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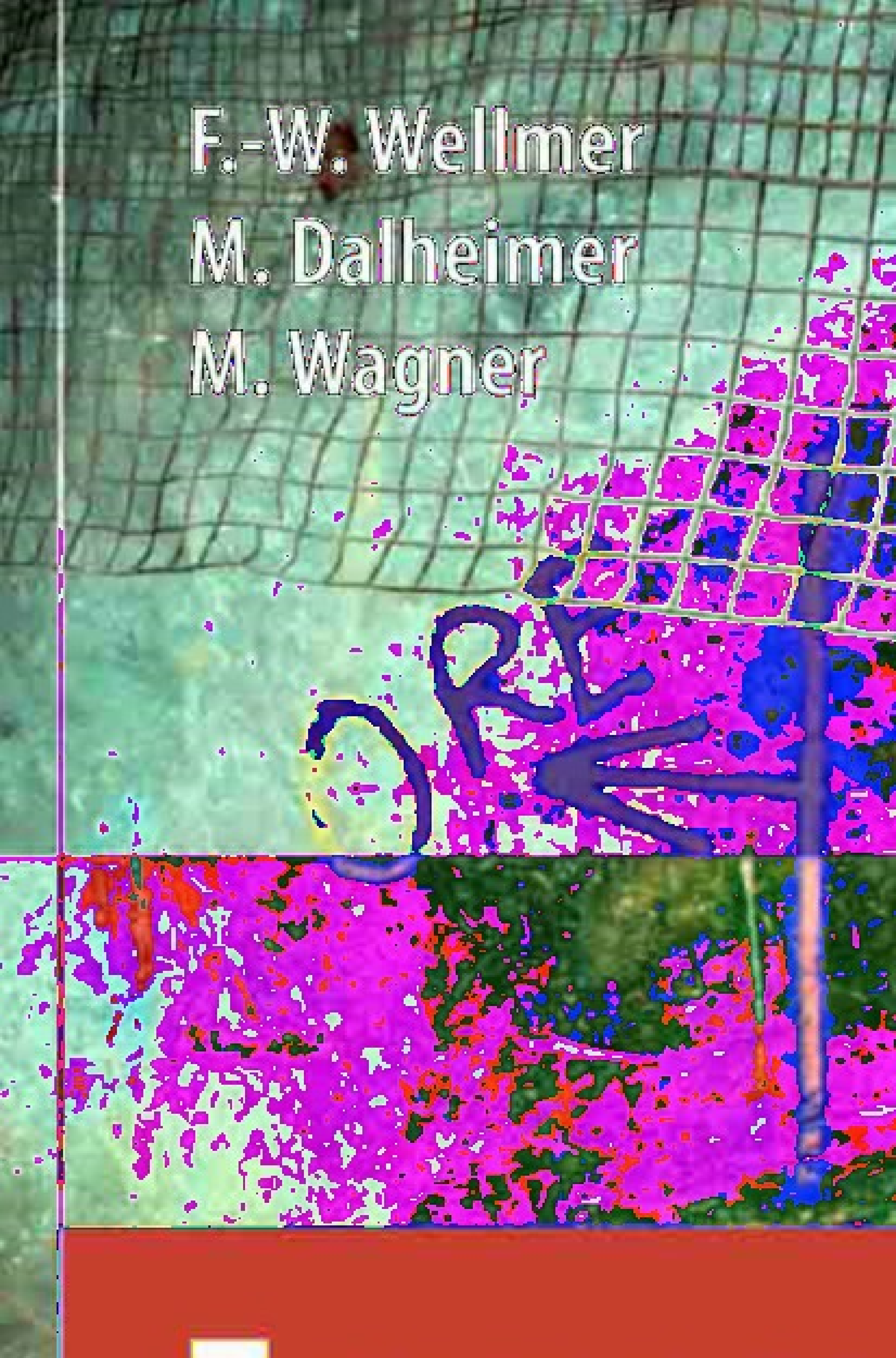

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Economic Evaluations in Exploration

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Economic Evaluations in Exploration

Second Edition

With 68 Figures and 61 Tables

Springer

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To Helgard and Georg,
who asked if I was writing a book about bear hugs

Preface to the Second Edition

This textbook, now in its second English edition, is originally a translation of the German textbook "Rechnen für Lagerstättenkundler und Rohstoffwirtschaftler, Teil 1", also translated into the Chinese and Russian languages. Compared to the previous English and German editions the chapters have been updated with new examples and in many cases amended.

The textbook is intended for the economic geologist who deals with the evaluation of deposits at an early stage of development. Once an exploration project has reached the feasibility stage, the exact calculations that are necessary for a comprehensive technical and economic assessment will be performed by a team of geologists, mining engineers, metallurgists, and economists. In the early stages of exploration, however, any evaluator of deposits must be able to cover the whole spectrum himself.

Since only order of magnitude parameters are available at this early stage, the calculations can only yield order of magnitude results. Precise calculations would even be misleading, since the evaluation does not yet aim at accurate economic assessment but at making the right decision: should the investigation be abandoned or should it be continued at higher costs and with more detailed methods.

Therefore, this textbook offers rules for quick and easy calculations based on the application of approximate data. It hopes to provide both the student and the geologist in the field with a complete set of rules and methods enabling to perform a quick initial evaluation of the deposit without the support of specialists or computers – even if he is left to his own resources. To support the "how to do"-approach all rules for calculations are illustrated with examples. The textbook also points out mistakes and pitfalls the authors encountered when working for the exploration industry or gave seminars.

In addition, it is intended as a compendium. Every calculation can be done by hand or by a calculator. Since cost data vary from country to country, absolute figures are only given as examples, but advice is offered on how to adjust the available data to any particular case.

Ultimately, these calculations do little more than transform initial geological data, like reserves and grades, into a simple economic model that can then be used to decide, before committing further funds to the venture, whether an occurrence of mineralization has, or does not have, the promise of economic viability. This transformation of preliminary geological data into the final economic model is merely a routine mechanical procedure. Of importance is the quality of input which depends on the correct initial geological evaluation of tonnage and grades, reserves and potential! Therefore quality control in sampling and analytical procedures is a crucial aspect in the evaluation of any exploration or mining project right from the start. This aspect is dealt with in the book Wellmer 1998 (Statistical Evaluation in Exploration for Mineral Deposits).

For a project evaluation frequently a geologist has to research data quickly. Here the internet is an invaluable tool. To help to find relevant data quickly often internet addresses are given in the text. In addition in Appendix F relevant possible sources of information with internet addresses are listed.

We should like to acknowledge our appreciation to B. Bognar, Friedberg, Germany and S. Schmidt, Cardiff, UK, who critically read the manuscript and made numerous suggestions for improvements including the spread sheet for density calculations in Appendix C, but shortcomings are, of course, the responsibility of the authors. We also thank E. Gschwindt, Luxembourg, M. Glasson, Perth, Australia, K.-H. Huck, Wolfach, Germany, P. L. Nelles, Bensheim, Germany, S. Schmidt, Cardiff, UK, and A. Schneider, Santiago, Chile, for support in up-dating the rules-of-thumb for interest rates and operating and capital costs, F. Barthel and H. Kaiser for information related to uranium, P. Buchholz, Hannover, Germany for research on various topics, U. and F. Dennert, Hannover for advice on probabilities, E. von der Linden, Dreieich, Germany, for advice on concentrate grades, W. Loer, Essen, Germany for uranium energy conversion factors, K. Stedingk, Halle, Germany, for the information of massive ore shoot grade control in the Grund mine, Germany and Mrs. B. Ogiolda, D. Lohmann and M. Zachcial, Bremen, Germany for information of sea freight rates. For technical support our special thanks are due to Mrs. D. Homberg, Mrs. M. Simon and Mrs. E. Westphale, Hannover.

Friedrich-Wilhelm Wellmer
Manfred Dalheimer
Markus Wagner Hannover, October 2007

Preface to the First Edition

This textbook is a translation of the German textbook "Rechnen für Lagerstättenkundler und Rohstoffwirtschaftler, Teil 1" published by the Ellen Pilger Publishing Company. Those passages in the German edition which were especially written for the German readership were transformed for English speaking readers. Compared with the German edition many chapters have been slightly amended. The main new additions in this English version are the chapter on linear optimization in Chapter 10.2 and Chapter 12 on the comparison of ore deposits.

The textbook is intended for the economic geologist who deals with the evaluation of deposits at an early stage of development. Once an exploration project has reached the feasibility stage, the exact calculations of the deposit, the technical and economic assessment will be performed by a team of geologists, mining engineers, metallurgists, and economists. In the early stages of exploration, however, any evaluator of deposits has to be able to cover the whole spectrum himself.

Since only order of magnitude parameters are available at this stage, the calculations can only yield order of magnitude results. Precise calculations would even be misleading, since the evaluation does not yet aim at accurate economic assessment but at making the right decision: should the investigation be abandoned or should it be continued at higher costs and with more detailed methods.

Therefore, this textbook offers rules for quick and easy calculations based on the application of approximate figures. It hopes to provide both the student and the geologist in the field with a complete set of rules and methods enabling him to perform a quick initial evaluation of the deposit without the support of specialists or computers – even if he is left to his own resources.

In addition, it is intended as a compendium. Every calculation can be done by hand or by calculator. Since cost data vary from country to country absolute figures are only given as examples, but advice is given on how to adjust the available data to any particular case.

Ultimately, these calculations do nothing but transform initial geological data like reserves and grades into an economic unit and decide if an occurrence of mineralisation can be regarded as an economically viable ore deposit. This transformation of preliminary geological data into the final economic unit is merely a routine mechanical procedure. Of importance is the quality of input which depends on the correct initial geological evaluation of tonnage and grades, reserves and potential!

I should like to acknowledge my appreciation to Dres. Bering (Hannover), Gschwindt (Bong Mine, Liberia), Kaiser (Erlangen), Kollwentz (Frankfurt), Sommerlatte (Zug) and Thalenhorst (Toronto) for initially reading the manuscript and making numerous suggestions for improvements, to G. Kater (Sydney) who supplied the niobium-tantalum data in Chapter 5.2., and to Dr. Heide (Meggen) for the advise on the Bond index. My special thanks are due to Mrs. U. Grawe (Melbourne) and Mr. B. Bognar (Frankfurt) for translating the German text into English.

Hannover, Spring 1989 FRIEDRICH-WILHELM WELLMER

Contents

Introduction

This book starts with conversions. Although the metric system is being adopted worldwide, older data from Anglo-American countries will always be non-metric. Any deposit evaluation should be based on solid historical research. Many deposits have a long exploration history. Meticulous investigation of older data often answers many questions and avoids mistakes.

In addition, uncommon units used in the raw materials field are introduced.

Thereafter the book generally follows the usual steps of an evaluation: Assessment of tonnage and grades; conversion of geological data into mining data; derivation of a commodity price; calculation of return per tonne of ore; determination of optimum mine capacity; estimation of capital and production costs; final economic evaluation as appropriate for early stage projects.

The book ends with four chapters dealing with aspects of general interest to economic geologists and mineral economists: Valuation of exploration projects without mineralization, comparison of deposits, calculations of growth rates and equity.

Conversions

Exploration geologists working at an international level always have to convert measurements, weights and prices into different units. Although many countries, such as Canada or Australia, have adopted the metric system, time and again the exploration geologist will come across non-metric units in old texts. This chapter deals mainly with the conversion of common measures of the "imperial" system used in the past or still in use in the Anglo-American environment.

Before dealing with conversions in detail a sound understanding of the way numerical figures are presented is necessary. Unfortunately, the continental European usage of the decimal comma and point is exactly the reverse of the Anglo-American usage. For the Anglo-Americans the continental European decimal comma is a point, whereas the comma is only used to separate units of thousands. An example:

3,451 in continental European usage is expressed in Anglo-American English three point four five one. (In the old-fashioned Anglo-American way of writing, the decimal point was in the middle, i.e. 3·451, as can often be found on older plans.)
3.451 in continental European usage corresponds to three thousand four hundred and fifty-one in Anglo-American English.

In Anglo-American English nought is frequently omitted before the decimal point, e.g. .5 stands for 0.5, or nought comma five in continental European usage.

The concept "billion" often tends to create confusion. One distinguishes the "long scale" and the "short scale". In Germany and in most countries where English is not the primary language billion is 10^{12} (long scale), in the USA, Canada, France it is 10^9 (short scale), i.e. one thousand million, the way it is used in the geochemical unit "ppb" (parts per billion), i.e. 1 part in 10^9 parts or 1 mg in 1 t. In Australia and the United Kingdom (UK) the usage varies. The Australian Macquarie Dictionary defines billion as 10^{12}, the Australian government in its annual budget understands billion as 10^9. In the UK billion as 10^{12} is still encountered, but in official documents and largely in journalism and finance it is now 10^9. So it is advisable always to check what is meant by the term "billion": thousand million (short scale) or million million (long scale).

Anglo-Americans use prefixes for units indicating the power of 10 such as kilo (10^3), mega (10^6) etc. more frequently than Europeans. The volume of reservoirs, for example, is often given in mega litres (10^3 m^3). Table D1 (Appendix D) lists these prefixes.

With the introduction of the metric system several unusual abbreviations have become common in the Anglo-American sphere. Instead of "100 km", "100 k" is used, even in writing. Occasionally the abbreviation “k” for kilo, i.e. 10^3, is also used in other, e.g. monetary contexts: US $50 k = US $50 000 or kmt for kilo metric tonnes (see Sect. 1.14).

In contrast to continental European usage, English speakers use the suffix “s” for the plural of units: lb = pound, becomes lbs in the plural (see Sect. 1.1.4). Applied to metric units, one may come across plural abbreviations such as kms or kgs.

Sometimes in Anglo-American texts one encounters odd abbreviations of metric units which are similar to abbreviations of imperial units but not correct in the international SI-convention (SI = Système International d’Unités), like gm for gram (g), kgm for kilogram (kg) or cm for cubic meter (m^3).

When converting, the following principle must be observed:

The accuracy of the converted quantity cannot be greater than that of the original one.

For example 115 feet equals 35 m and not 35.05 m or even 35.052 m. This would only be correct, if the original figure were given as 115 feet, 0 inch (or 115'0", see Sect. 1.1.1).

Assignment. Samples taken from an alluvial tin deposit 30 years ago gave the results as listed in Fig. 1.1a, with the mass units being related to a volume of 1 $yard^3$. What is the accuracy of the sampling? To what accuracy can the values be converted into metric units?

The accurate conversion factors are:

- 1 foot = 0.3048 m (see Sect. 1.1.1)
- 1 yard = 0.9144 m (see Sect. 1.1.1)
- 1 pound = 0.454 kg (see Sect. 1.1.4)
- 1 ounce = 28.3 g (see Sect. 1.1.4)

Obviously, the accuracy of the sample is 1/4 ounce, i.e. (28.3/4) g/$yard^3$. Converted to m^3 that is 9.3 g/m^3. According to the accuracy above, all mass units per m^3 must be rounded off to the nearest 10 g. A converted drilling profile would then look like Fig. 1.1b.

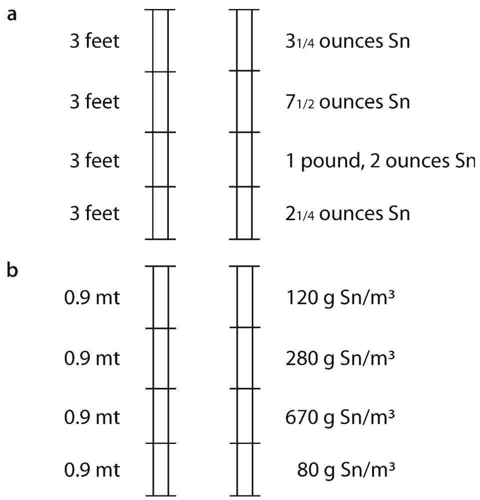


Fig. 1.1.
a Sampling of an alluvial tin deposit with “imperial” units. **b** Same sampling as in **a** with converted metric units

1.1
Conversion of Units

1.1.1
Measures of Length

Mile (abbreviation USA: mi; 1 mile = 1.6093 km). One mile is 5 280 feet or 1 760 yards or 80 chains.

A conversion diagram between miles and km is given in Appendix A, Fig. A1.

This is the mile commonly used in Anglo-American countries, and should not be confused with the nautical mile (1.852 km). To distinguish it from the nautical mile, it is also called "statute mile".

A handy rule-of-thumb for quick mental calculations reduces the conversion to mere doubling and subtracting. The number of miles is multiplied by 2 and 20% is subtracted from the result, e.g.

65 miles: $2 \times 65 \rightarrow 130 - 20\% \rightarrow 130 - 26 = 104$ km

The more precise value is 104.6 km, an error of less than 1%.

Chain (abbreviation: ch; 1 chain = 20.1168 m; 1 chain = 22 yards). In Anglo-American usage the unit of chain was formerly often employed as a mapping scale in the surveying of claims.

Yard (abbreviation: yd; 1 yard = 0.9144 m; 1 yard = 3 feet). A conversion diagram between yards and metres (m) is given in Appendix A, Fig. A1.

Foot (plural: feet) (abbreviation: ft or ', e.g. 3 feet = 3'; 1 foot = 0.3048 m; 1 foot = 12 inches). A conversion diagram between feet and metres is given in Appendix A, Fig. A1.
Note: For simplicity, the factor 0.3 is often used in practice which results in an error of almost 2%. If one divides by 3, however, the resulting error increases to 10%. While this error might still be acceptable for measurements of length in the field, it becomes totally unacceptable in volume calculations (reserve estimations) since here it is raised to the power of 3, resulting in an error of as much as 32%!

Inch (abbreviation: in or ", e.g. 4 inches = 4"; 1 inch = 25.40 mm). A conversion diagram between inches and cm is given in Appendix A, Fig. A1.

The subdivision of the unit "inch" is based on the 8 system, i.e. $n/8$, then $n/16$, $n/32$, $n/64$ etc. In the following steps these subdivisions are important for drilling diameters used worldwide, which are based on the inch system. The most common measures are listed in Table D2 (Appendix D).

Fathom (as a special measure of length) (1 fathom = 1.829 m; 1 fathom = 2 yards or 6 feet resp.). Fathom is a nautical unit. However, fathom was also used in alluvial mining, e.g. in Australian deep-lead gold mining. (Deep leads are old alluvial deposits covered by younger, barren alluvial deposits or basalt flows). Since the thickness of deep leads fluctuated considerably, the metal values were expressed in terms of average intensity: weight of gold per unit of area. Ten pennyweights per fathom e.g. meant 10 pennyweights per square fathom. Since 10 pennyweights equal 15.55 g (see Sect. 1.1.4) this would be equal to

$$15.55/(1.829)^2 = 4.65 \text{ g/m}^2$$

1.1.2
Square Measures

A conversion diagram between common square measures in the imperial and metric systems is given in Appendix A, Fig. A2.

In addition to the square measures derived from the above measures of length (e.g. 1 square foot = $[0.3048 \text{ m}]^2 = 0.0929 \text{ m}^2$), the acre is of importance: 1 acre = 4 047 m^2; rounded, 2.5 acres = 1 hectare (ha).

There are two additional archaic square measurements which are still used in the United Kingdom: 1 rood = 1 011.712 m^2 and 1 perch = 25.2929 m^2.

1.1.3
Cubic Measures/Dry Measures

A conversion diagram between the common cubic measures in the imperial and the metric system is given in Appendix A, Fig. A3.

Among the cubic measures derived from the above standard measures of length, the yard^3 (cubic yard) is still of major importance today, especially in reserve estimations for alluvial deposits or in giving the volume of loader shovels: 1 $\text{yard}^3 = (0.9144 \text{ m})^3 = 0.7646 \text{ m}^3$.

In colloquial English the "cubic" in cubic yard is often omitted, as in "a 10 yard bucket". Note that in Anglo-American writing cubic metre is sometimes abbreviated "cm" (as in centimetre). Another common abbreviation used in connection with alluvial deposits is "lcm" = loose cubic metre, i.e. a cubic measure for material not in situ but after mining. For the in situ cubic measures the expression bank cubic metre, abbreviated "bcm" or "BCM", is also used.

To make things even more confusing, the abbreviation bcm is also used for billions cubic meter in measuring natural gas volumes, corresponding to 10^9 m^3 in the USA. Another metric abbreviation for natural gas is tcm or trillion m^3 (in the USA 10^{12} m^3). In the Imperial system, the term tcf is used for trillion cubic feet (10^{12} cubic feet), and ncm is applied for norm cubic meter, the same as the continental European Nm^3, meaning a cubic meter of natural gas under standard conditions (0 °C and 1 atmosphere of pressure, 1 013 millibar).

Gallon. Liquids such as water and fuels are measured in gallons. Two different kinds of gallons need to be distinguished:

a 1 U.S. gallon = 3.785 L (litres)
b 1 imperial gallon = 4.546 L (litres)

The imperial gallon is or was, among other countries, used in Great Britain, Canada and Australia. Both the U.S. gallon and the imperial gallon are subdivided into quarts and pints:

1 gallon = 4 quarts, 1 quart = 2 pints

As a rule-of-thumb, 1 quart = 1 L (error of 6 or 12% for U.S. and imperial measures respectively).

Barrel. Liquids such as crude oil are measured in barrels (U.S. barrel or petroleum barrel):

1 barrel = 42 U.S. gallons = 34.974 imperial gallons = 158.984 L

'Barrel' is internationally the most common unit used for example in the quotation of oil prices. In continental Europe, however, the metric tonne is used in official statistics, i.e. a mass unit instead of a cubic measure. The conversion factor used is 7.35, so that

7.35 barrels $\triangleq$ 1 tonne of oil

The assumed density is 0.86 g/cm^3.[1]

Acre-foot. A hydrological unit, abbreviated ac.ft, which exploration geologists may come across when dealing with water resources and competing land use claims. It is a common volume unit used in connection with irrigation and defines the water volume which covers an area of one acre one foot deep. Since 1 acre = 4 047 m^2 (see Sect. 1.1.2) and 1 foot = 0.3048 m (see Sect. 1.1.1) we have

1 acre-foot = 4 047 × 0.3048 = 1 233.5 m^3

1.1.4 Mass Units

(Also wrongly designated as units of weight in colloquial English.)

Tonne. Three kinds of tons have to be distinguished:

a *Metric tonne* (abbr. t) 1 tonne = 1 000 kg
In British and American English the abbreviations "m.t." or sometimes capital T for metric tonneare also used. Unfortunately, the English way of writing is inconsistent: "tonne" is always metric tonne as against (short) ton or (long) ton. However, the version "metric ton" is also in use with the specification "metric" always added.

[1] The density of oil is dependent on its viscosity. It is standardized by "degree API" (API = American Petroleum Institute). The density of 0.86 g/cm^3 is equivalent to an oil of 37° API.

b *"Short ton"* (abbr. sh tn; 1 short ton = 907.185 kg)
One short ton has 2 000 pounds.

A conversion diagram between short tons and (metric) tonnes is given in Appendix A, Fig. A4.

This unit is predominantly used in North America (USA, Canada). Often "short" in "short ton" is omitted, whereas "long" in "long ton" never is. (An exception is the early gold mining era in Nevada and California when "ton" was also used for "long ton".) In U.S. or Canadian literature of a later date, whenever tonnages of for instance "3.5 million tons" are mentioned, these have always to be understood as short tons.

c *"Long ton"* (abbr. 1 tn; 1 long ton = 1 016.0470 kg).
One long ton contains 2 240 pounds.

A conversion diagram between long ton and (metric) tonne is given in Appendix A, Fig. A4.

This unit is predominantly used in Great Britain and countries under British mining influence outside North America, e.g. Australia and New Zealand. However, up to the beginning of this century it was also used in North America. Countries commonly using the long ton often omitted the term "long". Whenever older Australian literature refers to tonnages of for instance "3.5 million tons", these are normally *"long"* tons. Short tons were used only as an exception usually for mines under U.S. American ownership.

Internationally, this unit is still in use for bulk commodities such as iron ore and coal. Up to 1970, the quotations on the London Metal Exchange (LME) referred to long tons. After that they were replaced by metric tons.

Note: It is absolutely essential to distinguish between short tons and long tons, for example in the case of precious metals (see Sect. 1.2.3).

Deadweight tonne (abbreviation: dwt, sometimes tdw). When dealing with sea freights an exploration geologist will come across the unit dwt indicating the capacity of bulk carriers and other ships. The unit dwt is the unit for the mass of a ship minus the lightship mass, meaning the mass of the cargo, fuel, ballast, stores etc. We are only interested in the cargo capacity of a ship. For this also the term "deadweight cargo capacity, dwcc" is used. According to ship size the cargo capacity is 93 to 97% of the deadweight tonnage.

So for our practical work we take the deadweight tonnes as the cargo capacity of a bulk freighter.

The abbreviation dwt is the same as for the gold mass unit pennyweight (see below Precious Metal Units), but of course the cargo ship unit has nothing to do with the precious metal unit.

Hundredweight (abbreviation: cwt; 1 hundredweight = 50.80 kg). One hundredweight contains 112 pounds.

This unit used to be applied in small selective mining areas in Great Britain and Australia, usually in gold mining.

Pound (abbreviation: lb, plural, lbs; 1 pound = 0.4536 kg). A conversion diagram between pounds and kg is given in Appendix A, Fig. A4.

The unit pound is important internationally, since North American metal prices are quoted in pounds (see Sect. 1.2.7).

In later chapters metal prices in US $/lb will have to be converted into metal prices per 1%, i.e. per 10 kg (see this chapter "Unit" in Concentrates). The conversion factor for this is

$$\frac{10}{0.4536} = 22.046$$

Since the quotation on the London Metal Exchange today is US $/t for all metals quoted, but the unit used in the USA is still US ¢/lb, we also have to convert metric tonnes into lbs for which the conversion factor is 2 204.6 (see Assignment in Sect. 1.2.7).

Ounce (abbreviation: oz; 1 ounce = 28.35 g). The normal weight unit ounce, which is based on 1/16th of a pound, must be distinguished from the precious metal ounce or "troy ounce" weighing 31.103 g, i.e. 10% more (see below). Although the term ounce is commonly used for precious metals too, the correct designation in this case is "troy ounce", at 31.103 g. With other commodities, the designation is simply "ounce" or more correctly "avoirdupois-ounce", at 28.35 g. The metal content related to yard^3 in alluvial deposits, e.g. alluvial tin, is sometimes expressed in pounds and ounces per yard^3 (see example in the introduction to Chap. 1).

Precious Metal Units

a Precious metals are weighed in troy ounces (oz), but the prefix "troy" is usually omitted:
 - 1 troy ounce = 31.103 g
 - 1 troy ounce is subdivided into 20 pennyweights (abbr. dwt)
 - 1 pennyweight = 1.555 g
 - 1 pennyweight is subdivided into 24 grains (abbr. gr)
 - 1 grain = 0.0648 g

 A conversion table between troy ounce and g is given in Appendix A, Fig. A4.

 Instead of pennyweights, older literature often only refers to weights, e.g. "the ore ran 3 weights per ton". This should read "the ore ran 3 pennyweights per long ton". In metric terms this would be 4.6 g/t.

 Note: Two errors are common when converting precious metal grades:
 - The precious metal ounce (troy ounce at 31.103 g) is confused with the normal weight ounce (avoirdupois-ounce) at 28.350 g.
 - The abbreviation gr for *grain* is easily confused with g for gram, resulting in an inadvertent multiplication by the factor 15.4!!

b The proportion of pure gold in an amalgam is often expressed in terms of "fineness" or parts per 1 000. Pure gold is 1 000 fine. Also the unit carat is used. 24 carat equals 100% gold or a fineness of 1 000. Correspondingly, 12 carat equals 50% gold or a fineness of 500.

c A unit often used in connection with precious metals, particularly gold assays, is 1 *assay ton* (or assay tonne). This unit designates the mass of individual samples to be assayed. It dates back to pre-data processing times when assay chemists preferred to work with quantities indicating in convenient figures the wanted grade – in this case 1 ounce/short ton – without the need for lengthy conversions.

One assay ton (or assay tonne) amounts to an assay sample weight of about 30 g. (The exact amount is 907.2/31.103 = 29.17 g. Related to the metric ton, this would be 1 000/31.103 = 32.15 g.)

Under certain circumstances of grade it is necessary to use samples weighing at least one assay ton, sometimes even up to two assay tons, in order to obtain significant precious metal assays. There are simple rules-of-thumb to choose the sample size. It is, however, more advisable to work out the sample weight by statistical methods, taking into account the size of the gold particles and the expected gold grade (see e.g. Clifton et al. 1969, also Wellmer 1998, Stat. Eval., p. 101ff).

d Finally an additional precious metal weight unit should be mentioned, which has a certain regional importance. Through Indian traders the unit 1 "tola" spread from India to eastern and southern Africa: 1 tola = 11.6638 g

"Unit" in concentrates. A unit we frequently come across when evaluating deposits is 1 unit in concentrates. One "unit" (abbreviated 1 u) is always 1% of the contained metal in the concentrate. Today most prices refer to metric tons, i.e. 1 unit = 10 kg. However, they often used to refer to long tons: 1 unit = 22.4 pounds = 10.16 kg. In the case of short tons: 1 unit = 20 pounds = 9.07 kg. The abbreviation for 1 metric tonne unit is sometimes "m.t.u.".

Special Mass Units

a For gem stones the unit "carat" is also used, but in a different sense than with precious metals (see Sect. 1.1.4) because it is, in this special case, an absolute unit:
- 1 carat = 0.2 g
- 1 carat is subdivided into 20 points (pt; 1 pt = 0.05 carat or ct)

b Mercury is sold in "flasks", 1 flask = 34.473 kg
- 1 flask contains 76 pounds

c The Malaysian tin (Sn) price, which is of international importance, was until recently quoted as the price per "picul":
- 1 picul = 60.47899 kg

1.1.5 Other Units

Energy units. A conversion matrix for energy units is listed in Table D3a (Appendix D).

For coal mining the specific heating value is of importance. Countries with imperial measures used British thermal units per pound (abbr. Btu/lb; often the term "the Btu-value of coal" is used without a reference to the mass unit), whereas the metric unit most commonly used is calorie/kg (abbr. cal/kg). Since 1 Btu = 252.2 cal and 1 lb = 0.4536 kg (see Sect. 1.1.4).

$$\left(\frac{\text{Btu}}{\text{lb}}\right) = \frac{252.2}{0.4536} = 556.0\left(\frac{\text{cal}}{\text{kg}}\right) \quad \text{or} \quad 0.556\left(\frac{\text{kcal}}{\text{kg}}\right)$$

As a rule-of-thumb, one can divide the Btu/lb value by 2 and add 10% to obtain the value in kcal/kg.

Example: 12 000 Btu/lb = 12 000 : 2 → 6 000 + 10% → 6 000 + 600 = 6 600 kcal/kg. The more exact value is 6 672 kcal/kg which is an error of about 1%.

It should be pointed out that in the International System of Units (SI) the unit for heat quantity is 1 joule (abbr. J). 1 kilojoule (1 kJ) is approximately 1 Btu (1 Btu = 1.055 kJ).

For natural gas, a practical rule-of-thumb is that 1 m^3 of good quality natural gas has an energy content of roughly 10 kWh (exactly 8.82 kWh). In Appendix D, Table D3b factors are given for converting units of standard fuel into energy units at standard conditions.

If we want to convert the energy content of fossil fuels or nuclear fuels into the energy value of electricity, we have to take into account the efficiency of a power plant. This varies today between 33 and 49%, meaning 33 to 49% of the energy content of the fuel is converted into electricity, the remaining part into heat. If we assume an efficiency of 36%, we can work with round numbers:

At 36% efficiency 10 Mega Joules fuel energy is equivalent to 1 kWh electricity.

Concerning the special situation of uranium as fuel for nuclear power plants see Sect. 4.5.

Units of angles. In German geodetics and occasionally also in German geology the unit "Gon" or "Neugrad" (abbreviation, e.g. 30^g) is used. 100 Gon is a right angle (90°), i.e. $1^g = 0.9°$. This "metric angle unit" is sometimes also used in other classic metric countries like France.

Sieve units (screen sizes). A conversion table for the unit mesh (number of openings per linear inch) into mm aperture of the meshes is given in Table D4, Appendix D.

A convenient rule-of-thumb is

$$\frac{15\,000}{\text{mesh number}} \approx \text{aperture in microns}$$

In alluvial mining qualitative expressions like "coarse" or "fine" are used, e.g. coarse tin or fine gold. "Fine" is normally used to describe the size fraction which is difficult to recover by normal gravity methods. Because of improved recovery techniques, the definition "fine" is more and more applied for increasingly smaller grains. The size fractions described by these qualitative terms also vary widely from region to region. A few examples are given in Table D5, Appendix D.

Pressure units. In Anglo-American countries, the pressure unit is "pounds per square inch" (abbr. psi): 1 psi = 0.070 kg/cm^2.

Temperature units. In English-speaking countries, the Fahrenheit scale (°F) is still frequently used. The freezing point of water on this scale is at 32 °F, the boiling point at 212 °F, i.e. 100 degrees in centigrade correspond to 180 degrees in Fahrenheit. The conversion is accordingly

$$y\,^{\circ}\mathrm{C} = \frac{x\mathrm{F} - 32}{1.8} \quad , \quad \text{e.g.} \quad \frac{90\,^{\circ}\mathrm{F} - 32}{1.8} = 32\,^{\circ}\mathrm{C} \quad , \quad \text{i.e.} \quad 90\,^{\circ}\mathrm{F} \text{ equals } 32\,^{\circ}\mathrm{C}$$

A rule-of-thumb is

$$^{\circ}\mathrm{C} = (^{\circ}\mathrm{F} - 32) \times 0.5 + 10\%$$

In another rule-of-thumb calculation one does not use the freezing temperature of water (0 °C = 32 °F) as a fixed point but uses the fact that 10 °C equals 50 °F. This is the basis for a rule-of-thumb calculation:

Example: 92 °F

a 92 °F – 50 °C $\rightarrow$ 42 : 2 $\rightarrow$ 21 + 10% $\approx$ 23 °
b 23 ° + 10 °C = 33 °C

The exact value is 33.3 °C.

The two temperature scales intersect at –40 °. It should be pointed out that continental Europeans do not read °C as "degree centigrade" but "degree Celsius". There is, however, no difference between the units.

Purity of metals. To describe the purity of metals the convention of counting the first nines in the percentage grade is used. The designation for 99.95% Cu therefore is "three nine copper" or 3 N copper. The digit behind the last nine is of no importance. *Another example:* 4 N Zn equals 99.99x% Zn, whereby x can be a number from 0 to 8. (The convention of describing the purity or fineness of precious metals has already been dealt with in Sect. 1.1.4.)

1.2 Conversion of Derived Quantities

1.2.1 Map Scales (on the Basis of Mile, Chain, Feet)

(Some common scales[2] are given in Appendix G.)

---

[2] Sometimes there is a confusion concerning "small scale" and "large scale" maps. "Small" and "large" refer to the scale ratio, which can also be read as a fraction:

$$\frac{1}{5\,000} \text{ is larger than } \frac{1}{1\,000\,000}$$

So 1 : 5 000 is a large scale map and 1 : 1 000 000 is a small scale map.

Today metric map scales are widely used in Anglo-American countries. Common scales formerly used for geological maps include:

a *Quarter mile, half mile and one mile maps.* One inch = 0.25 mile or 1 inch = 0.5 mile or 1 inch = 1 mile respectively
Converted, this entails for the quarter mile map as an example:
- 1 inch = 0.25 mile
- 2.54 cm = 0.25 × 1 609.3 m
- 1 cm = 158.4 m = 15 840 cm
- i.e. 1 : 15 840 (or rounded up 1 : 16 000) in the metric system

Accordingly, 1 inch = 1 mile is four times as much, i.e. 1 : 63 358 or rounded down 1 : 63 000.

b *Four mile map.* The scale of 1 inch = 4 miles, frequently used for survey mapping, almost corresponds to 1 : 250 000 (exactly 1 : 253 433).

c *Scales with chains.* Since 80 chains equal 1 mile (Sect. 1.1.1), the scale of, for example
- 1 inch = 20 chains corresponds to the quarter mile map

d *Scales with feet.* Detailed geological maps have scales such as 1 inch = 100 feet (abbr. 1" = 100') or multiples of this:
- 1 inch = 100 feet equals 1: 1 200 in the metric system

In Anglo-American countries, scales of mine level plans or cross-sections used in reserve calculations are, as a rule, given in feet.

If one encounters scales on the basis of the number 12, e.g. 1 : 6 000 or 1 : 12 000, most probably these are feet scales, which were converted into metric scales; for the examples 1 : 6 000 and 1 : 12 000 it is 1 inch = 500 feet and 1 inch = 1 000 feet (1" = 1 000'). For the scales 1 : 1 200, 1 : 2 400 and 1 : 6 000 graphic scales are given in Appendix G.

1.2.2 Density Conversions

1.2.2.1 Density/Tonnage Factor

While the metric system requires the multiplication of volume by density to arrive at the tonnage, the imperial system uses the “tonnage factor”. The tonnage factor is the number of cubic feet of ore corresponding to 1 short ton (or long ton).

An example. Tonnage factor of 10 equals:

$$
\begin{array}{lcl}
10 \text{ cubic feet} & \hat{=} & 1 \text{ short ton} \\
10 \times (0{,}3048 \text{ m})^3 & \hat{=} & 907.2 \text{ kg} \\
0.2832 \text{ m}^3 & \hat{=} & 0.9072 \text{ t} \\
1 \text{ m}^3 & \hat{=} & 3.2037 \text{ t} \\
\text{i.e. a density of} & & 3.2 \text{ g/cm}^3
\end{array}
$$

A comparative table of density and tonnage factor is given in Table D6, Appendix D.

Note:

a Whereas one multiplies by density, one divides by tonnage factor!

b The tonnage factor can, of course, only be used, if the volume has been calculated in feet!

In the initial stages of exploration, while no exact density data are available, approximate values are used. Some helpful approximate values are listed in Table D7, Appendix D.

1.2.2.2 Dry Density/Wet Density

In the case of normal, consolidated hard rock deposits, the extremely low natural rock moisture is ignored and reserves are calculated by using the density determined for example in drill cores. In the earl stages, the approximate values thus obtained can safely be applied. If the deposit has a high level of porosity and moisture content (as in unconsolidated rocks), the moisture must be taken into account. Relative values such as assay results (percent, ppm etc.) always refer to the dry substance. (For determining grade estimates in unconsolidated rocks, see Sect. 1.2.3.)

Example. A reserve estimate is required for a tailings reprocessing project. The wet density of the material (in situ density) is 1.5 g cm^{-3}, the water content 20%.

What is the dry density for this material?

A volume of 1 m^3 is assumed. This cube has a mass of 1 500 kg with a density of 1.5 g cm^{-3}. 20% of this is water, i.e. 300 kg. Hence the mass of the dry substance is 1 200 kg. Expressed in a general formula:

$$\text{wet mass} - \text{wet mass} \times \text{rel. } H_2O\text{-content} = \text{dry mass}$$

Division by the volume results in the following ratios between densities:

$$\frac{\text{wet mass}}{\text{volume}}(1 - \text{rel. } H_2O\text{-content}) = \frac{\text{dry mass}}{\text{volume}}$$

or

$$\frac{\text{dry mass}}{\text{wet mass}} = \frac{1 - \text{rel. } H_2O\text{-content}}{1}$$

or expressed in percentage:

$$\frac{\text{dry mass}}{\text{wet mass}} = \frac{100 - H_2O\text{-content}}{100}$$

In the above example the dry density would be

$$\text{dry density} = 1.5 \times \frac{100 - 20}{100} = 1.5 \times 0.8 = 1.20 \left(\frac{g}{cm^3}\right)$$

1.2.3 Grades

Percentage is a relative term; hence it does not matter whether it refers to metric tonnes, short tons or long tons. Percentage values of analyses for grades refer to the weight (or better mass) of the sample material. The density of the material has to be taken into account in the weighting process for calculating an average grade from various samples (see Sect. 2.2).

It must be stressed that the percentage values of an analysis are *not volume percentages*. Therefore, it is absolutely wrong to calculate with these values a volume and then further to multiply with the density of the metal to obtain the metal content. The density of the metal or of the ore mineral *directly does not play any role*. The density of the ore minerals however does play a role for grade calculations through visual estimates as outlined in Sect. 2.2.2.

Normally, grades are given in percentages, which is a relative concept as explained above. In special instances, however, absolute quantities are used. These, however, are not absolute quantities in a true sense, but "alternative" ways of grade reporting, i.e. relative figures.

a Today precious metal grades are given in g/metric ton. The Anglo-American usage used to be "ounces/short ton" or "ounces/long ton":

$$1 \text{ ounce/short ton} = 31.103 \text{ g}/0.907 \text{ t} = 34.29 \text{ g/t}$$

$$1 \text{ ounce/long ton} = 31.103 \text{ g}/1.0164 \text{ t} = 30.61 \text{ g/t}$$

A conversion diagram between ounces/short ton and ounces/long ton respectively and g/t is given in Appendix A, Fig. A5.

Unfortunately, many texts only refer to "oz/ton" (or sometimes "opt"), without specifying whether short tons or long tons are meant. In Australia oz/ton always means *long* ton (except for a few mines with American ownership), in North America almost always *short* ton (only in literature up to 1900 could long tons have been meant). It is essential that this confusion is clarified. Since we are dealing with absolute grades, a confusion of short tons with long tons would result in an error of 12%!

b Grades of precious metals and trace elements are often quoted in ppm: ppm = parts per million. Since 1 ton contains 10^6 g, 1 ppm equals 1 g/t (but only in the metric system, not for short or long ton).

c Since up to 1971 the gold price used to remain steady for many years, it had become common usage to convert the gold grade directly into US $/ton. Hence, it must always be established to which gold price such US $-grades refer. At a price of 45 US $/oz (or US $45/31.103 g), a statement like "the grade was US $21 per ton worth of gold" would translate into

$$\frac{21.00}{45.00} \times \frac{31.103}{0.907} = 16.0 \left(\frac{\text{g}}{\text{metric ton}} \right)$$

In countries of the British Commonwealth, the gold price in English pound sterling (£) used to play an important role. Sometimes mines would directly report their grades as money values. These data are based on the old non-decimal pound system:

- 1 £ = 20 shillings (20s)
- 1 s = 12 pence (12d)

Examples for abbreviations:

- £1-5s-0d or
- £1/5/0 i.e. 1 pound, 5 shillings, 0 pence or
- 3s/4d or 3/4 i.e. 3 shillings, 4 pence

Assignment. A mine reports a gold grade of 1/9/4. What is the equivalent in g Au/t, assuming a gold price of £4-5s-0d per ounce (the gold price up to World War I)? The mine was located in Australia and used long tons.

- *Step 1:* The currency units should be reduced to the smallest common denominator, i.e. 1 pound = 240 pence, 1 shilling = 20 pence. Hence, the gold price of £4-5s-0d is equivalent to 1 020 d/oz.

  The gold grade of the mine of £1/9/4 equals 352d. Thus the gold grade per long ton is

$$\frac{352}{1\,020} = 0.35 \left(\frac{\text{oz}}{\text{l tn}}\right)$$

  Since 1 oz is 31.103 g (see Sect. 1.1.4), this value is equivalent to 10.7 g/l tn.

- *Step 2:* Since 1 long ton equals 1.016 t (see Sect. 1.1.4), converted into metric tonne this equals

$$\frac{10.7}{1.016} = 10.6 \left(\frac{\text{g Au}}{\text{t}}\right)$$

Historical gold prices in US $/oz and in £/oz and historical silver prices in US $/oz are given in Appendix D, Table D8 for periods in which prices stayed more or less constant for a longer interval.

d In placer deposits the weight of the precious metal refers to volume measures, i.e. oz/yard^3 or g/m^3. Accordingly, the measure g/cubic yard is

$$1\ \text{g/cubic yard} = 1\ \text{g}/(0.9144)^3 = 1\ \text{g}/0.7646 = 1.31\ \text{g/m}^3$$

A conversion diagram between troy ounces/yard^3 and g/m^3 is given in Appendix A, Fig. A5.

Grades in hard-rock samples always refer to analytically determined absolute values and dilution has to be taken into account when calculating minable grades. For alluvial deposits, on the other hand, usually only recoverable grades are given due to the different sampling methods peculiar to these deposits. Therefore, it is essential to distinguish between *overall, contained alluvial grades* and *recoverable grades* and it must always be stated what unit is used for the volume (e.g. m^3 or cubic feet) and whether the measurements refer to in situ (in place or bank) material or loose material after mining. If assays are expressed in weight percent, then 1.7 g/cm^3 can often be used for the density of alluvial material.

Example. 300 g/t Sn (or 300 ppm) are therefore 510 g/m^3. These are "loose m^3". If this is to be reconverted to the volume in situ, a swell factor of 1.25 can be applied, i.e. in our example the result would be 638 g/m^3 (in situ).

e Frequently the grade and quality of products, like concentrates or intermediate smelting products like ferronickel, is not expressed in percentage values but in figures for the absolute metal content. So this is an instance where grade and quantity are combined in statements like "a ferronickel plant produces 3 500 t of nickel in ferronickel". This could be for example a lateritic nickel mine with a ferronickel plant which produces 10 000 t of ferronickel with a grade of 35% nickel. In Anglo-American countries the wording could be "7.7 million lbs Ni in ferronickel". Another example could be a tin mine which produces 5 000 t of Sn concentrate with a grade of 65% Sn. The wording then could be: "The mine produces 3 250 t Sn in concentrates".

1.2.4 Accumulation Values/Intensity Factors

In reserve calculations an accumulation value has to be determined: the product of thickness times grade, also called "grade-thickness product" or *GT*-factor (see Sect. 2.2.1 and 2.2.3). Occasionally, this factor is used in geochemical exploration, e.g. in the exploration for Pb-Zn deposits of Mississippi-Valley type, and is then called intensity factor:

$$1\% \times \text{feet} = 0.348\% \times \text{m}$$

In South African gold mining the accumulation value inch pennyweight (abbr. inch dwt) is of major importance:

$$1 \text{ inch} \times \text{dwt} = 2.54 \text{ cm} \times 1.555 \text{ g} = 3.95 \text{ g cm}$$

1.2.5 Production

a In Anglo-American countries, water yield is measured in "gallons per minute" (GPM). Related to "U.S. gallons", this means: 1 gallon/minute = 3.785 litres/min (L/min) or 0.227 m^3/h. Expressed in "imperial gallon": 1 gallon/minute = 4.546 litres/min (L/min) or 0.273 m^3/h.

b Oil production in Anglo-American countries is quoted in barrels/day, in Germany in tonnes/year, i.e. the Anglo-Americans use a volume, the Germans a mass per time unit. The rule-of-thumb applied is that 1 m^3 (1 000 L) crude oil weighs 0.86 t and accordingly 1 barrel/day equals 50 t/year (see also Sect. 1.1.3 Barrel).

c Gas production in Anglo-American countries is expressed in 1 000 cubic feet/day, in continental European countries in m^3/year.

$$1 \text{ cubic foot/day} = (0.3048 \text{ m})^3 \times 365 = 10.34 \text{ m}^3/\text{year}$$

1.2.6
Waste to Ore Ratios

There are two principally different kinds of waste to ore ratios or stripping ratios:

a In layered fossil fuel deposits (lignite in Germany, hard coal in USA, Canada, Australia, South Africa etc., oil-shale, tar sands) the waste to ore ratio is stated as m^3 waste per tonne raw material, in the USA for example in open-pit coal mines the term cubic yard waste to short ton of coal in used frequently. One reason being that the machinery used for removing waste, such as bucket-wheel excavators in lignite mining or drag-lines in coal deposits, are intended for moving huge volumes of material while different machinery is employed in the selective excavation of the coal itself.

b In open-pit mining for metal ore the waste to ore ratio is normally expressed as tonne of waste per tonne of ore. In Russia, however, and in countries of the CIS also, the overburden in metal mining is normally given as a volume in m^3 and the ore in tonnes.

To be able to compare the two different ratios with each other, the ratio under *(a)* has to be multiplied by the density.

In each case it must be properly examined what is really understood by the waste to ore ratio.

A simple method to determine the waste to ore ratio is outlined in Sect. 9.3.2.4.

1.2.7
Specific Metal Prices

The following metal prices are of international importance:

a American prices quoted in US$/pound (US$/lb)

b prices of the London Metal Exchange (LME), quoted today in US$/metric tonne, were formerly quoted in £/long ton and since 1970 in £/metric tonne. In a period from 1988 to 1993 all quotations at the LME were converted to US$/metric tonne (see Table D12a in Appendix D). There remain only rare special quotations in the British metal market, which are still given in £/tonne, e.g. lead scrap

In those European Union countries, which use the Euro, prices are usually quoted in €/100 kg. Precious metals are quoted in US$/ounce, in Euro-countries in €/kg.

Assignment. The copper price in the USA is 90 US¢/lb. Convert this price into

a £/t and

b €/100 kg

Rates of exchange: 1 US$ = 0.85 €
1 £ = 1.45 €

a $\dfrac{0.90 \text{ US\$} \times 0.85}{1.45} = £0.538$

0.538 £/lb = 0.538/0.454 kg = 1.185 £/kg
or 1 185 £/tonne

b 0.90 US$ × 0.85 = 0.765 €
0.765 €/lb = 0.765 €/0.454 kg = 1.685 €/kg
or 168.50 € /100 kg

There are practical rules-of-thumbs for conversion from US$/t to US¢/lb and vice versa.

One tonne contains 2 204.6 lbs (see Sect. 1.1.4, Pound):

$$2\,000\left(\frac{\text{US\$}}{\text{t}}\right)=\frac{2\,000}{2\,204.6}=90.7\left(\frac{\text{US¢}}{\text{lbs}}\right)\approx 90\left(\frac{\text{US¢}}{\text{lb}}\right)$$

So 1 000 US$/t is about 45 US¢/lb. These two prices are relevant for copper and zinc.

1.3 Conversion of Chemical Compounds

Whereas grades or prices for some metals such as Cu, Fe, Zn are always related to the element, for other materials the composition can vary considerably. In the case of tungsten, the grades sometimes refer to elemental W, sometimes to WO_3. The unit WO_3 is also the basis for which concentrate prices are quoted (as explained in Sect. 1.1.4 "Unit in Concentrates" 1 unit is always 1% of the contained metal in the concentrates per mass unit).

Molybdenum or antimony are sometimes designated as sulphides MoS_2 or Sb_2S_3 respectively, sometimes simply as the element Mo or Sb. For comparison purposes it must therefore be calculated how much Mo is contained in MoS_2.

Conversion is done using atomic weights. The most important atomic weights are listed in Table D9 (Appendix D).

Example. Derive the factor needed to convert a percentage value of MoS_2 in a percentage value of Mo:

atomic weight Mo	95.95
atomic weight S = 32.06 → atomic weight S_2	64.12
	160.07

i.e. the Mo-part is

$$\frac{95.95}{160.07}=0.60$$

Thus the conversion factor for MoS_2 into Mo is 0.60, so that 1% MoS_2 equals 0.6% Mo metal.

Conversely, the conversion factor of Mo into MoS_2 is 1/0.60 = 1.67.

A table of common conversion factors is provided in Table D10, Appendix D.

First Estimates of Grade and Tonnages and Potential Grade and Tonnages

It is common practice in exploration to start with economic evaluations as early as possible and to update these evaluations in parallel with the physical exploration work with an ever improving data base. The purpose of this ongoing process is to have a ready base for go/no-go decisions after each exploration stage before proceeding to the next normally more expensive stage. An economic evaluation needs tonnage and grade information to work with. In an early stage, the geologist has only an tentative idea about expected grades and tonnages based on the initial geological concept and early concrete indications through observations from trenches or a limited number of drill holes. This early idea about grades and tonnages we will call grade potential and tonnage potential.

If the exploration of a possible deposit is well advanced, one can work with geostatistical methods, which take the spatial interdependence of drill hole data into account (see e.g. Wellmer 1998, Stat. Eval.) and are certainly the best way to arrive at the most reliable input data. At an early exploration stage, however, a sufficiently large data base is not available for geostatistical methods. Other cruder methods have to be applied to arrive at approximate estimates of grade and tonnage or potential grade and tonnage. Many exploration projects have a chequered history with many owners. Sillitoe (1995) examined the history of 53 Circumpacific producing base- and precious metal mines. Only a third went from discovery to the stage of producing mine in one go, meaning with one company, for the second third two attempts were necessary, and for the last third up to 11 different companies tried their exploration luck and only the last one was successful to bring the deposit into production. Consequently one frequently deals with a mixed bag of data sets. For example, there might be a property with some percussion hole data, some data from core drilling – some with good core recoveries, some with low core recoveries – some data from chip sampling in trenches and from bulk sampling in an exploration pit. Some holes might have been drilled at very oblique angles in an attempt to show large apparent thicknesses to a potential buyer or farm-in partner. One cannot afford to disregard low quality data. The competition for good exploration projects is fierce, and therefore the maximum information value has to be extracted from all data available, regardless of quality.

In this book, we are dealing only with first order-of-magnitude estimates for grade and tonnage (or the potential of both quantities) aimed at obtaining quick-and-ready economic assessments using any available data. This is common practice for exploration and mining companies at all stages of evaluation when go/no-go decisions are required. More advanced methods for larger data sets are dealt with in Wellmer 1998 (Statistical Evaluations in Exploration for Mineral Deposits) or other geostatistical textbooks for ore reserve estimation.

For this purpose of obtaining quick-and-ready-economic assessments we need in any case the true thickness from a drill hole intersection and a first idea about block sizes. Only this is briefly demonstrated in this book, primarily concerned with economic evaluations, with deriving blocks on cross sections and plan maps.

The advances of computer programmes makes three dimensional (3D-) modelling very easy. They shall not be discussed here. It should be pointed out, however, that with limited data at hand a first volume estimate based on a computer model is not "more correct" than the sectional or polygonal approach.

2.1 Estimation of Volume and Tonnage of Ore Deposits

2.1.1 Calculating the True Thickness

2.1.1.1 Drilling Perpendicular to Strike

This is the standard case. As a rule, a profile is drawn from which the true thickness can be graphically measured. For exact calculations, if the drill length is L_B (Fig. 2.1a), the true thickness (M_w) is given by

$$M_w = L_B \times \sin[180° - (\alpha + \beta)] = L_B \times \sin(\alpha + \beta)$$

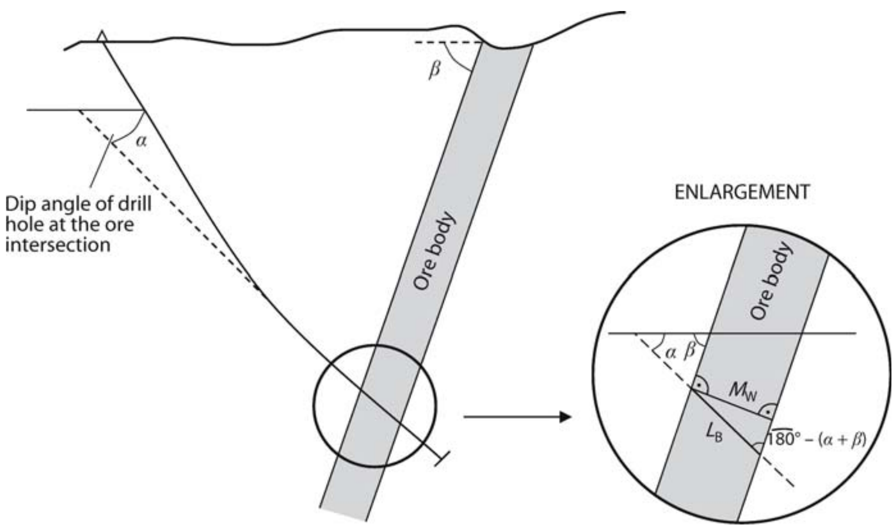


Fig. 2.1a. Vertical section to calculate the true thickness of a drill intersection

Where α is the inclination angle of the drill hole at the intersection of the drill hole with the ore body and β is the dip angle of the ore body. If the drill hole is perpendicular, i.e. perpendicular at the point of intersection, then α is 90° and the relationship will become (see also Sect. 2.2.3.3 and Fig. 2.9)

$$M_W = L_B \times \cos\beta$$

because $\sin(90° + \beta) = \cos\beta$

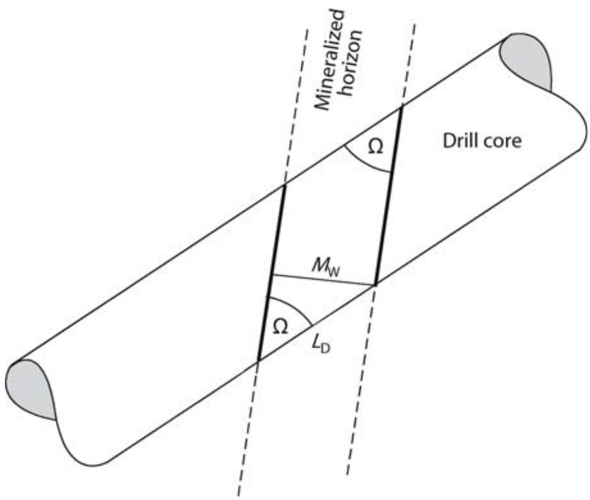
In Wellmer 1998 (Stat. Eval.) in Sect. 7.3, page 48ff and Fig. 18 about the law of perpetuation of errors, it is shown what effects errors in the angles α and β can have. If a drill hole does not intersect an ore body perpendicular, but at an oblique angle, the error for the true thickness increases dramatically at very oblique angles i.e. if the angle between ore body and drill hole is less than 30° or, respectively, more than 150°.

2.1.1.2
Drilling Oblique to Strike (see Appendix B)

The situation can be more complicated, if the drill hole runs oblique to strike. Spatial restrictions such as drilling underground or in mountainous areas often necessitate drilling oblique to strike. Sometimes, however, this method is used by promoters to give the impression of an exaggerated apparent thickness and disguise a low true thickness.

As long as one drills a stratabound horizon with clear hanging and foot wall contacts which are recognizable in drill core, the situation is simple. Let us do a thought experiment: We drill a stratabound deposit. Regardless under which angle you intersect the stratabound ore horizon you will get a core as shown on Fig. 2.1b. There is an angle between the core axis and the stratabound horizon, which we will call Ω. We do

Fig. 2.1b.
Example of a drill core which intersected a stratabound ore horizon at an oblique angle



not have to know anything about strike or dip of the ore horizon. With this angle Ω and the apparent thickness in the drill hole L_D we can determine the thickness of the ore horizon M_W, meaning the normal distance between foot and hanging wall measured at right angles, which is

$$M_W = L_D \sin\Omega$$

So Ω corresponds to the angle 180° – ($\alpha + \beta$) in the enlargement of Fig. 2.1a.

However, especially with vein or other epigenetic mineralizations, foot and hanging walls are frequently very irregular or blurred. Often core losses occur when the drill hole reaches mineralisation because of changes in rock competency. So one just knows in the drill core where the mineralisation starts and ends, but there are no obvious planes from which angles can be taken. We now have to calculate the true width from the known direction and dip of the drill hole in relation to the strike and dip of the mineralized body as best as this can be inferred.

α is the angle of inclination of the drill hole, β the angle of dip of the orebody, γ the angle between the horizontal projection of the drill hole and the dip direction (Fig. 2.2a). In addition, we need δ, the apparent angle of dip of the orebody along the drilling direction.

First we want to express the apparent dip angle δ in terms of the dip angle β and the profile angle γ via the depth h (Fig. 2.2b). The triangle AHG is oriented perpendicular to the strike of the orebody. So the angle between $\overline{\mathrm{AH}}$ and $\overline{\mathrm{GH}}$ is the dip angle β. Therefore

$$h = b \times \tan\beta \tag{2.1}$$

Now we consider the triangle AJG with the apparent dip angle δ. The relationship for h is

$$h = \mathrm{c} \times \tan\delta \tag{2.2}$$

combining Eqs. 2.1 and 2.2 we get

$$b \times \tan\beta = c \times \tan\delta \tag{2.3}$$

In the horizontally lying triangle AHJ the angle between b and c is γ, therefore

$$\frac{b}{c} = \cos\gamma \tag{2.4}$$

Combining Eqs. 2.3 and 2.4 we get

$$\tan\delta = \cos\gamma \times \tan\beta \tag{2.5}$$

To determine now the true thickness M_W we go back to Fig. 2.2a.

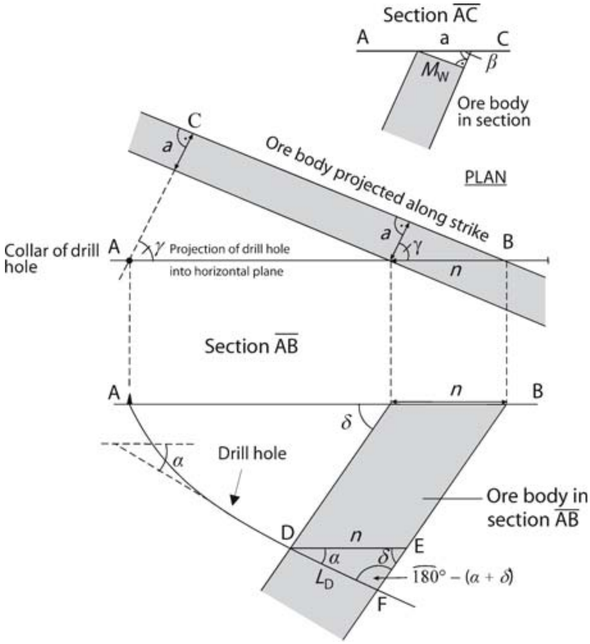


Fig. 2.2a.
Plan and section to calculate the true thickness from a drill hole running oblique to strike

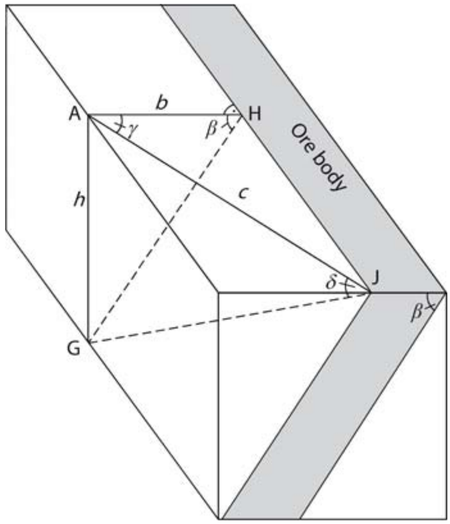


Fig. 2.2b.
Block diagram to calculate the apparent dip angle

From the profile $\overline{AC}$ (Fig. 2.2a) the true thickness M_W can be determined as

$$M_W = a \times \sin \beta \tag{2.6}$$

where a is the apparent horizontal thickness perpendicular to strike.

From the horizontal plan in Fig. 2.2a, with n being the apparent horizontal thickness in drilling direction AB, a can be determined:

$$a = n \times \cos\gamma \tag{2.7}$$

Equations 2.6 and 2.7 combined give

$$M_w = n \times \sin\beta \times \cos\gamma \tag{2.8}$$

n can be derived from the triangle DEF in profile $\overline{\text{AB}}$ (Fig. 2.2a) by using the sinus relation, with L_D being the length of the intersection:

$$\frac{L_D}{\sin\delta} = \frac{n}{\sin(180° - (\alpha + \delta))} = \frac{n}{\sin(\alpha + \delta)}$$

$$n = L_D \frac{\sin(\alpha + \delta)}{\sin\delta} \tag{2.9}$$

Substituting Eq. 2.9 for n in Eq. 2.8; the result is

$$M_W = L_D \frac{\sin(\alpha + \delta)}{\sin\delta} \sin\beta \times \cos\gamma \tag{2.10}$$

Replacing $\cos\gamma$ by the term in Eq. 2.5:

$$\cos\gamma = \frac{\tan\delta}{\tan\beta} = \frac{\sin\delta\cos\beta}{\cos\delta\,\sin\beta} \tag{2.11}$$

results in

$$M_W = L_D \frac{\sin(\alpha + \delta)}{\cos\delta} \cos\beta \quad \text{or} \quad M_W = L_D R_M \tag{2.12}$$

with

$$R_m = \frac{\sin(\alpha + \delta)}{\cos\delta} \cos\beta \tag{2.13}$$

or R_m expressed only with the directly observable angles α (angle of inclination of drill hole), β (angle of dip of the target) and γ (angle of profile between drill direction and dip direction), using Eq. 2.5 and thereby not using the auxiliary angle δ:

$$R_m = \cos\beta(\sin\alpha + \cos\alpha \times \cos\gamma \times \tan\beta)$$

R_m is the thickness reduction factor. In Appendix B, curve sets for R_m are given for various drill hole inclinations (Figs. B1 to B4). At the end of Appendix B, in addition, is a diagram showing at which angle to drill if an optimum length of the intersection is to be obtained when drilling oblique to strike (Fig. B5).

2.1.2
Reserve Estimations Based on Sections

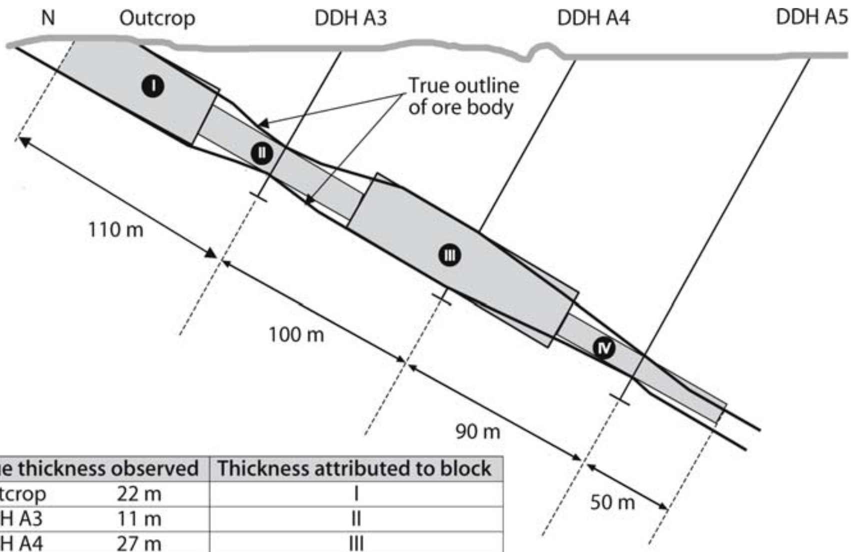
If a deposit has been systematically drilled on sections, e.g. on lines cut in the bush of northern Canada or in the rain forests of South America, reserve calculations will be based on cross-sections along these lines.

To each cross-section is assigned an area of influence corresponding to half the distance to the two adjoining sections. The limits of the blocks thus defined lie exactly halfway between the drill holes (see Fig. 2.3).

The surface area of the blocks on the section are given in Table 2.1.

If we assume the distance between neighbouring sections to be 50 m and the density of the ore to be 4.0 g/cm^3, we arrive at a tonnage on this profile of

$$T = 50 \times 4 \times 5\,595 = 1.119 \text{ million t}$$



True thickness observed		Thickness attributed to block
Outcrop	22 m	I
DDH A3	11 m	II
DDH A4	27 m	III
DDH A5	7 m	IV

Fig. 2.3. Cross-section for reserve calculations with blocks

Table 2.1.
Surface area of the blocks in Fig. 2.3

Block	Surface area (m^2)
I	$55 \times 22 = 1\,210$
II	$(55 + 50) \times 11 = 1\,155$
III	$(50 + 45) \times 27 = 2\,565$
IV	$(45 + 50) \times 7 = 665$
Total	5 595

The important question of how far one can extrapolate from the last drill hole can best be answered geostatistically, if enough data for a geostatistical evaluation are available (Wellmer 1998, Stat. Eval. p. 223). A rule-of-thumb from experience is to use half the distance between drill holes, but seldom more than 50 m. The resources beyond this limit should be considered as resource potential.

2.1.3
Reserve Estimations on the Basis of Plan Maps

Drilling in mountainous terrains or residential areas, where suitable sites for drill holes are restricted, will result in irregularly spaced intersections. Drill holes with significant hole deviations produce the same effect. In such cases, instead of using cross sections, it is better to work with plan maps for inclined tabular deposits or palinspastic maps for folded ones.

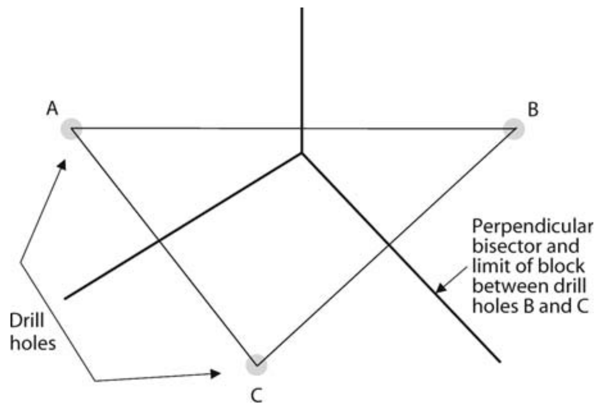


Fig. 2.4. Construction of equidistance lines

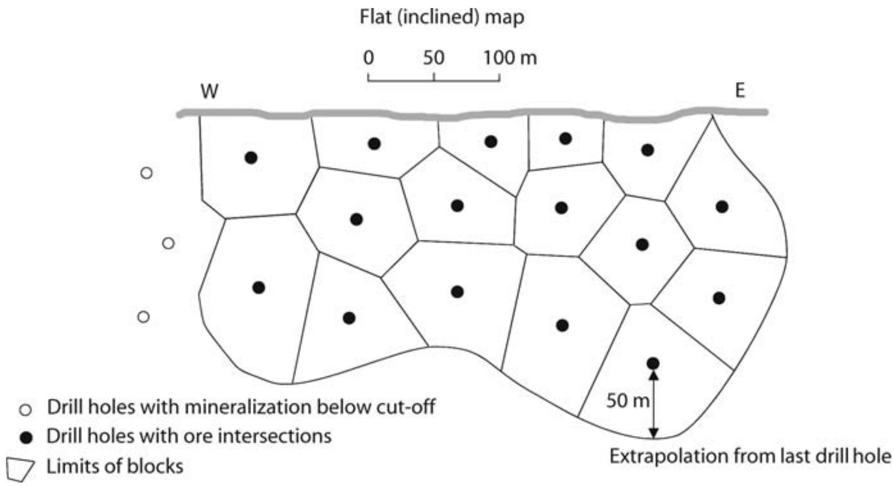


Fig. 2.5. Plan map for reserve calculation with blocks

Usually the blocks (see Fig. 2.4 and 2.5) are delimited by drawing equidistance lines to the adjoining drill holes. As Fig. 2.5 shows, applying this method creates polygons. That is the reason why this method is also called the polygon method. The block method of Sect. 2.1.2 and the polygon method definitely have weaknesses (Giroux 1990). If enough data are available and geostatistical tools can be applied, these are to be preferred (Wellmer 1998, Stat. Eval. Sect. 13.3). Block and polygon methods are, however, well suited for a first orientation. The surface area of the blocks is then multiplied by the thickness and density as in the example in Sect. 2.2.1. The construction of the equidistance lines is explained below and shown in Fig. 2.4.

By connecting adjoining boreholes with each other a net of triangles is created. The equidistance lines, perpendicular bisectors, halve the sides of these triangles and bound the polygonal area of influence centred on each hole. The western border of the deposit in Fig. 2.5 is defined by drill holes which encountered uneconomic mineralisation (grades below cutoff). How to determine cutoff limits will be dealt with in Sect. 10.1.

2.2 Grade Estimation and Weighting

Grade estimations will only be dealt with in this book if the calculations involve simple weighting with, for example, assay intervals in drill holes or with reserve block volumes. This is sufficient for a global estimate of a deposit, or potential deposit in the early stages of exploration. A global estimate is the estimate of grade (or tonnage) of the total deposit, contrary to a block estimate. As will be shown later in Chap. 11 we assume in our simplified economic calculations that the grades during each mining year are the same, meaning the grades of the global estimate. If one wants to model the deposit more in detail and simulate the change of grades from year to year, one has to use geostatistical methods for grade determinations of blocks (Wellmer 1998, Stat. Eval. Sect. 13.4).

In this chapter we will also deal with the problem of deriving grades from visual inspections. When there are old adits with visually recognizable mineralisation on a property offered for sale, it is possible to get a quick grade estimate as helpful preliminary information for a global estimate.

2.2.1 Weighting in Reserve Calculations

One of the most frequent calculations geologists have to do are weightings, e.g. for the calculation of the average grade of a drill hole from assay intervals of different lengths or of the average grade of a deposit from the combined grades of individual, unequal blocks.

If G_1 to G_n are the values whose weighted average is to be determined, and a_1 to a_n are the weighting factors, then the weighted average is $\overline{G}_w$:

$$\overline{G}_w = \frac{G_1 a_1 + G_2 a_2 + \cdots + G_n a_n}{a_1 + a_2 + \cdots + a_n} = \frac{\sum_{i=1}^{n} G_i a_i}{\sum_{i=1}^{n} a_i} \tag{2.14}$$

Assignment. The analytical results from unequal, but consecutive intervals are provided in Table 2.2.

What is the weighted mean?

The weighted mean is

$$\bar{G}_w = \frac{2.1 \times 1 + 8.4 \times 1.5 + 12.0 \times 0.75 + 10.2 \times 1.25}{1.00 + 1.50 + 0.75 + 1.25}$$

$$\bar{G}_w = \frac{36.45}{4.50} = 8.10\%\ \text{Pb}$$

Careful consideration must be given to the choice of the correct weighting factors. The weighting in the above example assumes that the densities are constant (or the difference in densities is negligible). If this assumption is not justified, as it often happens with vein deposits in which massive sulphide and disseminated ore occur together, then the density must also be allowed for in the weighting.

Assignment. Calculate the weighted mean for the drill intersections in a barite deposit presented in Table 2.3.

The weighted average is

$$\bar{G}_w = \frac{70 \times 1.5 \times 3.7 + 98 \times 2.8 \times 4.2 + 50 \times 1.0 \times 3.4}{1.5 \times 3.7 + 2.8 \times 4.2 + 1.0 \times 3.4}$$

$$\bar{G}_w = \frac{1\,771.0}{20.7} = 82.7\%\ \text{BaSO}_4$$

An additional exercise will show how important it is to perform the weighting correctly.

Table 2.2.
Analytical results from unequal, consecutive intervals

Analytical result (% Pb)	Sample interval (m)
2.1	1.00
8.4	1.50
12.0	0.75
10.2	1.25

Table 2.3.
Drill intersections in a barite deposit

Analytical result (% $BaSO_4$)	Sample interval (m)	Density (g/cm^3)
70	1.50	3.7
98	2.80	4.2
50	1.00	3.4

Assignment.

1. *Question:* Which mistake crept into the following reserve calculation and how big is it?
2. *Case Description:* A nickel laterite deposit has been sampled by pits. The pits are 25 m apart. Each pit has therefore an area of influence of 12.5 m to each side. The lines on which the pits are located are at a distance of 50 m so that an area of $50 \times 25 = 1\,250$ m is allocated to each pit. Two different types of ore with different densities were encountered in the pits (Fig. 2.6): the laterite (L) has an in situ density of 1.25, the decomposed serpentinite (ZS) has an in situ density of 1.0 g/cm^3.
   i. The average grades of the pits were determined by weighting with the lengths:

$$\text{Pit A:} \frac{4\times1.2+4\times2.9}{8} = 2.05\%\ \text{Ni}$$

$$\text{Pit B:} \frac{4\times1.1+3\times3.5}{7} = 2.13\%\ \text{Ni}$$

   ii. In addition, the densities were determined by weighting with the sample lengths:

$$\text{Pit A:} \frac{4\times1.25+4\times1.0}{8} = 1.125$$

$$\text{Pit B:} \frac{4\times1.25+3\times1.0}{7} = 1.143$$

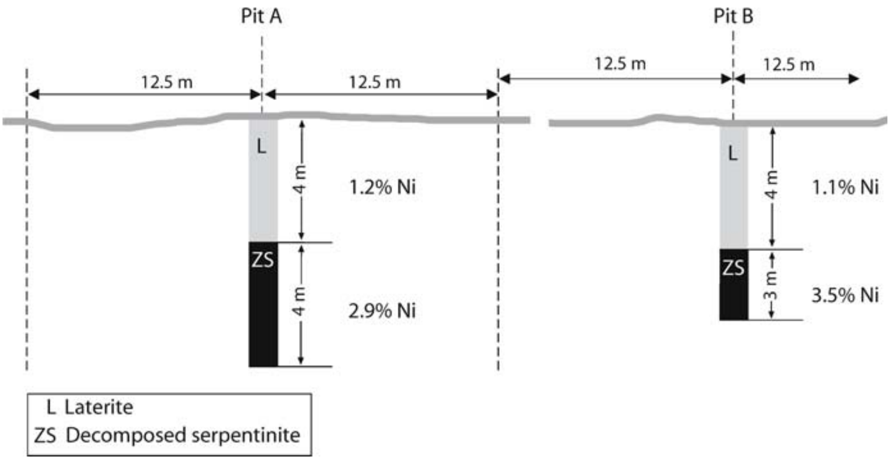


Fig. 2.6. Pit sampling in a nickel laterite deposit

iii. Since each pit has been allocated a surface area of 1 250 m^2 and the pits have a depth of 7 and 8 m respectively, the following tonnages were obtained:

Pit A: $1\,250 \times 8 \times 1.125 = 11\,250$ t with 2.05% Ni
Pit B: $1\,250 \times 7 \times 1.143 = 10\,000$ t with 2.13% Ni

iv. The nickel grade of the total tonnage was determined by weighting with the corresponding tonnages:

$$\frac{2.05 \times 11\,250 + 2.13 \times 10\,000}{21\,250} = 2.09\%\ \text{Ni}$$

3. *Correct Answer*: The following mistake was made in step *(i)*: the average grades of the individual pits were not determined by directly weighting with the densities. The correct procedure is

i. Pit A: $\dfrac{4 \times 1.25 \times 1.2 + 4 \times 1.0 \times 2.9}{4 \times 1.25 + 4 \times 1.0} = 1.96\%\ \text{Ni}$

ii. Steps *(iii)* and *(iv)* are correct. Using the correct grades step *(iv)* will result in

$$\frac{1.96 \times 11\,250 + 2.00 \times 10\,000}{21\,250} = 1.98\%\ \text{Ni}$$

The mistake leads to an overestimation of 6%. The mistake is unacceptably large for the purpose of reserve calculation, both from a purely mathematical as well as economic point of view.

2.2.2
Grade Calculations for Massive Ore Shoots

Determining grades through visual estimates is another example where correct weighting with densities is of importance. For vein-type ore deposits in which the ore occurs massive, visual grade control often plays a significant role.

Assignment. We are dealing with a steep vein which, for technical reasons, has to be mined at a minimum thickness of 1 m. In the vein a massive stibnite shoot occurs. How many percent antimony correspond to a band of 1 cm stibnite?

Stibnite has a density of 4.5 g/cm^3, the wall rock a density of 2.6 g/cm^3.

Theoretically stibnite (Sb_2S_3) contains 71.7% Sb. We assume 70%.

The thickness of the massive stibnite band has been measured at intervals of 1 m. We consider a vein surface of 1 m^2 and a mining width of 1 m.

1. With 1 m mining width and 1 cm stibnite band, the tonnage of the wall rock per 1 m^2 vein surface is

$$0.99\ \text{m} \times 1\ \text{m}^2 \times 2.6\ \text{t/m}^3 = 2.574\ \text{t}$$

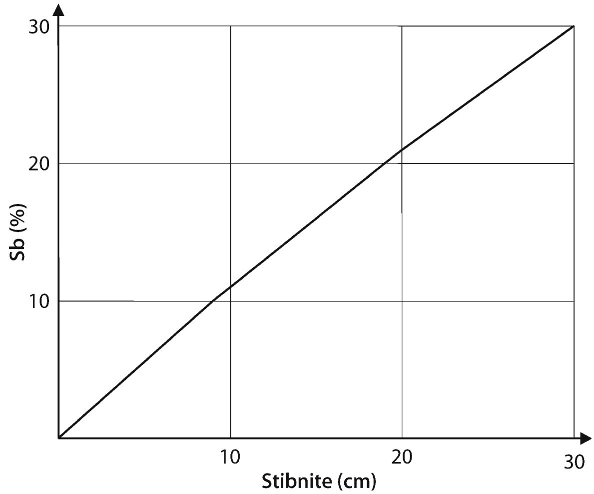


Fig. 2.7.
Graph for conversion of massive ore thicknesses (here, stibnite)

2. 1 cm stibnite per 1 m^2 vein surface corresponds to

$$1 \times 10^4\ (cm^3) \times 4.5 \left(\frac{g}{cm^3}\right) \hat{=} 45\ (kg)$$

   i.e. the total tonnage per 1 m vein surface is 2.619 t. With a conversion factor of 0.7 : 45 kg stibnite $\hat{=}$ 31.5 kg Sb
3. Conclusion: 1 cm stibnite $\hat{=}$ 31.5 / 26.19 $\hat{=}$ 1.2% Sb

Since the thickness of the lighter wall rock decreases with increasing thickness of the ore shoot, this conversion factor cannot be used as a linear function with greater ore thickness.

30 cm stibnite do not correspond with 36% Sb but with 29.8% Sb! It is better to construct a graph so that the grades can be quickly derived from the massive ore thicknesses (Fig. 2.7).

Although the ore phases often appear to be pure, a very fine intergrowth with gangue minerals is frequently revealed under the microscope. It is therefore advisable to check these conversion factors analytically and, if necessary, to correct them by means of a factor. A good example are the detailed analyses in the lead-zinc-vein mine Bad Grund in the Hartz mountains in Germany (Stedingk 2006). In the ore shoots the thicknesses of the sphalerite and galena bands were regularly measured optically and these measurements were the basis of grade control and mine planning. Whereas the predicted zinc grades agreed reasonably well with the grades of the run-of-mine ore, the lead grades were considerably overestimated. Microscopical studies showed an intimate intergrowth of galena with quartz and siderite gangue. This intimate intergrowth created the illusion of massive galena mineralisation. To bring predicted and realized grades into agreement coarse grained galena zones could be taken at face value, but the values of the visual measurements of the fine grained intergrown zones had to be divided by a factor of three. So in the mine the term "third-galena" was coined for this mineralogical phase.

2.2.3 Grade Determinations from Geophysical Downhole Logging

2.2.3.1 Introduction

In uranium exploration, it is common practice to use percussion holes, so no direct samples are obtained. However, because uranium and its radioactive decay products emit gamma radiation they can be detected and measured as counts per second "cps" in the drill holes by using down-the-hole gamma ray instruments[3]. In the evaluation of the geophysical measurements weighting plays an important role in determining grades.

Strictly speaking, uranium itself does not emit detectable amounts of gamma radiation. The gamma radiation is caused by the decay products of uranium, principally bismuth-214. In radiometric surveys, one assumes that the daughter products of the decay are in equilibrium. If this is not the case, one has to work with correction factors (see below Sect. 2.2.3.4 where correction factors are discussed). The procedure of determining uranium grades from gamma radiation cannot be used if other strong gamma emitters like thorium or potassium are present in significant amounts. Because the uranium is not measured directly, such values are not given as units of ppm or percent of U_3O_8 but as equivalent value. In the notation for this, an e is prefixed to signify that we are dealing with an equivalent value; for example, 150 ppm eU_3O_8.

2.2.3.2 Down-the-Hole Logs and Their Use

Grades are deduced from the gamma ray measurements. In consequence, it is common practice to diamond drill a hole with core after a certain number of percussion holes, usually 10, in order to be able to determine grades on core material by chemical analysis. This serves as the basis for calibration of the gamma-ray log results.

Drill hole logs are also used for other elements, such as lead, zinc, copper and iron. Fricke et al. (1987) describe a down-the-hole method which consists of introducing a radioactive source into the drill hole which induces a secondary radiation that can be measured with the help of an X-ray fluorescence device.

The following information can be determined from down-the-hole measurements:

a the thickness of the mineralized horizon
b the average grade of the mineralized horizon using the accumulation factor $G \times T$, i.e. the product of grade times thickness (see also Sect. 1.2.4)

This is illustrated with a gamma-ray log from an uranium exploration drill hole (Fig. 2.8). For a detailed explanation the reader is referred to handbooks available from the International Atomic Energy Agency (IAEA 1982, 1986).

[3] Gamma radiation is measured with crystal sensors which emit light flashes (scintillations) when they are hit by gamma particles. The light flashes are counted electronically in counts per second.

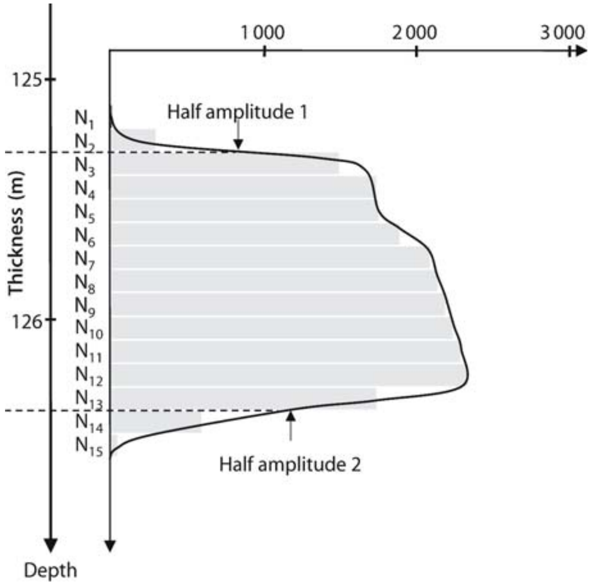


Fig. 2.8.
γ-log of an uranium exploration hole

2.2.3.3 Determination of Thickness

The thickness of the mineralized horizon normally is determined with the help of the called half-amplitude, where the measurements reach half of the value of the peak. It is more or less equivalent to the called half-width used otherwise in geophysics to interpret anomalies. For the log-curve in Fig. 2.8 the first peak occurs at 125.40 m. The log-value there is 1 760 cps (counts per second). Consequently the first half-value – half-amplitude – is 880 cps. At the lower end of the anomaly peak 2 occurs at 126.25 m. The log-value here is 2 440 cps. So the second half-value – the half-amplitude – is 1 220 cps. The half-value points should approximately coincide with the points of inflexion of the log-curve.

The two half-amplitude values are marked on the log-curve, and so the depth is determined. These are the lower and upper boundaries of the mineralisation which in the case of Fig. 2.8 occurs at 125.29 m and 126.4 m. So, in this case, the thickness is 1.1 m. We know from experience that the method works well when the thickness is at least 1.0 m. When the thickness is lower, corrections must be applied.

If the drill hole intersects the mineralization at right angle – for example, the drill hole is vertical and the mineralized zone horizontal – then the thickness obtained in this way is the true thickness M_w. If this is not the case, the thickness is the apparent thickness M_s which has to be multiplied by $\cos\beta$, whereby β is the dip angle of the mineralized horizon (see Fig. 2.9 and Sect. 2.1.1.1):

$$M_W = M_S \times \cos\beta$$

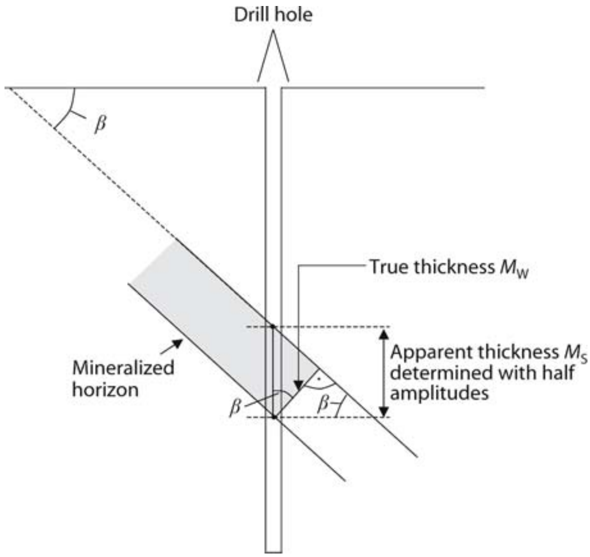


Fig. 2.9. Calculation of the true thickness from the apparent thickness in an uranium exploration drill hole

2.2.3.4 Determination of Grade

The grade is determined with the help of the accumulation factor $G \times T$, the product out of grade and thickness. The area under an anomaly F_A is proportional to the accumulation factor $G \times T$. Basically there are three methods for determining the accumulation factor $G \times T$ which differ in the treatment of the anomaly area outside of the two half-amplitude points:

- the total area method
- the tail-factor method and
- tails cutoff method

To compare these three methods the area of the anomaly is divided into three parts:

- area 1 is the tail-end area above the half-amplitude point 1 in Fig. 2.8, i.e. squares N_1 and N_2
- area 2 is the central anomaly area between the two half-amplitude points 1 and 2
- area 3 is the tail-end area below the half-amplitude point 2, i.e. squares N_{14} and N_{15}

All three methods determine the central anomaly area 2 between the two half-amplitude width the same way, as will be shown below. With the total area method the three areas, the two tail-end areas and the central area, are treated the same way. This is the example illustrated below. With the tail-factor method the tail-end areas are taken into account by multiplying the sum of the two half-amplitude points by an empirical tail-factor which is proportional to the width considered. With the tails cutoff method, used often in practice, the two tail-end areas are not considered at all because their contribution to the grade of a mineralised horizon is only minor and is also influenced by values in the hanging and footwall of the horizon under consideration, causing "dilution".

The factor of proportionality for determining the accumulation value $G \times T$ is called the K-factor in the literature. Frequently a correction factor F has to be applied to the K-factor. The K-factor assumes ideal conditions. In actual practice it is often necessary to apply a correction factor to the K-factor to take into account the real diameter of the drill hole, the influence of the drilling mud etc. For details, the reader is referred to the above mentioned IAEA handbooks. For the sake of simplicity we assume that the correction factor F is 1 in our example. In addition, we assume that uranium and its daughter products are in equilibrium (see Sect. 2.2.3.1).

So we have the equation

$$G \times T = K \times F_A$$

The area of the anomaly F_A theoretically has to be determined by integration under the anomaly curve. In praxis, it is determined by considering single segments of the anomaly. In the example of Fig. 2.8, we choose 10 cm long segments. Rectangles are constructed, which have the same area as the log curve in this segment. In the example of Fig. 2.8 these are the rectangles N_1 to N_{15}. For these segments the measurement values are determined from the log and multiplied by the width of the segment, in this case 0.10 m, so that for each segment we have a value with the unit (cps m). The results are listed in Table 2.4. All values are added then. In our case the sum is $F_A = 2\,330$ cps m. Now the sum has to be multiplied with the K-factor, which determines the relationship between the U_3O_8 content and the count rate. In our case the K-factor shall be 1.5 ppm eU_3O_8/cps. For our example this results in

$$G \times T = K \times F_A$$

$$G \times T = 1.5 \times 2\,330 = 3\,495 \text{ ppm } eU_3O_8 \times \text{m}$$

This value has to be divided now by the thickness in the drill hole as determined in Sect. 2.2.3.3 above (It is the apparent thickness M_s as encountered in the hole). In our example the thickness was 1.1 m. So the average grade of the mineralized horizon using the total area method is

$$G = \frac{3\,495}{M_s} = \frac{3\,495}{1.10} = 3177 \approx 3180 \text{ ppm } eU_3O_8$$

In modern γ-log instruments this calculation procedure is "built in", so after determination of the half-width the instrument calculates the eU_3O_8 grade automatically. In addition, manufacturers of modern equipment provide manuals describing the conversion of γ-log readings to eU_3O_8.

If we would have applied the tails cutoff method, we would consider only the squares N_3 to N_{13} in Fig. 2.8 and Table 2.4. The sum of the areas in cps × m would be 2 226. Multiplied with the K-factor of 1.5 and divided by the thickness of 1.10 m we would get 3 035 ppm eU_3O_8, a difference of less than 5%.

Table 2.4. Calculation of the anomaly area F_A

Area segment	Count rate/second	Depth interval	Area ((counts/s) m)
N_1	40	0.10	4
N_2	350	0.10	35
N_3	1510	0.10	151
N_4	1760	0.10	176
N_5	1780	0.10	178
N_6	1890	0.10	189
N_7	2050	0.10	205
N_8	2150	0.10	215
N_9	2200	0.10	220
N_{10}	2260	0.10	226
N_{11}	2380	0.10	238
N_{12}	2420	0.10	242
N_{13}	1860	0.10	186
N_{14}	600	0.10	60
N_{15}	50	0.10	5
Sum = Total area			2330

2.2.4 Grade Determination from Coverage Data Per Unit Area

For mineralization of large aerial extent and highly variable thickness, like the Deep Leads gold deposits in Australia, mentioned in Sect. 1.1.1 Fathom, and deposits like the nickel-, cobalt-, and copper-containing deep-sea manganese nodules for which thickness is insignificant, a coverage factor is given in kg metal per unit area. A coverage factor used also to be applied to the copper shale mines and uranium mines in the Erzgebirge in the former German Democratic Republic, the third largest uranium producer in the world in its time. There the term "spreading" was coined for such a grade intensity unit.

If it is necessary to calculate mining grades, the height of the necessary mining opening and the density of the extracted material have to be taken into account.

Example: In an area of the former copper shale mining district in eastern Germany the coverage (spreading) is 65 kg Cu/m^2; the density of the ore is 2.6 g/cm^3.

- *Case a:* The mining is planned to be conventional by drilling and blasting. The mining height will be 1.20 m. So, for 1 m^2 of the mineralisation the amount of run-of-mine ore will be

$$1 \times 1.20 \times 2.6 = 3.12 \text{ t} = 3\,120 \text{ kg}$$

with a coverage (spreading) of 65 kg Cu/m^2 the run-of-mine ore will have a grade of

$$\frac{65}{3120} = 0.021 \quad , \quad \text{i.e. } 2.1\%\ \text{Cu}$$

- *Case b:* The mine management decides to use a specialized mining tool, a shearer, which allows the mining width to be reduced to 0.30 cm. Hence, for 1 m^2 of the mineralized area only 780 kg of run-of-mine ore will be produced:

  $$1 \times 0.3 \times 2.6 = 0.78\ \text{t} = 780\ \text{kg}$$

  Consequently, the grade expected is

  $$\frac{65}{780} = 0.083 \quad , \quad \text{i.e. } 8.3\%\ \text{Cu}$$

Dealing with Data of Multi-Element Deposits

Generally speaking, multi-element deposits contain more than one metal as a significant source of revenue. This applies to most non-ferrous metal deposits. Complex volcanogenic sulphide deposits often contain five components: Cu, Pb, Zn, Ag, and Au. In this case, each individual component has to be weighted separately as shown in Sect. 2.2.1.

3.1 Metal Ratios

To show zoning in ore deposits, for example in isoline maps, frequently metal or element ratios are used. If these ratios have only a relatively narrow spread, the direct ratios can be used, e.g. the Au:Ag ratios in Au-deposits. In most deposits this ratio varies between 2 : 1 to 10 : 1.

If the ratios have a large spread, it is helpful to use a standardization procedure. Table 3.1 shows a list of Pb- and Zn-values of a Pb/Zn-deposit. If one uses Pb/Zn or Zn/Pb ratios, a large range has to be dealt with, which is difficult to handle on the normal scale of an isoline map. To simplify the calculation, the data is standardized by

$$\frac{\mathrm{Zn}}{\mathrm{Zn} + \mathrm{Pb}}$$

In case Zn = 0, the ratio is 0, in case Pb = 0, the ratio equals 1, so the ratio can only fluctuate between 0 and 1.

Table 3.1. Pb- and Zn-values of a Pb/Zn-deposit

Pb (%)	Zn (%)	Pb/Zn	Zn/Pb	Zn/Zn + Pb
1	10	0.1	10	0.91
10	1	10	0.1	0.09
5	0.1	50	0.02	0.02
0.1	5	0.02	50	0.98
6	0	8	0	0
0	6	0	8	1

The fineness of precious metals, which is dealt with in Sect. 1.1.4 (Precious Metal Units) is such a standardized metal ratio, which has only to be multiplied by 1 000:

$$\text{fineness} = \frac{\text{Au}}{\text{Au} + \text{Ag} + \text{Me}} \times 1\,000$$

where Me stands for other metal impurities.

3.2 Ternary Diagrams

As in petrology, ternary diagrams are often used to display the quantitative relationship between elements in multi-element deposits.

As an example, a deposit with 7% Zn, 2% Cu and 4% Pb has been chosen. The ratio of elements is demonstrated in a Cu-Pb-Zn diagram.

The construction of the diagram involves two steps:

- *Step 1*: The relative ratio of the elements is determined:

  Zn + Cu + Pb = 100% , i.e.

  Zn 54% relative

  Cu 15% relative

  Pb 31% relative

- *Step 2:* Plotting of the element ratios in the Cu-Pb-Zn diagram (Fig. 3.1a)

The corner points of the diagram represent ore which contain one metal only, that is, they correspond to 100% Zn or Cu or Pb grades respectively; in each case, the two other elements are not represented.

The side opposite the Zn-corner, between Cu and Pb, means: each ore that plots on this side contains no Zn, only Cu and Pb in proportions varying between 0 and 100%.

To find the point corresponding to our example, we first look for the 54% Zn-line in the diagram of Fig. 3.1a. Next we find the 15% Cu-line. The point of intersection is the wanted point. Since the three elements together have to add up to 100%, the Pb-grade can be read off automatically.

If, for example, all points taken from different parts of the same deposit or from a group of related deposits plot in the same corner of the ternary diagram, a convention is frequently used whereby only this section of the diagram is plotted in detail with the relative position of the selected corner schematically shown for the purpose of orientation in a small ternary diagram (see Fig. 3.1b).

If it becomes necessary to plot additional points which are, for the sake of argument, relatively Zn-rich, then using the above example the Zn-corner of the diagram in Fig. 3.1a could be selected with the other corners fixed at 50% Cu and 50% Pb, as shown in Fig. 3.1b.

Fig. 3.1a.
Pb-Zn-Cu ternary diagram

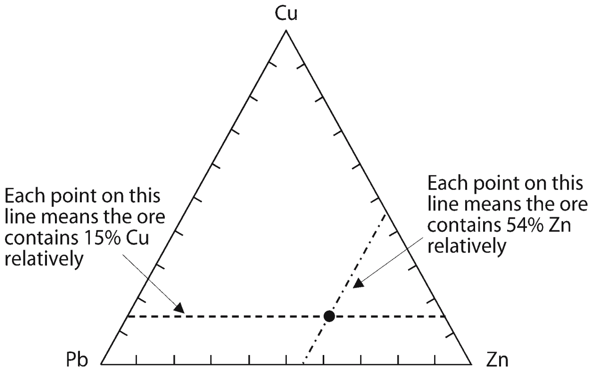
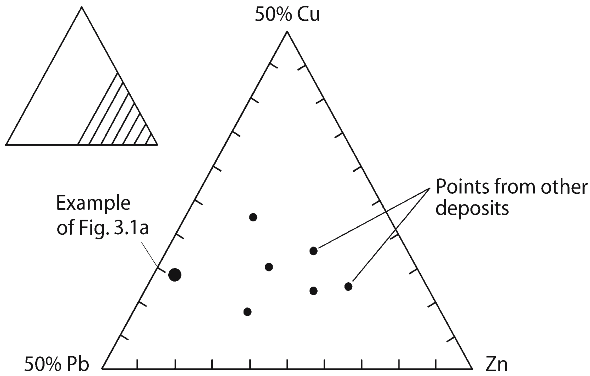


Fig. 3.1b.
Modified ternary diagram of Fig. 3.1a



3.3
Regression Analysis

Often it is worthwhile to examine how the major elements in a complex polymetallic ore deposit correlate with each other, or how these correlate with minor elements. This is achieved by means of regression analysis.

Of course it only makes sense to attempt a linear regression analysis if a linear correlation can be assumed. Frequently, however, the relationship is non-linear. Exponential relationships are important in cost relationships of different mines due to economics of scale, which will be dealt with in Sect. 9.2.2.

Strictly speaking there are two lines of regression, depending whether x is related to y or y related to x. This is illustrated in Fig. 3.2. If there is an ideal correlation, all data points lie on the line of regression and the correlation coefficient r, explained below in this chapter, equals one (Fig. 3.2c).

If no correlation exists at all, then the correlation coefficient r equals zero. If we relate y to x, the line of regression is $y = \overline{y}$, whereby $\overline{y}$ is the arithmetic mean of the y-values (Fig. 3.2a). Vice versa, if we relate x to y, in the case of zero correlation the line of regression has the equation $x = \overline{x}$, whereby $\overline{x}$ is the arithmetic mean of the x-values (Fig. 3.2a). Both lines of regression are perpendicular to each other.

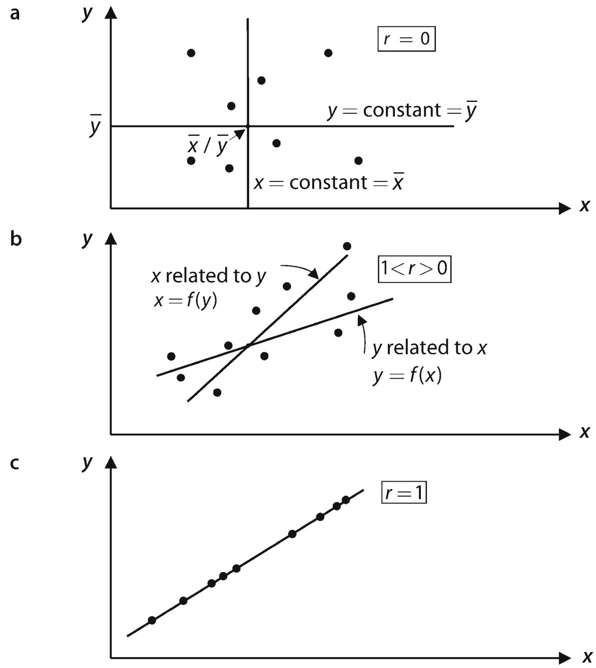


Fig. 3.2. Lines of regression related to x and y for **a** $r = 0$, **b** $1 < r > 0$, **c** $r = 1$

With increasing correlation both lines of regression move towards each other, meaning the angle between the two lines of regression decreases (Fig. 3.2b). This is the "regression scissor". Mathematically, the y-distances from the data points to the line of regression are minimised in the first case, the x-distances in the second case.

There are two cases to be distinguished: positive and negative correlation.

a *Positive correlation* (Fig. 3.3a): With increasing x, y increases too. Silver for example is often positively correlated with lead.
b *Negative correlation* (Fig. 3.3b): With increasing x, y decreases. In volcanogenic Cu-Zn deposits, for example, Cu and Zn are often negatively (or inversely) correlated.

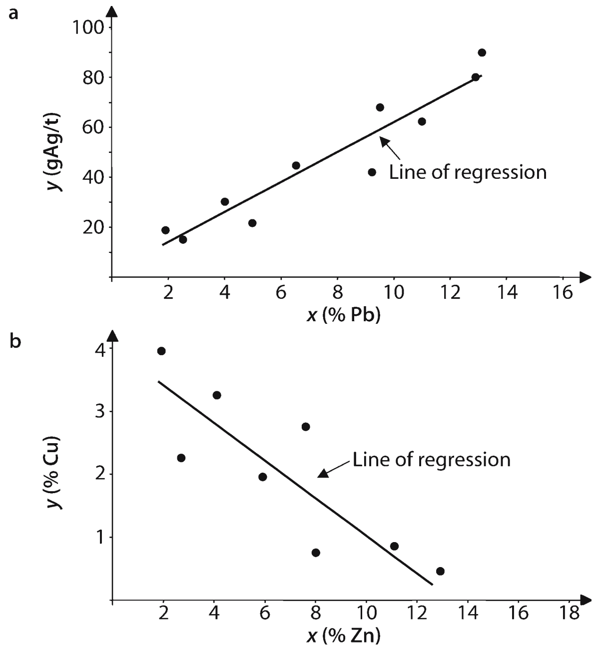
Regression follows the general equation of a straight line:

$$y = a \times x + b$$

The straight line is determined in a way that the distances of individual points (x_1, y_i) to the straight line are minimised, i.e. the straight line is the best fit to the points, with a and b being the regression coefficients. The equations for the regression coefficients are

$$a = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}$$

Fig. 3.3.
a Example of a positive correlation. **b** Example of a negative correlation



$$b = \overline{y} - a\overline{x}$$

with $\overline{x}$ and $\overline{y}$ being the arithmetic mean:

$$\overline{y} = \frac{\sum y_i}{n}$$

$$\overline{x} = \frac{\sum x_i}{n}$$

To determine the degree of correlation, the correlation coefficient r is calculated. $r = 0$ if there is no correlation at all, and $r = 1$ in case of a perfect correlation, i.e. when all points lie on the regression line.

The square of the correlation coefficient is determined in the following way:

$$r^2 = \frac{\left[\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}\right]^2}{\left[\sum x_i^2 - \frac{(\sum x_i)^2}{n}\right] \times \left[\sum y_i^2 - \frac{(\sum y_i)^2}{n}\right]}$$

r^2 is also called the coefficient of determination B. It is a measure of the degree of correlation. It indicates what percentage of the distribution can be explained by linear regression.

Assignment. During the first sampling of an alluvial columbium-tantalum deposit concentrates were panned at various locations on the concession. The samples contained the columbium and tantalum grades presented in Table 3.2. How are Cb and Ta correlated?

The Ta-grades are called x_i, the Cb-grades y_i. Next, the auxiliary values are computed.

$$a = \frac{7\,669.82 - \frac{186.7 \times 213.8}{5}}{7\,369.31 - \frac{(186.7)^2}{5}} = \frac{7\,669.82 - 7\,983.29}{7\,369.31 - 6\,971.38} = -0.79$$

$$b = 42.8 + 0.79 \times 37.3 = 72.3$$

The regression line therefore has the equation

$$Y_{(\text{Cb-Cont.})} = -0.79x_{(\text{Ta-Cont.})} + 72.3$$

The resulting correlation coefficient is

$$r^2 = \frac{\left[7\,669.82 - \frac{186.7 \times 213.8}{5}\right]^2}{\left[7\,369.31 - \frac{(186.7)^2}{5}\right] \times \left[9\,390.02 - \frac{(213.8)^2}{5}\right]}$$

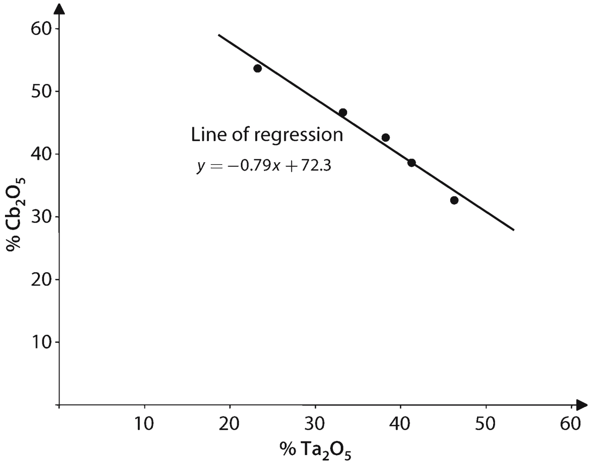
$$r^2 = \frac{[7\,669.82 - 7\,983.29]^2}{[7\,369.31 - 6\,971.38] \times [9\,390.02 - 9\,142.09]} = \frac{(-313.47)^2}{(397.93 \times 247.93)} = 0.996$$

This very high correlation coefficient, indicating an almost ideal negative correlation (compare Fig. 3.4 and Fig. 3.2c), suggests that Cb and Ta substitute each other in

Table 3.2. Columbium and tantalum grades of an alluvial Cb-Ta deposit

Concentrate	Ta_2O_5 (%)	Cb_2O_5 (%)	x_i	x_i^2	y_i	y_i^2	$x_i y_i$
C_1	22.3	54.5	22.3	497.29	54.5	2 970.25	1 215.35
C_2	47.9	33.8	47.9	2 294.41	33.8	1 142.44	1 619.02
C_3	33.4	45.8	33.4	1 115.56	45.8	2 097.64	1 529.72
C_4	43.7	38.5	43.7	1 909.69	38.5	1 482.25	1 682.45
C_5	39.4	41.2	39.4	1 552.36	41.2	1 697.44	1 623.28
Sum			186.7	7 369.31	213.8	9 390.02	7 669.82
			$\bar{x} = 37.3$		$\bar{y} = 42.8$		

Fig. 3.4.
Negative correlation of Cb and Ta in five concentrates



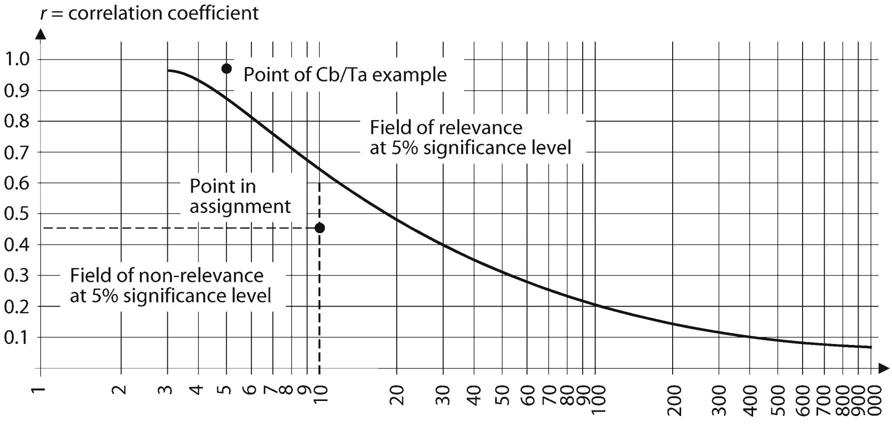


Fig. 3.5. Minimum correlation coefficients at a significance level of 5%

a mineral, here tantalocolumbite. Since r^2 (or B) = 0.996, the coefficient of determination is 99.6%, i.e. 99.6% of the distribution can be explained by linear regression.

If the correlation coefficient r is smaller, e.g. closer to zero than to one, the question arises whether the correlation between two elements is real or only apparent due to a chance distribution of a limited number of samples. We will only briefly discuss this statistical problem. For details the reader is referred to statistical textbooks (e.g. Weaver 1963, see also Wellmer 1998, Stat. Eval., p. 322).

To answer this question statisticians work with the called zero hypothesis. They select a number which is the percentage of the number of times they expect no correlation to exist.

This is called the significance level. A common selection for a significance level is 5%, meaning there is a 1-in-20 chance that there is no correlation and a 19-in-20 chance that the correlation is real. Minimum correlation coefficients then can be calculated as a function of the number of data pairs (Fig. 3.5).

Assignment. We have 10 analyses from zinc concentrates each for zinc and mercury. The correlation coefficient is $r = 0.45$. Is the correlation between zinc and mercury significant at the 5% significance level?

We use Fig. 3.5. For 10 sample pairs the minimum correlation coefficient at the 5% significance level is $r = 0.63$. At this chosen significance level the correlation is therefore not significant. However if we go back to our Cb/Ta-example above, for 5 sample pairs the correlation coefficient of $r = 0.998$ ($r^2 = 0.996$) certainly plots in the significant field.

3.4 Standardizations

If we have different parameters of a deposit, like the elements of a complex ore deposit, and we know a relationship between at least two of them, we can accentuate the relationships between the other parameters more clearly by standardizing with the help of the known relationship.

Example: In a mining district several Zn-Pb-Ag-ore bodies occur in carbonates with varying amounts of Hg and Ag. Through detailed investigations of the sphalerites and the production of pure zinc concentrates the following relationship has been established:

$$1\%\ \text{Zn} \triangleq 7\ \text{g/t Hg}$$

It is also known, however, that Hg is not only bound to sphalerite but also to fahlore and that fahlore does not only carry Hg but also Ag values. In addition, the Pb mineral galena also contributes to the Ag-content of the ore. So a simple relationship between Hg as a linear function of Zn and Ag in the ore is not to be expected. It is a case of multiple regression, which will not be dealt with here. Instead, via a standardization using the known Zn/Hg relationship, the relationship between

Table 3.3.
Analyses of ore lenses

Lens	Zn (%)	Hg (g/t)	Ag (g/t)
1	4.1	35	53
	3.5	26	48
	7.2	55	90
	6.8	56	102
2	3.8	45	66
	4.8	58	78
	6.4	79	110
	6.7	85	120

Ag and Hg can be made clearer graphically. The influence of Zn is practically eliminated in this way.

From the two ore lenses in this example we have the analyses presened in Table 3.3.

Applying now the relationship 1% Zn $\triangleq$ 7 g/t Hg the Hg-values are standardized against Zn-values (Table 3.4).

The differences between real Hg-values and standardized Hg-values are plotted as a function of the Ag-values in Fig. 3.6 and clearly show two distinct clusters.

Table 3.4. Standardization of Hg- against Zn-values

Lens	Zn (%)	Standardized Hg-value (g/t) from Zn-values	Real Hg-value	Difference between real and standardized Hg-value (g/t)	Ag (g/t)
1	4.1	29	35	6	53
	3.5	25	26	1	48
	7.2	50	55	5	90
	6.8	48	56	8	102
2	3.8	27	45	18	66
	4.8	34	58	22	78
	6.4	45	79	34	110
	6.7	47	85	8	120

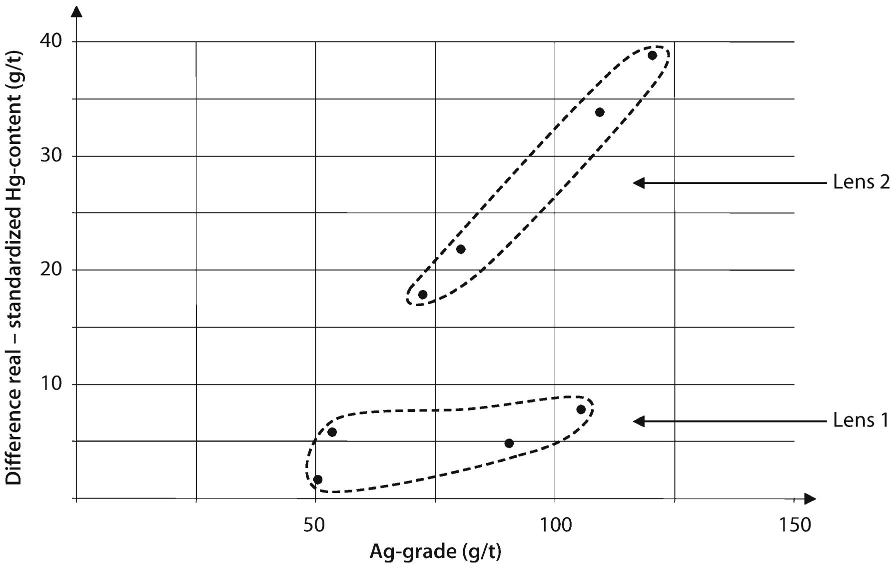


Fig. 3.6. Hg as a function of Ag-grades in a Zn-Pb-Ag-deposit, using Hg-values standardized against Zn

3.5 Calculating Metal and Value Equivalents

3.5.1 Introduction

If several elements contribute to the economic value of an ore deposit or one has to compare ore deposits with different economic components, the question arises how to find a common denominator for comparison. In the evaluation of the genesis of Pb-Zn-deposits Pb and Zn are frequently simply lumped together as Zn + Pb. This convention probably developed because for a long time in the past Pb and Zn-prices used to be quite similar (e.g. from 1947 to 1972 between 10 and 20 US¢/lb). Since we are looking at deposits from an economic point of view, it is important to allow for the difference in contribution to revenue by different metals, in this case lead and zinc, as a result of differences in their price. This is discussed in detail in Chap. 7.

If one wants to calculate value equivalents, one critically has to ask, what the common denominator should be. Different coal products are normally standardized using the Btu- or MTCE-values (metric tons of coal equivalent) (see Appendix D, Table D3a and b). If, however, we consider coal under the aspect of transport only we can add just tonnes. It does not matter, whether 1 t of hard coal with a high Btu-value is transported or 1 t of lignite with a low Btu-value. In the following, value equivalents are calculated for economic evaluations. In consequence, the common denominator is the revenue contribution of the individual components.

3.5.2 Calculating Metal Equivalents

For deposits with several economic components a metal equivalent is calculated as a rule to determine, for example, the cutoff grade (see Sect. 10.1). This is normally done in such a way that the operating costs are just covered. The metal equivalent is determined on the basis of the price the mine receives for the individual products, i.e. the components are "reduced to a common denominator".

Assignment. We have to evaluate a porphyry copper-molybdenum deposit. Molybdenum is to be converted into a Cu-equivalent.

1. First we have to make price assumptions. How to arrive at a reasonable price assumption will be dealt with in Chap. 6. For copper there is a metal exchange price. We assume a price of 0.90 US$/lb. This, however, is the price for refined copper. The mine does not produce any refined copper but copper concentrates. We must therefore deduct expenses for smelting and refining and for transporting the concentrates from the mine to the smelter (Chap. 7 will deal with this problem in greater detail). In this case, the deductions are 0.30 US$/lb so that the mine is left with US$0.90 – 0.30 = 0.60 US$/lb. This is the net smelter return received by the mine.

Since the return only applies to the actual metal value recovered, we have to take beneficiation losses into account. We assume 90% recovery. Thus the mine receives per lb Cu in the run of mine ore

$$0.9 \times \text{US}\,\$0.60 \mathrel{\hat{=}} 0.54\ \text{US}\,\$/\text{lb Cu}$$

2. The calculation for molybdenum is simpler because there is a concentrate price which we assume to be 4.75 US $/lb Mo in the MoS_2 concentrate. This is also the net smelter return for the mine. (Actually to make the concentrate price really comparable to the price of copper concentrate we would also have to consider transport charges to the smelter. Since the price is negligible in comparison to the value of the concentrate we simplify the calculation by omitting the transport charges). Mo recovery, which is normally lower than Cu recovery, must also be taken into account. It is assumed to be 80%.

   Hence, the mine receives per lb Mo contained in the run-of-mine ore

$$0.8 \times \text{US}\,\$4.75 \mathrel{\hat{=}} 3.80\ \text{US}\,\$/\text{lb Mo}$$

   The conversion factor from Mo to Cu-equivalent therefore is $K = 3.80/0.54 = 7.04$. The equation is

$$\text{Cu-equivalent CuE} = \%\,\text{Cu} + 7.04 \times \%\,\text{Mo}$$

Example. An ore body grades 0.40% Cu and 0.03% Mo. Applying the formula results in a

$$\text{CuE} = 0.4 + 7.04 \times 0.03 = 0.61\%$$

If grades had been given in MoS_2, they would have to be converted by the factor 0.6 into Mo (see Sect. 1.3). This can, of course, be included in the conversion factor KV. KV would then be $0.6 \times 7.04 = 4.22$.

Example: A block of ore has the grade of 0.40% Cu and 0.04% MoS_2. Thus

$$\text{CuE} = 0.40 + 4.22 \times 0.04 = 0.57\%$$

We will have to revert to multi-element deposits when dealing with break-even prices in Sect. 11.8.2. In later economic calculations price assumptions will also have to be varied (Sect. 11.7). The use of metal equivalents necessarily implies constant price relationships between metals. Generally this is seldom true.

It can be observed that recently reserve or resource data of multi-element occurrences are reported in company reports with metal equivalents only. The exploration geologist should avoid this and always report the original grade data. As obvious from the foregoing assignment various assumptions have to be made (price, recovery), which will (price) or can (recovery) change during the course of the exploration work. In

2007 the Australian Joint Ore Reserve Committee (JORC) which set the standard for reserve/resource classification, to be discussed in Chap. 4, published an update to the JORC Code 2004 dealing also with the use of metal equivalents and in effect discourages their use in reserve and resource reporting (Clark 2007).

3.5.3 Calculating Density Equivalents

In the second example of weighting in a barite deposit in Sect. 2.2.1, the grade was determined by weighting not only with the sample intervals but by density too. In metal-ore deposits with large variation in grades and very high spot values it may also be necessary to apply weighting by density. In complex deposits with several revenue components, it can be helpful to calculate a density equivalent of a main value component for each additional contributing phase present in the ore.

Assignment. A Pb-Zn-mineralisation in carbonates has to be evaluated. Zn occurs in sphalerite which is iron-poor, Pb in galena. The density of the barren carbonate gangue has been determined in 10 samples to be 2.7 g/cm^3; the density of galena (PbS) as 7.6 g/cm^3; the density of sphalerite (ZnS) as 4.0 g/cm^3. Galena contains 86.6% Pb, and sphalerite 67.1% Zn. We also have to consider porosity. Even apparently dense ore has a certain porosity; we assume 5%. Calculate the density equivalents for the sulphide phases.

- *Step 1:* We have to choose one of the sulphide phases as the reference. It is advisable to choose the heaviest component. Of the two sulphide phases galena is heavier.
- *Step 2:* We now convert Zn-grades into Pb-grades on the basis of density equivalents and construct a density curve as a function of Pb-grade (see Fig. 3.7). Two points on the graph of Fig. 3.7 are given by the density of the carbonate gangue, 2.7g/cm^3, and the density of galena, containing 86.6% Pb, 7.6 g/cm^3. Including 5% porosity the densities are 2.6 g/cm^3 for the gangue and 7.2 g/cm^3 for the pure galena. Now we have to find two other points to construct the curve, which is not a straight line as we have already seen in Sect. 2.2.2 and Fig. 2.7 about grade determinations from visual estimation in massive ore shoots.
- *Step 3:* To construct this curve we choose two additional intermediate points: we consider a cube of ore material of 1 cm^3 first with 25% galena and then 50%.

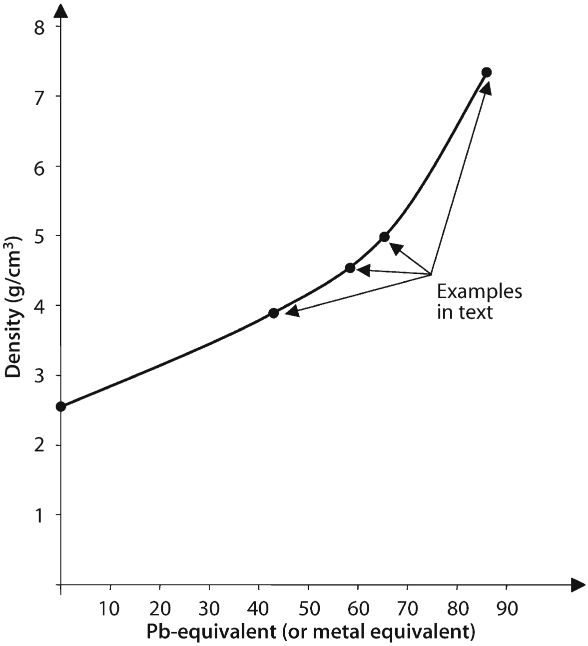
  If 25% of the cube consists of galena PbS by volume, then with a density of 7.6 g/cm^3 we have a weight of the galena of

  $$G_1 = 0.25 \times 7.6 = 1.9 \text{ g}$$

  75% by volume consists of carbonate. With a density of 2.7 g/cm^3 the carbonate weighs

  $$G_2 = 0.75 \times 2.7 = 2.025 \text{ g}$$

Fig. 3.7.
Density diagramme for Pb-equivalents (including 5% porosity)



The total weight of the 1 cm^3 cube, without porosity therefore is $G_1 + G_2 = 3.925$ g and consequently the density of the 1 cm^3 cube is 3.925 g/cm^3. Taking into account now 5% porosity we arrive at a density of 3.7 g/cm^3.

In G_1, the 1.9 g PbS contain $1.9 \times 0.866 = 1.645$ g Pb.

So the cube with 25 volume-% of galena and 75 volume-% of carbonate has a lead content of

$$\frac{1.645}{G_1 + G_2} = \frac{1.645}{3.775} = 0.414 \mathrel{\hat{=}} 41.4\%\ \text{Pb}$$

If we repeat the calculation for 50 volume-% galena we obtain a Pb-value of 63.8% Pb and a density of 4.9 g/cm^3 (including 5% porosity).

So we have two additional points to construct the curve in Fig. 3.7, which is certainly not linear; the density as a function of the Pb-content increases mainly in the upper part of the curve.

- *Step 4:* Now we want to calculate a density equivalent in general terms, so we have one Pb-equivalent value and can directly turn to the graph of Fig. 3.7 to read off the density, without the lengthy calculation of Step 3. It is not correct to use just the density ratio of galena and sphalerite. We have as a third component the density of the gangue, in this case carbonate. To arrive at the Pb-equivalent grade of the Zn-grade we can use the following formula, the derivation of which is given in Appendix C, Part 1. We determine a metal ratio for the density equivalent, in this case the equivalent ratio between Pb and Zn:

$$\frac{\mathrm{RG}_1}{\mathrm{RG}_2}=\frac{D_1\times\mathrm{MC}_1\times(D_2-D_\mathrm{G})}{D_2\times\mathrm{MC}_2\times(D_1-D_\mathrm{G})}$$

where:

RG_1 is the relative grade in percent of metal 1, in this case Pb
RG_2 is the relative grade in percent of metal 2, in this case Zn
D_1 is the density of the metal 1 mineral, in this case galena with 7.6 g/cm^3
D_2 is the density of the metal 2 mineral, in this case sphalerite with a density of 4.0 g/cm^3
D_G is the density of the gangue, in this case carbonate with a density of 2.7 g/cm^3
MC_1 is the metal content of the metal 1 mineral, in this case Pb in galena, which is 86.6%
MC_2 is the metal content of the metal 2 mineral, in this case Zn in sphalerite, which is 67.1%

If we substitute actual numbers into the general equation above, we get

$$\frac{\mathrm{RG}_\mathrm{Pb}}{\mathrm{RG}_\mathrm{Zn}}=\frac{7.6\times0.866\times(4.0-2.7)}{4.0\times0.671\times(7.6-2.7)}=\frac{8.556}{13.152}=0.65$$

Therefore, to obtain the density equivalent of Zn we have to multiply the Zn-grade by 0.65:

$$\text{Pb-equivalent}_\text{density} = \text{Pb} + \text{Zn}\times 0.65$$

Example: We have a high grade intersection with 35.2% Pb and 34.5% Zn. So

$$\text{Pb-equivalent}_\text{density} = 35.2 + 0.65\times 34.5 = 35.2 + 22.4 = 57.6\%\ \text{Pb-equivalent}$$

If we now go to the graph in Fig. 3.7 we can read off a density considering 5% porosity of 4.5 g/cm^3.

For more complex ore it might be helpful to use a spreadsheet. An example is given in Appendix C, Part 2 with a simple example of 5% Pb and 10% Zn.

Conversion of Geological Data into Mining Data for Ore Deposits

The following chapters will deal with revenues and costs, i.e. with costs during the mining stage and revenues for the mine after a saleable product has been put on the market. The geological grades will seldom correspond with the grades of the mill head ore. As a rule, the ore is diluted by inclusion of wall rock and losses occur during beneficiation. These factors have to be taken into account before revenues can be calculated.

These "modifying" factors convert the results of exploration which are mineral resources into ore or mineral reserves, denoting the part of the resources that can be economically extracted (Fig. 4.1). The today accepted definition for resources and reserves are the ones of the Canadian Institute of Mining and Metallurgy (CIM) Standing Committee on Reserve Definitions (2004) and of the Australian Joint Ore Reserve Committee (JORC 2004) which are shown in Fig. 4.1[4].

4.1 Dilution

The rate of dilution depends on the geometry and grade distribution in the deposit and on the nature of the mining method. Selective mining methods such as sublevel stoping with backfill or selective open-cut mining result in a lower rate of dilution than bulk mining procedures such as block caving. Elbrond (1994) compiled dilution and mining loss factors (see Sect. 4.2) for various mining methods based on intensive literature search. As a rule, dilution varies between 5 and 30%. We shall calculate with an average of 10%, which is appropriate at the exploration stage. In the English literature grades, are sometimes designated "ROM" which stands for "run-of-mine ore", meaning the grade after dilution. (For a more detailed study of dilution problems see Gunzert (1983) or Wright (1983). Geostatistical aspects of dilution are also dealt with by Wellmer (1998, Stat. Eval., p. 158).

[4] The Committee for Mineral Reserves International Reporting Standards (CRIRSCO) of the international Council of Mining and Metallurgical Institutions (CMMI) has developed an "International Reporting Template" as a guideline for developing reporting standards (www.crirsco.com/template.asp). The CIM and JORC standards, however, are still the base of all standards.

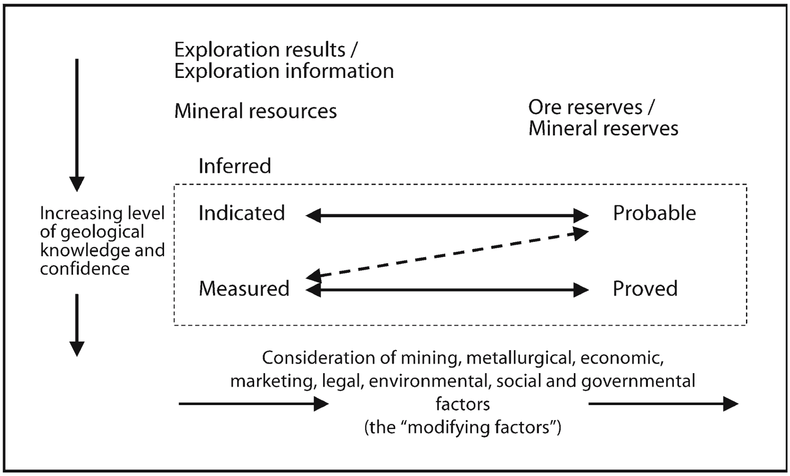


Fig. 4.1. General relationship between exploration results/exploration information, mineral resources and ore reserves/mineral reserves acc. to Canadian and Australian definitions (CIM 2004 and JORC 2004). (Ore reserves is the Australian term, mineral reserves the Canadian term) (originally published in "2004 Australasian Code for Reporting Exploration Results, Mineral Resources and Ore Reserves (The JORC Code)" and in the "CIM Definition Standards on Mineral Resources and Mineral Reserves". Reproduced with permission of the Australasian Institute of Mining & Metallurgy and the Canadian Institute of Mining, Metallurgy and Petroleum)

4.2 Mining Recovery of Tonnages or Loss of Tonnages Respectively

In underground mining a 100% recovery is virtually impossible. Pillars are often left, so that actual recovery depends on the particular mining method, and may range from below 70% for room and pillar operations to >90% for cut and fill operations. In many cases a recovery of 85–90% may reasonably be assumed, with complementary loss of ore or tonnages, i.e. a 90% mining recovery means a 10% loss of tonnages. Even for open pit mines one should not assume 100% recovery but allow for 5% loss, for example as ore that is to be left in the pit shell due to the open pit design[5].

4.3 Metal Recovery in the Beneficiation Plant

The varying rates of recovery during beneficiation have already been mentioned in Sect. 3.5.2. Recovery (ε) can be determined by the following formula:

[5] This has nothing to do with the problem of "vanishing" or "missing tonnes" which is widely discussed in geostatistical literature (see e.g. David, 1977) and which can be solved using proper geostatistical evaluation tools.

$$\varepsilon = \frac{\text{concentrate grade}}{\text{feed grade}} \times \frac{\text{(feed minus tailings grade)}}{\text{(concentrate minus tailings grade)}}$$

This is the metal recovery which has to be distinguished from the mass recovery (see Sect. 4.4).

Assignment. Determine the recovery for the following data from a zinc mine:

- feed grade = 10% Zn
- concentrate grade = 54% Zn
- tailings grade = 0.5% Zn

$$\varepsilon = \frac{54}{10} \times \frac{(10-0.5)}{(54-0.5)} = 0.96 \quad \text{or } 96\%$$

In the numerator of the above formula for ε we have the grade of the concentrate, in the denominator the difference between grade of concentrate and tailings grade. This quotient normally has a value close to 1. In the above example of the zinc mine the quotient is

$$\frac{\text{concentrate grade}}{\text{concentrate minus tailings grade}} = \frac{54}{54-0.5} = 1.01$$

There is a simplified alternative to obtain an approximate value for the recovery:

$$\varepsilon = \frac{\text{feed grade minus tailings grade}}{\text{feed grade}}$$

The numerator in this equation is simply the recovered metal grade and the concentrate grade is removed. Using the above example of the zinc mine again we have

$$\varepsilon = \frac{10-0.5}{10} = 0.95 \quad \text{or } 95\%$$

For an initial evaluation the difference is not significant.

In the technical literature and mining company reports, one often comes across information that, for example, state that an operation with an annual mining rate of say 5 million tonnes of ore with a grade of 0.8% Cu, produced 38 000 t of copper in concentrate. Such information can be used to calculate a rough estimate of recovery:

$$\frac{38\,000}{5\,000\,000 \times 0.008} = 0.95 = \varepsilon$$

It must be stressed that this can be used only as a round figure. The metal content in the concentrates is always rounded. One always says 38 000 t copper in concentrates but never 37 895 t, for example.

The recovery factor ε is very much influenced by the degree of intergrowth of the different mineral phases. For certain ore types, however, one can calculate in a first evaluation with rule-of-thumb factors. Typical recovery values are given in Table 7.1 in Sect. 7.2.2.

4.4 Concentration Factor and Mass Recovery

Recovery plays an important part in the determination of the concentration factor KF: i.e. how many tons of ore are needed to produce 1 ton of concentrate. This factor is essential for the valuation of in situ ore in a mine (see Sect. 7.2):

$$\text{KF} = \frac{\text{concentrate grade}}{\text{recovery} \times \text{ore grade}}$$

Example. A mine contains in situ ore grading 8% Pb. It produces a 65% Pb-concentrate, with a recovery rate of 95%. Hence, the concentration factor is

$$\text{KF} = \frac{65}{8 \times 0.95} = 8.55$$

i.e. 8.55 t of in situ ore are needed to produce 1 t Pb concentrate. The normal concentrate grades for various metals are given in Table D11 (Appendix D).

The mass recovery is the reciprocal value of the concentration factor KF:

$$\text{MR} = 1/\text{KF}$$

The mass recovery factor is of significance whenever transport costs are essential cost factors, i.e. with bulk products such as iron ore.

Example. An iron ore mine produces crude ore with 55% Fe and a concentrate with 66% Fe at a 90% metal recovery. The mass recovery factor therefore is

$$\text{MR} = \frac{1}{\text{KF}} = \frac{55 \times 0.9}{66} = 0.75$$

i.e. the mass recovery is 75%.

A mass recovery factor is also used in vein mining where the vein, e.g. in a barite or fluorite mine, has only a thickness of say 30 cm, but the minimum mining width is 1 m. Including the recovery at the beneficiation plant of 95%, the mass recovery then is

$$\text{MR} = \frac{0.3\ \text{m} \times 0.95}{1\ \text{m}} = 28.5\%$$

4.5 Special Case Uranium

Uranium is used almost exclusively as fuel in nuclear power plants. Of the three naturally occurring uranium isotopes only uranium-235 (U-235) is fissionable. Its relative content in relation to the other non-fissionable isotopes is, therefore, decisive in estimating primary tonnage requirements. (The calculation of the concentration factor KF during beneficiation follows the procedure in Sect. 4.4).

Natural uranium contains about 0.7% U-235, which decays with a half-life of 0.7×10^9 years. The remaining 99.3% of natural uranium consists of the non-fissionable isotope U-238, which decays with a much longer half-life of 4.5×10^9 years. Due to the difference in half-lifes, the ratio of these two isotopes in natural uranium has changed over geological time. About 2 billion years ago, for example, the content of fissionable U-235 was – at about 3.6% – much higher than today. At this concentration, which is comparable to the grades used in nuclear reactors, spontaneous nuclear fission can occur and be sustained. Indeed, relicts of natural reactors have been discovered in the Oklo uranium mine in Gabon.

During the process of nuclear fission a thermal neutron, that is, a neutron of suitable velocity is captured by the core of the U-235 isotope and causes fission. The nucleus of the resulting U-236 isotope is unstable and decays into two parts and emits 2 to 3 neutrons in the process. These neutrons in their turn can collide with other U-235 nuclei causing further fission, and so on. The resulting chain reaction is moderated in the reactor by the composition of the reactor charge, i.e. by control rods, and the geometry of the reactor. In the process of decay, binding energy is liberated. The resulting release of large amounts of heat is used in the thermal circuit of the nuclear power plant to generate electricity.

Unlike the U-235 isotope, U-238 is not fissionable by thermal neutrons. It can, however, absorb the excess neutrons released by the fission of U-235 nuclei in the reactor. This breeding process converts the U-238 isotope into fissionable plutonium Pu-239. After a longer residence in the reactor the Pu-239 isotope decays and releases thermal energy. The advantage of this process is that in a fast breeder reactor all of the uranium charge can be employed to generate electricity and not just the 3% or so U-235, increasing fuel supply in principle hundredfold.

The cost of fuel in nuclear power plants is low compared with the capital investment of a plant. Therefore, a high availability rate is attempted (for example 7 000 hours full operation per year) which is determined by the amount of U-235 in the fuel rods. It can be influenced by parameters of the plants and the operation itself.

The present commercial light water reactors work with enriched uranium (generally between 1.3 and 4 wt.-% U-235). The amount of natural uranium needed to produce enriched uranium depends on the U-235 residue which is left as tails (the "tail assay") in the depleted uranium during the enrichment process. The cheaper the enrichment, the more profitable it is to recover more of the U-235 from natural uranium. As a consequence, less natural uranium is needed.

However, if natural uranium is very cheap, not all of the U-235-isotopes are extracted, i.e. a higher depleted uranium grade is tolerated at the expense of higher natural uranium consumption. The ratio between enrichment costs and the natural uranium price determines the optimum level of tails assay at which the costs for enriched uranium can be minimised. In estimates, a tails assay of 0.2% U-235 is generally used.

As a rule, it can be assumed that 6 t of natural uranium are needed for 1 t of enriched uranium.

Since the energy sector is the only market for uranium (apart from military use), the demand for uranium can be determined from existing and planned nuclear plant capacities. The demand for uranium is (among other factors) not only dependent on the number of nuclear plants and the installed capacity, but also on the particular type of nuclear plant used or the type of nuclear fuel cycle adopted (with or without reprocessing). In addition, the overall uranium demand of an individual reactor is unevenly spread. At the start of the operation, a primary core has to be supplied. Each year about one-third of the core needs to be recharged, i.e. exchanged or replaced.

Total uranium demand for the period of the lifetime of each individual nuclear plant largely depends on its capacity utilisation so that estimates of fuel demand are subject to certain margins of error. Since a geologist makes forward estimates far into the future (from prospecting stage to production start there is a lead time of 10 to 15 years involved), such detailed individual calculations, requiring a host of individual assumptions, do not make sense. Approximate estimates are sufficient: for 1 000 MWe (megawatt electric) about 25 t enriched uranium or 150 t natural uranium p.a. are needed (see Keller et al. 1981). The proportion of the cost of natural uranium as part of the final electricity price is about 5% today.

Introduction to Economic Evaluations

Having dealt with grade and tonnage in the preceding sections, and having converted these into mine production data in Chap. 4, we now have all the necessary information to start our economic evaluation at whichever exploration stage we choose. The procedure we will adopt is shown in the "flow sheet" in Fig. 5.1.

1. Let us consider revenues first. For this we have to calculate the net smelter return which represents the financial return to the mine on the sale of its products, such as concentrates. This is done in Chap. 7. First, however, we have to assume a commodity or metal price, as the case may be. We will show in Chap. 6 how to derive a reasonable future metal price taking into account the fact that the planned mine is not likely to come on stream for 5 to 10 years.
2. From the point of view of revenues it does not matter if we have a small mine with perhaps 1 000 t/day productions or a large mine with say 150 000 t/day production.

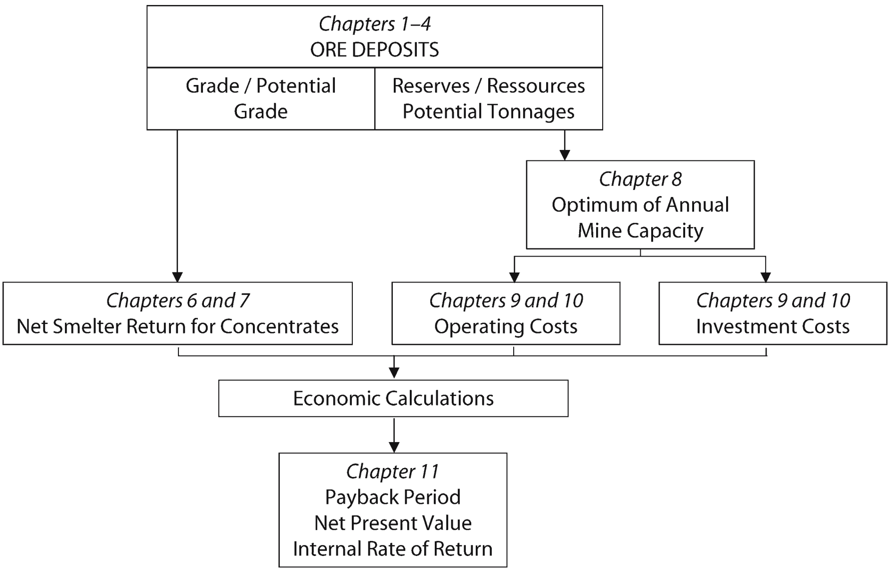


Fig. 5.1. Economic evaluations in exploration

The capacity, however, is very important when looking at operating and investment costs. Here the factor, "economies of scale", to be discussed later, has a large influence. The next step, therefore, is to determine the optimal capacity of a mining operation from the given reserves or potential of the indicated deposit to be evaluated. This is done in Chap. 8.

3. In Chap. 9 and 10 we will derive operating and investment costs, taking the capacities, determined in Chap. 8, into account.
4. For the economic calculations we then have to combine revenues, i.e. the net smelter return, with costs, i.e. investment and the operating costs, to derive factors like the payback period, the net present value or the internal rate of return, which will enable us to make a go/no-go decision or to compare the merits of different investment proposals. This is done in Chap. 11.

Metal Prices

6.1 Introduction

One of the most important assumptions to be made in any economic evaluation is that of prices. Commodity prices can be found in special publications. For metals (see Sect. 1.2.7) the London-based, twice-weekly "Metal Bulletin" is the standard reference. The "Engineering and Mining Journal" publishes monthly price surveys for metallic and non-metallic commodities. The journal "Industrial Minerals" is the best reference for prices of industrial minerals.

Unfortunately, most commodity prices are as constant as weather charts. Spot market prices that are subject to unpredictable swings can hardly be used in the evaluation of projects which has to look far into the future because years may elapse before production becomes possible. In times of booming prices most deposits would be judged economical. During a slump, however, even big, efficient producing mines find it difficult to operate economically.

Therefore the question arises: What is a reasonable average price? A very difficult question which, in the words of a Canadian mining director, "the experts are more often wrong than right" in answering. As a matter of rule, the historical development of the specific metal price (or in general commodity price) should be employed as the basis for projections. One reliable source for this is the *Metal Statistics* which up to 1993 was published annually by Metallgesellschaft AG, Frankfurt a. M., Federal Republic of Germany, and still the top reference book produced today by the World Bureau of Metal Statistics, Ware, Herts, England.

A few rules-of-thumb apply:

1. *Never* choose a price peak! Boom times do not last forever.
2. Some metals have producer prices, e.g. formerly the European producer price for zinc or the producer prices for Ni, Mo, and Al. Fluctuations of producer prices are less erratic than price movements at the metal exchanges. It is therefore advisable to choose a price close to the producer price.

6.2 Choice of Currency

Important is also the choice of currency in which the metal is quoted. Sometimes metal prices, formerly quoted in DM/t today in €/t, or £/t, changed purely because of changes in the rate of exchange to the US $, whereas in US $/t the price remained more or less steady. To eliminate or at least to reduce the exchange rate influence, the London Metal Exchange (LME), the world's most important metal exchange, has since 1993 been quoting all prices in US $/t and not anymore in £/t, acknowledging that the commodity world is a US $-world. The change-over dates are given in Appendix D, Table D12a. If one wants to calculate a long-term time series, therefore, rates of exchange between £ and US $ are required. These rates of exchange are given in Appendix D, Table D12b.

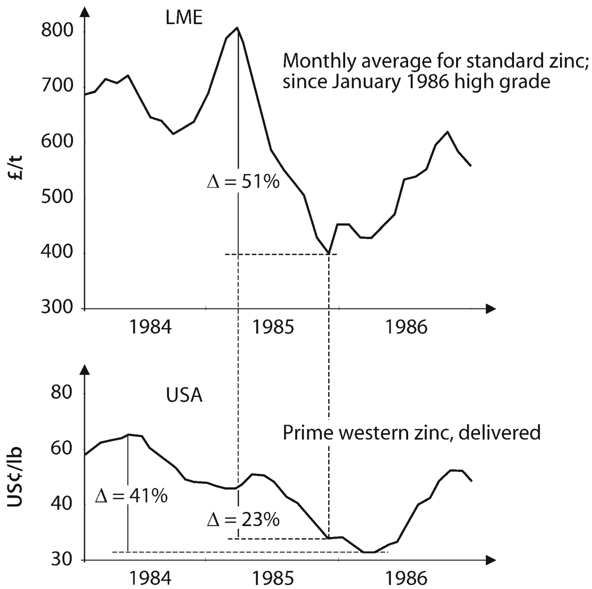
Again and again there are news that for political reasons countries want to abandon US $-quotations and try to change over into other currencies, the € for example (see Frankfurter Allgemeine Zeitung 2006). If one considers worldwide statistics about currency reserves it becomes obvious that the US $ is still the dominating currency in the world. For example in the fall of 2006 the COFER data bank (Currency Composition of Official Foreign Exchange Reserves) of the International Monetary Fund, IMF, indicated that still two thirds of the world currency reserves are held in US $.

To illustrate the stabilizing effect of quoting metal prices in US $/t, we compare the LME quotations for Zn during the period of three years between 1984 and 1986, when Zn on the LME was still quoted in £/t, with the U.S. price in US $/t (Fig. 6.1). It is obvious that the price variations over this period are smaller in US $ than in £. In addition, one can see that the price peaks and troughs in US $ and in £ occur at different times, well illustrating the effect of currency fluctuations on metal prices.

Consequently, for all economic calculations, regardless in which country the project is located, a commodity-price in US $ is chosen in this book.

To guarantee that in a group of companies all projects are evaluated on the same basis, some companies offer confidential price guides for evaluations. For the evaluator who is not in the fortunate position to have access to price guides and has to make his own assumptions, three methods of making reasonable price assumptions will be shown.

Fig. 6.1.
Fluctuations of the zinc price in the period 1984 to 1986 in (£/t at the London Metal Exchange (LME) (*top*) and in US ¢/lb in the USA (*bottom*)



Final economic assessments are normally based not on a single price, but on a price range. First a middle price is derived, which is considered the most likely approximation to the expected price trend. This is then varied up and down in a sensitivity analysis (see Sect. 11.7).

6.3 Calculation of Average Prices Adjusted for Inflation

6.3.1 Introduction

To calculate an average commodity price as the mean that is expected to be valid over years, inflation effects have to be allowed for. The correction is done by means of inflation indices. First an inflation index needs to be chosen.

Sometimes a price index is selected which is based on the mean value of various raw material prices (a "statistical breadbasket"). For price assumptions aimed at the economic evaluation of a mineral deposit this is the wrong choice! The simple graph in Fig. 6.2 will explain why.

If the metal price in Fig. 6.2 were deflated on the basis of the commodity price index, this would result in a price which over the years appears to increase steadily in real terms. Projecting this trend into the future would result in a rising metal price irrespective of periodic reversals caused by future economic setbacks. Mining revenues are not spent on raw materials, however, but have to pay for wages and consumer goods and finance ongoing and new investments. Therefore, an index for goods and services is a better choice. If the metal price in Fig. 6.2 is deflated accordingly, it will drop in real terms, i.e. a running mine would have to increase its production to be able to afford the same amount of goods and services. Its economic circumstances will have deteriorated.

In industrialized countries, producer price indices are therefore used as a rule. For projects in other countries, particularly developing countries, the World Bank's manufacturing unit value index, an index of international inflation based on unit values of manufactured goods to developing nations, would be the appropriate choice. A selection of indices is listed in Table D14 (Appendix D).

Indices always contain *relative* information. *The absolute value has no meaning.* An arbitrarily chosen year is set at 100 and the following years are related to this reference point in time. For the common indices the base year with the value 100 is adjusted in about every five years in order to have a reference year that is not too far away from the present.

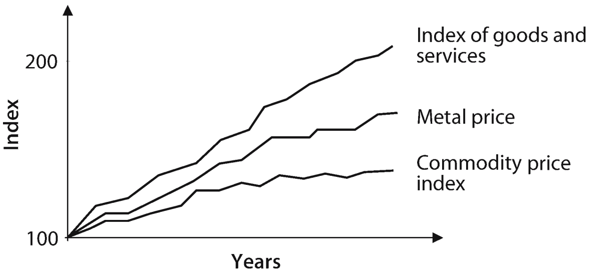


Fig. 6.2. Theoretical example of a metal price trend relative to a commodity price index and an index for goods and services

Table 6.1.
Canadian consumer price index

Year	Column I Index Base year 1995 = 100	Column II Index Base year 2000 = 100
1992	96.0	88.9
1993	97.7	90.5
1994	96.6	88.6
1995	100.0	92.6
1996	101.6	95.8
1997	103.6	95.9
1998	104.2	96.5
1999	106.0	98.1
2000	108.0	100.0
2001	117.7	109.0

Example. In the year 2002 the Canadian consumer price index had values for the last 10 years with the base year 1995 (1995 = 100) (Table 6.1, column I).

Now we assume the government decides to switch the base year for the value 100 to the year 2000, i.e. the value 108.0 in the table above becomes the value 100. Consequently all values have to be divided by 1.08 (Table 6.1, column II).

6.3.2 Correcting Prices for Inflation Effects

If we want to derive a reasonable metal price for our economic calculations from historic time series we have to transform the metal prices of each year, the nominal prices, into the metal price, or real price, of a reference year (also called constant money-value price). It is important to note that we can derive a reasonable metal price from a time series only if we compare real prices with the *same* reference year. This means for every year – with the exception of the reference year – we have to correct for the inflation effects. This is done by means of inflation indices as explained in Sect. 6.3.1 above.

Taking into account that indices are purely relative indicators, we have the relationship

$$\frac{\text{price at the money value of year } x}{\text{price in year } y} = \frac{\text{index in year } x}{\text{index in year } y}$$

$$\text{price at the money value of year } x = \text{price in year } y \times \frac{\text{index in year } x}{\text{index in year } y}$$

Theoretically every year can be taken as the base year for calculating prices in real values. Since, however, we want to look into the future when examining the economics of an exploration project, it is logical to take a year as the base year which is as close as possible to the present.

Table 6.2.
Cu-price

Year	Column I Cu-price (US $/t)	Column II US-producer price index (2000 = 100)	Column III Price in constant 2004 US $ (US $/t)
1995	2 934	94.0	3 449.01
1996	2 293	96.2	2 633.85
1997	2 276	96.2	2 614.32
1998	1 653	93.7	1 948.38
1999	1 572	94.5	1 838.16
2000	1 813	100.0	2 003.37
2001	1 591	101.1	1 738.93
2002	1 592	98.8	1 780.53
2003	1 779	104.1	1 888.37
2004	2 867	110.5	2 867.00

Assignment. Calculate the Cu-price in 2004-US $. In Table 6.2, column I, the respective Cu-price is given in US $/t, in column II the respective U.S. producer price index.

As an example the Cu-price of 1995 is adjusted for inflation and expressed in 2004-US $. In the above formula the price in year *y* (1995) is 2 934 US $/t. The index in year 1995 is 94.0, the index in year 2004 is 110.5. The result is

$$\text{price at 2004 money value} = 2\,934 \times \frac{110.5}{94.0} = 3\,449$$

Column III in Table 6.2 contains the prices in 2004-money values, i.e. adjusted for inflation. From these inflation adjusted prices a mean can be calculated. As mentioned in basic rule (1) in Sect. 6.1 it is advisable to avoid the peak price in 1995. With the peak price the mean is 2 276 US $/t, without the 1995-peak price the mean is 2 146 US $/t.

Since the mass unit used in the USA, the dominant market, is still the pound (lb), we have to convert into US $/lb. The conversion factor is 2 204.6 (see Sect. 1.1.4 Pound). Therefore

$$\frac{2\,146\left(\frac{\text{US\$}}{\text{t}}\right)}{2\,204.6} = 0.9733\left(\frac{\text{US\$}}{\text{t}}\right)$$

We always work in our economic calculations with round figures. If one is optimistic one chooses 1 US $/lb, if one is less optimistic one chooses 95 US ¢/lb.

6.4 Calculating Prices with Moving Averages

To smooth the erratic fluctuations of metal prices in a time series moving averages may also be used (see e.g. Wood et al. 1977), with the means calculated over a period of *x* years. Since boom and slump times alternate on an average of 4 to 5 years (see e.g. O'Leary and Butler 1978), it is advisable to calculate moving averages over a similar period.

Table 6.3. Five-year moving averages for Cu

Year	Column I Cu-price in 2004-US$ (US$/t)	Column II 5-year moving average
1995	3 449	
1996	2 634	
1997	2 614	2 497
1998	1 949	2 208
1999	1 838	2 029
2000	2 003	1 862
2001	1 739	1 850
2002	1 781	2 058
2003	1 888	
2004	2 867	

Assignment. Calculate the 5-year moving averages for Cu-prices in 2004-US$/t from Table 6.2.

For the 5-year moving average for the year 1997, for example the values for 1995, 1996, 1997, 1998, 1999 are added up and averaged, for the year 1998 the values for 1996, 1997, 1998, 1999 and 2000, and so on. The result is shown in column II, Table 6.3.

It is essential to scrutinize the result of such calculations closely. While the highest and second highest price in the years 1995 and 2004 have only been used once in the moving averages, the lowest prices in the year 2001 and 2002 were used three times, so the down side of the prices is exaggerated. Taking this into account it seems reasonable to work with a long-term price corridor of 2 000 to 2 100 US$/t, which can be used in sensitivity calculations, dealt with later in Sect. 11.7.

6.5 Deriving Prices from Cost Charts

Due to the erratic price fluctuations metals are subject to, many mining companies have come to employ a different method to arrive at price assumptions. From cost data for all producing mines they derive a minimum or break-even price (see Sect. 11.8) which is just sufficient to guarantee the economic survival of the mine.

In most cases it is difficult and time-consuming to determine the frequency distribution of breakeven data for all mines producing a particular mineral. Some companies have data banks from which this distribution can be determined. In recent years, however, cumulative frequency cost curves have started appearing in mining journals and in the grey literature, like abstracts of conferences. As will be outlined in Sect. 9.1 all exploration geologists who are required to carry out economic evaluations should add such frequency distribution of costs to their cost data collection. Normally one works with cumulative (or accumulated) frequency curves as shown in Fig. 6.3a. For the cumulative frequency the relative frequency of cost categories are simply added up from the lowest value upward. So the highest cost producer is the last mine considered, which brings the total to 100% (see e.g. Wellmer 1998, Stat. Eval. p. 31ff).

Two aspects can be pointed out with the copper examples in Fig. 6.3a and 6.3b:

1. Cost curves vary with time as is obvious from Fig. 6.3a and 6.3b. There is tremendous pressure in the mining industry to lower costs, so, in nearly all cases, one can observe that the frequency curves show an overall reduction of costs over time.
2. In Fig. 6.3a the median and the lower percentile of 33% are shown in the graph for copper. Normally the break-even point for a new investment has to lie below 33% ("lower third rule"). This is shown as an example in Fig. 6.3a by the mine Alumbrera, a new mine in Argentina. This has an obvious consequence: if new operations always lie below the mean of all operations the overall mean is lowered, thereby pushing older operations to the higher end of the cost curve. This explains the tremendous pressure, as mentioned above, for mining companies to rationalize and to stay

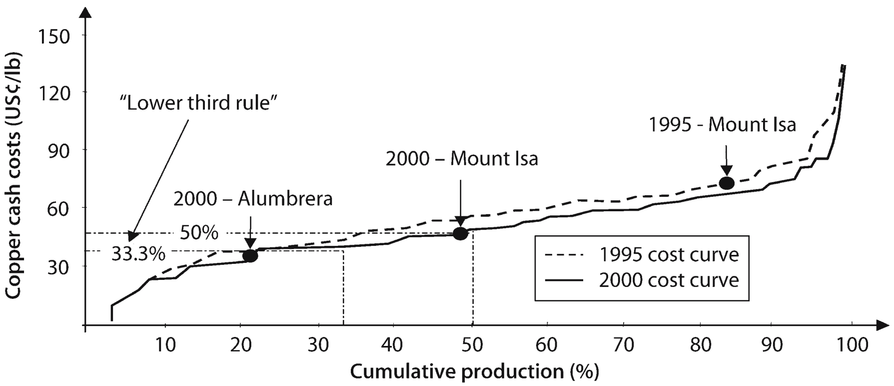
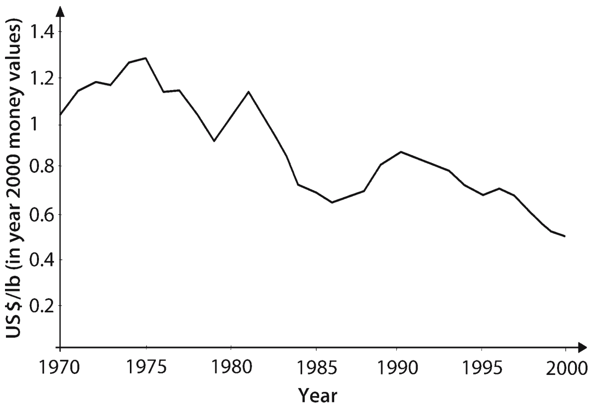


Fig. 6.3a. Cash costs to market of copper production 1995 and 2000 after by-product credits with 50% and 33.3 % percentile (= "lower third rule") (Source: Mount Isa Mines Ltd. 2001 modified)

Fig. 6.3b.
Cash break-even costs of the median mine in 2000-US $ money values (Source: Humphreys 2001) (with permission of the author)



away from the upper end of the cost curve where the threat of being pushed out of the market is ever present. Especially in times of depressed metal prices mines sometimes can slash costs considerably. Richmond and Blight (1986) give examples from the copper mining industry. Humphreys (2001) shows how the cash break-even costs of the median mine in 2000-US $ money-values decreased by about 40% from 1970 to 2000 (Fig. 6.3b). Figure 6.3a also shows such an example: The Mount Isa Mine in northern Queensland in Australia, discovered in 1923 and brought into production in 1942, was at the 85% percentile and in danger of being forced out of the market in 1995. By 2000, the mine had brought its costs down to the median of worldwide copper mine costs by ruthless rationalization.

Note: Formerly, such cost charts were commonly used to make metal price predictions in a different way to the method described above. Instead of applying the "lower third rule", the upper end of a cumulative frequency curve of operating or break-even costs used to be examined. From the graph, like the one in Fig. 6.3a, the operating costs of the last marginal producer – the supplier of the last increment that balances supply and demand – was identified. The costs of the marginal producer were assumed to be equivalent to the expected long-term price. As explained above, cost curves are not constant but are a function of time, and as such, are not suitable for long-term price forecasts. This method should therefore be avoided. The preferred method, one that is based on long term price trends at real prices, is outlined in Sect. 6.3. Such a method, using the upper end of a cost curve, is, however, appropriate for strategic considerations, for example about minerals availability (Tilton 2002).

Furthermore, the marginal-producer method assumes that mines will not produce at a loss over extended periods of time. Mines, however, are capital-intensive. Once an investment has been made, the capital costs must be paid back, regardless whether the mine is producing or closed. It makes sense for a mine to produce even at a loss as long as there is a big reserve basis, mere operating expenses are covered by the revenues and a fraction of the capital costs is earned. Besides this economic reason for continuing to operate at a loss there are social reasons in many countries which often prevail to keep uneconomic mines in operation.

Two final remarks:

1. For monomineralic deposits like gold or coal these cost curves represent directly the cost structures of the mines. In the case of polymetallic deposits, however, the mines are usually credited with by-products; in the case of copper, for example, with by-product credits for Mo, Au and/or Ag. To derive the real costs of a mine, one has to calculate backwards to eliminate these credits. This is shown in Sect. 9.1.2.
2. There is also a rule-of-thumb for deriving a long-term price using the cost charts: the long-term average price of a common mineral commodity is 1.5 times the average cost of production worldwide (McIntosh Engineering 2003). The normal geologists will not have a data bank at hand with cost data worldwide but only his collection of cost data and cumulative cost curves. Here again the problem of credits arises. It is, therefore, recommended to derive a long-term price from a time series as described in Sect. 6.3.2. Price data are always readily available.

Calculation of the Net Smelter Return (NSR) of a Mine

7.1 Simple Cases on the Basis of Prices Per Unit or Direct Concentrate Prices

As a rule, a mine produces concentrates. In rare cases it mines rich ore which can be shipped directly. In some cases price quotations for concentrates and ore are available, i.e. iron ore, tungsten, and antimony concentrates or "yellow cake"[6], U_3O_8, the end product of uranium mines. These quotations are supplied by the price lists of the weekly "Metal Bulletin", the "Engineering and Mining Journal", the "Mining Magazine", or numerous web pages. Generally these prices are quoted in "units", with 1 unit (1 u) being 1% of the metal in the concentrate (see Sect. 1.1.4). From this the net smelter return of the mine (abbreviated NSR) can easily be derived.

1. *Example:* For iron ore we assume a unit price of 0.50 US $/u. Accordingly, a mine producing high grade direct shipping ore of 64% Fe has a revenue of

$64 \times 0.50 = 32.00$ US $/t iron ore

To arrive at the return f.o.b. mine (free on board, see Sect. 9.4.1) freight costs have to be subtracted.

2. *Example:* For scheelite concentrates the price shall be 40 US $/unit WO_3. A deposit has grades of 0.8% WO_3. This ore has to be beneficiated first before yielding a saleable product. Recovery is assumed to be 85%. Hence the return from 1 t of in situ ore with 0.8% WO_3 is

$40 \times 0.8 \times 0.85 = 27.20$ US $/t

As in the case of iron ore, freight costs should be taken into consideration. While freight costs contribute considerably to the purchase price of iron ore for the steel plants (iron ore is a low-value bulk product), this is not true for tungsten concentrates which are a high-value product. For initial rough estimates the freight aspect can in such cases be neglected.

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[6] "Yellow cake" is actually sodium or ammonium diuranate, but the term is colloquially used for uranium oxide (see also Table D11, Appendix D).

For initial evaluations we assume that beneficiation will yield a saleable standard product, unless prior microscopic or beneficiation tests preclude this. If, however, only low-grade concentrates are produced, the mine has to accept penalties[7]. In such cases specialists should be consulted, since individual rules apply to each mineral. It is only for tin that the "Metal Bulletin" publishes smelter terms for low-grade concentrates (see Sect. 7.2).

Returning to the above scheelite example: The unit-price in Example 2 above applies to standard concentrate with a minimum quality of 65% WO_3. Here the rule applies that for each percent below 65% WO_3 1 US $/u is deducted, e.g. if the concentrate grade has only 60% WO_3, the price per unit will be $40 - (65 - 60) \times 1 = 35$ US $/unit.

7.2 Non-Ferrous Metals

7.2.1 Calculating with Smelter Formulae

For common non-ferrous metals[8] such as Cu, Al, Pb, Zn, Sn, or Ni, the situation is more complicated than for the examples given in Sect. 7.1. Quotations are available for the metals, i.e. the saleable end product, but not for the intermediate products. One might come across the information that a deposit of 100 million t containing 1% Cu at a price of 0.90 US $/lb has a value of almost US $2 billion. This is a totally misleading and *incorrect* estimation. The metal exchange quotation of 0.90 US $/lb refers to *refined copper*. The mine, however, produces *concentrates* (see Fig. 7.1).

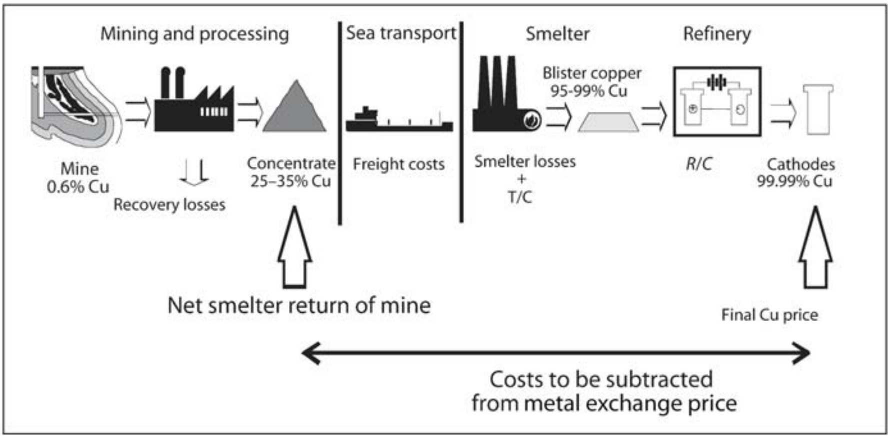

Fig. 7.1. Flow sheet for the recovery process copper

[7] Penalties are also incurred for deleterious elements in concentrates (e.g. As or Hg) see Table D14 (Appendix D).

[8] Definitions of the metal families commonly used in the industry are given in Table D15 (Appendix D).

To obtain the return for the mine we have to subtract from the price for refined copper every expense incurred at each stage in the production of refined copper from copper concentrates, the final mine product.

To determine the return from concentrates, particular formulae are used. For the sake of completeness, they are listed in Table D13 (Appendix D). Below, the example of copper is used to demonstrate the application of such a formula.

Since we are dealing with the evaluation of deposits in an early stage of development, rules-of-thumb will almost always be used which simplify the calculation considerably (see Sect. 7.2.2).

Example. We are to evaluate a porphyry copper deposit with an ore grade of 0.7% Cu. For the sake of simplicity, we assume that the Mo, Au or Ag grades, common in this type of deposit, are so low that these metals are not paid for in the concentrate. What is the revenue per tonne of ore?

For the calculation of revenues we have to make certain assumptions:

a Recovery in the beneficiation process: We assume 90%, i.e. of the 0.7% Cu, 0.63% Cu are recovered.
b Grade of concentrate: Grades of concentrates normally lie between 25 and 30% Cu. We assume 25% Cu.
c Freight for concentrates from mine to smelter is assumed to be 20 US $/t.
d Treatment charge (T/C) of the smelter: This refers to a tonne of concentrate. A reasonable assumption at present is T/C = 85 US $/t concentrate.
e Treatment losses: Since losses occur during treatment in the smelter, these losses are subtracted from the metal content of the concentrate. Treatment losses can vary, with copper they normally amount to 1 unit (u) (i.e. 1% Cu in the concentrate, see Sect. 1.1.4).
f Refining charge (R/C): This is based on the paid metal (minus treatment losses!) in the concentrate. A reasonable assumption at present is R/C = 8 US ¢/lb paid Cu.
g Metal price: This is the most important assumption. We assume 0.90 US $/lb (see Chap. 6).

The calculation is carried out as follows:

a The concentrate grade is 25%; from this we have to subtract the treatment loss of 1 u (= 1%), so that 24% Cu will be paid. 1% corresponds to 22.046 lb per tonne (see Sect. 1.1.4). Thus the gross value of the concentrate is

$$(25 - 1) \times 22\,046 \times 0.90 = 476.19 \text{ US \$/t}$$

b From this we subtract the treatment charge: T/C = 85 US $/t
c We also have to subtract the refining charge. If refers to the paid metal content. The R/C is: $(25 - 1) \times 22\,046 \times 0.08 = 42.33$ US $/t concentrate
d As a last step we have to subtract the freight

Summarised, the calculation method looks like this:

Gross value of the concentrate	476.19
–T/C	–85.00
–R/C	–42.33
–freight	–20.00
	= 328.86 US $/t concentrate

This is the net smelter return of the mine (NSR). However, we are not so much interested in the concentrate but in the net value of the ore: the ore has a grade of 0.7% Cu; we recover 90%, and the concentrate has a grade of 25% Cu.

Hence we need

$$\frac{25}{0.7 \times 0.9} = 39.68 \text{ t of ore}$$

to produce 1 t of concentrate.

Thus the concentration factor KF (see Sect. 4.4) is 39.68. From this we arrive at a net smelter return for the ore:

$$\text{NSR} = \frac{328.86}{39.68} = 8.29 \left(\frac{\text{US\$}}{\text{t}} \right)$$

which is of course rounded to 8.30 US $/t.

7.2.2 Calculating with Rules-of-Thumb

As shown in Sect. 7.2.1, a host of assumptions has to be made to obtain the net smelter return of the ore. In the early stages of evaluating a deposit which might take 10 years to reach production (common lead time at present) this calculation is overaccurate.

If metal prices rise, treatment and refining charges usually rise as well. An analysis of concentrate contracts shows that the mines receive a percentage of the final price of the end product which fluctuates only within a certain range (see Fig. 7.2 for Zn concentrates for example). For our estimates we can therefore work with approximate factors, i.e. in the copper example of Sect. 7.2.1 we replace the following assumptions and variables:

- concentrate content
- treatment charge
- treatment losses
- refining charge
- metal price

by a *single* variable, the metal price, and cover all other assumptions by one factor.

These factors are listed in Table 7.1. In combination with the recovery in the beneficiation process this is a very simple way to calculate the value of the ore.

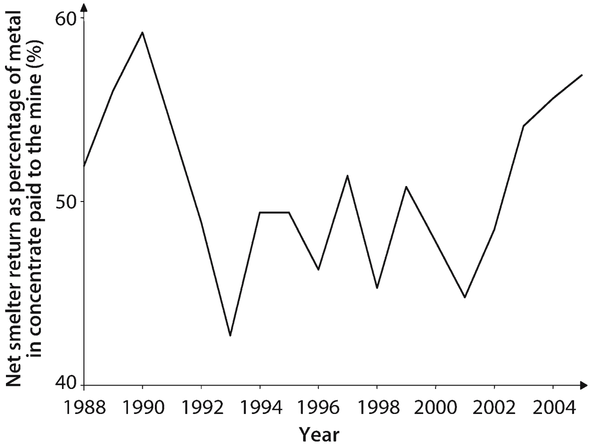

Fig. 7.2. Percentage of zinc in zinc concentrates paid to mine (net smelter return NSR) based on 50% Zn concentrates, yearly averages of daily LME quotations for Zn as published by Metal Bulletin and treatment charges T/C as published by Metals Economics Group Strategic Reports

Table 7.1. Fluctuation of mine returns

Element	Percentage net smelter return for mine NF (%)	Range of fluctuation of NF (%)	Recovery in beneficiation plant ε (%)
Cu	65 (Europe) 75 (Pacific Basin)	63–68 72–80	90 (92–85)
Zn	50	46–54	90 (92–85)
Pb	65	61–67	90 (92–80)
Ni	65	62–70	80 (75–80)
Sn	94	90–95	60 (50–65)
Au (in copper mines)	95	–	80 (75–85)
Au (in Au Mines, not heap leach operations)	98	–	90 (85–95)
Au (in heap leach operations)	98	–	40 (30–50)
Ag[a]	95	–	80 (75–85)

[a] For Ag it is assumed that it reports to Pb or Cu in the concentrate, not to Zn, as is normally the case. If it reports to Zn this factor is not applicable. In that case, the calculation has to be done with the standard smelting formula and terms given in Table D13 (Appendix D).

The range of values in Table 7.1 indicates the range within which mine returns fluctuate normally due to market changes. Recently one could observe larger fluctuations which are cannot be considered normal. The overall capacities of mines and smelters worldwide are seldom really in balance. In a buyer's market, when there is an abundance of concentrates and the buyer (the smelter) determines the market the lower values apply; in a seller's market, when concentrates are scarce and the mine determines the market, the higher values apply. Since in the initial stage of development of a deposit it is not possible to predict the behaviour of markets or the changes that might occur during the lifetime of the mine, it is justified to work with average values.

For the recovery in the beneficiation plant, ranges are given in brackets. Recovery is highly dependent on grain size and the degree of intergrowth. When dealing with complex, fine-grained ore one should therefore work with the low values.

Cu in Table 7.1 requires the following explanation: the Japanese copper market (as well as the South Korean, Taiwanese and Brazilian) is protected, allowing local smelters to offer mines very favourable terms. If an exploration geologist works for a European firm intending to ship the concentrates to Europe, he has of course to work with European terms. If his mine is looking for the best terms available worldwide, he will opt for the Japanese terms.

Assignment. A complex volcanogenic deposit contains 2% Cu, 1.5% Pb, 6% Zn, 1.3 oz/t Ag. What is the net smelter return of the ore? For Cu European terms are to apply. Price assumptions: Cu: 0.90 US $/1b; Pb: 0.35 US $/1b; Zn: 0.45 US $/1b; Ag: 5 US $/oz.

The factors from Table 7.1 will be used. The conversion factor of lb into % is 22.046 (see Sect. 1.1.4). The ore is good-natured, so that average recovery values can be expected.

$$
\begin{aligned}
&\text{Cu: } 2 \times 22.046 \times 0.65 \times 0.9 \times 0.90 \text{ US\$} &&= 23.21\\
&\text{Pb: } 1.5 \times 22.046 \times 0.65 \times 0.9 \times 0.35 \text{ US\$} &&= 6.77\\
&\text{Zn: } 6 \times 22.046 \times 0.5 \times 0.9 \times 0.45 \text{ US\$} &&= 26.79\\
&\text{Ag: } 1.3 \times 0.95 \times 0.8 \times 5 \text{ US\$} &&= 4.94\\
& &\Sigma &= 61.71 \text{ US\$/t} = \text{NSR/t ore}
\end{aligned}
$$

With regard to the precious metal content in base metal concentrates, it is recommended that the NF-factors for precious metals in Table 7.1 are cross-checked by carrying out a rough-and-ready comparison with the NF-factors for the base metal concentrates of Table 7.1 and corrected if found to be too high.

Example. We have to evaluate a Pb-ore with 8% Pb and 80 g/t Ag. A normal Pb-concentrate has a grade of 65% Pb (Table D11, Appendix D). We calculate with recovery values of Table 7.1, i.e. 90% for Pb and 80% for Ag. The concentration factor KF for the lead concentrate, therefore, is

$$\text{KF} = \frac{65}{8 \times 0.9} = 9.0$$

This concentration factor of 9.0 we apply now to Ag. With a recovery of 80%, Ag is enriched to

$$80 \times 0.8 \times 9.0 = 576 \text{ g/t in the concentrate}$$

With a smelter deduction of 50 g per tonne of concentrate (see Table D13 in Appendix D) the mine will be credited only with 526 g/t, meaning 91%. In such a case the NF-factor for Ag of Table 7.1 has to be reduced to 91% (or better rounded to 90%, since this is a rough estimate only).

Aluminium is a special case. The initial raw material is bauxite with Al_2O_3 grades between 35 and 50%. This is a low grade bulk raw material which cannot be further enriched by mechanical beneficiation, but is chemically processed into aluminium oxide Al_2O_3, also called alumina, at the refinery. This is the starting material for the aluminium plant turning out the finished industrial product. (Attention: the sequence 'mine-refinery-smelter' is the reverse of that for other metals.)

For a first rule-of thumb evaluation the "10-in-10-rule" is sometimes employed: 10 t of alumina has the value of 1 t of aluminium and 10 t of bauxite the value of 1 t of alumina. This ratio of course fluctuates as much as the aluminium prices itself. A recent analysis based on Australian customs statistics covering 23 years shows that the average value of 1 t of alumina was 12% of the price of aluminium with an annual average spread from 8 to 16% (Rowley 2006).

In evaluating bauxite deposits, freight costs to the refinery must not be neglected: they are a *crucial* cost factor.

Chapter 8

Production Lifetime

In the preceding chapters we demonstrated how net returns per tonne of ore can be calculated. They are, of course, dependent only on ore grades and not on the amount of ore mined (see also Chap. 5). When considering the production costs, the output per unit of time must also be taken into account. The greater the output per day or per year, the lower are the costs per tonne of ore. This effect is called "economies of scale", which was first described by Young (1928). The converse of an increased rate of production is a shorter life for the mine.

For the evaluation of a deposit the question therefore arises: What annual level of production should be assumed as optimal?

8.1 Rules-of-Thumb for the Lifetime of Deposits

8.1.1 General Rules

Some companies assume that the productive life of a mine should span at least 10 years so that any risk caused by cyclical price fluctuations can be compensated. Lately, however, one could witness many mines, especially gold mines, that were brought into production with much shorter lifetimes. Deposits with low grades but large reserves, such as porphyry copper deposits, which often require considerable investment for infrastructure, should have an operating lifetime of at least 20–25 years.

When evaluating a deposit in its initial stage, an estimate of the reserve potential is made, which is then divided by 10, if circumstances require a lifetime of 10 years. This is the annual capacity needed to determine costs (see Chap. 9).

8.1.2 Rules Based on Mining Experience

a Based on years of geological experience and knowledge of particular types of mineralisation, many mining camps have gathered practical values that can be applied in determining the optimal mining progress.

  In practice, these values should always override any theoretical considerations (see Sect. 8.1.3). In West Australian gold mining for example, an old historical rule-of-thumb says that the optimal mining progress per year is 100 feet, i.e. 30 m vertical. Thus a possible yearly capacity can be quickly determined from the strike length and the thickness of a vein deposit. This rule has been checked in 1999 over a period of 15 years by the Australian consulting company Australian Mining Consultants (AMC) (NN 1999). AMC concludes that the figure is now typically 50 vertical meters per annum (vm/a) for small to medium underground mines, but generally remains 30 to 35 vm/a for mines of 2 million t/a or more. For the smaller mines, 60 vm/a appears to be a practical upper limit. More ambitious mines generally suffer a collapse in production within 12 to 18 months when higher rates are attempted.

AMC checked also open pit mines and found that production rates are similarly constrained. For large pits a vertical advance of 30–35 vm/a is realistic, while small pits may achieve 50–60 vm/a, depending on the level of grade control and selectivity required. Surprisingly, the advance rates for open pits and underground mines seem largely independent of the geometry and attitude of the orebody.

In this context another rule-of-thumb is of interest which also originates in West Australian underground gold mining. If a mine has a yearly capacity of x t/a reserves of 3 times should have been blocked out through development and preparation work. As an example: if the target is 400 000 t/a reserves of 1.2 million t should have been blocked out.

b Interestingly enough, a similar rule concerning the vertical advance of underground mines exists in Canadian gold mining: the optimal daily production amounts to half the tonnage per vertical foot of reserves, i.e. with 300 working days per year this equals the tonnage contained within 150 vertical feet = 45.7 m, i.e. practically 45 vertical m of the deposit.

Assignment. A gold quartz vein has a strike length of 300 m and an average thickness of 1.5 m. What is the optimal production rate from this vein?

Since it is a quartz vein, we assume a density of 2.6 g/cm^3 (see Table D7, Appendix D) and apply the West Australian rule, updated by the consulting company AMC for small mines, that mining progress should be around 50 vertical m/a.

Hence the annual tonnage is $300 \times 50 \times 1.5 \times 2.6 = 58\,500$ t/a, i.e. 60 000 t/a.

8.1.3
Calculating the Optimal Lifetime

8.1.3.1
The Taylor-Formula for Calculating Optimal Lifetime

Taylor (1977) empirically arrived at a formula for the optimal lifetime of a deposit:

$$\text{lifetime } n \text{ (in years)} \approx 0.2\sqrt[4]{\text{total reserve tonnage expected}} \tag{8.1}$$

or

$$\text{lifetime } n \text{ (in years)} \approx 6.5\sqrt[4]{\text{tonnage (in million tonnes)}} \tag{8.2}$$

Taylor (1977) published Table 8.1.

Table 8.1. Lifetime of a deposit

Expected tonnage (reserves in 10^6 t)	Medium lifetime (yr)	Range of lifetime (yr)	Average daily production (t/d)	Range of daily production (t/d)
0.1	3.5	3 – 4.5	80	65 – 100
1.0	6.5	5.5 – 7.5	450	400 – 500
5	9.5	8 – 11.5	1 500	1 250 – 1 800
10	11.5	9.5 – 14	2 500	2 100 – 3 000
25	14	12 – 17	5 000	4 200 – 6 000
50	17	14 – 21	8 400	7 000 – 10 000
100	21	17 – 25	14 000	11 500 – 17 000
250	26	22 – 31	27 500	23 000 – 32 500
350	28	24 – 33	35 000	30 000 – 42 000
500	31	26 – 37	46 000	39 000 – 55 000
700	33	28 – 40	60 000	50 000 – 72 000
1 000	36	30 – 44	80 000	65 000 – 95 000

Assignment. What is the optimal mine capacity for a reserve of 8 million t of ore?

Formula (1): $n \approx 0.2 \times \sqrt[4]{8 \times 10^6} = 10.64$ years

Formula (2): $n \approx 6.5 \times \sqrt[4]{8} = 10.93$ years , i.e. the optimal lifetime is 11 years.

8 million t over 11 years corresponds to a production of 730 000 t/a.

8.1.3.2
Critical Examination of the Taylor-Rule

The theoretical Taylor-rule has been tested with real mine data. Wellmer (1979, 1981a) investigated Canadian basemetal mines at the stage of the investment decision, McSpadden and Schaap (1984) porphyry copper deposits world-wide (Fig. 8.1a,b). Although the data scatter widely, by and large there is a good agreement with the optimal production rates estimated by applying Taylor's formula (1977). Figure 8.1a shows the Canadian data, Fig. 8.1b the results for the porphyry copper deposits. In Fig. 8.1b the ratio between the real lifetime and the theoretical Taylor-lifetime is plotted on the x-axis. In industrial countries, the lifetimes follow the Taylor-rule more closely than in developing countries. Here the production rates are on average 20% higher than postulated by the Taylor-rule. This may, on the one hand, be caused by higher investment costs in developing countries, which require a higher throughput as compensation (see Introduction of Chap. 8 "economies of scale"), and on the other, it might reflect the desire of the mine operators to reduce the country risk by a shorter lifetime. We will return to this question in Sect. 11.1.3 where the related concept of the payback period is treated.

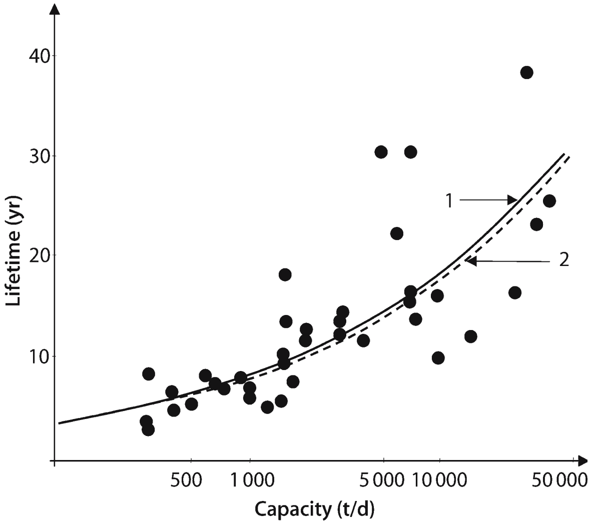

Fig. 8.1a.
Lifetime of Canadian base-metal mines at the time of production decision (1967–1977) (Wellmer 1979): 1. the relationship postulated by Taylor (1977): $y = 0.83x^{0.34}$; 2. interpolation of the real data points $y = 0.69x^{0.35}$

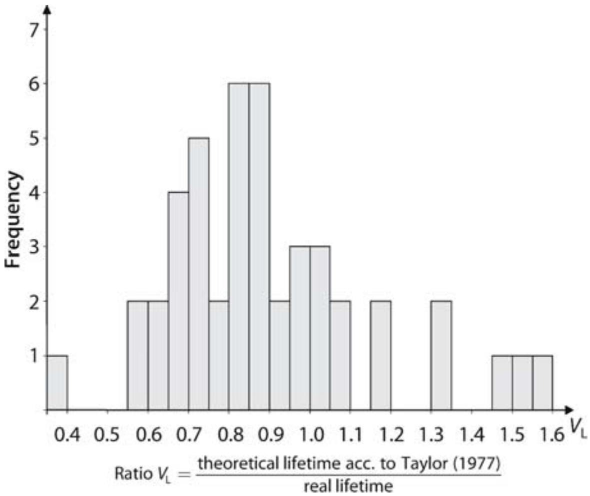

Fig. 8.1b.
Comparison of the lifetime postulated by Taylor (1977) and real lifetime of porphyry copper deposits (McSpadden and Schaap 1984)

These rules-of-thumb methods are adequate to determine the optimal lifetime of projects at the exploration stage. A more rigorous result can be achieved by applying purely economic methods, such as the net present value (NPV) described in Sect. 11.2.3, over a range of production capacities, or varying other economic parameters for different capacities. (See, for example, Cavender 1992.) But going into such detail is not necessary for economic evaluations at the exploration stage.

Wagner (1999) used economic methods to examine the potential impact of future exploration successes on the lifetime of a mine. He wanted to know whether any possible extensions to the life of a mine as a result of exploration success should be taken into account already at the planning stage. He based his work on the premise, inherent in the dynamic methods of economic evaluation which depend on the time value of money that the value of an asset changes with time. The later the reserves are mined the lower the present value of the resulting cash flow discounted to the start of the operation and therefore the lower their economic impact. Therefore, there are good economic reasons for mining an ore body in as short a time as possible, always taking technical constraints on extraction rates into consideration as discussed in Sect. 8.1.2.

In his paper, Wagner examined the potential conflict that can arise from the fact that exploration around a mine continues during its operational life. It is in the nature of any operation that as mining progresses the understanding of the deposit improves and the chances of finding additional reserves increase in line with the improved understanding. The more time there is for learning, the better the chance of finding additional reserves. Should one, therefore, prolong operational life of the mine in the expectation of additional discoveries, or speed up extraction from the start in the interest of economic efficiency? To find an answer to this dilemma, Wagner examined the mining history of a wide range of porphyry copper, Mississippi-Valley-type Pb-Zn, and polymetallic volcanogenic massive sulphide deposits and the relevant exploration successes for these deposits. He used the net present value method of Sect. 11.2.3 to evaluate the data and concluded that there is no economic justification for extending the lifetime of a deposit by decreasing the extraction rate to give exploration a better chance of success. (See also Wagner and Wellmer 1977.) Wagner's work confirms the validity of the Taylor-rule for determining the optimal lifetime of a mining operation. The learning effects of continued exploration while mining is going on has no bearing on the economics of a mine.

8.2 Market Barriers as a Determinant for a Mine Capacity

When trying to determine an optimum mine capacity with the above described rule-of-thumb methods we do not take market influence into consideration. For normal commodities like lead, iron, tungsten, or copper, we can always assume that there is a market to sell our product. The better the relative cost position of our potential mine, the better the chance to sell our product and to be successful in a competitive market.

Dealing with "high-tech"-commodities like rare earths or the "electronic metals" gallium, germanium, arsenic, selenium, indium, or tellurium (see Appendix D, Table D15), however, easy, unrestricted access to market cannot be readily assumed. The market is limited and often dominated by a few buyers. For such commodities, therefore, the market constraints have alsoto be considered in the early exploration stage. The preferable route is to form a joint venture with a producer who is already in the market for this specific commodity and let him decide the possible quantity which can be absorbed by the market. This then determines the capacity of a planned mine. Another route is to carry out a market research already at an early stage of exploration.

To illustrate the limitations one may encounter we will look as an example at gallium and germanium. The world consumption lies at 80 t gallium and 80–100 t germanium. Another constraint is that a mining operation must have a minimum size so it can be managed in a professional way and is able to carry the necessary overhead costs (see example in Sect. 9.3.2.4). Many companies consider the minimum sales volume, i.e. the net smelter return NSR of a mine (see Chap. 7), to be US $10 million.

These constraints of a minimum mining size and a limited market have the consequence that a new mine would have to conquer a large sector of the market. With a price of gallium of 450 US $/kg the world production of 80 t would have a value of US $36 million, i.e. the minimum size mine would cover 27.7% of the world market. Such a significant market share is impossible for a newcomer to attain. Wellmer et al. (1990) investigated this problem in detail. This is also the reason that these high tech commodities are mainly produced as a by-product commodity, gallium for example from bauxite and germanium from zinc concentrates. The only attempt to mine gallium and germanium ore as a direct product, Apex mine in Utah in 1987 and 1990, failed twice.

8.3 Lifetime Considerations in the Construction Minerals Industry

When determining the lifetime of metal deposits or that of high-value industrial minerals the data that is taken into consideration is restricted to the parameters of the deposit itself. The impact of the next upstream step in the chain of value creation, for instance the lifetime of a zinc smelter, is mostly neglected. This, however, is very important when contemplating lifetimes of deposits of bulk materials for the construction industry which are very sensitive to transport costs, like marl or limestone for a cement plant. Here the lifetime estimation has to go backwards from the processing stage. Based on a business plan, the lifetime and capacity of an operation is decided. This then determines the required resources of a construction mineral deposit. As a rule resources for a lifetime of 25 to 50 years have to be measured and indicated. Lütkehaus (1991) gives rules-of-thumbs for the required lifetime of resources for different raw materials (Table 8.2) which by applying the "modifying factors" (mining, metallurgical, economic, marketing, legal, environmental, social and governmental factors) can be converted into reserves (JORC 2004) (see also Fig. 4.1).

8.4 Ratio of Lifetime of Reserves

Lifetime of reserves considerations play also a role in project financing and shall be briefly discussed here. At the early exploration or prefeasibility stage we work with an ore potential or indicated resources, as for example in the Assignment of Sect. 8.1.3.1, to derive the optimal mine capacity. The feasibility study, as a basis for the final investment decision, however, has to be based on proven and probable reserves. At this stage, indicated resources and the resource potential count only as additional safety factors which may prolong the mine life. The general standards today for the definition of proven and probable reserves and indicated and measured resources are the Australasian Joint Ore Reserves Committee (JORC Code) and the Canadian CIM definition standards (http://www.jorc.org/main.php and http://www.cim.org), as already outlined in Chap. 4.

At the feasibility stage, the lifetime of the proven and probable reserves has to be long enough, with sufficient safety margin, to guarantee the payback of the loans. Therefore the ratio V_R is important. V_R is the ratio between the total lifetime of the deposit and the reserves necessary to guarantee the payback of loans.

Table 8.2. Quantitative minimum requirements for raw materials of the construction industry (Lütkehaus 1991)

Product	Required investment for processing plant[a] (million €)	Raw material	Raw material annual requirement		Mimimum required lifetime (yr)	Required resource base (million t)
			(m^3)	(million t)		
Cement	300	Limestone and marl (about 80% $CaCO_3$)	640 000	1.6	50	80
Lime	150	Limestone <95% $CaCO_3$	240 000	0.6	40	24
Gypsum for construction	20	Gypsum	45 000	0.1	25	2.5
Gypsum for cement		Anhydrite (about 40%)	45 000	0.1	50	5.0
Gravel	15	Gravel	200 000	0.4	25	10
Calcium silicate bricks	15 – 20	Sand	85 000	0.15	40	6

[a] Required investment for processing plant (Lütkehaus 1991). DM-values converted to € and inflated to 2004-values with machinery equipment index of German Federal Office of Statistics.

A feasibility study as the basis of an investment decision has to fulfil high requirements. For debt financing, it must be "bankable" if it is to enable the banks to decide whether they can finance the project with a loan as opposed to equity financing in which mining projects are financed by the owners directly from own funds. Cost data requirements for the feasibility study are discussed in Chap. 9. There are different types of debt financing. In mining, the most commonly used variety is project financing, in which the banks assume the risk of the loan after a certain defined period and accept as loan security the project itself. The collateral for the loan is at this point no longer the total assets of the mining company. This is called "nonrecourse financing". Before the bank takes over the risk of the loan the project has to be completed and a "completion test" has to be passed to ensure that the project is operating satisfactorily in accordance with the parameters of the feasibility study.

Banks will only be willing to take over the mining risk in a project financing if the necessary payback period for the loan is not too long (normally not longer than 10 years) and the proven and probable reserves are large enough with an adequate safety margin to guarantee the payback. Normally the banks require a ratio of at least 2:

$$V_R = \frac{\text{total lifetime of reserves}}{\text{time necessary to pay back the loans}} > 2$$

In Sect. 11.5 we will use this ratio V_R in our cashflow calculation.

Calculation of Cost Data

After having determined the capacity of the potential mine (Chap. 8), cost data will have to be calculated.

For a "bankable" feasibility study which can be presented to banks and used as a basis for financing and investment decisions, costs have to be determined "ab ovo": investment costs are based on real offers, operating costs are calculated directly from material consumption, salaries and wages, services, availability of machinery etc. This is the task of a team of engineers and not the subject of this book.

Prefeasibility studies and preliminary evaluations are based on indirect cost estimates, that is analogous costs. These costs are derived by comparison with existing or newly established plants. Overall costs are considered rather than specific costs for individual items (such as costs for head frame, mine hoist or the shaft itself), i.e. aggregated capital costs and operating costs for mining and beneficiation.

Example. A mine was commissioned 2 years ago at a capital cost of US $36 million. For the evaluation of a deposit with comparable capacity and similar mining conditions and without additional infrastructure requirements, the same investment costs are assumed, but inflated over 2 years (see Sect. 9.2.1.1).

9.1 Provision of Cost Data

9.1.1 Collection of Cost Data

All exploration geologists who are required to deal with economic evaluations should compile a reference collection of cost data which should be updated regularly. The following sources can be used:

- Company-owed mines.
- Information gathered during mine visits.
- Company reports: due to strict stock exchange regulations, Canadian and Australian mining companies in particular are required to publish detailed reports on their mines, cost breakdowns included. In the USA the called 10-K report for the stock exchange contains more technical information than the average annual report. Excellent sources are, therefore, company web-pages and the EDGAR- and SEDAR-webpages. EDGAR is

the **E**lectronic **D**ata-**G**athering, **A**nalysis, and **R**etrieval system of the U.S. Securities and Exchange Commission, SEC, which contains also the 10-K reports. SEDAR is the **S**ystem for **E**lectronic **D**ocument **A**nalysis and **R**etrieval operated by the Canadian Securities Administrators[9].

- Publications: International mining magazines (e.g. Engineering and Mining Journal, Mining Magazine, Mining Journal, International Mining, Canadian Mining Journal, Bulletin of the Australasian Institute of Mining and Metallurgy, Bulletin of the Canadian Institute of Mining and Metallurgy) regularly report on new mining projects and their respective capital costs. "Mining Journal" regularly publishes a supplement with individual data on South African gold mines. The "Canadian Mines Handbook" and the "Register of Australian Mining", both appearing annually, sometimes publish costs of new projects. A good source for operating costs of Canadian mines is the annual Canadian Mining Journal Mining Sourcebook. The Engineering and Mining Journal (E&MJ) publishes every year in its January edition a list of capital investment projects worldwide with investment costs. The Australasian Institute of Mining and Metallurgy in 1993 published a "Cost Estimation Handbook for the Australian Mining Industry" (Noakes and Lanz 1993) which in most cases is too detailed for our evaluations in the exploration stage. However, general cost information useful for our purposes can also be extracted. Rudenno (1998) published a list of capital costs and some operating costs of Australasian operations.
- Excellent sources of information for cost data are the publications "The Metals Economics Group Strategic Report" (formerly "Mine Development Bimonthly" and "Production Cost Update") of the Metals Economics Group (MEG) in Halifax (New Brunswick, Canada)[10]. They are, however, rather expensive and might not be readily available, except from libraries of mining companies. The same is true for data about commodities and mines of the Raw Materials Data Bank of the Raw Materials Group in Stockholm, Sweden[11]. There are also other mining consultants who publish and sell mining cost data studies, for example Western Mine Engineering, Inc., Spokane, Washington, USA (Mine Cost Service)[12]. Further information can be gathered from advertisements in the Mining Journal or more popular technical journals like the weekly "Northern Miner" (Canada).
- Research by stockbrokers: They regularly carry out investigations into the profitability and thus the cost structure of mining companies. Normally such studies are available to interested parties at no charge.
- Studies by economic consultants: They frequently conduct multi-client studies about raw materials and the cost structure of the producers. Good examples are Australian Mineral Economics (AME) in Sydney, Australia[13], or Roskill in London[14]. Such studies, however, are very expensive and normally only available if one works with a large mining company.

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[9] Internet addresses: http://www.sec.gov/edgar.shtml, and http://www.sedar.com/search/search_form_pc_en.htm.
[10] www.metalseconomics.com.
[11] www.rmg.se.
[12] www.westermine.com.
[13] www.ame.com.au.
[14] www.roskill.com.

- There are indirect sources for operating costs like information on cutoff grades or costs per unit metal which can be used to calculate operating costs per tonne of ore mined. This is shown in the following Sect. 9.1.2.

Note: Certain capital cost data should *not* be used in economic evaluations. Professional publications sometimes refer to specific costs per tonne of metal in a generalized manner without regard to mine and plant capacities. For the specific capital costs of a new mine the expression "new venture price" has been coined.

A typical case in point is the average cost of 5 000 US $/t copper metal to bring a new mine into production at present. As an example, we want to evaluate a copper mine with a production rate of 200 000 t of copper concentrates per annum. At a concentrate grade of 25% this amounts to 50 000 t copper content. At the above investment cost of 5 000 US $/t Cu the mine would cost US $250 million, regardless whether the mine had a grade of 0.5% Cu or 1.5% Cu. With a grade of 0.5% Cu and a beneficiation recovery of 90%, 11.1 million t ore/a or 37 000 t/day would have to be mined to produce 200 000 t of concentrates; with a grade of 1.5% Cu, however, only 3.7 million t ore/a or 12 000 t/day, i.e. only one-third of the above mentioned rate of production. Thus it becomes obvious that such strikingly different capacities must have considerable influence on capital costs.

Investment costs per ton metal are therefore *unsuitable* for the evaluation of an individual deposit. These figures are mainly useful for strategic considerations, e.g. how much capital has to be provided to guarantee future supply.

9.1.2 Indirect Cost Data Information

Whereas some mines are reluctant to give information on cost data, information on cutoff grades is usually available. Most mines which have been in operation for some years use an operating cost cutoff. That is the revenue obtained from ore with a cutoff grade chosen so that it just covers operating costs but not the capital costs (Sect. 10.1) or a profit margin. Hence with a known cutoff grade one can arrive at the operating costs by calculating backwards.

Assignment. A Zn-mine in carbonates of the Mississippi Valley-type uses a cutoff grade of 2.5% Zn. From technical discussions during a mine visit it can be ascertained that this is an operating cost cutoff grade. The mill recovery is 90%, i.e. $\varepsilon = 0.9$ (see Sect. 4.3).

The general price trend at the time of the mine visit is around 40 US ¢/lb Zn. What is the estimate for the total operating costs OPC?

For the calculation of the net smelter return for the mine we use the factors NF of Table 7.1 (Sect. 7.2.2). The conversion factor of lb into % is 22.046 (see Sect. 1.1.4). The revenues to the mine for 2.5% Zn, i.e. in this case the operating costs OPC, are

$$\text{OPC} = 2.5 \times 22.046 \times 0.5 \times 0.9 \times 0.40 = 9.92 \text{ US \$/t} \approx 10 \text{ US \$/t}$$

Based on the applied cutoff grade we estimate total operating costs of 10 US $/t.

Another example for a cutoff grade calculation is given in Sect. 10.1.1.

Cost data per unit metal can be converted into operating costs per tonne of ore mined. Sometimes companies publish cash operating costs for example for 1 oz of gold produced or 1 lb of copper after credits. With other data which are normally available the operating cost per tonne of ore can be calculated, as will be shown with two examples:

Example 1: An underground gold mine reports cash operating costs per ounce of US $240. Also in the annual report it is reported that the production was 530 000 ounces from a throughput of 3.6 million tonnes of ore. So the operating costs OPC are

$$\mathrm{OPC} = \frac{240 \times 530\,000}{3\,600\,000} = 35.33 \left(\frac{\mathrm{US\$}}{\mathrm{t}}\right)$$

If we are looking for cost data for an underground mine with an annual throughput of ca. 3.6 million tonnes for a preliminary evaluation we would take operating costs of 35 US $/t.

Example 2: An open pit copper mine reports cash costs after credits of 0.45 US $/lb Cu.

Porphyry copper mines usually receive smelter credits for gold values in the copper concentrates. Another possibility are credits for molybdenum values, which are recovered into a separate molybdenum concentrate. In this case we learn from relevant publications that the mine reports grades of 0.60% Cu and 0.01% Mo, however no gold values.

The mine is located in the Pacific Rim and we, therefore, assume that the concentrates are marketed there. So we assume an NF-factor of 0.75 of Table 7.1 in Sect. 7.2.2. We have to make assumptions about the recovery. A normal recovery for Cu is 90% (see Table 7.1, Sect. 7.2.2), for the low grades of Mo it is always lower. We take 80%. We also have to make an assumption for the Mo-price. At the time of publication the Mo-price fluctuated between 2.30 and 2.80 US $/lb Mo in MoS_2-concentrates. We assume 2.50 US $/lb Mo. The conversion factor from percent to lb is 22.046 (see Sect. 1.1.4 Pound).

So the cash costs per tonne of ore after credits are

$$0.6 \times 0.75 \times 22.046 \times 0.9 \times 0.45 \text{ US\$/t} = 4.02 \text{ US\$/t}$$

Now we have to add the credits from the molybdenum contribution, which is per tonne:

$$0.01 \times 22.046 \times 0.8 \times 2.50 \text{ US\$/t} = 0.44 \text{ US\$/t}$$

To arrive at the operating cost per tonne OPC we have to add the credits from molybdenum to that of copper:

$$\mathrm{OPC} = 4.02 + 0.44 = 4.46 \quad , \quad \text{meaning } 4.50 \text{ US\$/t of ore}$$

9.2 Processing of Cost Data

In most cases it will be necessary to modify the available data for each individual situation. Capital costs, for example, may date from earlier years and have to be adjusted for inflation for the current or for future years. Moreover, available cost data may not match the desired capacity.

9.2.1
Adjustment for Inflation of Capital and Operating Costs

Before making any interpolations (see Sect. 9.2.2) all data have to be brought to a common denominator and converted to money values of the current year.

Here the question arises: Why are costs adjusted for inflation to the current year when the potential mine to be evaluated will only be in production in a few years' time?

It is common practice to carry out economic calculations with constant prices and revenues. In Chap. 6 we have attempted to derive a realistic current metal price. This price has to correspond with realistic current costs.

We assume that future costs and revenues continue to develop in a parallel fashion. Only in a complete feasibility study would one work with differently inflated prices and costs.

9.2.1.1
Capital Costs

For the adjustment of capital costs for inflation, capital cost indices are used. If costs are to be calculated for the current year, the following formula is used:

$$\text{costs today} = \text{costs in year } x \times \frac{\text{index today}}{\text{index in year } x}$$

The calculation is done in the same way as in the exercise in Sect. 6.3.2.

Assignment. A mine in northern Australia was brought on stream in 2002 at a cost of AU $430 million producing 5.5 million t/a of steaming coal from an open pit. How much should this mine have cost in 2005?

We use the Marshall and Swift Mine and Mill Index (Table D14, Appendix D). For 2002 the index was 1 147.5; for 2005 it was 1 315.7. Thus capital costs in 2005 would have amounted to

$$430 \times \frac{1\,315.7}{1147.5} = \text{AU}\,\$493\,\text{million}$$

For capital cost calculations data, older than 5–6 years should be used with reservations; data older than 10 years not at all, unless later information for specific plants is unavailable. In the course of time, technical innovation usually renders data older than 10 years obsolete.

The best index to adjust capital costs for inflation for North American mines (USA, Canada) is the American Marshall and Swift Mine and Mill Index. It is listed in Table D14 (Appendix D). It is published together with other capital cost indices in the journal "Chemical Engineering" (see also Matley 1982).

For developing countries the index of international inflation, published by the World Bank as the manufacturing unit value index (unit values of manufactured goods to developing nations), already referred to in Sect. 6.3.1, can be used.

In many mining countries there exists a wealth of experience concerning investment costs in a specific country, which if possible should be taken into account. For example in the nineteen-eighties it was known as a matter of experience that capital costs in Australia were about 10 to 20% higher than in Canada. Also for capital cost increases there are sometimes empirical data based on practical experience which can be used.

Assignment. For a copper-nickel mine with a crude ore production of 360 000 t/a a feasibility study was made in 1999 which estimated capital costs at CA $80 million. How much would the mine cost in 2004, if we assume an annual cost increase of 3%?

Between 1999 and 2004 five years had passed. Since cost increases have a cumulative effect, the inflation factor is $(1 + 0.03)^5 = 1.16$. A cost estimate in 2004-CA $ would amount to

$$1.16 \times 80 = 92.8 \quad \text{, i.e. ca. CA \$93 million}$$

If a capital cost index is not available for a particular country, it can be composed from other indices. A rule-of-thumb is:

- 55% salary and wage costs
- 35% material costs
- 10% unchanged

Table 9.1.
Capital cost index

Year	I Wage index	II Material index	III Combined capital cost index
1991	100.0	100.0	100.0
1992	102.1	101.8	100.8
1993	102.3	104.2	102.7
1994	104.3	107.2	104.9
1995	107.0	110.3	107.5
1996	109.3	111.8	109.2
1997	112.8	113.6	111.8
1998	117.3	114.4	114.6
1999	118.2	115.3	115.4
2000	119.8	117.2	116.9
2001	123.0	117.9	118.9
2002	124.5	119.7	120.4
2003	127.1	122.0	122.6
2004	130.8	128.5	126.9

This covers productivity improvements and cost decreases arising from continuing technical innovations.

Assignment. Develop a capital cost index for the years 1991 to 2004 for a specific country for which wage and material cost indices are available, but not a capital cost index.

First the two indices, wage and material cost index, have to be related to the same basic year at the beginning of the period in question, i.e. 1991. This is best done by setting both indices at 100 for the year 1991, in the same way as in the example of the Canadian consumer price index in Sect. 6.3.1. The material and wage indices thus related to the basic year 1991 are listed in columns I and II of Table 9.1.

The calculation of our "derived capital cost index" is carried out in a simplified way with the following weighting (see also Sect. 2.2.1):

$$\frac{0.55 \times \text{wage index} + 0.35 \times \text{material index} + 0.1 \times 100}{0.55 + 0.35 + 0.1}$$

The result is listed in column III of Table 9.1.

9.2.1.2
Operating Costs

At the onset we have to realize that a relationship exists between operating costs and grades. A mine exploiting a very rich deposit can afford higher operating costs. The mine can afford to use more expensive methods to extract rich parts of the deposit. So, if we look in our economic evaluation for analogous data from operating mines we should be careful to select not only similar deposit types but also deposits with similar grades.

Example. In our exploration programme we discovered a gold mineralization with a grade of 10 g Au/t. Searching for operating cost data, we should select mines with similar grades and adjust the operating costs to capacity with methods described below in Sect. 9.2.2 or 9.3.2.2. Operating costs of mines with grades of or above 1 oz Au/t (31.103 g/t see Sect. 1.1.4), for example, should be avoided.

For the purpose of a rough-and-ready estimate of operating costs there are of course wage cost indices available (see exercise in Sect. 9.2.1.1) and we could follow the rule-of-thumb that about 50 to 60% of mine operating costs are salary and wage costs (see exercise in Sect. 9.3.2.3). However, mines run on modern management principles regularly increase their annual productivity and this increase has of course to be taken into account. The quality of management as an operating cost factor should not be underestimated. In Sect. 6.5 it has been shown for example how mining companies constantly try to move down the cost curve, i.e. trying to reduce costs.

To derive operating costs for our exercise we should try to study time series of operating mines. This enables us to adjust operating costs for inflation and also to recognize the impact of technological breakthroughs or price effects. As an example Fig. 9.1 shows time series of six Canadian operating mines. The data of the operating costs are from the 1990 to 2003 Mining Sourcebooks of the Canadadian Mining Jour-

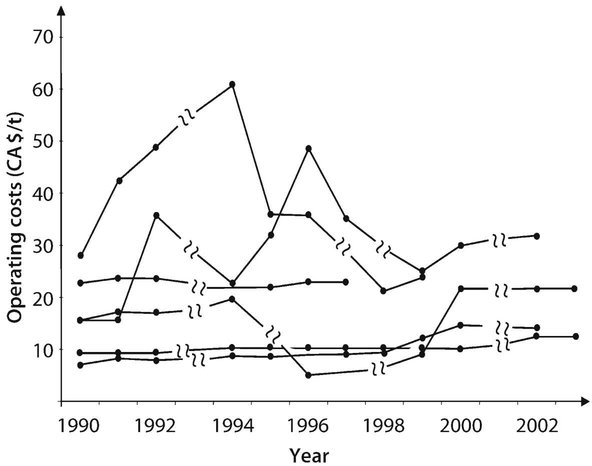


Fig. 9.1. Time series of six operating mines in Canada

nal, mentioned as a good source in Sect. 9.1. One sees some cost trends which slowly increase. If such mines are similar to the deposit type we want to evaluate we can use the method of geometric means, described below, to analyse the data. Other time series, however, show wide fluctuations. These can have various reasons: richer additional reserves are discovered allowing higher extraction costs; or a general price rise. If metal prices rise, operating costs normally rise too.

Helped by higher metal prices, cost intensive parts of a deposit (low grade veins or areas requiring more development) become economical and are integrated into the mining process. Since we want to work with constant costs and revenues (see opening remark in Sect. 9.2.1) we have to eliminate the effect of extra production on costs. Often there is a time lag, since the development of marginal parts of a deposit requires a certain period of preparation.

Although the following example is 25 years old it is given here because the effect of a price peak (silver speculation of the Hunt Brothers) is still an excellent example of how operating costs are influenced by singular events and how to deal with these peaks in a time series.

Example. The following time series for the operating costs of two silver vein deposits in Canada have been found in geological publications: Mine A in the Cordillera with a production of 40 000 t/a, mine B on the shield with a production of 85 000 t/a (Table 9.2).

If the rates of operating cost increase in Table 9.2 are plotted against the silver price in a chart (Fig. 9.2) it becomes obvious that the considerable increase in operating costs coincides with the silver price peak for 1980 (speculation of the Hunt brothers). It is also obvious that mine B reacted more quickly to the price slump than mine A. These "price peak effects" can be eliminated by taking only the 1981/1982 and 1982/1983 rates of increase for mine A and those of 1980/1981, 1981/1982 and 1982/1983 for mine B into account (Table 9.2).

The method used to determine the average rates of change per annum is the geometric mean (which is always lower than the arithmetic mean). The procedure is the same as for the calculation of average growth rates (see Sect. 14.1).

Table 9.2. Operating costs

Year	I Operating costs Mine A (CA $/t)	II Rate of increase *j* (%)	III Operating costs Mine B (CA $/t)	IV Rate of increase *j* (%)
1979	44.77	9.8	26.89	15.2
1980	49.16	18.4	30.99	3.9
1981	58.19	0.1	32.20	3.0
1982	58.27	4.9	33.17	4.1
1983	61.12		34.53	

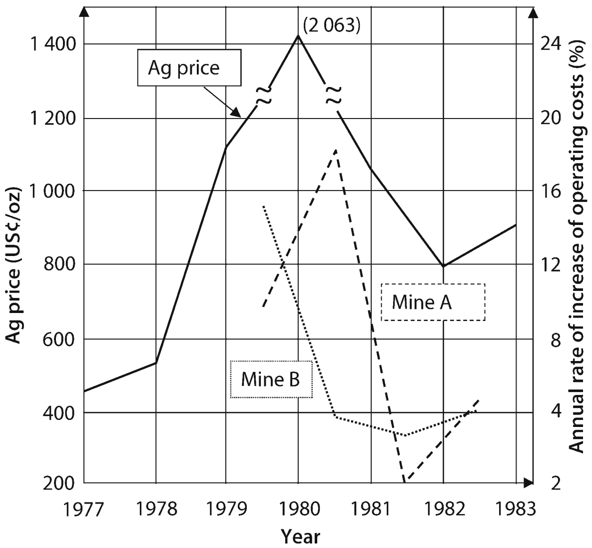


Fig. 9.2.
Development of the silver price and costs of two silver mine in Canada

The geometric mean is determined by the formula

$$W_G = \sqrt[n]{W_1 \times W_2 \times W_3 \cdots W_n}$$

First the annual rates of increase (columns II and IV) are determined in the time series for operating costs. Then the geometric mean is calculated with the rate of increase j in columns II and IV being expressed as $1 + j/100$:

Mine A: $\sqrt[2]{1.001 \times 1.049} = 1.025$

Mine B: $\sqrt[3]{1.039 \times 1.030 \times 1.041} = 1.037$

For these two values the arithmetic mean is found (unless we have technical or geological reasons to attribute a higher ranking to one mine than to the other):

$$\frac{1.025+1.037}{2}=1.031$$

i.e. an average annual increase of 3.1% in operating costs.

If a vein deposit is being evaluated and costs of 55 CA $/t for 1983 have been interpolated from a cost curve (see Sect. 9.2.2), these costs must be inflated for 1984:

$$55\times 1.031 = 56.71, \text{ i.e. } 57 \text{ CA \$/t}$$

Using the geometrical mean is a quick method, but has its drawbacks. This problem will be dealt with in Sect. 14.1 and 14.2.

9.2.2 Power Curves

To determine the interdependence between costs, adjusted to the same year (see Sect. 9.2.1), and corresponding operating capacities, non-linear functions are often used of the type

$$y = a\times x^b$$

where y are the costs, x the capacity and a and b are constants.

If the logarithmic expression for this equation is taken and the optimal power curve expressed by $y=a\times x^b$, then the constants a and b can be found via linear regression (see Sect. 3.3):

$$\ln y = \ln a + b\times \ln x$$

According to Sect. 3.3, the regression coefficients are

$$b=\frac{\sum(\ln x_1)\times(\ln y_1)-\dfrac{\sum(\ln x_1)\times\sum(\ln y_1)}{n}}{\sum(\ln x_1)^2-\dfrac{\left[\sum(\ln x_1)\right]^2}{n}}$$

$$a=\exp\left[\frac{\sum(\ln y_1)}{n}-b\frac{\sum(\ln x_1)}{n}\right]$$

Further, the square of the correlation coefficient r is

$$r^2=\frac{\left[\sum(\ln x_i)\times(\ln y_i)-\dfrac{\sum(\ln x_i)\times(\ln y_i)}{n}\right]^2}{\left[\sum(\ln x_i)^2-\dfrac{\left[\sum(\ln x_i)\right]^2}{n}\right]\times\left[\sum(\ln y_i)^2-\dfrac{\left[\sum(\ln y_i)\right]^2}{n}\right]}$$

Assignment. Determine the overall operating cost for a potential underground base metal mine in a massive sulphide deposit with a capacity of 3 000 t/day in Spain. Since cost data for Spain are not available to us for a quick evaluation in the exploration stage, we take as proxies data published in the Canadian Mining Journal's 2003 Mining Sourcebook (Table 9.3a). We select deposits of the same ore type, massive sulphides, with one exception: a lode gold mine is included because it also mines a relatively massive ore body.

A power curve in the form of $y = a \times x^b$ can be plotted interpolating the data points (Fig. 9.3). Here y stands for operating costs and x for capacity per day (Table 9.3a). To obtain the operating costs for the potential 3 000 t/day mine by interpolation, $x = 3\,000$ is inserted into the equation $y = 1\,445.0 \times x^{-0.52}$. (The calculation to arrive at this equation is shown below.) This results in $y =$ CA $22.48, which is rounded to 22.50 CA $/t. The same interpolation procedure using a power curve in the form of $y = a \times x^b$ can of course be applied to determine capital costs too.

A warning might be appropriate at this point with regard to extrapolation: Higher capacities pose no real problem, since the curve in Fig. 9.3 can safely be applied to up to 7 to 8 000 t/day. With capacities below the lowest data point, however, the extrapolation procedure becomes precarious. Since the curve rises steeply in this area, small variations in the production can result in disproportionate changes in costs. For capital costs in this range the "0.6-rule" is therefore preferable (see Sect. 9.3.1.2), for operating costs, the equivalent rule is described in Sect. 9.3.2.2.

The regression coefficients a and b and the correlation coefficient can be determined according to Table 9.3b.

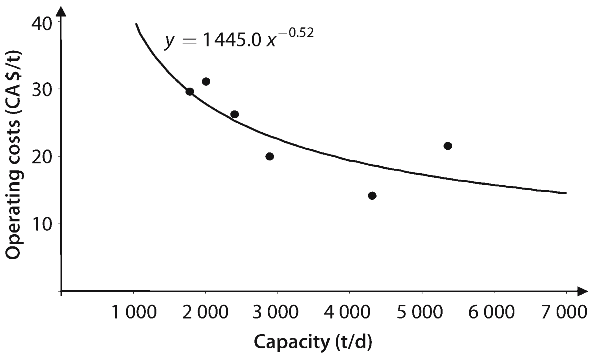


Fig. 9.3. Specific operating costs of various non-ferrous metal mines (Table 9.3a)

Table 9.3a. Operating costs of some selected mines

Mine	Capacity x (t/d)	Operating costs y (CA $/t)
Aur, Louvicourt	4 300	14.22
Barrick, Bousquet	2 400	26.33
Barrick, Holt-McDermott	1 775	29.72
Breakwater, Bouchard-Hebert	2 880	20.11
Newmont, Holloway	2 000	31.18
Hudson Bay, Ruttan	5 350	21.62

- *Step 1:* **Table 9.3b.** Data for calculation of regression and correlation coefficients

	$\ln x_i$	$(\ln x_i)^2$	$\ln y_i$	$(\ln y_i)^2$	$\ln x_i \ln y_i$
	8.366	69.990	2.655	7.049	22.212
	7.783	60.575	3.271	10.699	25.458
	7.482	55.980	3.392	11.506	25.379
	7.966	63.457	3.001	9.006	23.906
	7.601	57.775	3.440	11.834	26.147
	8.585	73.702	3.074	9.449	26.390
$\Sigma =$	47.783	381.479	18.833	59.543	149.493
$\Sigma/n =$	7.964	63.580	3.139	9.924	24.915

- *Step 2:* Regression coefficients

$$b = \frac{149.493 - \dfrac{47.783 \times 18.833}{6}}{381.479 - \dfrac{47.783^2}{6}}$$

$$b = \frac{149.493 - 149.983}{381.479 - 380.536} = -\frac{0.490}{0.943} = -0.520$$

$$a = \exp\left[\frac{18.833}{6} + 0.520 \times \frac{47.783}{6}\right]$$

$$a = \exp[3.139 + 4.137] = \exp 7.276$$

$$a = 1\,444.955$$

$$y = 1\,445.0 \times x^{-0.52}$$

- *Step 3:* Square of the correlation coefficient $= r^2$

$$r^2 = \frac{\left[149.493 - \dfrac{47.783 \times 18.833}{6}\right]^2}{\left[381.479 - \dfrac{47.783^2}{6}\right] \times \left[59.543 - \dfrac{18.833^2}{6}\right]}$$

$$r^2 = \frac{(149.493 - 149.983)^2}{(381.479 - 380.536) \times (59.543 - 59.114)} = \frac{-0.490^2}{0.943 \times 0.429} = 0.593$$

$$r^2 = 0.593; \; r = 0.770$$

i.e. 59.3% of the scatter of the data points can be explained by the linear regression of the logarithmic values.

9.3 Further Rules-of-Thumb

9.3.1 Rules-of-Thumb for Capital Costs

9.3.1.1 The Annualised Cost Per Tonne Rule

Frequently, in mining camps with a long production history, practical rules-of-thumb are available for capital costs of plants per tonne of annual plant capacity. They are usually adequate for a first estimate.

Example. In a gold mining camp with active mining operations past experience suggests that for a gold beneficiation plant, using carbon-in-pulp (CIP)-technology, capital costs are 10 AU $/t of annual capacity. Assuming that the mining operation will be contracted out, we are looking for a quick capital cost estimate of a 2 000 t/day beneficiation plant. The annual capacity is 700 000 t/a, and the capital costs would, therefore, be AU $7 million.

This annualised cost per tonne rule does not take into account economies of scale introduced at the beginning of Chap. 8, i.e. the higher the capacity the lower the specific costs. This aspect is taken into account in the 0.6-rule explained below.

9.3.1.2 The 0.6-Rule for Capital Costs

If available cost information is restricted to a single comparable plant, or if a plant is comparable in all but capacity, the 0.6-rule (or six-tenth rule) for capital cost estimates can be applied. The 0.6-rule was first described by Mular (1978). The formula of the 0.6-rule can be written:

$$\frac{\text{investment costs } x}{\text{investments costs } y} = \left[\frac{\text{capacity } x}{\text{capacity } y}\right]^{0.6}$$

According to Noakes (1993) the result of applying the 0.6-rule can be expected to have a margin of error of about 30%.

The 0.6-rule is a special case of the power curve $y = a \times x^b$ from Sect. 9.2.2 with $b = -0.4$, because as we will see later $0.6 = b + 1$. In this equation y represents capital costs per t of capacity and x stands for capacity. In the above formula of the 0.6-rule, however, not the specific costs per tonne of capacity are considered but absolute costs. The notation for these absolute costs shall be I. Therefore we have for a capacity of x_1

$$y_1 = \frac{I_1}{x_1} = a x_1^b$$

and for a capacity x_2

$$y_1 = \frac{I_2}{x_2} = a x_2^b$$

By division of the two equations we obtain

$$\frac{y_1}{y_2} = \frac{I_1}{I_2} \times \frac{x_2}{x_1} = \left(\frac{x_1}{x_2}\right)^b$$

$$\frac{I_1}{I_2} = \frac{x_1}{x_2} \times \left(\frac{x_1}{x_2}\right)^b = \left(\frac{x_1}{x_2}\right)^{1+b} = \left(\frac{x_1}{x_2}\right)^{0.6}$$

This proves that for the 0.6-rule $b = -0.4$.

Assignment. In 1999 a bucket line dredge with screen and pump was built for an alluvial deposit in Australia for a capacity of 150 t/h at a cost of AU $7.2 million. Estimate how much a comparable bucket line dredge with a capacity of 200 t/h would cost in 2004.

First the costs of 1999 have to be adjusted for inflation to those of 2004. Again we use the Marshall and Swift Mine and Mill Index (see Sect. 9.2.1.1 and Table D14, Appendix D):

$$\text{costs 2004} = \text{AU\$7.2 million} \times \frac{1\,232.6}{1106.0} = 8.02$$

Next, the 0.6-rule is applied:

$$\frac{\text{investment costs (200 t/h)}}{8.02} = \left[\frac{200}{150}\right]^{0.6}$$

Investment costs (200 t/h) = 8.02×1.19 = AU $9.5 million.

O'Hara (1980) used the 0.6-rule to derive a general rule for capital costs from predominantly Canadian data.

- Open cut mining with beneficiation plant: $I = A \times T^{0.6}$ (T = capacity in t/day)
- Underground mining with beneficiation plant: $I = B \times T^{0.6}$
- In the year 1980 the factor A was 400 000 for open pit mines and B = 800 000 for underground mines

O'Hara points out that the values thus derived are very rough guidelines only. Actual cases can deviate considerably. The capital cost structure of a gold mine with a carbon-in-pulp (CIP) processing plant differs from that of a sulphide deposit with flotation or a carnallite mine with a plant attached to separate the K-component from the K-Mg complex salt carnallite.

Updating these factors with a selection of typical base and precious metal projects in the feasibility stage or under construction in 2004 and 2005 in the classical mining countries Australia, Canada, South Africa, USA, other industrialised and South American countries (data from the Raw Materials Group data bank) results in mean values for A of roughly 750 000 and for B of roughly 1 000 000. Standard deviations are, however, very large: 700 000 for A and 900 000 for B. A and B are factors for very rough estimates of capital costs in US $.

Assignment. Calculate a rough investment cost estimate for a 1 000 t/day underground mine with processing plant:

$$I = 1\,000\,000\ T^{0.6} = 1\,000\,000 \times 1\,000^{0.6} = \text{US}\,\$63.1\ \text{million} \quad ; \quad \text{e.g. US}\,\$65\ \text{million.}$$

9.3.2
Rules-of-Thumb for Operating Costs

9.3.2.1
Rules-of-Thumb for Rough Calculations

For a quick estimate of the economics of a potential mine under normal mining conditions (i.e. no large-scale open pit mining or mines with extreme infrastructure requirements) the following rule-of-thumb can be applied: operating costs should be covered by *half* the paid metal content. The other half is normally sufficient to cover taxes, capital costs and yield a sufficient profit margin.

A worldwide study of economical gold deposits, for example, shows that the cutoff grade, in most cases defined as the grade just covering the operating costs (see Sect. 10.1), equals about half the average grade of the mines.

Example. If the operating cost calculations based on the interpolation of cost data of various mines (see Sect. 9.2.2) establish that a gold grade of 5 g/t is needed to cover the operating costs of the potential mine and if the recognizable potential grade of the deposit is 9–11 g/t, there is every chance that the deposit is economical. It is then justified to carry out more detailed analyses. Is the recognizable potential grade only 6–7 g/t, the deposit is likely to be only submarginal at best.

9.3.2.2
Working with Ratios of Exponents of the Power Curve Relationship of Sect. 9.2.2

If we have only one piece of information or just very few data for operating costs it does not make sense to derive an equation for an power curve as demonstrated in Sect. 9.2.2. In such a situation it is better to select operating costs from a similar deposit, use a coefficient of a power curve relationship from literature and apply a method of ratios similar to the 0.6-rule of Sect. 9.3.1.2.

Example: We have to estimate the operating costs for a Mississippi-Valley-Type Pb-Zn-deposit in carbonates. We have operating costs of 31 US $/t of a mine in a comparable deposit with a capacity of 2 000 t/day. An investigation to find the optimal lifetime using the relationship of Taylor (1977) (see Sect. 8.1.3.1) required a capacity of 1 500 t/day. How can we adjust the operating costs of the 2 000 t/day example to the capacity of 1 500 t/day?

Table 9.4.
Exponent b for different deposit types (Wagner 1999)

Deposit type	Exponent b
VMS	–0.34
MVT	–0.17
Porphyry	–0.30

Wagner (1999) investigated power curve relationships for operating and investment costs for volcanic massive sulphide (VMS), Mississippi-Valley type (MVT) and porphyry Cu and Mo deposits based on comprehensive data sets from 1989 to 1994. He obtained the values for the exponent b in the expression $y = a \times x^b$ for operating costs presented in Table 9.4.

We can now work with a ratio to find the relationship between capacity 1 and capacity 2:

$y_1 = a \times x_1^b$ and

$y_2 = a \times x_2^b$

Dividing these two equations we eliminate the factor a and obtain

$$\frac{y_1}{y_2} = \left(\frac{x_1}{x_2}\right)^b$$

In our equation $y_2 = 31$ US \$/t, $x_1 = 1\,500$ t, $x_2 = 2\,000$ t and $b = -0.17$ (from Table 9.4):

$$y_2 = 31\left(\frac{1\,500}{2\,000}\right)^{-0.17} = 32.55 \quad , \quad \text{i.e. } 33\left(\frac{\text{US\$}}{\text{t}}\right)$$

9.3.2.3 Rule-of-Thumb for Underground Operating Costs

Comparable mines (i.e. similar type of deposit, same mining method) in countries of the same industrial standard usually have comparable standards of underground efficiency. Experience shows that, as a rule, wage costs make up about 50–60% of underground operating costs. Therefore, estimates of overall mine costs can be made from the production per man and shift (t/M + S) and the sum of labour costs.

Example. A fluorite deposit in Italy is to be evaluated. It is a vein deposit. We assume that due to modern trackless mining methods and development through a ramp an output of 20 t/man and shift (M + S) can be achieved comparable to that of small German barite vein mines. Total labour costs (i.e. direct cost plus indirect labour costs such as insurance etc.) are estimated at 300 €/shift. Calculate the overall operating costs.

Since vein mining is relatively labour-intensive, we choose the upper end of the range 50–60%. With an efficiency of 20 t/M + S we arrive at specific labour cost per t of crude ore of 300/20 = 15 €/t. Assuming a 60% labour cost share, total underground operating costs amount to 15/0.6 = 25 €/t.

9.3.2.4
Rules-of-Thumb for Open Pit Operating Costs

Operating costs per tonne of ore in open pit mining depend to a very large extent on the waste/ore ratio (see Sect. 1.2.6), whereas costs per tonne of material moved are relatively constant. For small open pit mines (1 000–5 000 t/day) US$1.50/t can be expected, for big mines US$1.00/t. Loaders and heavy trucks have capacity limits. If the capacity in open pit mining is to be increased, more loaders and trucks have to be purchased.

Example. To illustrate this point, let us take an open pit mine with 10 000 t/day and a waste: ore ratio of 1 : 1. We have to move 1 t overburden for 1 t ore, i.e. a total of 2 t. At a cost of 1 US$/t of moved material the open pit operating costs amount to 2 US$/t. If the mine has a waste:ore ratio of 10 : 1, an additional 10 t of waste has to be moved for each 1 t of ore, i.e. a total of 11 t, and open pit operating costs become 11 US$/t of ore.

Assignment. Calculate the operating costs for a massive sulphide deposit to be mined to the level of –100 m in an open cut. A typical cross-section is given in Fig. 9.4. The open pit reserves amount to 6 million t. Envisaged lifetime of the mine is 10 years. The orebody dips at 45°. Density of the sulphide ore is 4, of the host rock 3, of the alluvial cover 2 g/cm^3.

- *Step 1:* The waste: ore ratio is calculated from the sectional areas. For the open pit wall an angle of slope of 45° is assumed:

  Area I = 5 949 m^2
  Area II = 4 900 m^2
  Area III = 1 980 m^2

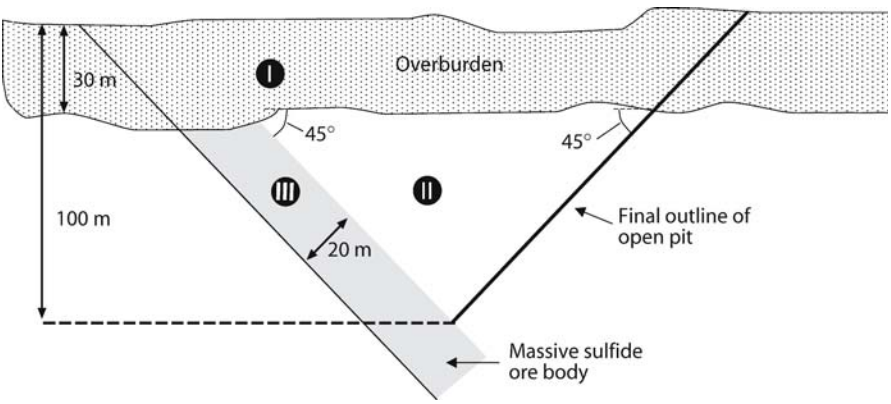


Fig. 9.4. Vertical section to calculate the waste:ore ratio

From these areas and the density we calculate the tonnage per m of section thickness:

Area I $= 5\,949 \times 2 = 11\,898$ t
Area II $= 4\,900 \times 3 = 14\,700$ t
Area III $= 1\,980 \times 4 = 7\,920$ t

Areas I and II represent waste. Hence our waste:ore ratio $W{:}O$ is

$$W:O = \frac{11\,898 + 14\,700}{7\,920} = 3.36:1$$

At both ends of the mine additional waste will occur in the direction of strike since the walls are at an angle. For a rough calculation the waste is increased by 10% to 3.7 : 1.

- *Step2:* Calculation of the operating costs of the open pit mine. The deposit in the open pit area has reserves of 6 million t to be mined over a period of 10 years, i.e. 600 000 t/a. With 300 working days/year this amounts to 2 000 t/day. With a waste: ore ratio of 3.7 : 1, a total of 4.7 t have to be moved for each tonne of ore, i.e. 9 400 t/day. This is a medium-sized open pit mine. Medium operating costs of 1.30 US $/t of moved material can be assumed. Thus, total operating costs amount to $4.7 \times 1.30 = 6.11$ US $/t ore.
- *Step 3:* Calculation of total operating costs. We assume to have arrived at beneficiation costs of 6.20 US $/t through interpolation (see Sect. 9.2.2) for our 600 000 t plant. (Beneficiation plants run 7 days a week so that the mill throughput would only be 1 650 t/day.)

  Combined costs are increased by 25% for administration, general expenses, technical services etc., the called overheads:

Open pit mining	6.11
Beneficiation	6.20
+25%	3.08
Total Operating Costs	15.39 US $/t

The final figure should be rounded to at least 15.50 US $/t or even to 16.00 US $/t.

9.3.2.5 Estimating Milling Costs

When evaluating a submission with known mineralization, mineralogical studies and some very preliminary beneficiation tests would normally have already been performed at the early stages of the exploration. Frequently, therefore, sufficient information is available at an early stage of an evaluation on the intergrowth of the ore minerals to be able to ascertain the grain size the ore has to be ground to in order to achieve a reasonable recovery; as well as data on the hardness of the ore.

In laboratory tests for hardness, the called Bond index (or the Bond work index) is determined. The index is a factor proportional to the kilowatt hours per ton required to grind a feed of theoretically infinite particle size to 80% passing 100 microns. There are equations to calculate the required energy per ton for grinding which, in turn, can determine the capital cost of mills (see Mular 1982). Such calculations are too detailed for the scope of this book. However, we can use them to adjust milling operating costs which have been interpolated from a number of mines according to the procedure described in Sect. 9.2.2.

Soft ore have a work index of under 12, medium hard ore of about 15 and hard ore of about 17. Direct grinding and milling costs (energy, balls, wear etc.) which are influenced by hardness and the required fineness of grind normally account for about one-third of the milling costs. If, therefore, a prospect is being evaluated for which preliminary tests indicate a hard ore and the necessity of fine grind (e.g. 80% passing 200 mesh, i.e. 0.074 mm or in technical short form K 80 = 74 µm or p 80 = 74 µm) and these characteristics are not reflected in the data from which the milling costs have been interpolated, then the interpolated milling costs should be increased by up to 20%.

Additional aspects which should be considered when dealing with milling costs are the following:

a Sometimes deposits are so high-grade that the ore can be exported without beneficiation (such ore is called “as is” or “tel quel” or “direct shipping ore”). In developing countries with low wages such directly saleable products can be achieved by hand sorting. If ore grades are just slightly lower than the lowest acceptable grade for direct shipping ore, a sharp increase of costs occurs due to the necessity for beneficiation. For iron ore for example the lowest grade is 62%, for barite the limit is at 96% $BaSO_4$. Sometimes preliminary beneficiation test may show that the necessary grades can be achieved by enrichment through removing a certain sieve fraction, when this fraction has a lower than average grade. Here the factor of mass recovery (Sect. 4.4) is of importance.

b If initial beneficiation tests were done using samples (in most cases split cores) which are of higher grade than the average grade to be expected, then it should be tested if the recovery thus achieved (see Sect. 4.3) can really be applied to the lower average grades of the deposit. It is often found that grades in the tailings will not go below a certain limit.
Example: First beneficiation tests were made with samples having a grade of 10 g/t. A recovery ε = 95% was achieved, corresponding to a tailings grade of 0.5 g/t. In the course of the exploration it becomes obvious that the average grade will be only around 5 g/t. As long as no new beneficiation tests on ore samples with an average grade of about 5 g/t are available, it is advisable to assume that the tailings grade will be constant at 0.5 g/t. With 0.5 g/t in the tailings and an ore grade of 5 g/t the recovered grade is 4.5 g/t and the recovery (Sect. 4.3)

$$\varepsilon = \frac{4.5}{5.0} = 0.9 \quad , \quad \text{i.e. } 90\%$$

c Sometimes the beneficiation characteristics of an ore deteriorate with lower grades. This can have an influence on the determination of the cutoff-grade (see also Sect. 10.1 and Wellmer 1998, Stat. Eval., Chap. 12). Mackenzie (1990) describes such a case history from a Pb-Zn-Ag mine, the ZC mine at Broken Hill in Australia, which is explained in Fig. 9.5a. A natural cutoff was 10 to 12% Zn, so only ore was mined with good beneficiation characteristics. By lowering the cutoff to 7 or even 5% Zn the tonnage could be increased significantly thereby achieving benefits due to economies of scale (see Chap. 8). Now, however, ore had to be mined with at best an average, or worse beneficiation characteristics. Instead of an improvement this resulted in an overall economic deterioration in the economics of the operation.

In Fig. 9.5b we show how recovery varies with changes in deposit grade at constant grades in the tailings. Let us assume, for example, an ore having a grade of 10% Zn. At a recovery of 90% 1% Zn remains in the tailings. If now ore grades decrease relatively by 30%, that is to 7% in absolute terms, and if the grade in the tailings stays constant at 1% Zn, the recovery drops to

$$\varepsilon = \frac{7-1}{7} = 0.857 \quad , \quad \text{i.e. } 85.7\%$$

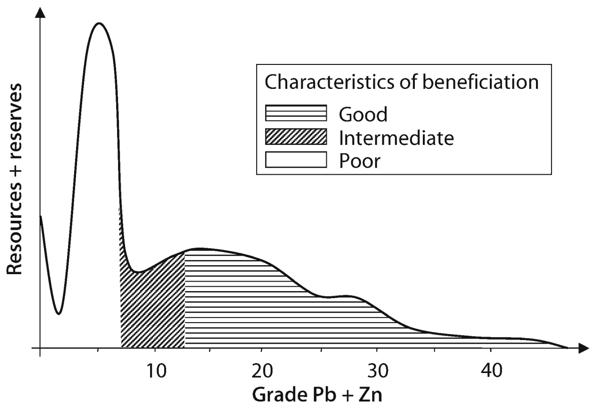


Fig. 9.5a.
Schematic diagram of resources/reserves versus grade of the ZC Mine, Broken Hill Australia, matched to qualitative metallurgical characteristics (modified from Mackenzie 1990)

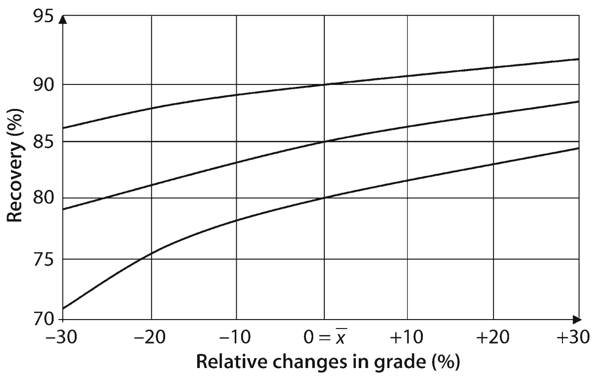


Fig. 9.5b.
Recovery as a function of changes of grade assuming a constant grade in the tailings (Wellmer 1981b)

9.3.2.6
Additional Aspects Concerning Operating Costs (Costs of Exploration, Cost of Grade Control)

Finally, we want to investigate how far the costs of exploration should be considered in an early economic evaluation at the exploration stage. The costs of detailed exploration for the development and preparatory work are already included in the operating costs used in the calculations in Sect. 9.3.2.1 to 9.3.2.4 (exception see below the case of very detailed grade control in selective gold mining), not however the cost of exploration to find the deposit itself and to find additional reserves during the lifetime of the mine. The former is normally included in the capital cost (Sect. 9.3.1.1 to 9.3.1.2). The costs of discovering new deposits that are necessary for the survival of a mining company are by their nature risk funds paid for out of profits (see for example Woodall 1984). How project economics and profitability are calculated in detail will be dealt with in Chap. 11. Consequently, at this early stage of economic appraisal we do not have to consider exploration costs at all.

Besides the task of detailed exploration for development and preparatory work the mine geologist has also the task of grade control, meaning the ongoing work to demarcate ore from waste according to a predefined cutoff limit (Sect. 10.1). As said above, the costs for ore development and grade control work are included in the operating costs. If, however, we consider a very selective mining operation like the ones in Australian open pit gold mines (see e.g. Wellmer 1998, Stat Eval. p. 148) with its very intensive grade control work, we should allow for this already in the early stages of evaluation. Dudley (1988) studied Australian selective gold mining open pits and concluded that the costs for grade control varied between 1 and 4 AU $/t of ore. In our rough economic evaluations at the exploration stage we should therefore add for an envisaged selective open pit mining operation the costs of ore development and grade control which we assume to be equivalent to the cost of moving 1 t of material (see Sect. 9.3.2.4).

Example. A selective gold open pit has a waste: ore ratio of 5 : 1. Because it will be a small open pit, we take mining costs to be 1.50 US $/t material moved (see Sect. 9.3.2.4). So, for 1 tonne of ore we have to move 6 tonnes in total. This means $6 \times 1.50 = 9$ US $/t. Because it is a selective open pit operation, we add for grade control the cost of 1 tonne material moved, i.e. 1.50 US $/t. So we work with total open pit costs of 10.50 US $/t.

9.4
Freight Costs

9.4.1
Abbreviations in the Shipping Industry like "fob" and "cif"

Before discussing freight costs, two abbreviations constantly used in connection with deliveries to customers and hence in connection with freight in general, have to be explained more in detail: "fob" and "cif":

- *fob* stands for "free on board" and implies that the producer delivers the product, e.g. concentrates, free on board the means of transport at a designated place, originally on board a ship but also other means of transport such as railway carriages or trucks (for which the abbreviations *for*: "free on rail" or *fot*: "free on truck" respectively are also used). This implies that freight from the producer to the consumer is paid for by the purchaser.
- *cif* stands for "cost, insurance, freight" and implies that costs such as customs, documentation, freight and insurance to the place designated are paid by the seller.

There are abbreviations other than the general ones like *fob* and *cif*, which come from INCO abbreviations (INCOTERMS or international commerce terms of the International Chamber of Commerce (ICC) in Paris), used in special cases. One example is *fid* meaning "free into container depot" which is used for price quotations for concentrates of rutile or zircon produced by Australian beach sand operations.

Generally it can be said (Fig. 9.6) that

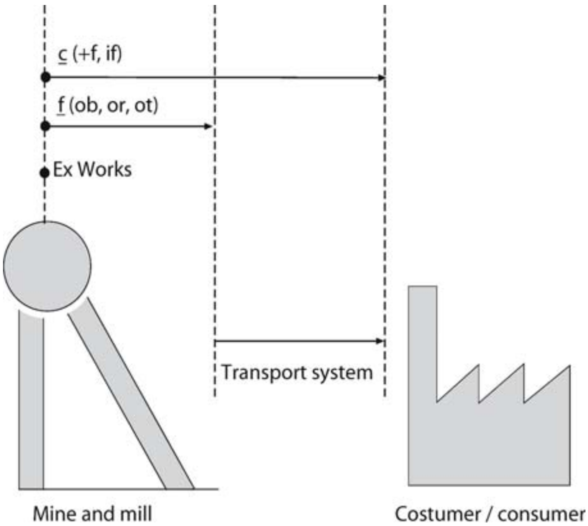
a if the abbreviation starts with an *f*, the product is delivered free to a defined point *before* the main transport starts.
b if the abbreviation starts with a *c*, the price generally includes costs and freight to a destination close to the buyer or customer respectively.

A list of the abbreviations for delivery terms in the transport business is given in Appendix D, Table D16.

An abbreviation frequently encountered in raw material bulk transport to Europe is ARA. It means the harbours Amsterdam, Rotterdam or Antwerpen.

For unusual abbreviations the Metal Bulletin's Prices and Data Book is a good reference source.

Fig. 9.6.
System of most common abbreviations in the shipping business



Example. A coal mining company in Alberta in Canada has a contract with a Japanese utility company to deliver steaming coal for 40 US $/t fob Vancouver. This means the mining company has to pay for the rail freight from the Alberta mine site to Vancouver and the loading on board of the ship. The sea freight to Japan and any land freight in Japan is paid by the Japanese customer.

9.4.2 Rules-of-Thumb for Freight Costs

For land freight costs, simple rules-of-thumb apply:

- Rail: 3 US ¢/mile/t
- Truck freight: 10 US ¢/mile/t

Rail freight costs apply to the normal transportation of bulk goods, not to the large standardized unit trains used for the shipping of iron ore or coal. For these, costs may drop to 1 US ¢/mile/t.

Assignment. A talc deposit situated in the interior of a country is to be evaluated. The consumer is located on the coast. The distance from the mine to the nearest railway station is 100 km, to the consumer by rail 560 km. What are the freight costs which have to be deducted from the return cif consumer?

100 km road freight cost $\frac{100 \times 0.10}{1.609} = 6.22 \left(\frac{\text{US\$}}{\text{t}} \right)$ [15]

560 km rail freight cost $\frac{560 \times 0.03}{1.609} = 10.44 \left(\frac{\text{US\$}}{\text{t}} \right)$ [15]

Loading processes at three points have to be added: loading at the mine, reloading at the railway station and unloading at the consumer's end. For each loading procedure 0.50 US $/t are added. Total freight costs are therefore

$6.22 + 10.44 + 3 \times 0.5 = 18.16$ or 18.50 US $/t

Sea freight rates fluctuate widely as can be seen from the Dry Bulk Freight Index[16] given in Table D17 (Appendix D). These fluctuations are very comparable to the fluctuations of commodity prices (Sect. 6.1). When we do an economic evaluation in the exploration stage we look far into the future. We do not know if we will hit the top or

[15] The factor 1.609 in the denominator is the conversion from miles to km (see Sect. 1.1.1).

[16] The index given in Appendix D, Table D17 is based on freight rates of Panamax vessels which are viewed by many in the freight industry as the best indicators of average dry cargo (or dry bulk) rates. For updates one can also study the journal Industrial Minerals which publishes various freight indices.

Table 9.5. Ship categories with typical bulk cargoes and freight costs

Ship category	Size	Typical dry bulk cargo	Rule-of-thumb for freight rates (US $/1 000 t-mile)
Capesize	Larger 100 000 dwt	Iron ore coal	1.00 US $/1 000 t-mile
Panamax	60–100 000 dwt typically 65 000 dwt	Iron ore Coal Bauxite Phosphate	2.00 US $/1 000 t-mile
Handymax	40–60 000 dwt	Coal Steel Cement Potash Gypsum	3.00 US $/1 000 t-mile
Handysize	10–40 000 dwt typically 30 000 dwt	Basemetal concentrates Sulphur Salt Industrial minerals Scrap	4.00 US $/1 000 t-mile (for 30.000 dwt) 6.00 US $/1 000 t-mile (for 20 000 dwt)

the bottom of the freight-rate cycle when our deposit that we are evaluating comes on stream. So it is advisable to work with round figures and rules-of-thumb which are based on long-term average freight rates. At present we have relatively high bulk freight rates due to the China boom, but these high freight rates must not be extrapolated far into the future.

Sea freight rates are dependent on the size of ship used. Table 9.5 gives the various ship sizes with typical dry bulk cargoes and rules-of-thumb for freight calculations. A freight rate of US $1 / 1 000 t-mile means that 1 000 metric tonnes can be moved for 1 nautical mile (see Sect. 1.1.1 1 nautical mile = 1.852 km).

Note: in the sea-freight business tonne-miles are nautical miles, *not* statute miles as for land-freight (see Sect. 1.1.1 Mile).

Assignment. Calculate the bulk freight rate for a potential chromite mine in south-eastern Turkey to Rotterdam.

- *Step a:* We choose the harbour of Iskenderun as the take-off point. From an atlas you determine the distance Iskenderun-Rotterdam via the Strait of Gibraltar: 6 300 km which you convert into nautical miles[17]:

$$\frac{6\,300\text{ km}}{1.852} = 3\,383 \text{ nautical miles, rounded } 3\,400$$

[17] There is an internet address to determine the distances between harbours: www.distances.com.

- *Step b:* We now calculate the freight rate with the rules-of-thumb of Table 9.5. Since it is envisaged that the chromite will be shipped in lots of 20 000 t we choose the freight rate of US $6 / 1 000 t-mile. Since the freight rate is related to 1 000 t and miles, we have to multiply the rate of US $6 / 1 000 t-mile with 20 ($20 \times 1\,000$ is the size of our lot 20 000 t) and the distance. So the freight rate for the total ship for one trip Iskenderun-Rotterdam will be

$$6 \times 3\,400 \times 20 = \text{US}\,\$408\,000$$

- *Step c:* This total trip rate we now have to divide by the tonnage, i.e. 20 000 t to derive at a dry cargo freight rate of

$$\frac{408\,000}{20\,000} = 20.40\left(\frac{\text{US\$}}{\text{t}}\right)\ , \quad \text{rounded to } 20.50\ \left(\frac{\text{US\$}}{\text{t}}\right) \text{ of chromite ore}$$

With low value bulk commodities, such as barite or fluorite, a simple freight cost estimate can single out those areas in which high freight costs alone would make mining uneconomical. This can simplify the search for deposits from the start (see example Sect. 13.6).

Additional Economic Planning Methods

Before going into economic calculations two economic planning methods shall be considered which can influence the economics of an ore deposit:

- determination of a cutoff grade, i.e. the lowest grade that will meet costs
- linear optimization when there are several development options

10.1
Calculation of Cutoff Grades

The economic optimization of cutoff grades, closely related to deciding optimal lifetime of a mine (see Sect. 8.1.3) is a complex problem. Extensive literature on this topic is available, e.g. von Wahl (1973), Taylor (1974), Lane (1988), Wellmer (1998), Slaby and Wilke (2005). In practice, however, operating cost cutoff grades that readily lend themselves to quick and easy determination are frequently used. In South African goldmines the term “pay limit” is frequently used instead of cutoff.

10.1.1
Normal Case of an Operating Cost Cutoff

Example. What is the operating cost cutoff grade in a gold deposit with operating costs of 55 US \$/t at a gold price of 400 US \$/oz? The recovery in the mill is 90% and mining dilution is 10%.

The operating costs are 55 US \$/t. At 31.103 g per ounce (see Sect. 1.1.4) US \$55 equal

$$\frac{55}{400}\times 31.103 = 4.28\left(\frac{\text{g Au}}{\text{t}}\right)$$

Recovery in the milling circuit is 90% (i.e. $\varepsilon = 0.9$, see Sect. 4.3), and mining dilution is 10%, so that the operating cost cutoff is

$$\frac{4.28}{0.9}\times 1.1 = 5.2\left(\frac{\text{g Au}}{\text{t}}\right)$$

10.1.2 Cutoff Calculations for Open Pits

10.1.2.1 Marginal Stripping Ratio

As shown in Sect. 9.3.2.4, open-pit operating costs largely depend on the waste:ore ratio. Figure 9.4 shows that the deeper the pit, the less favourable is the waste:ore ratio. The final depth of an open pit is often determined by an operating cost cutoff, the called marginal stripping ratio, which is defined as the maximum allowable waste:ore ratio beyond which the operation becomes uneconomic.

Assignment. A sandstone hosted uranium deposit (Fig. 10.1) dips at an angle of 45° and is to be developed as an open pit operation. Grades are 0.3% U_2O_3, dilution will be 20%, recovery in the beneficiation plant 85%. Beneficiation costs have been estimated at 27 US $/t; total overheads at 11 US $/t; open pit operating costs per tonne of moved material at 1.50 US $/t. Uranium price: 20 US $/lb U_3O_8.

What is the marginal stripping ratio per tonne of ore and the maximum sustainable depth of the open pit?

- *Step 1:* We calculate the revenue per t ore: The in situ ore grade is 0.3% U_3O_8, which in the course of open pit mining is diluted by 20% barren waste, i.e. to extract the same absolute amount of U_3O_8 we have to mine 1.2 t instead of 1 t. Of the in situ grade only 85% is recovered during beneficiation, so that the return per t ore at a uranium price of 20 US $/lb U_3O_8 (conversion factor of % → 22.046, see Sect. 1.1.4) is

$$\frac{0.3 \times 0.85}{1.2} \times 22.046 \times 20.00 = 93.70 \left(\frac{\text{US\$}}{\text{t ore}} \right)$$

- *Step 2:* From this gross return beneficiation costs and overheads have to be deducted. The remainder is

  93.70 – 27 – 11 = 55.70 US $/t

  The calculation of the marginal stripping ratio is therefore based on open pit operating costs of 55.70 US $/t, i.e. all the surplus money can be used for mining. With mining costs at 1.50 US $/t, this means that 37 t of material can be moved, i.e. a marginal waste:ore ratio of 36 : 1.
- *Step 3:* Now the maximum depth of the open pit in Fig. 10.1 can be determined: For the increment $\Delta h_E = 1$ m the corresponding waste area on the section in Fig. 10.1 is

$$h \times \sqrt{2} \times \sqrt{2} = 2h$$

  The ore surface area on the section of Fig. 10.1 is

$$8 \times \sqrt{2} \times 1$$

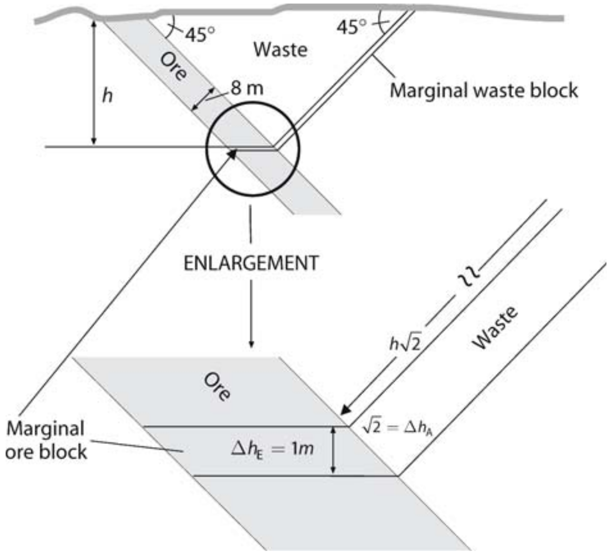


Fig. 10.1.
Vertical section for the calculation of a marginal stripping ratio

With a waste:ore ratio of 36 : 1 we obtain

$$\frac{A}{E}=\frac{36}{1}=\frac{2h}{8\sqrt{2}} \quad \text{or} \quad h=203.7\ \text{m} \quad , \quad \text{i.e. } 200\ \text{m}=h_{\max}$$

- *Step 4:* Thus the *average* stripping ratio is

Area of ore in cross-section of Fig. 10.1: $8\times\sqrt{2}\times 200=2\,262.7\ \text{m}^2$

Area of waste in cross-section of Fig. 10.1: $\dfrac{h\times 2\times h}{2}=h^2=200^2=40\,000\ \text{m}^2$

i.e. an average waste:ore ratio of

$$\frac{40\,000}{2\,262.7}=17.68 \quad , \quad \text{i.e. } \underline{18:1}$$

(Due to the low uranium content, the density of waste and ore is practically identical.)

10.1.2.2
Calculation of an Operating Cost Cutoff Grade in an Open Pit

When using an operating cost cutoff grade in open pit mining, the cutoff grade is actually a function of the depth, since mining costs increase with depth (see above, Sect. 10.1.2.1). However, for rough calculations it suffices to use the average waste:ore ratio.

Example. Referring to the assignment of Sect. 10.1.2.1:

Beneficiation costs:	27 US $/t
Overheads:	11 US $/t
Mining costs at stripping ratio of 18 : 1 and costs of 1.50 US $/t for moved material:	28.50 US $/t
Sum	66.50 US $/t
or rounded	67.00 US $/t

At 20 US $/lb uranium, a dilution of 20% and a beneficiation recovery of 85% the equivalent uranium grade is

$$\frac{67 \times 1.2}{20 \times 0.85} = 4.73 \left(\frac{\text{lb } U_3O_8}{\text{t}} \right)$$

Converted (conversion factor 22.046), the result is an operating cost cutoff grade of 0.21% U_3O_8.

This, however, is only the operating cost cutoff for the *foot* wall of the orebody.

For the definition of the *hanging wall cutoff*, we must not forget that the material, be it ore or waste, has to be extracted, loaded and transported at any rate, i.e. mining costs will accrue regardless of whether ore or waste is mined. Therefore an operating cost cutoff for the hanging wall takes only the *additional* costs into consideration, i.e. the beneficiation costs. In the above example they would be only 27 US $/t.

Converted by the same factors as above, this equals

$$\frac{27 \times 1.2}{20 \times 0.85} = 1.91 \left(\frac{\text{lb } U_3O_8}{\text{t}} \right) \quad \text{or} \quad 0.09\% \ U_3O_8$$

As will be demonstrated in Sect. 11.2.3.1, the overall economics of the mine largely depend on the cash flows of the initial operating years. Therefore different cutoff grades are sometimes applied, a higher one for the initial years to maximise average grades, and a lower one for the later years. Frequently enough low-grade ore (in our example ore with grades between the two cutoff points 0.21 and 0.09% U_3O_8) is put on stockpile and processed later, unless prices rise so sharply during the initial operating years that immediate processing of "low grade ore" becomes worthwhile.

By means of the operating cost cutoff grade, reserves in open pit (and underground) mining can be maximized. The operating cost cutoff grade of 0.21% U_3O_8 in our example is relatively high compared to the average grade of 0.3% U_3O_8 (compare Sect. 9.3.2.1 and Wellmer (1998), Chap. 12 and Table 39). The operating costs can only be effectively lowered by a reduction in the waste:ore ratio, i.e. by decreasing the final depth of the mine. In subsequent steps the calculations are repeated with progressively smaller open pits – and consequently lower reserves. As the size of the pit diminishes, the ore to waste ratio improves and with it costs. The exercise is repeated until the most economic pit is found.

10.2 Linear Optimization

When several ore deposits are located closely together, a central mill serving all the deposits is of advantage. Often more than one deposit has to be mined and milled at the same time, in order, for example, to improve the grindability of the ore by blending if one ore type is harder than the other. If three or more deposits are to be considered simultaneously, a mathematical method, called simplex algorithm, can be applied, but will not be discussed here (see e.g. Collatz and Wetterling 1966). However, to avoid operating at too many working sites at the same time, the number of deposits is usually restricted to two. To find an optimum schedule for these two deposits a simple graphical procedure can be followed which is best explained by an example.

Example. In a gold project a central mill is planned for the combined exploitation of various deposits. One deposit is an underground mine producing hard primary ore. There are several open pit possibilities from which soft oxidised ore can be mined. For practical reasons, one open pit will be in operation at a time. The grade of the underground ore is 10 g Au/t, the grade of the open pit material is 5 g Au/t.

The central mill can process either 100 000 t of primary (hard) ore or 150 000 t of oxidised (soft) ore or an equivalent combination of primary and oxidised ore. Mill recovery will be 90%.

The maximum rate of mining underground is considered to be 35 vertical metres per annum (see Sect. 8.1.2) which would mean an annual production of 70 000 t. The underground mine will be developed through a ramp using trackless LHD-equipment. The purchase of one full set of underground equipment would result in a minimum production rate of 35 000 t/a.

Mining in the open pit will be done by a contractor. He determined the maximum open pit mining rate at 80 000 t/a, but for reasons of equipment utilisation he requires a minimum of 20 000 t/a.

The assumed gold price is 400 US $/oz.

How can the optimum production rates of underground and open pit mines be determined?

Step 1: Designating y as the rate of underground mining and x as the rate of open pit mining, we arrive at the following relationships for mining:

$$80\,000 \geq x \geq 20\,000$$
$$70\,000 \geq y \geq 35\,000$$

If we take the maximum and minimum mining rates we can write the following four equations:

$$x_i = 20\,000$$
$$x_A = 80\,000$$
$$y_i = 35\,000$$
$$y_A = 70\,000$$

The four lines defined by these equations are plotted in a graph (Fig. 10.2a). They form a rectangle ABCD containing all the possible combinations which are allowed by the imposed mining constraints.

Step 2: For milling we have the upper limit of 100 000 t/a underground ore or 150 000 t/a of open pit ore, i.e.

$$y_{\text{max}} = 100\,000$$
$$x_{\text{max}} = 150\,000$$

If both points are plotted in Fig. 10.2a and connected with each other, one obtains a line containing all combinations for maximum mill utilisation. All the points below the line are allowed but underutilize the mill. No point above the line is allowed because of mill capacity constraints. Taking mining and milling constraints into account, only the points within the pentagon ABDFE, the *permitted area*, are allowed.

Fig. 10.2a.
Diagram with constraints for mining and milling

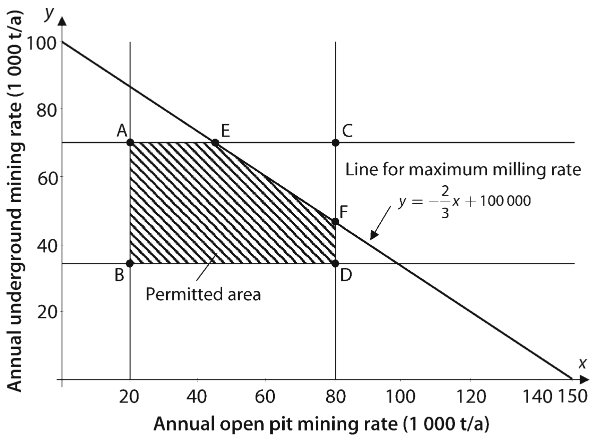
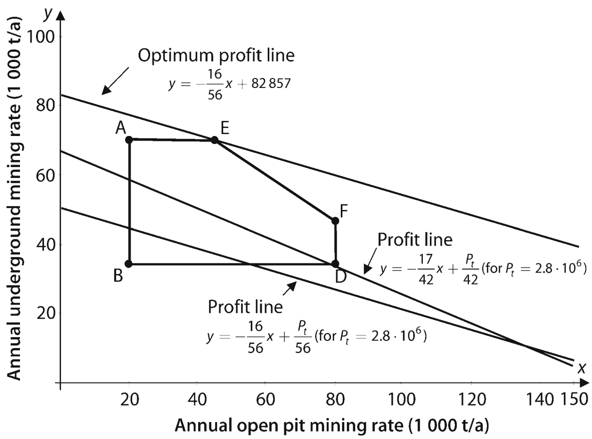


Fig. 10.2b.
Diagram of Fig. 10.2a with profit lines



The general equation of a line is

$$y = a \times x + b$$

For the line of maximum milling rate the rate of increase *a*, defining the slope of the line, is

$$a = -\frac{100\,000}{150\,000} = -\frac{2}{3} \quad \text{and} \quad b = 100\,000$$

The equation for the line of maximum milling rate is therefore

$$y = -\frac{2}{3}x + 100\,000 \tag{10.1}$$

Step 3:

1. We want to optimize the operating profit from mining and milling. An equation for the total operating profit P_t is

$$P_t = x \times P_o + y \times P_\mu \tag{10.2}$$

where P_o is the operating profit per tonne of open pit material and P_μ the operating profit per tonne of underground ore.

It becomes evident that Eq. 10.2 is the equation of a line with three unknowns, P_t, x, and y, if we consider P_o and P_μ as fixed and not variable. (As will be seen below, P_o and P_μ are a function of the throughputs x and y, but for the moment both P_o and P_μ are considered to be constants.) Then Eq. 10.2 can be written in the standard form of a linear equation:

$$y = -\frac{P_o}{P_\mu}x + \frac{P_t}{P_\mu} \tag{10.3}$$

The further the line can be pushed to the top and to the left, i.e. the larger x and y, the higher the operating profit P_t.

Looking at our permitted area ABDFE in Fig. 10.2a one can see that there are three possibilities for an optimum solution of the equation:

a the slope $-P_o/P_\mu$ is flatter than that of line EF, i.e. less than $-2/3$ (see Eq. 10.1). Therefore the maximum profit line must pass through point E, meaning E gives the optimum combination of underground and open pit production rates.

b the slope $-P_o/P_\mu$ is steeper than that of line EF, i.e. greater than $-2/3$.

In this case point F gives the optimum combination of underground and open pit production rates.

c the slope $-P_o/P_\mu$ is the same as that of line EF, i.e. it equals $-2/3$. Any point between E and F on line EF gives a combination of underground and open pit production rates with the same optimum profit.

2. We now have to analyze P_o and P_μ.
   The operating profit P is revenue (Rev) minus operating costs (Co):

$$P = \text{Rev} - \text{Co} \tag{10.4}$$

   Revenue per tonne is a function only of grade, recovery, and price, but not of throughput. As shown in Sect. 9.3.2.4, the operating costs for open pit mining are principally dependent on the waste:ore ratio which itself is not at all influenced by the rate of throughput. We can therefore consider the open pit operating costs as being constant. But the mill operating costs are always a function of the throughput. Mills are highly automated nowadays. All wages are fixed costs. The higher the throughput, the lower therefore are the costs per tonne of ore milled. Underground operating costs are also a function of the production rate (see example in Sect. 9.2.2).

   We therefore have to calculate the operating profits P_o and P_μ for the points E and F separately.

   The basic data for points E and F are:

Point E:	underground mining rate:	70 000 t/a
	open pit mining rate:	45 000 t/a
	milling rate:	115 000 t/a
Point F:	underground mining rate:	47 000 t/a (rounded)
	open pit mining rate:	80 000 t/a
	milling rate:	127 000 t/a

3. The revenues (Rev) at 90% recovery in the mill and at a gold price of 400 US $/oz are for underground ore at 10 g/t:

$$\frac{10 \times 0.9 \times 400}{31.103} = 115.74 \left(\frac{\text{US\$}}{\text{t}}\right) = \text{Rev}_\mu$$

   open pit ore at 5 g/t:

$$\frac{5 \times 0.9 \times 400}{31.103} = 57.87 \left(\frac{\text{US\$}}{\text{t}}\right) = \text{Rev}_\text{o}$$

   (31.103 is the conversion factor for troy ounces into g; see Sect. 1.1.4).
4. The contractor has calculated the operating costs for open pit mining at 22 US $/t, the same for both mining rates at 45 000 t/a and 80 000 t/a.
5. In analogy to other operations in the general area and with methods described in Chap. 9, the following operating cost for underground mining and milling are determined:

Point E:	underground mining rate 70 000 t/a:	40 US $/t
	milling rate 115 000 t/a:	20 US $/t
Point F:	underground mining rate 47 000 t/a	55 US $/t
	milling rate 127 000 t/a:	19 US $/t

6. We can now calculate the operating profits P_o and P_μ for points E and F with the profit P being (see Eq. 10.4):

   $P = \text{Rev} - \text{Co}$

   Point E: $P_o = 57.87 - 22 - 20 = 15.87$ ≈ 16 US$/t
   $P_\mu = 115.74 - 40 - 20 = 55.74$ ≈ 56 US$/t

   Point F: $P_o = 57.87 - 22 - 19 = 16.87$ ≈ 17 US$/t
   $P_\mu = 115.74 - 55 - 19 = 41.74$ ≈ 42 US$/t

7. We can now use Eq. 10.3 and derive the following relationship for the optimum profit line:

   Point E: $y = -\frac{P_o}{P_\mu}x + \frac{P_t}{P_\mu} = -\frac{16}{56}x + \frac{P_t}{56}$

   Point F: $y = -\frac{P_o}{P_\mu}x + \frac{P_t}{P_\mu} = -\frac{17}{42}x + \frac{P_t}{42}$

8. In both cases the rate of increase is lower than the slope of line EF which is –2/3 (see Fig. 10.2b). Therefore the optimum point (see l a above) maximising the operating profit is point E, i.e. an underground mining rate of 70 000 t/a and an open pit mining rate of 45 000 t/a. Total operating profit then is (see Eq. 10.2)

   $P_t = x \times P_o + y \times P_\mu = 45\,000 \times 16 + 70\,000 \times 56 = 4\,640\,000$ US$/a

Chapter 11

Economic Evaluations

Now that we know how to calculate the revenues (the net smelter return) for a potential mine from the grades of a deposit to be evaluated (Chap. 7); how to determine the lifetime of a mine (Chap. 8); and how to derive capital and operating costs from the capacity (Chap. 9), we have all the data required to carry out economic calculations.

In economic evaluations, *static* and *dynamic* methods are distinguished, although static methods are seldom applied nowadays. On an international level, economic assessment of deposits is done through dynamic methods which take the time factor for investments and returns, i.e. the *time value* of money, into account and are based on compound interest formulae. The following notations and abbreviations will be applied:

- Investments, abbreviated: *I*
- Revenues or sales, abbreviated: Rev
- Costs, abbreviated: Co
- Operating profit, abbreviated OP, i.e. the difference between revenues and costs: OP = Rev – Co; the operating profit is the cash flow before interest and taxes
- The cash flow *after* interest and taxes is the net cash flow: NC
- Interest or compound rate, abbreviated: *i*; for calculations the compound rate is expressed as a fraction of 1; e.g. the compound rate of 10% is $i = 0.1$
- The number of operating years, abbreviated *n*; an individual year is n_j

11.1 Static Methods

11.1.1 Profitability Quotient

A simple profitability quotient is the ratio operating profit OP : investment *I*. This applies when, instead of buying a deposit directly, someone buys shares in a company controlling the deposit and expects an annual dividend from the company. An abbreviation sometimes used in this context is ROCE, Return On Capital Employed.

Example. Shares issued by a company cost US $10 each and a minimum yearly dividend of 75 cents per share is expected. The profitability quotient qp, i.e. the yield, is

$$\mathrm{qp} = \frac{\mathrm{OP}}{\mathrm{I}} = \frac{0.75}{10} = 0.075 \quad , \quad \text{i.e. the yield is 7.5\%}$$

One can use this method as an approximation to quickly compare two projects.

Example. There are two projects to compare: the first one requires an investment of US $80 million, the second one US $100 million. The first project is expected to generate an annual profit of US $10 million, the second one an annual profit of US $20 million. So the profitability quotient of the first project is

$$\mathrm{qp}_1 = \frac{\text{profit}}{\text{investment}} = \frac{10}{80} = 0.13$$

and in the second case

$$\mathrm{qp}_2 = \frac{20}{100} = 0.2$$

So in a first rough calculation one would prefer the second investment project.

11.1.2 Calculation of Rent

As a rule, static methods are applied for the calculation of rent for equipment. The following factors are of importance:

- The lifetime or depreciation period (not necessarily identical), i.e. the number of years n over which the investment I is spread. The annual depreciation D is

$$D = \frac{I}{n}$$

- Maintenance and servicing costs M, usually assumed as percentage p of the initial investment (10–20% as a rule). Maintenance costs are $M = I \times p$.
- Payment of interest Pi at a given rate of interest i. Interest payment is calculated for the average of the capital tied up. As Fig. 11.1 shows, the average of the tied up capital is $I/2$. Hence the annual payment of interest is

$$\mathrm{Pi} = \frac{I}{2} \times i$$

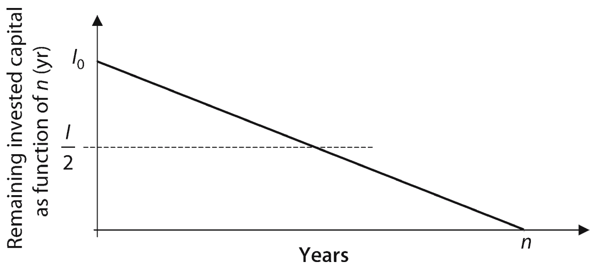


Fig. 11.1. Development of the capital tied up over a period of time

From these three elements the annual rent R can be expressed as

$$R = D + M + \text{Pi}$$

$$R = I(1/n + p + i/2)$$

Assignment. An automatic hydraulic roof support system for an underground coal mine costs €36 000 000. The depreciation period is 6 years, maintenance costs per year amount to 10% of the purchasing price, the compound rate of interest is 8%. What is the annual rent?

- Depreciation: $D = \frac{I}{n} = \frac{36\,000\,000}{6} = €6\,000\,000$
- Maintenance cost: $M = I \times p = 36\,000\,000 \times 0.1 = €3\,600\,000$
- Interest payment: $\text{Pi} = \frac{I}{2} \times i = \frac{36\,000\,000}{2} \times 0.08 = €1\,440\,000$

i.e. rent per year $R = 11\,040\,000$ €/a.

11.1.3 Payback Period

Strictly speaking, the calculation of the payback period, i.e. the number of years needed to repay the investments from the net cash flows, also falls under the heading static methods. For most normal mining projects payback periods lie between 3 and 8 years, as a rule. In high risk countries shorter payback periods are required than in stable countries (compare Sect. 11.2.3.5).

Example. A project has annual net cash flows available for the repayment of investments of €25 million, presented in Table 11.1.

The payback period is 3.3 years and the entire investment can therefore be repaid within year 4.

Table 11.1. Net cash flow

Year	Net cash flow (€ Mio)	Net cash flow accumulated (€ Mio)
1	6.5	6.5
2	7.5	14.0
3	8.5	22.5
4	8.5	31.0
5	8.5	39.5
6	7.0	46.5
7	6.0	52.5

11.2 Dynamic Methods

11.2.1 Introduction

As mentioned in the introduction of Chap. 11, dynamic methods take the *time value* of money into account.

Everybody knows intuitively what the time value of money is. If I invest US $1 000 today and after a year get US $1 200 in return, I consider this a good bargain. If I get the money after only 20 years, I would not invest the US $1 000, i.e. a profit of US $200 within a year has a considerably higher value than a profit of US $200 after 20 years.

The time value is calculated by means of the compound interest formula. If an investment of I = US $1 000 is made today at an interest rate of 10%, the value is

- after 1 year: $I \times (1 + i) = 1\,000 \times (1 + 0.1) =$ US $1 100
- after 2 years: $I \times (1 + i) \times (1 + i) = 1\,000 \times (1 + 0.1)^2 =$ US $1 210
- after 10 years: $I \times (1 + i)^{10} = 1\,000 \times (1 + 0.1)^{10} = 1\,000 \times 2.594 =$ US $2 594
- generally after n years: $I \times (1 + i)^n$

This procedure can also be reversed. At an interest rate of 10%, US $1 000 will be worth US $2 594 in 10 years. If R = US $2 594 is the target value my investment is to reach in 10 years time, I will have to invest

$$I = \frac{R}{(1+i)^{10}} = \frac{2\,594}{(1+0.1)^{10}} = \frac{2\,594}{2.594} = \text{US\$1\,000}$$

In other words: If I get US $2 594 in 10 years, the present value at an interest rate of 10% is US $1 000. Thus US $1 000 is the present value of US $2 594 at an interest rate of 10% over 10 years.

This, for example, is the principle of special government bonds in Canada, the called stripped bonds. They are stripped of their annual interest coupon. These interests are accumulated and paid out at the payback date of the bond. To take the example of above, if the interest rate would be 10% and the lifetime of the bond 10 years, one would pay CA $1 000 and get back CA $2 594 after 10 years.

To find out how much today's investment will be worth in n years, we have to compound by $(1 + i)^n$. The factors $(1 + i)^n$ are therefore called compounding factors. If, however, we want to project the value R into the future and want to know how much R is worth today, we have to discount R by

$$\frac{1}{(1+i)^n}$$

The factor

$$\frac{1}{(1+i)^n} \quad \text{or} \quad (1+i)^{-n}$$

therefore is called the discounting factor q^{-n}. As will be shown below, the discounting factor is the most important entity for our calculations. The values most frequently used are listed in Table D18 (Appendix D).

Two dynamic methods, also called DCF (Discounted Cash Flow) techniques, will be dealt with

- the calculation of the net present value (NPV),
- the calculation of the internal rate of return (IRR or IROR); it is also called the DCF rate or the earning power of a project.

The cash flow calculation is the summary of a feasibility or prefeasibility study. During the feasibility stage a team of geologists, mining engineers, metallurgists, economists etc. work together. The cash flow calculation, which can be very time-consuming, is usually prepared by economists.

This study will only deal with simple cash flow calculations a geologist or mining engineer will have to make at the prefeasibility stage of a project in order to establish whether an exploration project is worth pursuing. Once the principles are understood the actual calculation can be carried out using spread-sheet computer programmes.

11.2.2
Elements of Cash Flow Calculations

In cash flow calculations only true flows of money (of cash) of a project are considered. To better imagine what cash flow is, we imagine the mine as a "box" (Fig. 11.2) and ask ourselves, what money flows in and what money flows out of this "box". The money that flows in are the revenues which we calculated in Chap. 7. The money that flows out are first the operating costs (wages that have to be paid to the employees of the mine, money for energy, explosives etc.) which we calculated in Sect. 9.3.2. Further money that flows out are interest for loans, to be dealt with in Sect. 11.4, and taxes and royalties to be dealt with in Sect. 11.3. The difference between money inflows and outflows is the cash flow. One distinguishes between gross cash flow, i.e. the difference

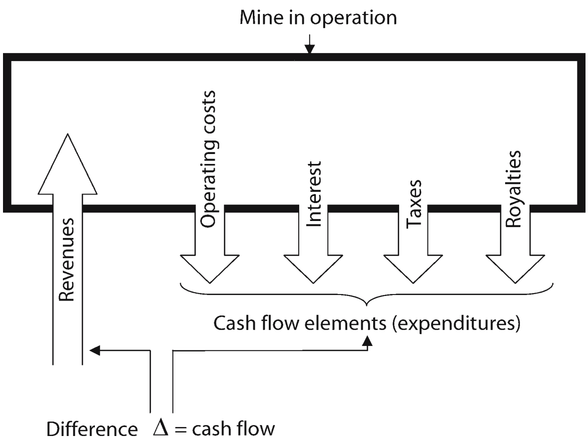


Fig. 11.2. Explanation of the term "cash flow"

between revenues and operating costs *before* interest, taxes and royalties (CF) and the net cash flow (NC) the same difference *after* interest, taxes and royalties.

Hence depreciations and periods of depreciation are of no direct importance. Depreciation is an accounting measure to calculate tax deductions. Periods of depreciation only have an indirect influence via tax charges; the latter are genuine annual payments and therefore are included in cash flows (see Fig. 11.2). However, if no taxes are paid as used to be the case in the Australian gold mining industry or if tax holidays are granted for the first years of operation of a project to encourage mining investments as is the custom in some developing countries, then depreciation for the purpose of cash flow calculations is altogether irrelevant.

Cash flow calculations are done in tables (see e.g. Tables 11.4a,b in Sect. 11.2.4.2 and Table 11.7 in Sect. 11.5) so that cash flows of a kind are compiled year by year. Generally, the simplifying assumption is made, which we will adopt, that all cash flows are due *at the end* of a specific year.

The elements of a cash flow calculation with the individual money streams are displayed in Fig. 11.3:

a Investment *I*

In a cash flow table (Fig. 11.3) the investment years are generally marked by negative figures, the production years by positive figures.

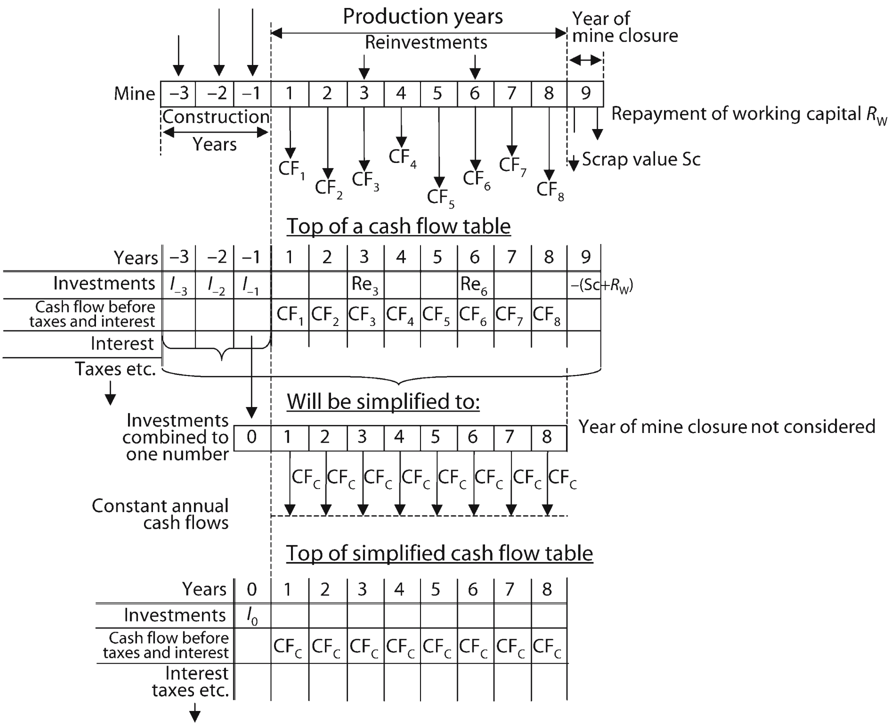


Fig. 11.3. Cash flows of a project

b Costs (Co) and revenues (Rev)
The difference between revenues and operating costs is the operating profit (OP) or gross cash flow (CF). If an investment is financed entirely by equity, only taxes and royalties have to be deducted to arrive at the net cash flow. If, however, outside capital has been borrowed then interest on this capital must also be deducted.

c Reinvestments (Re) or ongoing capital expenditures
As a rule, reinvestments will have to be made over the years. The operating lifetime of mining equipment such as loaders seldom corresponds with that of the entire mine.

d Recovery of working capital
In the initial stages of a mining operation, working capital has to be provided (as part of the initial investment) which flows back at the end of the mine's lifetime. An example: concentrates are regularly shipped to a port and put into storage awaiting shipment. The freighter, however, comes only every 3 months, i.e. the mine receives payments for these concentrates only every 3 months and hence needs working capital for at least 3 months in advance to be able to pay wages and finance the purchase of material in the meantime.

e Revenues from the salvage value of a mine
Has a deposit been mined out, the plant still has a salvage value. Equipment can be sold second hand or used in another company-owned plant. The closed mine will be credited with the amount.

Complete cash flow calculations can be complicated and time-consuming and are usually done on computers with spread-sheet programmes. For our estimates in the initial stage we will make the following simplifying assumptions (see Fig. 11.3).

a We assume that all pre-production capital investments are incurred in one year, year 0. We combine the investments of the individual investment years and add the interest during construction up to start-up of the mine (capitalization of the interest during the construction phase). This is, in fact, the figure which in practice is presented as the sum of total capital investment.

b We discount all cash flows to the end of year 0 (or to the beginning of year 1, the first production year, which is the same). This implies that the cash flow of the first production year (which we expect to be due at the end of the year, see above) is already discounted by q^{-1}.

c We disregard the special cash flows "recovery of working capital" and "revenues from salvage value" due at the end of a mine's life. As shall be seen in Sect. 11.2.3.1, the later in the future cash flows occur, the smaller is their influence on the economic parameters to be calculated. Nowadays, it is generally expected that the closure of a mine will entail additional investments for rehabilitation and other ecological measures. The environmental costs can be quite substantial, particularly in the case of uranium deposits. It is more than likely that any special revenues will be offset by these final investments.

d As will be demonstrated in Sect. 11.2.3.3, cash flow calculations can be considerably simplified if identical cash flows occur in each production year. To achieve this, we distribute reinvestments evenly from year to year and increase the operating costs by a value lying between 2 to 3% of actual operating costs.

11.2.3
Net Present Value (NPV)

11.2.3.1
Introduction

When applying the net present value method, the net cash flows (NC), are discounted at a given interest rate i and investments I deducted from the sum of the discounted net cash flows:

$$\mathrm{NPV} = \left[\sum (NC \times q_j^{-n})\right] - I$$

i.e. the net present value indicates to the investor the value of a potential investment in a deposit not yet in production by taking the following factors into consideration:

- investments I
- the individual annual net cash flows NC (cash flow after taxes and possibly interest)
- the date for the net cash flows determined by the discounting factors as a function of the year n, in which the cash flow is due
- the risk inherent in the investment at the chosen interest rate i (see below Sect. 11.2.3.5 and 11.2.3.6)

Example. A mining project requires an investment of €45 million, interest during construction included. The annual net cash flows of €10 million remain the same over 10 years and the discounting rate at 15%, i.e. $i = 0.15$, stays the same, too. The method of calculating the NPV is described in Fig. 11.4a. We calculate (or extract from Table D18, (Appendix D) the discounting factors q^{-n} for $q = 1 + i = 1.15$ and multiply them by the annual net cash flows NC.

The sum of the discounted net cash flows for years 1 to 10 in the last line is €50.3 million. According to the formula above, the net present value (NPV) is

$$\mathrm{NPV} = \left[\sum_{\mathrm{j=i}}^{10} (NC \times q_j^{-n})\right] - I = 50.3 - 45 = €5.3\ \text{million}$$

If the investor expects to earn an interest of 15% on his capital, he would value the project at €5.3 million before the investment is made.

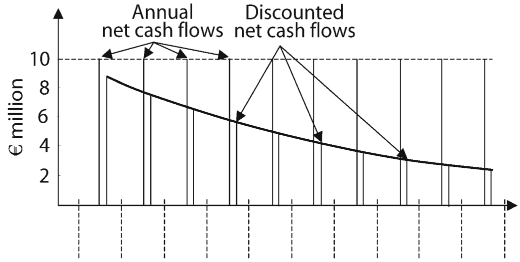
Of course, the net present value largely depends on the interest rate chosen. This question will be dealt with separately in Sect. 11.2.3.5 and 11.2.3.6.

Figure 11.4a clearly shows the rapid decrease of the net present value in the individual operating years. The factors

$$q^{-n} = \frac{1}{(1+i)^n}$$

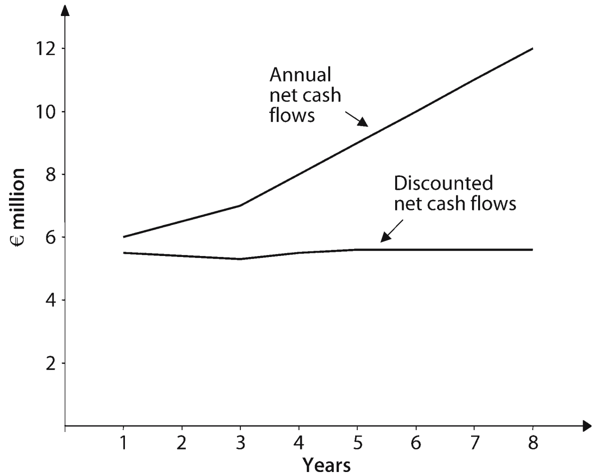
are the terms of a falling geometrical series. In the example above, the net cash flow of

Fig. 11.4a.
Procedure for calculating with the net present value method



Year n	1	2	3	4	5	6	7	8	9	10
Net Cash flows NC (€ million)	10	10	10	10	10	10	10	10	10	10
Discounting factors $q^{-n} = \frac{1}{(1+i)^n}$ for $i = 0.15$	0.87	0.76	0.66	0.57	0.50	0.43	0.38	0.33	0.28	0.25
Discounted net cash flows $q^{-n} \cdot$ NC (€ million)	8.7	7.6	6.6	5.7	5.0	4.3	3.8	3.3	2.8	2.5

Fig. 11.4b.
Annual cash flow and discounted cash flow of an operation with practically constant discounted cash flows



€10 million in year 1 has a net present value of €8.7 million, that of year 10, however, a net value of €2.5 million, i.e. only 29% of the net value of year 1. If a project runs over many years, e.g. 25 years, with unchanging net cash flows and an interest rate of 15%, the last 5 years (i.e. 20% of the total lifetime of the mine) contribute only 3% to the sum total of the net present value.

Hence the criticism that is occasionally levelled against the application of this method, particularly with regard to projects with a long lifetime. The reason is that rising geometrical series escalate slowly to start with and then accelerate rapidly, with falling series the opposite is true. In Chap. 14 on growth rates we will return to this subject. The little story in the Appendix E illustrates this problem convincingly.

The higher the interest chosen, the faster the decrease in later years. This raises the question which interest rate to choose. This will be discussed in Sect. 11.2.3.5 and 11.2.3.6.

Table 11.2. Net cash flow in relation to time value of money

Years n_j	1	2	3	4	5	6	7	8
Net cash flow NC (€ Mio)	6.0	6.5	7	8	9	10	11	12
Discounting factor on the basis of $i = 0.1$ (i.e. 10% interest rate)	0.909	0.826	0.751	0.683	0.621	0.564	0.513	0.467
Discounted net cash flow (€ Mio)	5.5	5.4	5.3	5.5	5.6	5.6	5.6	5.6

Another way of looking at the importance of the time value of money in economic evaluations is to see what annual increases in cash flows are required if we want to keep the corresponding discounted cash flows more or less the same. As an illustration we choose a case where net cash flow increases by €0.5 million in the second and third years and by €1 million in subsequent years up to year 8 and assume an interest rate of 10% (Table 11.2). Similar cash flow patterns could arise when a mine is struggling with start up problems as discussed later in Sect. 11.11, or when grades improve with depth as mining progresses and annual cash flows increase as a result. It can be seen in Fig. 11.4b that while annual net cash flows increase steadily, the discounted cash flows stay flat.

These two examples make obvious how important it is for dynamic economic evaluation methods to maximize the cash flows in the first years in which the discounting effect is not too strong. In Sect. 10.1.2.2 we dealt with the problem of calculating operating cost cutoffs in an uranium open pit and explained that sometimes different cutoff grades are applied, a higher one for the initial years to maximise average grades, and a lower one for the later years. Frequently, low-grade ore is put on stockpile and processed only later. The two examples above now give an explanation for this procedure from the discounted cash flow point of view.

There exists a rule-of-thumb to determine when the discounted annual cash flow reaches 50% of the not discounted cash flow: "the rule of 72". If one divides 72 by the discount rate it gives fairly accurately the time in years. For example: Discount rate 10%, 72 / 1 = 7.2 years. As shown in Table 11.2 the discounting factors for 10% decrease from 0.513 in year 7 to 0.467 in year 8, confirming this rule-of-thumb.

11.2.3.2
Calculations with Unequal Annual Cash Flows

Although the example in Sect. 11.2.3.1 Fig. 11.4a assumed the cash flows to be equal in order to illustrate the principle, the calculating procedure was carried out as if we had been dealing with unequal annual cash flows. The net present value (NPV) is

$$\mathrm{NPV} = \left[\sum (NC_j \times q_j^{-n})\right] - I$$

with $q = 1 + i$ and i being the given interest rate and n the specific year.

11.2.3.3
Calculations with Equal Annual Cash Flows

When the relevant cash flows are equal for each production year, the calculation can be simplified considerably.

In the example of Sect. 11.2.3.1 we chose the following calculating method:

$$\sum = 10 \times \frac{1}{(1+i)^1} + 10 \times \frac{1}{(1+i)^2} + 10 \times \frac{1}{(1+i)^3} + \cdots \frac{1}{(1+i)^n}$$

This can be expressed differently as

$$\sum = 10 \times \left[\frac{1}{(1+i)^1} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \cdots \frac{1}{(1+i)^n} \right]$$

The sum enclosed in the bracket is the sum of a geometrical series which can be summarised in the following way:

$$b_n = \frac{q^n - 1}{q^n \times (q-1)}$$

with q again being $1 + i$, i.e.

$$\sum = \mathrm{NC}_j \times b_n$$

The factor b_n is called annuity present value factor (also called discrete uniform present worth factor or series present worth factor). The most important of these factors are listed in Appendix D, Table D19 and graphically represented in Fig. D1 (p. 227).

Thus the formula for the net present value (NPV) with equal annual cash flows can be written as

$$\mathrm{NPV} = \mathrm{NC} \times b_n - I$$

Example. Returning to the example of Sect. 11.2.3.1 Fig. 11.4a with annual net cash flows of €10 million, an interest rate of 15%, a mine life of 10 years and the investment of €45 million, b can simply be calculated:

$$b_n = \frac{q^n - 1}{q^n \times (q-1)} = \frac{1.15^{10} - 1}{1.15^{10} \times (1.15 - 1)} = 5.02$$

and the net present value is

$$\mathrm{NPV} = \mathrm{NC} \times b_n - I = 10 \times 5.02 - 45 = €5.2 \text{ million}$$

(The difference between 5.3 in Sect. 11.2.3.1 and 5.2 here is due to rounding.)

11.2.3.4
Comparison of Projects Via Standardized Net Present Value (NPV)

If we want to compare projects it is not correct just to consider the net present value NPV based on the same interest rate and conclude that the project with the highest NPV is the best one. One has to consider the NPV in relation to the investment *I*. This can be done by standardizing, or normalizing, the NPV-value by the investment.

Example. Project 1 required an investment of US $100 million and has a NPV of US $60 million based on an interest rate of 10%. Project 2 required an investment of US $150 million and has an NPV based on the same interest rate of US $70 million. We standardize now the NPV-values against the investment:

$$\text{Project 1: } \text{NPV}_{\text{stand.}} = \frac{\text{NPV}}{I} = \frac{60}{100} = 0.60$$

$$\text{Project 2: } \text{NPV}_{\text{stand.}} = \frac{70}{150} = 0.47$$

Therefore the first project would be considered the better one.

The calculation is comparable to the one of the profitability quotient of Sect. 11.1.1.

11.2.3.5
The Influence of Country Risk on the Interest Rate for the Net Present Value (NPV)

As already pointed out in Sect. 11.2.3.1 the net present value NPV largely depends on the interest rate chosen. In many companies there are internal guidelines available. As a rule, government bonds are chosen as guidelines, i.e. long-term capital investments with the lowest risk. If these bonds yield an interest rate of 10%, as they did twenty years ago, the discounting factor for mining projects would have to be at least 15% in order to compensate for the risk involved in mining. Today with much lower interest rates of government bonds, around 4%, some companies choose interest rates of about 10%. Risk surcharges can be quantified by calculating the called β-factor. [A detailed analysis of the risk problem typical for mining projects would go beyond the framework of this book. However, attention is drawn to a number of comprehensive studies on the subject from the South African or Australian mining practice (see for example Gilbertson 1980)].

One problem an exploration geologist will frequently encounter when working internationally is how to deal with political, i.e. country risks. The higher the risk, the higher the required interest rate, and the shorter the acceptable payback period as shown in Sect. 11.1.3. This means that political risk will translate into a spread of interest rates for projects with otherwise comparable parameters. Ainsworth (1991) and Bhappu and Guzman (1995) investigated the required internal rate of returns of mining projects which we in turn can take as interest rates for NPV computations.

Ainsworth (1991) classed countries into four risk categories which are actually only three, because he considers the USA separately. According to Ainsworth, the rates required by industry for discounting in net present value calculations vary between 10 and 15% for countries with the lowest country risk, 15–20% for countries with intermediate country risk and 20–25% for countries with highest country risk.

Country risks change due to political circumstances. The Fraser Institute of Vancouver conducts an "Annual Survey of Mining Companies"[18], which are normally published in the February edition of the Engineering and Mining Journal. Also the political and economic risk map of the Aon Group, Inc., Oxford, UK, shall be mentioned[19].

Besides information from the Fraser Institute there are especially two other organisations which provide information that is especially helpful in establishing a country ranking: the non-governmental organisation Transparency International[20] which annually publishes the Corruption Perceptions Index and the World Bank[21] which publishes the Governance Indicators. The World Bank considers six dimensions of governance for 209 countries: voice and accountability, political stability and absence of violence, government effectiveness, regulatory quality, rule of law, and control of corruption. To each of these dimensions scores from –2.5 (most negative) to +2.5 (most positive) are assigned.

To derive a country risk indicator it is recommended to follow the rule of Ainsworth (1991) and class the countries into three risk categories and use information from the Fraser Institute or preferably the World Bank which is more comprehensive than the Fraser Institute. The Fraser Institute considers opinions of mining executives and, therefore, only countries in which mining executives already have experience.

Example. A gold project in Egypt is offered. Derive a risk category for Egypt to select a net present value interest rate for discounting. Egypt does not appear in the country ranking of the Fraser Institute.

- *Step 1*: We use the Fraser Institute list to establish the top and bottom ranking. In the 2005/2006 list for the Policy Potential Index (Fraser Institute 2006), the four top positions are taken up by Nevada, Alberta, Manitoba and Chile. (The Fraser Institute splits larger countries such as the USA, Australia, and Canada into separate provinces or states, mainly because provincial and state governments are individually responsible for their own independent mining and environmental legislation.) Since the World Bank Governance Index only covers entire countries, we take three countries for the top ranking into account: USA, Canada, and Chile.

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18 www.fraserinstitute.ca/admin/books/files/Mining20052006.pdf.
19 www.aon.com/politicalrisk.
20 www.transparency.de/Corruption-Perceptions-Index-2.810.0.html.
21 www.worldbank.org/wbi/governance/pdf/2004kkzcharts.xls.

- *Step 2*: We consult the Governance Index of the World Bank (World Bank 2005) and look up the scores for the six dimensions listed above for the USA:

– Voice and accountability:	+1.21
– Political stability:	+0.47
– Government effectiveness:	+1.80
– Regulatory quality:	+1.22
– Rule of law:	+1.58
– Control of corruption:	+1.83
Arithmetic average	+1.35

  We do the same for Canada and Chile:

– Canada:	+1.63
– Chile:	+1.16

  We now take the average of the score of the USA, Canada and Chile and thereby establish the upper limit: The average is +1.39.
- *Step 3*: We now do the same to establish a lower limit. According to the country ranking of the Fraser Institute the lowest scoring countries are the Democratic Republic of Congo (formerly Zaire), Papua New Guinea and Zimbabwe. According to Step 2 above we again calculate the average score according the World Bank Governance Index. The average scores are:

– Democratic Republic of Congo:	–1.87
– Papua New Guinea:	–0.72
– Zimbabwe:	–1.54

  Again we take the average as the lower limit which is: –1.38.
- *Step 4*: Now we have a score range from +1.39 to –1.38, i.e. 2.77 in total. This range we divide into three equal parts to establish our 3 risk categories with each risk category covering a score range of 0.92.

  We have now the results presented in Table 11.3.
- *Step 5*: Having established the risk categories and the score ranges according to the World Bank Governance Index we now calculate the average score for Egypt according to the procedure in Step 2. The average score for Egypt is: –0.46.

  This score lies at the border between category 2 (medium risk) and 3 (highest risk). So, according to the categories of Ainsworth (1991) (see above) a discount rate of 20% would be appropriate in a calculation of NPV for a project in Egypt.

Table 11.3.
Risk categories and the score ranges

Risk category	Score range
1 (lowest risk)	+1.39 to +0.47 (and more positive)
2 (medium risk)	+0.47 to –0.45
3 (highest risk)	–0.45 to –1.38 (and more negative)

11.2.3.6
Comparable-Transaction Analysis for Determining the Interest Rate for the Net Present Value (NPV)

If we can find enough of transactions involving of sales property or, alternatively royalty payments to the previous owner we should have enough data to mathematically derive an interest rate which the market demands for a net present value calculation. Stein (1991) describes a relevant case from the market for brick clay deposits in Germany. For this calculation we take the formula for the internal rate of return (IRR) for equal annual cash flows which will be discussed in detail in Sect. 11.2.4.1 and 11.2.4.3:

$$I = \mathrm{CF} \times b_n$$

I is the investment which occurs before the mine starts producing, CF is the equal annual cash flow during the mine operation and b_n is the annuity present value factor introduced in Sect. 11.2.3.3.

We now transform this formula with either outright sale or royalty payment in mind. In the above equation the selling price SP would be equivalent to the investment I and the annual royalty payments RP to the annual cash flow CF. So now our formula for calculating the required interest rate via the annuity present value factor b_n becomes

$$\mathrm{SP} = \mathrm{RP} \times b_n$$

The result for the transactions of clay deposits for brick making in Germany (Stein 1991) is shown in Fig. 11.5.

Example. In a country, the average selling price of clay deposits for brick making from which a production of 20 000 m^3 annually can be generated for 15 years is on average €140 000. For comparable deposits some owners may not be willing to sell outright, but ask for an annual royalty instead.

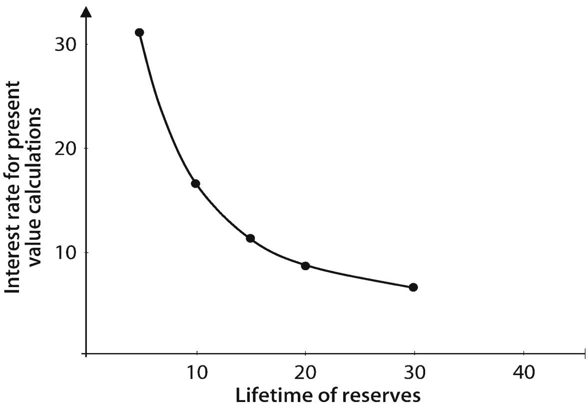


Fig. 11.5. Interest rates for present value computations calculated by comparing selling prices with royalty payments for clay deposits for brick making in Germany (after Stein 1991)

The average royalty is 1.40 €/m³, i.e. for a production of 20 000 m³/a the annual payments are €28 000. What is the required interest rate for present value calculations?

Substituting in the above equation we get

$$140\,000 = 28\,000 \times b_n \ , \quad \text{i.e.}$$

$$b_n = b_{15} = \frac{140\,000}{28\,000} = 5$$

From Table D19 in Appendix D or the graph in Fig. D1 (p. 227) we can see that for $n = 15$ yr the annuity present value must lie between 1.19 and 1.18. An interpolation results in 1.184. This means, the required interest rate for present value calculations would be 18.4%.

Figure 11.5 shows that the required interest rate is a function of the lifetime of a deposit. One can clearly see that the market requires a risk premium in the form of higher interest rates.

Shorter lifetime means higher risk. There could be start-up problems which frequently occur in mining operations and are dealt with in Sect. 11.11. If the deposit has a long life these initial problems can be offset in later years but only if the deposit is big enough. For metal deposits there is in addition a price risk. In Sect. 8.1.1 it was mentioned for example that some mining ventures, therefore, require a minimum life of 10 years to be able to compensate for cyclical price fluctuations.

11.2.4 The Internal Rate of Return (IRR or IROR)

11.2.4.1 Derivation of the Internal Rate of Return Method from the Net Present Value Method

The factor

$$q^{-n} = \frac{1}{(1+i)^n}$$

being part of a falling geometric series, the net present value (NPV) will decrease, the higher the rate of interest applied. The following assignment will serve as an example.

Assignment. A mining project requires investments of €50 million. The net cash flow over 12 years is €12 million/a. Calculate the net present value of this project at an interest rate of 10, 20 and 30%. Since the annual cash flows are constant, we follow the procedure in Sect. 11.2.3.3:

Step 1: The annuity net present value factor for $n = 12$ years and 10% [i.e. $i = 0.1$ and $q = (1 + i) = 1.1$] is

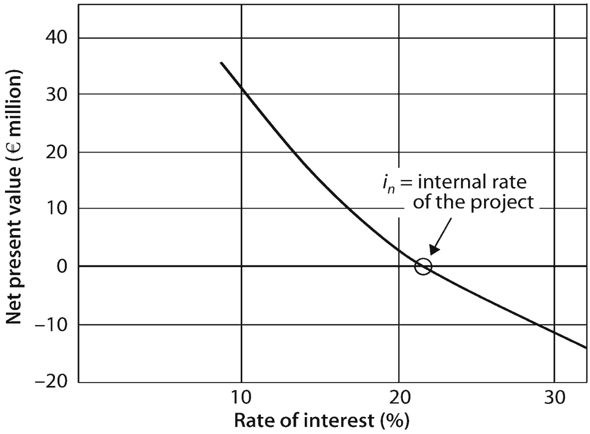


Fig. 11.6.
Derivation of the internal rate of return from the net present value method by interpolation

$$b_n = \frac{q^n - 1}{q^n \times (q-1)} = \frac{1.1^{12} - 1}{1.1^{12} \times (1.1 - 1)} = 6.81$$

Similarly, for 20%: $b_n = 4.44$
and for 30%: $b_n = 3.19$

Step 2: The formula for the net present value is (see Sect. 11.2.3.3)

$$\text{NPV} = \text{NC} \times b_n - I$$

Thus the net present value is

- for an interest rate of 10%: $12 \times 6.81 - 50 =$ €31.7 million
- for an interest rate of 20%: $12 \times 4.44 - 50 =$ €3.3 million
- for an interest rate of 30%: $12 \times 3.19 - 50 =$ €–11.7 million

In Fig. 11.6 this result is plotted in a graph.

The curve in Fig. 11.6 intersects the x-axis at 21.7%, i.e. the point where the net present value (NPV) equals 0. This point is the *internal interest return, i.e. the internal rate of return (IRR or IROR) of the project.* It is also called the DCF (discounted cash flow) rate or the earning power of a project.

With the internal rate of return method that rate of return is chosen at which the sum of the discounted net cash flows (NC) or, in special cases, the discounted gross cash flows (CF) just equal the investments. Based on the equations from Sect. 11.2.3.2 for unequal and from Sect. 11.2.3.3 for equal annual cash flows we arrive at the following terms:

- unequal cash flows: $I = \sum(\text{NC}_j \times q_j^{-n})$ or in special cases $I = \sum(\text{CF}_j \times q_j^{-n})$
- equal cash flows: $I = \text{NC} \times b_n$ or in special cases $I = \text{CF} \times b_n$

11.2.4.2
Calculating with Unequal Annual Cash Flows

If a project has unequal annual net cash flows, the internal rate of return cannot be calculated directly but has to be determined via an iteration process. There are calculators available today programmed for automatic iterations. In case the iteration process has to be done by hand, the following simple example will demonstrate the calculating method.

Example. A small vein mining project required an investment of €28 million, interest during construction included. The net cash flows are listed in Table 11.4a. For the first test a rate of return of 20% is chosen, i.e. $i = 0.20$.

Thus

$$\Sigma(q^{-n} \times \text{NC}) = 30.50$$

i.e. the equation $I - \Sigma(q^{-n} \times \text{NC}) = 28 - 30.50 = -2.5$ is negative.

The net cash flows have not been discounted enough. In the next step we therefore choose 25%, i.e. $i = 0.25$ (Table 11.4b).

Table 11.4a. Net cash flows for a small vein mining project

Year n_j	0	1	2	3	4	5	6	7	8
Investment (€ Mio)	28								
Net cash flow NC (€ Mio)		6.5	9.5	9.0	8.5	8.0	7.5	7.0	6.5
Discounting factor q^{-n} for $i = 0.20$ (20%)		0.833	0.694	0.579	0.482	0.402	0.335	0.279	0.233
Discounted net cash flow $q^{-n} \times$ NC (€ Mio)		5.41	6.59	5.21	4.10	3.22	2.51	1.95	1.51

Table 11.4b. Discounted net cash flows with $i = 0.25$

Years n_j	0	1	2	3	4	5	6	7	8
Discounting factor for $q^{-n} = \frac{1}{(1+i)^a}$ for $i = 0.25$		0.800	0.640	0.512	0.410	0.328	0.262	0.210	0.168
Discounted net cash flow $q^{-n} \times$ NC (€ Mio)		5.20	6.08	4.61	3.49	2.62	1.97	1.47	1.09

Thus

$$\Sigma(q^{-n} \times \mathrm{NC}) = 26.53$$

i.e. the equation $I - \Sigma(q^{-n} \times \mathrm{NC}) = 28 - 26.53 = +1.47$ is positive.

In this case the net cash flows have been discounted too much. Thus, the internal rate of return, at which the equation $I - \Sigma(q^{-n} \times \mathrm{NC})$ is just zero, lies between 20 and 25%, i.e. closer to 25%, since the difference in the second trial is smaller than in the first. The solution can now be found through a simple interpolation, either graphically or by calculation. The graphic solution is shown in Fig. 11.6 and Fig. 11.7.

By calculating, the interpolation is as follows:

$$\frac{28-26.53}{30.5-26.53} \times (25-20) = 1.85$$

Thus, the point looked for is 25 – 1.85 = 23.15, i.e. the internal rate of return at which the sum of the discounted net cash flows (NC) equals the investment I is

$$i = 23.2\%$$

A more exact calculation which reduced the difference between I and NC to < 0.01 was 23.01%. For the purpose of our preliminary evaluations such interpolations as shown above are sufficiently accurate.

These calculations once again show how critical the first production years are in determining the financial viability of a project expressed, in our case, in terms of the IRR described in Sect. 11.2.3.1 on NPV. This is well illustrated in Fig. 11.8 with a geothermal project.

Example. A geothermal energy project for heating needs an investment of I = US $25 million. It competes against a conventional heating system. In a model calculation two variables are considered:

a The amount of water obtained from the well. The amount of water is directly proportional to the amount of geothermal energy which can be harvested, i.e. the energy savings against using conventional energy. For the model calculation in Fig. 11.8, therefore, three savings rates are implied: Sr_1 = 2.15 million US $/a, Sr_2 = 2.5 million US $/a and Sr_3 = 3.0 million US $/a.

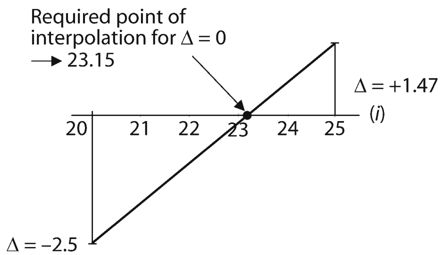


Fig. 11.7.
Interpolation to determine the internal rate of return

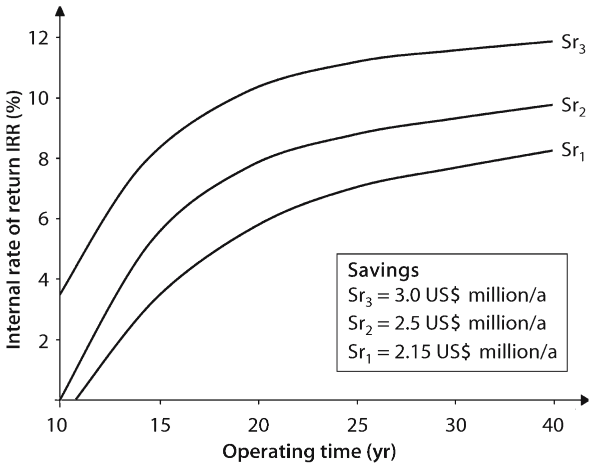


Fig. 11.8. Internal rate of return (IRR) of a geothermal project as function of operating time and annual energy savings achieved

b The number of years the geothermal well can be used. This is the *x*-axis in Fig. 11.8. For an internal rate of return IRR of 0, meaning no interest is earned, this is achieved for example for $Sr_2 = 2.5$ million US $/a in

$$\frac{25}{2.5} = 10 \text{ years}$$

This is identical with the payback period of Sect. 11.1.3.

In Fig. 11.8 one can clearly see how the curves for the IRR steeply rise in the first years after payback has been achieved, but then flatten after year 30, so that the remaining years hardly have an effect.

11.2.4.3 Calculating with Equal Annual Cash Flows

The tedious calculation procedure of the previous chapter can be avoided in the initial stages of a deposit evaluation by calculating with equal annual cash flows. The average grade of the deposit can be roughly estimated at an early stage. In the absence of more precise information about the distribution of grades it would be pedantic and wrong to assume different grades for each year. For capital and operating costs, figures based on experience and precedents are available. We assume the operating costs to be constant over the entire period of production (for a justification of this assumption see Sect. 9.2.1). From the difference between revenues (Rev) minus costs (Co) we obtain the operating profit (OP). It is accepted practice in the early stages of a project evaluation to determine, via the operating profit, the internal rate of return *before* tax under the assumption that the project is 100% equity financed, so that no interest for debt will accrue. If constant annual tax payments are also assumed (see Sect. 11.3), an internal rate of return after tax can be determined by employing equal annual cash flows (i.e. with net cash flows, NC).

Since interest payments and, as a consequence, taxes generally always occur unequally distributed over the years, unequal annual cash flows are inevitable in case debt financing is involved in the evaluation of a project. In such cases the procedure described in Sect. 11.2.4.2 should be applied.

If operating profits (OP) can be assumed to be constant, the internal rate of return is determined by the equation

$$I = \mathrm{OP_c} \times b_n \quad , \quad \text{i.e.}\ b_n = \frac{I}{\mathrm{OP_c}} \tag{11.1}$$

The value b_n, the annuity present value factor, can be taken from Table D19 (Appendix D) or can be interpolated from the tabulated figures (Fig. D1, p. 227).

Assignment. Investments for a mine project amount to I = US $40 million, annual operating profits (OP) = US $12.5 million. The mine has a life of 10 years. What is the internal rate of return *i*?

$$b_n = \frac{I}{\mathrm{OP_c}} = \frac{40}{12.5} = 3.2$$

Table 11.5 presents values we take from Table D19 (Appendix D) for $n = 10$.

Interpolating again (see example at the end of Sect. 11.2.4.2) we obtain $i = 0.289$, i.e. an internal rate of return of 28.9% (the exact value is 28.8, an insignificant difference). For an approximate interpolation the diagram of the annuity present value factors in Appendix D, Fig. D1 (p. 227) can also be used.

Computations with constant cash flows and the annuity present value factors enable quick back-of-an-envelope arithmetics to arrive at an IRR value. One calculates the ratio between investment *I* and annual cash flow CF. For a lifetime of a project of 10 years the internal rate of returns IRR are given in Table 11.6. The cash flow CF can be either the gross or net cash flow depending whether it is calculated before or after taxes.

Table 11.5.
Values taken from Table D19 (Appendix D) for i and b_n

i	b_n
0.3	3.09
0.25	3.57

Table 11.6.
Internal rate of returns IRR

I/CF	Internal rate of return IRR (%)	IRR (%), rounded for back-of-an-envelope calculations
3	31.11	30
3.5	25.66	25
4	21.41	20
4.5	17.96	17.5
5	15.10	15
5.5	12.66	12.5
6	10.56	10

Calculations with annuity present value factors are ideally suited for quick breakeven estimates (see Sect. 11.8) or for the calculation of a minimum target. The following assignment serves as an illustration of how to define a minimum target.

Assignment. In a gold exploration programme there are indications that an orebody of 250 000 t can realistically be expected. Mining will be underground. Mining recovery is estimated at 85%. The lifetime of the mine should be at least 8 years. Preliminary beneficiation tests indicated a recovery of 90%. Investment costs were estimated at US $12 million, operating costs at 90 US $/t.

What must be the minimum average gold grade if a base gold price of 400 US $/oz is assumed and the mining company expects a minimum internal rate of return of 24% before tax?

Step 1: With a mining extraction of 85% and a lifetime of 8 years tonnages of 250 000 t will yield an annual production of

$$\frac{250\,000 \times 0.85}{8} \approx 27\,000 \left(\frac{\mathrm{t}}{\mathrm{a}}\right)$$

Step 2: The annuity present value factor for 24% ($i = 0.24$) is

$$b_n = \frac{q_n - 1}{q_n \times (q-1)} = 3.42$$

with $q = 1 + i = 1 + 0.24 = 1.24$ and $n = 8$.

Step 3: With investments of US $12 million the annual operating profit must be

$$I = \mathrm{OP} \times b_n$$

$$\mathrm{OP} = \frac{I}{b_n} = \frac{12}{3.42} = \mathrm{US\$}3.51 \text{ million}$$

Step 4: With an annual tonnage of 27 000 t/a this is per tonne of ore:

$$\frac{3.51 \times 10^6}{27\,000} = 130 \left(\frac{\mathrm{US\$}}{\mathrm{t}}\right)$$

Step 5: The operating costs of 90 US$/t have to be added to the operating profit of 130 US$/t to arrive at the minimum revenue, i.e. minimum revenue = 90 + 130 = 220 US $/t.

Step 6: At a gold price of 400 US $/oz, 220 US $/t = 0.55 oz/t or 17.1 g/t (1 oz = 31.103 g, see Sect. 1.1.4).

Step 7: With a beneficiation recovery of 90% ($\varepsilon = 0.9$, see Sect. 4.3) the ore in situ must have the following minimum grade so that requirements of minimum profitability are fulfilled:

$$\frac{17.1}{\varepsilon} = \frac{17.1}{0.9} = 19 \left(\frac{\text{g Au}}{\text{t}}\right) \text{ in situ ore}$$

Such rough calculations allow the geologist to quickly decide

- whether a particular grade can be obtained and if so, which procedures are required to prove this grade, or
- should this grade be unrealistic, whether the tonnage can be increased to guarantee a minimum profitability in spite of lower grades. In analogy to calculations of minimum grade in the above assignment, the minimum tonnage can also be calculated if a fixed maximum grade is given.

If the evaluation shows that the minimum required grade cannot be attained for the given tonnage potential nor the minimum tonnage for a given potential grade, then further exploration can be abandoned (no-go decision).

11.3 Aspects of Taxation

11.3.1 Introduction

Geologists tend to be of the opinion that questions of taxation do not concern them. Ultimately, however, taxes are costs which the mine has to bear. With high taxes the grades of a deposit have to be higher than with low or no taxes. It is not indifferent for the required metal content whether I have to pay 30% taxes as in Indonesia or 46% as in South Africa under certain circumstances. Often royalties apply independent of profits, which can add considerably to costs.

To encourage new mining investments several countries have introduced tax incentives such as depletion allowances (e.g. in the USA, see also Sect. 11.3.3 and 11.5), tax holidays for start-up years, or accelerated and increased depreciation rates. A detailed analysis would go beyond this book. When the preliminary evaluation of a deposit requires the tax regime to be taken into consideration, it might be advisable to consult the tax information supplied by big international auditing and tax consulting companies such as those of the called "Big Five", Arthur Andersen, Ernst & Young, PriceWaterhouseCoopers PwC, KPMG and Deloitte & Touche, who publish information on most countries, generally free of charge. Helpful information are also the Sectoral Notes – Industry and Mining – of the World Bank, which can be accessed in the internet.

Royalties to governments[22] are determined by national mining legislation. For information on the mining legislation of foreign countries one has to consult the embassies or publications of the above mentioned international auditing firms. The company

[22] Royalties might also be payable to property owners. These royalties are of course determined by contract negotiations.

PricewaterhouseCoopers[23] regularly publishes summaries of foreign mining taxation rules (Comparative Mining Tax Regimes – A Summary of Objectives, Types and Best Practices in 1998 or Effective Tax Rates Comparison of the Global Mining Industry 2005).

The tax issue requires a comment in principle. In Anglo-Saxon countries great emphasis is put on the annual budget. Together with the budget, annual changes in tax or depreciation rates are published. Since our evaluation of a potential mine has to make assumptions far ahead into the future, the tax and royalty terms valid at the time the project is likely to be realized are unknown. One should therefore always work with basic tax regulations. If a project appears to be of interest only because a special concession or a loophole in the present tax system is favourable, it is hardly worth pursuing at all. Tax incentives can quickly be abolished.

11.3.2 Depreciation

In every country the tax burden can be reduced by deducting a certain proportion of the capital investments from the basic tax. The deduction is called depreciation. Depreciation rates are part of the tax legislation. As a rule, details can be obtained from the above mentioned publications of the international tax consulting companies.

There are two basic ways of calculating depreciation: linear and non-linear depreciation:

a In the case of linear depreciation the investment is simply divided by the number of years of the depreciation period. The result is the annual depreciation rate. If we have invested US $60 million and depreciate this amount over the working lifetime of the mine, i.e. 8 years, the depreciation rate is 60 : 8 = 7.5 million US $/a.

b With *non-linear* depreciation, also called depreciation on a declining balance, the rates deducted decrease from year to year (Table 11.7).

Example. The investment sum is again US $60 million. The rate of depreciation is 20% on a declining balance. The annual amounts depreciated develop progressively (Table 11.7).

Table 11.7. Depreciation rates

Year	Basis for calculating depreciation (Mio US $)	Depreciation (Mio US $)
1	60.0	12
2	60 – 12 = 48.0	9.6
3	48 – 9.6 = 38.4	7.7
4	38.4 – 7.7 = 30.7	6.1
5	30.7 – 6.1 = 24.6	4.9
	etc.	

23 http://www.pwc.com/extweb/pwcpublications.nsf/docid/2ab77c96280216758525727a0080661e.

Table 11.7 shows that through non-linear depreciation higher amounts can be depreciated during the first three operating years than with linear depreciation; it is only from year 4 onwards that the amounts become smaller. As we have observed in Sect. 11.2.3.1, the cash flows of the initial years have a particularly strong effect on the overall economics of the project. Higher rates of depreciation decrease the tax burden, thus raising the net cash flows and improving the economics of the project.

Since non-linear depreciations never attain 100%, from a particular year on linear depreciation is applied for the remainder, or the remainder is depreciated over the last year.

11.3.3
Depletion Allowances

In addition to depreciation, mining ventures in certain countries, for example the USA, benefit from depletion allowances which further diminish the tax base. The depletion allowance is based on the concept that a mining operation uses up a non-renewable resource. Unlike normal industrial operations, mine operators have to bear an additional risk due to the necessity of having to replace mined out reserves by exploring for and discovering new deposits. In general a certain percentage of the net smelter return NSR (see Chap. 7) is deductible from the tax base up to a specified maximum limit of the taxable income. This is called percentage depletion in contrast to cost depletion. Cost depletion relates to the recovery of the taxpayer's investment to the proportion that the current unit sales of mineral products bear to the total anticipated products from the property. An example of percentage depletion is given at the end of Sect. 11.5. In the USA, depletion allowances are calculated either as a percentage – normally varying between 14% and 22% – of net smelter return after royalty payments or as the upper limit of 50% of taxable income, whichever is the lower of the two. Information on the depletion percentages for the USA can be gathered from the annual publication of the U.S. Geological Survey, Mineral Commodity Summaries, or from information of the above mentioned tax consulting companies.

11.4
Equity and Debt Financing

Another financial aspect we have to consider in economic evaluations is the ratio of equity to debt financing, the called gearing ratio or leverage. This is important because the interest on debt is paid out of earnings and is therefore a project cost and, as such, an integral element of cash flow (see Sect. 11.2.2 and Fig. 11.2).

The following example will explain the meaning of these terms.

Example. A project has an internal rate of return (entire project) of 15%. If the capital ratio is 1/3 equity and 2/3 debt and the latter requires an interest payment of only 10%, then the internal rate of return on the equity increases. The difference between the 15% interest earned and the 10% spent on servicing the debt can be added to the equity, thus increasing the rate of return by approximately $2 \times 5\%$, to 25%.

Naturally, this gearing ratio can also have the reverse effect. If a project has an internal rate of return of only 6%, with 2/3 debt financed at an interest rate of 10%, then the

10% interest must, of course, be paid in spite of the low internal rate of return and the difference of 10 – 6 = 4% must be subtracted from the equity return (1/3 of the entire capital). Consequently, the rate of equity return is roughly speaking 6 – (2 × 4) = –2%.[24] The project is therefore running at a loss!

The example also shows that variable ratios of equity/debt financing can considerably improve the economics of equity, in the extreme case (equity almost zero) they can achieve an infinitely high return rate, provided the internal rate of return is higher than the expenses for debt capital.

It is therefore advisable to first calculate the rate of return without gearing, and to determine the equity return later. In industrialized mining countries, Canada and the USA in particular, it is possible to 100% debt-finance mining projects. In developing countries, banks will require equity from mine owners. The terms may vary, but a standard ratio is 3 : 1, i.e. 25% equity, 75% loan capital. If a project is 100% debt-financed, the calculation of an internal return on equity is sensu strictu superfluous. In this case the net present value (NPV) serves as an economic criterion (see Sect. 11.2.3).

A very critical factor for the ratio of equity to debt financing is the ability of the project to pay back the loan with a high degree of certainty, based on proven and probable reserves only. For debt financing in general a cash flow to debt coverage ratio of at least 1.2 to 1.5 is required, calculated as the ratio of cash flow to the sum of debt plus interest (e.g. Gschwindt 1991). Even more important for mining projects is the coverage ratio of the reserves which was described in Sect. 8.4. If the condition that total proven and probable reserves have a lifetime of at least twice the period required to pay back the loan is not fulfilled, the bank will more than likely reduce the share of debt financing in proportion to the shortfall in reserves.

Critical for the economics of a mining project is the interest rate. In the eighties and beginning of the nineties, a period of high interest rates, one had to calculate with about 12% interest. Today, interest rates are much lower. Many loans for project financing nowadays do not carry a fixed rate but are oriented at the LIBOR rate (LIBOR = London Interbank Offered Rate). The premium above the LIBOR rate takes into account the risk of a mining project as well as the country risk already discussed in Sect. 11.2.3.5. What has been said in Sect. 6.1 about fluctuating commodity prices is also valid for interest rates to a certain degree. For an economic evaluation in the exploration stage we have to make a reasonable assumption for a future interest rate. To be on the safe side, we will use an interest rate of 8% in our example of a cash flow calculation.

11.5 Example of a Cash Flow Calculation

For cash flow calculations *annual cash flows* are used. The simplified examples in Fig. 11.4a and Table 11.4a and b, in which annual net cash flows and operating profits were considered, already represent basic cash flow calculations. As explained in Sect. 11.2.2, for simplicity's sake we assume that all cash flows are due *at the end* of a year.

[24] Since the ratio is non-linear (see example, Sect. 11.5) real cash flow calculations will yield different values. The above example only serves to explain the principle.

There is no standard method for doing cash flow calculations. The individual steps depend largely on the intricacies of calculating taxes and the mode of financing.

The following example is a step-by-step illustration of the general procedure used in cash flow calculations. The sums in Table 11.8 are given in million US $.

Example. A base metal project requires investments of US $55 million for an underground mine development (also called "capex" for capital expenditures). It is a newly discovered ore body close to an existing mining camp, so an already existing mill can be used and the ore trucked to this mill and toll-milled (or custom-milled). Planned production is 300 000 t / a. Operating costs, including processing costs, amount to 40 US $ / t. The mine receives a net smelter return of US $120 per tonne of ore (see calculation procedure Sect. 7.2).

The investment is 100% debt-financed, i.e. by bank loans. The interest rate is 8%. The state where the mine is located imposes a royalty of 3% on revenues; the government of the country charges 35% taxes on the profit. Royalties are not tax deductable, interest, however, is. The investments can be linearly depreciated over the lifetime of the mine.

The following steps are necessary:

Step a: From the annual production and the revenue per tonne of ore – the net smelter return (NSR) – we obtain the gross revenue:

$300\,000 \text{ t} \times 120 \text{ US\$/t} = 36 \text{ million US\$/a}$

Step b: From the annual production and the cost per tonne of ore we obtain the overall operating costs per year:

$300\,000 \text{ t} \times 40 \text{ US\$/t} = 12 \text{ million US\$/a}$

Step c: From the difference between gross revenue and overall operating costs per year we obtain the operating profit (OP) per year (or cash flow *before* interest and taxes):

$36 \text{ million US\$/a} - 12 \text{ million US\$/a} = 24 \text{ million US\$/a} = \text{OP}$

Step d: Interest has to be paid from the operating profit, i.e. in year 1 (the first year of production) 8% on US $55 million, i.e. US $4.4 million. In year 2 interest will be lower, since the net cash flow (see Step h in Table 11.8) is used to repay bank loans as quickly as possible. Interest payment will decrease from year to year. After repayment of the entire loan (i.e. the payback period in this case, see Sect. 11.1.3) interest will be nil, since the investment was 100% debt-financed.

Step e: The tax basis, on which the profit tax of 35% is charged, can be reduced by deducting depreciation (see Sect. 11.3.2). Since the investments can be depreciated linearly over the 8-year lifetime of the mine, 55 / 8 = US $6.9 million can be deducted each year.

Step f: The federal government receives a 35% profit tax after interest and deductions for depreciations.

Table 11.8. Cash flow calculation

Line	Calculation factor	Description in the text	Calculating method	Year n								
				0	1	2	3	4	5	6	7	8
1	Investment I or outstanding loan financing		Amount in year n: year $(n-1)$ in line 1 less year $(n-1)$ in line 10	55	55	40.9	26.1	10.6	0	0	0	0
2	Gross revenue	a)			36	36	36	36	36	36	36	36
3	Operating costs	b)			12	12	12	12	12	12	12	12
4	Operating profit OP	c)	Line 2 – line 3		24	24	24	24	24	24	24	24
5	Interest (8%) on outstanding loan	d)	8% on amount line 1		4.4	3.3	2.1	0.8	0	0	0	0 $\Sigma = 10.6$
6	Depreciation	e)			6.9	6.9	6.9	6.9	6.9	6.9	6.9	6.9
7	Tax basis: Operating profit minus interest minus depreciation	f)	Line 4 less – line 5 – line 6		12.7	13.8	15.0	16.3	17.1	17.1	17.1	17.1
8	Taxes: 35% on operating profit after interest and depreciation (line 7)	f)	35% on amount in line 7		4.4	4.8	5.3	5.7	6.0	6.0	6.0	6.0
9	Royalty: 3% on gross revenues (line 2)	g)	3% on amount in line 2		1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1
10	Net cashflow NC (cashflow after interest, tax and royalty)	h)	Line 4 less – line 5 – line 8 – line 9		14.1	14.8	15.5	16.4	16.9	16.9	16.9	16.9 $\Sigma = 128.4$
11	Remaining cashflow after repayment of loan financing (line 1)		Line 10 less – line 1, if positive		0	0	0	5.8	16.9	16.9	16.9	16.9

Table 11.8. *Continued*

Evaluation of cashflow stream
A Net present value NPV

12	Discounted net cashflow NC, discount rate, $i = 12\%$		Line 10 × Line 13		12.5	11.8	11.0	10.5	9.6	8.6	7.6	6.8 $\Sigma = 78.9$
13	Discounting factors for $i = 0.12$ (see Sect. 11.2.1)		$q^{-n} = \frac{1}{(1+0.12)^n}$		0.89	0.80	0.71	0.64	0.57	0.51	0.45	0.40

NPV (at 12%) $= \Sigma (q^{-n}$ NC$) = 78.9 - 55 = 23.9 = 23.9$ million US $

Step g: The state receives a royalty of 3%, i.e. US $/a 1.1 million on the annual gross revenue of 36 million US $/a (see Step a).

Step h: From the annual operating profit of US $24 million (Step c in Table 11.8) interest (Step d), taxes (Step f) and royalty (Step g) have to be deducted. The remainder is the net cash flow after interest and tax, available for the repayment of the loan or for dividends to the owners of the mine after the payback period.

The cash flow table appears as in Table 11.8 with all amounts indicated in million US $. The results of the cash flow calculation in Table 11.8 are the following:

A NPV (at 12%) = $\Sigma(q^{-n}\text{NC})$ = 78.9 – 55 = 23.9 = US $23.9 million.
B Payback period (see line 1). At the end of year 3 or the beginning of year 4 respectively, there is a residual debt of US $10.6 million. Since a net cashflow of US $16.4 million was achieved in year 4 (see line 10), a fraction of (10.6 / 16.4) = 0.65 of year 4 are needed for the repayment of the remaining dept, i.e. a payback period of 3.65 years. Payback is therefore 3.7 years in round numbers.
C Internal rate of return. Does not apply since equity financing is nil. Sometimes, nevertheless, an internal rate of return on the capital invested is calculated from the net cashflow after tax and interest. This is a project return after tax on the entire capital invested. Since at 12% the net present value is US $23.9 million (see above *A*), the rate of return must be higher than the discount rate used. It is 22.9%.
D Since the project is 100% debt financed the payback period is identical with the time required to pay back the loan. Total lifetime of the project is 8 years. So the ratio of total lifetime of the project to time necessary to pay back the loan, the payback period, is V_R = 8 / 3.65 = 2.2 i.e. the requirement of sufficient debt coverage is fulfilled (Sect. 8.4).
E Cash flow to debt coverage ratio (Sect. 11.4). The sum of debt plus interest payments (see line 5) = 55 + 10.6 = 65.6. The sum of the cash flows of the eight production years is 128.4 (see line 10). So the requirement of a cash flow to debt coverage ratio of at least 1.2 to 1.5 is more than fulfilled.

Finally, we will show how a depletion allowance, in countries where it is available, will influence the cash flow of an entire project by its affect on the tax base and therefore on taxes paid. We will take from Table 11.8 only the money streams and calculations of year 1 as an example. We assume percentage depletion (see Sect. 11.3.3). In deciding which of the two possible depletion allowance rates is applicable, we have to carry out two calculations in our example. In Alternative A, we assume a depletion allowance of 15% of net smelter return after royalty payments of 3% as before. In Alternative B, the depletion allowance is calculated on the basis of a maximum of 50% of taxable income. The actual depletion allowance that is applicable in determining the tax base for cash flow purposes is the lesser of the two alternatives.

If we compare the taxes paid in Table 11.8 (line 8) and 11.9 (line 13a) it becomes obvious how much the tax burden is reduced by the depletion allowance and how much the project is helped by increasing the net cash flow. In Table 11.8 the net cash flow in year 1 was US $14.1 million (line 10). In the case of Table 11.9 it would be US $15.9 million after depletion.

Table 11.9. Cash flow calculation

Line	Calculation factor	Description in the text	Calculating method	Example year 1
1	Investment *I* or remainder of loan finance			55
2	Gross revenue	a)		36
3	Operating costs	b)		12
4		c)	Line 2 – line 3	24
5a	Royalty: 3% based on revenue (net smelter return) line 2	g)	3% of amount in line 2	1.1
Alternative A				
6a	Basis for depletion allowance		Line 2 – line 5a	34.9
7a	Maximum depletion allowance 15%		15% of line 6a	5.2
Alternative B				
8a	Interest (8%) on the remainder of loan fincancing	d)	8% of amount line 1	4.4
9a	Depreciation (= line 6)	e)		6.9
10a	Tax basis before depletion allowance: Operating profit minus interest minus depreciation	f)	Line 4 less – line 5 (Table 11.8) – line 6 (Table 11.8)	12.7
11a	Maximum depletion allowance: 50% of tax basis line 10a		50% of line 10a	6.35
Because the amount of line 7a is lower than the amount of line 11a this amount is taken as the depletion allowance to calculate the tax basis and the tax:				
12a	Tax basis after depletion allowance: Operating profit minus interest, depreciation and depletion allowance		Line 4 less – line 5 (Table 11.8) – line 6 (Table 11.8) – line 7a	7.5
13a	Tax: 35% on tax base		35% on line 12a	2.6

11.6 The Concept of Profit

We have dealt now with three criteria to be used in economic evaluations: the payback period in Sect. 11.1.3, the net present value (NPV) in Sect. 11.2.3 and the internal rate of return (IRR), the earning power of a project, in Sect. 11.2.4 (see also Fig. 5.1 in Chap. 5). According to an investigation by Bhappu and Guzman (1995) about mineral investment decision making by mining and exploration companies these are the main criteria. When the authors of this text book conducted seminars about economic evaluation of ore deposits in formerly centrally planned economies they were frequently asked, why the term profit does not appear. So far we have used the term "operating profit" only (e.g. in the introduction to Chap. 11, in Sect. 11.2.4.3 or Sect. 11.5 and Table 11.8), but the term "profit" alone had only been used in the context of the profitability quotient in Sect. 11.1.1. Of course, a mining company wants to make a profit from mining a deposit – otherwise it is by definition not a deposit with ore reserves but only a mineralization. Profit, however, is more a concept to measure the performance of a company or a mining operation on a yearly or, nowadays, even on a quarterly basis. In economic evaluations of a potential deposit in the exploration stage we are not looking at a single year in isolation. We are interested to evaluate a deposit over its entire life and to compare the input, or investment with the output, or net cash flow. For this input/output calculation payback period, net present value (NPV) and internal rate of return (IRR) are more relevant.

Returning to Table 11.8 in Sect. 11.5, the profit Pr would be the net cash flow NC in line 10 minus the depreciation of line 6. So for the year 1 the profit Pr would be

$$Pr = 14.1 - 6.9 = \text{US}\,\$7.2 \text{ million}$$

Although not relevant for our evaluations in the exploration stage, two abbreviations should be mentioned in this context for completeness sake. They regularly occur in business journals reporting about company performances concerning their annual or quarterly reports: Ebitda and Ebit.

Ebitda means Earnings before interest, tax, depreciation and amortisation. This is practically identical with our operating profit OP in Table 11.8, line 4.

Ebit then is Earnings before interest and tax.

11.7 Sensitivity Analysis

The cash flow calculations carried out in the preceding chapters clearly indicate the impact of metal prices. If the metal price in the example in Sect. 11.5 is increased by 10%, the net smelter return will also rise by 10% from 120 US $/t to 132 US $/t or, for the whole year, from 36 million US $/a to 39.6 million US $/a.

Since the operating costs are not influenced by metal prices an operating profit (OP) of 39.6 – 12 = 27.6 million US $/a has been made as compared with 24 million US $/a in our base case, i.e. an increase of 15%. If we now turn to the net cash flow and repeat the calculation listed in Table 11.8 using a metal price which is 10% higher, the net cash flow will increase by 16% as compared with the base case, i.e. an increase (and, vice versa also a decrease, of course!) in metal prices makes itself disproportionally felt in the final economic result.

In Chap. 6 we demonstrated how to obtain reasonable price assumptions. Price assumptions are, however, notoriously fraught with uncertainties. It is, therefore, unwise to work with a single price and we should test the sensitivity of projects to price changes by varying our assumptions within a range of prices. Economic calculations for the individual values of this price spectrum, generally 10 and 20% of the base case, are carried out repeatedly and the results plotted in a sensitivity diagram.

For complete feasibility studies it is essential that additional parameters such as ore grades, reserves, operating costs, capital costs etc. be also varied, since even in a detailed study (see Wellmer 1981) these factors may have an accuracy of only 10–20%. However, price variations usually have the most significant impact (Wellmer 1986). Therefore, a sensitivity analysis for a range of prices should already be done at the prefeasibility stage.

Assignment. We have been offered a gold deposit in Australia. Reserves would allow a 25 000 t/a operation over 8 years. Grades after dilution were estimated at 25 g Au/t. Beneficiation tests yielded a recovery of 80%. Operating costs were estimated at 120 AU $/t, capital costs (interest during construction included) at AU $10 million. Investment is 100% equity-financed. Gold mining in Australia has until recently been tax-free (to keep things simple, a tax-free case is taken).

Carry out a sensitivity analysis and make a sensitivity diagram.

The base case assumes a gold price of 400 AU $/oz, with variations of 300 AU $/oz and 500 AU $/oz.

1. Base case 400 AU $/oz Au

   At a gold price of 400 AU $/oz, a conversion factor of 31.103 g for 1 oz (see Sect. 1.1.4), a recovery of 80% (€ = 0.8) and an annual production of 25 000 t/a, the annual revenues of the mine are

$$400 \times \frac{25}{31.103} \times 0.8 \times 25\,000 = 6.430 \text{ million} \left(\frac{\text{AU\$}}{\text{a}}\right)$$

   The annual operating costs are: $120 \times 25\,000 = 3.000$ million AU $/a. Thus the operating profit of the mine is = 3.430 million AU $/a.

   Since the project is equity financed and we assume no tax, the operating profit equals the net cash flow. Therefore a simple cash flow table can be calculated as in Table 11.10.

Table 11.10. Cash flow

Year	0	1	2	3	4	5	6	7	8
Investment I	(10)								
Remaining capital		6.57	3.14						
Cash flow		3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43

From the cash flow in Table 11.10 a payback period of 2.9 years is derived. Since the annual cash flows are equal, the simple procedure from Sect. 11.2.4.3 can be followed to calculate the internal rate of return:

$$\frac{I}{\mathrm{OP}} = \frac{10}{3.430} = 2.92 = b_n \text{ (annuity present value factor)}$$

From Table D19 (Appendix D) an internal rate of return of IRR = 30.1% is obtained. To determine the net present value (NPV) with a rate of return of 15% chosen by us (see Sect. 11.2.3.3) the simple method for equal annual cash flows can again be applied:

$$\mathrm{NPV} = \mathrm{OP} \times b_n - I$$

where b_n over 8 years and with $i = 0.15$ is 4.49 (Table D19, Appendix D).

Hence NPV = 3.42 × 4.49 – 10 = AU $5.4 million.

2. Variations 300 AU $/oz and 500 AU $/oz
   The same steps as taken in the base case are now repeated for gold prices of 300 AU $/oz and 500 AU $/oz respectively.

   For 300 AU $/oz the operating profit is 1.823 million AU $/a

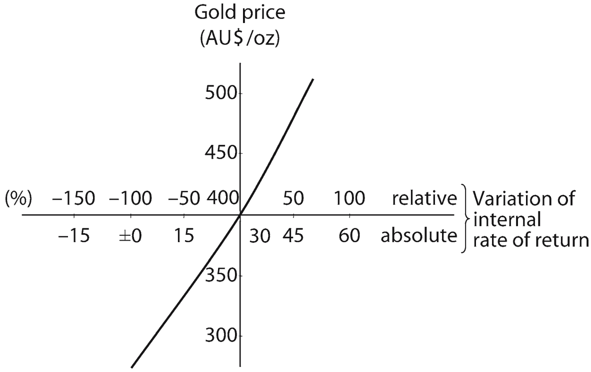
   For 500 AU $/oz it is 5.038 million AU $/a.

3. Result
   The result is summarized in Table 11.11.

Table 11.11. Result of the sensitivity analysis

Gold price (AU $/oz)	Payback period (yr)	IRR (%)	NPV at 15% (Mio AU $)
300	5.5	9.2	–1.8
400	2.9	30.1	5.4
500	2.0	48.2	12.6

Fig. 11.9. Sensitivity diagram



As a rule, the result is plotted in a graph. Figure 11.9 gives a graphic example of the internal rate of return (IRR). From such a graph it can be interpolated with sufficient accuracy what effect a change in the gold price from 400 AU $/oz to 450 AU $/oz would have, i.e. a relative change of 30% leading to an internal rate of return of about 39%.

11.8 Breakeven Calculations

"Breakeven" is the metal content or metal price just covering project costs. Unfortunately, the definition is not used uniformly.

As a rule, breakeven means that all operating and capital costs, taxes and interest payments (if the project is debt-financed) are covered. In this case the payback period (see Sect. 11.1.3) is identical with the lifetime of the mine (see assignment below).

In some cases breakeven means that the internal rate of return on equity equals the inflation rate or the government bond rate. Then the method of "reverse economic calculation" described in Sect. 11.2.4.3 should be applied.

11.8.1 Breakeven Calculations for Mono-Metallic Deposits

Assignment. We want to invest in a gold deposit in Western Australia. Investments were calculated at AU$30 million, to be equity financed. Production will be 200 000 t/a. Reserves will last for 10 years. Operating costs were estimated at 55 AU $/t. What is the breakeven price of the project, if the grade after dilution of 10% is 9.5 g Au/t and the mill recovery 90%?

Step 1: Investments are AU $30 million. Over the 10-year operating lifetime of the mine, 2 000 000 t of ore will be produced, i.e. per t of ore an amount of

$$\frac{30\times10^6}{2\times10^6}=15\left(\frac{\text{AU\$}}{\text{t}}\right)$$

must be earned for capital repayment.

Step 2: Due to equity financing, no interest is paid. Since breakeven means there is no profit, no taxes are paid. Breakeven therefore requires operating costs plus capital repayment of

$$55 + 15 = 70 \text{ AU\$/t}$$

Step 3: The beneficiation recovery is 90%, $\varepsilon = 0.9$, i.e. from the 9.5 g Au/t in the ore $0.9 \times 9.5 = 8.55$ g Au/t will be recovered. This equals (see Sect. 1.1.4)

$$\frac{8.55}{31.103}=0.27\left(\frac{\text{oz Au}}{\text{t}}\right)$$

Step 4: If 0.27 oz Au/t are to yield 70 AU $/t (see Step 2), a breakeven price of

$$\frac{70}{0.27}=260\left(\frac{\text{AU\$}}{\text{oz}}\right) \text{ is required}$$

11.8.2
Breakeven Calculations for Multi-Element Deposits

Following the preceding section (11.8.1), one could calculate a breakeven price for a metal equivalent based on the procedure in Sect. 3.5.2, where it is assumed that the price ratio on which the calculation of the metal equivalent is based does not change. This, however, is seldom the case.

Thus, we do not have a fixed breakeven price, but variable breakeven functions. For a deposit with two payable metal components, A and B, and an assumed price for A, a breakeven price for B can be calculated. If the price assumption for A is changed, the breakeven price for B will also change. Therefore such calculations for multi-element deposits can very well be compared with the sensitivity analysis in Sect. 11.7.

Assignment. A Pb-Zn-Ag deposit produces run of mine (ROM) ore with 10% Zn, 5% Pb, 130 g Ag/t. Recovery during beneficiation is 90% for Zn, 90% for Pb and 80% for Ag. Breakeven costs (operating costs, costs for servicing capital, interest, taxes) are 70 US $/t.

Develop a set of breakeven curves (there are three metals involved and hence three prices have to be varied).

Step 1: We use the net smelter return factors from Table 7.1, Sect. 7.2.2, to calculate the net smelter return of the mine: 50% for Zn, 65% for Pb and 95% for Ag. Taking recoveries after treatment in the mill into account, the paid metal content is: 4.5% Zn, 2.93% Pb and 98.8 g Ag/t.

Step 2: We want to plot a set of curves for a given Zn-price in a Pb and Ag price diagram. For the first breakeven curve we assume a Zn price of 45 US ¢/lb. The conversion factor lb into % is again 22.046 (see Sect. 1.1.4). Hence the Zn revenue is

$$4.5 \times 22.046 \times 0.45 = 44.64 \text{ US\$/t}$$

This amount is subtracted from the breakeven costs of 70 US $/t so that 70 – 44.64 = 25.36 US $/t must be covered by Pb and Ag.

Step 3: We assume a Pb price of 30 US ¢/lb. The Pb revenue is

$$2.93 \times 22.046 \times 0.30 = 19.38 \text{ US\$/t}$$

This leaves for Ag

$$25.36 - 19.38 = 5.98 \text{ US\$/t}$$

Since 1 ounce contains 31.103 g, 98.8 g Ag/t equal 3.18 oz/t. The breakeven price is

$$5.98/3.18 = 1.88 \text{ US\$/oz}$$

Step 4: We assume a second Pb price of, e.g., 15 US¢/lb. According to Step 2, the revenue for Pb is only 9.69 US$/t. This leaves 25.36 – 9.69 = 15.67 US$/t to be obtained from Ag. The breakeven price for Ag is

$$15.67/3.18 = 4.92\ \text{US\$/oz}$$

Now we can plot the first straight line in our diagram (Fig. 11.10). Since the breakeven curves are straight lines, two points suffice to define the line. To be on the safe side, an additional third intersection should be calculated.

Step 5: We assume different Zn prices, e.g. 40 and 35 US¢/lb respectively, and repeat the calculations of steps 2 to 4, and plot the results as shown in Fig. 11.10.

The results of the graph in Fig. 11.10 can also be illustrated in a "conversion matrix". A question often asked is for example: by how much must the price for Pb or Ag rise to compensate a drop in the Zn price of 10 US¢/lb?

In our example 10 US¢/lb for Zn is

$$4.5 \times 22.046 \times 0.10 = 9.92\ \text{US\$/t}$$

Since the paid Pb content is 2.93%, this equals a price variation of 15.4 US¢/lb for Pb or of 3.12 US$/oz for Ag (3.18 oz paid). A "conversion matrix" would appear as in Table 11.12.

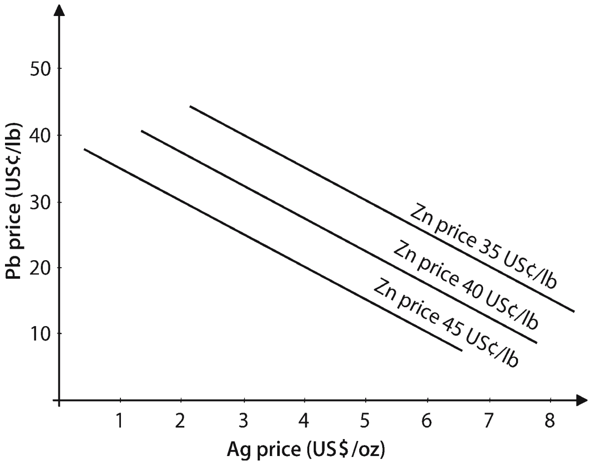


Fig. 11.10.
Set of breakeven curves

Table 11.12.
Conversion matrix

	–Δ (US-cts/lb)	Zn (US-cts/lb)	Pb (US-cts/lb)	Ag (US $/oz)
Zn (US-cts/lb)	–10	+10	+15.4	+3.12
Pb (US-cts/lb)	–10	+6.5	+10	+2.03
Ag (US $/oz)	–1	+3.2	+4.9	+1

11.9
The Expected Monetary Value (EMV) Method

In various chapters we dealt with political risk when considering a mining investment in a specific country. For example in Sect. 11.1.3 it was pointed out that in high risk countries shorter payback periods are required than in stable countries. In Sect. 11.2.3.5 it was discussed how political risks influence the interest rate for the computation of the net present value NPV. There are other risks that can be considered in an economic calculation, for example the exploration risk. The problem is the lack of reliable statistical probability factors in the exploration for ore deposits. In the hydrocarbon or the geothermal industry with far better possibilities to quantify exploration success, the method of the expected monetary value (EMV) is regularly applied for decision-making (e.g. Harbaugh et al. 1977 or Caldwell and Johnston 1985). Wellmer (1998, Stat. Eval., Chap. 21) discusses this concept in more detail. However, in special cases it might be possible to do a rough estimate, making purely intuitive decision-making a bit more rational.

The EMV method essentially compares the monetary reward weighted by the probability of success with the expenditure of risk capital weighted by the chance of a failure. The final total must be positive in order to justify a positive decision for the expenditure of risk capital (for example, the drilling of a hole). If p is the probability of success, then the probability of failure is

$$q = 1 - p$$

If the monetary success, or risk reward, is B, and the risk capital that is invested is Ri, then the EMV is

$$\mathrm{EMV} = B \times p - \mathrm{Ri} \times (1 - p)$$

The final reward, B, is the monetary success in terms of the cash value at the time of the drilling decision, in other words the net present value, NPV (see Sect. 11.2.3). With two examples, one from the hydrocarbon industry, and one with a rule-of-thumb computation from the metal deposit exploration industry, the application of the EMV method will be demonstrated.

Example 1. A basin analysis suggests that, with a probability of 50%, an offshore oil field could contain sufficient oil to generate an NPV of US $200 million when brought into production.

The probability of success is assumed to be 30%, which is about the average probability of success for wild cat drilling.

The offshore drilling is expected to cost US $10 million. Is it justified to drill this hole?
Answer. There are actually two probabilities in this problem that determine the chances of success:

a the 50% probability, p_1, that oil worth an NPV of US $200 million could actually be proven; and b the about 30% worldwide probability of success for wild cat drilling[25], p_2 (Sandrea and Sandrea 2007).

Both probabilities are independent of each other, and both probabilities must be multiplied together in order to derive the overall probability.

c Since the sum of the risk of failure and the probability of success has to be 1 as shown above ($q + p = 1$), the risk of failure in this case has to be

$$q = 1 - p_1 \times p_2$$

In consequence, the equation of above for the EMV becomes now

$$\text{EMV} = B \times p_1 \times p_2 - \text{Ri} \times (1 - p_1 \times p_2)$$

$$\text{EMV} = 200 \times 0.5 \times 0.3 - 10 \times 0.85$$

$$\text{EMV} = 30 - 8.5 = \text{US}\,\$21.5 \text{ million}$$

The EMV is therefore positive, and it is justified to drill the hole.

Example 2. Based on a compilation of North American exploration successes from many sources Sames and Wellmer (1981) calculated average probabilities of success to find an economic deposit. For virgin discoveries of a mineralization the success-to-failure ratio was found to be 1 : 16, for discoveries with a certain tonnage the probability was found to be 1 : 3. We assume we reached the stage of exploration on a volcanogenic massive sulphide occurrence so far that we can see that our discovered mineralization is not merely a local occurrence but an economically significant tonnage can be envisaged.

We can further quantify the probabilities of success using ore deposit model studies which give probability distributions of tonnage and grade. We take the data from Cox and Singer (1986) which are presented as cumulative frequency curves for tonnages and grades. (For an explanation of cumulative frequency curves see Wellmer (1998), Stat. Eval., p. 31ff.) We take as a model the example of a potential ore body which could sustain the 300 000 t/a operation for 8 years of our cash flow example in Sect. 11.5, Table 11.8, i.e. a potential ore body of 2.7 million t inclusive 10% mining losses (see Sect. 4.2).

For volcanogenic deposits we conclude from the relevant frequency distribution of Cox and Singer (1986) that 35% have at least the required size of 2.7 million t. So p_1 would be 0.35. We calculate that we need an exploration budget of US $4.5 million to outline proven and probable reserves (see Sect. 8.4) and to bring the deposit to the stage of a bankable feasibility study.

The probability p_2 in the above equation equals 1, because we have already discovered a significant mineralization.

[25] A wild cat well is a well drilled on a geological feature not yet proven to be productive.

We can now use the above formula for the EMV of Example 1 to calculate what is the necessary net present value NPV and therefore revenue which has to be generated by the potential deposit, i.e. what is *B* in our equation in Example 1. For a borderline case the EMV is 0:

$$\text{EMV} = B \times p_1 \times p_2 - \text{Ri} \times (1 - p_1 \times p_2) = 0$$

Then

$$B \times p_1 \times p_2 = \text{Ri} \times (1 - p_1 \times p_2)$$

Substituting the numbers of our example, we get

$$B \times 0.35 \times 1.0 = 4.5 \times 0.65$$

$$B = \frac{2.925}{0.35} = \text{US\$8.4 million}$$

So now we need to decide, from a geological point of view, how good the chances are of finding grades to generate a NPV of greater than US $8.4 million.

This exercise can be repeated for other potential tonnages. Let us assume that there is also a chance of outlining an ore body of 10 million t. The exploration expenditures would, of course, be higher. We estimate US $7 million. The accumulated frequency curve of Cox and Singer (1986) gives a probability of 0.2 that a discovery has at least this size. Repeating the exercise from above we now get for *B*, i.e. the required minimum NPV

$$B \times 0.2 \times 1.0 = 7 \times 0.65$$

$$B = \frac{4.55}{0.2} = 22.75 \quad \text{or, in round numbers, US\$23 million}$$

So we have practically quadrupled the tonnage but the minimum NPV is only increased by a factor of 2.7. This means the minimum grades can be lower, again an example of the economies of scale (see Sect. 9.2.2).

11.10 The Option Pricing Method

The option pricing method would normally go beyond the scope of evaluation tools a geologist applies in an early exploration stage. However, also an exploration geologist should be aware what tools are available to optimize the economics of a project.

Two important aspects of investment have not yet been addressed by any of the evaluation methods described:

1. being able to respond to fluctuating world market prices by postponing investments
2. being able to exercise operational flexibility by temporarily halting operations

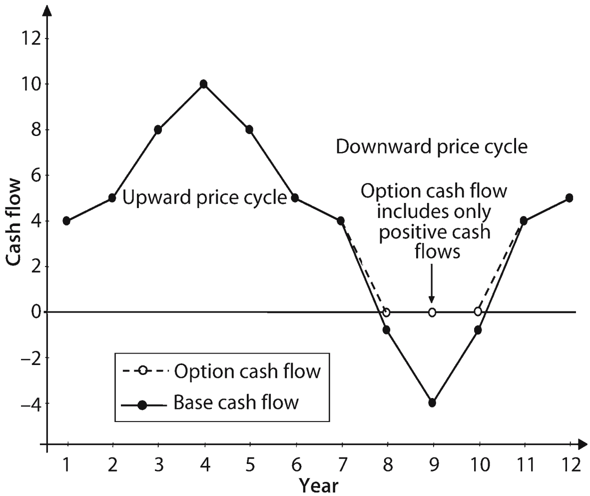
Ignoring these options in the analysis may substantially undervalue mining assets. The concept of using option pricing methods to evaluate mineral investment opportunities attempts to take into account the cyclical nature of mineral prices. The forecast of prices, which we have done so far, is replaced by forecasting the variability. Option pricing also addresses the question, what value to assign to a project which is not economic at the moment.

A financial option is the right – not the obligation – to buy or sell a commodity at a certain price, (the exercise price or the strike price) at some time in the future. Since this right has a value, options have a price. Therefore, purchasing an option for the promise of future delivery of a commodity is comparable to purchasing a mineral project with the intention to sell the mine production at some time in the future. Therefore the valuation of financial options can be transferred to mineral property evaluation. The higher the expected volatility of the commodity price, the greater is the value of the option. Hence, option pricing is based on the awareness of the expected variability of prices in the future. The basic method was developed by Black and Scholes (1973). Myron Scholes received the 1997 Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel for their work; Fischer Black died in 1995. The adaptation to natural resource investments was presented by Brennan and Schwartz (1985).

The diagram shows a typical cyclical cash flow (base cash flow) which is negative for a certain period, in this case during the years 8, 9, and 10. The option cash flow reflects the modification in case the operator exercises management flexibility and chooses to put the operation on care and maintenance when running into a negative cash flow.

Suppose there is the opportunity to purchase 1 oz of gold for US $400 from a third party in exactly one year; hereafter the option expires. This contract is known as an European call option on an ounce of gold with an exercise or strike price of US $400. American options that may be exercised at any time before the expiry date are more difficult to value, and a choice of models is available (e.g. binomial options model, Monte

Fig. 11.11.
Cyclical cash flow pattern depicting conventional evaluation and option cash flow



Carlo Model) which will not be dealt with further. We will only deal with the simpler European model. As – in this chosen scenario set-up (July 2003) – gold has recently been trading at US $350, the option is currently "out of the money". However, since the price of gold may rise above US $400 by the opportunity to transact, the option has a value. It is the present value of the expected payoffs from possibly exercising the option.

For example, you may believe that there is a 5% chance that gold will be priced at US $450 in 1 year's time. In which case we would profit US $50 by exercising the option (call option) and selling the commodity on the spot market. Conversely, you expect a 95% chance that gold will remain below US $400; in this case you will not benefit from your call option. The option is worth the present value of 5% of US $50, or about US $2.38:

$$50 \times 0.05 \times 0.952 = 2.38$$

whereby 0.952 is the net present value factor for $i = 0.05$ and 1 year, which is identical to the discounting factor for year 1 (see Appendix D, Table D18).

Taking into account all the probabilities of the gold price rising to specific prices above the strike price within the next year and considering the derivable payoffs will result in the present value or the price of the option. Sophisticated option pricing techniques incorporate all these probabilities and variables. The simplest and most widely applied option pricing formula to value European call options is called the Black-Scholes model which we just repeat here and do not discuss further. Accordingly, the present value of the call, C, is

$$C = S\mathrm{e}^{-\delta T} N(d_1) - X\mathrm{e}^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - \delta - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

and

$$d_2 = d_1 - \sigma\sqrt{T}$$

where

S = the current commodity price (US $/oz)
X = the exercise price (US $/oz)
T = the remaining time of the contract (years)
σ = the standard deviation of the change in the gold price over 1 year (% expressed as decimal)
r = the chosen risk-free interest rate (%/a expressed as decimal)
δ = the convenience yield on gold which is inversely related to inventory levels and reflects the cost of keeping stocks in warehouses (%/a expressed as a decimal)
$N(x)$ = the cumulative probability distribution function for a standardized normal variable, e.g. 5% in the example

Example. We take the case of above. The input values are $S = \text{US}\,\$350$ (e.g. monthly average July 2003), $X = \text{US}\,\$400$, $T = 1$ year, $\sigma = 0.14$ (by computing monthly averages for the year 2002), $r = 0.15$ (a standard interest rate in mineral economics), $\delta = 0.01$ (for simplification), and $N(x) = 0.05(x)$. Introducing these values into the Black-Scholes model renders a price of US $2.35 for this European call option:

$$C = 350\mathrm{e}^{-0.01\times 1}0.05(d_1) - 400\mathrm{e}^{-0.15\times 1}0.05(d_2) = -9.0752 + 11.4266 = 2.35148$$

where

$$d_1 = \frac{\ln\left(\frac{350}{400}\right) + \left(0.15 - 0.01 - \frac{0.14^2}{2}\right)\times 1}{0.14\sqrt{1}} = -0.5238$$

and

$$d_2 = -0.5238 - 0.14\sqrt{1} = -0.6638$$

11.10.1 Mine Production As an Option on Future Delivery

Mineral production can be interpreted as providing such an option, and hence mineral properties can be valued using the same technique as for valuing financial options. Consider a developed gold mine that has only 100 000 oz of reserves remaining. Daily production is 320 oz and the mine operates at 312 days a year. Hence, there is 1 calendar year of production left. Assume that the mine receives payment for its output and must pay its bills simultaneously at the end of the year. The mine uses contractors for mining and milling. So operating costs are locked through these contracts to 400 US $/oz, but the gold price – currently at 350 US $/oz – fluctuates and is unhedged. Unhedged means that the future production is not already sold (foreward selling) to protect against gold price fluctuations.

Before proceeding to demonstrate the option pricing approach, consider the traditional NPV analysis first (compare Sect. 11.2.3): It begins with the assumption that production of the remaining reserves will take place immediately with no chance for delay. In the above example we assume that the net cash flow would equal US $ –5 000 000 and based on the risk-adjusted discount rate of 15%, the NPV at the beginning of the year is US $ –4 347 826:

$$-5\,000\,000 \times \frac{1}{(1+0.15)^1} = -4\,347\,826\ \text{US\$}$$

Based on this analysis, the mine has a negative NPV – that means a zero value – and should rather be shut down than producing at a loss. If sold on the market, the 100 000 developed oz of gold in the ground would, however, have a positive value.

What the DCF analysis misses is that the mine owners have the option to delay producing their reserve until prices improve. Note that in this chapter the exemplified costs of extracting an ounce of gold are the same as the exercise price in the call option described above. Of course, because the option is currently "out of the money", its owner would choose not to exercise the option today. In other words, if free to choose, the mine owners will not mine their reserve until the gold price rises.

Now that the mine valuation problem is established as an option pricing problem, the next step is to price one of the identical 100 000 ounces as an option. This is normally a difficult task and can seldom be solved by analytical formulas. In this example, the valuation problem is set up in analogy in pricing of the call option with the Black-Scholes model. We go back to the above example, in which the option price for 1 oz of gold was calculated at US $2.3514. Given that the mine contains remaining reserves of 100 000 oz, the total option value of the mine is 100 000 × US $2.35 = US $235 148. Such pricing techniques demonstrate that no matter how uneconomical current production is, a mineral property has some speculative value. This makes sense given that uneconomical properties do trade for positive values.

For profitably operating mines where the cost of extracting a unit of reserves is less than the mineral price, NPV valuation is satisfactory but will somewhat undervalue the mine. A rule-of-thumb as to the degree of undervaluation is to add about 8% to the NPV to take into account the various managerial decisions and options associated with production (Davis 1998).

11.10.2 Assessing Undeveloped Properties by Option Pricing

Option pricing is also applied for the valuation of undeveloped reserves. The South African gold mining industry developed a methodology for valuing its deep-level mineral rights for gold that are uneconomic given the low gold prices prevailing during the mid 1990s (Mining Journal 1996). The method does not apply the Black-Scholes model, but instead uses empirical values on the following approach to equate NPV of projects to equivalent option premiums:

- The selected properties must be under the company's effective control so that it can decide on the timing of the investment.
- The geological model must be well understood and the reserves delineated.
- Mine plans are drawn up at various cutoff grades in order to construct cash flow models.
- A target gold price is chosen that would allow the mine to operate economically.
- The company then looks for specific gold call options at prices above the target price.

The intrinsic value of the options would then be equivalent to the expected option value of the undeveloped property. The sensitive point with this method is the need to find suitable option listings that are out of the money on the capital markets. Another possibility is to apply the Black-Scholes model.

Assume that the property is explored and the reserves are outlined but remain to be developed. Yet, there is no obligation for immediate construction and development. The valuation exercise can be viewed as an option pricing problem in which the owners have the option to pay the development costs (the exercise price) for receiving developed reserves that they may or may not decide to extract immediately. For example, assume that the same 100 000 oz property discussed above is undeveloped and has a total construction cost of US $10 million. We assumed above a negative cash flow of US $–5 000 000 with a NPV of US $–4 347 826 yielding a total NPV of US $–14 347 826.

However, when viewing this property as an option on developed reserves, a different result is reached. Note that paying the development cost is the same as paying the exercise price of an option. As shown above the developed resources currently have an option value of currently some US $235 148 based on a spot market price of US $350 for an ounce of gold. Thus, the option to develop the reserves is currently "out of the money". However, unexpired options always have a positive value. In this case we assume the property owners have a 20-year European option to develop a mine. Under these conditions the option value of the undeveloped reserve may be estimated to be about US $20 745 or US $0.21 per ounce. This is calculated using $S = 235\,148$, $X = 10\,000\,000$, $T = 20$ years, $\sigma = 0.14$ (year 2002), $r = 0.15$, and $\delta = 0.01$ in the Black-Scholes model. In the current example, S is the current value of the property as calculated above (equivalent to the current market price), and X are the development costs which have to be recovered (equivalent to the exercise price).

$$C = 235\,148 e^{-0.01\times 20} 0.05(d_1) - 10\,000\,000 e^{-0.15\times 20} 0.05(d_2)$$

$$= -186\,541 + 207\,296 = 20\,745$$

where

$$d_1 = \frac{\ln\left(\dfrac{235\,148}{10\,000\,000}\right) + \left(\dfrac{0.15 - 0.01 - 0.14^2}{2}\right)\times 20}{0.14\sqrt{1}} = -2.1495$$

and

$$d_2 = -0.5238 - 0.14\sqrt{20} = -2.7756$$

Hence, again the NPV analysis undervalues undeveloped properties, and option pricing techniques should be used instead.

Option pricing techniques can also be important for marginal undeveloped properties where DCF is less than the development cost but higher than zero. Fortunately, to simplify the approach, NPV analysis can still serve as a basis. Empirical work shows that pricing marginal undeveloped mineral properties as options adds as a rule-of-thumb a value equivalent of 5% to the DCF to reach the option value of a property.

As stated at the beginning of Sect. 10.10 the option pricing method is a method to refine a cashflow calculation. For an normal project approach for a go/no-go decision in the exploration stage the standard NPV calculation as explained and applied to in the previous chapters is sufficient.

11.11 Dealing with Start-up Problems in Economic Evaluations

In Fig. 11.4a,b in Sect. 11.2.3.1 about the net present value and in Fig. 11.8 in Sect. 11.2.4.2 about the internal rate of return it was demonstrated how in these dynamic economic evaluation methods the first production years are decisive. If there are start-up problems and therefore a deficit of cash flows in the first years this can have a major impact on the economics of our project. This is illustrated in Fig. 11.12a with a cash flow model.

The model project has an internal rate of return of 25% and is operating 10 years. We assume 10 progressively deteriorating cases, numbered with Roman numbers I to X. From each case to the next, there is a decrease in cash flow of 5% which is then improved in each succeeding year by 5% relatively:

Case I: cash flow in year 1 only 95%, in year 2 return to the base case cash flow of 100%
Case II: cash flow in year 1 only 90%, in year 2 only 95%, in year 3 100% of base case
Case III: cash flow in year 1 only 85%, in year 2 only 90%, in year 3 only 95%, in year 4 finally the base case of 100%
Case IV: etc.

In the graph in Fig. 11.12a there are two cases considered:

Case a: the reduced cash flow is due to a reduced mining rate. So the lifetime of the mine is prolonged
Case b: the reduced cash flow is due to other factors, for example a reduced recovery in the mill, so the lifetime of the mine remains 10 years

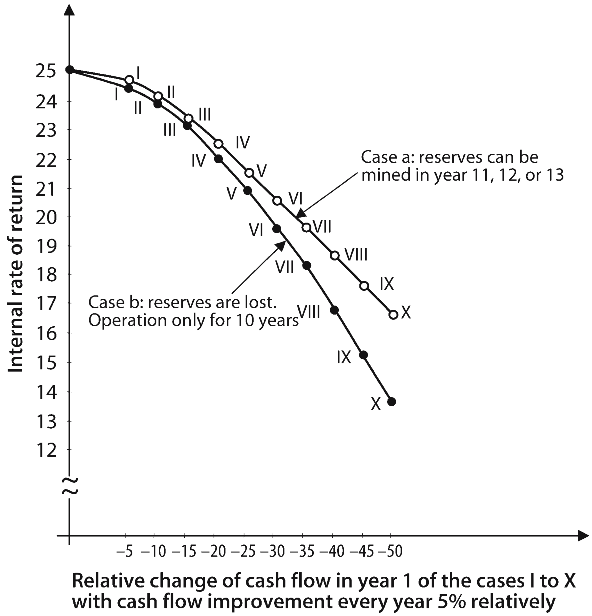


Fig. 11.12a. Changes of the internal rate of return for cash-flow models I to X

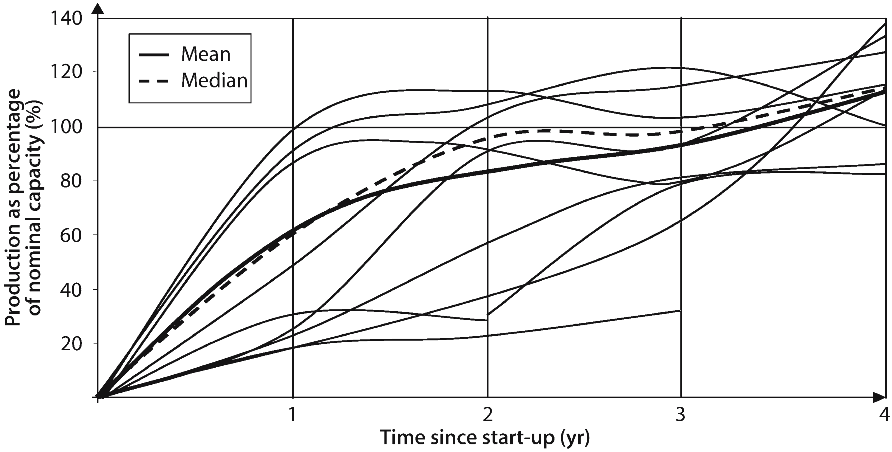


Fig. 11.12b. Production as percentage of nominal capacity of new base metal mines during the first four years after start-up (Source: Charles River Assoc. Inc. 1979, Agarval et al. 1984, modified by Wagner 1999)

In Fig. 11.12b real case histories are shown from a 1979 investigation by the Boston consulting group Charles River Associates Inc. who examined the start-up phases of 15 mine-mill and processing plants of copper, lead, zinc and nickel.

It is obvious how important it is to reach the assumed targets for our economic evaluation rapidly. The conclusion for our economic evaluations in the exploration stage is to work whenever possible with standard methods and standard figures, and not to be overly optimistic. This cautious approach has, for example, been adopted for the recovery values in Table 7.1, Sect. 7.2.2.

If, for example, a new technology is to be introduced, like a pressure leach method for a complex gold deposit or a lateritic nickel deposit, one should ask a milling engineer what learning curve, i.e. improvement per year, one can expect.

Chapter 12

Quantitative Valuation of Exploration Projects without Known Mineralization

12.1 Introduction

Exploration geologist will have to deal with the valuation of exploration projects without mineralization mainly in two cases:

a to decide whether an exploration programme should be undertaken outside of a known mining camp with known grades and tonnages which are being mined economically
b when properties without defined mineralization are offered for farm-in by prospectors or other exploration companies

In case a one will conduct an economic evaluation to define a minimum target according to the methods described in Chap. 8, 9, 10, and 11 and then decide if this target can realistically be expected. If, for example in a remote area far away from infrastructure, such a model study results in a minimum target of 20 million t with 20% combined Pb + Zn for sedimentary-exhalative (SEDEX) deposits and the potential grade of all known deposits in the same geological province does not exceed 10% combined Pb + Zn, this would be a no-go criterion. The decision should only be reversed if the exploration geologist can convincingly demonstrate with a new ore deposit model that higher grades can be expected in the selected area. For example, a deposit with additional Ag-values which can result in much higher Zn- or Pb-equivalent values (see Sect. 3.5.2).

The second case (b) will be dealt with in the following section.

12.2 Valuation of Properties without Known Exploitable Reserves

The value attributed to an exploration property without known mineable reserves is of course much influenced by subjective opinions and determined by the market situations (Goulevitch 1991; Thompson 2002). Much higher prices can, for example, be obtained in an exploration boom than during a lull in exploration activity. So we have periods of buyer's and seller's markets for exploration properties quite comparable to the market for concentrates described in Sect. 7.2.2. In consequence one has to analyse the market and carry out a comparable-transaction analysis (Lawrence 2002). Sources are stock exchange reports of exploration companies, mainly juniors, trading in such properties, or journals, like the Northern Miner in Canada.

Thus an exploration geologist has not only to compare transactions of properties without mineable reserves but also compare properties without mineable reserves itself using, for example, the geoscience factor method (Thompson 2002). Here a system of relative value scores for four main categories (location, inclusion of valuable mineralization, inclusion of geophysical and/or geochemical targets, and inclusion of geological targets) and 19 subcategories suggested by Kilburn (1990) might be helpful.

1 Location of property to be valued with respect to known off-property mineral occurrences (1.1–1.6) or geological, geophysical and/or geochemical targets (1.7–1.8).
   1.1 Interesting but sub-ore grade material that has been measured in two horizontal dimensions.
   1.2 Ore grade material that has been measured in two horizontal directions. Such mineralisation need not necessarily be economically exploitable. As explained in Chap. 8 and Sect. 9.2.2 costs decrease with increasing tonnage (economies of scale), so the required ore grade also decreases.
   1.3 Interesting but sub-ore grade material that has been measured in three dimensions.
   1.4 Ore grade material that has been measured in three dimensions (but not yet shown to be economically exploitable).
   1.5 A mine – past or present producer.
   1.6 A major mine – past or present producer.
   1.7 One such target (geological, geophysical and/or geochemical) exists or two such targets, that although based on different methods, correlate with one another.
   1.8 Three or more such targets that correlate with one another.
2 Grade of mineralization on the property to be valued.
   2.1 Interesting but sub-ore grade material that has been measured in two horizontal dimensions.
   2.2 Ore grade material that has been measured in two horizontal directions. Such mineralization need not necessarily be economically exploitable, see 1.2.
   2.3 Interesting but sub-ore grade material that has been measured in three dimensions.
   2.4 Ore grade material that has been measured in three dimensions (but not yet shown to be economically exploitable).
   2.5 A mine – past or present producer.
   2.6 A major mine – past or present producer.
3 Geophysical and or geochemical targets on the property to be valued. These targets are similar to those indicative of known exploitable mineral deposits.
   3.1 One such geophysical/geochemical target.
   3.2 2 or 3 such geophysical/geochemical targets which correlate with each other.
   3.3 4 or more such geophysical/geochemical targets which correlate with each other.
4 Geological patterns on the property to be valued. These are geological features like rock types, their size, shape and contacts, their alteration, their structural features etc. which frequently have been recognized to be closely associated with certain mineral deposits.
   4.1 One or two such patterns.
   4.2 Three or more such patterns.

These 19 subcategories must be assigned a relative importance with respect to each other. Kilburn (1990) recommends the following prioritization:

2.6 > 2.5 > 2.4 > 1.6 = 2.3 > 1.5 > 3.3 > 1.4 = 2.2 = 3.2 = 4.2 > 1.3 > 1.2 = 2.1 = 3.1 = 4.1 > 1.1 = 1.8 > 1.7

As a next step we have to assign weighting factors to each subcategory which of course have to reflect the priority assigned previously. For example the weighting factor for subcategory 2.6 must be larger than for 2.5 and the weighting factors for subcategories 1.6 and 2.3 must be equal. Kilburn (1990) proposes the following weighting factors (Table 12.1):

In this context, a rule for property transactions described by Kilburn (2005) for Canada is of interest: the $1/9^{th}$ rule. He discovered that the money value of expenditure commitments on the property entered into by the buyer was about nine times the cash payments for the property. He suggests a 50 : 50-balance between expenditure commitments and cash payments (including payments in shares) and, therefore, for a fair market value FMV the formula:

FMV = cash payments + 1 / 9 of expenditure commitments

After doing a comparable-transaction analysis as described above and quantifying the differences to the offered property with the weighting factors of Table 12.1 one can then structure the fair market value FMV according to this formula.

Table 12.1. Weighting factors proposed by Kilburn (1990)

Subcategory	Weighting factor
1.1	1.5
1.2	2.0
1.3	2.5
1.4	3.0
1.5	4.0
1.6	5.0
1.7	1.3
1.8	1.5
2.1	2.0
2.2	3.0
2.3	5.0
2.4	6 – 8
2.5	7 – 8
2.6	9 – 10
3.1	2.0
3.2	3.0
3.3	3.5
4.1	2.0
4.2	3.0

Example: Our comparable-transaction analysis shows that in a typical transaction a similar property was sold at CA $20 000 cash payment and a work commitment of CA $100 000. So the FMV base price would be

$$FMV = 20\,000 + 100\,000/9 \approx \text{CA}\,\$30\,000$$

The property contains an interesting geochemical target (subcategory SC 3.1) in favourable geological setting (subcategory SC 4.1).

So the property, according to Table 12.1, would be valued at

$$FMV_1 = \text{base price} \times \text{SC } 4.1 \times \text{SC } 3.1 = \text{base price} \times 2 \times 2 = \text{base price} \times 4$$

The property offered to us contains also an interesting geochemical anomaly in a similar geological setting, but in addition, it contains an interesting, but submarginal mineralization which is only known from surface, i.e. in two dimensions (SC 2.1). So, according to Table 12.1 this property would be valued at

$$FMV_2 = \text{base price} \times \text{SC } 2.1 \times \text{SC } 4.1 \times \text{SC } 3.1 = \text{base price} \times 2 \times 2 \times 2 = \text{base price} \times 8$$

So the ratio between FMV_1 and FMV_2 would be 2. Now we can "upscale" our FMV_1 = CA $30 000 from above to FMV_2 of CA $60 000.

Although the final price will always be determined by the market conditions prevailing at the time, as outlined above, this procedure can nevertheless assist in defining a quantitative value.

Comparison of Deposits

Both in preliminary evaluations and final feasibility studies the deposit under investigation is frequently compared to other deposits which are either in production or still in the preproduction stage. The best comparison is an economic one. This can be done via the payback period (Sect. 11.1.3), the earning power of a project expressed as the internal rate of return IRR (Sect. 11.2.4) or the standardized net present value NPV (Sect. 11.2.3.4).

In the preliminary stage, however, when only order of magnitude parameters are available, other "semi-economic" methods of comparison are also used.

13.1 Comparison of Deposits Via the Metal Content

If a deposit under investigation at an early stage of evaluation is to be compared with other deposits, usually of different tonnage and grade, a grade-tonnage diagram is often used. The grades are plotted on the x-axis, the tonnages on the y-axis (Fig. 13.1a,b). Either linear or logarithmic scales are used on both axes. Logarithmic scales are preferable since lines connecting points with the same metal content would be straight lines in a logarithmic diagram.

The grade in percent is x, the tonnage is y. Thus the total metal content M_t is

$$M_t = \frac{x \times y}{100} \quad \text{or}$$

$$y = \frac{M_t \times 100}{x} \tag{13.1}$$

This is the equation of a hyperbola. Thus, if the scales are linearly divided, the lines of equal metal content in a grade-tonnage diagram are hyperbolas.

If the grade-tonnage diagram has logarithmic axes, then Eq. 13.1 has to be transformed into logarithms:

$$\ln y = \ln (M_t - 100) - \ln x$$

This is the equation of a straight line in a logarithmic diagram.

The lines of total metal content help to illustrate the relative position of the deposit under investigation.

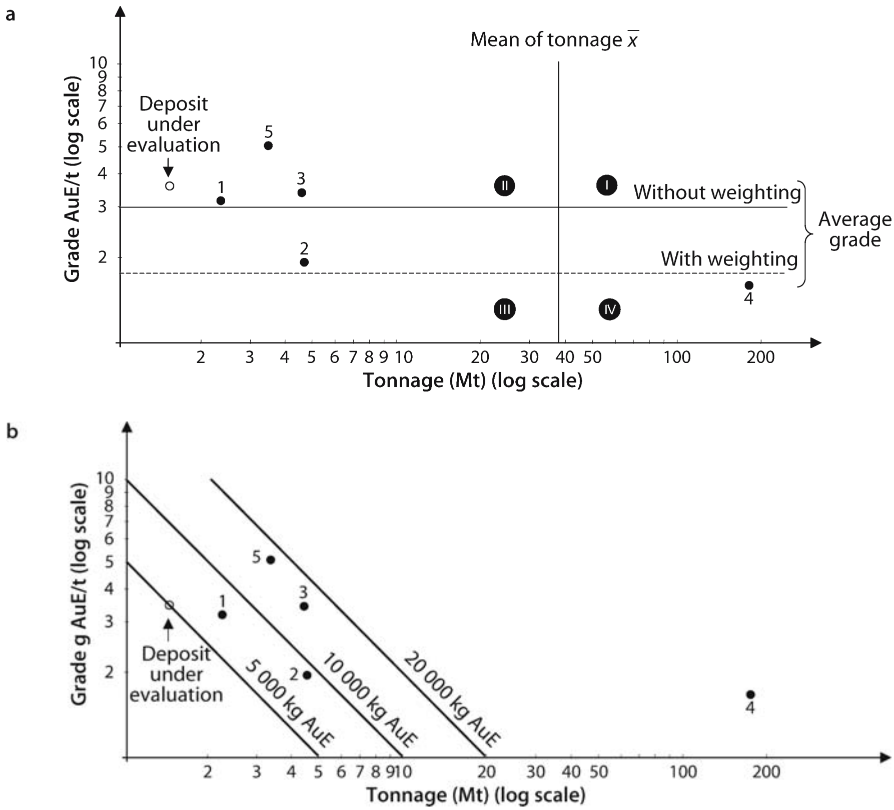


Fig. 13.1. a Grade-tonnage diagram with data of Table 13.1. **b** Grade-tonnage diagram of a with lines of average grades and tonnages

Example. An epithermal gold deposit with potential resource potential of 1.5 million tonnes containing 3.5 g Au/t is to be compared with other epithermal gold deposits in Nevada (Table 13.1).

The silver grades are converted into gold equivalents by the following equation (see Sect. 3.5.2):

$$\text{Au-equivalent AuE} = \frac{\text{g Au}}{\text{t}} + \frac{\frac{\text{g Au}}{\text{t}}}{50}$$

These values are plotted into a grade-tonnage diagram with logarithmic axes (Fig. 13.1a). (Normally, a larger geographic area or larger region and more deposits are considered for a meaningful comparison. This case only illustrates the principle involved.)

We now plot the lines of absolute metal content. If we start at the point of 5 g AuE/t on the y-axis, it corresponds to a tonnage of 1 million tonnes. Therefore, the metal content is

$$1 \times 10^6 \times 5 \text{ g AuE} = 5\,000 \text{ kg AuE}$$

Table 13.1.
Epithermal gold deposits in Nevada

No.	Deposit name	Tonnage (Mio t)	Gold equivalent grade (g Au/t)
1	Borealis	2.3	3.2
2	Buckhorn	4.6	1.9
3	Hasbrouck	4.5	3.4
4	Round Mountain	175.0	1.6
5	Sleeper	3.4	5.0

The same metal content is obtained at the point of 5 million tonnes of the x-axis which correlates to the grade of 1 g AuE/t. The metal content is

$$5 \times 10^6 \times 1 \text{ g AuE} = 5\,000 \text{ kg AuE}$$

If we connect both points, we obtain a line on which all deposits falling on this line have a metal content of 5 000 kg AuE, i.e. 5 000 kg gold equivalent. The lines for 10 000 kg AuE and 20 000 kg AuE metal content in Fig. 13.1b are similarly constructed.

It has to be pointed out that this is a comparison which does not say much about the relative economic merits of the mines. As discussed in Sect. 9.1.1, it makes a big economic difference whether, for example, the same metal content of 50 000 t Cu is mined in 11.1 million t of ore or, with the grade higher, in 3.7 million t of ore.

Another way of organising a grade-tonnage diagram is to use the mean of tonnage and grade. The mean is the arithmetic average $\overline{x}$:

$$\overline{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n} = \frac{1}{n} \times \sum_{i=1}^{n} x_i$$

For the tonnage the mean ($\overline{x}$) is

$$\overline{x} = \frac{2.3 + 4.6 + 4.5 + 175 + 3.4}{5} = \frac{189.8}{5} = 38.0$$

a value which is of course heavily biased by the outlier value of x_4.

Concerning the grade, there are two alternatives:

1. An average grade or mean value without weighting. In this case the mean would be

$$\overline{y} = \frac{y_1 + y_2 + y_3 + y_4 + y_5}{5}$$

$$\overline{y} = \frac{3.2 + 1.9 + 3.4 + 1.6 + 5.0}{5} = \frac{15.1}{5} = 3.0$$

2. An average grade weighted with the tonnage x_i. In analogy to the Eq. 2.1 in Sect. 2.2.1, the average grade y_w would be

$$y_w = \frac{x_1 y_1 + x_2 y_2 + x_3 y_3 + x_4 y_4 + x_5 y_5}{x_1 + x_2 + x_3 + x_4 + x_5}$$

$$y_w = \frac{3.2 \times 2.3 + 1.9 \times 4.6 + 3.4 \times 4.5 + 1.6 \times 175 + 5.0 \times 3.4}{2.3 + 4.6 + 4.5 + 175 + 3.4}$$

$$y_w = \frac{328.40}{189.8} = 1.73 \left(\frac{\text{g AuE}}{\text{t}} \right)$$

Again this value is very much influenced by the tonnage x_4.

As shown in Fig. 13.1a, the grade-tonnage diagram can now be subdivided into four sectors:

Sector I: grade and tonnage higher than average
Sector II: grade higher, tonnage lower than average
Sector III: grade and tonnage lower than average
Sector IV: grade lower, tonnage higher than average

A deposit lying in Sector I has, of course, a higher chance of being economically viable than a deposit lying in Sector III. Of course, the plots of Fig. 13.1a,b can be combined.

Sometimes the opinion is voiced that large deposits have lower grades than small deposits. If all known deposits of one type are plotted, it becomes obvious, however, that there is no correlation at all between grade and tonnage in a grade-tonnage diagram (see e.g. Singer and DeYoung 1980).

13.2 The Borderline of Viability

If mines which are operating and projects with known viability or non-viability are plotted into a grade-tonnage diagram, a borderline of viability can be constructed. Of course, such a borderline does not take into account special circumstances like special infrastructure situations or price peaks that benefited or helped a specific project along. It can be said, however, the further away a deposit under investigation is from the borderline of viability, the higher is the chance of it being an economic deposit (Fig. 13.2).

Example. This example of a comparative nickel deposit study from 1986 is given because it clearly shows which deposits one should consider and which not. In Fig. 13.2 operating nickel mines are plotted. The cobalt grades are converted into Ni-equivalent grades (see Sect. 3.5.2). One realizes that point 1 (Riddle, Oregon) lies well outside the main field and has to be considered a special case. Keeping this in mind, one can now construct a hyperbola-like curve as the borderline of viability.

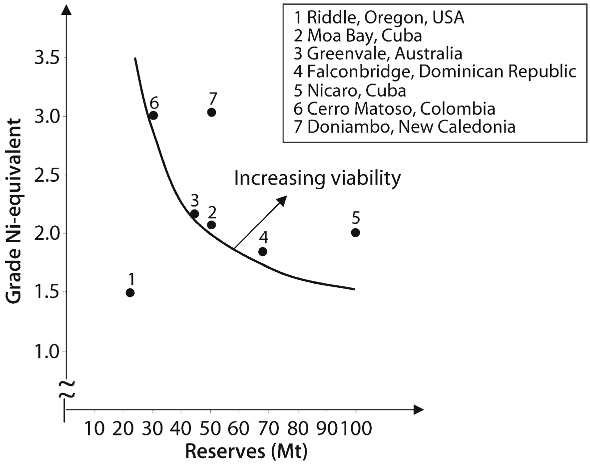


Fig. 13.2. Grade-tonnage diagram for nickel laterite mines (modified after Derkmann and Jung 1986)

13.3 The Breakeven Curve in a Grade-Capacity Diagram

A grade-capacity diagram with a breakeven curve of operating mines is a diagram comparable to the one discussed in Sect. 13.2. But instead of the tonnage, the capacity of a mine is plotted on the x-axis.

Sometimes breakeven costs (for breakeven, see Sect. 11.8) are published in annual company reports, e.g. in Australia or Canada. If companies have only one operating mine, the breakeven costs can be calculated from the profit and loss statements in the balance sheet of a company's annual report.

Example. A Pb-Zn mine mines 7 500 t/day, i.e. 2.7 million t/a. From the consolidated profit and loss statement in the annual report of the company the following figures in US $ are taken (Table 13.2a).

From Table 13.2a we can now calculate the breakeven cost.

Step 1: According to Sect. 11.8, the definition of breakeven implies that all operating and capital costs and taxes are covered. Since the debt to equity ratio (see Sect. 11.4) varies from project to project, we will calculate a breakeven for equity financing, that is, we are not going to consider interest and debt expenditures.

Capital costs are covered by depreciation and amortisation. Therefore only younger operations which are not working with equipment already written off should be taken into consideration.

We learn from the annual report that the company also conducted exploration outside the mining lease to which the exploration expenditures are related. The exploration expenditures in the mine are already included in the operating expenditures. For our breakeven calculation we therefore do not include the exploration expenditures of Table 13.2a.

Foreign exchange losses are dependent upon the marketing arrangements. We consider these as special costs not to be taken into account.

Table 13.2a.
Values taken from the consolidated profit and loss statement

Consolidated profit and loss statement	US $
Revenues	118 724 000
Operating expenditures	56 659 000
Depreciation and amortisation	11 823 000
Interest and debt expenditures	10 204 000
Exploration expenditures	751 000
Foreign exchange losses	1 081 000
Income tax and royalties	21 526 000
Net income	16 680 000

Table 13.2b.
Values for breakeven calculation

Cost item	US $
Operating expenditures	56 659 000
Depreciation and amortisation	11 823 000
Income tax and royalties	21 526 000
Total costs to be considered	90 008 000

For the breakeven calculation we therefore consider the cost items of Table 13.2b. Dividing the amount of US $90 008 000 by the annual production of 2.7 million tonnes, we obtain the breakeven cost/t:

$$\frac{90\,008\,000}{2\,700\,000} = 33.34\left(\frac{\text{US\$}}{\text{t}}\right)$$

Step2: We want to plot this into a grade-capacity diagram. We therefore have to convert the breakeven cost of Step 1 into a breakeven metal grade. For this purpose we have to assume a metal price. We want to convert everything into Zn-grades, i.e. Zn-equivalent grades, which means that Pb-grades have to be converted into Zn-equivalents (see Sect. 3.5.2). We take a Zn price of 45 US ¢/lb Zn. For calculating the net smelter return of the mine (see Sect. 7.2) we calculate with the rule-of-thumb figures of Table 7.1. For Zn we take the percentage net smelter return for the mine NF = 50% from Table 7.1. The recovery in the mill is assumed as 90%, i.e. $\varepsilon = 0.9$. The conversion factor from percent to lb is = 22.046 (see Sect. 1.1.4).

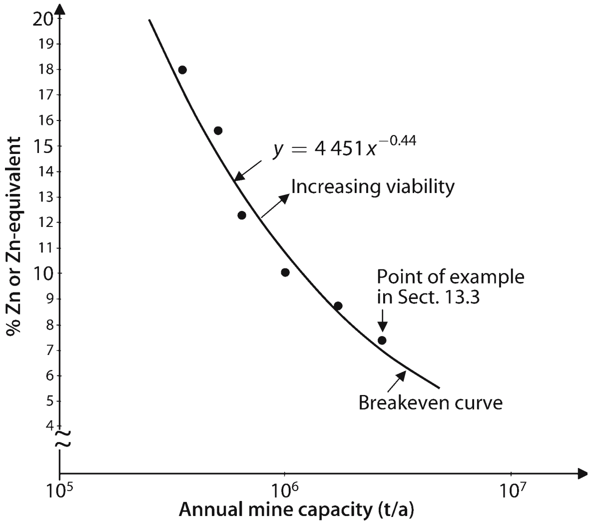
Therefore the breakeven Zn grade g_b in percent is

$$g_b \times 22.046 \times 0.5 \times 0.9 \times 0.45 = 33.34 \text{ US\$/t}$$

$$g_b = 7.5\% \text{ Zn}$$

This value can now be plotted into a grade-capacity diagram (Fig. 13.3). After having derived other points for the diagram in the same manner, a power curve can be calculated as an optimal fit through the points according to the procedure in Sect. 9.2.2 (Fig. 13.3).

Fig. 13.3.
Grade-capacity diagram with breakeven curve



Step 3: If, for example, one wants to compare a deposit with a potential of 10 million tonnes with 13% Zn, the potential has to be converted into the production capacity of a possible mine. We choose the formula for the optimal lifetime (in years) from Taylor (1977), the Taylor-rule (see Sect. 8.1.3.1, Eq. 8.1):

$$\text{lifetime } n \approx 0.2\sqrt[4]{\text{total tonnage expected}}$$

$$\text{lifetime } n \approx 0.2\sqrt[4]{10\,000\,000} = 0.2 \times 56.23$$

$$\text{lifetime } n \approx 11.25$$

A lifetime of 11.25 years for a deposit of 10 million tonnes means an annual production rate of 890 000 t, i.e. 900 000 t.

We now plot this value of 900 000 t/a and a grade of 13% Zn in the diagram of Fig. 13.3. The same rule as in the diagram of Fig. 13.2 applies: the further away a deposit under investigation plots from the breakeven curve, the better its chance of being viable.

13.4 Grade-Capacity Diagram with Lines of Equal Economic Parameters

Grade-capacity diagrams with lines of equal economic parameters are comparable to the diagram with a breakeven curve as described in Sect. 13.3.

In this case power curves for operating and investment costs are calculated from researched data. As shown in Sect. 9.2.2, power curves have the equation

$$y = a \times x^b$$

where y represents the operating or investment costs, x is the capacity and a and b are constants.

After calculating such curves, one can construct the lines of equal economics by calculating data points for various capacities.

Example. Going back to 1983, data from Australian underground gold mine power curves were used to obtain operating and investment costs by interpolation. For a capacity of 50 000 t/a, pre-production costs of AU $11.9 million and operating costs of 96 AU $/t were interpolated.

For a gold price of 500 AU $/oz data points are to be calculated for an internal rate of return (IRR) of 10%, 15% and 20% respectively.

We assume 100% equity financing for which no interest and no taxes are paid, since gold was at that time tax-free in Australia[26]. We assume a working life of 8 years for the mine. Since we can calculate with equal annual cash flows, we use the method of calculating with annuity present value factors as described in Sect. 11.2.4.3.

The following calculations are performed:

Step 1: We calculate the amount of annual cash flow for the operating profit necessary to obtain an internal rate of return (IRR) of 10, 15 and 20%.

According to Eq. 11.1 in Sect. 11.2.4.3:

$$I = \mathrm{OP_c} \times b_n \tag{13.2}$$

whereby I is the investment, OP the operating profit (or cash flow) and b_n the annuity present value factor.

In our case I is AU $11.9 million. The factors b are looked up for 8 years in Appendix D, Table D19. They are shown in Table 13.3.

According to Eq. 13.1 above and Table 13.3, we obtain

$$\mathrm{OP_c} = \frac{I}{b_n}$$

$$\text{for IRR} = 10\%: \mathrm{Op_c} = \frac{11.9}{5.335} = \text{AU}\,\$2.23 \text{ million}$$

$$\text{for IRR} = 15\%: \mathrm{Op_c} = \frac{11.9}{4.487} = \text{AU}\,\$2.65 \text{ million}$$

$$\text{for IRR} = 20\%: \mathrm{Op_c} = \frac{11.9}{3.837} = \text{AU}\,\$3.10 \text{ million}$$

Step 2: Total revenues (Rev) needed per year are the total operating costs plus the operating profits calculated in Step 1.

Total operating costs are $96 \times 50\,000 = 4.8$ million AU $/a.

[26] Again a tax-free case is taken to explain the principles more clearly.

Table 13.3.
IRR and b_n taken from Table D19 (Appendix D)

IRR (%)	b_n
10	5.335
15	4.487
20	3.837

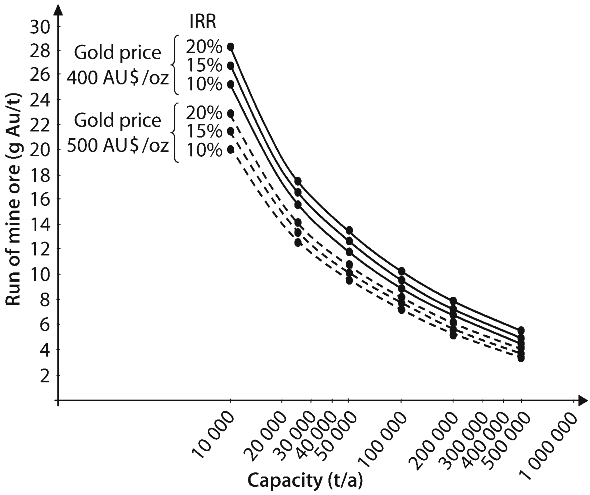


Fig. 13.4.
Grade-capacity diagram with lines of equal internal rate of return

Total revenues (Rev) required are therefore:

for IRR = 10%: Rev = 4.8 + 2.23 = 7.03 million AU $/a

for IRR = 15%: Rev = 4.8 + 2.65 = 7.45 million AU $/a

for IRR = 20%: Rev = 4.8 + 3.10 = 7.90 million AU $/a

Step 3: We now calculate the gold grade required. We assume a recovery in the mill of 90%. For the required gold grade G_g and a gold price of 500 AU $/oz we can establish the following relationship (31.103 is the conversion factor of ounces into grams, see Sect. 1.1.4):

$$\frac{G_g \times 0.9 \times 50\,000 \times 500}{31.103} = \text{Rev} \qquad (13.3)$$

with Eq. 13.2 and the revenue data of Step 2 we obtain

for IRR = 10%: G_g = 9.7 g Au/t

for IRR = 15%: G_g = 10.3 g Au/t

for IRR = 20%: G_g = 10.9 g Au/t

Step 4: In the same manner, data points are calculated for other capacities and plotted into a grade-capacity diagram as shown in Fig. 13.4. The same calculation can also be done for other gold price assumptions, as shown in Fig. 13.4.

To compare a deposit under investigation with other deposits, these deposits can now be plotted in the same diagram which will provide a set of economic parameters as standards of comparison.

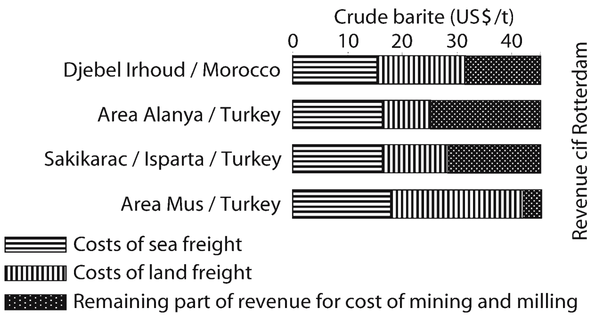
13.5 Comparison of Deposits with Cost Charts

Cost charts displaying breakeven costs for all mines producing a particular mineral or metal were dealt with in Sect. 6.5. Such charts are an ideal means for comparing deposits. As outlined in Sect. 6.5 and in Fig. 6.3a,b such cost charts vary with time. So, for a comparison of deposits, one has to take care to have up-to-date cost charts at hand. If we have to evaluate a copper deposit for example, and the result of a preliminary economic evaluation in the year 2002 is a breakeven price of 0.50 US $/lbs Cu (see Sect. 11.8), the conclusion can then be drawn from the 2000 cost chart in Fig. 6.3a that about 60% of all mines have a better economic position, while the other 40% would be worse off.

13.6 Comparison of Deposits with Auxiliary Criteria

Frequently there are auxiliary criteria available for comparing deposits. For raw materials that are sensitive to transport costs, the transport costs to the market of the saleable product are such a criterion (see also Sect. 9.4). For example, in Fig. 13.5 the transport costs for land and sea transport to Rotterdam, one of the ARA harbours (see Sect. 9.4.1), are shown for various barite deposits in the Mediterranean area. These are useful figures if one would want to bring these deposits into production and would try to receive the shown revenues cif Rotterdam. From the graph in Fig. 13.5, one can realize how much surplus remains for mining and milling costs and which deposits have to be considered uneconomic right from start, before even going further into detail, or see how much higher the grade would have to be to render the deposit economically viable. Concerning the remaining costs for mining and milling it was pointed out in Sect. 9.3.2.5 under *(a)* that there is an abrupt break in costs at 96% $BaSO_4$. Barite above 96% can be shipped and sold directly, below 96% it has to be beneficiated.

Fig. 13.5. Comparison of transport costs for barite deposits in the Mediterranean area



Calculating Growth Rates

A strategic question is often raised in connection with the evaluation of a deposit: What will the future demand for the metal (or metals) in the deposit under consideration be like? Does an analysis of the planned mining projects reveal shortfalls in satisfying demand?

First the historical development of consumption is analyzed and an average growth rate established. Data might again be taken from "Metal Statistics", up to 1993 published annually by Metallgesellschaft AG, Frankfurt a. M., Federal Republic of Germany, today produced by the World Bureau of Metal Statistics, Ware, Herts, England. Again the internet is an invaluable source of information for statistics of metal consumption.

14.1 Calculating Growth Rates Using the Geometrical Mean

As discussed in Sect. 9.2.1.2, the usual way to determine average growth rates is the geometrical mean:

$$W_g = \sqrt[n]{W_1 \times W_2 \times W_3 \times \cdots \times W_n}$$

Assignment. Calculate the average growth rate for zinc from 1994 to 2004.

When the above formula is applied to column IIb of Table 14.1, the result is

$$W_g = \sqrt[10]{1.079 \times 0.999 \times 1.016 \times \cdots \times 1.051} = 1.041$$

i.e. over a period of 10 years the average growth rate was 4.1%.

In this calculation the increment from 1994 (7.029 million t) to 2004 (10.415 million t) was quite simply evenly distributed. The intermediate years could just as well be disregarded and a simple formula set up:

$$\sqrt[10]{\frac{10.451}{7.029}} = \sqrt[10]{1.487} = 1.04 \quad \text{(the difference is due to rounding)}$$

The example shows that that the choice of the initial and final year in this kind of calculation is crucial. If the year 2000 had been used as the initial year and again 2004 as the final year then the average growth rate would have been

Table 14.1. Growth rate calculation for zinc

Year	I World consumption (× 1000 t) (y)	IIa Growth (%)	IIb Rates (j) (as $1 + j/100$)	III Years (x)	IV $\ln y = y_a$
1994	7029			0	8.858
1995	7584	7.9	1.079	1	8.934
1996	7536	–0.1	0.999	2	8.927
1997	7659	1.6	1.016	3	8.944
1998	7997	4.4	1.044	4	8.987
1999	8448	5.6	1.056	5	9.042
2000	9069	7.3	1.073	6	9.113
2001	9368	3.3	1.033	7	9.145
2002	9768	4.2	1.042	8	9.187
2003	9912	1.5	1.015	9	9.202
2004	10415	5.1	1.051	10	9.251

$$\sqrt[5]{\frac{10.415}{9.096}} = \sqrt[5]{1.145} = 1.027$$

i.e. a 2.7% growth rate, only about two thirds of the one calculated above.

The extrapolation of realistic growth rates is a very complex problem which will be discussed in the next chapter. For a more reliable calculating procedure a different method should be adopted which takes every year equally into account and not just the final. Such a method is described in Sect. 14.2.

14.2 Calculating Growth Rates with Logarithmic Values and Linear Regression

In order to take each year into account equally, a line is plotted through the points representing annual consumption figures by applying the method of linear regression.

Example. Again we use the example of world zinc consumption from 1994 to 2004 in Table 14.1 to plot a graph. A line of best fit defined by the equation

$$y = a \times (1 + j)^n$$

is to be drawn through the points on the graph.

This is the equation of a geometrical series, since growth rates always have an accumulative effect. With a constant growth rate over a particular period of time, consumptions y are the terms of a geometrical series, whereby j is the growth rate and n the year, starting from the base year 0. This equation can be expressed in its logarithmic form as

$$\ln y = \ln a + n \times \ln (1 + j)$$

If one compares this with the straight line equation:

$$y_n = a_n + b \times x$$

one can replace

- $\ln y$ by y_n
- $\ln a$ by a_n
- n by x
- $\ln(1+j)$ by b

and apply the formula of linear regression from Sect. 3.3.

To avoid high numbers for years that are cumbersome to work with, we assume the starting year as 0 (see above) and count from this year onwards (column III in Table 14.1). The individual consumption values of column I are transformed into logarithmic values (column IV).

The regression analysis results in

$$a_n = 8.856 \text{ (antilogarithm: 7 017)}$$

$$b = 0.0395 \text{ (antilogarithm: 1.040)}$$

Since b equals $\ln(1+j)$, j becomes 0.040, i.e. a growth rate of 4.0% which means that the difference to the geometrical mean is insignificant. Initial and final years to determine the geometrical mean were well chosen. The line of best fit has the equation

$$y = 7\,017 \times (1 + 0.04)^n$$

It must be pointed out that growth rates cannot be extrapolated indefinitely, since there is no unlimited growth (see story in Appendix E). In addition, growth rates are influenced by structural changes in the industry. Growth rates follow sigmoidal learning curves. A theoretical learning curve is shown in Fig. 14.1a: First the slope of the learning curve is flat, then it steepens and at the end it flattens again. Such learning curves are given for the world-wide steel and zinc consumption in the 20th century (Fig. 14.1b). It is obvious that the curves steepened after World War II and flattened after the first oil crisis 1973. It can be observed that with structural changes new learning curves can start. This is taking place now with the increasing demand of the newly industrialized countries, especially the so called BRIC-countries (Brazil, Russia, India, China). If one averages the growth rates over periods of 5 years it becomes obvious how, for example, the growth curve for steel in Fig. 14.1c goes into a new phase of higher increases.

New technologies and environmental considerations can also influence consumption patterns and thereby growth rates. Good examples are cadmium and fluorspar. 30 years ago Cd was desirable by-product of Zn-ore, providing additional values due to credits in the concentrates. It was used in NiCd-batteries. Nowadays, however, there are environmental concerns about the use of cadmium. Better nickel-based batteries

Fig. 14.1a.
Ideal type of a learning curve

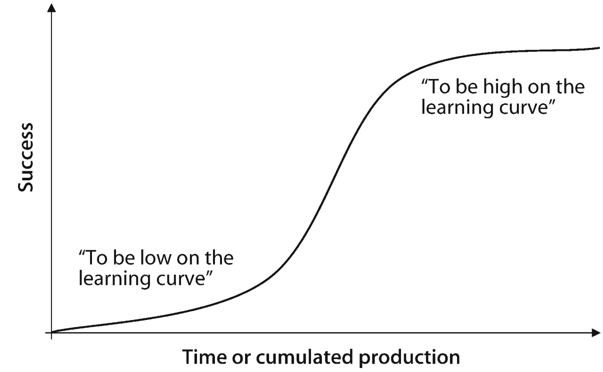


Fig. 14.1b.
Worldwide production of steel and consumption of zinc 1900 to 2000

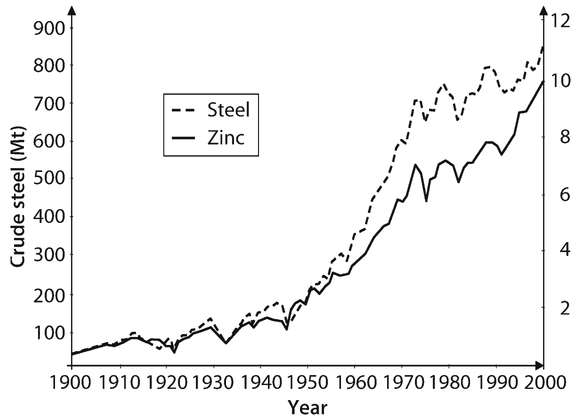
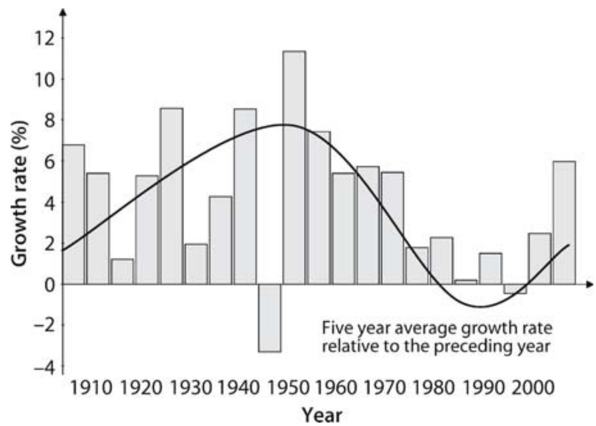


Fig. 14.1c.
Five years average growth rates of steel in percent from 1900 to 2005



– the Cd-free nickel meta hydride (NiMH) batteries – have been developed and cadmium has become a penalty element in Zn-concentrates. Fluorspar was considered a commodity with severe shortages in the seventies, then fluor was banned due to environmental concerns, now it is a sought-after commodity again.

14.3 Doubling Periods

Sometimes growth rates are described by the period of time it takes for consumption to double.

According to the formula in Sect. 14.2:

$$y = a(1 + j)^n$$

y then equals $2a$.

With growth rate j given, n is unknown. As a result:

$$n = \frac{\ln 2}{\ln(1 + j)}$$

Table 14.2 indicates the doubling periods for different growth rates.

Table 14.2. Doubling periods for different growth rates

Growth rate *j* (%)	Doubling period (yr)
1	70
2	35
3	23
4	18
5	14
6	12
7	10
8	9
9	8
10	7

Chapter 15

Equity Calculations

Equity calculations will be considered under two aspects:

- equity calculations with several partners
- calculations of possible foreign equity in countries where foreign investment is limited

15.1 Equity Calculations with Several Partners

Often exploration projects are run by syndicates in order to spread the risk. As a rule, each partner pays his share of exploration expenditure pro rata. If a partner decides to withdraw from the project while the others continue exploration, his equity is diluted. Generally, his equity is also calculated pro rata in the dilution phase.

Assignment. Three partners join in an exploration venture, each bearing a third of the cost. Expenses during the first phase are US $6 million. At the end of Phase 1, partner A decides to withdraw and accept a dilution of his equity. Expenses in Phase 2 are also US $6 million. What is the equity of partner A at the end of Phase 2?

At the end of the second phase US $12 million will have been spent of which partner A has paid US $2 million. Hence his equity is

$$\frac{2}{12} \times 100 = 16.7\%$$

Partners B and C have an equity of 41.7% each.

In the reverse case, the called farming-in concept, calculations are often more complex. Partner A has carried out an exploration project with promising results. Since exploration expenses increase progressively with each stage, he is looking for a partner in order to reduce his financial exposure. Since he financed the initial and riskier phase himself, he will ask for a premium and demand from partner B a disproportional share of the exploration expenses until the latter has caught up, i.e. earned his equity (catch-up point).

Assignment. Partner A has spent US $5 million on a potash project and indicated a promising find. The next step requires a systematic drilling programme on a grid. Partner A wants to reduce his financial burden and invites partner B to join the project. Partner B can purchase a minimum equity of 40% by paying a premium (common practice with promising projects) and by financing a higher share of further exploration costs during the farming-in phase. In this phase partner A only contributes 20% of the costs, while partner B pays 80%.

Case A: Calculate the catch-up point at which, related to the farming-in date, a premium of 200% has to be paid.
Case B: Determine the catch-up point at which a premium of 50% is paid for gaining a 40% equity.

Case A: Up to the farming-in date partner A has spent US $5 million. The 40% equity which partner B can gain equals $0.4 \times 5 =$ US $2 million.

Partner B must pay a premium of 200%, i.e. for his 40% equity he must pay a total of US $6 million. Since up to the catch-up point partner A only contributes 20% of the exploration costs, this date will be reached after further expenditures of

$$\frac{6}{0.8} = \text{US\$7.5 million}$$

Since US $5 million have already been spent and partner A paid 20% of exploration costs during the farming-in phase (i.e. the expense ratio of partner A to partner B is 1 : 4), total project expenses are: 6.0 + 5 + 1.5 = US $12.5 million, of which partner A paid 52% and partner B 48%. From the catch-up point onwards each participant pays according to his equity, i.e. at a ratio of 60 : 40.

Case B: Payments from partner A are called PA, payments from partner B PB; total project costs at the catch-up point are *T*.

Equation 15.1: The ratio of payments by partners A and B during the farming-in phase is

$$\frac{\text{PA}}{\text{PB}} = \frac{1}{4} \tag{15.1}$$

Equation 15.2: At the catch-up point the equation is

$$T = 5 + \text{PA} + \text{PB} = 5 + 0.25\ \text{PB} + \text{PB} \tag{15.2}$$

Equation 15.3: At the catch-up point B must have paid a 50% premium, i.e.

$$\text{PB} = 1.5 \times (0.4\ \text{T}) \text{ or } T = 1.667\ \text{PB} \tag{15.3}$$

Inserting Eq. 15.3 into Eq. 15.2 we obtain the following result:

$$1.667\ \text{PB} = 5 + 1.25\ \text{PB}$$

i.e. PB = US $12 million.

The catch-up point is reached after partner B has paid US $12 million. Total exploration costs amount to

$$12 + 5 + 3 = \text{US \$20 million}$$

An example of dilution and farming-in with restricted foreign equity participation is given at the end of Sect. 15.2.

15.2 Calculation of Foreign Equity in Exploration and Mining Projects

Limitations on foreign equity participation in mining projects are imposed by several mining countries. This is one of the crucial aspects in the evaluation of a potential deposit in a country where foreign equity is limited and where local partners able to finance a mining investment on a larger scale are often almost impossible to find. This problem is often encountered in developing countries where companies are forced to finance 100% of the investment, but receive only 49% of the profit. As a consequence, minimum grades of a deposit worth pursuing must be significantly higher.

Two fundamental cases need to be distinguished:

- In some countries, e.g. Canada, a mining company is considered local as long as foreign equity does not constitute the majority. Thus a company or a mine is either foreign or local, without any intermediate steps.
- In other countries, most significantly in Australia, foreign equity is graded and the involvement of subsidiaries and their daughter companies has a bearing on the calculation of foreign or local equity (Fig. 15.1). The following example is still relatively simple:

Company C is 50% controlled by local interests, 50% by foreigners. Company C controls 60% of company B. Hence the foreign equity in company B is: $0.5 \times 0.6 = 0.3$, i.e. 30%. Company B controls 60% of a Cu deposit; accordingly, the complete foreign equity is $0.3 \times 0.6 = 0.18$, i.e. 18%. With a 50% foreign equity restriction, foreigners could acquire only a maximum of an additional 32% equity in the Cu deposit.

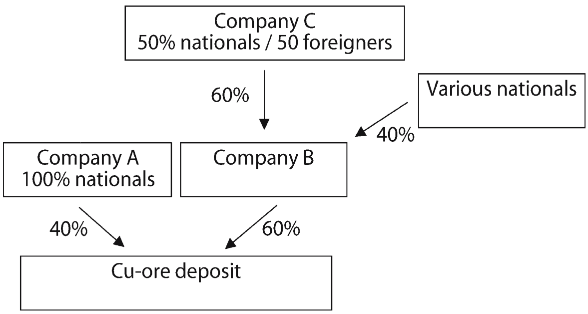


Fig. 15.1. Ownership in a Cu deposit

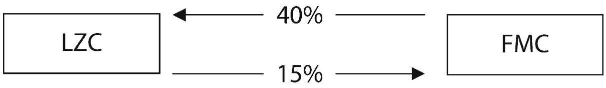


Fig. 15.2. Cross-holdings of Australian local and foreign company

The matter becomes complicated when companies are interconnected (Fig. 15.2), as the following example shows.

Example. A local zinc company LZC is 40% controlled by a foreign mining company FMC. LZC, in turn, has a 15% equity in FMC.

What is the foreign equity in LZC?

Step 1:

- Local equity in LZC will be called lz
- local equity in FMC lm
- foreign equity in LZC fz
- foreign equity in FMC fm

As a result, the following equations are valid:

$$1 = lz + fz \quad , \quad \text{and}$$

$$1 = lm + fm$$

Step 2: Local equity in FMC is

$$lm = 0.15lz$$

and foreign equity in LZC is

$$fz = 0.4fm$$

Step 3: Now we have four equations with four unknowns. By mutual substitution we obtain

$$fz = 0.4fm = 0.4(1 - lm)$$

$$fz = 0.4(1 - 0.15\,lz)$$

$$fz = 0.4(1 - 0.15[1 - fz]) = 0.4 - 0.06 + 0.06fz$$

$$0.94fz = 0.34$$

$$fz = 0.362$$

i.e. foreign equity in the Local Zinc Company is 36.2%. With a foreign equity ceiling of 49%, a 100% foreign owned company could still acquire 12.8%.

The following assignment again concerns a farming-in case.

Assignment. A company is 75% locally and 25% foreign controlled. There is a 50% foreign equity restriction. Another foreign partner is to join. The old partners are evenly diluted. What is the maximum equity the new partner can gain?

We will call the old local equity LA, the old foreign equity FA and the new foreign equity FN. We obtain

$$\text{LA} + \text{FA} + \text{FN} = 1$$

Local equity must not drop below 50%, i.e. LA = 0.5. Since the ratio

$$\frac{\text{LA}}{\text{FA}} = \frac{3}{1}$$

does not change with dillution, FA is

$$\text{FA} = \frac{\text{LA}}{3} = \frac{0.5}{3}$$

This results in

$$\text{FA} = 0.5 - \frac{0.5}{3} = 0.333$$

i.e. 33.3 is the maximum equity the new partner may acquire.

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Appendices

Appendix A
Diagrams for Conversion between Imperial and Metric Units

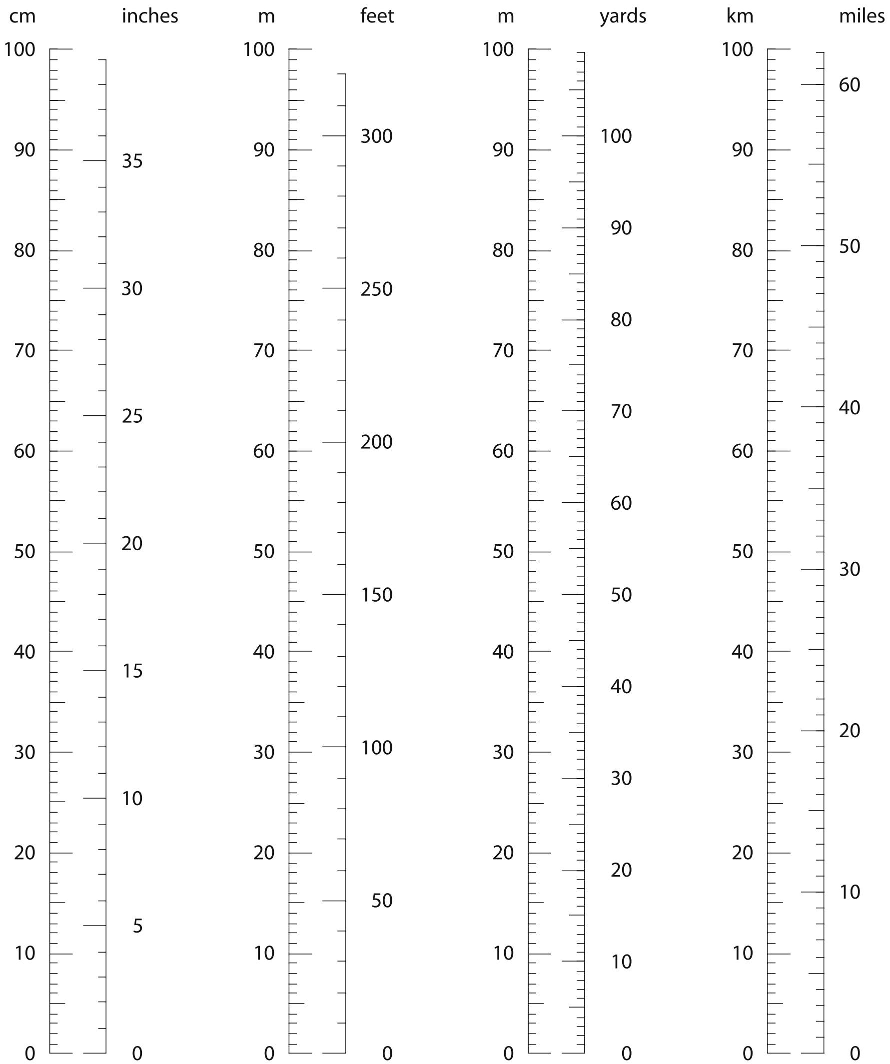


Fig. A1. Conversion between common length measures

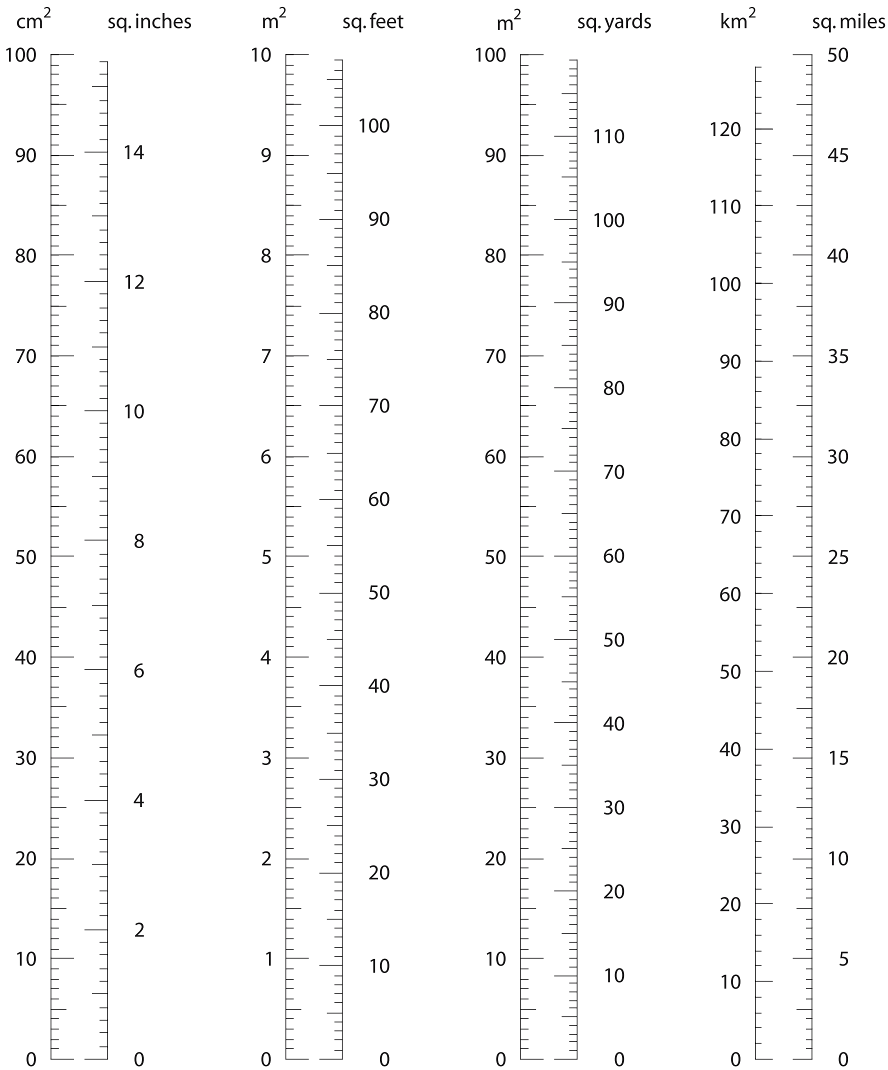


Fig. A2. Conversion between common square measures in the imperial and metric systems

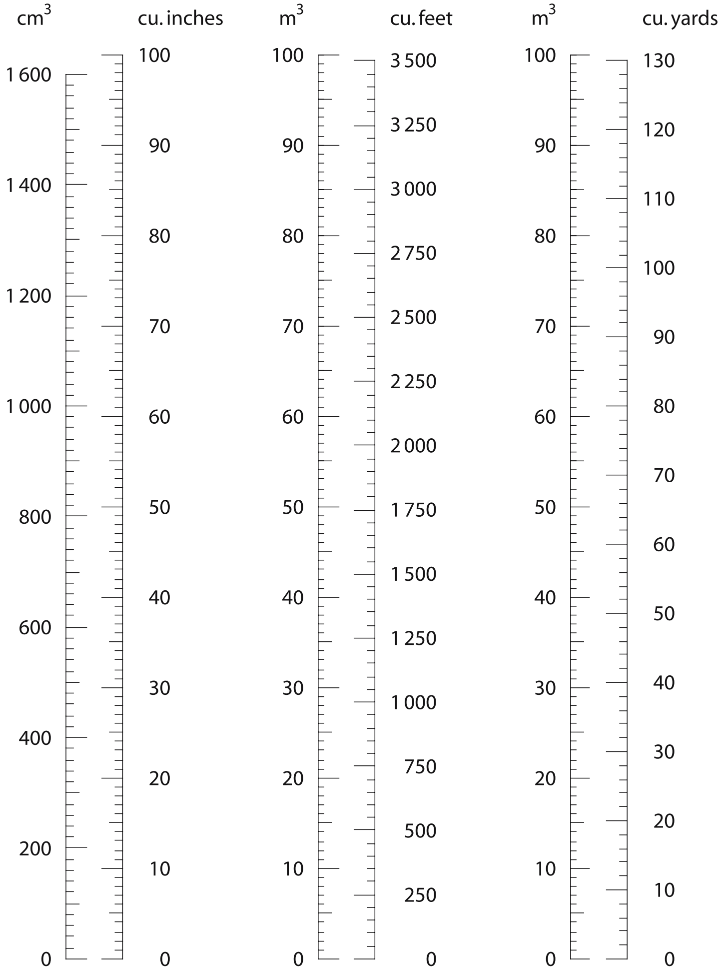


Fig. A3. Conversion between common cubic measures in the imperial and the metric system

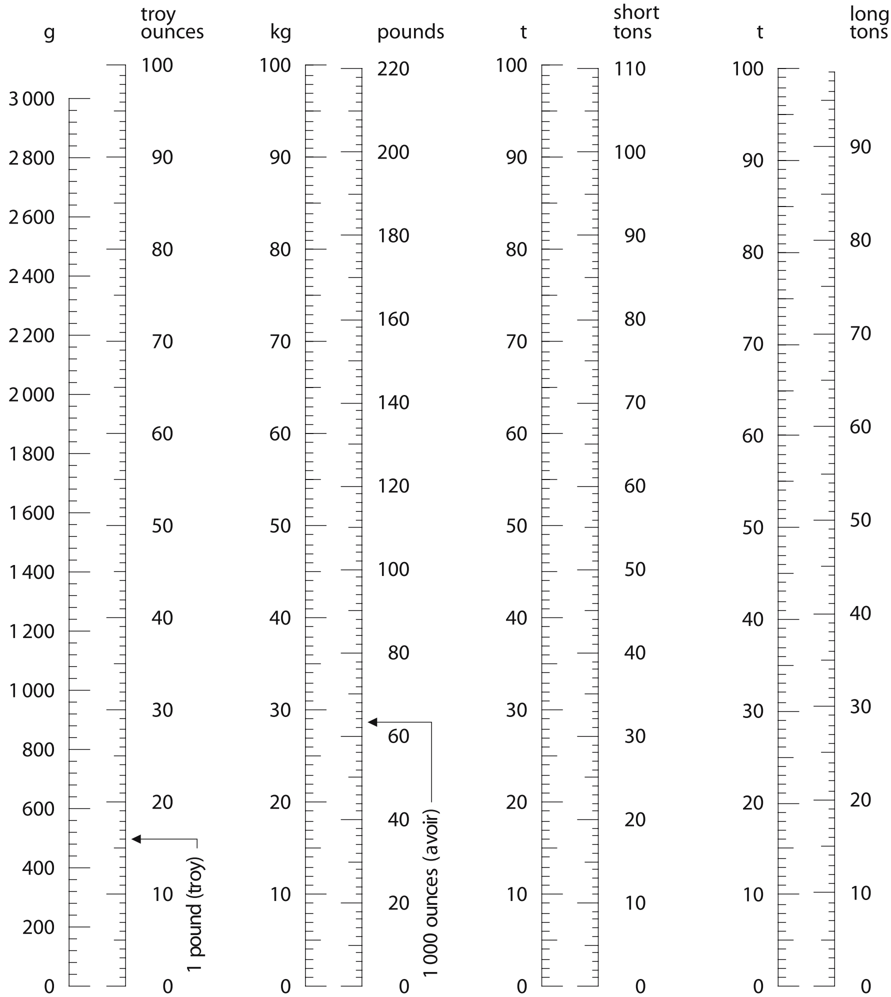


Fig. A4. Conversion between common mass measures

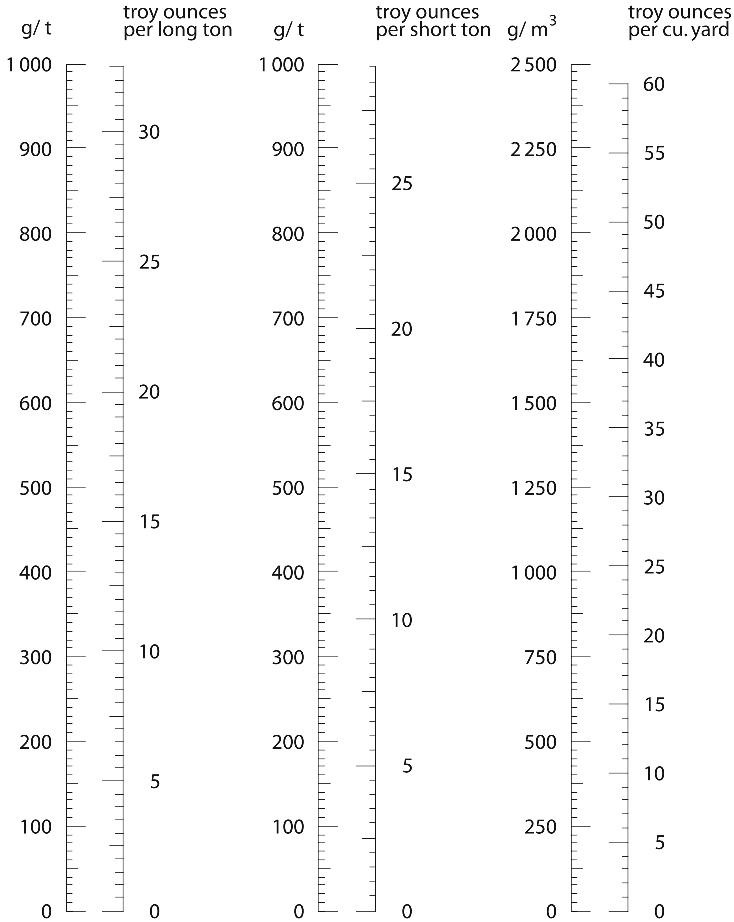


Fig. A5. Conversion between some concentration measures

Appendix B
Diagrams to Determine the Thickness Reduction Factors for Drilling Oblique to Strike and Dip at Different Angles of Inclination and Diagram to Determine the Optimal Angle of Inclination of Drill Holes for Drilling Oblique to Strike

Determination of the thickness factor R_m as a function of the drill inclination α, the dip of beds β and the profile angle γ:

- α = angle of inclination of drill hole
- β = angle of dip of the target (geological strata or ore deposit)
- γ = angle of profile between drill direction and dip direction (see Fig. 2.2b, Sect. 2.1.1.2)

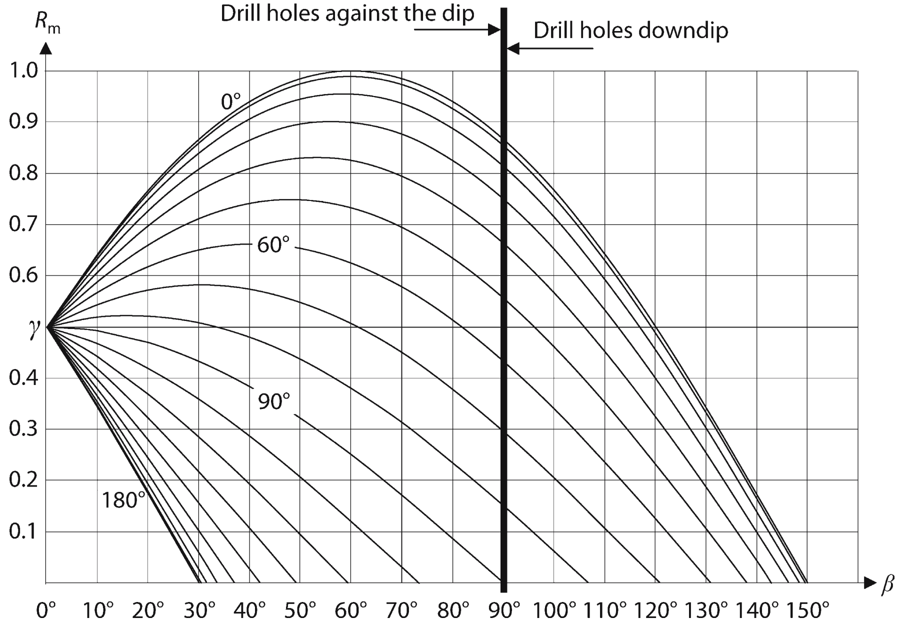


Fig. B1. $\alpha = 30°$

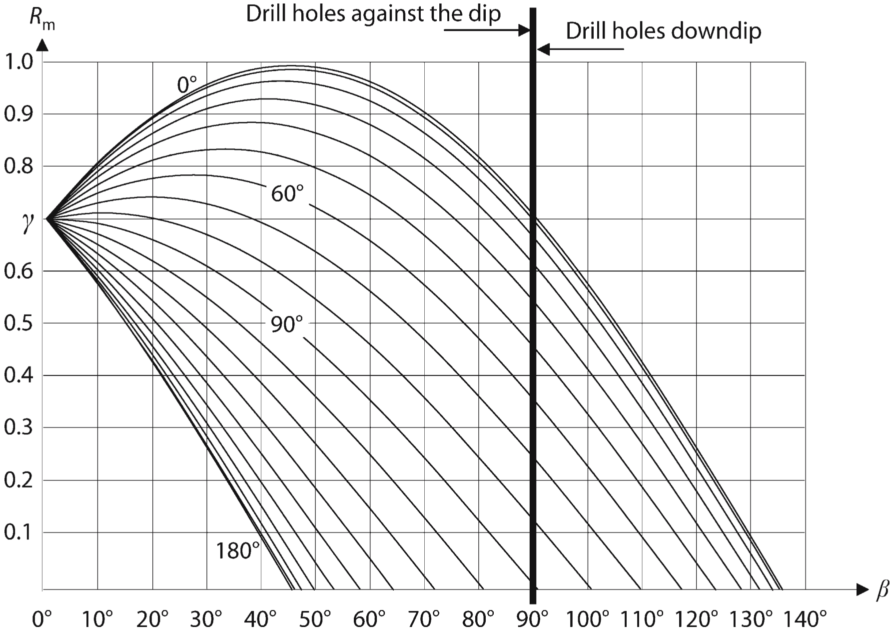


Fig. B2. $\alpha = 45°$

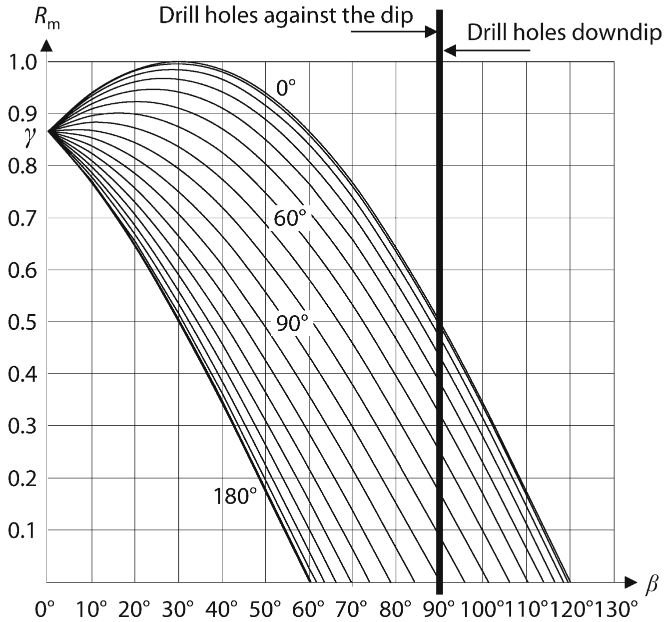


Fig. B3. $\alpha = 60°$

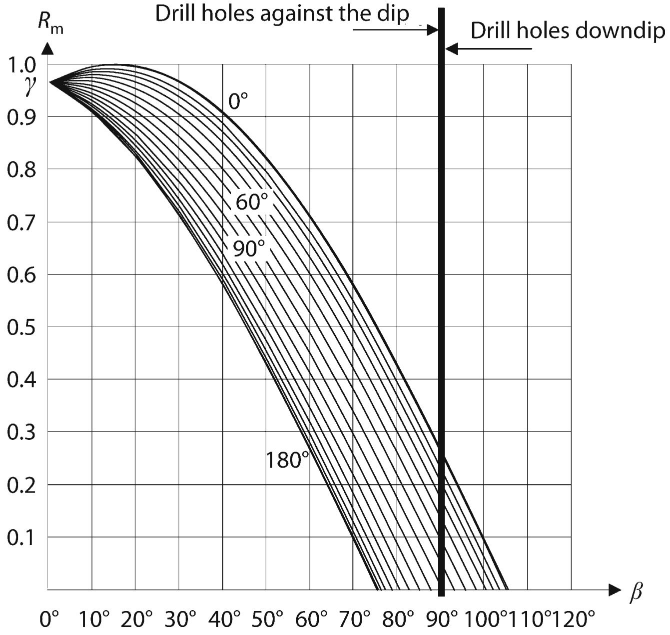


Fig. B4. $\alpha = 75°$

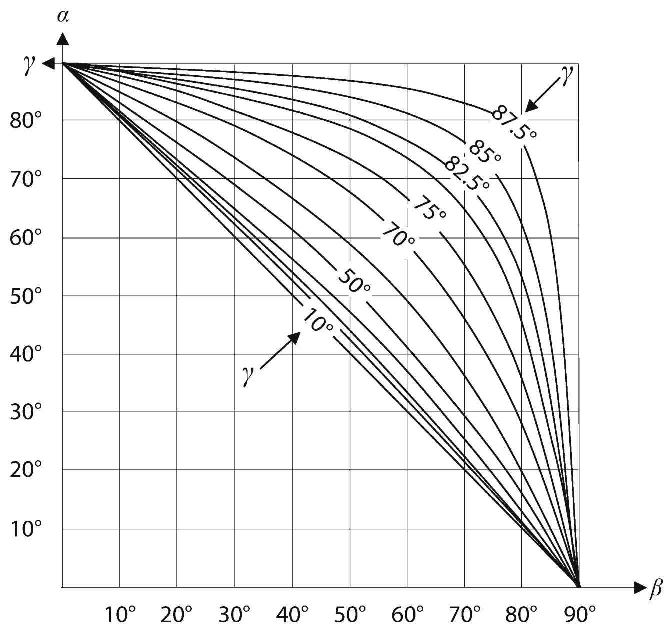


Fig. B5. Angle of inclination of drill holes for drilling oblique to strike to achieve optimal drill intersections (summary diagram)

Appendix C

Part 1
Derivation of the Formula for Calculating a Density Equivalent of Sect. 3.5.3

In Sect. 3.5.3 in Step 4 the formula to calculate the conversion factor between two ore components with different densities and a third density for the gangue was given:

$$\frac{RG_1}{RG_2} = \frac{D_1 \times MC_1 \times (D_2 - D_G)}{D_2 \times MC_2 \times (D_1 - D_G)}$$

where

- RG_1 is the relative grade in percent of metal 1, in this case Pb
- RG_2 the relative grade in percent of metal 2, in this case Zn
- D_1 the density of the metal 1 mineral, in this case galena with 7.6 g/cm^3
- D_2 the density of the metal 2 mineral, in this case sphalerite with a density of 4.0 g/cm^3
- D_G the density of the gangue, in this case carbonate with a density of 2.5 g/cm^3
- MC_1 is the metal content of the metal 1 mineral, in this case Pb in galena, which is 86.6%, and
- MC_2 is the metal content of the metal 2 mineral, in this case Zn in sphalerite, which is 67.1%

To derive the above formula we use Fig. C1. We need the following additional notations:

- V for the unit volume of 1 cm^3 of the ore in the example
- AC_1 for the absolute metal content of metal 1 in the ore volume of 1 cm^3
- AC_2 for the absolute metal content of metal 2 in the ore volume of 1 cm^2
- W for the weight of 1 cm^3 of ore
- D_0 for the density of the ore in the volume of 1 cm^3

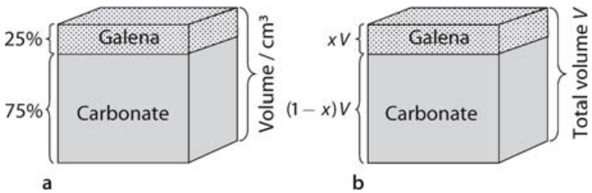


Fig. C1.
Determining volumes for the calculation of the density equivalent

Since we want to consider a case with two metals, in this example Pb and Zn respectively, we have to work with two ore densities, which we will call

- $D_{0,1}$ and $D_{0,2}$

We generalize the procedure of Step 3 in Sect. 3.5.3.
As shown in Fig. C1 the absolute metal content in the ore mineral 1 (in this case galena) is

$AC_1 = x \times V \times D_1 \times MC_1$

To obtain the relative content, or grade RG_1 of metal 1, the absolute metal content AC_1 has to be divided by the weight W of the ore cube of 1 cm^3. The weight W is

$W = V \times D_{0,1}$ consequently

$$RG_1 = \frac{x \times V \times D_1 \times MC_1}{W} = \frac{x \times V \times D_1 \times MC_1}{V \times D_{0,1}}$$

$$RG_1 = \frac{x \times D_1 \times MC_1}{D_{0,1}}$$

So we can calculate the density of the ore $D_{0,1}$:

$$D_{0,1} = \frac{x \times D_1 \times MC_1}{RG_1} \tag{C.1}$$

The Pb-content in our ore is known from the analyses. However, besides the density of the ore $D_{0,1}$ we have another unknown in the above equation, which is x. The unknown x can be derived from the density of the ore $D_{0,1}$ which can be calculated from the density of the metal 1 – mineral D_1 and the density of the gangue D_G:

$$D_{0,1} = x \times D_1 + (1 - x) \times D_G$$

So we get for x

$$D_{0,1} = x \times D_1 + D_G - x \times D_G$$

$$D_{0,1} - D_G = x \times (D_1 - D_G)$$

$$x = \frac{D_{0,1} - D_G}{D_1 - D_G} \tag{C.2}$$

Substituting Eq. C.2 for x in Eq. C.1 we get

$$D_{0,1} = \frac{D_1 \times MC_1 \times (D_{0,1} - D_G)}{RG_1 \times (D_1 - D_G)} \tag{C.3}$$

From Eq. C.3 we now can calculate the density of the ore $D_{0,1}$, which occurs on both sides of the equation. Solving it for $D_{0,1}$ we get

$$D_{0,1} = \frac{D_1 \times MC_1 \times D_G}{D_1 \times MC_1 - RG_1 \times (D_1 - D_G)} \tag{C.4}$$

Now we can use Eq. C.4 for Zn, for which sphalerite is the ore mineral. So we get

$$D_{0,2} = \frac{D_2 \times MC_2 \times D_G}{D_2 \times MC_2 - RG_2 \times (D_2 - D_G)} \tag{C.5}$$

To calculate the density equivalent $D_{0,1} = D_{0,2}$ which results in

$$\frac{D_1 \times MC_1 \times D_G}{D_1 \times MC_1 - RG_1 \times (D_1 - D_G)} = \frac{D_2 \times MC_2 \times D_G}{D_2 \times MC_2 - RG_2 \times (D_2 - D_G)} \tag{C.6}$$

If now Eq. C.6 is solved for the ratio RG_1 / RG_2, we arrive at the equation in Sect. 3.5.3, Step 4:

$$\frac{RG_1}{RG_2} = \frac{D_1 \times MC_1 \times (D_2 - D_G)}{D_2 \times MC_2 \times (D_1 - D_G)}$$

Part 2
Spreadsheet to Calculate Densities of Complex Ore

The calculations are based on one kg of ore sample. A simple example is given in Table C1 with a carbonate hosted Pb-Zn ore without pyrite with 5% Pb and 10% Zn.

Table C1. Density calculation of one kg ore sample

Ore sample = 1 000 g								
Element	**Weight-% in sample**	**Absolute weight in sample (g)**	**Mineral**	**% of element in mineral**		**Weight of mineral in sample (g)**	**Density of mineral or sample (g/cm³)**	**Volume of mineral in sample (cm³)**
Pb	5.0	50	Galena	86.6	50 / 0.866 =	57.7	7.6	7.6
Zn	10.0	100	Sphalerite	67.1	100 / 0.671 =	149.0	4.0	37.3
					Total ore mineral in sample =	206.8		
					Gangue = calcite → deduct weight of ore in sample: 1 000 – 206.8 =	793.2	2.7	293.8
					Total volume of minerals in sample =			338.7
					Density at zero porosity: 1 000 / 338.7 =		2.95	
					Assume 5% porosity: 1 000 / (338.7 × 100 / (100 – 5)) =		**2.80**	

Appendix D
Tables

Table D1.
Prefixes for units

Prefix	Symbol	Multiple
exa-	E	10^{18}
peta-	P	10^{15}
tera-	T	10^{12}
giga-	G	10^{9}
mega-	M	10^{6}
kilo-	k	10^{3}
hekto-	h	10^{2}
deka-	da	10
dezi-	d	10^{-1}
centi-	c	10^{-2}
milli-	m	10^{-3}
micro-	µ	10^{-6}
nano-	n	10^{-9}
pico-	p	10^{-12}
femto-	f	10^{-15}
atto-	a	10^{-18}

Table D2.
Dimensions used in wire line drilling (notations and standards of *DCDMA* = Diamond Core Drill Manufacturers Association)

Norm/type	Diameter of core		Diameter of hole	
	(inches)	(mm)	(inches)	(mm)
AQ	1 1/16	27.0	1 57/64	48.0
BQ	1 7/16	36.5	2 23/64	59.9
NQ	1 7/8	47.6	2 63/64	75.7
HQ	2 1/2	63.5	3 25/32	96.0
PQ	3 11/32	85.0	4 53/64	122.6

Table D3a. Matrix for the conversion of energy units

Unit[a]	kJ	MTCE	kcal	kWh	Btu	TOE
1 kJ	1	0.034×10^{-6}	0.2388	0.0003	0.95	0.024×10^{-6}
1 MTCE	29.3×10^{6}	1	7×10^{6}	8 140	27.8×10^{6}	0.67
1 kcal	4.1868	0.14×10^{-6}	1	0.001163	3.968	0.09×10^{-6}
1 kWh	3 600	0.123×10^{-3}	860	1	3 412	0.086×10^{-3}
1 Btu	1.055	0.036×10^{-6}	0.252	0.000293	1	0.025×10^{-6}
1 TOE	41.8×10^{6}	1.5	10.8×10^{6}	11 600	39×10^{6}	1

[a] Other units in use: 1 Therm = 100000 Btu; 1 MBDOE (millions of barrels per day of oil equivalent) $= 50 \times 10^{6}$ TOE. *Abbreviations:* kJ = kilojoules; *MTCE:* metric tons of coal equivalent; (in Germany also called SKE: "Steinkohleeinheit"); *kcal:* kilocalories; *kWh:* kilowatthours; *Btu:* British thermal units; *TOE:* tons of oil equivalent (the equivalent German unit (Rohöleinheit RÖE) based on kg does not differ by a factor of 10^3, but by 1.084×10^3).

Table D3b. Conversion factors from standard fuel units to energy units under standard conditions, depending on the efficiency of conversion to usable energy

1 t of hard coal	≙	1 MTCE
1 t of lignite	≙	0.3 to 0.5 MTCE
1 t of petrol	≙	1.5 MTCE
1 000 Nm^3 of natural gas	≙	1.083 MTCE
1 t of natural uranium	≙	14 000 to 23 000 MTCE[a]

Nm^3: norm cubicmeter = 1 m^3 at 1 013 millibar and 0 °C.
[a] Example: for German nuclear power plants in 2007: 1 t of natural uranium = 16 500 t MTCE.

Table D4. Sieve (screen) units: conversion of mesh

Mesh (Tyler Standard Scree Scale Sieve Series)	Aperture *W* (mm)	Mesh (Tyler Standard Scree Scale Sieve Series)	Aperture *W* (mm)	Mesh (Tyler Standard Screen Scale Sieve Series)	Aperture *W* (mm)
1.05"	26.9	7	2.83	48	0.297
.883"	22.6[a]	8	2.38	60	0.250[a,b]
.742"	19.0	9	2.00[a,b]	65	0.210
.624"	16.0[a,b]	10	1.68	80	0.177[a]
.525"	13.5	12	1.41[a]	100	0.149
.441"	11.2[a]	14	1.19	115	0.125[a,b]
.371"	9.51	16	1.00[a,b]	150	0.105
2 ½	8.00[a,b]	20	0.841	170	0.088[a]
3	6.73	24	0.707[a]	200	0.074
3 ½	5.66[a]	28	0.595	250	0.063[a,b]
4	4.76	32	0.500[a,b]	270	0.053
5	4.00[a,b]	35	0.420	325	0.044[a,b]
6	3.36	42	0.354[a]	400	0.037

[a] Corresponds to ISO-Norm 3310/1 (international standard).
[b] Corresponds to DIN 4188 (German standard).

Table D5. Qualitative description of the grain size of gold particles (Boericke 1947; Faulkner 1986; Giusti 1986; Moen and Huntting 1975; Wang and Poling 1981; West 1971)

Description with grain size		
Description	**Location**	**Grain size**
Very fine gold	USA (lower 48 states)	<0.42 mm (<35 mesh)
Minute gold	USSR for – 0.037 the term "dispersable gold" is used	<0.10 mm >0.037 mm (<150 mesh >400 mesh)
Fine gold	Alaska	<0.210 mm (<65 mesh)
	Canada (mainly British Columbia)	
	Older convention	<1.5 mm >0.35 mm (<12 mesh >42 mesh)
	More recent convention	<0.074 mm (<200 mesh)
	USA (lower 48 states)	<0.84 mm >0.42 mm (<20 mesh >35 mesh)
Medium gold	USA (lower 48 states)	<1.6 mm >0.84 mm (<10 mesh >20 mesh)
Coarse gold	USA (lower 48 states)	>1.6 mm (>10 mesh)

In Canada, mostly in British Columbia there is a convention of describing how many particles will make up 1 troy ounce of gold	
Term	**Number of particles/troy ounce of gold**
Nugget (rarely "rattler")	up to 200
Coarse	up to 500
Medium	up to 2000
Fine	up to 12000
Very fine	up to 40000
Flour gold	over 40000

Table D6.
Tonnage factors converted into metric densities

Tonnage factor for short ton	**Density (g/cm^3)**	**Tonnage factor for long ton**	**Density (g/cm^3)**
7	4.58	7	5.13
8	4.00	8	4.49
9	3.56	9	3.99
10	3.20	10	3.59
11	2.91	11	3.26
12	2.67	12	2.99
13	2.46	13	2.76
14	2.29	14	2.56
15	2.14	15	2.39

Table D7.
Rule-of-thumb for densities

Mineral or rock	Density (g/cm^3)
Au-quartz vein without sulphides	2.6
Au-quartz reef, South Africa	2.8–2.85
Massive sulphides, pyrite most important component	4.0–4.5
Semi-massive sulphides as frequently encountered in magmatic Cu-Ni deposits	3.3–3.6
Hematitic iron ore	4.3
Barite	4.0
Fluorspar	3.1
Porphyry copper ore	2.3–2.6
Basic intrusives and extrusives	2.8–3.0
Acid intrusives and extrusives	2.6–2.7
Limestone	2.6
Dolomite	2.8
Sandstone	2.6
Slate	2.8
Greywackes	2.7
Bauxite	1.4
Ni-laterite	1.25
Decomposed serpentinite	1.0
Gravel, sand	1.7
Bituminous coal	1.3–1.5
Lignite	1.2

Table D8.
Historic gold and silver prices

Period	Price	
	in U.S.$/oz	in English £/oz
Gold: longer periods of constant prices		
1700–1717		4 £-ls-0d
1717–1918		4 £-5s-0d
1792–1834	19.39	
1834–1934	20.67	
1934–1971	35.00	
1972	38.00	
Silver: nearly constant over a longer period		
1792–1873	1.29–1.35 fluctuating	

Table D9. Selection of the most important atomic weights relative to oxygen, O = 16

Symbol	Element	Atomic weight	Symbol	Element	Atomic weight
Al	Aluminium (Aluminum)	26.97	Mo	Molybdenum	95.95
Sb	Antimony	121.76	Ni	Nickel	58.69
As	Arsenic	74.91	Nb	Niobium (Columbium)	92.91
Ba	Barium	137.36	N	Nitrogen	14.008
Be	Beryllium	9.02	Os	Osmium	190.20
Bi	Bismuth	209.00	O	Oxygen	16.00
B	Boron	10.82	Pd	Palladium	106.70
Br	Bromine	79.92	P	Phosphorus	30.98
Cd	Cadmium	112.41	Pt	Platinum	195.23
Ca	Calcium	40.08	Pu	Plutonium	239.00
C	Carbon	12.01	K	Potassium	39.096
Ce	Cerium	140.13	Ra	Radium	226.05
Cs	Cesium (Caesium)	132.91	Re	Rhenium	186.31
Cl	Chlorine	35.46	Rh	Rhodium	102.91
Cr	Chromium	52.01	Rb	Rubidium	85.48
Co	Cobalt	58.94	Ru	Ruthenium	101.70
Cb	Columbium (Niobium)	92.91	Sc	Scandium	45.10
Cu	Copper	63.57	Se	Selenium	78.96
Eu	Europium	152.00	Si	Silicon	28.06
F	Fluorine	19.00	Ag	Silver	107.88
Ga	Gallium	69.72	Na	Sodium	22.997
Ge	Germanium	72.60	Sr	Strontium	87.63
Au	Gold	197.20	S	Sulphur (Sulfur)	32.06
Hf	Hafnium	178.60	Ta	Tantalum	180.88
He	Helium	4.003	Te	Tellurium	127.61
H	Hydrogen	1.008	Tl	Thallium	204.39
In	Indium	114.76	Th	Thorium	232.12
I	Iodine	126.92	Sn	Tin	118.70
Ir	Iridium	193.10	Ti	Titanium	47.90
Fe	Iron	55.85	W	Tungsten (Wolfram)	183.92
Pb	Lead	207.21	U	Uranium	238.07
Li	Lithium	6.94	V	Vanadium	50.95
Mg	Magnesium	24.32	Y	Yttrium	88.92
Mn	Manganese	54.93	Zn	Zinc	65.38
Hg	Mercury	200.61	Zr	Zirconium	91.22

Table D10. Conversion table for the most frequent chemical compounds in raw material evaluations

Element/compound	Factor	Element/compound	Factor	Element/compound	Factor
Al from Al_2O_3	0.529	Mo from MoS_2	0.599	Ti from TiO_2	0.600
BaO from $BaSO_4$	0.657	Mo from MoO_3	0.667	U from UF_6	0.676
Be from BeO	0.360	Cb from Cb_2O_3	0.699	U from U_3O_8	0.848
Cr from Cr_2O_3	0.684	P[a] from P_2O_5	0.436[a]	V from V_2O_5	0.560
F from CaF_2	0.487	Sb from Sb_2S_3	0.717	W from WO_3	0.793
K_2O from KCl	0.632	Sn from SnO_2	0.787	Zr from ZrO_2	0.740
Li from Li_2O	0.464	Ta from Ta_2O_5	0.819		

[a] Grades in phosphate ore or concentrates are sometimes given in BPL or TPL grade. BPL means bone phosphate of lime; TPL triphosphate of lime. BPL and TPL are both equivalent to $Ca_3(PO_4)_2$. The conversion factor is % BPL (TPL) × 0.4576 = % P_2O_5.

Table D11. Typical concentrate grades (BRGM 1997; Codner 1993; Metal Bulletin, own data)

Element/mineral	Product	Grade
Al	Bauxite – uncalcined – calcined Alumina – calcined	25% Al 45% Al 52.9% Al
Cb	Columbite ore	65% $Cb_2O_5 + Ta_2O_5$ (ratio 10 : 1)
Cr	Concentrates and lump ore	40–48 Cr_2O_3
Cu	Concentrates	25–30% Cu
Fluorspar	Concentrates – acid grade – ceramic grade – metallurgical grade	97% CaF_2 93–95% CaF_2 >70% CaF_2
Graphite	Concentrates (crystalline large flakes)	90% C
Mo	Sulphide concentrates	50% Mo
Li	Spodumene concentrate Petalite concentrate	7.25% Li_2O 4.2% Li_2O
Mn	Concentrates and lump ore	45–50% Mn
Ni	High-grade sulphide concentrates Normal sulphide concentrates	20–24% Ni 9–12% Ni
Pb	Concentrates	65% Pb
Phosphate	Concentrates	62–75% BPL (see Table D10)
Sb	Concentrates	60% Sb
Sn	High-grade concentrates Medium-grade concentrates Low-grade concentrates	70–75% Sn 50–60% Sn 30% Sn
Ta	Tantalite	25–40% Ta_2O_5
Ti	Ilmenite concentrates Rutile concentrates	50–60% TiO_2 95% TiO_2
U	Chemical concentrate "yellow cake"	75% U
V	Chemical concentrates	98% V_2O_5
W	Scheelite concentrates Wolframite ore	70% WO_3 65% WO_3
Zn	Concentrates	50% Zn
Zr	Concentrates	65% ZrO_2

Table D12a. Quotation of the London Metal Exchange (LME)

Metal	Unit of quotation	Change over from £/t to U.S.$/t
Zinc	U.S.$/t	01.09. 1988
Lead	U.S.$/t	01.07. 1993
Copper	U.S.$/t	01.07. 1993
Aluminium	U.S.$/t	01.01. 1989
Nickel	U.S.$/t	01.02. 1988
Tin	U.S.$/t	since 01.06. 1989 24.10. 1985–01.06. 1989 quotation suspended before 24.10. 1985 in £/t

Table D12b. Rates of exchange between £ and US $, yearly averages (Source: Metallgesellschaft 1987 and subsequent editions) (To use the table an example: In 1955 £1 had as a yearly average a value of US $2.7917)

Year	Rate	Year	Rate	Year	Rate
1950	2.8000	1965	2.7962	1980	2.3267
1951	2.7824	1966	2.7932	1981	2.0285
1952	2.8100	1967	2.7467	1982	1.7507
1953	2.8109	1968	2.3939	1983	1.51622
1954	2.8089	1969	2.3902	1984	1.33635
1955	2.7917	1970	2.3958	1985	1.29552
1956	2.7959	1971	2.4435	1986	1.46683
1957	2.7935	1972	2.5016	1987	1.63858
1958	2.8098	1973	2.4521	1988	1.78060
1959	2.8089	1974	2.3399	1989	1.63951
1960	2.8077	1975	2.2218	1990	1.78592
1961	2.8023	1976	1.8066	1991	1.76798
1962	2.8078	1977	1.7456	1992	1.76595
1963	2.7999	1978	1.9195	1993	1.50220
1964	2.7925	1979	2.1214	1994	1.53210
				1995	1.57830

Table D13. Common smelter terms. Current smelting or treatment charges (T/C) and refining charges (R/C) are published in Metal Bulletin. For those elements which are environmentally sensitive, like As or Hg, it has to be pointed out that the acceptance of concentrates with appreciable amounts of these deleterious elements depends very much on the quality mix of the concentrates being treated. If a smelter is treating clean concentrates as a base load, it is then able to accept some "dirty" concentrates to blend them in. In contrast, if the smelter already treats concentrates with relatively high values of these detrimental elements, then it will probably accept only very limited amounts concentrates with relatively high impurity levels, or none at all. The consequence is that it is often very difficult to market unclean concentrates at all

Deductions		
1. Cu-concentrates (1 u = 1 metric unit, see Sect. 1.1.4)		
Cu	– 1 u to 1.3 u; T/C related to 1 t of concentrate; R/C related to paid metal content	
Au	– 1 g, remainder fully paid	
Ag	– 25 to 35 g, remainder fully paid	
Typical penalties	As	above 0.2%
	Hg	above 5 to 10 ppm
	Bi	above 0.1%
2. Zn-concentrates		
Zn	– 8 u, but not more than 85% of the metal content of the concentrate is paid for; only T/C related to 1 t of concentrate	
Ag	– 3 oz, remainder fully paid	
Typical penalties	Fe	above 8%
	Hg	above 20–30 ppm
3. Pb-concentrate		
Pb	– 3 u, but not more than 95% of the metal content of the concentrate is paid for; only T/C related to 1 t of concentrates	
Ag	– 50 g, remainder fully paid	
Au	– 1 g, remainder fully paid	
Typical penalties	As	above 0.1%
	Hg	above 30 ppm
	Sb	above 0.1%
	Bi	above 0.01%
4. Mixed concentrates		
Pb	– 3 u, but not more than 90% of the Pb-content of the concentrate is paid for	
Zn	– 7 u, but not more than 80% of the Zn-content of the concentrate is paid for	
Cu	– 1–5 u, but not more than 25% of the Cu-content of the concentrate is paid for	
Ag	– 75 g, but at the maximum only 95% of the Ag-content of the concentrate is paid for; T/C related to 1 t of concentrate; R/C related to the paid amount of each metal	
Typical penalties	Hg	above 100 ppm
5. Tin concentrates		
Sn	– 1 u; T/C related to 1 t of concentrate; T/C depends on the concentrate grade	
6. Ni-concentrates		
Ni	– 0.7 u; T/C related to 1 t of concentrate; R/C related to paid metal content; credit for Co about 15% of metal content	
Typical penalties	As	above 200 ppm

Table D14. Inflation indices

Year	Marshall & Swift[a] equipment cost index mining, milling (1926 = 100)	Index of manufactured[b] goods exports by developed market economies (1990 = 100)	US-producer[c] price index (all commodities) (2000 = 100)	Canada consumer[d] price index (all items) (2000 = 100)	South Africa[e] production price index (all groups) (2000 = 100)	Australia[f] consumer price index (8 capital cities)[g] (2000/01 = 100)	Year
1960	241.0	25.6	23.9	16.3	3.1	10.2	1959/60
1961	239.0	26.7	23.8	16.5	3.2	10.6	1960/61
1962	240.0	26.7	23.9	16.7	3.2	10.7	1961/62
1963	240.0	26.7	23.8	16.9	3.2	10.7	1962/63
1964	243.0	27.9	23.8	17.3	3.3	10.8	1963/64
1965	245.0	27.9	24.4	17.7	3.4	11.2	1964/65
1966	253.0	27.9	25.1	18.3	3.6	11.6	1965/66
1967	264.0	27.9	25.1	19.0	3.6	12.0	1966/67
1968	273.0	27.9	25.8	19.7	3.7	12.4	1967/68
1969	286.0	27.9	26.8	20.6	3.8	12.6	1968/69
1970	303.0	29.1	27.8	21.3	3.9	13.1	1969/70
1971	321.0	31.4	26.7	21.9	4.1	13.6	1970/71
1972	332.0	33.7	30.0	23.0	4.4	14.6	1971/72
1973	343.0	39.5	33.9	24.5	5.0	15.5	1972/73
1974	394.0	47.7	40.3	27.5	5.9	17.4	1973/74
1975	451.0	53.5	44.1	30.4	6.9	20.4	1974/75
1976	483.0	53.5	46.1	32.7	8.0	23.0	1975/76
1977	521.0	59.3	48.9	35.3	9.0	26.1	1976/77
1978	565.0	67.4	52.7	38.4	9.9	28.7	1977/78
1979	619.0	76.7	59.3	41.9	11.4	31.0	1978/79
1980	684.0	86.0	67.7	46.2	13.3	34.1	1979/80
1981	740.0	80.2	73.8	51.9	15.1	37.3	1980/81
1982	780.0	79.1	75.3	57.5	17.2	41.3	1981/82
1983	799.0	75.6	76.3	60.9	19.0	46.0	1982/83
1984	817.0	73.3	78.1	63.5	20.6	49.2	1983/84
1985	823.0	74.4	77.7	66.1	24.1	51.3	1984/85
1986	827.0	88.4	75.5	68.8	28.8	55.6	1985/86
1987	837.0	98.8	77.4	71.8	32.8	60.8	1986/87

Table D14. *Continued*

Year	Marshall & Swift[a] equipment cost index mining, milling (1926 = 100)	Index of manufactured[b] goods exports by developed market economies (1990 = 100)	US-producer[c] price index (all commodities) (2000 = 100)	Canada consumer[d] price index (all items) (2000 = 100)	South Africa[e] production price index (all groups) (2000 = 100)	Australia[f] consumer price index (8 capital cities)[g] (2000/01 = 100)	Year
1988	870.0	105.8	80.6	74.7	37.1	65.3	1987/88
1989	914.0	104.7	84.6	78.4	42.8	70.0	1988/89
1990	940.0	116.3	87.6	82.3	47.9	75.6	1989/90
1991	959.0	116.3	87.8	86.8	53.4	79.7	1990/91
1992	976.0	119.8	88.3	88.1	57.8	81.2	1991/92
1993	999.0	112.8	89.6	89.7	61.7	82.0	1992/93
1994	1 028.0	115.1	90.7	89.9	66.7	83.5	1993/94
1995	1 058.0	125.6	94.0	91.8	73.0	86.2	1994/95
1996	1 072.0	123.3	96.2	93.3	78.1	89.8	1995/96
1997	1 089.0	115.1	96.2	94.8	83.6	91.0	1996/97
1998	1 097.0	110.5	93.7	95.7	86.6	91.0	1997/98
1999	1 106.0	105.8	94.5	97.4	91.6	92.1	1998/99
2000	1 124.0	100.0	100.0	100.0	100.0	94.3	1999/00
2001	1 130.3	98.0	101.1	102.6	108.4	100.0	2000/01
2002	1 147.5	99.0	98.8	104.8	123.8	102.9	2001/02
2003	1 169.6	107.0	104.1	107.8	125.9	106.1	2002/03
2004	1 232.7	115.8	110.5	109.8	126.7	108.5	2003/04
2005	1 325.9	118.0	118.6	112.1	130.6	114.8	2004/05
2006	1 391.9	122.0[h]	124.2	114.4	140.6	118.4	2005/06

[a] Marshall & Swift equipment cost index located in back of issues of Chemical Engineering (Current Busi-ness Indicators, Chemical Engineering Plant Cost Index, and Vatavuk Pollution Control Cost Index) are also located on the same page in the same journal. http://www.che.com/pindex/index.php?che.
[b] UN Statistics Division, Monthly Bulletin of Statistics Online, World exports by commodity classes and by regions: developed economies. http://unstats.un.org/unsd/mbs/.
[c] Producer Price Indexes/Bureau of Labor Statistics – http://stats.bls.gov/ppi/ and L2.61 Gov Docs (please note: codes 06, etc. are for chemical and allied products).
[d] Statistics Canada, Consumer Price Index (CPI). http://www.statcan.ca/menu-en.htm.
[e] Statistics South Africa, Production Price Index (PPI). http://www.statssa.gov.za/keyindicators/ppi.asp.
[f] Australian Bureau of Statistics, Consumer Price Index (CPI). http://www.abs.gov.au.
[g] Calculated from Consumer Price Index (CPI); all groups, weighted average of eight capital cities.
[h] 1.–3. quarter.

Table D15. Definition of metal families. Some families like the noble or precious metals or the rare earth elements are well defined, whereas others like base metals, minor or special metals are only loosely defined and overlapping

1. Precious (noble) metals (abbreviation PM)			
Gold (Au)	Silver (Ag)	Platinum group (see point 2)	
2. Platinum group metals or elements (abbreviation PGM or PGE)			
The platinum group metals are subdivided into the light and heavy platinum groups			
▪ Light platinum group with a density around 12 g/cm^3			
Palladium (Pd)	Rhodium (Rh)	Ruthenium (Ru)	
▪ Heavy platinum group with a density around 22 g/cm^3			
Platinum (Pt)	Iridium (Ir)	Osmium (Os)	
3. Rare earth elements or lanthanides (abbreviation REE)			
The rare earth elements are subdivided in light and heavy rare earth elements In an even wider sense all elements in the transition group 3b of the periodic table, meaning besides yttrium, the lanthanides and also scandium, are called rare earth elements			
▪ Light rare earth elements			
Lanthanum (La)	Praseodymium (Pr)	Promethium (Pm)	Europium (Eu)
Cerium (Ce)	Neodymium (Nd)	Samarium (Sm)	
The light subgroup is sometimes also called cerium subgroup			
▪ Heavy rare earth elements			
Gadolinium (Gd)	Dysprosium (Dy)	Erbium (Er)	Ytterbium (Yb)
Terbium (Tb)	Holmium (Ho)	Thulium (Tm)	Lutetium (Lu)
The heavy subgroup is sometimes also called yttrium subgroup. Although yttrium itself does not belong to the yttrium subgroup the name is given, because the heavy REE tend to occur together with yttrium (Y), for example, in the mineral xenotim Sometimes yttrium is included with the rare earth elements. It is chemically similar and almost always occurs in association with REE			
4. Base or non-ferrous or heavy metals			
Copper (Cu)	Zinc (Zn)	Tin (Sn)	Bismuth (Bi)
Lead (Pb)	Cadmium (Cd)	Antimony (Sb)	Mercury (Hg)
For the chemist heavy metals are metals with a density higher than 5 g/cm^3			
5. Older major metals			
Copper (Cu)	Lead (Pb)	Zinc (Zn)	Tin (Sn)

Table D15. *Continued*

6. Light metals			
Aluminium (Al)	Titanium (Ti)	Beryllium (Be)	Strontium (Sr)
Magnesium (Mg)	Lithium (Li)	Cesium (Cs)	
For the chemist light metals are metals with a density less than 5 g/cm^3			

7. Steel industry (steel alloy) metals		
Iron/steel (Fe)	Cobalt (Co)	Columbium (Cb) = niobium (Nb)
Nickel (Ni)	Molybdenum (Mo)	Tantalum (Ta)
Manganese (Mn)	Tungsten (W)	
Chromium (Cr)	Vanadium (V)	

8. Minor (special) metals		
Titanium (Ti)	Tantalum (Ta)	Arsenic (As)
Zirconium (Zr)	Lanthanides (La) (see point 3)	Selenium (Se)
Halfnium (Hf)	Gallium (Ga)	Tellurium (Te)
Columbium (Cb) = niobium (Nb)	Germanium (Ge)	

9. Electronic metals		
Cadmium (Cd)	Mercury (Hg)	Selenium (Se)
Gallium (Ga)	Indium (In)	Tellurium (Te)
Germanium (Ge)	Rhenium (Re)	Silicon (Si)

10. Nuclear metals		
Uranium (U)	Cesium (Cs)	Lanthanides (La) (see point 3)
Zirconium (Zr)	Rubidium (Rb)	
Hafnium (Hf)	Beryllium (Be)	

11. Strategic minerals
There is no generally accepted definition of strategic minerals, metals or commodities. The geological survey organisations of Australia, Canada, Germany, South Africa, USA, United Kingdom, and USA set up an informal group "International Strategic Minerals Inventory", ISMI. In their commodity reports, for example Krauss et al. 1989, they write: "The term 'strategic minerals' is imprecise. It generally refers to mineral ore and derivative products that come largely or entirely from foreign sources, that are difficult to replace, and that are important to a nation's economy, in particular its defense industry. Usually, the term implies a nation's per-ception to vulnerability to supply disruptions and of a need to safeguard its industries from the repercussions of a loss of supplies. Because a mineral that is strategic to one country may not be strategic to another, no one list of strategic minerals can be prepared. Tin is a good example. It is considered strategic in the USA, but not in Germany."

Table D16. Abbreviations used in the shipping business

1. INCOTERMS[a]	
Group E – Departure	
exw	ex works (named place)
Group F – Main Carriage Unpaid	
fca	free carrier (named place)
fas	free alongside ship (named loading port)
fob	free on board (named loading port)
Group C – Main Carriage Paid	
cfr	cost and freight (named destination port)
cif	cost, insurance and freight (named destination port)
cpt	carriage paid to (named destination port)
cip	carriage and insurance paid to (named destination port)
Group D – Arrival	
daf	delivered at frontier (named place)
des	delivered ex ship (named port)
deq	delivered ex quay (named port)
ddu	delivered duty unpaid (named destination place)
ddp	delivered duty paid (named destination place)
2. Other abbreviations used in the shipping business of raw materials	
fobst	free on board, stowed and trimmed
fot	free on truck
for	free on rail
fid	free into (container) depot
fis	free in store
ciffo	cost, insurance, freight, free out
c&f	cost and freight

[a] Devised and published by the International Chamber of Commerce (ICC) in Paris and endorsed by the United Nations Commission on International Trade Law (UNICITRAL).

Table D17. Dry Bulk Freight Index (derived from 1980s Panamax 1 year timecharter rates, Shipping Review Database, Clarkson Research Services, London (www.crsl.com)), year 2000 = 100

Year	Index	Year	Index	Year	Index	Year	Index
1984	60	1990	111	1996	94	2002	74
1985	56	1991	123	1997	98	2003	134
1986	48	1992	100	1998	66	2004	234
1987	73	1993	108	1999	72	2005	187
1988	121	1994	108	2000	100	2006	114
1989	134	1995	142	2001	77		

Table D18. Discounting factors

$q = 1 - i$	Years																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1.03 (3%)	0.971	0.943	0.915	0.888	0.863	0.837	0.813	0.789	0.766	0.744	0.722	0.701	0.681	0.661	0.642	0.623	0.605	0.587	0.570	0.554
1.04 (4%)	0.962	0.925	0.889	0.855	0.822	0.790	0.760	0.731	0.703	0.676	0.650	0.625	0.601	0.577	0.555	0.534	0.513	0.494	0.475	0.456
1.05 (5%)	0.952	0.907	0.864	0.823	0.784	0.746	0.711	0.677	0.645	0.614	0.585	0.557	0.530	0.505	0.481	0.458	0.436	0.416	0.396	0.377
1.06 (6%)	0.943	0.890	0.840	0.792	0.747	0.705	0.665	0.627	0.592	0.558	0.527	0.497	0.469	0.442	0.417	0.394	0.371	0.350	0.331	0.312
1.07 (7%)	0.935	0.873	0.816	0.763	0.713	0.666	0.623	0.582	0.544	0.508	0.475	0.444	0.415	0.388	0.362	0.339	0.317	0.296	0.277	0.258
1.08 (8%)	0.926	0.837	0.794	0.735	0.681	0.630	0.583	0.540	0.500	0.463	0.429	0.397	0.368	0.340	0.315	0.292	0.270	0.250	0.232	0.215
1.09 (9%)	0.917	0.842	0.772	0.708	0.650	0.596	0.547	0.502	0.460	0.422	0.388	0.356	0.326	0.299	0.275	0.252	0.231	0.212	0.194	0.176
1.10 (10%)	0.909	0.826	0.751	0.683	0.621	0.564	0.513	0.467	0.424	0.386	0.350	0.319	0.290	0.263	0.239	0.218	0.198	0.180	0.164	0.149
1.11 (11%)	0.901	0.812	0.731	0.659	0.593	0.535	0.482	0.434	0.391	0.352	0.317	0.286	0.258	0.232	0.209	0.188	0.170	0.153	0.138	0.124
1.12 (12%)	0.893	0.797	0.712	0.636	0.567	0.507	0.452	0.404	0.361	0.322	0.287	0.257	0.229	0.205	0.183	0.163	0.146	0.130	0.116	0.104
1.13 (13%)	0.885	0.783	0.693	0.613	0.543	0.480	0.425	0.376	0.333	0.295	0.261	0.231	0.204	0.181	0.160	0.141	0.125	0.111	0.098	0.087
1.14 (14%)	0.877	0.769	0.675	0.592	0.519	0.456	0.400	0.351	0.308	0.270	0.237	0.208	0.182	0.160	0.140	0.123	0.108	0.095	0.083	0.073
1.15 (15%)	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247	0.215	0.178	0.163	0.141	0.123	0.107	0.093	0.081	0.070	0.061
1.16 (16%)	0.862	0.743	0.641	0.552	0.476	0.410	0.354	0.305	0.263	0.227	0.195	0.168	0.145	0.125	0.108	0.093	0.080	0.069	0.060	0.051
1.17 (17%)	0.855	0.731	0.624	0.534	0.456	0.390	0.333	0.285	0.243	0.208	0.178	0.152	0.130	0.111	0.095	0.081	0.069	0.059	0.051	0.043
1.18 (18%)	0.847	0.718	0.609	0.516	0.437	0.370	0.314	0.266	0.225	0.191	0.162	0.137	0.116	0.099	0.084	0.071	0.060	0.051	0.043	0.037
1.19 (19%)	0.840	0.706	0.593	0.499	0.419	0.352	0.296	0.249	0.209	0.176	0.148	0.124	0.104	0.088	0.074	0.062	0.052	0.044	0.037	0.031
1.20 (20%)	0.833	0.694	0.579	0.482	0.402	0.335	0.279	0.233	0.194	0.162	0.135	0.112	0.093	0.078	0.065	0.054	0.045	0.038	0.031	0.026
1.21 (21%)	0.826	0.683	0.564	0.467	0.386	0.319	0.263	0.218	0.180	0.149	0.123	0.102	0.084	0.069	0.057	0.047	0.039	0.032	0.027	0.022
1.22 (22%)	0.820	0.672	0.551	0.451	0.370	0.303	0.249	0.204	0.167	0.137	0.112	0.092	0.075	0.062	0.051	0.042	0.034	0.028	0.023	0.019
1.23 (23%)	0.813	0.661	0.537	0.437	0.355	0.289	0.235	0.191	0.155	0.126	0.103	0.083	0.068	0.055	0.045	0.036	0.030	0.024	0.020	0.016
1.24 (24%)	0.806	0.650	0.524	0.423	0.341	0.275	0.222	0.179	0.144	0.116	0.094	0.076	0.061	0.049	0.040	0.032	0.026	0.021	0.017	0.014
1.25 (25%)	0.800	0.640	0.512	0.410	0.328	0.262	0.210	0.168	0.134	0.107	0.086	0.069	0.055	0.044	0.035	0.028	0.023	0.018	0.014	0.012
1.26 (26%)	0.794	0.630	0.500	0.397	0.315	0.250	0.198	0.157	0.125	0.099	0.079	0.062	0.050	0.039	0.031	0.025	0.020	0.016	0.012	0.010
1.27 (27%)	0.787	0.620	0.488	0.384	0.303	0.238	0.188	0.148	0.116	0.092	0.072	0.057	0.045	0.035	0.028	0.022	0.017	0.014	0.011	0.008
1.28 (28%)	0.781	0.610	0.477	0.373	0.291	0.227	0.178	0.139	0.108	0.085	0.066	0.052	0.040	0.032	0.025	0.019	0.015	0.012	0.009	0.007
1.29 (29%)	0.775	0.601	0.466	0.361	0.280	0.217	0.168	0.130	0.101	0.078	0.061	0.047	0.037	0.028	0.022	0.017	0.013	0.010	0.008	0.006
1.30 (30%)	0.769	0.592	0.455	0.350	0.269	0.207	0.159	0.123	0.094	0.073	0.056	0.043	0.033	0.025	0.020	0.015	0.012	0.009	0.007	0.005

Table D19. Annuity present value factors

$q = 1 + i$		Year							
		5	**8**	**10**	**12**	**15**	**20**	**25**	**30**
1.03	(3%)	4.580	7.020	8.530	9.954	11.938	14.877	17.413	19.600
1.04	(4%)	4.452	6.733	8.111	9.385	11.118	13.590	15.622	17.292
1.05	(5%)	4.329	6.463	7.722	8.863	10.380	12.462	14.094	15.372
1.06	(6%)	4.212	6.210	7.360	8.384	9.712	11.470	12.783	13.765
1.07	(7%)	4.100	5.971	7.024	7.943	9.108	10.594	11.654	12.409
1.08	(8%)	3.993	5.747	6.710	7.536	8.559	9.818	10.675	11.258
1.09	(9%)	3.890	5.535	6.418	7.161	8.061	9.129	9.823	10.274
1.10	(10%)	3.791	5.335	6.145	6.814	7.606	8.514	9.077	9.427
1.11	(11%)	3.696	5.146	5.889	6.492	7.191	7.963	8.422	8.694
1.12	(12%)	3.605	4.968	5.650	6.194	6.811	7.469	7.843	8.055
1.13	(13%)	3.517	4.799	5.426	5.918	6.462	7.025	7.330	7.496
1.14	(14%)	3.433	4.639	5.216	5.660	6.142	6.623	6.873	7.003
1.15	(15%)	3.352	4.487	5.019	5.421	5.847	6.259	6.464	6.566
1.16	(16%)	3.274	4.344	4.833	5.197	5.575	5.929	6.097	6.177
1.17	(17%)	3.199	4.207	4.659	4.988	5.324	5.628	5.766	5.829
1.18	(18%)	3.127	4.078	4.494	4.793	5.092	5.353	5.467	5.517
1.19	(19%)	3.058	3.954	4.339	4.611	4.876	5.101	5.195	5.235
1.20	(20%)	2.991	3.837	4.192	4.439	4.675	4.870	4.948	4.979
1.25	(25%)	2.689	3.329	3.571	3.725	3.859	3.954	3.985	3.995
1.30	(30%)	2.436	2.925	3.092	3.190	3.268	3.316	3.329	3.332
1.35	(35%)	2.220	2.598	2.715	2.779	2.825	2.850	2.856	2.857
1.40	(40%)	2.035	2.331	2.414	2.456	2.484	2.497	2.499	2.500

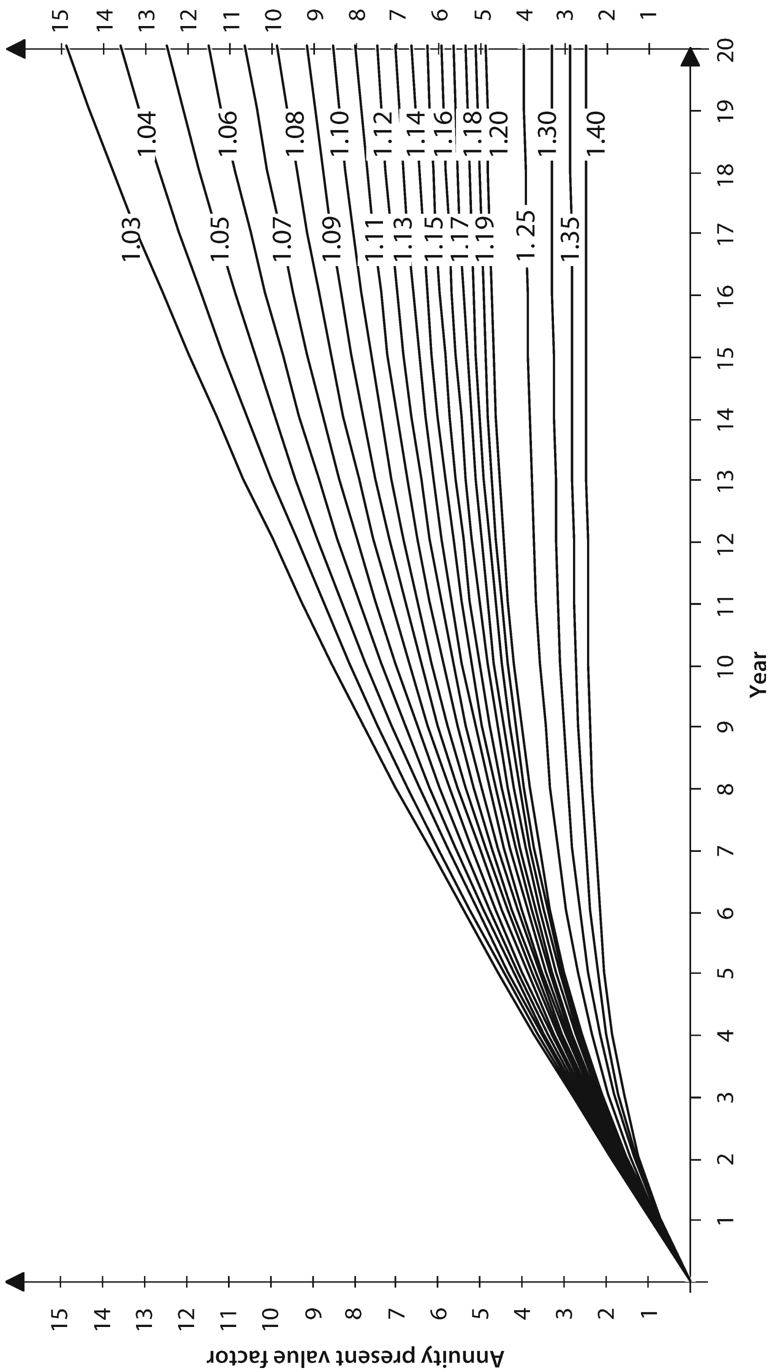


Fig. D1. Annuity present value factors

Appendix E
Problems Created by the Application of Geometrical Series

Dynamic economic calculations (see Sect. 11.2.1 and 11.2.3.1) and the calculation of growth rates (see Chap. 14) are based on the geometrical series:

$$y = (1 + i)^n$$

In this equation i is the growth rate or rate of return, n equals the years.

It is characteristic for geometrical series to increase slowly at first with later values increasing exponentially. As an example, we assume $i = 0.06$ (i.e. a growth rate of 6% per year). For $n = 1, y = 1.06$; for $n = 2, y = 1.12$; for $n = 10, y = 1.79$ and so on: however, for $n = 25, y = 4.29$.

Often people are not aware how quickly these geometrical series increase over long periods of time so that growth rates are projected into the future ad infinitum which leads to completely absurd absolute figures.

The following example shall demonstrate the accelerating effect of geometrical series: A book was published in 1982 concerning the most unfortunate decisions in the world (D. Frost: I Could Have Kicked Myself), from which the following story is taken:

> "This is my land.
> The territorial expansion of the United States owed much to the courage and pioneering spirit of its citizens, but the contribution made by other people's mistakes was also considerable. One of the victims claimed by the talent for making a good deal which the New World inspired in its settlers was an anonymous Indian chief who, in 1626, sold the island of Manhattan to Governor Peter Minuit for US $24 worth of axes, kettles and fabric. A city block in mid-town Manhattan passes hands these days for around US $80 million. Even allowing for inflation, Governor Minuit got himself a bargain."

Did he indeed?

Assignment. Determine how good the bargain actually was by calculating the rate of return of the investment of US $24 under the assumption that there are 1 000 city blocks in Manhattan.

- *Step 1:* In the equation $y = (1 + i)^n$, i is the unknown; n, the number of years, is $n = 1\,984 - 1\,626 = 358$.
- *Step 2:* 1 000 city blocks have a value of US $\$8 \times 10^{10}$. This value must be divided by 24, since the above formula is related to the compound rate of return or the increase of a quantity with the value 1. Hence y is

$$y = 3.33 \times 10^9$$

- *Step 3:* The equation now is

  $$3.33 \times 10^9 = (1 + i)^{358} \quad \text{or}$$

  $$(3.33 \times 10^9)^{-358} - 1 = i$$

  From this we derive:
  $i = 0.063$ or an interest rate of 6.3%, i.e. US $24 at an interest rate of 6.3% per year would result in a sum of US $$8 \times 10^{10}$!

To answer the question how good the bargain actually was: a mining investment with an internal rate of return of 6.3% (see Sect. 11.2.4) would be considered submarginal.

Appendix F
Sources of Information, Internet Addresses, Abbreviations, Conversions

Sources of Information and Internet Addresses

Table F1. Information sources

Name	Mining, market, development	Country	Web
AME, AME Mineral Economics	Market analysts in metals, mining and minerals	Australia	www.ame.com.au
AMM, American Metal Market	World Metals Information Network	USA	www.amm.com
Baltic Exchange, Indices	Baltic Dry, Capesize, Panamax, Supramax Indices	GB	www.balticexchange.com
Bertelsmann Stiftung, Bertelsmann Transformation Index	Global ranking of the transformation processes	Germany	www.bertelsmann-transformation-index.de
Bloomsbury Minerals Economics	Metal market analysis and metal price modelling	UK	www.bloomsburyminerals.com
CRU, CRU International	Analysis and market reports on mining, metals, minerals	UK	www.crugroup.com
CIA, Central Intelligence Agency	World Factbook: maps and facts on countries	USA	www.cia.gov/cia/publications
EduMine	Online education and technical reference for Mining	Canada	www.edumine.com
EIC, Energy Information Center	Experts for industrial/ commercial energy users	UK	www.eic.co.uk
EITI, Extractive Industries Transparency Initiative	Transparency in transactions between governments/companies	UK	www.eitransparency.org
Fraser Institute	Annual survey of mining companies	Canada	www.fraserinstitute.ca/admin/ books/files/Mining20052006.pdf
InfoMine	Information and links of mining and mineral exploration	Canada	www.infomine.com

Table F1. *Continued*

Name	Mining, market, development	Country	Web
ILO, International Labour Organization of the UN	Sectoral activities: mining, small-scale mining, oil/gas	Switzerland	www.ilo.org
Maritime Research	Reports charter fixtures and indexes for the tramp market	USA	www.maritime-research.com
MBendi, Information for Africa	World mining, oil and gas, trade, companies	South Africa	www.mbendi.co.za
MEG, Metals Economics Group	Global minerals exploration and acquisition activity	Canada	www.metalseconomics.com
MII, Mineral Information Institute	Source for free teaching materials	USA	www.mii.org
MEI online, Minerals Engineering International	Mineral processing and extractive metallurgy	UK	www.min-eng.com
Mining Communications	Mining Journal, Mining Magazine, Coal Magazine	UK	www.mining-journal.com
MiningLife	Information, reference, resource site, web portal	Australia	www.mininglife.com
RMG, Raw Materials Group	Raw materials data: mining industry database	Sweden	www.rmg.se
Roskill, Roskill Information Services	International reports on metal or mineral markets	UK	www.roskill.com
Times Atlas of the World, (Collins Bartholomew)	Maps on countries, exotic destinations, islands, oceans	UK	www.bartholomewmaps.com
Transparency International	Corruption Perceptions Index	Germany	www.transparency.de/Corruption-Perceptions-Index-2.810.0.html
World Bank Group, Oil, Gas, Mining, and Chemicals Department	Investments policy and reform, extractive industries review	USA	www.worldbank.org www.ifc.org
World Bank Group, World Bank Institute	Comparison of governance and socio-economic indicators	USA	www.worldbank.org/wbi/governance/pdf/2004kkzcharts.xls
World Mine Cost, Data Exchange	Operating cost information for mining industry analysts	USA	www.minecost.com
WEC, World Energy Council	Energy information, resources, efficiency, policies	UK	www.worldenergy.org

Table F2. Statistical sources

Name	Information and Statistics	Country	Web
ABMS, American Bureau of Metal Statistics	Non-Ferrous Metal Yearbook	USA	www.abms.com
BGS, British Geological Survey	World Mineral Statistics	UK	www.bgs.ac.uk
BMWA, Bundesministerium für Wirtschaft und Arbeit	Welt Bergbau Daten/World Mining Data	Austria	www.bmwa.gv.at
EIA, Energy Information Administration	U.S./International Energy Statistics, Country Analysis Briefs	USA	www.eia.doe.gov
Eurostat, EU Statistical Office	Europe in Figures – Eurostat Yearbook	Europe	www.eurostat.ec.europa.eu
FAO, UN Food and Agriculture Organization	Agricultural FAOStat	Italy	faostat.fao.org
GFMS, Gold Fields Mineral Services	Gold, silver, and platinum and palladium surveys	UK	www.gfms.co.uk
IM, Industrial Minerals	Industrial minerals prices database	UK	www.indmin.com
IAEA, International Atomic Energy Agency	"Red Book" Uranium Reserves/Resources	Austria	www.iaea.org
IEA, International Energy Agency	World Energy Outlook	France	www.iea.org www.worldenergyoutlook.org
IMF, International Monetary Fund	World Economic Outlook, producer/consumer prices foreign exchange reserves	USA	www.imf.org
ISSB, Iron and Steel Statistics Bureau	World Steel Statistics	UK	www.issb.co.uk
JM, Johnson Matthey	Platinum group metals data on price, production	UK	www.platinum.matthey.com
MB, Metal Bulletin	Mineral and metal industry directories and metalprices	UK	www.metalbulletin.plc.uk
OECD, Org. for Econ. Co-operation and Developm.	Economic outlooks, country surveys and statistics	France	www.oecd.org
Silver Institute	World Silver Survey (Produced by GFMS)	USA	www.silverinstitute.org
UNCTAD, UN Conference on Trade and Development	Handbook of (World Mineral Trade) Statistics	Switzerland	www.unctad.org
UN Statistics Division	Consumer Price Index	USA	www.unstats.un.org
U.S. Department of Labor, Bureau of Labor Statistics	Producer and Consumer Price Index	USA	www.bls.gov
USGS, U.S. Geological Survey	Mineral Information, Year-book, Commodity Summary	USA	minerals.usgs.gov/minerals
WBMS, World Bureau of Metal Statistics	Data resource for the global metals industry	UK	www.world-bureau.com
WTO, World Trade Organiz.	International trade statistics	Switzerland	www.wto.org

Important Abbreviations and Notations

Table F3. Abbreviations and notations

£	British pound	NF	Percentage of metal in concentrate paid to mine
ε	Recovery in mill	n_j	Individual year
AU$	Australian dollar	NPV	Net present value
b_n	Annuity present value factor (also called discrete uniform present worth factor or series present worth factor)	NSR	Net smelter return
CA$	Canadian dollar	OP	Operating profit
CF	Cash flow	Pr	Profit
Co	Operating costs	q	Defined as $1 + i$, whereby i is the interest or compound rate (see above)
DCF	Discounted cash flow	R/C	Refining charge
DM	Deutsche Mark	Rev	Revenue
EMV	Expected monetary value	R_m	Thickness reduction factor
€	Euro	t/a	Tonnes per year
i	Interest or compound rate	T/C	Treatment charge
I	Investment	u	1 unit = 1% of the metal in 1 t of concentrates
IROR	Internal rate of return	U.S.$	U.S. dollar
IRR	Internal rate of return	vm	Vertical meter
KF	Concentration factor	W:O	Waste to ore ratio
KV	Conversion factor in metal equivalent calculations	x_i	Individual values of x
MR	Mass recovery factor	$\bar{x}$	Arithmetic mean of values x_i
n	Number of years	y_i	Individual values of y
NC	Net cash flow	$\bar{y}$	Arithmetic mean of values y_i

Conversion Table (for the Field Book)

Table F4. Conversion table

Original unit	Convert to	Multiply by
Acre	Square metre	4 047
Acre	Hectare	0.4047
Barrel	Litre	158.99
Btu	Kilocalorie	0.252
Btu	Kilojoule	1.055
Calorie	Joule	4.1868
Carat (precious stones)	Gram	0.2
Chain	Metre	20.1168
Cubic foot	Cubic metre	0.028317
Cubic inch	Cubic centimetre	16.39
Cubic yard	Cubic metre	0.76456
Foot	Metre	0.3048
Flasks (mercury)	Kilogram	34.473
Gallon (imperial)	Litre	4.546
Gallon (US)	Litre	3.785
Grain (precious metals)	Gram	0.0648
Hectare	Acre	2.471
Hundredweight	Kilogram	50.8
Long ton	Kilogram	1 016.047
Mile (statute)	Kilometre	1.6093
Mile (nautical)	Kilometre	1.853
Ounce (avoirdupois)	Gram	28.35
Ounce (troy for prec. met.)	Gram	31.103
Pennyweight (precious met.)	Gram	1.555
Pint (imperial)	Litre	0.568
Pint (US)	Litre	0.473
Pound	Kilogram	0.4536
Pound per square inch	Kilogram per square centimetre	0.07031
Percent value in one metric tonne	Pound (lbs)	22.046
Quart (imperial)	Litre	1.137
Quart (US)	Litre	0.946
Short ton	Kilogram	907.2
Square foot	Square metre	0.0929
Square inch	Square centimetre	6.452
Square mile (statute)	Square kilometre	2.59
Square yard	Square metre	0.8361
Yard	Metre	0.9144

Appendix G
Scales (for the Field Book)

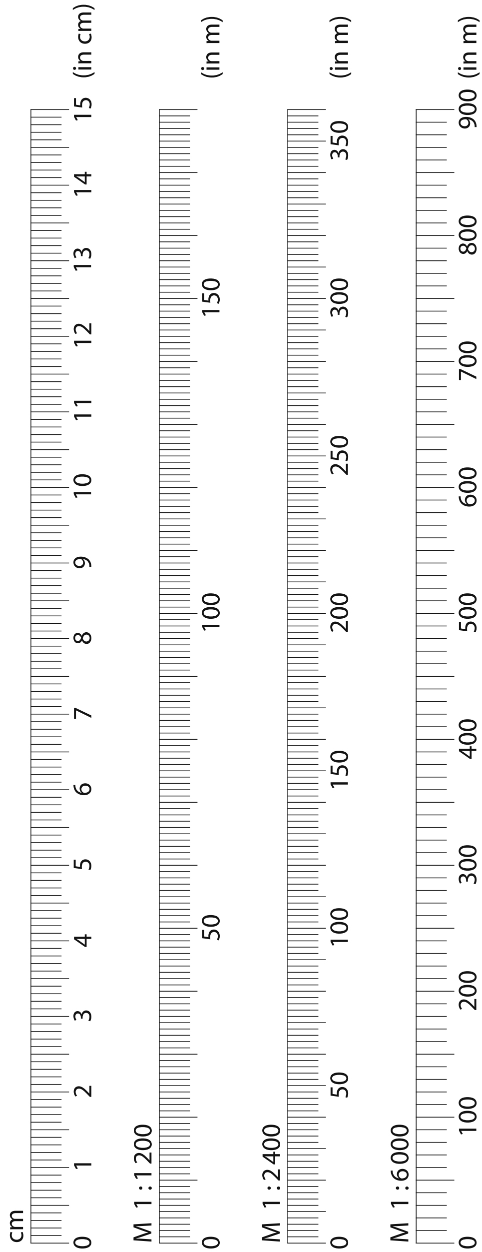
These scales can be cut out, glued on thicker paper and then they can be a practical aid in the field.

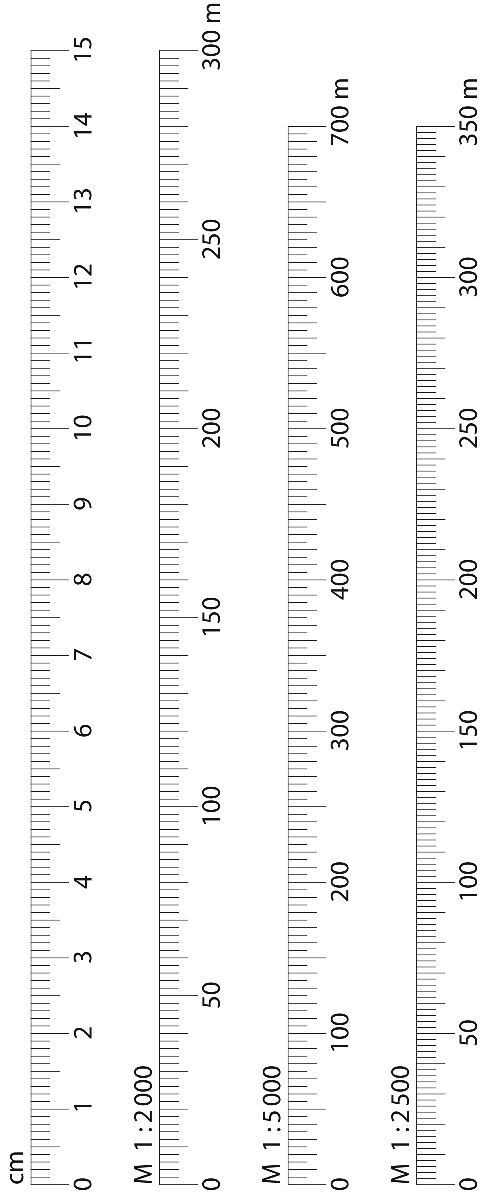
From the metric scales one can directly read the metre values. Are the scales a multiple of the given scales, e.g. 1 : 25 000, then the metre value of the 1 : 2 500 has to be multiplied in this case by 10.

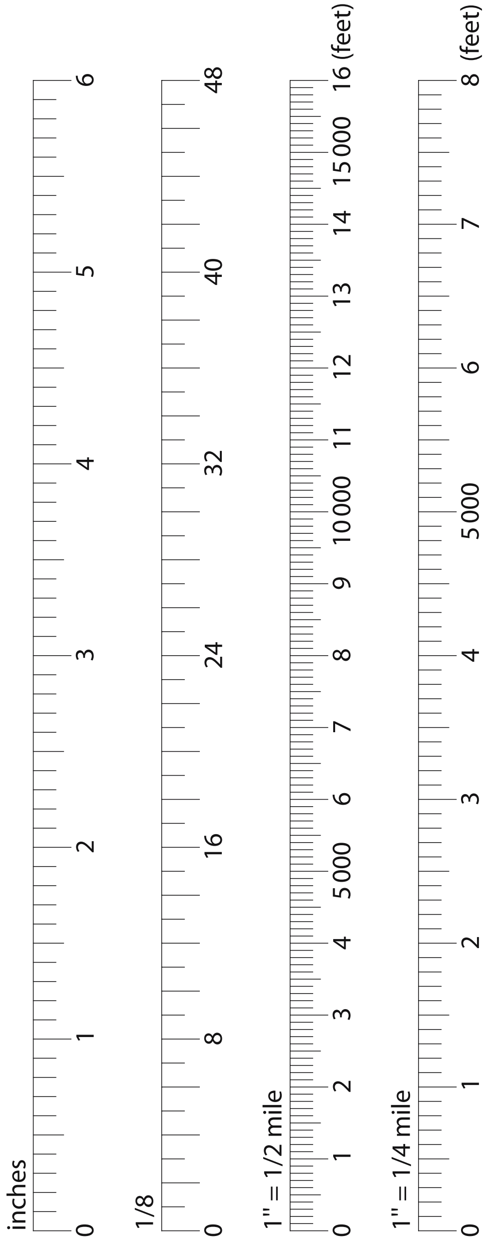
From the mile scales, e.g. 1 inch (or 1") = 1 / 4 mile, one can read directly the feet value.

If one encounters scales with the basis of 12, e.g. 1 : 6 000 or 1 : 12 000 in most cases these are scales based on feet, which are numerically converted into metric scales. The scale of 1 : 6 000 is the same as 1 inch = 500 feet and 1 : 12 000 is equivalent to 1 inch = 1 000 feet.

In such a case it is best to use the feet scale, determine the feet value, and then to convert to metres by multiplying with the conversion factor of 0.3.


cm
(in cm)
M 1 : 1 200
(in m)
M 1 : 2 400
(in m)
M 1 : 6 000
(in m)


cm
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
M 1 : 2000
0 50 100 150 200 250 300 m
M 1 : 5000
0 100 200 300 400 500 600 700 m
M 1 : 2500
0 50 100 150 200 250 300 350 m


inches
0 1 2 3 4 5 6
1/8
0 8 16 24 32 40 48
1" = 1/2 mile
0 1 2 3 4 5000 6 7 8 9 10000 11 12 13 14 15000 16 (feet)
1" = 1/4 mile
0 1 2 3 4 5000 6 7 8 (feet)

Index

I

J

K

L