

# FUNDAMENTALS OF SWITCHING THEORY AND LOGIC DESIGN

# Fundamentals of Switching Theory and Logic Design

A Hands on Approach

by

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## Preface

Information Science and Digital Technology form an immensely complex and wide subject that extends from social implications of technological development to deep mathematical foundations of the techniques that make this development possible. This puts very high demands on the education of computer science and engineering. To be an efficient engineer working either on basic research problems or immediate applications, one needs to have, in addition to social skills, a solid understanding of the foundations of information and computer technology. A difficult dilemma in designing courses or in education in general is to balance the level of abstraction with concrete case studies and practical examples.

In the education of mathematical methods, it is possible to start with abstract concepts and often quite quickly develop the general theory to such a level that a large number of techniques that are needed in practical applications emerge as “simple” special cases. However, in practice, this is seldom a good way to train an engineer or researcher because often the knowledge obtained in this way is fairly useless when one tries to solve concrete problems. The reason, in our understanding, is that without the drill of working with concrete examples, the human mind does not develop the “feeling” or intuitive understanding of the theory that is necessary for solving deeper problems where no recipe type solutions are available.

In this book, we have aimed at finding a good balance between the economy of top-down approach and the benefits of bottom-up approach. From our teaching experience, we know that the best balance varies from student to student and the construction of the book should allow a selection of ways to balance between abstraction and concrete examples.

Switching theory is a branch of applied mathematics providing mathematical foundations for logic design, which can be considered as the part

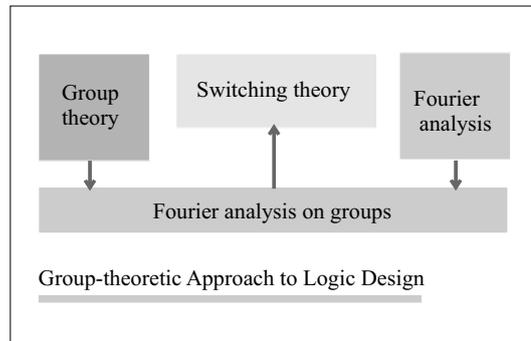


Figure 1. Switching theory and Fourier analysis.

of digital system design concerning realizations of systems whose inputs and outputs are described by logic functions. Thus, switching theory can be viewed as a part of Systems Theory and it is closely related to Signal Processing.

The basic concepts are first introduced in the classical way with Boolean expressions to provide the students with a concrete understanding of the basic ideas. The higher level of abstraction that is essential in the study of more advanced concepts is provided by using algebraic structures, such as groups and vector spaces, to present, in a unified way, the functional expressions of logic functions. Then, from spectral (Fourier-like) interpretation of polynomial, and graphic (decision diagrams) representations of logic functions, we go to a group-theoretic approach and to optimization problems in switching theory and logic design. Fig. 0.1 illustrates the relationships between the switching theory and Fourier analysis on groups. A large number of examples provides intuitive understanding of the interconnections between these viewpoints.

Consequently, this book discusses the fundamentals of switching theory and logic design from a slightly alternative point of view and also presents links between switching theory and related areas of signal processing and system theory. In addition, we have paid attention to cover the core topics as recommended in IEEE/ACM curricula for teaching and study in this area. Further, we provide several elective lectures discussing topics for further research work in this area.

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## Acronyms

ACDD	Arithmetic transform decision diagram
ACDT	Arithmetic transform decision tree
BDD	Binary decision diagram
BDT	Binary decision tree
BMD	Binary moment diagram
BMT	Binary moment tree
*BMD	*Binary moment diagram
DD	Decision diagram
DT	Decision tree
DTL	Decision Type List
EVBDD	Edge-valued binary decision diagram
EVBDT	Edge-valued binary decision tree
ExtDTL	Extended Decision Type List
FFT	Fast Fourier transform
FDD	Functional decision diagram
FDT	Functional Decision tree
FEVBDD	Factored edge-valued binary decision diagram
FPGA	Field-programmable gate array
FPRM	Fixed-polarity Reed-Muller expression
KDD	Kronecker decision diagram
KDT	Kronecker decision tree
LUT	Look-up-table
MPGA	Mask programmable gate array
MTBDD	Multi-terminal binary decision diagram
MTBDT	Multi-terminal binary decision tree
PKDD	Pseudo-Kronecker decision diagram
PKDT	Pseudo Kronecker decision tree
PLA	Programmable logic array
PPRM	Positive-polarity Reed-Muller expression
POS	Product-of-Sum expression
RAM	Random-access memory
ROM	Read-only memory
SBDD	Shared binary decision diagrams
SOP	Sum-of-Product expression
STDT	Spectral transform decision tree
STDD	Spectral transform decision diagram
TVFG	Two-variable function generator
ULM	Universal logic module
WDD	Walsh decision diagram
WDT	Walsh decision tree