h , for nitrogen and argon at various states of operation

ΔT_h		
$T = 300 \text{ K}$		$P = 40 \text{ MPa}$
$P \text{ [MPa]}$	Nitrogen	Argon
5	−9.8	−17.5
10	−18.5	−34.3
15	−25.5	−49.6
20	−30.8	−62.7
30	−36.9	−81.4
40	−38.6	−91.5
50	−36.8	−95.6
60	−33.3	−95.7
		$T \text{ [K]}$
		Nitrogen
		Argon
		330
		320
		310
		300
		290
		280
		270
		260

indeed obtain a solution; however, it is quite surprising that he did not realize, or at least did not mention, its coincidence with the differential inversion curve. A couple of decades later, Meissner,⁴⁵ applied the equation of state proposed by Kamerlingh Onnes for hydrogen, also with the goal of determining the states that would enable a Linde-Hampson machine to produce a maximum liquefaction yield.

4.5.2 ΔT_h in the Domain of $T \geq T_{CD}(P)$

4.5.2.1 The Relationship Between ΔT_h and Δh_T

Since both ΔT_h and Δh_T comprise expressions of the same Joule-Thomson effect, it is natural to inquire about the relationship between the two terms. Following Koeppel,^{46,47} let us observe the trends of various isobars in the (h, T) plane as illustrated in Fig. 4.28.

The slopes of the isobars, $(\partial h / \partial T)_P$, express the “local” specific heat capacities, c_P . The zero pressure (or as low as 0.1 MPa) isobar is quite linear, almost of a constant slope, a feature that is indicative of the heat capacity of an ideal gas,

$$\tan \alpha = c_{PO} = (\partial h / \partial T)_P \quad @ \rightarrow 0 \quad (4.62)$$

Proceeding from state A to B represents the isothermal Joule-Thomson expansion process, so that $AB = \Delta h_T$. Similarly, $AC = -\Delta T_h$, hence the desired relation becomes,

$$\Delta T_h = -\Delta h_T / c_{PO} \quad (4.63)$$

The minus sign on the right side indicates that in the expansion from high to low pressure, ΔT_h and Δh_T have opposite signs. It may be stated alternatively, that a positive cooling content, Δh_T , is associated with a negative

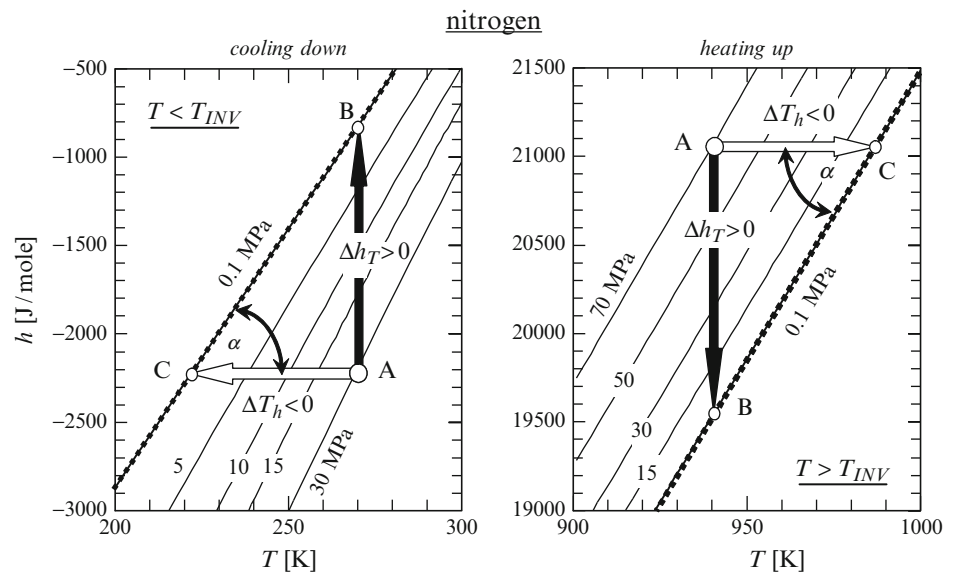


Fig. 4.28 The ΔT_h and Δh_T are interrelated in the (h, T) plane through c_{PO}

temperature change, ΔT_h , and vice versa. Gustaffson⁴⁸ proved this relation while applying strictly a formal mathematical approach.

4.5.2.2 Remarks

1. Obviously, this relation holds as well for the warming that occurs as a result of isenthalpic expansion, that is, Δh_T is negative for the same pressure drop that produces a positive value of ΔT_h , in the case where $T > T_{INV}$.
2. The central condition for which Eq. 4.63 holds, is that the expanded gaseous (single phase) states exhibit a constant value of c_P (the straight low pressure isobar in Fig. 4.28). Even if the initial states are in the two-phase regime, but the central condition is fulfilled, Eq. 4.63 is still legitimate.

4.5.2.3 Demonstrating the Relationship Between ΔT_h and Δh_T

Consider, for example, nitrogen at initial conditions of 300 K and 20 MPa. Using the property data available from NIST-12³² we have,

$$c_{PO} \approx c_P(300 \text{ K}, 0.1 \text{ MPa}) = 29.15 \text{ J/mole}$$

By Eq. 4.12 for nitrogen of $\kappa = 7/5$,

$$c_{PO} = \frac{\kappa}{\kappa - 1} R = \frac{7}{2} R = 29.10 \text{ J/mole}$$

$$\begin{aligned} \Delta h_T(300 \text{ K}, 20 \text{ MPa}) &= h(300 \text{ K}, 0.1 \text{ MPa}) - h(300 \text{ K}, 20 \text{ MPa}) \\ &= 8716 \text{ J/mole} - 7807 \text{ J/mole} \\ &= 909 \text{ J/mole} \end{aligned}$$

Hence, through the above relation,

$$\begin{aligned} \Delta T_h(20 \text{ MPa} \rightarrow 0.1 \text{ MPa}) &= -\frac{\Delta h_T}{c_{PO}} = -\frac{909}{29.15} \\ &= -31.18 \text{ K} \end{aligned}$$

On the other hand the direct calculation of the temperature at the lower pressure, but with the same enthalpy as the initial state, gives

$$T_2(h = 7807 \text{ J/mole}, 0.1 \text{ MPa}) = 268.8 \text{ K}$$

Thus the isenthalpic temperature change, resultant from the same pressure drop, is,

$$\Delta T_h(20 \text{ MPa} \rightarrow 0.1 \text{ MPa}) = 268.8 \text{ K} - 300 \text{ K} = -31.2 \text{ K}$$

fairly close indeed to the value obtained through Eq. 4.63. Similarly, for argon at 350 K and 45 MPa,

$$c_{PO} \approx c_P(450 \text{ K}, 0.1 \text{ MPa}) = 20.82 \text{ J/mole}$$

$$\Delta h_T = 1453 \text{ J/mole}$$

Hence, from Eq. 4.63,

$$\Delta T_h = -69.79 \text{ K}$$

It is left for the readers to verify this figure by direct calculation of ΔT along the isenthalpic curve.

4.5.3 ΔT_h in the Domain of $T < T_{CD}(P)$

The states of this domain fall within the two-phase region after the isenthalpic expansion and the final temperature is clamped at the value of T_{BOIL} . Therefore,

$$\Delta T_h = T - T_{BOIL} \quad @T < T_{CD}(P) \quad (4.64)$$

ΔT_h is independent of pressure. The dependence on temperature is trivial. As such, values of ΔT_h are not related to Δh_T .

In the previous domain, above T_{CD} , the general trend was that both Δh_T and the associated ΔT_h , increase in magnitude as the inlet temperature decreases. However, in the present domain, below T_{CD} , the trend reverses and the magnitude of ΔT_h begins to decrease as the inlet temperature decreases, although Δh_T continues its monotonic increase. Figure 4.29 displays both Δh_T and ΔT_h for nitrogen and clearly demonstrates this point.

In this region liquefaction is achieved by a single expansion. Hence, ΔT_h loses its interpretation as the driving potential for cooldown toward liquefaction.

4.5.4 The Dependence of ΔT_h on Molecular Structure, $T > T_{CD}(P)$

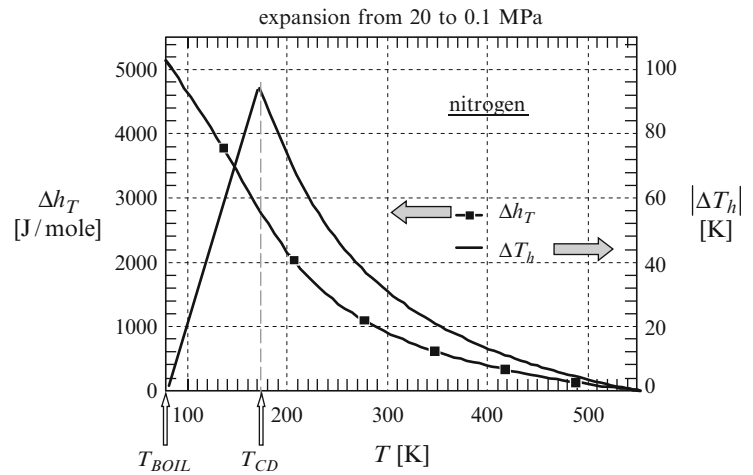
In order to demonstrate the significance of c_{PO} as regards ΔT_h , two cases are introduced.

4.5.4.1 Different ΔT_h for Identical Δh_T

Another aspect of the difference between Δh_T and ΔT_h is reflected in the fact that the “proportionality factor”, c_{PO} , varies from gas to gas. The molar specific heat capacity at zero pressure is a measure of the atomic complexity of a molecule. According to the kinetic theory of gases, the heat capacity is related to the number of degrees of freedom, n , of a molecular structure,⁴⁹ by

$$\frac{c_{PO}}{R} = \frac{\kappa}{\kappa - 1} = \frac{n + 2}{2} \quad (4.65)$$

Fig. 4.29 Demonstration of the non-equivalence of the integral Joule-Thomson effects, ΔT_h and Δh_T



$$\kappa = \frac{c_{PO}}{c_{VO}} = \frac{n+2}{n} \quad (4.66)$$

A more complicated molecule possesses more degrees of freedom. The simplest configuration belongs to the noble gases, with symmetric (“round” that is, zero acentricity) molecules, and $n = 3$ for the three translational degrees of freedom. Therefore,

$$c_{PO} = 2.5 R = 20.80 \text{ J}/(\text{mol} \cdot \text{K}) \quad @\kappa = 5/3 \quad (4.67)$$

For a more complicated molecule such as a diatomic N_2 or O_2 , with $n = 5$,

$$c_{PO} = 3.5 R = 29.10 \text{ J}/(\text{mol} \cdot \text{K}) \quad @\kappa = 7/5 \quad (4.68)$$

which is quite close to the value for real gases at low pressure. Note that c_{PO} for a real gas may be temperature dependent, although the dependence is quite moderate. Therefore, the low pressure specific heat capacity may be considered a constant, with the value of c_{PO} , over the entire temperature range associated with the expansion. Consequently, the same value of Δh_T may be converted into various values of ΔT_h for different gases as one may observe in Table 4.8.

4.5.4.2 Gases with Similar Values of T_C But with Different Molecular Structures

Gases with similar values of T_C but with different molecular structures, such as monatomic or diatomic, might belong to the same low acentricity factor (ω) group. Such gases should have similar values of Δh_T . However, the c_{PO} values deviate by a factor of about $29.10/20.80 = 1.4$ thereby creating a large difference between the corresponding ΔT_h values. This behavior may be demonstrated by comparing values of Δh_T

Table 4.8 Different ΔT_h values for the same temperature, 300 K, and the same Δh_T of 1,000 J/mole for gases with various molecular structures

	ΔT_h [K]	P [MPa]	c_{PO} [J/(mole · K)]
Nitrogen	34.30	24.5	29.20
Argon	48.00	14.4	20.79
Oxygen	34.1	13.7	29.37
Methane	28.03	6.35	35.80
Krypton	47.83	5.74	20.80
R-14	16.03	3.04	61.80
Xenon	47.45	2.44	20.81
Ethane	19.3	1.64	52.60
R-13	14.94	1.24	67.28
Carbon dioxide	27.07	2.34	37.25

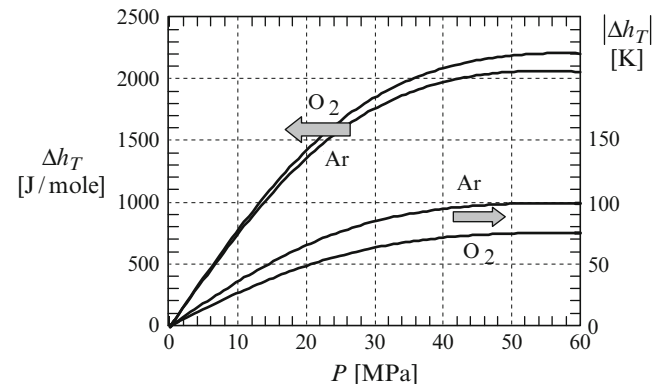


Fig. 4.30 Demonstration of the non-equivalence of ΔT_h and Δh_T ; oxygen has higher Δh_T than argon but lower ΔT_h

and ΔT_h for oxygen and argon, as shown in Fig. 4.30. Oxygen has a somewhat higher value of T_{BOIL} and Δh_T . Even so, its values of ΔT_h are lower than those of argon. Figure 4.31 provides even a sharper demonstration of this behavior,

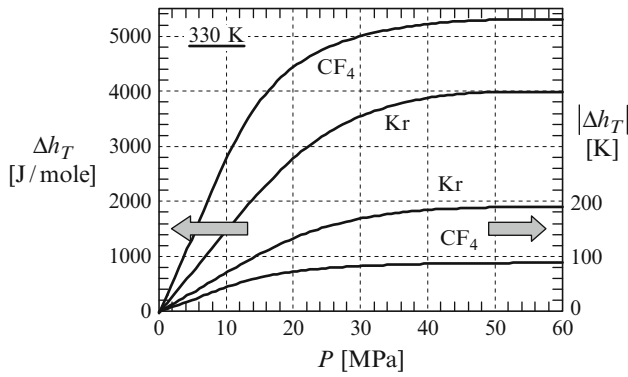


Fig. 4.31 Demonstration of the non-equivalence of ΔT_h and Δh_T ; R-14 has higher Δh_T than krypton but lower ΔT_h

comparing CF_4 and Kr. CF_4 has higher values of T_{BOIL} and Δh_T . However its c_{PO} is also significantly larger and therefore exhibits smaller values of ΔT_h .

4.5.4.3 Remarks

1. One should recall that coolers operating with the same Δh_T are characterized by a similar molar gas consumption, $\dot{n}$, in order to produce a given cooling power $\dot{Q}$, since $\dot{Q} = \dot{n} \cdot \Delta h_T$. This fact remains true, even though each of the above cases are driven by a different potential, ΔT_h .
2. It seems as the isothermal effect, Δh_T , is more universal when observing the space of gases than the adiabatic, ΔT_h . Different gases still have similar values of Δh_T at similar corresponding states.

4.5.5 The State Dependent ΔT_h Variation, $T > T_{\text{CD}}(P)$

4.5.5.1 Pressure Dependence

Differentiating both sides of Eq. 4.63 by pressure at constant temperature, and recalling that $c_{PO} = c_{PO}(T)$ and $(\partial h^{IG}/\partial P)_T = 0$, leads to,

$$\begin{aligned} \left(\frac{\partial}{\partial P} \Delta T_h \right)_T &= -\frac{1}{c_{PO}} \left[\frac{\partial}{\partial P} (h^{IG} - h) \right]_P \\ &= \frac{1}{c_{PO}} \left(\frac{\partial h}{\partial P} \right)_T \end{aligned} \quad (4.69)$$

Combining with Eq. 2.8, leads to the first relation,

$$\left(\frac{\partial}{\partial P} \Delta T_h \right)_T = -\mu \frac{c_P}{c_{PO}} = \frac{\phi}{c_{PO}} \quad (4.70)$$

4.5.5.2 Temperature Dependence

Differentiating both sides of Eq. 4.63 by temperature at constant pressure, and assuming a negligible dependence of c_{PO} on temperature in the relevant range, and since,

$$(\partial h^{IG}/\partial T)_P = c_{PO} \quad (4.71)$$

we get,

$$\left(\frac{\partial}{\partial T} \Delta T_h \right)_P = \frac{c_P - c_{PO}}{c_{PO}} = \frac{c_{PO}^R}{c_{PO}} \quad (4.72)$$

It is equivalent to Eq. 4.30. See geometrical interpretation of ΔT_h in Fig. 3.4.

4.5.5.3 State Derivatives of ΔT_h and μ

After eliminating the heat capacities, we finally derive,

$$\mu = -\frac{\left(\frac{\partial}{\partial P} \Delta T_h \right)_T}{1 + \left(\frac{\partial}{\partial T} \Delta T_h \right)_P} \quad (4.73)$$

as reported by Vortmeyer in 1966.¹⁷

4.5.5.4 Mapping of ΔT_h

The zero pressure heat capacity, c_{PO} , characterizes the molecular structure of the gas and may vary from one gas to another. For a group of gases that share a similar molecular structure, and therefore also a similar heat capacity, ΔT_h depends on the corresponding states and boiling point exactly in the same manner as Δh_T , as extensively discussed in the previous Sect. 4.3. A higher boiling point gas exhibits a larger value of ΔT_h .

For any given temperature, ΔT_h also vanishes at the integral inversion pressure and peaks at the corresponding differential inversion pressure, so that,

$$\Delta T_h^{\text{MAX}} = -\frac{\Delta h_T^{\text{MAX}}}{c_{PO}} \quad (4.74)$$

4.5.6 The Highest Attainable ΔT_h , the $\Delta T_h(\text{MAX})$

4.5.6.1 A Given Gas

A given gas exhibits its highest attainable integral temperature drop by beginning its expansion at $T_{\text{CD}}^{\text{MAX}}$ and reaching the normal boiling point after expansion. Below $T_{\text{CD}}^{\text{MAX}}$ the temperature drop decreases since T_{BOIL} has a fixed value.

$$\Delta T_h(\text{MAX}) = -(T_{\text{CD}}^{\text{MAX}} - T_{\text{BOIL}}) \quad (4.75)$$

This is the same value as of $\Delta T_h(\text{XY})$, as calculated in Eq. 4.52. Inserting previously derived T_{CD}^{MAX} by Eq. 4.58,

$$-\frac{\Delta T_h(\text{MAX})}{T_{\text{BOIL}}} = \frac{1}{2} \left(1 + 42.6 \frac{\kappa - 1}{\kappa} \right)^{1/2} - \frac{1}{2} \quad (4.76)$$

One should recognize the difference between $\Delta T_h(\text{MAX})$ and ΔT_h^{MAX} of Eq. 4.74. Each gas has a single value of $\Delta T_h(\text{MAX})$ which is the largest ΔT_h value of all values of $\Delta T_h^{\text{MAX}}(T)$; each one associated with another T . Put another way,

$$\Delta T_h^{MAX}(T) = -\frac{1}{c_{PO}} \Delta h_T^{MAX}(T) \quad (4.77)$$

However,

$$\begin{aligned}\Delta T_h(\text{MAX}) &= \Delta T_h^{\text{MAX}}(T_{CD}^{\text{MAX}}) \\ &= -\frac{1}{c_{pO}} \Delta h_T^{\text{MAX}}(T_{CD}^{\text{MAX}})\end{aligned}\quad (4.78)$$

is unique since ΔT_{CD}^{MAX} is unique.

For instance, Fig. 4.32 displaces ΔT_h^{MAX} for $T = 350$ and 430 K for some gases, demonstrating as well, the unique position of the noble gases.

4.5.6.2 The Space of Gases

For the purpose of examining $\Delta T_h(\text{MAX})$ in the space of all gases, through Eq. 4.76, it is re-expressed in the form,

$$\Delta T_h(\text{MAX}) = -f(\kappa)T_{BOIL} \quad (4.79)$$

$\Delta T_h(\text{MAX})$ depends on T_{BOIL} and on κ . For a given value of κ a gas with a higher value of T_{BOIL} is therefore one with a higher value of $\Delta T_h(\text{MAX})$. One may ask, which gas

exhibits the largest ΔT_h^{MAX} ? Is ΔT_h^{MAX} bounded at all, or does it increase with each gas that exhibits a higher boiling point and critical temperature?

The proportionality factor, $f(\kappa)$, increases for gases with larger values of κ , that is, for simpler molecules with less degrees of freedom. However, a counter trend is also observed. As we proceed towards the gases of higher T_{BOIL} , they begin to have more complicated molecules and therefore lower values of κ .

This is graphically demonstrated in Fig. 4.33, in the plane of $[\Delta T_h(\text{MAX}), T_{BOIL}]$. Gases of constant k are located on a straight line with a slope of $f(\kappa)$. As T_{BOIL} increases, their slope decreases.

As a result of the two competing trends, the highest $\Delta T_h(\text{MAX})$ does not belong to the noble gases with higher values of k , nor to gases with much larger values T_{BOIL} but with lower values of k . The peak choice of this analysis belongs to the material HI, with a normal boiling point of 237.7 K. The highest achievable $\Delta T_h(\text{MAX})$ is about 305 K.

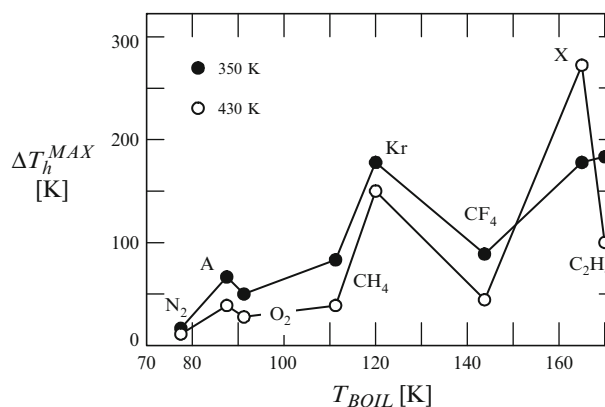


Fig. 4.32 The maximum isenthalpic integral Joule-Thomson effect for some gases as function of the boiling point

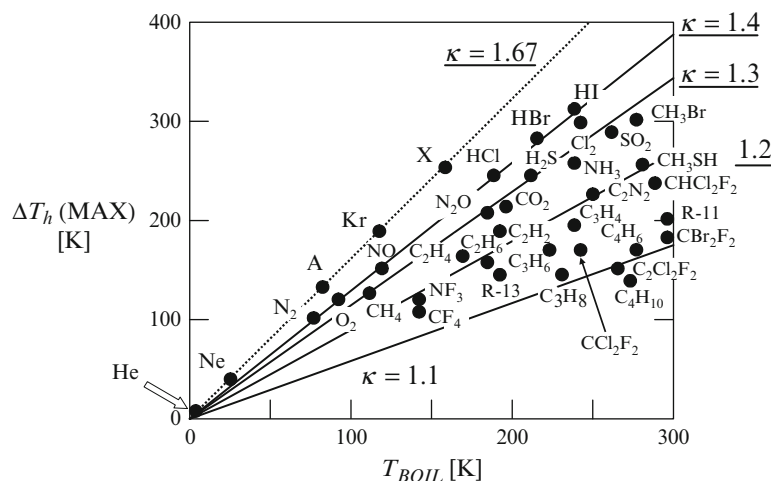


Fig. 4.33 $\Delta T_h(\text{MAX})$ as function of gases' boiling point. Gases of constant κ are located on *straight lines*

This result has a universal significance. It is the largest integral temperature drop attainable by a Joule-Thomson expansion, for all gases and states. However, if we restrict our consideration solely to non-toxic gases, it is xenon that has the highest value of $\Delta T_h(\text{MAX})$, about 275 K.

4.5.7 Evaluation of $\Delta T_h(P, P_{OUT})$ Through the Equation of State

So far we have discussed the temperature change, ΔT_h , accompanying an isenthalpic pressure drop down to 0.1 MPa, as defined by Eqs. 2.9 and 4.61. It was determined with the aid of Δh_T and c_{PO} through Eq. 4.63. In the general case, the final pressure, after expansion, could be any arbitrary value, $P_{OUT} \neq 0.1$ MPa, while starting at P ,

$$\Delta T_h(P, P_{OUT}) = T(P_{OUT}, h) - T(P, h) \quad (4.80)$$

By the definition of the Joule-Thomson coefficient through Eq. 2.4,

$$\mu dP = dT \quad @ \quad dh = 0 \quad (4.81)$$

thus,

$$\Delta T_h(P, P_{OUT}) = \left[\int_{P_{OUT}}^P \mu dP \right]_h \quad (4.82)$$

One may employ any of the expressions for μ of Sect. 2.2.3, like of Eq. 2.10, and get,

$$\Delta T_h(P, P_{OUT}) = \int_{P_{OUT}}^P \frac{1}{c_P} \left[T \left(\frac{\partial v}{\partial T} \right)_P - v \right] dP \quad (4.83)$$

while,

$$c_P = c_{PO} - T \int_0^P \left(\frac{\partial^2 v}{\partial T^2} \right) dP \quad (4.84)$$

Schluender and Zemlin⁵⁰ mentioned this procedure for pure gases.

Chou et al.⁵¹ applied an equation of state for nitrogen using the c_P data and calculated values of μ in some intervals of pressure. They found that accurate values of ΔT_h could be obtained by integrating with only a few intervals of pressure, and a few steps of μ . As reported elsewhere⁵² Eq. 4.44 can also be used to obtain values of ΔT_h , through Δh_T . The approach only requires data for c_{PO} rather than values of c_P .

4.5.8 ΔT_h for Mixtures

4.5.8.1 Evaluating a Mixture's Δh_T and c_{PO}

Equation 4.63 holds as well for mixtures in the single-phase region. An equation of state for a mixture is obtained by adapting a “mixing rule” for the equations of state of the components and the composition. It enables one to determine Δh_T for the mixture. Using the c_{PO} data and applying Eq. 4.63 leads to ΔT_h of the mixture. Koeppe^{46,47} used this procedure by applying the virial equation of state to calculate Δh_T for mixtures of A-O₂, A-N₂, N₂-H₂ and N₂-He. The c_{PO} of a mixture is obtained through a composition-weighted linear combination of the various components' values of c_{PO} . Sobanski⁵³ used the Bettie-Bridgman equation of state and derived a closed form solution for ΔT_h . He applied this approach for air by considering it a mixture of nitrogen and oxygen.

4.5.8.2 Mixing of Components' ΔT_h Values

Schluender and Zemlin^{50,54,55} proposed to determine ΔT_h for a mixture of components A and B in terms of $\Delta T_h^{(A)}$ and $\Delta T_h^{(B)}$ and the molar fractions $x^{(A)}$ and $x^{(B)}$. The partial pressure of each component, $P^{(A)}$ and $P^{(B)}$, is determined as if they were alone, and is calculated on the basis of their equations of state or compressibilities, $Z^{(A)}$ and $Z^{(B)}$, and the composition, by

$$\frac{P}{P^{(A)}} = 1 + \frac{Z^{(B)}}{Z^{(A)}} \left[\frac{1}{x^{(A)}} - 1 \right] \quad (4.85)$$

$$\frac{P}{P^{(B)}} = 1 + \frac{Z^{(A)}}{Z^{(B)}} \left[\frac{1}{x^{(B)}} - 1 \right] \quad (4.86)$$

One may also obtain the individual values of $\Delta T_h^{(A)}$ and $\Delta T_h^{(B)}$. For the mixture,

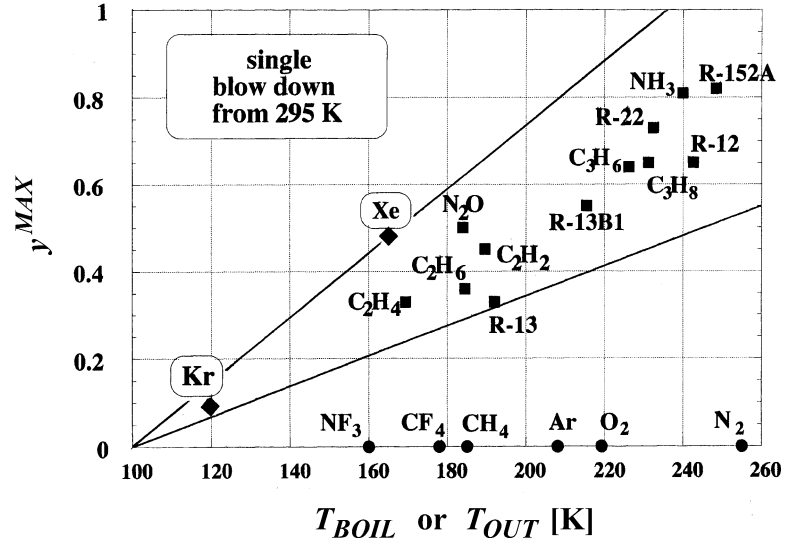
$$\Delta T_h^{MIX} = \frac{c_P^{(A)} x^{(A)} \Delta T_h^{(A)} + c_P^{(B)} x^{(B)} \Delta T_h^{(B)}}{c_P^{(A)} x^{(A)} + c_P^{(B)} x^{(B)}} \quad (4.87)$$

When $\Delta T_h^{(A)}$ and $\Delta T_h^{(B)}$ have opposite signs (as with nitrogen and helium over a wide range of temperatures) it is possible to approach a zero value of ΔT_h^{MIX} over a wide range of temperatures as has been achieved elsewhere.⁵⁶

4.6 Direct Blow Down Yield of Liquefaction

If the gas temperature at the inlet to an isenthalpic expansion is below T_{CD} , its temperature at the outlet (at atmospheric pressure) reaches a constant value, that is the boiling point, which is solely dependent on the low outlet pressure. The flow

Fig. 4.34 Gases of T_{CD} above the room temperature: higher boiling point gases have larger yield of liquefaction under a single isenthalpic expansion down to atmospheric pressure



at the outlet is a partially liquefied (two phase) stream with the fraction y of liquid (or instead, with the quality, $x = 1 - y$). With reference to the inlet conditions, (P, T) , y is determined by the enthalpy, h , at the inlet through energy balance,

$$h(P, T) = y h_F + (1 - y) h_G = h_G - y L \quad (4.88)$$

After differentiating and algebraic re-arrangement, the pressure dependence of y at a given constant temperature is

$$\left(\frac{\partial y}{\partial P}\right)_T = -\frac{1}{L} \left(\frac{\partial h}{\partial P}\right)_T \quad (4.89)$$

Since $(\partial y / \partial P)_T$ vanishes when $(\partial h / \partial P)_T$ does, therefore, y peaks on the differential inversion curve (at the associated inversion pressure) where $(\partial y / \partial P)_T = 0$. This feature is visualized in Fig. 3.3. The peaks of the isotherms in the p - h plane are also associated with the peak values of y . Johnston et al.⁵⁷ presented the measured values of y for hydrogen as a function of pressure for various isotherms in the range of 64–80 K, and indeed these exhibit a peaking behavior.

Figure 4.34 focuses on gases with values of T_{CD} above the room temperature. As previously seen, the monatomic and simple molecule gases have relatively higher values of T_{CD}^{MAX} than the multi atom molecule gases when the basis of comparison is T_{BOIL} . This feature translates into a relatively higher fraction of liquefaction by a direct blow down.

In general, gases with higher boiling points have larger values of y (in analogy to higher values of Δh_T). Notice that the uniqueness of krypton in this regard is demonstrated again in Fig. 4.34. The independent variable in this figure,

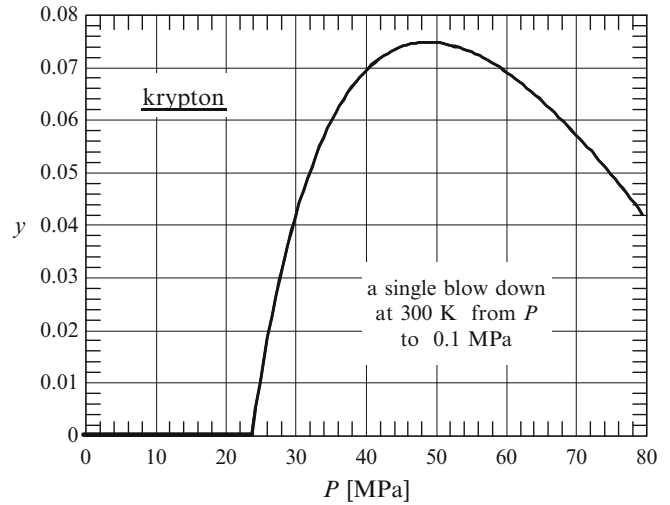


Fig. 4.35 Pressure dependence of the yield of liquefaction under a single expansion of krypton down to atmospheric pressure

T_{BOIL} or T_{OUT} , represents the outlet temperature in the two cases that are determined by whether liquefaction is achieved as a result of a direct blow down or not. Krypton enables the lowest achievable temperature under a single expansion from room temperature. Furthermore, when expanded, krypton will include both liquid and vapor phases.

The gas CF_4 , often applied as a pre-coolant, has a higher normal boiling temperature than krypton. Despite this, it does not liquefy and does not reach as low of a temperature as krypton does, under a single expansion. The yield of liquefaction for krypton under a single expansion at each pressure is shown in Fig. 4.35.

Ambrose and Hsu^{58,59} utilized a direct blow-down approach to achieve immediate liquefaction for cooling a sensor.

4.7 Compressibility of Coolants

The compressibility of a gas, defined by

$$Z = \frac{P}{RT\rho} \quad (4.90)$$

is the most direct way to express its deviation from the ideal gas model. In the context of Joule-Thomson cryocoolers, the relevance of compressibility is evident when one considers how much gas can be ‘packed’ under pressure in a vessel. Stated another way, the compressibility of a gas determines how much of it can be stored in the process of compressing the gas from P_1 to P_2 , or how much can be released when the vessel discharges from P_2 to P_1 . A cryocooler, fed by a pressure vessel, will run longer when more gas, in terms of moles, is stored in a similar pressure range. For this application, we are therefore more interested in $\Delta\rho/\Delta P$ (and not in Z itself), or in the differential form, $(\partial\rho/\partial P)_T$. The interest in the differential form along an isothermal trajectory is explained by the fact that when cryocoolers are operated over an extended period of time, the pressure vessel remains at a fairly constant temperature. The expression may be normalized by its zero pressure (ideal gas) value to obtain the non-dimensional parameter,

$$Y = \frac{(\partial\rho/\partial P)_T}{[(\partial\rho/\partial P)_T]_{P=0}} \quad (4.91)$$

which may be interpreted and referred to as the *normalized differential compressibility*. For Joule-Thomson cryocoolers, gases with higher values of Y are desirable.

Since

$$\left[\left(\frac{\partial\rho}{\partial P} \right)_T \right]_{P=0} = \frac{1}{RT} \quad (4.92)$$

therefore

$$Y = RT \left(\frac{\partial\rho}{\partial P} \right)_T = \left[\frac{\partial}{\partial P} \left(\frac{P}{Z} \right) \right]_T \quad (4.93)$$

or in the differential form,

$$dY = d \left(\frac{P}{Z} \right) \quad @ \quad dT = 0 \quad (4.94)$$

The differential compressibility Y is related to the isothermal compressibility,⁶⁰ K_T , by Eq. 2.133,

$$Y = R T \rho K_T = \frac{P}{Z} K_T \quad (4.95)$$

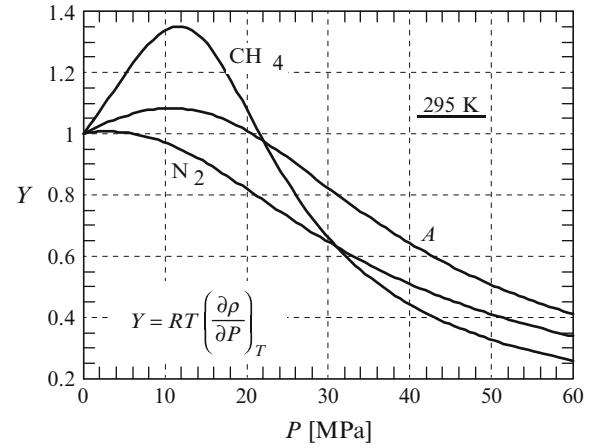


Fig. 4.36 Pressure dependence of the normalized differential compressibility at room temperature for N_2 , Ar and CH_4

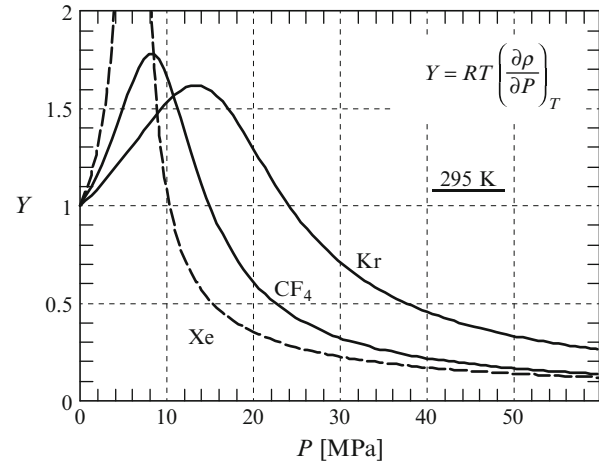


Fig. 4.37 Pressure dependence of the normalized differential compressibility at room temperature for Xe, CF_4 and Kr

In terms of the compressibility, Z ,

$$Y = \frac{1}{Z} \left[1 - \left(\frac{\partial \ln Z}{\partial \ln P} \right)_T \right] \quad (4.96)$$

Along an isotherm in the range of reduced pressures $[\bar{P}] > 10$, Z increases monotonically as a function of pressure, therefore, $Y < 1/Z$.

At room temperature and with a pressure as low as atmospheric, a 0.1 MPa pressure elevation adds about one standard liter of gas to a 1 l volume vessel. At any higher pressure, P , an additional increase of 0.1 MPa will contribute a number of Y standard liters of gas. This interpretation was introduced by Longworth.⁶¹

The pressure dependence of Y at room temperature for some gases is displayed in Figs. 4.36, 4.37, and 4.38. Figure 4.36 displays this information for nitrogen, argon and methane. The same dependence for gases with higher

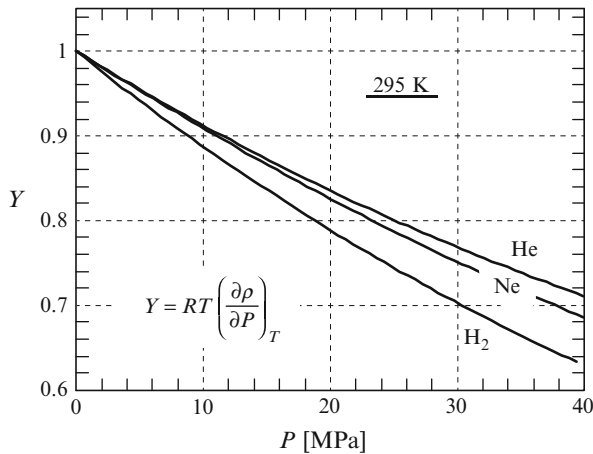


Fig. 4.38 Pressure dependence of the normalized differential compressibility at room temperature for H_2 , Ne and He

boiling points such as krypton, CF_4 and xenon, is shown in Fig. 4.37, and for the quantum gases, helium, hydrogen and neon, in Fig. 4.38. As a general trend, Y decreases at elevated pressures, however, attention should be paid to the peaking behavior where $Y > 1$, at the low pressure range. The integral under the Y curve between two pressures is indicative of the change in density, $\Delta\rho$, and represents the total amount of gas. As an example, let us compare two 1 l vessels at 40 MPa, one containing nitrogen gas and, the other, argon. Upon lowering the pressure to 39.9 MPa, only 0.51 standard liters of nitrogen are released, but 0.65 l of argon are released.

Finally, note that the noble gases exhibit better values of Y with respect to their boiling points. At room temperature, neon and argon have correspondingly higher values of Y than do hydrogen and nitrogen.

4.8 The Cooling Potential of a Pressure Vessel

In this section, we investigate the ideal cooling capacity of a pressure vessel, of volume V , discharging isothermally or adiabatically through a perfect Joule-Thomson cryocooler. That is,

$$dQ = \Delta h_T \cdot dn = V \cdot \Delta h_T \cdot d\rho \quad (4.97)$$

4.8.1 The Isothermal Discharge of a Pressure Vessel

When the discharge rate of a pressure vessel is sufficiently slow (as is very often the case); the vessel succeeds to be in thermal equilibrium with the environment, so that

Table 4.9 The cooling capacity, Q/V [kJ/Lit.] of an isothermal pressure vessel

	70 to 0 MPa		40 to 0 MPa	
	300 K	340 K	300 K	340 K
N_2	14.5	9.3	9.2	5.9
Ar	28.6	20.3	16.7	10.6
CH_4	48.2	37.0	32.1	22.8
Kr	68.6	53.9	48.1	34.0
CF_4	72.8	56.3	54.9	38.8
			85 K	90 K
Ne			17.4	14.6
H_2			7.3	6.3

practically, the content of the vessel remains at a constant temperature throughout the discharge process.

4.8.1.1 The Cooling Capacity Per Unit Volume of the Vessel

The cooling capacity per unit volume of the vessel while it discharges from a pressure, P , to zero is given by

$$\frac{Q}{V} = \left[\int_0^P \Delta h_T \cdot d\rho \right]_T = \frac{1}{RT} \left[\int_0^P \Delta h_T \cdot d\left(\frac{P}{Z}\right) \right]_T \quad (4.98)$$

or in terms of Y ,

$$\frac{Q}{V} = \left[\int_0^P \Delta h_T \left(\frac{\partial \rho}{\partial P} \right)_T dP \right]_T = \frac{1}{RT} \left[\int_0^P Y \Delta h_T dP \right]_T \quad (4.99)$$

Equation 4.98 combines the specific cooling capacity of the gas, Δh_T , with its compressibility data, in terms of Z . In Eq. 4.99 Y is the weighting function for the integration of Δh_T . Lower values of Δh_T can be compensated by higher values of Y to improve Q/V . The upcoming figures display the integration as a function of the instantaneous pressure, P , in the vessel, starting at 70 MPa, for a variety of gases. The curves of Q/V exhibit an inflection point at a pressure that satisfies the condition

$$\left[\frac{\partial}{\partial P} (Y \cdot \Delta h_T) \right]_T = 0 \quad (4.100)$$

Were it not for the involvement of Y , the inflection would occur at the inversion pressure, but in general it occurs below that point. Table 4.9 summarizes some Q/V values for various gases.

Figure 4.39 shows values of Q/V for nitrogen and argon at 300 K and 340 K ambient temperatures. One should compare the potential of nitrogen with the actual performance by air, in Fig. 5.4. Similar curves of ideal operation

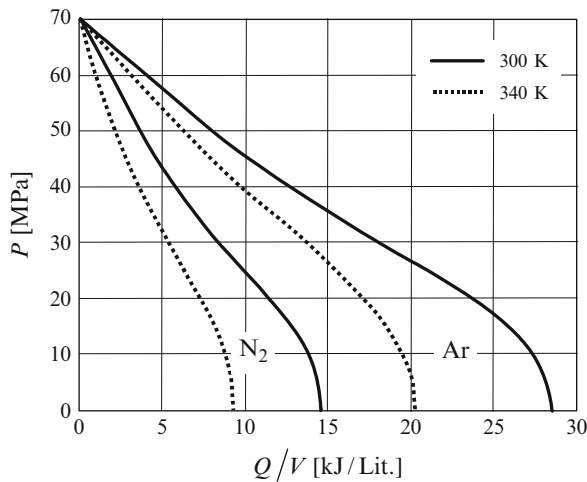


Fig. 4.39 Cooling capacity, Q/V , of nitrogen and argon by a single liter pressure vessel isothermally discharged through an ideal Joule-Thomson cryocooler

with argon are also shown elsewhere.⁶² For example, an argon vessel at 300 K, discharging from 70 to 62.5 MPa can provide 3.02 kJ per 1 l vessel. As it continues to discharge down to 27.5 MPa, 19.61 kJ/l may be obtained. Hence, 16.59 kJ/l are available between 62.5 and 27.5 MPa. The cooling potential of a vessel charged with argon is remarkable, being about twice that one charged with nitrogen in similar conditions. The maximum Δh_T at 300 K is only 1.5 times larger for argon (1.8 against 1.2 kJ/l). However, the higher normalized differential compressibility, Y , raises the ratio to 2. The penalty in terms of an elevated boiling point is only about 10 K. This is what makes argon so attractive and widely used, whenever it is tolerable.

Figure 4.40 shows values of Q/V for methane, krypton, and CF_4 at 340 K. It demonstrates the outstanding capability of krypton. In comparison to CF_4 , krypton has a lower boiling point by about 25 K and a lower peak value of Δh_T . However, due to the compensation provided by large values of Y , it exhibits a cooling potential very similar (or even larger in some pressure ranges) to the other two gases.

Figure 4.41 presents the same type of information for hydrogen and neon. Since these are liquefied by being precooled by nitrogen, their cooling capacities are of interest at 85 and 90 K. The critical pressures of the two gases are relatively low; therefore, in general, pressurization over 40 MPa is not practical. Furthermore, elevating the charging pressure of hydrogen above 20 MPa is less “cost effective” due to a reduced marginal gain in terms of Q/V .

4.8.1.2 The Loss of Cooling Potential Due to Void Volume

Very often in practice, the gas in a pressure vessel (of volume V) initially expands into a void volume (V_V) before it feeds the cryocooler. As a result, the gas effectively

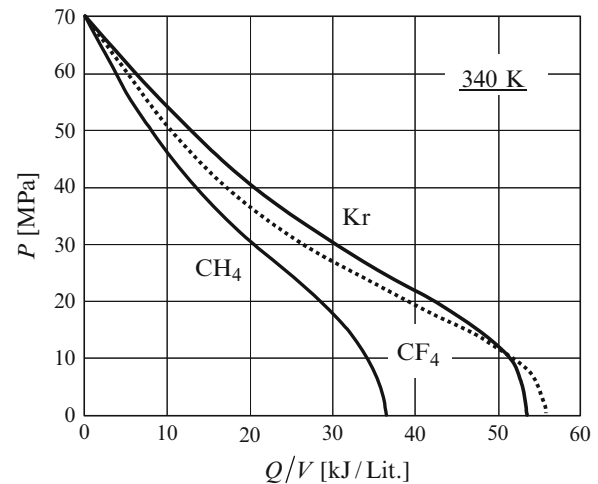


Fig. 4.40 Cooling capacity, Q/V , of methane, krypton and R-14, by a single liter pressure vessel isothermally discharged through an ideal Joule-Thomson cryocooler

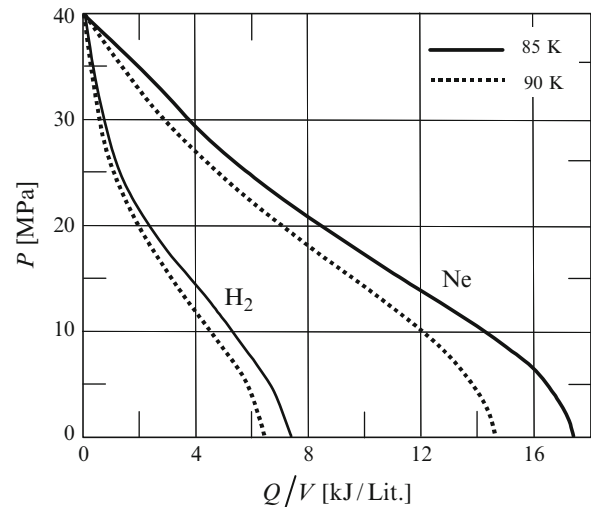


Fig. 4.41 Cooling capacity, Q/V , of neon and hydrogen by a single liter pressure vessel isothermally discharged through an ideal Joule-Thomson cryocooler

occupies a larger pressure vessel with a volume ($V + V_V$) but at a reduced pressure. The void volume V_V might represent the tubing and ducts between the vessel and the cryocoolers, and/or the void volume inside a desiccant cartridge and valves. The reduction of pressure lowers the cooling capacity of the gas, and the potential total cooling Q is reduced due to the existence of the void volume. For example, consider a vessel pressurized with nitrogen to 70 MPa at 295 K, with two different values of the void volume so that, V_V/V is 0.5 or 1. Under an isothermal expansion the total volume ($V + V_V$) reaches pressures of 34.3 MPa or 23.45 MPa, respectively. Due to the pressure dependence of Q/V (as displayed in Fig. 4.39) the fraction of

the total cooling capacity $Q(V_V)$ compared to that obtained with a zero void volume $Q(V_V = 0)$ in each of the two cases is 0.86 and 0.67. For argon under a similar condition, the intermediate pressure values are 35.3 and 24.83 MPa and the cooling capacity Q shrinks by the factors of 0.77 and 0.61, respectively.

4.8.1.3 The Cooling Capacity Per Unit Weight of the Vessel

For any given V , the content of gas and the associated cooling capacity $Q(P)$ are monotonically increasing functions of the charge pressure, P (as long as $\Delta h_T > 0$). However, higher values of P also monotonically increase the mass, $m(P)$, of the pressure vessel including the gas. The vessel itself requires a larger thickness to withstand the high P . Gromov, Landa, et al.⁶³ examined the ratio Q/m as a function of P by a similar procedure of isothermally integrating Eq. 4.98. They found that an optimal charging pressure exists, that maximizes this ratio. For nitrogen, the optimum pressure is 50 MPa. According to their experience designing titanium vessels, the optimum value of Q/m is approximately 10 kJ/kg.

4.8.2 The Isothermal Discharge Pattern

The term Q/V may be translated into a time scale for discharging a pressure vessel as $Q = t \cdot \dot{Q}$, under three conditions. First, the cryocooler must be subjected to a constant heat load $\dot{Q}$. Second, its recuperator must be completely efficient ($\varepsilon = 1$, by Sect. 3.5.2). Third, the instantaneous flow rate must be precisely adjusted to balance the heat load (as discussed in paragraph 5.3.3.3). With such conditions, the graphs of P as function of Q/V may be interpreted as the time dependence, $P(t)$, of the discharge pressure. Buller⁶⁴ made such an attempt to determine the time dependence of the pressure discharge for nitrogen and argon. However, his estimation did not include the variable compressibility of the remaining gas in the vessel even when allowing a pressure as high as 65 MPa. Similarly, Maytal⁶⁵ derived an expression based on parabolic approximation for the pressure dependence of Δh_T up to 60 MPa. So that,

$$\Delta h_T = K \Psi (1 - \Psi) \quad (4.101)$$

where K is a scaling factor and Ψ is the pressure normalized by P_K so that,

$$\Psi = P/P_K \quad (4.102)$$

Since, according to Eq. 4.101, the value of Δh_T peaks at $\Psi = 1/2$, the value of P_K represents twice the inversion pressure of the gas at the constant temperature. The two

parameters K and P_K change from gas to gas. For nitrogen at 295 K the two are 4,480 J/mole and 80 MPa. For argon, the respective values are 8,215 J/mole and 100 MPa. The cooling capacity while discharging from the initial pressure noted by 1 down to the final pressure noted by 2, is given by,

$$Q/V = \frac{K \cdot P_K}{2R \cdot T \cdot \bar{Z}} \left[\left(\Psi^2 - \frac{2}{3} \Psi^3 \right)_1 - \left(\Psi^2 - \frac{2}{3} \Psi^3 \right)_2 \right] \quad (4.103)$$

where $\bar{Z}$ is the average compressibility in the relevant range of discharge pressures (P_1, P_2). This formulation was experimentally verified in reference.⁶⁵

Desyatov et al. conducted a similar study, below the inversion pressure values.⁶⁶ Therefore, they could utilize a monotonic approximation for $\Delta h_T(P)$, in the form of a logarithmic function. They also neglected the effect of compressibility.

4.8.3 The Adiabatic Discharge of a Pressure Vessel

The gas from a vessel, undergoing an adiabatic discharge through a cryocooler, is modeled by a isentropic process. Along with the pressure, the temperature of the gas drops as well. In this case,

$$\frac{Q}{V} = \left[\int_0^P \Delta h_T \cdot d\rho \right]_s \quad (4.104)$$

Such an integration along an isentropic trajectory, for argon and nitrogen, beginning at the initial state of 40 MPa and 300 K, is displayed in Fig. 4.42 along with the corresponding isothermal discharge trajectory.

In general, the cooling capacity of an adiabatic discharge process (represented in Fig. 4.42 by the $s = \text{const}$ curve) is smaller than that of the isothermal process (the $T = \text{const}$ curve) for a common pressure range.

At the very low pressure range, the adiabatic discharge appears to have a larger cooling capacity Q/V . However, at such low pressure, a very large aperture of the throttle would be required in order to satisfy the ideal flow rate demand.

As the pressure decays, the gas enters the cryocooler at a lower and lower temperature. As a result, the exhausted gas drops significantly below the initial and ambient temperature. One may regard this feature as a waste of cooling capacity.

Due to its larger value of κ , the above described effects associated with the isentropic expansion are more dramatic

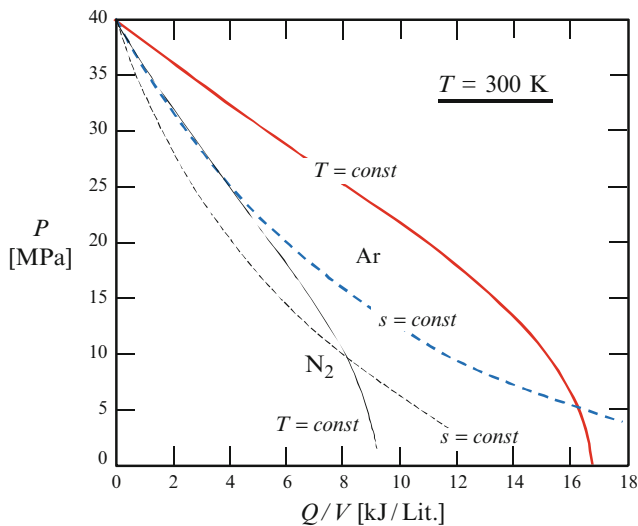


Fig. 4.42 Cooling capacity, Q/V , of argon and nitrogen for an adiabatic and isothermal discharge of a pressure vessel through an ideal Joule-Thomson cryocooler

for argon than for nitrogen. Figure 4.42 therefore displays a larger departure between the adiabatic and isothermal discharge for argon than for nitrogen.

4.9 Monatomic and Other Coolants: Closing Remarks

4.9.1 Characteristics of the Monatomic Gas Family

The closing section of this chapter is an appropriate occasion for identifying the group of noble gases as the favorite coolants for Joule-Thomson cryocooling.⁴⁰ This fact is solely due to their monatomic micro structure which manifests itself through the macro thermodynamic parameters such as the lowest value of c_{PO} (approximately $20 \text{ J mole}^{-1} \text{ K}^{-1}$) and the highest value of κ , that is $5/3$. The uniqueness of this group when compared to other gases of similar boiling points is:

- (a) The noble gases are completely inert and unable to interact chemically, which is an advantage for handling and operation. (Even though, one should pay attention to the fact that Ar, Kr and Xe are heavier than air so they may fill a cavity and cause a dangerous deficiency of oxygen).
- (b) The driving potential, ΔT_h , of the monatomic gases is larger than of other gases of similar T_{BOIL} .
- (c) The cooldown range, ΔT_{CD}^{MIN} , of the monatomic gases is smaller than of other gases of similar T_{BOIL} and demonstrated in Fig. 4.27.

The ΔT_h and ΔT_{CD} are responsible for faster cooldown rate and shorter cooldown period. Section 7.4.4

discusses the parameter of cooldown tendency, TND, which is the ratio of the two, $|\Delta T_h|/\Delta T_{CD}$, for various gases

- (d) The noble gases “are satisfied” by a smaller recuperative heat exchanger than other gases. Their extent of recuperation,

$$\delta h^{MAX} = c_{PO}(T_{AMB} - T_{BOIL}) \quad (4.105)$$

for similar T_{BOIL} is smaller, since their c_{PO} is smaller.

Section 3.7.6 discussed the parameter Σ , which is a measure of the required heat exchanging area. For a given T_{BOIL} , the noble gases exhibit larger values of ΔT_h , hence, by Eq. 3.96, smaller Σ values, and thus smaller heat exchangers. Longworth⁶⁷ and Bonney and Stubbs⁶⁸ discussed values of Σ for various gases. In general, the lower boiling point gases need larger heat exchangers. However, argon whose normal boiling temperature is quite close to that of nitrogen, requires even less the half of the heat exchanging area. Krypton, with its significantly lower normal boiling point (119 K) than that of CF_4 (145 K) requires only half of the heat exchanger compared to that required by the warmer CF_4 .

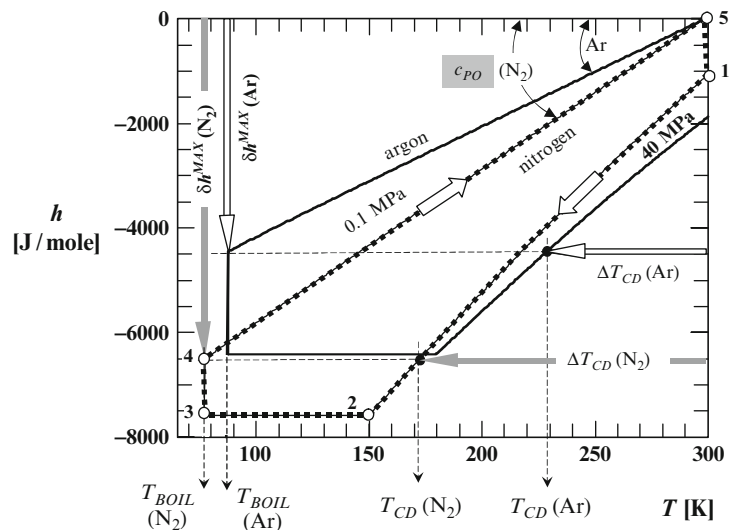
Figure 4.43 displays the h - T diagrams of nitrogen and argon cycles as two gases whose boiling temperatures, T_{BOIL} , are close to each other. The figure illustrates how the smaller value of c_{PO} for the noble gases reduces their associated values of δh^{MAX} and ΔT_{CD} in comparison to other gases.

- (e) While expanding at temperatures below T_{CD} , the noble gases experience the higher portion of liquefaction, y , again, for a given value of T_{BOIL} .
- (f) Noble gases are transparent to infrared radiation. As monatomic gases, their atomic energy transition does not include energy levels associated with the infrared range. This feature may become an issue whenever the exhausted low pressure gas has to cross any optical path, as discussed by Pope⁶⁹ regarding krypton.
- (g) The last thermodynamic feature of the noble gases cannot be regarded as an advantage in view of the practical operation of the Joule-Thomson cryocoolers. It is the relatively small gap between the normal boiling point and their triple point. By lowering the ambient pressure, the operating boiling temperature can be only slightly depressed. That is the reason that hydrogen is often preferred instead of neon.

4.9.2 Particular Identity of Each Noble Gas

Unfortunately, the group of noble gases includes only few members. Cook et al.⁷⁰ prepared a comprehensive collection of data on the “rare gases”. Hence, it is worthwhile to scan

Fig. 4.43 The smaller c_{p0} of argon is reflected by a smaller δh^{MAX} and ΔT_{CD} in comparison to nitrogen



the individual characteristics of each gas and identify their respective niche of applications:

4.9.2.1 Helium

Helium has the lowest critical temperature and pressure. ^3He is the gas with the lowest boiling point. The more common gas is ^4He , but ^3He has also been used for Joule-Thomson cryocooling. ^3He has a higher vapor pressure than ^4He and requires less pumping power to significantly depress its boiling point. Furthermore, unless one is operating in the millikelvin temperature regime, ^3He enables one to avoid concern regarding superfluidity.

4.9.2.2 Neon

1. It is a coolant for the 30 K range. Neon also requires a significantly smaller recuperative heat exchanger than hydrogen.
2. Liquid neon is almost twice as dense as liquid helium and liquid hydrogen in terms of moles per liquid liter.

4.9.2.3 Argon

1. Argon is the strong competitor to the widely utilized nitrogen gas. Although its normal boiling point is 10 K higher than that of nitrogen, other properties make argon a significantly promising substitute.
2. A given pressure vessel of argon has a larger cooling capacity than nitrogen because of the superior Joule-Thomson effect and compressibility behavior.

4.9.2.4 Krypton

1. In a broad sense, it is an outstanding candidate for Joule-Thomson cryocooling. In answer to the general question regarding the lowest possible temperature achievable by a single Joule-Thomson expansion from room temperature, we find that it is 119.9 K, and that it is enabled by krypton (by the aid of Fig. 4.34). Furthermore, the expansion with

krypton produces a two-phase fluid with a few percent of liquid, which enhances its cooling capability.

2. Krypton is an excellent precoolant for nitrogen, argon^{67,69,71} and their mixtures. It is much more efficient than the traditional precoolant as CF_4 . As shown in Table 4.5, krypton has a higher value of T_{CD} although its boiling point is lower.
3. Krypton is used as a coolant for physi-sorption cryocoolers in the range of 120–160 K.^{72,73}

4.9.2.5 Xenon

Xenon has analogous features to room temperature krypton but at elevated ambient temperatures.

1. Xenon reaches temperatures as low as 166 K through a single expansion and without any recuperator, even for ambient temperatures up to about 440 K.
2. Of all the non toxic gases, xenon exhibits the largest ΔT_h , which is about 274 K.
3. Xenon's potential niche, however is for precooling nitrogen and argon from as high ambient temperatures as 500 K.

Two applications of xenon^{32,74} in Joule-Thomson cryocooling are reported. The first is a three-stage sorption type system patented by Bard and Jones.⁷⁵ This case provides an appropriate example within the present context since two noble gases, xenon and krypton, are employed to precool oxygen. The second application is the MEMS sorption compressor^{76,77} of Burger, Wiegner et al.

Currently, xenon is considered as an expensive gas.

4.9.3 Other Gases

To complete our summary remarks on coolants, we include a few characteristics of other important, and useful, polyatomic gases with their typical applications:

4.9.3.1 Nitrogen

Nitrogen is the gas with the lowest normal boiling point that is still liquefied by a non-pre-cooled single stage Linde-Hampson process.

4.9.3.2 Oxygen

Oxygen enables cooling below 70 K with a single stage by pumping on its vapor pressure, because of its low temperature of solidification. Oxygen is also used as a coolant for chemisorption J-T cryocoolers in the range of 60–100 K.^{78,79}

4.9.3.3 Air

Air is very similar to nitrogen for any practical purpose. The fact that it is a mixture (about 80% nitrogen and 20% oxygen) is irrelevant. The sole justification for using air is its ubiquitous availability (in the atmosphere). Instead of being supplied in pressure vessels, it is compressed and purified before being fed into a cryocoolers, and then it is released back into the atmosphere.

4.9.3.4 R-14

R-14 (CF₄) is often used as a precoolant for nitrogen and argon.

4.9.3.5 Methane

Methane (CH₄) is often used as a precoolant for nitrogen and argon. Additionally, it serves as a coolant for a sorption J-T cryocooler for the range of 110–150 K.⁸⁰

4.9.3.6 Nitrous Oxide

Nitrous oxide (N₂O) is often employed in the practice of cryosurgical probes. Liquefaction (at 194 K) is achieved under a single expansion without any recuperator.

4.9.3.7 Carbon Dioxide

Carbon dioxide (CO₂) is also employed in the practice of cryosurgical probes, and as well for general purpose cooling devices. It is characterized by a high triple point, which is above atmospheric pressure. Therefore, the expanded stream resulting from a non recuperated single expansion will include solid particles instead of liquid.

4.9.3.8 Hydrogen

Hydrogen (H₂) is the coolant for the range of 15–30 K.

4.9.4 The Role of the Differential Inversion Curve

Throughout this chapter, the differential inversion curve has been mentioned in many contexts. Indeed, the curve is significant in many regards to the Linde-Hampson process.

Therefore it is worthwhile to concentrate on its unique and relevant features.

For any given temperature, the following entities reach their maximum values on the differential inversion curve (or are somehow related with the curve), that is, at the inversion pressure associated with the given temperature:

- (a) Δh_T , the specific cooling content,
- (b) ΔT_h , the integral isenthalpic Joule-Thomson effect,
- (c) $y^{(1)}$ and $y^{(2)}$, yield of liquefaction of a cooler and a liquefier,
- (d) Yield of liquefaction under a single blow down in the case where $T < T_{CD}$,
- (e) h , trajectories of constant enthalpy,
- (f) The highest T_{CD} value occurs on the differential inversion curve,
- (g) The COP at any given T peaks at a pressure which is lower than the inversion pressure associated with T .

4.9.5 The Quantum Gases: ³He, ⁴He, H₂, D₂ and Ne

Some features of the quantum gases have already been discussed under previous headings. Nevertheless, as a unique group it deserves special attention at this point.

Except for neon, which behaves quite similarly to a low accentricity gas, the quantum gases deviate sharply from the principle of corresponding states. The behavior is related to quantum effects, and therefore the proposed corrections must be derived through quantum mechanics.

With regard to Joule-Thomson cryocooling, a very relevant property of the quantum gases (besides the low boiling points), is that their maximum inversion temperatures fall below room temperature. Consequently, they do not liquefy spontaneously and require precooling.

Only the gases ³He, ⁴He, H₂ and D₂ exhibit a maximum inversion pressure in the integral sense as discussed in Sect. 2.6.3 and observed in Fig. 2.19. While expanding from a higher pressure (down to atmospheric pressure) and even if the initial temperature is very low, the outlet temperature increases.

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Part III

Practical Aspects

5.1 Introduction

A variety of operating modes are possible for basic Joule-Thomson cryocoolers. These differ according to their component configuration and application, but may be generally classified on the basis of the following criteria:

- (a) The manner of pressurization, both in terms of configuration (open system, closed system, or open cycle), and with respect to the pressure source,
- (b) The purpose and expected performance; whether for a short run and especially if including a *fast cooldown*, or for long run,
- (c) The manner in which the cooling power of the Joule-Thomson cryocooler is enhanced; whether by another Joule-Thomson cryocooler, referred to as *staging*, or by a different kind of cryocooler (typically a closed cycle system) in which case it is referred to as a *hybrid* cryocooler.

These criteria are not independent and may indeed overlap. For example, staging is a possible strategy for achieving a faster cooldown. A continuously operating cryocooler may be pressurized by either an open or a closed system.

5.2 Pressurizing Alternatives

Joule-Thomson cryocoolers are driven by a supply of compressed gas. They consume the gas in a continuous (DC) flow, in contrast to other kinds of cryocoolers, which are driven by periodic changes of pressure and alternating (AC) flow. The present section focuses on the three common methods for pressurizing a Joule-Thomson cryocooler as displayed in Fig. 5.1 and the associated pressure sources.

5.2.1 The Open System

The most common manner of operating Joule-Thomson cryocoolers is in the open system configuration, with pure

coolants. As discussed in Sect. 8.10, it is also feasible to use mixed coolants in an open system configuration. Such systems enjoy simplicity and high reliability while avoiding the use of any moving elements.

5.2.1.1 The Layout

A cryocooler is fed by a pressure source that in most cases is a charged reservoir, as shown in Fig. 5.1a. An activation valve is usually located between the pressure source and the cooler and very often, a small container filled with desiccant is included. If the cooler is staged (Sect. 5.4) this layout is doubled; with separate components for each coolant.

The low-pressure stream is released to atmosphere, a characteristic that defines the system as open. As such, the down stream pressure of expansion, P_D , is determined by the atmospheric pressure at the location of the cooler but exists slightly above it. The boiling point of the coolant may vary at different altitudes, unless special attention is paid, as Bard et al.^{1,2} did, to actively control the exhaust pressure.

The reservoir is initially charged to as high as about 10–15 times the critical pressure. While the reservoir discharges, cooling persists as long as the pressure exceeds about twice the critical pressure. For any subsequent use, the vessel has to be replaced by a freshly charged one. Typically, logistics must be established to recharge the vessels and to supply them to the site of operation.

Note that a high pressure is essential for the open system mode of operation in order to store more gas in the reservoir and prolong the run period. A low-pressure reservoir storing the same quantity of gas would be unacceptably large.

5.2.1.2 The Pressure Source

The pressure source is very often a cylindrical vessel.³ High strength steel alloys are used to withstand the high charging pressure, which may sometimes be as large as 65 MPa, or even larger at elevated ambient temperatures. The weight of the storage vessel may be reduced to about half by constructing a spherical shaped reservoir³ or by employing

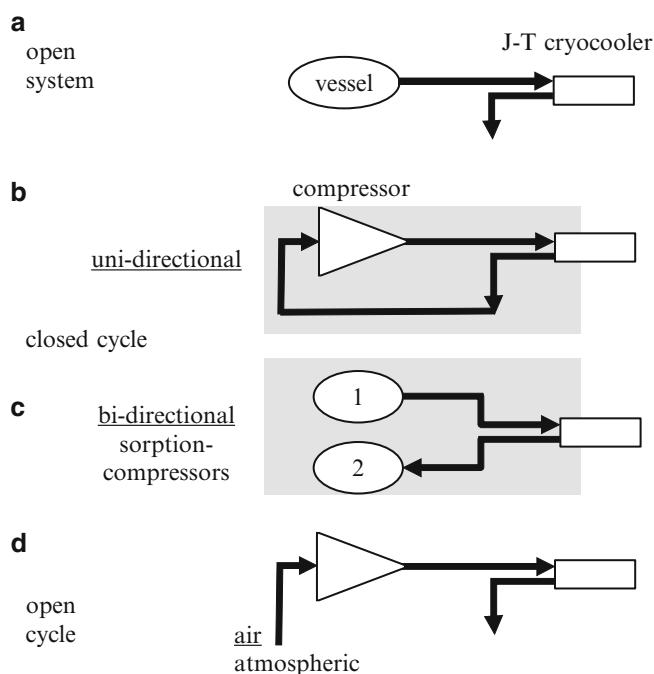


Fig. 5.1 The principal layouts and modes to pressurize a Joule-Thomson cryocooler

low-density high strength materials such as titanium^{4,5} instead of steel or composite materials.

Proposals to replace the pressure vessel with a gas generator are discussed elsewhere.^{4,6,7} Examples of active materials used to release nitrogen or oxygen are metal azides and chlorates or perchlorates of alkali or alkaline earth metals. Here, storage is accomplished at low pressure in order to reduce the weight of the system. When activated upon demand, a chemical reaction releases a large amount of gas within a confined volume, thereby elevating the pressure to feed the Joule-Thomson cryocooler. To the best of our knowledge, these represent the only published reports describing such cryocoolers.

5.2.2 The Closed Cycle

5.2.2.1 Configuration

In a closed cycle system, a pressure generator feeds the cryocooler, and the low-pressure exhaust stream returns to the suction port for subsequent compression, as shown in Fig. 5.1b. A fixed amount of coolant, initially charged into the closed cycle system circulates in a loop and undergoes cyclic changes in the domain of thermodynamic states. A significant distinction exists between the operation of the closed cycle and the open system. In the closed system, downstream conditions and properties affect the upstream

behavior, as will be discussed in Sect. 8.7. Such a dependence is impossible in the open system since the flow through the orifice is choked.

From the early stages of development, attempts were made^{8,9,10,11,12,13,14,15} to overcome the disadvantages of an open system by closing the cycle of flow around a Joule-Thomson cryocooler. In such a case, the cooling capacity may be controlled even with a fixed orifice by simply adjusting the compressor's speed^{16,17} rather than by trying to regulate the flow at the cold end of the cryocooler.

The less common, bi-directional closed cycle configuration as shown in Fig. 5.1c, operates with two (1 and 2) opposing sorption compression chambers.^{18,19} The content of chamber 1 is released through the cryocooler and accumulates in chamber 2. Subsequently, chamber 2 discharges through the cryocooler in the opposite direction. Such bi-directional operation is enabled by the symmetric character of the JT expander.

5.2.2.2 The Potential Advantages

The potential advantages of a closed cycle in comparison to the open system are:

1. The effort of replacing pressure vessels is eliminated. Instead, cooling is maintained by a continuous supply of electricity or heat to the pressure generator.
2. Especially for long runs, the incorporation of a continuously operating pressure generator (compressor) enables a more compact system. However for short runs, the option of a pressure vessel may be more compact.
3. The coolant is recycled instead of being wasted. Preserving the coolant becomes a significant issue especially for rare gases such as helium-3, as discussed by Duant and Lerner²⁰ and Wilkes.^{21,22} Geist and Lashmet^{11,12} and Tward et al.²³ built closed cycle cryocoolers for helium-4 temperatures, and Koeppe²⁴ for neon.
4. It enables the use of coolants that should not be released to the atmosphere, such as hydrogen, because of its flammability.^{9,10,25}
5. System failure due to clog formation is less likely. The fixed amount of coolant and an efficient cleaning procedure for the hardware typically suppresses the level of contamination.
6. The ambient atmospheric pressure is irrelevant to the operation of a closed cycle. The downstream pressure, and therefore the operating temperature of any coolant, is dependent on the parameters of the closed loop but not on the ambient pressure.

5.2.2.3 Two Versions

Two versions of closed cycle cryocoolers are common. The first employs a pure coolant such as nitrogen, and requires a double or triple staged compression to reach values of at least 10 MPa (or about $3P_C$). For example, Lester,

Levenduski et al.^{26,27,28,29,30,31,32,33} developed a nitrogen space-borne system. It is discussed in more details in Sect. 5.4.9. The nitrogen cooler of Hedegard³⁴ provides another example.

The second version of closed cycle systems is based on mixed refrigerants that exhibit sub critical operating pressures as low as 2 MPa, and specific cooling content comparable to those of pure coolant systems operating at much higher pressures. The low operating pressure is a clear advantage in any sense and can be handled by a single stage compressor. Chapter 8 is entirely devoted to this category of cryocoolers.

5.2.2.4 The Pressure Generator

The Mechanical Compressor

The majority of compressors used in closed-cycle systems are of the mechanical kind, driven by an electrical motor, with either a rotary or linear (“free”) piston without or with flexure springs.³⁵ The rotary type may be lubricated to reduce wear, cost, and prolong the lifetime. However, it requires proper management of the lubricant.

The Sorption Compressor

Some applications such as those in space are much more demanding in terms of lifetime and reliability. For this purpose, sorption compressors, which do not have moving elements, are a proper solution. These are broadly discussed elsewhere.^{36,37,38,39,40} Chemical sorption is the primary mechanism recommended^{41,42} to run a single stage cryocooler, but other mechanisms, such as physical sorption, or both arranged for multi-stage cryocooling,^{43,44,45} and mixed coolants⁴⁶ are also utilized. The successive strokes of sorption and de-sorption of a given amount of coolant in a confined volume, driven by periodic warming operations are analogous to the charge and discharge strokes of a mechanical compressor. The level of compression is typically somewhat below the critical pressure. The parameters of a sorption compressor are: the adsorption capacity of any adsorbent with respect to any coolant, the desorption temperature, the number of pairs of sorption chambers, the amount of adsorbent, and the void volume.

The Electrochemical Compressors

The electrochemical compressors, driven by voltage, are another type of pressure generator without moving elements. One may find examples of such systems in the literature where the pressurization of hydrogen or oxygen is accomplished by a ceramic electrolyte membrane,^{47,48} proton conductive membrane⁴⁹ polymer electrolyte membrane,⁵⁰ sintered ceramic membrane⁵¹ and even two stage Joule-Thomson cryocooling. In these, the compressed oxygen pre-cools the compressed hydrogen.⁵²

5.2.3 The Open Cycle

In contrast to the previous versions of pressurization, the open cycle operates strictly with atmospheric air, which is a free, ubiquitous coolant. The mechanical compressor of the open cycle system takes in atmospheric air, elevates its pressure and feeds a cryocooler. The expanded air is released to the atmosphere, while the fresh supply of incoming air is continuously compressed. Benson⁵³ describes such a system installed in an air-to-air missile launcher. It includes a purification unit for removing water vapor and carbon dioxide. The operating pressure is as high as 29 MPa, since air, as a coolant is similar to nitrogen. In this example, the inlet pressure is a function of the altitude, and the achievable flow rate is sharply reduced during the high altitude portion of the flight. A similar commercial system is described elsewhere.⁵⁴

Chorowski et al.⁵⁵ compressed atmospheric air to about 1 MPa. The compressed air stream was pre-cooled to about 101 K, which is below T_{CD} , and then throttled without undergoing a recuperating process, similar to the free jet release described in Sect. 5.4.7. The pre-cooler utilized a mixed refrigerant closed cycle as described in Sect. 8.3.5.6.

Thornton⁵⁶ proposed the use of a double cassette desiccant with a self-cleaning procedure to enable continuous operation, a feature of great significance for the open-cycle cryocooler since the air is available in unlimited quantities from the atmosphere.

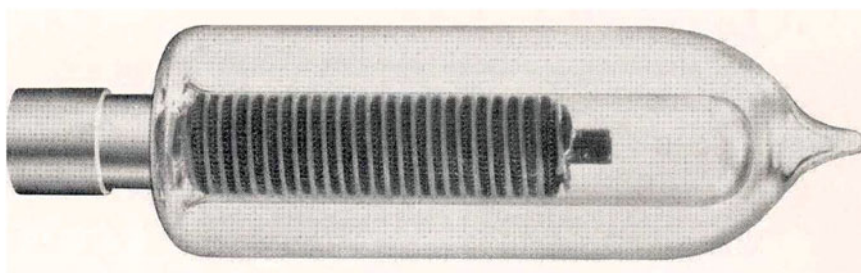
5.3 Continuous Operation Cryocoolers

5.3.1 Introduction

Miniature cryocoolers that are designed for long-term operation and that can be turned on and off many times may be categorized as continuous operation cryocoolers. The central design feature of these cryocoolers is their extremely low gas consumption. They accommodate a slower cooldown process to promote a reduction of the steady state flow rate. The key features necessary for extending the run time of a given pressure vessel are twofold. First, the heat load of the dewar and cryocooler assembly should be minimized. Second, one must carefully consider the consumption of gas in order to satisfy heat load requirements. As discussed below, there are three main strategies for supplying the flow rate:

- A variable flow rate, adjusted according to the instantaneous operating conditions (pressure in the vessel and heat load requirement),
- A constant flow rate (over the entire range of pressure in the vessel), at the lowest value which is still large enough to balance the maximum heat load, and,

Fig. 5.2 A miniature Joule-Thomson cryocoolers inside a glass dewar, The Hymatic Engineering Co., Ltd., Redditch, Worcestershire, UK



(c) A periodic flow rate used to charge a solid-cryogen thermal storage device.

5.3.2 Characteristics

The following typical features characterize this category of miniature Joule-Thomson cryocooling:

5.3.2.1 Pressurization

Pressurization of this kind of miniature cryocoolers is very often (though not always) maintained by a rechargeable vessel of a few liters, designed to run for not more than about 10 h.

5.3.2.2 A Highly Evacuated Dewar

A highly evacuated dewar encapsulates the cryocooler and cooled object to minimize convective heat loads. Very often dewars are made of glass as photographed in Fig. 5.2, with the cooler directly inserted into the glass construction to minimize the conductive heat load. A cross section of such an assembly used for cooling an infrared detector is shown in Fig. 5.3. The overall thermal load for a typical cold finger with a 5–8 mm diameter and operating between 300 K ambient and 80 K is in the range of 150–300 mW and is generated by both the cold finger of the dewar and the cryocooler itself.

5.3.2.3 A Long Heat Exchanger

A long heat exchanger is used to provide enough heat transfer area to ensure a high effectiveness, ε . This way, the efficiency, η , is higher as explained in Sect. 3.5.3 and the specific cooling capacity is higher. Therefore, the gas consumption decreases. The room temperature effectiveness of the heat exchanger is typically higher than 95%. However, at elevated ambient temperatures such as 350 K, the required extent of recuperation becomes larger and the effectiveness decreases.

5.3.2.4 Small Heat Capacity

Small heat capacity is sacrificed for the sake of reducing heat leaks. The thermal capacity of the dewar and cryocooler

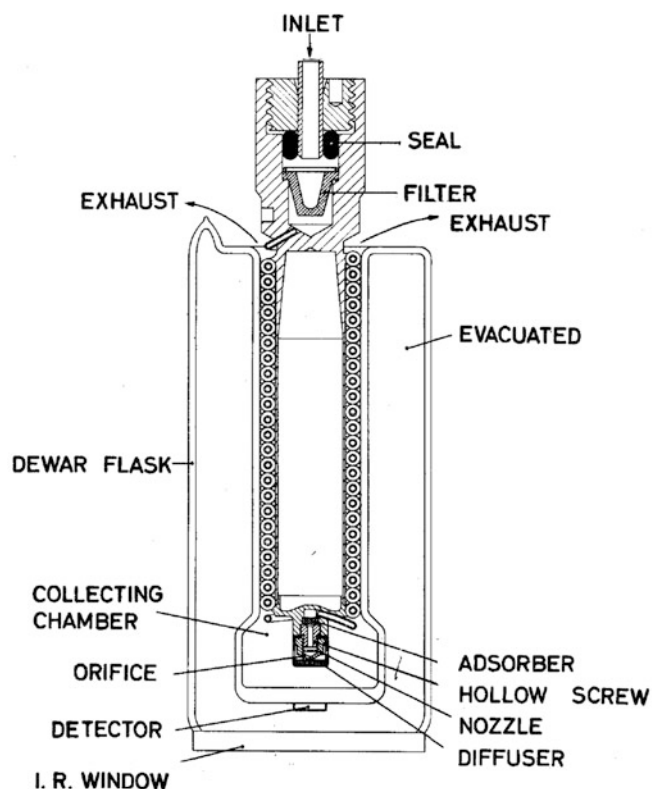


Fig. 5.3 An assembly of a Joule-Thomson cryocoolers with a dewar for cooling an infrared sensor (The Hymatic Engineering Co., Ltd., Redditch, Worcestershire, UK)

assemblies is relatively high, at least in comparison to the fast Joule-Thomson cryocoolers. Typically, for 5–8 mm diameter cryocoolers, values fall in the range of 150–400 J and are affected by the ambient temperature.

5.3.2.5 Cooldown Periods

Cooldown periods are relatively long, at least when compared with the best values achievable using Joule-Thomson coolers. However, this feature may be regarded as a sacrifice for the sake of reduced gas consumption. System level designers of continuous operation cryocoolers are tolerant of cooldown periods lasting about 25 s for argon systems and even up to 60 s for nitrogen systems. Nevertheless, these Joule-Thomson cryocoolers cool down faster, at least by an

order of magnitude, than any other closed cycle cooling machines.

5.3.2.6 Coolants

Coolants mostly common in practice are argon and nitrogen. Deeper cooling, below the boiling point of nitrogen, necessitates a binary mixture of nitrogen with neon⁵⁷ or with helium, but this reduces the run time below that of a pure nitrogen cooler.

5.3.2.7 High Purity Gases

High purity gases have to be supplied for a reliable, long run and to sustain a stable, non-fluctuating temperature in the evaporator and on the payload.

5.3.3 Flow Regulation by Adjusting the Throttle Size

Very often, cryocoolers that incorporate a flow regulation mechanism are referred to in the literature as *flow demand* coolers. The term highlights the fact that flow is adjusted actively and dynamically to match the demand rather than being pre-determined. The required flow rate is adjusted by changing the size of the throttle, for example, by changing the position of a needle in a hole. Long-term reliable operation of such a cryocooler is strongly influenced by its capability to correctly regulate the flow rate.

The most common flow regulated cryocoolers are configured as a cylindrical finned tube heat exchanger of 5–10 mm diameter with a bellows type regulator installed at the cold end, as described in Sect. 6.2.2. Practical aspects of flow regulation mechanisms are listed in Sect. 6.2.1.

5.3.3.1 Performance Criteria

Flow is regulated to compensate for varying operation conditions. Three aspects are generally of special concern:

1. Gas consumption,
2. Temperature stability, and
3. Capability to mitigate clog formation. Here a rapid response for removing any frozen particulate is desirable to avoid even the smallest temperature perturbations.

5.3.3.2 Operating Conditions

Operating conditions affect the consumption of a given coolant through a combination of the following parameters:

1. The quality, or vacuum integrity, of the dewar,
2. The ambient temperature, inasmuch as it affects the heat load, and,
3. The instantaneous pressure and temperature of the feed coolant, which determine its specific cooling content, Δh_T .

5.3.3.3 The Ideal Run

The ideal run is the longest one for which the flow rate, $\dot{n}$, is perfectly adjusted so that it exactly satisfies the basic energy balance equation of the cryocooler, $\dot{Q} = \dot{n} \cdot \Delta h_T$, for any instantaneous pressure in the discharging vessel, and at any environmental temperature and heat load, $\dot{Q}$. The flow rate, $\dot{n}$, at any inlet conditions (T, P), is inversely proportional to Δh_T , at a given $\dot{Q}$, namely,

$$\frac{\dot{n}_1}{\dot{n}_2} = \frac{\Delta h_T(T, P_2)}{\Delta h_T(T, P_1)} \quad (5.1)$$

Therefore, at higher pressures (which still are below the inversion pressure), lower flow rates are expected. Note that for systems with a fixed orifice, higher pressures produce higher flow rates. In a regulated system, the flow rates increase as the pressure decreases, and the rate of pressure decay also increases.

The total cooling potential of a 1-l pressure vessel charged with various gases was determined in Sect. 4.8. For example, a 1-l vessel of nitrogen at 40 MPa and 300 K may provide 8.9 kJ of refrigeration (Table 4.9) while it discharges down to ambient pressure. If, for this case the required cooling rate is 0.2 W, the vessel will last for 12.36 h. Practically, cooling ceases when the pressure decreases to about 4–8 MPa, but 12.36 h is still a fair estimate for the maximum obtainable run time. At 40 MPa and 340 K, the 1 l vessel will suffice for only 8.05 h of operation. However, by using argon under similar conditions, the same cooler will run for 14.6 h, longer even than the system with nitrogen at 300 K.

For the same cooling power of 0.2 W, the instantaneous flow of nitrogen at 30 MPa will increase from 0.24 to 0.37 SLPM as the temperature of the nitrogen increases from 25°C to 50°C. If the cryocooler and dewar are also at 50°C, the heat load can increase to 0.25 W, with a resulting flow demand of 0.46 SLPM.

Maytal⁵⁸ derived a closed form approximation for the cooling potential of a pressure vessel in the case of ideal flow regulation while the pressure vessel discharges through a cryocooler. The analysis applies for pressures below 60 MPa and is explained in Sect. 4.8.2. The actual run time is bounded by the expression,

$$t < \frac{K P_K V}{2 R T \dot{Q}} \left[\left(\frac{P}{P_K} \right)^2 - \frac{2}{3} \left(\frac{P}{P_K} \right)^3 \right] \equiv t^{MAX} \quad (5.2)$$

where P is the initial charging pressure, V is the volume of the vessel. K and P_K are parameters specific to each gas that describe the approximately parabolic pressure dependence of Δh_T , as discussed in Sect. 4.8.2 by Eq. 4.103. For nitrogen

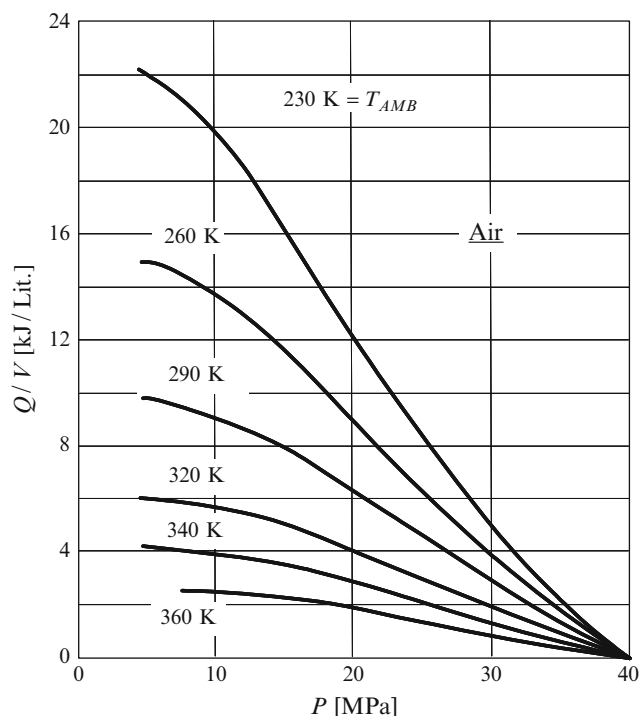


Fig. 5.4 Total cooling capacity of a self regulating cryocooler fed by a 1-l pressure vessel of air (Hart,⁵⁹ The Hymatic Engineering Co., Ltd., Redditch, Worcestershire, UK)

at 295 K these two parameters are 4,480 J/mole and 80 MPa respectively. For argon, under the same conditions, K and P_K are respectively 8,215 J/mole and 100 MPa.

5.3.3.4 Actual Gas Consumption

Actual gas consumption remains within 90% or better of the ideal run values. Figure 5.4 displays the measured performance of a commercial regulated cryocooler as described by Hart⁵⁹ at various ambient temperatures while being fed by a 1-l pressure vessel filled with air. Note that the limit of such curves with perfect flow regulation is discussed and derived in Sect. 4.8 and plotted for nitrogen in Fig. 4.39. Air has a slightly elevated cooling content in comparison with nitrogen. More about the discharge of a vessel through a self-regulated cryocooler may be found elsewhere.^{60,61}

In practice, some gas may be wasted if the flow regulating mechanism is slow to reduce the high flow rates associated with the cooldown process after the desired temperature is achieved.

5.3.3.5 Argon Versus Nitrogen

In practice, the following rule is generally accepted: nitrogen is used if it is necessary to reach the 80 K range, otherwise argon is used whenever the cooled object tolerates the 90 K range. Argon's normal boiling temperature, 87.3 K, is only 10 K above that for nitrogen. Therefore, it is worthwhile to

Table 5.1 Comparison of N₂ and Ar at 300 K

	$P = 70 \text{ MPa}$				$P = 40 \text{ MPa}$		
	N ₂	Ar			N ₂	Ar	
ΔT_h [K]	27.9	93.1	($\times 3.3$)		38.5	92.3	($\times 2.4$)
$y = \Delta h_T/L$	0.15	0.29	($\times 1.9$)		0.20	0.29	($\times 1.5$)
T_{CD} [K]	161	224			172	229	
ΔT_{CD} [K]	139	76	($\times 0.6$)		128	71	($\times 0.6$)
Δh_T [kJ/mole]	0.81	1.94	($\times 2.2$)		1.12	1.92	($\times 1.7$)
$\Delta \rho$ [mol/Lit]	13.5	16.6	($\times 1.2$)		8.92	10.91	($\times 1.2$)
from P to 10 MPa							
Q/V [kJ/Liter]	13.7	27.5	($\times 2.0$)		8.09	14.83	($\times 1.8$)
from P to 10 MPa							

summarize the significant benefits of argon as listed in Table 5.1.

The specific cooling content, Δh_T , and the superior compressibility of argon result in a total volume-specific cooling capacity, Q/V , about 70% higher than that of nitrogen. Therefore, the duration of a flow-regulated system as determined by a given vessel size charged with argon is correspondingly increased. In addition, argon cools down faster than nitrogen.

The cut off pressure is another factor that influences the operating duration of a cooler supplied with gas from a fixed-size pressure vessel. The cut off pressure for argon is lower than that for nitrogen, thus prolonging the operating period. With similar flow rates and encapsulation, a lower pressure of argon will adequately support the same heat leak and still maintain cooling. This is merely a reflection of the higher value of Δh_T for argon at a given pressure, or the lower pressure for a given value of Δh_T .

5.3.3.6 Flow Rates

Flow rates of nitrogen or argon consumed under steady state operation range from 0.2 standard liters per minute (SLPM) as an extremely low regulated value, up to 1.0 SLPM as a high one. Cooldown flow rates are significantly higher than the steady state regulated values, in order to enable cooldown within a reasonable period of time. A typical flow rate to cool down a miniature cryocooler is about 20 SLPM. In terms of the overall system performance, a useful rule of thumb is that each cooldown process is equivalent to a steady state run of about 30 min. Therefore, when anticipating an interruption of cooling demand one has to be aware of the risk of turning off the cooler since the subsequent cooldown might shorten the mission by about 30 min.

5.3.3.7 Temperature Stability

Temperature stability is influenced by the gain factor of the flow regulator. A moderate gain factor is desirable. Such a

feature mitigates overshooting or under-shooting of the flow, that would otherwise result in temperature fluctuations on the payload. A degree of proportional regulation is preferred, rather than a pure on-off response, sometimes referred to as a bang-bang regime. In some applications, a stability limit of 1 K is required on the payload, this being regarded as a severe constraint. Maintaining a temperature control in the range below 3 K stability is quite normal.

5.3.3.8 Temperature of Operation

Temperature of operation is slightly above yet quite close to the normal boiling temperature of the applied coolant. A relatively low backpressure is induced by the well-regulated flow rates. In practice, the increase of boiling temperature is observed to be less than 2 K.

5.3.3.9 Precooling

Precooling is another aspect associated with minimized gas consumption and the reduction of the weight and volume of the whole system. An additional improvement would be to involve another gas of elevated cooling content and boiling point (such as CF_4 or Kr) to precool the final coolant (nitrogen or argon). One gains a higher cooling content of the final coolant, which in turn allows for lower regulated flow rates. Giles,⁶² Nicholds,⁶³ and Pierson⁶⁴ presented such a system that also regulated the precooling flow rates. The benefit becomes more significant at higher heat loads such as are associated with a two-stage cryocooler. This topic will be discussed in Sect. 5.4.

5.3.3.10 Technology of Heat Exchangers

The vast majority of continuously operating coolers incorporate traditional finned tube heat exchangers rather than utilizing the newer technology developments such as photolithography,^{65,66,67,68} sintering by Bodio^{69,70} and Chorowski,⁷¹ or a wire-mesh matrix by Steyert⁷² and by Longworth et al.⁷³ Traditionally, the mostly commonly used heat exchangers are cylindrical with either a single diameter, or stepped with two diameters. Alternative technologies have not been adapted for flow demand miniature cryocoolers for the following reasons:

1. Complications arise in trying to install the regulating mechanism,
2. The self-induced heat leak of alternative configurations and technologies is often higher.

5.3.3.11 The Cut Off Pressure

The cut off pressure is the lowest pressure that still maintains the temperature of operation before payload starts warming up. Its value is associated with the actual heat load and the highest attainable (non regulated) flow rate. Due to the vacuum isolation, it is relatively low and typically stays in the range of 4–6 MPa.

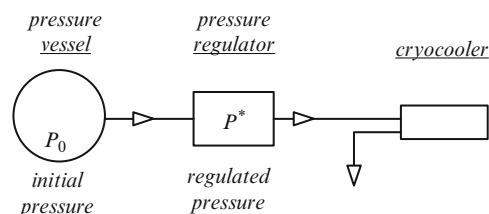


Fig. 5.5 Layout of a pressure regulated operation of a cryocooler fed by a vessel

5.3.4 Constant Flow Rate Discharge

In the constant flow rate configuration, the cryocooler is separated from the vessel by a pressure regulator as shown in Fig. 5.5. The flow rate is constant because the nozzle opening of the cryocooler is fixed and the cooler is driven by the constant pressure of the regulated value, P^* , that is independent of the instantaneous pressure in the reservoir. The level of flow rate is the minimum necessary value to balance both the highest possible heat load and, at the same time, the highest possible temperature of the gas in the reservoir. Some coolant is wasted whenever the circumstances are not at such extremes.

This strategy is discussed by Buller⁷⁴ and Longworth.⁷⁵ Hughes and Herr⁷⁶ describe such a Joule-Thomson system, that was the first cryocooler mission in space on Mariner 6 and 7, in July-August 1969. In this case Hydrogen is precooled by nitrogen and both coolants operate with regulated pressures. Hydrogen with an initial charge pressure of 37 MPa is reduced to 11.5 MPa, and nitrogen with an initial charge pressure of 40 MPa is regulated to 18.3 MPa. Ross reports⁷⁷ that the nitrogen stage on the Mariner-6 system failed to operate.

5.3.4.1 The Cooling Capacity

The cooling capacity defined in terms of the total heat rejected, $\dot{Q}$, by a pressure vessel of volume, V , depends on the regulated pressure, P^* . The operation terminates when the instantaneous pressure in the vessel reaches P^* . Since this is the lowest pressure of the lowest Δh_T , the flow rate has to be properly chosen to satisfy the energy balance of the heat load,

$$\dot{n} = \frac{\dot{Q}}{\Delta h_T(P^*)} \quad (5.3)$$

Actually, this condition dictates the necessary size of the fixed orifice. Following Sect. 4.8, formulating the specific cooling potential of a pressure vessel initially charged to P_0 takes the form,

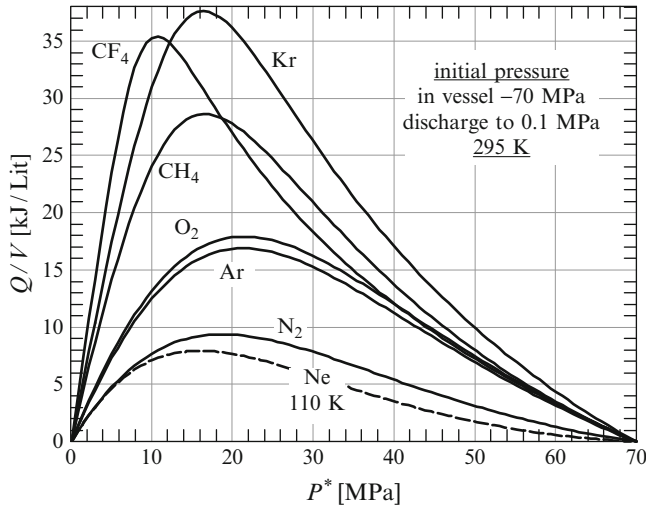


Fig. 5.6 Total available cooling capacity obtained by a single liter of pressure vessel when the Joule-Thomson cryocooler is fed by regulated pressure, P^* , for various gases

$$\frac{Q}{V} = \left[\int_{P^*}^{P_0} \Delta h_T \cdot d\rho \right]_T = \left[\int_{P^*}^{P_0} \Delta h_T(P^*) \cdot d\rho \right]_T \quad (5.4)$$

Since

$$\Delta h_T(P^*) = \text{const}$$

then,

$$\frac{Q}{V}(P^*) = \Delta h_T(P^*) \cdot [\rho(P_0) - \rho(P^*)]_T \quad (5.5)$$

5.3.4.2 The Optimal Regulated Pressure

One may expect an optimal value of the regulated pressure that maximizes Q/V , since it is a product of two terms, both of which are dependent on P^* , but while one increases, the other decreases. The optimal behavior is demonstrated in Fig. 5.6. For an initial pressure of 70 MPa, the optimum regulated pressures, P^* and the associated integral cooling capacities are listed in part (a) of Table 5.2. Only about half of the theoretical cooling potentials are utilized by the pressure regulated mode.

One should note that the optimum value of P^* depends on the initial charge pressure. A lower value of P_0 results in lower optimum values of P^* at the same ambient temperature. Buller⁷⁴ focused on this issue; however, his analysis did not include the pressure dependence of compressibility.

Table 5.2 Optimal regulated pressure, P^* , and the associated specific cooling capacity, Q/V , compared with the ideal flow regulated regime

(a) Initial charging pressure at 300 K is 70 MPa

	Pressure regulated		Flow regulated
	P^* [MPa]	Q/V [kJ/Lit]	Q/V [kJ/Lit]
Nitrogen	19.2	8.9	14.5
Argon	22.1	16.3	28.6
Methane	17.1	22.1	48.2

(b) Initial charging pressure of 40 MPa

	Pressure-regulated		Flow-regulated
	P^* [MPa]	Q/V [kJ/Lit]	Q/V [kJ/Lit]
Nitrogen	300 K 14.2	5.3	8.9
Argon	300 K 16.3	9.2	15.9
Hydrogen	85 K 9.1	5.3	7.2
	90 K 9.3	4.7	6.3
Neon	85 K 11.9	9.7	14.6
	90 K 12.2	8.4	17.4

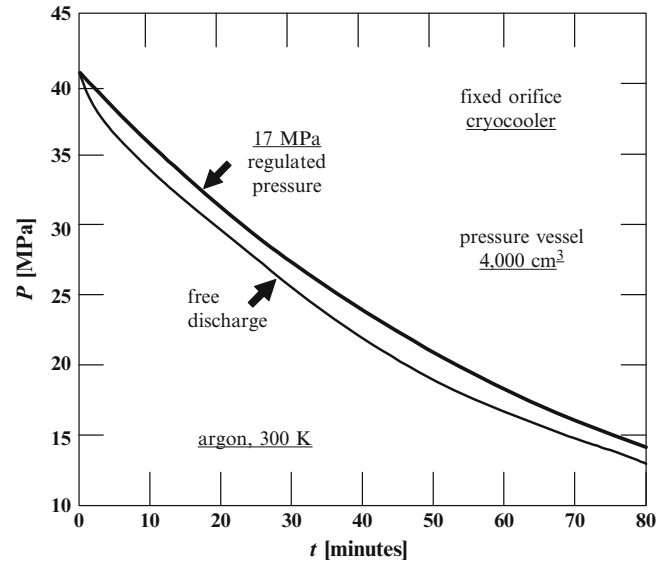


Fig. 5.7 Comparison of the free discharge pattern of a pressure vessel (40 MPa) to the discharge by regulated pressure (17 MPa) of the same fixed orifice Joule-Thomson cryocooler after Maytal⁷⁸

5.3.4.3 Comparison with Non-regulated Discharge

Comparison with non-regulated discharge (direct blow down). The operating mode with the highest gas consumption is that without any regulation, in which operation occurs by direct discharge (blow down) of the pressure vessel through the fixed orifice of a cryocooler. Eliminating the pressure regulator provides simplicity. In addition, as Maytal⁷⁸ shows, the benefit of regulating at a constant and relatively low pressure, as compared to the direct blow down approach is minor. Figure 5.7 displays two discharge patterns $P(t)$, with the same fixed orifice cooler and the same vessel.

The first employs pressure regulation and the other one does not. One may observe that the two lines are quite close. The regulated pressure generates a constant flow rate and therefore its time dependent behavior, $P(t)$, resembles a linear decay. The instantaneous flow rates associated with the free discharge are driven by the instantaneous pressure. Even so, the associated pressure decay is not exponential because the recuperative flow rates are not proportional to the instantaneous pressure. At elevated pressures, the flow rate grows less strongly than proportional with pressure (see Fig. 9.4), as discussed in Sect. 9.2.3.

5.3.4.4 The Cooldown

The cooldown period for a fixed orifice and heat load depends on the regulated value, P^* . The low values of P^* that optimize the run time are typically associated with low values of $\dot{n}$, Δh_T , and T_{CD} . The resulting cooldown periods can also become unacceptably long. Therefore, one should seek a compromise by regulating the pressure above the optimal value in order to accelerate the cooldown process.

Hughes⁷⁹ dealt with this issue by supplying a higher elevated pressure only during cooldown. He employed a second pressure regulator for each gas of his two-stage nitrogen/hydrogen system to provide a higher pressure during cooldown. Hughes then switched to the primary regulator to utilize the economically optimum lower value, P^* during the remainder of the operation.

5.3.5 Periodic Flow Rate and a Thermal Storage Device

Thermal storage devices provide a strategic benefit in cases where a large (enough) difference exists between the average heat load and a peak instantaneous cooling demand. In such a case one may size the system for the average demand, and utilize the thermal storage device to ‘shave’ the peaks. Without a thermal storage device, one must provide a system sufficiently large to satisfy the peak demand. However, most of the time it would have to maintain a continuously adjusted very low flow rate and cooling power. Thermal storage is accomplished by solidifying a bulk of cryogen that can subsequently provide cooling on demand. Flow is supplied for a certain period until the storage reservoir is full, and then is stopped until the next refill.

The evaporator of an open cycle Joule-Thomson cryocooler is replaced by a porous element that serves as a phase separator. As the expanding jet of the two-phase cryogen exits the nozzle into the evaporator, the liquefied portion is trapped in a porous media while the gaseous phase flows through and returns to recuperate. As the liquid accumulates and fills up the porous reservoir, the high-pressure incoming flow is stopped. Then, the reservoir is

exposed to a low pressure environment so that the saturated vapor pressure is pumped down, below the triple point value. This results in solidification of the liquid plenum. Cryocooling is maintained by sublimation instead of evaporation. Reported coolants for this purpose are argon, nitrogen, hydrogen, and neon.

The solid cryogen thus obtained “stores” the excess of cold that will be subsequently utilized to support the continuous heat load. As the solid absorbs heat, it turns into vapor, or sublimates. The temperature remains stable (below triple point) and is solely determined by the corresponding pressure, a process completely analogous to the cooling effect associated with evaporation, when the boiling temperature is determined by the imposed pressure.

An alternate version of the same idea utilizes the cooling power of a Joule-Thomson cryocooler to condense a separate flow stream from another reservoir, possibly even of another gas. The condensed bath is exposed to a vacuum and solidified as described above. Figure 5.8 displays a schematic of the two versions, each of which has its own advantages and drawbacks as explained in reference.⁸⁰

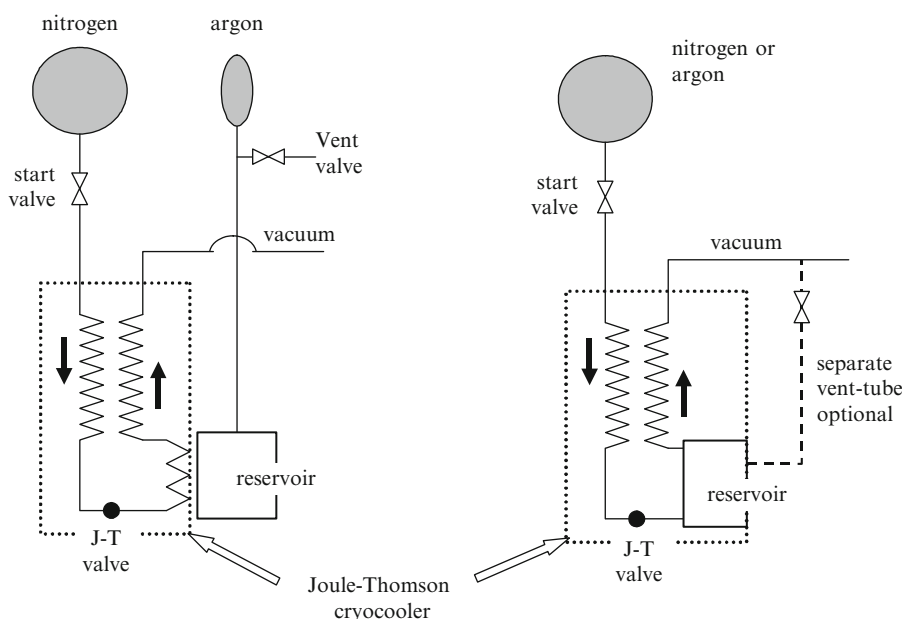
Space applications enjoy an unlimited capability for extensive vapor pumping, simply by exposing the vent port to outer space. They provide the natural (and utilized) niche for applying the thermal storage idea.^{80,81}

The matrix within a thermal storage unit is specially constructed⁸² as a wick for a liquid cryogen, based on the surface tension of liquid. Longworth explains the construction of a collecting system⁸³ and the associated venting arrangement.⁸⁴

Warner et al.⁸⁵ proposed to cool an infrared detector of a missile guidance system via the thermal capacity of a solid cryogen. The cryocell is filled with a metal foam of aluminum and copper. A Joule-Thomson cryocooler lowers the temperature of the structure to the cryogen’s triple point. For a similar purpose, Pflibsen⁸⁶ describes a “Solid cryogen cooling system for focal plane arrays” of infrared detectors. The internal pressure vessel that initially maintains the cryogen in the liquid state, discharges after launch resulting in a lower pressure and thereby creating a solid bulk of cryogen (in a beryllium container) which maintains cooling throughout the entire mission.

The above discussion has focused on open cycle cryocooling. However, closed cycle systems may also drive similar thermal storage units. Arkhipov, Bondarenko and others^{87,88,89} describe a space application in which a mixed coolant Joule-Thomson closed cycle (discussed in Chap. 8) or Stirling cooler⁹⁰ is used to solidify a cryogen inside a thermal storage unit. Here the closed cycle operates periodically instead of running continuously throughout the mission. A similar attempt is reported with a pulse-tube^{91,92} as well as other cryocoolers.⁹³ One may find additional information regarding the use of thermal storage units elsewhere.^{94,95,96,97}

Fig. 5.8 Two versions to accumulate solid cryogen by Joule-Thomson liquefaction process used to cool by sublimation after Longsworth⁸⁰



5.4 Multi-stage Cryocoolers

5.4.1 Introduction

5.4.1.1 Chronological Note

The two experiments associated with the beginning of the discipline of cryogenics, both occurring in December 1877, were based on staging.⁹⁸ It was Caillietet who precooled oxygen to 244 K using SO_2 , and Pictet who used a precooling cascade of SO_2 and CO_2 . Six years later, Wroblewski and Olszewski employed ethylene to precool air at 137 K. The pioneers of cryogenics were already aware of the advantages afforded by staging with Linde-Hampson liquefiers. For example Carl von Linde's original machine (1895) was a staged dual pressure liquefier⁹⁹ with two steps of expansion. Professor Dewar precooled an air liquefier by an additional stage of carbon dioxide⁹⁹ in 1896 to increase yield. Kamerlingh Onnes precooled his air liquefier with a cascade of methyl chloride, ethylene and nitrogen. Keesom¹⁰⁰ in 1933 analyzed the efficiency of a three stage Joule-Thomson cryocooler. The final methane stage was precooled by the sequential liquefaction of ammonia and acetylene; each stage operating as a compressor-driven closed cycle (see Fig. 8.19). Keesom showed that such a system liquefies nitrogen more efficiently than the Linde cycle precooled by ammonia, and even more than the Claude cycle, that incorporates an expander to extract external work. The initial process for liquefying natural gas utilized the same idea, employing a three-stage cascade process, and successively liquefying at each stage, propane, ethylene

and methane. Staged cooling systems have evolved further to include the use of mixed coolants instead of a series of pure coolants; as discussed in Chap. 8, with the autocascade approach of Kleemenko,¹⁰¹ Fuderer,¹⁰² and Missimer.¹⁰³

5.4.1.2 The Regions of Precooling

Two or more Joule-Thomson cryocoolers may be combined together in a cascade so that each one precools the next one. The schematic notation of a Joule-Thomson cryocooler in Fig. 5.9a displays its pair of recuperative streams, the expansion nozzle and the evaporator with the bath of liquefied cryogen. The precooling temperature, T_{PRE} , of the incoming stream of stage 2 results from the operation of a prior stage 1. Part (b) of the figure illustrates the regions used for precooling. At least one of the two regions of the precooling stage (No. 1) is used to reduce the temperature of the high-pressure stream of the next stage (No. 2):

Region A: The bath of the liquefied precoolant in the evaporator, and,

Region B: The returning expanded cold stream of the precoolant (1), counter flowing and cooling the high-pressure stream (2).

In a unique and fairly rare example described elsewhere,¹⁰⁴ a bath of cryogen (region A) precools the low pressure returning cold stream instead of the incoming stream (also discussed in Sect. 6.4.5.1).

Staging is further discussed in Sects. 5.5.5.2 and 5.5.5.3, and in Sects. 6.3 and 6.4. Some practical examples of miniature staged cryocoolers are available elsewhere.^{9,10,68,105,106,107,108,109,110,111,112}

Fig. 5.9 Schematic notation of a Joule-Thomson cryocooler in (a) and the scheme of precooling in (b). The gray shapes note the possible regions for precooling, A and B

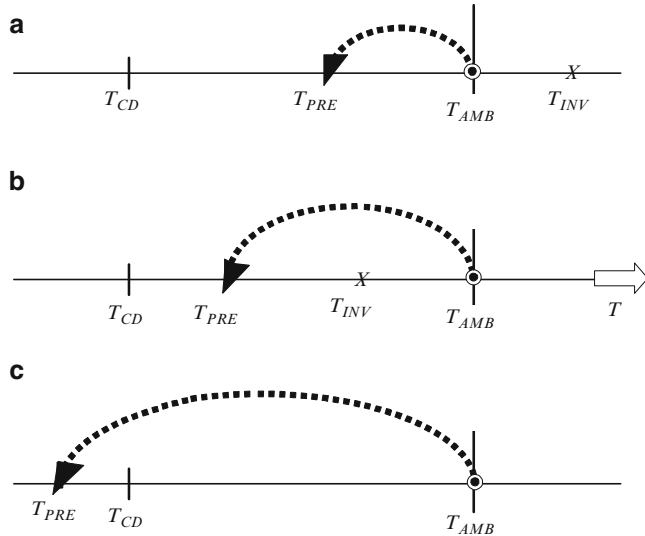
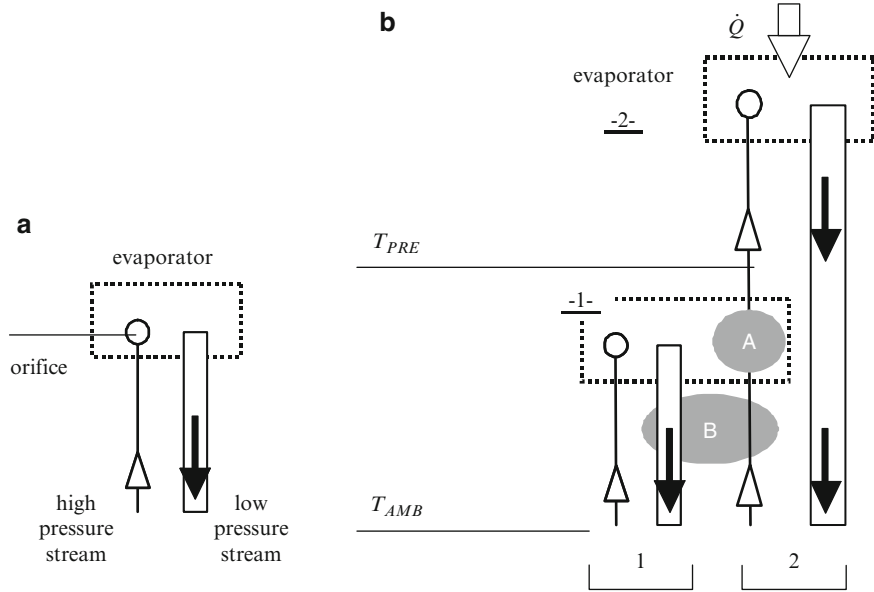


Fig. 5.10 Three categories of staged Joule-Thomson cryocoolers with the following equivalent effects: (a) merely lowering T_{AMB} , and lowering T_{AMB} below (b) T_{INV} or below (c) T_{CD}

5.4.2 Categories of Staging Joule-Thomson Cryocoolers

There are three significant temperatures associated with the coolant of a Joule-Thomson stage, which are from high to low: the ambient, the inversion and the cooldown temperatures, T_{AMB} , T_{INV} , and T_{CD} , as discussed in Chap. 4. The relation of the precooling temperature, T_{PRE} , and these three temperatures, may serve as the basis for classifying the variety of combinations of staging in three categories (Fig. 5.10) as follows.

5.4.2.1 $T_{PRE} < T_{AMB} < T_{INV}$; Operational Benefits

Simply lowering the effective ambient temperature of the final stage enables a series of significant improvements:

1. The efficiency of cryocooling (COP) is elevated, due to two reasons. To understand, let the boiling point of the precooling stage be $T_{BOIL, 1}$, which is higher than that of the next (final) stage, $T_{BOIL, 2}$. First, the task of precooling down to the intermediate temperature, $T_{BOIL, 1}$ is accomplished by the precoolant which, having a higher boiling temperature will also have a higher COP as explained in Sect. 3.13.1 and demonstrated in Figs. 3.29 and 3.32. Second, the final coolant enters the recuperator at a lower temperature (T_{PRE}), which is somewhat above $T_{BOIL, 1}$, thus gaining a larger value of Δh_T , as explained in Sect. 4.3.6 and Figs. 4.17 and 4.18.
2. The cooldown phase is accelerated (discussed in Sect. 5.5),
3. The weight and volume of the entire system is reduced,
4. The operating pressure (or the compression ratio in the case of a closed cycle) is potentially reduced,
5. Cooling is provided at various intermediate temperatures,
6. The size of the cold end, and the back pressure at that location are reduced.

5.4.2.2 $T_{PRE} < T_{INV} < T_{AMB}$;

Reaching Lower Temperatures

$T_{PRE} < T_{INV} < T_{AMB}$; reaching lower temperatures and possible liquefaction of the quantum gases. A single stage Joule-Thomson cryocooler, beginning at room temperature, can produce as low a temperature as approximately the boiling point of nitrogen (77.3 K) or somewhat lower, if employing mixtures. Only liquefaction of the quantum gases (helium,

hydrogen, and neon) enables a Joule-Thomson cooler to reach temperatures below 30 K. These gases do not liquefy “spontaneously” at room temperature by a single stage, but rather require a precooling stage or two in order to bring them below their respective inversion curves and thereby enjoy the positive Joule-Thomson effect.

5.4.2.3 $T_{PRE} < T_{CD} < T_{AMB}$

No Recuperator at the Final Stage

Pre-cooling below the cooldown temperature, T_{CD} , enables the production of a two-phase, free jet of cryogen and/or a very compact final stage. The free jet of cryogen, useful for example for impingement cooling, is produced when a separate pressurized stream of the coolant at a temperature below T_{CD} expands through a Joule-Thomson nozzle. Since this stream leaves the cooler as a free jet, the returning stream that would otherwise sustain the recuperative heat exchanger and cool the incoming high-pressure stream, does not exist. Therefore, the precooling stage is necessary to bring the separate stream below T_{CD} .

5.4.2.4 Remarks

1. Staging is naturally associated with a multiplicity of coolants. However, for some purposes the same coolant may be utilized for both stages. These cases are discussed in Sects. 5.4.7, 5.4.8, and 5.4.9.
2. The idea of staging is not unique to Joule-Thomson cryocooling. Attempts to use staging in the sense of “one on the shoulders of the other” has been utilized by the developers of most other kinds of cryocoolers, such as Stirling, Gifford-McMahon, pulse-tube and thermoelectric.
3. The strategy of precooling a Joule-Thomson cryocooler by another type of cooler such as a Stirling, Gifford-McMahon, or thermoelectric cooler is regarded as a *hybrid* cryocooler (in focus of Sect. 5.6) rather than as a staged Joule-Thomson cryocooler.
4. The possibility of building a multistage Joule-Thomson cryocooler is not limited to a certain type of heat exchanger technology. Indeed one can find multistage cryocoolers based on finned tubes, photolithography and porous heat exchanger construction. Nevertheless, the staging schemes are influenced by the technology actually employed, as discussed in the next section.
5. Staging is a powerful strategy for accelerating cooldown, as is also discussed in the next section.

5.4.3 Steady State Analysis

5.4.3.1 The Schematic Layout

The schematic layout of a two-stage cryocooler incorporating two separate flow circuits is shown in Fig. 5.11. The precooling stage is marked by subscript 1

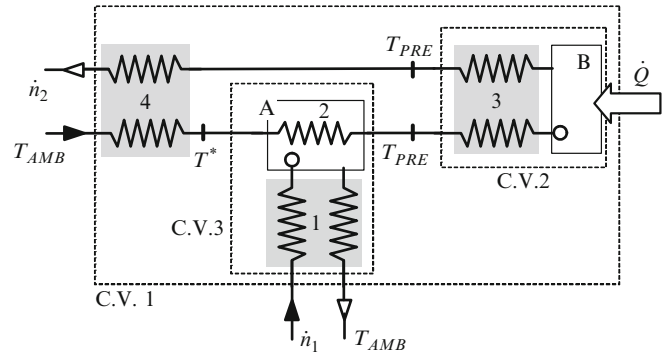


Fig. 5.11 A schematic layout of an idealized two-stage cryocooler

and the final stage is marked by subscript 2. The precoolant boils at $T_{BOIL,1}$ in evaporator A and the final coolant boils at $T_{BOIL,2}$ in evaporator B. In this example, the sole location for precooling the high pressure stream of the final stage is in evaporator A. The various sections of the heat exchangers are labeled 1 through 4.

The entire heat load, $\dot{Q}$, is supported by the evaporator (marked by B) of the final coolant at $T_{BOIL,2}$. The counter flow heat exchangers are assumed to be completely efficient. The high pressure stream of the final coolant deposits heat at an intermediate temperature, T^* , and leaves C.V.3 at the designated precooling temperature, T_{PRE} . In practice, T_{PRE} is close to, but above the precoolant's boiling point,

$$T_{PRE} = T_{BOIL,1} + \Delta T_{HE} \quad (5.6)$$

where ΔT_{HE} is the temperature difference at the outlet of evaporator A, resultant from the ineffectiveness of heat exchange process at this site.

5.4.3.2 The Energy Balance

The energy balance equations applied on the control volumes as marked in Fig. 5.11 provide the relations for determining the various states in the staged cryocooler. Applying an energy balance to control volume No. 1 (C.V.1), that includes the entire system leads to the relationship,

$$\dot{n}_1 \cdot \Delta h_{T,1}(T_{AMB}) + \dot{n}_2 \cdot \Delta h_{T,2}(T_{AMB}) = \dot{Q} \quad (5.7)$$

It should be mentioned that $\Delta h_{T,1}(T_{AMB})$ may be negative, as in the case of the quantum gases: helium, hydrogen, and neon. Control volume No. 2 (C.V.2) encompasses the 2nd stage of the cryocooler where the temperature of the inlet and outlet flow are the same and equal to T_{PRE} , thus,

$$\dot{n}_2 = \frac{\dot{Q}}{\Delta h_{T,2}(T_{PRE})} \quad (5.8)$$

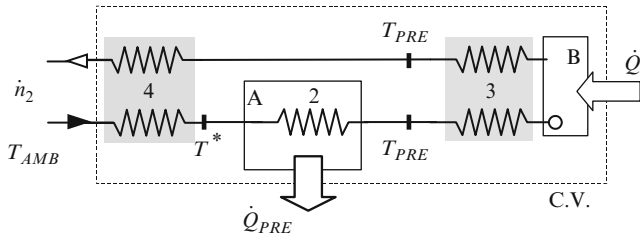


Fig. 5.12 Control volume on the final coolant circuit: scheme of precooling

An energy balance around control volume No. 3, which encompasses the precooling stage, produces the expression:

$$\begin{aligned} \dot{n}_1 \cdot \Delta h_{T,1}(T_{AMB}) &= \dot{n}_2 \cdot [h_2(T^*) - h_2(T_{PRE})] \\ &= \dot{n}_2 \cdot \bar{c}_{P,2} \cdot (T^* - T_{PRE}) \end{aligned} \quad (5.9)$$

where $\bar{c}_{P,2}$ is the isobaric heat capacity of the main coolant averaged over the temperature range (T^*, T_{PRE}) . Combining Eqs. 5.7 and 5.8 results in an expression for the ratio of flow rates as function of T_{PRE} ,

$$\frac{\dot{n}_1}{\dot{n}_2} = \frac{\Delta h_{T,2}(T_{PRE}) - \Delta h_{T,2}(T_{AMB})}{\Delta h_{T,1}(T_{AMB})} \quad (5.10)$$

The numerator may be identified as the enhancement of cooling capacity obtained through precooling.

Figure 5.12 defines a control volume that focuses on the primary flow circuit. In this view, precooling is represented by the heat extraction, $\dot{Q}_{PRE}$. The energy balance for this control volume leads to,

$$\dot{Q} = \dot{n}_2 \cdot \Delta h_{T,2}(T_{AMB}) + \dot{Q}_{PRE} \quad (5.11)$$

or

$$\begin{aligned} \dot{Q}/\dot{n}_2 &= \Delta h_{T,2}(T_{AMB}) + \dot{Q}_{PRE}/\dot{n}_2 \\ &= \Delta h_{T,2}(T_{AMB}) + \frac{\dot{n}_1}{\dot{n}_2} \cdot \Delta h_{T,1}(T_{AMB}) \end{aligned} \quad (5.12)$$

where

$$\dot{Q}_{PRE} = \dot{n}_1 \cdot \Delta h_{T,1}(T_{AMB})$$

The term $\dot{Q}_{PRE}/\dot{n}_2$ represents the additional specific cooling capacity at the final stage due to the precooling, in comparison to the case of no precooling.

5.4.3.3 Comments

1. The more one pre-cools, that is, the larger the value of $\dot{Q}_{PRE}$, the more cooling capacity is gained at the final stage,

2. An increase in precooling, $\dot{Q}_{PRE}$, implies that more heat is extracted in the pre-cooler and the system reaches a lower temperature T_{PRE} . Hence, lowering $T_{BOIL,1}$ will increase the cooling power at stage 2.
3. A dilemma then arises; pre-coolant cycles that produce a lower $T_{BOIL,1}$ are less efficient, having smaller values of Δh_T . Thus, the expected gain in stage 2 may be obtained at the expense of a less efficient stage 1.
4. The precooling capacity $\dot{Q}_{PRE}$ is rejected at $T_{BOIL,1}$. In return, the same cooling capacity, $\dot{Q}_{PRE}$, is gained at the final stage, being rejected from the cooled object, but this time in “harder circumstances”: that is at a lower temperature, $T_{BOIL,2}$. Therefore, the overall system energy balance should show a net benefit in terms of an elevated COP.
5. Another view of observation (4): $\dot{Q}_{PRE}$ is extracted by the precoolant, that is a warmer gas having a higher COP and Δh_T . Therefore, less gas is required than if the same result would have been accomplished by the primary flow coolant itself.
6. The baths of the two coolants have to be sufficiently separated because of the relatively large difference between $T_{BOIL,1}$ and $T_{BOIL,2}$, unless the precoolant flow serves only for the cooldown and ceases during steady state operation.
7. Precooling the high-pressure stream of the final coolant, reduces the ΔT along the final recuperator, marked as 3, (between T_{PRE} and $T_{BOIL,2}$) as compared to the case of no precooling (as discussed elsewhere¹¹³). In keeping with the discussion presented in Sect. 1.2.4, it reduces the generation of entropy.

5.4.4 COP Considerations of Staging

5.4.4.1 The Serial and Parallel Staging Configurations of Closed Cycle Cryocoolers

The serial and parallel staging configurations of closed cycle cryocoolers are displayed in Fig. 5.13. The serial configuration (of three stages) is shown in part (a) of the figure, while its representation in the $T-s$ plane is shown in part (b) following the notation of Quack¹¹⁴. The first stage rejects heat to the high temperature reservoir at T_H , (generally the surroundings at T_{AMB}) while the last one absorbs heat at the low temperature, T_L . Each stage (i) extracts heat $\dot{Q}_{L,i}$ at a temperature of $T_{L,i}$ from the lower stage ($i+1$) and releases heat $\dot{Q}_{H,i}$ at the temperature of $T_{H,i}$ to the upper (warmer) stage ($i-1$). The intermediate temperature is $T_{L,i} = T_{H,i+1}$. A power of $\dot{W}_i$ (whether mechanical or of any other kind) is invested in each stage, accumulating with $\dot{Q}_{L,i}$ so that,

$$\dot{W}_i + \dot{Q}_{L,i} = \dot{Q}_{H,i} \quad (5.13)$$

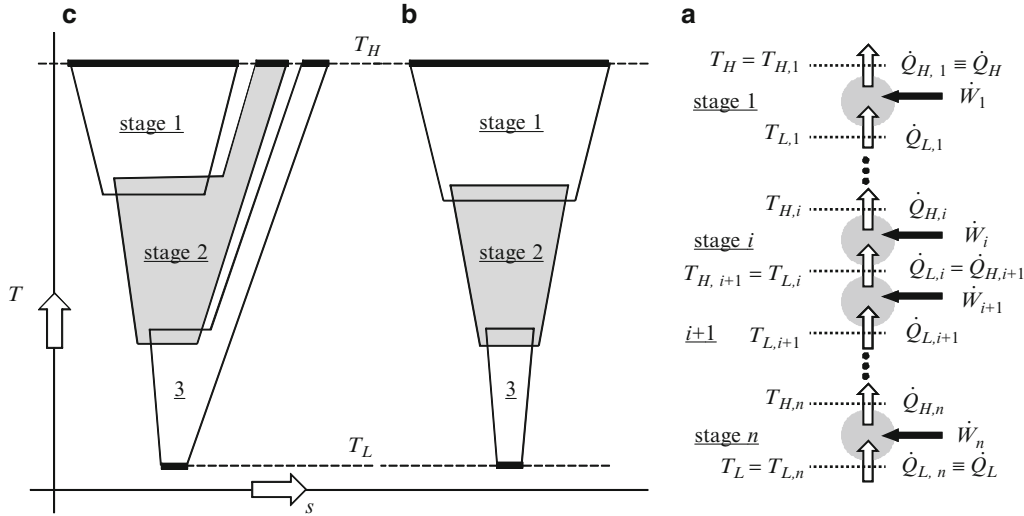


Fig. 5.13 Multi staged cryocooling: (a) and (b) depict serial staging, while (c) depicts parallel staging following the notation of Quack¹¹⁴

The configuration for multi-stage thermoelectric coolers exactly follows this arrangement.

The parallel configuration is displayed in Fig. 5.13c on the $T - s$ plane. One may expect in general, that any warmer stage of the parallel configuration will be thermally less loaded than in the serial configuration (which accumulates all the $\dot{W}_i$ terms from the “previous” colder stages). Hence, the parallel configuration has the potential to be more efficient than the serial cascade¹¹⁴.

Pulse tube cryocoolers, for instance may be staged in a series or parallel arrangement. Mitsubara and Gao¹¹⁵ showed that the parallel staging configuration is superior. They were able to reach 4 K using three stages in parallel, but could not attain the same temperature with the serial arrangement.

The multi-stage configuration can be optimized by arranging the staging to maximize the overall coefficient of performance, COP. The relevant optimization parameters are:

1. The number of stages, n , to cover the temperature range (T_L, T_H) ,
2. The intermediate temperature of each stage, and,
3. The characteristic performance of each stage in terms of its deviation from the ideal, or Carnot, cycle.

If each stage would exhibit the Carnot ideal coefficient of performance, $\text{COP}_{\text{CARNOT}}$, the optimization would become irrelevant. For any number of stages and any set of staging temperatures, the overall COP would remain as $\text{COP}_{\text{CARNOT}}$ according to Eq. 1.75, which is the highest attainable COP when refrigerating from T_H down to T_L . However, in practice, the stages are not ideal and each stage generates entropy at the rate of $\dot{S}_i$. The extent of the degraded performance, as a measure of irreversibility, is described by the magnitude of the inequality, $\text{COP}_i < \text{COP}_{\text{CARNOT}, i}$. The deviation of COP_i

from its maximum value, $\text{COP}_{\text{CARNOT}, i}$ has been treated below in two ways for the serial configuration:

5.4.4.2 The Serial Configuration with Stages Having the Same FOM

The serial configuration with stages having the same FOM (Sect. 1.4.1.4) was analyzed by Jeon and Smith,¹¹⁶

$$\frac{\text{COP}_i}{\text{COP}_{\text{CARNOT}, i}} \equiv \text{FOM}_i = \text{Const} = \varphi \quad (5.14)$$

By Eq. 1.79 that is equivalent to the assumption that the irreversibility at each stage, i , is given by,

$$\left(\dot{W}^{\text{LOST}} / \dot{W}^{\text{MIN}} \right)_i = \text{Const} \quad (5.15)$$

Jeon and Smith studied the condition for minimum entropy generation, to maximize the overall COP, and showed that for this purpose the intermediate temperatures for each stage (i) should fulfill,

$$T_{H,i} / T_{L,i} = (T_H / T_L)^{1/n} \quad (5.16)$$

Any two adjacent stages have the same ratio of temperatures. Thus, on the logarithmic scale, the interval (T_L, T_H) is equally divided among all stages. Put another way, any inter-stage temperature is the geometric mean of its two adjacent ones. The associated overall coefficient of performance is,

$$\frac{1}{\text{COP}_{\text{OPT}}} = \left[1 + \frac{(T_H / T_L)^{1/n} - 1}{\phi} \right]^n - 1 \quad (5.17)$$

As φ approaches unity, the overall COP increases and finally reaches the highest value of $\text{COP}_{\text{CARNOT}}$ as $\varphi = 1$, for any n .

5.4.4.3 The Serial Configuration with Stages of the Same Relative Entropy Generation, $(\dot{S} T_H / \dot{Q}_L)_i$

The serial configuration with stages of the same relative entropy generation, $(\dot{S} T_H / \dot{Q}_L)_i$, which by Eq. 1.78 is equivalent to the condition,

$$\frac{1}{\text{COP}_i} - \frac{1}{\text{COP}_{\text{CARNOT}, i}} = \left(\frac{\dot{S} T_H}{\dot{Q}_L} \right)_i = \left(\frac{\dot{W}^{\text{LOST}}}{\dot{Q}_L} \right)_i = \text{Const} = \vartheta \quad (5.18)$$

The serial configuration under this assumption was optimized by Xuan.¹¹⁷ The analysis results in the same rule of staging, or choice of intermediate temperatures, as in the previous case with the constant FOM, as defined by Eq. 5.16. However, the associated overall coefficient of performance is different,

$$\frac{1}{\text{COP}_{\text{OPT}}} = \left[(T_H / T_L)^{1/n} + \vartheta \right]^n - 1 \quad (5.19)$$

In this case, the overall performance is the same as the ideal Carnot cycle, for any number of stages, when $\vartheta = 0$.

5.4.4.4 The Influence of the Number of Stages in the Serial Configuration

In both cases of the serial configuration discussed above, an increase in the number of stages degrades the overall COP_{OPT} . For example, in the case of a constant FOM at each stage, for $T_H / T_L = 3$ and $\varphi = 0.6$, the overall COP_{OPT} decreases from 0.254 to 0.225, as n increases from 2 to 4 (but at the same time the total temperature span of the staged cascade extends from $T_H / T_L = 9$ to 81). However, this decrease is bounded as there is a non-vanishing COP_{OPT} even at an infinite number of stages,

$$\text{COP}_{\text{OPT}} = \left[(T_H / T_L)^{1/\phi} - 1 \right]^{-1} \quad (5.20)$$

In the above example the limiting value of COP_{OPT} is 0.191. In the second case, with constant relative entropy generation, $(\dot{S} T_H / \dot{Q}_L)_i$, at each stage, the COP_{OPT} approaches zero as the number of stages becomes very large.

The two results of COP_{OPT} by Eqs. 5.17 and 5.20 are not comparable since the irreversibilities in the two models are not defined on the same basis.

5.4.4.5 Staging of Closed Cycle Joule-Thomson Cryocoolers

In most cases, staged, closed cycle JT cryocoolers are arranged in a parallel configuration, as shown in Fig. 8.19a. Each stage is driven by a separate “warm” compressor that rejects its heat of compression to the surroundings. Consequently, the stages interact not only with the two adjacent stages (as in Fig. 5.11) but also reject heat to the atmosphere (T_H). Serial staging of Joule-Thomson cryocoolers is also possible, but demands “cold” compressors; their heat of compression is rejected into the precooling stages and, except for the first compressor, they do not reject heat to the ambient surroundings.

In the case where a single Joule-Thomson stage can bridge a given temperature span (T_H, T_L), one may question why multiple stages are used at all, if any additional stages reduce the overall COP. The reason, as discussed in Sect. 3.13, is that each additional Joule-Thomson precooling stage with its successively higher boiling point will have a higher FOM than the precooled stage. A higher FOM means lower irreversibility, thus elevating the total COP. For example, a single nitrogen stage (77.3 K) operates ideally with the highest FOM of 0.075. However, methane’s precooling cycle at 111 K has a higher figure of merit, like $\text{FOM} = 0.262$. A similar trend holds for thermoelectric stages; the FOM is higher as the average temperature, $(T_H + T_L)/2$, is increased.^{116,118} Finally, the overall COP increases as more stages of warmer coolants are involved. However, each additional stage would involve another coolant, a new liquefaction cycle, and additional hardware. This naturally leads to a more complex system.

5.4.4.6 Staged Cooling of a Stream

Staged cooling of a stream is another application of cooling by a cascade that is somewhat related to the one discussed above. The finite temperature difference along a cooled stream is the major source of irreversibility associated with such a process. This is of practical concern in the large-scale liquefaction of a stream of natural gas. The least efficient approach for liquefaction would be to extract heat by a single stage to cool the stream down to the boiling point of natural gas. Higher efficiency may be obtained by utilizing at least a few stages, extracting heat at successive intermediate temperatures, gradually reducing the temperature and finally liquefying the entire stream. In practice, the intermediate stages are of the Linde-Hampson kind employing coolants as propane, ethylene, and methane. Ait-Ali and Wilde¹¹⁹ studied the optimum inter-stage temperatures for minimizing the invested work. For a stream of constant heat capacity, the quest for the maximum COP led to the same rule of intermediate staging temperatures as in the serial scheme of staging, given by Eq. 5.16.

Knudsen and Ganni¹²⁰ investigate an ideal Claude liquefier that uses a cascade of stages to precool a stream of gas

(helium). Each stage incorporates a closed cycle reverse Brayton cryocooler. They derive the same optimal intermediate staging temperatures, of equal steps on the logarithmic scale (“Carnot Step”), that preserve the ratio of $T_{H,i}/T_{L,i}$.

Closely related is the “dynamic isolation” optimizing scheme by Chen et al.¹²¹ with the same rule of logarithmic mean for the adjacent stages. Nevertheless, this case is different from the staged cryocooler of the scheme described in Fig. 5.13. Because the finite temperature difference is dominant in this case, more stages reduce the invested work and make the liquefaction process more efficient.

5.4.4.7 Miscellaneous

Cain¹²² developed a numerical code to obtain the input power of a multistage cryocooling system, given the ambient and staging temperatures, heat loads, and efficiencies at each stage. Kirkconnell^{123,124} discusses optimization procedures for multi-staged cryocoolers. Interested readers are recommended to more advanced literature on exergy analysis.^{125,126,127} A review of other staging schemes may be found elsewhere.¹²⁸

Staging of endo-reversible cryocoolers is discussed elsewhere.^{129,130}

5.4.5 Reduction of System Weight and Volume

The major component of a continuously operating cryocooler system is its pressure supply unit, whether in the form of a store of compressed gas in vessels, or as a compressor based unit. Very often, the weight and volume of the pressure supply unit become the most significant parameters of the entire cryocooler system. Utilizing a higher boiling point gas to precool nitrogen or argon within a two-stage cooler may remarkably reduce weight and volume. Alternatively, one might achieve longer run time while using equipment of similar size and weight. This is a reflection of the elevated COP and FOM that may be obtained through precooling.

The potential cooling content of a pressure vessel per unit volume (Q/V) and per unit mass of the vessel, (Q/m), when discharged through an ideal Joule-Thomson cryocooler was discussed in Sect. 4.8. As an example of the per-volume advantage afforded by pre-cooling, let us assume that CF₄ is used to precool N₂ so that the pressure does not exceed 40 MPa at 300 K. As shown in Table 4.9, a single liter of compressed N₂ or CF₄ possesses respective cooling capacities of 8.9 kJ or 54.9 kJ. Put another way, 1 kJ of heat is rejected by 112 cm³ of N₂ or 18 cm³ of CF₄ (each being rejected at the associated boiling point). Therefore, any fraction of single stage N₂ storage, if replaced by

Table 5.3 Volume and weight comparison of nitrogen operated open cycle systems with and without precooling at ambient temperature of 343 K after Pierson⁶⁴ and Stephens¹³¹

Fixed orifice mini cooler	1 W/12 h	0.2 W/2 h
Bottle capacity (liters)	92.5	14.0
Bottle weight (kg)	134.0	20.5
System weight (kg)	170.0	28.5
<i>Self-regulating mini cooler</i>		
Bottle capacity (liters)	24.0	2.0
Bottle weight (kg)	35.5	2.5
System weight (kg)	45.5	5.5
<i>Pre-cooled by R-12 (CCl₂F₂) self-regulating mini cooler</i>		
Bottle capacity (liters)	5.6	0.7
Bottle weight (kg)	7.3	1.3
System weight (kg)	13.6	4.5

pressurized CF₄, results in a volume reduction by a factor approaching 54.9/8.9, or 6.2.

At elevated environmental temperatures, the advantage gained by involving a precooling stage is even more visible. Again, for N₂ and CF₄ at 40 MPa but at 340 K, the respective cooling capacities are 5.8 kJ and 38.8 kJ (Table 4.9). The ideal factor of volume reduction becomes 6.9. Another parameter adding advantage to the higher ambient temperatures case is that the cut-off pressure remains low due to the effect of precooling. In this way, a larger fraction of the stored gas is utilized at higher temperatures.

The major part of the total weight of a pressurized gas storage is the metal vessel (“bottle”) itself. The weight fraction of the gaseous content is usually quite small. One may regard it as a metal envelope for the volume occupied by the compressed gas. For a given shape and material, a smaller volume has a smaller surface, and is thus associated with a lower weight. The shape indeed plays a serious role. For example, a long cylindrical pressure vessel has almost double the weight of a spherical one with the same volume.

A reduction in volume (by a factor N) is almost directly proportional to a reduction in the weight of pressure vessel. For a spherical vessel, the surface is reduced by a factor of $N^{2/3}$ and the thickness by $N^{1/3}$ for a combined reduction factor of N . The study of Wolfe and Zissis³ shows that the weights of cylindrical vessels are proportional to $V^{1.087}$ and spherical vessels with $V^{0.86}$.

An investigation motivated precisely by these arguments has been developed and reported by Stephens¹³¹ and Pierson.⁶⁴ They present a quantitative treatise regarding the benefits of using a standard self-regulated nitrogen mini cooler enhanced by a R-12 precooling stage at elevated ambient temperature to 343 K. The weight comparison of three strategies of an entire open cycle systems is displayed in Table 5.3. The configuration consuming the largest amount of gas is the single stage nitrogen cooler with a

non-regulated flow system. The other two are regulated alternatives, one with nitrogen alone and the other with R-12 pre-cooling as Proposed by Campbell.¹³² Two cooling objectives are considered: The larger one providing 1 W of cooling for 12 h, thereby rejecting a total of 43.2 kJ of heat. The shorter task providing only 0.2 W of cooling for 2 h, thereby rejecting a total of 2.88 kJ of heat. The larger task enjoys a weight reduction by a factor of 45.5/13.6, i.e. 3.3 via precooling. Pierson⁶⁴ concludes that these precooling attempts:

...enable miniature Joule-Thomson cooling systems to be considered for applications for which they were previously considered unsuitable on the ground of weight, complexity, and size.

When compressors supply the gas for both stages, a smaller size may be used for each stage than would be needed for a single stage. Since a compressor merely provides a mechanism to displace volume, and smaller volumes of compressed gas are involved in a precooled system, the compressors can either provide smaller displacements or operate at a lower speed of rotation. In either case, the precooling approach provides a more compact system.

The comparison would remain incomplete without mentioning the fact that a staged system is more complicated, since it includes two units of pressure supply: two species of gas, and two kinds of pressure vessels or two compressors. Obviously, as the required period of operation increases or a larger cooling power at higher ambient temperature is needed, the advantage, and attractiveness of precooling increase as well.

5.4.6 Liquefaction of Quantum Gases:

³He, ⁴He, H₂, D₂ and Ne

A single stage Joule-Thomson cryocooler fed by room temperature quantum gases will warm up. However, it is possible, by precooling these gases, to relocate the “initial” state to a location below the inversion curve, so that a self-sustained cooldown can proceed spontaneously until they liquefy.

5.4.6.1 Candidate Precoolants

Candidate precoolants have to have a boiling point below the corresponding zero pressure inversion temperatures, 34 K, 43 K, 201 K, 203 K and 220 K for ³He, ⁴He, H₂, D₂, and Ne. This condition would be necessary but in some cases insufficient. To provide a substantial integral Joule-Thomson effect, the precoolant pressure must be elevated to the range of about 10P_C and even higher. At elevated pressure, the corresponding integral inversion temperature decreases to a point below the zero pressure values. Additional precooling is needed since a temperature margin below the

integral inversion temperature is required for the cooldown process to progress in a reasonable time.

Regarding the depth of precooling, one should be aware of the existence of a maximum inversion pressure for ³He, ⁴He, H₂ and D₂. As discussed in Sect. 2.6.3, their integral inversion curve displays a maximum pressure. In the case of “over pressurizing”, no matter how deep we precool the gas, it will still warm up. For ³He, the maximum pressure is 3.11 MPa and for ⁴He it is 9.45 MPa. These moderate values are due to their low critical pressures. For H₂ and D₂, the respective maximum inversion pressures are significantly higher, at 51.55 and 53.76 MPa.

Each quantum gas has only a few recommended pure precoolants:

1. ³He can be precooled by ⁴He, or with careful but marginal design, H₂ might also be acceptable,
2. ⁴He can be precooled by H₂ and, albeit with reduced efficiency, Ne,
3. H₂ and D₂ can be precooled by commonly used gases such as nitrogen, air, argon and oxygen
4. Ne can use all the candidate precoolants of H₂, and also the higher boiling point gases such as CH₄, Kr and even CF₄.

From the discussion above, it is clear that at least two stages are required for liquefying Ne and H₂. In view of Sect. 5.4.4., the maximum COP for liquefaction of hydrogen by logarithmic mean for the adjacent stages will occur at an intermediate temperature of, $300(20.4/300)^{1/2}$ K = 77.5 K. Hence, nitrogen is the preferred pre-coolant. In order to liquefy neon, the optimal intermediate temperature is accordingly, $300(27.1/300)^{1/2}$ K = 90.2 K, pointing to argon as the pre-coolant of choice.

Since ⁴He has to be precooled by Ne or H₂, at least three stages are required for its liquefaction. The possible staging sequences are for example N₂-H₂-⁴He or N₂-Ne-⁴He. Note that in each of these, N₂ can be replaced by Ar or O₂. The ideal optimum temperatures of staging are then, $300(4.2/300)^{1/3}$ K = 17.4 K, and 72.3 K by $300(4.2/300)^{2/3}$ K, which are closer to the normal boiling points of N₂ and H₂.

If ³He is precooled by ⁴He, four stages are required for its liquefaction. ⁴He indeed provides a substantially deep precooling, and in fact it is possible to liquefy ³He by only three stages, with precooling provided by H₂ or Ne, and Ar or N₂. Returning once again to the logarithmic mean for the adjacent stages temperature distribution for a four stage process, we find that $(300/3.2)^{1/4}$ K = 3.12 K, thereby defining optimum intermediate temperatures of 96 K, 32 K and 10 K. Similarly, for a three stage process, we find that $(300/3.2)^{1/3}$ or 4.54 K, thereby defining intermediate temperatures in that case of 14.5 K, and 66 K.

5.4.6.2 Example Cryocoolers

Fabricators of early Joule-Thomson cryocoolers as Pastuhov and Zimmermann¹⁰⁵ in 1956, Geist and Lashmet^{9,10,133} in 1960, Maybury,¹⁰⁶ LeFranc¹⁰⁶ in 1966, and Yelukhin et al. in 1967¹³⁴ utilized staging for liquefying hydrogen and helium. Hsu et al.¹³⁵ and Inaguchi Takashi¹³⁶ liquefied neon by precooling with various gases. Little et al.⁶⁸ built a neon cryocooler precooled by nitrogen.

A three-stage autonomous helium liquefier with three compressors for nitrogen, hydrogen and helium and its finned tube heat exchangers was described¹³⁷ in *Machine Design Journal* already in 1963.

A unique system with four stages to produce liquid helium was used for cooling Josephson Junctions as reported by Tward and Sarwinskiin.²³ They employed the three precoolants CF_4 , N_2 and Ne, although a combination of only N_2 and Ne would have sufficed as well.

A double stage miniature cryocooler for the Mariner space mission is described by Hughes and Herr.⁷⁶ It liquefied hydrogen that was precooled by nitrogen. The dual gas pressure system included two vessels. Nitrogen was stored at 40 MPa and regulated at the inlet of the cooler to 18 MPa. Hydrogen was stored at 37 MPa and reduced to 11 MPa. Cao et al.¹³⁸ optimized the staging of a hydrogen-nitrogen microcryocooler for 28 K; the expanded pressure of both stages was about 0.6 MPa.

Doomink et al.¹³⁹ and Burger et al.^{140,141,142} built a two stage cryocooler for space applications establishing 10 mW at 4.5 K. Two sorption compressors for hydrogen¹⁴³ and for helium were heat sunk at two passive radiators.

5.4.6.3 Miniature Laboratory Liquefiers

Miniature laboratory liquefiers of quantum gases are closely related prior art devices. In 1953, Chester and Jones^{144,145} liquefied ^4He by precooling with hydrogen, which in turn was precooled using liquid oxygen. Parkinson¹⁴⁶ built a hydrogen and ^4He liquefier in 1954. Chelton et al. in 1960¹⁴⁷ analyzed the parameters of a helium refrigerator, precooled by a liquid hydrogen stage and a bath of liquid nitrogen. The high pressure stream of helium undergoes a series of two throttling expansions, from 3 MPa to about 1 MPa and then finally to about 0.1 MPa. As dictated by the Second Law, the smaller temperature differences along the recuperators produce a higher COP. Currie^{148,149} describes a small helium temperature refrigerator that uses a supply of liquid nitrogen to precool both the intermediate stage of hydrogen and the inflowing stream of helium.

A small continuous cryocooler liquefying ^3He and supported by liquid ^4He is reported first by Daunt and Lerner²⁰ and later by Wilkes.^{21,22} Doband¹⁵⁰ built a unique “double stage helium sorption cooler”; using a stage of ^4He precooled by a stage of ^3He .

Hydrogen liquefiers were reported by Parkinson¹⁵¹ in 1954 and by Galuszewski et al.¹⁵² A neon liquefier was built by Koeppe.²⁴

5.4.7 Free Jet Release

A high pressure stream that is precooled below its T_{CD} by a Joule-Thomson cryocooler (see Sect. 5.4.2.3) and subsequently throttled down to its T_{BOIL} can be released as a two phase (“mist” like) jet. Since T_{BOIL} is higher than T_{CD} , at least in principle, the precoolant may be the same gas as the coolant. This principle takes us back to the origin of cryogenics as notified by Hands.¹⁵³ Raoul Pictet in 1877 liquefied oxygen for the first time using a similar approach. He precooled down to 140 K a 47 MPa stream of oxygen and then throttled it to the atmosphere.

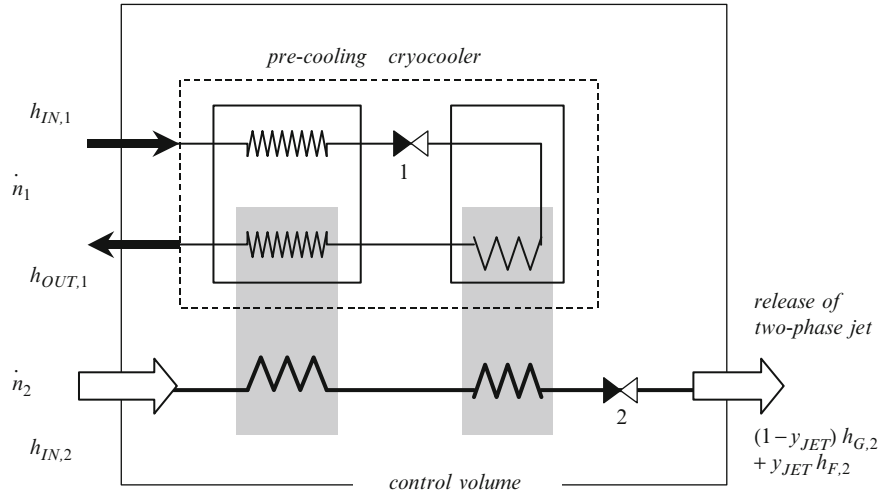
5.4.7.1 Motivation

In some applications, adequate cooling may be obtained simply from a free jet of two-phase cryogen impinging on the cooled object as an alternative to the traditional plenum of cryogen maintained beneath the object. In this way, one may tolerate a flexible interface between the cooled object and the cooler. Hingst¹⁵⁴ describes an application where the cooled object is mounted in an assembly on a couple of gimbals. The cooler releases the free jet towards the object, yet remains fixed and separate from the gimbals’ mechanism. Such a configuration can provide significant advantage. The alternative of loading the cooler on the gimbals results in a more complicated system. It demands larger operating torque, and flexible tubing to feed the cryocooler while the gimbals change position. A low temperature jet for impinging living tissues may also serve cryo-therapeutic purposes (see Sect. 9.4.6.1).

5.4.7.2 The Model

The schematic of the free jet system is displayed in Fig. 5.14. As in Sect. 5.4.3, the precooling stage is noted by the index 1. Coolant 1 has the flow rate of $\dot{n}_1$, inlet conditions at pressure P_1 and T_{AMB} with enthalpy $h_{IN,1}$ and expands through throttle 1. Assuming an ideal recuperator, the outlet stream is at the same temperature, T_{AMB} , at the pressure 0.1 MPa with an enthalpy of $h_{OUT,1}$. The final and precooled stream of coolant 2, enters at (P_2, T_{AMB}) with the enthalpy of $h_{IN,2}$. It expands through the throttle 2 down to 0.1 MPa and exits the system so that a fraction, y_{JET} of its flow rate $\dot{n}_2$ is liquid at its normal boiling temperature. In a broad sense, the arrangement depicted in the control volume of Fig. 5.14 effectively resembles a liquefier (Sect. 3.6) since part of the supplied coolant, $y_{JET}\dot{n}_2$, leaves the system as a liquid phase. However, there is also an additional fraction, $(1 - y_{JET})\dot{n}_2$ of

Fig. 5.14 The pre-cooling cryocooler, 1, is employed to lower the temperature of a pressurized stream, 2, below its T_{CD} prior to being expanded in throttle 2. The released jet includes a fraction of liquefied coolant



saturated gaseous phase that leaves the control volume, a feature that does not exist in a liquefier.

Balancing the energy flow of the control volume in Fig. 5.14 and assuming ideal heat exchangers, leads to the expression,

$$y_{JET} = \frac{\dot{n}_1}{\dot{n}_2} \cdot \frac{\Delta h_{T,1}}{L_2} - \frac{h_{IN,2} - h_{G,2}}{L_2} \quad (5.21)$$

where the latent heat of vaporization of the final stage is, $L_2 = h_{G,2} - h_{F,2}$. Since the final coolant releases its heat to the precoolant, the precoolant needs to satisfy the condition,

$$T_{BOIL,1} < T_{CD,2} \quad (5.22)$$

In fact, the precoolant's boiling point has to be sufficiently below T_{CD} of the final coolant in order to obtain a large fraction of liquid in the expanded free jet. For example, nitrogen may be precooled by nitrogen itself or by argon, methane, krypton, and R-14.

5.4.7.3 Common Inlet Conditions for Both Stages

Common inlet conditions for both stages means that the precooler, 1, and the non recuperated final stage, 2, are fed by the same pressure source; the same coolant and at the same pressure and temperature. The total flow rate entering the control volume of Fig. 5.14 is,

$$\dot{n} = \dot{n}_1 + \dot{n}_2 \quad (5.23)$$

In contrast to a liquefier, this system includes an additional design parameter, which is the ratio of the flow rates, $\dot{n}_1/\dot{n}_2$, or $\dot{n}_1/\dot{n}$, of the two stages. There is a minimum necessary ratio of flow rates to ensure that the normal boiling point of stream 2 is obtained at the outlet. It is found by

noting that the beginning of liquefaction (of zero yield) is defined by the condition,

$$\left(\frac{\dot{n}}{\dot{n}_2}\right)^{MIN} = \frac{1}{i^{MAX}} \equiv \frac{h_{OUT,2} - h_G}{\Delta h_T} \quad @ \quad y_{JET} = 0 \quad (5.24)$$

where the maximum ineffectiveness, i^{MAX} , is a state function of the common incoming coolant, as discussed in Sect. 3.5.2.3. Only at a larger flow rate ratio, some fraction of $\dot{n}_2$ is liquefied, by the extent,

$$y_{JET} = \frac{\Delta h_T}{L} \left[\frac{\dot{n}}{\dot{n}_2} - \left(\frac{\dot{n}}{\dot{n}_2}\right)^{MIN} \right] \quad (5.25)$$

The yield of liquefaction may be related to the total flow rate, $\dot{n}$, then,

$$y_{TOTAL} = y_{JET} \frac{\dot{n}_2}{\dot{n}} = \frac{\Delta h_T}{L} \left[1 - \frac{(\dot{n}/\dot{n}_2)^{MIN}}{(\dot{n}/\dot{n}_2)} \right] \quad (5.26)$$

In the case of complete liquefaction of $\dot{n}_2$, the present arrangement coincides with the traditional Linde-Hampson liquefier as discussed in Sect. 3.6.1.3. One should check and show that in this case, indeed the liquefied fraction does not exceed that of a liquefier, by Eq. 3.66,

$$\left(\frac{\dot{n}_2}{\dot{n}}\right)^{MAX} = y^{(2)} \equiv \frac{\Delta h_T}{h_{OUT,2}(0.1 \text{ MPa}, T_{AMB}) - h_F} \quad @ \quad y_{JET} = 1 \quad (5.27)$$

which is a check of consistency. Note however, that the free jet release arrangement of Fig. 5.14 while regarded as a

liquefier, is always less efficient than the traditional liquefier displayed in Fig. 3.1 and discussed in Sect. 3.6.1.

5.4.7.4 The General Case

The general case, allowing different coolants and inlet conditions, is described by,

$$\left(\frac{\dot{n}_1}{\dot{n}_2}\right)^{MIN} = \left(\frac{1}{i_2^{MAX}} - 1\right) \frac{\Delta h_{T,2}}{\Delta h_{T,1}} \quad (5.28)$$

For a given y_{JET} of a given $\dot{n}_2$, but different precoolant and its inlet conditions,

$$\dot{n}_1 \propto \frac{1}{\Delta h_{T,1}} \quad (5.29)$$

If one's interest is merely the free jet of cryogen, being less concerned with gas consumption, the precoolant may be the same gas as the final coolant.^{155,156,157} However, if the cryogen were nitrogen for example, higher boiling point precoolants such as argon, methane, or krypton, would be advantageous. Since the value of $\Delta h_{T,1}$ for these gases is higher (see Sect. 4.3.8) one obtains, by Eq. 5.29 a lower consumption of $\dot{n}_1$ for a given $\dot{n}_2$. Nevertheless, the constraint imposed by Eq. 5.22 avoids the possibility of employing "too" effective of a precoolant, that is one with too high of a boiling point. On the other hand, the use of a common coolant for both stages simplifies the entire assembly, as both stages share a common high-pressure vessel. The additional precoolant vessel and its operating valves are eliminated. Hingst¹⁵⁸ employed methane at 20–35 MPa to precool and release a free jet of argon (20–50 MPa). His patent shows a typical layout for heat exchangers in a miniature Joule-Thomson cryocooler; argon is precooled as it flows through a finned tube that is immersed in liquefied methane, and also by the cold returning flow of argon.

5.4.8 Cold End Benefits

5.4.8.1 Reducing the Size of the Cold End

Precooling can dramatically decrease the size of the final stage heat exchanger in a Joule-Thomson cooler. As the precooling temperature is reduced, a smaller final stage heat exchanger can be tolerated. If the precooling temperature is reduced below T_{CD} , no heat exchanger is required at all, and the only remaining component required at the cold tip is the expansion nozzle. If the cold end diameter is less than 2.5 mm, then to reach temperatures below 100 K, incorporation of a precooling stage is unavoidable. Without precooling, the small space does not provide sufficient heat

exchange area. For example, Dobak et al.¹⁵⁹ built a Joule-Thomson cryosurgical catheter for treating heart arrhythmia. The key feature is its tiny size. They were able to achieve the desired cooling in such a small size by precooling the final stage with another coolant. Varney¹⁶⁰ also proposed a flexible catheter, in which the same coolant was used for both final and precooling stages.

5.4.8.2 Reducing Back Pressure

Precooling allows lower flow rates of a final coolant to provide a given heat load. Therefore, for a given size of heat exchanger, the backpressure induced by the final stage coolant flow will also be reduced, as compared to that of a flow stream that is not pre-cooled. The associated boiling point and operating temperature of the cooler are also reduced. In some special applications, such a small decrease of temperature, becomes a highly desirable feature. The same objective may be obtainable by precooling with the same gas as used in the final stage.

5.4.8.3 Modularity

For any cooling capacity and its related flow rate there is a corresponding size of recuperative heat exchanger and an associated size and shape of cryocooler. Prentice et al.^{161,162} suggested a modular construction of extended cryocoolers, based on a common construction unit. The principle is staging of identical cryocoolers of the same flow rate, so that each one precools the next one of the same construction. More modules and more stages increase the total flow rate and refrigeration power. Each additional stage amplifies the precooling and results in a larger fraction of liquefaction at the final stage. In the system proposed by Prentice, all stages are fed by the same coolant, which is argon.

5.4.9 Staging by Pressure with Double Expansion: The Ball Aerospace Joule-Thomson Cryocooler

A space-borne closed cycle cryocooler developed at Ball Aerospace Systems Division, launched in August 1997, and described by Lester, Levenduski, Scarlotti and Fernandez,^{26,27,28,29,30,31,32,33} is referred to as the Cryogenic On-Orbit Long Life Active Refrigerator (COOLLAR). Being unique in many aspects, it is hard to classify under a single heading, and deserves a special treatment of its own. However, in that its dominant attribute is the staging employed, both in temperature and pressure, this section is the most suitable site to introduce the cryocooler.

In all previous examples, staging by temperature was obtained by involving the liquid phase of higher boiling

Fig. 5.15 Layout of staging by pressure with double expansion: The Ball Aerospace Joule-Thomson cryocooler, as reported by Lester, Levenduski Scarlotti and Fernandez.^{26,27,28,29,30,31,32,33} States notation corresponds to Fig. 5.16

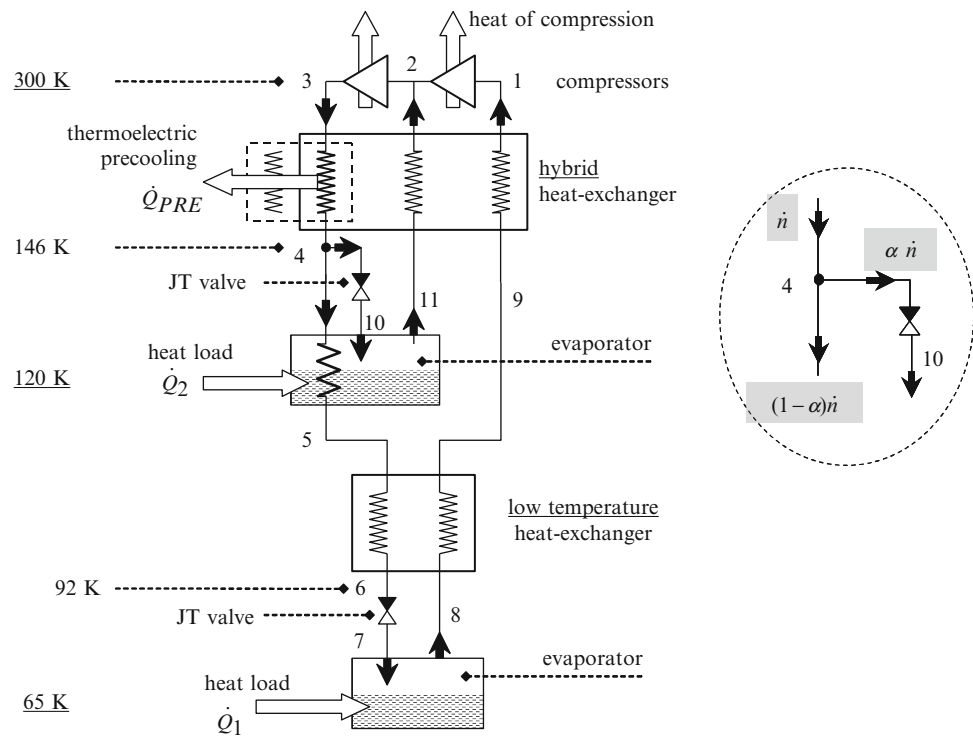
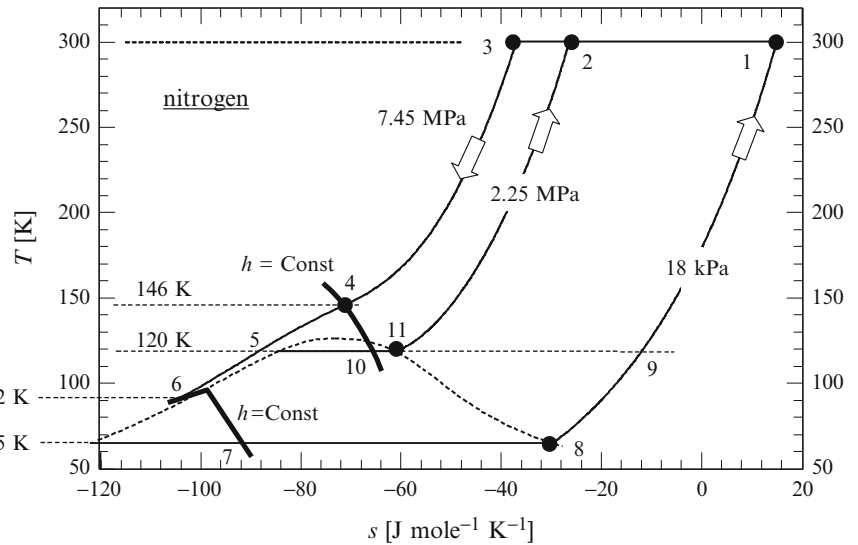


Fig. 5.16 Thermodynamic description of the double stage precooled system: The Ball Aerospace Joule-Thomson cryocooler, as reported by Lester, Levenduski Scarlotti and Fernandez.^{26,27,28,29,30,31,32,33} State notation corresponds to Fig. 5.15



point gases. In the COOLAR, staging is accomplished by a single coolant undergoing two successive expansions. Following the general scheme in Fig. 5.15, the high-pressure (7.45 MPa) nitrogen flow splits, at junction 4, into two streams. The corresponding thermodynamic states are displayed in Fig. 5.16. Each stream circulates in its own loop, undergoing its Joule-Thomson expansion and recuperative heat exchange process, subsequently liquefying and boiling in a separate evaporator.

Only a small fraction of the entire flow expands down to the lowest pressure. The first and largest portion, $\alpha = 0.98$, of the total flow expands to an intermediate subcritical pressure of 2.25 MPa. It liquefies in the first stage evaporator at the saturation temperature (120 K) corresponding to the intermediate pressure (state 10).

The remaining $1 - \alpha = 0.02$ of total flow is recuperative precooled to 92 K (state 6), expands down to the lowest pressure associated with compressor's suction (state 7),

that being actively regulated by a flow control valve, at 18 kPa. The cold stream liquefies in the final evaporator at the associated saturation temperature of 65 K. Therefore, the two streams of the same species have two different boiling points (120 K, 65 K), in analogy to the classical staging configuration generally accomplished with two gases of different boiling points.

To complete this description, it should be noted that the two loops are thermally coupled. The higher boiling point evaporator precool the second stream before expansion, in the manner that is traditionally accomplished with the aid of another higher boiling point coolant. It should also be noted that the near critical conditions in the first evaporator enhance the convective heat transfer in the precooling process, as discussed elsewhere.¹⁶³ The heat exchanger identified in Fig. 5.15 as “hybrid” provides the second site of the two streams’ interaction. Each of the two returning cold streams is in a recuperative thermal relation with the counter flowing high-pressure total stream. Such a three-stream heat exchanger is discussed elsewhere.⁹⁸

Another objective of this “hybrid” heat exchanger is to incorporate thermoelectric elements to extract more heat, as precooling $\dot{Q}_{PRE}$, from the total flow stream. Nevertheless, this aspect of the cooler is discussed separately in Sect. 5.6.2. The associated gain in Δh_T by the thermoelectric-induced temperature reduction enables operation at lower pressures with a simpler and more reliable compressor. It is shown that the energy invested in the thermoelectric pre cooler indeed pays off, as the overall COP is increased by its presence in the system. Because of the two combined cooling mechanisms, the nitrogen is precooled to 146 K (state 4).

The COOLAR thus uses pressure staging to create a temperature staged system. Two benefits of the dual pressure system are readily identifiable:

- (a) Improved efficiency of the cooling process with special relevance for space applications, and,
- (b) Heat extraction at two temperatures, $\dot{Q}_1 = 3.5$ W at 65 K and $\dot{Q}_2 = 5$ W at 120 K.

The expectation for a higher COP may be explained by the following argument. The isothermal compression work of an ideal gas is proportional to the logarithm of the compression ratio, $R T \ln(P_{OUT}/P_{IN})$. On a logarithmic scale, the distance from 1 to 10 MPa is the same as from 0.1 to 1 MPa. However, the Δh_T value resulting from an expansion from 10 to 1 MPa is reduced by an insignificant amount compared to that resulting from an expansion to 0.1 MPa. Hence, almost the same cooling effect is obtained with only about half of the invested work, a fact that may equivalently be identified as an elevated COP. This is in analogy with the traditional staged system that involves a higher boiling point coolant of higher COP to elevate the overall COP.

The energy balance determines the total cooling capacity at both stages,

$$\begin{aligned} \frac{\dot{Q}}{\dot{n}} &= \frac{\dot{Q}_1 + \dot{Q}_2}{\dot{n}} = (h_3 - h_1) - \alpha (h_2 - h_1) + \frac{\dot{Q}_{PRE}}{\dot{n}} \\ &= \Delta h_T^{(3 \rightarrow 1)} - \alpha \Delta h_T^{(2 \rightarrow 1)} + \frac{\dot{Q}_{PRE}}{\dot{n}} \end{aligned} \quad (5.30)$$

The higher flow rate stream is of a lower compression ratio (from state 2 to 3), while the higher compression ratio (from state 1 to 2) circulates less flow. Thus, the power required for the two compressors is somewhat balanced.

The COOLAR provides an enhanced reliability through its use of an oil-lubricated compressor running at a low rotation speed, thereby performing fewer revolutions and lasting longer. However, it requires an oil management system.

The central idea of the COOLAR configuration is closely related to the dual pressure liquefier of Carl Linde, 1898,⁹⁹ with two successive isenthalpic expansions, $P_3 \rightarrow P_2 \rightarrow P_1$. (The Ball dual pressure cryocooler has two parallel isenthalpic expansion, $P_3 \rightarrow P_2$ and $P_3 \rightarrow P_1$). It is described in classical cryogenics books,^{164,165,166} where it is referred to as the dual pressure or dual expansion cycle. Barron¹⁶⁷ discusses the optimal intermediate pressure that minimizes the power consumption of a dual pressure liquefier for any split flow ratio. The yield of such a liquefier is smaller than of a simple Linde-Hampson liquefier but the COP is higher. Nevertheless, dual pressure Joule-Thomson cryocoolers are quite rare.

5.5 Fast Cooldown Cryocooling

5.5.1 Introduction

The most widely recognized and unique feature of open system Joule-Thomson cryocoolers is their ability to provide rapid cooldown. Furthermore, it is a performance criterion where Joule Thomson cryocoolers are unbeatable by any other kind of cryocooler. These small devices are still the fastest cryo-machines ever built for pumping heat to room temperature. For such an application, gas consumption is not of primary importance, although it remains a significant concern. In fact, with respect to gas consumption, the design strategy for rapid cooldown is counter to that adapted with the previously discussed continuous operation mode. While one designs the continuous operation version for minimum consumption, the fast cooldown version is designed with high instantaneous consumption, because the excess flow accelerates the cooldown. The primary design goal is rapidly attaining the coolant’s liquefaction temperature. Depending on capacities and ambient conditions, a few seconds are typically all that are required to achieve 80 K. From the

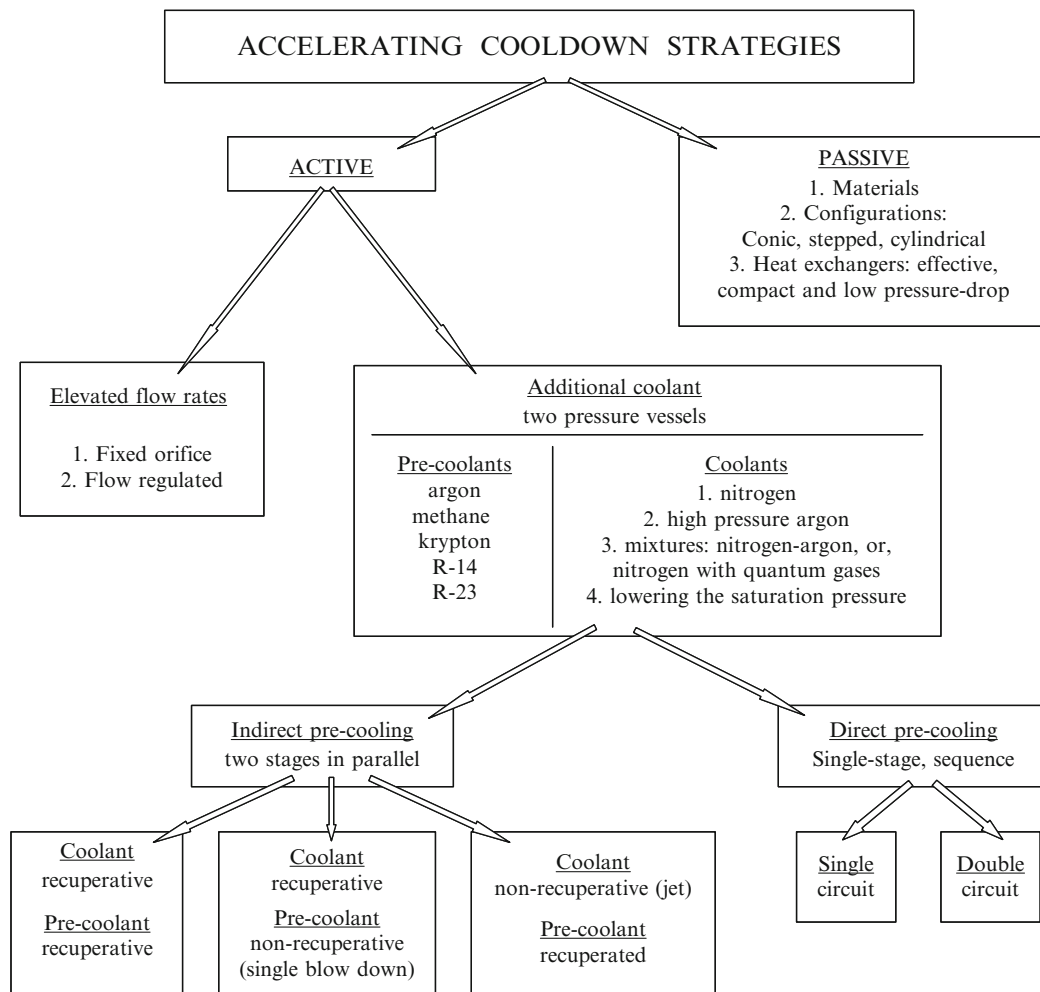


Fig. 5.17 Classification of means for achieving fast cooldown after reference¹⁶⁹

view of the system designer, this type of cooling is essential for standby equipment that requires immediate readiness upon activation. Longworth⁷³ describes a classic example of such an application,

Fast cooldown J-T coolers are ideally suited for cooling applications in tactical weapons that may be stored for long periods of time but must provide cooling within a few seconds of being activated and then maintain cooling for periods of a few seconds to a few minutes. Most of the applications are for cooling sensors on missiles or projectiles where environmental conditions are harsh, (high ambient, shock, and vibrations), space and weight must be minimized and low cost in large quantities is required. . .

Bonney and Stubbs¹⁶⁸ similarly describe the typical scope of applications. The basic analysis of fast cooldown conditions naturally relates closely with the study of transient behavior presented in Chap. 7. The variety of ways to achieve fast cooldown may be organized as displayed in Fig. 5.17, following Maytal.¹⁶⁹ In the broad sense, one may identify two categories, the passive, and the active. These are discussed in more detail at a later point in this chapter.

5.5.2 Characteristics

Fast cryocoolers, typical examples of which are shown in Fig. 5.18, share some remarkable characteristics that are discussed in detail below:

5.5.2.1 Cooldown

To about 85 K is typically required and accomplished within 3–5 s for ‘small’ thermal masses (50 J) or within 10 s for ‘large’ objects (400 J).

5.5.2.2 A Short Run Time

Is tolerated, being consistent with the fast cooldown requirement. One typically does not expect a fast cooldown system to run for a long time, although in principle the two issues are independent. Put another way, a device designed for long-term operation might be turned on well in advance of the time its use is required. A very short run, such as that by Pahler, Maier et al.^{170,171} might last about 15 s, while a long one might last up to 150 s.



Fig. 5.18 Typical collection of fast cooldown cryocoolers characterized by short heat exchangers (The Hymatic Co., Redditch, UK)

5.5.2.3 A Small Pressure Vessel

Of 20–100 cm³ will typically suffice for the entire run.

5.5.2.4 High Flow Rates

Necessary to generate the higher cooling power that is needed to shorten the cooldown period. Since the run time is relatively short, the supply vessel can be quite small.

5.5.2.5 Non Evacuated Encapsulation

For a fast cryocooler, the heat leak during the cooldown period does not significantly affect the process. The total heat leak accumulated during the short period of cooldown is negligible in comparison to the applied high cooling capacity. Longworth⁷³ describes the test of a fast cooler encapsulated in a shrink-fit thin plastic sleeve. Non evacuated encapsulations for fast cooldown cryocooling are also described elsewhere.^{172,173}

Practical values for the correspondingly elevated heat loads obviously depend on the actual sizes of the cryocooler, as well as on the cold tip payload and shape. An 8 mm diameter cryocooler assembled with a metal encapsulation will have a corresponding thermal mass of approximately 50 J. Alternatively, the heat load generated in the case of the same cooler without a thermally isolating vacuum might be about 3 W, while for a 5 mm size the associated heat leak would be only 1 W. In these cases, the total energy load is only 15–45 J for a 15-s operation. For comparison, the cooling power in such typical rapid cooldown applications is about 20 W to 40 W.

5.5.2.6 Constant Area Orifice

Very often, such fast cooldown cryocoolers do not include flow regulating mechanisms. Any such temperature sensitive adjusting device would contribute significantly to the overall heat capacity, resulting in a reduction of the cooldown rates. At first glance, sacrificing the benefit of flow reduction following cooldown might be regarded as a waste of gas. However, elevated cooling power and gas consumption will be realized in any event because of the high heat loads associated with the non-evacuated encapsulation. Furthermore, on the system level, designers are tolerant with extended flow rates, since the run times are so short, and the integral amount of gas saved in absolute terms would be small. In addition, the goals of a fast cooldown as well as simplicity, reliability and lower cost are achieved. The typical range of flow rates for miniature cryocoolers are 20–50 standard liters per minute of either argon or nitrogen.

5.5.2.7 Short Heat Exchanger

Typically, fast cryocoolers have shorter heat exchangers than the continuously operating versions. A representative length for an 8 mm diameter cryocooler is 20 mm. The shorter length indeed reduces the effectiveness and efficiency of the heat exchanger. However, this design feature enables the higher rates of cold production (Sect. 3.9) which in turn results in the rapid cooldown. Argon tolerates a smaller heat exchanger than nitrogen.

5.5.2.8 The Cutoff Pressure

Is usually high due to the lack of vacuum insulation that is typical of a fast cooldown cooler. Consequently, cooling terminates at relatively high pressures, even higher than twice the values associated with the continuous-operation dewar and cryocooler assembly, of about 8–12 MPa.

5.5.2.9 Integrated Assembly

Very often, although unnecessarily, these coolers are assembled with a small pressure vessel, as shown in Fig. 5.19, and allocated for a single shot mission. Hermetic metal sealing ensures long-term storage, and discharge is initiated by a build-in electrically operated pyrotechnic device.

5.5.2.10 Clogging

Fast cooldown cryocoolers are significantly less sensitive to the purity of the coolants. The combination of the following reasons results in an operation that is much more tolerant of gas contamination:

1. The diameter of the orifice is large because of the high flow rates that accelerate the cooldown.
2. The duration of operation is short.



Fig. 5.19 Rapid cooldown conic cryocooler assembly incorporating a pressure vessel and an initiating pyrotechnic valve (The Hymatic Co., Redditch, UK)

Since the process of building up a clog to the point of completely blocking an orifice happens gradually, this enables a wider margin of gas purity.

5.5.2.11 Temperature of Operation

The typical high flow rates associated with these coolers induce an elevated backpressure, which in turn results in an elevated boiling temperature. This effect directly translates into an undesirable higher temperature of operation, as recorded in the evaporator and on the payload. In practice, the increase in temperature may be as large as 10 K. The exact functional relationship between the backpressure and the resultant increase of the boiling point is simply defined by the saturation curve in the (P , T) plane for each gas, as discussed in Sect. 4.2. As a rough average for nitrogen and argon, each increase of 10 kPa results in a corresponding increase of about 1 K.

5.5.2.12 System Level Considerations

Play a central role in determining the combination of a cooler and a pressure vessel. There is a strong interdependence between the relevant factors, so it is inadequate to consider performance based on the cryocooler alone.

5.5.3 Coolants: Argon Versus Nitrogen and Their Mixtures

Argon is recognized as the preferred coolant for rapid cooldown purposes below 100 K. The superiority of argon for steady state continuous operation was previously discussed in Sect. 5.3.3.5. This section focuses on its features for rapid cooldown purposes.

As studied by Maytal¹⁷⁴ and verified experimentally,^{175,176} argon enables a cooldown rate that is about three times faster than nitrogen and a total cooldown period even four times shorter. These advantages may be explained in terms of the two pertinent thermodynamic features: ΔT_h , which as displayed in Table 5.1, is about two to three times larger for argon, and the ΔT_{CD} , that is only about half that of nitrogen.

One may also note from Table 5.1 that at steady state the yield of liquefaction, y , for argon is 1.5–1.9 larger than for nitrogen. Thus, a higher fraction of argon flow is liquefied in the evaporator than of nitrogen for similar conditions. More liquid phase in the evaporator enhances the convective heat transfer coefficient thereby further accelerating the cooldown process of the object.

When one considers the duration that a fixed orifice cooler can operate based on the storage capacity of a fixed-volume pressure vessel, again there exists a preference for argon over nitrogen. A superior compressibility profile (Sect. 4.7 and Fig. 4.36) results in more stored moles of argon for a given vessel and charging pressure. Consequently, during a discharge process, the pressure of argon in the vessel decays more slowly.

Next, consider the influence of pressure on the cooldown time. The pressure dependence of ΔT_h displays a maximum; therefore the cooldown period behaves in a similar fashion. Table 5.1 shows how over-pressurizing suppresses ΔT_h and Δh_T for nitrogen but the same pressure change increases the same entities for argon. For example, by increasing the pressure from 40 to 70 MPa, ΔT_h for argon still increases (though moderately from 92.3 to 93.1 K) while for nitrogen it decreases significantly from 38.5 to 27.9 K.

A higher pressure is desirable, as it provides for a longer discharge. For nitrogen, no gain in the cooldown period is achieved when its pressure exceeds 40 MPa. Furthermore, for pressures as large as 60 MPa, the cooldown period is significantly longer. However, the cooldown period for argon continues to decrease up to pressures of about 70 MPa, and even at 85 MPa only minor increases are realized. Kunitomo,¹⁷⁷ Pahler et al.,^{170,171} and Skertic¹⁰⁹ have all taken advantage of this feature. In summary, for a given storage vessel volume, a larger quantity of gas can be stored, the cooling can be achieved more quickly, and cooling is maintained for a longer period of time when argon is used instead of nitrogen. For nitrogen, the highest recommended pressure is 45 MPa, while for argon it may reach 70 MPa.

Mixtures of nitrogen and argon exhibit intermediate performance between the two pure gases. Cooldown with a nitrogen-argon mixture is slower and the run time is shorter than with pure argon. However, a significant benefit of including nitrogen in the mixture is that the temperature of operation^{73,178} is lowered. Thus, in spite of the mixture's

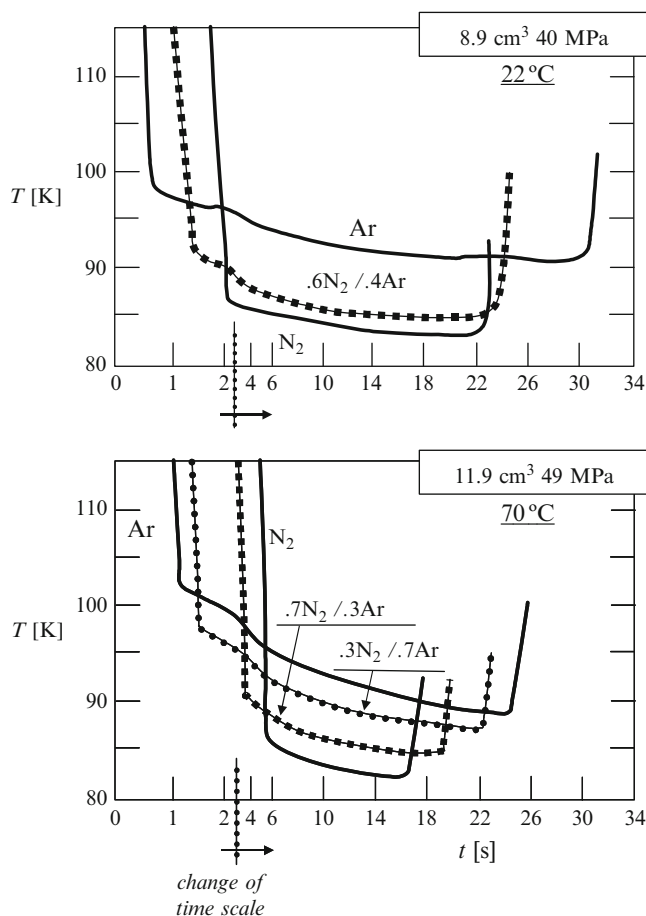


Fig. 5.20 Tests results for finned tube heat exchangers, 3.3 mm diameter and length of 35 mm with argon, nitrogen and mixtures at 22 C and 70 C, inside a non-evacuated stainless steel sleeve of 0.08 mm thick, pressurized by miniature vessels, after Longworth and Steyer⁷³

slower cooldown, the temperature of liquid argon may be obtained more quickly. Whenever such an advantage becomes relevant, the use of mixtures may be justified. Following Longworth et al.,⁷³ Figure 5.20 displays the relationship between the relevant parameters, which are temperature of operation, cooldown rate, and duration of run. The faster mixture operates at a higher temperature and for a longer run. Real et al.¹⁷⁹ built a cryocooler for a nitrogen-argon mixture in a way that most of the returning stream is recuperated, but only a fraction cools the payload without being recuperated. The configuration enables a further reduction of the boiling point by lowering the backpressure, since the larger total flow rate through the payload cooling channel would induce a higher backpressure.

To further lower the operating temperature below that of nitrogen while yet taking advantage of the large cooling power of a precoolant, the final coolant may be nitrogen mixed with a quantum gas such as with 15% neon¹⁰⁹ or with 30% hydrogen.¹⁸⁰ A combined mixture offers another

Table 5.4 Thermal diffusivity at 300 K,¹⁸³ α , measured in cm²/s

Ag	Au	Cu	Al	Si	Mg	W	Mo	Fe
1.75	1.30	1.18	0.95	0.88	0.86	0.68	0.57	0.25

option, such as 0.1Ne-0.45 N₂-0.45 Ar.¹⁸¹ However, Longworth comments that in spite of the initial high cool-down rate, 80 K is achieved within essentially the same time as if pure nitrogen were used. Another drawback exists regarding the inconstancy of the operating temperature. Effects related to mixture boiling cause changes in the operating temperature. For example, after the lowest temperature is reached, the temperature increases towards the boiling point of the higher temperature ingredient.

The preferred combination of the performance parameters has to be determined at the higher system level in the light of a target function that weighs each factor. For example, one may be more interested in a longer run even if the temperature of operation is somewhat elevated, while in another case, the cooldown period may be the key factor. This type of a system approach is further discussed in the next section.

Gong et al.¹⁸² discussed hydrocarbon mixtures for fast cooldown applications.

5.5.4 Passive Techniques

Various passive techniques utilize component configurations and materials with superior relevant thermal properties in order to:

- Accelerate the rate of recuperation, and,
- Minimize the thermal delay between the bath of cryogen and the cooled object.

5.5.4.1 Materials

The following thermo-physical material properties are relevant for promoting rapid cooldown:

Thermal diffusivity, α ,¹⁸³ is the single most important parameter for determining the transient thermal behavior as governed by heat conduction in solids,

$$\alpha = \frac{\lambda}{\rho \cdot c_p} \quad (5.31)$$

Materials that increase the cooldown rate have a larger value of α . This explains why Phaler et al.¹⁷¹ used silver instead of copper fins as listed in Table 5.4 and Maytal¹⁸⁴ used silver coating instead of tin. The thermal diffusivity represents the ratio between the conducted energy that is driven by a given temperature gradient and the volumetric heat capacity that has to be removed (or supplied) in order to

maintain the temperature gradient. In general, thermal diffusivity increases at lower temperatures, and this because of the rapid decrease in the heat capacity of solids at low temperatures. Thus, energy transfer by conduction is enhanced as a cooldown process proceeds.

Specific heat capacities, c_p ,^{185,186} of materials determine the actual thermal mass that should be rejected, in terms of Joules, in order to reduce the temperature of the material. Lower heat capacities guarantee that by extracting a given amount of heat, lower temperatures are achieved. Since α is derived from the non-dimensional formulation of the Fourier equation,^{187,188} it determines the evolution of the transient phase in a relative manner and does not reflect the actual amount of extracted heat. For example, suppose we reduce the temperature of one end of a solid bar by 10 K and wait until the other end will cool down by 9 K. The same period of time will be required when cooling one end by 100 K in order to obtain a 90 K reduction on the other end. Nevertheless, the heat extracted in the second case will be ten times larger.

Thermal conductivity, λ ,^{189,190} is responsible for conductive thermal impedance or, equivalently, the induced temperature drop under a given heat flux. From one perspective, larger thermal impedances introduce larger dissipative mechanisms in a cooling system. Therefore, along path lengths where heat is being extracted, it is preferable to incorporate highly conductive materials. On the other hand, a thermal shunt that short-circuits a cryogen plenum to the room temperature environment introduces a heat leak to the low temperature environment. In such a case, a low λ is preferred.

5.5.4.2 The Recuperator

The magnitude of the rate of heat exchange in a recuperator affects the cooldown rate. The rate of heat exchange is determined by the compactness of the recuperator, that is, the heat-transfer area per unit thermal mass of the heat exchanger. This parameter is primarily a function of the technology used to fabricate the heat exchanger, such as finned tube, stacked screens, or micro-channels. The total heat capacity and end-to-end conduction of the recuperator by itself are also relevant as are the hydrodynamic flow impedance, especially along the low-pressure return path. This last parameter determines the lowest attainable (boiling) temperature. Many of these aspects are further discussed in Sect. 6.3 and 7.4.2.

5.5.4.3 Non-evacuated Encapsulation

Instead of a heavy glass dewar, fast cooldown devices utilize only a thin metal sleeve cover, or in some cases include a foamed thermal isolation as described by Peffley.¹⁹¹ In this way, the thermal mass contributed by the encapsulation is reduced. Actually, the specific heat capacity of glass is lower than steel; however, a metal cover may be prepared with a thickness of only 0.1–0.2 mm, whereas a glass tube must be

at least 0.7 to 1 mm thick. Consequently, the total heat capacity of the cold finger is reduced.

5.5.4.4 Thermal Interface to Payload

The designation of fast cooldown is, in practice, only of interest with respect to the cooled object, rather than to the coolant in the evaporator. Thus, a rapid liquefaction but with delayed cooling to the object is not acceptable for fast cooldown applications. Such a situation can occur when the interface between the evaporator and the cooled object has a high thermal impedance. Clark et al.¹⁷⁸ focuses on various aspects that define the significance of successful thermal coupling:

1. The thermal impedance at the mounting interface of the cooled object,
2. Factors that enhance the convective heat transfer coefficient, such as
 - (a) A higher flow rate and associated velocity,
 - (b) Yield or amount of liquid in the impinging jet.

Usually the payload is mounted on a substrate that is immersed in, and therefore cooled by, the two-phase cryogen. The substrate plays the central role in defining the time delay in a two-fold manner: through its thermal diffusivity, α , and its thickness, Δx . One should expect a thermal delay time constant approximated by,

$$\tau \approx \frac{(\Delta x)^2}{\alpha} \quad (5.32)$$

This relationship displays the direct and sensitive dependence of the time delay on the substrate thickness, by virtue of the quadratic dependence on Δx , and the inverse dependence on α . The same observation is highly relevant to the cold shield, which is conductively cooled¹³⁵ as subsequently discussed in Sect. 5.5.7.8. Although the shield tolerates a much higher target temperature than the sensor, its characteristic dimension is relatively large, and can introduce a significant thermal time constant. This becomes important for example, if its length of 5–15 mm has to reach 200 K^{192,193} before the infrared sensor mounted on the substrate reaches 80 K.

The next most important factor influencing the thermal conductance of the interface is the convective heat transfer coefficient, h , on the immersed side of the substrate. Since the Nusselt Number, Nu , increases monotonically with the Reynolds Number,¹⁹⁴ like $Re^{0.8}$, a higher flow rate will produce a higher value of h . A higher flow rate will therefore both elevate the overall cooling production, and locally improve the thermal response of the payload. Experiments^{195,196} verify that a direct impingement of a two-phase jet at the outlet of an expanded nozzle in a Joule-Thomson cryocooler results in an enhanced heat transfer process.

5.5.5 Active Techniques

5.5.5.1 Higher Flow Rates

Higher flow rates generally increase the rate of cooldown. As shown by the analysis in Sects. 3.9.1 and 3.9.2, higher flow rates improve the rate of cold production. This is the elementary active strategy displayed as part of the classification chart shown in Fig. 5.17. However, elevating of the cooling rate by increasing the flow rate also reduces the efficiency of the process. Therefore, the penalty is quite clear. The cost of cold production increases in terms of invested flow rate. Put another way, larger flow rates result in a shorter run time for a given size supply vessel, or require a flow regulating mechanism to reduce the flow after cooldown.

Simon¹⁹⁷ in 1958, Wurtz¹⁹⁸ in 1962, and Longworth¹⁹⁹ in 1970 already recognized these two conflicting trends and adopted a compromise configuration using two parallel and concentric recuperators fed by the same pressure source. Two fixed orifices determine the flow rates through each recuperator; a large one for cooldown and a small one for steady state cryocooling. As the cooldown process draws to a close, the larger stream stops and the smaller flow maintains the steady state cooling. Alternatively, one may employ a flow regulating mechanism to operate in a continuous mode as discussed in Sect. 5.3, or by switching between two differently sized orifices.

5.5.5.2 Incorporating an Additional Higher Boiling Point Coolant

This robust and highly effective strategy can be realized in two versions by incorporating either a single stage cooler or a double stage one. The additional gas, cooling some part of the primary flow assembly from T_{AMB} down to its boiling point $T_{BOIL, 1}$, is referred to as the *precoolant*. The primary gas referred to as the *final coolant*, provides the remaining cooling, from $T_{BOIL, 1}$ down to its lower boiling point, $T_{BOIL, 2}$. The precoolant possesses a stronger drive for cooldown, ΔT_h , and a higher cooling content, Δh_T , than the final coolant as previously discussed in Chap. 4. Obviously, two separate pressure chambers for the two gas species are required.

The benefit of this approach is quite clear and may be summarized in two points:

- The pre-coolant brings the temperature from T_{AMB} to $T_{BOIL, 1}$ substantially faster than would be possible with the final coolant. Precoolants may even have values of T_{CD} above the ambient temperature, and may therefore achieve liquefaction immediately.
- Typically, the duty of the final and “less energetic” coolant becomes less demanding for two reasons:

- The temperature difference between the boiling points $T_{BOIL, 1} - T_{BOIL, 2}$ is only about half of $T_{AMB} - T_{BOIL, 1}$, and,
- The specific heat capacities tend to decrease at lower temperatures.

5.5.5.3 Precoolants

Precoolants commonly used are argon, CF_4 and krypton.^{111,109,200,201,202} Less popular alternatives are CH_4 ,^{154,158} R-23,^{135,203,204} or nitrogen – hydrocarbon mixtures (such as 28.8% N_2 , 39.5% CH_4 , 27.7% C_2H_6 and 4% C_3H_8) as reported by Little.⁶⁷

Krypton would appear to be a superior precooling alternative, and may be identified as the preferred precoolant, at least for fast cooldown purposes. Within the group of all gases that liquefy immediately, that is, through the first and single Joule-Thomson expansion, krypton is the one with the lowest boiling point, 120 K. As such, it only requires a very minute recuperator. The final coolant also requires only a small recuperator because of the deep precooling by krypton. Section 7.4.4 compares the cooldown tendency of gases and demonstrates the uniqueness of krypton in Fig. 7.10.

A precoolant may be introduced in two different procedures, as illustrated in Fig. 5.21, either via direct or indirect precooling.

5.5.5.4 Direct Precooling of Payload; Sequential Precooling

In the first procedure, a bath of the precoolant is created directly beneath the cooled object as illustrated in Fig. 5.21a. The approach is motivated by the desire for faster exposure of the cooled object to a lower temperature. However, for further cooling, one has to replace the precoolant bath with a bath of the final coolant in order to expose the object to the lower temperature $T_{BOIL, 2}$. The substitution is accomplished by switching the coolants, which is why the process is referred to in the literature as *sequential precooling*.

A disadvantage of this approach is the necessity to completely remove the warmer precoolant before the final coolant can be added. Having both coolants in the same location is problematic for two reasons: firstly, any remnant of the warmer precoolant imposes an unnecessary additional thermal load to be cooled by the final coolant's bath. Secondly, in some cases, the remnant of the precoolant may freeze upon contact with the final coolant leading to either clogging or at the least an additional thermal resistance between the cooled object and final coolant. Special attention is recommended in the case of the monatomic precoolants, that exhibit high freezing points, as discussed in Sect. 4.2.

The time development of a sequential cooldown using nitrogen pre-cooled by methane^{154,158} is qualitatively shown

Fig. 5.21 (a) Direct cooling of the object by a single stage cooler: sequential operation of the precoolant and the final coolant. (b) Indirect precooling of the object. Precooling stage rejects heat from the stream of final coolant

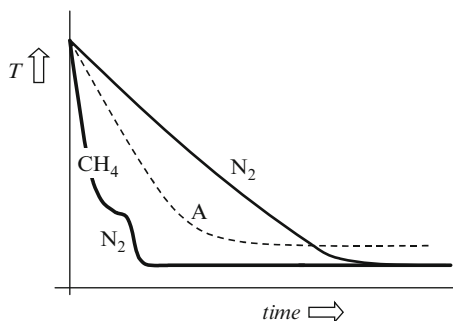
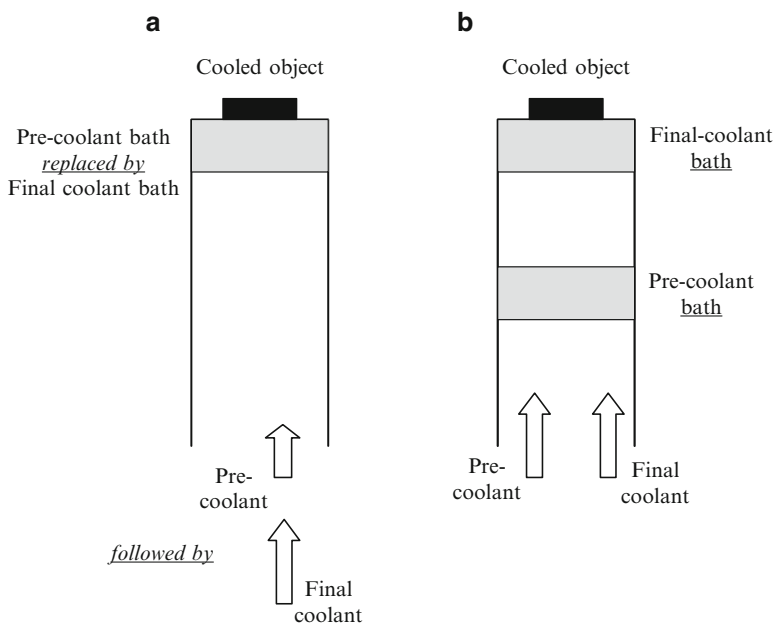


Fig. 5.22 Cooldown by a single coolant, nitrogen, versus sequential precooling by methane. Argon is shown for reference

in Fig. 5.22. Some delay is observed when nitrogen takes over, but the sequence is still much faster than with nitrogen alone.

The sequenced procedure also requires a somewhat more complicated pressure supply system. One possible arrangement uses separate vessels, or a double chamber single bottle, with two sequentially synchronized activating valves. Bytniewski et al.²⁰⁵ proposed a system with a single activating valve. Discharging the precoolant induces a pressure difference between the two chambers. When the pressure difference becomes sufficiently large, it opens a port to discharge the final coolant. In an alternate suggestion,^{206,207} a diaphragm that separates the coolant chambers is designed to burst as the precoolant discharges completely.

Another version of this type includes two flow circuits in parallel, each fed by a different coolant. It includes two separate recuperative heat exchangers, each with its own evaporator, both operating in sequence. It should be emphasized that each evaporator has direct thermal contact

with the cooled object. While somewhat more complicated, this approach is not encumbered by frozen precoolant residue as the final coolant takes over, since they flow in different channels. Liston²⁰⁸ in 1967 proposed such a fast cooldown cooler for infrared detection. The precoolant was argon (itself pre-cooled by carbon dioxide) and the final coolant was a mixture of freons, R-14 and R-23.

Another variation of a sequential cooldown scheme is provided by Haas et al.²⁰⁹ Here a sequence of three gases are supplied from vessels arranged in a series. The methane vessel, situated next to the cooler, discharges first. Then the next vessel, filled with argon, discharges through the methane vessel into the cooler. Finally, nitrogen from the third vessel passes through both vessels and feeds the cooler. In this way, there are three activating valves, but a single tube entering the cooler.

The serial assembly also extends the run time. The final coolant, nitrogen, expands prior to entering the cooler, occupying the volume of all three gas supply vessels. Therefore, its pressure becomes lower, the associated flow rates are reduced, and cooling is maintained for a longer period. Haas²¹⁰ also provides a second version of such a sequential vessel configuration.

An advantage of the sequenced technique, pointed out by Longworth,⁷⁵ is that it requires only a single inlet tube, while the double circuit option requires two tubes to feed the cooler. This issue becomes significant in the case where the cooler is mounted on gimbals.

5.5.5.5 Indirect Precooling of a Payload

Indirect precooling of a payload is accomplished by a staged cryocooler as described in Sect. 5.4. In this case, what is

actually precooled is the high pressure stream of the final coolant by the scheme of Fig. 5.9b. The cooled object is only exposed to the final coolant's plenum as shown in Fig. 5.21b. The two coolants may flow simultaneously, in parallel channels, during the whole cooldown process. The strategy of this approach is to:

1. Continually precool the inlet stream of the final refrigerant, thereby amplifying the cooldown driving factor, ΔT_h , and,
2. Remove the thermal capacity of the hardware assembly down to $T_{BOIL, 1}$.

Since separate channels for each coolant are employed, this approach requires special hardware construction in the form of two stage cryocoolers, as previously discussed in Sect. 5.4.

In the most typical configuration, both stages incorporate recuperative heat exchangers^{109,135,200,201,211,212,213} wherein the returning expanded coolant absorbs heat from the high pressure incoming stream, providing a positive feedback as in the traditional Linde-Hampson cycle. However, one may also find a non-recuperative precoolant stage^{135,203} where the expanded precoolant is released and is not in thermal contact with the high pressure feed. Additionally, there may be a non-recuperative final stage^{158,154} where the expanded final coolant is released as a jet.

It is possible to combine the direct and indirect modes of precooling. As proposed by Skertic and Hlava¹⁰⁹ the cryocooler is initially fed by argon in both stages so that the precoolant argon operates directly on the object. Subsequently, the argon in the final stage flow path is replaced by nitrogen and the cooler operates in its indirect mode.

5.5.5.6 Simon's Cooling Effect

Simon's cooling effect^{214,215,216,217} involves sequential precooling and "explicitly" enables a fast cooldown mode, because it involves discharging the final gas from a small pressure vessel at a relatively high flow rate. It shortens the cooldown period and may extend the duration of discharge.

The fluid that remains in a discharging pressure vessel does work on the exiting gas and therefore undergoes an isentropic expansion, although it is sometimes referred to as Joule expansion.¹⁶⁵ Consequently, its temperature may drop all the way down to the boiling point. If thermal interaction with the vessel does not occur, the initial and final temperatures of an ideal gas of the remaining portion inside the vessel are related to the corresponding pressures,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{(\kappa-1)/\kappa} \quad (5.33)$$

and the fraction of remaining gas is given by,

$$\frac{m_2}{m_1} = \left(\frac{P_2}{P_1} \right)^{1/\kappa} \quad (5.34)$$

This cooling effect takes us back to the very beginning of cryogenics science. Cailletet^{99,218} applied it in 1887, compressing oxygen to 30 MPa and precooling with sulphur dioxide at -29°C . After expansion, he observed the fog of oxygen inside the vessel. However, the technique carries the name of Simon,²¹⁴ who utilized the same principle in 1932 to obtain liquid helium for laboratory consumption. Expanding the pressurized helium gas from a vessel precooled to the triple point of hydrogen liquefies a large fraction of the remaining helium inside the vessel. Analyses may be found elsewhere.¹⁶⁷

Stuart²¹⁹ suggested applying Simon's method alone to cool infrared detectors, without any recuperator. He considered discharging a 120 MPa bottle of nitrogen until it reached its boiling point inside. Because the efficiency of the method is degraded by heat transfer from the warm container to the gas, Stuart proposed a layer of thermal isolation to cover the inner surface of the vessel. The detector cooldown obtained in this fashion is indeed "immediate" or, at least as fast as the rate of fluid release. However, the remaining amount of liquefied nitrogen is small and suffices only for a short run.

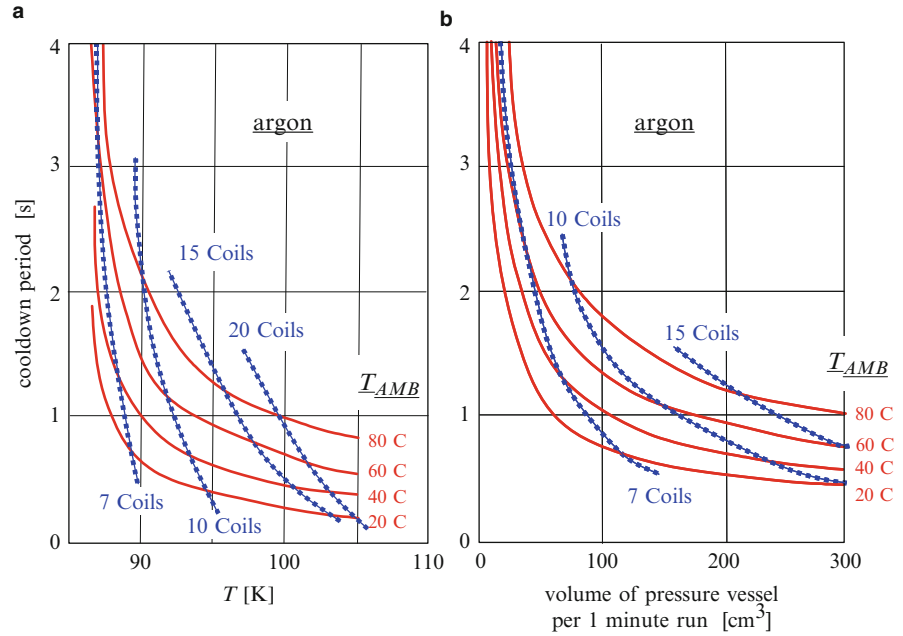
Buller^{220,221} combines Simon's cooling approach (enhanced by a similar isolation of the vessel's inner fluid) to assist the liquefaction process of a fast cooldown Joule-Thomson cryocooler. He provides an example of a 65 MPa vessel of argon used to feed a cryocooler with a thermal mass of 75 J and an orifice, which generates an initial flow rate of 50 SLPM. The gain associated with Simon's effect is manifested by reducing the cooldown time to 1.4 s instead of 2.2 s and by prolonging the run time to 30 s instead of 15 s. The improvement results from successively decreasing the cooler's supply temperature and from reducing the cut off pressure.

Longworth²²² discusses a short-term mission in which a sensor was cooled to 10 K. He proposed a two or three stage Joule-Thomson system to cool helium below 25 K. A Simon expansion cell was constructed and tested to provide the final temperature reduction to 10 K.

5.5.6 System Approach

If a cryocooler was reported to liquefy within a short period, such as 0.3 s, one might ask whether or not it was a fast cryocooler. How might such a question be evaluated? It will be shown below that this question is inadequately defined; the duration of cooldown period is only one part of a broader picture to be considered. In fact it is quite easy to get immediate liquefaction (cooldown in zero seconds) with

Fig. 5.23 Coolers are of 30° conical shape finned tube heat exchangers, optimized for minimum cooldown period with a non evacuated stainless steel encapsulation: the achievable temperature and the associated vessel per 1 min run¹⁷⁵



a single higher boiling point gas like krypton. However, it is significant that the temperature achieved is not below 120 K. The salient point is that the cooldown period should always be reported in conjunction with the achieved temperature.

Furthermore, it is also significant to inquire regarding the flow rate used to achieve a high cooling rate. If it is too high, a large pressure vessel may be required, and the total result will not be satisfactory, even though the cooldown is fast. Another issue of relevance is the ambient temperature.

The important point from the above consideration is that a system approach is unavoidable, and any design must take into account more than simply the cooldown period, t_{CD} .

5.5.6.1 Optimized Cryocoolers

Reference¹⁷⁵ focuses on this very issue, applying a system approach to a group of *optimized* cryocoolers; each cryocooler with its encapsulation is optimized for the shortest cooldown period (as discussed later in Sect. 7.6). All of the coolers incorporate 30° conical shaped finned tube heat exchangers (as shown in Figs. 5.18 and 5.19), encapsulated in a vacuum isolated thin sleeve. A heat capacity of 30 J is reported for the assembly of the 15 coil heat exchanger with its encapsulation, (about 2 J per coil) in addition to the 1.3 J ceramic substrate at the cold end. For any length, L , of heat exchanger (indicated by the number of coils) the proper flow rate (and corresponding fixed orifice), is chosen to minimize the cooldown period. The optimized orifice is determined in this study by the storage volume required per 1-min run. The ambient temperature is also a relevant parameter for the study of cooldown periods.

The results for argon are summarized in Fig. 5.23 by two graphs, and define a relationship between the examined parameters of the form,

$$f(t_{CD}, L, T, V, T_{AMB}) = 0 \quad (5.35)$$

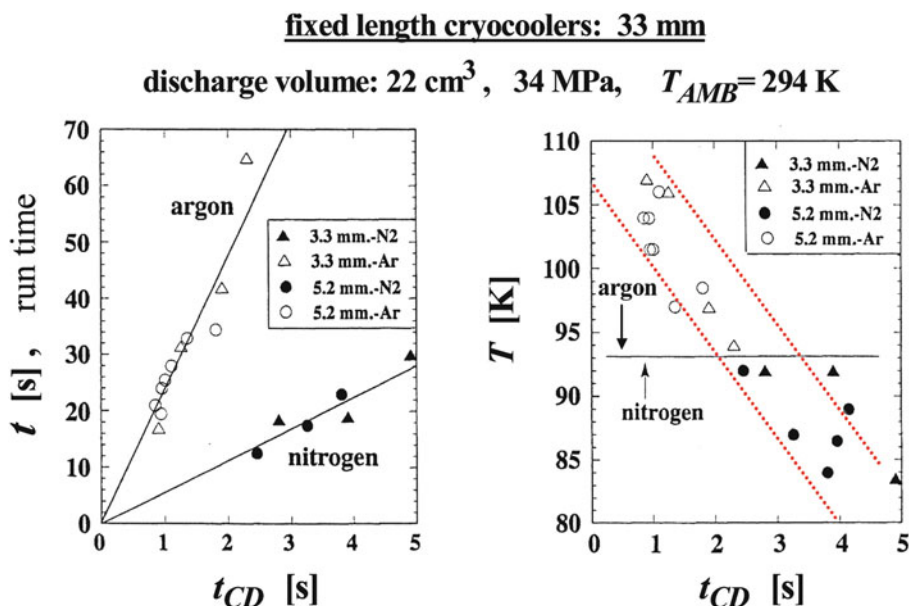
Each point on the graph represents one optimized cryocooler. One should observe the following trends:

1. A shorter cooldown time requires a larger pressure vessel and is accompanied by an elevated temperature of operation.
2. A longer cryocooler might have a shorter cooldown period. However, the shorter period will still require a larger vessel and elevated temperature. A longer heat exchanger will have a larger hydrodynamic impedance and therefore elevate the back pressure, for a given flow rate. Thus, the operating temperature will also be elevated.
3. A lower environmental temperature is beneficial for faster cooldown and prolonged run time.

Finally, the point of the work is to enable trade-off analyses related to fast cooldown. For example, in the above case, one might first explore whether it is acceptable at all to use a longer cooler with 15 coils instead of 10 coils. If it is possible, the next question would be whether, in order to shorten t_{CD} at 60 C from 1.2 to 0.7 s, it is worthwhile to enlarge the pressure vessel from 130 to 300 cm³.

The charge pressure of the vessel, although not specified in reference¹⁷⁵ also plays a role in the design process. On the one hand, increasing the pressure can accelerate the cooldown and prolong the run. However, over-pressurizing can also extend into regions of poor compressibility,

Fig. 5.24 Given size cryocoolers with different flow rates: cooldown period and run-time, after Longworth²²³



resulting in only a minor addition of gas and degrading the cooldown.^{174,176}

5.5.6.2 Fixed Length Cryocoolers

Quite often, the length is the primary parameter used in determining the size and mechanical design of the entire device. In practice, one may consider a compromise of gaining a shorter configuration with a somewhat lower temperature, but be willing to sacrifice the cooling rate to some extent. That might explain why Longworth and Steyert²²³ investigated a group of fast cooldown finned tube cylindrical cryocoolers with a constant length (33 mm) but with two different diameters, 5.2 and 3.3 mm. All of the coolers used a non-evacuated encapsulation constructed of 0.076 mm thick stainless steel. Figure 5.24 displays the results with nitrogen and argon. In this group of fixed length cryocoolers, only one may be optimized for shortest t_{CD} . Hence, all examined cryocoolers employ a smaller flow rate than the optimal one. The rational would be: let us degrade the t_{CD} and examine if the benefit (run time, temperature, diameter) is worthwhile.

Here the run time, t , provided by flow from a pressure vessel of 22 cm³ initially charged to 34 MPa is displayed rather than the volume necessary for a 1-min run. Implicitly, the run time sets the size of the fixed orifice. The reported experiments do not include the influence of higher ambient temperatures as in the previous study. One may observe the following trends:

1. For a given run time, t , argon cools down about four times faster than nitrogen, and,
2. A faster cryocooler has a higher temperature of operation.

Another study of Longworth and Steyert⁷³ with a fixed length cryocooler was introduced in Sect. 5.5.3. A cryocooler

of 35 mm long and 3.3 mm diameter was tested with a variety of mixtures of nitrogen and argon. Such mixtures of coolants with close boiling points, provide new solutions with properties unattainable by any pure coolant (Fig. 5.20). In this way, a continuous range of intermediate values of the relevant parameters of performance may be obtained: operating temperatures (between 78 and 87 K), cooldown periods, sizes, sensitivity to elevated ambient temperature, and gas consumption. The same trend also holds for these mixtures: the mixture with the lower boiling temperature produces the slower cooldown and consumes less gas.

5.5.7 Special Examples

This section introduces three examples of staged cryocoolers and five configurations that promote fast cooldown.

5.5.7.1 Staged, Porous and Flow Regulated Fast Cryocooler

A unique development, reported by Pope,¹¹¹ is a system used for cooling a Mercury Cadmium Telluride focal plane detector array. The system cools a relatively large thermal mass, along with a massive, highly evacuated dewar, within about 5 s, from an ambient temperature of 343 K down to 80 K. A few of its features are as follows:

1. Pope reports the use, for the first time (to the best of the author's knowledge), of the ultimate precoolant, that is krypton.
2. The system utilizes a matrix heat exchanger as a non-direct precooling stage. Nitrogen flows through the final stage at the same time that krypton flows through the precooling stage.

3. The system incorporates a flow regulator for nitrogen. Krypton is involved only for precooling.

5.5.7.2 Staged Wire Screen Compact Heat Exchanger

Some versions of these cryocoolers as introduced by Steyert and Lonsworth²⁰⁰ are discussed in Chap. 6. The key factor for accelerating cooldown is the compactness parameter (heat transfer area per unit thermal mass) and its low induced backpressure.

5.5.7.3 Photolithographic Precooled Fast Cooler

This special and unique technology developed by Prof. W. Little utilizes the manufacturing process of microelectronics, which is separately discussed in Sect. 6.4.3.3, to produce cryocoolers. The prototype of reference⁶⁷ incorporates a two-stage heat exchanger and a number of interesting features:

1. The pre-coolant is a hydrocarbon mixture (28.8% N₂, 39.5% CH₄, 27.7% C₂H₆ and 4% C₃H₈) at a pressure of 30 MPa and a flow rate of 4.3 Standard liters per minute, rather than the traditional pre-coolant, CF₄.
2. Nitrogen in final stage enters at a relatively low pressure: 12.9 MPa and a flow rate of 0.8 SLPM (Standard liters per minute).
3. A miniature Venturi pump is used, driven by the high flow rate mixture. It reduces the outlet pressure and the boiling point of nitrogen. From the view of a system approach, it is an original method to address the typical relationship between the cooldown period and ultimate temperature. In some cases, closing the gap of the last 3 K at cooldown occurs much slower than the average cooldown rate. Therefore, the contribution made by the pump to lower the final attainable temperature is non-negligible.

5.5.7.4 Fast Cooldown System with High Shock Resistance

An outstanding and unique system from any point of view (construction, architecture, thermal design and thermodynamic approach) was developed in Germany and reported by Pahler et al.¹⁷⁰ The exploded schematic of the construction detail and cut-away photograph of the cooler are shown in Fig. 5.25. The system is an integrated assembly of compact design, includes a cooler and a 20 cm³ pressure vessel, and is expected to achieve cooldown within 0.5 s and a run time of at least 10 s. Some of the remarkable features are as follows:

1. The assembly is built to withstand mechanical shocks of 40,000 g.

2. The high-pressure heat exchanger tube employs silver fins, rather than the traditional choice of copper. Indeed, its thermal conductivity is higher and its specific heat capacity is lower. Altogether, the choice results in a higher thermal diffusivity, thereby providing a faster thermal response.
3. The construction allows argon charging pressures as high as 80 MPa, thereby taking advantage of the fact that argon still gains speed in this pressure range.
4. The unique flat coil construction results in a recuperator with the same diameter as the small vessel.

5.5.7.5 The “Inverse” Cryocooler

The “inverse” cryocooler is analogous to a Copernican revolution: so simple, so obvious but never before proposed. Dunn and Gowlett²²⁴ inverted the cooler, mounting the infrared sensor at the cold end, in the opposite direction, so that radiation passes through the heat exchanger, as shown in Fig. 5.26. The core of the heat exchanger serves as a cold shield (see sect. 5.5.7.8) and cools the cold filter. The entire construction is shorter and faster since no additional time delays are involved. It is a higher-level integrated assembly, as the cooler attaches to the pressure vessel with an activating valve.

5.5.7.6 A Single Non-recuperative Expansion

Gases of higher boiling points are capable of partially liquefying just by passing through a single Joule-Thomson expansion. The liquefaction occurs instantaneously and without a heat exchanger. This property was utilized by Ambrose and Hsu^{135,203,225,226,227} for fast cooldown of sensors, using methane, krypton, propane, and R-23. They applied the same idea in a three-stage cryocooler, where the first precooling stage used the direct expansion of R-23. The final coolant was neon.

5.5.7.7 Thermal Isolation Between the Cooler and Its Encapsulation

Maier¹⁷² proposed covering the fast conical heat exchanger with layers of a thermal super-isolator, held in position as the cooler is pressed against the cold finger. In this way, the expanded cold exiting gas is not exposed to the warming effect of the cold finger, and its entire cooling power is used to pre-cool the incoming high-pressure warm gas. The approach intends to accelerate the self-sustained positive thermal feedback and ultimately leads to a faster cooldown. The super-isolator is a multi layer foil made of polyimide or a metal with oxidized surface of low thermal capacity and thermal conductivity. By a similar approach, Webster²²⁸ reported that such thermal isolation in a cylindrical cryocooler suppressed its cut off pressure.

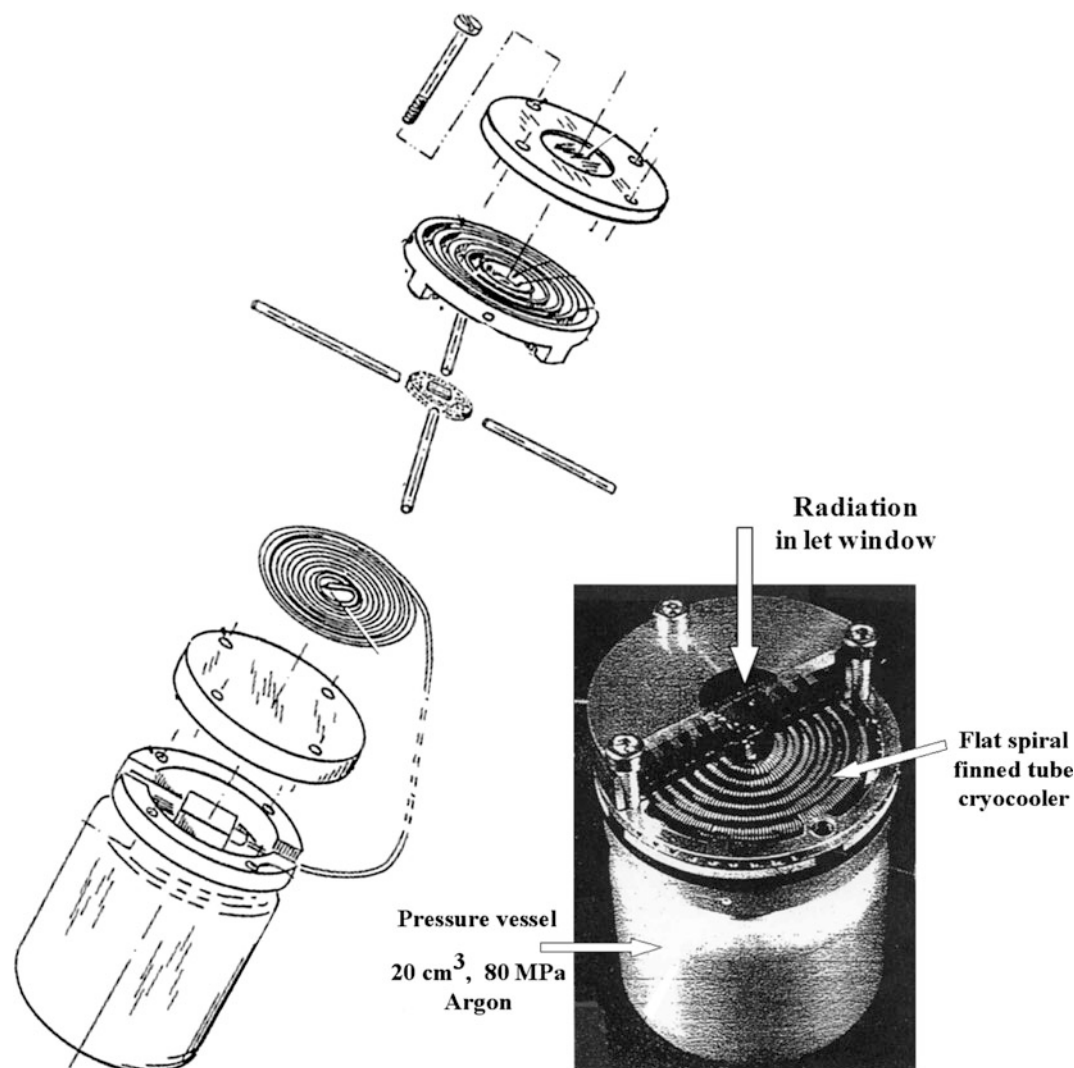


Fig. 5.25 An integrated assembly of a miniature pressure vessel with a flat finned tube heat exchanger after Pahler, Maier et al.^{170,171} for fast cooldown and withstanding high mechanical shocks

5.5.7.8 Faster Cooldown of the Cold Shield of an Infrared Detector

Cooling an infrared sensor includes the simultaneous cooling of its cold shield and detector. The cold shield protects the detection process by avoiding the collection of parasitic radiation. The cooldown process is completed when the sensor reaches about 80–95 K while at the same moment, the warmer end of the cold shield must reach about 200 K as described by Hsu et al.¹³⁵ Even if the sensor is rapidly cooled, the task of cooling the infrared sensor remains unaccomplished until the cold shield temperature goes below 200 K. Therefore, the thermal impedance of heat extraction from the thermal shield deserves the same attention as that of the infrared sensor itself. Figure 5.27 displays a schematic of (a) the traditional cooling configuration of the thermal shield and (b) an enhancement proposed by Eneim¹⁹² for fast cooldown. The former is a serial heat

extraction approach, cooling the shield through the sensor's assembly. The latter provides cooling in parallel; both the shield and the sensor are in direct thermal contact with the bath of cryogen. In this approach, the sensor cools down faster, but the major gain is in shortening the cooldown time of the shield. In addition, by parallel cooling and proper design, both shield and sensor may reach their target temperatures at nearly the same time. Loiseau²²⁹ suggested an additional method to accelerate the cooldown of the cold shield. Rather than conductively cooling the shield, the cold gas of the Joule-Thomson cryocooler simultaneously cools down the cold shield together with the detector. Loiseau mentions two versions; one with a single cryocooler and another with an additional Joule-Thomson cryocooler dedicated to the cooled shield. This issue is also discussed elsewhere.²³⁰ Kunimoto and Eneim¹⁹³ combined the parallel heat extraction with an additional cryocooler.

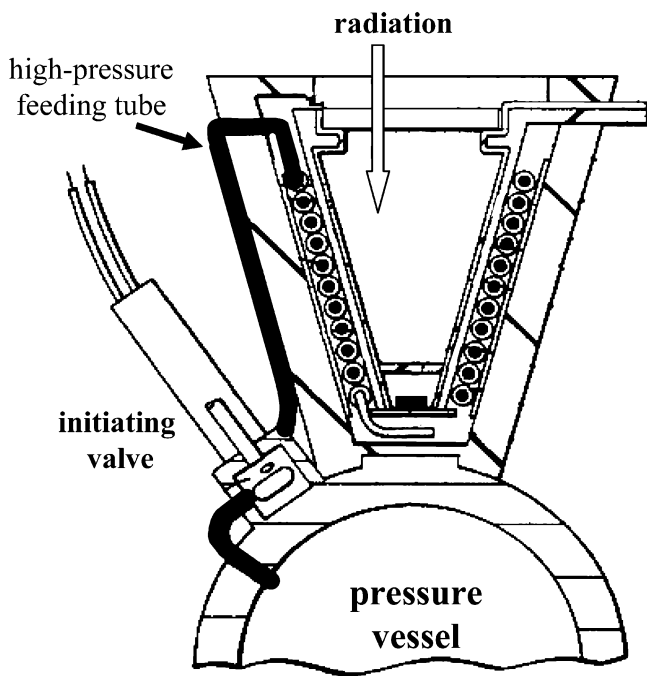


Fig. 5.26 The “inverse” cryocooler after Dunn and Gowlett²²⁴. The core of the heat exchanger serves as a cold shield and cools the cold filter

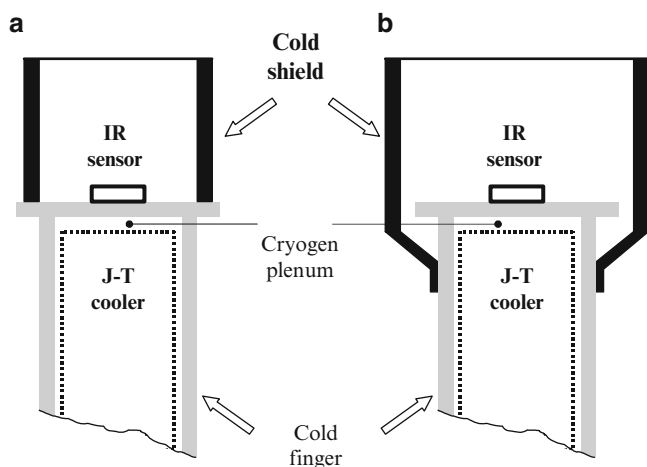


Fig. 5.27 (a) The traditional cooling of the thermal shield by serial heat extraction, and, (b) the enhancement proposed by Eneim et al.,¹⁹² and Kunimoto et al.¹⁹³ for fast cooldown of the thermal shield in parallel with the sensor, both in direct thermal relation with the bath of cryogen

5.5.7.9 Xenon or Krypton in a Non-evacuated Encapsulation

Xenon or krypton in a non-evacuated encapsulation may partially liquefy or even solidify while a nitrogen JT cryocooler cools down. Such a possibility may prolong the run time to some extent, as reported elsewhere.²³¹

5.6 Hybrid Joule-Thomson Cryocoolers

5.6.1 Introduction

Previous sections demonstrated the potential benefits of adding a precooling stage to a Joule-Thomson cryocooler. A system that precools the high-pressure stream with the use of a thermodynamic cycle of any type other than a Joule-Thomson cycle, is regarded as a hybrid cooler. Heat may be extracted at a single or multiple intermediate temperatures. Therefore, in a broad sense, a hybrid cryocooler may be conceived as a special category of a staged cooler. Designers of hybrid cryocoolers are motivated by the same concerns of those designing staged cryocoolers (Sect. 5.4.1) and apply similar considerations for optimal interstaging temperatures.

The most common type of precoolers is the regenerative Gifford-McMahon (GM) and Stirling cryocoolers. Below 20 K the effectiveness of regenerators is generally poor and the efficiency of the corresponding (AC) cryocoolers decrease significantly in this range. On the other hand, the size of a recuperative system bridging the temperature span from ambient down to temperatures below 20 K can grow very large. Therefore, the hybrid system combines the benefits of both the compact regenerative systems in the high temperature region and those of the recuperative system in the low temperature region to target applications whose temperatures extend even into the pumped liquid helium region from 2.5 to 4.5 K. Typically a recuperative Joule-Thomson cycle is used for the lower temperature final stage. In these, a high-pressure stream of helium is typically precooled at two intermediate temperature stages; the first in the range of 80 to 120 K and the next at about 10–20 K.

The hybrid coolers operate in general as closed cycles. In this way, cryogen free cooling is supplied for a variety of applications such as for recondensing liquid cryogen boil-off, superconductive magnets, SQUID devices, computers, receivers, Maier amplifiers, cryopumps, and electromagnetic sensors on space applications.

Thermoelectric precooling is also a viable option for both an open system that releases the exhaust gas into the atmosphere and for a closed cycle system.

5.6.2 Thermoelectric Precooling

Precooling via the thermoelectric (Peltier) effect is highly attractive because of its simplicity, since it does not incorporate any moving parts. Thermoelectric coolers are electric solid-state devices driven solely by a supply of electric current. As with other thermodynamic cycles it is still necessary to reject some heat at the warm end of the thermoelectric element. And as with other precooler options,

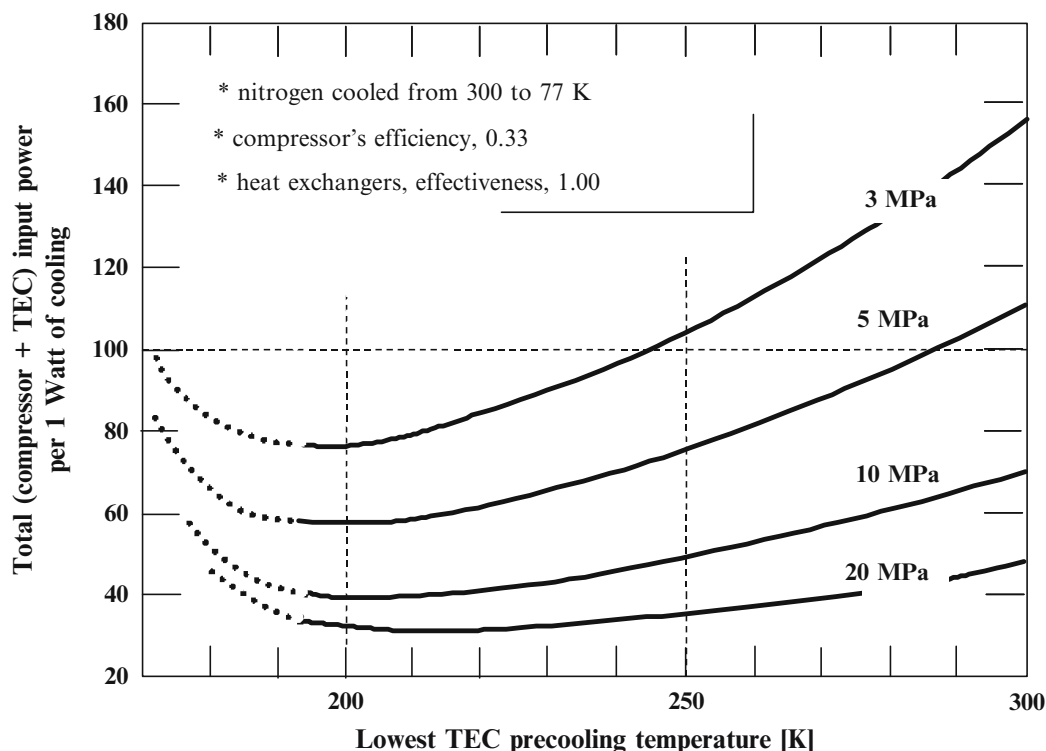


Fig. 5.28 Efficiency improvement of a Joule-Thomson cryocooler while being precooled by thermo-electric coolers, as reported by Lester, Levenduski, Scarlotti, and Fernandez.^{26,27,28,29,30,31,32,33}

thermoelectric precoolers prolong the run time afforded to an open Joule-Thomson system by a given size pressure vessel, thus gaining less frequent exchange of vessels. Furthermore, they enable a reduction of the flow rates in the Joule-Thomson cooler and/or lower the cutoff pressure, both benefits extending the period of operation. In the case of closed cycle, precooling with a thermoelectric cooler reduces the temperature difference, ΔT , along the recuperator and improves the overall COP.

A heat exchanger is required to bring the high-pressure stream of the Joule-Thomson cycle into thermal contact with the cold surface of the thermoelectric element. The precooling heat exchanger may be completely separated from the cooler as proposed by Baily²³² for a heat seeking missile and by Hingst.^{233,234} Hingst²³⁵ also suggested a novel arrangement of the thermoelectric elements – in a peripheral shape around the helical winding of the high-pressure tube and adjacent to the cryocooler at the inlet (or outlet). The arrangement provides a very convenient means of utilizing the exhausted low-pressure stream, flowing directly over the warm end of the thermoelectric elements, to provide the necessary cooling for those elements. This elegant solution avoids any other means of forced convection to remove heat at the warm end.

Benedict, Lester and Linenberger²³⁶ attach thermoelectric elements directly onto the recuperative heat exchanger

of a Joule-Thomson cooler, eliminating the need for an additional heat exchanger. Six stages of thermoelectric coolers extract heat at the intermediate temperatures of 300 K, 270 K, 240 K, 215 K, 190 K, 170 K, and 165 K through copper plates in thermal contact with the high-pressure tube.

This method of precooling was also employed in the Ball Aerospace closed cycle Joule-Thomson cryocooler, previously discussed in Sect. 5.4.9, as reported by Lester, Levenduski, Scarlotti and Fernandez.^{27,28,29,30,31,32,33}

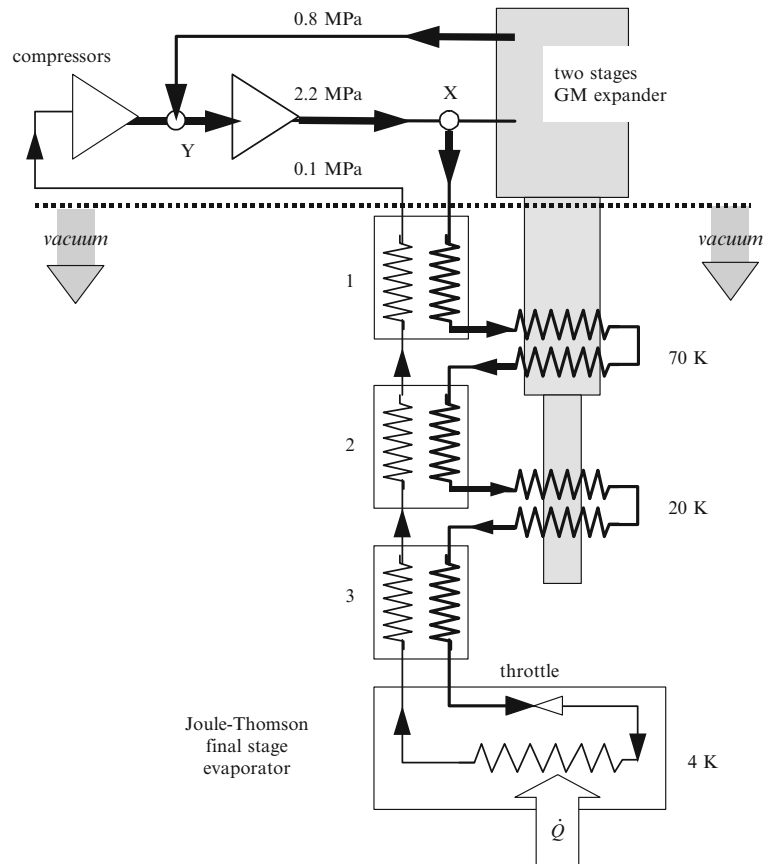
Figure 5.28 demonstrates the associated power gain. The total power consumption for both compressor and thermoelectric elements is cut in half by precooling to about 200 K.

A MEMS based microcryocooler for a terahertz sensor,^{237,238} developed by Bradley, Radebaugh, et al. employed a thermoelectric stage that provides precooling down to 240 K.

One should be aware of a common challenge associated with thermoelectric cooling, which is a relatively high power of heat rejection at the warm end of the element. The thermodynamic efficiency of thermoelectric coolers is quite low, and the entire electrical power input together with the cold-end heat load must be extracted at their warm end.

A complementary review of hybrid cycles using throttle and thermoelectric coolers may be found elsewhere.²³⁹

Fig. 5.29 Hybrid cryocooling system: a GM double stage cryocooler is combined with a Joule-Thomson final stage and precools it at about 70 K and 20 K. The working fluid of the GM cryocooler serves as the coolant for the Joule-Thomson stage. A small portion of the high pressure helium splits at junction X and returns at Y. The sections 1, 2, and 3 comprise the Joule-Thomson recuperator



5.6.3 Gifford-McMahon (GM) and Joule-Thomson (JT) Hybrids

5.6.3.1 The Combined Helium Cycle of GM and JT Cryocoolers

The compression unit of a Gifford-McMahon cryocooler generates a continuous (DC) flow. A rotary valve, typically mounted on the top of the GM cold-head, converts the DC flow into an AC flow to feed the reciprocating expander. Note that a fraction of the DC flow may be used to feed a Joule-Thomson cryocooler. In this way, the GM system can be combined with the JT system, both sharing a common compression unit.^{240,241}

The double expansion nitrogen system of Ball Aerospace, presented in Sect. 5.4.9, serves as a good basis to illustrate the idea of a GM/JT hybrid as displayed in Fig. 5.29. In Ball Aerospace's dual pressure cryocooler, the majority of the flow circulates in the precooling stage through the GM expander between the high and intermediate pressure. The smaller complementary portion of the flow splits at junction X (in Fig. 5.29), circulating in the final JT stage (through the recuperator of sections 1, 2, and 3) expanding from the highest pressure down to the lowest pressure, and joining the main stream again at junction Y.

A central difference between the Ball Aerospace hybrid cooler and a typical two-stage JT cooler is related to the coolant and the manner in which it provides cooling. Although the GM/JT hybrid employs helium, the work extracted by the GM expander from the gas is independent of the real gas properties of the coolant. In practice, the GM expander incorporates two or three stages, and the precooling at each stage of the GM expander enhances the cooling power of the recuperative JT loop. Typically, the final JT stage only consumes about 5–10% of total flow and the rest feeds the GM expander. The GM expander operates between 2.2 and 0.8 MPa while the JT loop expands from 2.2 down to 0.1 MPa. The intermediate temperatures of precooling are 50–80 K and 15–20 K. One may find details about the precooling heat exchangers for the JT stage elsewhere.²⁰¹

5.6.3.2 Miscellaneous

Not all GM/JT hybrids have a combined cycle. Other types use two separate compression cycles and flow loops; one for the GM expander and the other for the JT circuit of helium-4 for 4 K,^{242,243,244,245,246} for 2.5 K,²⁴⁷ for 1.3 K^{248,249} or of neon²⁵⁰ for 25 K. The JT cycle

is precooled at the two or three stages of the GM cryocooler realizing the same benefit as in the case of the combined cycle.

A retrospective review of GM/JT hybrid coolers was presented by Walker²⁵¹ and Longworth.²⁴⁰ The latter comprehensively compares a variety of GM expanders, compressors, and types of heat exchangers. Following the pioneering work of Gifford, Hoffman, and Stuart et al.,^{252,253,254,255} many different GM/JT hybrid machines have been built in the USA,^{256,257,258,259,260,261,262,263,264} in Japan,^{246,265,266,267,268,269,270,271,272} and in France.^{273,274,275,276,277}

Hybrid coolers are frequently used for re-condensing the helium vapor evolving from a liquid bath that cools a superconductive magnet of an MRI machine^{278,279,280} or other general-purpose device.^{281,282,283} Longworth and Steyert²⁸⁴ studied the influence of a magnetic field on the cryocooler and the potential interference of the refrigerator on the homogeneity of magnetic field.

High reliability is a unique feature of the GM coolers. It was already recognized and reported in 1976²⁸⁵ that these may run with an MTBF of 20,000 h and operate for a million hours. Therefore, one may expect that a 4 K hybrid cooler would indeed be a reliable machine.

5.6.4 The Stirling and Joule-Thomson Hybrids

In contrast to the Gifford-McMahon cryocooler, Stirling cryocoolers operate solely with an AC flow. Therefore, any direct interaction of flow between the Stirling and the Joule-Thomson cycle is impossible. Hence, Stirling coolers are typically used to precool the high pressure stream of a JT loop through a separate heat exchanging interface. In principle, the configuration is completely similar to that used with a non-combined GM/JT hybrid cryocooler as described in the first paragraph of Sect. 5.6.3.2.

In the early versions of Prast,²⁸⁶ Haaruus,²⁸⁷ and Salomonovich et al.²⁸⁸ two separate Stirling cryocoolers were used to precool the high-pressure helium flow stream in the JT cycle. The first Stirling cooler precooled the helium to 80 K, and the second to 20 K. The final helium temperature was close to its normal boiling point, 4.4 K. However, a single multi-stage Stirling cryocooler is capable of maintaining both intermediate temperatures at each of its stages. Such improved configurations are described by Prast²⁸⁹ and later by Orlowska and Bradshaw^{290,291,292} with additional constructive details in reference.²⁹³ Continued improvements by Jones and Ramsay²⁹⁴ resulted in a system capable of lifting 5 mW at 4 K. The Stirling cooler used by Jones and Ramsay was developed at the Rutherford Appleton Laboratory (RAL), UK.²⁹⁵ The special features of the expanders in these coolers that ensured long,

reliable operation were developed earlier for the Oxford University Stirling cooler,²⁹⁶ and were also employed in the design of the compressor for the Joule-Thomson circuit. Penswick and Lewis²⁹⁷ further integrated the Stirling and JT hybrid cooler by using a common linear drive motor to provide power for both the Joule-Thomson helium compressor and for the pressure wave generator of the two stage Stirling precooler.

Hybrid Stirling/JT coolers have been developed for space applications in the range of 4–12 K by utilizing a three stage Stirling cryocooler to precool the separate JT helium stream at each of its stages. See references^{298,299,300,301,302,303,304} and others reviewed elsewhere.³⁰⁵ The first use of this kind of hybrid in space occurred already in 1981.³⁰⁶ In some cases, the Stirling precooler operates periodically together with a thermal storage unit interfacing it to the Joule-Thomson loop.^{307,308,309,310} Another hybrid version for the liquid helium temperature range that incorporates a three-stage pulse-tube in order to precool the JT loop and provide remote cooling is presented by Raab et al.³¹¹ and Durand et al.^{312,313} Wang et al.³¹⁴ used a two-stage coaxial pulse tube to precool a helium sorption JT loop; this cooler has no moving parts.

5.6.5 The Final Joule-Thomson Helium Stage Enhancements

Being precooled by a GM or Stirling cryocooler to 10–20 K, helium is still above its T_{CD} . Therefore, a recuperative heat exchanger is required in such a system to provide further cooling before the final Joule-Thomson expansion in order to produce liquid. Usually the upstream helium pressure is quite moderate, typically ranging from 1.0 to 2.0 MPa due to the deep precooling and because of the low critical pressure of helium. Some attempts have been made to improve the performance of such a helium cycle.

5.6.5.1 An Ejector Expander for the Final Joule-Thomson Stage

Cooling with a JT cycle below 4 K demands special consideration. In such a system, the vapor pressure of ^4He at the downstream side of the expansion valve is determined by the suction pressure maintained at the compressor inlet. For temperatures below 4 K the vapor pressure of ^4He becomes significantly lower than 100 kPa. It is less than 50 kPa for 3.5 K and about 10 kPa for 2.5 K. Rietdijk^{315,316} showed that to approach 3.5 K, the size of the compressor must be doubled in comparison to what is required to achieve 4.2 K. Cooling down one more degree, to 2.5 K, increases the required size and power consumption by about an order of magnitude. Such observations have motivated the search for more sophisticated solutions to the final Joule-Thomson helium circuit; ones that would enable a lower boiling point

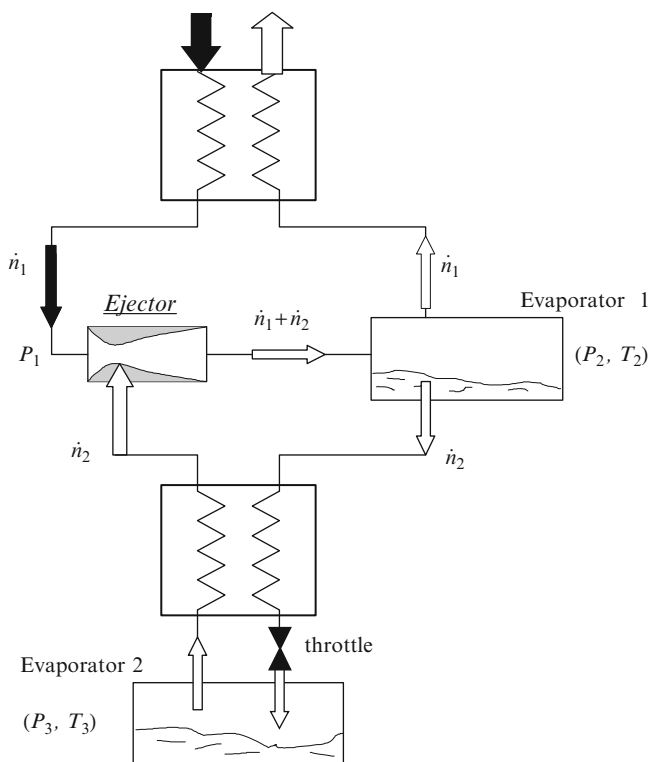


Fig. 5.30 Incorporating an ejector for pumping on the outlet pressure of the final Joule-Thomson stage

than the saturation temperature associated with compressor's suction pressure.

The first solution, proposed by Rietdijk himself in 1966 and further studied by Haisma and Roozendaal^{317,318,319,320,321,322} replaces the traditional throttling device with an expansion ejector, as shown in Fig. 5.30. The ejector performs two functions. First, it reduces the pressure of the molar flow stream $\dot{n}_1$ from P_1 to P_2 as in a standard supersonic nozzle, restricting the flow, and increasing the velocity while maintaining the total (or stagnation) enthalpy constant. The pressure P_2 in evaporator 1 is a result of the suction capability of the compressor. Further expansion of the liquid phase in evaporator 1 occurs through a Joule-Thomson valve, from P_2 down to P_3 , with an associated temperature reduction down to T_3 . The second, and conventional, function of the ejector is to pump on evaporator 2, generate the flow stream $\dot{n}_2$, and maintain the pressure P_3 below P_2 . In this way, the compressor's suction pressure, P_2 , is higher than the vapor pressure, P_3 . Between the two evaporators, the pressure is decreased through a standard Joule-Thomson valve, as shown in Fig. 5.30.

The Rietdijk cycle has the potential of exhibiting a higher thermodynamic efficiency than a simple JT cycle. One might say that the expansion ejector recovers part of the kinetic energy for pumping work, which would otherwise have been dissipated by a standard throttle. On the other hand, the cycle

introduces another irreversibility; that associated with the mixing of two streams. The combined expansion through the ejector nevertheless generates less entropy than a pure throttle. In addition, the Joule-Thomson valve in this scheme throttles a high density (liquid phase) stream, which generates less entropy than a low-density gaseous phase expansion. The lowest temperature achieved by Rietdijk was 2.3 K. Note that the expansion ejector does not include any moving mechanisms, hence in this sense it remains as reliable as a pure throttle.

Prast³²³ adapted the same idea for a cooler nominally operating at 3.5 K. Applying a special ejector, he obtained 1.74 K as the lowest temperature but without the recuperative heat exchanger at the cold end.

Agapov et al.^{324,325} utilized the pumping capability of the expansion through an ejector to assist circulation of cryogen from a bath to a cooled object.

5.6.5.2 Cooling with ^3He

Orlowska, Bradshaw and Hieatt^{326,327} applied an alternative solution to bypass the problem of the low suction pressure required to reduce the boiling temperature below 4 K; they changed the coolant. Instead of ^4He , they employed ^3He , which has higher saturation vapor pressure at any given boiling temperature. For example, at 100 kPa ^3He has its (normal) boiling point 3.2 K. A pressure of 35 kPa would be required with ^4He to achieve the same boiling temperature. Maintaining a suction pressure of 44 kPa produces a saturation temperature of 2.5 K with ^3He while a pressure of 10.3 kPa is required with ^4He to produce the same temperature. Orlowska, Bradshaw and Hieatt built two versions of the helium circuit. One, operating with ^4He absorbed 11 mW at 4.5 K. The other, with a smaller orifice, operating with ^3He at 2.46 K produced a cooling capacity of 3 mW. The system is quite unique, existing as one of only a few Joule-Thomson cryocoolers that use ^3He as a coolant.

So far, no attempt has been reported to combine these two solutions, employing both an expansion ejector and ^3He . The advantage in both cases is lower power consumption and smaller compressors. In addition, the elevated pressure of the return flow stream facilitates the construction of a heat exchanger.

Shimazaki et al.³²⁸ precooled a closed-cycle ^3He cooler with a GM cryocooler. They reached 0.65 K with a molar flow rate of $160 \mu\text{mole} \cdot \text{s}^{-1}$, while providing 1 mW of cooling capacity.

5.6.5.3 Serial Double Throttling

Serial double throttling of a high pressure helium stream can enhance the specific cooling capacity of the final stage as observed by Chelton et al.¹⁴⁷ and Mann, et al.,³²⁹ already in 1960. Let us examine a typical stage, precooled to about 10 K, with a discharge pressure of 1.6 MPa and suction

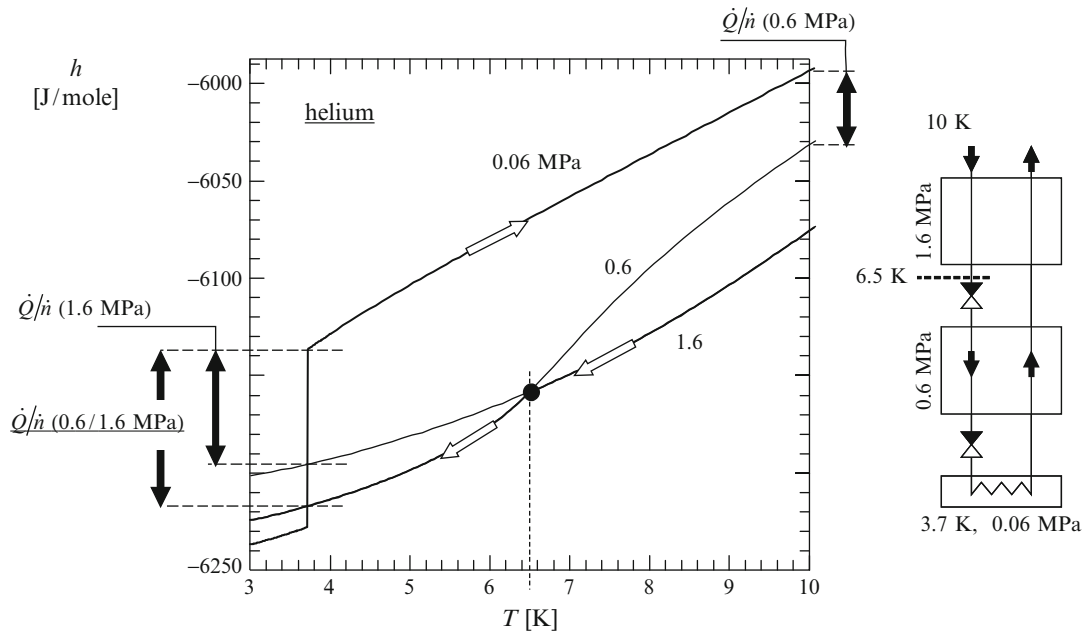


Fig. 5.31 A final Joule-Thomson stage of helium with an “additional” serial throttling from 1.6 to 0.6 MPa at the intermediate temperature of 6.5 K. The final throttling expands the stream down to 0.06 MPa and 3.7 K. This method enhances the cycle’s specific cooling capacity

pressures of 10 kPa²⁴¹ or 60 kPa.³³⁰ Figure 5.31 displays the enthalpy along the isobars of 1.6, 0.6, and 0.06 MPa, for a JT system providing cooling at 3.7 K. When applying a discharge pressure of 1.6 MPa, a pinch point occurs at the cold end of the recuperator, in contrast to most of other pure coolant cryocoolers.

The specific cooling capacity, noted by $\dot{Q}/\dot{n}(1.6 \text{ MPa})$ approaches 58 J/mole, while the value of Δh_T at the warm end is about 80 J/mole. With an upstream pressure of 0.6 MPa, the pinch point moves to the warm end and the specific cooling capacity, $\dot{Q}/\dot{n}(0.6 \text{ MPa})$, is only 38 J/mole, although the value of Δh_T at the cold end is about 80 J/mole. Therefore, switching from 1.6 to 0.6 MPa one elevates the specific cooling capacity, $\dot{Q}/\dot{n}(0.6/1.6 \text{ MPa})$, from 58 J/mole to about 80 J/mole. The switch is achieved by a serial isenthalpic expansion from 0.6 to 0.006 MPa. Collins et al.^{331,332} discussed a similar final Joule-Thomson section for helium; a 1.5 MPa stream starting at 7 K and reaching 2 K. He directed the high pressure stream to:

...pass through additional sections of heat exchangers and through four expansion valves. If only the final expansion valve were used, the high pressure stream could not recover all of the refrigeration potential available in the outgoing vapor. ... To maintain optimum conditions for exchange of heat between the countercurrent streams in the heat exchanger, the pressure of the high-pressure stream should be continuously adjusted for minimum enthalpy at each temperature level. Fortunately, this entails a reduction of the pressure.

Muhlenhaupt and Strobridge³³³ applied such a sequence of throttle steps in a Claude liquefier of helium with a single

expander. It would be interesting to compare their system with an opposite strategy employed by Quack³³⁴ who used a cold compressor in order to elevate the pressure of the expanded helium stream below 9 K from 0.1 to 1.2 MPa.

5.6.6 Special Examples of Hybrid Cryocoolers

5.6.6.1 Subcritical ($P_U < P_C$) Methane JT Cycle Precooled by a Stirling Cooler

The Stirling/JT and GM/JT hybrids are generally utilized to reach temperatures below 50 K. An interesting Stirling hybrid used in a space application to reach about 110 K was described by Willen et al.³³⁵ and Nieczkoski et al.³³⁶ The Stirling cryocooler precools the high-pressure stream of methane before it enters the throttle of a closed cycle Joule-Thomson system. Due to the deep precooling by the Stirling cooler, the JT cycle is able to accommodate an upstream subcritical pressure as low as 190 kPa. The throttling process to 80 kPa decreases the temperature by about 10 K. The hybrid configuration is used for two reasons: first, to accommodate the shape of the cooled object and second, to transfer the cooling potential of the Stirling cryocooler across the space structure’s gimbals.

5.6.6.2 A JT Cryocooler with an Additional Ejector

Expansion ejectors for cryogenic temperatures, may be used with other fluids besides helium and in locations other than the final stage of a Stirling precooler as discussed above in

Sect. 5.6.5.1. Lacaze and Gianese³³⁷ used expansion ejectors for liquefying helium and hydrogen. Daney et al.³³⁸ used a nitrogen ejector to replace the Joule-Thomson throttle in a nitrogen precooling stage of a hydrogen Joule-Thomson refrigerator. They report an increase of about 34% in cooling capacity. The 16.6 g/s flow at the inlet to the ejector was at 20 MPa, and 161 K. A suction pressure of 27 kPa was obtained, and the associated saturation temperature was lowered to 67.7 K.

Yu³³⁹ analyzed the performance of a small Joule-Thomson cryocooler operating at approximately 80 K. Part of the high-pressure (8 MPa) flow in the closed cycle is diverted into an ejector, which then pumps on the evaporator of the cryocooler. The approach enables a higher expansion pressure and requires less work for compression, thereby resulting in a higher COP. The coolant in Yu's analysis was either nitrogen or a mixture of nitrogen and methane. The same principle was adapted for miniature open cycle plain JT cryocoolers by Nicholds,³⁴⁰ by Little³⁴¹ and by Alexeev et al.³⁴² in order to lower the exhaust pressure below the ambient (see Sect. 6.5.3.1). Additional information regarding the use of ejectors in refrigeration technology, may be found elsewhere.^{343,344,345,346}

5.6.6.3 Supplying a DC Flow to a JT Cooler by Rectifying an Oscillating Flow

Nellis, Maddocks, Skye, et al.^{347,348,349} supply a DC flow to a JT cooler by rectifying an oscillating flow. The AC flow at the cold end of a Stirling or pulse tube cryocooler is rectified with the aid of two counter-acting check valves and associated buffer volumes. The first check valve enables flow into the JT cryocooler during the positive part of the AC cycle, while the second check valve allows flow out of the JT cooler during the negative part of the cycle. The rectified DC pressure is significantly lower than the typical pressure of a pure coolant JT system. However, this disadvantage is offset by the deeply reduced temperature of the rectified stream. Compared to the Stirling precooled JT systems described in Sect. 5.6.4, the rectifying system requires two additional cold valves but employs only a single compression unit and eliminates the precooling heat exchangers. The cold and rectified flow can also feed a Brayton stage instead of a JT.³⁵⁰

Another variation of a rectified AC flow system was proposed by Nast et al.³⁵¹ of Lockheed Martin Advanced Technology Center, in Palo Alto, CA., for the Mid-Infrared Instrument (MIRI), one of the James Webb Space Telescope instruments. The object requiring cooling to 4.5 K was located approximately 10 m from the cryocooler. A pair of reed valves rectified a fraction of the pulsating flow at room temperature at the outlet of the AC compressor. (The pressurized DC flow was successively precooled by each of the four stages of the same pulse tube but without a Joule-Thomson expansion. The coolant was ³He.)

Gao et al.³⁵² conducted experiments to rectify a warm pulsating flow of nitrogen and helium. They measured the DC mass flow rate and heat transport as a function of the frequency, from 10 to 25 Hz. The charge pressure ranged from 1 to 2.5 MPa.

5.6.6.4 A Brayton-JT Hybrid Cryocooler

A Brayton-JT hybrid cryocooler was proposed by Zagarola and Chen³⁵³ for future astronomical observatories and surveillance satellites. The final JT stage for 3–5 K is precooled by a reverse Brayton stage operating at 15 K. Both stages are supplied in parallel from three serially connected centrifugal (DC flow) compressors.³⁵⁴ Such a purely DC system is expected to be vibration free. The ³He coolant exhibits a significant JT effect even at modest pressures (due to its low critical pressure, P_c) and pressure ratios consistent with those used in reverse Brayton cycle cryocoolers.

5.6.6.5 A Sequence of an Open Cycle JT Cooler and an Expander

Traditionally, expanders are incorporated within a recuperative throttle cycle for purposes of large-scale liquefaction and cryocooling. The expanders are typically installed in a parallel manner, as taught elsewhere,^{355,356} separating off part of the flow and bridging the entire available pressure range as shown in Fig. 1.7b. Kundig,³⁵⁷ Quack et al.,^{358,359,360} Ziegler and Quack,³⁶¹ and Fredrich et al.³⁶² also proposed using expansion through a sequence of turbines along the high pressure stream without splitting the flow; as depicted in Fig. 1.7c. Ultimately, the entire flow is throttled at the cold end. A small-scale version of this idea was adapted by Bessendorf³⁶³ who focused on miniature open cycle JT cryocoolers. He proposed using a MEMs based expander as described in reference³⁶⁴ between the pressure vessel and the miniature cryocooler so that the stream expands to an intermediate pressure prior to entering the JT cooler. On the one hand, due to restricted flow through the expander, the stream enters the cooler at a lower pressure than anticipated, however, it is accompanied by a decrease in the temperature of the stream. Consequently, as analyzed by Maytal,³⁶⁵ such a combination enables (or allows) elevating the charge pressure above the inversion pressure and one may expect a faster cooldown and prolonged run for a given size pressure vessel.

5.6.6.6 Precooling Helium Sorption Compressor Stage

Duband³⁶⁶ studied the combination of a GM/JT hybrid system driven by a helium sorption compressor. In another report, Duband et al.³⁶⁷ attempt to precool multiple stages of a JT sorption cooler with a pulse tube. Starting from the 4 K obtained by the pulse tube, a ⁴He JT stage precools a subsequent ³He JT stage, which in turn cools down to

260 mK. The complete helium system cools down all the way from 300 K to 260 mK.

5.6.6.7 A Radiant Refrigeration Stage

A radiant refrigeration stage was suggested by Annable³⁶⁸ to precool the high-pressure incoming stream of a Joule-Thomson cryocooler to about 150 K. In a similar proposal, a closed cycle hydrogen cooler is precooled by a passive radiator to space. The hydrogen cycle then precools a helium cycle.³⁶⁹ Doornink, Burger and ter Brake^{139,140,141} combined a Joule-Thomson sorption cooler with passive radiant cooling. Zhang et al.³⁷⁰ compared passive coolers with hydrogen or neon JT coolers to precool a helium JT cryocooler for deep space infrared imaging applications.

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The thermodynamic principles discussed in the previous chapters describe the potential for Joule-Thomson cryocooling, and define the constraints and conditions within which they are viable cryocoolers. However, it is the detail of their actual construction that transforms the potential into reality. Many constraints associated with the practical aspects of construction and configuration are not addressed in the domain of thermodynamic principles, and yet when considered, lead us to a wide variety of structures. Design concerns take us back to the earliest stage of cryogenics, in the year of 1895 when Linde and Hampson proposed different construction approaches for the recuperators in their inventions. This chapter complements the previous one concerning modes of operation. The details of configuration and construction enable the various modes of operation.

6.1 Joule-Thomson Expansion Valves

6.1.1 The Model of a Joule-Thomson Valve

6.1.1.1 Shock and Expansion Waves: The Ultimate Throttling Mechanism

Most coolants entering a Joule-Thomson valve are compressible and expand significantly as they exit into the low pressure zone inside the evaporator. In practice, the inlet to outlet pressure ratio is significantly larger than the critical ratio for choked flow. Therefore, the expanding gas reaches the speed of sound and becomes supersonic. Downstream, the flow undergoes a shock wave or a series of shock waves, and a series of Prandtl-Meyer expansion waves, gradually equilibrating to the ambient pressure. This phenomena is explained in textbooks on compressible gas dynamics.^{1,2,3}

The initial flow acceleration occurs isenthalpically and without any entropy generation. However significant dissipation develops across the shock waves, most likely located outside the valve, dropping the pressure at constant enthalpy. (Therefore, the term “Joule-Thomson valve” is preferred as a description of the supersonic compressible flow rather

than “Joule-Thomson throttle”). The integral isenthalpic temperature drop, ΔT_h , is measured, with respect to the inlet temperature, after the expansion through the shock waves and in the expanded pressure domain.

The flow characteristics of the throttling mechanism are demonstrated by a direct experimental study of Chen et al.⁴ They examined the flow pattern at the exit of a capillary tube that served as a kind of valve in a miniature Joule-Thomson cryocooler. Its inner diameter varied between 0.18 and 0.10 mm along its length of 5 cm. An inlet pressure of 2.5 MPa created a series of 9 expansion and compression waves, with the familiar diamond shape, beyond the exit of the tube. However, an inlet pressure of 7.5 MPa produced a single and strong normal shock wave close to the exit of the capillary.

6.1.1.2 The Choked (Molar) Mass Flux

The choked (molar) mass flux, G , is completely determined by the upstream stagnation inlet conditions, P_0 and T_0 , and is independent of the outlet pressure.

Ideal Gas

The molar mass flux for an ideal gas in choked flow conditions may be calculated by

$$G^I(T_0, P_0) = \dot{n}A = \kappa^{1/2} \left(\frac{2}{1 + \kappa} \right)^{\frac{\kappa+1}{2(\kappa-1)}} \frac{P_0}{\sqrt{MRT_0}} \quad (6.1)$$

where A is the minimum cross section area of the passageway. As an example, corresponding molar and mass flux values for helium, nitrogen and argon with inlet conditions of 10 MPa and 295 K are respectively: 2.32, 0.83, 0.73 mole · mm² · s⁻¹, and 9.27, 23.13, 29.32 g · mm² · s⁻¹.

Real Gases

Maytal,⁵ and later Maytal and Elias,⁶ derived expressions for the choked mass flux, G , of real gases, allowing the stagnation temperature drop as low as T_C . Section 9.2.2 provides

Fig. 6.1 Choked mass flux of argon compared to the ideal gas prediction, $\Gamma = G/G^{IG}$, as function of the inlet stagnation pressure and temperature (Maytal et al.^{5,6})

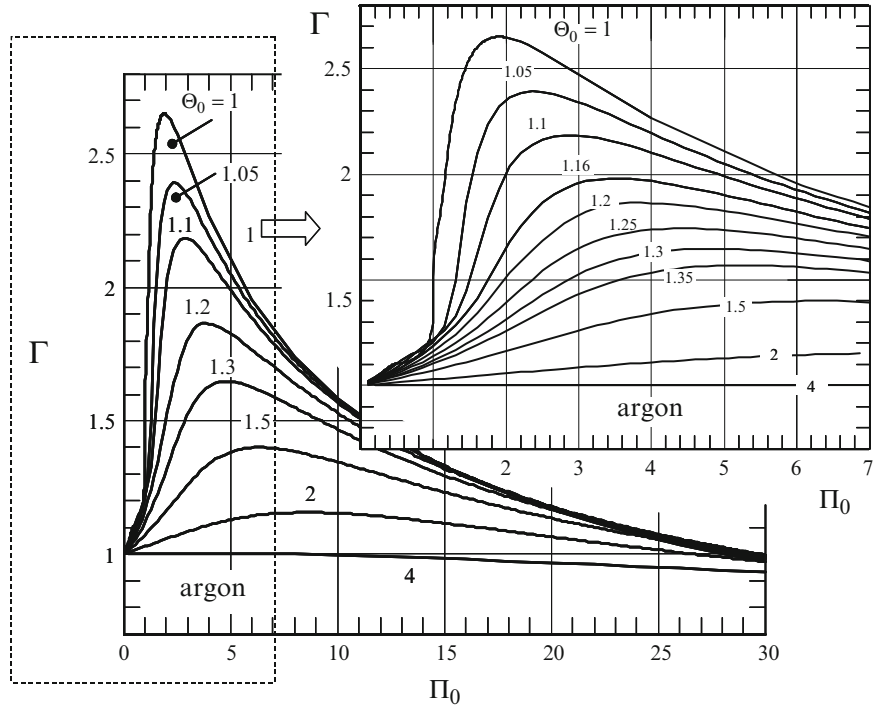
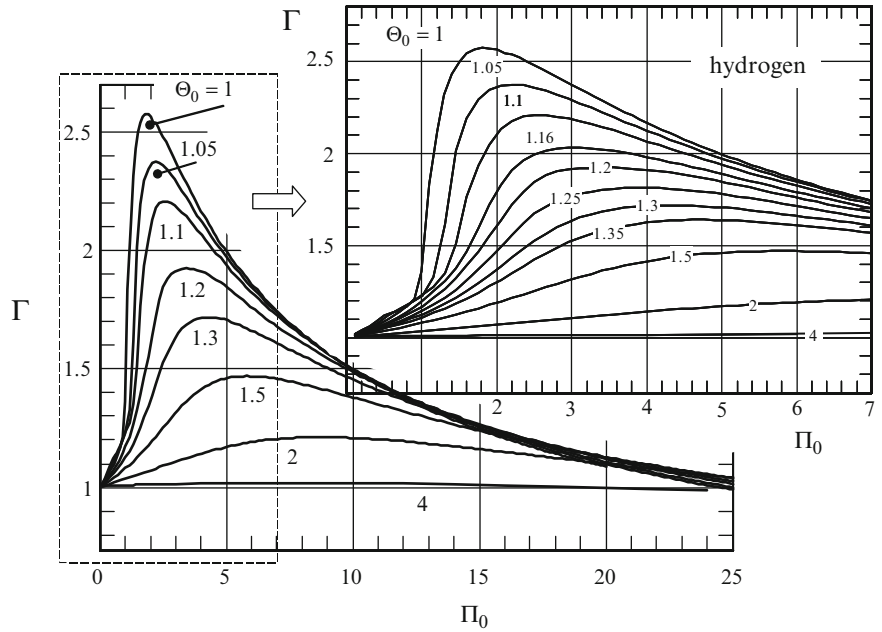


Fig. 6.2 Choked mass flux of hydrogen compared to the ideal gas prediction, $\Gamma = G/G^{IG}$, as function of the inlet stagnation pressure and temperature (Maytal^{5,6})



further information regarding choked flow rates for real gases. Figures 6.1 and 6.2 display, for argon and hydrogen, the deviation factor, Γ , from ideal gas behavior, which enables one to obtain the mass flux for a real gas,

$$G = G^{IG} \cdot \Gamma(T_0, P_0) \quad (6.2)$$

The deviation factor, Γ , is introduced as a set of trajectories at constant reduced inlet stagnation temperature,

$\Theta_0 = T_0/T_C$, as function of the reduced stagnation pressure, $\Pi_0 = P_0/P_C$. The trajectories display a maximum for Π_0 ranging from 5 to 10, depending on Θ_0 , with values larger than unity, so that the actual mass flux may become significantly larger than the ideal gas value.

At steady state operation of a Joule-Thomson cryocooler, the high pressure stream enters the valve at a temperature below T_{CD} , as discussed in Chap. 4. A reasonable estimation of this inlet temperature is approximately $\Theta_0 = 1.35$ for

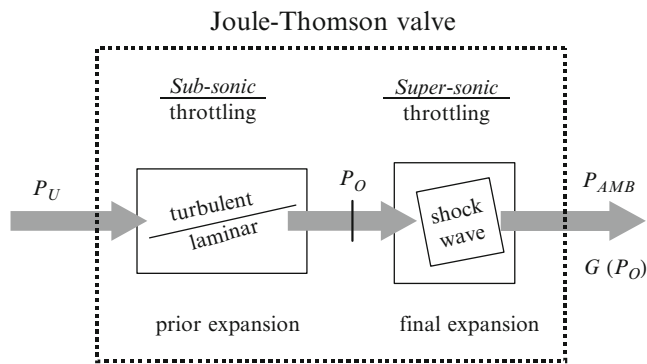


Fig. 6.3 A conceptual model of a Joule-Thomson valve model

argon and hydrogen. These two trajectories of constant Θ_0 peak at about $\Pi_0 = 5$ so that the real mass flux for nitrogen and argon exceeds the ideal gas predictions by about 60%, at the same pressure.

6.1.1.3 Subsonic Expansion

As shown in Fig. 6.3, the model of a Joule-Thomson valve includes two sites where dissipation occurs. The supersonic site, at the very outlet of the valve, as discussed above, is the final one. This mechanism, accompanied by shock waves, occurs due to the large ratio of inlet to outlet pressures. However, additional dissipation is very often associated with the laminar or turbulent throttling that occurs upstream of the supersonic flow region; it decreases the stagnation pressure from the inlet value, P_U , down to the value of P_O , at the inlet of the supersonic expansion element. This dissipation also reduces the choked mass flux.

6.1.1.4 The Passageway Area of a Joule-Thomson Valve

The passageway area of a Joule-Thomson valve is of great interest because of its sensitivity to clog formation. Contaminants in the coolant freeze out and gradually block the passageway of the valve (see Sect. 9.1). A “clog free” valve is highly desirable, and a larger passageway generally exhibits a lower sensitivity to clogging and a slower rate of associated flow degradation. Hence, a desirable feature of any Joule-Thomson valve is to have a larger passageway for a given flow rate. The conceptual model of a Joule-Thomson valve presented below provides a mechanism for analyzing this issue.

With increased dissipation in the upstream section of the valve, the stagnation pressure at the inlet of the supersonic section, P_O decreases. Since the choked mass flux, G , increases monotonically with P_O (even linearly for an ideal gas), a lower value of P_O means a lower value of G . Put another way, for a given mass flow rate, a lower value of P_O enables a larger passageway area, A . Following this model, the variety of

Table 6.1 Classifications of Joule-Thomson valves according to the kind of dissipation at the prior expansion

Joule-Thomson valve	Laminar dissipation	Turbulent dissipation
Circular short duct	No prior expansion	
Capillary tube		X
Labyrinth		X
Vortex		X
Annular duct	X	
Porous plug	X	

Joule-Thomson valves may be classified according to the character of their upstream expansion (since all share in common the final, supersonic, expansion), as displayed in Table 6.1.

6.1.1.5 Remarks

The upstream pressure drop is desirable since it reduces the threat of clogging in the valve by solidified contaminants. However, the larger sizes associated with viscous and turbulent throttling processes might be too large for miniature cryocoolers. Other highly desirable features for Joule-Thomson valves are adjustable flow regulation, and ease of manufacture.

6.1.2 The Short Duct: The Highest Mass Flux

The simplest and the most frequently used Joule-Thomson expansion valve is just a minute hole in a thin plate, that is, a knife-edged orifice (Fig. 6.4). In terms of the previously described conceptual model, it does not have any upstream dissipation, and therefore the upstream pressure P_O is also the inlet stagnation pressure, P_U . The throttling is purely supersonic, and consequently generates the highest possible mass flux for the given cross section area, A . Thus, one may regard it as a metering device. Put another way, for any given flow rate, this valve provides a smaller cross sectional area for the passageway than any other Joule-Thomson valve. In some applications this hole might be so small that being difficult to manufacture. In addition, one should expect it to be the most sensitive to clogging by solidified gaseous impurities. However, these are the sole drawbacks of this type of orifice. Fixed orifices providing less than 2 standard liters per minute of nitrogen at room temperature under a pressure of about 10 MPa become smaller than 0.05 mm.

Such a circular short hole may be drilled directly through the wall of a high-pressure tube as shown schematically in Fig. 6.4a. This construction affords the advantage of being very compact. The diameter of such fixed orifice valves is typically less than 0.15 mm. In addition, one may easily convert such a valve into an adjustable orifice by the addition of a conical needle as displayed in Fig. 6.4b. In this case, the diameter of the hole may be as large as 0.2 mm with a needle

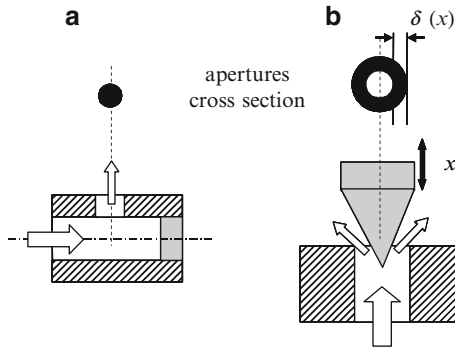


Fig. 6.4 The short hole orifice; the fixed flow orifice may be drilled directly in the high pressure tube (a). The adjustable orifice (b) has a larger diameter drilled in a part, which accommodates a needle

of about 0.5 mm (maximum diameter) and a plate thickness that is approximately one to four times larger. Such a needle valve regulates the effective annular cross section and the associated flow rate. The longitudinal position, x , sets the width of the annulus, δ , and defines the cross sectional area of the annular short hole. Most of the flow-demand cryocoolers embody this kind of adjustable valve.

6.1.3 The Circular Long Duct: The Capillary Tube

6.1.3.1 Adiabatic Compressible Flow with Turbulent Friction

Adiabatic compressible flow with turbulent friction in a duct of constant area (Fanno flow) provides our second example of a Joule-Thomson valve. This type typically incorporates a minute diameter (d) tube, that generates a longitudinal pressure gradient and increases the specific volume of the gas. Therefore, the flow velocity increases, and if the tube is longer than a critical length, L^* , the speed of sound is finally achieved at its outlet as discussed elsewhere.^{1,2,3} A fraction of the stagnation pressure drops along the capillary by friction through turbulent dissipation. In terms of the conceptual model described above, one may consider this pressure drop as an upstream expansion. The remaining pressure drop occurs supersonically, across the shock wave. The mass flux through this type of valve is smaller than that through the short hole. Equivalently, a larger diameter is required in a long circular duct to produce the same flow rate as provided by a short hole. Therefore, one may expect this valve to be less sensitive to clog formation, as demonstrated in examples given below. The critical length, L^* , can be determined for an ideal gas,

$$\frac{L^* f}{2d} = \frac{1 - \text{Ma}^2}{\kappa \text{Ma}} + \frac{\kappa + 1}{2\kappa} \ln \frac{(\kappa + 1)\text{Ma}^2}{(\kappa - 1)\text{Ma}^2 + 2} \quad (6.3)$$

where Ma stands for the Mach Number at the entrance to the tube and f is the friction factor as defined in Sect. 3.12.3. A gas of a larger κ is choked at a shorter L^* . Choking the flow by using a tube that is longer than the value L^* reduces the mass flux. For any given minute diameter tube, the flow rate is set by the length of the capillary; it may be precisely set by successively cutting the length of the capillary. Alternatively, one may compensate for an increased diameter by stretching L while still preserving the flow rate.

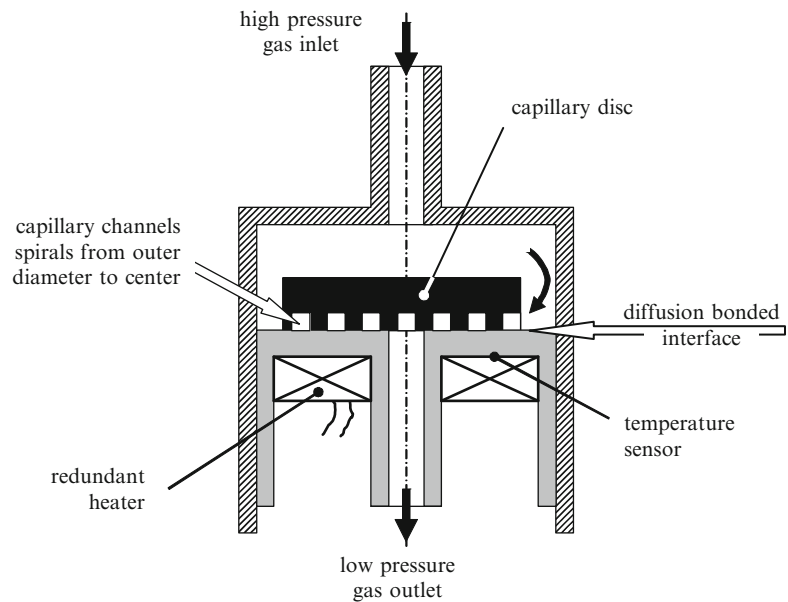
6.1.3.2 Practical Examples

Because of the tiny diameter, even a long tube will not occupy much space and practically, may fit within the overall dimensions of a miniature cryocooler's cold tip. Skertic and Hlava⁷ used a capillary size of 0.127 and 0.254 mm internal and external diameters. Sherman^{8,9} used a similar size capillary, which supplied about 38 standard liters per minute under a pressure of 34 MPa. Xue et al.¹⁰ employed a capillary of 0.1 mm inner diameter. Hood et al.¹¹ built a cryosurgical apparatus based on a miniature Joule-Thomson cryocooler and also used a capillary tube as the throttling device.

It is important to keep in mind that this type of valve is incapable of incorporating a flow adjusting mechanism. Therefore, its niche is limited to applications that tolerate a fixed orifice, or at the most for dual orifice configurations, as applied by Longsworth.¹² In this case two Joule-Thomson valves are used in parallel one configured as a regulating needle valve and the other as a capillary tube, which is permanently open.

The Ball Aerospace Joule-Thomson cryocooler^{13,14,15,16} is part of a closed cycle nitrogen system for space applications (Cryogenic On-Orbit Long Life Active Refrigerator); where the long term reliable clog free operation plays a central role (see Sect. 5.4.9). For this reason the designers chose a long capillary channel for the expansion valve, the construction of which is displayed in Fig. 6.5. A fine, smooth spiral groove is machined on the face of the upper circular plate, conducting the flow from the outer diameter towards the center. The rectangular cross section of the channel is about 0.05–0.15 mm and about 10 cm long. Another circular plate covers the spiral groove, being diffusion bonded with the first one. Through this approach, they obtain an internal, spiral capillary. The pressure inlet and outlet ports are positioned at the center of both plates. The resulting flow area is more than 10 times larger than one would need for a short hole orifice with the same flow rate. The heater shown in the figure serves to defrost any frozen vapor contaminants that may block the spiral groove. Based on Lester's and Benedict's prior study,¹⁷ controlled pulses of heat are released when any clog forms, affecting the flow rate. Bartlett et al.¹⁸ also applied heating, simply for the sake of higher reliability.

Fig. 6.5 An expansion orifice as a spiral capillary channel of the Ball Aerospace Joule-Thomson system.^{13,14,15,16} It enables larger flow area for similar flow rate than the short orifice. The heater serves to defrost frozen vapor contaminants that block the spiral groove



The interested reader is also encouraged to explore the use of capillary tubes as a throttle in refrigerating systems.¹⁹ There, the expanding stream enters in the liquid phase. As the pressure drops along the tube, the flow transitions to two phases at saturation conditions.

6.1.3.3 The “Open Tube” Cryocooler

When high flow rates are required (as in a precooling stage) the high-pressure tube of the heat exchanger itself provides adequate restriction and no additional Joule-Thomson expansion valve is required. Sometimes this configuration is referred to²⁰ as an “open tube” cryocooler, as in Peterson et al.²¹ Part of the pressure drops along the tube due to the large flow rate and velocity, and the rest drops through the shock wave. Both the heat interaction (cooling or heating) and friction take place all along the high-pressure tube. The situation is similar to the ideal gas flow related by Rayleigh,^{1,2,3} where choking is achieved by thermal interaction (cooling or heating) along the tube but without friction. Arp²² formulated the more general case, including heat interaction, friction, the local Joule-Thomson effect and the speed of flow. His formulation was established for long lines of flowing helium. Dean et al.²³ report on experiments with helium and analyze their results in terms of the above relations. They verified, as is typical for helium (below 20 K), that the temperature may increase or decrease along the stream, depending on inlet parameters and the provided heat flux. Further discussion of this work exists elsewhere.²⁴

6.1.3.4 The Long Duct with Laminar Friction

Almost all cryocoolers discussed in the section regarding very narrow channel heat exchangers (Sect. 6.4) incorporate a downstream expansion valve, which is a long capillary duct of smaller cross section than the heat-exchanging

channel, and which is prepared by the same technology as the entire heat exchanger.

6.1.4 Porous Media Valve

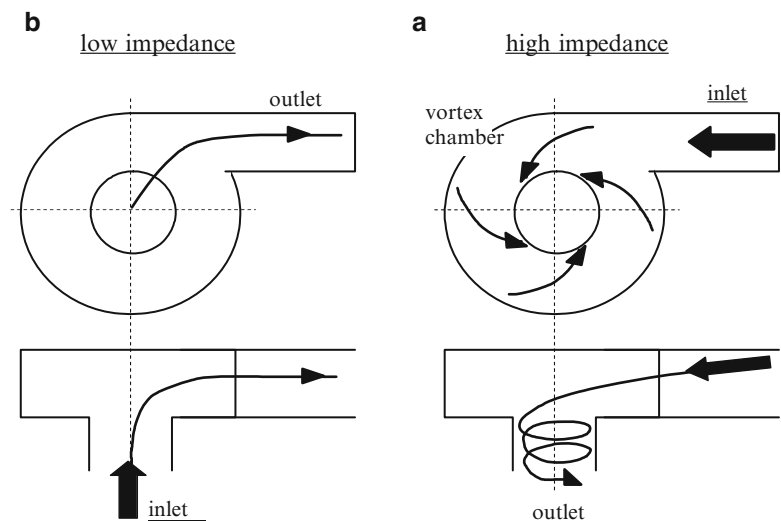
The expansion valve in the original experiment of Joule and Thomson was a porous plug made of cotton with an inlet pressure below 1 MPa. Today, porous valves are made of sintered metals, and withstand about two orders of magnitude higher pressures. The upstream expansion (in Fig. 6.1) in a porous plug reduces the stagnation pressure through laminar dissipation. Also it provides a larger total cross section area than the short hole valve for the same flow rate, and is therefore expected to be less sensitive to plug formation.

Levy and Wade²⁵ used a sintered metal porous valve for a hydrogen Joule-Thomson cryocooler driven by a sorption compressor. The cryocooler was used in a long duration balloon mission^{26,27} and was designed with special emphasis on high reliability. For comparison, the diameter of a short hole valve to provide the same 100 mW cooling capacity, would be about 0.02 mm. Obviously, the size of the equivalent passage-way in the porous valve is larger. Paine²⁸ characterized a porous sintered material as a Joule-Thomson expansion valve for the Planck sorption cooler operating at 18 K.

Burger et al.^{29,30,31} developed a small capacity helium sorption cooler for 4.5 K delivering 4.3 mW. The Joule-Thomson valve was also made of sintered steel, precisely calibrated for the correct flow rate at 4.5 K.

Porous flow restrictors are commercially available,³² in a variety of porosities, shapes, sizes, materials and hydrodynamic impedances. Marquardt and Radebaugh³³ used a sintered copper porous plug in a minute cryosurgical cardiac catheter.

Fig. 6.6 The principle of vortex throttle operation is demonstrated by its behavior as a flow diode. The tangential inlet circulates the flow (a) thus evokes high hydrodynamic impedance. The central inlet (b) faces small restriction



Hill et al.³⁴ used a porous valve for pure coolants, and for mixed refrigerants in a closed cycle in order to reduce the flow rate after cooldown. Their approach is further discussed in the context of flow regulation (Sect. 6.2.8.2) and mixed coolants (Sect. 8.3.6.3).

Gier³⁵ employed a porous valve for Joule-Thomson expansion of He-I in order to produce superfluid helium (He-II), trying to reduce the risk of clogging.

Steyert³⁶ used a flattened tube as a flow restrictor. In addition, he packed it (and the high pressure tube) with a fine fibrous cotton-like material as a porous media that further impedes the flow. The packed media also served to absorb frozen contamination particles.

6.1.5 The Vortex Throttle

The upstream expansion occurring in this valve is associated with an intensive swirl-induced turbulent dissipation. To the best of the authors' knowledge, only a single prototype of a miniature Joule-Thomson cryocooler was constructed and tested with a vortex throttle.^{37,38} It was already suggested in 1966 by Cowans³⁹ for use in a cryocooler. However, the idea of flow-throttling by a vortex was already known and applied in a variety of large-scale applications like wastewater engineering⁴⁰ and nuclear facilities.⁴¹ Fluidic circuits incorporate a version of this throttle functioning as a flow diode. Its principle of operation is displayed in Fig. 6.6a. The incoming stream enters tangentially into the round vortex chamber, rotates and exits at the center perpendicular to the plane of rotation. The entire contents of the chamber tend to circulate. Hence, the velocity close to exit, being inversely proportional with the radius, increases remarkably. It produces the high turbulence and intense dissipation⁴² that throttles the flow.

The flow rate induced in this manner is significantly lower than what would have been obtained by the metering effect with the same channel cross section but without swirl. Figure 6.6b focuses on this issue. As the stream enters in the counter direction, the hydrodynamic impedance in the center is very low.

Since the free passage of the vortex throttle is larger than that of an equivalent short hole, it should be less sensitive to blockage by the formation of a frozen vapor plug. This was the target of Maytal,³⁷ who provided an experimental demonstration that such an expectation is indeed justified. He used the commercially available product of The Lee Company (Westbrook, Connecticut, USA) carrying the trademark "Visco-Jet", and originally dedicated for high pressure pneumatic and hydraulic systems. It is sufficiently compact for installation in a 8.3 mm diameter finned tube cryocooler as displayed in Fig. 6.7. The passageway of this product has a cross sectional area that is 25 times larger than a short hole metering the same flow rate. The clog retarding capability of this throttle is discussed in Sect. 9.1.4.4. It should be noted that the Visco-Jet incorporates a series of multiple vortex-throttles, and that a part of the '25 times larger area' is enabled by the additional upstream expansion associated with the labyrinth effect.

Papell et al.,⁴³ LeBar,⁴⁴ Jurns et al.,^{45,46} also used a Visco-Jet as a Joule-Thomson throttle to decrease the pressure of a stream of liquid hydrogen propellant. The purpose was to reduce its temperature as part of a special thermodynamic vent system. A Visco-Jet was also applied to throttle liquid nitrogen⁴⁷ and liquid methane.⁴⁸

6.1.6 The Annular Duct

An annular passageway of a miniature valve is a thin clearance between two concentric and relatively large

Fig. 6.7 A 8.3 mm diameter finned tube Joule-Thomson cryocooler employing a vortex throttle (of Lee Company Westbrook, Connecticut, carrying the trade mark “Visco-Jet”) (Maytal^{37,38})

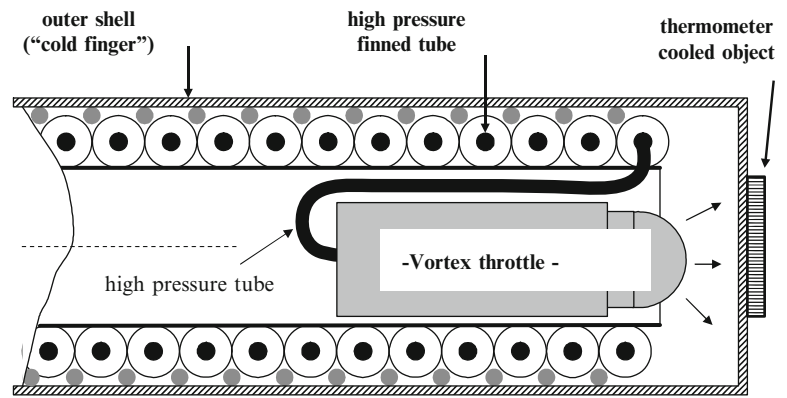
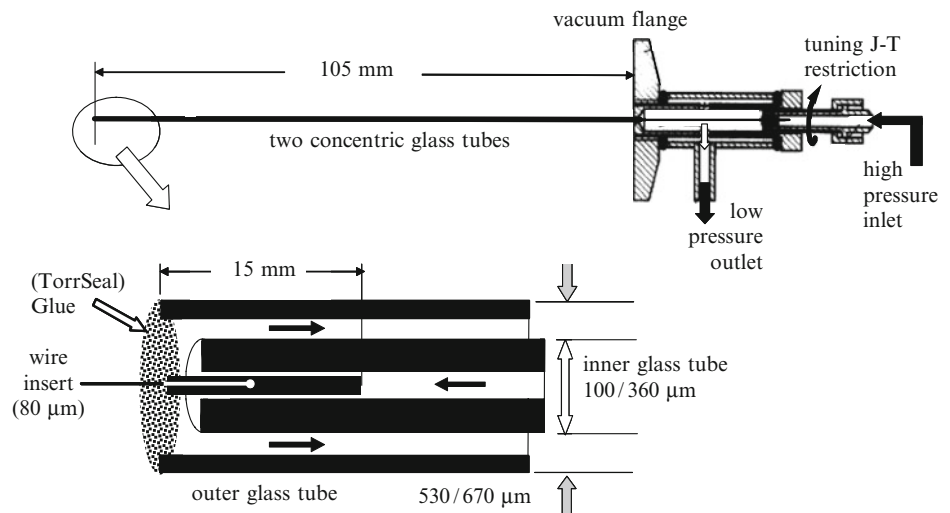


Fig. 6.8 A miniature Linde type Joule-Thomson heat exchanger of two concentric glass tubes; throttling occurs at the fine annulus between the inner tube and an inserted wire (Holland et al.⁴⁹)



elements; one element is hollow, allowing the second to fit inside. The flow along such a thin channel is laminar. It is a prior expansion in the sense of Fig. 6.3, that lowers the stagnation pressure of the stream.

The annular passageway also provides a convenient constriction that can be used to adjust the cross sectional area for the flow through the valve. Changing the thickness of the annulus is accomplished by a relative longitudinal shift of the two elements. The equivalent diameter of a round short hole to produce the same flow rate might be less than 0.01 mm and difficult to produce.

6.1.6.1 A Cylindrical Insert

A cylindrical insert in the form of a wire inside a tube of close inner diameter (0.10 mm) is shown in Fig. 6.8, after Holland et al.,⁴⁹ for a low capacity cryocooler. The annular gap along the wire becomes a long (in terms of length per equivalent diameter) capillary expansion valve, serving a low capacity cryocooler with a small flow rate. However, it is a laminar capillary in contrast to the circular long duct of

Sect. 6.1.3, which is turbulent. It is possible, by an adjustment at the warm end, to change the length of the annular capillary, by modifying the extent to which the wire is inserted. Other configurations utilizing a cylindrical insert are described by Reale et al.,⁵⁰ Albagnac et al.⁵¹ as well as one with a spherical insert⁵² in a round hole.

6.1.6.2 A Conical Annular Valve

A conical annular valve, created by the annular gap between two concentric conic surfaces, is mainly used for adjusting the cross sectional area as displayed in Fig. 6.9 after Walker et al.^{53,54,55,56,57,58,59} The relative longitudinal displacement of the two cones regulates the width of annular clearance and results from the thermal expansion of the connecting metal rods (paragraph 6.2.3.2).

The example shown in Fig. 6.9 includes peripheral grooves on the inserted cone. These have the effect of a labyrinth thereby intensifying the process of prior expansion (in the sense of Fig. 6.1), although it is a turbulent dissipation. The inventors also claim that this valve is “clog resistant”.^{58,59}

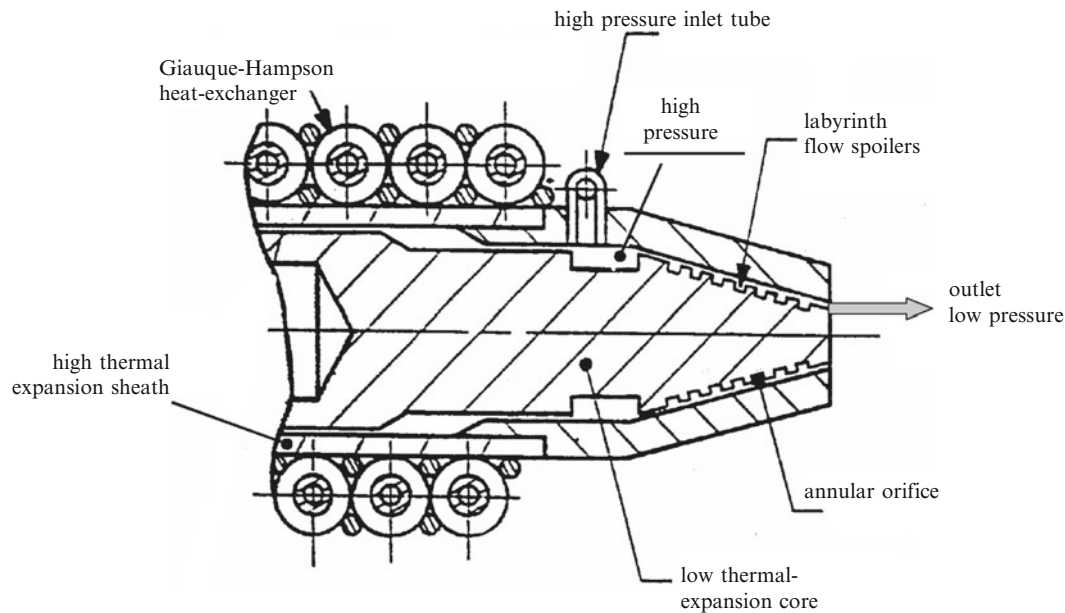


Fig. 6.9 A throttle as an annular gap between two concentric conic surfaces (Walker et al.^{53,54,55,56,57,58,59}) and enables flow adjustment. The labyrinth on the inner cone aims to reduce plug formation and clogging of the orifice (see element (e) in Fig. 6.32)

Other examples of conical throttles are that used by Real⁶⁰ which was adjusted by an external vapor bellows at the warm end, and that used by Anikeev.⁶¹ Curlier⁶² applied a similar throttle mechanism for a hydrocarbon mixture closed cycle Joule-Thomson cryocooler.

6.2 Flow Adjustment

6.2.1 Introduction

Flow adjustment is required for two reasons:

- (a) To cut down the flow rate after cooldown is completed, and,
- (b) To set the proper flow rate under varying conditions of operation, taking into account the heat load and the thermodynamic state of the coolant.

Various aspects of flow adjustment and the associated performance criteria were already discussed in Sect. 5.3.3. A reliable adjusting mechanism is one that prevents the waste of gas, but maintains a reasonably stable temperature and avoids payload warm up even for a short period. A broad variety of adjusting mechanisms are described in patents and their features will be reviewed below.

The earliest report regarding the idea of a self-regulating miniature Joule-Thomson cryocooler, proposing to utilize the thermal expansion effect, belongs to Jepsen et al.⁶³ in 1965, at Air Products and Chemicals, Inc., Philadelphia, PA. One year later, Hansen⁶⁴ at the Santa Barbara Research Center of Hughes Aircraft Company, Culver City, California, proposed the same objective, but through the

use of a gas charged bellows. The gas content of the bellows (communicating thermally with the cold object) condenses and shrinks at lower temperatures.

6.2.1.1 Classification

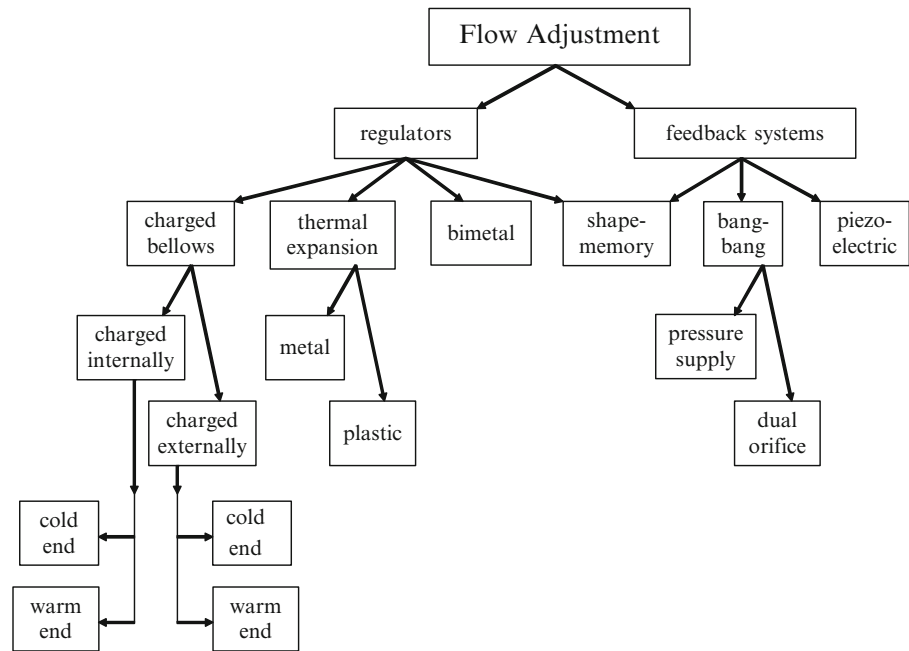
The flow adjusting devices are organized in Fig. 6.10 under two categories: regulators and feedback (servo) systems. Regulators utilize a single element for both sensing the temperature and actuating the corresponding response. Regulators are distinguishable through their method of temperature measurement; whether is it a gas bulb, thermal expansion of metallic or plastic materials, bimetals or shape memory alloys. Put another way, a regulator is a device that changes its state in a desirable manner, but there is no flow of information that diminishes the existing error as in the case of a feedback system. A servo system, on the other hand, uses a separate sensor and actuator. The sensed signal feeds a processor, which estimates the error and generates a command for the actuator to reduce the instantaneous error. The strategy of control may be a bang-bang or a continuous type. Nevertheless, for practical reasons, most flow adjustments are accomplished by regulators.

6.2.1.2 Characteristics of Flow Adjustment

The main parameters that distinguish the various flow adjusting devices are:

1. The principle of temperature sensing,
2. Whether the temperature is sensed locally, such as at the cold end, or as an integral of the temperature distribution along the heat exchanger,

Fig. 6.10 Classification of the wide variety of flow adjusting devices operating and proposed for Joule-Thomson cryocoolers



3. Method of actuation for accomplishing the regulation,
4. The extent of inducing additional heat load and thermal mass,
5. The influence of the ambient temperature, and the range of ambient temperatures over which the regulation remains adequately efficient,
6. Capability to adjust the flow rate according to the instantaneous feed pressure and temperature of the coolant,
7. Preserving controllability under high acceleration conditions,
8. Capability to reduce the flow as cooldown nears completion,
9. Capability to regulate the flow at the warm end during operation, and the degree to which such regulation conflicts with heat leak reduction.

6.2.1.3 Sensing the Heat Load and Temperature

The desire to match the flow rate with an external heat load rather than with the temperature at the cold end deserves some explanation. Flow regulators are temperature sensitive and often located on the cold end of the cryocooler, partially immersed in the two phase cryogen. A complete description of their operation is more complex than simple temperature tracking, since the temperature of the saturated boiling cryogen is quite constant and stable, even if the heat load changes. The fraction of liquid content in the evaporator also affects the regulator's response. Convective heat transfer is an order of magnitude larger through a liquid phase interface than through a gaseous one. Thus, the effective temperature that is sensed, and that drives the actuation, is dependent on a combination of both the temperature and quality of the cryogen. In this respect, the regulating

mechanism is in fact heat load sensitive. A decreased heat load results in an increased fraction of liquid and thereby reduces the mean effective temperature sensed by the regulating mechanism. Since the typical function of a regulator is to increase flow in response to sensing an increasing temperature and to reduce the flow in response to sensing a decreasing temperature, the net effect of a decreased heat load is a reduction in flow rate, in spite of the unaltered cryogen temperature in the evaporator.

6.2.1.4 Flow Regulators for Rapid Cooldown Cryocoolers

The rapid cooldown mode of cryocooling is characterized by a short run, as discussed in Sect. 5.5.2. This situation is less demanding in terms of minimum gas consumption and in fact tolerates flow rates that are larger than required for the actual heat load. However, after the cooldown is completed, the flow rate is drastically reduced.

6.2.2 Charged Bellows Flow Regulator

The majority of flow demand cryocoolers in operation utilize a charged-bellows to regulate the position of a needle valve, and thereby regulate the flow. This approach indeed provides a number of favorable features, such as a very sensitive response, low thermal mass, insensitivity to ambient temperature, fast response with large gain, relatively low cost, and some (although not complete) clog resistance. A metallic, miniature, longitudinally elastic pressurized bellows is the heart of this mechanism. It was first proposed by Peterson and Nagy⁶⁵ at the Santa Barbara Research Center (SBRC), CA, in 1964. Hansen⁶⁴ at the Hughes Aircraft Company

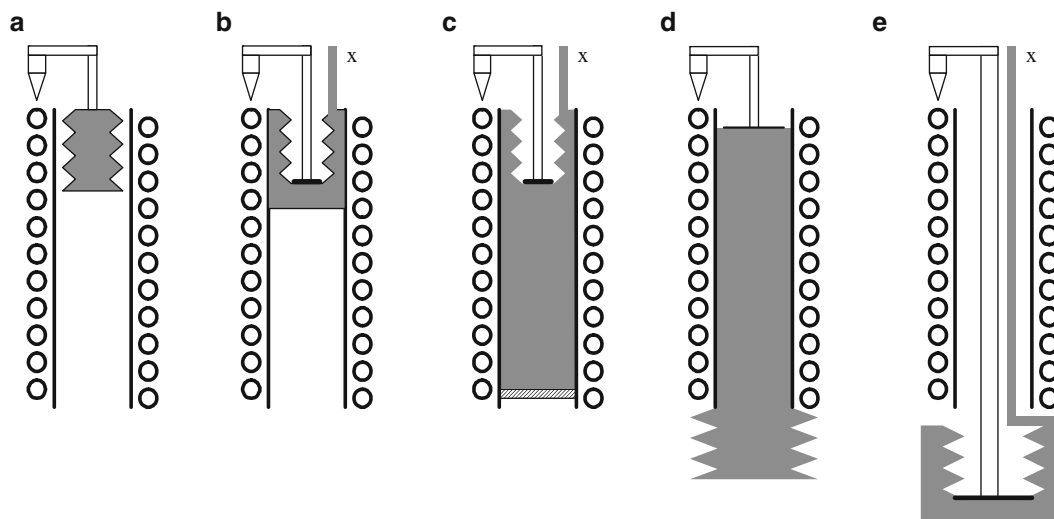


Fig. 6.11 Various flow regulators based on actuation by a charged bellows. Bellows may be located at the cold end, (a), (b) and (c), and charged internally or externally. In (d) and (e) the bellows is located at the warm end

employed a bellows in 1966 as an actuator to adjust a throttle through the relative positioning of a ball with respect to a valve seat. Subsequently, in 1967 Nicholds^{66,67,68} and Stephens^{69,70} of the Hymatic Engineering Company in UK, in 1969 Buller^{71,72} at the SBRC and in 1971 Longworth⁷³ at Air Product Chemicals, PA, employed a bellows to actuate a needle valve.

While the refrigerator cools down, the pressure and volume of the fixed amount of gas enclosed in the bellows decreases. By virtue of the elasticity of the bellows, its length also decreases. Because of its mechanical link to the needle, the shortening of the bellows induces a movement in the needle of the valve that further restricts the flow.

The earliest report (according to the authors' best knowledge) directly addressing the performance of the bellows type regulator is given by Nicholds, the Technical Manager of The Hymatic Engineering Co. Ltd., UK, in 1967⁶⁶:

The principle is that cold production is controlled to maintain a liquid level. Temperature is sensed when the desired level is reached at a vapour filled bulb, and the reduction in pressure is used to close the expansion orifice, thus reducing flow. There is an additional advantage likely that microphony may be reduced.

(underlined by the authors and not by Nicholds)

In 1971, Buller of the Santa Barbara Research Center, California, discussed the reduced gas consumption resulting from such flow adjustment, and compared it with the gas consumption associated with the use of an optimal constant orifice while regulating the pressure supply.⁷¹

The bellows actually performs two functions: sensing the gas flow temperature and actuating accordingly, in order to instantaneously adjust the flow. The variety of embodiments based on the elastic bellows as displayed in Fig. 6.11, are

distinguished by three parameters. First is the location of the bellows, whether at the cold end or at the warm end. Second, is whether the charging gas occupies the space, interior to, or exterior of, the bellows. The third parameter is the actual size of the gas bulb (which is shaded in the figure); and whether it is localized at the cold end or distributed along the heat exchanger.

6.2.2.1 Cold End Bellows

When the bellows is located at the cold end, it senses the local temperature as close as possible to the site of liquefaction. This, the most popular version, is depicted in the schematic layouts (a) and (b) in Fig. 6.11. Note that a tube used to charge the bellows with a fixed amount of gas (identified in the figure by x) is crimped at the supply end to seal the pressurized fluid inside the bellows. In fact, the gas contained in the charging tube influences the response of the gas bulb to some degree. For proper operation, it is recommended⁷⁴ that the regulator be filled with the same gas as the working fluid in the cryocooler.

In configuration (a) as proposed by Buller⁷² and Longworth,⁷³ the charging tube is oriented away from the cold end, and therefore exposed to the warmer part of the cryocooler. In order to direct the charging tube towards the bath of cryogen, as shown in scheme (b), one must pressurize the bellows externally instead of internally. The charged gas bulb occupies the space between the bellows and an external cylindrical mandrel. Such an embodiment, after Campbell⁷⁵ and Brown,⁷⁶ may offer some advantages over the previously described internal version. In this approach, the bulb gains a more intimate thermal connection with the cold end and responds more quickly.

The schematic presentations in Fig. 6.11 locate the adjustable orifice and the needle at the periphery of the heat exchanger, as is indeed the common practice.⁶⁴ However, in this configuration the expanded jet that impinges on the needle exerts an eccentric force, and an associated torque on the bellows mechanism, which is proportional to the eccentricity. Since the sliding element has to support this torque, lateral forces are introduced, and opposed by friction. An alternative design, demonstrated by Brown⁷⁶ and Chevet et al.,⁷⁷ locates the needle and the orifice concentric with the actuating bellows. Such a mechanism is more complicated but may provide more precise control and higher reliability.

Configuration (c) in Fig. 6.11, after Campbell^{78,79} and Fox,⁸⁰ displays another configuration of an externally pressurized bellows. The gas bulb completely occupies the available internal space along the heat exchanger. In this case, the gas bulb temperature is not uniform anymore, since at the one end it senses the warm ambient temperature, while at the other end it is exposed to the boiling cryogen bath. One may deduce that such a regulator responds to the integral of the entire temperature distribution along the heat exchanger rather than to a locally sensed temperature. On the one hand, the integral average temperature is less sensitive to changes at the cold end. On the other hand, the much larger volume of the gas bulb enables larger displacements and thus a higher gain of response.

Campbell⁸¹ describes a bellows type flow regulator involved in a two-stage cryocooler.

6.2.2.2 Warm End Bellows

When the bellows is located at the warm end it is typically installed outside of the heat exchanger. Here too, the bellows may be pressurized internally, as in scheme (d) of Fig. 6.11, adopted by Longworth⁷³ or externally, as in scheme (e) adopted by Real⁶⁰ and Campbell et al.,⁸² and Guichard et al.⁸³

In configuration (d), the gas bulb of the bellows extends along the entire length of the cooler up to the cold end, and therefore responds to the longitudinal integral temperature distribution. The flexible component, located at the warm end, necessitates a long connecting rod to actuate the flow adjusting needle valve, and may therefore increase the thermal load of the cryocooler. This less common configuration may be beneficial for very short cryocoolers, and provides the possibility for lower thermal mass at the cold end.

In configurations (d) and (e) the gas in the bulb has a much higher average temperature than if the bulb were located at the cold end. Therefore, the charge gas has to be suitably chosen in order to respond at a higher temperature range.

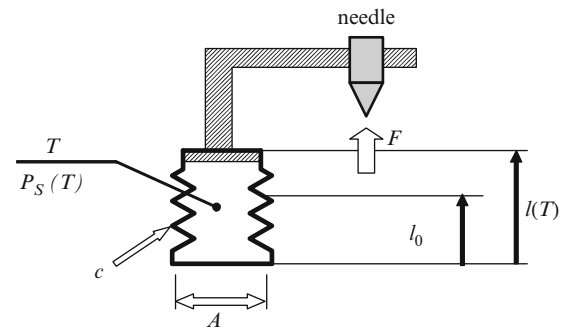


Fig. 6.12 Balance of forces on the bellows type flow regulator

6.2.2.3 Principle of Operation

The specific gas, and its initial charge pressure in the bellows, is chosen to ensure the coexistence of vapor and liquid phases after cooldown. The pressure and temperature in the bellows becomes sub critical at steady state operation. If the bellows is located at the cold end, close enough to the plenum of liquefied cryogen, the gas in the bellows may be the same kind as the operating refrigerant. When the bellows is installed far from the cold end, its charging gas is of a higher boiling point and critical temperature than the working refrigerant.

The two phase coolant inside the bellows enables its sensitive regulation. Its response is driven by the heat transfer between the bellows and its surrounding even before any sensible temperature changes occur. Simply the extraction or addition of heat drives the phase transition in the bulb respectively toward one direction (saturated liquid) or the other (saturated vapor). The large difference in the specific volume of the two phases is translated into an actuating and correcting signal.

Even a minor increase in the amount of liquefied cryogen in the evaporator extracts a small amount of heat from the bellows, thus evoking a contracting response. Therefore, the bellows bulb is often regarded as a means of maintaining a constant level of cryogen in the evaporator.

In the two-phase region, the temperature is solely dependent on pressure. Hence, any expansion or contraction results in a variation of pressure, and is related with the coefficient of elasticity, c , of the bellows. (Elsewhere,⁶⁵ the contraction of the bellows is assisted by an opposing spring of adjustable load). The temperature inside the bellows is associated with the new pressure of saturation, $P_S(T)$.

6.2.2.4 The Balance of Forces on the Bellows

The balance of forces on the bellows at steady state, is schematically introduced in Fig. 6.12, and leads to the expression,

$$F + A[P(T) - P_{AMB}] = c[l(T) - l_0] \quad (6.4)$$

Table 6.2 The slope, $\partial P_S/\partial T$, in kPa/K, along the saturation line of different gases

	N ₂	CO	Ar	O ₂	CH ₄
100 K	52.53	40.32	24.92	20.68	3.34
120 K	124.53	94.33	66.18	58.53	13.27
T_{BOIL} [K]	77.35	81.66	87.29	90.20	111.67

where A is the effective cross section area of the bellows, l_0 is its length before being pressurized, $l(T)$ is its length at operating conditions, T is the temperature of the fluid content and P_{AMB} is the pressure on the outside of the bellows. The external force exerted by the expanded gas jet impinging the needle, is represented by F . All these quantities are combined together through the coefficient of the bellows' elasticity, c . Sometimes an additional spring is incorporated⁷² in the flow regulator acting in parallel with the elasticity of the bellows. Both counterbalance the pressure inside the bellows. In this case, c represents the equivalent elasticity. Differentiating the above equation by T leads to:

$$\frac{dl}{dT} = \frac{A}{c} \cdot \frac{dP_S}{dT} \quad (6.5)$$

Here, dl/dT is the sensitivity coefficient of the regulator's response. It is directly related to the slope, dP_S/dT , of the saturation (vapor pressure) line. Bonney⁸⁴ reports some typical values for the miniature bellows regulators. For example, the range of the elasticity parameter at two different temperatures is:

$$\text{@300 K} \quad \frac{dP}{dl} = 5,000 \text{ to } 20,000 \frac{\text{MPa}}{\text{m}} \quad (6.6)$$

and the response coefficient:

$$\text{@100 K} \quad \frac{dl}{dT} = 2 \text{ to } 2.5 \frac{\mu\text{m}}{\text{K}} \quad (6.7)$$

It is interesting from the perspective of control to analyze the various physical parameters influencing the regulation sensitivity. One can draw the following general guidelines:

- The slope, dP_S/dT , increases with temperature. In this regard, a warmer temperature for the same sensing fluid provides a larger gain, and a more sensitive response. As listed in Table 6.2, a bellows pressurized with nitrogen, while sensing a 120 K temperature, will exhibit almost two and a half times larger gain, $\partial P_S/\partial T$, than at 100 K. On the other hand, a bellows located at the warm end of the heat exchanger, causes a delayed response and lower sensitivity to changes that occur at the cold end.
- A similar trend holds for other gases. For a given temperature sensed by the bellows, the slope, dP_S/dT ,

increases for gases with lower boiling points. For example, at 120 K, dP_S/dT for nitrogen is almost ten times larger than for methane (Table 6.2). One is therefore driven to use a gas with as low a critical temperature as possible. In the range of above 77 K operation, nitrogen is the preferred choice.

Warm end bellows typically employ higher boiling point gases, such as methane, krypton and carbon dioxide and ethylene, all of which also exhibit higher values of T_C . However, the lower associated values of $\partial P_S/\partial T$ are usually offset by a larger volume of gas, which one can quite freely locate at the warm end. Hence, the total gain, dl/dT , remains within satisfactory limits.

So far, we have considered only the gas bulb itself. However, in that its purpose is to regulate the needle's location within the flow-restricting valve, it is also of interest to inspect the interaction of the needle and valve cross section. Chien et al.⁸⁵ analyzed the opening of the valve as a function of the average temperature in a bellows charged by nitrogen at room temperature to the pressure of 2.76 MPa. At each temperature, the needle finds a new position, thereby exposing a different cross sectional area of the orifice. They determined the two phase condition of each state in the bellows through the use of the van der Waals equation of state in conjunction with the force balance equation.

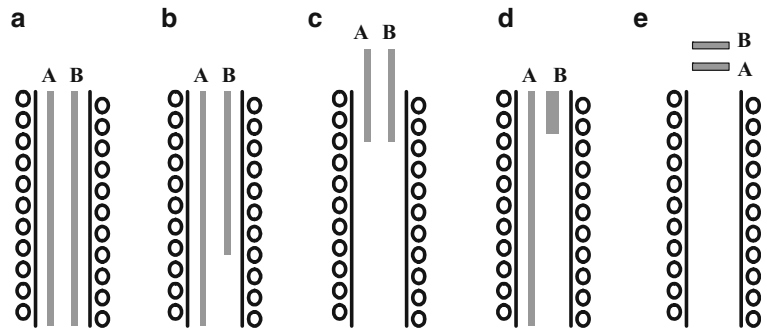
6.2.2.5 Two Phase Versus Single Phase Bellows' Content

Chien et al.⁸⁶ simulated the performance of a regulated cryocooler and focused on the response of a two-phase bellows. They applied the Benedict-Webb-Rubin equation of state and showed that the bellows displays a more sensitive response when filled with a two-phase, rather than a single phase, fluid.

Nevertheless, Chovet et al.⁷⁷ explored the use of single-phase, rather than two-phase, gas bulbs. They charged the bellows with gases such as helium, hydrogen, neon or their mixtures and used them in the temperature range of the associated refrigerants, such as nitrogen or argon, where the bellows gases do not condense. In this case, the bellows behaves like a gas bulb thermometer, and exhibits variations in volume (or displacement of the actuator) that are only linearly related to the variations of the sensed temperature. As a result, the configuration displays a low gain, in terms of displacement per unit temperature.

The needle valve in these studies, also operates in parallel with a minute fixed orifice. This arrangement is discussed later in Sect. 6.2.5 as a dual orifice. Through this approach, the lowest achievable flow rate is never zero. Drastic changes in the regulated flow rate are not required, and the lower gain is acceptable.

Fig. 6.13 Range of variations of flow regulators by thermal expansion of solid bars, A and B, and their position along the heat exchanger, which actuated and drive the required correction. Versions (a) through (d) utilize longitudinal expansion while (e) uses transverse expansion. Version (d) employs a short plastic bar of high thermal expansion coefficient. The *upper side* is the cold end



6.2.3 Solid Thermal Expansion Flow Regulators

6.2.3.1 Classification

The pioneering attempt to control flow rate by utilizing a metal's thermal expansion belongs to Jepsen et al.⁶³ in 1965. The simplicity of the governing principle is quite attractive and offers promising reliability. One might expect it to regulate in a linear fashion, and exhibit a smooth and stable response. However, such response is less common in practice, especially for cases that require very fine-tuning of flow rates.

The operative mechanism results from the difference in the thermal expansion coefficients of materials, $\alpha_1 - \alpha_2$. Two elements of similar initial length, l , that experience the same temperature change, ΔT , generate a relative longitudinal displacement between them,

$$\Delta l = l \cdot (\alpha_1 - \alpha_2) \cdot \Delta T \quad (6.8)$$

The relative displacement, Δl , increases if either $\alpha_1 - \alpha_2$ or l increase. The associated change in the cross sectional area of the elements is irrelevant to their linear thermal expansion (but very relevant for heat leak). The range of differential contraction applications in JT coolers is organized in Fig. 6.13 in terms of the length of the expanding elements, going from the longest elements in (a) to the shortest in (e). Versions (a) and (b) typically employ metallic expanding materials while the other shorter versions use non-metallic expanders.

6.2.3.2 Metal Expanding Elements

Jepsen et al.⁶³ utilized the differing thermal expansion coefficients of Invar and stainless steel to produce a value of $\alpha_1 - \alpha_2$ of about $3 \cdot 10^{-6}$ 1/K. Such low values are in fact typical for metals. This inherent deficiency is somewhat compensated by the length, l .

Version (a) displays expansion elements that occupy the entire length of the cooler. One should be aware of various associated effects that can be problematic at the least and, in some applications, intolerable:

1. An elevated heat leak results from thermal conduction through the expanding elements,
2. The net expansion of each element responds to the temperature distribution along the entire heat exchanger rather than simply to that at the cold end. Note that the longitudinal temperature pattern does not immediately rearrange itself when the cryogen yield is reduced or the temperature in the evaporator warms up. Therefore, one may expect a delayed response and a less efficient flow adjustment.
3. Frozen contaminants tend to clog the expansion orifice. Only a rapidly responding and sensitive flow regulator is able to impede the formation of a massive blockage. Consequently, one should expect these flow regulators to be quite susceptible to clog formation.
4. The thermal mass is increased. These cryocoolers will cool down slower or consume more gas for cooldown.
5. This configuration enables a warm end adjustment of the flow rate while the cooler is operating inside the dewar.

Walker^{53,54} applied a version of configuration (a) to control the annular gap between the two tapered surfaces in his special Joule-Thomson valve as described in Sect. 6.1.6.2. The linearly expanding metal elements are shaped as a thin sleeve and a bar⁵³ or two thin concentric sleeves.⁵⁴ A delicate micrometer (element c in Fig. 6.32) modifies the longitudinal position between the two expanding elements. The micrometer provides a valuable feature in many applications for precisely adjusting the desired flow rate, while the cooler operates under specific conditions.

Each of the tapered surfaces connects to one of the expanding sleeves. The flow is released between the tapers. Therefore, the narrow space between the two sleeves is exposed to the high supply pressure and has to be handled accordingly. More about this type of cryocoolers and their application may be found elsewhere.^{55,56,57,58,59}

Bodio et al.⁸⁷ built and tested similarly designed cryocoolers with two metal expanders. The first metal, an Invar rod starting at the warm end, is sharpened at the cold end to comprise the needle itself. The orifice is attached to a steel mandrel comprising the second expander. Albagnac⁸⁸

proposed another version of the same idea in which the position of one of the expanding elements is adjustable at the warm end.

Version (b) employs shorter thermal expanding elements that do not extend to the warm end; in effect, a refinement of version (a). Albagnac⁸⁹ applied a thin thermally expanding metal rod, shorter than the entire heat exchanger, incorporated more towards the cold end of the cryocooler, and not exposed to the warm end temperature. At one end, it senses the cold operating temperature, while on the other it senses the temperature near the middle of the heat exchanger. A miniature thread on the needle of the valve allows a delicate adjustment of flow rate. However, the flow cannot be regulated while the cryocooler is operating, in contrast to configuration (a). Nevertheless, one may still easily calibrate the non-recuperative flow.

Version (c) uses an extended evaporator and includes longer expanding elements. Such a configuration is described by Cramer et al.⁹⁰ The response, ΔI , is dominated by the quality of the two-phase content in the evaporator. The needle includes a thread for fine adjustment of its position. The adjustment is accomplished at the cold end, which means one has to interrupt the run and extract the cryocooler from the dewar in order to adjust the flow.

Configuration (c) moves the nozzle away from the cooled object. The two phase expanded stream does not directly impinge on the target surface and the heat rejection process is less intensive. The bath of cryogen and its thermal inertia is enlarged; therefore the response of the regulator to changes in the thermal balance and liquid level is slower. Another similar construction was proposed by Campbell.⁹¹

6.2.3.3 Plastic and Other Non-metallic Expanding Elements

Berry et al.⁹² introduced the idea in 1967 of enlarging the difference $\alpha_1 - \alpha_2$, by making one of the expanding elements out of a plastic instead of a metal, thereby also reducing the heat leak. They employed a long rod of Nylon or Delrin in a metal sleeve, both extending from the cold to the warm end, such as version (a) in Fig. 6.13. The adjusting thread is incorporated into the warm end of the plastic rod.

Longworth⁹³ also took advantage of materials with much higher thermal expansion coefficients than metals, such as polyamide polymer (PPMI) or polycarbonate (PC). Typical average values for α of these and some other candidates are listed in reference⁹³ at various temperature ranges. Through a judicious selection of plastics and metals, a differential expansion, $\alpha_1 - \alpha_2$, of about $100 \cdot 10^{-6} \text{K}^{-1}$ can be achieved. Additional and comprehensive information regarding the thermal expansion of metallic elements and alloys, and nonmetallic solids, may be found elsewhere.^{94,95}

Two configurations of metal-plastic bi-material expanders are possible. The first arrangement employs a

metallic sleeve to fix the orifice location, and a plastic rod for the needle. The second version uses a metallic rod to position the needle, while the orifice is fixed to the plastic mandrel. Both configurations enable flow rate adjustment at the warm end, even while the cryocooler operates.

Bonney^{84,96,97} adopted the same approach as Longworth and employed materials of especially high thermal expansion such as polyethylene, see version (d) in Fig. 6.13. He succeeded to shrink a flow control mechanism below the length of a gas-charged bellows regulator. Another design also based on a short plastic rod at the cold end is described elsewhere.⁹⁸

Bi-material flow-regulating mechanisms may also be installed in matrix type heat exchangers.⁹⁶ Another example of such an approach is given by Pope⁹⁹ who incorporated a thermal expansion mechanism to regulate the flow of nitrogen gas through the final stage of a krypton-precooled, rapid cooldown cryocooler. A bar passing through the porous matrix heat exchanger and liquid krypton bath of the precooling stage extends into the second, or nitrogen, stage. After cooldown is accomplished, it contracts and actuates a needle valve.

Version (e) utilizes differential thermal expansion in order to change the size of the orifice itself as function of the low temperature rather than to generate a displacement for actuation. Albagnac et al. use the annular clearance between two concentric and closely fitting elements as a restricting orifice. The closely fitting elements are a sphere⁵² or a cylindrical⁵¹ core in a round hole. As the temperature is reduced, the clearance, and resulting flow rate, decreases. Such a mechanism is very compact and may be installed at the cold end in a fashion similar to a bellows type regulator. The pairs of contracting materials used by Albagnac et al. were steel and glass, brass and sapphire, and nickel and Invar. Chazot¹⁰⁰ also discussed a cylindrical clearance. For a range of pressures up to 20 MPa he proposed using plastic materials such as polyamids or Teflon for the expanding elements while for a higher pressure range, the materials of choice are aluminum and copper or copper alloys.

In the review entitled “Technical level and ways of improvement of Joule-Thomson minicoolers” Abakumov et al.¹⁰¹ summarize the status of Joule-Thomson cryocooler development in Russia. Referring to the work of Landa,¹⁰² they describe a thermal expansion flow regulator based on plastic elements installed in the core of the heat exchanger. Because of its high thermal expansion coefficient, the regulator responds to temperature changes in the evaporator. The plastic component displays long-term dimensional stability. A second element is used to sense the ambient temperature and compensates the response according to variations in that temperature. Abakumov et al. report that the flow regulators perform well with pure gases, such as nitrogen or argon, and with mixed refrigerants.

Landa, Gromov, et al.^{103,104} proposed a design with a central rod made of Covar within a plastic sleeve, a version (a) configuration but of a hybrid type, employing metal and non metal expanders.

6.2.3.4 Operation of a Plastic Expander Versus a Charged Bellows Regulator

Bonney⁸⁴ discusses the valuable accumulated experience gained from operating the bi-material controlled demand flow cryocoolers and compares their performance with coolers regulated by gas-charged bellows. The central topics are:

1. Bi-material controlled flow demand Joule-Thomson coolers exhibit better proportional flow control stability, and thus better cold end temperature stability during steady state operation, than gas charged bellows controlled coolers.
2. The increased flow-regulation stability reduces gas consumption, and extends the steady state run time for a given quantity of refrigerant, without compromising temperature stability,
3. However, it responds less effectively to plug formation. When such a cooler becomes clogged due to excessive contamination, it will typically require a more extensive warming process to enable recovery, than would a gas charged bellows controlled cooler. This can be explained by the lower temperature sensitivity as compared to the gas-charged bellows. A bi-material's largest values of dl/dT , occurring around 100 K, are approximately 1–1.5 $\mu\text{m/K}$, only about half as large as those afforded by a bellows. Furthermore, the total response of the bi-material controller over the range of 300–80 K, $\Delta l/\Delta T$ is similar. Hence, one should expect a slower dynamic response under varying operation conditions and larger fluctuations of temperature.
4. Bi-material controllers have a very large spring constant, so that the valve actuation force is several times greater than that of the bellows type,
5. In general, while a regulated Joule-Thomson cooler is typically designed to operate with a given refrigerant, for a bi-material controlled cooler, a higher regulated flow rate can be obtained with a higher boiling point refrigerant. Such flexibility is less easy to achieve with gas charged bellows regulators,
6. The simplicity of the bi-material design enables rapid adaptation to new products or custom applications,

6.2.4 Bimetal Flow Regulators

If two rectangular bars with differing thermal expansion coefficients are attached to each other along their long surface, relative movement as a result of linear expansion in that long direction is inhibited. As a result, when they undergo a temperature change, one of them will be in tension

while the other is compressed, and the composite structure will bend. Such behavior differs from the linear thermal expansion mechanism described in the previous section. There, each member expanded independently. The deflection of a bending cantilever resulting from the attached configuration is known as the bimetal response. For similar stresses, the bending deflection of a bar is much larger than the change of length resulting from pure compression or tension. In other words, the resulting deflections are larger than the displacements obtained through the linear thermal expansion mechanism described in the previous section. Consequently, the bimetal response can provide a more sensitive flow regulation mechanism, that is, a larger deflection for a smaller temperature change. On the other hand, the complete actuating mechanism is more complicated and embodies many parts.

Herrington¹⁰⁵ employs a bimetal mechanism that extends along the heat exchanger, from the warm end towards the cold end. The deflection of the bimetal cantilever is transmitted to the needle of the Joule-Thomson valve by the aid of an additional rotating lever. One sets the desired flow rate by adjusting the length of the bimetal that is free to bend, an adjustment that can be accomplished at the warm end of the cryocooler without interrupting its operation.

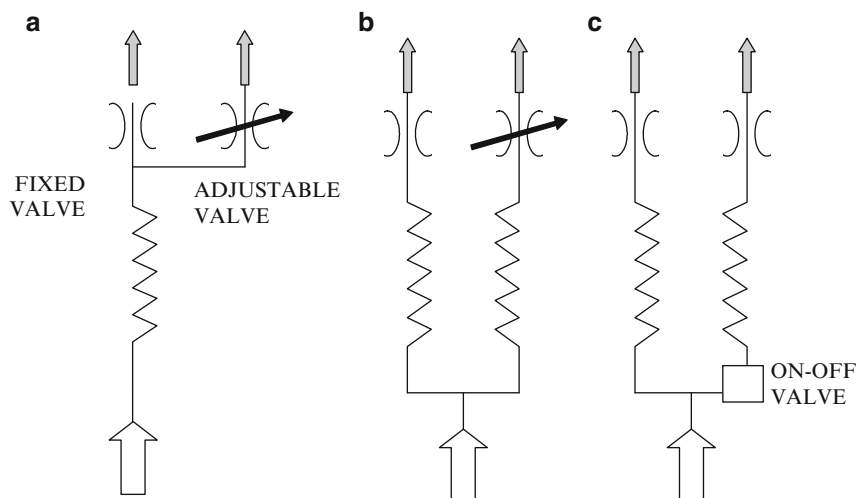
Bulat, Dray and Sollami^{106,107} installed a miniature Joule-Thomson cryocooler in the tip of a cryosurgical apparatus operating with nitrogen. To reduce gas consumption, they proposed a flow control element consisting of a series of bimetal fingers oriented in the radial direction to support a disc above the outlet orifice. When the fingers bend as the temperature is reduced, the disc approaches the orifice and restricts the flow.

Buelov et al.¹⁰⁸ used a double layer bimaterial snap disc for flow control. It is located at the cold end and directly actuates the needle of an expansion valve. A similar arrangement based on a snap disc was discussed by Mangano.¹⁰⁹ It cannot provide continuous regulation, since the snap disc provides only two positions. Nevertheless, it may be useful for fast cooldown applications, where the short period of operation allows a somewhat elevated flow rate.

6.2.5 Dual Joule-Thomson Valve

The configuration of the dual Joule-Thomson expansion valve enables a flow regulation strategy: a large fixed orifice is active during cooldown and then the regulating effect switches to a small fixed orifice for steady state operation. An alternative version incorporates one of the previously reviewed adjustable flow orifices operating in parallel with a fixed orifice. In this approach, even if the regulated orifice blocks the flow, the other one maintains a minimal desirable flow thereby excluding, even instantaneously, a zero total flow condition. As a

Fig. 6.14 Layouts of the high pressure channel of cryocoolers with dual Joule-Thomson expansion valves. Part of the flow is regulated by a self adjusting valve as seen in (a) and (b) or part of the flow is switched (c). The total flow never vanishes



result, more moderate and less demanding flow adjustments are possible, and the configuration eliminates on-off flow fluctuations during steady state operation.

There are three ways to realize this idea as described in Fig. 6.14, depending on whether the entire flow splits, as depicted in (a), at the cold end or, as in (b) and (c), at the warm end. In both cases, each of the valves is exposed to a similar pressure.

As proposed by Longworth et al.,¹² version (a) incorporates a single helical high-pressure tube. They used a short delicate capillary in a finned tube heat exchanger as a parallel fixed orifice, completely bypassing the adjustable needle valve at the cold end. Bonney⁹⁶ applied a similar idea within a matrix heat exchanger. Other configurations providing flow splitting at the cold end include a short hole in parallel with the needle valve, a hole inside the needle of the valve,⁷⁷ a few minute channels along the tapered surface of the needle or the seat of the needle¹¹⁰ or a hole inside any other type of adjustable valve as shown elsewhere.¹¹¹ However, such fixed-dimension short holes typically have a tiny diameter, supply a minute flow rate and are therefore both difficult to fabricate and highly susceptible to flow blockage. Real and Faure⁵⁰ constructed a dual orifice comprised of two fixed orifices. The first provided by a short hole and the other by the radial clearance between a plunger and a tightly fitting sleeve. A combination of a linear electromagnet and a spring, both at the cold end, position the plunger inside or outside of the sleeve. The same reference describes a rotational version for switching from one orifice to another.

Version (b) incorporates a pair of high-pressure finned tube heat exchangers helically wound in parallel on the central support mandrel of a cooler. The outlet of the first is restricted by an adjustable needle valve. The other stream flows through an “open tube” and expands solely along the tube, without the help of any orifice at the cold end.¹²

The third configuration, after Longworth,¹¹² is shown in Fig. 6.14c and has two constant orifices. An on-off valve at the warm end of one of the branches controls flow through that branch. After cooldown, the valve is closed and halts the flow in that branch.

The dual orifice configuration can also enable a fast cooldown strategy.^{113,114} Initially, a large flow rate is supplied through both orifices; subsequently one orifice is completely closed during steady state operation.

6.2.6 Temperature Dependent Shape Memory Alloys

The shape memory alloys (SMA) belong to the relatively modern category of smart materials. This technology provides unique engineering opportunities^{115,116,117,118} in a variety of fields. Displacements and actuation are accomplished due to an austenite-martensite (metallurgical phase) transition. Such a transition may be either stress induced or temperature induced. Designs utilizing these materials are based on the fact that the alloy element of a given shape “remembers” its designated shape. At a desirable temperature of transition, the element “recalls” its original shape and returns to it.

To some extent, their use in coolers resembles that of the thermal expansion regulators. Here again, flow regulation results from the passive response of the cooler component (in this case the bulk material itself) rather than from an active control of the gas flow. However, the response of the shape memory alloys may be much more pronounced. Indeed, if compared to other mechanisms such as elasticity of metals or thermal expansion or electromagnets, the new alloys enable much larger deflections.

The first SMA based flow regulator for a Joule-Thomson cryocoolers appears to have been proposed in 1974 by Arbusova et al.¹¹⁹ and tested by Desyatov et al.¹²⁰ A bended leaf, made of Nitinol SMA, counterbalanced by a regular helical spring, positions the delicate needle of the flow regulator at the cold end of the cryocooler. At the low temperature, the SMA element loses its stiffness, and the helical spring pushes the needle thereby reducing the aperture of the needle valve. Yermakov and Kaner^{121,122} employed a regulating orifice in a disc made from a bronze-aluminum shape memory alloy. During cooldown, and at a low transition temperature, the shape memory effect contracts the size of the orifice, thereby decreasing the associated flow rate, and providing flow regulation.

The shape memory element may be configured as a bi-stable device switching from one state to another as the temperature crosses a certain border. As such, it fits very well within the dual orifice concept. Reale and Faure⁵⁰ demonstrated this idea, showing that one orifice can replace the other as the memory transition occurs. In their cooler, the high-pressure flow discharges through the metering orifice, then passes through a wide slot in a plunger, which does not restrict the flow. However, when the plunger undergoes a slight rotation, the flow expanding through the orifice is blocked and is forced to pass through a precise radial clearance between the plug and the sleeve. In this case, the flow encounters two restrictions in series, further reducing the flow rate.

A set of adjacent metal leafs attached to a plunger comprise the actuator. One of them, the shape memory alloy element, remembers its twisted position. At the end of cooldown, the leaf reaches its transition temperature, twists back, and rotates the plunger. The other leaf provides a normal elastic restoring mechanism that enables the shape memory leaf to return to its straight position as the component warms up.

Guichard et al.⁸³ describe a dual flow cooler where switching is enabled by the flexing of a washer constructed of a shape memory alloy. Guille and Real¹²³ used another shape memory bi-stable element, configured as a thermally sensitive washer in a “V” profile. It is counterbalanced by a spring, and changes the size of the orifice when it changes its own shape.

Another flow regulating system, also based on shape memory alloys, but which is a continuous transducer, is introduced in Sect. 6.2.7.5.

6.2.7 Active Feedback (Servo) Systems

6.2.7.1 Description

The flow regulators described here may be distinguished from those reviewed up to this point, by the functions of

sensing and correcting actuation. In active feedback systems, the temperature measured at the cold end, or at the payload itself, generates a feedback signal that when processed by a controller, results in a command to correct an error. The command may drive a change to the location of a needle in the Joule-Thomson valve, or the pressure supply in the case of an open cycle system, or change a compressor’s speed of rotation in the case of a closed cycle.

The simplest feedback loop is the bang-bang system. The bang-bang mode of regulation has been widely discussed in text books^{124,125} and is often applied. It represents the most elementary non-linear control approach. In some cases, it even embodies the optimal strategy. The correcting response vector generated by such a controller includes only two possible discrete commands: open, or close (bang-bang). In contrast, the continuous control system produces a variety of responses, derived from the deviation of the system from a defined set point. The signal-processor generates a multiplicity of commands associated with a desirable control policy instead of the simple bang-bang options. Hence, the bang-bang policy is expected to provide less potential for economic gas consumption. On the other hand, it remains attractive because of the simplicity and reliability of the associated hardware.

6.2.7.2 Motivation

In some applications, the required sensitivity to flow control is not adequately satisfied by the performance of the passive flow regulators described in the previous sections. Westhoven and Smith¹²⁶ precisely summarized the background of this situation,

... Many applications, such as missile applications, require low sustaining flow rates to achieve long required run time. during low heat load conditions such as low environmental temperatures, cryogen flow diminishes to very low values and the needle valve becomes almost close.....flow can be interrupted and the controlled temperature increases unacceptably before the actuator responds and flow is resumed

The need exist for a Joule-Thomson cryostat which affords rapid cooldown, reliable demand (low) sustained flow rates, and operation with multiple cryogen types a self cleaning cryostat offering robust performance handling a range of cryogen impurities at low cryogen flow rates ...

The central issues are underlined (not by the inventors). From their summary, one can appreciate the need for an active, rather than passive, feedback system. Actively adjusting the orifice enables the system to avoid temperature instabilities, even at the very low flow rates that are problematic for the dual orifice strategy described in Sect. 6.2.5. An active feedback approach can also overcome clog disturbances, and successfully handle different coolants (see Sect. 6.2.8.1).

Active feedback systems can also mitigate the effect of shocks on temperature stability. When a cooler is subjected

to a longitudinal acceleration, the liquid cryogen may be displaced from the cooled object towards the temperature-sensing element such as a charged bellows. In such a scenario, the cooled object actually warms up while the sensing element detects increased cooling. Therefore, the “corrective” response tends to reduce the flow rate, possibly even interrupting it, and thereby fails to maintain a stable temperature. An active feedback system can avoid such a shock-induced inherent instability because it directly senses the temperature on the cooled object. As discussed by Campbell and Richardson,¹²⁷ one possible response by an active feedback system in such a situation would be to change the compressor’s speed of rotation. Pahler¹²⁸ proposes another version of an active feedback system for a similar purpose in which actuation is accomplished by heating up a silicon oil bath so that its expansion drives the needle valve.

6.2.7.3 A Bang–Bang Pressure Supply

A bang-bang pressure supply represents the simplest feedback mechanism for flow regulation and is accomplished by turning the gas supply valve on and off. Peterson et al.²¹ utilized this approach in the very early stages of regulated cryocooler development. It is attractive for its simplicity. In this particular case, the cooler uses an open-end tube without any special nozzle. The pressure supply valve, (necessary in any case for initiating cooldown or turning off the cooler), is operated in an on or off mode, and the cooler, with its fixed orifice, maintains a constant flow. The temperature measured at the payload provides the feedback signal. While the temperature remains within a reasonable margin around a target value, the flow and resulting cooling exists either at a maximum, or zero, value. Due to various thermal delays and the typically more stringent temperature stability requirements, this control loop usually provides less than satisfactory performance. An improvement may be obtained when the on-off switching is controlled by a Proportional-Integral-Differential (PID) regulator which analyses the target temperature signal as described elsewhere.¹²⁹ The resulting gas consumption is higher than what is achievable by a bellows regulator and the controlled temperature is less stable. On the other hand, since the gas always expands through a large orifice, one may expect a lower susceptibility to clogging.

6.2.7.4 Piezoelectric Actuation

Piezoelectric actuation is accomplished by a piezoelectric transducer in the form of a bending cantilever at the warm end that pushes a bar along the heat exchanger to move the needle. Such a flow regulator was patented by Feger¹³⁰ and described by Curler.¹³¹ The piezoelectric element is a solid-state device that translates a voltage into a very delicate displacement. The controller in such a system measures the temperature mounted directly on the payload at the focal plane, and generates an associated electric command that

drives the actuator. A dither, superimposed on the actuation signal, enables the system to avoid hysteresis and sticking of the delicate mechanism. Highly accurate temperature stability is achieved. Within the ambient temperature range of -40°C to $+70^{\circ}\text{C}$, payload temperature fluctuations did not exceed a rate of 0.1 K/s. Another embodiment of piezoelectric actuation is described by Hipper.¹³²

However, such a mechanism introduces a thermal short-cut, connecting the warm and cold ends, thus resulting in a somewhat elevated heat leak. In addition, it requires a special electronic driver for the piezoelectric actuator.

6.2.7.5 Shape Memory Alloy Based Transducer

Hipper¹³² proposed the use of a shape memory longitudinal element made of NiTi, CuZnAl or CuAlNi alloys to govern the location of the needle in a restricting valve. Similarly, Skertic, Hlava and Brunton¹³³ used a transducer based on a shaped memory alloy wire, and counter balanced by a bellows spring. The heat required for the metallurgical phase transition is controlled by the electrical current supplied to a heater, while the temperature sensed at the payload, provides the feedback signal to the controller. Actuation is obtained by the changes in length resulting from the shape memory effect, thus positioning the needle within the flow restricting valve.

6.2.7.6 A Reactive-Thermo Elastic Transducer

A reactive-thermo elastic transducer was proposed by Westhoven et al.¹²⁶ Here, the thermal expansion properties of elastomers provide the actuating mechanism for the control loop. Westhoven et al. proposed the use of elastomeric polymers such as epoxies, silicones, and polyurethanes at a size similar to a bellows regulator, and also located at the cold end of the cryocooler. The element includes a thermal heating wire that controls the temperature of the elastomer and thereby adjusts the location of a needle inside a valve.

6.2.8 Miscellaneous

To end this review on flow adjustment, we present a few related aspects and special ideas.

6.2.8.1 Flow Adjustment of Different Coolants

It would be of advantage, for example with the case of nitrogen and argon, to design a regulated cooler that could operate with two different fluids. However, design challenges arise. For example, a regulator designed for use with nitrogen but operated with argon will “feel” warmer and thus try to enlarge the orifice. However, because the specific cooling capacity of argon is higher, in fact a smaller

orifice would suffice. Skertic⁷⁴ proposed overcoming this inherent contradiction by using the primary coolant to also fill the bellows of the regulator. Thus a bellows filled with the proper content will regulate the corresponding coolant. However, construction of the pneumatic system becomes a bit more complicated. A feedback flow regulator was proposed by Westhoven et al.¹²⁶ for this purpose (as one of multiple purposes). It requires adjusting the target temperature until it is stabilized at the corresponding boiling point value, but the hardware remains unaltered.

6.2.8.2 A Self-Adjusting Effect for a Porous Plug

A self-adjusting effect for a porous plug was identified by Hill et al.³⁴ They used a sintered porous plug as the Joule-Thomson expansion valve primarily for mixed coolants, but also for pure coolants. During the cooldown process, the instantaneous temperature at the inlet of the porous plug decreases, causing the fluid viscosity to increase. The resulting increase to the effective hydrodynamic impedance through the expansion valve in turn decreases the fluid flow rate. The behavior therefore provides a self-adjusting effect to the flow without the aid of moving parts. Lerou, ter Brake et al.^{134,135} developed a throttle using nano-scale parallel ducts with laminar flow as in a porous plug. They measured the flow rate during the cooldown process and displayed it as function of the instantaneous temperature. In this case, the flow rate does increase during the cooldown, but begins to decrease as the temperature approaches 140 K.

6.2.8.3 Flow Regulation Induced by Pressure of the Vessel

Mangano et al.¹³⁶ installed a mechanism incorporating a diaphragm and a spring at the warm end of cryocooler, that is sensitive to the instantaneous pressure in the supply reservoir. When the pressure falls below a given value, the mechanism actuates a longitudinal rod that reduces the area of the orifice at the cold end. The mechanism is utilized in a rapid cryocooler with large flow rates during cooldown. At a preset value of pressure corresponding to the desired termination of cooldown, the flow rate is reduced. Another example of the same idea is presented elsewhere.¹³⁷

6.2.8.4 A Floating Needle in a Needle Valve

In order to obtain very low flow rates through a needle valve, one would like to locate the needle symmetrically within the orifice hole. However, because such precision is difficult to obtain in practice, very low flow rates are usually not attainable. In addition, large sticking, or frictional forces must be overcome. Instead of a rigid attachment, Sagev et al.¹³⁸ proposed to allow the needle to float so that its position would be adjusted to match that of the valve seat, thereby improving the reliability of the adjustment.

6.2.8.5 Flow Regulation by Liquid–Solid Transition

The ultimate example of a passive, yet direct control mechanism is perhaps one associated with the liquid–solid transition. In this approach, with a fixed orifice, excess cooling beyond a desired value (below a desired temperature) freezes the coolant, thereby blocking the orifice and halting the flow. Subsequently, the temperature in the evaporator warms up, and flow is renewed as the blockage thaws. Markum^{139,140} and Chorowski¹⁴¹ studied such an approach using mixed coolants. The basic coolant, such as nitrogen, is combined with another component with a higher boiling point and a higher temperature of solidification, such as CF₄. However, this approach results in an unstable evaporator temperature that can fluctuate with a relatively large amplitude, depending on the actual composition.

6.2.8.6 A Manually Adjustable Flow Regulator

A manually adjustable flow regulator is used to adjust the coolant flow rate while the cryocooler is operating by means of a mechanism positioned at the warm end of the cryocooler. Walker^{54,55,56,57,58,59} and Bodio⁸⁷ employed a delicate micrometer, that enabled manual adjustment of the longitudinal position between a couple of thermally expanding elements. The adjustment changes the equivalent orifice and allows one to set a new desired flow rate. Ellison¹⁴² modified such a cryocooler by employing an actuating solenoid. This valuable feature enables a precise setting of the desired flow rate. Furthermore, the real-time adjustment can mitigate clog formation, by effectively opening the orifice thereby avoiding flow reduction and flushing out the frozen contamination.

6.2.8.7 A Mechanism to Squeeze the Tube

A mechanism to squeeze the tube that serves as a Joule-Thomson valve, in order to reduce its cross section and further restrict the flow during the cooldown, is described by Chan et al.¹⁴³ The effect results from the differential thermal expansion between two components, one of brass and the other of titanium

6.2.8.8 The Pressure Dependence of Flow Rates

The pressure dependence of flow rates through a given fixed orifice cryocooler were examined by Maytal,¹⁴⁴ and is worthwhile mentioning at the end of this section on flow adjustment. Flow through an orifice does not vary proportionally with the supply pressure as that pressure increases. Instead, its rate of growth is attenuated (Sect. 9.2.3.2), a feature that could be used to facilitate the task of flow adjustment.

6.3 Heat Exchangers

6.3.1 Introduction

6.3.1.1 Classification

The counter flow heat exchanger, providing the central role of the amplifying “interchanger,” is an essential element in any Joule-Thomson cryocooler. Its inlet channel is exposed to high pressures, typically about 50–500 times larger than in the counterflow channel. Consequently, the volumetric flow rate at the low pressure outlet is correspondingly higher than at the high pressure inlet.

There are two traditional versions of this heat exchanger. When the two streams flow counter-currently at any intermediate location along the entire flow path, we have the Linde type^{145,146,147} counter-flow heat exchanger, originally proposed in 1895 by Carl von Linde.

The other configuration, proposed by Hampson¹⁴⁸ also in 1895, and further developed by Giauque, is often referred to in the literature as a *coiled tube* heat exchanger.¹⁴⁹ A finned tube with high-pressure flow through its inside is helically wound over a cylindrical core so that all its fins are positioned almost perpendicular to the returning low-pressure flow. Such a heat exchanger embodies a counter-flow configuration in the global sense but is comprised of a series assembly of cross-flow elements.

Figure 6.15 schematically shows the general layout of the two configurations. These define the basis for organizing the entire family of heat exchangers for Joule-Thomson cryocoolers as displayed in Fig. 6.16.

Hampson’s concept has been modified by incorporating wire mesh or porous sintering to extend the surface area on the low-pressure side, while the high-pressure channel remains a coiled tube. Other versions of compact heat exchangers, based on perforated plates or narrow channels provide variations of Linde’s idea.

6.3.1.2 Parameters of Construction

Parameters of construction and technology that are relevant for a comparison of candidate heat exchangers for miniature Joule-Thomson cryocoolers are as follows:

1. Thermal compactness, in terms of heat transfer area per unit thermal mass, and the total thermal mass, that influence the cooldown rate.
2. Longitudinal thermal conductivity, since it impacts the heat load^{150,151,152} and degrades the effectiveness of the heat exchanger.^{150,153}
3. The hydrodynamic impedance of each channel; it induces a distributed JT effect due to the pressure gradient, and may also degrade the effectiveness of the recuperator as discussed elsewhere.¹⁵⁴ This observation is consistent with Walker’s²⁰ and Hudson’s¹⁵⁵ comment that they

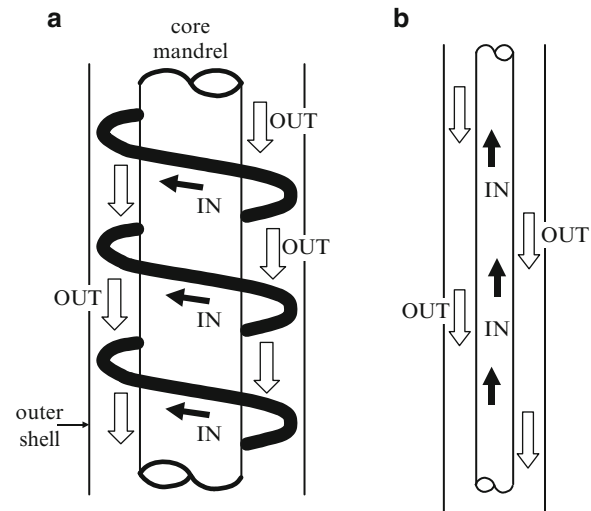


Fig. 6.15 The two basic configurations of counter flow heat exchangers for Joule-Thomson cryocoolers, (a) Giauque-Hampson, and (b) Linde. IN represents the high-pressure inlet and OUT the low-pressure outlet

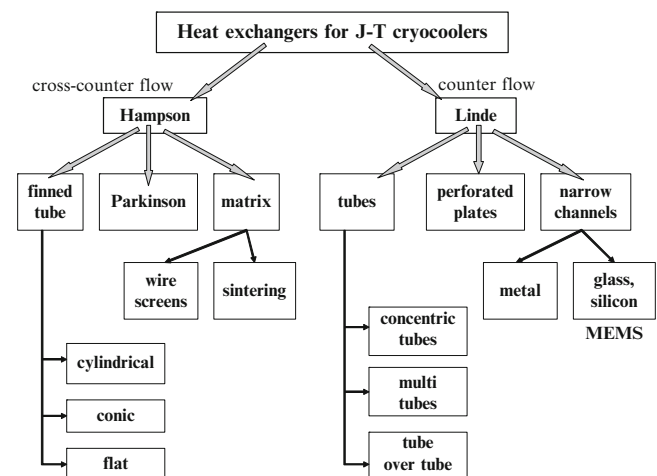


Fig. 6.16 Classification of candidate heat exchangers for Joule-Thomson cryocoolers

would prefer a Joule-Thomson valve at the end of a larger diameter high-pressure tube and avoid the longitudinal pressure drop.

4. The hydrodynamic impedance of the low-pressure channel has an additional significance; for a given cooling power and the associated flow rate, it dictates the elevation of the boiling point and the operating temperature. This feature becomes more relevant in the case when high flow rates are used to accelerate the cooldown process.
5. The viability and ease of incorporating a flow regulation mechanism.
6. The feasibility of staging.

7. The feasibility of forming a heat exchanger into a desirable shape, for example a long cylinder or a flat disk, affects configuration considerations.
8. The stiffness.

Additional parameters of relevance are cost and performance insensitivity to high acceleration.

The comprehensive review of heat exchangers for Joule-Thomson cryocoolers by Longworth¹⁵⁶ is recommended for further reading.

6.3.2 Finned Tube Heat Exchangers

Finned tube heat exchangers represent the most widely applied heat exchanger technology for miniature Joule-Thomson cryocooling. Abadzic and Scholz¹⁴⁹ utilized the effectiveness-NTU relationships presented by Kays and London¹⁵⁷ for cross flow and counter flow heat exchangers to compare the effectiveness of a coiled tube heat exchanger (referred to by the authors as cross-counter flow heat exchangers) with that of a true counter flow heat exchanger. They concluded that for more than 14 windings the effectiveness of the two types become very similar, if the capacity rates are equal, that is $\dot{C}_L = \dot{C}_H$. (In practice, the heat exchangers in most cryocoolers exceed this number of coils.) However, in case of only 5 windings, the coiled tube heat exchanger loses 5–10% of the effectiveness in comparison to a pure counter flow heat exchanger.

The following description of finned tube heat exchangers will focus separately on (a) the finned high-pressure capillary tube itself and on (b) the configuration and arrangement of the finned tube.

6.3.2.1 Finned High-Pressure Tube

Figure 6.17 shows a continuous fin with a rectangular cross section helically wound around the high-pressure capillary tube, and oriented nearly perpendicular to the tube. It comprises the basic element of the heat exchanger. The parameters of interest in the design of such a heat exchanger are: material and size of both the pressure tube and fin, the pitch of the helix or equivalently, the density of fins, and the method of attaching the fin to the high pressure tube. Typically the ratio of the external finned surface area to internal surface area is about 8–12, depending on the size and density of the wound fins. In general, the range of inside and outside tube diameters is 0.25–0.8 mm and 0.4–1.2 mm.

Pressure Tubes

The preferred material for the pressure tube is copper^{158,159} or a copper nickel alloy,⁷ because of its high thermal conductivity.

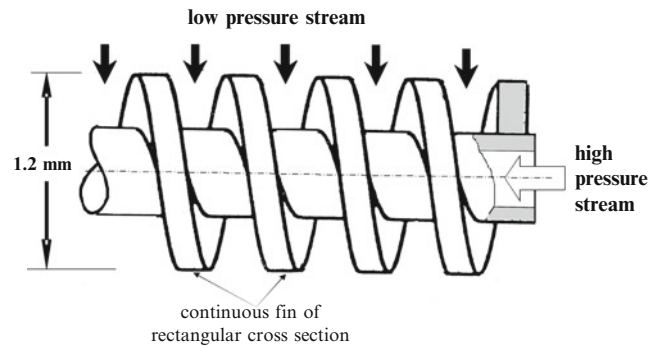


Fig. 6.17 Continuous fin, of a rectangular cross section, is helically wound around the high pressure tube. The local flow directions are shown in this elementary segment of a finned tube heat exchanger

Table 6.3 Inside (d) and outside (D) high pressure tube diameters, rectangular copper fins' sizes and helix pitch

	Tube	Fin size	Fines per cm
1.	$d = 0.254 \text{ mm (0.010")}$ $D = 0.508 \text{ mm (0.020")}$	$0.2540 \text{ mm (0.010")}$ $0.0762 \text{ mm (0.003")}$	35
2.	$d = 0.508 \text{ mm (0.020")}$ $D = 0.762 \text{ mm (0.030")}$	$0.2540 \text{ mm (0.010")}$ $0.0762 \text{ mm (0.003")}$	35
3.	$d = 0.508 \text{ mm (0.020")}$ $D = 0.762 \text{ mm (0.030")}$	$0.4572 \text{ mm (0.018")}$ $0.0762 \text{ mm (0.003")}$	35
4.	$d = 0.686 \text{ mm (0.027")}$ $D = 0.934 \text{ mm (0.037")}$	$0.4572 \text{ mm (0.018")}$ $0.0762 \text{ mm (0.003")}$	22
5.	$d = 0.914 \text{ mm (0.036")}$ $D = 1.168 \text{ mm (0.046")}$	$0.5842 \text{ mm (0.023")}$ $0.1270 \text{ mm (0.005")}$	19
6.	$d = 0.39 \text{ mm}$ $D = 0.65 \text{ mm}$	0.33 mm	
7.	$d = 0.305 \text{ mm (0.012")}$ $D = 0.508 \text{ mm (0.020")}$	$0.001"$	

1 through 5: Geist and Lashmet¹⁶⁰

6: Xie¹⁶¹

7: Skertic and Hlava⁷

However, stainless steel tubes,^{160,161} perform quite well, too, though somewhat less efficiently and with a slower cool down. Table 6.3 lists sample dimensions reported for capillary pressure tubes and rectangular copper fins. LeFranc¹⁶² proposed using high pressure tubes made of nickel but for a pressure of only 200 bar.

Fins

Certainly the most widely used material for the fin structure is copper of rectangular cross section. However, silver is an excellent alternative to the copper fins due to its higher thermal conductivity (420 against $380 \text{ W m}^{-1} \text{ K}^{-1}$) and diffusivity (1.7 against $1.1 \text{ cm}^2 \text{ s}^{-1}$). Pahler et al.¹⁵⁸ conducted a comparative study focusing on rapid cooldown performance in which they examined tube/fin material combinations such as: stainless-steel/copper, copper-alloy/copper and copper-alloy/silver. The most successful pair

was the copper alloy tube with silver fins. Actually, the use of silver fins for a Joule-Thomson cryocooler had already been suggested a couple of decades earlier by Le Franc.¹⁶² Depending on actual sizes, the density of fins ranges from 15 to 40 windings per centimeter along the axis of the pressure tube.

Bodio and Sujak¹⁵⁹ built heat exchangers with circular copper fins instead of ones with a rectangular cross section. The fins were defined by a 0.5 mm diameter wire, wound with a pitch of 1.5 mm on the outside of a copper tube with inner and outer diameters of 0.4 and 1.4 mm, respectively.

Coating

Beyond the thermal properties, size and density of fins, a crucial factor for an efficient finned tube heat exchanger is the extent of intimate thermal contact between the fins and tube. Tin solder⁷ is often used for this purpose as well as to preserve the position of the fins along the tube. A thin layer of a few micrometers (μm) of tin covers the fins and the pressure tube. However, the thermal conductivity ($64 \text{ W m}^{-1} \text{ K}^{-1}$) and diffusivity ($0.4 \text{ cm}^2 \text{ s}^{-1}$) of tin are significantly inferior to that of the copper comprising the majority of the fins. Thus, the structure may be regarded as a highly conductive heat exchanger covered by a insulating layer, although the layer is very thin indeed.

This observation may explain the idea and experience of Xie¹⁶¹ who eliminated the use of any coating at all. Xie developed a special process to produce a finned tube without any solder. In the process, the stainless steel pressure tube undergoes a vacuum annealing process before the copper fins are tightly wound onto the bare tube. The winding process slightly crushes the tube surface under the tension of the windings, thus resulting in a good thermal contact. Since the presence of tin leads to a less efficient heat exchange and a lower cool down rate, Xie reported that removing the tin solder provided a noticeable advantage.

Maytal¹⁶³ has suggested the use of silver in place of tin as a coating since it provides superior thermal conductivity and diffusivity. Furthermore, it has mechanical advantages compared to tin because of aspects of electrolytic coating associated with the mobility of silver ions.

Active Fin

For completeness in this survey, we include an unusual construction investigated by Bodio, Dudek et al.¹⁶⁴ They utilized the idea of Landa and Morozow,¹⁶⁵ referred to by them as an active fin, in which the traditional wire fin is replaced by a capillary tube. When activated, the capillary tube carries a stream of the same coolant at the same high pressure as in the primary tube. However, closer inspection reveals that the configuration merely operates as two parallel tube windings, and completely lacks any fins at all.

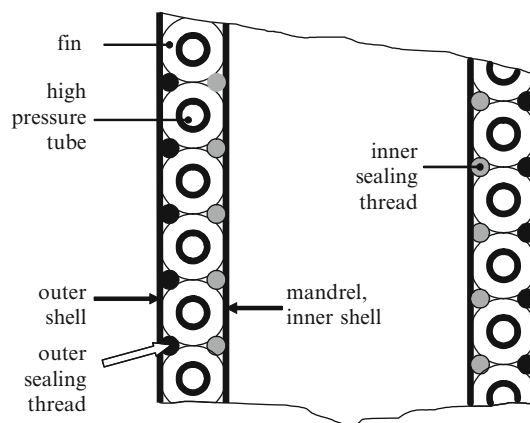


Fig. 6.18 Cylindrical configuration of the Hampson type heat exchanger obtained by winding the finned pressure tube on a cylindrical core. The returning low-pressure stream flows between the concentric cylindrical shells (Geist and Lashmet¹⁶⁶)

Nevertheless, the close contact of the tubes has an effect somewhat like an extended surface, and a minor improvement of cooling power and cooldown was observed in comparison with passive fins. Overall, the configuration may be regarded simply as an incorporation of another finer and longer parallel tube. A multi-layer bare tube configuration, as discussed in the following paragraphs, could have been even more fruitful.

6.3.2.2 Configuration of Finned Tube Heat Exchanger

The various configurations of finned tube heat exchangers are modifications of: (a) the shape of the annular space, whether cylindrical, stepped, conic or flat, and, (b) number of finned tubes and their arrangement into single or multi layers.

Cylindrical Shape

The cylindrical shape displayed in Fig. 6.18 is the most widely used heat exchanger configuration in Joule-Thomson cryocoolers. Here, the annular space between two cylindrical shells accommodates the coiled tube and provides a channel for the expanded return flow stream. In practice, it is convenient to pack the cooled object of a JT system in a separate assembly, very often within an evacuated enclosure. The outer shell identified in Fig. 6.18, though essential for the heat exchanger, is actually not a part of the cryocooler, but belongs to the dewar. The associated jargon refers to it as the “cold finger.” The inner shell is the core of the heat exchanger.

The cylindrical core provides the stiffness for the heat exchanger and cooler and the structure around which the high-pressure finned tube is helically wound. A short heat

exchanger for fast cooldown applications includes at least 15 turns, while a long one, operating at elevated ambient temperature might approach as many as 40 turns. In general, the core is a thin metallic mandrel, sometimes filled with an insulating material like perlite¹⁶⁶ but also as likely to be constructed of a round bar of plastic, like nylon.^{7,162,167} The inner space of the core is generally unoccupied, unless it accommodates a flow regulator.

While the incoming stream simply flows through the helical pressure tube, the flow regime of the low-pressure stream is somewhat more complicated. The annular conduit, outside of the core and inside the cold finger, has a larger cross section area, as necessary for the large volume of the expanded stream. The high-pressure helical tube is immersed in, interferes with, and mixes the low-pressure return flow. This is the essence of the Giauque-Hampson type heat exchanger.

The return flow is restricted and forced to pass between the fins, rather than finding a shortcut to the outlet. This task is well accomplished by two sealing wires as shown in Fig. 6.18. Geist and Lashmet¹⁶⁶ describe the employment of a gasket material to keep the gas flowing through the fins. The first seals against the “cold finger” cylinder, and is helically wound on the outer diameter of the heat exchanger, with the same pitch as the finned pressure tube. Similarly, the second wire seals against the core. Various fiber materials have been used as sealing wires or strings. For example, Webster¹⁶⁸ utilized Terylene, Peterson²¹ employed “cord material,” Kuo et al.¹⁶⁹ used Dacron, while Walker²⁰ mentions using cotton and plastic. In any case, the material should remain flexible at low temperatures to maintain the seal. Instead of a fibrous thread, Campbell¹⁷⁰ suggested using an outer sleeve made of an elastomer, that fits well into the windings of the heat exchanger and provides the sealing effect.

Because of the moderate helical angle of the pressure tube winding along the core, the cold stream flowing between the fins flows almost perpendicular to the warm stream enclosed in the tube.

While a typical miniature Joule-Thomson cryocooler is the size of a human finger, Croft et al.^{171,172} built a very similar copper finned tube heat exchanger for use with helium, but at a 45 times larger scale: 220 cm long and 18 cm diameter. It has a solid nylon core and uses a nylon cord for sealing and to fill up the dead space. The large finned tube heat exchanger was analyzed by Croft and Tebby¹⁷³ and optimized by Gupta et al.¹⁷⁴ Gupta et al.¹⁷⁵ experimentally studied the pressure drop characteristics of such cross-counter flow coiled finned tube heat exchangers.

Figure 6.19 displays a cryocooler with a cylindrical finned tube heat exchanger, and a needle valve controlled by a bellows type actuator.

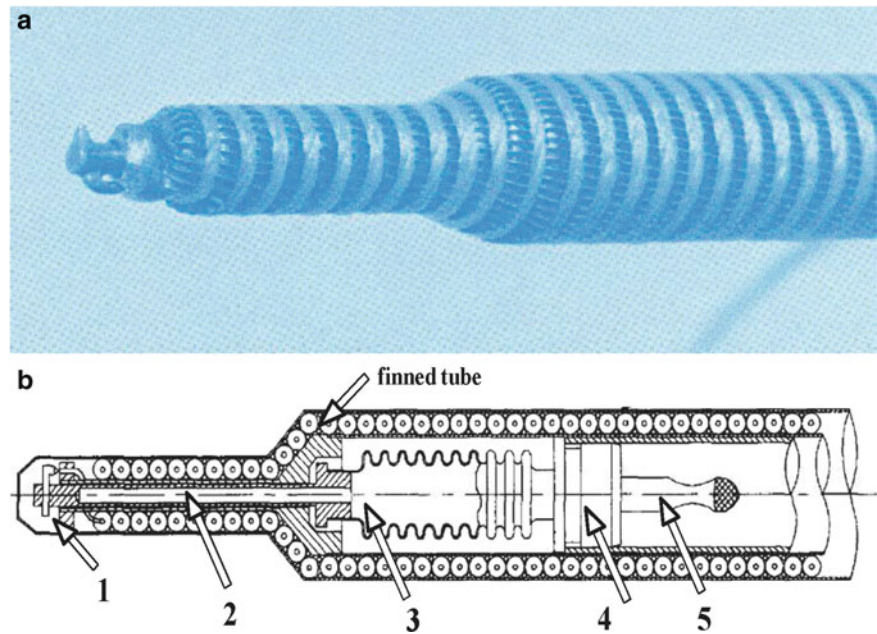


Fig. 6.19 Exploded view of a flow regulated cryocooler of Hymatic Engineering Company, Reddich, England, UK, (a) the inlet nipple, (b) the inner thin mandrel, (c) the finned tube heat exchanger wound on this mandrel, (d) the flow regulating mechanism based on a bellows, including the delicate needle of the valve, and (e) the final assembly

A Stepped Shape Heat Exchanger

A stepped shape heat exchanger is a dual diameter cylindrical heat exchanger as displayed in photograph and cross section drawing format in Fig. 6.20, parts (a) and (b) respectively. The corresponding flow regulating mechanism, based on a bellows regulator, is shown in part (c) of the same figure. It has an elongated sensing tube in order to extend through the reduced diameter step of the recuperator. This approach reduces the crucial thermal mass at the cold end thereby enabling a faster cooldown, as discussed by Longworth.¹⁷⁶ The somewhat more complicated installation of the flow regulator presents a slight disadvantage since

Fig. 6.20 (a) A stepped cryocooler constructed by a finned tube wrapped in the form of two cylinders, with a needle valve at the cold tip to regulate the flow rate (APD Cryogenics, Allentown, PA.). (b) A cross section drawing of a stepped cryocooler including a bellows type flow regulated mechanism⁸⁵: (1) needle valve, (2) sensing tube, prolonged for the reduced diameter cold section of the recuperator, (3) bellows, (4) support set, and, (5) gas filling tube



it cannot be located close to the bath of cryogen, at the tip of reduced diameter. In addition, the cold finger of the dewar must also have a correspondingly stepped shape.

A Conical Shaped Heat Exchanger

A conical shaped heat exchanger is obtained by using a cone shaped core (possibly made of a very low thermal conductivity material¹⁷⁷ such as polyurethane¹⁷⁸) as a base for the finned tube winding. For a given length of finned tube, this configuration provides two advantages. First, the conical shape results in a shorter cryocooler than the cylindrical one. In some applications, the length of the cryocoolers is a key parameter that dictates the layout of the entire assembly, and any shortening is significant. Second, the conical configuration reduces the hydrodynamic impedance of the low-pressure stream. Along the path of the returning flow, the cross sectional area becomes wider and wider according to the angle of the cone. Because the induced pressure drop associated with the return flow stream elevates the backpressure, the boiling point of the cryogen, and therefore the cryocooler operating temperature, the conical configuration can enable lower operating temperatures. This shape is typically adapted for fast cooldown applications as already shown in Figs. 5.18 and 5.19. Maier¹⁷⁹ and Dunn et al.¹⁸⁰ and Nicholds¹⁸¹ each have demonstrated a rapid cooldown cooler with a conical shape.

Flat Shape Heat Exchanger

The coin-like shape of a flat finned tube heat exchanger is a very unique configuration.¹⁸² It may be regarded as a cone with the extreme angle of 180°. Alternatively, one may

consider it as a flat spiral configuration, confined between two parallel round plates. It is the shortest and stiffest configuration, but because of its enlarged diameter, it is beneficial for certain applications. This may be the reason why it was chosen by Pahler et al.¹⁵⁸ for high acceleration applications. Their cryocooler, shown in Fig. 5.25, is positioned on a miniature cylindrical pressure vessel. The returning low-pressure stream flows in the radial direction. (A flat version Parkinson's heat exchanger is described in Sect. 6.3.4)

6.3.2.3 Pressure Tube Arrangement: The Single Stage

In most cases, a single finned pressure tube is used. However, there are circumstances that dictate applying more than one tube wound in parallel, and fed by the same pressure source.

Single Layer, Double Thread

When constructing a dual orifice cryocooler as suggested by Longworth,¹² each orifice requires access via a separate tube to the pressure source. Hence, two tubes are co-wound in a single layer around the core of the cooler. In this case, one must carefully consider the length of core, since each pressure tube can occupy only half of the length otherwise available in a single tube arrangement.

Multi-Layering for High Flow

High flow rates are required for very large cooling power demands. In such situations, rather than utilizing any special orifice the tube itself provides the necessary restriction,²⁰ and its length meters the flow rate. If the flow rate obtained

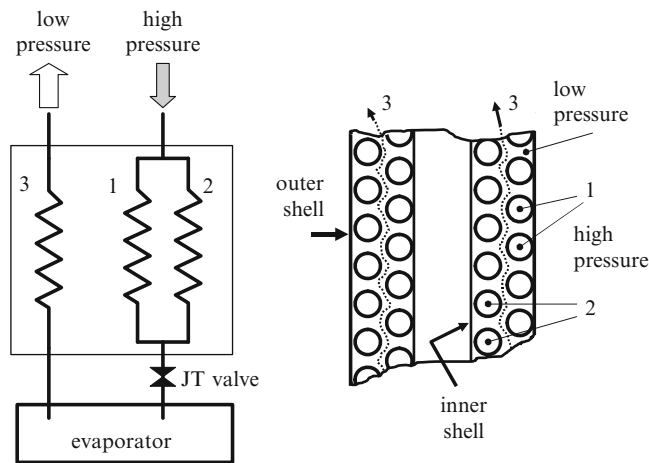


Fig. 6.21 Double layer coiling of finned tube. The two layers are connected in parallel and the entire flow expands through a single orifice (L'Air Liquide Company^{111,183})

by this approach is inadequate, additional parallel pressure tubes are used in a multi-orifice arrangement. All are pressurized in parallel by the same source. These tubes are co-wound on the same core, layer by layer.

Multi-Layer Short Cryocooler

Often, higher-level system considerations limit the length of a cooler. For example, the single layer heat exchanger may be “too short” or the flow rate “too high”, resulting in insufficient heat transfer area. In these cases, the multi layer configuration may provide the answer, as shown in Fig. 6.21. Here, instead of extending the length of the cooler, its diameter is increased. The two layers are arranged in parallel, each branch carrying only one half of the total flow rate. The heat transfer area per unit flow rate is doubled. The additional layer, enlarging the diameter, compensates for the short cooler. Such bi-tubular cryocoolers were introduced in the brochure of L'air Liquide Company¹¹¹ and presented by Viennot.¹⁸³ Such a design is also compatible with a dual orifice cryocooler like the single layer, double thread arrangement described above in paragraph (1).

Also, the multi-layers configuration provides the advantage of lowering the operating temperature because the additional layers increase the width of the low pressure channel, thereby reducing its hydrodynamic impedance. Hence, the induced back pressure and the associated boiling temperature are reduced. For the liquid hydrogen range, an application incorporating a 6 and 10 layer configuration is described elsewhere.¹⁸⁴

Multi-Layer Effectiveness

Bodio and Sujak¹⁵⁹ studied the efficiency of a multi-layer heat exchanger in comparison with a single-layer one. They

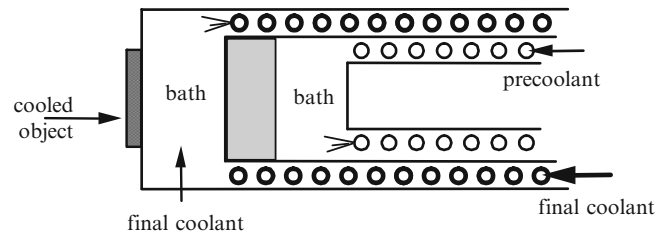


Fig. 6.22 A simple arrangement of staging; the precooling stage is inserted into the final stage

constructed the heat exchangers of their Joule-Thomson cryocoolers with round wire fins, which as previously mentioned, are typically not very efficient. However, by maintaining the same total flow rate but allowing some of the flow to pass through a single additional layer almost tripled the specific cooling power. The improved efficiency was achievable only by a multiplication of the available heat transfer area. Interestingly, the addition of a third layer did not contribute further improvement, and the fourth and fifth layers deteriorated the specific cooling power.

It should be clarified that if the efficiency of the single layer heat exchanger would have been sufficiently high (large enough area for a given flow rate), the additional layers would not yield such a gain in efficiency.

Hong et al.¹⁸⁵ compared the effectiveness of the double layer arrangement with that of the single layer and double thread. They concluded that the latter configuration is somewhat more effective. However, it induces a larger back pressure which elevates the boiling point of the coolant.

6.3.2.4 Pressure Tube Arrangements for Two Stages

Two concentric windings of high-pressure finned tubes, as schematically displayed in Fig. 6.22, are commonly used for two stage cryocoolers according to the schematic layout in Fig. 5.9. Each winding is fed by a separate pressure vessel, these typically containing different species of gas. The two windings may be separated by a full screen,¹⁸⁶ by a perforated screen¹⁸⁷ or without any separating screen.¹⁸⁸

6.3.2.5 Heat Leaks and Stiffness

The core of the finned tube heat exchanger connects the cold end and the warm room-temperature end of the cryocooler; hence, it provides a path for an undesirable heat leak. The thermal conductance of a rod is represented in Fourier's law by $\lambda A/L$, where λ is the integrated average thermal conductivity over the spanned temperature. Therefore, a thermal designer would prefer a long heat exchanger with a smaller diameter. However, a mechanical designer concerned about the stiffness of the column that supports the cooled object mounted at the cold end would prefer a column with a low

A/L ratio. In the end, a compromise must be reached between the two competing factors to form a practical solution.

The helical pressure tube itself also contributes a heat leak. However, the aspect ratio A/L is much smaller than that of the core mandrel. Nevertheless, the multilayer configuration obviously multiplies this heat leak according to the number of layers.

The same arguments are applicable to the “cold finger” column in the dewar, which is part of the Hampson type heat exchanger and supports the cooled object. It too is a thin tube of either metal or glass. The various modes whereby heat leaks into the packing of an infrared detector that is cooled by a miniature Joule-Thomson cryocooler, are analyzed elsewhere.¹⁸⁹

6.3.3 Matrix Heat Exchangers

6.3.3.1 Introduction

Compact heat exchangers¹⁵⁷ incorporating high-density heat transfer area have to be considered as a candidate technology for miniature cryocoolers where size is a limited and valuable resource. These heat exchangers are typically associated with an elevated thermal effectiveness, reduced hydrodynamic impedance and innovative production techniques. For example, the heat transfer area per cubic meter of heat exchanger volume reaches values as high as $6,000 \text{ m}^2/\text{m}^3$ as compared to values of only $1,000 \text{ m}^2/\text{m}^3$ for the traditional tube and shell configuration. The initial progress in this direction appeared in the form of perforated plates,^{190,191,192} followed by wire screens as demonstrated by Lins and Elkan,¹⁹³ and porous sintered materials, these all being characterized by higher surface area density. The technology of porous sintering, or powder metallurgy, for low temperature heat exchangers utilizes a variety of materials including stainless steel,¹⁹⁴ silver,^{195,196,197} and copper.^{198,199,200,201} Berg²⁰² compared the performance of sintered heat exchangers using silver or copper.

Efforts continue in order to improve the technologies for manufacturing the matrix itself, such as incorporating longitudinal thermal isolators and improving bonding techniques. Venkatarathnam²⁰³ provides a broad and detailed literature survey on the subject. Additional analytical works^{204,205,206,207,208,209} are also highly recommended for further reading.

One of the more convenient transitions from conventional to compact heat exchanger designs is to utilize a capillary tube as the high-pressure channel, and surround it with a matrix that efficiently increases the heat transfer area for the returning low pressure stream. In this approach, the matrix functions as a replacement of the fins along the pressure tube for extending the heat transfer area on the low-pressure side. However, the surface area in the matrix is significantly larger than that with fins.

Utilizing a matrix heat exchanger provides new possibilities and advantages as compared to the finned tube technology. For example:

1. Size reduction of the cryocooler, which is very beneficial for packaging and assembly of delicate mechanisms. Very often, the length itself is an important parameter,
2. Easy manufacturability into any desirable shape such as rectangular, flat disc, etc.,
3. Thermal mass reduction, since less material is required to obtain a similar heat transfer area, thereby enabling the possibility for fast cooldown applications,
4. Reduced hydrodynamic impedance for the returning stream,
5. Potential cost reduction, especially for mass production.

On the other hand, the matrix may induce a higher thermal leak due to its shorter length and higher longitudinal thermal conductance.

6.3.3.2 Wire Mesh Matrix

Wire screens are commercially manufactured in a remarkable variety of wave modes, geometries, meshes, wire sizes and materials.

Heat exchangers are fabricated from screens by cutting and stacking them in a column to form a porous matrix. In general, wire screen heat exchangers have the potential to provide a lower hydrodynamic impedance than a sintered porous matrix. Steyert and Stone,^{210,211} confirmed this expectation by conducting a comparative study of the two technologies of heat exchangers (though for current leads). In addition, the contact resistance between the multiple separate layers of wire screens has the effect of reducing the undesirable longitudinal thermal conductivity, thus reducing the ineffectiveness of such a heat exchanger.

These advantages may explain why Steyert and Longworth²¹² chose the wire screen technology to build their fast Joule-Thomson cryocoolers. They used a copper wire screen of 100 and 150 mesh, stacked in 100 and 150 layers, respectively, and arranged alternately in a parallel and a 45° orientation. Previous experience using copper wire screens was explored and reported elsewhere.^{213,214} The high-pressure tube was made of stainless steel, with outer and inner diameters respectively of 0.013" and 0.007" and with an orifice sized for a nitrogen flow rate of 2.5 standard liters per minute at 6.8 MPa. A schematic of their 0.13" diameter by 0.4" long cryocooler is shown in Fig. 6.23. The expanded low-pressure stream flows through the matrix. The thermal contact between the cylindrical stacked-screen matrix and the surrounding high-pressure helical tube provides the recuperative heat exchange.

Another version with a larger diameter (0.326") and longer length (0.6") incorporated a second annular column of stacked screens outside the coiled tube.

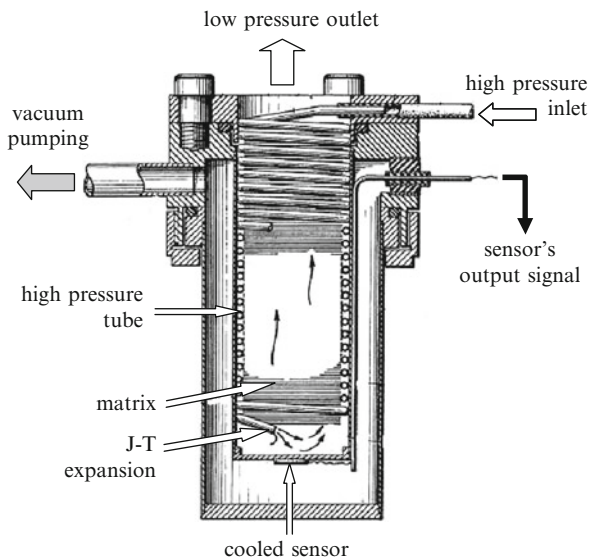


Fig. 6.23 Matrix heat exchanger as a stack of wire screens (Steyert and Longworth²¹²)

These cryocoolers were pressurized by 40 MPa of argon or nitrogen. The same reference²¹² describes two other configurations: a flat cryocooler and a two-stage cryocooler suitable for liquefying nitrogen, nitrogen/neon mixtures or argon, precooled by Freon. In addition, the cryocooler can be used with hydrogen or neon, precooled by air or argon. Longworth^{113,114} reports on the performance of various matrix cryocoolers with special attention to fast cooldown applications. In another report,²¹⁵ he utilizes the advantage of their compactness for a cryosurgical probe. An additional example of a compact construction is provided by Harada et al.²¹⁶

Wire screen heat exchangers are compatible with continuous flow regulators. Similarly, as shown by Longworth,¹¹³ a dual orifice type control mechanism with a cooldown valve may be installed in a matrix heat exchanger. However, the short length of this compact heat exchanger elevates the heat leak to such an extent that it is not attractive for continuous run applications with low gas consumption.

6.3.3.3 Porous Sintered Matrix

A second technology for creating matrix heat exchangers is the porous sintering of metals. In this option, powder-like, micron-scale metal spheres, tightly packed in a confined space and shape, are heated to a temperature near, but below their melting point in an inert atmosphere. An accelerated diffusion process, and in some cases even a partial melting, occurs at their interfaces, bonding the spheres and creating a rigid bulk of porous material.

A schematic layout of a sintered Joule-Thomson cryocooler was described by Chorowski, Bodio et al.²¹⁷ The helical capillary tube for high pressure is surrounded

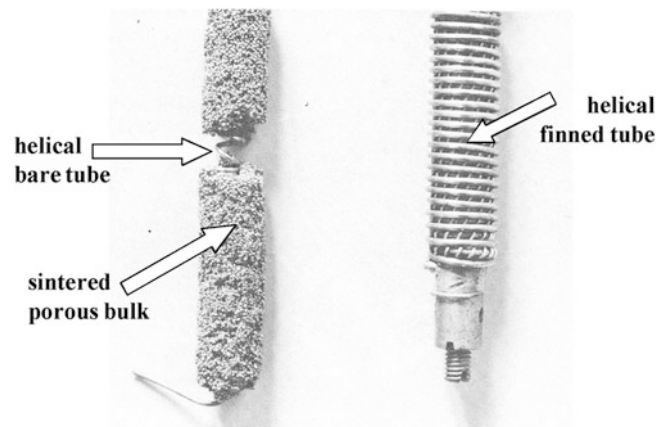


Fig. 6.24 A sintered cryocooler and a finned tube cryocooler, both 8 mm diameter (Chorowski et al.²¹⁷)

by the metallic powder, which in turn is vibrated for better packing, and then sintered together. The resulting porous bulk is metallurgically attached to the tube and replaces the fins. The low-pressure stream then flows through the multiple small passages in the porous plug. The typical porosity of the sintered block ranges between 50% and 80%. The outer shape of the cryocooler is defined by the die of the sintered powder; thus, there is a possibility of obtaining a round or even a rectangular cryocooler. Actually, the sintering die may take on any desired shape.

The hydrodynamics of porous media is ruled by Darcy's law,²¹⁸ which defines a linear relationship between the volumetric flow rate, $\dot{V}$, and the associated pressure drop, ΔP ,

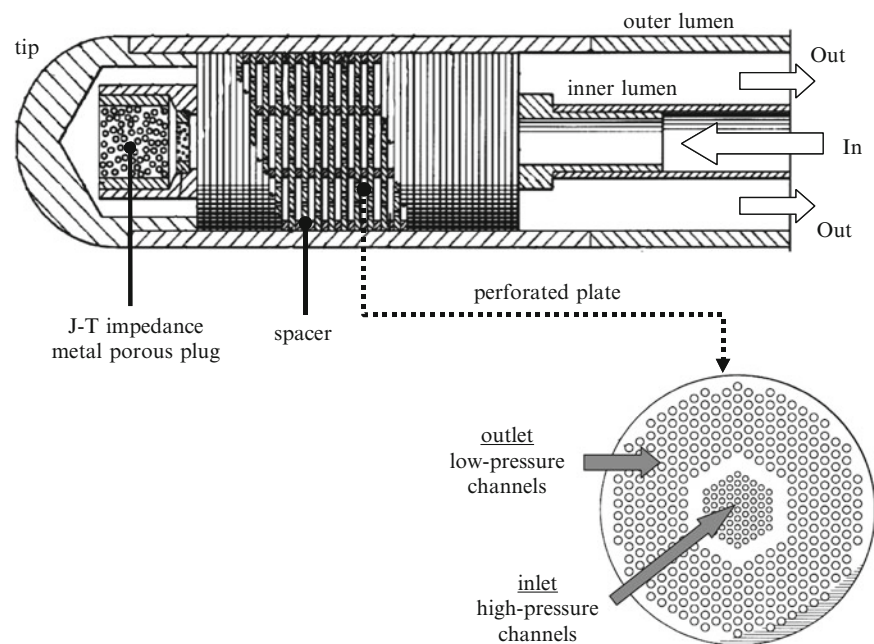
$$\frac{\dot{V}}{A} = \frac{k}{\mu} \cdot \frac{\Delta P}{l} \quad (6.9)$$

where A is the cross sectional area of the porous sample, perpendicular to the flow vector, and l is its length. The fluid is characterized by its viscosity, μ , and the porous media by its permeability, k . The corresponding relationship for compressible flow becomes more complicated, as discussed elsewhere.²¹⁸

The design of a porous matrix heat exchanger depends upon a variety of, at times competing, parameters. These include: thermal conductivity and diffusivity, strength and stiffness, and porosity and hydrodynamic impedance.

Bodio, Chorowski et al.^{217,219} in Wroclaw, Poland, built cryocoolers by sintering phosphor bronze powder, ranging in size from 0.4 to 0.8 mm, as shown in Fig. 6.24. The strength of the matrix was improved as compared to a copper version, however, the thermal conductivity and diffusivity were degraded. They used a high-pressure copper tube with respective external and internal diameters of 0.6 and 0.4 mm and a length of 400 mm to construct a heat exchanger with overall dimensions of 8 mm diameter and

Fig. 6.25 Perforated plate heat exchangers for Joule-Thomson cryocooler. The bonding of the plates provides the sealing between the high and low pressure streams (Dobak, Radebaugh, and Marquardt).²²⁴ It was implemented for a miniature catheter of 3 mm diameter³³



40 mm length. They recommend adding an amount of porophor, which is ceresine, to the metal powder in order to obtain the proper porosity.

Bodio and Chorowski et al. experimentally characterized the performance of their sintered heat exchangers with argon and nitrogen gas. However, the associated pressures did not exceed 16 MPa, which is low compared to common optimal values. Bragard, Chazot and Real^{220,221} of the French company L'Air Liquide, patented porous metal Joule-Thomson cryocoolers prepared via a special powder metallurgy technique. They sintered metal spheres coated with tin, which enabled them to reduce the sintering temperature. However, tin has poor thermal conductivity and diffusivity. Viennot¹⁸³ and the brochure¹¹¹ of the same company, presented additional sintered cylindrical and flat cryocoolers.

Pope⁹⁹ describes a fast two-stage cryocooler, utilizing a porous heat exchanger in the first stage, to liquefy krypton. The high-pressure pre-coolant tube is wound around the porous core (similar to the wire mesh screen design of Fig. 6.23) and bonded to it in order to provide good thermal contact. Nitrogen, at the final stage is recuperatively liquefied by a finned tube heat exchanger.

6.3.3.4 Perforated Plate Heat Exchanger

Perforated plate heat exchanger designs have benefited from the progress made in metal perforation techniques and in the bonding of conducting and isolating layers.^{222,223} Various reports describe attempts at building miniature Joule-Thomson heat exchangers based on this technology.

Compared to the traditional Hampson type heat exchanger, perforated plate exchangers utilize the entire available space more effectively, including the inner core space.

Marquardt, Radebaugh and Dobak^{33,224} developed a 3 mm diameter flexible cryogenic catheter for treating cardiac arrhythmias, by installing a miniature Joule-Thomson cryocooler at its tip. The cooler is fed by a 2.5 MPa nitrogen-hydrocarbon gas mixture operating in a closed cycle. A miniature heat exchanger was fabricated from a stack of diffusion bonded copper plates with interleaving layers of poor thermally conducting steel as shown in Fig. 6.25. The length of the prototype heat exchangers ranged from 5 to 15 mm. The same reference²²⁴ describes a variety of configurations of such stacks. In this application, the technology of bonded perforated plates is more than simply an alternative fabrication procedure, but rather a technique for miniaturization.

Luo, Zhou and Gong^{225,226} also built a perforated plate heat exchanger for a mixed refrigerant Joule-Thomson cryocooler. This embodiment is unique because of its use of a thin steel tube to separate the two perforated stacks; an inner one for the high pressure stream and the outer one for the low pressure return stream as shown in Fig. 6.26. The tube prevents any leakage from the high-pressure to the low pressure channel. Luo et al. compared its performance with a traditional Hampson cryocooler of the same flow rate, discharge pressure and overall dimensions: 14 mm diameter and 120 mm long. The perforated cooler reached a lower temperature (67.5 K instead 70 K) and developed a higher cooling power at 80 K, 2.4 W versus 2.0 W.

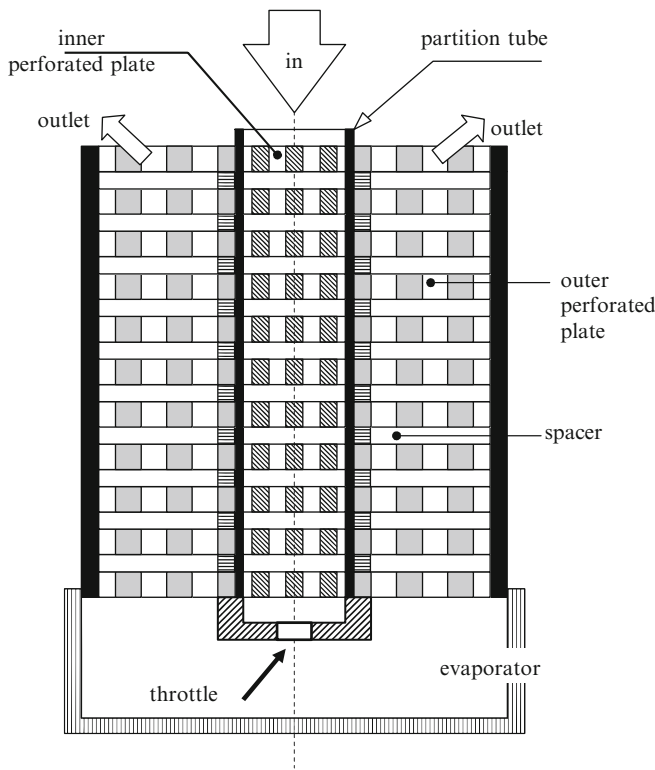


Fig. 6.26 Perforated plate heat exchangers for Joule-Thomson cryocooler. A thin metallic partition cylinder separates the high and low pressure streams (Luo, Zhou and Gong^{225,226})

6.3.4 Parkinson's Heat Exchanger

Another variation of a Hampson style heat exchanger was proposed by Parkinson.^{227,228} Parkinson wound several small-diameter, bare (finless) helical coils in parallel around a cylindrical core of larger diameter. Parkinson applied the design to a small scale liquefier for hydrogen and helium precooled by liquid nitrogen. Galuszewski et al.²²⁹ constructed two heat exchangers, one a Hampson type and the other a Parkinson type, both for the purpose of liquefying hydrogen (with a liquid nitrogen pre-cooling stage) and compared their performance.

Parkinson's version extends the length of the high-pressure tube but does not benefit from the extended surface of the fins. In addition, the potential pressure gradient along the high pressure tube might degrade the effectiveness of the heat exchanger.¹⁵⁴

Patents by Lennard²³⁰ in 1957, and Simon²³¹ in 1958 already defended the "coiled coil" configuration for a small size Joule-Thomson cryocooler. However, the early attempt to build a miniature Joule-Thomson cryocooler based on Parkinson's heat exchanger (using nitrogen as the working fluid) is reported by Webster¹⁶⁸ and by Bodio.²³² A 0.8 mm tube was wound onto a 3 mm coil in Bodio's cryocooler. Three such coils were wound around a 5 mm core.

He mentions the following advantages of the Parkinson heat exchanger: low heat capacity, low backpressure induced in the return flow channel, and simplicity of construction due to the absence of fins.

The interest in Parkinson's heat exchangers for Joule-Thomson cryocoolers was revived a few decades later by Chazot and Cotteareau,²³³ and with Guichard.⁸³ They adopted the principle of Parkinson's heat exchanger to reduce the cost of miniature Joule-Thomson cryocoolers for infrared detectors. They built a disc-like rapid cooldown cryocooler, similar to the arrangement described in paragraph 6.3.2.2(4). The coil was helically wound in a spiral channel formed in a flat disc of about 20 mm diameter and 6 mm thick. It was pressurized by a 55 cm³ argon vessel charged to 75 MPa and was able to reach 120 K within 2.7 s.

6.3.5 Linde Type Heat Exchanger

6.3.5.1 The Tube in Tube Heat Exchanger

The tube in tube heat exchanger was originally developed by Linde himself, for his gas liquefying machine^{145,146,147} as described in Scurlock's review on the history of cryogenics.²³⁴ The heat exchanger is comprised of two concentric tubes with the inner one carrying the high pressure stream, and an orifice at its end. Gas expands into the annular space between the two tubes. As shown in Fig. 6.15, the low-pressure stream flows counter-currently around and directly cools the high-pressure tube without any fins. In 1959, at the very early stage of developing miniature Joule-Thomson cryocoolers, Frame and Coles²³⁵ proposed a coiled Linde type heat exchanger.

The two concentric tubes are co-wound into a coil of diameter D and located inside an evacuated vessel. The helical configuration enhances the heat transfer process in comparison to a straight tube¹⁴⁹ by the factor,

$$\frac{h(\text{curved})}{h(\text{straight})} = 1 + 3.6 \cdot \left(1 - \frac{d}{D}\right) \left(\frac{d}{D}\right)^{0.8} \quad (6.10)$$

where d is the diameter of the tube and h represents the heat transfer coefficient. On the other hand, the helical configuration elevates the pressure drop along a curved tube by the factor,

$$\frac{\Delta P(\text{curved})}{\Delta P(\text{straight})} = 1 + 0.0823 \left(1 + \frac{d}{D}\right) \left(\frac{d}{D}\right)^{0.53} \text{Re}^{0.25} \quad (6.11)$$

Curie²³⁶ used a Linde type heat exchanger in a three stage helium liquefier. Steyert³⁶ also constructed a complete Joule-Thomson cryocooler with its dewar utilizing such a heat exchanger. The inner and outer diameters of the

high-pressure tube were respectively 0.24 mm (0.015") and 0.56 mm (0.022"), and of the low-pressure tube respectively 0.75 mm (0.03") and 1.00 mm (0.04"). Bradshaw and Orłowska²³⁷ also used the same concept.

The Linde type heat exchanger may also be classified as a multi tube heat exchanger, as reviewed by Timmerhaus and Flynn.²³⁸ In this respect, the outer low pressure tube accommodates more than a single high-pressure tube. For example, Alexeev et al.²³⁹ used a multi tube heat exchanger for a closed cycle mixed refrigerant Joule-Thomson cryocooler.

Glass tube-in-tube heat exchangers with an outer diameter of 0.67 mm are discussed later in paragraph 6.4.5.1.

6.3.5.2 The Parallel Wrapped Tube Type Heat Exchanger

The parallel wrapped tube type heat exchanger is a modification of Linde's idea as constructed by Longworth.^{156,240,241,242} This is indeed an exact embodiment of a Linde type heat exchanger in the sense that the two parallel tubes carry the low and high-pressure streams. However, in this case the tubes exist side by side, with external thermal contact, instead of in a concentric configuration. The arrangement comes in two versions; in the first one, the high-pressure tube is sandwiched between two low-pressure tubes, and in the second one, the high-pressure tube is helically wrapped around a bundle of low pressure tubes. The paired tubes in both cases are flattened to improve the thermal contact between the tubes.

These heat exchangers have been primarily developed for use in helium based Joule-Thomson cryocoolers that are precooled by multi-staged Gifford-McMahon cryocoolers. To compensate for the changes in density and other properties of helium as its temperature decreases, the cross sections of both the high and low-pressure flow channels may be reduced in a gradual manner towards the lower temperatures.

6.3.5.3 Narrow Channel Heat Exchangers in Diffusion Bonded Metal Plates

Mikulin et al.²⁴³ constructed adjacent narrow channels as grooves in a flat metallic body to function as a planar Joule-Thomson heat exchanger. The heat exchangers were fabricated using photolithographic masking and chemical etching techniques on stainless steel plates. After coating the stainless steel surfaces with micro-layers of nickel or copper, the plates are aligned and hermetically sealed by diffusion welding in a vacuum. The plane rectangular cryocooler includes two steel plates, each 0.3 mm thick, sandwiching a 0.2 mm thick copper plate. The high-pressure zigzag duct, etched in the steel plates is rectangular in cross section, 0.65 mm wide, 0.13 mm deep and 20 cm long. The low-pressure duct comprises two parallel zigzag channels,

2.25 mm wide and 0.12 mm deep. A smaller rectangular channel, 2 mm long, serves as a Joule-Thomson restriction. However, its performance is not too encouraging. With a flow rate of about 5 standard liters per minute of nitrogen, fed at a pressure of 18–22 MPa, the cooler did not reach a temperature lower than 100 K within 60–80 s. The warm end temperature difference was as high as 18–20 K. The associated effectiveness is approximately $\varepsilon = 0.92$, while the minimum necessary value for liquefaction is $\varepsilon^{MIN} = 0.86$. Hence, the efficiency, η , of such heat exchanger does not exceed 0.4.

Another attempt with a round flat (disc-like) cryocooler incorporates a similar approach. The incoming channel is fashioned in the shape of an Archimedean spiral groove with a 1.24 mm pitch. Its rectangular cross section is 0.5 mm wide by 0.135 mm deep, and the total length of the groove is 62.5 cm. A restrictive outlet is comprised of a 35.5 mm long capillary in the center of the disc, with a cross section of 0.2 mm wide by 0.12 mm deep. The low-pressure stream passes through the dividing plate into another plane and undergoes a radial expansion through a fine steel wire screen: 0.18 mm diameter and 0.45 mm cells.

Mikulin et al. do not address the inherently high thermal loads originating through the highly conducting metal plates. However, these may explain the low effectiveness and a long cooldown period.

6.3.5.4 Hampson's Versus Linde's Heat Exchangers

Most of the operating and proposed miniature Joule-Thomson coolers are based on Hampson's heat exchanger. However, the Linde type does provide several advantages. The following is a list of compared features:

Hampson's: Strongly Coupled with Its Dewar

The core mandrel and the cold finger of the dewar together comprise the annular channel of the return flow stream. Therefore the dewar is constrained to fit tightly to the size and shape (conic or cylindrical) of the heat exchanger. The Linde type enables large structural flexibility.

Linde's: More Readily Adaptable for Hybrid Precooling

Linde's: more readily adaptable for hybrid precooling, especially as precoolers for Gifford-McMahon or Stirling coolers. The coil is simply wound around the cylindrical face of the precooling stage.

Linde's: Potentially Provides a Lower Heat Leak

In a Hampson style heat exchanger, both the core mandrel and the cold finger of the dewar are much shorter than the corresponding tubes of a Linde style heat exchanger, thereby inducing a higher thermal leak.

Hampson's: Enables a Simpler Flow Adjustment

It is more complicated to install a flow adjusting mechanism in a Linde type recuperator, while in a Hampson type, the core mandrel is very suitable for positioning the regulator at the cold end.

Hampson's: Potentially More Compact

6.4 Mems Cryocoolers

6.4.1 New Emerging Opportunities

The technology used for building Micro Electro Mechanical Systems (MEMS) is also being applied to Joule-Thomson cryocoolers. Various aspects of size reduction for micro scale cryocoolers was discussed elsewhere.^{244,245,246} Such Joule-Thomson cryocoolers are characterized by the following possible features:

6.4.1.1 Size Reduction, in Terms of Length and Volume

Size reduction, in terms of length and volume for miniature and low capacity cryocoolers, especially below 2 W,

6.4.1.2 Cooling Capacity Below 20 mW

Cooling capacity below 20 mW, a feature that would be difficult to realize by scaling down traditional technologies used to construct heat exchangers,

6.4.1.3 Flat and Rectangular Shape

Flat and rectangular shape, instead of cylindrical,

6.4.1.4 Advantages for Integrating

Advantages for integrating the cryocooler with cooled electronics objects, especially valuable for the high flux values associated with very large scale integration elements (VLSI) circuits,

6.4.1.5 Cost Reduction

Cost reduction due to mass production techniques,

6.4.1.6 Fixed Orifice

To date there are no reports of attempts to incorporate a flow regulating mechanism in a MEMS-scale cooler.

6.4.2 Glass Versus Silicon

MEMS technology routinely employs thin wafers of glass or silicon. In MEMS scale Joule-Thomson coolers, the recuperator is typically fashioned from two parallel channels of about 10–50 μm height and various widths. Thus, they

may be classified as the Linde type heat-exchangers discussed in Sect. 6.3.5. The recuperator channels are fabricated as etched grooves on one or both sides of the glass or silicon plate and carry the high and low pressure counter flowing streams. The wall separating the channels can be as thin as 100–250 μm , a feature that is very significant especially for glass wafers. It enables an acceptable thermal conductance between the channels in spite of the low thermal conductivity of glass, $\sim 1 \text{ W}/(\text{mK})$. The low thermal conductivity of glass does provide the advantage of reducing the longitudinal heat leak.

The thermal conductivity of silicon is 3 orders of magnitude larger than that of glass. Therefore the magnitude of heat exchanged between the etched silicon channels is larger. However, it is accompanied by large longitudinal conduction, frequently resulting in an unacceptably large heat leak. As demonstrated below, the recuperator design requires careful consideration. Zhu et al.^{247,248} combined the advantages of both materials by bonding the silicon plate to a glass plate spacer; The approach enables an architecture of perforated plates such that the transverse thermal conductance between the streams is like that of silicon and the longitudinal conduction is like that of glass.

The MEMS-scale heat exchangers make up their own category of compact heat exchangers. The narrow channels reduce the flow rate to the point that it becomes laminar. However, because the surface area to volume ratio increases as the channel dimensions decrease, the size reduction has an effect similar to extending the heat transfer surface area per unit of mass flow rate.

6.4.3 Superconducting Electronics: Stanford University, CA, USA

6.4.3.1 William Little

William Little, professor of physics at Stanford University, pioneered the development of miniature Joule-Thomson cryocoolers in the 1980s. Building upon the prior experience of Terry,²⁴⁹ also of Stanford University, who built a gas chromatograph on a silicon wafer, Professor Little used the chemical etching and photolithographic techniques, further reported elsewhere,^{250,251,252} to etch channels in silicon wafers sealed in a Pyrex enclosure. Little adapted the idea to develop a micro heat exchanger with a Joule-Thomson expansion valve. Some of the earlier experimental cryocoolers investigated by Hollman during his Ph.D. study²⁵³ were operated with carbon dioxide and ethylene. These relatively high temperature cryocoolers accommodated the use of silicon substrates, in spite of their high thermal conductivity. However, cooling down to liquid nitrogen temperature necessitated the replacement of silicon with glass plates.

Fig. 6.27 Typical layout of a single stage, single layer, MEMS technology, narrow channels Joule-Thomson cryocooler. The throttle itself is constructed as a narrow channel, capillary following the idea of Little.²⁵⁷

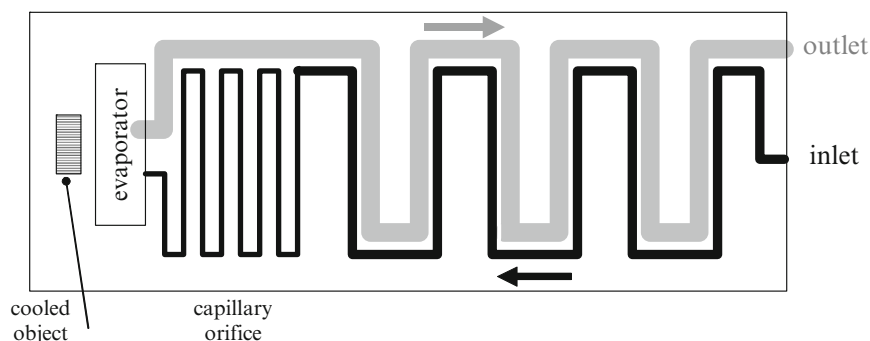
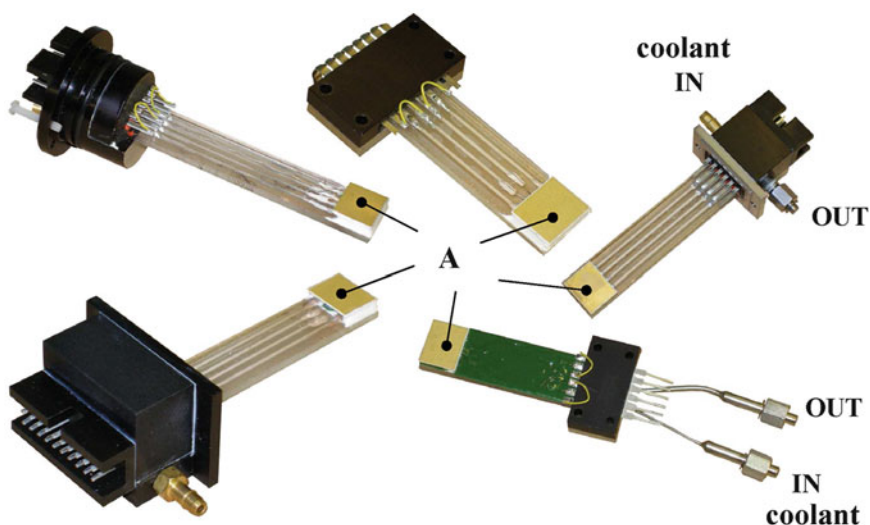


Fig. 6.28 An integrated narrow channel flat cryocooler built by MEMS technology (Little^{257,267}). The cooled objects are marked by A. The typical length of a flat heat exchanger is about 30 mm (MMR Technologies, Mountain View, CA, USA)



Little's fabrication approach utilized photolithographic masking techniques and fine particle sandblasting, an abrasive etching process, to produce the grooves in the glass and silicon plates. He formulated various scaling relations for constructing micro-miniature Joule-Thomson cryocoolers²⁵⁴ and introduced design considerations for laminar-flow narrow-channel heat exchangers.²⁵⁵ Little founded the company MMR (MicroMiniature Refrigerators) Technologies, Inc. in Mountain View, California, which manufactures a variety of products based on this technology; some more ideas as reviewed elsewhere.²⁵⁶

6.4.3.2 Single Stage Narrow Channel Devices

A typical layout of Little's single stage cooler²⁵⁷ is described in Fig. 6.27. A pair of high and low pressure heat exchanger channels run parallel in a zigzag pattern to obtain a longer pathlength for a given volume. A typical cross section is about 250 μm wide by 50 μm deep. The Joule-Thomson expansion valve is created from a long shallow capillary groove, 10 μm deep by 125 μm wide. The evaporator, in the same figure, marks the location of the low temperature bath adjacent to the cooled object. The grooves in the etched plate

are covered by another plate, thus creating the narrow channel. The plates are bonded to withstand pressures up to 20 MPa.

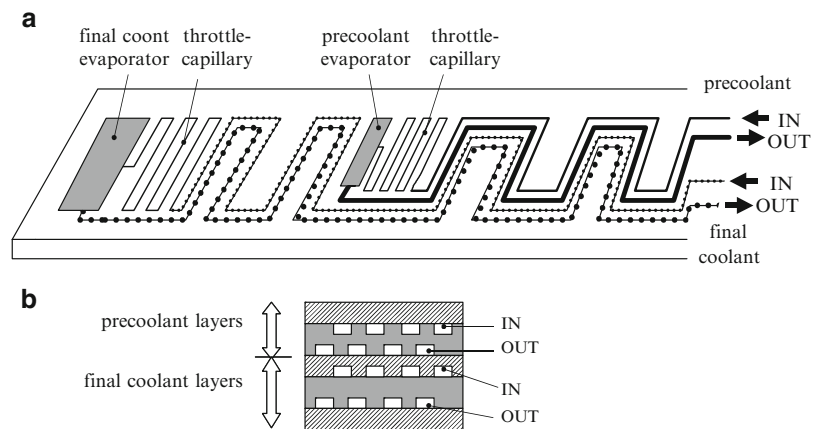
An alternative layout with a multiplicity of parallel straight channels employs a cover plate over traverse thermally conducting strips,²⁵⁷ which enhance the recuperative heat transfer process.

The natural configuration of the etched-plate cryocoolers is rectangular in cross section, in contrast to the cylindrical shape of finned tube and other heat exchangers described earlier in this chapter. Some final integrated products are shown in Fig. 6.28. To maintain a low heat leak, the assembled cooler is installed in an evacuated environment.²⁵⁸ Note that it may also be easily shaped in the form of a round disc.²⁵⁹

A variety of military and civil applications, design considerations and additional constructive remarks regarding Little's Joule-Thomson coolers may be found elsewhere.^{260,261,262,263,264,265,266,267,268,269,270,271}

Analyses and experimental investigation of heat transfer coefficients and friction factors within these narrow channel heat exchangers, accomplished by Little and Peiyi, are reported elsewhere.^{272,273,274}

Fig. 6.29 Typical layout of a double stage Joule-Thomson narrow channel heat exchanger; the precooling interaction occurring in a single layer (a), or across two separate layers (b)



6.4.3.3 Multi-Staging

Multi-staging the narrow channel heat exchangers may be accomplished in two manners.

Common Layer Strategy

In this approach the channels of the final and precooling stages are etched into the same plate. As displayed in Fig. 6.29a the precooling stream (nitrogen) runs close to the inlet stream of the final stage and in the same layer. The counter-flow heat exchanger lowers the temperature of the final coolant (hydrogen or neon).

Separate Layers for Each Stage

As displayed in Fig. 6.29b the other configuration locates the two stages on separate, but adjacent layers: one layer for the inlet and return stream of nitrogen, and the other layer for the two streams of hydrogen. Multiple examples have been constructed and reported using nitrogen in the final stage, precooled by a hydrocarbon gas mixture,²⁷⁵ liquefying neon in the, final stage, precooled by nitrogen,²⁷⁶ and nitrogen in the final stage precooled by nitrogen or a nitrogen-hydrocarbon mixture.²⁷⁷ The flow rate of the precooling stream is increased to enable significant reduction in final stage flow thus lowering the back pressure and associated boiling point.

6.4.4 Missile Application: Segmented and Isolated Silicon Layers

The configuration displayed in Fig. 6.30 represents a strategy for optimizing heat exchange between the channels, which is beneficial for fast cooldown applications, while at the same time avoiding the elevated heat leak associated

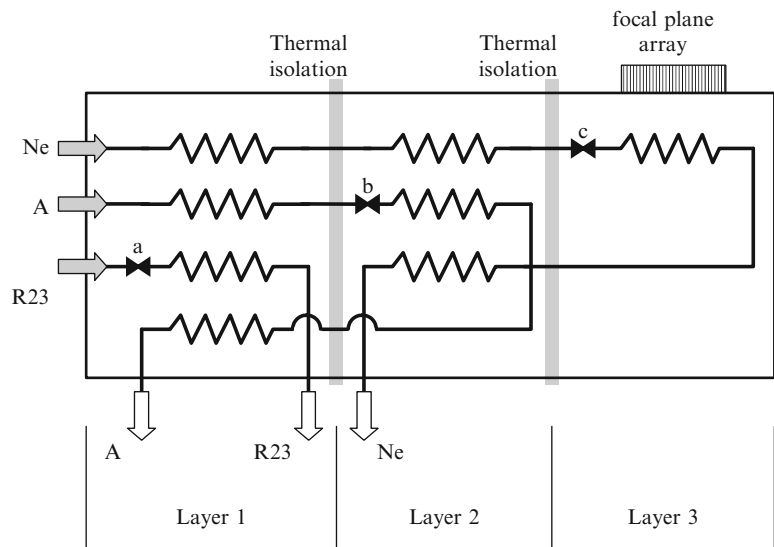
with the high thermal conductivity of silicon. The objective is achieved by inserting a thermal isolation layer between stacked silicon layers. The idea was patented in 1995 by Hsu, Wu, and Ombrose^{278,279,280} of Lockheed-Martin Inc. to develop double and triple-stage crycoolers for the rapid cooldown of an infrared focal plane array detector. However, the same principle may be adapted for a single stage cooler as well. As shown in Fig. 6.30, each layer accommodates segments from all the stages, rather than being devoted to a single state. Figure 6.30 displays the specific example used for rapid liquefaction of neon (or hydrogen) by a three stage cascade with a cold end temperature range between 20 and 40 K. One may observe that the inlet stream of neon passes through all layers. The auxiliary precoolants are R23 and argon.

The depth of the channels is 100 μm and the width ranges from 1.2 to 1.6 mm. The wall separating the channels is 150 μm thick, while their length depends on the layout, whether straight and parallel or a long serpentine. The occupied area by any layout is a square of approximately 25 mm on a side.

A rectangular channel of 65 by 115 μm cross section and 5.4 mm long serves as a Joule-Thomson valve. Thermal isolation between the silicon layers is achieved through the use of short rods of about 2 mm diameter and made of G10 fiberglass, PTFE, or epoxy.

Such a two stage MEMS cryocooler was used to cool an infrared HgCdTe focal plane array operating at 95 K and a long cold shield cooled to about 200 K. Cooldown was accomplished within 15 s. A 75 cm^3 reservoir of argon and a 50 cm^3 reservoir of R23, both pressurized to 60 MPa supplied gas to the cooler. These volumes provided cooling for about 40 s after the cooldown was complete. Measurements with this kind micro heat exchanger are discussed elsewhere.²⁸¹

Fig. 6.30 Triple stage narrow channel Joule-Thomson cryocooler for liquefaction of neon (Hsu, Wu and Ombrose in references^{278,279,280}). Each silicon plate carries only a segment of any stream or stage. Plates are thermally isolated. The *a*, *b*, and *c* represent Joule-Thomson valves



6.4.5 Space Applications: Twente University, The Netherlands

A project initiated during the late 1990s by ter Brake, Burger, Rogalla, Lerou, Holland, Jansen, Elven-spook^{282,283,284,285,286,287} utilizes MEMS-based cryocoolers for space applications. As an important part of the project they also developed a small, highly reliable sorption compressor to simultaneously feed multiple cryocoolers and sensors remote from the compressor.²⁸⁸

6.4.5.1 Concentric Glass Tube Heat Exchanger

In 1998 Holland et al.⁴⁹ constructed a unique nitrogen cryocooler with a cooling capacity of 10–150 mW by using two straight concentric tubes of fused silica glass, 105 and 270 mm long (for smaller and larger flow rates) as a Linde type heat exchanger. The inner/outer diameters of the first tube carrying the high pressure stream (7–10 MPa) are 0.10/0.36 mm respectively while the diameters of the other tube carrying the exhausted stream were 0.53/0.67 mm respectively. The heat exchanger was previously displayed in Fig. 6.8 in the context of its expansion valves. The low thermal conductivity of glass minimizes the longitudinal heat leak. Wiegerinck²⁸⁹ built a similar cryocooler, 150 mm long, with the same outer tube, but incorporating an inner tube with inner/outer diameter of 0.25/0.36 mm, and capable of withstanding a supply pressure of 5–10 MPa. By immersing part of the outer tube in a precooling bath of liquid nitrogen, this cryocooler was able to liquefy neon, thereby reaching 30 K.

6.4.5.2 Silicon Wafer Heat Exchanger for Two Phase Streams ($P_U < P_C$)

In the early 2000s Burger, Holland et al.^{290,291} developed a cryocooler with a 200 mW capacity at 165 K. The cryocooler

utilizes a small sorption compressor^{292,293} pressurizing xenon or ethylene as the working fluid from 0.1 MPa up to 2.0 MPa and producing a flow rate of approximately 0.5 mg/s. Since the high-pressure stream is below the critical pressure (see a sub-critical methane cryocooler in Sect. 5.6.6.1), and precooled by a thermoelectric cooler, the colder section of this stream exits as a two-phase flow, gradually condensing the entire stream at the saturation temperature of 244 K (corresponding to 2 MPa). The returning low pressure stream includes two phases in its colder section at 169 K (for ethylene). Subcritical JT cryocoolers with condensation inside a Linde type recuperator are analyzed elsewhere.²⁹⁴

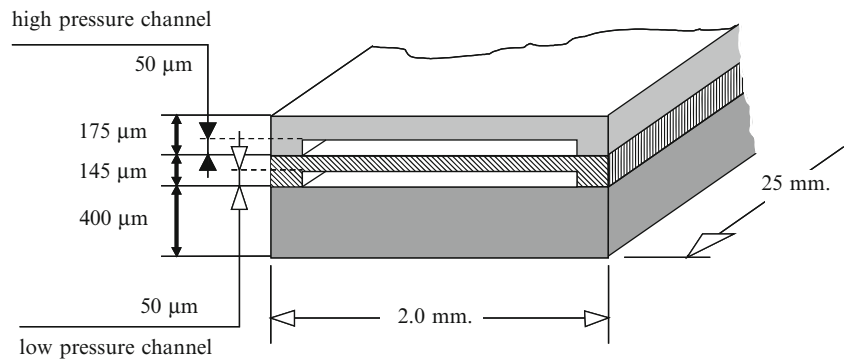
The recuperator of this cryocooler is constructed of straight concentric glass tubes as described in the above paragraph. However, it also incorporates two sections made of silicon, an intermediate section as a condenser and the evaporator at the cold end. Each of the silicon sections has one stream with a phase change at constant temperature (169 and 244 K). The high-pressure channel of the condenser follows a serpentine path, similar to that of Little's etched channel regenerators, as described in the previous section, while the low-pressure channel has a larger cross section and includes a few straight sections.

The Joule-Thomson expansion valve is defined by a 1 μ m shallow channel, 4 mm wide and 3 mm long located at the inlet of the evaporator section. It was etched into the same silicon section that also included the evaporator that absorbs heat from the cooled object.

6.4.5.3 Optimized Wide Channels

Additional progress by the Twente University group was reviewed²⁹⁵ in 2004 and advanced by Lerou et al.^{135,296,297} The more recent effort targets a nitrogen cooling system,

Fig. 6.31 Optimized size of the heat exchanging micro channel etched in a wafer of glass for minimum generation of entropy²⁹⁶



providing 10–25 mW of cooling at 96 K as characterized elsewhere.²⁹⁸ The system pressures range from 8 to 0.6 MPa. To compensate for increased heat leak at the lower operating temperatures, the Twente group optimized the size of the micro-channel heat exchangers etched in a wafer of glass, in order to minimize entropy generation. The resulting design provides a straight and relatively short (25 mm) channel that is 2.0 mm wide. As displayed in Fig. 6.31 it is made of three wafers. The depth of each channel is 50 μm . The wall separating the channels is only 95 μm thick. Structural pillars were added in the channels to reduce stresses; 200 μm diameter in the high pressure channel and 50 μm diameter in the low pressure channel. These optimized sizes are very similar to those used by Hsu, Wu, and Ombrose at Lockheed-Martin Inc.^{278,279}

A shallow channel of 0.3 μm height, 83 μm length and 1 mm width serves as an expansion valve. Fabrication was based on photolithographic masking techniques but also employed wet chemical etching, using hydrogen fluoride (HF) or KOH.

Cao et al.²⁹⁹ optimized the staging of a hydrogen-nitrogen microcryocooler for 28 K.

6.4.6 On Chip Cooling of Terahertz Sensor: NIST/CU Program

Terahertz radiation of about 1 mm wavelength penetrates through clothing and skin, and may therefore be used to detect nonmetallic hidden weapons and to spectroscopically identify chemical and biological materials.³⁰⁰ In January 2006, NIST and the University of Colorado (CU), both in Boulder, CO, initiated a collaborative effort to develop a micro scale closed cycle, autonomous, cryocooler; the university focused on the micro compressor.

6.4.6.1 The System

The system^{301,302,303} is expected to provide 9 mW of gross cooling, with about one third of that amount cooling the detector at approximately 80 K, and consuming less than

280 mW of input power. It incorporates a mixed refrigerant Joule-Thomson closed cycle, precooled to 240 K by a thermoelectric element. The mixed refrigerant flow rate is approximately 6 $\mu\text{mole/s}$, (or approximately 0.15 Standard cm^3/s) which is less than 0.01 SLPM. Its specific cooling capacity is 1.43 kJ/mole. The entire system of cryocooler, sensor and compressor is targeted to occupy less than 10 cm^3 . The same cooler was employed in another system for 200 K.^{304,305}

6.4.6.2 The Cryocooler

The cryocooler,^{306,307} in its initial embodiment, includes a 2 cm long Linde type recuperator. The inlet, or high pressure, flow channel is comprised of six capillary glass tubes, each with an inner and outer diameter of 75 and 125 μm . These tubes are installed in a single larger tube of glass, with inner and outer diameters of 536 and 617 μm respectively. The manifolds at each end are prepared using MEMS techniques.

6.4.6.3 The Compressor

The compressor³⁰⁸ has a positive displacement of only 2 mm^3 , and is driven by a piezoelectric stack at a frequency of 100 Hz, with a stroke of 28 μm . The device produces a compression ratio somewhere between 16:1 and 25:1, and operates typically in the range of 0.1–1.6 MPa. Another piezoelectric compressor^{304,305} was built to feed a 200 K system with a compression ratio of 4:1.

6.4.7 Miscellaneous

Pradeep, Narayanan, and Venkatarathnam³⁰⁹ analyzed the performance of miniature heat exchangers with micro scale narrow channels.

6.4.7.1 Micro-Size Heat Pipes

Micro-size heat pipes, installed between the counter flow channels, enhance the recuperative heat exchange through glass, as described by Musinski.^{310,311} The heat exchanger

arrangement includes an intermediate plate with a plurality of hollow spherical shells filled with a volatile fluid.

6.4.7.2 A Cryosurgical Probe

A cryosurgical probe based on MEMS technology was developed as a joint effort of the University of Wisconsin-Madison, and the University of Michigan. Hoch, Nellis, Zhu, et al.^{312,313} and White, Nellis, Zhu, et al.^{314,315} describe the development of a MEMS micro heat exchanger for cryosurgical applications. The heat exchanger is constructed from a stack of perforated plates that are fabricated using MEMS techniques. For use in a closed cycle, the mixed gas Joule-Thomson cooler must withstand a pressure of 1.4 MPa.

6.4.7.3 A Radio-Frequency Coil

A radio-frequency coil, used as a detector in an imaging system, was proposed³¹⁶ to be cooled down to about 80 K by a flat micro-miniature Joule-Thomson cryocooler comprised of laminated glass plates. Such a coil has been demonstrated³¹⁷ by Wright et al.

6.5 Accessories and Special Arrangements

Here we briefly discuss a few additional elements that are often employed in the practical construction and operation of cryocoolers. These accessories are not crucial for liquefying cryogenic gases, but still improve reliability and refine the performance.

6.5.1 Filtration

6.5.1.1 A Filter at the Warm End

A filter at the warm end is usually employed in every cryocooler. Often, the very fine mesh filters are made of porous metal or packed wire screens and are located at the entrance to the cryocooler as one may observe elsewhere.^{13,73,93} These prevent solid particles as small as 5 μm ¹⁵⁵ or even 3 μm ³¹⁸ from reaching and blocking the delicate fixed orifice. This filter also makes a significant contribution to the long-term reliable operation of a flow regulated cryocooler. Since minute regulated flow rates are typically involved, the effective size of the adjustable orifice can be very small indeed. Therefore, even minute particles can disturb the operation of the flow regulator and result in excessive flow rates.

6.5.1.2 A Filter at the Cold End

A filter at the cold end is less common in practice, but when used, is located as close as possible to the expansion orifice.^{319,320,321,322} Its location at the cold end enables the filter to capture frozen particles of gaseous contamination (like

carbon dioxide or water), thereby avoiding clog formation at the orifice itself. In addition, such a location provides the last opportunity for stopping any particles that may be released from the inner surface of the pressure tube.

6.5.2 Enhancing the Cooling Effect of a Cryogen Bath

6.5.2.1 Liquid Absorbent Materials

Liquid absorbent materials are often located inside the evaporator and adjacent to the substrate that carries the cooled object in order to keep the liquid cryogen close to the cooled object. The arrangement avoids direct impingement of the two-phase expanding jet on the cooled object, and maintains a permanent presence of liquid in intimate relation with that object. Kunimoto³²³ was able to attenuate cold end temperature fluctuations through this approach. The liquid absorbent material may be a low density felt³²³ or rolled blotter paper.²¹ Others have proposed supporting the absorbent material with a disc of wire gauze.^{320,321} An additional embodiment of the same idea uses a porous body of sintered metallic gauze.³²²

6.5.2.2 Fluid Deflection Using a Skirt

As proposed by Stabbs,³²⁴ a skirt can be used to shield a portion of the evaporator close to the border of the cooled object, from jet impingement of the expanded fluid. As a result, a larger amount of the liquid coolant is collected and brought into contact with the cooled object. This approach increases cooling rates, and provides more spatially and temporally uniform temperatures.

6.5.3 Controlling the Outlet Pressure

6.5.3.1 An Ejector

An ejector is occasionally employed to reach a lower temperature of operation. In such an approach, part of the refrigerant stream is shunted from the pressure supply to an ejector, which induces a suction effect at the gas exit, as proposed by Nicholds.³²⁵ As a result, the saturation pressure of the coolant is lowered below the ambient pressure, and the associated boiling point decreases. However, the pumping effect consumes additional mass flow. Little²⁷⁵ utilized the same idea, referring to it as a Venturi pump. He produced the effect through the use of three glass laminates, each 0.25 mm thick. The throat of the Laval nozzle was 0.28 mm and accelerated the flow to a Mach number of 3.18.

Ejectors are typically utilized in this fashion to satisfy stringent requirements for fast cooldown. The approach can significantly shorten the time required to reach a desired

temperature close to the normal boiling point, noting that it is otherwise approached in a slow, asymptotic fashion.

6.5.3.2 Active Servo Control of the Expanded Pressure

Active servo control of the expanded pressure is utilized in the confined space where the expanded gas is released. In this case, the pressure is measured and compared to a target value. Variations in the pressure are minimized by changing the rate of venting from the confined volume, as proposed by Bard et al.^{326,327} or the rate of pumping as first applied in Joule-Thomson refrigeration schemes for subcooled helium II by Claude.³²⁸ The approach provides an “ultra-high temperature stability Joule-Thomson cooler” even if the ambient pressure does vary.

6.5.3.3 An Absolute Pressure Controller

An absolute pressure controller for regulating the plenum pressure was proposed by Mangano et al.^{329,330} It is a significantly simpler device than the previous example of the active servo controller but less accurate and to some extent susceptible to the external atmospheric conditions. However, it provides an adequate solution for many applications.

6.5.4 Collecting the Outgoing Gas

In general, the outgoing coolant (of an open cycle) is released into the atmosphere, all around the opening of the recuperator’s low-pressure channel. Hedegard^{55,56} and Walker^{53,54} constructed a cooler with an exit port for the low pressure coolant exhaust. An o-ring sealing the cooler’s flange against the dewar enables the expanded gas to be collected, as shown in Fig. 6.32.

Motivation for such a collection feature may arise from the following factors:

- (a) Safety issues associated with a coolant’s flammability or toxicity,
- (b) Incompatibility of a coolant exhaust stream with nearby elements,
- (c) Closed cycle operation requiring a means to conduct the expanded coolant back to the compressor (regardless of its type: sorption, mechanical etc.). Chapter 8 broadly discusses the most common type of closed cycle operation employing mixed coolants and a mechanical compressor.

Alvarez³³¹ reports on the successful operation of such a cryocooler in a closed cycle Joule-Thomson system using a sorption compressor.

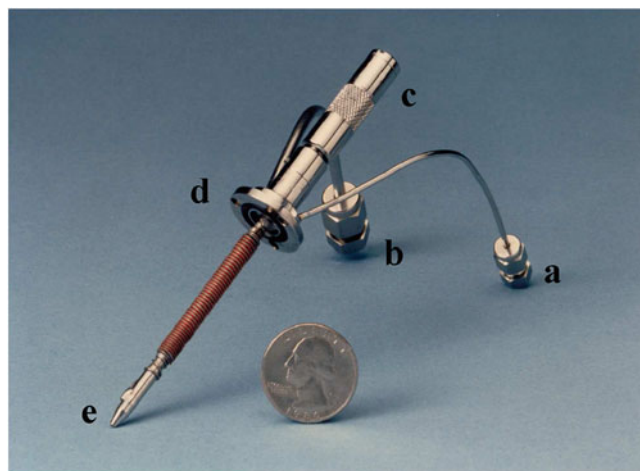


Fig. 6.32 A Joule-Thomson cryocooler of General Pneumatic Co., Scottsdale, AZ., USA; (a) inlet port, (b) outlet port, (c) micrometer for adjusting the flow rate at the warm end, (d) a flange that seals the exhausted stream with an o-ring, and (e) the throttle of two concentric conic surfaces (details in Fig. 6.9) (Walker et al.^{53,54,55,56,57,58,59})

6.5.5 Matching a Cooled Object’s Shape

6.5.5.1 Cooling a Rectangle

In order to cool a rectangular shape, for example a large sensor array, Stabbs³³² proposed employing a set of two adjacent cryocoolers sharing the same outer “cold finger” and sharing the same skirt (in the sense discussed in Sect. 6.5.2.2). In this manner, the entire object is cooled by a bath of cryogen. Alternative cooling options, for example by a single round cold finger of a Stirling cryocooler, require conductively cooling the object through a mediating element, and can be less effective than by a boiling bath of cryogen.

6.5.5.2 Cooling an Annular Payload

Kline and Ross³³³ addressed the challenge of cryocooling infrared up-converters, that is “solid state electronic arrays that receive an infrared image on one surface and emit a corresponding image at a selected wavelength from another surface”. They intended to cool a disc-shaped up-converter, but only at its peripheral annulus thereby staying out of the way of the incoming and outgoing radiation. For this purpose, they describe a Joule-Thomson cryocooler in the shape of a hollow pipe. Helical grooves on the inside wall of the tube comprise the recuperator, leaving a “clear aperture cryostat for an infrared detector”. Similar experience is described elsewhere.^{182,334}

6.5.6 Miscellaneous

Typically the exhaust gas leaves a Joule-Thomson cryocooler at a temperature lower than ambient by about 5–10 K. Hingst³³⁵ utilized this potential for cooling electronic elements mounted on a heat exchanger and located adjacent to the exhaust flow of the expanded gas. The exhaust stream flowing through the heat exchanger cooled the electronic elements.

Wayte et al.³³⁶ used a JT cryocooler, fed by oxygen at 16 MPa to cool a specimen for a scanning electron microscope. Lennad²³⁰ shows how few cryocoolers may be arranged to cool a single object (“electron discharge device”), simultaneously fed by the same pressure source.

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7.1 Introduction

The transient behavior of Joule-Thomson cryocoolers has received less attention than their steady state performance, and its description occupies the shortest chapter in this book, with the shortest list of references. Most of the present chapter is devoted to the cooldown phase because of its practical relevance for miniature Joule-Thomson cryocoolers. The significance of the cooldown phase can be recognized in two regards:

- (a) The cooldown period and its dependence on operating conditions is very often a significant performance parameter, and,
- (b) The ability to rapidly cool down to cryogenic temperatures is recognized as a unique niche of Joule-Thomson cryocoolers.

The evaporator, the location where cryocooling occurs, is schematically displayed in Fig. 7.1. The notation used for the temperatures at the inlet and outlet of the expansion valve, $T_2(t)$ and $T_3(t)$ correspondingly, and of the returning stream temperature, $T_4(t)$, remain consistent with the previous notation used for the Linde-Hampson cycle shown in Figs. 3.3, 3.4 and 3.5.

Note that during a transient regime it is possible for the cooled object to remain at a constant temperature, $dT/dt = 0$, while the temperature at the inlet to the expansion valve, $T_2(t)$, and the associated yield of liquefaction, $y(t)$, are not steady. Another parameter of interest during the cool down process is the instantaneous amount of liquid, n_{LIQ} , accumulated in the cryocooler.

7.2 Regimes of Transient Behavior

7.2.1 The Heat Load

7.2.1.1 The External Heat Load

The external heat load, $\dot{Q}$, is comprised of two components with independent origins,

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 \quad (7.1)$$

$\dot{Q}_1$ originates from the cooled object and is released directly into the evaporator. $\dot{Q}_2$ represents the heat leak from the surroundings that enters through the structure of the cryocooler and dewar assembly, and is typically distributed along the entire cooler. A fraction of this parasitic heat leak flows into the evaporator. $\dot{Q}_1$ is usually turned on after the cooldown is completed. $\dot{Q}_2$ is zero at the start of cooldown and increases to its full value at the end of the cooldown as the driving temperature difference, $T_{AMB} - T(t)$, reaches its highest value, $T_{AMB} - T_{BOIL}$.

7.2.1.2 The Latent “Internal Heat Load”

In addition to $\dot{Q}_1$ and $\dot{Q}_2$, during the cool-down there is an internal heat load, due to the heat capacity of the cryocooler and dewar assembly. This heat load appears as soon as the first expansion of the coolant generates colder gas because it, in turn, cools the warmer hardware. Starting at T_{AMB} , the temperature of the hardware decreases $dT/dt < 0$, and continues to drop as the gas cools down. Removing the heat from the cryocooler and dewar hardware “consumes

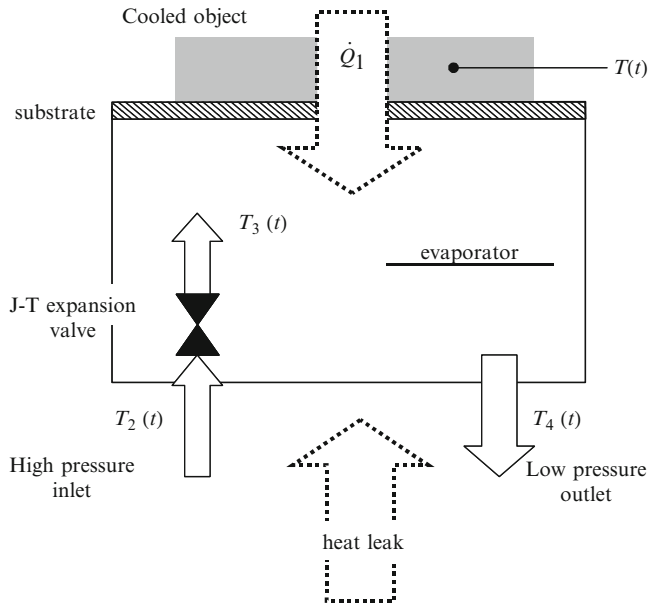


Fig. 7.1 Schematic layout of the evaporator of a miniature Joule-Thomson cryocooler

cold” and as such, the extracted heat is equivalent to an average heat load,

$$\bar{Q}^{CD} = \frac{C_{EFF}}{t_{CD}} \quad (7.2)$$

where C_{EFF} is the effective heat capacity, determined later in Sect. 7.5.2, and t_{CD} is the cool-down time. The coolant absorbs heat (“thermal mass”) from the surrounding hardware at the rate that is dictated by the local heat transfer coefficients. At the end of the cool-down $\bar{Q}^{CD}$ disappears.

7.2.1.3 Changes of Heat Load

Any heat load, $\dot{Q}$, within the range accommodated by the cryocooler will induce a cooling rate of $\eta \dot{n} \Delta h_T$, that matches the heat load at an associated steady state, such that $0 \leq \eta \leq 1$, as discussed in Sects. 3.3.2 and 3.5.3. The system adjusts itself to changes in the heat load, moving from one steady state to another, each with their own corresponding value of η . The mechanism is as follows: a higher $\dot{Q}$ elevates the outlet temperature, T_5^* , which in turn results in a higher efficiency, higher η and larger cooling rate. The opposite occurs when the heat load $\dot{Q}$ is decreased.

7.2.2 The Cooldown Process

The evolution of cool down is the result of the positive feedback mechanism associated with the “interchanging” process of Siemens, as discussed in Chapter 1¹; thus the

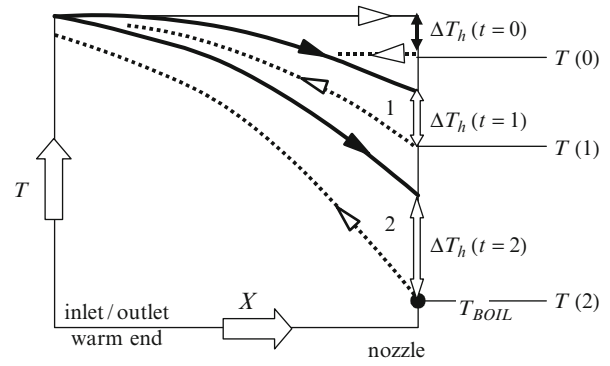


Fig. 7.2 Qualitative representation of transient behavior: instantaneous temperature distribution along the heat exchanger of both low and high pressure streams at three successive moments

temperature at the warm outlet of the interchanger ($T_{H, OUT}$ or T_2 in Figs. 1.1, 1.2, and 3.6), or equivalently, at the inlet to the expansion valve, is reduced. The fundamental cooling mechanism results from the integral Joule-Thomson effect, ΔT_h , experienced by the coolant as it flows through the expansion valve located at the cold end of the interchanger. This mechanism plays the role of δT (paragraph 1.1.2). The evolution of the cooldown is illustrated in Fig. 7.2, where the instantaneous temperature distributions, $T(X, t)$, along the heat exchanger of both the low and high pressure streams at the sequence of time points marked as: 0, 1, and 2 are displayed. The temperatures $T(t=0)$, $T(t=1)$, $T(t=2)$ at each moment are a result of the recuperation process and the driving isenthalpic temperature drop at the valve, $\Delta T_h(t=0)$, $\Delta T_h(t=1)$ and $\Delta T_h(t=2)$. In this figure, $T(2)$ is also the boiling point obtained at the end of the cooldown process. The steady state longitudinal temperature distribution was also discussed in Sect. 3.11.5.

For the purpose of discussing the cooldown process, we will assume that the Joule-Thomson valve has a fixed size aperture. Even in a regulated cryocooler the expansion valve, and consequently the flow rate, is adjusted only when the cooldown is almost completed. Hence, the valve remains fixed during the large majority of the cooldown process.

7.2.3 The Surplus of Cold Production

Joule-Thomson cryocoolers frequently experience an imbalance between the rate of cold production, $\eta \dot{n} \Delta h_T$, and the actual heat load, $\dot{Q}$. Typically, periods of excess cooling result from larger than necessary flow rates. As demonstrated in Sect. 3.9.3, a cryocooler with a given recuperator size will generate a larger cooling rate as the flow rate increases (but does not exceed an upper limiting value). Since the flow rate during cooldown for a small cryocooler is typically much larger than that required to balance the steady state heat load,

a variety of transient effects typically occur at the end of the cooldown. These are as follows:

7.2.3.1 De-stablizing Effects

Continuous Accumulation of Liquefied Coolant

When the rate of evaporation, driven by the heat load, is surpassed by the rate of liquid yield, the accumulated content of liquid cryogen, n_{LIQ} , inside the cryocooler increases at the rate,

$$\frac{d}{dt}(n_{LIQ}) \propto \dot{n}(t)y(t) - \frac{\dot{Q}}{L} \quad (7.3)$$

In such a case, the liquefied cryogen floods the evaporator and penetrates into the returning low-pressure channel. State 4, of the returning low pressure stream at the inlet to the interchanger (see Fig. 3.6), is no longer a saturated vapor, but transitions to state 4d, (Fig. 3.6d) that is, the over recuperated case; it includes some portion of liquefied cryogen.

Temperature Decrease at the Inlet to the Nozzle

The accumulated liquid in the return-flow channel causes a continued decrease in the nozzle inlet temperature T_2 , resulting in a positive feedback situation: The decreasing temperature at the inlet to the nozzle, $dT_2/dt < 0$, is accompanied by a further increase in the yield of liquefaction, $dy/dt > 0$. Furthermore, the decrease of T_2 increases the flow rate, $\dot{n}$ (for a give inlet pressure and fixed orifice). Thus, the rate of increase for n_{LIQ} also grows, as governed by Eq. 7.3.

Alternatively, the same behavior can be explained by the interchanging process described in Sect. 1.2.3. The heat capacity of the liquid cryogen is effectively infinite. Hence, the balance of capacity rates becomes $C_H < C_L$ along the section that contains some liquid, and the temperature remains at T_{BOIL} . This section contributes a large magnification index, I_M , by Eq. 1.41, which manifests itself by the decrease of T_2 . Alternatively, one may regard the decrease of T_2 due to the enhanced heat rejection from the high pressure stream because of the existence of liquid in the counterflowing low pressure stream.

The temperature front where $T = T_{BOIL}$ proceeds from the evaporator towards the warm end of the recuperator. If the recuperator were very (“infinitely”) long, this trend would continue; more and more liquid would be generated and accumulated in the recuperator as T_2 would continually decrease and approach the value of T_{BOIL} . At such a limit, the entire returning flow does liquefy, resulting in a cooling power given by $\dot{n}L$. As a rough estimation, the flow rate $\dot{n}$

would double. Furthermore, the latent heat $L = h_G - h_F$ is about 5 times larger than Δh_T (for nitrogen), thereby enabling $\dot{Q}$ to grow by an order of magnitude.

In the case of a liquefier such an instability is impossible since the entire yield of liquid is removed and does not enter the return flow channel of the recuperator.

7.2.3.2 Passive Stabilizing Effects

Two factors contribute to limit the accumulation of liquefied coolant described in the previous paragraph and these are associated with the finite size of the recuperator. As the liquid region increases, so does the heat load. At the same time the cooling power is suppressed, both effects continuing until the heat load matches the cooling power.

Elevation of Heat Leak

As the temperature front where $T = T_{BOIL}$ approaches the warm end of dewar-cryocooler assembly, the section in which the temperature change from T_{AMB} to T_{BOILL} occurs becomes shorter, and its thermal impedance shrinks. Consequently, the growing parasitic contribution, $\dot{Q}_2$, elevates the heat load.

Suppression of the Cooling Capacity

As the liquid zone in the return flow path increases, the efficiency, η , of the recuperator decreases and the temperature difference between the recuperating streams, at the warm end, ΔT^* , increases (Sect. 3.5.). When the temperature front where $T = T_{BOIL}$ reaches a certain location a limiting condition occurs at the warm end outlet, $\Delta T^* = |\Delta T_h|$; the effectiveness of the interchanger is minimal, ($\eta = 0$) and the cooling capacity vanishes, by Eqs. 3.28 and 3.52. Subsequently the system will warm up and the front where $T = T_{BOIL}$ will recede to a steady location where the new (“artificial”) heat load $\dot{Q}$ matches the degraded cooling power, $(\eta\dot{n}\Delta h_T)$.

7.2.3.3 Active Stabilization: Reducing Excess Flow

When the cooldown process is complete, the cooldown load $\overline{Q}^{CD}$ vanishes, the parasitic heat load $\dot{Q}_2$ reaches its full value, and the primary active load $\dot{Q}_1$ is initiated. In practice, the cooldown heat load, $\overline{Q}^{CD}$ is larger by at least an order of magnitude than the sum of the other two loads. Hence, at the end of the cooldown process a large excess flow rate exists and $\dot{Q}/\dot{n}$ becomes significantly smaller than Δh_T , which transitions the cooler into a highly gas consuming (less cost effective) regime. Therefore, at the end of the cooldown process a flow regulating mechanism will typically be used to reduce the flow rate; it increases $\dot{Q}/\dot{n}$ so that it approaches the value of Δh_T . Adequate steady state flow rates are about one to two orders of magnitude smaller than the average flow

rates during cooldown. Flow rates are not regulated when the run period is relatively short.

7.2.3.4 Bang-Bang Control

The undesirable features of poor temperature stability and excessive coolant waste can result when the gain of the flow regulating mechanism is too large. In such a case, the “on” state produces an unnecessarily large flow rate, the amount of n_{LIQ} continues to grow and liquid penetrates the recuperator. Finally, the flow regulator responds, switches to the off state and drastically reduces the cooling power. Then, n_{LIQ} decreases and the temperature front where $T = T_{BOIL}$ recedes until the regulator returns to on state. It is possible to observe such an abrupt fluctuation of the liquid front through a transparent glass dewar; from a state where multiple windings of the finned tube are wet, to the state where only one winding is wet.

7.2.4 Warming

When the net heat load $\dot{Q}$ becomes larger than the maximum cooling power, $\dot{n}\Delta h_T$, the instantaneous liquid content in the evaporator boils off and consequently, the cooled object begins warming up, $dT(t)/dt > 0$.

The most common warming situation occurs when the pressure inside a discharging vessel decreases below the cut-off value, as discussed in Sects. 5.3.3.5 and 5.5.2., and as calculated in Sect. 9.2.5. by Eq. 9.9. The instantaneous pressure becomes too low; and consequently the product of the pressure dependent molar flow rate $\dot{n}$ and Δh_T falls below the magnitude of the heat load.

Insensitive flow regulating mechanisms can also cause warming, for example if their response results in an insufficient flow rate.

As long as $T_2(t) < T_{CD}$, a non-zero liquid yield will occur and the temperature inside the evaporator will remain at T_{BOIL} . However, as $T_2(t)$ increases, the warming rate of the cooled object will be determined by the thermal inertia and the difference between $\dot{Q}/\dot{n}$ and $\eta \cdot \Delta h_T$. It is also possible to achieve an intermediate steady temperature above T_{BOIL} ; in such a case the cooling process does not include evaporation.

7.3 Characteristic Cooldown Behavior of the Cooled Object

7.3.1 The Expanded Stream

Let us focus on the time dependent shape of the temperatures $T_2(t)$ and $T_3(t)$, after Maytal² as qualitatively displayed in Fig. 7.3. The vertical separation between the two curves is determined by the integral isenthalpic Joule-Thomson

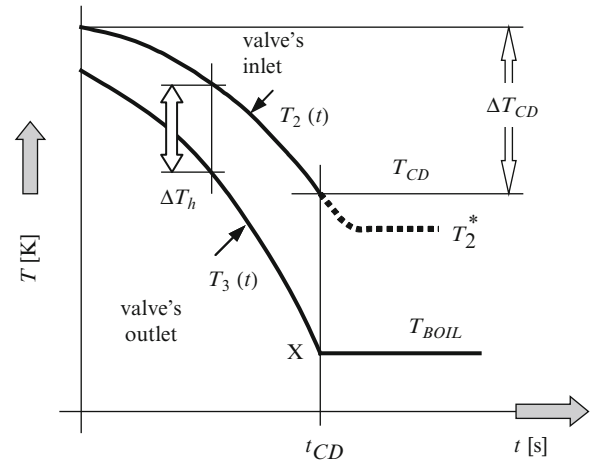


Fig. 7.3 Instantaneous temperatures during cooldown at the inlet and outlet of the nozzle; the shape of the outlet temperature follows that of the inlet because it will always be lower by the instantaneous ΔT_h , after Maytal²

effect, ΔT_h , associated with each value of $T_2(t)$ and depends on the operating pressures.

The cooldown characteristics of a cryocooler are most strongly defined by the time dependent decrease of $T_3(t)$, that is the temperature inside the evaporator. However, the temperature $T_3(t)$, at the outlet of the nozzle responds immediately to the temperature $T_2(t)$ at the nozzle inlet. There is no delay between any instantaneous change of $T_2(t)$ and the resulting change to $T_3(t)$.

At the same moment that $T_2(t)$ reaches the temperature of T_{CD} (Sect. 4.4), the expanded stream reaches the boiling point, T_{BOIL} , although at that instant, the yield of liquefaction is still zero. Subsequently, $T_2(t)$ decreases below T_{CD} , as shown by the dashed line in Fig. 3.7, down to the steady state value T_2^* . However, once below T_{CD} it no longer influences the value of $T_3(t)$. The temperature in the evaporator is fixed at T_{BOIL} , but the quality of the expanded stream changes: the yield of liquefaction increases from zero to its steady state value, y , as discussed in Sect. 3.8.

Perhaps one the more intriguing observations regarding the cooldown process is that the heat exchanger itself does not lower the system temperature down to T_{BOIL} . In fact, the transient temperature reduction by the heat exchanger does not go below T_2 . The remaining temperature drop, down to $T_3(t)$, occurs naturally and at the same time simply by the isenthalpic expansion process through the valve.

The shape of $T_2(t)$ and $T_3(t)$, as reflected in Fig. 7.3, is indeed expected to be convex. That is, the cooldown rate increases with time – a feature that is unique to Joule-Thomson cryocoolers, as already mentioned in paragraph 1.3.2d. and Fig. 1.8. The positive feedback mechanism in

the recuperator becomes amplified as the cooldown process proceeds due to three factors that combine to reduce $T_2(t)$:

- (a) The driving potential ΔT_h , increases (for a pure fluid) as the temperature is reduced,
- (b) The molar flow rate increases, as will be explained in Sect. 7.5.1, which means that during the cooldown process, the instantaneous cooling power increases, and,
- (c) The temperature reduction becomes “easier” since the c_p of the cooler and its encapsulation decrease as the temperature decreases.

The temperature $T_2(t)$ gradually approaches the steady state value, T_2^* , as noted in Fig. 7.3. However, the temperature, $T_3(t)$, approaches the steady state temperature of T_{BOIL} in an “abrupt” or non-gradual manner, with a discontinuous derivative at point X in Fig. 7.3. This feature results from the external constraint imposed in form of the environmental pressure (in the evaporator) which prevents any further decrease of $T_3(t)$ below T_{BOIL} .

In order to derive the exact time dependence of $T_2(t)$ one would need a detailed numerical simulation of the transient behavior of the cryocooler as discussed in Sect. 9.3. The simulation would have to include all the relevant parameters of the heat exchanger such as geometry, materials, heat transfer coefficients, configuration, flow rate, pressure, and gas species.

7.3.2 The Cooled Object

In practice, the more relevant temperature is the one measured at the location of the cooled object, denoted as $T(t)$, rather than the evaporator temperature, $T_3(t)$. The object is generally mounted on a substrate adjacent to the evaporator (Fig. 7.2) and therefore indirectly cooled by the two-phase cryogen bath. The cooldown behavior of the object, $T(t)$, reflects a somewhat delayed response compared to the driving coolant temperature, $T_3(t)$, and can be described in terms of a first order delay satisfying the expression

$$\tau \frac{dT(t)}{dt} + T(t) = T_3(t) \quad (7.4)$$

Here τ is the thermal time constant of delay associated with the substrate, depending on its thickness, its thermal diffusivity, and the convective heat transfer coefficient. This issue is very significant for rapid cooldown cryocoolers (Sect. 5.5.4.4.). Clark³ focused on the conditions for minimizing τ .

The cooldown behavior, $T(t)$, can take on two different forms. The first one is initially concave and then transitions into a convex shape through a point of inflection, before returning to the concave shape through an inflection point

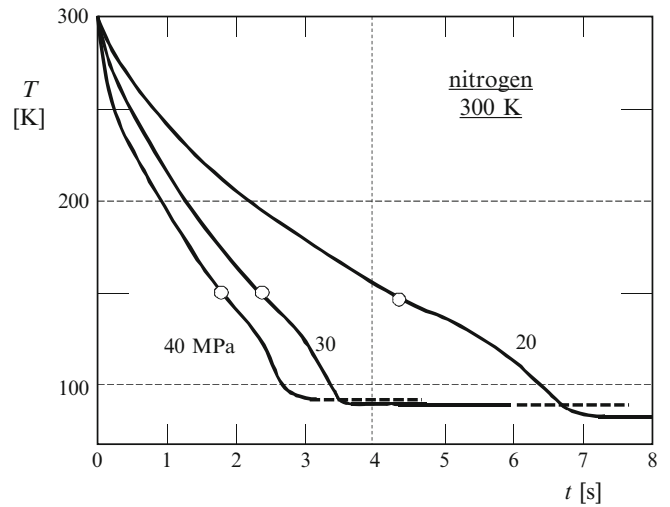


Fig. 7.4 Measured cooldown temperatures pattern of a fast 3.3 mm diameter cryocooler with metal non-evacuated encapsulation, so that, $\tau \ll t_{CD}$. The circles mark the existing inflection points, after Maytal.²

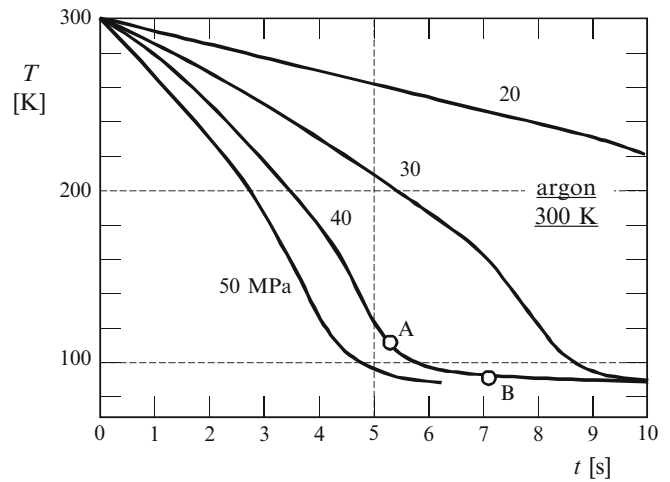


Fig. 7.5 Measured cooldown temperatures pattern of a 5.2 mm diameter cryocooler within a glass dewar. The concave shape is due to the condition of $\tau \gg t_{CD}$, after Maytal.²

as shown in Fig. 7.4. In this case, τ is relatively short because of the thin metal substrate supporting the cooled object.

The second type begins as a convex curve before transitioning to the final concave shape as shown in Fig. 7.5. The value of τ in this case is larger, due to a much thicker glass substrate of lower thermal diffusivity. Therefore, the final approach to the steady state temperature is quite slow. Such a behavior is typical of a long running cryocooler with a heavy dewar and a glass cold finger.

The origin of the difference between the two cases is explained with the aid of Fig. 7.6 in terms of τ and its relation to the cryocooler’s cooldown period, t_{CD} . The

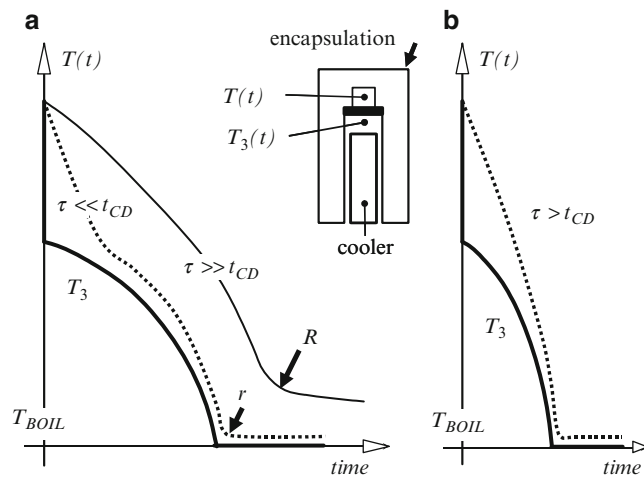


Fig. 7.6 Cooldown patterns displaying, $T(t)$, of the cooled object driven by the nozzle exit temperature, $T_3(t)$. Part (a) depicts a rapid cryocooler with a long cooldown period, t_{CD} , as if operated with nitrogen, satisfying the condition $\tau \ll t_{CD}$, and exhibiting an inflection point. Part (a) also displays the opposite case, when $\tau \gg t_{CD}$. Part (b) depicts the case of a shorter cooldown period as if operated with argon

curve of $T_3(t)$ in Fig. 7.6 represents the temperature history in the evaporator and it is convex. The temperature of the cooled object responds to this thermal driving force and, with the most rapid response, it approaches the same pattern. Part (a) of Fig. 7.6 displays a cryocooler with relatively long t_{CD} . The dotted curve represents the case where $\tau \ll t_{CD}$, that is, when τ is much shorter than the cooldown period, t_{CD} . Note that at the very first moment, that is when $t = 0$, the temperature in the evaporator drops by ΔT_h . Because of the rapid response associated with the small thermal resistance $T(t)$ follows the step change quite sharply with only a minor delay and a rapid approach to steady state – as depicted by “r” in Fig. 7.6a. This explains the initial concave shape of $T(t)$. Subsequently $T(t)$ adjusts itself to the shape of $T_3(t)$ and also becomes convex.

The second curve in Fig. 7.6a marked by $\tau \gg t_{CD}$, represents the case when the time constant τ is much longer than the cooldown period, t_{CD} . The initial response of $T(t)$ to the step change ΔT_h is very moderate and therefore remains convex throughout the entire cooldown process, exhibiting a more gradual approach to steady state – see the feature “R” in Fig. 7.6a.

As depicted in Fig. 7.4, a rapid cryocooler operated with nitrogen, which has a relatively long value of t_{CD} , typically satisfies the condition $\tau \ll t_{CD}$ and therefore exhibits an inflection point in the cooldown pattern. However, for the same cryocooler with the same τ , when operated with argon the value of t_{CD} becomes much shorter. In this case the cooldown pattern follows the second form now fulfilling $\tau > t_{CD}$. The pattern begins with a convex shape, but

moderately convex so that at times it is almost linear. Figure 7.6b provides a qualitative description of such occurrence. Practically, nitrogen-operated rapid cooldown patterns are more likely to display the two inflection points while argon-operated rapid cooldown patterns are almost linear with only a small curvature.

In case where t_{CD} is very large, it is most often true that $\tau \ll t_{CD}$. Such is the case with the measurements of the MEMS type cryocooler reported by Lerou et al.⁴ where t_{CD} is approximately 750 s; indeed, these display an inflection point.

7.3.3 Miscellaneous

7.3.3.1 The Cooldown Behavior of a Flow Demand Cryocooler

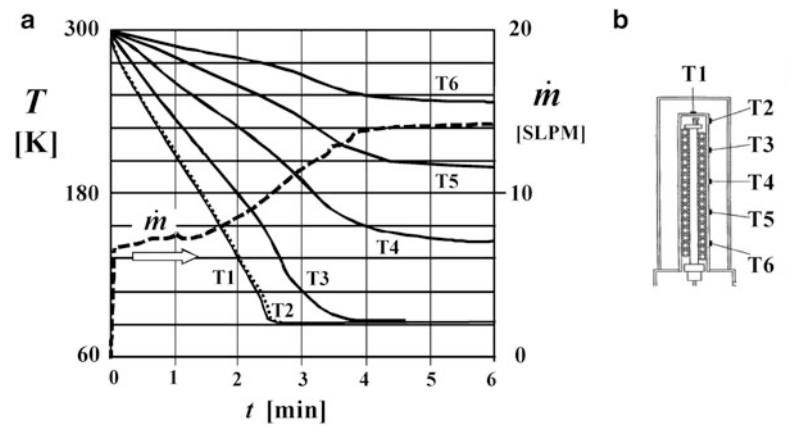
The cooldown behavior of a flow demand cryocooler is affected by the response of the flow regulator. By sensing the temperature inside the evaporator, it drastically reduces the flow rate when a certain target temperature is reached. From that point on, the cooldown process is slowed and the temperature smoothly approaches the boiling point. In this case, the temperature at which the drastic reduction of flow rate occurs becomes a highly relevant parameter. For example, if the flow reduction occurs at a too high temperature, the final section of the cooldown process will continue very slowly. However, such a flow reduction does provide another, desirable effect, that is lowering the backpressure and the associated boiling point in the evaporator.

7.3.3.2 Termination of Cooldown

Termination of cooldown often occurs when a certain target temperature is reached at the cooled object. Typically such a temperature does not exceed the normal boiling point by more than 10 K. As observed in Fig. 7.5, the object approaches the lowest target temperature asymptotically. Thus it becomes very significant whether termination occurs before (point A) or after (point B) the “knee”. The small temperature difference between points A and B may be magnified into a large difference in cooldown periods. The precise selection of the termination temperature usually sparks a debate between those designing the dewar (responsible for the temperature gradient of the cold object’s mounting) and those designing the cryocooler (responsible for the coolant, its flow rate and back pressure).

Alternatively, the cooldown may be terminated when liquid is first produced in the evaporator rather than conditioning the action on the temperature of the cooled object. However, obtaining such information would require an additional temperature sensor in the evaporator.

Fig. 7.7 The cooldown behavior at six locations along the cryocooler, from T1 at the cold end to T6 close to the warm end as measured by Hong et al.⁵ The dashed line is the measured transient flow rate; its rate of growth increases as the temperature gets lower. The operating pressure is 10 MPa



7.3.3.3 Cooldown Behavior at Various Locations Along the Cryocooler

Cooldown behavior at various locations along the cryocooler was measured by Hong et al.⁵ His investigation used a fixed orifice cryocooler pressurized by 10 MPa nitrogen. Figure 7.7. displays the behavior at six locations along the cryocooler, from T1 at the cold end to T6 close to the warm end. Since the t_{CD} is quite long, about 140 s, this case is characterized by $t_{CD} \gg \tau$, and therefore, the behavior exhibits an inflection point; the curves begin concave and turn convex before finally returning to a concave shape, as shown in Fig. 7.4. The temperature diodes are attached at the metal cold finger thus indicating the temperatures of the expanded low pressure stream. Figure 7.7 also displays the increasing flow rate realized during the cooldown. Elsewhere, Hong et al.,⁶ conducted similar measurements but with different sized vessels and with charging pressures ranging from 8 to 12 MPa.

7.4 Correlations and Similarity of Cooldown Periods

Similarity analyses provide scaling relationships by combining appropriate physical parameters, typically into dimensionless numbers. For example, similarity relations derived for the JT cooldown process would enable one to scale the cooldown period at new conditions based on knowledge of the same parameter at any reference condition. A significant part of this task involves identifying the relevant scaling parameters.

7.4.1 Empirical Correlations

Bodio and Sujak⁷ described their measurement of cooldown periods for nitrogen, air and argon with the empirical correlation,

$$\ln \frac{T_{AMB} - T_{BOIL}}{\dot{m} t_{CD}} = xP + y \quad (7.5)$$

where x and y are characteristic coefficients depending on the coolant, and the construction and materials of the cryocooler and dewar assembly. Bodio and Wilczek⁸ developed correlations for a widely distributed population of cooldown period measurements in the form,

$$\ln(t_{CD}) = a - 2.82P \quad (7.6)$$

where a depends on whether the coolant was nitrogen or argon. However, these results have limited validity, since they are only based on experiments with pressures below 15 MPa. This is a particularly narrow and less relevant range of operation at least for rapid cooldown applications.

Bodio and Sujak⁷ also introduce a scaling parameter of relevance to the cooldown period of a cryocooler, which is its mass per unit of heat transfer area for the low pressure stream, m/A . This parameter provides a measure of compactness, reflecting the configuration and fabrication technology for the cryocooler and dewar assembly. The scope of their observations included multiple parallel heat exchanging layers, and led to the following conclusion, in the last sentence of the paper:

the liquefaction time is not only dependent on the initial pressure and the kind of the gas but also on the mass of heat exchange system to the area of heat exchanging surface.

Using the techniques of dimensional analysis and Buckingham pi theory, Bodio, Chorowski and Wilczek^{9,10} derived three dimensionless groups of parameters, and proposed the following correlation,

$$\left(\frac{\dot{m} t_{CD}}{zA} \right) = B_0 \left(\frac{T_{AMB}}{T_{BOIL}} \right)^{B_1} \left(\frac{zPA^{3/2}}{\dot{m}} \right)^{B_2} \quad z = \frac{m}{A} \quad (7.7)$$

On the basis of their collected data, at operating pressures below 15 MPa, they determined values for the coefficients:

$B_0 = 7.46$, $B_1 = 2.71$ and $B_2 = -0.14$. Additional related reports by Bodio and coauthors^{11,12,13,14} and others^{15,16} have been published in Polish and Russian.

7.4.2 The Semi Analytical Similarity Model for Rapid Cooldown Cryocoolers

The above results were developed completely on an empirical level, and lack any physical justification for the choice of the dimensionless groups and the associated correlations. One may suggest, for example that the normal boiling temperature T_{BOIL} and pressure P could represent relevant parameters for correlating cooldown periods. Gases with higher boiling temperatures, such as like krypton, and at an appropriate pressure, liquefy immediately through a single expansion, that is “no time” ($t_{CD} = 0$). Pressure represents a relevant scaling parameter since, at elevated values, the Joule-Thomson effect degrades, which may result in a longer t_{CD} . The mass of the cryocooler also appears to be a relevant parameter, however, not in terms of absolute mass, m , measured in kilograms but rather in terms of thermal mass, C , measured in Joules.

The approach described in the previous section did not include such physical considerations. However, the semi analytical model of Maytal² does provide such a capability. It focuses on similarity relations for the cooldown periods of rapid cooldown cryocoolers (the category discussed in Sect. 5.5). Because there is no time delay associated with the isenthalpic temperature drop, the relevant temperature gap that must be closed by the recuperator in order for liquefaction to occur (following Sect. 4.4.1) is simply,

$$T_{AMB} - T_{CD} \equiv \Delta T_{CD} \quad (7.8)$$

This temperature span better represents the actual cooldown range than does T_{AMB}/T_{BOIL} or $T_{AMB} - T_{BOIL}$. For example, when $T_{AMB} = 300$ K, the ΔT_{CD} for nitrogen and argon are approximately 125 and 75 K respectively, differing by a ratio of 0.6. Such a consideration highlights the faster cooldown potential of argon more clearly than if one were to consider the difference of only 10 K in T_{BOIL} of the two gases, in which case the ratio of $T_{AMB} - T_{BOIL}$ is 0.95. The pressure dependence of T_{CD} (or ΔT_{CD}) is discussed in Sect. 4.4.2. Figure 4.27 displays the minimum values ΔT_{CD}^{MIN} for various gases as a function of their boiling points.

Maytal² begins his model with the assumption that the cool down time t_{CD} is proportional to the temperature gap divided by the rate of closing this gap,

$$t_{CD} \propto \frac{\Delta T_{CD}}{(dT/dt)_{t=0}} \quad (7.9)$$

Secondly he assume that the cooldown rate is driven by the instantaneous value of ΔT_h , that being dependent on the instantaneous temperature $T(t)$, according to the relationship,

$$C \frac{dT}{dt} = -U A |\Delta T_h| \quad (7.10)$$

where U is the overall heat transfer coefficient, A is the heat transfer area of the returning stream, and C is the heat capacity of the cryocooler and dewar assembly. Combining (7.9) and (7.10), the semi analytical model gives,

$$t_{CD} \propto \frac{\Delta T_{CD}}{U |\Delta T_h|} t_{CD} \propto \frac{\Delta T_{CD}}{U |\Delta T_h|} \left(\frac{C}{A} \right) \quad (7.11)$$

The parameter C/A reflects the compactness of the recuperator in terms of thermal mass per unit of heat transfer area. Bodio et al.⁷ introduced and examined the same parameter on an intuitive basis. However, here C/A is a direct result of the model.

7.4.3 The Pressure Dependence for the Cooldown Period of a Given Cryocooler

7.4.3.1 Formulation

For a given cryocooler, with fixed size and geometry for the heat exchanger and orifice, the pressure dependence of the cooldown time t_{CD} can be examined from Eq. 7.11 on the basis of fluid properties and flow characteristics² since the pressure will not affect the value of C/A . For internal flow through the cryocooler's channels¹⁷ we have,

$$\text{Nu} \propto \text{Re}^{0.8} \text{ and } \text{Re} \propto \dot{n} \quad (7.12)$$

therefore,

$$U \propto \dot{n}^{0.8} \quad (7.13)$$

Thus, for a given cryocooler, the pressure dependence of t_{CD} at any value of T_{AMB} becomes,

$$t_{CD} \propto \frac{\Delta T_{CD}}{\dot{n}^{0.8} \Delta T_h} \quad (7.14)$$

All three parameters, ΔT_{CD} , ΔT_h and $\dot{n}$ are pressure dependent and may be estimated from their values at the beginning of a cooldown. Similarity relations, based on Eq. 7.14, and scaled by the value of t_{CD}^{REF} , at the reference conditions of 30 MPa and 295 K for nitrogen,

$$t_{CD}^{REF} = t_{CD}(\text{N}_2, 30 \text{ MPa}, 295 \text{ K}) \quad (7.15)$$

provide an estimate of t_{CD} at any value of (P, T_{AMB}) .

Fig. 7.8 Cooldown time represented by similarity ratios for nitrogen and argon at room temperature. Dots and curves represent the respective results from experiments and the semi analytical similarity model, after Maytal.^{2,18}

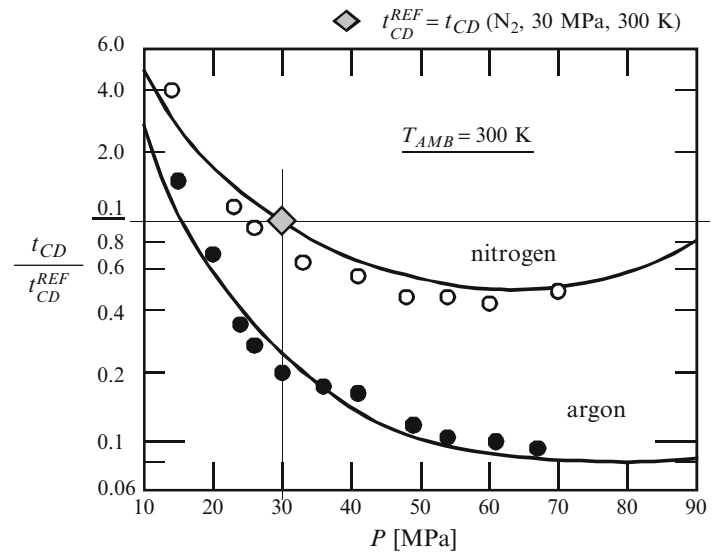
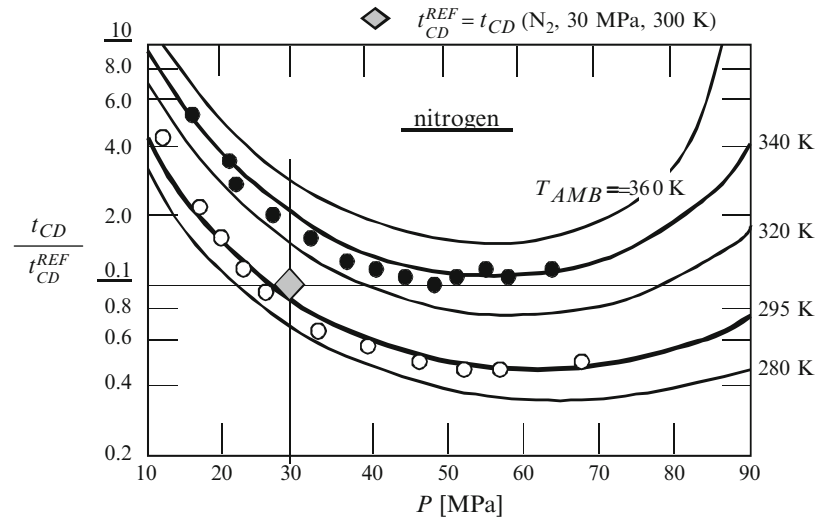


Fig. 7.9 Cooldown time similarity ratios for nitrogen at various ambient temperatures; The open and solid dots represent measurements at 295 K and 340 K respectively. Curves represent the semi analytical model, after Maytal.^{2,18}



7.4.3.2 Similarity Results

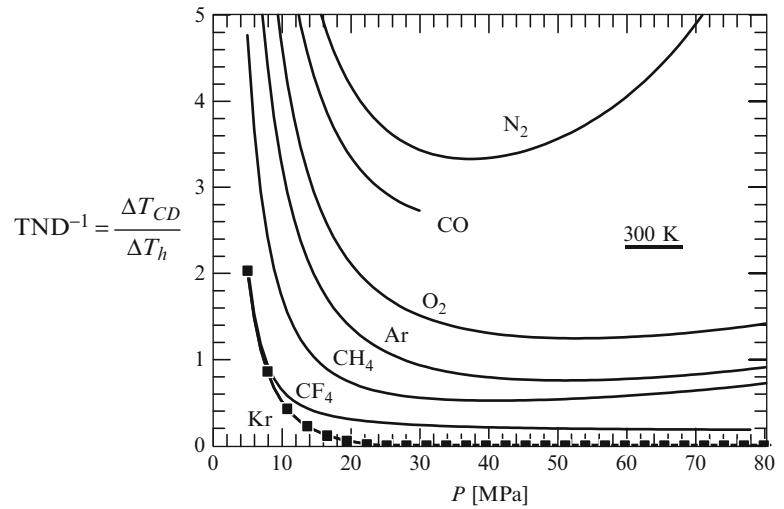
Figure 7.8 displays the corresponding similarity relations as curves of t_{CD}/t_{CD}^{REF} as a function of pressure for a single rapid cooldown cryocooler operated with nitrogen and argon at 295 K along with data that verify the relations.¹⁸ Fig. 7.9 extends the comparison of measurements with the similarity model to elevated ambient temperatures for nitrogen.

For any given value of T_{AMB} , the pressure dependence of t_{CD} exhibits a minimum. Cooldown periods decrease quite significantly as P increases but above a certain value it begins to increase again. The minimum is a direct result of the peaking behavior of ΔT_h with P . For nitrogen at room temperature, the minimum of t_{CD} occurs between 50 and 60 MPa.

However, the decrease in t_{CD} for argon extends even up to 75 MPa. This is the reason for applying higher pressures for fast cooldown applications with argon. However, for nitrogen, increasing the pressure up to 65 MPa, increases the value of t_{CD} by a factor of 1.6 times compared to its value at 40 MPa. Thus, Fig. 7.9 provides the experimental justification for prolonged nitrogen cooldown at the ‘high enough’ pressure of 40 MPa.

One may correctly expect that at higher values of T_{AMB} , the cooldown processes last longer. While it is true that the pressure of a fixed charge of gas in a vessel at room temperature will increase at higher T_{AMB} , the pressure elevation will not compensate for the degradation caused by the elevated temperature. Section 4.3.9. (Fig. 4.22) demonstrated that the

Fig. 7.10 The cooldown tendency of gases as function of pressure at 300 K



value of Δh_T decreases with temperature for a constant density process.

7.4.3.3 Approximate Similarity Relations

Approximate similarity relations are also derived² for the pressure range of $P = 10$ MPa to 60 MPa and ambient temperatures between $T_{AMB} \equiv T = 290$ K to 360 K. The relations, derived by choosing a representative value of T_{CD} and by developing curve fits for the values of $\Delta T_h(P, T)$, are given as,

$$\frac{t_{CD}(N_2, P, T)}{t_{CD}^{REF}} = \frac{4.27}{P^2} \frac{T - 168}{1 - P/80 - 0.15(T - 300)/P} \quad (7.16)$$

$$\frac{t_{CD}(Ar, P, T)}{t_{CD}^{REF}} = \frac{1.68}{P^2} \frac{T - 220}{1 - P/1120 - 0.15(T - 300)/P} \quad (7.17)$$

with P measured in MPa and T in Kelvin.

At an ambient temperature of 300 K, the successive values of pressures that reduce t_{CD} to half the value at the previous pressure are, for nitrogen: 10, 15, 25, 45 MPa, and for argon: 10, 15, 20, 30, 50. In the temperature range of 280 K to 360 K and for pressures below 50 MPa, t_{CD} values may be scaled by the pressure of operation,

$$\frac{t_{CD,1}}{t_{CD,2}} = \left(\frac{P_2}{P_1}\right)^{1.5} \quad \text{for nitrogen} \quad (7.18)$$

$$\frac{t_{CD,1}}{t_{CD,2}} = \left(\frac{P_2}{P_1}\right)^{1.75} \quad \text{for argon} \quad (7.19)$$

7.4.4 Cooldown Tendency of Gases, TND

The general expression for t_{CD} as given in Eq. 7.11 can be cast in terms of the parameter,

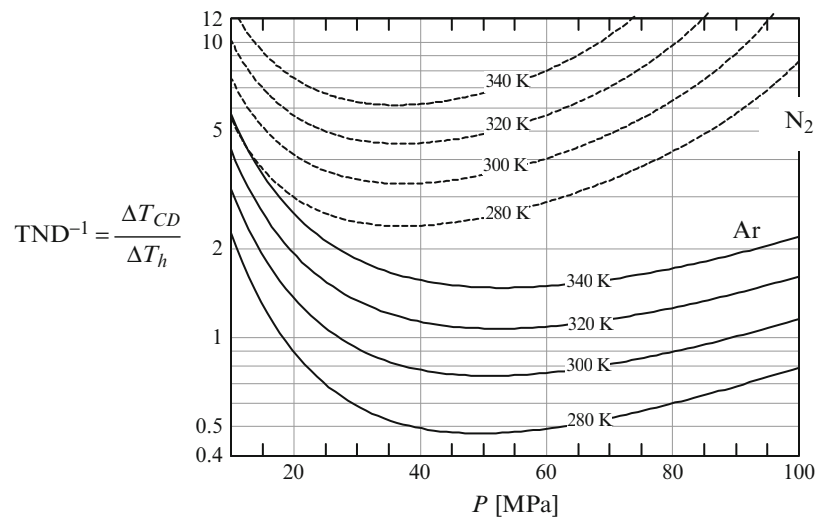
$$TND \equiv \frac{|\Delta T_h|}{\Delta T_{CD}} \quad (7.20)$$

that depends solely on the species of gas and its operating conditions (P, T_{AMB}). The term reflects the tendency (therefore designated as TND) or potential of the gas to undergo liquefaction in the Linde-Hampson process. Note that TND^{-1} therefore indicates the impedance of a gas to undergo the same process. Put another way, cryocoolers that share a common flow rate, value of C/A , and U , satisfy the relationship:

$$t_{CD} \propto TND^{-1} = \frac{\Delta T_{CD}}{|\Delta T_h|} \quad @ \quad \dot{n}, C/A, U = Const \quad (7.21)$$

As the value of TND increases, t_{CD} decreases (an indication that a smaller heat exchanger would suffice for producing liquid). Figure 7.10 displays TND^{-1} for some coolants at 300 K as a function of pressure. As a general trend, TND^{-1} sharply decreases as the pressure is elevated from low values towards 20 to 30 MPa. For nitrogen, TND^{-1} begins to increase above 40 MPa. For the higher boiling point gases such as oxygen, argon, methane, and CF_4 , the increase of TND^{-1} above 40 MPa is very moderate. Up to 80 MPa it remains almost constant, preserving its value at 25 MPa. The general trend for nitrogen and argon is similar to that of Fig. 7.8. Note that the order of the curves in Fig. 7.10 corresponds to the normal boiling points of the gases. Note also that krypton displays a rather distinctive feature.

Fig. 7.11 The cooldown tendency of nitrogen and argon at different temperatures, as function of pressure



The TND^{-1} of krypton vanishes at pressures above about 22 MPa (a feature that is consistent with Fig. 4.35). One may find elsewhere¹⁹ the same criteria of TND^{-1} applied as a comparison for the expected values of t_{CD} for different coolants and states.

Figure 7.11 displays values of TND^{-1} for nitrogen and argon at various temperatures.

7.5 The Integral Model for the Cooldown Periods

The integral model uses the average flow rate, $\bar{n}_{CD}$ and cooling power, $\bar{Q}^{CD}$ during the cooldown period and the effective heat capacity, C_{EFF} (to be defined below). The integral model is semi-empirical but enables one to relate the cooldown period of a given cryocooler with the size of the expansion valve (in terms of flow rate), the materials of construction, the coolant and its pressure and temperature. The predictive capabilities of the model are limited to simply providing a good approximation but the procedure is quite simple.

7.5.1 Cooldown Flow Rates

The cooldown process of a fixed orifice Joule-Thomson cryocooler is accompanied by a transient development of the flow rate, as the system responds to the changes of temperature. At any given pressure, the flow rate depends on the temperature, T_2 , at the inlet to the orifice. Therefore, the transient history of the flow rate, $\dot{n}(t)$, corresponds to the transient behavior of $T_2(t)$. As the cooldown proceeds, the orifice inlet temperature decreases due to the heat

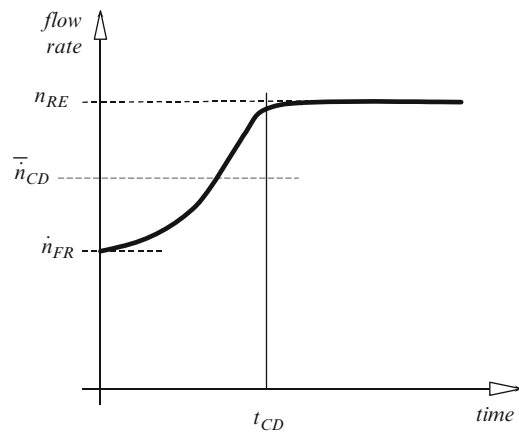


Fig. 7.12 During cooldown, the flow rate increases from the initial value $\dot{n}_{FR}$ and approximately doubles by the end of the process, reaching the value of $\dot{n}_{RE}$. The average flow rate during cooldown is $\bar{n}_{CD}$

exchange (“interchanging”) process. The flow rate increases as the temperature decreases (see Sect. 9.2.2.). Furthermore, since the rate at which T_2 decreases becomes larger and larger, the rate at which $\dot{n}(t)$ grows also becomes larger and larger as qualitatively illustrated in Fig. 7.12. Such a transient behavior of the flow rate was measured by Hong et al.⁵ and is displayed in Fig. 7.7. The measurements of Lerou et al.²⁰ on a two stage cryocooler also demonstrate the same behavior.

At the very first moment when gas is supplied to the cryocooler, its flow rate is unaffected by the heat exchange process, and is determined solely by the ambient temperature and the initial pressure. The flow rate under such conditions will be referred to as the *non recuperated flow rate*. On occasion it is also identified as the *warm* or “free” flow

rate and denoted by $\dot{n}_{FR}$, as for example when the cryocooler (with a Hampson type heat exchanger) is extracted from its encapsulation and the recuperative counterflow heat exchanger is eliminated. Such a case is discussed later in Sects. 9.2.3 and 9.2.4. The steady state value, denoted by $\dot{n}_{RE}$, is referred to as the “cold” or *recuperated flow rate* and is determined by the steady state value of $T_2(t)$. Figure 7.12 displays the transition from $\dot{n}_{FR}$ to $\dot{n}_{RE}$ during cooldown. It is of practical interest to know how $\dot{n}_{RE}$ compares with $\dot{n}_{FR}$. For nitrogen and argon, the approximate or representative value is,

$$\dot{n}_{RE}/\dot{n}_{FR} \approx 2 \quad (7.22)$$

More accurate and pressure dependent values are discussed in Sect. 9.2.4. However, for the present we may observe that the average flow rate during the cooldown process is approximately,

$$\bar{n}_{CD} \approx 1.5\dot{n}_{FR} \approx 0.75\dot{n}_{RE} \equiv 0.75\dot{n} \quad (7.23)$$

7.5.2 The Effective Heat Capacity of a Cryocooler

Let the total heat capacity of a complete cryocooler and dewar assembly, C , be the energy that has to be extracted in order to lower its temperature from T_{AMB} down to the liquefaction temperature, T_{BOIL} , that is,

$$C = \int_{X=0}^{X=L} dm \int_{T_{BOIL}}^{T_{AMB}} c_P(T) dT \quad (7.24)$$

where X is the longitudinal coordinate along the assembly of the cryocooler and its encapsulation.

In the common case of a Hampson type heat exchanger, C is a sum of four components, each obtained by integration along their respective lengths:

- C_1 -the high-pressure duct and its outer fin-like extended surface,
- C_2 -the inner mandrel, whether a sleeve or a rod,
- C_3 -the cold finger of the encapsulation, a glass or a metal tube, and,
- C_4 -the thermal mass of the cooled object located at the cold end that typically includes a substrate, an infrared sensor, and the cold shield.

The high pressure finned tube, the inner mandrel and the cold finger of the encapsulation are not entirely cooled down to T_{BOIL} . Instead, a steady state temperature

distribution exists along each of them after the cooldown. The effective heat capacity, C_{EFF} , is determined from the actual energy extracted during the cooldown phase. Each element located at X cools down from T_{AMB} to its local temperature, $T(X)$,

$$C_{EFF} = \int_{X=0}^{X=L} dm \int_{T(X)}^{T_{AMB}} c_P(T) dT < C \quad (7.25)$$

It is simple to determine the value of C from knowledge of the materials, T_{AMB} and T_{BOIL} . However, it is more difficult to calculate C_{EFF} since it must be determined in conjunction with the solution for $T(X)$. However, on an empirical basis after Longworth and Steyert^{21,22} it may be approximated by

$$C_{EFF} = \frac{C_1 + C_2 + C_3}{N} + C_4. \quad (7.26)$$

For argon $N = 3$ and for nitrogen $N = 2$. That is, the effective heat capacity of the elements that are subjected to the longitudinal temperature distribution is only a third or a half of their total heat capacity. The difference between the distributions with argon and nitrogen is due to the larger Joule-Thomson effect of argon, and is qualitatively displayed in Fig. 3.29. The cooled object at the cold end is assumed to be entirely cooled from T_{AMB} to T_{BOIL} and in practice it is a small part of C_{EFF} .

7.5.3 The Average Cooling Power During Cooldown

The cooling power changes during the cooldown process. For nitrogen or argon the average cooling power, $\bar{Q}^{CD}$, corresponds with the average flow rate during cooldown so that,

$$\bar{Q}^{CD} \approx \eta_{CD} \cdot \bar{n}_{CD} \cdot \Delta h_T \approx 1.5\eta_{CD} \cdot \dot{n}_{FR} \cdot \Delta h_T \quad (7.27)$$

Here η_{CD} is the average efficiency of the heat exchanger during the cooldown period which is typically lower than the steady state η values. For rapid cooldown applications the average efficiency is about 0.4 to 0.6. Continuous or long-running applications utilize a longer and more efficient heat exchanger, so that η is about 0.6 to 0.8.

The steady state cooling capacity at the end of the cooldown process, $\dot{Q}^{SS}$, before (or without) adjusting the flow rate, is $\eta\dot{n}\Delta h_T$ (Sect. 3.5.3.), or about $2\eta\dot{n}_{FR}\Delta h_T$. Therefore, the two cooling rates are related to each other by,

$$\bar{Q}^{CD} \approx 0.75 \dot{Q}^{SS} < \dot{Q}^{SS} \quad (7.28)$$

(Note that in case of a closed cycle operation with a large fraction of liquefaction, as described in Sect. 8.7.4., the “self regulating effect” reduces the cooling power during cooldown so that, $\dot{Q}^{SS} < \bar{Q}^{CD}$).

7.5.4 Energy Balance During Cooldown

The central idea of the integral model^{21,22} is that in order to extract the necessary energy, C_{EFF} , within the cooldown period t_{CD} the required average cooling power matches the value calculated above using the average flow rate, so that following Eq. 7.2,

$$t_{CD} \approx \frac{C_{EFF}}{\bar{Q}^{CD}} \approx \frac{C_{EFF}}{1.5 \eta_{CD} \dot{n}_{FR} \Delta h_T} \quad (7.29)$$

Alternatively, Eq. 7.29 enables one to determine the necessary flow rate in order to accomplish the cooldown within any desirable period, t_{CD} .

For a given cryocooler the parameters $\dot{n}_{FR}$ and Δh_T are dependent on the operating conditions, P and T_{AMB} . The value of C_{EFF} depends on T_{AMB} and T_{BOIL} . For argon values of C_{EFF} will be smaller than for nitrogen. The value of Δh_T for argon is also larger than for nitrogen. Thus, according to Eq. 7.29 argon is a “faster” coolant.

7.5.5 Gas Consumption for Cooldown

The amount of gas consumed in a single cooldown process can be approximated by,

$$\Delta n_{CD} = \bar{n}_{CD} t_{CD} \approx 1.5 \dot{n}_{FR} t_{CD} \quad (7.30)$$

which, when combined with Eq. 7.29 yields

$$\Delta n_{CD} \approx \bar{n}_{CD} \cdot t_{CD} \approx \frac{C_{EFF}}{\eta_{CD} \cdot \Delta h_T} \quad (7.31)$$

At elevated values of T_{AMB} , C_{EFF} increases for a given cryocooler and Δh_T becomes smaller, both factors contributing to an increased amount of coolant, Δn_{CD} , required for cooldown.

The pressure dependence of Δn_{CD} is inverse with that of $\Delta h_T(P)$. More gas is consumed at lower pressures because although $\bar{n}(P)$ is lower, the values of t_{CD} are larger and

dominate the magnitude of Δn_{CD} . At elevated pressures, the increased values of $\bar{n}(P)$ dominate and Δn_{CD} increases again. At intermediate pressures near the inversion pressure, the values of Δn_{CD} reach their minimum values. In their numerical modeling of a miniature JT cryocooler, Hong et al.²³ calculate values of $\Delta n_{CD}(P)$. At 10, 30, and 50 MPa the corresponding amounts of nitrogen are 24, 11, and 14 Standard Liters.

As a rule of a thumb, the value of Δn_{CD} for a miniature flow regulated JT cryocooler is equivalent to the gas consumed during a 25 min run, an estimate that should be valid for any pressure of operation. Some applications require periodic cryocooling, or an interruption in the demand for cryocooling. According to the above rule then, one should not turn off the pressure supply for any interruptions that are shorter than about 25 min. From the perspective of economic gas consumption, the subsequent cooldown would consume the same amount as if the cryocooler would have run without interruption.

7.6 Classifications of Rapid Cooldown Cryocoolers

In the following sections we examine rapid cooldown cryocoolers according to three different classifications; each classification includes a different set of expectations, demands and emphasis.

7.6.1 Optimized Cryocoolers

For each specific heat exchanger in a rapid cooldown Joule-Thomson cryocooler there is a corresponding optimal nozzle size such that the associated flow rate minimizes the cooldown time for that particular cryocooler. The existence of an optimal nozzle size and flow rate may be understood qualitatively as follows. On the one hand, a larger flow rate for a given coolant has the potential to both enhance the heat transfer coefficient and produce a higher refrigeration power, thereby accelerating the cooldown process. On the other hand, a larger flow rate will reduce the NTU for a given heat exchanger, effectively making it perform like a “smaller” heat exchanger. Therefore, the effectiveness, ε , and the efficiency, η , (Sects. 3.5.2 and 3.5.3.) also both decrease, and a lower fraction of the potential cooling is realized. These two conflicting mechanisms result in an optimum flow rate that maximizes the cooling power and minimizes the cooldown period.

The qualitative discussion given above is strongly linked to the steady state analysis for the maximum cooling capacity

that can be obtained with a given size heat exchanger. Indeed Sect. 3.9.1 shows that the same two competing mechanisms described in the above paragraph also determine the optimum steady state performance and the associated optimal flow rate. Note that the last moment of the cooldown phase is already the first moment of the steady state operation. Qualitatively, it is reasonable to expect that the two optimal cases, the transient and the steady state, are closely related, that is, that the cryocooler with the highest cooling power at steady state will also have the shortest cooldown period. The integral model also supports this expectation, as explained in Sect. 7.5. Following Eqs. 7.28 and 7.29,

$$t_{CD} \approx \frac{4}{3} \frac{C_{EFF}}{\dot{Q}^{SS}} \quad (7.32)$$

indicating that the cooldown period is directly (though inversely) related to the steady state cooling power.

The predicted steady state optimum occurs at relatively “short” values of heat exchanger NTU such as 1.25 for argon and 3.5 for nitrogen. In practice, the rapid cryocoolers are indeed short and have relatively low values of effectiveness. Following the steady state analogy, an “optimized” cryocooler that is too short will exhibit a sharp increase in the cooldown time because (or in spite) of an excessive flow rate.

A complementary perspective of optimum steady state performance that is closely related to the optimization of cryocooler cooldown time is described in Sect. 3.9.2. For a given flow rate, one may derive the size of heat exchanger that maximizes the ratio, $\dot{Q}/(UA)$, namely, the cooling power per unit of area of heat exchanger (A). Note that more heat transfer area, A , means a larger heat exchanger and thus a larger value of C_{EFF} . Hence, the quantity $\dot{Q}/(UA)$ corresponds with the parameter $\dot{Q}^{SS}/C_{EFF}$ which by Eq. 7.32 is inversely proportional to t_{CD} .

Figure 7.13 is a quantitative display of flow optimization for fixed heat exchangers. Note that the nitrogen flow rate that optimizes a given heat exchanger is smaller (by about half) than the optimum argon flow rate for the same heat exchanger. Put another way, one may increase the argon flow rate much more than that of nitrogen and still not reach the condition of a heat exchanger, that is “too short” in that it begins to degrade the cooling. Based on this comparison, one may say that the thermodynamic parameters of nitrogen are inferior; for optimum performance at the same flow rate, nitrogen requires a larger heat transfer area than argon. Therefore, the cooldown rate with nitrogen is attenuated “earlier”, or at a lower flow rate than with argon. Furthermore, the cooldown time for an optimized nitrogen cooler is a few times longer than that of an optimized argon cooler.

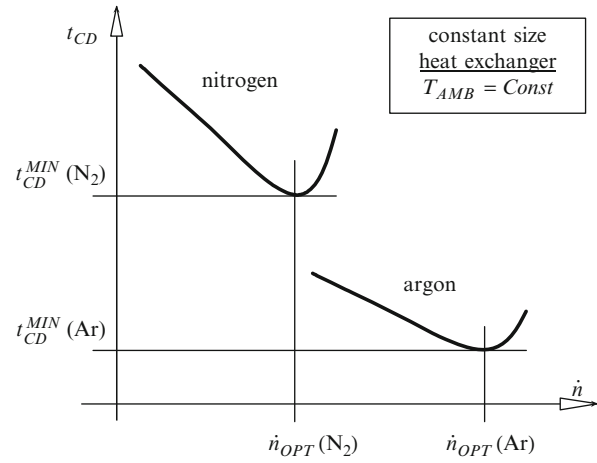


Fig. 7.13 Flow rate dependence of cooldown times for cryocoolers sharing the same size heat exchanger. Optimized rapid cooldown cryocoolers are those that employ the optimal flow rate

A study of optimized fixed-orifice rapid cooldown cryocoolers²⁴ was already discussed in Sect. 5.5.6.1. In general, cryocoolers optimized for use with nitrogen are larger than those optimized for use with argon, although the nitrogen cryocoolers have longer cooldown times, t_{CD} .

7.6.2 Cryocoolers with a Common Efficiency and Backpressure

In the second classification, all cryocoolers share a common efficiency, η , and back pressure in the evaporator, ΔP . The first parameter affects the values of $\dot{Q}$, t_{CD} , sizes, and cutoff pressure, while the second one determines the operating temperature through its dependence on the increased saturation pressure above the boiling point. Little²⁵ derived the following scaling relations for this group of cryocoolers:

$$t_{CD} \propto \dot{n}^{0.6} \quad (7.33)$$

$$D \propto \dot{n}^{0.5} \quad (7.34)$$

$$L \propto \dot{n}^{0.6} \quad (7.35)$$

Here L , and D , are the length and diameter respectively, of the heat exchangers. Longworth²¹ developed matrix-tube heat exchangers with the expectation that they would enable a more rapid cooldown than the traditional finned tube cryocoolers. As displayed in Fig. 7.14, Longworth grouped the calculated values of t_{CD} , L , D , and weight as functions of the steady state cooling power. Since all the investigated cryocoolers share a common supply pressure (34 MPa) and efficiency ($\eta = 0.9$), the cooling power actually represents the flow rate. The constant back- pressure of 135 kPa

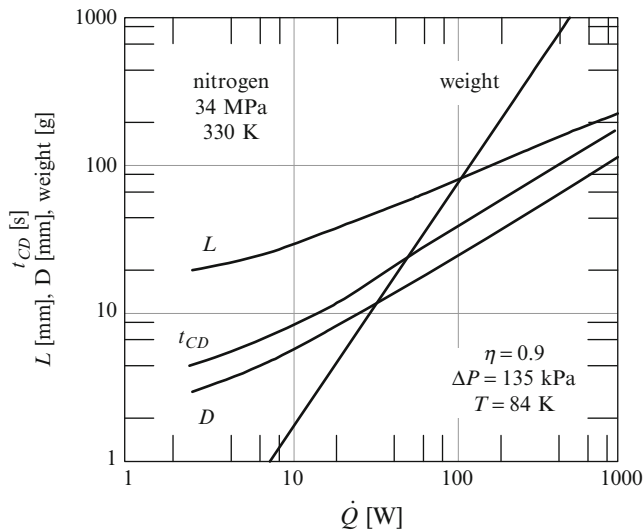


Fig. 7.14 Matrix-tube cryocoolers, optimized for fast cooldown, sharing the same efficiency, η , and back pressure, ΔP , elevating the temperature of operation by nitrogen, to 84 K, after Longworth²¹

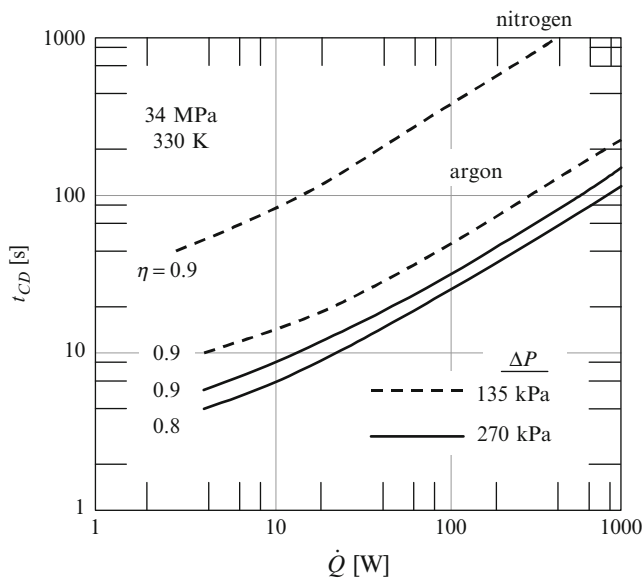


Fig. 7.15 Matrix-tube cryocoolers, optimized for fast cooldown, sharing the same efficiency, η , and back pressure elevation, ΔP , operated with argon, after Longworth²¹

corresponds to a saturation temperature of 84 K for nitrogen and 96 K for argon. Similarly, Fig. 7.15 displays the values of t_{CD} in an argon driven cooler with two different efficiencies, 0.8 and 0.9, and two different backpressures, 135 kPa and 279 kPa. One may observe that coolers with lower efficiency are faster.

Longworth found that the scaling relations as predicted by Little by Eqs. 7.33, 7.34, and 7.35 provided a very close fit to his measurements displayed in Figs. 7.14 and 7.15.

Within this group of cryocoolers, the smaller sizes and the associated lower flow rates enable the cryocooler to cool

Table 7.1 Comparison of three classifications for rapid cooldown cryocoolers, sharing the same initial conditions of supply pressure and ambient temperature. Here the relationships between an increasing (up arrow) or decreasing (down arrow) value of $\dot{n}$, the temperature of cryocooling, T , and the length of the cryocooler, L , with a shortening cooldown period, t_{CD} are shown

Classification of fast cooldown cryocoolers	t_{CD}	$\dot{n}$	T	L
Optimized cryocoolers	↓	↑	↑	↑
$\eta, \Delta P = \text{Const}$	↓	↓	—	↓
$L = \text{Const}$	↓	↑	↑	—

down faster. However, the same trend also produces a smaller and smaller cooling power. Nevertheless, some discussion is required to explain why a larger flow rate is not associated with a faster cooldown. The key to the puzzle is associated with the constraint imposed on the group of parameters (t_{CD} , L , D , and weight) in order to preserve constant values of η and ΔP . If the flow rate $\dot{n}$ is increased, the cryocooler must grow in size in order to preserve the efficiency and backpressure. Consequently, the thermal mass expands, which in turn increases the cooldown time, t_{CD} . Figure 7.14 clearly displays this feature, noting that $\dot{Q}$ comprises an alternative scale for the flow rate. Figure 7.15 displays that the cooldown time for a given cryocooler can be shortened by increasing the backpressure (270 instead of 135 kPa) or lowering the efficiency (0.8 instead of 0.9).

7.6.3 Comparison of Cryocooler Classifications

In the third and final classification, previously discussed in Sect. 5.5.6.2, the cryocoolers are characterized by a fixed recuperator length, but with different nozzles (and flow rates). In this case, each of the relevant parameters, which include $\dot{n}$, L and the temperature of cryocooling, T have a different affect on the cooldown time t_{CD} . Table 7.1 summarizes the trends which accompany the decrease in t_{CD} .

With the optimized cryocoolers (for minimum t_{CD} , see Sects. 7.6.1 and 5.5.6.1), one must chose a longer cryocooler in order to obtain a shorter value of t_{CD} . While this change will also increase the cooling power, it will also increase gas consumption and raise the operating temperature due to the larger ΔP . However, within the second classification, (constant η and ΔP), the trend is opposite; a smaller cryocooler, accompanied with less gas consumption, is the faster one. Within the third classification, the cooldown time is shortened when the flow rate increases.

Note from Table 7.1 that $\dot{n}$ and L relate in a similar fashion to changes in t_{CD} . The cryocooled temperature, T , exhibits an inverse dependence on t_{CD} ; shorter values of t_{CD} are accompanied by larger values of ΔP (unless ΔP is preserved) and correspondingly higher values of T .

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Compared to the operation and performance of pure coolant Joule-Thomson cryocoolers, the mixed coolant versions introduce new and previously unattainable possibilities. In particular, mixed coolants enhance the performance of JT coolers by enabling equivalent (even improved) performance at reduced operating pressures, increasing the specific cooling capacity, and achieving lower temperatures.

8.1 Introduction

8.1.1 Classification of Mixed Refrigerant Joule-Thomson Cryocoolers

The three main categories, as displayed by columns A, B and C, in Fig. 8.1 are:

8.1.1.1 The Linde-Hampson Mixed Coolant Closed Cycle Cryocooler

Remarkable improvements and opportunities emerge for Linde-Hampson cryocooling merely by replacing the pure coolant with a mixture of gases. This may be regarded as an evolution of the traditional Linde-Hampson cycle. The change enables a dramatic decrease in the operating pressure while preserving the capability of reaching temperatures as low as 75 K. A single stage compressor can suffice to generate the reduced operating pressure of only about 2.5 MPa. Hence, the entire closed cycle cryocooler becomes a reliable system. This category of cryocoolers (often operating below 100 K) and its associated mixtures are the central subject of the present chapter as discussed in Sects. 8.2, 8.3, 8.4, 8.5, 8.6, and 8.7.

8.1.1.2 The Auto-Cascade Closed Cycle Cryocooler

The auto-cascade closed cycle cryocooler presents another method of employing mixed refrigerants at reduced operating pressures of about 2.5 MPa. Quite often, it is referenced in the literature as Kleemenko's cycle. It includes

phase separators (one or more), throttle valves (at least two) and corresponding recuperators. The phase-separators remove the liquid phase from the stream at various junctions along the high-pressure channel. Since the liquid phase is enriched with higher boiling point components, its removal is a means of altering the composition of the coolant. Indeed, in contrast to the Linde-Hampson cycle, which maintains a constant composition along the recuperator, the auto-cascade cycle maintains a composition gradient along the recuperator; the coolant is enriched with lower boiling point components towards the lower temperature end of the recuperator. Section 8.8 discusses this type of cryocooler and compares the auto-cascade cycle to the Linde-Hampson closed cycle.

Section 8.9 describes typical applications of both kinds of the Joule-Thomson closed cycle cryocoolers and compares them with the compact Stirling type cryocooler.

8.1.1.3 The Linde-Hampson Mixed Coolant Open Cycle

The Linde-Hampson mixed coolant open cycle, incorporates a pressurized gas reservoir, and represents the high-pressure version of mixed coolant cryocoolers. In this case, the potential of the mixed refrigerants is translated into an increased specific cooling capacity or is utilized for a faster cooldown. The last Sect. 8.10, focuses on this category of mixed coolant cryocoolers.

8.1.2 The Synergy of Mixed Coolants

Before coming to the key advantage of mixed coolants, it should be clarified that it is not simply the fact that the Joule-Thomson effect for a mixture is enhanced compared to that of a single component coolant. Indeed, adding gases of a higher boiling point to any gas makes a mixture with a higher integral Joule-Thomson effect than the original lower boiling point gas. However, this is not the primary advantage since

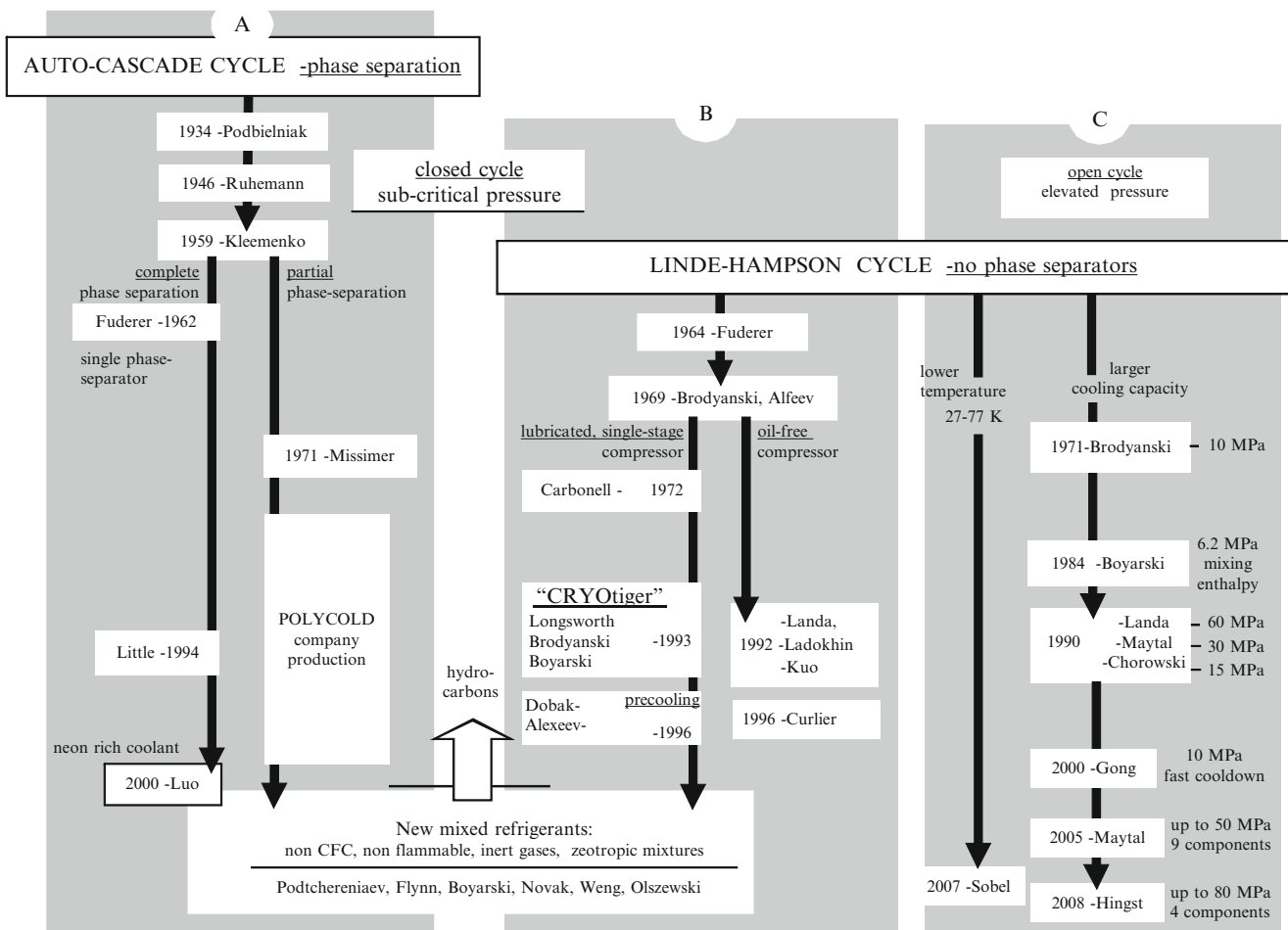


Fig. 8.1 Classification and chronology of throttle cryocooling cycles with mixed refrigerants. *Column A:* The earlier trend is the closed cycle incorporating phase separators, as originally proposed by Podbielniak¹ but which carries the name of Kleemenko.² *Column B:* The next version is a derivative of the traditional Linde-Hampson cycle but with mixed coolants and without phase-separators. This kind of closed cycle was originally proposed by Fuderer.⁵ *Column C:* Open cycle operation at high pressure and mixed coolants is another class of the same version. The names in this chart corresponds to the first authors of the associated publications

no real synergy is associated with this enhancement of the integral Joule-Thomson effect. Indeed, the Joule-Thomson effect of a mixture is comparable with the linear combination of the Joule-Thomson effect in the individual gases weighted by their compositions (as discussed in Sect. 2.2.6).

The most significant advantage of a mixture is associated with its boiling point; it remains quite close to that of the lowest boiling point component instead of being elevated as a result of the presence of higher boiling point components. This feature results when the liquid phase of the mixture splits into two or more liquid phases, in equilibrium at the temperature of the lower boiling point component. Then, even if the fraction of the lowest boiling point component is small, the mixture may still boil at this low temperature.

The general trend as previously displayed in Sect. 4.3.8, relating T_{BOIL} and Δh_T , is not valid for mixed gases. It is possible to have a mixture and a pure gas with a similar

value of T_{BOIL} , and yet the mixture may exhibit a higher value of Δh_T at the same pressure.

Put another way: the added higher boiling point components create a mixture with a “larger” two phase dome and a higher critical temperature than that of the lowest boiling point gas. The normal boiling point of pure gases, T_{BOIL} , is about $0.6 T_C$ as discussed in Sect. 4.2 by Eq. 4.1. The synergy of mixtures manifests itself with a suppressed value of T_{BOIL} which is significantly below the value of $0.6 T_C$. However, the enhancement of the integral Joule-Thomson effect does correspond with the elevated T_C of the mixture, as consistent with the principle of corresponding states, and no special synergy is associated with that feature. The capability to reach the mixture’s suppressed value of T_{BOIL} via the interchanging process is evidence of reduced irreversibilities, $\dot{I}/\dot{n}$, and an elevated FOM for the mixed coolant closed cycle in comparison to pure coolants.

8.1.3 Miscellaneous

8.1.3.1 Chronological Notes

A chronological account of the three categories of mixed coolant cryocoolers is displayed in Fig. 8.1. The auto-cascade cycle (column A) is associated with the earliest date, being proposed already in 1934 by Podbielniak¹ and reinvented by Kleemenko² and Fuderer.^{3,4} The Linde-Hampson closed cycle (column B) is introduced at a later date, being first proposed by Fuderer⁵ in 1964. Conceptually and practically, this cycle resembles the traditional Linde-Hampson cycle of pure coolants since it does not employ any phase separator. Nevertheless, the evolution of mixed coolant closed cycle cryocoolers started with the auto-cascade cycle. Open cycle cryocooling, which takes advantage of mixtures at high-pressure, is the last and the least studied category (column C).

8.1.3.2 Interchanging with Mixed Coolants

With regard to the unified view of cryocoolers as introduced in Chap. 1, the mixed coolant cycles provide opportunities for less common ratios of capacity rates. Pure coolants have in general a high pressure capacity rate, $\dot{C}_H$, that is larger than that of the low pressure flow, $\dot{C}_L$. In the case of mixed coolants it is also possible to produce values of $\dot{C}_L$ larger than $\dot{C}_H$. Furthermore, this relation may change along the recuperator; beginning with larger values of $\dot{C}_H$ and gradually changing to larger values of $\dot{C}_L$, or vice versa.

8.2 The Mixed Coolant Linde-Hampson Cycle

8.2.1 Chronological Perspective

Although the development of mixed-gas Joule-Thomson cryocoolers have progressed through various complex, sophisticated routes, in the end the cycle may be simply characterized as a modification of the traditional Linde-Hampson cycle. A review article⁶ regarded it “a renaissance in the Joule-Thomson refrigerators, used first by Linde in 1895 to liquefy air continuously”. The simple substitution of a pure coolant by a mixture of gases results in a very significant improvement. Therefore, this cycle will be subsequently referred to as the *mixed coolant Linde-Hampson cycle*, although Linde and Hampson only intended to liquefy pure gases. Linde mentions liquefaction of mixtures for the purpose of air separation to produce oxygen rich air. This cycle is also known in the literature as a “mixed gas Joule-Thomson cryocooler” or just a “throttling cycle cryocooler”.

Fuderer⁵ appears to be the first to suggest a low-pressure Joule-Thomson cryocooler using mixed coolants. He filed a patent in Germany to defend the idea in as early as 1964 and

it was finally granted in 1969. However, to the best of authors’ knowledge, he did not publish any further studies or subsequent references on the topic. This may explain why Fuderer’s proposal did not attract the proper attention it really deserves in the cryocooler community.

In 1971 Brodyansky, Gresin, Gromov, Yagodin, Nikolsky and Alpheev from the former Soviet Union presented a short paper⁷ at the 13th Congress on Refrigeration in Washington DC, describing the theoretical basis for, and some experimental results of “The use of mixtures as the working gas in throttling ‘Joule-Thomson’ cryogen refrigerators”. The quotation marks on Joule-Thomson reflect the fact that traditional Joule-Thomson refrigerators do not apply mixed coolants. The paper became strongly identified with this new class of cryocoolers and was primarily cited in this context. The other frequently cited article is the patent of Alfeev, Brodyansky, Yagodin, Nikolsky and Ivatsov⁸ filed during the same year in the UK. Alfeev, Brodianski, Yagodin and Nikolsky filed a Russian patent on mixed coolants already in 1969⁹ and in 1970.¹⁰ Early reports in Russian on this subject may be found elsewhere.^{11,12,13,14} Other former USSR contributors are Boiarski, Abakumov, Zacharov, Landa, Yudin, and Ladokhin. In contrast to Fuderer, the Russian group actively reported their experiments and theoretical studies, and thereby evoked interest and gained recognition. Bodio¹⁵ published a comprehensive survey of these activities, including the principles and applications of the mixed coolant Linde-Hampson cryocoolers, in 1985 in Polish. Additional information in the form of reviews may be found elsewhere.^{16,17,18}

An interesting attempt was reported by Carbonell et al.¹⁹ in 1972. Recuperation of the mixed coolant was accomplished using multiple regenerators and the DC flow of the compressor was altered by a four-way electromagnetic valve, as already discussed in Sect. 1.3.6.1.

Development of mixed-gas throttle cycle coolers was further promoted by Professor William Little from Stanford University^{20,21} and the founder of MMR Technologies Inc. in Mountain View, CA. He focused attention in the United States on the potential of Alfeev’s patent⁸ and verified the reported performance of some of the proposed mixtures. Little describes elsewhere²² how further development became possible only after the fall of the Iron Curtain. Chan²³ also discussed the potential of mixed coolants. Subsequently, a collaboration was established^{16,24,25} between Brodyansky and Boyarsky with Longworth and Kharti of APD Cryogenics, Inc. in Allentown, PA.

Additional contributions to this field, include those by Radebaugh and Marquardt of NIST in Boulder, Colorado, Dobak in San Diego, California, Nellis in Madison, Wisconsin, Alexeev, Quack and Haberrstroh in Dresden, Germany, Oellrich in Karlsruhe, Germany, Curlier and Ravex in Sassenage,

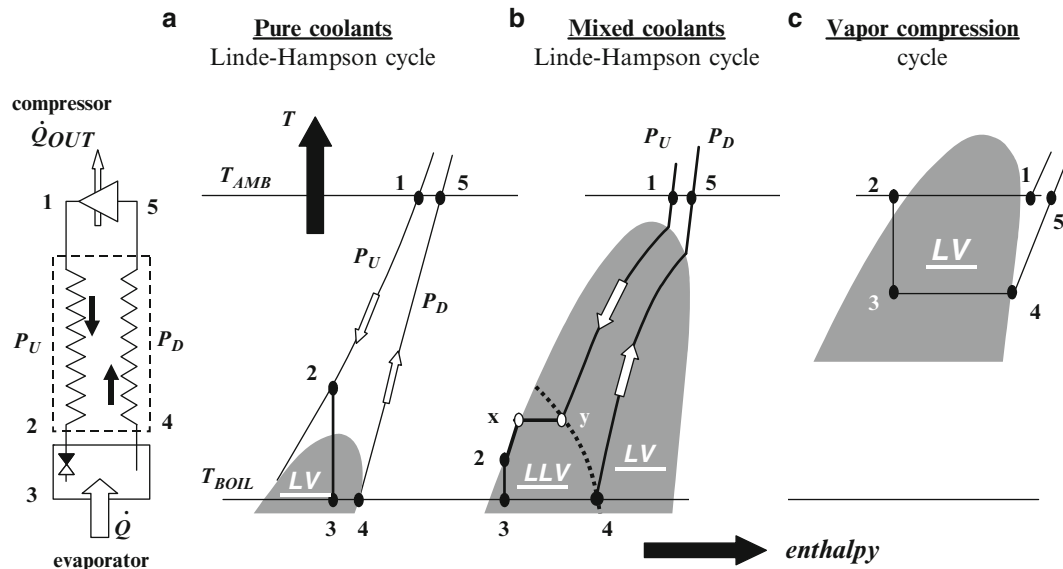


Fig. 8.2 Comparison of throttle cycles: pure coolant Linde-Hampson cycle, mixed coolant Linde-Hampson cycle, and pure coolant vapor-compression cycle

France, Luo, Liang, Gong, Wu, and Zhou at the Chinese Academy of Sciences, Chorowski and Bodio in Wrocław, Poland, Venkatarathnam, Kumar, and Murthy in Madras, India, and Atrey in Bombay, India.

8.2.2 Description of the Mixed Coolant Cycle

Figure 8.2 provides a qualitative display in the T - h plane of the two versions of the Linde-Hampson cycle. The first diagram (a) is the traditional pure coolant cycle and the second diagram (b) is that of a mixed coolant. The notation defining the various states is similar for both cases and remains consistent with those in Chap. 3. Comparison of the two enables one to explore some characteristics of the mixed coolant version. One may clearly observe the general difference: most of the mixture cycle occurs inside the two-phase (L-V) dome in contrast to the pure gas cycle. Similarly but differently, this issue also characterizes the traditional vapor compression cycle of pure coolants without any recuperation, as shown for reference in Fig. 8.2c.

Note the following features of the Linde-Hampson mixed coolant closed cycle:

- The two-phase, liquid-vapor (L-V) dome of the mixed coolant is much “higher” (in critical temperature) and “broader” (in latent heat) than that of the pure gas.
- The recuperated enthalpy, $h_1 - h_2$ or $h_5 - h_4$, of the mixed coolant cycle is larger than that of the pure coolant, and thus may require more heat transfer area in the recuperator.
- The isobars inside the L-V dome of the mixture are not isotherms (thus not horizontal all along the T - h coordinates).

- Most of the isobaric trajectories are inside the L-V dome of the mixture. This indicates that the *upstream pressure*, P_U , is *sub-critical*. Hence, the applied pressure of the mixture is quite low in comparison with the pure coolant cycle where the applied pressure can be as high as about $10 P_C$.
- Most of the recuperated enthalpy (Sects. 1 and 2) of the mixture is located inside the L-V dome. Thus, the counter flow channels in the heat exchanger conduct mainly two-phase streams. The high-pressure stream begins to condense (at point x) close to the inlet. The complete evaporation of the low-pressure exhaust stream occurs only (at point y) close to the outlet.
- Only part of the liquefied mixture in the evaporator boils off. The returning downstream flow (state 4 in diagram b) includes a large fraction of liquefied mixture that boils along the recuperation path. This is in contrast to the pure coolant cycle, where the high-pressure stream is entirely a single phase. Two phases exist only after expansion where liquefaction takes place. However, the flow leaving the evaporator is essentially liquid free.
- The preferred mixture for more stable operating temperatures has a composition such that the liquid phase (L) actually splits into two (or more) coexisting liquid phases, (L1, L2) at the low temperature. Inside the two-phase, LV, dome, Fig. 8.2b shows a smaller LLV dome where the two liquid phases are immiscible. The isobars in this zone are isotherms (horizontal Sects. x-y and 3-4), indicating that such a mixture boils like a pure substance at a discrete temperature (see Sect. 8.5.4.4). Brodianski et al.²⁶ observed this split of the liquid phase into two liquid phases while studying mixtures of N_2 - C_2H_6 - C_3H_8 .

- (h) Conservation of mass, for each separate component, dictates that the average composition of the circulating coolant at any location around the entire cycle at steady state remains the same. The composition may vary during the cool down, or during any transient occurrence, as a component of the liquefied coolant accumulates or boils off at some point in the loop.

8.2.3 Characteristic Features

In comparison to cryocooling with pure coolants, one may note the following features of closed cycles with mixed coolants:

8.2.3.1 Low Pressure of Operation

Even at about 2.5 MPa the mixed coolants can provide a specific cooling content, Δh_T , which is comparable to that of pure nitrogen at pressures as high as about 30–40 MPa.

8.2.3.2 Suppressed Boiling Point of the Mixed Coolant

The boiling point of the mixture remains close to that of the lower boiling point component and holds steady, by virtue of splitting the liquid phase into two (or more) coexisting immiscible liquid phases in mutual equilibrium with the vapor phase. Even if a glide exists, it is quite moderate.

8.2.3.3 Balanced Recuperation

The pair of two-phase recuperative streams have similar specific heat capacities (and capacity rates); much more alike than in the case of pure coolants. In the T - h diagram, this feature produces the nearly parallel isobaric Sects. (1–2 and 4–5), and results in smaller temperature differences, $\Delta T = T_U - T_D$, between the streams then for pure coolants as discussed in Sect. 3.11.3.

8.2.3.4 Reduced Entropy Generation

Reduced entropy generation at the expansion valve and in the recuperator due to:

1. The isenthalpic expansion at the nozzle: less entropy is generated because the throttled fluid is colder and more dense, according to Eq. 3.153.
2. Smaller temperature differences, $\Delta T = T_U - T_D$, across the heat exchanger and between the streams, generates less entropy, according to Eq. 1.44.
3. Furthermore, the longitudinal distribution of ΔT along the heat exchanger is such that smaller values occur towards the cold end and larger values exist at the warm end. This is demonstrated in Fig. 8.3 by Brodianski et al.¹¹ measurements of temperatures T_U and T_D along a heat exchanger operated once with pure nitrogen and once

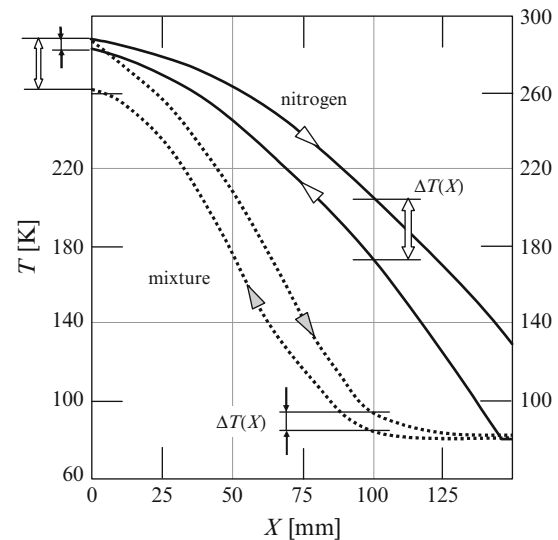


Fig. 8.3 Typical temperature distribution along the recuperator of a Linde-Hampson cryocooler operated by a pure nitrogen and by a mixture of nitrogen, Freon and neon, as measured by Brodianski et al., in reference.¹¹

with a mixture. Such a dependence of ΔT on T_U , exhibiting a pinch point at the cold end, means that $\dot{C}_H/\dot{C}_L < 1$ at the lower temperature range. This behavior is closer to the optimal case²⁷ of minimum entropy generation (Sect. 1.2.4.4) than if ΔT would increase toward the cold end as in the case of pure coolants.

8.2.3.5 The Temperature at the Entrance to the Nozzle

The temperature at the entrance to the nozzle (state 2), is lower than that of a pure coolant (with a similar boiling point as that of the mixture). Most of the mixture, or even all of it, at state 2 is liquid. Consequently:

1. A large fraction of the stream remains liquid, after expansion at state 3,
2. The Joule-Thomson temperature drop at the nozzle of the expanding dense phase, $T_2 - T_3$, is significantly smaller than in the case of a pure coolant.
3. The temperature drop, $T_2 - T_3$, plays the role of the elementary cooling mechanism, δT , in the sense of Sect. 1.1.1. The reduced value of δT is compensated by an increased value of I_M , due to the more balanced interchanging process by mixed coolants (Sect. 8.2.3.3).

8.2.3.6 Possible Non-choked Flow Through the Throttle

Possible non-choked flow through the throttle with mixed coolants provides another point of distinction versus compared to the pure coolant cryocooler.²⁸ Recall that non-choked flow conditions are determined both by the up stream and down stream pressures.

8.2.3.7 Increased Thermodynamic Efficiency

A lower operating pressure means less work of compression for an equivalent production of cooling. The similar values of Δh_T elevate the COP according to Eqs. 1.88 and 3.168. Lowering the mixture's boiling temperature, T_{BOIL} enables a higher FOM according to the corollary in Sect 1.4.1.4 or Eq. 3.200. If the boiling temperature T_{BOIL} is not sufficiently low, the ideal coefficient of performance, COP_{CARNOT} will be high and the FOM will not increase even if the COP does. This observation also reflects the lower rate of entropy generation as explained in Sect. 8.2.3.4, that is the explicit reason for the increased FOM. For cryocooling at the 80 K range, typical values of the FOM for mixed coolants range from 30% to 50% at a pressure of about 2.5 MPa. Such values are comparable with those of vapor compression coolers operating near 250 K.²⁹ Pure coolants reach maximum FOM values of about 20% at a pressure of about 30 MPa.

8.2.3.8 The Distribution of Exergy Losses

The distribution of energy losses for pure coolant cryocoolers was determined in Sect. 3.13.4. The major destroyer of availability in that case is the throttling process, that destroys about 60%; another 20% is lost in the recuperator. However, for a mixed coolant³⁰ cycle, the irreversibility generated in the JT valve is less than 10% while that in the recuperator is about 50%.

8.2.3.9 A Larger Heat Transfer Area

A larger heat transfer area is required to accomplish the recuperation of mixed coolants. Because the capacity rates are more balanced, a larger NTU is required for a given desired effectiveness of the recuperator. This feature may also be recognized as the result of both a smaller driving potential for heat exchange, $\Delta T = T_U - T_D$, between the mixed coolants as compared to a pure coolant, and because the recuperative enthalpy, δh , to be exchanged is significantly larger than that for a pure coolant (as demonstrated in Fig. 8.2). On the other hand, the large liquid portion in each stream of the mixed coolant system enhances the heat transfer process between the streams. Nevertheless, the combination of the aforementioned factors results in a larger heat transfer area, typically $NTU > 50$, as discussed by Alexeev et al.³¹

8.3 Mixed Coolant Linde-Hampson Cryocoolers

8.3.1 Introduction

Most mixed coolant Linde-Hampson cryocoolers are pressurized by a single-stage, lubricated compressor and use a mixture of hydrocarbons and nitrogen as the coolant.

Lubricated compressors require an oil management system. In contrast to the oil-free compressors, the lubricated type have a longer lifetime and a lower price. However, the oil-free compressors avoid the complication of the oil management system, tolerate coolants with mixed halogenated hydrocarbons, and are insensitive to orientation.

The main parameters that distinguish the various types of closed cycle cryocoolers are the temperature of operation, type of compressor, mixture composition, whether the coolant is single phase or double phase at room temperature, whether the cryocooler has a pre-cooling stage, its rate of cooldown, the recuperator size, and the thermodynamic efficiency.

8.3.2 Oil Free, Two-Stage and Single Stage Compression

The earlier versions of mixed-coolant Linde-Hampson cryocoolers employed two-stage, oil-free diaphragm compressors and produced pressures in the range of about 4.5–7.5 MPa. The mixtures of Alfeev et al.^{8,14} are compressed to 5 MPa, and therefore require a two-stage compression process. Landa et al.³² used mixtures of freons, N_2 -R13-R14-Ne, compressed at steady state to 7.1 MPa.

Kuo³³ built a two-stage linear compressor with a dual opposing collinear reciprocating pistons each reacting against elastic resonating elements. The electromagnetic drive includes a permanent magnet and a coil that moves within the fixed magnetic field. The compressor is oil-free, but a self-lubricated composite covers the pistons and their guides. The cryocooler has a fixed orifice and a finned tube heat exchanger operating within a vacuum jacket (dewar). When pressurized with a hydrocarbon mixed coolant at 5.45 MPa, the flow rate at steady state is about eight SLPM. The configuration and size of the heat exchanger are similar to cryocoolers used for infrared detectors, which employ pure gases.

Additional attempts with hydrocarbon mixtures were reported by Ladokhin et al.,³⁴ Curlier,³⁵ and Luo et al.³⁶ The latter used a two stage compressor that was truly oil free, but operated at 2.5–6.5 MPa.

Curlier³⁷ used a single stage oil-free compressor, with a 1 cm³ swept volume, and with a compression ratio of 4, discharging at 2.4 MPa. The cooler pumped 0.5 W from 140 K to an ambient temperature of 70°C, while consuming 10 W of electrical power.

8.3.3 Lubricated, Single-Stage Compression

The viability of constructing a closed cycle cryocooler for the 80 K temperature range, based on a single-stage,

Table 8.1 Four mixtures examined by Longworth, Brodianski et al.^{17,41} at 5 and 2 MPa when the discharge pressure was 0.1 MPa. Mixtures 1, 2, 3, and 4 are of by Alfeev et al.⁸ while mixture 5 is of Boyarsky et al.⁴²

Mixture No.	Composition, % by volume											
	1		2		3		4		5			
Ne					20							
N ₂	40		30		25		30		30			
CH ₄			30		25		35		15			
C ₂ H ₆	30		20		15		35		25			
C ₃ H ₈	30		20		15				30			
Performance												
Pressure [MPa]	5		2		5		2		5		2	
Min. temperature [K]	78		78		80		80		67		67	
At load temperature [K]	78		78		99		80		71		70	
Δh_T [J/mole]	940		480		1,700		610		560		360	
FOM [%]	30		19		37		23		19		16	
									29		16	
									34		31	

lubricated compressor represents a further technical advance, no less significant than the original invention of the mixed coolant cycle itself. One may view it as the last section of a long race: it could not occur without the prior pioneering efforts, but the mission would remain incomplete without the final phase. Some of the key ideas for this technology grew out of the prior experience of developing a highly reliable oil-lubricated helium compressor for the long-term operation of 4 K GM/JT refrigerators.^{38,39} However, the advance was only realized subsequent to the following two enabling conditions:

- The possibility of reducing the operating pressure down to the range of 2 MPa, and,
- The ability of the associated mixtures of hydrocarbon components to tolerate a higher level of oil vapor without clogging the orifice.

Longworth et al.^{25,40,41} at APD Cryogenics, Allentown, Pennsylvania, and Brodianski et al.¹⁷ investigated the performance of Alfeev's mixtures⁸ at the lower pressure of 2 MPa, instead of 5 MPa as originally proposed, and of a mixture used by Boyarsky et al.⁴² The results are displayed in Table 8.1. The Δh_T values obtained in these investigations were about one to two thirds of the values obtained at the higher pressures, but the temperatures remained the same or dropped even lower. After some adjustments of mixture composition, Longworth was able to demonstrate and patent a "Cryogenic refrigerator with a single-stage compressor"⁴³ that was adequate for achieving a pressure up to about 2.5 MPa. Luo et al.³⁶ also measured the influence of a pressure reduction from 4.5 to 2.5 MPa on performance, although they carried out their investigation with a two stage, oil free compressor. It is interesting to note that about 30 years earlier, Fuderer⁵ suggested reducing the pressures down to two and even 0.7 MPa, albeit for operation at higher temperatures.

In each of their activities, Longworth, Boiarski and Klusmier^{25,44} and Khatri^{24,45} concentrated on using

hydrocarbon mixtures, which exhibit a reduced sensitivity to contaminating oil vapors that would otherwise form particulates and block the expansion nozzle. This remarkable feature is due to the higher solubility of the aforementioned solid contaminants in the liquid phase of the mixed coolants. Marquardt et al.⁴⁶ point out that impurities up to a level of 2% oil vapor remain safe and do not freeze out. The reduced sensitivity to oil vapor, enabling operation based on standard lubricated compressors and standard oil separators, provides a significant advantage for the hydrocarbon mixtures, and has facilitated their widespread use. Note that the use of an oil separator is still essential; without it the system could not cool below the lubricant's freezing point, that is about 215 K as discussed elsewhere.⁴⁷

Longworth's demonstrated use of a single stage lubricated compressor, described for example in the publication "80 K throttle cycle refrigerator cost reduction"^{25,48} is a remarkable accomplishment. The oil-lubricated compressors, manufactured to supply the world's residential air-conditioning and refrigeration demand, are produced in large quantities with automatic dedicated tools, thereby dramatically reducing their price. Furthermore, the lubricated compression technology guarantees reliable operation for more than 20,000 h. "Cryotiger" is a commercial cooling system based on this technology, developed by APD Cryogenics, Inc.

Oil separation compressors also typically employ an absorbing element^{48,49,50,51,52} to collect oil vapors, H₂O and CO₂. The life time of the entire cooling system is very often dictated by the absorbing capacity of this element. Figure 8.4 displays the scheme of such a cycle. Furthermore, sometimes the mixture has to be replaced since the higher boiling point components are better absorbed by the absorbent and therefore the composition of the mixture gradually changes. It deteriorates the performance of the system being unable to provide the expected cooling power as taught elsewhere.⁵³ Instead of

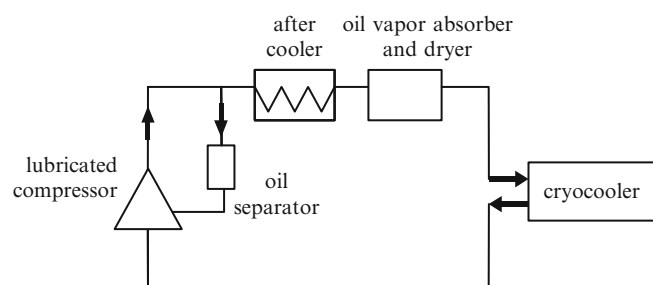


Fig. 8.4 The mixed refrigerant Linde-Hampson cryocooling system includes a lubricated compressor, an oil-separator, an absorber of oil and water and carbon dioxide vapor, and a cryocooler

mechanically separating only liquid particles, Quack and Alexeev^{50,51} focused on the extraction of oil vapor with an additional gaseous separator. The next section discusses the use of a precooler for the same purpose.

Additional studies focusing on the use of lubricated, single-stage compression are reported by Alexeev, Quack, and Haberrstroh.^{30,54,55,56} Also Luo, Zhou, and Gong^{57,58} explored further reductions, from 2.5 MPa down to 1.2 MPa as well as associated applications.⁵⁹

8.3.4 Precooled Mixed-Coolant Closed Cycles

The precooling cycle associated with mixed-gas systems is typically comprised of a vapor compression cycle using either a pure coolant or another mixed refrigerant, at an intermediate temperature.

8.3.4.1 Advantages

The benefits of a precooled cycle are the same as those discussed for multi-stage cryocoolers in Sect. 5.4. The auxiliary loop may also operate with a single stage lubricated compressor much like the main loop. The expanded stream of the precooling cycle cools the counter flowing warm stream of the mixed coolant main loop, with the aid of a heat exchanger, marked as HE1 in Fig. 8.5.

Alexeev et al.⁶⁰ identified additional potential advantages of precooling, which are especially profitable for mixed coolant closed cycles^{61,62,63,64}:

Enhanced Oil Removal

A portion of the precooling stream is throttled (down to about somewhat below zero Centigrade) and utilized to condense oil vapor at the discharge of the mixed coolant stage with heat exchanger two. In this way, the oil entering the oil separator is mainly liquid, thus being more efficiently separated from the stream. For another strategy of oil removal, see Sect. 8.3.8.4.

Eliminating the Use of an Absorbent

Eliminating the use of an absorbent that must be periodically replaced, and that might affect the composition of the mixture.

Eliminating Higher Boiling-Point Components

Eliminating higher boiling-point components that would be used to enhance the cooling capacity of the mixture. Since the precooling stage provides such an enhancement, it,

- (a) Reduces the chance of solidification and clogging, and,
- (b) Avoids the use of mixtures, with components that exist as two phase fluids at room temperature, (LRS kind, see Sect. 8.3.8.1)

Alternatively, note that the Kleemenko cycle, with its use of phase separators, produces a similar effect to that of precooling, while using only a single compressor.

8.3.4.2 Precooling with a Closed Cycle Vapor Compression System

Alexeev et al.⁶⁴ proposed a precooled system using a closed cycle vapor compression system that employs refrigerant R-507. The primary cycle circulated a mixture of hydrocarbons (CH_4 - C_2H_6 - C_3H_8) with nitrogen as the lowest boiling point component and was optimized for the highest specific cooling content in the temperature range of 95–100 K. They reported an efficiency about 1.5 times higher efficiency than a Gifford-McMahon cryocooler, which is indeed impressive for any throttling cryocooler. Skye, Nellis, and Klein^{65,66,67} discuss the use of an R-22 vapor compression cycle to precool a closed cycle in which a mixture of N_2 , Ar, CH_4 , C_2H_6 , C_3H_8 , C_4H_{10} , C_5H_{12} is used as the fluid in the primary loop.

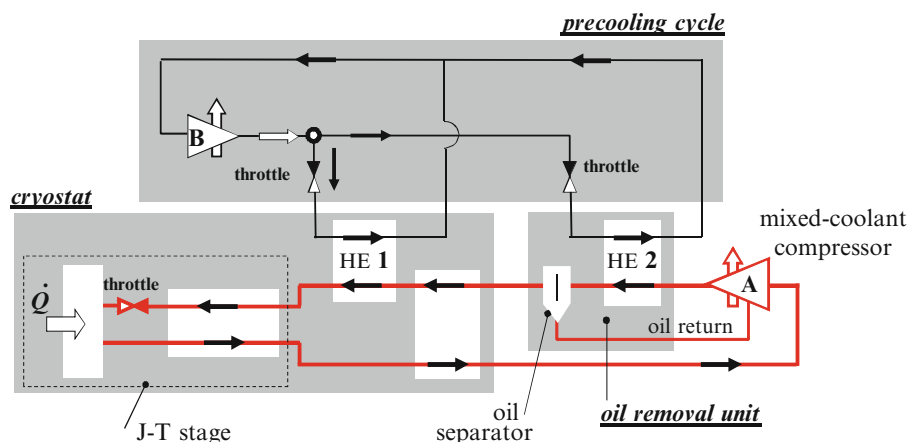
8.3.4.3 Thermoelectric Precooling of a Microcryocooler

Thermoelectric precooling of a microcryocooler.^{68,69} Section 6.4.6 describes the use of a thermoelectric element to precool a mixed-refrigerant microcryocooler that produces about 9 mW of cooling power for a terahertz sensor.

8.3.4.4 Reducing the Size of the Cold End of a Two Stage Cryosurgical Probe

Dobak et al.^{70,71,72} describe methods to reduce the size of the cold end of a miniature cryosurgical probe driven by a closed cycle mixed-gas cooler. For this purpose, they utilize a recuperative closed cycle precooler, with refrigerant R-13b1. The primary cycle, cooling the tip of the probe, utilizes a compressed mixture of freons (R-22, R-23 and R-116) with krypton as the lowest boiling point component. The system achieved temperatures below 183 K (but not down to 120 K).

Fig. 8.5 A cascade of two closed cycles with lubricated compressors; the precooling stage is arranged to condense oil vapor of the compressed main stream (Alexeev et al., in references.^{60,61,62,63,64}) (A and B represent the compressors, with their after-coolers, of the mixed-coolant and the precooling cycle, respectively)



Van der Walt et al.⁷³ describe a similar system where the precoolant is R410a and the primary coolant includes R116. Their system utilizes lubricated compressors, but they propose a preparatory process in which the final stage compressor is baked in a vacuum at about 100 C for approximately 1 week before being filled with oil, as an effort to reduce the chance for blockage. Skye, Nellis, and Klein⁶⁵ also report the use of a precooling stage to reduce the size of a cryoprobe.

8.3.5 Mixed Coolant Closed Cycle for Precooling Pure Coolants

8.3.5.1 Scope

The mixed coolant closed cycle provides notable advantages for the task of precooling. Compared to the traditional precoolants such as CF₄ or Kr (for cryocooling in the range of 80–100 K), the mixed coolant closed cycle can provide deeper precooling, down to about 90 K, at a lower pressure than CF₄ or Kr and with a comparable COP. Such deep precooling significantly reduces the pressure of the final coolant. This feature is especially relevant for pure coolants in the final stage, that would otherwise require elevated pressures in order to provide an optimum Joule-Thomson effect. Instead, their pre-cooled state enables a sufficiently large Joule-Thomson effect simply due to the reduced (pre-cooled) temperature. For the purpose of this discussion, natural gas (NG) which includes about 85% of methane (the rest is mainly ethane and propane), and air (about 80% nitrogen) may be regarded as pure gases.

In most cases, precooling is accomplished in region B of the staging scheme as displayed in Fig. 5.9b. That is, the returning cold stream of the mixed coolant pre-cools a counter-flowing warm, high-pressure feed stream of an open or closed cycle. In this arrangement the transferred heat becomes a distributed load for the precooling closed cycle mixed coolant cryocooler.

Examples of closed cycle mixed coolant cryocoolers used for precooling purposes are presented here, arranged according to the pre-cooled species:

8.3.5.2 Nitrogen

The attractiveness of a precooling stage for nitrogen was described by Longworth.⁷⁴ Goloubev et al.^{75,76,77} used mixed coolant closed cycles to pre-cool a closed cycle of nitrogen that provided refrigeration for current leads. The deep precooling allowed a pressure as low as 1.2 MPa in the nitrogen loop. They present two versions of the nitrogen cycle: with and without recuperation. A patent of Alexeev et al. protects a similar idea.⁷⁸ Their nitrogen cycle includes an ejector to further reduce the discharge pressure of the nitrogen. Apparao, Podtcherniaev, Flynn et al.⁷⁹ proposed a mixed coolant closed cycle to pre-cool a closed loop of circulating nitrogen (or helium) in a chilling system down to about 120 K.

On a larger scale of liquid nitrogen production, a mixed refrigerant closed cycle was proposed for precooling a Linde-Hampson liquefier by Boiarskii et al.⁸⁰ and Olszewski et al.⁸¹ A similar arrangement for precooling Kapitza's nitrogen liquefier was proposed by Olszewski et al.⁸² and Brostow et al.⁸³

8.3.5.3 Oxygen

Longworth and Hill⁸⁴ employed a mixed coolant closed cycle to pre-cool a final closed cycle oxygen stage down to 100 K. The oxygen cycle was driven by a sorption compressor and reached 65 K.

8.3.5.4 Quantum Gases

Liquefying quantum gases that begin at room temperature requires precooling. Arkhipov et al.⁸⁵ report such closed cycle systems to pre-cool the final stage of neon or hydrogen liquefiers. The mixed refrigerant precooling stage operates at about 85–90 K while the final stage runs at 25–30 K. Cryocooling for a similar range of temperatures is discussed elsewhere.⁸⁶

8.3.5.5 Vapor-Liquid Cycle of Higher Boiling Point Gases

Boiarski and Podcherniaev⁸⁷ intended to increase the cooling power of the liquid-vapor cycle of R12 or NH₃ through the use of a mixed coolant closed cycle. The precooling stage utilizes a mixture of R22, R142b and R123.

8.3.5.6 Nitrogen in an Open Cycle

Nitrogen, separated from compressed atmospheric air (as described in Sect. 5.2.3) and pressurized to only about 0.5 MPa, was precooled by Chorowski et al.^{88,89} with the aid of a mixed coolant closed cycle. The low pressure was essential in order to feed a membrane module used to separate the oxygen, moisture and carbon dioxide. The stream of nitrogen reached a temperature of 101 K and did not require any recuperation; in fact a fraction of the nitrogen was liquefied simply through the throttling process. The open cycle system serves as an on-demand liquefier of nitrogen for small quantities, such as 0.2–0.5 l/h. It also resembles the case of a free jet release as discussed in Sect. 5.4.7. Potapov et al.⁹⁰ precooled a stream of nitrogen for a small-scale liquefier, with a closed cycle mixed refrigerant system. Their optimization study identified 110 K as the preferred temperature.

Sankar and Venkatarathnam⁹¹ cooled a stream of nitrogen with a closed cycle mixed refrigerant cryocooler, down to about 145 K. However, the nitrogen was first precooled by a closed cycle of R22, down to about 240 K. The stream of precooled nitrogen was not throttled, and therefore not liquefied.

8.3.5.7 Precooling a Natural Gas Liquefier

Natural gas is not pure, but most of it is methane. Piotrowska and Chorowski⁹² describe the fabrication and use of a mixed refrigerant cycle to precool a small scale natural gas liquefier. Both the precool, and primary cycles operated at about 1 MPa. For a large production scale, Maher et al.⁹³ proposed precooling a 4 MPa stream of NG with a mixed coolant closed cycle to a sufficiently low temperature (about 113 K) so that by throttling the stream, it would yield a certain fraction of liquefied NG. A similar proposal by Swenson⁹⁴ suggests extracting a portion of liquefied propane and heavier hydrocarbons from the precooled NG stream through the use of an intermediate phase separator. Elsewhere are described a mixed refrigerant single stage⁹⁵, and a two stage closed cycle for precooling the NG feed.⁹⁶

8.3.6 Accelerated Cool-Down Cryocoolers

8.3.6.1 Enlarged Orifice at Cool-Down

The cool down rate of a closed cycle cryocooler may be increased by utilizing a larger orifice at the beginning of

cool-down than that required for steady state circulation. This approach elevates the density in the suction line. Each stroke of the compressor therefore circulates more coolant and the flow rate increases. The larger orifice is accompanied by a smaller pressure drop across the orifice. Nevertheless, the modification results in a larger cooling capacity, and an accelerated rate of cooldown. A subsequent discussion in Sect. 8.7, provides further detail regarding these issues.

As the cooldown proceeds, the orifice must gradually become smaller, finally approaching the size required for steady state operation. Otherwise, two non-desirable effects result:

1. The flow rate through the large orifice will sharply increase, and the discharge pressure will be lower than originally intended. The large flow rate results from the increased density of the coolant at the inlet of the orifice as the temperature decreases. The coolant may even be completely in the liquid phase at the end of cooldown.
2. The suction pressure increases and consequently the boiling point of the coolant is elevated.

Boiarski et al.⁴⁹ successfully accomplished the above task with a mechanism positioned at the cold end that automatically adjusts the flow area of the throttle. The regulation trend resembles that of an open cycle cryocooler. However, its response is more gradual and less of an on-off type actuation, as explained in the same reference. Khatri and Boiarski⁹⁷ report two attempts to adjust the throttle, with a solenoid and with a bimetallic device.

Additional information is available regarding flow regulated closed cycles^{98,99} and systems that provide a boost for the cooldown.^{100,101}

8.3.6.2 Mixtures with Helium and a Fixed Orifice

Longsworth et al.⁴¹ proposed a method to shorten the cool-down period of the closed cycle while still employing a fixed orifice without any moving elements. The relevant parameter for realizing such a solution is the ratio of the densities of the coolant at the inlet to the orifice at the end of the cool-down (or at steady state run) and at the beginning of the process. When this ratio is relatively high (above 20), it is beneficial to adjust the orifice as mentioned above. However, by adding 3–25% of helium this ratio may be maintained below 17% for any mixture. In this way, a faster cooldown is obtained and it is still possible to maintain a fixed orifice. On the other hand, it is associated with some reduction of the specific cooling capacity Δh_T of the mixture.

8.3.6.3 Porous Plug Throttle

A similar effect is obtained without any moving parts simply by throttling through a porous plug. Laminar flow in the porous media is dominated by viscosity, which is temperature dependent. The high velocity associated with flow through a single miniature orifice would result in turbulent flow.

However, due to the multiplicity of flow paths through the porous media, and the viscous interaction with the many walls the flow remains laminar. Furthermore, as the fluid cools, a larger fraction of the incoming stream is liquefied, and thus the vapor flow faces a larger effective restriction. Consequently, at the same pressure and with the same pores, the flow gradually decreases as the cooldown progresses. Hill et al.¹⁰² introduced this idea and report experiments with a porous element of 10, 12, 22, and 40 μm . Elsewhere⁹⁷ the use of a throttle combining a capillary and a porous cartridge is reported.

8.3.6.4 Pressure Vessel Assistance

The Russian book of Sulsov, Gorshkov and Maslakov¹⁰³ describes a closed cycle cryocooling system that utilizes a pressure vessel dedicated specifically to drive the cooldown process. Its discharge is halted when the cooldown is complete. Cooling down an open system with the use of a pressure vessel can be significantly faster than that of a similar approach with a closed cycle system. Since the cooldown is accelerated by the additional pressure vessel, the size of the compressor may be appropriately reduced so that it just satisfies the lower demand of the steady state cooling capacity. Another closed cycle cryocooling system employing pressure vessels for cooldown is described by Bovin.¹⁰⁴

8.3.7 The Heat Exchanger

The recuperator plays a central role in closed cycle mixed refrigerant cryocoolers. It influences the cryocooler's performance even to a larger extent than in the case of pure coolants. The following are some typical issues of a mixed coolant recuperator:

- Each channel contains, along a substantial portion of its length, a two-phase fluid. Boiling occurs not only in the evaporator, but also in the down stream, low-pressure channel.¹⁰⁵
- A pinch point might occur not only at the warm end, but also at an intermediate temperature or at the cold end. Alexeev et al.,³¹ Gong et al.,^{105,106} Keppler et al.,¹⁰⁷ and Shpitzer¹⁰⁸ discuss the longitudinal temperature profile of heat exchangers in this sense for different mixtures. Venkatarathnam^{109,110} addresses the general conditions for pinch point formation as it depends on the composition of binary and ternary mixtures.
- Furthermore, the pressure drop along each of the channels may shift the location of the pinch point (for some compositions of mixtures) from the cold end to the warm end; possibly resulting in a significant decrease of the specific cooling capacity, as analyzed elsewhere.¹¹¹

- The relatively small temperature difference between the streams necessitates a large heat exchanger surface and a large NTU. The performance of the heat exchanger is therefore more sensitive to the actual size of heat exchanger, as discussed elsewhere.³¹
- The thermodynamic behavior of mixtures enables a linkage between the actual heat load and the operating temperature. Luo et al.,^{58,112} Khatri,⁴⁵ and Venkatarathnam et al.¹¹³ compare the performance of mixtures with two different heat exchangers.
- Gresin and Landa¹¹⁴ identify the heat transfer coefficient of the returning stream as the key parameter that determines the behavior of the entire heat exchanger.

The most frequently reported configuration is the tube in tube recuperator. Gong et al.¹¹⁵ and Ardhapurkar et al.,¹¹⁶ measured the heat transfer coefficients associated with such a heat exchanger operating with mixed coolants. Their investigation utilized three different mixtures, each suitable for a different range of cooling temperatures. The local heat transfer coefficients of non-azeotropic hydrocarbon mixtures as measured by Hughes et al.^{117,118} indicate their sensitive dependence on mass flux, quality and temperature, and a minor influence of pressure. A typical upper value was $14,000 \text{ W m}^{-2} \text{ K}^{-1}$. Little¹¹⁹ compared these results with various analytical predictions.

Dobak, Marquard et al.^{46,70,120} focused on the compactness of heat exchangers in order to minimize their size at the cold end. This topic is further discussed in Sect. 8.5.4.2.

Anikeev, Gresin and Zakharov¹²¹ suggest incorporating porous linings around the high-pressure tube in a miniature heat exchanger and if possible, between the coils along the low-pressure channel of the returning stream. The capillary pores are expected to capture the more dense fractions of the streams and, in this way, guarantee a lower sensitivity to the orientation of the cryocooler with respect to gravity, and thereby increase the thermodynamic efficiency.

8.3.8 Miscellaneous

8.3.8.1 Single or Double Phase Charged Refrigerant

Cryocoolers may be differentiated by whether the mixed refrigerant is completely gaseous or partially condensed at the relevant ambient temperature and at the high discharge pressure at the inlet to the recuperator. Accordingly, the two categories are referenced as systems of *Gas Refrigerant Supply* (GRS) or *Liquid Refrigerant Supply* (LRS). Alexeev et al.^{30,54} and Boyarski et al.^{122,123} compared the performance of these two classes.

The LRS system must be charged with a two-phase refrigerant. Because it circulates as a two-phase stream, it

is subject to problems associated with non-homogeneous flow. In this sense, the GRS system is definitely simpler. However, the LRS system provides the advantages of: (a) higher efficiency, (b) faster cooldown, and (c) tolerating a smaller recuperating surface. One may comment that precooling may be considered as an alternative strategy to the LRS system while avoiding partial condensation at room temperature. More about these two strategies may be found in Sect. 8.5.3.

8.3.8.2 Cryocooling Temperatures

Closed cycle mixed gas cryocoolers have the potential to operate in the temperature range of liquid nitrogen and even at somewhat lower temperatures. However, many applications accommodate higher temperatures, in the range of 120–150 K, as described by Naer and Rozhentsev.¹²⁴ As will be discussed later, the operating temperature depends on the composition of the coolant and especially on the lowest boiling point component.

8.3.8.3 Centrifugal Compressor

Chen and Zagarola¹²⁵ present a unique closed cycle mixed refrigerant JT system in which a centrifugal compressor is used as part of a space born cryocooler that distributes 10 mW of cooling at 150 K to each of multiple miniature cold heads. The high pressure is only 0.2 MPa and the compression ratio is only 2.4. Nevertheless, the high rotating speed of 8,000 rev/s makes the centrifugal compressor a compact flow source in comparison to traditional positive displacement compressors.

8.3.8.4 Recuperation by Regenerators

Carbonell et al.¹⁹ incorporated the principle of counter flow recuperation with the aid of two regenerators, as discussed in Sect. 1.3.6.1 and shown in Fig. 1.12. The valves that switch the direction of the flow operate at room temperature. It was an early attempt (before 1972) to use a hermetic lubricated compressor in a closed cycle Joule-Thomson cryocooler with mixed coolants such as, nitrogen, hydrocarbons, neon and helium. Carbonell et al. point out that:

An important advantage of the regenerator unit resides in the use of air tight lubricated compressors, in that the lubrication does not hinder the operations of the refrigerator. With this type of compressor, a certain quantity of lubricating oil is carried off by the cooling mixture when compressed. But the oil, which is simply a heavy hydro-carbon behaves in the same way as the other components of the cooling mixture in the regenerator, and is left behind in the hot part of these regenerators. There is consequently no oil filtration device and no need to clean the regenerators periodically.

8.3.8.5 Thermal Ballast

Solid cryogenics, combined with a mixed refrigerant closed cycle cooler^{85,126,127} may increase the run time of a

cryocooling system. In such systems, the closed cycle cooler typically solidifies a pure cryogen and stores it in an accumulator for subsequent use upon demand. Single stage cryocoolers are sufficient to solidify argon or nitrogen. However, a two-stage cryocooler is needed to solidify neon.

8.3.8.6 Integral Closed Cycle Cryocooler

Curlier¹²⁸ proposed a Joule-Thomson cryocooler in which the single stage compressor (of about 3 MPa) and the cold-head are integrated into a single block. The orifice is constructed as an annular passageway between two conical surfaces. Longitudinal positioning of the two surfaces enables a delicate adjustment of the orifice size. The overall configuration resembles that of an integral Stirling cryocooler with a compression unit and an adjacently mounted cold finger.

8.3.8.7 Flammable Versus Nonflammable Coolant Cryocoolers

Flammable versus nonflammable coolant cryocoolers were discussed by Boiarski et al.¹²⁹ and Khatri et al.¹³⁰ From the perspective of almost any cryocooler performance parameter a price must be paid to enjoy the non-flammability feature of a coolant: the cooling capacity decreases by about half, the required size of the heat exchanger increases by about 50%, and the mass flow rate decreases unless the orifice is enlarged. In addition to flammable and non flammable options, Walimbe et al.¹³¹ also discussed a group of semi-flammable components.

8.3.8.8 Sorption Compression for a Multi Component Gas

Sorption compression for a multi component gas is an attractive option to the same extent as for pure coolants, as described in Sect. 5.2.2.4 (2). Attempts to build a mixed coolant sorption cryocooler^{132,133} reveal that since the adsorption capacity is species dependent, the structure and composition of the sorbent requires special attention.

8.4 Thermodynamic Performance of the Mixed Coolant Cycle

The essential differences between pure and mixed coolant cycles may be described in the following terms:

- (a) In the case of pure coolants the lowest attainable temperature for a given pressure $P < P_C$ in the evaporator, that is the boiling point (vapor liquid equilibrium), remains fixed at T_{BOIL} as long as liquid is maintained in the evaporator. As discussed in Sect. 8.4.1, the case of mixtures presents a more complicated set of possibilities:

1. The boiling temperature, T_{BOIL} might be non accessible, and,
 2. T_{BOIL} may be dependent on operating conditions such as the actual heat load and the extent of recuperation (i.e., the size of the recuperator).
- (b) The maximum capacity cannot be determined solely by the inlet pressure and temperature as in the case of pure coolants (Sect. 8.4.2).

The next two sections discuss the principles of analysis, which are further illustrated by the examples provided in Sect. 8.4.3.

8.4.1 Temperature of Operation

8.4.1.1 The Temperature in the Evaporator and the Operating Line

The *operating line* is a useful means for determining the actual temperature of operation, which is somewhat below but close to the temperature T_4 at the outlet of the cryocooler's evaporator (noted in Figs. 3.3 and 3.4 and as $T_{L,IN}$ in Fig. 1.1). It depends on the composition of the mixed coolant and on the running mode in terms of the actually supported heat load. For a pure coolant the operating temperature is known since the cooler always operates at the lowest attainable temperature, that is the boiling temperature associated with the pressure in the evaporator.

The idea of the *operating line* was previously introduced in Sect. 3.11.4, assisting the analysis of the pure coolant Linde-Hampson cycle. It was especially useful for obtaining the temperature difference, ΔT , between the recuperative streams in the h - T plane. The *operating line* is generated by displacing the high-pressure isobar (towards the low pressure isobar) by an amount of Δh , which represents the actual specific cooling content for any particular heat load per unit flow rate, $\dot{Q}/\dot{n}$. The horizontal separation between the operating line and the downstream isobar describes the ΔT between the high and low pressure streams.

To remain consistent with the Second Law, heat is not transferred across the recuperator at the point where $\Delta T = 0$. This condition sets the lowest achievable steady state temperature in the system, $T_4 = T_4^{MIN}$. The location where $\Delta T = 0$ is defined by the intersection of the operating line with the low-pressure isobar, and is often referred to as the *pinch point*, fulfilling the condition,

$$h(P_D, T_4^{MIN}) = h(P_U, T_4^{MIN}) + \dot{Q}/\dot{n} \quad (8.1)$$

Achieving the lowest possible value of T_4 , that is T_4^{MIN} , requires both that the pinch point occur at the cold end of the recuperator, and that the recuperator is infinitely large. Hence, in practice, T_4^{MIN} is a lower bound, and for any finite

size recuperator, $T_4 > T_4^{MIN}$. This condition defines a correlation between the temperature of operation and the size of the recuperator in case of mixed coolants. For a smaller recuperator, the value of T_4 will be higher.

When the pinch point is at the cold end of the recuperator, the condition $\Delta T = 0$ also defines the lowest temperature at the inlet to the JT valve, $T_2^{MIN} = T_4^{MIN}$. In the general case, the resulting temperature T_3 after the isenthalpic expansion, and at the inlet to the evaporator, is lower than T_4 . Then, the operating temperature cannot be less than T_3 .

The recuperative process is sharply attenuated in the vicinity of the pinch point temperature and further heat transfer-induced reductions in temperature below this point do not occur. Below the pinch point temperature, the warm stream would have to become colder than the cold stream, which is not possible.

Maytal et al.¹³⁴ were the first to introduce the use of the non-negative ΔT condition into the analysis associated with mixed coolants. This condition is obviously valid for both pure and mixed coolants. The operating temperature also depends on P_U as one may observe in Table 8.1. The high pressure influences the shape and curvature of the isobar, especially as P_U changes from subcritical to above the critical value.

8.4.1.2 The Boiling Point Versus the Pinch Point Temperature

The lowest attainable temperature by any Joule-Thomson cryocooler is the boiling point of the coolant. Since a mixture might (but not necessarily) boil over a range of temperatures (between a dew point and a bubble point), the temperature in the evaporator might rise due to the absorbed heat from the cooled object. This elevation depends on the amount of absorbed heat, the fluid composition, the number of liquid phases at the T_{BOIL} of the mixture, **and** on the boiling temperature, $T_4 = T_{BOIL}$.

Pure coolants always boil at a fixed temperature, which remains unaffected by the heat load. However, for mixed coolants the curvature of the isobars in the h - T plane is typically larger and dependent on P_U . Therefore, the pinch point under various heat loads and pressures may occur at a higher temperature than the heat load dependent boiling point, and $T_4 > T_{BOIL}$.

In such a case, the recuperation process is not able to reach the lowest attainable temperature, which is the mixture's T_{BOIL} , at the pressure in the evaporator.

8.4.1.3 Equivalent Specific Heat Capacities and the Pinch Point Occurrence

In the case of a mixture of various components with two or more phases in equilibrium, the slope of the isobar, in the h - T plane, $(\partial h / \partial T)_P$ represents the (local) *equivalent* specific heat capacity. The term *equivalent* emphasizes the fact that it is not exactly the traditional c_P , since an incremental

enthalpy change may produce a change of phase (evaporation or condensation) as well as a change of temperature. Nevertheless, with this fact in mind, the term $(\partial h/\partial T)_P$ will still be regarded as c_P . Inside the two phase dome, at a quality x of mixed liquid and vapor phases, it may be described in terms of the respective heat capacities, $c_{P,F}$ and $c_{P,G}$ for each phase by,

$$c_P = \left(\frac{\partial h}{\partial T} \right)_P = \left(\frac{\partial x}{\partial T} \right)_P (h_G - h_F) + x c_{P,G} + (1-x) c_{P,F} \quad (8.2)$$

As one approaches the pinch point from above, ΔT decreases as the temperature is reduced. Therefore, following the discussion in Sects. 3.11.3 and 3.11.5, there should be a temperature interval along which the condition $c_{P,D} > c_{P,U}$ is satisfied, namely, the low pressure heat capacity is larger than the high pressure one. Such a condition is commonly encountered in mixtures but rarely with pure coolants.

8.4.2 The Cooling Capacity

8.4.2.1 The Limiting Cooling Capacity, $\Delta h_T(T_{IN})$

The limiting cooling capacity, $\Delta h_T(T_{IN})$, or the maximum specific cooling content of a candidate refrigerant for a Joule-Thomson cryocooler is determined by inspecting the energy balance of the flow. In the ideal case, as described in Sect. 3.2.4, it is represented by the function of the warm end states, $\Delta h_T(T_{IN})$. At a steady state, the cooler can support any heat load smaller than $\dot{Q}^{MAX}$, satisfying the condition,

$$\frac{\dot{Q}^{MAX}}{\dot{n}} = \Delta h_T(T_{IN}) \quad (8.3)$$

In the case of pure coolants, the relative magnitude of the specific heat capacities of the heat exchanging streams guarantees that the cooldown process can proceed down to the boiling point, T_{BOIL} , which is the lowest attainable temperature and is independent of the heat load.

8.4.2.2 The Actual Cooling Capacity, Δh_T^{MIN}

With a mixed coolant, it is possible, although not always true, that a pinch point “barrier” (of the Second Law) will prevent cooling below the pinch temperature, and thereby inhibit the fluid from reaching the lowest possible temperature of the mixture, that is, T_{BOIL} . In order to overcome this barrier and reach T_{BOIL} , one has to reduce the specific cooling power at least to a value consistent with the *local* minimum separation Δh_T^{MIN} between the isobars so that,

Table 8.2 Cooling capacities and the associated temperatures of operation for a mixed coolant cycle of throttling and recuperation

(a) $\Delta h_T^{MIN} = \Delta h_T(T_{IN})$		
$\frac{\dot{Q}^{MAX}}{\dot{n}} = \Delta h_T(T_{IN})$	$T = T_{BOIL}$	
(b) $\Delta h_T^{MIN} < \Delta h_T(T_{IN})$		
$\frac{\dot{Q}^{MAX}}{\dot{n}} = \Delta h_T^{MIN}$		
Specific heat load $\dot{Q}/\dot{n}$		Attainable temperature
If $\Delta h_T^{MIN} < \frac{\dot{Q}}{\dot{n}} < \Delta h_T(T_{IN})$	then	$T > T^{MIN} > T_{BOIL}$
If $\frac{\dot{Q}}{\dot{n}} < \Delta h_T^{MIN}$	then	$T = T_{BOIL} < T^{MIN}$

$$\begin{aligned} \frac{\dot{Q}}{\dot{n}} &= \Delta h_T^{MIN} \equiv \min_T [h(P_U, T) - h(P_D, T)] \\ &\equiv \min_T [\Delta h_T(T)] \end{aligned} \quad (8.4)$$

Here, the minimum (MIN) condition is the smallest constant temperature enthalpy difference between the high and low pressure streams over the entire temperature range of cooldown, from the inlet temperature, T_{IN} , which is often T_{AMB} , down to the lowest attainable temperature, T_{BOIL} . Thus, the entire distribution by temperature of $\Delta h_T(T)$ becomes relevant, instead of only the warm end function, $\Delta h_T(T_{IN})$, as in the case of pure coolants.

The value of Δh_T^{MIN} is realized at a certain intermediate temperature, denoted by T^{MIN} . At this temperature, according to the ideas presented in Sect. 3.11.3,

$$\begin{aligned} @T^{MIN} \quad c_{P,D} &= c_{P,U} \quad \frac{d(\Delta T)}{dT_D} = 0 \quad \text{and} \\ \frac{d(\Delta T)}{dT_U} &= 0 \end{aligned} \quad (8.5)$$

This issue is further discussed elsewhere.^{105,109,110} It should be emphasized that the occurrence of the local minimum, Δh_T^{MIN} , may appear in some compositions of mixtures but not in others.

If a local minimum, Δh_T^{MIN} , does appear that is larger than $\Delta h_T(T_{IN})$, the highest specific cooling capacity remains limited by $\Delta h_T(T_{IN})$ and the fluid will still be able to reach T_{BOIL} even while applying $\dot{Q}^{MAX}$. In other words, for this case $\Delta h_T(T_{IN})$ plays the role of Δh_T^{MIN} .

However, if the local minimum, $\Delta h_T^{MIN} < \Delta h_T(T_{IN})$, the resulting operating temperatures, as determined by the actual heat load, are summarized in Table 8.2. Therefore, in the case of mixtures, it might become necessary to specify not only $\Delta h_T(T_{IN})$, but also the other relevant measure for

Fig. 8.7 $\dot{Q}/\dot{n}$ cannot exceed $\Delta h_T^{MIN} = 0.58$ kJ/mole in order to operate below 80 K. For higher values of $\dot{Q}/\dot{n}$ the temperature of operation exceeds $T^{MIN} = 163.5$ K

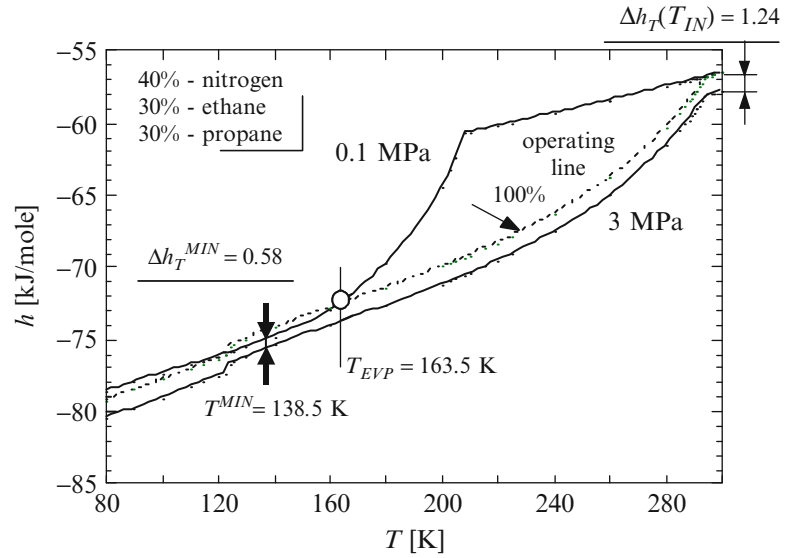
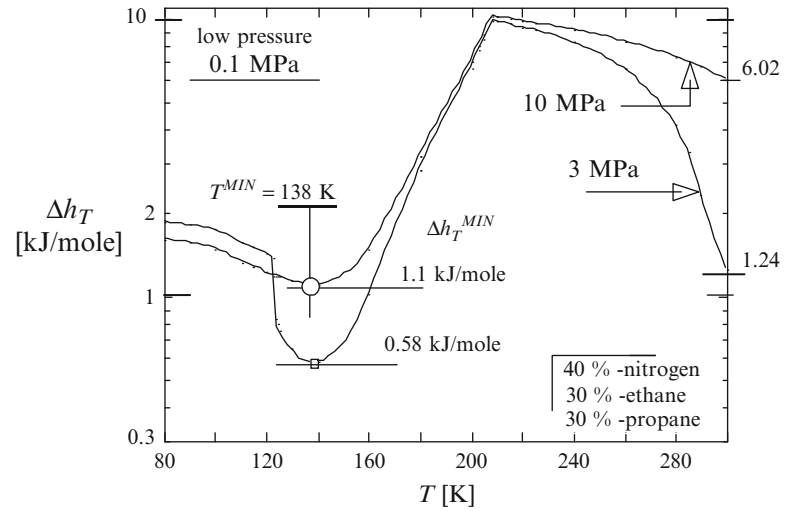


Fig. 8.8 By elevating the pressure from 1 to 3 MPa, the cooling potential $\Delta h_T(T_{IN})$ increases almost by a factor of 5 (from 1.24 to 6.02 kJ/mole). However, when operating below 80 K the cooling capacity, Δh_T^{MIN} is only doubled (from 0.58 to 1.15 kJ/mole)



below 80 K to above the $T^{MIN} = 138$ K. In this case, the operating line of 25% of $\Delta h_T(T_{IN})$, representing a specific cooling capacity of 1.25 kJ/mole, would elevate the temperature in the evaporator to at least 160.6 K.

Figure 8.7 analyzes the same mixture but at the lower pressure of only 3 MPa. Now, the highest specific cooling capacity, $\Delta h_T(T_{IN})$, at 300 K is only 1.24 kJ/mole and the lowest maintainable temperature with such a specific cooling load is 163.5 K. However, in order to operate below 80 K, $\dot{Q}/\dot{n}$ cannot exceed $\Delta h_T^{MIN} = 0.58$ kJ/mole.

Figure 8.8 directly displays the separation between the isobars, that is, $\Delta h_T(T)$ for the same mixture and for both pressures. Indeed, as the pressure increases from 3 to 10 MPa the maximum cooling capacity, $\Delta h_T(T_{IN})$, increases from 1.24 to 6.02 J/mole, which means, by a factor of 5. However, while operating below 80 K the

allowable cooling capacity, Δh_T^{MIN} , is only doubled: 1.15 instead of 0.58 kJ/mole.

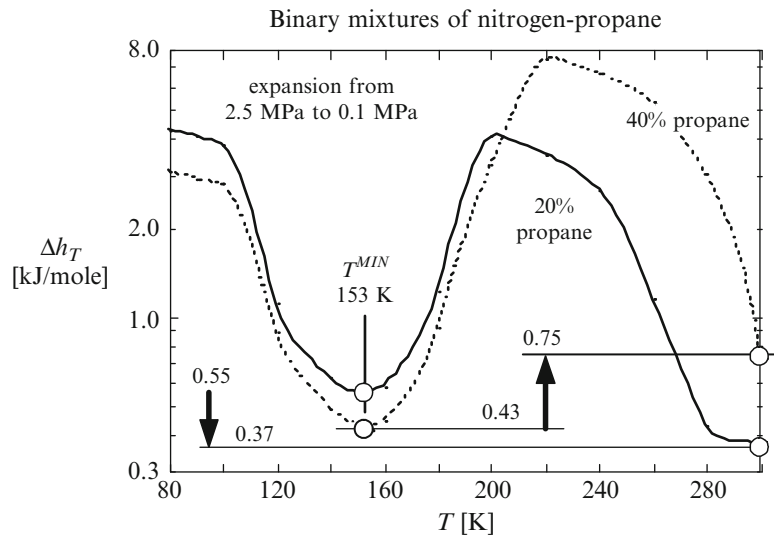
8.4.3.2 The Binary Mixtures of N_2 with 20% and 40% of C_3H_8

The binary mixtures of N_2 with 20% and 40% of C_3H_8 are described in Fig. 8.9. Each cycle operates in the pressure range of 2.5 and 0.1 MPa. In both cycles a local minimum, Δh_T^{MIN} , appears once at the warm end and once at a lower temperature,

$$\begin{aligned} 0.6 - N_2 \\ 0.4 - C_3H_8 \end{aligned} \quad \Delta h_T^{MIN} > \Delta h_T(T_{IN}) \quad (8.7)$$

$$\frac{\dot{Q}^{MAX}}{\dot{n}} = \Delta h_T(T_{IN}) = 0.37 \text{ kJ/mol}$$

Fig. 8.9 Doubling propane's concentration (20–40%) doubles the maximum cooling capacity (from 0.37 to 0.75 kJ/mole). However, to operate below 80 K, the cooling capacity is limited to only, $\Delta h_T^{MIN} = 0.43$ kJ/mole which is a raise of only 17%



0.6 – N₂
0.4 – C₃H₈

$$\Delta h_T^{MIN} < \Delta h_T(T_{IN}) \quad (8.8)$$

$$\frac{\dot{Q}^{MAX}}{\dot{n}} = \Delta h_T(T_{IN}) = 0.75 \text{ kJ/mol}$$

In this case, doubling the propane concentration (from 20% to 40%) doubles the maximum cooling capacity from 0.37 to 0.75 kJ/mole. However, to operate below 80 K, the cooling capacity is limited to only, $\Delta h_T^{MIN} = 0.43$ kJ/mole which is an increase of only 17%. Put another way, the potential of propane to elevate $\Delta h_T(T_{IN})$ is only fractionally translated into a higher actual specific cooling capacity, $\dot{Q}/\dot{n}$.

8.5 Aspects of Mixed Coolant Composition

A composition of mixtures is defined by the species (components) and their concentration. Three aspects of the composition are relevant to the operation of a Joule-Thomson cryocooler:

- The distribution of Δh_T by temperature, which determines the cooling capacity and the temperature of operation,
- Solid–liquid–vapor equilibria to ensure “freezing before liquefaction” for clog free operation, and,
- Liquid–liquid–vapor for more stable temperature of operation.

Below will be shown why including components with high boiling points, which in principle enhance the Joule-Thomson effect of the mixture, can induce a counter effect that in the end may reduce the specific cooling capacity of the mixture.

8.5.1 The Δh_T of Components and of Their Mixture

8.5.1.1 The Δh_T of Pure Gases at Subcritical Pressure

The distribution of Δh_T over the entire cooldown temperature range is relevant for studying the cooling capacity of a closed cycle system as explained in the previous chapter. One can begin the study by examining the $\Delta h_T(T)$ profile for the appropriate pure components at their respective subcritical pressures.

The shape of $\Delta h_T(T)$ (as displayed in Fig. 8.10 for nitrogen and ethane) can be described as a single high, wide peak spread between the saturation temperature, $T_S(P)$ or the boiling point associated with the given subcritical pressure, (P) and the normal boiling point, T_{BOIL} . For example, the interval for nitrogen would be (77.3, 120.0 K) and for ethane (184.4, 275.0 K) both with a pressure range of (2.5, 0.1 MPa). Approaching the peak from the right, Δh_T moderately increases as the temperature is lowered towards the saturation value. Continuing to the left of the peak, Δh_T exhibits slightly negative values, gradually decreasing toward the temperature of solidification. Gases with a higher normal boiling point (T_{BOIL}) have a more pronounced peak.

8.5.1.2 The Linear Superposition of Enthalpies

Let us examine the relation between the Δh_T values of the components and those of its mixture. By mixing components at subcritical conditions, the resulting mixture is not strictly an ideal solution. Therefore, the linear combination of enthalpies by molar fractions, z^i , is but an acceptable qualitative approximation for the Δh_T of the mixture,

Fig. 8.10 Temperature distribution, $\Delta h_T(T)$ for nitrogen and ethane at subcritical pressures; each one significantly spreads along its typical temperature range and becomes lower out of this range

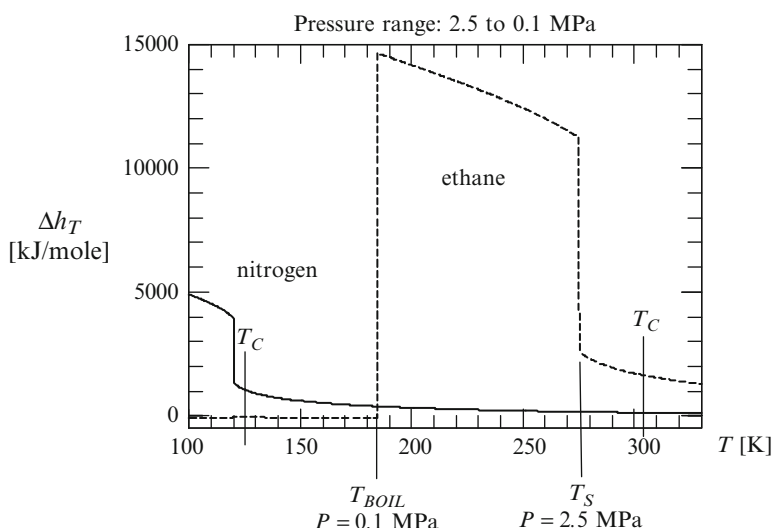
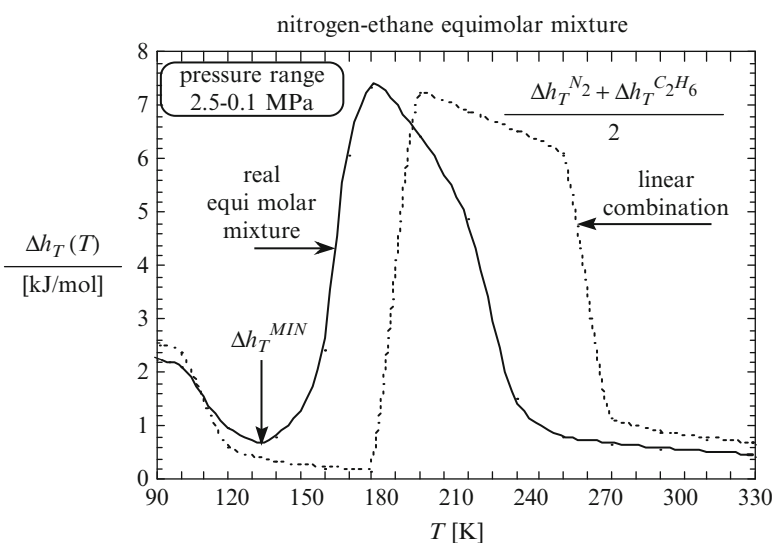


Fig. 8.11 Comparison of the actual $\Delta h_T(T)$ of a binary mixture with the weighted-linear-combination of the particular $\Delta h_T(T)$ values of the components



$$\Delta h_T^{MIX} \approx \sum_i z^i \cdot \Delta h_T^i \quad (8.9)$$

Figure 8.11 compares the approximate and actual values of Δh_T for an equimolar mixture of nitrogen-ethane. In both cases the mixture exhibits a couple of smaller peaks spread over the temperature intervals (T_S , T_{BOIL}) correspondingly for each component. By increasing the ethane fraction, its peak becomes higher but remains spread over a similar temperature interval, as shown in Fig. 8.12. However, replacing ethane by propane or by butane moves the peak on the right to higher temperatures, as demonstrated in Fig. 8.13 for a constant nitrogen fraction of 0.6.

The above examples of superposition include two phase regions in the temperature interval of (T_S , T_{BOIL}). However,

in the case of a mixture of single-phase components, the superposition of enthalpies provides an improved and even quantitatively acceptable estimation for the resulting mixture.

8.5.2 Functional Groups of Components

8.5.2.1 Reducing the Operating Pressure of the Mixture

Reducing the operating pressure of the mixture becomes possible by including components whose critical temperatures are characterized by $T_C > 300$ K. Figure 4.17 displays the values of Δh_T^{MAX} (the peak values of Δh_T as a function of pressure) at various temperatures. As a general trend, the maximum

Fig. 8.12 While increasing the share of a component in a mixture, the range of distribution of $\Delta h_T(T)$ of the mixture remains similar

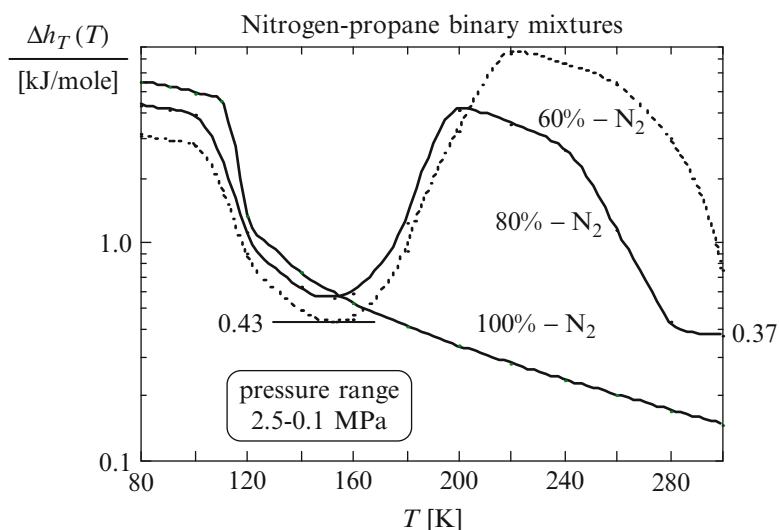
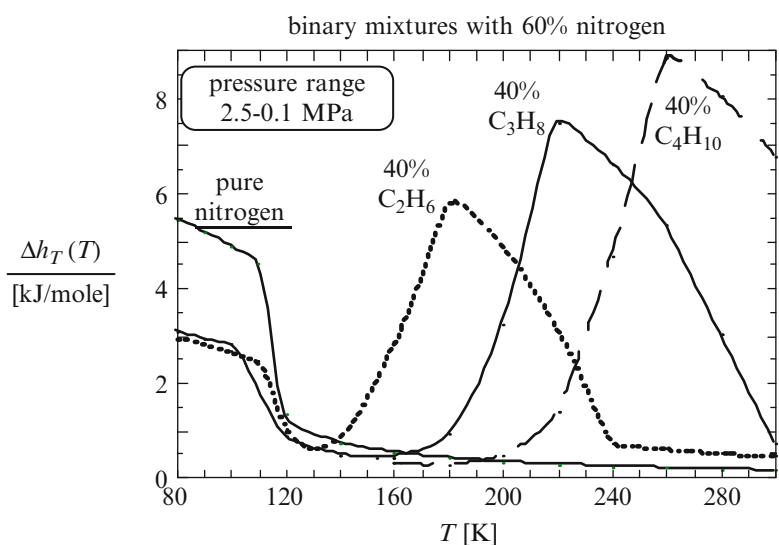


Fig. 8.13 Replacing a lower boiling point component by another one of higher boiling point, changes the distribution $\Delta h_T(T)$ of the mixture. The peak moves towards higher temperatures



value of Δh_T increases at lower temperatures. Values of Δh_T that are almost as large as the peak values are obtained at pressures below P_C only if the temperature is also somewhat subcritical (see also Fig. 4.18). In order to guarantee subcritical temperatures in an environment of approximately 300 K, one has to choose gases with values of T_C above 300 K. Within this group of candidate components, those with higher values of T_C exhibit higher maximum values of Δh_T . Thus, including gases with higher critical temperatures serves a twofold purpose: (a) it enables larger values of Δh_T , and, (b) these are realized at a lower operating pressure. It is therefore easy to understand why higher temperature components such as, C_2H_6 , C_3H_8 , and C_4H_{10} are commonly included in mixtures.

Simply for comparison, one should recall that if instead of subcritical, the environmental temperatures increase to

approximately $2 T_C$, the pressure corresponding the maximum value of Δh_T is as high as about $10 P_C$.

8.5.2.2 Components for Suppressing a Mixture's Boiling Point

Mixtures involving solely components with critical temperatures, $T_C > 300$ K, do liquefy at temperatures above 160 K. However, in order to target the temperature range of 80 or 90 K, one must include nitrogen or argon components to suppress the boiling point of the mixture.

The least volatile component in the mixture (excluding quantum gases, if involved) is considered the *primary component* (denoted by the superscript A). The higher boiling point components (referred to as the “warmer” gases and denoted by B, C, D, etc.), are regarded as *additives*,

$$T_{BOIL}^{(A)} < T_{BOIL}^{(B)} < T_{BOIL}^{(C)} < T_{BOIL}^{(D)} < \dots \quad (8.10)$$

The temperature of operation or the temperature of liquefaction of the mixture, in general, will not extend below $T_{BOIL}^{(A)}$.

Very often, the primary component is nitrogen which sharply lowers the boiling point of the mixture, to a value closely approaching its boiling point, $T_{BOIL}^{(A)}$. Grezin and Zacharov¹³⁷ point out that an equimolar mixture of C_3H_6 and C_3H_8 (T_{BOIL} of 226.2 and 231.2 K) requires a fraction of nitrogen as small as only 0.006 in order to remain as a liquid at 77.4 K. Similar behavior is observed in the ternary mixture of C_2H_6 , C_3H_8 and CH_4 , or binary mixture of the freons R-13 and R-14 at temperatures of about 75 K. Unlike enthalpies, the boiling point of these mixtures cannot be determined even roughly, by a linear combination of components' individual boiling temperatures.

8.5.2.3 Bridging Components

Bridging components, used to elevate the value of Δh_T^{MIN} are those with critical temperatures less than the ambient temperature, that is $T_C < 300$. As previously explained, a mixture's distribution of $\Delta h_T(T)$ along the cool down temperature range, is a superposition of a series of peaks across the successive temperature ranges associated with each component. Any two separate peaks define a "valley" in between, which in turn defines a local minimum of $\Delta h_T(T)$, or Δh_T^{MIN} . A temperature gap will typically exist between the components used to lower the mixture boiling temperature (T_{BOIL}) and those components which have $T_C < 300$ K. One may observe in Fig. 8.13, that as a result of including the higher boiling point components, the values of Δh_T^{MIN} become smaller. The constraint defined by Δh_T^{MIN} limits the cooling capacity of the mixture. However, by including gases from a group of components, mainly CH_4 , CF_4 , and C_2H_4 , a bridging effect can be realized that in fact may elevate Δh_T^{MIN} towards values of $\Delta h_T(T_N)$. Although these components have critical temperatures below 300 K, they cause a more uniform distribution of $\Delta h_T(T)$ and thereby elevate the cooling capacity of a mixture of components that all have higher boiling temperatures.

Mostitski et al.¹³⁸ point out the improved efficiency of hydrocarbon mixtures by employing the component CF_4 . The composition of their mixture is N_2 (10–20%), CH_4 (10–20%), CF_4 (5–10%), C_2H_6 (10–15%), C_3H_8 (10–25%), with the balance being made up by iC_4H_{10} .

Luo et al.^{139,140} focused on a uniform distribution of $\Delta h_T(T)$ even in a broader sense. They recommended including both CH_4 and CF_4 in any mixture. The remaining components should be comprised of one from each of the following groups: (C_2H_6 , C_2H_4 , R-13, R-23, R-116) and

(C_3H_8 , C_3H_6 , R-22, R-12, R-218) and (iC_4H_{10} , iC_4H_8) and (iC_5H_{12} , iC_5H_{10}). Wickemann and Oellrich¹⁴¹ also discussed using CF_4 as a bridging additive.

8.5.2.4 Quantum Gases

A mixture's boiling point can be further suppressed below that of nitrogen (or any other primary component) by including the quantum gases, primarily either neon or helium, as components in the mixture. The boiling points of the quantum gases are significantly below 70 K, and because of their inert nature, they do not interact with the other components at the temperatures above 70 K. One may regard their addition as though the partial pressure of (the rest of) the mixture is reduced. The boiling point of the rest of the mixture is then suppressed by the effective reduction of its ambient pressure. Interestingly, the lower boiling point is achieved, even though at the same time the cooling capacity of the closed cycle cryocooler may be increased as a result of the elevated down stream pressure.

8.5.2.5 Miscellaneous

Sections 8.5.2.3, and 8.5.2.4, highlighted recommended components with critical temperatures, T_C , below 300 K. Section 8.5.2.1 explained the need for components with $T_C < 300$ K. In addition to the problematic possibility of reducing Δh_T^{MIN} , the next two sections explain two other disadvantages of using "warm" additives. These are:

1. Clog formation: in general, warmer additives solidify at higher temperatures and thus may plug the cryocooler, and,
2. Two phase flow: two-phase refrigerants are more difficult to circulate around a closed cycle, and in order to prolong its lifetime, it is especially important that the mixture not exist as a two-phase fluid inside the compressor.

8.5.3 Clog Free Operation and Solid-Liquid-Vapor Phase Equilibria

8.5.3.1 Introduction

"Warmer" gases tend to solidify "earlier," or at higher temperatures than "colder" gases. This observation is significant for Joule-Thomson cryocoolers because they are especially intolerant of any solid phase precipitation in the low temperature regions of the cycle, which may block the orifice and interrupt the cooling mechanism. For a single component fluid, solidification typically occurs below the normal boiling point, $T_{BOIL}^{(A)}$, as one may observe on the solid-liquid-vapor (SLV) phase equilibria diagram. However, for a mixture, the challenge is to know the extent to which additives are allowed to be "warmer" and in what concentration without causing solid precipitation at the temperature of operation.

In the case where all the melting points, T_M , of the additives are below the boiling point of the mixture for the prime component,

$$T_M^{(B)}, T_M^{(C)}, T_M^{(D)} \dots < T_{BOIL}^{(A)} \quad (8.11)$$

then plug formation would not be expected. However, such a case is very rare and essentially non-existent in the case where nitrogen is the primary component. Most gases begin solidifying at temperatures above nitrogen's boiling point, except oxygen and some non-applicable toxic gases such as fluorine and carbon monoxide.

Alfeev et al.⁸ focused on generalizing the conditions that guarantee clog free operation with components having their solification temperature T_M above the boiling temperature of the primary component (such as nitrogen). Such an occurrence is possible due to two mechanisms: solubility and eutectic composition. They identified three candidate groups of mixtures:

8.5.3.2 Mixtures of Soluble Additives

Mixtures of soluble additives are such that the solid phases of the additives are soluble in the liquid phase of the primary component at its boiling point. The soluble additives in liquid nitrogen are argon, methane and CF_4 . Although each component solidifies at a higher temperature than 77.3 K, in the mixture with nitrogen, and even at high concentrations, they still remain liquid (possibly with vapor) at 77.3 K. Alfeev et al.⁸ mentions an equimolar mixture of nitrogen and methane that provides cooling at 82 K.

In general, the boiling points of soluble additives are not significantly higher than that of the primary component. Therefore, their contribution to elevating the mixture's $\Delta h_T(T)$ and efficiency is moderate.

8.5.3.3 Eutectic Composition of Insoluble Additives

These mixtures begin solidifying at temperatures below the freezing points of any of the individual additives. More importantly, a eutectic (or close) composition may even suppress the temperature of any solid precipitation below $T_{BOIL}^{(A)}$. Table 8.3 lists some eutectic compositions and their associated eutectic temperatures, T_{EU} , of solidification according to calculations of Maytal et al.¹³⁴ and Robinson.¹⁴² For instance, the eutectic composition of the binary mixture, (0.68 CH_4 – 0.32 C_2H_6) freezes at about 70 K while each component alone would freeze at about 90 K. Ternary and quaternary mixtures have the potential for suppressing the eutectic temperature of solidification even below that of the binary mixtures.

Maytal et al.¹³⁴ introduced the term of complete “liquefaction before solidification”. Figure 8.14 is a triangular

Table 8.3 Mixtures of eutectic compositions (percents) and temperatures as calculated by Maytal et al.¹³⁴ and Robinson¹⁴²

N_2	CH_4	C_2H_6	C_3H_8	iC_4H_{10}	nC_4H_{10}	Ne	T_{EU} [K]
68	32						69
68			32				69
86				14			81
		48	52				76
		80		20			86
			84	16			83
67	33						70
68			32				72
		48	32				75
68	32						49
10						90	24
			3			97	22
53	25	22					66
60	36	5	3				52
92					8		85
					10		87
			92		8		84

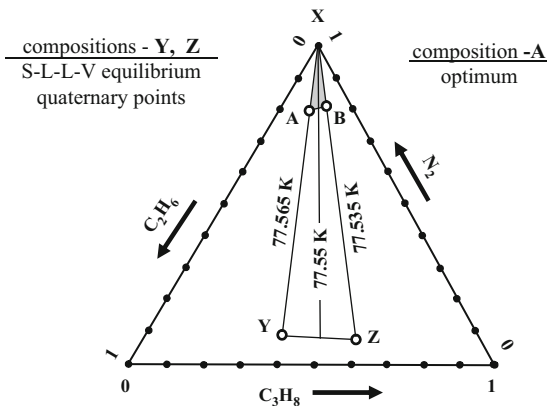


Fig. 8.14 Composition diagram of nitrogen-ethane-propane mixtures; the triangle XYZ represents the mixtures that completely liquefy before solidification. Only the compositions in the triangle XAB have their pinch point at the cold end of 80 K. The optimum state A is discussed in Sect. 8.10.3

composition diagram of the ternary mixture: nitrogen with two insoluble additives, ethane, and propane (also discussed elsewhere¹⁴³). The diagram displays the compositions represented by the triangle XYZ, that depicts this feature. It is a region of liquid-liquid-vapor (LLV) triple points bounded by four-phase solid-liquid-liquid-vapor coexistence. In the same reference, one can find similar composition diagrams for (N_2 , C_3H_8 , $1-C_4H_8$) and for (N_2 , CH_4 , C_2H_6).

8.5.3.4 Additives of Transitive Solubility

This group of mixtures enables a further increase in efficiency while compromising slightly on the operating

temperature. Let at least one additive, say B, be soluble in A at $T_{BOIL}^{(A)}$. Additives C, D, etc. are not soluble in A but are soluble in B. This transitive relation of solubility enables one to incorporate “warmer” additives while postponing their solid precipitation. In this case, the number of coexisting liquid phases may be 2–4, as reported by Alfeev et al.,⁸ each phase including all of the components. For example, consider the possible mixtures of (N₂-CH₄-C₂H₆). Methane is the soluble additive in nitrogen. Ethane is insoluble in nitrogen but soluble in methane. A composition of (0.30, 0.35, 0.35) may raise the cycle efficiency by a factor of 6–9 compared to that for pure nitrogen, but the minimum operating temperature will be limited to 79.5 K. By incorporating a “warmer” additive such as propane, the mixture of (N₂-CH₄-C₂H₆-C₃H₈) with the composition of (0.3, 0.3, 0.2, 0.2) may increase the efficiency by a factor of 10–12 compared to that of pure nitrogen, but the operating temperature will not be able to go below 80 K.

8.5.3.5 A Conservative Approach

A conservative approach for estimating the melting point of any candidate mixture would be to use the mole fraction-weighted average of the melting points of the N components, that is,

$$T_M^{MIX}(pseudo) = \sum_{i=1}^N z^i \cdot T_M^i \quad (8.12)$$

The associated requirement for the boiling point of the mixture would then be,

$$T_M^{MIX}(pseudo) < T_{BOIL}^{MIX} \quad (8.13)$$

Very often, the melting point of a mixture is reduced below the melting point of each component. Thus there are many mixtures for which the condition defined by Eq. 8.13 would not be met but yet they would still be feasible. Eq. 8.13 therefore defines a smaller composition space than the actual space of candidates. Keppler, Nellis and Klein¹⁰⁷ applied such an approach while trying to optimize mixtures of seven components.

8.5.3.6 Miscellaneous Lubricants for Compressors

Lubricants for compressors require special attention so that they are compatible with the particular mixed coolant. The vapor of the lubricant becomes an additional component, which has to be controlled within the composition of the refrigerant to avoid clog formation; the composition of suitable oils and their percentage of the mixture composition is discussed elsewhere.^{129,144}

Propane

Propane presents special considerations. It may be regarded as a high boiling point component ($T_{BOIL} = 231.2$ K) with a relatively low triple point temperature, that is 85.5 K. Hence, it has the potential to elevate the cooling capacity of a mixture and suppress its freezing point.

8.5.4 Aspects of Liquid–Vapor Phase Equilibrium

8.5.4.1 Condensation Inside the Compressor

One must take care not to include additives that are either “too warm” or in too high a dose since, depending on the inlet conditions, they may result in partial condensation of the mixture as the pressure is elevated within the compressor. Such a problem is more likely to occur if the compression process approaches being isothermal. This undesirable behavior is the parameter that limits the addition of components that are “too warm.”

8.5.4.2 Condensation at Ambient Temperature

Another issue to consider regarding mixtures with components that are “too warm” is whether the mixed coolant is completely gaseous or partially condensed at the relevant ambient temperature and at the high discharge pressure at the inlet to the cryocooler. This distinction differentiates between the GRS or LRS systems already introduced in Sect. 8.3.8.1. As discussed by Alexeev et al.^{54,30} and Boyarski et al.^{122,123} the second category enables larger concentrations of higher boiling point components such as propane, butane and pentane. This elevates the mixture’s integral Joule-Thomson effect and the associated specific cooling capacity. On the other hand, homogeneous circulation of a two-phase fluid is more complicated than that of a single gaseous phase.

8.5.4.3 The Temperature Inside the Evaporator: Miscible Additives

An additive may be completely soluble in the liquid phase of the primary component at $T_{BOIL}^{(A)}$. For example, as previously mentioned, methane is soluble in liquid nitrogen. The phase diagram in such a case is described in terms of dew point and bubble curves. These two lines define the (LV) region of a single liquid phase coexisting in equilibrium with a vapor phase, that is, the transition zone between the completely gaseous and the completely liquid domains. The bubble point curve represents the lowest achievable temperatures in the evaporator that include both vapor and liquid phases. The extent of evaporation, and thus the temperature too, will depend on the magnitude of the applied heat load. The lowest achievable temperature in the evaporator is higher than $T_{BOIL}^{(A)}$ and depends on the curvature (or flatness) of the dew point locus.

For instance, the lowest temperature of an equimolar mixture of nitrogen and methane may be 82 K. Typically, soluble additives satisfy the relationship: $T_{BOIL}^{(B)} - T_{BOIL}^{(A)} < 70$ K.

8.5.4.4 The Temperature Inside the Evaporator: Partially Miscible Additives

In general an additive will be only partially miscible in the liquid phase of the primary component when a large gap exists between its boiling point and that of the primary fluid, that is $T_{BOIL}^{(B)} - T_{BOIL}^{(A)} < 70$ K. Many cryogenic mixtures are of this kind.¹⁴⁵ For example, one may find experimental evidence for partial miscibility of ethane in nitrogen^{146,147,148} or propane in nitrogen^{149,150} at cryogenic temperatures. These additives are “warmer” than gases that are soluble in the primary component, (liquid nitrogen) and thus the $\Delta h_T(T)$ of such a mixture may be higher. The phase diagram of a binary mixture of ethane in nitrogen was discussed by Maytal et al.¹³⁴ The horizontal isotherm at about 77.3 K represents a coexistence region (a triple point) of three phases in equilibrium: liquid-liquid-vapor (LLV). The liquid phase of this mixture at temperatures below 77.3 K splits into two coexisting phases: the first, almost pure nitrogen, and the other, rich with ethane. The transition from liquid to vapor resembles that of a pure material; the temperature remains constant until the nitrogen-rich phase boils off. The split, or coexistence of the two liquid phases at the lower temperature is typical and affords advantages for this kind of mixtures. In particular, the LLV phase equilibrium guarantees:

- (a) A constant and stable temperature in the evaporator, and,
- (b) A temperature of operation that practically coincides with $T_{BOIL}^{(A)}$.

In some mixtures, the temperature remains constant even when applying the maximum cooling power. However, in other mixtures the second liquid phase may begin to evaporate and consequently, the temperature rises up to the dew point.

8.5.5 Miscellaneous

Phase equilibria of solid, liquid and vapor are discussed in fundamental texts such as Walas¹⁵¹ and Prausnitz.¹⁵² Maytal et al.¹³⁴ used the Peng-Robinson equation of state to study both the SLV and LLV phase equilibria associated with mixed coolant cycles. Robinson^{142,153} focused on eutectic compositions and solubility limits of the solid phase. A comprehensive discussion regarding the relevance of SLV and LLV phase equilibria for Joule-Thomson coolants may be found in the patent of Hingst¹⁵⁴ about the relevance of SLV and LLV phase equilibrium for Joule-Thomson coolants.

Xu et al.¹⁵⁵ and Luo et al.¹⁵⁶ studied the liquid-vapor equilibrium with hydrocarbons and freons.¹⁵⁷ Luo et al.^{139,156} determined equilibrium regimes for binary and ternary solid-liquid mixtures. The freezing points for similar mixtures were studied by Hwang et al.¹⁵⁸

Complementary publications on candidate mixtures and their thermodynamic properties in other languages are recommended, such as those in Russian by Boyarski^{143,159,160} and others,^{13,161,162} in Chinese and Japanese by Luo^{163,164,165} in German by Alexeev.⁵⁰ Chorowski reports, in Polish^{166,167} on the LV and LLV phase equilibria of N₂-CH₄ and N₂-R13 mixtures.

Thermophysical properties of mixtures with hydrocarbons may be obtained by various commercially available numerical codes.^{168,169,170,171} Optimized compositions for various objectives are discussed in the next section.

8.6 Reported Mixtures

8.6.1 Species and Concentration

A broad array of mixtures and mixture components have been examined either experimentally or analytically, and these provide both an interesting as well as a practically useful data base.^{3,4,5,8,10,20,30,32,33,34,42,43,53,57,72,75,76,87,95,120,124,134,138,144,172,173,174,175,176,177,178,179,132,180,181,182,183,184,185,186,187,188,189,190} Table 8.4 chronicles the evolution of mixtures for Linde-Hampson closed cycle cryocoolers. High-pressure mixtures utilized in open cycles are discussed later in Sect. 8.10 but their compositions are also displayed in Table 8.6. In view of the variety of proposed mixtures in literature, the following general comments may be helpful:

8.6.1.1 Primary Components

1. Nitrogen is adapted in most cases as the primary component for mixtures operating in the temperature range of 80–100 K.
2. Argon is the primary component used to reach temperatures in the range from 90 to 120 K.
3. For higher temperatures, such as 120–185 K the primary component may be methane,^{20,37,43,124,183,184} krypton⁷² or CF₄.^{113,183,184,141}

8.6.1.2 Hydrocarbons

Hydrocarbons represent the first group of components used as additives. The family of hydrocarbons (HC) has been very popular and remains very much in common use “...due to good thermodynamic properties, good solubility with

[illegible]

compressor oils, and zero depletion potential.”^{129,191} The most widely utilized hydrocarbons are the light (CH_4 , C_2H_6) and heavier (C_3H_8 , C_4H_{10}) hydrocarbons. Less common are C_2H_4 , C_3H_4 , C_3H_6 , C_4H_8 and the heavier hydrocarbons (C_5H_{12} , C_6H_{14}). In comparison to the later proposed families of additives, the HCs have relatively low melting points, low molecular weights, and are compatible with mineral lubricants. However, the HC's are flammable. Although closed cycle systems designed to provide a few watts of cooling contain only a limited amount of refrigerant (less than 200 g), nevertheless their flammability represents a definite disadvantage.

8.6.1.3 Flammability Retardant for Hydrocarbons

The refrigerant R-13b1 (the “halon” CF_3Br , bromium trifluoromethane) is an additive with the capability to extinguish the flammability of some hydrocarbon mixtures. Vogt¹⁷⁹ claims that a concentration of less than 10% (by volume) CF_3Br in a mixture with as high as a 70% concentration of hydrocarbons (CH_4 - C_2H_6 - C_3H_8) in nitrogen, will provide a complete flame retardant. Furthermore, Little²⁰ used a concentration of only 4.7% of this additive in a mixture of only hydrocarbons, i.e. without nitrogen. It should be emphasized that Vogt's¹⁷⁹ claims relate solely to the three mentioned hydrocarbon additives. However, the same component does not guarantee flame retardation with other additives such as oxygen or the heavier hydrocarbons. Hingst¹⁵⁴ proposed replacing R-13b1 with the flame-resistant agent heptafluoropropane in a concentration of 5–15% by volume, either in addition to, or in combination with a 3–20% concentration of CF_4 (R-14).

It should be mentioned that the low capacity closed cycle systems include a relatively small charge of mixture, and thus a very limited amount of flammable material is actually involved. Furthermore, if there are no leaks and the mixture is not exposed to oxygen, the content is not flammable.

8.6.1.4 Halogenated Derivatives of Hydrocarbons

Halogenated derivatives of hydrocarbons, commercially known as freons and helons are nonflammable candidate additives. In these components, some of the hydrogen atoms of any hydrocarbon are replaced by fluorine, chlorine or bromine. Typical members of this group are:

1. Hydro-chloro-fluoro-carbons (HCFC) like CHClF_2 (R-22),
2. Chloro-fluoro-carbons (CFC) like, CClF_3 (R-13), CCl_2F_2 (R-12),
3. Fluorocarbons (FC) like, CF_4 (R-14), C_2F_6 , C_3F_8 ,
4. Hydro-fluoro-carbons (HFC) like, CHF_3 (R-23), C_2HF_5 (R-125).

The enthalpy of nitrogen-freon mixtures was studied by Zakharov et al.¹⁹²

8.6.1.5 Fluoro-Ethers

Fluoro-ethers include gases such as $\text{CF}_3\text{-O-CF}_3$, $\text{CHF}_2\text{-O-CF}_3$, and $\text{CH}_2\text{F-O-CF}_3$, as well as others. Such additives are utilized by Novak et al.,^{183,184} Achurya et al.,^{193,194} Olszewski et al.^{81,82} in zeotropic mixtures to accommodate distributed and variable loads over a wide temperature range. In a single compression stage cycle, they employed non ozone-depleting components from the groups of fluorocarbons, hydrofluorocarbons and fluoroethers.

8.6.1.6 Inert Gas Additives

Inert gas additives, as proposed by Flynn et al.,¹⁸⁹ include nitrogen, argon, krypton, xenon, neon, and helium. They also investigated mixtures including gases from the group: carbon dioxide, oxygen, and nitrous oxide (N_2O).

8.6.1.7 Ozone Depleting Additives

The halogenated derivatives of hydrocarbons with chlorine or bromide atoms contribute to the depletion of the ozone layer. The Montreal Protocol convention defined low-ozone-depleting materials as those having an ozone depleting potential less than 0.15. For reference, the ozone depleting potential of R-12, which is CCl_2F_2 is 1.0. In the early stage of their development, there was little awareness regarding the ozone-depleting potential of these gases and they were not regulated or controlled. Modern refrigerants exclude the CFC components. Inert gases and hydrocarbons have very low ozone depleting potential.

8.6.1.8 Oxygen

In too high of a concentration, the quantum gases may cause two undesirable effects: (a) a reduced Joule-Thomson effect with an associated lower efficiency and slower cooldown, and (b) solidification within the mixture thereby interrupting operation of the cycle. For example Landa et al.³² describe an experiment with a N_2 -Ne mixture closely approaching the triple point of nitrogen (63 K) that displayed such problems. However, oxygen provides benefits as an additive that counterbalance these drawbacks. It elevates the Joule-Thomson effect and its low melting point (54.4 K) retards the mixture's solidification. Anikeev, Grezin and Zakharov¹⁷³ added a substantial fraction of oxygen to a mixture of N_2 , R14, R13 and Ne in order to reach temperatures as low as 50 K.

8.6.1.9 Quantum Gases

Helium, hydrogen, and neon are incorporated as mixture additives for the following purposes:

1. Lowering the boiling temperature, T_{BOIL} . Alfeev et al.⁸ used the quantum gases in each of their proposed mixtures for the 80–100 K range. Since the quantum gases are immiscible at these temperatures, their presence is equivalent to lowering the partial pressure of the

remaining mixture components. In this way, the operating temperature is reduced below the normal boiling point, $T_{BOIL}(A)$, of the primary component. For example²⁰ the mixture of (N_2 - CH_4 - C_2H_6 - C_3H_8 -Ne) with the composition of (0.25, 0.25, 0.15, 0.15, 0.20) is able to reach 68 K. Weckemann, Wilke and Oellrich^{141,195} demonstrated that increasing the molar concentration of neon in a mixture of nitrogen and hydrocarbons up to 8.2% suppresses the boiling point by about 18 K in a no load operation. Any further impact at higher neon concentrations is minor. Additional examples may be found elsewhere.^{33,182,186,196}

2. Accelerating the cooldown of a closed cycle by employing a fixed orifice without any regulating mechanism, as proposed by Longworth et al.⁴¹ In this approach, the ratio of the densities of the coolant at the inlet to the orifice at the end of cooldown (or at steady state run) compared to the beginning of the process is reduced, as discussed in Sect. 8.3.6.2.
3. One would expect that the addition of helium to any mixture operating in the temperature range of 80–120 K would decrease its Δh_T and the associated COP. However, Boiarski et al.¹⁸¹ point out that when such an addition also elevates the suction pressure up to 0.3 MPa, the efficiency of the cycle may increase.

8.6.1.10 Miscellaneous

1. From Table 8.4, one may observe that the most commonly used components in the mixtures are N_2 , CH_4 , C_2H_6 , and C_3H_8 . Next come Ar, C_4H_{10} , and CF_4 .
2. The gas 1-butene (an isomer of C_4H_8) has a unique feature. Its melting temperature, $T_M = 87.8$ K, is relatively low in comparison to its elevated boiling temperature, $T_{BOIL} = 266.9$ K (employed in references³¹² and³²²).
3. Brodianski et al.¹⁷² show that a mixture of neon and hydrogen may be superior to each component for cryocooling below 35 K.
4. Dobak et al.^{70,120} explored a wide variety, (16), of less common components such as C_3H_4 and the toxic NF_3 . Their goal was to use a mixed coolant for the final stage of a precooled JT system, in which precooling was provided by a pure refrigerant, such as R13b1. Therefore, the majority of their mixtures were considered for a cooling range below 270 K.
5. Boyarski et al.⁸⁷ considered adding a precooling stage to a vapor-compression refrigeration cycle based on coolants such as R12 and NH_3 . They suggested using a zeotropic mixed refrigerant for the auxiliary stage, such as a mixture of R22-R142b-R123.
6. Bradley and Radebaugh¹⁹⁷ used a mixture with a compression ratio of only 4:1 for the 200 K temperature range. Its optimized composition, that is 8% methane,

46% ethane, 14% propane, 4% butane, and 26% pentane, possess a minimum enthalpy difference of 4 kJ/mole.

8.6.2 Optimized Mixtures

To define a ‘preferred’ composition for a mixture, one has to first specify the objective of the cooling cycle. This will in turn dictate the optimization criteria for the mixture.

8.6.2.1 The COP of a Closed Cycle Cryocooler

The COP of a closed cycle cryocooler represents the most common optimization goal. The standard basis for comparison must include the minimum allowable ΔT between the recuperating streams along the heat exchanger and the cold-end, or lowest operating temperature. There are two strategies for maximizing the COP:

1. Maximizing the specific cold production, $\dot{Q}/\dot{n}$, for any given pressure range and work of compression,^{54,31,50,107,123,139,198,199,200} or,
2. Minimizing the entropy generated in the heat exchanger by maintaining as small and constant a ΔT , as possible, along the entire recuperator. Podtcherniav et al.²⁰¹ demonstrate how the cooling capacity and COP do in fact increase as the minimum ΔT decreases. The optimal composition will be the one that equalizes the specific heat capacities of the recuperating streams. One of the earliest attempts of optimizing a three-component mixture belongs to Grezin, Gromov and Zakharov.^{137,202} Little¹⁸⁰ introduced a combined objective function, which accounts for both parameters: Δh_T and the extent to which the specific heat capacities are indeed similar over the relevant temperature range. The optimal composition maximizes the function,

$$\frac{\Delta h_T^{MIN}}{const + \sigma[\Delta h(T)]} \quad (8.14)$$

where $const > 0$ and where $\sigma[\Delta h_T(T)]$ denotes the standard deviation, σ , of the distribution of $\Delta h_T(T)$ over the temperature span. In the case of a vanishing difference between the heat capacities at all temperatures, there is no deviation of $\Delta h_T(T)$ and $\sigma = 0$. Such a condition maximizes the objective function of Eq. 8.14.

Wang et al.⁴⁷ optimized the composition of a zeotropic binary mixture along with various other parameters of a closed cycle for operation at 215 K.

8.6.2.2 The COP of a Precooled Cryocooler

The COP of a precooled cryocooler (discussed in Sect. 8.3.4) was optimized by Alexeev et al.⁵³ The precooled temperature, or inlet to the final stage heat exchanger, is 233–243 K,

and the optimized composition of the mixture is: N_2 (0.26–0.36), CH_4 (0.2–0.38), C_3H_8 (0.06–0.20), C_2H_6 the rest.

8.6.2.3 The COP for a Distributed Load Cycle

The load in this case (as discussed in Sect. 8.3.5) is distributed along the cold returning stream. Piotrowska and Chorowski⁹² precooled a 0.8 MPa stream of natural gas within a liquefier. The composition of the precoolant was optimized within the group of the candidates N_2 , CH_4 , and C_2H_6 . The COP was maximized by the composition: 0.76- CH_4 and 0.24- C_2H_6 . Gong et al.²⁰³ optimized a mixed coolant with five components for the same purpose of condensing natural gas; the primary (lowest boiling point) component was methane. Nellis, Pfothauer et al.^{204,205} focused on cooling the current leads for a superconducting magnet, absorbing both the distributed load generated by electrical resistive heating along the lead as well as a concentrated load on the cold end. They optimized the composition of hydrocarbons in order to obtain a maximum $\dot{Q}/\dot{n}$ or COP. Distributed heat loads are also discussed elsewhere.²⁰⁶

One may find analyses of numerous examples of optimized compositions in a book, which is entirely dedicated to “cryogenic mixed refrigerant processes.”²⁰⁷ It deals with cryocoolers, precooled cryocoolers, precooling cryocoolers with distributed loads, and natural gas liquefiers. In general, the composition within an optimized cryocooler has a larger fraction of nitrogen in the mixture than a precooling cryocooler with a distributed load.

8.6.2.4 Compactness of the Cold End

In other cases, the central issue of concern is reducing the size of the cold end, which means minimizing the size of the heat exchanger. Dobak, Marquardt et al.^{46,70,120} used such an approach to build a cryosurgical invasive probe for cardiac arrhythmia. In this application, ‘small’ is crucial. They compared numerous mixtures based on their demand for recuperation and heat transfer area, reflected by the parameter, Σ , defined by Eq. 3.95, as discussed in Sect. 3.7.6.

A direct method for optimizing the recuperator size was adopted by Keppler, Nellis and Klein.¹⁰⁷ Their optimization scheme varied mixture composition in order to maximize the heat load per unit of heat exchanger conductance, namely, $\dot{Q}/(U \cdot A)$. Applying advanced optimization techniques, their investigation included seven components, once with hydrocarbons and once with synthetic refrigerants. Skye et al.⁶⁵ optimized the mixture composition of a precooled cycle as function of a precooling temperature, using the same criteria. Nellis, Pfothauer et al.^{204,205} also optimized $\dot{Q}/(U \cdot A)$ for the cryocooled current leads. Note that compositions that optimize the size of a heat exchanger in

terms of $\dot{Q}/(U \cdot A)$ are not the same as those that optimize $\dot{Q}/\dot{n}$.

8.6.2.5 Cooldown

Gresin and Landa¹¹⁴ point out the fact that the largest demands for cooling are typically realized during the cooldown process. Hence, any optimizing procedure should primarily focus on satisfying the demands of the cooldown process. Khatri⁵² proposed mixture compositions that are capable of producing an elevated cooling power during the cooldown process; approaching 85% of the highest cooling power at the end of the cooldown.

8.7 Aspects of Closed Cycle Operation

Closed loop Joule-Thomson cryocooling is most frequently associated with mixed coolants, and for this reason we introduce the topic in the present chapter. Various reports regarding closed cycle operation with pure coolants also exist. However the results with pure coolants also hold for mixed coolants and, various additional typical aspects of mixed coolant behavior can also be identified.

8.7.1 Closed-Loop Parameters

8.7.1.1 The Amount of Coolant and the Volume of the Loop

Figure 8.15 displays a schematic of a closed cycle cooler. The entire loop is divided into two sections that are separated by points of pressure discontinuity, those being the compressor and the isenthalpic expansion orifice. The high pressure (discharge, or upstream section) and the low-pressure (suction, or downstream) section occupy the volumes V_U and V_D , correspondingly. The total volume of the closed

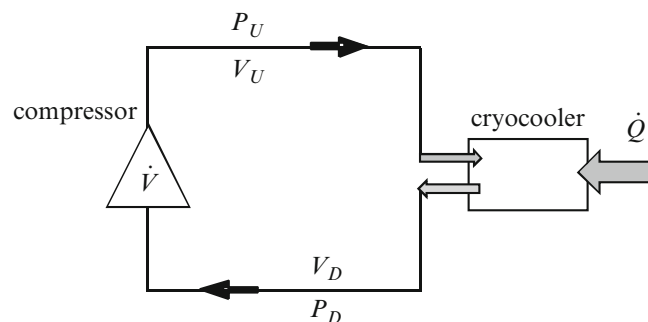


Fig. 8.15 Functional layout and the relevant parameters that affect the behavior of the closed-cycle Joule-Thomson cryocooler; displacement, volumes, throttle departure, pressures, and heat load. The V_U is the volume of the upstream high-pressure section of the closed loop and so V_D is the volume of the low pressure section. The rate of compressor's volumetric displacement is $\dot{V}$

system, $V_U + V_D$, is charged with a molar amount of gas, n_0 , which brings the initial, charging pressure to the value of P_0 . The Joule-Thomson expansion orifice has a cross section area of a .

8.7.1.2 The Relationship Between the Up and Down-Stream Pressures

The relationship between the up- and down-stream pressures is unique to closed cycles and not present in open cycle systems. When the compressor begins running, the pressure at the upstream section, P_U , which is the discharge pressure of the compressor, rises above P_0 . The pressure of the down-stream section, P_D , which is the suction pressure, drops below P_0 . The molar mass, n_0 , of the coolant is distributed between the two sections; n_U and n_D , correspondingly. In contrast to the open loop, in a closed loop P_U and P_D are not independent. Any change in one of these as a response to varying operating conditions (such as the adjustment of the orifice's size) is accompanied by a corresponding change in the other one. The linkage between the two pressures results from two aspects associated with the conservation of mass and continuity, that are inherently satisfied by a circulating coolant operating at steady state in a closed loop:

1. The high-pressure mass flow rate has to match the low-pressure mass flow rate. This aspect relates the two pressures through the size of the orifice,
2. The total amount of coolant, n_0 , in the entire loop remains unaltered although the distribution between the two sections, n_U and n_D , may alter. This aspect is associated with the actual pressure values and the volumes V_U and V_D , and,
3. In the case of a mixed coolant, the amount of each species is conserved separately.

8.7.1.3 The Compressor

The compressor removes gas from the low pressure, or suction side, and displaces it into the high-pressure section of the cycle.

The Displacement

The displacement, d , is the swept volume during a single stroke a compressor, measured in cm^3 . It characterizes the size of any compressor.

The Rate of Volumetric Displacement

The rate of volumetric displacement, $\dot{V}$, measured in $\text{m}^3 \text{s}^{-1}$, depends on f , the frequency of the strokes, so that,

$$\dot{V} = d \cdot f \quad (8.15)$$

Without the flow restriction introduced by the orifice, only a negligible pressure drop would be generated through

the piping, so that essentially $P_0 = P_U = P_D$. In such a case, the compressor simply transfers gas from the suction side to the discharge side, albeit at the highest achievable flow rate.

The Rate of (Molar) Mass Displacement

The rate of (molar) mass displacement, $\dot{n}$, that is generated by the compressor when a non-zero flow restriction exists, is determined by a couple of factors. When a restriction, such as an orifice is introduced, the suction pressure drops below P_0 and the inlet density is correspondingly reduced. At the same time, the discharge pressure P_U rises above P_0 . The (molar) mass flow rate, $(\dot{n}) \dot{m}$, which is a constant around the complete loop, can be determined from the density and volumetric flow rate at the inlet, or suction side of the compressor,

$$\dot{n} = \dot{V} \cdot \rho(P_D) \cdot \eta_V \quad (8.16)$$

where η_V is the volumetric efficiency of the compressor, a value less than unity.

The Volumetric Efficiency

The volumetric efficiency, η_V , is the fraction of the swept mass, $\dot{V} \cdot \rho(P_D)$, which is actually transferred and discharged into the high-pressure wing. A variety of factors influence the volumetric efficiency of a compressor including its void volume, the content of which is compressed but not discharged, internal leakages induced by the discharge and suction pressure difference, $P_U - P_D$, and at very high flow rates in reciprocating compressors, the inertial impedance of flow into and out of the cylinders. The volumetric efficiency is a moderately decreasing function of the pressure ratio. It is sometimes approximated as a linear function of the pressure ratio, such as in references^{181 208}, and for various mixtures as given by Boiarski et al.,¹²⁹

$$\eta_V = 0.71 + (0.35 - 0.34 \kappa) \cdot \left[(P_U/P_D)^{1/\kappa} - 1 \right] \quad (8.17)$$

Here, $\kappa = \sum_i z^i \cdot \kappa^i$, where z^i and κ^i are respectively the molar fraction and specific heat ratio, κ for each component, i . Khatri and Boiarski¹³⁰ emphasize that when evaluating the volumetric efficiency, η_V , the values of P_U , and especially P_D , must accurately represent the suction and discharge pressures at the respective ports of the compressor itself; that is, they must not include the pressure drops along the lines and through the oil separator.

8.7.1.4 The Specific Cooling Capacity of the Coolant

The specific cooling capacity of the coolant, in the case of an open loop operation, is characterized by the cooling power per unit molar flow rate, $\dot{Q}/\dot{n}$, which is Δh_T , typically given

in terms of J/mole. Thus, a gas with a larger value of Δh_T may require a smaller pressure vessel for a given cooling objective. In contrast, the comparison of coolants in a closed loop system is based on a similar volumetric displacement rate, $\dot{V}$, which is associated with the size of compressor. The cooling power per $\dot{V}$, in a closed loop, that is the *volumetric cooling content*, then becomes,

$$Q_V = \frac{\dot{Q}}{\dot{V}} = \frac{\dot{n} \cdot \Delta h_T(P_U, P_D)}{\dot{V}} = \rho(P_D) \cdot \Delta h_T(P_U, P_D) \equiv \rho_D \cdot \Delta h_T \quad (8.18)$$

and is measured in terms of J/m³.

The volumetric cooling content enables one to compare the combined influence of the coolants and different operating (suction) pressures.⁴² In an open cycle, the downstream pressure P_D has only a minor effect on the specific cooling capacity, $\dot{Q}/\dot{n}$. For example, if P_D is doubled (0.2 instead of 0.1 MPa) the flow rate will change very little due to the choked flow conditions at the orifice, and the value of Δh_T will in fact decrease slightly. However, in the case of a closed cycle and a given coolant, by merely changing P_D for example by doubling its value, the circulating mass flow rate is also approximately doubled by the same compressor with the same $\dot{V}$. The resulting cooling capacity may thereby also be significantly altered as reflected by the magnitude of Q_V which is $\rho_D \cdot \Delta h_T$.

8.7.1.5 The Liquefied Amount, n_{LIQ}

The portion of the total number of moles of gas n_0 in a charged system that is liquefied and accumulated at the cold end during the operation of a closed cycle cryocooler (n_{LIQ}) is determined by two factors: (a) the transition from cooldown to steady state operation, and, (b) variations in the applied heat load. Of course, $\dot{n}_{LIQ}$ may be positive or negative. The behavior of a closed cycle is affected by n_{LIQ} , and whether it is significantly smaller than or rather comparable to n_0 .

8.7.2 Simplified Analysis

8.7.2.1 Assumptions

In order to provide a simplified formulation of steady state operation, a number of assumptions must be included. However, these must preserve the essence of the closed loop behavior. After Maytal,²⁰⁹ the following assumptions are applied:

1. The liquefied molar amount, n_{LIQ} , is relatively minor. That is,

$$n_{LIQ} \ll n_0 \quad (8.19)$$

2. Pressure drops along the upstream and downstream piping volumes, V_U and V_D , are negligible in comparison to that across the orifice,
3. The volumetric efficiency of the compressor, η_V , has a constant value, independent of P_U or P_D .
4. The high-pressure flow rate $\dot{n}_U$, being driven through the orifice by P_U , is choked. Assuming that at steady state the inlet temperature is constant, then $\dot{n}_U$ is solely dependent on P_U , through a linear relationship with a proportionality factor, K .
5. The compression is isothermal at T_{AMB} .
6. The effectiveness of the recuperator is 100%, so that its outlet temperature is very close to that of the inlet. Hence, the contents of V_U and V_D remain at the same temperature, T_{AMB} .
7. The gaseous contents of V_U and V_D behave as an ideal gas, since the pressure level is relatively moderate.
8. Liquefaction occurs only inside the cryocooler (heat exchanger and evaporator) and both V_U and V_D contain only gaseous phases.
9. The combined volume of the heat exchanger's channels and the evaporator is significantly smaller than V_U or V_D .

8.7.2.2 The Compression Ratio

The mass continuity equation is used to match the separate flow rates in the high and low-pressure sections. The intake flow rate, given in terms of compressor's displacement rate, is expressed as,

$$\dot{n}_D = \rho_D \cdot \dot{V} \cdot \eta_V = \dot{V} \cdot \eta_V \cdot \frac{P_D}{R \cdot T_{AMB}} \quad (8.20)$$

The high-pressure flow rate through the orifice is expressed as,

$$\dot{n}_U = K \cdot P_U \cdot a \quad (8.21)$$

Equating the two molar flow rates then gives,

$$\frac{P_D}{P_U} = a \left(\frac{\dot{V} \cdot \eta_V}{K R T_{AMB}} \right)^{-1} \equiv \bar{a} \quad (8.22)$$

where $\bar{a}$ is the non dimensional cross section area of the orifice.

8.7.2.3 Mass Conservation

Mass conservation in the two sections of the loop also provides a second relationship between P_U and P_D ,

$$n_U + n_D = \frac{P_U V_U}{R T_{AMB}} + \frac{P_D V_D}{R T_{AMB}} = \frac{P_0 (V_U + V_D)}{R T_{AMB}} = n_0 \quad (8.23)$$

8.7.2.4 The Absolute Values of the Pressures

The absolute values of the pressures, are obtained by combining the equations of continuity and mass conservation, expressed with the aid of the volume ratio parameter,

$$\alpha = \frac{V_U}{V_D} \quad (8.24)$$

$$\frac{P_D}{P_0} = \bar{a} \frac{\alpha + 1}{\alpha + \bar{a}} \quad \text{and} \quad \frac{P_U}{P_0} = \frac{\alpha + 1}{\alpha + \bar{a}} \quad (8.25)$$

$$\begin{aligned} \frac{\Delta P_U}{P_0} &= \frac{P_U - P_0}{P_0} = \frac{1 - \bar{a}}{\alpha + \bar{a}} \quad \text{and} \\ \frac{\Delta P_D}{P_0} &= \frac{P_0 - P_D}{P_0} = \alpha \frac{1 - \bar{a}}{\alpha + \bar{a}} \end{aligned} \quad (8.26)$$

$$\frac{\Delta P}{P_0} = \frac{\Delta P_U + \Delta P_D}{P_0} = \frac{P_U - P_D}{P_0} = (1 + \alpha) \frac{1 - \bar{a}}{\alpha + \bar{a}} \quad (8.27)$$

As α becomes larger, the charging pressure approaches the upper limit of P_U , and vice versa. Several issues are impacted by the magnitude of the charging pressure. For example, a closed system with a large charge pressure will be more susceptible to induced leakage over time. On the other hand using a low charging pressure for a desired P_U , may require a large suction volume or small discharge volume.

As the size of the orifice opening, a , increases, both P_U and the pressure drop ΔP decrease. This is purely a closed cycle effect. Additionally in this case, the suction pressure P_D will increase and elevate the boiling temperature.

8.7.2.5 The Distribution Ratios of a Coolant's Mass and Pressure

The above equations enable us to derive a number of useful expressions relating the pressure and volume ratios to the molar and pressure distributions:

$$\frac{n_U}{n_D} = \frac{\alpha}{\bar{a}} \quad \text{or} \quad \frac{n_U}{n_0} = \frac{\alpha}{\bar{a} + \alpha} \quad (8.28)$$

$$\frac{\Delta P_D}{\Delta P_U} = \alpha \quad (8.29)$$

Note that the mass and pressure are equally divided between the upstream and downstream sections when $\alpha = \bar{a}$ and $\alpha = 1$, correspondingly.

8.7.2.6 The Circulating Flow Rate

The circulating flow rate, may also be expressed in terms of the volume and pressure drop ratios:

$$\frac{\dot{n}}{\dot{n}_{MAX}} = \bar{a} \frac{\alpha + 1}{\alpha + \bar{a}} \quad (8.30)$$

where

$$\dot{n}_{MAX} = \frac{\eta_V \cdot \dot{V} \cdot P_0}{R \cdot T_{AMB}} \quad (8.31)$$

For a larger a , the mass flow rate increases since P_D and the suction density $\rho(P_D)$ increase, thus more material is displaced by each stroke of the compressor.

8.7.2.7 Cooling Power

One may develop an expression for the cooling power by starting with the assumption that in the relevant low pressure range of the closed cycle, the specific cooling power, can be linearized in the form,

$$\Delta h_T = Y \cdot (P_U - P_D) \quad (8.32)$$

Then, the non dimensional representation of the cooling power becomes,

$$\dot{Q} \left(\frac{\eta_V \cdot \dot{V} \cdot Y \cdot P_0^2}{R \cdot T_{AMB}} \right)^{-1} = \bar{a}(1 - \bar{a}) \left(\frac{\alpha + 1}{\alpha + \bar{a}} \right)^2 \quad (8.33)$$

There is an optimum size of the orifice, which maximizes the cooling power; its size is,

$$\bar{a}_{OPT} = \frac{\alpha}{2\alpha + 1} \quad (8.34)$$

inducing a flow rate of,

$$\dot{n}_{OPT} = \frac{1}{2} \dot{n}_{MAX} \quad (8.35)$$

By physical argumentation, one may expect an optimum value of $\dot{Q}$. A very small orifice elevates P_U which, according to this model, would result in a higher cooling content, Δh_T . However, the associated vanishing flow rate through the minute orifice results in a vanishing cooling power, $\dot{Q}$. On the other hand, allowing the orifice to grow too large, with the associated large flow rate, decreases the discharge pressure P_U and suppresses both Δh_T and $\dot{Q}$. Therefore, an intermediate and optimal size of a exists. In other words, broadening the orifice in a certain range, may enable an increase of cooling power. This feature can be observed in the patent of Boyarski et al.^{101,181} in a mixed coolant closed cycle.

The value of the maximum attainable cooling power is,

$$\dot{Q}^{MAX} = \left(1 + \frac{1}{\alpha} \right) \frac{\eta_V \cdot \dot{V} \cdot Y \cdot P_0^2}{4 R \cdot T_{AMB}} \quad (8.36)$$

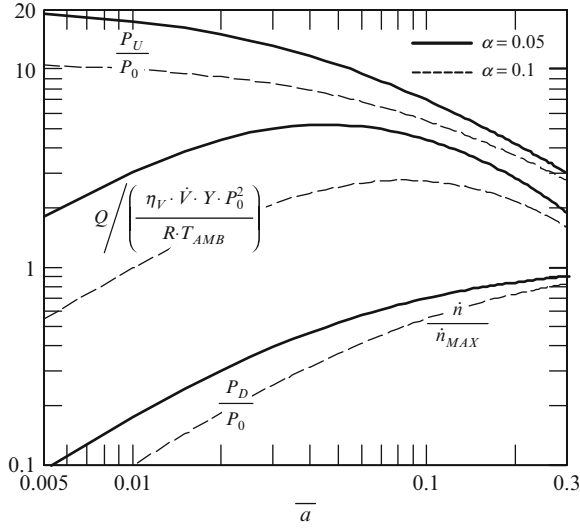


Fig. 8.16 The cooling power together with the discharge and suction pressures displayed as function of the non dimensional size of the orifice, $\bar{a}$, for $\alpha = V_U/V_D = 0.05$ and 0.1

and in terms of the maximum allowable pressure, P^{MAX} and the pressure ratio,

$$\dot{Q}_{MAX} = \frac{4 \eta_V \cdot \dot{V} \cdot Y \cdot (P^{MAX})^2}{R \cdot T_{AMB}} \cdot \left(1 - \frac{P_D}{P^{MAX}}\right) \cdot \frac{P_D}{P^{MAX}} \quad (8.37)$$

Figure 8.16 displays the cooling power together with the discharge and suction pressures as function of the size of the orifice.

8.7.2.8 Warming Capability

Skye et al.⁶⁵ identify a convenient method for warming a cryosurgical probe with a closed cycle JT system. By employing a widely opened orifice, through which the flow is unrestricted, the flow path essentially provides zero pressure drop, that is $P_U = P_D$, and therefore no Joule-Thomson effect. In this case, the compressor circulates the flow at the highest rate, $\dot{h}_{MAX}$, but no cooling power is generated. The coolant approaches room temperature and warms up the cooled object.

8.7.2.9 The Hydrodynamic Time Constant of the Closed Cycle

During the transient start up of a closed cycle system, the compressor gradually removes gas from the low pressure section, V_D , and transfers it into the high-pressure section, V_U , at the volumetric rate of $\eta_V \cdot \dot{V}$. Simultaneously, a portion of gas is transferred through the orifice from V_U to V_D at a rate which is driven by the instantaneous pressure, $P_U(t)$.

The rates of the two streams are not equal and therefore the pressure gradually increases on the discharge side of the compressor. The upstream pressure $P_U(t)$ starts at P_0 and increases to its steady state value while at the same time the downstream pressure, $P_D(t)$ decreases. The transient link between the two pressure changes is,

$$\begin{pmatrix} \dot{P}_U \\ \dot{P}_D \end{pmatrix} = \begin{bmatrix} -\bar{a}/\tau_U & 1/\tau_U \\ \bar{a}/\tau_D & -1/\tau_D \end{bmatrix} \begin{pmatrix} P_U \\ P_D \end{pmatrix} \quad (8.38)$$

$@t = 0, P_U = P_D = P_0$

where

$$\tau_D = \frac{V_D}{\dot{V} \cdot \eta_V} \quad \text{and} \quad \tau_U = \frac{V_U}{\dot{V} \cdot \eta_V} \quad (8.39)$$

The solution is,

$$\frac{P_U(t)}{P_0} = \frac{\alpha + 1}{\alpha + \bar{a}} + \frac{\bar{a} - 1}{\bar{a} + \alpha} \exp\left(-\frac{t}{\tau}\right) \quad (8.40)$$

and

$$\frac{P_D(t)}{P_0} = \bar{a} \frac{\alpha + 1}{\alpha + \bar{a}} - \alpha \frac{\bar{a} - 1}{\bar{a} + \alpha} \exp\left(-\frac{t}{\tau}\right) \quad (8.41)$$

where

$$\frac{1}{\tau} = \frac{1}{\tau_D} + \frac{\bar{a}}{\tau_U} \quad (8.42)$$

or,

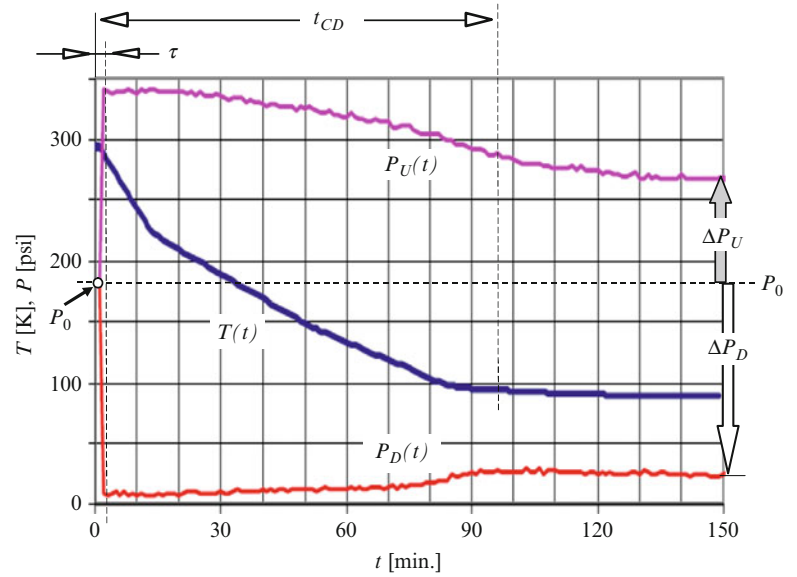
$$\tau = \tau_D \cdot \frac{n_U}{n_0} \quad (8.43)$$

τ is the hydrodynamic time constant of the cycle and its significance is demonstrated in the next paragraph. Pressure build up is faster as the orifice is wider open and the volumetric displacement of the compressor is larger. Larger volumes V_U and V_D prolong the process. As the fraction of n_U is smaller the transient response is faster.

8.7.3 The Hydrodynamic Behavior During Cooldown of a Closed Cycle

Typical measurements¹³⁰ of the up and down stream pressures, $P_U(t)$, $P_D(t)$ and the temperature $T(t)$ during the transient startup and cooldown are shown in Fig. 8.17. The first feature to note, is the significance of the hydrodynamic time constant, τ . As $T(t)$ decreases, so does the temperature

Fig. 8.17 Measurements of the hydrodynamic behavior during cooldown¹³⁰ demonstrating the significance of the hydrodynamic time constant, τ



at the inlet to the JT valve and therefore, the circulating flow rate increases. In terms of the hydrodynamic behavior, it is equivalent to assuming that $\bar{a}$ increases during cooldown. Since in general, $\tau \ll t_{CD}$, the changes in pressure may be considered as quasi-static; thus the behavior should obey the previously derived formulation as summarized in Fig. 8.16. While $\bar{a}$ increases, P_U decreases and P_D increases, until a time, t_{CD} , when steady state values of ΔP_U and ΔP_D are achieved, as shown in the figure. In the same example $\Delta P_U < \Delta P_D$ which means that $V_U > V_D$. Consequently, the compression ratio is large at the beginning of cooldown, and gradually decreases approaching a steady state value at the end of cooldown. The behavior is consistent with Eq. 8.22 if one adopts the interpretation of effectively increasing $\bar{a}$ during the cooldown.

In general, the development from the start-up, through time τ , and up to time t_{CD} , results in a peak value of $P_U(t)$ and a minimum value of $P_D(t)$. This feature is accompanied by increased power demand from the electrical motor that drives the starting phase of the closed cycle cryocooler, as discussed elsewhere.²¹⁰

8.7.4 The Self-Regulating Effect of a Substantial Liquefied Fraction

8.7.4.1 Description of the Self-Regulating Effect

In the case that n_{LIQ} is a significant and non-negligible portion of n_0 , the closed cycle exhibits a self regulating or a self-balancing effect. Therefore, a system of smaller sizes and smaller n_0 is more likely to experience the self-regulating effect. Self-regulation of a miniature cryocooler occurs in two respects:

- The rate of cooling, $\dot{Q}$, at the end of cooldown is reduced to the steady state value, and,
- The rate of cooling adjusts to the instantaneous heat load.

The amount of liquid, n_{LIQ} accumulates at the cold end of the cryocooler so that part of it boils off or condenses as the heat load increases or decreases. Landa et al.^{32,98} identified and investigated the self regulating effect. In their experiments, the liquid is captured in a porous element in the evaporator and in the low-pressure channel.

An open system, with a fixed orifice and pressure supply does not enable this feature. Instead, frequent flow demand adjustments are included as discussed in Sect. 6.2.

8.7.4.2 Pure Coolant Closed Cycle

The Mechanism of Self-Regulation: Adjustment of ΔP

Liquefying a significant fraction the circulating fluid reduces the amount that remains as a gas. The result is equivalent to reducing n_0 and P_0 of the closed cycle. This in turn also reduces P_U , P_D , ρ_D and ΔP , thus lowering the Δh_T . Hence, the specific cooling capacity, Q_V , of the coolant becomes smaller and with a given compressor, the cooling rate is reduced. The opposite scenario is also true: reducing n_{LIQ} will cause an elevation of $\dot{Q}$.

Cooldown Versus Steady State Cooling Capacity

An amount n_{LIQ} is accumulated during the cooldown process and maintained during the steady state (SS) operation. Landa et al.³² expressed the reduction of the cooling capacity at steady state, $\dot{Q}^{SS}$, as compared to that during the cooldown, $\dot{Q}^{CD}$, by,

$$\frac{\dot{Q}^{CD}}{\dot{Q}^{SS}} = 1 + \frac{n_{LIQ}}{n_D} \quad (8.44)$$

where n_D is the molar content of coolant in the volume V_D . In the case of an open cycle, the trend is opposite. The cooling capacity increases as the cooldown process proceeds, and continues to increase until a steady state is reached.

The Self-Regulating Response to Heat Load Variation

As the instantaneous heat load increases, a part of the accumulated liquid will evaporate. This releases more coolant into the circulating stream, elevating ρ_D and ΔP , resulting in a higher cooling capacity, $\dot{Q}_V$ and $\dot{Q}$. In short, the increase of the applied heat load produces an increase in the cooling capacity. On the other hand, an excess of cooling power, $\dot{Q}$ increases the amount of liquid n_{LIQ} . However, a larger amount of liquid reduces $\dot{Q}$. This is the essence of the stabilizing effect; a negative feedback that opposes the cause of the imposed change.

Landa et al.³² demonstrated the self-regulating effect by measurements with pure nitrogen, once during cooldown and secondly, as a function of applied heat load. The values of V_U and V_D were 270 and 4.5 cm³, correspondingly; which means $\alpha = 0.017$. The upstream pressure P_U changed from about 6 to 10 MPa as the heat load increased from 0 to 3 W. One should recall that the associated change of P_D is accompanied by a change of the boiling temperature.

8.7.4.3 Mixed Coolant Closed Cycle

The Mechanism of Self-Regulation:

Adjustment of the Composition

The above conclusions with pure coolants remain valid for mixtures too. However, in the case of mixed coolants, the self-regulating effect may be enhanced by an additional mechanism. The accumulated condensate, n_{LIQ} , changes, not only by the amount, but also by the composition of the remaining and the circulating coolant. This mechanism has a similar stabilizing response.

Since n_{LIQ} is very rich with the less volatile components, the remaining gaseous part of n_0 becomes enriched with the lower boiling point components. In this way, the composition of the circulating gas at steady state is different from the originally charged mixture, n_0 . However, when a mixture becomes enriched with volatile components, a twofold result occurs; first, Δh_T is reduced, and second, the boiling point is decreased (for a given P_D).

Cooling Power at Steady State Versus Cooldown

Landa et al.³² proposed an expression for the extra cooling capacity of a mixture during cooldown,

Table 8.5 The self regulating effect of a mixed refrigerant closed cycle, after Landa et al.³² The initial composition of the coolant changes as cooldown proceeds to steady state

Mixture	Composition, % by volume	
	Charged	Steady state run (post cooldown)
N ₂	43.4	64.1
R-14 (CF ₄)	15.1	12.4
R-13 (CClF ₃)	38.9	18.3
Ne	2.6	5.2
Performance regime		
	Cooldown	Steady state run
High pressure [MPa]	15	7.1
Low pressure [MPa]	0.20	0.11
Δh_T [J/mole]	3,867	942
$\dot{n}$ [SLPM]	4.9	3.0

$$\left(\frac{\dot{Q}^{CD}}{\dot{Q}^{SS}}\right)^{MIX} = \left(\frac{\Delta h_T^{CD}}{\Delta h_T^{SS}}\right)^{MIX} \cdot \frac{M^{CD}}{M^{SS}} \cdot \left(1 + \frac{n_{LIQ}}{n_D}\right) \quad (8.45)$$

The molecular mass, M , of the circulating coolant at steady state is smaller than that of the charging composition. The more volatile components in the remaining mixture are in general lighter and of higher concentration, hence,

$$\frac{\Delta h_T^{CD}}{\Delta h_T^{SS}} > 1 \quad \frac{M^{CD}}{M^{SS}} > 1 \quad (8.46)$$

The Change of Composition During Cooldown

The measurements displayed in Table 8.5 compare compositions of the circulating coolant at the beginning of cooldown and at steady state. The more volatile components, nitrogen and neon, increase their concentration as the cooldown approaches steady state while the less volatile components R-13 and R-14 become less concentrated.

The same mechanism of changing composition enhances the self-regulating effect when a change occurs in the external heat load. A reduced heat load converts the excess cooling power into further accumulation of coolant, which in turn leads to a decrease in the cooling capacity of the cycle. Likewise an elevated heat load results in a larger cooling power.

One may also observe the self-regulating hydrodynamic effect (not related with composition). Because of the substantial value of n_{LIQ} at steady state, decreases are observed in P_U (in the above example, from 15 to 7.1 MPa), P_D (from 0.2 to 0.1 MPa) and $\dot{n}$ (from 4.9 to 3.0 SLPM). This explains why Δh_T of the circulating mixture decreases from 3,867 to 942 J/mole.

8.7.5 Additional Closed Cycle Cryocoolers

8.7.5.1 Compressor Output Regulation

A compressor's output may be regulated by continuously measuring the absolute suction pressure, P_D , or instead, by directly measuring the payload's temperature. The rotation speed of the compressor is then regulated according to an algorithm that adjusts one of the two measured functions. Landa²¹¹ explains how the regulation of a compressor's output may expand its life from 5,000 to 20,000 h.

8.7.5.2 Composition Changes During Cooldown

Gong et al.²¹² measured the composition changes of a circulating coolant during cooldown. The measurements were gathered with three different mixtures to accommodate different ranges of cooling temperatures. In addition, they compared the composition at three locations along the loop, and studied the composition shift of the coolant due to differences in the solubility of the compressor's lubricant²¹³ or, in the case of oil free compressors, due to the effects of liquid phase holdup in the two-phase flow.²¹⁴ The shift of composition is caused by the different compositions in the liquid and vapor phases. Lakshmi and Venkatarathnam^{215,216} discussed the inverse problem, that is, determining the necessary charging composition in order to guarantee a desirable composition of the circulating stream.

8.7.5.3 The Influence of the Orifice

Luo et al.⁵⁷ showed that a larger orifice accelerates cycle cooldown, but elevates the steady state temperature of operation. However, Walimbe et al.,²¹⁷ report that a cycle with a longer capillary restrictor, which is equivalent to a smaller orifice, demonstrates a somewhat faster cooldown than with a short capillary (wider orifice).

8.7.5.4 Operating Parameters Versus Heat Load

A common experiment that reflects the nature and applicability of examined mixtures is one that measures the dependence of the operating temperature on the applied heat load. Longworth et al.²⁵ find, by adjusting the flow impedance at the orifice that the slope of the cooling power versus temperature increases as the no load temperature increases. Both increases result from the *higher* suction pressure associated with a smaller pressure drop across the orifice when it is more open. On the other hand, Lakshmi and Venkatarathnam²¹⁸ demonstrate experimentally, with seven different hydrocarbon mixtures, that with higher heat loads (up to 20 W), and correspondingly higher operating temperatures, a given charge in a closed cycle produces a higher discharge pressure, but also a *lower* suction pressure, lower compressor volumetric efficiency, and therefore a lower mass flow rate. Their data reveal that conditions

associated with lower no-load temperatures provide higher cooling power at a given temperature. They point out that the lower suction pressure at high heat load results from a larger pressure drop in the capillary expander due to an increased vapor fraction in the flow.

8.7.5.5 Inlet and Outlet Temperatures of a Capillary Tube Throttle

Inlet and outlet temperatures of a capillary tube throttle were measured by Qi et al.,^{219,220} Venkatarathnam et al.¹¹³ and Reddy et al.²²¹; once during the cooldown, and again under the transient conditions caused by a changing heat load. The instantaneous temperature difference between the inlet and outlet of the throttle has the significance of the elementary temperature drop, $\delta T(T)$, discussed in Sect. 1.3.2. Reddy et al.²²¹ correlated this value of $\delta T(T)$ with the $h - T$ characteristics of the tested mixtures.

8.7.5.6 The Influence of the Charging Pressure

The influence of the charging pressure was demonstrated by Walimbe et al.²¹⁷ and Venkatarathnam et al.¹⁸⁶ For a given throttle position, the cycle with a higher charging pressure exhibited a somewhat faster cooldown.

8.7.5.7 Miscellaneous

Miscellaneous. Weckemann and Oellrich¹⁴¹ examined the effect of a buffer volume in the high-pressure section (enlarging V_U and α) in order to reduce the upstream pressure. Measuring the temperatures along the recuperator, they observed that the lowest temperature during cooldown in the evaporator is achieved significantly before the development of the steady state temperature distribution along the recuperator. Operation a mixed JT cycle at extreme ambient temperatures and for a period of a few months is reported elsewhere.^{130,222}

8.8 Kleemenko's Cycle and Coolers

8.8.1 Introduction

As discussed in Sect. 5.4, staged Joule-Thomson cryocoolers are comprised of a series of linked Joule-Thomson cryocoolers. Staging is obtained typically with two or three cycles, by successively throttling different coolants, or by using a single coolant with sequentially decreasing pressures at each stage (Sect. 5.4.9).

The cycle of Kleemenko, also known as the single flow cascade or the Auto-Refrigerating Cascade (ARC) cycle, exhibits a *staging*-like effect by changing the *composition* of the single pressurized stream of mixed gases and vapor.

This stream travels through a series of phase separators, combined with throttling expanders. At each stage the liquid phase, being rich in the higher boiling point components, is separated from the remaining (main) gas flow, and is then throttled and mixed with the returning gas flow. The main pressurized gas flow, extracted from the top of the phase separator, is precooled by the expanded and low pressure flow, and continues to the next lower temperature stage. As such, the separated liquid portion functions in the same fashion as a precooling stage with another coolant.

Missimer, who deserves primary credit for developing and building this kind of cryocooling system, regarded the cycle “as the combination of vapor compression and absorption refrigeration”. It should be mentioned that the separation of liquid and vapor phases had already been utilized in 1891 by Linde in his air liquefier. Abadzic and Scholz²²³ note that by using this approach, Linde produced “oxygen rich air”.

8.8.2 Chronological Notes

Podbielniak¹ was the first to identify and utilize the above mechanism in order to “attain exceedingly low temperatures” as described in his U.S.A. patent filed in 1934. He incorporated a multicomponent “refrigerant fluid, which may suitably be a fluid mixture such as refinery gas or natural gas” but also chlorinated and fluorinated hydrocarbons “where low inflammability is of importance”. It included a single driving compressor and three phase separators. His lowest boiling point component was methane, thereby limiting the lowest attainable temperature to 103 K when the compressor’s inlet, or suction, was at a “reduced pressure of say 20-in. vacuum”.

Ruhemann²²⁴ in 1947 discussed “a mixed refrigerant consisting preferably of a mixture of two or more hydrocarbons having different boiling points” as “ethylene and propane” or “ethylene and methane” for a vapor-compression cycle. He explained the benefit of separating the phases as follows: In prior art “. . . the lowest temperature obtainable is the boiling point at the lowest attainable pressure, of the mixed refrigerant having the composition at which it enters the machine”. However, “. . . if. . . the refrigerant is separated into a liquid phase richer in the less volatile component and a vapor phase richer in the more volatile component than the initial mixture, the lowest temperature obtainable is now the boiling point at the lowest obtainable pressure of this vapor phase, which in general will be lower than that of the initial mixture”. More of Ruhemann’s contributions may be found elsewhere.^{225,226}

Kleemenko,² in the former USSR, proposed the auto-cascade cooling process for liquefying natural gas. He

presented an analysis at the 10th International Congress on Refrigeration, Copenhagen, Denmark, in 1959. A similar system in principle was proposed by Kennedy and Smith²²⁷ in a 1957 patent filing (patented only in 1960) entitled “Refrigeration system especially for low temperatures,” and also described elsewhere.²²⁸ The system employed a binary mixture of R12 and ethylene, and the patent was assigned to the United States Atomic Energy Commission. Chaikovsky et al.²²⁹ built an auto-cascade system employing R12 and R13. Because of their significant impact on this subject, the contributions of Fuderer and Missimer are separately discussed in Sect. 8.8.4.

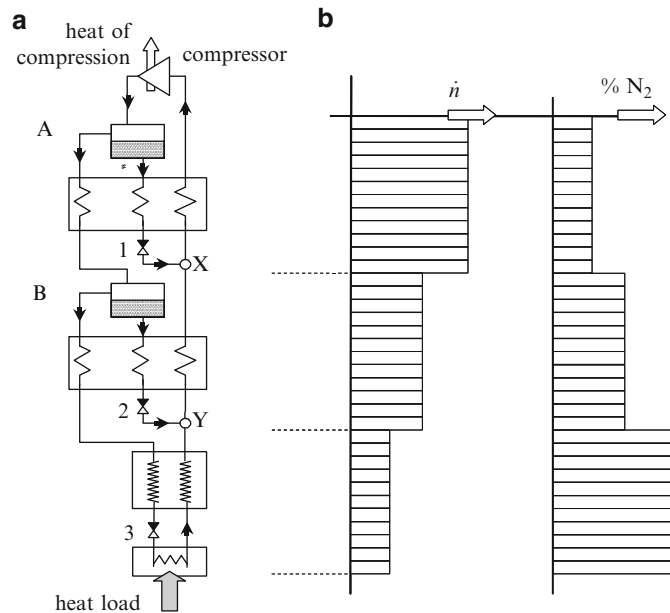
The potential of this idea was recognized and adopted by the industry of natural gas liquefaction. The prior art of natural gas liquefaction was based on the classical cascade cycle as one may find in various books.^{230,231} The classical cascade cycle was accomplished by liquefying progressively lower boiling point coolants each with its separate loops and compressors. Each stage pre-cooled the subsequent stage. Already in 1933 Keesom²³² originally proposed and analyzed a cascade system including ammonia, ethylene and methane, showing that it might liquefy nitrogen twice as efficiently as the Linde cycle and about 1.5 times more efficiently than the Claude cycle even with its expansion engine process. Barber and Haselden²³³ showed in 1957 that for the purpose of natural gas liquefaction a cascade of ammonia and ethylene is generally more efficient than cycles using expansion engines. However, the latter may be simpler. More of Haselden’s contributions may be found elsewhere.²³⁴

Typically, the cascade chain in modern plants includes propane, ethylene, and methane. Often, each of these cooling loops is further staged with intermediate expansion pressures. Two or three more intermediate temperatures (of evaporation corresponding with the intermediate pressure of expansion) are therefore obtained. Such a pressure-staged loop incorporates interconnected circuits driven by two or three compressors.²³⁵ Gaumer²³⁶ reviewed the development of cascade cycles for the liquefaction of natural gas from the classical version, up to the mixed refrigerant cycle.

8.8.3 Description

Kleemenko designed his cycle for use as both a liquefier and a cryocooler, and incorporated three phase separators. The central idea, displayed in Fig. 8.18 with the aid of only two phase separators, is as follows: the high-pressure recuperating stream of mixed vapor is interrupted by a series of phase separators, each one removing from the high-pressure stream the entire liquid fraction that is enriched with the higher boiling point components. The liquid portion

Fig. 8.18 (a) Layout of Kleemenko's cycle with two phase separators, A and B, and three JT valves 1, 2, and 3, similar to Carbonell et al.¹⁹ and Little et al.²⁴⁰ Each phase separator is a junction along the high-pressure stream, while X and Y are the corresponding junctions along the low-pressure returning stream. (b) Variation of the circulating flow rate and the concentration of the low boiling point component (nitrogen) displayed along the path of recuperation



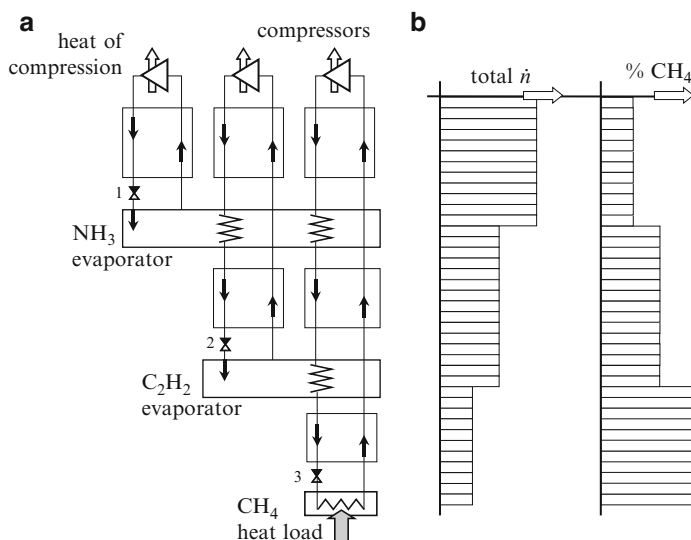
at each stage is throttled and mixed into the channel of the returning low pressure stream, in order to more effectively precool the counterflowing high-pressure stream. The remaining high-pressure vapor stream flows on to the next phase-separator. Park²³⁷ suggested replacing the throttles with ejectors (as previously discussed in Sect. 5.6.5.1).

This closed cycle is characterized by several typical features:

- In contrast to the classical multi-cycle cascade, it circulates a single stream driven by a single compressor. The title of Kleemenko's original paper² was somewhat perplexing for the first readers since it included the phrase "One flow" on the one hand, yet also referred to a "...cascade cycle", on the other. In fact, the vapors of all components are mixed at the compressor inlet.
 - The phase separators provide the mechanism for altering the mixture composition down-stream because coexisting liquid and vapor phases of a multi-component refrigerant have different compositions. This is a feature of a *zeotropic* mixture. An *azeotropic* mixture behaves like a pure gas and the composition of coexisting vapor and liquid phases remain the same. In such a case, phase separation would not change the composition of the stream. For a zeotropic mixture, the liquid phase has a higher concentration of the higher boiling point components than the initial mixture.
 - After being separated, the liquid is throttled down to the low pressure and sent (forward to a colder spot) into the low-pressure returning stream of the recuperative heat exchanger. In this way, the remaining high pressure
- The high pressure stream is further pre-cooled and partially condensed, repeating the previous process, according to the number of phase separators. Once again, the liquid phase is extracted from the next phase separator and expanded to a lower intermediate temperature.
 - Each phase separator noted by A and B in Fig. 8.18a constitutes a junction where a fraction of the high-pressure stream splits off. These fractions join the low-pressure stream at a series of junctions, noted by X and Y, in the same figure. In addition to change of flow, the composition also changes at each junction.
 - At the warmer end of the heat exchanger, the flow rate becomes higher than at the cold end and the coolant is enriched with the higher boiling point components. Towards the cold end, the coolant becomes enriched with the lowest boiling point component. This is illustrated in Fig. 8.18b. A similar behavior exists in the classical cascade cycle as observed in Fig. 8.19. At the warm end the total flow rate is higher because at this point three streams are circulating (although of different gases). At the cold end only the lowest boiling point coolant is circulating.
 - In this sense, each successive recuperating region, between the compressor and A, between A and B, and from B to the cold end, has a different (composition of)

vapor phase is enriched with the more volatile components, and is further conducted towards the lower temperature end of the system. The boiling point of this vapor is lower than that of the initial mixture.

Fig. 8.19 (a) A classic cascade of three separate closed cycles arranged as successive stages; each with another coolant as originally proposed by Keesom²³² in 1933. (b) The total flow rate and the concentration of the low boiling point component (methane) change along the path between the high and low temperatures in a similar way to Kleemenko's cycle



coolant. Thus, any description of the cycle in one of the thermodynamic planes becomes a superposition of different thermodynamic diagrams of the various coolants.

- (h) Following Eq. 3.153, the entropy generation associated with throttling in Kleemenko's cycle is reduced because of two reasons:
 1. Since the phases are separated, only the dense (high ρ) liquid phase is throttled.
 2. Various portions of the stream are throttled at higher intermediate temperatures.
- (i) Only in the final stage, is vapor the primary phase that is throttled, and this with a very high concentration of the most volatile component.
- (j) Each phase separator is a vessel that has to withstand the high pressure.
- (k) In Fig. 8.18a, the separated liquid is precooled in a heat exchanger prior to being throttled. Actually, this feature is an enhancement reported by Etzbach et al.,²³⁸ Simon et al.²³⁹ and Little et al.^{240,241,242} In the original proposals of Podbielniak¹ and Kleemenko,² the separated liquid phase was directly throttled without any precooling.

In summary, the approach used by Kleemenko to manage the mixed coolants produced a twofold result. First, by distilling the lowest-boiling-point-component, the vapors of all the higher boiling point components are prevented from reaching the cold end where they might freeze. Secondly, the intermediate throttling process resulted in a staged sequence of precooling. This might explain why the cycle is often referenced in the literature as an *auto refrigerant cascade* (ARC) or *mixed refrigerant cascade* (MRC) cycle.

Kleemenko² developed an optimal mixture composition of 0.65 CH₄ – 0.20 C₂H₆ – 0.15 nC₄H₁₀, at 6–6.5 MPa in order to reach 117 K, which is close to the boiling point of methane.

8.8.4 Fuderer and Missimer

In 1962 Fuderer^{3,4} suggested building a cooler with a single-phase separator according to Kleemenko's cycle. Remarkably for its early point in development, he envisioned the following crucial parameters for such a cycle:

- (a) The pressure ratio should range from 3:1 to 5:1; thereby enabling the use of a single-stage compressor, although he also refers to a double stage option.
- (b) The components of a mixed coolant should have adequately distinct boiling points, so that the difference between the higher and lower boiling point preferably ranges from 60 to 100 K. (Note that one might consider this as a general rule of a thumb to guarantee that the liquefied mixture at the cold end splits into various liquid phases).
- (c) The after-cooler, at the compressor discharge, should condense the oil into the liquid phase in the phase separator. In this way, the vapor component that later expands to a lower temperature is almost oil free and is therefore less susceptible to clogging.
- (d) The group of hydrocarbons (HC) and their halogen derivatives (CFC) provide good candidates as mixture components.
- (e) The cycle can self-balance itself through the use of an auxiliary gas tank that is connected to the discharge line by one valve and to the suction line by another, in such a way that only one of the two valves can be open at a time. In this way, depending on the operating conditions, the circulating stream receives lower or higher boiling point components.

Fuderer^{3,4} reports on the performance of single stage compression (2.5 MPa) system with an equimolar mixture of R12 and R14 operating at 157 K and double stage compression system with 0.8 R12 – 0.2 CH₄ (by weight) operating at 111 K.

However, it is Missimer who converted this invention into a commercial product and developed many of its crucial features such as the control system, the inflammability of the coolants, compatibility of the coolant with international regulations and more. Furthermore, the Polycold company established by Missimer in 1974 became a manufacturing facility for many commercial cryocooling systems of this kind. Missimer's comprehensive overviews of the ARC systems are available elsewhere.^{243,244,245,246}

8.8.5 Cryocoolers and Coolant Compositions

8.8.5.1 Missimer's Multi-throttling Cryocoolers

Missimer's multi-throttling cryocoolers manufactured at Polycold Inc. are the sole commercialized product of its kind to the best of the authors' knowledge. The enhancements developed by Missimer^{247,248} provide example of the necessary changes that a fruitful idea (that of Kleemenko and Fuderer) must undergo in order to become a commercialized and reliable long-term product.

Missimer devoted attention to controllability, stable operation through self-balancing, and the time required for cool-down. Another major operational concern was "the fact that a system utilizing separators could not be restarted after it had been shut down due to excessive start up pressures". By separating, throttling and returning only a fraction (about 60%) of the liquid phase back to the suction line, the system gained a faster cooldown. Hence, what remains from Kleemenko's idea is the *multi-throttling* process that results from successively splitting the single flow, and thereby distilling the higher boiling point component. However, the phase separators are eliminated.

Missimer further developed the use of halocarbon components instead of hydrocarbons in the so-called Polycold cycle. For a cycle of three stages with a compression ratio of about 10:1 and a maximum pressure of 2.5 MPa, he suggests two different mixtures of argon, R11, R12, R13, and R14. The first mixture for operation at 127 K includes the corresponding molar portions: 0.164, 0.225, 0.298, 0.15, 0.163. The second mixture with molar respective molar portions of 0.077, 0.35, 0.231, 0.182, and 0.16 operates at 155 K with a charge pressure of 0.6 MPa.

Missimer continued to develop other versions of the product to provide a rapid defrost feature²⁴⁹ combining a sorption compressor in the auto-cascade²⁵⁰ and to enable recovery of multi-component vapor mixtures.²⁵¹ Subsequent collaborators explored methods to prevent clogging,²⁵² to control cooldown and warm up²⁵³ and to incorporate a scroll compressor.²⁵⁴ In addition, they developed mixed refrigerants that are nonflammable,¹⁴⁴ free of R-22,¹⁸⁷ nonchlorinated and enable fast cooldown¹⁸⁸ or even include only inert components.¹⁸⁹ Other non-CFC auto-cascade

refrigerants were proposed by Weng of the General Signal Corporation^{255,256} in 1994.

8.8.5.2 The Enhanced Phase Separation

The enhanced phase separation that results from judicious cooling with even just a single-phase separator can increase the reliability of the system by ensuring that the higher boiling point components will not reach the final and coldest throttling device. This was the central idea used by Professor Little, Sapozhnikov et al.^{22,240,257,258,259,260,261} to reduce the cost in their version of Kleemenko's cryocooler. Employing a single-stage oil lubricated compressor, they focused on the effectiveness gained by separating the oil drops, oil vapor, and higher boiling point components through a "self cleaning" feature. By filling the phase separator with packing material and cooling it with the cold expanded stream, the oil vapor and higher boiling point components condense and join the liquid phase that is throttled and returns to the compressor. In order to reach 173 K at 2 MPa they used²⁴⁰ an argon based mixture of 0.08 Ar, 0.45 R12, 0.32 R13, and 0.15 R14. Another mixture²⁵⁷ used to reach 141 K at only 1.7 MPa was comprised of 0.07 N₂, 0.12 Ar, 0.08 C₃H₈, 0.08 nC₄H₁₀, 0.185 R14, 0.145 R134a, 0.175 R23, and 0.145 R123.

Wu et al.²⁶² adapted the technology of dephlegmation, that is utilized in the gas separation industry to recover hydrocarbons.²⁶³ While the standard phase separation process is based on partial condensation, the enhancement provided by the dephlegmation principle results from heat removal by an additional flow stream. This type of phase separation, combining heat transfer with mass transfer, was analyzed and tested by Wu et al. Gong et al.²⁶⁴ tested other configurations of the cycle with the same phase-separator.

Stenhouse²⁶⁵ enhanced the phase separation process in a Kleemenko cryocooler through the use of a coalescent filtration membrane. The membrane is comprised of ultra fine fibers (polysulphone/borosilicate pyrex) or a low density centered matrix (polymer or a metal powder), and exhibits a high capture efficiency, exceeding 98%, in comparison to efficiencies of only about 85% for other approaches.

8.8.5.3 Reaching Low Temperatures

Reaching low temperatures around 50 K may be possible with Kleemenko's cycle by including a substantial fraction of a quantum gas such as neon in the refrigerant's blend. Through multiple successive throttling steps, the distillation process can produce a highly neon-enriched mixture at the cold end. Luo et al. utilized this unique approach^{266,267} in their attempt to liquefy neon with the aid of a single-phase separator, and by mixing neon with (molar concentrations of) nitrogen (15%), argon (10%) and oxygen (35%). They expected the first stage to cool below 90 K after which the phase separation process would provide almost pure neon in

the final expansion thereby obtaining cooling at about 30 K. However, the lowest reported temperature was 51 K. Nevertheless, reaching such a low temperature by any other throttling cycle would necessitate a double stage cascade system. A Kleemenko cryocooler using two phase separators for the 70 K range (actually reaching a temperature as low as 65.3 K) was described by Wang et al.²⁶⁸

8.8.6 Kleemenko Cycle Versus the Linde-Hampson Cycle with Mixed Coolants

Kleemenko's cycle might appear as the next step in the evolution of mixed coolant Linde-Hampson cycles, including the additional complication of the phase separators. However, from a chronological perspective, Kleemenko's cycle was first. This fact was mentioned by Brodianski et al.¹⁷ in their discussion of the "retrospective of mixed-refrigerant technology". Here it should be mentioned again that it was Fuderer^{3,4,5} who actually dealt with both types of mixed coolant cryocoolers; those with, and those without phase separators.

Nevertheless, as a summary, it would be interesting to compare the two mixed gas cycles, the one without phase separators, sometimes referred to as the single stage cycle, and the Kleemenko cycle, including the phase separators:

8.8.6.1 The Thermodynamic Efficiency

The thermodynamic efficiency that may be achieved with or without phase separator cycles are quite similar. The study by Gong et al.^{269,270} concludes that the mixture composition represents the key distinguishing factor for efficiency. In fact, any efficiency of one cycle may be achieved with the other cycle by a proper choice of composition. Furthermore, each type of cycle may be optimized through its own separate process. For example, Gurudath Nayak and Venkataratham²⁷¹ optimized mixtures of nitrogen or argon with hydrocarbons in a Kleemenko cryocooler of the GRS type with two throttles and with a single gas separator. Their investigation also includes an exergy analysis of such a cycle. They devoted an additional study to optimize LRS mixtures.²⁷²

Podtcherniaev et al.²⁰¹ even conclude that the simplest cycle without any phase separator may be more efficient than complex cycles. On the one hand, they argue, Kleemenko's cycle indeed generates lower irreversibility through its sequenced throttling. However this advantage is counterbalanced by the irreversibility associated with mixing.

Alexeev et al.^{273,274} categorized three kinds of mixtures according to their pinch point locations; the warm end of the heat exchanger, somewhere at the middle, and at the cold end. In the last case, the efficiency of a Kleemenko cycle may be higher than that of the mixed-coolant

Linde-Hampson cycle. With the first mixture group, the two cycles have the same efficiency, and for the second group, the Linde-Hampson cycle is superior.

8.8.6.2 Temperature Stability

Temperature stability at the cold end, that is, inside the final stage evaporator, is expected to be higher in the case of Kleemenko's cycle. Due to the fractionating of components at higher temperatures, the fluid stream in the final stage is highly enriched with the low boiling point component, thereby producing less glide in the final evaporator. The cold end temperature is therefore less sensitive to changes in the associated heat load.

8.8.6.3 Flexibility to Include Higher Boiling and Melting Point Components

Higher boiling point components may be included in Kleemenko cycle mixtures than in the cycles with no phase separators. The concern about solidifying the "warmer" components is less severe in this case. These components are adequately condensed "early" at the intermediate temperature of the phase separator, and by returning back to the compressor, avoid exposure to temperatures that would result in their solidification. This feature also provides a larger flexibility to configure nonflammable mixtures.²⁰¹

8.8.6.4 Capability to Support Distributed Load

In some applications, the heat load is distributed along the cryocooler's heat exchanger. For example, this occurs when one coolant stream liquefies another.^{123,275} Gong et al.²⁰⁶ simulated the liquefaction of a stream of methane and concluded that the Kleemenko's cycle is preferred over a single stage cooler for such a task.

8.8.6.5 Construction and Operation

1. Kleemenko's cycle may operate without any absorbent.²² It tolerates a larger amount of contamination in a coolant, since the contamination is captured in the phase separator and is returned before reaching the cold end throttle. On the other hand, a similar effect may be obtained by adding a vapor compression cooling cycle to the single stage system in order to condense out the impurities.^{60,61,62,63,64}
2. Kleemenko's cycle enables a reduced area for heat transfer, since the flow rate becomes smaller towards the cold end.
3. The hydrodynamic impedance of the recuperative heat exchanger is reduced as successive fractions of the flow are returned to the compressor before reaching the cold end. The reduced impedance in turn reduces the energy losses required to maintain a given pressure.
4. The hardware associated with a Kleemenko cycle is somewhat more complicated, because of the phase separator. The phase separator can also be sensitive to

gravitational orientation. St. Pierre²⁷⁶ therefore proposed a “zero gravity” (position insensitive) phase separator for the same cycle.

5. Following a leak, the steps required to partially recharge a Linde-Hampson cryocooler would be simpler since the refrigerant composition is uniform and not influenced by the leak. In a Kleemenko cryocooler there would be various zones with different compositions. The composition of the leak would depend on its location and it might be necessary to replace the lost gas with one different from the original composition.

8.9 Closed Cycle Applications

8.9.1 Comparison of Closed Cycle Mixed Coolant Joule-Thomson Coolers with Closed Cycle Stirling Coolers

The steady development of closed cycle mixed gas Joule-Thomson coolers has occurred during a time in which the manufacture of Stirling cryocoolers was already a mature technology and the development of pulse tube cryocoolers has been receiving increased attention. Nevertheless, the preference for utilizing Joule-Thomson cryocoolers can be justified in various niche applications where the associated requirements cannot be satisfied by the Stirling cycle cryocoolers. The following list contrasts unique features provided best by Joule-Thomson or Stirling coolers:

8.9.1.1 Very Low Level of Vibrations at the Cold End

The Stirling cryocooler typically generates cold end vibrations from three different sources (only the last two sources remain valid for pulse tubes):

1. The displacer, extending down to the cold finger, usually the point of interest, produces mechanical impulses at the frequency of the Stirling cycle, and
2. The pulsating gas flow at the same frequency, typically in the range of 20–50 Hz, and,
3. The compression unit (which is actually a pulse generator) is most often operating in very close proximity to the cold finger. Due to efficiency considerations, the integral compressor & cold-head configuration is preferred and predominantly utilized. The alternate, split-Stirling configuration, separates the compression unit from the cold finger via tubing that serves as a transmission line for the pulses. However, the tube length remains limited in order to reduce the dead volume and fluid inertance of the line. Fluid-power transmission lines, and inertance, are discussed elsewhere.²⁷⁷ Consequently, if the tubing connecting the compressor and cold head exceeds

25 cm, a significant reduction of efficiency is experienced.

Hill^{278,279} and Longworth⁴⁸ experimentally verified the low level of vibrations being induced at the cold end of an operating Joule-Thomson system. A further reduction of vibrations at the interface to the load itself was obtained by using a porous plug¹⁰² as a throttling element instead of a capillary tube. The flow through the pores is at a lower velocity and therefore remains laminar instead of transitioning to turbulent flow at the exit of the orifice. Hence, the momentum of the stream is sharply dissipated and provides a lower impact on the cooled object. The comparative study led to the conclusion that the reaction forces produced by a Stirling cooler are more than an order of magnitude higher than those produced by the Joule-Thomson cooler. The Gifford-McMahon cryocooler also produces significantly stronger vibrations than the Joule-Thomson cryocooler.

Hohmann et al.²⁸⁰ specifically chose a mixed gas throttle system to successfully operate a SQUID magnetometer for a portable material-defect detector. They observed that temperature fluctuations at the cold end and mechanical vibrations provided only minor sources of disturbance. However, for their application, sensitivity was most strongly affected by the electromagnetic disturbances through the metal gas line.

Alexeev, Haberstroh and Quack^{54,56} proposed the use of non metallic (nylon) pressure lines, noting their compatibility with the low operating pressure. Such lines reduce the electromagnetic noise and maintain electrical separation. In addition, their softer mechanical properties may also suppress the transmission of mechanical vibrations from the compressor.

8.9.1.2 Large Separation Between the Compressor and the Cold End

Mixed gas JT coolers also enable large distances between the compressor and the cooled object. Such a feature becomes especially significant when the cold end is positioned on a delicate mechanism such as a gimbal, that cannot structurally support a compressor. Boiarski et al.¹⁹⁹ simulated the operation of JT system in which the compressor and the various cold ends operating in the temperature range of 70–120 K were separated by 33 m.

8.9.1.3 Flexible Connection Between the Cold End and the Compression Unit

When the cooled object is mounted on a movable mechanism it is important to have a flexible connection between the compressor and cold head. Various gimbal mechanisms described elsewhere^{281,282,283} provide example of this requirement. Even if it were possible to locate the

compressor for a split-Stirling cooler on a gimbal, the tube connecting the cold finger to the compression unit of a split Stirling cooler is not flexible enough to follow the large movements of the rotating gimbals. However, the inlet and outlet tubes of a low pressure closed cycle Joule-Thomson system can be very long and flexible, for example by being wound into a helix. The example referenced above²⁸¹ uses a Stirling cryocooler to precool a closed cycle Joule-Thomson cooler utilizing methane, pressurized only to 0.2 MPa, as the working fluid.

8.9.1.4 Distribution of Cryocooling Potential

A variety of disadvantages, that may collectively be identified as added thermal resistance, present themselves when one attempts to cool multiple and separated objects with a single Stirling cold finger. However, a single compressor, even at a distant location, may drive several separate Joule-Thomson cryocoolers without any added thermal penalty.

8.9.1.5 Cryocooling of a Large Surface

As in the above mentioned example,^{281,282} Joule-Thomson systems enable distributed cooling over large areas. In this case the evaporator is configured as the large hollow structure of the cooled object into which the two phase cryogen is injected through the throttle.

8.9.1.6 Magnetic Interference at the Cold End

Ter Brake et al.²⁸⁴ measured the magnetic interference present in a Stirling cooled high T_c SQUID. They found the Stirling cryocooler unsuitable because of “the remnant magnetization of the displacer assembly” which was “much too high”. The Joule-Thomson system, lacking any moving elements at the cold end, avoids such an interference.

8.9.1.7 The Rejected Heat Flux at the Payload

The latent heat associated with a boiling bath of cryogen may provide a larger heat sink than the cold gas resident in a Stirling or pulse tube cryocooler. One may also expect a higher degree of temperature stability, and a lower dependence of the cold head temperature on heat load. Ellsworth et al.²⁸⁵ reported an attainable heat flux exceeding 5 W/cm² with a Joule-Thomson cryocooler. This feature likely explains (to the best of author’s knowledge) the absence of any cryosurgical devices operated with a Stirling cryocooler. Because of the higher heat fluxes allowed, the diameter of a Joule-Thomson “cold finger” may be smaller.

8.9.1.8 Heat Rejection at the Warm End of the “Cold Finger”

In the case of the Stirling cryocooler operating near 80 K, up to about 10% of the entire input power is rejected at the

warm end of the cold finger and about 90% is rejected at the compressor. For low capacity cryocoolers this dissipation is accomplished through a simple small heat exchanger and via free convection. However, for higher cooling capacities such heat rejection requires special consideration. Closed cycle Joule-Thomson coolers also require heat rejection, but only at the compressor.

8.9.1.9 Advantages of the Stirling Cryocooler

1. Although the thermodynamic efficiency, reflected in the COP, is significantly larger with mixed coolant, than with pure coolant, JT cycles, the COP for Stirling cryocoolers is consistently higher than with any JT systems.
2. The “compressor” for Stirling cryocoolers is typically simpler than those employed with JT coolers. For example, the compression ratio is significantly smaller, such as 1.2–2 instead of 5–10. In addition, compressors for Stirling coolers do not require valves for suction and discharge. This latter feature means that the Stirling ‘compressor’ may be more accurately classified as a pressure wave generator (with a single gas port) and not as a more standard compressor with separate inlet and outlet gas ports,
3. The Stirling cryocooler is typically more compact.
4. While the low operating pressure of the Stirling cooler could formerly be identified as an advantage, that fact is no longer true, since the pressures utilized with mixed gas cycle JT coolers are comparable to, and in some cases even lower, than those used in Stirling coolers.

8.9.2 Typical Experience

Kleemenko’s central idea has been implemented in the gas liquefaction industry (primarily the LNG industry) as reflected in various patents from the period of 1966 to 1976, see for example those of Grenier et al.,²⁸⁶ Perret,²⁸⁷ Gaumer et al.,²⁸⁸ Linnet,²⁸⁹ Darredeau,²⁹⁰ Coers et al.,²⁹¹ and Rojey.²⁹² For the same purpose, Baek et al.,²⁹³ proposed and analyzed a hybrid system using a reverse Brayton and a Kleemenko cycle; they replaced the last JT expansion stage with a turbine expander.

A low temperature environmental test chamber based on Kleemenko’s cycle was proposed as early as 1968 by Myre²⁹⁴ in which he employed two compositions of halocarbonated refrigerants. Naer et al.¹²⁴ also utilized a hydrocarbon mixture for the same purpose. Boiarski et al.¹⁰¹ proposed a cooling system for thermal analysis based on a Linde-Hampson mixed refrigerant closed cycle with a flow regulated cryocooler, to accelerate the cooldown and to adjust the cooling power.

Kuo³³ and Luo et al.^{59,267} employed a closed-cycle Joule-Thomson system to cool infrared detectors using the same

size and configuration as is employed for high-pressure open systems. The compactness of their system is constrained by the large heat exchanger required for the mixed coolant cycle at low pressure. The capacity of their cooler ranges from 0.2 to 5 W at temperatures around 80 K.

Water trapping and cryopumping are perhaps the most established commercial application of the mixed refrigerant throttling cycle. Missimer^{245,295} utilized the Kleemenko cycle extensively for this purpose at temperatures below 120 K. The Linde-Hampson kind of mixed refrigerant closed cycle has also been utilized for water trapping by Longworth and Lotz,¹⁸² De Simon,²⁹⁶ and Khatri et al.²⁹⁷

The reduced operating pressure of the Kleemenko cycle makes it very attractive for medical applications, especially for cryosurgical probes,^{59,259,260,267} because of safety concerns. A cryogenic catheter for heart arrhythmia as proposed by Dobak, Marquardt and Radebaugh^{46,298,299,300} provides just such a safety feature. Other examples of miniature cryoprobes, and cryosurgical applications based on Kleemenko's cycle are provided by Gong et al.,¹⁸⁵ Little et al.,²⁶¹ Naer et al.,¹²⁴ and Van der Walt et al.⁷³ A design method for mixed coolant Joule-Thomson cryosurgical probes is discussed elsewhere.³⁰¹

Novak^{183,184} proposed utilizing the mixed coolant cycle to cool food or pharmaceuticals in an isolated enclosure employing zeotropic refrigerants. Another freezer for preserving biological substances at cryogenic temperatures with the aid of a pre-cooled mixed refrigerant cycle was presented by Alexeev et al.³⁰² Gong et al.³⁰³ built a biological freezer based on Kleemenko's cycle.

CMOS based computer processors can operate at higher speeds if they are cooled to sub-ambient temperatures. Ellsworth²⁸⁵ studied the feasibility and performance of a computer cooled by a mixed-gas throttle cycle. High reliability, high efficiency, and low cost are mandatory for this application. One of the most significant challenges in such a computer is the competing demand of modularity and sufficient thermal isolation to avoid condensation on the electrical components.

Various attempts to incorporate mixed refrigerant cycles for cooling currents leads^{86,304} are reported. Golubev et al.^{75,76} and Alexeev et al.⁷⁸ pre-cooled a closed cycle two-phase flow of nitrogen along a current lead at 1.2 (and 0.8 MPa), so that it was (partially) liquefied without any recuperation. Nellis, Pfotenhauer and Klein²⁰³ proposed cooling a set of current leads directly with the returning cold stream of a Linde-Hampson mixed refrigerant cycle and optimized the mixture composition for this purpose.

Other candidate applications include: reducing the boil-off of an MRI system,³⁰⁵ cooling an X-ray radiation detector mounted at the cold end of a vacuum insulated^{306,307} probe, portable MRI systems, gas chromatography, gamma detectors, and more.

Mixed coolant cycles combined with solid cryogen thermal accumulators have supported various space missions.^{85,126,127,308}

Podtcherniaev et al.²⁰¹ describe a number of industrial applications for multi-stage auto-cascade refrigeration systems, such as film deposition, thermal processing, electronic testing, water vapor cryo-trapping and semiconductor technology. They note how such refrigeration systems offer valuable functionality such as rapid cooldown, accelerated defrost mode, and thermal stand-by at predetermined temperatures.

In yet another potential application for closed cycle Joule-Thomson cryocoolers Notardonato and Haddad³⁰⁹ describe their use to manage the boil-off of cryogenic propellants in space missions. Here they suggest using multiple Joule-Thomson cryocoolers located along the pipes of flowing cryogen to extract heat. The design utilizes a common compressor to drive all of the cryocoolers in parallel, and demonstrates the capability of distributing the cooling power to multiple locations and over large distances.

Lee et al.³¹⁰ designed a high efficiency mixed refrigerant pre-cooled system for cooling high temperature superconducting (HTS) cables for urban subterranean electrical transmission lines.

8.10 Open Cycle Cryocooling by Mixed Gases

8.10.1 Introduction

This last section of Chap. 8 is devoted to the third category of Joule-Thomson cryocooling that employs mixed refrigerants, as displayed in Fig. 8.1, column C. Up to this point, we have discussed the twin categories of closed cycle coolers; those with phase separation (column A) and those without (column B). In general, the use of mixtures in open cycles has drawn less attention than that in closed cycles. The following aspects distinguish the open cycle from closed cycle operation:

8.10.1.1 The Necessity for High Pressure

Open cycles typically operate at pressures significantly above the critical pressure of the coolant. So far we have highlighted the advantage of mixed coolants in sharply reducing the operating pressure of closed loop cycles while at the same time elevating their COP. In addition, the associated low-pressure compressors are significantly simpler and more reliable. However, the open cycle cooler is supplied by a pressure vessel and re-compression is not an issue. Furthermore, if the supply pressure were low, the amount of gas stored in the supply vessel would be limited, or the volume of the vessel would be excessively large for a

given task. Put another way, the run time is prolonged by providing a high supply pressure.

8.10.1.2 Preference Parameters

Preference parameters: The advantage of using mixed coolants in open cycle applications is determined by comparing their performance to that of the same cycle using a pure coolant at the same ambient temperature, charge pressure and supply vessel volume. The parameters of performance are: the (boiling) temperature of operation, T_{BOIL} , the cooldown period, t_{CD} , and the duration of run time, Δt , for a fixed orifice or regulated cryocooler. The unconditionally preferred mixture is the one with a lower T_{BOIL} , shorter t_{CD} , and longer Δt . However, the superiority of a mixture might be conditional if for instance T_{BOIL} is lower, t_{CD} is shorter but Δt is also shorter. In such a case, one has to consider the advantages versus the drawbacks for each particular application.

8.10.1.3 The Composition of High Pressure Mixed Coolant

The composition of high pressure mixed coolant: Generally the composition of the open cycle mixture is different from those that are optimized for low pressure operation and broadly discussed in the previous sections. The example in Fig. 8.6 simulates a low pressure candidate mixture (with 60% of hydrocarbons) operated at an elevated pressure (10 MPa). Typically, elevating the pressure significantly increases the lowest attainable temperature if the cycle still operates at the highest cooling capacity, that is, $\dot{Q}^{MAX}/\dot{n} = \Delta h_T(T_{IN})$.

8.10.2 High Pressure Operation and Proposals

Various earlier studies have explored the use of mixed coolants at the intermediate and low pressure range. Brodyanski et al.¹¹ in 1971 used mixed-coolants at 10 MPa to experimentally demonstrate that the cooldown period for a given volumetric flow rate can be reduced by a factor of 8 compared to that of a system using pure nitrogen under the same operating conditions. A Russian patent³¹¹ of 1971 proposed a mixture of $N_2 - R13 - R14$ pressurized to 15 MPa to provide cooling at 70–100 K, as cited in the patent of Anikeev et al.³¹² in 1974. Anikeev enhanced the previous mixture by adding 1 – C_4H_8 (1-butene) and C_3H_6 . Table 8.4 includes the compositions of these mixtures. Boiarski et al.³¹³ in 1986 describe an experiment with an open cycle flow regulated cryocooler fed by a mixture at 6.2 MPa (utilizing the mixing enthalpy as discussed in the

next section). Chorowski^{166,167} in 1990 discussed the operation of an open cycle with mixtures of $N_2 - CH_4$ and $N_2 - R13$ at pressures up to 14 MPa.

In 1990, independently, and at the same conference and session, Little and Paugh³¹⁴ and Maytal, Van Sciver and McMahon¹³⁴ presented their studies on mixtures operated at elevated pressures, in both cases, as high as 30 MPa. In the first study, the mixture included 28.8% nitrogen balanced by methane, ethane, and propane. Its flow rate of 4.3 SLPM was used to precool a final pure nitrogen stage of 0.8 SLPM at 12.4 MPa and to accelerate the cooldown of the nitrogen to 70 K. In fact the experimentally measured cooldown was accomplished within 14 s. The second study by Maytal et al.¹³⁴ provided a theoretical optimization of a three component mixture as discussed in the next section.

In 1994 Abakumov et al.¹⁶ reviewed the “Technical level and ways of improvement of Joule-Thomson minicoolers” in Russia. The authors cite an experiment conducted by Landa et al.³¹⁵ in 1990, based on a pressure vessel charged up to as high as 60 MPa with a mixture of nitrogen and freons. The 1.7 kg system fed a flow-regulated Joule-Thomson cryocooler and provided 5 h of cryocooling at 80 K. Unfortunately, the authors of this book have not been able to trace the Russian reference.

Gong et al.³¹⁶ in 2000, following the approach of Luo et al.,³¹⁷ discuss the use of nitrogen and hydrocarbon mixtures to accelerate the cooldown of miniature open cycle Joule-Thomson cryocoolers for infrared devices. The highest pressure used in their model of the cooldown period was 10 MPa but their experimental verification was limited to only 5 MPa. Nevertheless, they were able to conclude that for similar cooldown rates, the mixture enables operation at a lower pressure in comparison to that required when using pure nitrogen.

Vetrovec³¹⁸ proposed the use of a high pressure (up to 100 MPa) open-cycle Joule-Thomson thermal management system in a series of short single-shot expansions to cool a high-average power solid state laser. His approach would replace nitrogen or argon with a mixture of 83% nitrogen supplemented with hydrocarbons, as in reference¹³⁴

In order to provide cooling in the temperature range of 27–77 K, which does not include the normal boiling point of any pure gas, Sobel proposed³¹⁹ the use of a group of binary mixtures of argon, with molar fraction of 19–40%, and neon. In a second proposal³²⁰ he incorporates the addition of nitrogen, carbon monoxide or a hydrocarbon. However, although these mixtures normally boil below about 50 K, their inversion temperatures are below room temperature. Therefore, a precooling stage is also necessary in order to reach the boiling points in this temperature range. Table 8.4 lists the typical compositions.

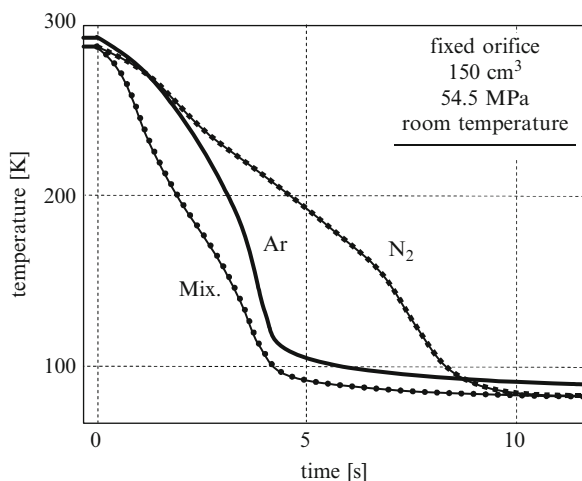


Fig. 8.20 A ternary mixture (Mix.) of nitrogen and hydrocarbons that cools down faster than argon but reaches a temperature of boiling nitrogen

Hingst¹⁵⁴ adapted the compositions as proposed by Alfeev et al.⁸ of nitrogen with up to three hydrocarbon additives, however at elevated pressures up to 80 MPa. He describes experiments of a pressure vessel charged to 34.5 MPa at three ambient temperatures, -54 , 22 , and 70 C, feeding a miniature flow regulated cryocooler. He reports a significantly prolonged run time. However, at low ambient temperatures the pressure in the vessel sharply decreases and therefore Hingst suggests maintaining the temperature of the vessel at the warmer temperatures. In addition, Hingst proposed a mixture of argon and methane, still operating below 100 K.

Maytal³²¹ demonstrated experimentally how the use of a mixture can provide a faster cool down than with argon but still enable the system to reach the boiling temperature of nitrogen. Figure 8.20 displays the measurements gathered with a room temperature supply. The same study also reports on the system performance when the gas is discharged from a supply at 85 C. The cooler incorporated a ternary mixture of nitrogen, ethane and propane, with corresponding fractions of each component as 0.83, 0.10, and 0.07. The 150 cm^3 supply vessel is pressurized up to 54.5 MPa and feeds a fixed orifice, rapid cool-down cryocooler encapsulated in a dewar.

Section 5.5.3 mentions open cycle coolers using high-pressure mixtures of nitrogen with argon (and of neon with hydrogen) for rapid cooldown applications. However, these do not really belong to the present discussion since such mixtures provide no significant advantages. Because the boiling points of the two components are close to each other, these mixtures only exhibit somewhat average cool-down rates and boiling temperatures between those of nitrogen and argon (and neon and hydrogen).

8.10.3 Optimized High Pressure Mixtures

The first reported attempt to optimize a mixture at an elevated pressure as high as 30 MPa belongs to Maytal, Van Sciver and McMahon.¹³⁴ They examined and optimized three component mixtures of nitrogen with two hydrocarbons at 30 MPa, that still maintain the temperature of liquid nitrogen.

Following the discussion in Sect. 8.5.3.3 and Fig. 8.14, the triangle XYZ includes the relevant states of liquefaction before solidification. The triangle XAB represents a group of mixtures that when pressurized to 30 MPa can cool down to temperatures below 80 K, that is, without having a pinch point above 80 K. Point A is the optimal mixture with the highest specific cooling capacity. The presumed specific cooling capacity, Δh_T , of the mixtures associated with state A is approximately twice as large as that of pure nitrogen. The optimized composition for (N_2 , C_3H_8 , $\text{i-C}_4\text{H}_{10}$) is (0.87, 0.08, 0.05) respectively, and for (N_2 , C_2H_6 , C_3H_8) is (0.80, 0.12, 0.08) respectively.

Another comprehensive study of high-pressure mixtures and their open cycle operation was accomplished by Maytal et al.³²² in 2006. The question posed in this publication represents a very general one with regard to mixed coolants for Joule-Thomson cryocoolers at 80 and 90 K: for any number of species and any level of pressurization, what are the highest achievable values of specific cooling capacity, $\dot{Q}/\dot{n}$ and COP, and what are the associated mixture compositions? In order to answer this question the authors optimized the composition at each value of pressure, up to 50 MPa, all at 290 K. The largest number of components was nine. The resulting optimized values of $\dot{Q}/\dot{n}$ for (mainly) hydrocarbon additives as function of pressure are displayed in Fig. 8.21. In addition, this figure shows optimized mixtures of 3, 6, 7, and 8 components. A larger number of components enable larger values of $\dot{Q}/\dot{n}$ up to the limit obtained with eight components. The contribution of the eighth component is only a few percent while the gain contributed by the ninth component is insignificant. However, significantly lower values are achieved with only three components.

The curve generated with the optimized mixtures displays a maximum resembling that of the pressure dependent values of Δh_T for pure gases. It represents the instantaneous specific cooling capacity during the isothermal discharge of a reservoir. However, one should note that the optimized mixtures curve does not describe the results obtained with a single mixture, nor is it a discharge curve. Furthermore the pressure where $\dot{Q}/\dot{n}$ is maximized is not the inversion pressure. Rather, the curve represents an aggregate of optimized mixtures; each of its points represents a different composition, optimized for the particular pressure. This observation

Fig. 8.21 Optimized specific cooling capacity of various hydrocarbon mixtures as a function of the pressure of optimization.³²² Each curve indication represents the mixture by the number of its components. Nitrogen, argon, and methane are shown for reference.

The following compositions belong to each curves: **3A** (N₂, C₂H₆, C₃H₈) **3B** (N₂, C₃H₈, C₄H₁₀) **3C** (N₂, CH₄, C₂H₆) **6** (like 7 without C₅H₁₂) **7** like 8 without CF₄ **8** (like 9 without C₂H₄) **9** (N₂, A, CH₄, CF₄, C₂H₄, C₂H₆, C₃H₈, C₄H₁₀, C₅H₁₂)

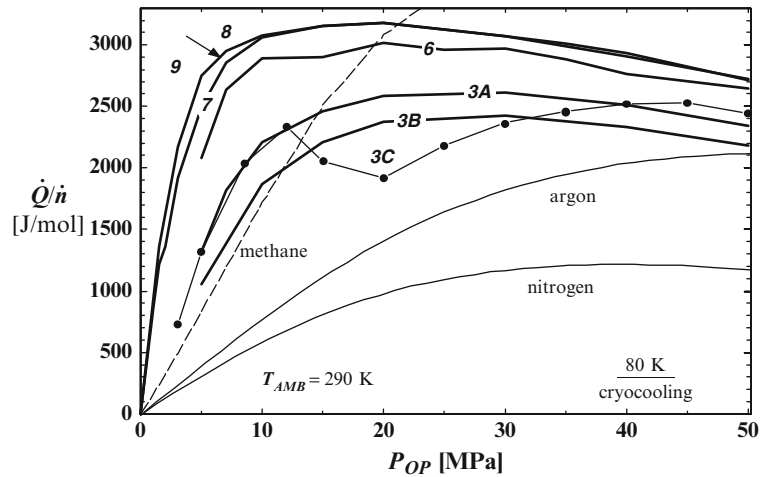


Table 8.6 Composition of mixtures (percent mole fractions) of 8 hydrocarbon components optimized³²² at some values of pressures at 290 K for maximum $\dot{Q}/\dot{n}$. The highest attainable $\dot{Q}/\dot{n}$ value occurs at 20 MPa

P [MPa]	$\dot{Q}/\dot{n}$ [J/mole]	N ₂	A	CH ₄	CF ₄	C ₂ H ₆	C ₃ H ₈	C ₄ H ₁₀	C ₅ H ₁₂
1.5	1,464	54.1	3.4	10.5	12.1	5.2	5.2	0.0	9.5
3	2,171	55.2	0.2	7.6	13.7	13.3	0.7	0.0	9.3
5	2,751	60.4	0.1	5.0	11.1	11.6	0.8	3.0	8.0
7	2,947	61.2	4.3	5.0	5.5	11.4	2.1	3.8	6.7
10	3,080	64.6	2.0	9.8	1.7	9.0	5.0	0.6	7.3
20	3,180	70.2	2.3	8.6	1.3	7.2	2.2	1.2	7.0
30	3,070	74.6	0.0	7.4	2.5	6.2	2.0	1.0	6.3
40	2,930	75.5	2.3	6.0	2.6	4.7	2.1	1.7	5.1

leads us to the answer for the general question posed above. The largest value of $\dot{Q}/\dot{n}$ obtained with a cryocooler operating at 80 K, and with an ambient temperature of 290 K is 3180 J/mole, achieved at a pressure of about 20 MPa (versus only about 1200 J/mole obtained with pure nitrogen at about 40 MPa). The associated COP is 0.17. The composition of this mixture and various other optimized mixtures is shown in Table 8.6 for a few pressures. In addition to nitrogen and argon the mixtures also include mainly hydrocarbons. The highest COP (0.19) is associated with another optimized mixture, at a pressure of 5 MPa and with a composition as given in the same table. For nitrogen, the highest COP is only about 0.08, that being reached at about 30 MPa (see Table 3.10).

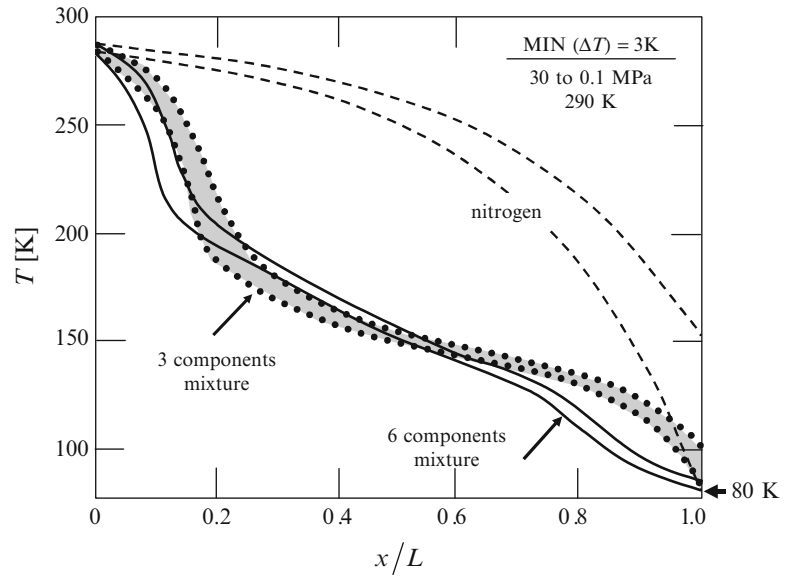
With the aid of halogenated hydrocarbons the highest value of $\dot{Q}/\dot{n}$ for cryocooling at 80 K with an ambient temperature of 290 K is somewhat smaller; 2700 J/mole at 35 MPa. The composition of components (N₂, A, R14, R23, R116, R32, R125, R134a) by percent is correspondingly, (73.5, 3.6, 7.9, 6.6, 0.2, 3.8, 0.5, 3.9). Below 35 MPa the values of $\dot{Q}/\dot{n}$ for these mixtures are significantly smaller than those for the more simple hydrocarbon mixtures.

The same reference³²² presents other typical features of the high pressure optimized mixtures, which are:

- The high-pressure optimized mixtures include a larger fraction (approximately 0.75) of the primary component, that is nitrogen, than the low-pressure mixtures.
- As more components are involved and the value $\dot{Q}/\dot{n}$ becomes higher, a larger recuperator is required, and the COP increases. For example, the recuperator for an eight component mixture has an NTU above 40. The optimized mixtures with only three components require a recuperator only about half that size due to the reduced ΔT between the streams as more components are involved. Figure 8.22 displays the longitudinal profiles $T_u(X)$ and $T_d(X)$ for pure nitrogen versus optimized mixtures of 3 and 6 components.
- The value of $\dot{Q}/\dot{n}$ decreases sharply if the cycle operates with an ambient temperature that is higher than that assumed in the optimization process. The sensitivity to elevated ambient temperatures becomes small only if the mixture is optimized at the highest possible ambient temperature.
- Optimizing the mixtures for operation at 90 K or even 95 K instead of 80 K increases the value of $\dot{Q}/\dot{n}$ by only a few percent. This behavior is in contrast to the large difference in $\dot{Q}/\dot{n}$ between pure nitrogen (78 K) and pure argon (88 K).
- The compactness parameter, $\dot{Q}/(UA)$, of the recuperator has also been examined for each coolant. For operation at 80 K the optimized mixtures enable a somewhat more compact recuperator than for nitrogen. However, for operation at 90 K, argon tolerates a much smaller recuperator than that required for the optimized mixtures.

As displayed in Fig. 8.22, the temperature at the inlet of the expansion nozzle is significantly lower than that associated with pure nitrogen. Therefore, for a similar JT valve, the mass flux of the mixture in choked flow conditions is larger than that with nitrogen.

Fig. 8.22 Temperature distribution along the heat exchanger for optimized mixed coolants of nitrogen with 2 and 5 higher boiling point components, versus that of pure nitrogen. The minimum allowed separation between the recuperating streams is 3 K



8.10.4 Mixing On-Site

$$\dot{Q}_2 = h_0^{MIX} \cdot \dot{n} - h^{MIX} \cdot \dot{n} = \Delta h_T^{MIX} \cdot \dot{n} \quad (8.49)$$

8.10.4.1 The Mixing Enthalpy

The mixing enthalpy is the heat released (either positive or negative) when the components are mixed adiabatically, that is, thermally isolated from the surroundings. The quantity enables one to compare the performance of a cycle in which a mixture is fed directly into an open cycle cryocooler, with one in which the individual components of the mixture are admitted and allowed to mix inside the heat exchanger of the cryocooler. One might expect that the in-situ mixing process would enhance the cooling capacity of the resultant coolant.

For such a study we must compare the following two cases: first, beginning with the components and second, starting with the mixture. The components, in the first case, and the mixture, in the second, enter the cryocooler at the same conditions, (T, P) . In both cases it is a mixture at $(T, 0.1 \text{ MPa})$ which exits the cryocooler.

The energy balance when starting with separate components is given by,

$$\dot{Q}_1 = h_0^{MIX} \cdot \dot{n} - \sum h^i \cdot \dot{n}^i \quad \dot{n} = \sum \dot{n}^i \quad (8.47)$$

where h^i and $\dot{n}^i$ are the enthalpy and flow rate of component number i , and $\dot{n}$ is the total flow rate (of the mixture). The molar enthalpy of the exhaust mixture is, h_0^{MIX} . The inlet and outlet conditions, respectively are then:

$$h^i = h^i(P, T) \quad h_0^{MIX} = h_0^{MIX}(0.1 \text{ MPa}, T) \quad (8.48)$$

Similarly, for the case when the high pressure mixture is fed into the cryocooler,

where $h^{MIX} = h^{MIX}(P, T)$ is the molar enthalpy of the mixture at the same initial conditions and Δh_T^{MIX} is the isothermal Joule-Thomson effect of the mixture at the same (P, T) conditions. Then,

$$\begin{aligned} \dot{Q}_1 - \dot{Q}_2 &= h^{MIX} \cdot \dot{n} - \sum h^i \cdot \dot{n}^i \\ &= \left(h^{MIX} - \sum h^i \cdot z^i \right) \dot{n} \end{aligned} \quad (8.50)$$

where z^i is the fraction of the i -th component. But,

$$\Delta h_T^{MIXING}(P) \equiv h^{MIX}(P, T) - \sum z^i \cdot h^i(P, T) \quad (8.51)$$

is exactly the definition³²³ of the *isothermal mixing enthalpy*, therefore,

$$\frac{\dot{Q}_1}{\dot{n}} = \frac{\dot{Q}_2}{\dot{n}} + \Delta h_T^{MIXING} = \Delta h_T^{MIX} + \Delta h_T^{MIXING} \quad (8.52)$$

Thus, the specific cooling capacity of the mixture, Δh_T^{MIX} , is enhanced exactly by the extent of the *mixing enthalpy*, Δh_T^{MIXING} , that results when the supply of separate components are mixed in situ. The mixing process results in lower gas consumption, lower cut-off pressure, and longer run time. Of course, for this purpose, the actual mixing process has to be exothermic so that $\Delta h_T^{MIXING} > 0$. The associated temperature change due to the adiabatic mixing, in analogy to Eq. 4.63, would be,

$$\Delta T_h^{MIXING} = - \frac{\Delta h_T^{MIXING}}{c_{PO}} \quad (8.53)$$

where c_{PO} refers to the mixture.

8.10.4.2 Examples

Such an attempt to gain the mixing enthalpy in an open system is described by Boiarski et al.³¹³ and by Bodio¹⁵ following the proposals of Altunin et al.³²⁴ and Boiarski et al.³²⁵ A flow demand cryocooler, is initially fed by a single 700 cm³ pressure vessel containing a mixture of nitrogen and hydrocarbons. In the second case, two 350 cm³ vessels are used, one with pure nitrogen and the second with the hydrocarbons (HC), and the two streams are mixed inside the cryocooler. The mixing occurs below ambient temperature, since the mixing chamber is located at an intermediate point along the recuperator. Prior to the mixing point, the high pressure streams of N₂ and HC are precooled by the returning low pressure mixture of the two. The initial pressure in each of the vessels was 6.2 MPa. An on-off flow regulation system was used so that each pressure vessel had its own operating valve to also enable mixing on demand. Indeed, the run duration was extended to 3255 s instead of 2172 s, and the cut-off pressure was decreased to only 1.6 MPa instead of 2.4 MPa.

According to the same reference,³¹³ the increase of molar cooling capacity, $\Delta h_T^{MIXING} / \Delta h_T^{MIX}$ for nitrogen mixtures with hydrocarbons or freons may be about 20–30% in the pressure range of 25–30 MPa. At the lower pressure range of 4–6 MPa where Δh_T^{MIX} is smaller, the relative enhancement by ΔT_h^{MIXING} is even larger, by about 100–200%.

Mixing effects of real gases and candidates for such a cooling scheme were studied by Gong et al.³²⁶ in two forms: the isothermal mixing effect, ΔT_h^{MIXING} , (measured in J/mole) and the adiabatic mixing effect, ΔT_h^{MIXING} . They focused on the influence of pressure, temperature and composition on the extent of both mixing effects for various binary mixtures of seven substances (nitrogen, methane, carbon tetrafluoride, ethane, propane, isobutene, isopentane and helium).

One should be aware of two practical difficulties in utilizing the mixing enthalpy:

1. The number of pressurized storage vessels is doubled; a separate pressure vessel is required for each group of components prior to their mixing, and,
2. As a general trend, the relative contribution of the mixing enthalpies is relatively small at elevated pressures. Thus, the idea is less attractive for open cycle cryocoolers since the elevated pressure is crucial for their operation as explained above in Sect. 8.10.1, and,
3. The cryocooler is more complicated because of the intermediate mixing zone.

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This chapter deals with some topics of general relevance to all formerly discussed aspects of operating Joule-Thomson cryocoolers: the necessary purity of the coolant, consumed flow rates and numerical modeling of cryocoolers. Attention is dedicated to the unique field of cryosurgical probes and devices that utilize miniature Joule-Thomson cryocooling. The closing section discusses the negative Joule-Thomson effect and its application for progressive warming.

9.1 Gas Purity and Clogging

The need for high purity gas supply to a Joule-Thomson cryocooler is a main concern for long reliable operation. The effort associated with the supply and maintenance of high purity for the coolant is a necessary price to pay in order to gain the advantages of the Joule-Thomson miniature cryocooling. The same concerns about gas purity are relevant not only to miniature Joule-Thomson cryocoolers but as well to any liquefier or cryocooler which employs a final Joule-Thomson stage.

9.1.1 The Origin of Impurities

9.1.1.1 Vapor Phase Contaminants

Vapor phase contaminants are the most problematic type of impurities because they come mixed in with the gaseous phase of the pressurized coolant supply. The problematic impurities are those substances that have higher boiling points than the coolant itself, and can therefore solidify during the cooldown process or during steady state operation at the cold end of the cryocooler. Such precipitation gradually clogs the passageway of the Joule-Thomson expansion valve. Consequently, the flow rate decreases and may even reach values that are insufficient to maintain a stable cryogenic operating temperature.

The most relevant impurities are the vapor type that freeze out “earlier”, such as water, carbon dioxide and hydrocarbons. Only by ensuring a very low concentration of such components, typically below a few parts per million (PPM) by volume, may one prevent interruptions from clogging. For the temperature range associated with continuous cooling via liquid nitrogen, Bonney and Longworth¹ of APD Cryogenics defined the critical upper concentration for these three respective contaminants that still guarantees continuous operation: 2 PPM, 2 PPM and 3 PPM by volume. Thornton² of British Aerospace mentions similar figures. This level of purity has become a standard requirement, as evidenced for example in the British Defense Standard.³

An example from a lower temperature range is provided by Jurns, et al.⁴ in which a Joule-Thomson valve used to throttle liquid hydrogen in order to control the pressure and temperature of a propellant tank, was clogged due to a suspected trace amount of neon in the hydrogen. Another example was reported⁵ regarding the final stage of a helium system precooled by hydrogen. In this case a blockage formed due to traces of 40 PPM of hydrogen instead of the planned 0.1 PPM impurity concentration, and interrupted the helium stage cooling.

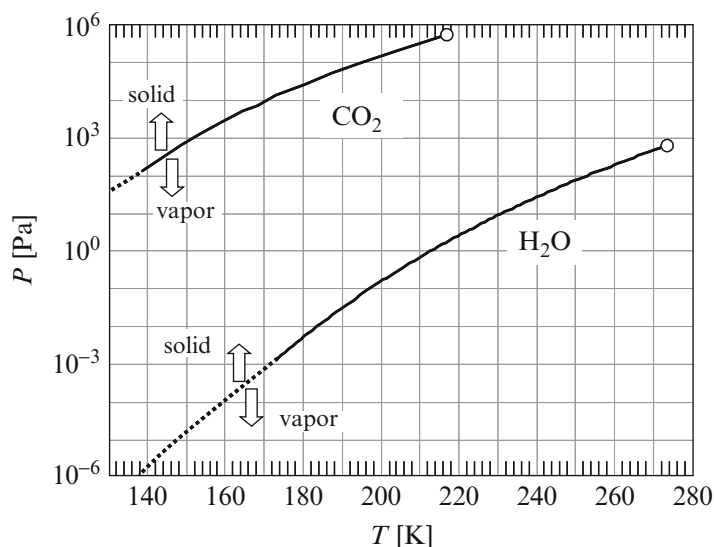
9.1.1.2 Minute Solid Particles

Minute solid particles are the other common and problematic impurity. These can become lodged in the orifice, blocking it or interrupting the flow adjustment process. However, such particles can be successfully handled by an inlet filter with a mesh size of a few micrometers and are collected before reaching the minute expansion orifice, as discussed in Sect. 6.5.1.

9.1.1.3 Cleanliness of the Pressure Supply Components

Cleanliness of the pressure supply components (tubing, connections and vessels) is essential; otherwise, even a

Fig. 9.1 Curves of solid-vapor equilibrium, up to the triple point, marked by a circle. The curve of water is based on reference (The standard IS/ DIS 7183.2) and of carbon dioxide on reference (*Perry's Chemical Engineer's Handbook*, edited by L.H. Perry, McGraw-Hill Book Company, 1973, table 3-8)



clean gas supply may become contaminated by flowing through the channels. Necessary cleanliness standards may be achieved only by tight process control and procedures in manufacturing in order to avoid flux residue and small solder particles, as stated elsewhere.^{1,6}

9.1.2 Parameters of Clog Formation

9.1.2.1 Vapor-Solid Phase Transition

At adequately low temperatures, de-sublimation is the physical mechanism that results in the deposition of solid impurities onto surfaces and that eventually can clog flow passages. Sublimation is the opposite process and occurs as the solid absorbs heat. Below the triple point pressure and temperature the coexistence of solid and vapor phases is viable. At any temperature below the triple point, there is a solid-vapor pressure of equilibrium, P_{S-V} , that is connected to the rate of solid deposition. Such vapor pressure curves, $P_{S-V}(T)$, for water and carbon dioxide are displayed in Fig. 9.1. For example, at 200 K, the vapor pressure of water $P_{S-V} = 0.16$ Pa reflects the equilibrium pressure, where in the absence of any other solid surfaces, the first crystal of ice is nucleated. However, if the temperature is lowered to 160 K, a pressure of only 0.001 Pa could suffice to initiate formation of ice crystals. It should be emphasized here that the $P_{S-V}(T)$ curve describes states of thermodynamic equilibrium, where solid formation is possible but also might not occur for non-equilibrium conditions.

9.1.2.2 Nucleation and the Rate of Deposition

In practical JT refrigerators, the vapor is a part of a high pressure flow stream and is not necessarily at equilibrium. Furthermore, initiating precipitation of the solid phase

requires that the molecules are incident on, and captured by, the channel surface. The dynamics of crystal growth, at cryogenic temperatures as for cryo-pumping are discussed elsewhere.⁷ Higher flow rates in a JT system are typically associated with a higher flux of convected impurities. However, at the same time, larger flow rates lower the chance for capturing vapor particles into the solid phase.

The absolute partial pressure, $P_P(X)$, of low concentration vapor impurities, X , can be represented by,

$$P_P(X) = z(X) P \quad (9.1)$$

where $z(X) \ll 1$ is the molar (volume) fraction of the vapor impurities X and P is the absolute total pressure of the coolant. For example, $z = 2$ PPM and $P = 20$ MPa means $P_P = 40$ Pa.

At any given temperature, when the partial pressure, P_P , of the vapor is higher than P_{S-V} , the probability of nucleating the vapor molecule into the solid phase increases significantly. Nucleation becomes more likely because the higher partial pressure enables smaller nuclei to initiate the process of crystal growth. At any given temperature, T , the tendency of the vapor to convert into the solid phase can be expressed as a function of the excess above the saturation vapor pressure, $P_P - P_{S-V}(T) > 0$. Alternatively, one can note that at higher partial pressures, the vapor can solidify ("earlier") at a higher temperature and the potential for clogging flow passages is higher.

9.1.2.3 Viability of Clog Formation in the Heat Exchanger

The lowest temperature of the upstream flow along the heat exchanger is T_2 at the inlet to the JT valve. For a nitrogen feed pressure near 40 MPa, T_2 is about 150 K (Fig. 3.16).

Water Vapor Contamination

Let us consider an intermediate temperature of 200 K. The minimum partial pressure of water that will precipitate solid formation at this temperature, P_{S-V} (200 K) = 0.16 Pa, reflects an impurity level of $z = 4.10^{-3}$ PPM in a 40 MPa feed. Potentially, even such a low impurity could solidify inside the heat exchanger at the longitudinal location corresponding to 200 K in the upstream flow. However, practical considerations^{1,2,3} place the upper impurity limit at $z = 2$ PPM. Thus, the gap between the potential clog formation concentration and the practical one is almost 3 orders of magnitudes.

Carbon Dioxide Vapor Contamination

The saturation pressures for CO₂ at a given temperature are higher than those of water. Thus, higher partial pressures of CO₂ can be tolerated without solid formation. For example, with P_{S-V} (150 K) = 1,000 Pa, a nitrogen feed of 40 MPa could tolerate a CO₂ impurity concentration up to 250 PPM before a solid would precipitate, and clog the inside of the heat exchanger. The practical solid-contamination limit of at $z = 2$ PPM would correspond to the CO₂ saturation pressure of $P_P = 80$ Pa all the way down at 130 K, a condition which might only occur inside the expansion valve.

9.1.2.4 The Level of Contamination and the Feed Pressure

It is important to note that a given concentration of some impurity does not define an absolute partial pressure for that impurity. For a given z , a higher total pressure results in higher values of the absolute partial pressure for each impurity, and a higher potential for nucleation and clogging. Indeed, the study of Bonney and Longworth¹ supports this rational indicating that the critical level of moisture content (concentration) becomes lower and more severe at higher coolant pressures. At any given level of contamination, a solid plug is more likely to form as the total pressure of the gas gets higher.

9.1.2.5 Flow Rate and Size Dependence

Bonney and Longworth¹ demonstrate that by broadening the passageway of a valve, and thereby increasing the associated nominal flow rate to such an extent that the run time for a given pressure vessel is cut in half, the system will tolerate a water contamination level of 4 PPM instead of the previous limit of only 2 PPM. The same effect is evidenced by the severe limitation experienced when the orifice diameter for a 6.7 MPa flow of nitrogen is reduced below 0.07 mm, and the associated flow rate is less than 0.5 SLPM. In this case purity levels even smaller than 2 PPM are required, levels which are not common in practice. Furthermore, MEMS size cryocoolers exhibit even higher sensitivity^{8,9} as will be discussed below.

9.1.3 Experimental Study of Clog Formation

9.1.3.1 Evolution of Plug in a Sonic Expansion Valve

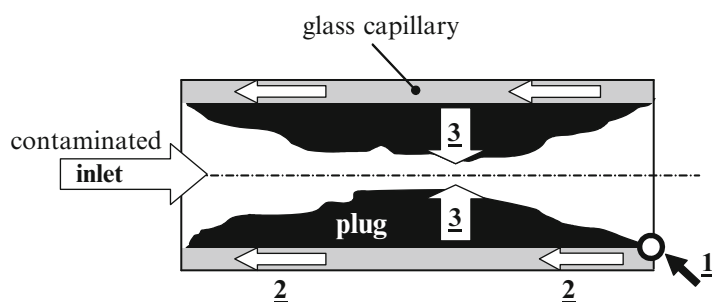
The experiment of Wade et al.¹⁰ provides a close observation of plug formation in a Joule-Thomson valve. Here valves were comprised of transparent glass tubes with diameters of 0.2, 0.4 and 0.6 mm, each with a length to diameter ratio of 10. A microscope traced the formation of the plug. In most cases, the concentration of water vapor in the nitrogen was 70 PPM. The temperature of the inlet stream ranged from 230 K to 255 K and was held steady, while the pressure of the stream ranged from 2.5–4.5 MPa resulting in choked flow conditions with mass flow rates of 0.3–2 g · s⁻¹.

The essence of the experiment may be summarized by the following conclusions:

1. The lowest partial pressure of water contamination was $P_P = 175$ Pa. Even at the highest inlet temperature of 255 K the associated saturation pressure ($P_{S-V} = 125$ Pa) is lower than the partial pressure. Thus, the water vapor could solidify in the high pressure supply tube, prior to reaching the valve. However, the solid plug was formed only in the orifice itself,
2. The evolution of plug deposition follows the process illustrated in Fig. 9.2.
 - (a) The solid contaminant first precipitates at the far end of the orifice, just before the exit (marked by 1 in the same figure),
 - (b) Nucleation and growth of the plug proceeds upstream in the direction opposite to the pressure gradient, from low to high pressure. At the end of this phase, a thin layer of solid covers the inner surface of the tube (marked by 2 in Fig. 9.2),
 - (c) The solid plug begins growing in the radial direction, filling the inner space of the orifice, and reducing the diameter of the passageway (marked by 3 in Fig. 9.2).
3. As a result of the plug formation, the associated flow rate decreases to about 40–80% of the maximum undisturbed value,
4. When the plug becomes sufficiently large, it breaks away from the wall so that the flow rate is restored to its maximum value,
5. Only about 0.05% of the water vapor that passes through the orifice is captured and trapped in the solid phase,
6. The plug is a snow-like porous material.

Wade et al.¹² explain that the coldest location along the flow path in the expansion tube is indeed at the orifice exit, where the freezing begins. The complete process of isenthalpic expansion through the nozzle, may be broken into two sections. In the first section, the sonic flow isentropically expands to about 53% of the inlet pressure, while at the same time, the temperature decreases to about 83% of the

Fig. 9.2 The steps of plug formation (1,2,3) in a Joule-Thomson expansion nozzle in the form of a tube, after Wade et al.¹² Plug nucleation begins at the farthest downstream site (1). It proceeds against the pressure gradient, covering a thin layer as marked by *arrow 2*. Finally, it grows in the radial direction (*arrow 3*)



inlet temperature, for an ideal diatomic gas. For sonic isentropic expansion the velocity increase exactly balances the enthalpy decrease in the gas, all the while preserving the total stagnation enthalpy. In the second section, the stream undergoes further expansion through a series of shock waves down to the ambient pressure. The flow decelerates and warms up until the enthalpy of the stream returns to the value at the inlet to the expansion tube. The intermediate temperature drop due to the isentropic expansion is larger than the net temperature drop which is determined by the adiabatic integral Joule-Thomson effect.

9.1.3.2 Clog Formation in the Heat Exchanger

As noted in the previous paragraphs, it is possible to develop clogs within the heat exchangers of JT coolers. Lerou, ter Brake, et al.^{10,11} at the University of Twente in the Netherlands (Sect. 6.4.5.) have experimentally verified this fact. They traced plug formation within an operating MEMS cryocooler made of transparent glass and visually observed plugs forming at the cold end of their heat exchanger. The pressure and mass flow rate of the gas at the inlet to the heat exchanger were 8 MPa and about 1 mg/s respectively. The gas expanded to 0.6 MPa, corresponding to a boiling temperature of 96 K. As reported, a solid plug began to appear in the colder part of the heat exchanger, at a temperature of 200 K. The solid's precipitation was consistent with the estimated partial pressure of about 1 Pa, which is greater than the saturation pressure, P_{S-V} (200 K) = 0.16 Pa. However, it is interesting to note that in this case the level of contamination was relatively low, only 0.12 PPM. Particles of ice migrated into the very shallow restricted channel of 300 nm and blocked it. The flow rate sharply decreased, but subsequently recovered as the cooler warmed up and sublimated the clog. Apparently, the narrow channels of the recuperator and the MEMS fabrication technique provided the nucleation conditions for clogging even below the 2 PPM impurity level.

9.1.3.3 Water Versus Carbon Dioxide Clog Formation

Water versus carbon dioxide clog formation was compared by Maytal.¹¹ In this study, a cryocooler with a short-hole

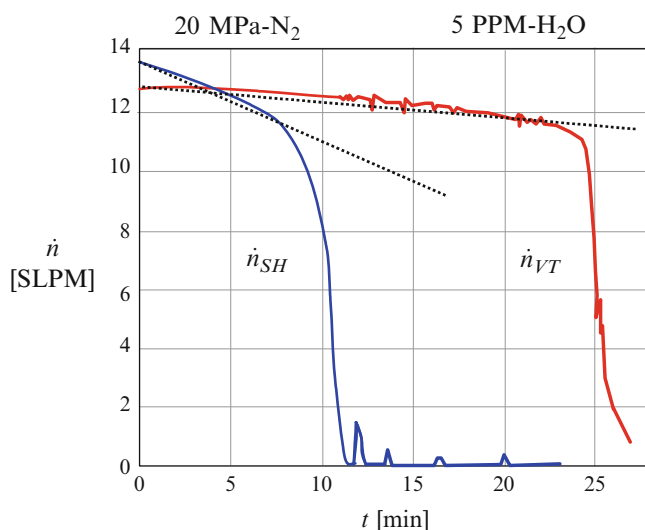


Fig. 9.3 Comparison of the traditional short hole (SH) and the vortex throttle (VT) clog retard. The flow rates, ($\dot{n}_{SH}$, $\dot{n}_{VT}$), in Standard Liters Per Minute, decrease in response to plug formation¹³

expansion valve was supplied with nitrogen at 20 MPa. The nitrogen was contaminated, in one test by 5 PPM (100 Pa) of water and in another by 80 PPM (1,600 Pa) of carbon dioxide. Water solidifies in a manner that continuously blocks the orifice and gradually suppresses the flow rate, as displayed in the later example of Fig. 9.3; the flow rate does not recover, even after many minutes. As carbon dioxide participates, the flow rate fluctuates over a wide range. It may rapidly fall to very low values but will recover almost immediately to the original value; both the sublimating and the freezing processes appear to occur readily. Furthermore, the temperature of the payload is only slightly affected prior to a complete blockage.

The difference between these behaviors can be traced to the vapor pressure curves displayed in Fig. 9.1. Since at any given temperature solid carbon dioxide has a higher vapor pressure than water, it can be present at a higher concentration without precipitation. At 150 K the vapor pressure, P_{S-V} , of water is less than 0.001 Pa. Hence, the partial pressure of 100 Pa in the experiment is five orders of magnitude above the equilibrium vapor pressure. Therefore the tendency to solidify is large; it proceeds monotonically and accumulates

until the passageway is completely blocked. However, P_{S-V} (150 K) for CO₂ is about 1,000 Pa, quite close to the partial pressure of 1,600 Pa. Therefore, the drive for clog formation is low and the clog sublimates easily. The transition from a blocked to an open passageway is fast and consequently, the flow rate fluctuates.

9.1.4 Aspects of Impeding Clog Formation

9.1.4.1 Proposed Clog Retarding JT Valves

Cowans¹² in 1964 proposed a vortex throttle valve based on the observation that a larger passageway or even an open tube without any restriction at its end^{13,14} would have a lower chance of being plugged by solid formation. Maytal,^{13,15} used a JT valve based on a series of vortex throttles (Fig. 6.7). Walker^{16,17} developed a clog resistant orifice (Fig. 6.9.) constructed in the form of a conical converging annulus with peripheral grooves. These interrupt the expanding stream so that a mixing effect opposes the growth of crystals. The grooved orifice was adapted for an extremely high thermal stability cooler¹⁸ and for a sorption type cooler.¹⁹ Lester and Benedict²⁰ used a long capillary (Fig. 6.5) with a larger diameter than a corresponding localized orifice in order to introduce the same pressure drop while reducing the likelihood of clogging.

9.1.4.2 Mechanisms to Mitigate Clog Formation in Heat Exchangers

A variety of approaches have been explored to suppress the migration of frozen particles into the JT valve. Evers²¹ in 1959 built a recuperator with a pair of parallel high pressure tubes instead of a single one leading to a common expansion valve. He reported that the modified design was less sensitive to contaminations and ran for a longer period. McInroy²² in 1960 installed a porous plug adjacent to the expansion orifice, at its inlet, in order to capture impurities that solidified in the heat exchanger. Smith et al.²³ in 1965 proposed coating the inner surface of the high pressure supply tube and the orifice with Teflon. In this way they intended to eliminate tubing irregularities that could serve as nucleation sites and promote plug formation and crystal growth. Steyert²⁴ packed elongated fibrous materials such as cotton, silk or other hydrophilic fibers into the helical high pressure tube. This is expected to prevent migration of contaminating species towards the orifice at the cold end.

9.1.4.3 Orifice Heating

Smith et al.²⁵ proposed heating the region near the orifice above the crystallization temperature with the use of an electrical element in order to avoid clog formation. This idea was carefully examined about two decades later by Lester and Benedict²² in 1984 at Ball Aerospace System

Division. They designed a “Joule-Thomson valve defroster” for a “long term in space cryocooler” in which an electrical heater is used to sublimate any clog. Indeed, this technique was adapted and utilized in the Ball Aerospace closed cycle dual pressure Joule-Thomson cryocooler, as described in Sect. 5.4.9. for operation in space. In a prior study²⁵ they released various controlled pulses of heat whenever a developing plug affected the flow rate. The prior study verified the effectiveness of the approach and demonstrated that the cooling system could recover its intended operation once the plug was removed. Bartlett et al.²⁶ also proposed using a heater to mitigate clog formation, as did Lerou et al.^{10,10} with their MEMs based cryocoolers for space applications.

9.1.4.4 The Vortex Throttle

Maytal^{13,17,13} reports on experiments using a vortex throttle in order to avoid clog formation. Here two different cryocoolers were constructed for comparison. The first one incorporated a compact series of vortex throttles, as described in Sect. 6.1.5. with its associated data identified by the subscript VT. The second one had a similar size recuperator and flow rate but used a short-hole throttle; its associated data are identified by the subscript SH. Both were fed in parallel from the same pressure supply with a very precisely controlled level of water (5 and 17 PPM) or carbon dioxide vapor (80 PPM) contamination. Figure 9.3 displays the measured instantaneous flow rates ($\dot{n}_{SH}$, $\dot{n}_{VT}$), using a nitrogen stream of 20 MPa and contaminated by 5 PPM of water vapor. The slower flow degradation of the vortex throttle cooler and its longer run time indicates a lower effective rate of blockage. The payload warms up as the flow decreases and the associated cooling power become too small. With regard to the carbon dioxide contamination, the vortex throttle is almost “clog-free”; the instantaneous flow rate was very moderately affected by the carbon dioxide contamination.

9.1.4.5 Desiccants

The most effective means for sustaining high purity is to remove the vapor impurities by a desiccant and absorbent chemicals. The molecular sieve kind of absorbent desiccant,^{1,15,27} type 4A or 5A,² entraps water vapor and carbon dioxide very efficiently. Hydrocarbon vapor and other contaminants are handled by activated carbon^{2,22} A pressure vessel, packed with a bed of the absorbent chemicals is installed upstream, close to the Joule-Thomson cryocooler. A molecular sieve, if not contaminated by hydrocarbons, may be reversibly recycled thereby releasing the absorbed water and carbon dioxide and removing it completely from the system. The process is accomplished by purging with pure gas, preferably warm and under as low a pressure as possible. Thornton² proposed utilizing the low suction pressure of a compressor that drives the Joule-Thomson

cryocooler. Hingst²⁸ suggested a warm purging process through the use of a thermo-electric element which also would serve on its opposite side for precooling the incoming stream.

9.1.4.6 Fixed Orifice Versus Flow Demand Valves

An adjustable orifice operates with a significantly smaller opening at steady state than a typical fixed orifice. Nevertheless, it is less affected by freezing contaminants. As solid particulate begins to form, the flow regulator can respond by enlarging the orifice, thereby compensating for the reduced area. However, in such a case it is useful to examine the response time of the various kinds of flow regulators. Bonney²⁹ points out that the bellows type of regulator reacts better, being less affected by clog formation than the thermally expanding solid actuator, as discussed in Sect. 6.2.3.2. Ellison³⁰ proposed an actively controlled flow regulator that enlarges the orifice when required to expel any clog formation.

9.1.5 Monitoring Gas Purity

9.1.5.1 Laboratory Equipment

The delicate task of contamination control requires the use of accurate instrumentation. Some examples of these are:

The Gas Chromatograph

The gas chromatograph is recommended by the British Defense Standard³ for carbon dioxide and hydrocarbons.

The hygrometer

The hygrometer is used to indicate the concentration of water vapor in terms of the corresponding dew point at 0.1 MPa. For example, a concentration of 2 PPM (by volume) corresponds to a dew point of -72°C . Concentrations of 1, 4 and 30 PPM correspond respectively to dew points at 0.1 MPa of -76 , -67 and -52°C .

9.1.5.2 Gas Purity Testers

Gas purity testers are often used^{1,31} to verify the contamination level of the entire gas supply system and determine whether it exceeds the acceptable limits. In principle, such devices incorporate a fixed orifice cryocooler fed at a constant specified pressure and equipped to indicate the actual instantaneous temperature and flow rate. The run time until the flow rate reduces to a certain fraction of the maximum value, like $50\%^1$ is correlated with the level of contamination. Alternatively one could measure the elapsed time until the device warms up, being unable to maintain a steady low temperature. Both procedures correlate a longer run time with a higher purity level.

If a purity monitor becomes contaminated by an impure gas, the impurity could affect the results of subsequent tests with other pressurized sources. In view of this concern Galinka et al.³³ included an on-off valve in their monitor in order to bypass the Joule-Thomson expansion orifice at the beginning of a new test. In this way the entire tubing can be purged at room temperature with the next gas before it is cooled and liquefied.

9.2 Flow Rates

9.2.1 Introduction

9.2.1.1 Three Kinds of Flow Rates

Three kinds of flow rates are associated with the operation of a cryocooler. The first two are the initial and warm, or “free” flow rate, $\dot{n}_{FR}$, and the cold or recuperated flow rate, $\dot{n}_{RE}$, both introduced in Sect. 7.5.1. The recuperated flow rate, $\dot{n}_{RE}$ is reached at the end of the cooldown process and sustained by the steady state operation of a fixed orifice cryocooler. The third kind of flow rate is associated with the flow demand cryocooler as discussed in Sect. 5.3. A regulating mechanism reduces the flow rate towards the end of the cooldown process so that at steady state the instantaneously regulated flow rate is lower than $\dot{n}_{RE}$ and $\dot{n}_{FR}$ at least by an order of magnitude. This flow rate depends on the cooling potential of the gas (Sect. 4.8) and on the character of the regulating mechanism.

9.2.1.2 The Flow of a Joule-Thomson Cryocooler Is Choked

The inlet or driving pressure typically ranges from three times the critical pressure up to twenty times the critical pressure P_C (corresponding to a reduced pressure range of $\Pi = 3\text{--}20$) while the outlet pressure is generally below $P_C/15$. Since the expanded, or low pressure determines the target temperature for boiling and cryocooling, it is typically reduced as low as possible. Consequently, the inlet to outlet pressure ratio is quite high and above the critical pressure ratio for choked flow, even at the low temperatures that exist at the inlet to the expansion valve. Therefore, it is very likely that the flow of any operating Joule-Thomson cryocooler is choked.

9.2.1.3 The Unit of “Standard Liters Per Minute”

The unit of “Standard Liters Per Minute” (SLPM) is adapted in practice to measure the volumetric flow rate, $\dot{V}_{ST}$, at the standard (ST) conditions, $T_{ST} = 295\text{ K}$, $P_{ST} = 100\text{ kPa}$. It represents a mass (molar) flow rate of,

$$\dot{n} = \dot{V}_{ST}/v_{ST} \quad (9.2)$$

where for all gases,

$$v_{ST} = 22.4 \text{ Lit mole}^{-1} \quad (9.3)$$

The volume of V_{ST} , measured in SL (Standard Liters) is often used to specify the amount (by mass or moles) of gas. For example, the value of $V_{ST} = 30$ SL for any gas is the mass of $30/22.4 = 1.34$ moles (37.5 g for nitrogen). The flow rate of $\dot{V}_{ST} = 30$ SLPM is 1.34 moles per minute or 0.0223 moles per second. It is equivalent to 0.625 g per second of nitrogen or 0.892 g per second of argon.

The mass flow rate represented in this form is independent of the pressure. However, assuming a constant temperature, the pressure dependent volumetric flow rate is obtained from $\dot{V}_{ST}$, by scaling with the ratio of the elevated pressure, P , to the standard pressure, according to:

$$\dot{V}(P, T_{ST}) = \frac{\dot{V}_{ST}}{(P/P_{ST})} Z(P, T_{ST}) \quad (9.4)$$

Thus, for a standard volumetric flow rate $\dot{V}_{ST} = 30$ SLPM of nitrogen, the associated volumetric flow rate at 30 MPa and 295 K is reduced to only 0.1141 l per minute (since $Z = 1.141$). Notice that for most JT cryocoolers expanding to 100 kPa, it is appropriate to use $\dot{V}$ and $\dot{V}_{ST}$ to represent the up- and down-streams' volumetric flow rates respectively.

9.2.2 Choked Flow Rates of Real Gases

9.2.2.1 The Deviation from the Ideal Gases Model

The condition for choked mass flow with an ideal gas was already introduced through Eq. (6.1). Conditions for choked flow with a real gas were derived by Maytal³² based on an isentropic expansion until the speed of sound is achieved, but not penetrating into the two phase dome. This study was enhanced by Maytal and Elias,³³ by allowing the stagnation temperature to extend down to T_C and allowing the choked state (at the “throat”) to occur inside the two phase dome or on its boundary of saturated states. The choked flow rates were obtained for homogenous flow and with slip flow conditions. The results were discussed in Sect. 6.1.1.2. and displayed in Figs. 6.1. and 6.2. for argon and hydrogen in terms of the deviation factor, $\Gamma = \Gamma(T_0, P_0)$, from the ideal gas (molar) mass flux, by Eq. (6.2). References 34 and 35 also discuss the residual mass flux, G^R , which is the absolute deviation of the choked mass flux for a real gas from that of an ideal gas, $G - G^{IG}$. Some earlier studies on the choked mass flux for real gases were provided by Johnson^{34,35} and Schmidt, et al.^{36,37}

9.2.2.2 The Pressure Range of Significant Deviations

For inlet temperatures below a reduced temperature value of 2, the (relative) deviation coefficient, $\Gamma = \Gamma(T_0, P_0)$, for choked flow rates of real gases displays a maximum and exceeds the ideal gas values, mostly in the reduced pressure range of 3–12. This pressure range is also that which is most relevant for the operation of Joule-Thomson cryocoolers. For nitrogen and argon, the deviation coefficient approaches a factor of 2 which means that the predictive power of the ideal gas model is quite limited for the practice of Joule-Thomson cryocooling. The absolute deviations of the real gas model from ideal gas behavior are spread over a fairly broad reduced pressure range.

9.2.2.3 Inlet Temperature Dependence

In the relevant pressure range and at the lower temperature range where the deviations from ideal gas behavior peak, the real gas choked flow rate, $\dot{n}$, increases as the temperature decreases, more strongly than $T_0^{-0.5}$. Since, by Eq. (6.2),

$$\dot{n}(T_0, P_0) \propto P_0 \frac{\Gamma(T_0, P_0)}{\sqrt{T_0}} \quad (9.5)$$

depending on the particular pressure, the contribution of $\Gamma(T_0, P_0)$ causes $\dot{n}$ to increase at lower temperatures significantly stronger than $T_0^{-0.5}$. Bradshaw et al.³⁸ measured the value of $\dot{n}(T_0)$ of helium for T_0 between 20 and 5 K at $P_0 = 1$ MPa. The value of $\dot{n}(T_0)$ increased by a factor of about 3.5, larger than $\sqrt{20/5} = 2$. According to reference³⁵ this ratio cannot exceed the ideal value of about 4.

9.2.2.4 The Principle of Corresponding States

The principle of corresponding states can be used to predict the choked mass flux rates for various gases. The normalized mass flux,

$$\bar{G} = G/G_C \quad (9.6)$$

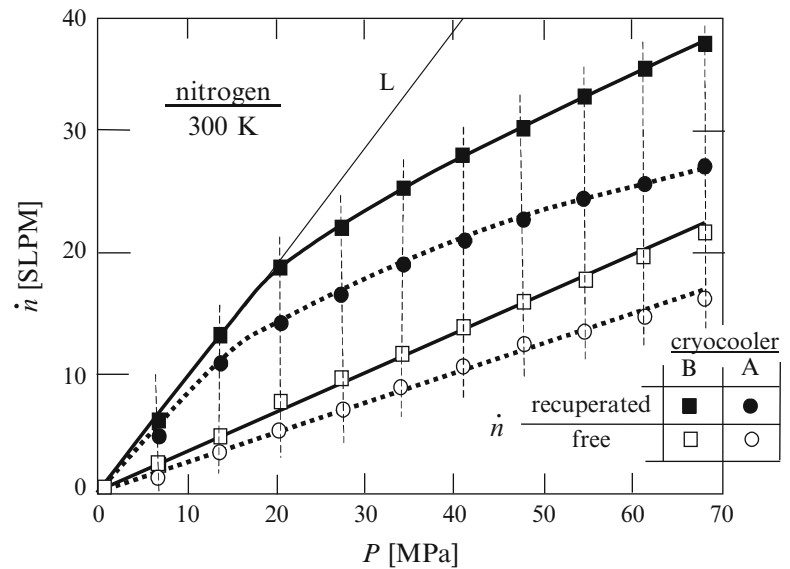
for common reduced stagnation conditions, (Π_0, Θ_0) , exhibits a very uniform dependence when normalized by,

$$G_C = \frac{P_C}{\sqrt{M \cdot R \cdot T_C}} \quad (9.7)$$

Hendricks and Simoneau^{39,40,41} first normalized the mass flux G by the group $P_C/(Z_C \sqrt{T_C})$.

The various gases of similar acentricity factors, ω , at the same reduced temperature (Θ) and pressure (Π) exhibit the same deviation factor, $\Gamma(\Theta, \Pi)$. This issue is demonstrated in the previously mentioned study³⁴ by examining groups of gases with similar values of ω like nitrogen, oxygen and methane or the group of noble gases.

Fig. 9.4 Recuperative and non-recuperative measured flow rates of nitrogen for two cryocoolers (A and B) as function of pressure, at 300 K ambient, both having a fixed orifice, after reference 45. The non recuperated flow rate, $\dot{n}_{FR}$, at 10 MPa for A and B are 2.5 and 3.5 SLPM



Gases of lower acentricity factor, ω , systematically have a larger $\Gamma(\Theta, \Pi)$ and their choked flow rate deviates more from the ideal gas. For instance, argon deviates more than nitrogen, and nitrogen more than carbon dioxide but less than helium. Helium exhibits the highest deviation.

9.2.2.5 The Critical Pressure Ratio

An ideal gas flow rate is already choked when the inlet to outlet pressure ratio is about 2, independent of the inlet conditions of the fluid. The previously mentioned studies^{34,35} show that as the inlet temperature becomes lower and lower, and the inlet pressure grows higher, the critical pressure ratio for real gases becomes higher and may reach values of about 20. Hendricks et al.⁴² measured the critical ratio for nitrogen above and below the critical temperature. Nevertheless, it is not likely that the flow in a practical Joule-Thomson cryocooler will ever be un-choked either with pure or with mixed coolants.

9.2.3 Pressure Dependence of a Cryocooler's Flow Rates

9.2.3.1 The Free Flow Rate, $\dot{n}_{FR}$

The pressure dependence of $\dot{n}_{FR}$ reflects the total hydrodynamic impedance of the high pressure channel (tube with the expansion valve at its end) at room temperature. Very often, the orifice contributes the main impedance, and $\dot{n}_{FR}$ therefore primarily reflects the characteristics of the expansion valve.¹ For nitrogen and argon, the free flow rate is directly proportional to the inlet pressure as verified experimentally by Maytal.⁴³ The slope of this linear dependence represents the hydrodynamic impedance of the channel. Therefore, it is sufficient to specify the free flow rate at any single pressure

value such as 6.8 MPa (1,000 psi)^{1,44} or 10 MPa.⁴⁵ Maytal⁴⁵ used two cryocoolers with overall dimensions of 8.3 mm diameter and 20 mm length. The free flow rates for nitrogen are 2.5 and 3.5 SLPM, respectively, at 10 MPa for cryocoolers A and B as displayed in Fig. 9.4.

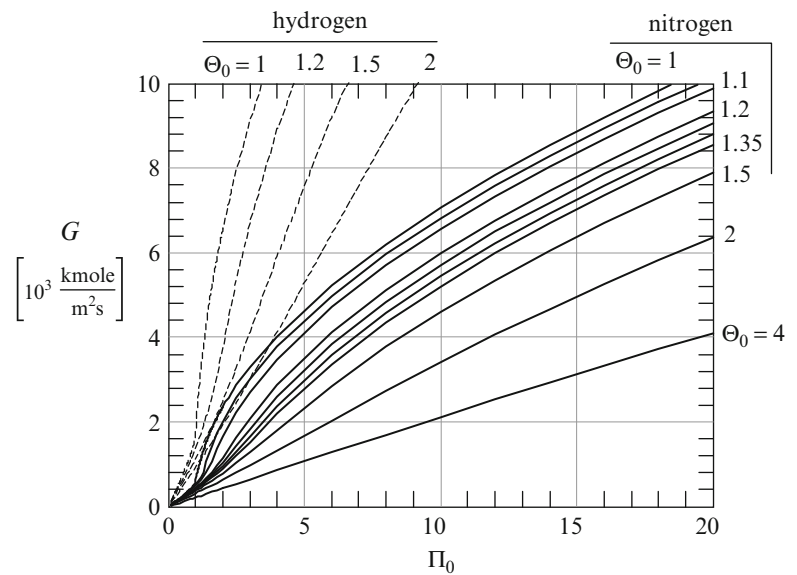
This linear pressure dependence of $\dot{n}_{FR}$ is consistent with the prediction of the real gas choked flow rates. Room temperature corresponds to a reduced temperature of about 2.4 for nitrogen and 2 for argon. Following the real gas prediction³⁴ the deviation coefficient, Γ , at these reduced temperatures is quite close to unity. Therefore, the choked free flow rates of nitrogen and argon behave as expected of an ideal gas and display a linear pressure dependence.

However, this is not the case for gases with higher values of T_C and T_{BOIL} . For such gases, room temperature is equivalent to a reduced temperature less than 2 and the associated $\Gamma(T, P)$ is no longer close to unity. Consider for example krypton at room temperature, 295 K, for which $\Theta = 1.4$. By similarity, through the principle of corresponding states, the values of $\Gamma(T, P)$ for argon (Fig. 6.1.) are applicable for krypton too. The deviation from ideal gas for krypton at room temperature approaches values of 1.4, a substantial departure from unity. This deviation at room temperature results in a choked flow rate about three times larger than that of argon and about 10 times larger than that for nitrogen. Therefore, one may expect that the pressure dependence of $\dot{n}_{FR}$ for krypton will not be linear as it is for argon or nitrogen.

9.2.3.2 The Recuperated Flow Rate, $\dot{n}_{RE}$

The recuperated flow rate, $\dot{n}_{RE}$, corresponds to lower temperatures at the inlet of the valve than those associated with the free flow rates. Hence, even for lower boiling point gases such as nitrogen and argon, the inlet temperatures are about $\Theta = 1.3$ and below.

Fig. 9.5 Real gas choked molar flux of nitrogen and hydrogen for various inlet reduced temperatures and pressures, after references^{34,34}



The pressure dependence of $\dot{n}_{RE}$ can therefore be non-linear as indeed the measurements shown in Fig. 9.4 with the same two cryocoolers display. The recuperated flow rate grows quickly with P at the lower pressure range but less quickly as P increases. At the highest pressures, the increase of $\dot{n}_{RE}$ with pressure is closer to a square root dependence than a linear one. Put another way, at the higher pressure range $\dot{n}_{RE}$ decreases more moderately as the pressure is exhausted than does the pressure itself. The line L in Fig. 9.4 displays, for reference, a linear dependence on P . Hence, at the lower pressure range, up to about 20 MPa, the recuperated flow rate tracks very proportionally with P .

Figure 9.5 displays the calculated real gas choked flow rates (molar flux) for nitrogen and hydrogen at various inlet temperatures and pressures. Notice the pressure dependence at the reduced inlet temperatures of 1.2 or 1.3 for nitrogen. These could represent the actual temperature of nitrogen at the inlet to the expansion valve. At least qualitatively they resemble the measured pressure dependence of $\dot{n}_{RE}$ of Fig. 9.4. The next section provides a somewhat more quantitative verification of real gas choked flow rates. Hendricks et al.⁴⁴ measured choked flow rates for nitrogen.

9.2.4 The Ratio of Cryocooler Flow Rates, $\dot{n}_{RE}/\dot{n}_{FR}$

It is relatively easy to measure a cryocooler's free flow rate since it is not necessary to produce liquefaction. One may simply extract the cryocooler (of the Hampson type heat exchanger) from the dewar in order to avoid recuperative cooling, and then measure the flow rate at any relatively low pressure. This single measurement may then be scaled

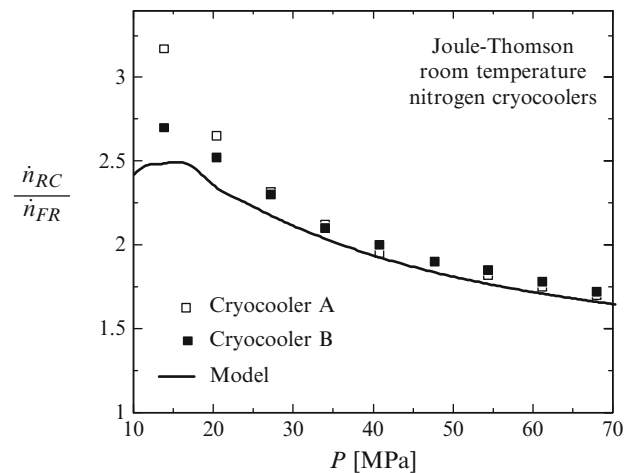


Fig. 9.6 Ratios of recuperated and non-recuperated flow rates of two cryocoolers (A and B) as a function of pressure, at 300 K ambient, both having a fixed orifice. The measured values were reported in reference 45 and the prediction is based on real gas choked flow rates in reference 34 assuming a reduced inlet temperature of 1.2

through the linear pressure dependence for nitrogen and argon in order to obtain the free flow rate at the operating pressure of the cryocooler. However, it is typically more important to know the actual flow rate, $\dot{n}_{RE}$, during the (non regulated) operation of the cryocool as this will more closely reflect the actual gas consumption during cooldown. Therefore it is beneficial to explore the ratio $\dot{n}_{RE}/\dot{n}_{FR}$ in order to convert values of $\dot{n}_{FR}$ into values of $\dot{n}_{RE}$.

Figure 9.6 displays the ratio $\dot{n}_{RE}/\dot{n}_{FR}$ from measurements with nitrogen. At the higher pressure range of 70 MPa, the ratio is as low as 1.7 while at the lower pressure-range of 14 MPa, it is above 3. (The measurement of Lerou et al.¹¹ show a ratio which is close to 3 at 8 MPa). Figure 9.6 also

displays the predicted $\dot{n}_{RE}/\dot{n}_{FR}$ ratio³⁴ in terms of the deviation coefficients,

$$\frac{\dot{n}_{RE}}{\dot{n}_{FR}}(P) = \frac{\Gamma(T_2, P)}{\Gamma(T_{AMB}, P)} \left(\frac{T_{AMB}}{T_2} \right)^{1/2} \quad (9.8)$$

Calculation of this ratio depends on T_{AMB} and on the proper estimation of T_2 at the inlet to the nozzle. At ambient room temperature the reduced temperature for nitrogen is $\Theta_{AMB} = 2.4$ and assuming that a representative reduced value of T_2 over the entire pressure range is $\Theta_2 = 1.2$ (that is about 151.5 K), the prediction of $\dot{n}_{RE}/\dot{n}_{FR}$ is reasonably close to the experimentally derived ratios as displayed in Fig. 9.6.

In the same case, with $T_{AMB}/T_2 = 2$ and since $\Gamma(T_2, P)$ and $\Gamma(T_{AMB}, P)$ both have values close to unity at elevated pressures, the ratio $\dot{n}_{RE}/\dot{n}_{FR}$ has a lower limit that is approximately $\sqrt{2}$. However, for gases with higher boiling temperatures such as krypton or CF_4 , the values of T_{CD} and T_2 are higher and closer to T_{AMB} and therefore the ratio $\dot{n}_{RE}/\dot{n}_{FR}$ is closer to 1.

The values of $\dot{n}_{RE}/\dot{n}_{FR}$ for argon, as obtained by measurements in the same reference⁴⁵ are somewhat lower; about 1.5 at the higher pressure range of 70 MPa, and about 2.4 at 15 MPa.

9.2.5 Discharge of a Pressure Vessel Through a Fixed Orifice Cryocooler

9.2.5.1 Discharge Pattern of a Fixed Versus Adjustable Orifice Cryocooler

Section 4.8 discussed the cooling potential of an isothermal pressure vessel and the corresponding discharge period. It was assumed that the instantaneous pressure dependent flow rate is adjusted by changing the size of the orifice in order to match the instantaneous heat load. At higher pressures, since the isothermal enthalpy change Δh_T is larger, the adjusted flow rates are smaller than at lower pressures (see paragraph 5.3.3.3.). However, in the case of a fixed orifice, the flow rates are larger at higher pressures and the potential cooling power is larger than the heat load. In fact the only point during the discharge where the heat load, $\dot{Q}$, is balanced by the potential cooling power (so that $\eta = 1$) is at the last moment of cryocooling (the first moment of warm up), that is, at the final, or cut-off pressure, $P(t) = P_{CUT}$.

9.2.5.2 A Simplified Model

A simplified model exists that enables a closed form solution of the discharge behavior and run time.⁴⁵ Here, it is assumed that the specific cooling capacity at the relatively low cut off pressures can be described as,

$$\Delta h_T(P_{CUT}) = \alpha P_{CUT} \quad (9.9)$$

where α is a constant. Eq. 9.9 is a reasonable approximation for nitrogen and argon when P_{CUT} does not exceed about 20 MPa. In general, one would like to have a lower cut-off pressure in order to utilize more coolant from the reservoir. For nitrogen and argon the corresponding values of α at 300 K are 50 and 70 J mole⁻¹ MPa⁻¹. At the cut off moment,

$$\dot{Q} = \dot{n}(P_{CUT}) \cdot \Delta h_T(P_{CUT}) \quad (9.10)$$

The content of the reservoir is assumed to behave like an ideal gas, so that,

$$\dot{P}(t) \cdot V = -\dot{n}(P) \cdot R \cdot T(t) \quad (9.11)$$

where V is the volume of the pressure vessel. The initial charging pressure is P_0 , and the associated amount of gas is n_0 . In order to determine the explicit time dependence for the pressure by integrating Eq. 9.11, one has to assume a pressure dependence of the flow rate, $\dot{n}(P)$, and the thermal regime of the vessel, as described in the next two paragraphs.

9.2.5.3 The Instantaneous Flow Rate

The instantaneous flow rate for a fixed orifice cryocooler is the recuperated flow rate, $\dot{n}_{RE}$; its dependence on pressure, $\dot{n}(P)$, was discussed in Sect. 9.2.3. For the low range of charging pressures, up to about 30 MPa, the flow rate may be approximated as proportional to pressure, so that,

$$\dot{n}(P) = K_1 P \quad (9.12)$$

For higher values of charging pressure, a better approximation is,

$$\dot{n}(P) = K_2 \sqrt{P} \quad (9.13)$$

where K_1 and K_2 are the respective discharge coefficients of a fixed size orifice.

9.2.5.4 The Thermal Interaction of the Pressure Vessel

The thermal interaction of the pressure vessel at its boundaries with the surroundings may be assumed to provide a constant gas temperature at the ambient temperature, $T(t) = T_0 = \text{Const}$ due to the large surface area involved or in the case of “sufficiently slow” discharge that enables the temperature of the vessel to equalize with that of the surroundings. The opposite extreme case is the adiabatic discharge; it occurs in cases of a rapid and short discharge or if the boundaries are thermally isolated. Any other thermal

condition for the discharge between the isothermal and adiabatic may be described by a polytropic expression,

$$P v^n = P T^{n/(n-1)} = \text{Const} \quad @ \quad 1 \leq n \leq \kappa \quad (9.14)$$

9.2.5.5 The Optimal Orifice Size

When the orifice and the associated flow rates are large, the pressure in the vessel decays rapidly, but the associated flow provides cooling down to the cutoff pressure, P_{CUT} , and for the run period Δt . A small orifice discharges more slowly but terminates at a higher cutoff pressure, P_{CUT} . Since each orifice has a corresponding cut off pressure, the parameter,

$$\beta = (P_0/P_{CUT}) \quad (9.15)$$

provides an alternative description of the orifice size. An optimum orifice size exists that maximizes the run time to a value of Δt^{MAX} . Solving the simplified model determines the optimal orifice size in terms of its initial flow rate, $\dot{n}_{OPT}(P_0)$, or in terms of β_{OPT} and the corresponding Δt^{MAX} (displayed in Table 9.1). The value of β_{OPT} corresponds with the cutoff pressure for an optimal orifice. Note that β_{OPT} is independent of any other parameters of the system such as T_0 , P_0 , α , V and the heat load $\dot{Q}$. The optimum size of the orifice is such that its cut off pressure is between one third and one half of the initial pressure. Because an optimal orifice size is typically small, a practical orifice is at least few times larger than the optimal one.

9.2.5.6 The Run Time Δt

The run time Δt , expressed in the form of a fraction, $\Delta t/\Delta t^{MAX}$, is derived as function of the actual size orifice size normalized by the ratio,

$$\gamma = \frac{\dot{n}(P_0)}{\dot{n}_{OPT}(P_0)} \quad (9.16)$$

Values of β and γ are related through β_{OPT} . Values of $\Delta t/\Delta t^{MAX}$ as function of β or γ are displayed in the same table. The quantity $\Delta t/\Delta t^{MAX}$ sharply decreases as the orifice size is reduced below its optimum value, $\gamma < 1$, but for $\gamma > 1$ it decreases much more moderately. One half and one third of the longest run time is obtained respectively by an orifice with values of γ about 5 and 10. The adiabatic and any polytropic discharge durations are shorter than the isothermal one.

9.2.5.7 Different Coolants Discharging Through the Same Cryocooler

In the case of an isothermal discharge with the flow rate in a square root regime as described by Eq. 9.13, one obtains,

$$\Delta t = 2 \frac{n_0}{\dot{n}_0} \left[1 - \left(\frac{\dot{Q}}{\dot{n}_0 \alpha P_0} \right)^{1/3} \right] \quad @ \quad \dot{n}_0 \equiv \dot{n}(P_0) \quad (9.17)$$

This functional relation for an ideal gas can serve to estimate the trend of diverse values of Δt for various real gases at the same initial condition (P_0 , T_0), in terms of $\dot{n}_0$, n_0 and α . The run time, Δt , becomes shorter as $\dot{n}_0$ is larger (given $\gamma > 1$). Different gases may have different choked flow rates, $\dot{n}_0$. Additionally, the initial amount of stored gas, n_0 , is a function of the particular compressibility of each gas. Gases with larger values of α have the potential to prolong their run time Δt . In the case of a short discharge (small vessel), the particular yield of liquefaction and the size of the liquid plenum (that is, the stored, non utilized cooling potential) become relevant as well.

9.3 Modeling the Joule-Thomson Cryocooler

9.3.1 The Scope of Modeling

9.3.1.1 Description of the Model

The general case of modeling time dependent performance is associated with integrating a set of partial differential equations of energy and momentum balance for the different parts of the heat exchanger. The thermo-physical and transport properties are temperature and pressure dependent. Such a model can be solved only by numerical integration.

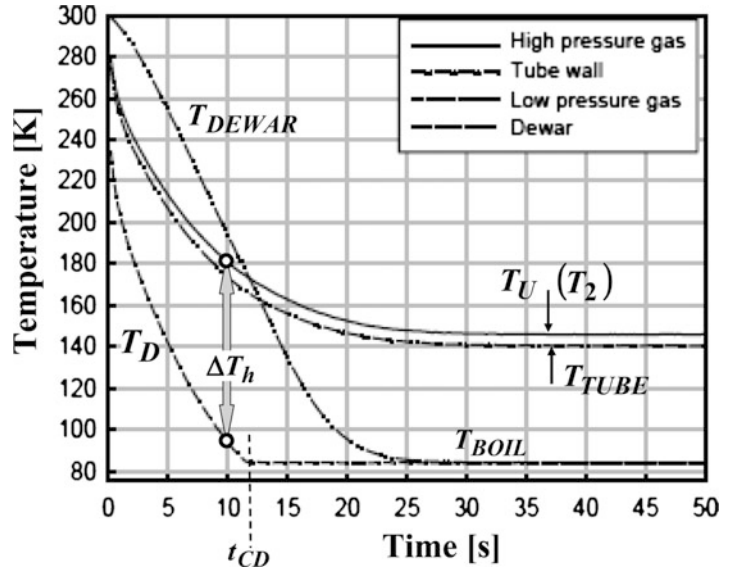
The model discussed below refers to the Hampson kind of heat exchanger, which is the most commonly used type in miniature Joule-Thomson cryocoolers. The model provides temperature distributions along the heat exchanger for both the high and low pressure counterflowing streams, $T_U(t, x)$ and $T_D(t, X)$. In addition, the longitudinal temperature distributions are derived along the high pressure tube, $T_{TUBE}(t, x)$, along the dewar cold finger, $T_{DEWAR}(t, X)$, and along the mandrel, $T_{MANDREL}(t, X)$. The longitudinal coordinate x , runs along the center line of the helically coiled high pressure tube. On the other hand, X is the longitudinal coordinate along the center line of the inner mandrel and the whole heat exchanger or the mandrel. The two are related through the diameter of the helix and its thread angle. Consistency of the model can be examined by comparing the steady state difference of the temperature profiles, $T_U(X) - T_D(X)$, as function of $T_U(X)$ with the predicted values in Sect. 3.11. by the energy balance.

Parameters of geometry, flow, and heat transfer which refer to the high pressure (up) stream or low pressure (down) stream are indicated by their respective subscript U or D. The instantaneous flow rate, whether determined by a fixed

Table 9.1 Discharge regimes of a pressure vessel through a fixed orifice cryocooler, for two flow rates pressure dependences: $\gamma = \dot{n}(P_0)/\dot{n}_{OPT}(P_0)$

	$T = Const$		$Polytropic P \cdot \gamma^n = Const$	
	$\dot{n} = K_2 \sqrt{P}$	$\dot{n} = K_1 \sqrt{P}$	$\dot{n} = K_1 \sqrt{P}$	for $n = \kappa = 5/3$
$\beta_{OPT} = (P_0/P_{CUT})_{OPT}$	$\left(\frac{4}{3}\right)^2 = 1.77$	$\sqrt{e} = 1.65$	$\left(\frac{4n}{1+3n}\right)^{2n/(n-1)}$	$\left(\frac{10}{9}\right)^5 = 1.69$
$\beta(\gamma)$	$\gamma^{2/3} \beta_{OPT}$	$\gamma^{1/2} \beta_{OPT}$	$\gamma^{1/2} \cdot \beta_{OPT}$	
$\dot{n}_{OPT}(P_0) / \left[\frac{\dot{Q}}{\alpha \cdot P_0} \right]$	$\left(\frac{4}{3}\right)^3 = 2.37$	$e = 2.72$	$\left(\frac{4n}{1+3n}\right)^{3n/(n-1)}$	$\left(\frac{10}{9}\right)^{15/2} = 2.20$
$\Delta P^{MAX} / \left[\frac{V \cdot \alpha \cdot P_0^2}{R \cdot T_0 \cdot \dot{Q}} \right]$	$\frac{1}{2} \left(\frac{3}{4}\right)^2 = 0.21$	$\frac{1}{2e} = 0.18$	$2 \left[\frac{(1+3n)^{3n+1}}{(4n)^{4n}} \right]^{1/(n-1)}$	$\frac{1}{3} \left(\frac{9}{10}\right)^{10} = 0.12$
$\Delta P^{MAX} / \left[\frac{n_0}{\dot{n}_{OPT}(P_0)} \right]$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{1+3n}$	$\frac{1}{3}$
$\frac{\Delta t}{\Delta P^{MAX}} \left(\beta = \frac{P_0}{P_{CUT}} \right)$	$\frac{4^4}{3^3} \cdot \frac{\sqrt{\beta} - 1}{\beta^2}$	$2e \cdot \frac{\ln \beta}{\beta^2}$	$\frac{1}{n-1} \left[\frac{(1+3n)^{3n+1}}{(4n)^{4n}} \right]^{1/(n-1)} \cdot \left(\beta^{-\frac{3n+1}{2n}} - \beta^{-2} \right)$	$\frac{1}{4} \left(\frac{9}{10}\right)^{10} \cdot \left(\beta^{-9/5} - \beta^{-2} \right)$
$\frac{\Delta t}{\Delta P^{MAX}}(\gamma)$	$\frac{4}{\gamma} - \frac{3}{\gamma^{4/3}}$	$\frac{1 + \ln \gamma}{\gamma}$	$\frac{4n}{n-1} \gamma^{-\frac{3n+1}{4n}} - \frac{3n+1}{n-1} \gamma^{-1}$	$\frac{10}{\gamma^{9/10}} - \frac{9}{\gamma}$

Fig. 9.7 Temperatures at the cold end via a numerical simulation of a miniature JT cryocooler: during cooldown: T_U and T_D , temperatures of the coolant at the inlet and outlet of the JT valve, and T_{TUBE} and T_{DEWAR} are the temperatures of the high pressure tube and the dewar.⁵⁹



orifice or by an adjusting mechanism, defines an additional variable parameter for the model.

9.3.1.2 Reported Modeling Experience

In 1979 Oren and Gutfinger^{46,47} simulated the steady state behavior, $T(X)$, for various flow rates and heat loads. In 1986 Longworth and Steyert⁴⁸ simulated the transient response of rapid cooldown cryocoolers operated with argon. They demonstrated the benefit of higher NTU values as determined by the low and high pressure flow streams. Chou, Chien et al.^{49,50} utilized a numerical model to characterize a stepped diameter cryocooler. Chien et al.⁵² enhanced the simulation by including a bellows type flow regulator and the characteristics of a needle valve. They compared the response of the flow regulator when the bellows was charged with a single gaseous phase to its response when charged with two coexisting phases.^{52,51} The steady state and transient behavior of a fixed orifice cryocooler was simulated and verified experimentally by Xue, Ng, Wang, Chua et al.^{52,53,54,55} In another report, Hong et al. simulate the performance of a heat exchanger and its effectiveness for various flow rates⁵⁶ and cryocoolers.^{57,58} Kauschke and Quack⁵⁹ simulate the transient behavior of a Joule-Thomson cryocooler fed by helium below 20 K.

Derking, ter Brake, et al.,⁶⁰ modeled a MEMS based cryocooler, including longitudinal conduction along the body of the heat exchanger and radiation at its outer surface.

Hong, et al.⁶¹ applied CFD analysis on an entire miniature Linde-Hampson miniature. The overall diameter was 5 mm, the diameter of the helical high pressure tube was 0.5 mm, and the fin thickness was 0.08 mm. They found that the heat transfer coefficient and the Fanning friction factor both were larger than the values obtained through common empirical correlations.⁶²

Figure 9.7 shows a typical result of the cooldown simulation of a miniature JT cryocooler,⁵⁹ displaying the temperature history at the cold end ($X = L$) as characterized by $T_U(t)$, $T_D(t)$, $T_{DEWAR}(t)$, and $T_{TUBE}(t)$. The cryocooler is about 40 mm long and 5 mm in diameter. The inner and outer diameter of the high pressure tube is 0.3/0.5 mm. The coolant is nitrogen supplied at 50 MPa.

9.3.2 The High Pressure Coolant Stream

9.3.2.1 The Governing Equations

The governing equations in their general form for time dependent compressible internal flows are:

Mass Conservation (Continuity)

$$\frac{\partial \rho}{\partial t} + \frac{\partial G}{\partial x} = 0 \quad (9.18)$$

Where the mass flux of the upstream flow is,

$$G_U = \dot{m}/A_U = (\rho V)_U \quad (9.19)$$

and V is the flow velocity.

Energy Conservation

Energy conservation governs the recuperation process,

$$\frac{\partial(\rho h)}{\partial t} + \frac{\partial(G h)}{\partial x} = -\frac{p_U}{A_U} h_U (T_U - T_{TUBE}) \quad (9.20)$$

where p_U is the wetted perimeter of the high pressure channel, A_U is the cross section area perpendicular to the

direction of flow, h_U is the local convective heat transfer coefficient, and h is the local instantaneous enthalpy,

$$h = h(P, T) \quad (9.21)$$

Momentum conservation

Momentum conservation governs the pressure gradient along the channels,

$$\frac{\partial G}{\partial t} + \frac{\partial(G^2/\rho)}{\partial x} = -\frac{\partial P}{\partial x} - \frac{2 G^2}{\rho d_{TUBE}} f \quad (9.22)$$

where d_{TUBE} is the diameter of the channel, P is the fluid pressure, and f is the friction factor.

Practically, the solution of the momentum equation with the time derivative is quite close to the steady state solution as explained elsewhere.⁶³ In addition, because the cross section of the flow channel is typically constant, G is not a function of x . Therefore, Eq. 9.22 may be replaced by,

$$\frac{G^2}{\rho^2} \frac{d\rho}{dx} = \frac{\partial P}{\partial x} + \frac{2 G^2}{\rho d_{TUBE}} f \quad (9.23)$$

9.3.2.2 Geometric Parameters of the Model

For a cylindrical channel,

$$p_{U, TUBE}/A_U = 4/d_{TUBE} \quad (9.24)$$

Chou, Chien et al.^{51,52} employed the local convective heat transfer coefficients after Timmerhaus and Flynn,⁶⁴

$$h_U = 0.023 c_P G \cdot \text{Re}^{-0.2} \text{Pr}^{-2/3} \left(1 + 3.5 \frac{d_{TUBE}}{d_{HELIx}} \right) \quad (9.25)$$

where d_{HELIx} is the coil diameter of the high pressure tube. Oren and Gutfinger^{48,49} used a correlation of Mori and Nakayama^{64,65,66} for h_U .

The Reynolds Number is,

$$\text{Re} = \frac{V d}{\nu} = \frac{G d_{TUBE}}{\mu} \quad (9.26)$$

The Fanning friction factor, f , as defined in Eq. 3.164, may be obtained by an empirical correlation for the straight tube⁶⁷ and corrected for the curvature of coiled tube⁶⁴ as in case of the Hampson type heat exchanger,

$$\frac{\Delta P(\text{curved})}{\Delta P(\text{straight})} = 1 + 0.0823 \cdot \left(1 + \frac{d_{TUBE}}{d_{HELIx}} \right) \left(\frac{d_{TUBE}}{d_{HELIx}} \right)^{0.53} \cdot \text{Re}^{0.25} \quad (9.27)$$

9.3.3 The Low Pressure Stream of Coolant

The downstream flow thermally interacts not only with the high-pressure tube (over the outer finned surface) but also with the inner mandrel and the cold finger of the dewar. Each interaction is associated with its own convective heat transfer coefficient, h_D . At the outer surface of the helical finned tube, as the flow is almost perpendicular to the tube, there is $h_{D, TUBE}$. At the inner surfaces of the dewar it is $h_{D, DEWAR}$, and at the outer surface of the mandrel there is $h_{D, MANDREL}$. Hence accordingly, the downstream energy equation becomes,

$$\begin{aligned} \frac{\partial(\rho h)}{\partial t} + \frac{\partial(G h)}{\partial X} = & -h_{D, TUBE} \frac{\eta_f \cdot p_{D, TUBE}}{A_D} (T_D - T_{TUBE}) - \\ & -h_{D, DEWAR} \frac{p_{D, DEWAR}}{A_D} (T_D - T_{DEWAR}) - \\ & -h_{D, MANDREL} \frac{p_{D, MANDREL}}{A_D} (T_D - T_{MANDREL}) \end{aligned} \quad (9.28)$$

where A_D is the cross section area of the downstream channel passageway. The terms $p_{D, DEWAR}$ and $p_{D, MANDREL}$ are, correspondingly, the wetted perimeters of the heat transfer interfaces between the downstream flow and the cold finger of the dewar or of the mandrel. Because the temperature profile of the finned surface will be influenced by its material and geometric configuration, the wetted perimeter of the tube, $p_{D, TUBE}$, must be multiplied by the fin efficiency, η_f . Fin efficiency terms are defined in standard heat transfer heat transfer textbooks (see for example, Nellis & Klein).⁶⁸

For all heat interactions of the downstream flow, Chou, Chien et al.^{51,52} adapted a single h_D correlation for all elements, the tube, the dewar, and the mandrel, which was proposed⁶⁴ for an helical tube,

$$h_D = 0.26 c_P G \text{Re}^{-0.4} \text{Pr}^{-2/3} \quad (9.29)$$

The Re for the down stream flow is based on the *hydraulic diameter* of the cross section defined by,

$$d^{HYD} = 4 A_D / p_D \quad (9.30)$$

where p_D is the wetted perimeter of the entire heat transfer interface in the down stream annular channel. Oren and Gutfinger^{48,49} used for h_D a correlation of Yelukhin et al.⁶⁹

The mass flux in the returning, down stream channel is, $G_D = \dot{n}/A_D$. Because of the larger specific volume (lower pressure and temperature) of the returning exhausted stream, A_D is typically larger than A_U . Since both channels share the same flow rate, $\dot{n}$, $G_U > G_D$. The Re is calculated separately for the up- and down-stream flows. At steady state, the Reynolds's number changes along the heat exchanger and is affected by the local pressure and temperature, but G

remains constant along each stream. During the cooldown phase, a change of G accompanies the change of $\dot{n}$.

The momentum equation for the down-stream flow is as given by Eq. 9.23 but with the down stream channel geometry characterized by its own value of d^{HYD} .

9.3.4 Other Components of the Heat Exchanger

The following elements interact thermally with one or both of the recuperating streams:

9.3.4.1 The High Pressure Tube

The energy balance for this component is given by,

$$\begin{aligned}
 (\rho \cdot c)_{TUBE} \frac{\partial T_{TUBE}}{\partial t} = & -h_U \frac{p_{U,TUBE}}{A_{TUBE}} (T_{TUBE} - T_U) - \\
 & -h_{D,TUBE} \frac{\eta_f \cdot p_{D,TUBE}}{A_{TUBE}} (T_{TUBE} - T_D) \\
 & + \lambda_{TUBE} \frac{\partial^2 T_{TUBE}}{\partial x^2}
 \end{aligned} \quad (9.31)$$

where the last term accounts for the heat conducted along the tube. $(\rho \cdot c)_{TUBE}$ is the average specific heat capacity of the tube and its fins and A_{TUBE} is the effective cross section of the metal finned tube itself (looking in the direction of tube's axis). $p_{U,TUBE}$ and $p_{D,TUBE}$ are the wetted perimeters of interface between the up- and down-stream flows with the high pressure tube.

9.3.4.2 The Cold Finger Encapsulation

The energy balance for the encapsulation, or dewar, must account for the convective cooling by the downstream flow and the warming due to conduction heat leak. Xue, Ng and Wang^{54,55} also included a term to account for heat exchange via thermal radiation, in which case the energy balance equation becomes,

$$\begin{aligned}
 (\rho \cdot c)_{DEWAR} \cdot \frac{\partial T_{DEWAR}}{\partial t} = & \lambda_{DEWAR} \cdot \frac{\partial^2 T_{DEWAR}}{\partial X^2} \\
 & - h_{D,DEWAR} \cdot \frac{p_{D,DEWAR}}{A_{DEWAR}} \cdot (T_{DEWAR} - T_D) \\
 & - h_R \cdot \frac{p_{D,DEWAR}}{A_{DEWAR}} \cdot (T_{DEWAR}^4 - T_{AMB}^4)
 \end{aligned} \quad (9.32)$$

where h_R is the coefficient for heat transfer by radiation. In the case of a non-evacuated encapsulation of the cryocooler, the energy balance should include a term to account for convective heat flux from the surroundings.

9.3.4.3 The Mandrel

The energy balance for the mandrel includes terms that account for convective cooling via the down-stream flow, and conduction heat flow through the mandrel itself,

$$\begin{aligned}
 (\rho \cdot c)_{MANDREL} \cdot \frac{\partial T_{MANDREL}}{\partial t} = & \lambda_{MANDREL} \cdot \frac{\partial^2 T_{MANDREL}}{\partial X^2} \\
 & - h_{D,MANDREL} \cdot \frac{p_{D,MANDREL}}{A_{MANDREL}} \cdot (T_{MANDREL} - T_D)
 \end{aligned} \quad (9.33)$$

9.4 Cryosurgical Devices

9.4.1 Introduction

Miniature Joule-Thomson cryocoolers provide a good fit with the practical size requirements of cryo-surgical probes used to treat human organs, and at the same time are capable of maintaining the necessary high heat removal rates. Cryosurgery is a medical technique to devitalize undesirable living tissues. Sufficiently lowering the temperature of biological tissues causes cryo-necrosis. Two other possible effects often utilized in ophthalmology are: inflammatory response for reattaching retinas and cryo-adhesion for extraction of cataracts or lens delivery.

The first experience with cryo-ablation of cancerous living tissues was reported in 1865 by the English physician James Armott.⁷⁰ Modern cryosurgery starts perhaps with the pioneering work of a New York surgeon named Cooper in about 1960. Fifteen years later he reviewed⁷¹ the “rapid expansion of applications of controlled freezing to problems in the various surgical specialties” like neurosurgery, ophthalmology, cardiology, otolaryngology, dermatology, gynecology, urology, immunology, and veterinary medicine. In addition, tumors are cryo-treated in the rectum, liver, breast, bones and other organs. Various entire books focus on cryobiology^{72,73} and cryosurgery^{74,75,76,77} and are recommended for further reading.

In terms of their engineering and operational aspects, cryosurgical devices are categorized on the basis whether they are initially cold or warm before the operation. A ‘cold’ machine is cooled with a low temperature agent, such as liquid nitrogen, and a large part of the machine begins at a low temperature. Joule-Thomson cryosurgical devices belong to the warm group.

9.4.2 Elements of Cryobiology

It is important both for the producers and the users of cryosurgical devices to be acquainted with the elements of

cryobiology. The lethal temperature for frozen tissue is generally accepted as about -40°C . In many cases, cryo ablation is obtained even at a higher temperature than the freezing point of water. However, below the lethal temperature, the chance of survival is very low. The surface of a frozen “ice-ball” is always at zero Centigrade, but an inner isothermal surface at -40°C defines the cryo-ablated zone in which most of the tissue remains below the lethal temperature.

The mechanism associated with the cryo destruction of cells and tissues is actually complex. The rate at which the temperature is reduced plays a role, but the rate of thawing is also relevant. Very low or very high cooling rates result in a more effective cryo-ablation. However, there are intermediate cooling rates for which the tissue may very well survive or even undergo a process of preservation. Indeed, *Cryo-preservation* is another branch of cryobiology. Cell cultures, tissues and vascular structures exhibit varying responses from exposure to cryogenic temperatures.

The most frequently applied method for enhancing the cryo-ablative effect uses a cyclic process of freezing and thawing. Additional cycles reduce the survival chance for non desirable tissues. At each successive freeze and thaw cycle, the frozen tissue reaches a lower temperature and the size of the frozen bulk increases, approaching a limit after six or seven cycles.⁷⁸

Cryo-promoters are materials that assist the ablative effect of freezing.^{79,80} *Cryo-protectives*⁸¹ have the opposite effect and are applied in order to avoid damage in adjacent healthy tissues.

A freezing process when accompanied by ultrasound promotes nucleation of ice inside the cells, thereby further enhancing the destructive process, as discussed elsewhere.^{82,83,84}

In the practice of cryosurgery, tissues and cells are often cooled deeply below the lethal temperature. For example, liquid nitrogen is frequently used as the cooling agent, imposing temperatures below 100 K. The primary purpose of this approach is to generate a large ice-ball, and to freeze and ablate a larger bulk, such as a large organ. A secondary objective of the approach would be to accelerate the freezing process.

9.4.3 “Cold” Cryosurgical Machines

This category of cryosurgical machines provide cooling through a phase transition of a cold agent. Most often the phase transition involves boiling, but melting and sublimation are also utilized.

9.4.3.1 Cooling by a Boiling Agent

The vast majority of devices employ a dewar filled with liquid nitrogen which occupies a substantial fraction of its

volume. Liquid nitrogen is transferred through isolated hoses to the tips of cryo-probes, where it boils at 77.3 K. However, the actual temperature of the probe is higher since vapor bubbles are created and flow along with the liquid, thereby attenuating the heat transfer coefficient because of the resulting film of vapor (Leidenfrost effect). In addition, local evaporation generates large vapor bubbles due to the extreme difference in the specific volume of the two phases. The resulting “vapor lock” impedes further flow of fresh liquid cryogen.

There were two strategies to improve this behavior. The first by Rubinsky, Baust et al.^{85,86,87} reduced the vapor pressure inside the dewar, thereby lowering the temperature below the normal boiling point, approaching 65 K. The second strategy attempted to eliminate vapor bubbles completely by operating “near critical nitrogen” as proposed by Littrop et al.^{88,89,90} In this approach the reservoir of liquid nitrogen is pressurized up to the critical pressure. The temperature increases significantly up to the critical value of about 126 K. However, near the critical state the liquid and vapor phase are of similar density. Therefore, there is essentially no driving potential for large vapor bubbles to form in the bulk liquid.

The designer and operator of liquid nitrogen based machines should be aware of the following necessary features:

1. Providing sufficient thermal isolation, since a significant fraction of the machine remains at a cryogenic temperature,
2. Due to boil-off, the system should be recharged no longer than a few hours before operation, in order to begin operation at full capacity,
3. The hoses that transport the liquid nitrogen from the dewar can freeze and lose their flexibility, in which case they must remain in the same position during the entire surgery,
4. Care must be taken to avoid any contact between the frozen hoses and the patient.

Levin^{91,92,93} built a cryosurgical machine which rapidly transmits a series of small drops of liquid nitrogen from a dewar to the probe. The frequency of the transmission is controllable.

9.4.3.2 Cooling by Melting Agent

Levin⁹⁴ built a device based on a solid cooling agent, such as ethylene glycol, that gradually melts during the treatment. It is recharged by an external refrigeration device which freezes the agent. The device transfers the cooling to the tissue through a rod shaped heat pipe that internally circulates a low boiling point coolant such as R13 or R23. This special model maintains a temperature of about 200 K for less than an hour.

9.4.3.3 Cooling by a Sublimating Agent

Cooling by a sublimating agent is based on “dry ice”, that is solid carbon dioxide.^{95,96,97} This approach was first utilized in 1964 by Bellows.

Finally, cold may be stored just through the thermal inertia of a solid tool. Here, a block or needles are precooled prior to operation, as proposed by Gao.⁹⁸

9.4.4 “Warm” Cryosurgical Machines

The second category of cryosurgical machines includes the warm type, which remain completely at room temperature until they are used. Then, cooling is produced upon demand, at the necessary rate and at the designated site, which is at the cryo-tip. These devices can be stored indefinitely and made available for immediate use. Only minor thermal isolation is required. The following effects are utilized on site in order to reduce the temperature:

9.4.4.1 The Joule-Thomson Effect

The Joule-Thomson effect, both in open and closed cycle devices. An open cycle system incorporates vessels containing a pure coolant, that is released to the atmosphere during operation. A closed cycle system compresses a mixed coolant on site (as discussed in Chap. 8), and therefore requires a compressor and an electric power supply. The next section discusses this group of cryosurgical devices.

9.4.4.2 The Peltier (or Thermoelectric) Effect

The Peltier (or thermoelectric) effect represents a promising approach for cryosurgical probes. These compact, solid-state devices require no moving parts, only an electrical power supply.^{99,100,101,102,103} The highly desirable function of active thawing is elegantly obtained simply by reversing the polarity of the power supply. However, the largest temperature reduction that can be obtained with a single stage is only about 60 K, a value that decreases as the applied heat load increases. Nevertheless, such low duty thermoelectric probes are suitable for ophthalmology and dermatology, for which only relatively small volumes must be frozen.

Rowland¹⁰⁴ used a double stage cascade thermoelectric device at the probe for reaching lower temperatures. In order to further reduce the temperature, this device was pre-cooled by a circulating coolant (a mixture of ethylene glycol and water) to maintain the heat sink temperature at about -5°C .

9.4.4.3 The Blow-Down of a Pressure Reservoir

The blow-down of a pressure reservoir reduces the temperature of both the exhausted stream and the remaining content in the vessel. This cooling mechanism is similar to Simon’s effect, introduced in paragraph 5.5.5.6. However, in the present application the container includes two phases of a

relatively high boiling point coolant. The typical coolants that are employed in this approach include freons such as R-12, R-22, R-500 and R-502 pressurized to about 0.5 MPa, at room temperature

About four decades ago, a small pencil-like disposable device based on this approach was introduced that provided cooling for a few minutes^{101,105,106,107,108,109,110,111,112,113,114} at a temperature of not less than -50°C . The gaseous phase occupies less than a quarter of the volume. Upon demand by a controlled release to atmosphere, part of the content is exhausted from the container. In general, the released stream cools the object. Sometimes the cold residual fluid was utilized¹¹² to cool an immersed metal rod that functioned as the tip of the probe. The lowest achievable temperature is the normal boiling point of the coolant.

9.4.4.4 Stirling and Pulse Tube Closed Cycle Cryocoolers

Stirling and Pulse Tube closed cycle cryocoolers are rarely harnessed for cryosurgical purposes, primarily because of the following reasons:

1. It is difficult to miniaturize the cold end of these coolers, or provide any concentrated cooling power. As a result, any localized cooling (attainable heat flux) would be insufficient to form an effective ice-ball.
2. Separation of the cold end (cryo-probe) from either the compressor or a valve configuration is problematic. The closest configuration to such an idea is the split Stirling cooler, but even then the necessary displacer/regenerator components leave the cold head with a rather bulky geometry.

Any attempts to harness this kind of closed cycle cryocooler are rarely reported. A pulse-tube version has been suggested,¹¹⁵ and even built and tested¹¹⁶ but operated only above -90°C .

9.4.5 Joule-Thomson Probes and Machines

Longworth starts his “Considerations in applying open cycle JT cryostats to cryosurgery”¹¹⁷ with the following remark:

Some of the same characteristics of open cycle Joule-Thomson cryostats that have made them attractive for military applications also make them attractive for cryosurgical applications. These include their small size, ability to cool fast, long term storage and potentially small and flexible connecting lines.

The Joule-Thomson cryoprobe employs a miniature cryocooler (or even only an expansion nozzle), installed at the tip of a cryosurgical probe. A cold bath of cryogen is generated beneath the thin metal surface which is exposed to the target living tissues. A typical collection of probes is shown in Fig. 9.8. The probe is connected through a flexible gas supply tube to the main machine that incorporates the

Fig. 9.8 A collection of Joule-Thomson cryo-probes. The cylindric one at the top serves for liver operation while the other have flat cold surface designated for dermatology, after reference¹²⁰



pressure source, that is, either pressure vessels for an open system, or a compressor for a closed cycle. It also includes elements of control, display, signal processing and monitoring.

9.4.5.1 Nitrogen and Argon Devices

Hood and Simon¹¹⁸ in 1965 appear to be the first to employ a recuperator at the tip of a cryo-probe for “progressive cooling” of nitrogen. However, their goal was not to provide enough cooling to liquefy nitrogen. The lowest temperature they mention is 122 K. Hood and Simon installed a valve on the probe to divert a selectable fraction of the return flow of the recuperative stream into the atmosphere. By using this approach, they suggested the possibility of controlling a temperature of about -100°C . Hood and Simon had already incorporated several of the functions that are still in use today: fast thawing, and the use of thermocouples and vacuum isolation in the probe.

Bulat, Dray and Sollami^{119,120} in 1972, intending to liquefy nitrogen, added a bimetal temperature sensitive flow regulator in order to minimize nitrogen consumption.

For the next couple of decades, it is hard to find any report on recuperative cryosurgical probes for the temperature range of about 100 K. The interest in recuperative probes was renewed shortly after 1990, but this time with argon gas instead of nitrogen.^{120,121,122,123,124,125,126} The cooling power required to create an ice ball on the order of a few centimeters diameter is quite substantial, typically ranging from 10 to 20 W. The superiority of argon (as summarized in Table 5.3) in terms of Δh_T and ΔT_h translates significantly into a larger and more rapidly generated ice ball. Furthermore, a longer run time is obtained for a given vessel size and pressure, since argon enables lower flow rates, lower cut-off pressure and lower compressibility. The 10 K elevation of argon’s boiling point (above nitrogen) is of minor significance, even when the performance is compared to that of a liquid nitrogen machine.¹²⁷

Various versions of these gas based machines have been employed: with a plurality of probes,^{129,128,129} with special probes for shaping the ice-ball,¹³⁰ with two stage probes^{131,132} or with wire mesh compact heat exchangers.¹²⁹

A significant concern related to this class of devices is the relatively high pressure of operation, in the range of 20 MPa to 30 MPa or even higher, and the logistics of replacing the pressure vessels.

9.4.5.2 Carbon Dioxide and Nitrous Oxide Machines

In situations when a smaller cooling power is required, or a higher cooling temperature can be tolerated, the use of carbon dioxide (sublimating at 195 K) or nitrous oxide (boiling at 184 K) may be considered. Since the values of T_{CD} for these gases are above room temperature it is unnecessary to include any recuperators in such coolers, and they are therefore good candidates for very compact and precise probes. Hence, they are attractive for very accurate procedures such as in ophthalmology or neurology and invasive catheters. At times a recuperator is added^{133,134} to increase the liquefaction yield, or a precooler is added^{135,136} in order to further reduce the size of a catheter. In comparison with the recuperative types, these probes are less cold but their charge pressure, below 5 MPa, is significantly lower than that of the recuperative type. Widyaparaga et al.^{137,138} built a miniature “flexible wire-type” JT probe of 0.6 mm diameter, fed by carbon dioxide at 1.5 MPa.

The South African ophthalmologist Amoils focused on the extraction of cataracts.^{139,140} In the same year as Hood and Simon, but about a month later, he filed a patent in South Africa for a non recuperative cryo-probe using carbon dioxide.^{141,142,143} Later on, many non-recuperative cryosurgical devices were developed^{144,145,146,147,148,149,150,151,152,153,154,155,156} with carbon dioxide and with nitrous oxide.

9.4.5.3 Closed Cycle, Mixed Refrigerant Coolers

Closed cycle, mixed refrigerant coolers (of Chap. 8) have also been utilized for cryosurgical applications. Their cooling capability combines the compactness of Joule-Thomson cryo-probes on the one hand, with a low operating pressure (below 3 MPa) on the other, while still avoiding the supply and replacement of pressure bottles. Besides the operational advantages, the reduced pressure is a valuable safety feature for applications like the heart arrhythmia catheter developed by Radebaugh, Dobak and Marquardt et al.^{157,158,159,160} and Gong, Luo, et al.¹⁶¹. The trans-vascular cardiac catheter is 3 mm in diameter and about 1 m long with a bending radius of about 3 cm. A design method for mixed coolant Joule-Thomson cryosurgical probe is discussed elsewhere¹⁶². However, as evidenced by the following list of activities, the primary effort for this application focused on miniaturizing the tip of the catheter:

1. Optimizing the mixture composition for minimum recuperative heat transfer area.^{163,164,165,166,167}
2. Proposing to add a precooling stage,^{135,169,168,169} to further reduce the size of final stage at the tip of the probe, (see Sect. 8.3.4.4), and,
3. Developing a compact heat exchanger based on diffusion bonded perforated plates¹⁷⁰ or stacked together by MEMS techniques.^{171,172,173,174}

While the above references all utilized the mixed gas closed cycle Linde-Hampson cycle, Little et al.¹⁷⁵ employed a Kleemenko type cycle with a single phase separator and a double expansion. He also proposed building a flexible counter flow heat exchanger connecting the compression unit with the probe.¹⁷⁶ A similar attempt with the Kleemenko cycle is described by Naer and Rozhentsev.¹⁷⁷ One may find information about two commercial cryo-medical products elsewhere.^{178,179}

The designer of any gas machine has to be aware of the potential leakage that can occur when probes are connected and disconnected either for replacement or for sterilization. This is especially true since closed systems are initially charged with a finite, specified and necessary amount of gas. A simple alternative, to avoid disconnecting the probes for sterilization, is by using a disposable sheath.¹⁸⁰ Admittedly, this option is more difficult for the case of a multi-probe machine with a variable number of simultaneously operating probes.

9.4.6 Miscellaneous

The following additional features are occasionally utilized in the practice of cryosurgical probes and systems:

9.4.6.1 Open End Probes

Open end probes that release a two-phase jet of cryogen^{181,182,183} are desirable for some particular applications. These are used with liquid nitrogen machines and possibly also with Joule-Thomson probes,¹⁸⁴ but in that case a precooling stage is necessary. The direct impingement of a cryogen on the tissue is an alternative procedure for ablating tissues. It can be more effective than a stiff metal probe, especially in cases of a cavity such as in a bone cavity¹⁸⁵ or an irregular surface.

9.4.6.2 Multi-probe Devices

Multi-probe devices are used to meet the freezing demands of larger bulk tissue or complicated forms that are hard to cover with a single ice-ball. Typically, in such arrangements, each probe has its own temperature control and display.^{87,128,129,186} Another use involves high resolution observation of a desirable contour^{187,188,189} in order to avoid freezing adjacent healthy tissue especially in potentially high risk cases.

9.4.6.3 Active Warm-up

Active warm-up capability is a highly desirable feature that is actually possessed by most cryosurgical devices. The diversity of warm-up techniques is impressive. Warming is a complementary means for controlling the extent of the freezing process. Furthermore, pure heating probes are occasionally incorporated to shape the form of an ice ball generated by multiple cryoprobes, or to protect a nearby blood vessel,¹⁹⁰ or a urethra as in the case of cryo-ablation of prostate tumors.¹⁹¹

Following the freezing period, a warming process enables the cryoprobe to detach from the adhering tissues thereby releasing the probe. A stronger warming capability can also be used to accelerate the thawing of an ice ball. In many cases, the warming process is accomplished with the use of a minute electrical resistor at the tip of the probe, as was first employed in the recuperative probe of Hood and Simon.¹²¹ Amoils¹⁴⁵ also used the same approach as did many others subsequently.^{136,151,192,193} Longsworth¹²⁹ warmed up the probe by sharply reducing the supply pressure. In this way the Joule-Thomson effect practically vanishes and room temperature coolant floods the tip and warms it up. Morinets et. al.,¹⁹⁴ utilized the inverse Joule-Thomson effect for warming up the probe. Maytal,¹²⁴ Mikus and Crockett^{132,133,195} enhanced the basic warming process with the use of a flow of helium through the recuperator. Because of its uniqueness, this approach is discussed separately in the next section.

Systems using CO₂ and N₂O begin warming at the same moment when the exhaust is halted, a phenomenon known as

“exhaust blocking”.^{142,147,148,150} However, this process exposes the entire probe, including its metal shell to the higher pressure of the source, which is typically about 5 MPa.

Warming may also be obtained with the use of a room temperature stream of the coolant, a technique referred to as “reverse flow”^{136,152,196,197} or by forcing a stream of warmed gas through the circuit.^{198,199,200,201,202}

9.4.6.4 MRI Compatibility

MRI (Magnetic Resonance Imaging) assisted surgery is regarded as a very promising therapeutic technique. Rubinsky et al.^{203,204,205} reviewed the benefits of conducting cryosurgical procedures with the supporting involvement of MRI. The combined processes can be beneficial at any phase of the treatment: estimating the shape and size of the tumor, tracking the position of the probes, indicating the temperature distribution around the ice-ball and more.^{206,207,208,209,210} However, locating a cryosurgical machine in such a high magnetic field environment (1.5–2 T for open magnetic loop and 0.5 T for the closed magnetic loop) is quite challenging.

The gas based machine can be adapted to operate in the high magnetic field of an MRI.^{211,212} The main frame is located outside the high field region and only the probes are mounted at the surgical site. In this case high pressure tubing is used to connect the probes with the main frame machine. The probe and tools materials have to be compatible with high magnetic fields in order to avoid artifacts in the MRI image.

9.4.6.5 Krypton

Krypton is a promising coolant for cryo-probes. Its liquefaction temperature is sufficiently low (120 K) for cryosurgical applications. For the 80 K range of cryocooling, krypton may serve as the ultimate pre-coolant,²¹³ as discussed in Sect. 5.5.5.3. Longworth¹¹⁹ demonstrated by calculations and experiments the superiority of krypton for operating a cryo-probe even in comparison with argon, the boiling point of which is lower by about 30 K. He used both a finned tube and a matrix form for the recuperators of the probes. Krypton enables a higher cooling capacity and a more compact heat exchanger or smaller probe.

Since krypton may reach liquefaction even without any heat exchanger, Maytal²¹⁴ proposed building a thin (below 1.5 mm diameter) and flexible cryoprobe operated with krypton.

9.4.6.6 Accessories

A thermocouple is often installed inside the cold end of each probe to indicate the temperature that is produced. In fact,

this practice was even used in the early work of Hood and Simon.¹⁰⁶ Needles with a thermocouple installed at their tip are often inserted into the treated tissue, prior to freezing, in order to monitor the temperature distribution around the probe.

Heat pipes are also frequently employed to efficiently absorb and transfer heat from the treated tissue to the cold source.^{96,102,215,216}

9.5 The Warming Joule-Thomson “Cryocooler”

Somewhat ironically, in the last section of this book, dedicated to Joule-Thomson cryocooling, we present the warming that can be produced by the Joule-Thomson effect.

9.5.1 Recuperative Heating

9.5.1.1 The Positive Feedback of a Temperature Increase

Only the quantum gases, helium, hydrogen and neon have their Joule-Thomson inversion temperature below room temperature (Fig. 2.7). When one of these gases, in a pressurized Linde-Hampson cryocooler, expands – just at the very first expansion, the isenthalpic temperature change (ΔT_h) will be positive, that is the temperature will increase. Successive parcels of the incoming high-pressure stream will be heated, thereby further increasing the temperature. A positive feedback mechanism occurs, magnifying the elementary temperature increase at the nozzle, which is the same process of interchanging as introduced in Sect. 1.1. In a similar way as in the cool down process, ΔT_h may be interpreted as the driving potential for further heating.

Figure 9.9 shows the dependence of ΔT_h on temperature and pressure for helium. The adiabatic integral warming effect, ΔT_h of helium at 30 MPa and room temperature, is about half of the cooling effect, $|\Delta T_h|$, of nitrogen. However, the warming effect of helium increases with pressure at a rate of about 0.5 K per each MPa, hence the ΔT_h at 70 MPa reaches a similar value as the cooling effect of nitrogen at 30 MPa. One may observe that the positive values of ΔT_h at constant pressure display a maximum as a function of temperature as shown in Fig. 9.9, similar to the maximum negative values of ΔT_h for Joule-Thomson cooling. This feature is a reflection of the fact that there is a maximum integral inversion pressure as discussed in Sect. 2.6.2 and shown in Fig. 2.19.

An interesting report is given of heating in a high pressure hydrogen storage tank by a low pressure stream that warmed as a result of Joule-Thomson expansion.^{217,218} The effect was utilized to transfer heat to the gas remaining in the tank.

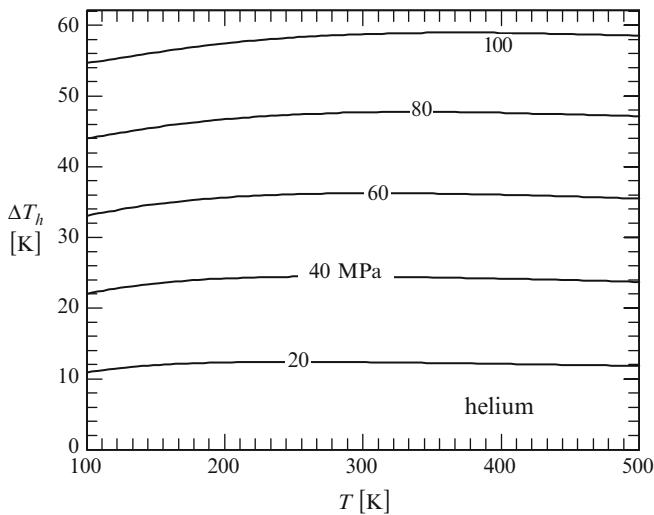


Fig. 9.9 The integral warming effect of helium at different temperatures (100–500 K) and pressure ($P = 20$ –100 MPa) as a result of an expansion at constant enthalpy down to 0.1 MPa

9.5.1.2 The Thermodynamic Relations

The thermodynamic relations as introduced in Sects. 3.2.4 and 4.5.2 are based on an energy balance and are therefore generally valid and applicable for the warming process as well,

$$\dot{Q} = \dot{n} \cdot \Delta h_T \quad (9.34)$$

In this case, $\dot{Q}$ is the rate that the “cooler” warms (or releases energy to) the payload. It means that the value of $\dot{Q}$ is negative and it is consistent with the fact that warming is obtained if $\Delta h_T < 0$, such as in case of helium at room temperature. For example, at 50 MPa and 300 K, the specific warming potential is $\dot{Q}/\dot{n} = \Delta h_T = -632$ J/mole, about half the specific cooling potential of nitrogen. At 100 K it is $\Delta h_T = -573$ J/mole.

Similarly, it is true that,

$$\Delta T_h = -\Delta h_T / c_{P0} \quad (9.35)$$

so that $\Delta T_h > 0$ if $\Delta h_T < 0$. For instance, helium at 300 K and 60 MPa, warms up to 336.8 K ($\Delta T_h = 36.8$ K) under a single isenthalpic expansion down to 0.1 MPa. Even at a much lower initial temperature such as 100 K, the ΔT_h is only slightly decreased to the value of $\Delta T_h = 33.5$ K.

9.5.1.3 Transient Behavior

The self-perpetuating warming process is analogous to the positive feedback mechanism characteristic of the more common Joule-Thomson cool down. However, the similarity is only partial; as the two processes proceed, the cool down intensifies while the warm up attenuates. The flow rate is inversely proportional to the nozzles’ inlet temperature at any given pressure. Therefore, the recuperative warming process

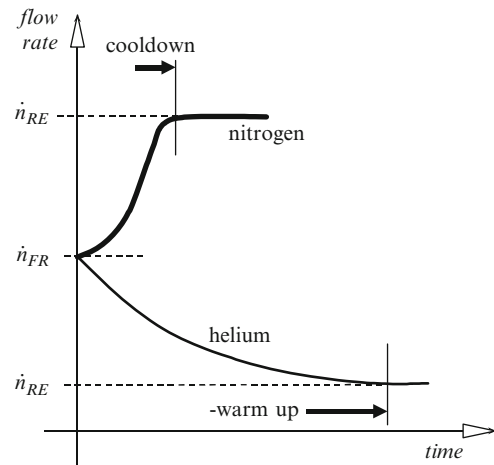


Fig. 9.10 The opposite trends of the transient recuperative flow rates, $\dot{n}_{RE}$, during cooldown and warm up, starting at the same free flow, $\dot{n}_{FR}$

decreases along with the flow rate during the transient temperature increase, as qualitatively displayed in Fig. 9.10. The warming behavior contrasts noticeably with the increasing flow rate exhibited during cooldown, which for example approximately doubles for nitrogen or argon. The warming transient includes an inherent element of negative feedback, or stabilizing effect, that opposes further warming. Therefore, the instantaneous temperature approaches a steady state value as the rate of warming gradually vanishes.

The positive feedback mechanism associated with the warming process develops slower than it does during the cooldown process due to two other factors. First, as the temperature rises, the value of ΔT_h remains fairly constant and may even decrease slightly. In the case of cooling, ΔT_h significantly increases as the temperature decreases. And second, the specific heat, c_P , of the “cooler” and its encapsulation increases at higher temperatures, during the warm up, but decreases during the cooldown.

The rate of warming increases at higher pressures due to the increase of both the flow rate and the values of ΔT_h as demonstrated by Maytal et al.²¹⁹

It is perhaps interesting to note that helium, the gas with the lowest boiling point and lowest critical temperature, has the highest warming potential as characterized by the integral Joule-Thomson effect. Figure 9.11 compares the ΔT_h values of the three quantum gases at 30 MPa between 100 K and 350 K. Indeed, hydrogen and neon exhibit lower values than helium.

9.5.1.4 The Final Steady State Temperature

The final steady state temperature is another distinction between cooling and warming modes. At the end of a cool-down the refrigerant is liquefied.

The attainable temperature is well known a priori and unaffected by the ambient temperature or heat load.

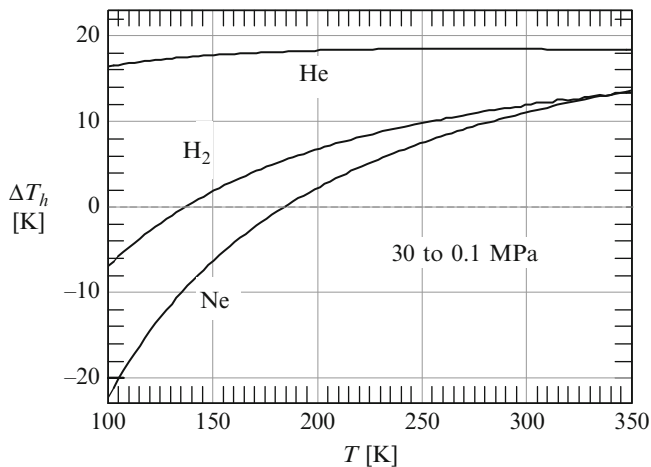


Fig. 9.11 The warming up effects of helium, hydrogen and neon as function of temperature; helium is superior as it may warm up when hydrogen and neon still do cooldown

However, the warming up process does not have any limiting upper limit temperature. In the ideal case of no heat leaks, the temperature rise is theoretically unlimited. Practically, the temperature rise stops when it is counterbalanced by the effect of cooling to the surroundings, or when,

$$\dot{Q} = \dot{n} \cdot \Delta h_T = U \cdot A \cdot (T_{AMB} - T) < 0 \quad (9.36)$$

Here T is the steady state temperature obtained, and U is the total heat transfer coefficient from the “cryocooler” to the surroundings.

9.5.2 Thawing a Cryosurgical Probe

Joule-Thomson cryosurgical probes, as well as other types of probes, generally include the capability to warm up, as discussed in Sect. 9.4.6.3. Maytal^{124,120} connected the cryocooler of a Joule-Thomson probe to two pressure sources in parallel, one of argon for cryocooling and the other of helium, for heating up, as illustrated in Fig. 9.12. Two fast electromagnetic on-off valves initiate the flow of each of the gases and two check valves protect the gases from mixing. At the end of the freezing phase, the argon flow is halted and the helium flow takes over, feeding the same counter flow heat exchanger and expansion nozzle at the tip of the probe. Using this approach, the probe is not only returned to room temperature, but hypothermic temperatures may also be achieved.

Morinets et al.¹⁹⁷ proposed a similar arrangement of cooling and warming gases. However, he used a non-recuperative probe. Consequently, the single expansion at

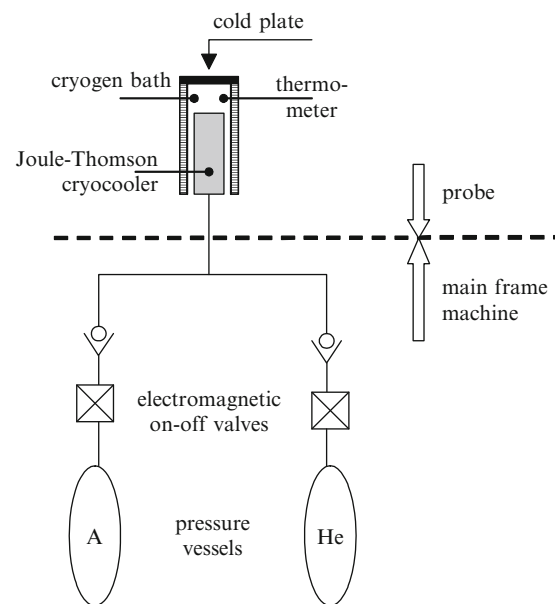


Fig. 9.12 Thawing a cryoprobe by the inverse Joule-Thomson effect; a miniature cryocooler is installed at the tip of the probe. It is pressurized by argon for cooling and then by helium for thawing. Successive repetition of the procedure generates cryo-cycles, after Maytal^{124,120}

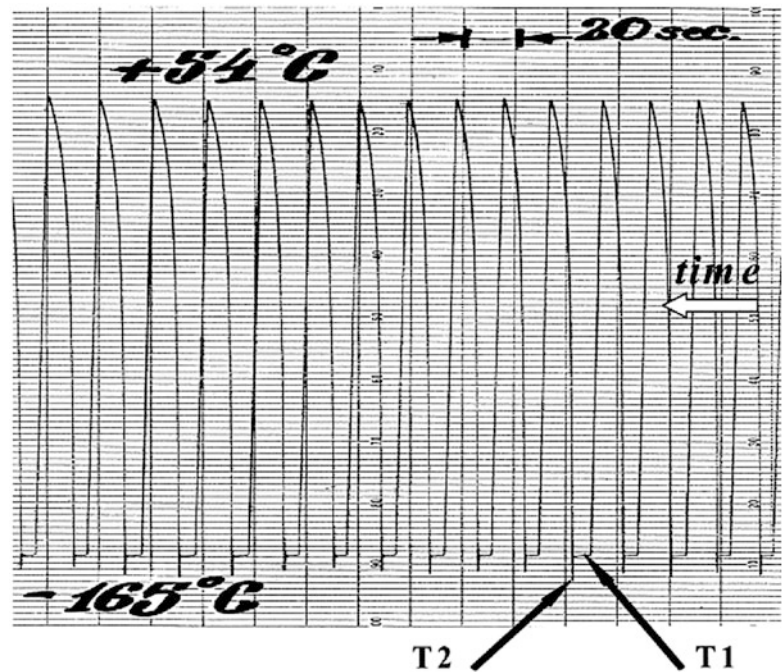
room temperature, from 30 MPa to 0.1 MPa has a fairly limited warming effect, with a temperature rise of only 18 K for helium, and 13 K for hydrogen. Crockett²²⁰ suggested a recuperative cryo-probe with two heat exchangers wound in parallel. Each heat exchanger has its own inlet port and is separately connected to the two pressure sources. Both streams exclusively expand through two nozzles, one for cooling and the other for warming.

To the best of the authors’ knowledge these are the sole practical and commercial machines taking advantage of the negative (warming) Joule-Thomson effect.

9.5.3 Cryo-Cycling

A dual gas supply of argon and helium enables rapid and significantly large temperature cycling (cryo-cycles) as demonstrated in Fig. 9.13. A 5.2 mm cryocooler within a non-evacuated encapsulation was successively pressurized by 30 MPa of argon and then helium. The recuperative warming portion of the cycle is slightly delayed when helium begins flowing through the cryocooler, which occurs at a temperature of about 100 K. Since the temperature is significantly above the inversion temperature of helium, the warming effect is not interrupted, and indeed the temperature cycles between 105 K and 327 K within a period of about 20 s. One should observe that the upper limit of the cycle is above room temperature.

Fig. 9.13 Rapid cryo-cycling with a 5.2 mm flat surface probe, loaded by touching a tissue of a potato; Automatic cryocycling with 20 s period; 10 s of cooling followed by 10 s of heating. Temperature measured in the tissue adjacent to probe, after Maytal¹²⁰



At the end of the cooldown phase resulting from the flow of argon, a temperature T1 (Fig. 9.13) is achieved and maintained until the warming phase takes over. However, at the moment of switching from argon to helium, for a short while, the temperature drops to a level of T2. The behavior results from a reduction of the back pressure, since at the moment the gases are switched, the instantaneous flow rate is zero.

Helium is the superior candidate for cryo-cycling, compared with hydrogen or neon, as one may deduce from Fig. 9.10. The integral inversion temperatures associated with expansion from 30 to 0.1 MPa are about 140 K for hydrogen and 175 K for neon. Below these temperatures hydrogen and neon cool down in response to an isenthalpic expansion, so the warming process is delayed. However, warming eventually takes over, since the incoming stream contributes a warming effect because it enters at room temperature, which is significantly warmer. Obviously, the safety concern of hydrogen play a role as well, and would not likely be approved for surgical applications.

Cryo-cycling has also been proposed^{221,222,223} to enhance the cryo-ablative process.

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Appendix A1: The British Patent No. 2064, of Dr. Charles Williams Siemens, 1857: “Refrigeration Apparatus”



A.D. 1857 N° 2064.

Refrigerating Apparatus.

(This Invention received Provisional Protection, but notice to proceed with the application for Letters Patent was not given within the time prescribed by the Act.)

PROVISIONAL SPECIFICATION left by Charles William Siemens, at the Office of the Commissioners of Patents, with his Petition, on the 29th July 1857.

I, CHARLES WILLIAM SIEMENS, of John Street, Adelphi, in the County of Middlesex, do hereby declare the nature of the said Invention for "IMPROVEMENTS IN REFRIGERATING AND PRODUCING ICE, AND IN APPARATUS OR MACHINERY FOR THAT PURPOSE," to be as follows:—

The Invention relates to freezing and refrigerating by the expansion of air or elastic fluids. The air is first compressed by a cylinder or pump of any suitable construction, by which its temperature is raised, and it is cooled while in the compressed state, and it is then allowed to expand in a cylinder or engine of any suitable construction by which its temperature is lowered. The air thus cooled is brought in contact with the articles to be cooled or frozen, and is then conducted through an interchanger or apparatus by which it is made to cool the compressed air, which enters the interchanger in the opposite direction. The pump for compressing the air is worked by a steam engine or other power, and the engine in which the air expands is made to assist in working the pump. I prefer to connect the steam engine and the air engine and the pump to the same rod or moving part, so as to transmit the power directly from one to the other. If the atmosphere be at 70° Fahrenheit, and the air be heated 50° by the compression, and be then cooled to 70°, and allowed to expand, its temperature will be lowered to about 20°, more or less. This cold air will then enter the interchanger and cool the

Siemens' Improved Apparatus for Refrigerating and Producing Ice.

next portion of compressed air, say to 30°, and will itself be heated say to about 110°, and will pass away into the atmosphere. The compressed air at 30° being allowed to expand, will be cooled to about 20° below zero, and in passing through the interchanger will cool the next portion of compressed air, say to 10° below zero. This on expanding will be reduced to about 60° below 5 zero, and so on. These temperatures are mentioned not as the absolute temperatures, but to shew that the principle of the Invention is adapted to produce an accumulated effect or an indefinite reduction of temperature.

I construct machinery or apparatus for carrying out the Invention by immersing the engine in brine or a solution of chloride of calcium or other 10 suitable fluid which will not congeal at the temperature which it is required to produce. I cause the air, after expanding in the engine, to pass into a series of tubes immersed in the brine or other fluid. These tubes are perforated on their under sides, so that the air passes through the brine in a multitude of streams or bubbles and thus cools the brine; or the air is 15 admitted into a casing surrounding the brine vessel, and allowed to pass through perforations in the sides. The vessel which contains the brine is covered, and the cold air is conveyed from it through the interchanger, which consists of a series of tubes inclosed in a case. The cold air is conveyed through the tubes, while the compressed air passes in the contrary direction 20 through the case or vice versa. The compressed air thus enters warm and passes out cold, while the expanded air enters cold and passes out warm. The water or other liquids or articles to be cooled or frozen, are immersed in the aforesaid brine or other fluid; thus the heat which is continually abstracted from the articles to be cooled or frozen, is carried off by the heated 25 air. This current of heated air may be allowed to escape into the atmosphere, or it may be employed for any purpose for which such a current of heated air may be required. The construction of the interchanger may be varied, or two or more regenerators or respirators filled with wire gauze or metallic plates, or other suitable materials may be employed alternately for the compressed 30 and expanded air. The water or other liquid to be cooled or frozen may also be conducted through tubes in the interchanger, or exposed to the contact of the cold air, so as to cool it down before it enters the vessels in which it is to be frozen. When it is required to supply cold air to a cellar or chamber of any sort, the cold expanded air is conducted into such chamber, and on 35 leaving the chamber it passes through the interchanger as before.

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Appendix A2: Equations of State

A2.1. Van der Waals Equation of State

The Van der Waals equation of state is mainly applicable for obtaining closed form solutions, and for describing the basic behavior, trends and phenomena, if one is willing to tolerate less accurate quantitative results.

$$P = \frac{R \cdot T}{v - b} - \frac{a}{v^2} \quad (\text{A2.1})$$

$$Z = 1 + \frac{b \cdot \rho}{1 - b \cdot \rho} - \frac{a \cdot \rho}{R \cdot T} \quad (\text{A2.2})$$

$$\Pi = \frac{8\Theta}{3\Phi - 1} - \frac{3}{\Phi^2} \quad (\text{A2.3})$$

$$a = \frac{27}{64} \frac{R^2 T_C^2}{P_C} = \frac{9}{8} R T_C v_C = 3 P_C v_C^2 \quad (\text{A2.4})$$

$$b = \frac{1}{8} \cdot \frac{R \cdot T_C}{P_C} = \frac{v_C}{3} \quad (\text{A2.5})$$

$$P_C = \frac{a}{27b^2} \quad (\text{A2.6})$$

$$T_C = \frac{8a}{27bR} \quad (\text{A2.7})$$

$$Z_C = \frac{3}{8} \quad (\text{A2.8})$$

$$Z^3 - \left(\frac{\Pi}{8\Theta} + 1 \right) Z^2 + \frac{27\Pi}{64\Theta^2} Z - \frac{27\Pi^2}{64\Theta^3} = 0 \quad (\text{A2.9})$$

A2.2. Peng-Robinson Equation of State

The Peng-Robinson equation of state provides a cubic EOS which may be considered as an advanced enhancement of the Van der Waals EOS. It also enables closed form expressions with much higher quantitative accuracy.

$$P = \frac{R \cdot T}{v - b} - \frac{a(T)}{v \cdot (v + b) + b \cdot (v - b)} \quad (\text{A2.10})$$

$$a(T) = 0.45724 \frac{R^2 \cdot T_C^2}{P_C} \cdot \alpha(T) \quad (\text{A2.11})$$

$$b = 0.07780 \frac{R T_C}{P_C} \quad (\text{A2.12})$$

$$\alpha^{1/2} = 1 + m \cdot \left[1 - (T/T_C)^{1/2} \right] \quad (\text{A2.13})$$

Formally, the acentricity factor of a gas is defined on the basis of the saturation curve as proposed by Pitzer,

$$\omega = -1 - \log_{10} \frac{P_S(T/T_C = 0.7)}{P_C} \quad (\text{A2.14})$$

where P_S is the saturation pressure. On the logarithmic scale, it describes the extent that the saturation pressure deviates from that of the non-quantum noble gases argon, krypton and xenon, for which it vanishes, $\omega = 0$. The acentric factor is implemented into the equation through,

$$m = 0.37464 + 1.5422 \omega - 0.26992 \omega^2 \quad (\text{A2.15})$$

$$A = \frac{a \cdot P}{R^2 \cdot T^2} \quad (\text{A2.16})$$

$$B = \frac{b P}{R T} \quad (\text{A2.17})$$

A proper rearrangement leads to the expressions,

$$T \cdot \frac{da(T)}{dT} = -m \cdot a(T) \cdot \left(\frac{T}{\alpha \cdot T_C} \right)^{1/2} \quad (\text{A2.18})$$

$$Z^3 + C_2 \cdot Z^2 + C_1 \cdot Z + C_0 = 0 \quad (\text{A2.19})$$

$$C_1 = A - 3B^2 - 2B \quad (\text{A2.20})$$

$$C_2 = B - 1 \quad (\text{A2.21})$$

$$C_3 = B^3 + B^2 - A \cdot B \quad (\text{A2.22})$$

$$Z_C = 0.307 \quad (\text{A2.23})$$

The van der Waals equation of state expanded by density is,

$$B = b - \frac{a}{RT} \quad (\text{A2.30})$$

$$C = b^2 \quad (\text{A2.31})$$

$$D = b^3 \quad (\text{A2.32})$$

A2.3. Virial Equation of State (Proposed by K. Onnes in 1912)

There are two versions of the Virial EOS, one expanded by pressure and the other expanded by density.

$$\begin{aligned} Z = \frac{P \cdot v}{R \cdot T} &= 1 + B \cdot \rho + C \cdot \rho^2 + D \cdot \rho^3 + \dots \equiv \\ &\equiv 1 + B' \cdot P + C' \cdot P^2 + D' \cdot P^3 \dots \end{aligned} \quad (\text{A2.24})$$

The coefficients of these two, $B(T)$, $C(T)$, $D(T)$ etc. are solely temperature dependent, as are $B'(T)$, $C'(T)$, $D'(T)$ etc., which are related to each other by the equations,

$$B' = \frac{B}{R \cdot T} \quad (\text{A2.25})$$

$$C' = \frac{(C - B^2)}{(R \cdot T)^2} \quad (\text{A2.26})$$

$$D' = \frac{D + 2B^3 - 3B \cdot C}{(R \cdot T)^3} \quad (\text{A2.27})$$

$$\frac{dB'}{dT} = \frac{1}{R \cdot T} \left(\frac{dB}{dT} - \frac{B}{T} \right) \quad (\text{A2.28})$$

$$\frac{dC'}{dT} = \frac{1}{(R \cdot T)^2} \left(\frac{B^2 - C}{T} + \frac{dC}{dT} - 2B \frac{dB}{dT} \right) \quad (\text{A2.29})$$

A2.4. Truncated Virial Equation of State

$$Z = \frac{P \cdot v}{R \cdot T} = 1 + B' P = 1 + \frac{B P}{R T} \quad (\text{A2.33})$$

A2.5. Interrelating the Third Parameter of the Principle of Corresponding States

$$Z_C = 0.29056 - 0.08775 \cdot \omega \quad (\text{A2.34})$$

The quantity Z_C ranges for most of the substances from 0.22 to 0.30. A more complex molecule with a larger positive value of ω has a smaller value of Z_C .

Riedel proposed another acentricity factor based on the saturation line, but at the critical state

$$\alpha_C = \frac{d(\log P_S)}{d(\log T_S)} \quad @ (T_C, P_C) \quad (\text{A2.35})$$

where P_S is the saturation pressure corresponding to the saturation temperature, T_S .

Riedel's acentricity factor is related to Pitzer's acentricity factor by,

$$\alpha_C = 5.811 + 4.919\omega \quad (\text{A2.36})$$

Appendix A3: Parameters of Gases

		M [g/mol]	T_C [K]	P_C [MPa]	T_{BOIL} [K]	T_M [K]	ω	Z_C
Helium-3	^3He	3.017	3.3	0.1188	3.2		−0.473	0.302
Helium-4	^4He	4.003	5.2	0.227	4.2		−0.365	0.302
Hydrogen	H_2	2.016	33.0	1.30	20.4	13.8	−0.214	0.306
Deuterium	D_2	4.032	38.3	1.66	23.6	18.6	−0.176	0.312
Neon	Ne	20.18	44.4	2.68	27.1	24.6	−0.029	0.311
Nitrogen	N_2	28.01	126.2	3.34	77.3	63.3	0.037	0.290
Carbon monoxide	CO	28.01	132.8	3.45	81.7	68.1	0.051	0.295
Fluorine	F_2	37.997	144.4	5.22	85.0	53.5	0.054	0.288
Argon	Ar	39.948	150.8	4.87	87.3	83.8	0.001	0.291
Oxygen	O_2	32.00	154.6	5.08	90.2	54.4	0.025	0.288
Methane	CH_4	16.04	190.6	4.60	111.7	90.7	0.011	0.286
Krypton	Kr	83.30	209.4	5.51	119.9	115.9	−0.002	0.288
R-14	CF_4	88.005	227.7	3.74	145.2	89.3	0.178	0.276
Xenon	Xe	131.30	289.7	5.82	165.1	161.3	0.004	0.287
Ethylene	C_2H_4	28.05	282.7	5.11	169.4	104.0	0.087	0.280
Ethane	C_2H_6	30.07	305.5	4.89	184.5	89.9	0.099	0.288
Nitrous oxide	N_2O	44.013	309.7	7.22	184.2	182.3	0.165	0.274
Acetylene	C_2H_2	26.04	308.0	6.20	189.6	192.34	0.184	0.270
R-13	CClF_3	104.46	302.1	3.82	192.0	91.6	0.170	0.278
Carbon dioxide	CO_2	44.02	304.2	7.37		215.7	0.281	0.274
R-13B1	CF_3Br	148.91	340.2	3.96	215.4	105.2	0.171	0.275
Propylene	C_3H_6	42.079	364.9	4.62	225.5	87.9	0.141	
Propane	C_3H_8	44.09	369.8	4.25	231.1	85.5	0.152	0.281
Isobutane	iC_4H_{10}	58.124	408.2	3.64	261.4	113.6	0.185	0.278
1-butene	$\text{1-C}_4\text{H}_8$	56.11	419.6	4.02	266.9	87.8	0.191	
n-butane	nC_4H_{10}	58.124	425.2	3.80	272.7	134.8	0.201	

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