Mathematical Problems from Applied Logic I

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Mathematical Problems from Applied Logic I

Logics for the XXIst Century

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Mathematical Problems from Applied Logic I
New Logics for the XXIst Century

Two volumes of the *International Mathematical Series* present the most important thematic topics of logic confronting us in this century, including problems arising from successful applications areas such as Computer Science, AI language, etc. etc.

Invited authors — world-known specialists in the field of logic — were asked to write a chapter (in the form of a survey, a specific problem, or a point of view) basically outlining

WHAT IS ON MY MIND AS MOST STRIKING/IMPORTANT/PRESSING NEED TO BE DONE?

Main Topics

- Nonstandard inferences in description logics; an overview of the modern state, open problems, and perspectives for future research
- Logic of provability and a list of open problems in informal concepts of proof, intuitionistic arithmetic, bounded arithmetic, bimodal and polymodal logics, Magari algebras and Lindenbaum Heyting algebras, interpretability logic and its kin, graded provability algebras
- Logical dynamics: a survey of conceptual issues and open mathematical problems emanating from the recent development of various "dynamic-epistemic logics" for information update and belief revision. These systems put many-agent activities at the center stage of logic, such as speech acts, communication, and general interaction
- The continuing relevance of Turing's approach to real-world computability and incomputability, and the mathematical modeling of emergent phenomena. Related open questions of a research interest in computability theory.
- Door to open: Mathematical logic and cognitive science
- Door to open: Semantics of medieval Arab linguists
- What logics do we need? What are logical systems and what should they be? What is a proof? What foundations do we need?
- Applied logic: characterization and relation with other trends in logic, computer science, and mathematics

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- Editor of journals and series ∘ *Autonomous Agents* ∘ *Studia Logica* ∘ *Journal of Applied Non-Classical Logics* ∘ *Journal of Logic, Language & Information* ∘ *F News* ∘ *Handbook of Logic in Computer Science* (with S. Abramsky and T. Maibaum), ∘ *Handbook of Logic in AI and Logic Programming* (with C. Hogger and J.A. Robinson), ∘ *Handbook of Philosophical Logic* (with F. Guenthner) ∘ *Handbook of the History of Logic* ∘ *Handbook of the Philosophy of Science* ∘ *Studies in Logic and the Foundations of Mathematics*, etc.

Scientific interests: Logic and computation, dynamics of practical reasoning, proof theory and goal-directed theorem proving, non-classical logics and non-monotonic reasoning, labelled deductive systems, fibring logics, logical modelling of natural language

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Scientific interests: Knowledge representation and reasoning, modal and temporal logics, description logics, spatial and temporal reasoning, automated theorem proving, intuitionistic and intermediate logics

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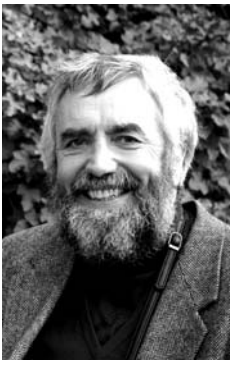

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Content

† The endless cycle of death and rebirth to which life in the material world is bound. (OED)

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